MAT 305 GRAPH THEORY AND COMBINATORICS

Quiz Proof G.1 Resubmit. Let G be a non trivial connected graph that is not bipartite. Prove that G contains two adjacent vertices u and v such that $deg(u) + deg(v)$ is even.

Prove by Contradiction. Let G be a connected bipartite graph, such that G contains no two adjacent vertices whose degree sum is even. Then, all the edges in G would connect to a vertex with an odd degree on the one end, and a vertex with an even degree on the other end. Rearrange the orientation of each edge, and let A denote the set of even degree vertices in G, and let B denote the set of odd degree vertices in G. Then, since adjacent vertices pair will not have both even degrees or both odd degrees, vertices in set A will be adjacent to only vertices in set B and vise versa. Because there is no edge connecting vertices within each set and the graph is nontrivial connected graph, A and B are disjoint bipartite set and then G has to be bipartite, contradicting with the assumption that G is a connected bipartite graph. Thus, prove by contradiction, if G is a nontrivial connected graph that is not bipartite, then G contains two adjacent vertices u and v such that $deg(u) + deg(v)$ is even. □