# MAT 305 GRAPH THEORY AND COMBINATORICS

**Exercise 1.4.3.13 (b).** Let n be a positive integer. Prove that $K_{n,2n,3n+1}$ is not Hamiltonian.

*Prove by Contradiction.* Let $n \in \mathbb{N}$, and let $G = K_{n,2n,3n+1}$. Let $X, Y, Z$ denotes the three partite sets of $G$ such that $X$ contains $3n+1$ vertices, $Y$ contains $2n$ vertices, and $Z$ contains $n$ vertices. Assume, for contradition, that $G$ is Hamiltonian. By definition, $G$ would have a Hamiltonian cycle denoted as $C$. Then, for $3n+1$ vertices in $X$, there will be correspondingly $3n+1$ edges on $C$ that connect them. By definition of a partite graph, there is no edge connecting from vertices in $X$ to other vertices in $X$, so the $3n+1$ edges have to come from the set $Y \cup Z$. There are $max\{2n, n, 2n+n\} = 3n$ number of vertices adjacent to $X$, providing $3n$ edges in the cycle. Since $3n < 3n+1$, it cannot provide enough edges to connect all the vertices in $X$. Thus, it contradicts with the assumption that $G$ contains a Hamiltonian cycle, and therefore, $K_{n,2n,3n+1}$ is not Hamiltonian. $\square$