

MAT 305 COMBINATORICS PORTFOLIO

Stars and Bars Exercise 4. How many positive integer solutions are there to $x_1 + x_2 + x_3 + x_4 < 50$

Proof. Because we are considering only integers, the inequality is equivalent to

$$x_1 + x_2 + x_3 + x_4 \leq 49$$

Then, this question can be converted to a stars and bars problem by considering distributing 49 1s as stars to 4 variables as boxes. Since we want positive integer solutions, $x_i \geq 1$, equivalent to distributing a 1 in each variable beforehand. Then, there will be $49 - 4 = 45$ 1s left. Let s be the sum of all four variables x_1, x_2, x_3, x_4 . Considering the remaining $45 - s$ 1s to be distributed to another variable x_5 , then, the question is equivalent to distributing 45 1s to 5 variables. Applying the stars and bars technique, for $n = 45$ and $k = 4 + 1 = 5$, we get

$$\binom{n+k-1}{n} = \binom{45+5-1}{45} = \binom{49}{45} = 211876$$

Thus, there will be 211876 positive integer solutions to this inequality. □