Discounted Cash Flow Valuation - Refresher

You own an oil pipeline that will generate a $3.9 million cash return over the coming year. The pipeline's operating costs are negligible, and it is expected to last for a very long time. Unfortunately, the volume of oil shipped is declining, and cash flows are expected to decline by 6.0% per year. The discount rate is 10%.

a) What is the PV of the pipeline's cash flows if its cash flows are assumed to last forever?

b) What is the PV of the cash flows if the pipeline is scrapped after 19 years?

a) $PV = \frac{C}{r-g} = \frac{\$3,900,000}{0.10-(-0.06)} = \$24,375,000$

b) There are different ways to solve this problem. One approach is the following two step process:

- Calculate the PV as of year 19 by assuming the cash flows last forever. That number is $PV_{19} = \$7,522,702$. Use the cash flow in year 20, $C_{20} = 1,203,632$ to calculate PV_{19}.
- Because the pipeline is scrapped in 19 years, subtract the present value of the above number from the perpetuity value. Thus, $PV_{Cash\ flows\ 1-19} = \$24,375,000 - \frac{PV_{19}}{(1+r)^{19}}$

$$PV_{Cash\ flows\ 1-19} = \$24,375,000 - \frac{\$7,522,702}{(1.1)^{19}} = \$23,144,978$$

Amortizing Loan Problem

Suppose that you take out a $200,000, 20-year mortgage loan to buy a condo. The interest rate on the loan is 6%, and payments on the loan are made annually at the end of each year.

1. What is your annual payment on the loan?

2. Construct a mortgage amortization table in Excel, showing the interest payment, the amortization of the loan, and the loan balance for each year.

3. What fraction of the loan has been paid off after 10 years? Why is the fraction less than half?

Bond Valuation

FINA 3301 Corporate Finance
Northeastern University

Kandarp Srinivasan

Example: Tesla

$1,800,000,000 5.30% Senior Notes due 2025

TESLA

August 11, 2017

Tesla Agrees to Issue and Sell $1.80 Billion of Senior Notes

PALO ALTO, Calif., Aug. 11, 2017 (GLOBE NEWSWIRE) -- Following the announcement of Tesla's proposed $1.50 billion senior notes offering on August 7, 2017, Tesla today announced that it has agreed to issue and sell $1.80 billion in aggregate principal amount of senior notes due in 2025 (the "Notes"), representing a 20% upsize. The Notes will have an annual interest rate of 5.30%, and the transaction is expected to close on August 18, 2017, subject to customary closing conditions.

Tesla intends to use the net proceeds from this offering to strengthen its balance sheet during this period of rapid scaling with the launch of Model 3 and for general corporate purposes.

- **Par value** is the stated face value of a single bond
- **Aggregate principal amount**: Par value × # of bonds
- **Coupon**: the interest the company pays each year. Fixed at the time of bond issue.
- **Maturity**: when the bond principal is due

Source: Tesla Investor Communication

What is yield to maturity?

$1,800,000,000 5.30% Senior Notes due 2025

Issuer:	Tesla, Inc., a Delaware corporation
Guarantor:	SolarCity Corporation, a Delaware corporation
Title of Securities:	5.30% Senior Notes due 2025 (the "Notes")
Placement:	144A/Regulation S for life
Offering Size:	$1,800,000,000
Gross Proceeds:	$1,800,000,000
Net Proceeds to Issuer (After Expenses):	Approximately $1.77 billion
Maturity:	August 15, 2025
Issue Price:	100% of face amount
Coupon:	5.30%
Yield to Maturity:	5.30%

Source: SEC Prospectus

It is the opportunity cost for the bond investor

- We will value bonds by using discounted cash flow techniques

- The value of a bond (or any financial asset) is the present value of future cash flows

- The rate at which we discount a bond's cash flows is called **yield to maturity** (YTM), sometimes denoted by y or r_d.

Find the price of a bond with the following data

- Par Value = $100
- Coupon Rate = 5.3%
- Yield to maturity (YTM) , y = 6%
- Maturity = 5 years
- Coupon payments = Once a year

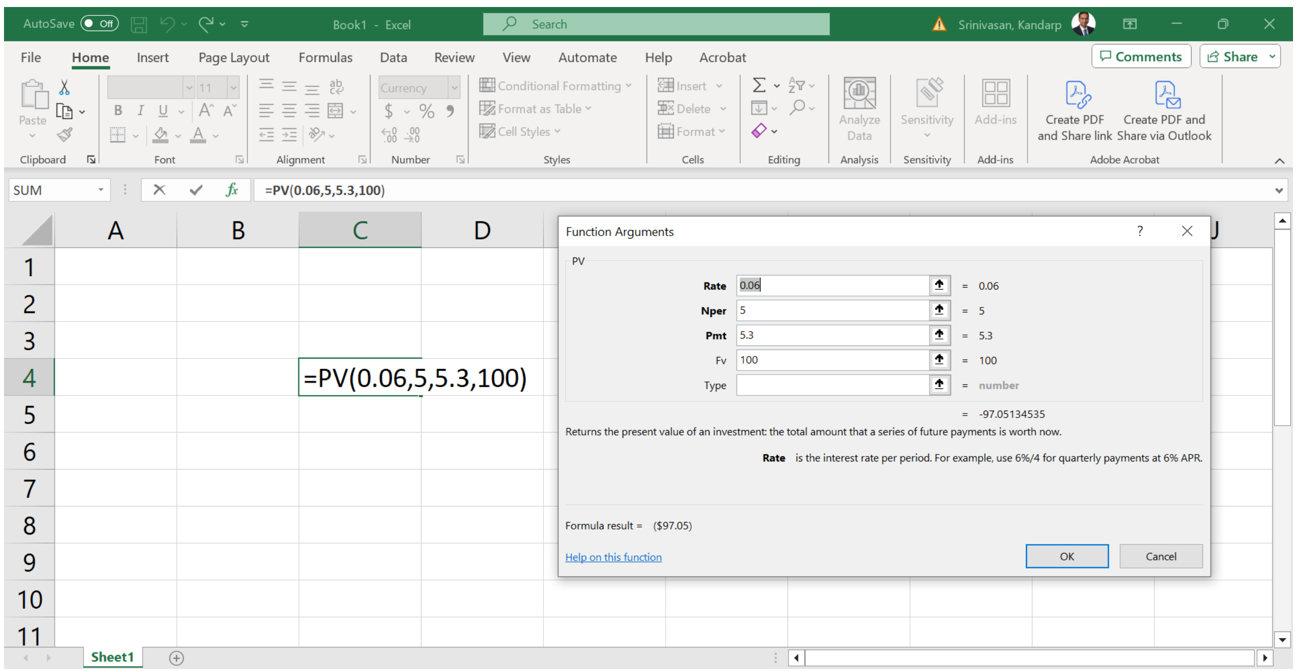
Pricing Bonds (Financial Calculator and Excel)

INPUTS

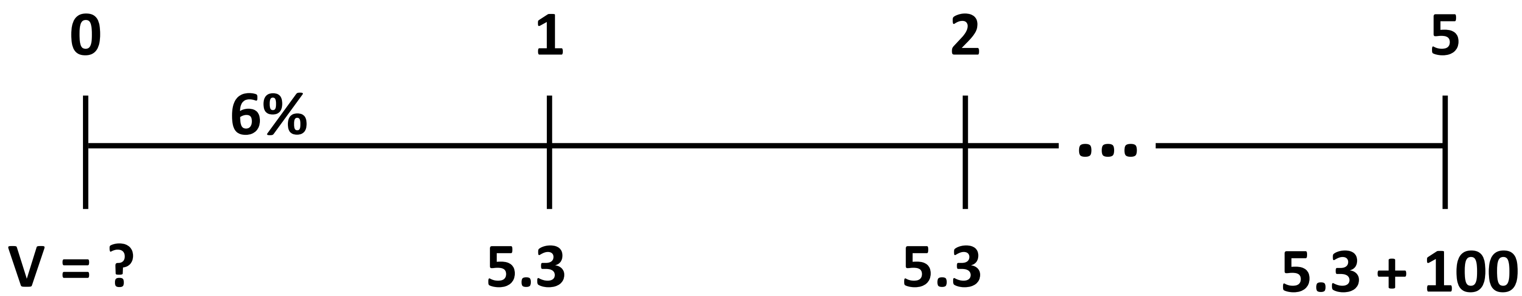
N	I/YR	PV	PMT	FV
5	6		5.3	100

OUTPUT

PV: **-97.05**



Remember that both these methods are ultimately performing a discounted cash flow valuation



$$Value = \frac{5.3}{(1.06)} + \frac{5.3}{(1.06)^2} \dots + \frac{105.3}{(1.06)^5}$$

$Value =$ 5.00 + 4.72 + ..+ 78.69

$$Value = \$97.05$$

Note that there are **two** components to bond value

Value of a 5-year, 5.3% coupon bond if y = 6%

PV annuity	=	$22.325
PV maturity value	=	$74.726
Value of bond	=	$97.051

Discount Rate (Yield)	Coupon Rate	Price
6%	5.3%	$97.05
4.8%	5.3%	$102.18

- If coupon rate < y, bond trades below par (**discount** bond)
- If coupon rate > y, bond trades above par (**premium** bond)
- Homework: for the Tesla bond, extend this table by changing values for discount rates and see how prices change.

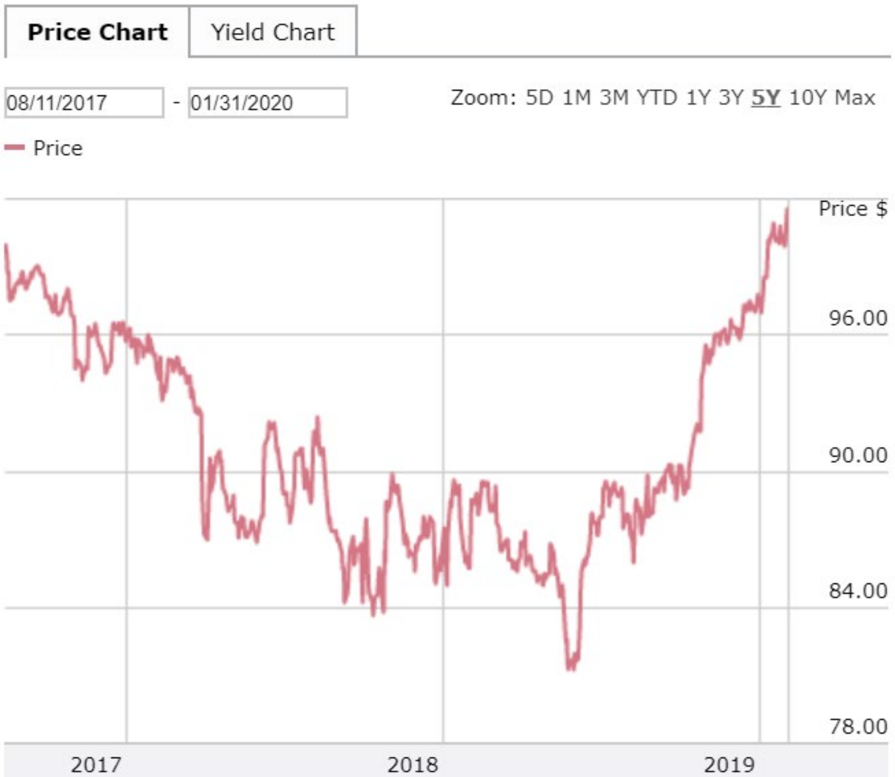
What drives bond prices?

- The **price** of a bond (or, equivalently, its **yield**) is subject to market forces of demand and supply

- Bonds are more valuable to investors if they are perceived to be less risky

- As these risk perceptions change, both the price (value) of the bond and its yield will change. The coupon rate, however, always remain fixed.

When prices drop, yields go up and vice versa



https://finra-markets.morningstar.com/BondCenter/Results.jsp

Prices and Yields are two sides of the same coin….

- …as in, you cannot change one without changing the other
- The yield can be interpreted as the discount rate that justifies the market price of the bond
- Industry practitioners use bond yields as a convenient way to quote the price

If prices fluctuate so much, how does Tesla know it will actually raise $1.8bn?

At the time of bond issue, the coupon rate is set at y

$1,800,000,000 5.30% Senior Notes due 2025

The information in this pricing term sheet supplements Tesla, Inc.'s preliminary offering circular, dated August 7, 2017 (the "Preliminary Offering Circular") and supersede extent inconsistent with the information in the Preliminary Offering Circular. In all other respects, this term sheet is qualified in its entirety by reference to the Preliminary Offi the respective meanings as set forth in the Preliminary Offering Circular.

Issuer:	Tesla, Inc., a Delaware corporation
Guarantor:	SolarCity Corporation, a Delaware corporation
Title of Securities:	5.30% Senior Notes due 2025 (the "Notes")
Placement:	144A/Regulation S for life
Offering Size:	$1,800,000,000
Gross Proceeds:	$1,800,000,000
Net Proceeds to Issuer (After Expenses):	Approximately $1.77 billion
Maturity:	August 15, 2025

Homework: Set the discount rate to 5.3% (same as the coupon rate) in the Tesla bond example and see what the price is

Valuing Meta's bonds offered in August 2024

$10,500,000,000

Meta Platforms, Inc.

$1,000,000,000 4.300% Senior Notes due 2029
$1,000,000,000 4.550% Senior Notes due 2031
$2,500,000,000 4.750% Senior Notes due 2034
$3,250,000,000 5.400% Senior Notes due 2054
$2,750,000,000 5.550% Senior Notes due 2064

We are offering $1,000,000,000 of our 4.300% senior notes due 2029 (the "2029 notes"), $1,000,000,000 of our 4.550% senior notes due 2031 (the "2031 notes"), $2,500,000,000 of our 4.750% senior notes due 2034 (the "2034 notes"), $3,250,000,000 of our 5.400% senior notes due 2054 (the "2054 notes") and $2,750,000,000 of our 5.550% senior notes due 2064 (the "2064 notes" and, together with the 2029 notes, the 2031 notes, the 2034 notes and the 2054 notes, the "notes").

We will pay interest on the notes on February 15 and August 15 of each year until maturity, beginning on February 15, 2025. The notes will be our unsecured obligations and will rank equally with all of our other unsecured senior indebtedness from time to time outstanding. The notes of each series will be issued only in registered form in minimum denominations of $2,000 and integral multiples of $1,000 in excess thereof.

Source: https://www.sec.gov/Archives/edgar/data/1326801/000119312524197096/d874457d424b2.htm

Valuing Meta's bonds

In 2024, Meta Platforms offered 1,625,000 bonds and raised \$3.25bn. The bonds are expected to mature in 2054.
The coupons are semi-annual and the coupon rate is 5.4%. If bond investors expect 5.2% returns today, what price will they be willing to pay? (Note that all interest rates are quoted as APRs.)

FV = \$3.25bn/1,625,000=2000
N = 30*2 = 60
I/Y = 5.2/2 = 2.6
PMT = 5.4%/2 * 2000 = 54

CPT PV = \$2060.43

Current yield and capital gains yield

Think of YTM as the total income per dollar invested in a year

$$\text{Rate of return} = \frac{\text{total income}}{\text{investment}}$$

$$\text{Rate of return} = \frac{\text{coupon income} + \text{price change}}{\text{investment}}$$

$$y = \frac{C}{P_0} + \frac{P_1 - P_0}{P_0} = CurrentYield + CapitalGainsYield$$

Can you write a formula for bond price in terms of its characteristics?

Price of a bond (P_0) , Par value (Par), coupon payments (C), time to maturity (N), discount rate or "yield to maturity" y

$$P_0 = \frac{C}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{Par}{(1+y)^N}$$

Interest Rate Risk

PIMCO California Flexible Municipal Income Fund

Use of Proceeds

The Fund's Investment Objectives and Strategies

Use of Leverage

Principal Risks of the Fund

How the Fund Manages Risk

Management of the Fund

Plan of Distribution

Information Regarding State Escheatment Laws

Interest Rate Risk

Interest rate risk is the risk that fixed income securities and other instruments in the Fund's portfolio will fluctuate in value because of a change in interest rates. For example, as nominal interest rates rise, the value of certain fixed income securities held by the Fund is likely to decrease. A nominal interest rate can be described as the sum of a real interest rate and an expected inflation rate. Interest rate changes can be sudden and unpredictable, and the Fund may lose money as a result of movements in interest rates. The Fund may not be able to effectively hedge against changes in interest rates or may choose not to do so for cost or other reasons.

A wide variety of factors can cause interest rates or yields of U.S. Treasury securities (or yields of other types of bonds) to rise including, but not limited to, central bank monetary policies, changing inflation or real growth rates, general economic conditions, increasing bond issuances or reduced market demand for low yielding investments. Risks associated with rising interest rates are heightened under current market conditions given that the U.S. Federal Reserve (the "Federal

43 **Prospectus** | PIMCO California Flexible Municipal Income Fund

Source: PIMCO Fund Prospectus, April 2023

Discuss answers in a group. Don't perform calculations, focus on intuition

You and your friend are both holding a $1000 par value bond which pays annual coupons of 10%. Your friend's bond matures in 3 years whereas your bond matures in 20 years.

Is one of you more exposed to risk due to changes in interest rates?

You and your friend are both holding two annual bonds of maturity 7 years. Par value is $1000 for both. Suppose your bond pays a coupon of 9% but your friend's bond pays 3%.

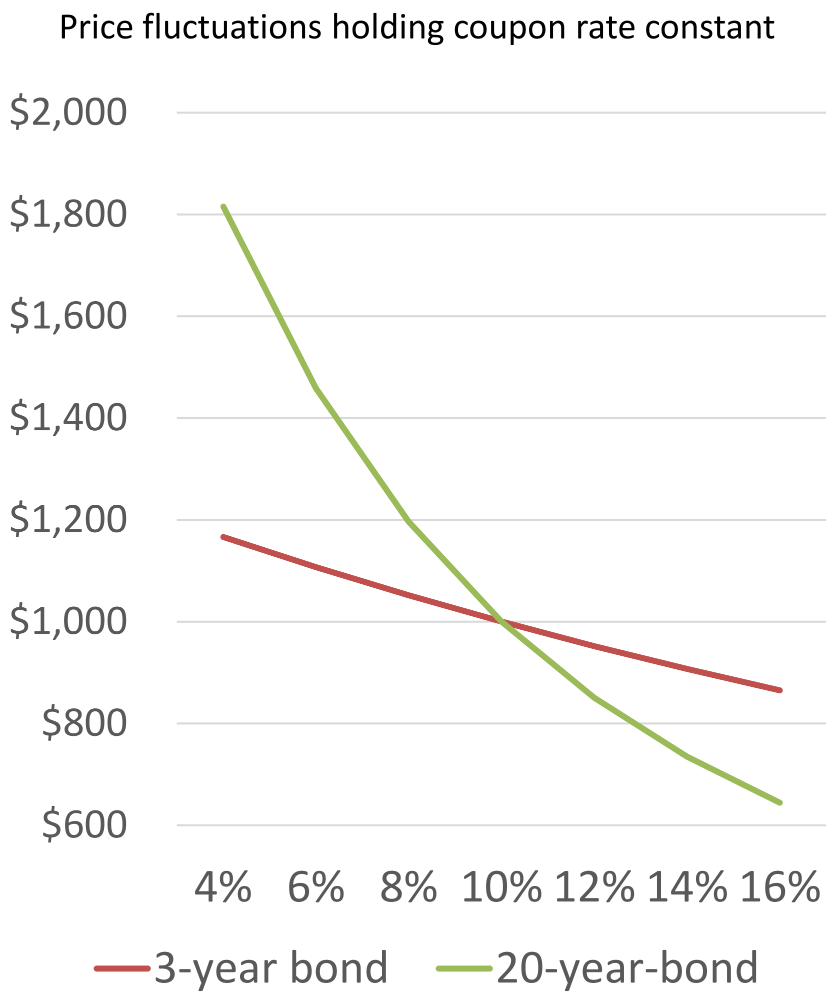
Is one of you more exposed to risk due to changes in interest rates?

What is the intuition? Why is longer maturity or lower coupons associated with greater risk?

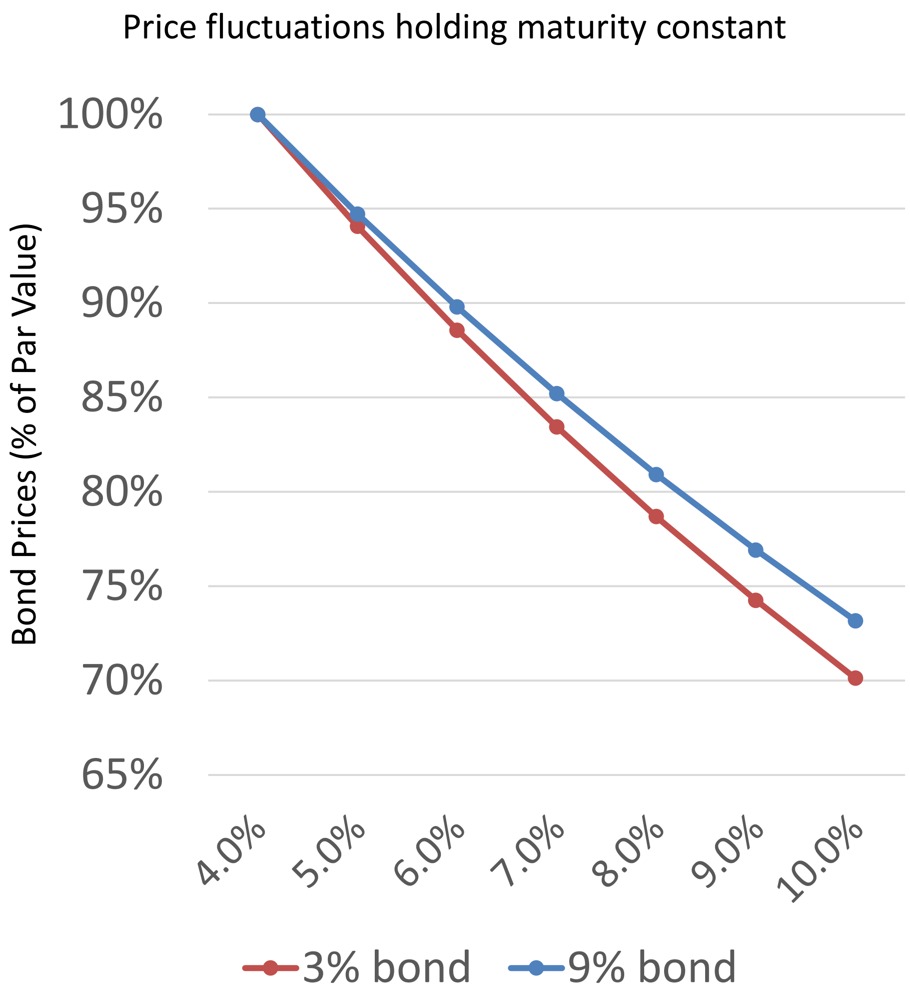
We discount cash flows in a non-linear fashion

Changes in interest rates have a modest impact on near-term cash flows but a greater impact on distant cash flows

So if cash flows are realized “faster” (in present value terms), the risk will be lower



Price fluctuations holding coupon rate constant
$2,000
$1,800
$1,600
$1,400
$1,200
$1,000
$800
$600
4%
6%
8%
10%
12%
14%
16%
3-year bond
20-year-bond



Price fluctuations holding maturity constant
Bond Prices (% of Par Value)
100%
95%
90%
85%
80%
75%
70%
65%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
3% bond
9% bond

Intuition is good, but we also need some way to quantify risk

- For instance, can we intuitively determine which of these bonds is more exposed to interest rate risk?
  - 3% coupon with maturity 6 years
  - 9% coupon with maturity 7 years

- Ideally, we need a quantitative measure that incorporates information on both maturities and coupons

- Such a measure would solve a practical problem – i.e., help investors compare bonds across different maturities and coupons

Let’s compute one such measure, using an example

Suppose a $1000 par value bond with 9% coupon and 7 year maturity has a YTM of 4%. First, let’s look at each cash flow, **in present value terms**, since the faster cash flows are realized, the lower the interest rate risk

Coupon	Price	Year 1	Year 2	….	Year 7
9%	$1300.10	$90	$90	…	$1090

We can compute what fraction of the bond's total value is realized each year

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Payment	\$90	\$90	\$90	\$90	\$90	\$90	\$1090
PV@4%	\$86.54	\$83.21	\$80.01	\$76.93	\$73.97	\$71.13	\$828.31
Fraction in PV terms	6.7%	6.4%	6.2%	5.9%	5.7%	5.5%	63.7%

$$Fraction\ in\ PV\ terms = \frac{PV(CF_T)}{P_o}, where\ P_0 is\ bond\ price$$

We can now weight these fractions because the longer we wait, the higher the interest rate risk

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Payment	\$90	\$90	\$90	\$90	\$90	\$90	\$1090
PV@4%	\$86.54	\$83.21	\$80.01	\$76.93	\$73.97	\$71.13	\$828.31
Fraction in PV terms	6.7%	6.4%	6.2%	5.9%	5.7%	5.5%	63.7%

$$WeightedAverage = \sum_{T=1}^{N} T \times \frac{PV(CF_T)}{P_o}$$

The average time to receive each cash flow, in present value terms is 5.69 years

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Payment	\$90	\$90	\$90	\$90	\$90	\$90	\$1090
PV@4%	\$86.54	\$83.21	\$80.01	\$76.93	\$73.97	\$71.13	\$828.31
Fraction in PV terms	6.7%	6.4%	6.2%	5.9%	5.7%	5.5%	63.7%

$$\sum_{T=1}^{N} T \times \frac{PV(CF_T)}{P_o} = 1 \times 6.7\% + 2 \times 6.4\% + 3 \times 6.2\% + \cdots + 7 \times 63.7\% = 5.69\ years$$

Summary: The main idea was to think of the "average time" it takes to receive a bond's cash flow, in present value terms

- The average time to receive each cash flow payment, in present value terms, has a special name
- It is called the **duration** of a bond
- Why is this number important?
- Because it is a handy measure of interest rate risk
  - A change in interest rates has a greater effect on the price of long-duration bonds relative to short-duration bonds

- (Homework) Use the same parameters as above and find the duration for a 3% bond instead of 9%. You should get a number higher than 5.69 years.

- So the 3% bond is more exposed to interest rate fluctuations than the 9% bond.

- If interest rates rise, we expect the drop in value to be greater for the 3% bond relative to the 9% bond

- (Homework) Suppose the 3% bond had a shorter maturity (say, 6 years) you can use duration to compare the risk of the two bonds

Modified Duration ("Volatility")

- Using calculus, we can directly quantify by how much the price of a bond changes when its yield moves. The derivative of bond price with respect to yield is known as modified duration or volatility.

$$ModifiedDuration = \frac{dP}{dy}$$

- It can be shown that:

$$ModifiedDuration = \frac{dP}{dy} = -\frac{Duration}{1+y}$$

- Ignore the negative sign when reporting actual numbers.