Matching Tenants and Landlords in the Turkish Housing Market: A One-to-One Matching with Contracts Algorithm

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1 Introduction

In the current Turkish rental market system, matching is conducted informally via agents and negotiations. Landlords typically hire real estate agents (realtors) to find tenants, and the renter pays a commission equal to a month's rent upon signing a lease as a finder's fee. We do not observe an algorithmic approach to matching landlords with tenants. However, given the success of matching theory in addressing similar allocation problems (school choice, college admissions, hospital residency matches, etc.), it is natural to ask whether a stable matching mechanism could improve the rental market. In contrast to the classical house allocation problems such as Top Trading Cycles, serial dictatorship mechanisms or probabilistic serial mechanisms, we will create a matching mechanism with contracts, as we believe that such a mechanism is ideal to incorporate the rent as part of the match while still taking preferences over respective partners into account. We place particular importance on landlords' preferences over prospective tenants, given that in recent years, non-monetary factors have gained importance due to the rising number of court cases regarding rental contracts. This creates an urgent need for a mechanism to optimize the market which will take non-monetary factors into account when looking at preference relations instead of the current market assumption that only the price matters. A landlord might prefer to rent to a slightly lower-paying tenant who is trustworthy and clean, rather than a higher-paying tenant who might cause problems. Conversely, a tenant might prefer a certain apartment and be willing to pay a bit more for it, compared to a cheaper but less desirable flat.

We seek to improve the market by designing a matching mechanism that produces optimal, and stable allocations of tenants and landlords, while preserving the role of realtors as facilitators. We propose a one-to-one matching with contracts framework tailored to the landlord–tenant market. In this framework, a contract specifies a landlord (house), a tenant, and the terms of lease (e.g. rent price). Both tenants and landlords will have a preference over whom they match with as well as the contract terms (the price of the rent). Our mechanism will be a deferred acceptance algorithm, where tenants propose contracts to the landlords. This approach builds on the foundational work of Gale and Shapley (1962), who introduced the deferred acceptance algorithm for stable matching in the context of men and women in the stable marriage problem). In their work, they proved when all participants have strict preferences, a stable matching always exists. This inherently suggests the stability of our algorithm. The deferred acceptance algorithm, yields an outcome optimal for the proposing side, in our context this will suggest tenant-optimal matching. Our model extends the classical two-sided matching to allow contracts, following the framework of Hatfield and Milgrom (2005). Hatfield and Milgrom's matching with contracts model generalizes Gale-Shapley to settings where matches involve additional terms like wages or prices. We adapt their insights to landlords and tenants, which translates to landlords regarding different prospective tenants or contract offers as substitutes in our context. This property will mean that having more options will not induce a landlord to reject a previously acceptable tenant. Under this condition (and a technical condition discussed later), a stable outcome is guaranteed to exist.

Given this landscape, we can view the process of pairing tenants with landlords as a two-sided matching problem. On one side we have a set of tenants, searching for a house, and on the other side a set of landlords with a property to lease. It is important to note that, while a landlord might own multiple houses, our mechanism will treat each house as a separate "agent" on the landlord side, which will translate to each property being considered as an independent landlord, to maintain a one-to-one matching structure. We argue that treating multi-house landlords as multiple agents will not impact outcomes, as the landlords would have separate preference relations for each house, which aligns with the idea that each vacant house will be rented to the best available tenant for that house. In the rest of the paper we will formally describe the model, the agents, their preferences, and the properties of the mechanism.

2 Model and Proposed Mechanism

2.1 Agents, Preferences, and Contracts

We consider a set T of tenants and a set L of landlords. Each landlord $l \in L$ is associated with a different house. As discussed, if a single landlord owns multiple houses, we treat each house as a distinct landlord in L, for the matching. The outcome of the matching process will be a set of contracts between tenant-landlord pairs.

A contract x is a triplet (t, l, π) specifying a tenant $t \in T$, a landlord $l \in L$, and a rental agreement π. The term π includes only the rent price. Other contractual details e.g., whether the apartment is furnished, allowed pets, etc. will be assumed to be impactless for the landlord and will not affect their preferences, as they will be predetermined by the landlord for each house and will remain the same for all prospective tenants. These non-monetary elements are included in the tenant's preference list. A tenant t and landlord l can only be matched if there is some contract $x = (t, l, \pi)$ that both find mutually acceptable. We say a contract $x = (t, l, \pi)$ is feasible if t is eligible to rent from l under terms π (a rent price both parties accept) and l is available to t. We also include a null "contract" representing being unmatched (no tenant for a landlord, or no landlord for a tenant).

Each tenant t has a preference ordering over all contracts that involve t (as well as the option of not renting any house). These preferences are determined by both monetary and non-monetary factors. For example, a tenant may prefer a contract with a lower rent ceteris paribus, a better location, larger size, and so on. We assume each tenant's preferences are private information to the tenant, and we require that they are transitive and strict (no indifferences, for simplicity). An illustrative example of a tenant's preference:

$$(t, l_1, \text{rent} = 50000) \succ_t (t, l_2, \text{rent} = 40000) \succ_t \text{unmatched},$$

meaning tenant t prefers renting from landlord l_1 at 50000 over renting from l_2 at 40000, and prefers that over not renting at all. This could occur if l_1 offers a much nicer apartment worth the extra cost to t. This model observes that, tenants will trade off price against quality and other non-monetary attributes; and we include them in the ranking of contracts. We impose that for any given landlord l, a contract with a lower rent is preferred by t to one with a higher rent, assuming monotonicity in money, and likewise, for a fixed rent level, the tenant has a strict ranking of houses (effectively landlords).

Each landlord l likewise has preferences over contracts that involve l or being unmatched (vacant). A landlord's preference order depends on the rent and on the tenant. Landlords care about tenant characteristics (non-monetary factors) and the rent price. For instance, a landlord might prefer a tenant with a stable job or without pets, etc., if such qualities affect expected maintenance costs or reliability. We assume each landlord l has a strict preference ranking $\succ_l$ over all contracts (t, l, π) that it could sign, plus the option of leaving l vacant. We impose that for any given tenant t, a contract with a higher rent is preferred by l to one with a lower rent, assuming monotonicity in money, and likewise, for a fixed rent level, the landlord has a strict ranking of tenant types, similar to our assumption with tenants. This ensures that their preference follows intuitive economic incentives.

Importantly, we assume that each landlord considers each potential tenant independently, i.e., there are no complementarities across different houses or tenants. Our assumption has its roots in our initial decision to treat each house as a separate landlord: this gives us the freedom to argue for independent choice functions for each house. This is formalized by the substitutes condition: adding more potential contracts cannot make a landlord accept an offer of a tenant that she would have rejected when fewer options were available. This substitutes property is crucial for stable matching existence in contracts models, and we will assume it holds for all landlord preferences in our setting (we revisit this in Section 3.5). Under this assumption, each landlord's choice behavior can be described by a choice function $C_l(\cdot)$ that selects, from any set of contract offers, the most preferred contract (at most one, since a landlord can only sign one contract per house) and rejects all others. The substitutes condition can be stated as: if from a set of offers Y landlord l rejects a contract x (i.e. $x \notin C_l(Y)$), then for any larger set $Y' \supseteq Y$, we must have $x \notin C_l(Y')$. In other words, any contract x that is rejected from some set Y will also be rejected from any superset of Y containing x. This formalizes the idea that contracts (tenants) are substitutes rather than complements for the landlord.

2.2 Tenant-Proposing Deferred Acceptance Algorithm with Contracts

We now describe the matching mechanism. We design an algorithm, inspired by Gale-Shapley deferred acceptance, in which tenants "propose" to houses. The mechanism operates in rounds, processing contract offers and tentative acceptances, and terminates in a stable matching. We assume each tenant has preferences over which property (and landlord) they would like to rent (from), considering factors like location, size, quality of the apartment, neighborhood, etc., as well as the price of the rent. Similarly, each landlord has preferences over potential tenants, considering factors such as the tenant's reliability, income, family size, and how well they might care for the property, in addition to the rent that tenant would pay. Each of these aspects affects how they rank the contracts respectively. We assume that each landlord informs their realtors about their predetermined priority ranking or choice function, and the realtors accept or reject the offers on behalf of the landlords as an intermediary.

Initially, all tenants are unmatched and all houses are vacant (landlords are unmatched). Each tenant is assumed to have a list of acceptable contracts, as a result of a choice function taking both monetary and non-monetary aspects into account, ranked from most preferred to least preferred. Landlords have their choice functions as described.

Step 1: In each round, each unmatched tenant t proposes a contract for the *most preferred* landlord l for which t has not yet been definitively rejected.
Step 2: Upon receiving proposals, each landlord l considers all contracts proposed to herself in this round, together with any contract she may have tentatively accepted in the previous round. The landlord then chooses the best contract among these according to their preference (using C_l). That chosen contract with tenant t^* is tentatively accepted by l, and any other proposals l received are rejected.
Step 3: Any tenant who is rejected in Step 2 remains unmatched and moves on to their next choice. Return to step 1 for another round of proposals from all currently unmatched tenants who still have remaining options.

The algorithm terminates when every unmatched tenant has no further acceptable landlord to propose to, or equivalently, when no landlord receives new offers. All tentatively held contracts at that point are finalized and signed (matched), and all other tenants remain unmatched.

Each tentative acceptance in this algorithm can be viewed as a landlord holding onto a tenant while waiting to see if a more preferred tenant comes along in a later round. A tenant may be rejected either because the landlord they gave an offer to does not find them acceptable at all, or because the landlord found a preferable tenant. A tenant who is rejected will then try the next landlord on their preference list. This procedure is very much like the classical Gale-Shapley algorithm, but altered to handle contracts.

The outcome of the algorithm is a set of matches μ between certain tenants and landlords. We denote $\mu(t)$ as the landlord tenant t is matched to (or $\mu(t) = \varnothing$ if unmatched), and $\mu(l)$ as the tenant matched to landlord l (or $\varnothing$ if l remains unmatched).

To illustrate our algorithm consider:

- Tenants: $T = \{\{t_1, t_2\}$
- Landlords: $L = \{l_1, l_2\}$
- Each tenant can propose to both landlords at varying rents.
- A contract x is a tuple $(\text{tenant}, \text{landlord}, \text{rent})$.

$$
\begin{aligned}
x_1 &= (x_1, l_1, 60000) \\
x_2 &= (x_1, l_2, 55000) \\
x_3 &= (x_2, l_1, 50000) \\
x_4 &= (x_2, l_2, 58000)
\end{aligned}
$$

$$
\begin{aligned}
\textbf{Tenant } t_1\textbf{'s Preferences:} &\quad x_2 \succ x_1 \succ \text{unmatched} \\
\textbf{Tenant } t_2\textbf{'s Preferences:} &\quad x_4 \succ x_3 \succ \text{unmatched} \\
\textbf{Landlord } l_1\textbf{'s Preferences:} &\quad x_1 \succ x_3 \succ \text{unmatched} \\
\textbf{Landlord } l_2\textbf{'s Preferences:} &\quad x_4 \succ x_2 \succ \text{unmatched}
\end{aligned}
$$

Tenant Proposing Deferred Acceptance Algorithm with Contracts
Round 1:

- t_2 offers $x_2 = (t_2, l_2, 55000)$
- t_1 offers $x_4 = (t_1, l_2, 58000)$
- l_2 receives both x_2 and x_4: Tentatively accepts x_4, rejects x_2

Round 2:

- t_2, after getting a rejection, offers $x_1 = (x_2, l_1, 60000)$
- l_1 receives x_1: accepts (only offer)

Final Matching

Tenant	Landlord	Contract (Rent)
t_2	l_1	60000
t_1	l_2	58000

2.3 Role of Realtors

In this mechanism, realtors serve an intermediary role. They can be thought of as the agents who collect information and facilitate proposals. In practice, a realtor could run the algorithm as a centralized authority. Landlords provide the realtors with their rankings of the contracts. The realtor then processes the proposals made by the prospective tenants in rounds as described. Realtors can also assist in the preference elicitation of the landlords by helping them define their ranking, perhaps by screening tenants, and giving information about the market. Realtors can continue to earn a commission for successfully matched contracts, but the critical difference is that the matching is determined by the algorithm's stable outcome. This preserves the realtor's role (showing houses, advising on terms, processing paperwork) in the current market. The commission could still be paid by the tenant or split in some way, but since the allocation is fair, stable, and tenant-optimal, tenants are not at a disadvantage of having to bid excessively or rushing to the first available house. Moreover, with a stable matching mechanism, a tenant will not lose a desired house to another tenant who is less preferred by that landlord, eliminating justified envy in the market. The mechanism does not eliminate the realtor, however it transforms the realtor's function into operating a mechanism.

3 Properties of the Mechanism Outcome

We now analyze several key properties of the matching produced by the tenant-proposing deferred acceptance mechanism with contracts. We assume that all tenants and landlords participate honestly in the mechanism (we will examine strategic aspects shortly). Given the preference profiles of all agents, the algorithm yields a matching μ.

We will demonstrate the following desirable properties: individual rationality, stability, optimality for tenants, strategy-proofness for tenants, fairness, and pareto efficiency. We also clarify the technical conditions needed on preferences: the *substitutability* and *irrelevance of rejected contracts* conditions, following the results of Hatfield & Milgrom and others.

3.1 Individual Rationality

A matching μ is **individually rational** if no agent is matched in a way that is worse than being unmatched. In our context, this means: every tenant t who is matched to some landlord $l = \mu(t)$ must weakly prefer that contract to being unmatched (not renting at all), and every landlord l that is matched to tenant $t = \mu(l)$ must weakly prefer that contract to leaving the house vacant. Our mechanism ensures individual rationality by construction. In the deferred acceptance process, tenants only propose to landlords they find *acceptable*. If a tenant would rather remain without a house than accept some contract (for example, if the rent is too high or the location undesirable), that contract would be ranked below the unmatched option in their preference list, so the tenant would never propose it. Thus, a tenant t will only end up matched to a landlord l if l was on t's acceptable list and was higher-ranked than being unmatched. Similarly, landlords only tentatively accept proposals that are acceptable. If a particular tenant t has rent or attributes that l considers worse than keeping the house empty, then t's proposal will be rejected immediately. Landlords never agree to a contract that leaves them worse off than vacancy. Therefore, when the algorithm terminates, any match (t, l) that exists is mutually acceptable to both t and l. By definition this means neither t nor l prefers the unmatched status over their match. Individual rationality holds. This is a crucial requirement for practical viability: no tenant or landlord is forced into an undesirable lease. Participation in the mechanism is voluntary in the sense that if a tenant cannot find any house that they consider at least as good as not renting, they will simply end up unmatched (and likewise for a landlord).

3.2 Stability and the Absence of Blocking Pairs

We now turn to **stability**, the central solution concept in matching theory. A matching μ is called *stable* if (i) it is individually rational (already established) and (ii) there are no blocking pairs: no tenant t and landlord l who are not matched to each other would both prefer to be matched together (under some contract) rather than their outcomes under μ. In other words, there is no incentive for any tenant-landlord pair to deviate and form a contract bilaterally. In our one-to-one matching with contracts, a blocking pair would mean there exists a contract $x = (t, l, \pi)$ not in the matching such that t is either unmatched or prefers x to $\mu(t)$, and l is either unmatched or prefers x to $\mu(l)$, and moreover x is feasible for both. Stability requires that no such x exists.

Stability of μ produced by the DA mechanism: It is a well-known property from Gale-Shapley (1962) that the deferred acceptance algorithm yields a stable matching. We briefly argue why that holds in our context with contracts. Suppose, for the sake of contradiction, that the outcome μ from our tenant-proposing algorithm is not stable. Then there is some tenant t and landlord l that form a blocking pair.

Consider t and l who form a blocking pair. Then, they are unmatched in μ, but prefer each other over their current matches. Then t must have sent an offer to l at some point since t prefers l to being unmatched and also ranks l higher than her current match. The algorithm would only leave them unmatched if either l rejected t or t never proposed because l was matched with someone they liked better. If l rejected t, it means l had a better option. If l ended unmatched but rejected t, it implies l preferred being unmatched to accepting t, both of these scenarios are contradicting the assumption that l prefers t. If l is matched to someone else in μ, then l clearly prefers its match over t, or else l would not have rejected t in favor of that match. So t and l cannot both prefer each other to their outcomes.

This intuitive argument formalized in the classic proof shows that deferred acceptance yields a stable matching. The contract setting adds the twist that the landlord might compare not just which tenant, but also the price of the rent. If t and l would like to deviate under some contract π, that contract must have been among the options considered during the algorithm. The fact that they did not end up together means either t never offered π to l, implying t had a better option or l was unacceptable or l rejected it in favor of a better contract. Either way, a mutually preferable deviation cannot exist at the end.

Therefore, μ is stable. There is no landlord-tenant pair who could exit the mechanism and arrange a lease that both find preferable to the official outcome. Stability is highly desirable in our market: it ensures, that both tenants and landlords are happy with their match. And furthermore, it implies no justified envy. For example, that if a tenant A covets the apartment that tenant B got, it must be that the landlord of that apartment prefers B over A, so A has no justified claim to it. We next elaborate on this fairness aspect.

3.3 Fairness

Stability in matching markets often corresponds to the fairness criteria. In the context of school choice, for instance, stability with respect to schools' priority rankings is equivalent to two conditions: *no justified envy* and *non-wastefulness*, together with individual rationality. A similar interpretation holds in our tenant-landlord setting if we view landlords' preferences as "priority" rules for who gets the house. We define these terms and argue how the stable outcome from our algorithm satisfies them.

Non-Wastefulness: An assignment is non-wasteful if there is no house left vacant while there is some tenant who would be happy to rent it and whom the landlord finds acceptable. In other words, we cannot have a situation where l is unmatched and there is a tenant t unmatched such that both t and l would prefer to be matched together. Any such situation would constitute a blocking pair, violating stability. Thus, a stable matching inherently avoids waste: if a house is empty, it must be that every unmatched tenant either does not want that house or is unacceptable to that landlord. Our algorithm's outcome μ is non-wasteful by virtue of stability. In the DA algorithm, if a landlord remains unmatched, it means they rejected all offers, finding them unacceptable; if a tenant remains unmatched, it means they were rejected by all landlords they found acceptable. It cannot be that an acceptable pair failed to match when they both would have preferred so, or they would have matched. So no mutually beneficial opportunity is left unrealized.

No Justified Envy: This concept adapts the idea of fairness in priority-based allocation. Formally, there should not exist two tenants t and t' and a landlord l such that t is not matched with l but l is matched to t', t prefers l to their own match, and l would rather have t than t'. If this occurred, t would have a justified envy towards t' for landlord l. In a stable matching, this cannot happen: if t preferred l to their match, t would have sent an offer to l before the less-preferred landlord; and if l actually preferred t to t', then l would not have accepted t''s contract over t's. The stability condition directly implies that there is no tenant who can claim that they "deserved" a house more than the person who got it, where deserving is based on the landlord's preferences. Therefore, no justified envy holds. The tenant-optimal stable mechanism is fair because whenever a tenant doesn't match with a landlord they wanted, it's only because that house went to someone the landlord ranked higher. This satisfies the no justified envy criterion.

In summary, the outcome of our mechanism can be seen as fair. All houses that could be filled with acceptable tenants are filled (no needless vacancies), and no tenant is overtaken by another of lower "priority" in matching with a landlord they desire. These properties are essentially an unpacking of stability.

3.4 Optimality and the Lattice of Stable Matchings

It is known from classic matching theory that when preferences are strict and the substitutes condition holds, the set of stable matchings form a **lattice** under the partial order of preferences. There is a unique *tenant-optimal* stable matching and a unique *landlord-optimal* stable matching in this lattice. Our algorithm yields the tenant-optimal stable matching. This means that in the outcome μ, each tenant gets the best house and the landlord among those they consider acceptable that they could possibly obtain in any stable matching. The tenant-optimality of a tenant-proposing DA is a standard result, Gale-Shapley proved that no other stable matching can make any tenant better off without making some tenant worse off. If a tenant t were matched to a better landlord in some other stable matching μ', then in the DA algorithm t would have proposed to that house and, through the iterative improvement property, t would not have been rejected unless that landlord obtained someone it prefers to t. But if μ' is stable and gives t that landlord, it implies that the landlord did not have a preferable tenant in μ'. This leads to a contradiction. Therefore, μ is tenant-optimal stable.

For landlords, tenant-optimal stable matching is not the optimal stable matching, potentially incentivizing landlords to strategize, as discussed below. However, we emphasize that μ is still individually rational for landlords and stable. Tenant-optimality means that if there were multiple stable outcomes, μ is slanted in favor of tenants. This is a deliberate design choice: since tenants are typically the less powerful side (facing high rents and fees), a tenant-optimal stable mechanism ensures they are not shortchanged. Moreover, tenant-proposing DA mechanism is the most similar to the current system, which can provide a more seamlessly applicable solution to the tenant-landlord problem.

3.5 Strategy-Proofness

A crucial incentive property of the deferred acceptance mechanism is its strategy-proofness for the proposing side. In our mechanism, tenants have a dominant strategy to truthfully reveal their preferences (Dubins-Freedman, 1981). In other words, a tenant cannot benefit by misrepresenting which houses or contracts they prefer. This result is analogous to the classic Dubins-Freedman (1981) and Roth (1982) findings for stable matchings: under the Gale-Shapley algorithm, no proposer can gain from manipulating their preference list. The intuition is that since the algorithm is tenant-proposing and yields the best stable outcome for tenants, a tenant's honest ranking leads to the most preferred landlord they can get while respecting stability. If a tenant lies (for example, by not listing a certain landlord or by reordering their choices), they risk losing a house they could have obtained, or getting a worse one. Any attempt to strategically game the system by a tenant can only result in the tenant being matched with an outcome that is worse or equal to the truthful outcome. It cannot improve their match, because the mechanism already gives them the top feasible match.

Formally, consider any tenant t. Suppose t submits a false preference order $\succ'_t$ instead of the true $\succ_t$. Run the DA algorithm with $\succ'_t$ (others truthful) and get outcome μ'. Now, compare with the truthful outcome μ with $\succ_t$. If $\mu'(t)$ is some landlord l, there are two cases: either t actually prefers $\mu'(t)$ to $\mu(t)$ under $\succ_t$, or not. If t does prefer $l = \mu'(t)$ to $\mu(t)$, then l was an option under truthful preferences that t did not get. But since μ was the tenant-optimal stable match, the only reason t didn't get l in μ must be that l was matched to someone l prefers over t. If t truly prefers l, they would have sent an offer and l would have rejected it in favor of a preferable tenant, meaning t couldn't get l in any stable matching. So $\mu'(t) = l$ would violate stability or individual rationality if l prefers someone else. Thus this case can't actually yield a better outcome. In fact, it is possible to show $\mu(t) \succ_t \mu'(t)$ truth-telling dominates in all scenarios.

Hence, tenants do best by listing their preferences in true order of desirability and they do not need to engage in strategic behavior. However, the mechanism is *not* strategy-proof for the landlord side, a landlord might wish to misreport their preference if they thought it could get them a better tenant. Since the matching is tenant-optimal, the landlords could strategize to tweak the outcome in their favor. This could be done through providing the realtors with altered lists where the landlords could change their reported preference orders or not list a tenant as acceptable entirely. Through strategizing, the landlords could match with tenants who are higher in their preference rankings. Note that no stable mechanism can be strategy-proof for *both* sides simultaneously. We thus settle for strategy-proofness on the tenant side, which is the side facing more information asymmetry and costs in the current scenario.

To summarize, the tenant-proposing DA mechanism removes the confusion for tenants. A tenant should list all landlords they find acceptable, in true order. They will then get the best possible landlord they could under any stable arrangement. This is a significant improvement over the current system, where tenants often have to strategize (e.g., accepting an offer quickly or waiting for a better apartment, risking ending up with nothing). Here, the mechanism itself finds the optimal stable outcome, and tenants cannot do better by trying to game the timing or misinforming preferences.

3.6 Pareto Efficiency

The concept of **Pareto efficiency** means that there is no other feasible outcome in which some individuals could be better off without making someone else worse off. A proposer DA matching is Pareto efficient among all matchings that respect individual rationality. In fact, it can be argued that any stable matching is Pareto-efficient: if you could make one tenant-landlord pair better off without hurting anyone, that pair would have been a blocking pair to begin with or it would violate non-wastefulness.

More concretely, consider the matching μ from our mechanism. Suppose ν makes at least one tenant t better off and no one worse off. For t to match with a strictly better landlord l in ν, l must either be the same or better off in ν as well (since no one is worse off). If l prefers t in ν to its tenant in μ, then (t, l) was a blocking pair to μ. Stability of μ rules that out. If l had the same tenant in μ and ν, and t somehow got a different better landlord, then some other tenant got worse, the one who lost their match to t, contradicting the assumption. Essentially, any attempt to Pareto-improve upon μ would need to reassign some landlord to a more preferred tenant without hurting anyone but the originally assigned tenant of that landlord would then be worse off, or the landlord would be worse off if they get a less preferred tenant. The only scenario is if some house was vacant and an unmatched tenant could take it which would be a Pareto improvement, but stability already ensured no such mutually beneficial arrangement was left unused.

Thus, μ is Pareto efficient among the set of all matchings which are individually rational, or in other terms, where no one is forced into a worse-than-unmatched situation. We can confidently state that any stable mechanism is both Pareto efficient and individually rational. This is an important reassurance: the mechanism is not leaving money on the table or failing to match someone when it could have done so beneficially.

3.7 Substitutability and Irrelevance of Rejected Contracts

Up to now, we have assumed certain conditions on preferences to guarantee the desirable properties of our mechanism. We briefly highlight two key conditions identified in the matching with contracts literature:

Substitutability: We introduced this earlier, each landlord's choice function satisfies the substitutes condition. This condition is essential for the existence of a stable matching and for the deferred acceptance algorithm to work properly in many-to-one or contracts settings. Hatfield and Milgrom (2005) proved that if all firms (landlords) have substitutable preferences and satisfy the Law of Aggregate Demand, then the set of stable allocations is non-empty and forms a lattice. In our one-to-one setting, substitutability is easier to satisfy because each landlord fills only one position, intuitively, any standard strict preference ordering over individual contracts is substitutable when only one contract can be accepted. The subtlety arises more in many-to-one settings; by modeling each landlord as controlling a single unit, we avoid that complexity. Nevertheless, the concept remains important: we rule out situations such as a landlord who is willing to rent *only if* two particular tenants jointly accept two different apartments: such complementarities would violate substitutability and lead to instability or the absence of any stable matching. By treating each landlord separately and assuming independent preferences, we ensure compatibility with substitutability.

We can formalize substitutability in our context as follows: Let $C_l(Y)$ denote landlord l's chosen contract from a set Y of offers. Contracts are substitutes for l if for any sets Y' and Y with $Y' \subseteq Y$, we have

$$C_l(Y) \cap Y' \subseteq C_l(Y'),$$

meaning that if a contract x is chosen from the larger set Y and $x \in Y'$, then x must also be chosen from the smaller set Y' whenever x is available in Y'. Equivalently, the condition can be restated as follows: if a contract x is rejected from a set Y, then the addition of another contract x' which is also rejected, to the set of offers will not cause x to become acceptable.

Our mechanism satisfies this condition, assuming that each landlord's behavior is consistent and can be rationalized by an underlying strict preference ordering over individual contracts.

Irrelevance of Rejected Contracts (IRC): This condition, introduced by Aygün and Sönmez (2013), further refines the model of choice in matching with contracts. IRC demands that if a contract is rejected from a given set of offers, then removing other rejected contracts from the offer set should not affect the outcome. Formally, a choice function C_l satisfies IRC if for any set of contracts Y and any contract $z \notin C_l(Y)$, we have

$$C_l(Y \setminus \{z\}) = C_l(Y).$$

That is, if z is rejected from Y, then the removal of z itself does not alter the chosen contract. Intuitively, the landlord's decision to reject z is final and not contingent upon the presence of other rejected offers. This rules out pathological cases where a rejected contract becomes acceptable only because another rejected offer is no longer present. In well-behaved choice models, where selections are derived from a strict and complete preference ordering, IRC holds automatically. Aygün and Sönmez observed that Hatfield and Milgrom's stability results implicitly assume IRC through their assumption of choice functions arising from strict preferences. If a choice function were to violate IRC, then some key stability results would no longer hold.

In our setting, we assume that each landlord has a strict and complete preference ordering over contracts. Through their communications with their realtors, the realtors notify landlords of the current market conditions as they assist in the preference elicitation process. This assumption ensures landlords' choices are stable under new information (new offer). As a result, the induced choice function satisfies IRC by construction. A landlord rejects a contract either because they strictly prefer another one or deem it unacceptable; the presence or absence of other rejected offers does not affect this judgment. Thus, IRC holds in our model.

In summary, the theoretical foundation of our mechanism depends critically on these two conditions: substitutability and IRC. Under these assumptions, the existence of a stable matching is guaranteed. Specifically, Theorem 1 of Aygün and Sönmez (2013) proves that if every landlord's choice function satisfies substitutability and IRC, then a stable matching exists and the deferred acceptance algorithm finds one. Our mechanism, by assuming strict preferences over one-to-one contracts, satisfies these conditions and thus the matching found by our algorithm is guaranteed to be stable.

4 Conclusion

This paper has presented a mechanism to match tenants and landlords in the Turkish housing market using a one-to-one matching with contracts framework. By viewing the rental matching problem from a matching theory perspective, we reach results from Gale-Shapley's stable marriage to Hatfield-Milgrom's matching with contracts that guarantee a stable and fair outcome. The proposed tenant-proposing deferred acceptance algorithm with contracts produces a matching that is individually rational, stable, and Pareto efficient, while favoring tenants. We showed that stability in this context corresponds to an absence of any tenant-landlord pair who would rather be matched with each other, ensuring that no tenant has a justified envy and no landlords' opportunity is wasted.

Our mechanism improves on the current system in several ways. First, it aligns incentives properly: tenants can honestly rank houses without fear of regret, and landlords engage with the process knowing they will get the best available tenant who wants their house. Second, it removes the randomness from the search process: currently, a tenant might miss out on a great apartment due to the chaos of the market. In our system, as long as the tenant values that apartment and the landlord prefers that tenant, they will end up matched. This leads to a sense of fairness and transparency that is currently lacking. Third, it can reduce vacancies and mismatches: no house that could profitably be rented to an eager tenant will remain empty, and no tenant will remain homeless while an acceptable house sits idle. Stability implicitly achieves a form of market efficiency in that regard. Importantly, the role of realtors is not removed but rather repurposed. Realtors can continue to earn commissions by facilitating the matching process: they will help the landlords to articulate their preferences, and guide both the tenants and the landlords through the deferred acceptance process. The mechanism could even be seen as a realtor-managed "renting exchange" or clearinghouse. By supporting an organized market, realtors could potentially match clients more quickly and with higher satisfaction, thereby possibly increasing volume and their overall earnings in the long run.

In conclusion, using a one-to-one matching with contracts approach for the Turkish housing rental market offers a theoretically sound and practical way to improve allocations. It guarantees **optimal and stable matches**: no tenant or landlord can do better by deviating, and every match is as good as possible for tenants given those constraints. It also satisfies **fairness**: the outcome respects both sides' preferences without bias or unnecessary costs, eliminating justified envy and preventing avoidable vacancies. Compared to the current intermediary-driven system, our mechanism would result in higher overall satisfaction, less frustration, and potentially more trust in the market process. We have thus successfully implemented matching theory mechanisms to a real-world problem, showing how classic results like Gale-Shapley's algorithm, and modern extensions, can be applied to design a better housing market that protects tenants, respects landlords' rights, and still leaves room for realtors to play a constructive role.

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