```
(a) Write out these formulae for the graph ({0, 1}, {{0, 1}}). (You can abbreviate color(0, Red) as r0, color(0, Blue) as b0, and similarly for other atoms.)
Variables:
r0: Vertex 0 is colored with red b0: Vertex 0 is colored with blue.
r1: Vertex 1 is colored with red. b1: Vertex 1 is colored with blue.
Domain: The available colors \rightarrow {Red, Blue}.
Constraints:
a. Each vertex must be assigned to a color (red or blue): (r0 V b0) \( \text{(r1 V b1)} \)
Vertex 0 should be red or blue
Vertex 1 should be red or blue
b. Each vertex must be assigned to one color: \neg (r0 \land b0) \land \neg (r1 \land b1)
Vertex 0 cannot be red and blue
Vertex 1 cannot be red and blue
c. Adjacent vertices must be assigned different colors:
                                                            \neg (r0 \land r1) \land \neg (b0 \land b1)
Vertex 0 and 1 cannot be same color (red)
Vertex 0 and 1 cannot be same color (blue)
Objective: Finding a satisfying assignment to the variables that satisfies the constraints.
Propositional logic representation for the graph (\{0, 1\}, \{\{0, 1\}\}):
((r0 \ V \ b0) \ \Lambda \ (r1 \ V \ b1)) \ \Lambda \ (\neg(r0 \ \Lambda \ b0) \ \Lambda \ \neg(r1 \ \Lambda \ b1)) \ \Lambda \ (\neg(r0 \ \Lambda \ r1) \ \Lambda \ \neg(b0 \ \Lambda \ b1)))
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(b) Transform these formulae into CNF format. The CNF format has following characteristics:

Disjunction of Literals: Each clause is a disjunction (OR/v) of literals. A literal can be either a variable or its negation.

Conjunction of Clauses: The CNF formula is a conjunction (AND/ Λ) of multiple clauses. Each clause represents a condition or constraint.

 $\neg (r0 \land r1) \land \neg (b0 \land b1)$

- Standard Form: The CNF formula is usually written in a specific standard form where each clause is enclosed in parentheses, and literals 3. are separated by OR symbols.
- a. Each vertex must be assigned to a color (red or blue): (r0 V b0) \(\text{(r1 V b1)} \) O should be red or blue Vertex 1 should be red or blue
- b. Each vertex must be assigned to one color: $\neg (r0 \land b0) \land \neg (r1 \land b1)$
- $(\neg r0 \ V \ \neg b0) \ \Lambda \ (\neg r1 \ V \ \neg b1)$
- Vertex 0 cannot be red and blue
- Vertex 1 cannot be red and blue
- c. Adjacent vertices must be assigned different colors:

- $(\neg r0 \ V \ \neg r1) \ \Lambda \ (\neg b0 \ V \ \neg b1)$
- Vertex 0 and 1 cannot be same color (red) Vertex 0 and 1 cannot be same color (blue)

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(c) Transform the CNF formulae into DIMACS CNF format.
                                                                                                                                                 120
                                                                                                                                                3 4 0
a. Each vertex must be assigned to a color (red or blue):
                                                                                  (r0 V b0) \(\Lambda\) (r1 V b1)
                                                                                                                                                -1 - 20
Vertex 0 should be red or blue
                                                                                                                                                -3 -40
Vertex 1 should be red or blue
                                                                                                                                                -1 - 30
b. Each vertex must be assigned to one color: \neg (r0 \land b0) \land \neg (r1 \land b1)
                                                                                                                                                -2 -40
                       (\neg r0 \lor \neg b0) \land (\neg r1 \lor \neg b1)
Vertex 0 cannot be red and blue
                                                                                                                                                minisat/
Vertex 1 cannot be red and blue
c. Adjacent vertices must be assigned different colors:
                                                                                   \neg (r0 \land r1) \land \neg (b0 \land b1)
                                                                                                                                           Load a predefined example:
                                                                                                                                           simple
                     (\neg r0 \ V \ \neg r1) \ \Lambda \ (\neg b0 \ V \ \neg b1)
Vertex 0 and 1 cannot be same color (red)
                                                                                                                                             simple v3 c2.cnf
                                                                                                                                           p cnf 4 6
Vertex 0 and 1 cannot be same color (blue)
                                                                                                                                           3 4 0
                                                                                                                                           -1 -2 0
                                                                                                                                           -3 -4 0
                                                                                                                                           -1 -3 0
                                                                                                                                           -2 -4 0
Assigning integer values to variables: r0: 1, b0: 2, r1: 3, b1: 4
Combined together:
(r0 \ V \ b0) \ \Lambda \ (r1 \ V \ b1) \ \Lambda \ (\neg r0 \ V \ \neg b0) \ \Lambda \ (\neg r1 \ V \ \neg b1) \ \Lambda \ (\neg r0 \ V \ \neg r1) \ \Lambda \ (\neg b0 \ V \ \neg b1)
Transforming the CNF formulae into DIMACS CNF format:
 (1 \ \ \textbf{V} \ \ 2) \ \ \textbf{\Lambda} \ \ (3 \ \ \textbf{V} \ \ 4) \ \ \textbf{\Lambda} \ \ (-1 \ \ \textbf{V} \ \ -2) \ \ \textbf{\Lambda} \ \ (-3 \ \ \textbf{V} \ \ -4) \ \ \textbf{\Lambda} \ \ (-1 \ \ \textbf{V} \ \ -3) \ \ \textbf{\Lambda} \ \ (-2 \ \ \textbf{V} \ \ -4) 
                                                                                                                                           Verdict: SATISFIABLE
                                                                                                                                           SATISTIABLE
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Using: http://logicrunch.it.uu.se:4096/~wv/ Verdict: $-1\ 2\ 3\ -4\ 0 \rightarrow satisfiable$ or enter a problem in DIMACS:

Check Your problem is sent to the server. Please be a little patient for the answer...

⊞ Running Minisat

-1 2 3 -4 0