(a) Write out these formulae for the graph ({0, 1}, {{0, 1}}). (You can abbreviate *color*(0, *Red*) as r0, *color*(0, *Blue*) as b0, and similarly for other atoms.)

Variables:

r0: Vertex 0 is colored with red **b0**: Vertex 0 is colored with blue.

r1: Vertex 1 is colored with red. **b1**: Vertex 1 is colored with blue.

Domain: The available colors → {Red, Blue}.

Constraints:

a. Each vertex must be assigned to a color (red or blue): (r0 ∨ b0) ∧ (r1 ∨ b1)

```
Vertex  0 should be red or blue
Vertex 1 should  be red or blue
```

b. Each vertex must be assigned to one color: ¬(r0 ∧ b0) ∧ ¬(r1 ∧ b1)

```
Vertex 0 cannot be red and blue
Vertex 1 cannot be red and blue
```

c. Adjacent vertices must be assigned different colors: ¬(r0 ∧ r1) ∧ ¬(b0 ∧ b1)

```
Vertex 0 and 1 cannot be same color (red)
Vertex 0 and 1 cannot be same color (blue)
```

Objective: Finding a satisfying assignment to the variables that satisfies the constraints.

Propositional logic representation for the graph ({0, 1}, {{0, 1}}) :

((r0 ∨ b0) ∧ (r1 ∨ b1)) ∧ (¬(r0 ∧ b0) ∧ ¬(r1 ∧ b1)) ∧ (¬(r0 ∧ r1) ∧ ¬(b0 ∧ b1)))

(b) Transform these formulae into CNF format.

The CNF format has following characteristics:

1. Conjunction of Clauses: The CNF formula is a conjunction (AND/∧) of multiple clauses. Each clause represents a condition or constraint.
2. Disjunction of Literals: Each clause is a disjunction (OR/∨) of literals. A literal can be either a variable or its negation.
3. Standard Form: The CNF formula is usually written in a specific standard form where each clause is enclosed in parentheses, and literals are separated by OR symbols.

a. Each vertex must be assigned to a color (red or blue): $(r0 \lor b0) \land (r1 \lor b1)$

```
Vertex  0 should be red or blue
Vertex 1 should  be red or blue
```

b. Each vertex must be assigned to one color: $\neg(r0 \land b0) \land \neg(r1 \land b1)$

→ $(\neg r0 \lor \neg b0) \land (\neg r1 \lor \neg b1)$

```
Vertex 0 cannot be red and blue
Vertex 1 cannot be red and blue
```

c. Adjacent vertices must be assigned different colors: $\neg(r0 \land r1) \land \neg(b0 \land b1)$

→ $(\neg r0 \lor \neg r1) \land (\neg b0 \lor \neg b1)$

```
Vertex 0 and 1 cannot be same color (red)
Vertex 0 and 1 cannot be same color (blue)
```

(c) Transform the CNF formulae into DIMACS CNF format.

a. Each vertex must be assigned to a color (red or blue): $(r0 \lor b0) \land (r1 \lor b1)$

Vertex 0 should be red or blue

Vertex 1 should be red or blue

b. Each vertex must be assigned to one color: $\neg(r0 \land b0) \land \neg(r1 \land b1)$

$\rightarrow$ $(\neg r0 \lor \neg b0) \land (\neg r1 \lor \neg b1)$

Vertex 0 cannot be red and blue

Vertex 1 cannot be red and blue

c. Adjacent vertices must be assigned different colors: $\neg(r0 \land r1) \land \neg(b0 \land b1)$

$\rightarrow$ $(\neg r0 \lor \neg r1) \land (\neg b0 \lor \neg b1)$

Vertex 0 and 1 cannot be same color (red)

Vertex 0 and 1 cannot be same color (blue)

Assigning integer values to variables: r0: 1, b0: 2, r1: 3, b1: 4

Combined together:

$(r0 \lor b0) \land (r1 \lor b1) \land (\neg r0 \lor \neg b0) \land (\neg r1 \lor \neg b1) \land (\neg r0 \lor \neg r1) \land (\neg b0 \lor \neg b1)$

Transforming the CNF formulae into DIMACS CNF format:

$(1 \lor 2) \land (3 \lor 4) \land (-1 \lor -2) \land (-3 \lor -4) \land (-1 \lor -3) \land (-2 \lor -4)$

p cnf 4 6

1 2 0

3 4 0

-1 -2 0

-3 -4 0

-1 -3 0

-2 -4 0

Using: http://logicrunch.it.uu.se:4096/~wv/minisat/

Verdict: -1 2 3 -4 0 → satisfiable