

CS412 Machine Learning - 2023

HW3- Gradient Descent

50pts

- 1) **50pts** - We are trying to minimize a function $F(x) = x^2 - 10x + 5$ with respect to its parameter x . In other words we want to find the x for which $f(x)$ is minimum.

Starting from the point $x=10$ use steepest descent algorithm *for TWO steps* to find the local minimum of the function around this point. You should use a “step size” of $\alpha = 0.1$ in update, as: $x = x - \alpha \nabla$

We will use a subscript to indicate the subsequent values of x , starting from x_0 .

Worksheet:

$$F(x_0) = 10^2 - 10 \cdot 10 + 5 = 5 \text{ (since the starting point is } x = 10 \text{ indicated as above)}$$

: Just to note at what F value we start (5pts)

$$\nabla F = \text{Take the derivative of } F(x) = 2x - 10$$

: Compute the gradient (10pts)

Note: Even though F is a function of a single variable, you can still write/think of the gradient as a vector of size one.

$$\nabla F \mid x_0 = \text{we need to plug in } x_0 = 10 \text{ in } \nabla F = 2x - 10 \Rightarrow 2 \cdot 10 - 10 = 10$$

: This is the gradient **evaluated** at x_0 (10pts)

$$x_1 = 10 - 0.1(2 \cdot (10) - 10) = 9 \text{ (} x = x - \alpha \nabla \text{ used this formula: } x = 10, \alpha = 0.1, \nabla = 2x - 10 \text{)} \quad \text{:Update } x_0 \text{ to find } x_1 \text{ (5pts)}$$

$$F(x_1) = \text{plug in } x_1 = 9 \text{ in the } F(x) \Rightarrow 9^2 - 10 \cdot 9 + 5 = -4$$

: just checking to see if we are indeed minimizing

Now do the 2nd step similarly and write your results below (no partial so be careful please): :20pts

ANSWER: x_2 (x after 2 steps of gradient descent) = $x_1 - a \nabla F(x_1) \Rightarrow 9 - 0.1 \cdot (8) = \mathbf{8.2}$

$F(x_2)$ = plug in $x_2 = 8.2$ into $F(x) \Rightarrow (8.2)^2 - 10 \cdot (8.2) + 5 = \mathbf{-9.76}$

Submission: Write the ANSWER line as inline submission to homework and attach the filled page as a pdf document to Sucourse.