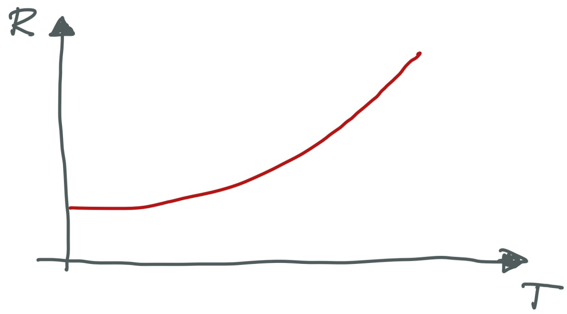
SUPERCONDUCTIVITY

FIRST MEASUREMENT:

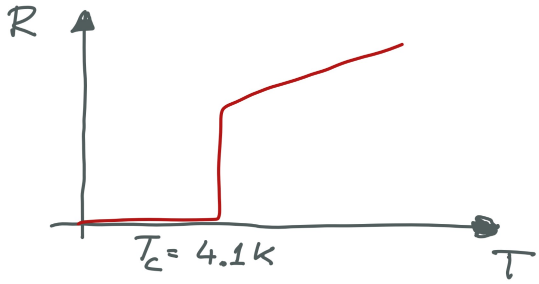
- 1911 KAMERLINGH ONNES (LEIDEN)

  MEASURING RESISTANCE OF METALS IN LIQUID ^4He



GOLD, PLATINUM

(PHONONS FREEZE OUT
RESIDUAL RESISTANCE)
LINEAR DEPENDENCE



MERCURY

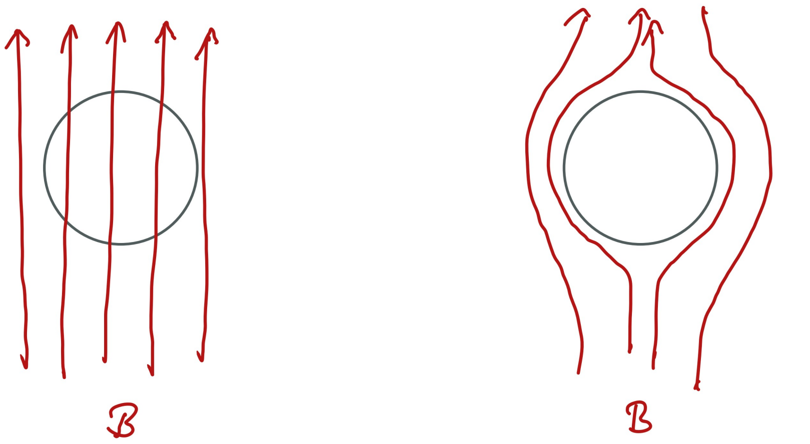
FOR EXAMPLE, Al ~1.2K
Nb ~9K
Ta ~4K

① ZERO ELECTRICAL RESISTANCE BELOW T_c

"SECOND" MEASUREMENT

- 1933 MEISSNER & OCHSENFELD

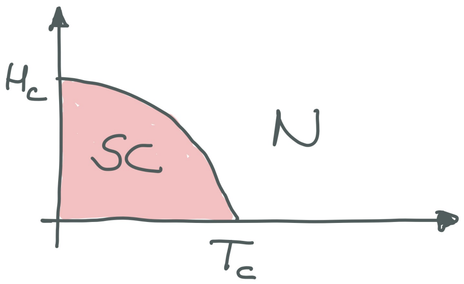
NORMAL STATE — SUPERCONDUCTING STATE



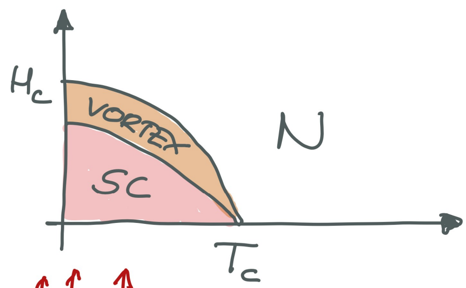
② THE MAGNETIC FIELD IS EXPELLED FROM SC.

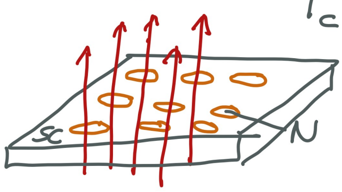
→ IT BREAKS DOWN ABOVE $H_c(T)$

PHASE DIAGRAM OF TYPE I SC



PHASE DIAGRAM OF TYPE II SC





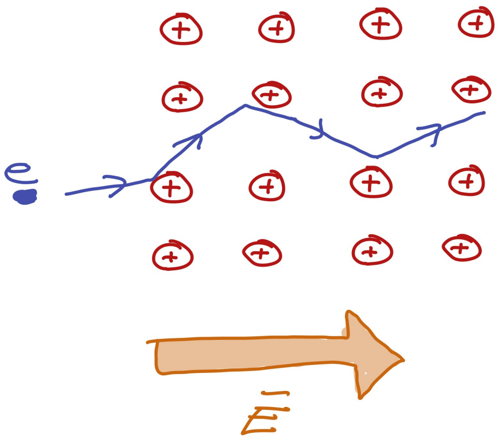
EACH VORTEX CORE CARRIES $\Phi_0 = h/2e$ FLUX.

LONDON MODEL (1935)

OHM'S LAW $\quad I = \frac{1}{R} \cdot U = G \cdot U$

$\downarrow$ MICROSCOPIC VERSION

$$j = \sigma E$$



DRUDE MODEL:

$$m \frac{dv}{dt} = eE - mv/\tau$$

STEADY STATE: $\quad v = \frac{eE\tau}{m}$

$$j = nev = \underbrace{\frac{ne^2\tau}{m}}_{\sigma} \cdot E$$

IN A SC, $\tau \to \infty$ (NO SCATTERING)

TWO TYPES OF CHARG CARRIERS, n_S $\quad \tau \to \infty$

n_N $\quad \tau$ FINITE

$$\frac{dv}{dt} = \frac{e}{m} \cdot E \quad \to \quad \boxed{\frac{dj}{dt} = \frac{n_S e^2}{m} \cdot E} \quad (1)$$

- FARADAY'S LAW:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\frac{d}{dt}(\nabla \times j) = \frac{-n_s e^2}{m} \frac{dB}{dt}$$

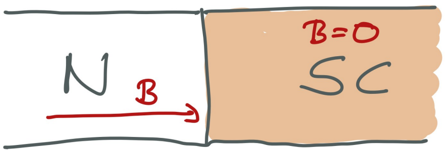
$$\boxed{\nabla \times j = -\frac{n e^2}{m} B} \qquad (2)$$

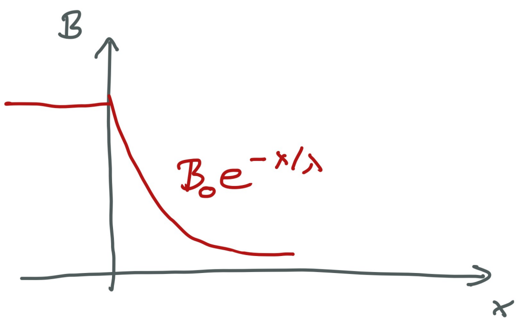
- AMPERE'S LAW

$$\nabla \times B = \mu_0 j$$

$$\nabla^2 B = \frac{1}{\lambda_L^2} \cdot B \qquad \lambda_L = \frac{m}{\mu_0 n_s e^2}$$

LONDON PENETRATION DEPTH.





$\lambda_L \approx 50 - 500$ nm

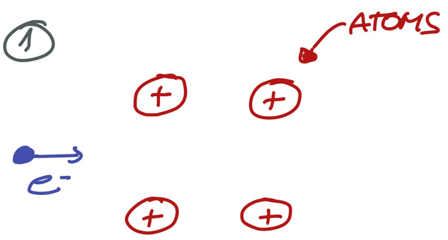
DIAMAGNETIC STATE

BCS THEORY (1956) BARDEEN, COOPER, SCHRIEFFER

MICROSCOPIC THEORY OF SUPERCONDUCTIVITY

- HOW CAN ELECTRONS ATTRACT EACH OTHER ??

1



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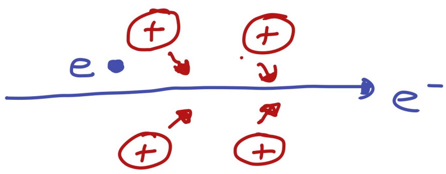

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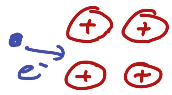
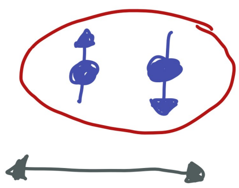


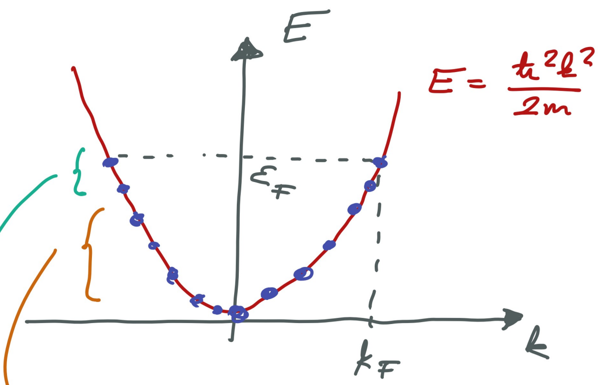
LATTICE - MEDIATED ELECTRON - ELECTRON INTERACTION

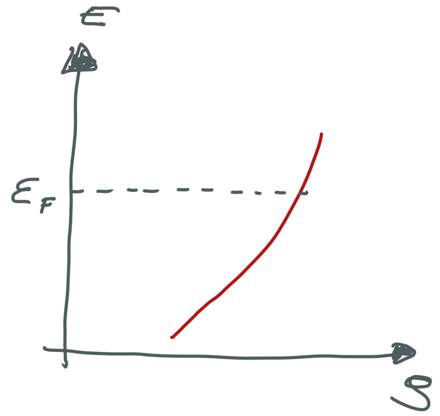


COOPER PAIRS (COMPOUND BOSONS)

ξ_0 COHERENCE LENGTH.

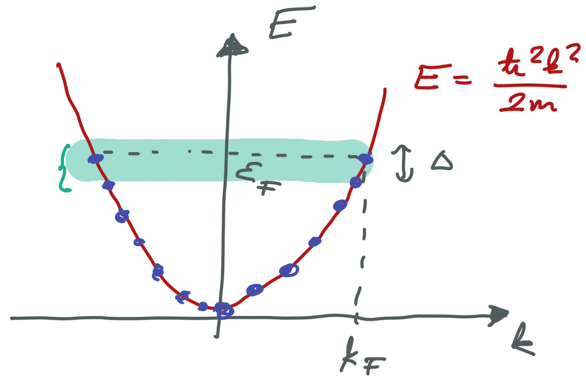
WHICH ELECTRONS FORM COOPER PAIRS IN A METAL?

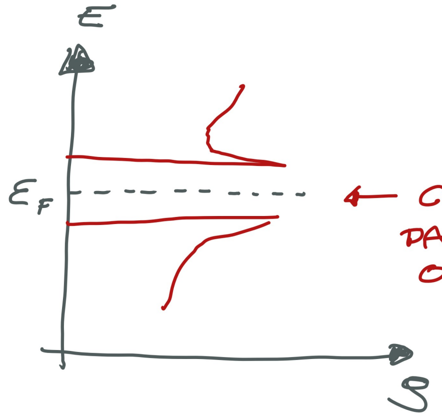




ELECTRONS ARE FROZEN HERE

ELECTRONS ARE MORE FREE HERE → THEY MOVE MORE EASILY





COOPER PAIRS ARE FORMED AROUND THE FERMI SURFACE (Δ), $\Delta = 1.76\, k_B T_C$

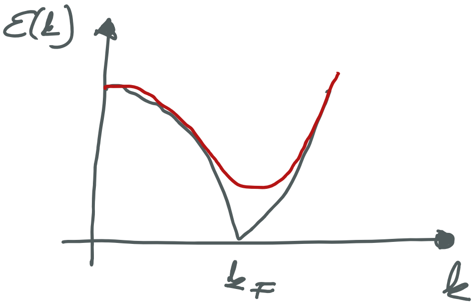
$(k, \uparrow) \leftrightarrow (-k, \downarrow)$

GAP IS FORMED → CAN NOT GIVE ENERGY DURING SCATTERING.

→ MAKE A QUBIT

→ HOW CAN WE PUT ENERGY LEVELS INTO IT? QUANTUM CIRCUITS.

EXCITATIONS:
$$\varepsilon(k) = \begin{cases} \frac{\hbar^2 k^2}{2m} - E_F & \text{ELECTRON-LIKE} \\ E_F - \frac{\hbar^2 k^2}{2m} & \text{HOLE-LIKE} \end{cases}$$



MACROSCOPIC WAVEFUNCTION (DOES NOT FEEL ATOMIC IMPURITIES)

CONDENSATE WAVEFUNCTION: $|\psi\rangle$

$$|\langle\psi\rangle|^2 = n_s/2$$

N: THE NUMBER OF COOPER PAIRS

ϕ: THE PHASE OF THE CONDENSATE

$$\Delta\phi \cdot \Delta N \gtrsim h$$

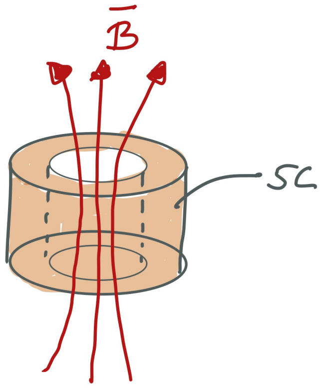
THE CURRENT CARRIED BY THE WAVEFUNCTION

$$j_s = \frac{e^*}{2m^*}\left[\psi^* p \psi + \psi p^+ \psi^*\right], \quad \psi = \sqrt{\frac{n_s}{2}}\, e^{i\phi}$$

$$\hat{p} = -i\hbar\nabla - e^*\bar{A}$$

$$j_s = -\frac{e^2 n_s}{m}\left(\bar{A} - \frac{\hbar}{2e}\nabla\phi\right)$$

FLUX QUANTIZATION



$$\oint j_s \, dl = -\frac{e^2 n_s}{m}\left[\underbrace{\oint \bar{A}\, d\bar{l}}_{\phi} - \frac{\hbar}{2e}\cdot 2\pi n\right]$$

INSIDE THE SC $j_s = 0$

WHY?

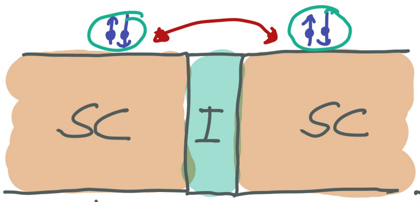
$\nabla \times \bar{B} = \mu_0 \bar{j}$

$\bar{B} = 0 \Rightarrow \bar{j} = 0$

$$\phi = \underbrace{\frac{h}{2e}}_{\phi_0} \cdot n$$

FLUX QUANTUM

JOSEPHSON EFFECT (1962)



GAP
OXIDE LAYER

$\psi_1 = \sqrt{n_{S1}/2}\, e^{i\phi_1}$ $\qquad$ $\psi_2 = \sqrt{n_{S2}/2}\, e^{i\phi_2}$

$$\hat{\psi} = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} \qquad H = \begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix}$$ HYBRIDIZATION

$$i\hbar \partial_t \hat{\psi} = H \hat{\psi}$$

$$\Rightarrow \quad \frac{dn_{s1}}{dt} = -\frac{dn_{s2}}{dt} \propto K \sin(\phi_2 - \phi_1)$$

$$\boxed{I = I_c \sin \Delta\phi}$$

CURRENT - PHASE RELATIONSHIP

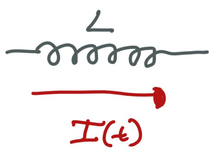
DC JOSEPHSON EFFECT

IF VOLTAGE IS APPLIED : $H = \begin{bmatrix} eV & K \\ K & -eV \end{bmatrix}$

$$\boxed{(\Delta \dot{\phi}) = \frac{2e}{\hbar} \cdot V}$$

AC JOSEPHSON EFFECT

JOSEPHSON INDUCTANCE:



VOLTAGE ACROSS

$$V = L \frac{dI}{dt}$$

FLUX :

$$\phi = LI$$

FOR JJ: $\frac{dI}{dt} = I_c \cos\phi \cdot \dot{\phi} = I_c \cos\phi \frac{2e}{\hbar} \cdot V$

NON-LINEAR INDUCTOR: $L(\phi) = \frac{\hbar}{2e} \frac{1}{I_c \cos\phi}$

$$\boxed{L(\phi) = \frac{\Phi_0}{2\pi I_c \cos\phi}}$$

JOSEPHSON ENERGY

$$\Delta E = \int_{t_1}^{t_2} I(t)V(t)\,dt = \int_{t_1}^{t_2} I_c \sin\phi \cdot \left[\frac{\hbar}{2e}\cdot\dot{\phi}\right] dt =$$

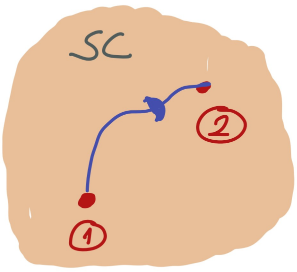
$$= -\frac{\Phi_0 I_c}{2\pi}\left[\cos\phi_2 - \cos\phi_1\right]$$

$$\boxed{\begin{array}{l} E = -E_J \cos\phi \\ E_J = \dfrac{\Phi_0 I_c}{2\pi} \end{array}}$$

SIMPLEST SC DEVICES: SQUID (SC QUANTUM INTERFERENCE DEVICE)

$$j_s = -\frac{e^2 n_s}{m}\left(\bar{A} - \frac{\hbar}{2e}\nabla\phi\right)$$

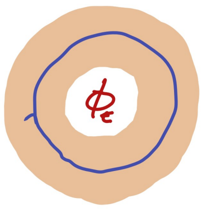
INSIDE A SC IS ZERO.



$$\bar{A} = \frac{\hbar}{2e}\nabla\phi$$

$$\int_1^2 A\,dl = \frac{\Phi_0}{2\pi}\int_1^2 \nabla\phi = \frac{\Phi_0}{2\pi}\left[\phi_2 - \phi_1\right]$$

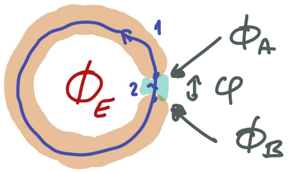
SC LOOP → QUANTIZED FLUX



$$\oint A\,dl = \frac{\Phi_0}{2\pi}\oint\nabla\phi = \frac{\Phi_0}{2\pi}\cdot 2\pi n \qquad n\in\mathbb{Z}$$

$$\boxed{\Phi_E = \Phi_0 \cdot n}$$

SC LOOP WITH ONE JUNCTION (RF-SQUID)

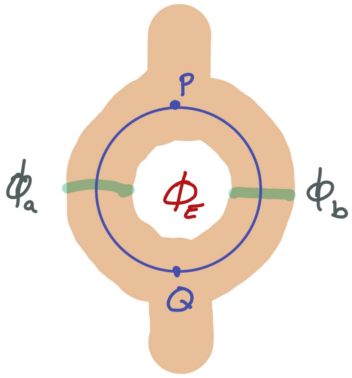


$$\oint\nabla\phi = 2\pi n$$

$$\oint\nabla\phi = \int_{\substack{A\\1}}^{B}\nabla\phi + \int_{\substack{B\\2}}^{A}\nabla\phi = \frac{2\pi}{\Phi_0}\underbrace{\int_A^B A\,dl}_{\approx\Phi_E} + \varphi$$

$$\boxed{2\pi n = \frac{2\pi}{\Phi_0}\phi_E + \varphi}$$

SC LOOP WITH TWO JUNCTIONS (DC-SQUID)



$$\phi_Q - \phi_P = \phi_a + \frac{2\pi}{\Phi_0}\int_P^Q \bar{A}\,d\bar{l} = \phi_b + \frac{2\pi}{\Phi_0}\int_P^Q \bar{A}\,d\bar{l}$$

$$\phi_a - \phi_b = \frac{2\pi}{\Phi_0}\oint \bar{A}\,d\bar{l} = \frac{2\pi}{\Phi_0}\phi_E$$

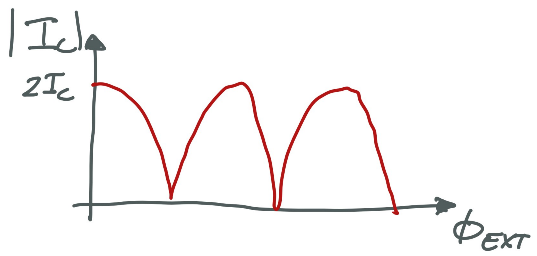
$$I_{TOTAL} = I_c\left[\sin\phi_a + \sin\phi_b\right]$$

$$\phi = \phi_a - \pi\frac{\phi_E}{\Phi_0}$$

$$\phi = \phi_b + \pi\frac{\phi_E}{\Phi_0}$$

$$\boxed{I = 2I_c\cos\left[\pi\frac{\phi_E}{\Phi_0}\right]\sin\varphi}$$

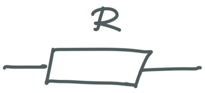
THE CRITICAL CURRENT IS MODULATED BY THE EXTERNAL FIELD



CLASSICAL ELECTRONIC CIRCUITS

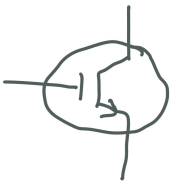
CAPACITOR

INDUCTOR



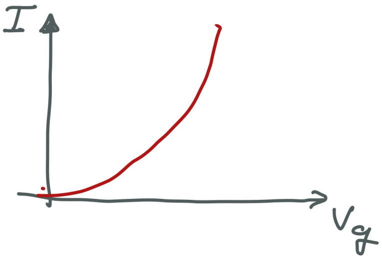
RESISTOR

} LINEAR ELEMENTS



TRANSISTOR

NON LINEAR ELEMENT



WE DESCRIBE IT WITH CLASSICAL VARIABLES;

- U VOLTAGE
- I CURRENT
- Q CHARGE
- ϕ FLUX

FOR EXAMPLE:

$I = \dot{Q}$

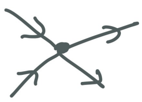
$R = U/I$

$C = Q/U$

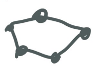
$\phi = L \cdot I$

...

→ KIRCHOFF'S LAWS:



$\sum I_n = 0$



$\sum U_n = 0$

QUANTUM ELECTRONIC CIRCUITS

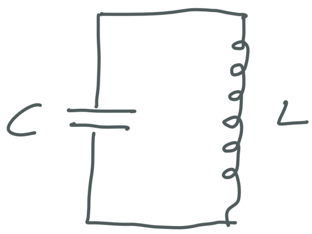
CAPACITOR

INDUCTOR

JOSEPHSON JUNCTION

NO RESISTOR:
- DISSIPATIVE (NON-UNITARY EVOLUTION)
- SC $\rightarrow$ $R=0$

SIMPLEST CIRCUIT: LC OSCILLATOR



CAPACITIVE ENERGY: $\frac{1}{2} C V_c^2$

INDUCTIVE ENERGY: $\frac{1}{2} L I^2 = \frac{1}{2} \frac{\phi^2}{L}$

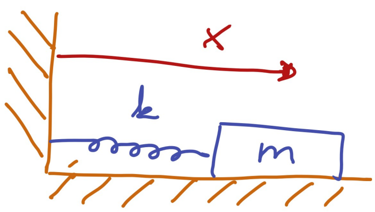
$\phi = LI$

KIRCHOFF'S: $V_L = -V_C$

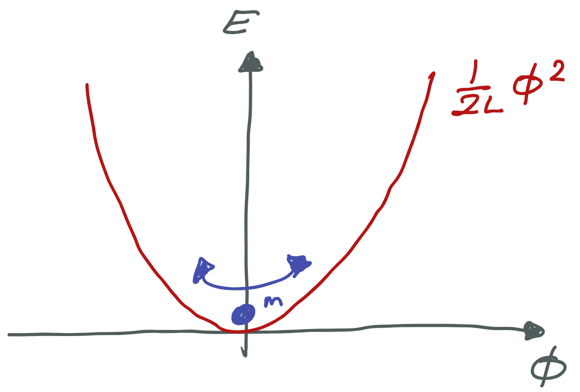
$$V_L = -L\dot{I} = -\dot{\phi}$$

$$E_{TOT} = \underbrace{\frac{1}{2} C \dot{\phi}^2}_{\text{KINETIC ENERGY}} + \underbrace{\frac{1}{2L} \phi^2}_{\text{POTENTIAL ENERGY}}$$

OF A PARTICLE WITH COORDINATE ϕ AND MASS OF C



$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$$



LAGRANGIAN OF THE CIRCUIT:

$$\mathcal{L} = K - U = \frac{1}{2} C \dot{\phi}^2 - \frac{1}{2L}\phi^2$$

→ LEGENDRE TRANSFORMATION

$CU = Q$

MOMENTUM $\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = C\dot{\phi} = Q$

$$\mathcal{H} = p\dot{\phi} - \mathcal{L} \Rightarrow \frac{Q^2}{2C} + \frac{1}{2L}\phi^2$$

$$\mathcal{H}(Q,\phi) = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

EQUATIONS OF MOTION:

$$\left.\begin{aligned} \dot{\phi} &= \frac{\partial \mathcal{H}}{\partial Q} = \frac{Q}{C} \\ \dot{Q} &= -\frac{\partial \mathcal{H}}{\partial \phi} = -\frac{\phi}{L} \end{aligned}\right\} \quad \ddot{\phi} + \underbrace{\frac{1}{LC}}_{\omega_0}\phi = 0$$

$$\phi(t) \propto \cos(\omega_0 t)$$
$$Q(t) \propto \sin(\omega_0 t)$$

QUANTUM LC CIRCUIT:

MECHANICAL OSCILLATOR:

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega_0^2 \hat{x}^2$$

$$[\hat{x}, \hat{p}] = i\hbar$$

$$\hat{p} = -i\hbar\,\partial_x$$

$$\hat{H}(\hat{Q}, \hat{\Phi}) = \frac{1}{2}\frac{\hat{Q}^2}{C} + \frac{1}{2}\frac{\hat{\Phi}^2}{L}$$

$$\boxed{[\hat{\Phi}, \hat{Q}] = i\hbar}$$

$$\hat{Q} = -i\hbar \frac{\partial}{\partial \Phi}$$

$$\boxed{\begin{aligned} \hat{Q} &= 2e\,\hat{n} \\ \hat{\Phi} &= \frac{\Phi_0}{2\pi}\cdot\hat{\varphi} \end{aligned}} \qquad \left(\Phi_0 = \frac{h}{2e}\right)$$

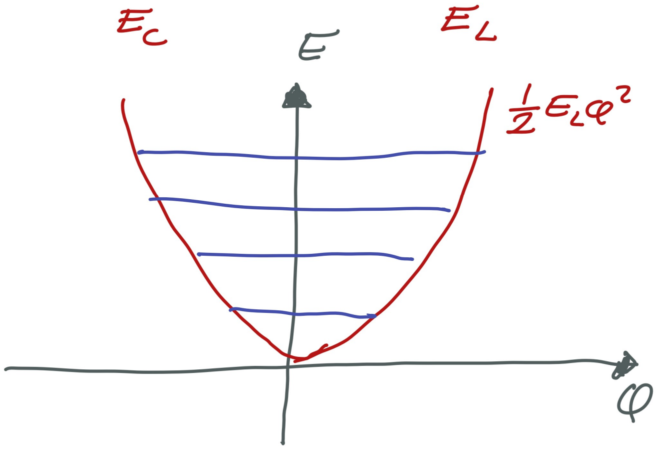
$\hat{n}$: NUMBER OF COOPER PAIRS

$\hat{\varphi}$: PHASE OF THE WAVEFUNCTION

$$\frac{\Phi_0}{2\pi}\cdot 2e\,[\hat{\varphi}, \hat{n}] = i\hbar$$

$$[\hat{\varphi}, \hat{n}] = i \longrightarrow \hat{n} = -i\,\partial_\varphi$$

$$\hat{H} = 4\underbrace{\frac{e^2}{2C}}_{E_C}\hat{n}^2 + \frac{1}{2}\underbrace{\frac{Q_0}{2\pi L}}_{E_L}\varphi^2$$

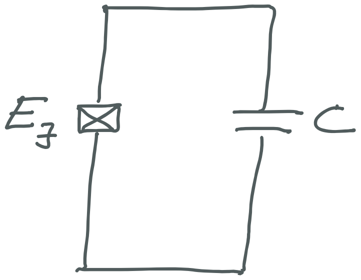


$$\omega_0 = \sqrt{8E_C E_L}/\hbar$$

$$E_n = \hbar\omega_0\left(n + \frac{1}{2}\right)$$

NOT A QUBIT!
(YET)

LET'S USE A NON-LINEAR INDUCTOR:



NON-LINEAR INDUCTANCE OF JJ:

$$L(\varphi) = \frac{\Phi_0}{2\pi} \frac{1}{I_c \cos\varphi} = L_0 \frac{1}{\cos\varphi}$$

$$L_0 = \frac{\Phi_0}{2\pi I_c}$$

ENERGY OF THE JOSEPHSON JUNCTION:

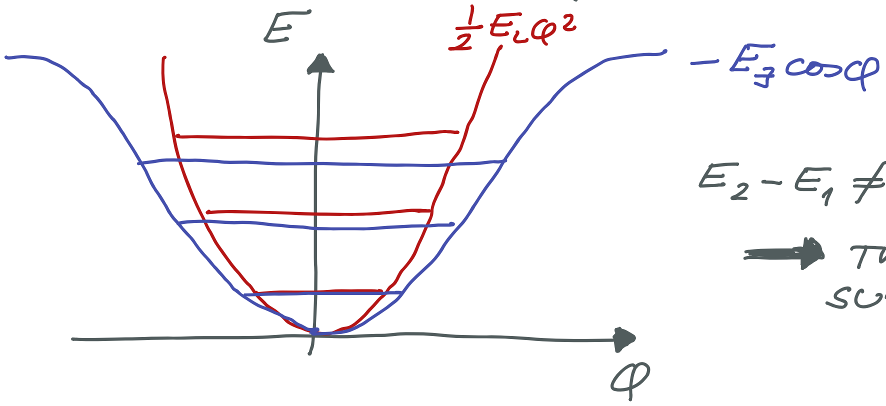
$$-E_J \cos\varphi =$$

$$= -\frac{\Phi_0 I_c}{2\pi}\left(1 - \frac{1}{2}\varphi^2 + \frac{1}{24}\varphi^4 + \dots\right) \approx \frac{1}{2}\frac{\Phi_0 I_c}{2\pi}\varphi^2 =$$

$$= \frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2 \cdot \frac{1}{L_0}\varphi^2 = \frac{1}{2L_0}\cdot\Big[\underbrace{\frac{\Phi_0}{2\pi}\varphi}\Big]^2$$

"EFFECTIVE FLUX" ACROSS THE NON-LINEAR INDUCTOR

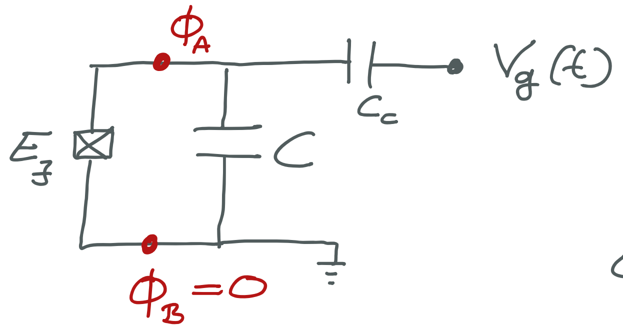
$$\rightarrow \quad H = 4E_c n^2 - E_J \cos\varphi$$



$$E_2 - E_1 \neq E_1 - E_0$$

$\rightarrow$ TWO-LEVEL SUBSPACE.

BUT THIS IS NOT THE FULL STORY, THE QUBIT IS EMBEDDED INTO AN ENVIRONMENT:



GENERALIZED

$\left.\begin{array}{l}\phi_A \\ \phi_B\end{array}\right\}$ NODE FLUX

$\phi_A - \phi_B$: BRANCH FLUX

$$\dot{\phi}_A = -V_A(t)$$
$$\phi_A = -\int_{-\infty}^{t} d\tau\, V_A(\tau)$$

$$\mathcal{L} = \frac{1}{2} C \left[\dot{\phi}_A - \phi_B\right]^2 + \frac{1}{2} C_c \left[\dot{\phi}_A - V_G\right]^2 + E_J \cos\left[\phi_A - \phi_B\right]$$

$\phi_B = 0$ (GROUND) $\qquad \phi = \phi_A$

$$\mathcal{L} = \frac{1}{2}\left[C + C_c\right]\dot{\phi}^2 - C_c \dot{\phi} V_G + E_J \cos\phi =$$

$$= \frac{1}{2}\left[C + C_c\right]\left[\dot{\phi} - \frac{C_c}{C + C_c} V_G\right]^2 + E_J \cos\phi$$

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \left[C + C_c\right]\dot{\phi} - C_c V_G$$

$$\longrightarrow \quad \dot{\phi} = \frac{Q + C_c V_G}{C + C_c}$$

$$\mathcal{H} = Q\dot{\phi} - \mathcal{L} = Q\left[\frac{Q + C_c V_G}{C + C_c}\right]^2 - \frac{1}{2}\left[C + C_c\right]\left[\frac{Q}{C + C_c}\right]^2$$

$$- E_J \cos\phi =$$

$$= \frac{1}{2} \frac{Q^2}{C + C_c} + \frac{2QC_cV_G}{C + C_c} - E_J \cos\phi =$$

$$= \frac{1}{2} \frac{[Q - Q_g]^2}{C + C_c} - E_J \cos\phi$$

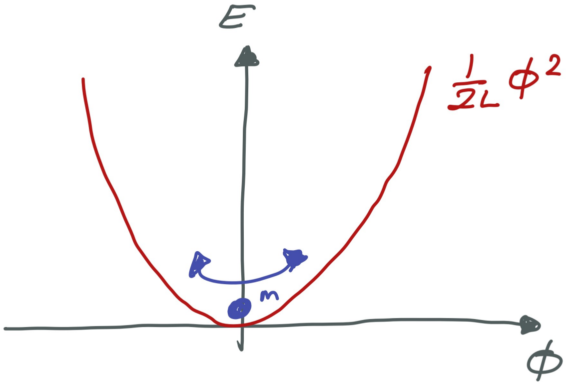
$$Q_g = - 4 C_c V_G$$

$$Q = 2en$$

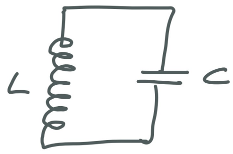
$$\boxed{H = 4E_c (n - n_g)^2 - E_J \cos\phi .}$$

$$E_c = \frac{e^2}{C + C_c} \qquad n_g = - 8eV_G C_G$$

PHASE SPACE VS CHARGE SPACE



QUANTUM LC OSCILLATOR



$$H = 4E_c n^2 + \frac{1}{2}E_L \phi^2$$

$$[\hat{\varphi}, \hat{n}] = i$$

THE PHASE EIGENSTATES:

$$\hat{\varphi}|\varphi_0\rangle = \varphi_0|\varphi_0\rangle \longrightarrow \text{THE PARTICLE IS LOCATED AT } \varphi = \varphi_0$$

$$\langle\varphi_1|\varphi_0\rangle = \delta(\varphi_1 - \varphi_0)$$

THE CHARGE EIGENSTATES:

$$\hat{n}|n_0\rangle = n_0|n_0\rangle \longrightarrow \text{THE PARTICLE HAS MOMENTUM OF } n_0$$

RELATION BETWEEN THE TWO SPACE $\longrightarrow$ FOURIER TRANSFORMATION

$$\hat{n}|n\rangle = n|n\rangle$$

$$\langle\varphi|\hat{n}|n\rangle = \langle\varphi|n|n\rangle = n\langle\varphi|n\rangle$$

$$\langle\varphi|-i\partial_\varphi|n\rangle = n\langle\varphi|n\rangle$$

$$-i\partial_\varphi\langle\varphi|n\rangle = n\langle\varphi|n\rangle$$

$$\boxed{\langle\varphi|n\rangle = \frac{1}{\sqrt{2\pi}}e^{i\varphi n}}$$

$$|n\rangle = \int d\varphi\,|\varphi\rangle\langle\varphi|n\rangle = \frac{1}{\sqrt{2\pi}}\int d\varphi\, e^{i\varphi n}|\varphi\rangle$$

$$\boxed{\begin{aligned} |n\rangle &= \frac{1}{\sqrt{2\pi}}\int d\varphi\, e^{i\varphi n}|\varphi\rangle \\ |\varphi\rangle &= \frac{1}{\sqrt{2\pi}}\int dn\, e^{-i\varphi n}|n\rangle \end{aligned}}$$