21.09.22

# QUANTUM INFORMATION PROCESSING

## History

QM 1900 – 1935

Classical Information – Shannon – 1948

"Information is physical" - Landauer & others - 1960

"Quantum Physics has smt. to Inf. Proc." - Benioff, Feynman, Bennett - Early 1980's

Question: Is a QM good resource?

QM offers new resources w.r.t classical processing.

## Syllabus

Intro : Interference experiments
- Double slit exp.
- Moch - Zehndan Interferometer
- SQUID

} Superposition principle & first notion of quantum state

CH2: Introduction to quantum bit

CH3:

CH4: Manipulation of spin (qubit) by magnetic fields (Rabi Oscillations)

CH5: QKD

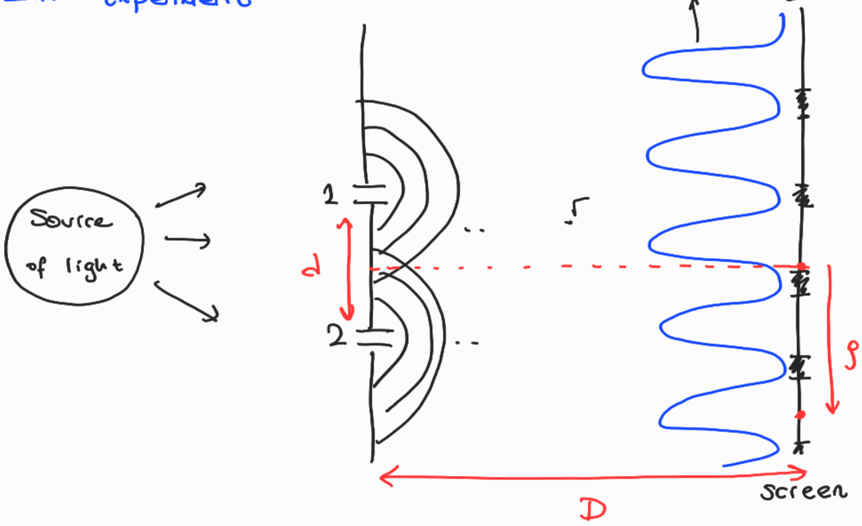
CH6 : Entanglement (Intrication) form of non-classical correlation between distant partners that share

CH7 - Density Matrix

CH8 - Von Neumann Entropy

## Interferometer Experiment

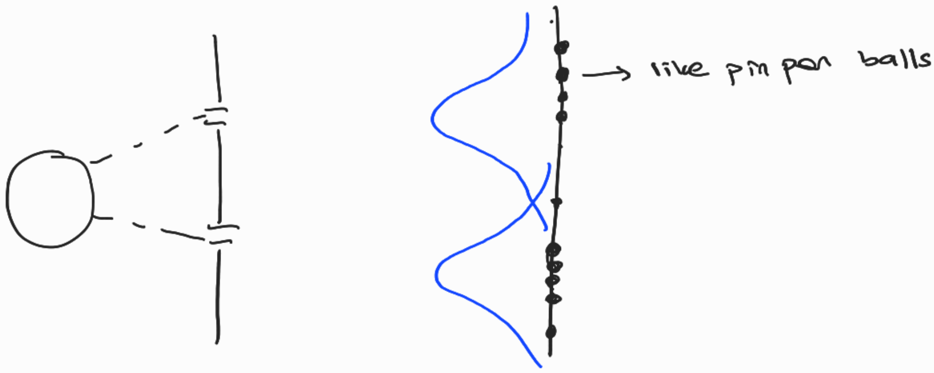
Double Slit Experiment → Young ~1800's




Wave Theory of Light

$\frac{d}{D} \ll 1$

According to Newton Physics, it should have been:




$$\Psi_{tot}(r) = \Psi_1(r) + \Psi_2(r) \quad \text{where} \quad \Psi_1(r) \,\&\, \Psi_2(r) \in \mathbb{C}$$

$$|\Psi_{tot}(r)|^2 = \frac{4A^2}{D^2}\left[\cos\left(\frac{\pi}{\lambda}\frac{d}{D}\rho\right)\right]^2$$

MESA PRINCIPLE

Young ordinary light $\lambda \simeq 700-900$ $\underbrace{nm}_{10^{-9}\text{ meters}}$

1900's Low Intensity Light

1960's Source of unique "photons" "C60 Molecules"

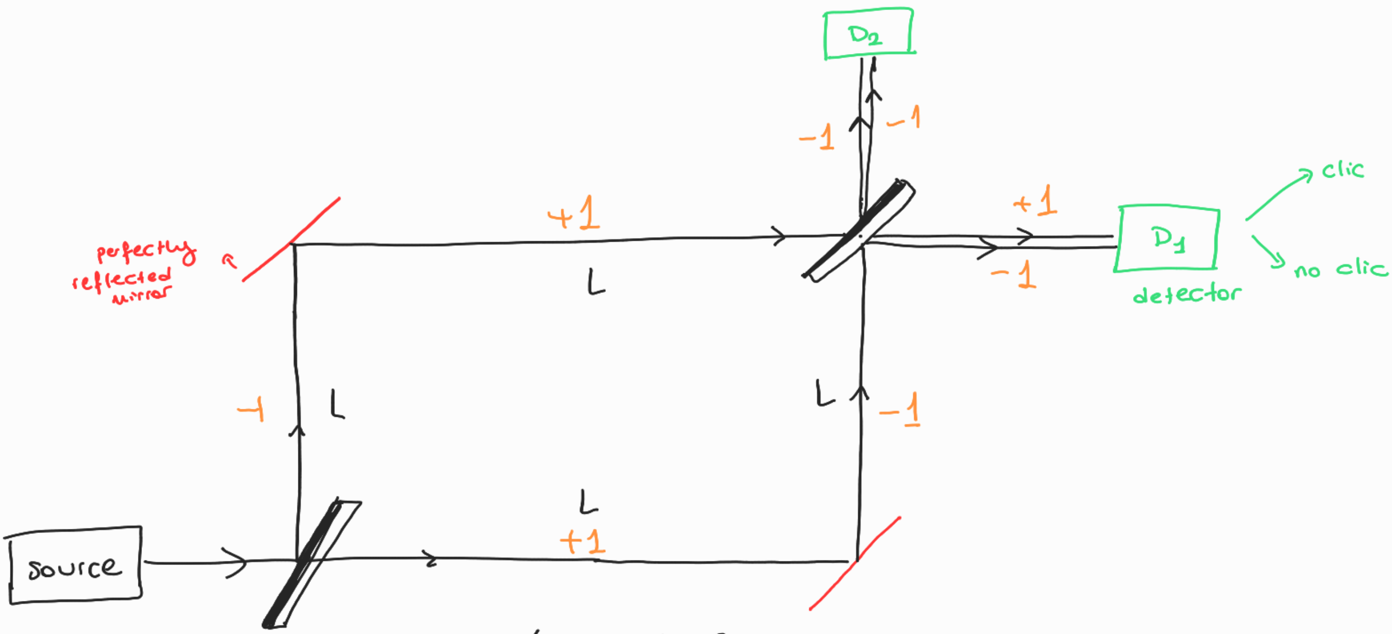
$|\Psi\rangle = |\Psi_1\rangle + |\Psi_2\rangle$

State vector

probability to find the particle at $r$ $= |\Psi(r)|^2$

# MZ interferometer

22.09.22




semitransparent mirror / beam splitter

classical ball $\begin{cases} \text{Prob}(D_1, \text{click}) = 1/2 \\ \text{Prob}(D_2, \text{click}) = 1/2 \end{cases}$

## Wave Theory of Light

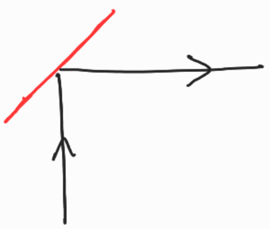
Wave has an amplitude → complex number → $\rho e^{i\varphi}$

modulus $\mathbb{R}_+$ ; phase $[0, 2\pi]$

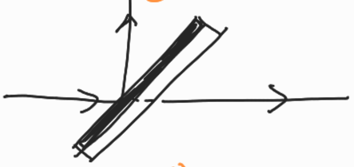
$e^{\frac{i 2\pi L}{\lambda}}$

$\lambda$ = wavelength

$L$ = length of the path

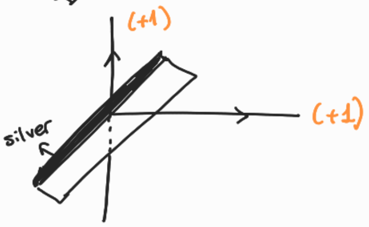


$e^{i\pi}$



$e^{i\pi} = -1$

$e^{2i\pi} = +1$

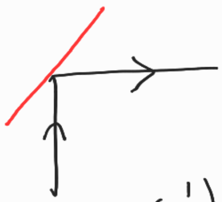


$$D_1 = \left\{(+1)(-1)(+1)\, e^{i\frac{2\pi L}{\lambda}2}\right\} + \left\{(-1)(-1)(+1)\, e^{\frac{i2\pi\lambda}{L}2}\right\} = 0$$

Quantum

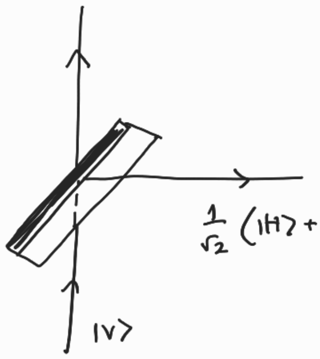
$$\alpha|H\rangle + \beta|V\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}, \quad \alpha, \beta \in \mathbb{C}^2$$

Rules



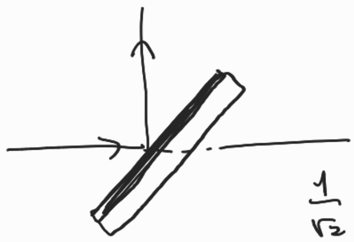
$-|H\rangle = \begin{pmatrix}-1\\ 0\end{pmatrix}$

$|V\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}$



$\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$

$|V\rangle$



$|H\rangle$

$\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$

Initial state $|H\rangle \xrightarrow[\text{beam splitter}]{} {}_{1^{st}\ BS}$ $\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle) \longrightarrow \frac{1}{\sqrt{2}}(-|V\rangle + |H\rangle)$

$$\downarrow$$

$$\underbrace{-\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle) + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)}_{-|V\rangle}$$

$D_2$ will click

$D_1$ won't click

Prob to observe a photon in $D_2 = 1$

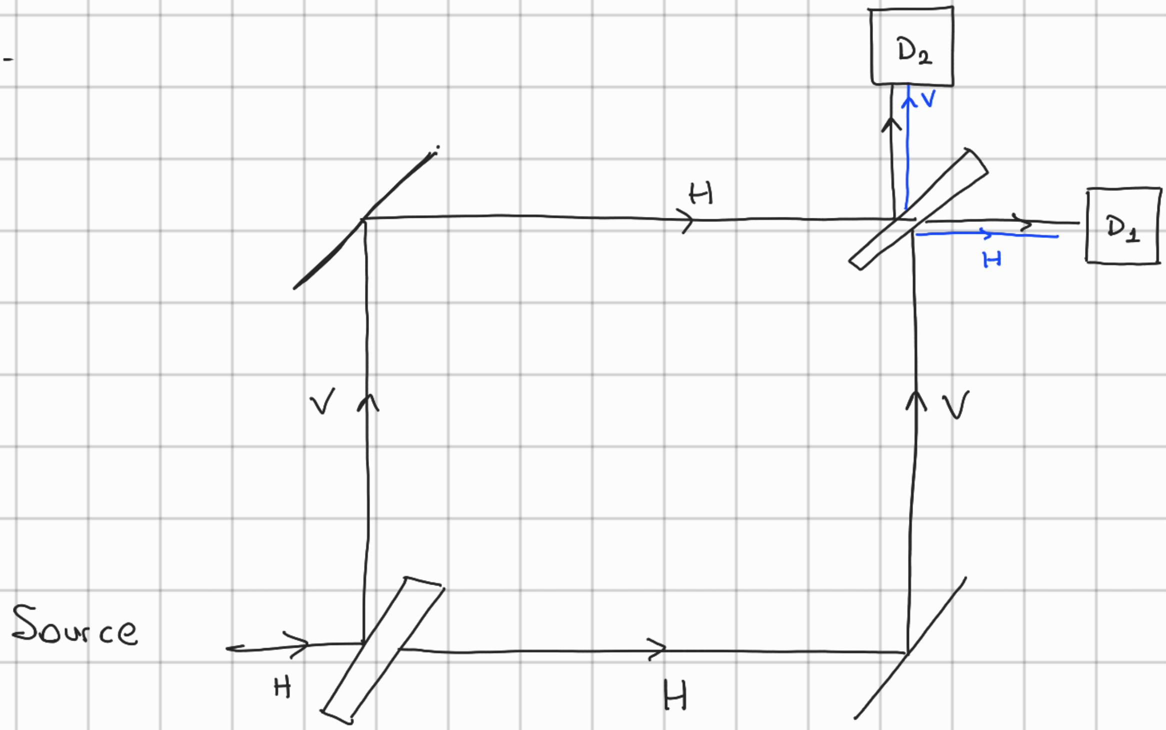
Rule $\alpha|H\rangle + \beta|V\rangle$

$\text{prob}(\text{abs}\,|H\rangle) = |\alpha|^2$

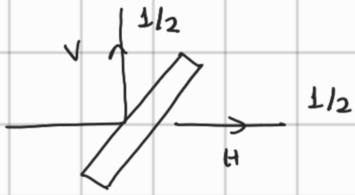
$\text{prob}(\text{abs}\,|V\rangle) = |\beta|^2$

From 2021 videos

# MZ Interferometer




## ① Billiard Ball Case ( Classical)




(semi-transparent)
mirror

$$\text{prob}(H) = 1/2$$

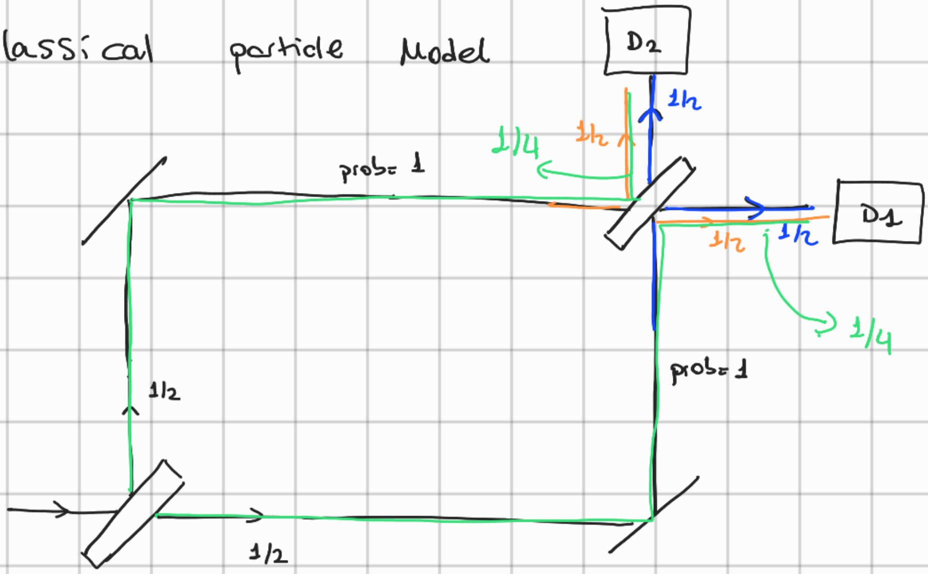
$$\text{prob}(V) = 1/2$$

$$\text{prob}(H \to V) = 1$$

## ② Electromagnetic classic

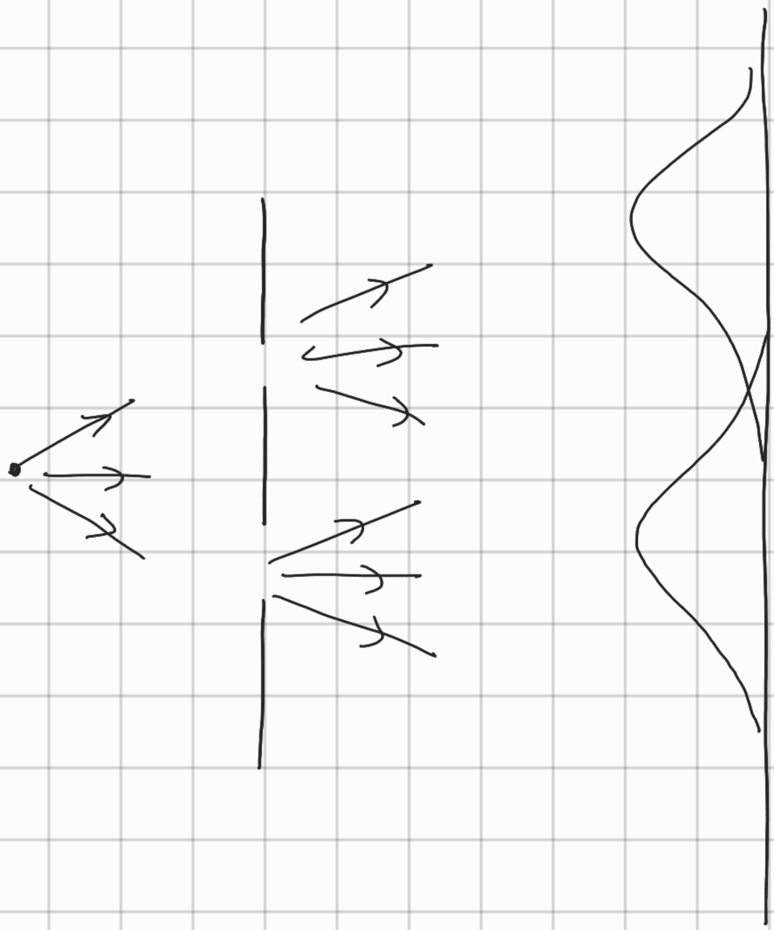
## ③ Quantum Particle → Photon

① Classical particle Model

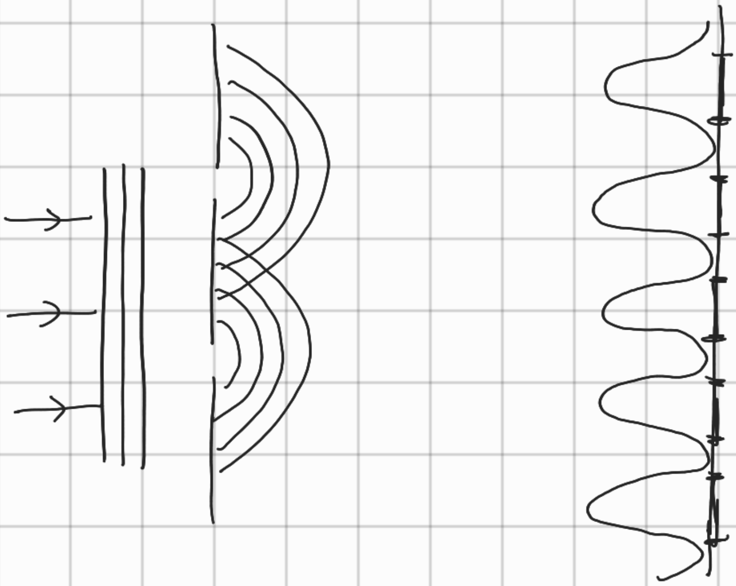



prob (D1) = 1/2 = 1/4 + 1/4

prob (D2) = 1/2 = 1/4 + 1/4

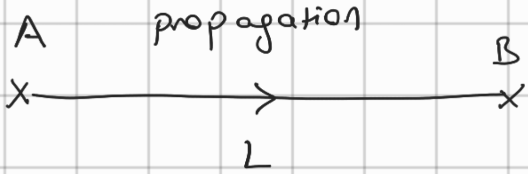


② Electro Magnetic classical

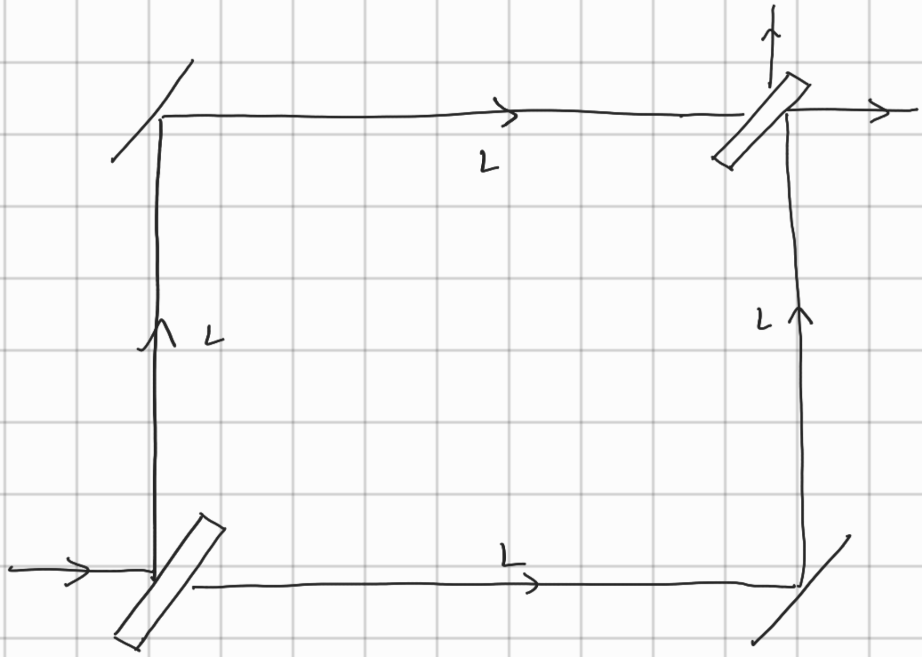


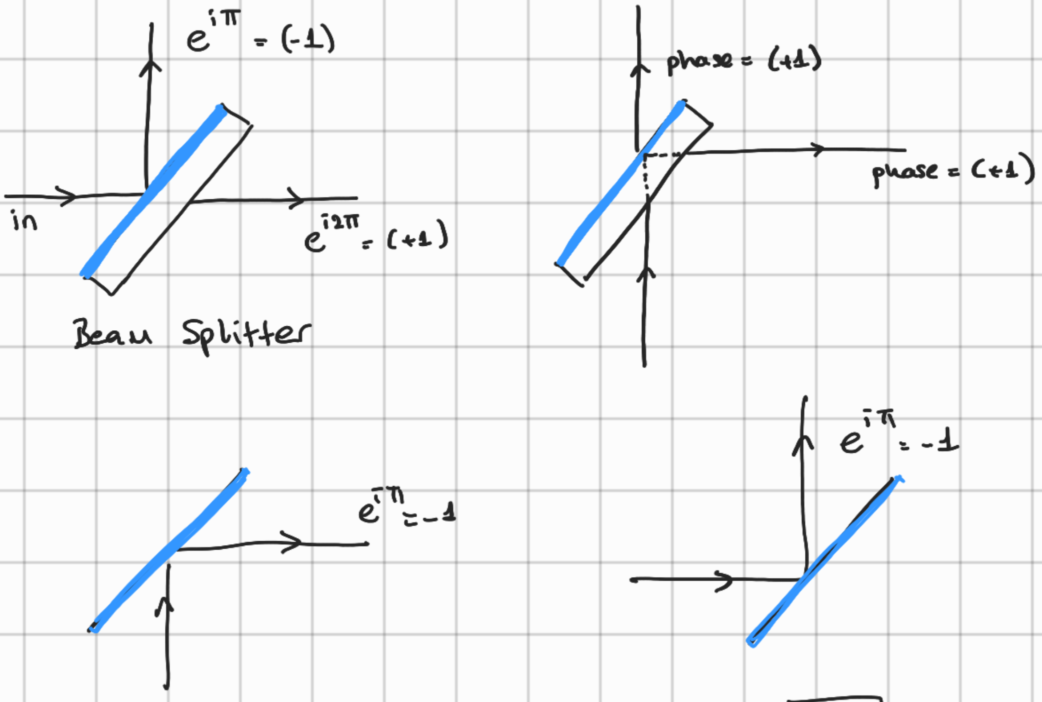
phase $[0, 2\pi]$

$$Z = \rho e^{i\varphi}$$

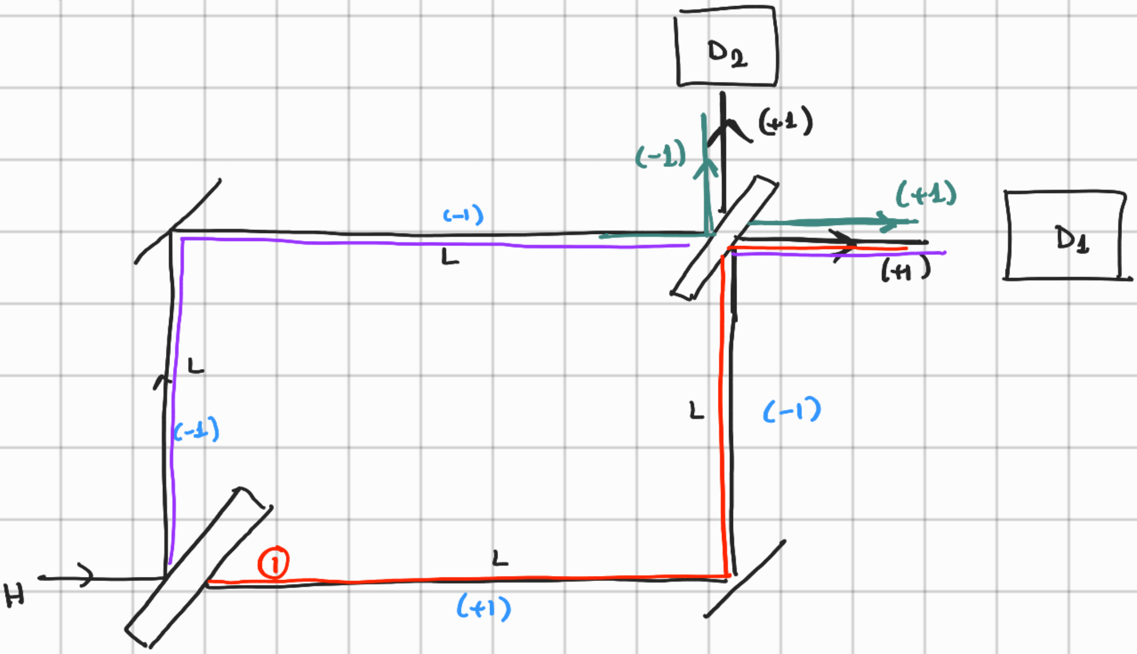



$$e^{i\frac{2\pi L}{\lambda}}$$






Beam Splitter




$D_1$ $\begin{cases} \text{Trajectory 1:} & (+1)\cdot(-1)\cdot(+1)\, e^{\frac{2\pi i L}{\lambda}\cdot 2} = -e^{\frac{4\pi i L}{\lambda}} \\ \text{Trajectory 2:} & (-1)\cdot(-1)\cdot(+1)\, e^{\frac{2\pi i L}{\lambda}\cdot 2} = +e^{\frac{4\pi i L}{\lambda}} \end{cases}$

$$-e^{4\pi i L/\lambda} + e^{4\pi i L/\lambda} = 0$$

↓

destructive Interference

$D_2$ $\begin{cases} \nearrow (+1)\,(-1)\,(+1)\, e^{\frac{2\pi i L}{\lambda}\cdot 2} \\ \searrow (-1)\,(-1)\,(-1)\, e^{\frac{2\pi i L}{\lambda}\cdot 2} \end{cases}$ $\Big\}\; -2\, e^{\frac{4\pi i L}{\lambda}}$

$$\left| -2e^{4\pi i L/\lambda} \right|^2 = 4$$

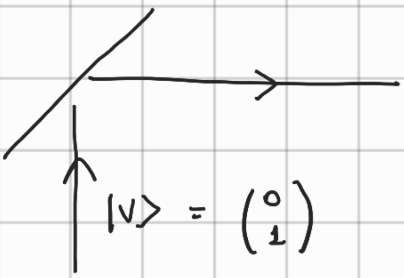
# 3) Photon Model

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha \underbrace{\begin{pmatrix}1\\ 0\end{pmatrix}}_{\substack{\text{Direction } H\\ |H\rangle}} + \beta \underbrace{\begin{pmatrix}0\\ 1\end{pmatrix}}_{\substack{\text{Direction } V\\ |V\rangle}}$$

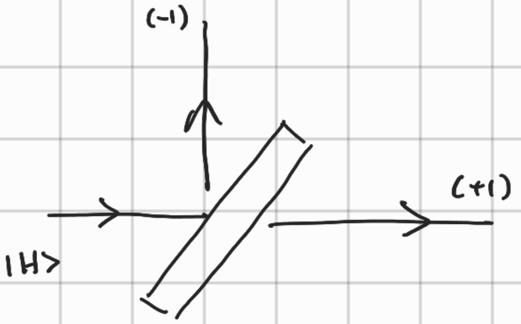
$$\alpha |H\rangle + \beta |V\rangle \quad , \quad \alpha, \beta \in \mathbb{C}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

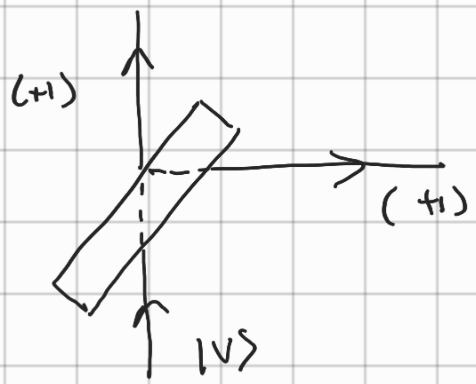
$$\alpha\alpha^* + \beta\beta^* = 1$$



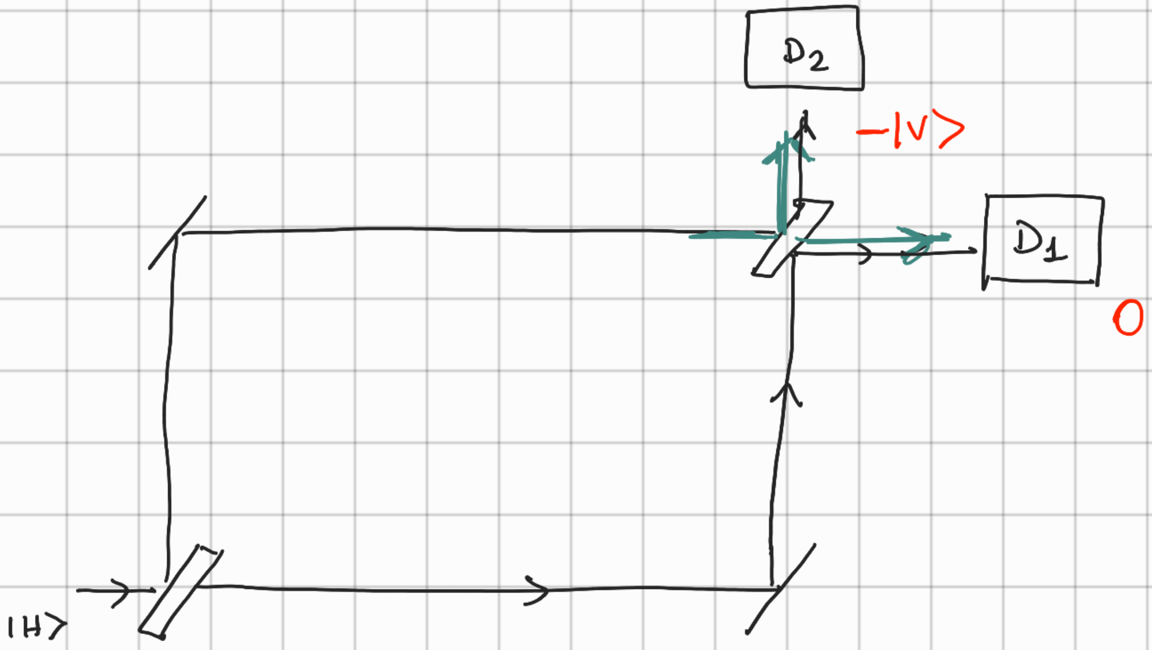

$$e^{i\pi} |H\rangle = -|H\rangle = \begin{pmatrix}-1\\ 0\end{pmatrix}$$




$$\frac{1}{\sqrt{2}} \left( \overbrace{|H\rangle - |V\rangle}^{\text{in superposition}} \right)$$




$$\frac{1}{\sqrt{2}} \left( \overbrace{|H\rangle + |V\rangle}^{\text{in superposition}} \right)$$




$1^{st}$ beam splitter $\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$

Mirror : $\frac{1}{\sqrt{2}}\left( (-)|V\rangle - (-)|H\rangle \right) = \frac{1}{\sqrt{2}}\left( -|V\rangle + |H\rangle \right)$

$2^{nd}$ beam splitter :

$$-\frac{1}{\sqrt{2}}\left( \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle) \right) + \frac{1}{\sqrt{2}}\left( \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle) \right)$$

$$= \cancel{-\frac{1}{2}|H\rangle} - \frac{1}{2}|V\rangle + \cancel{\frac{1}{2}|H\rangle} - \frac{1}{2}|V\rangle = -|V\rangle$$

$$\text{Prob}(D_1) = \left| \langle H | \text{Detection of photon} \rangle \right|^2$$

$$\text{prob}(D_2) = \left| \langle V | \text{Detection of photon} \rangle \right|^2$$