Particle Physics II
Lecture 4: The weak interactions of leptons

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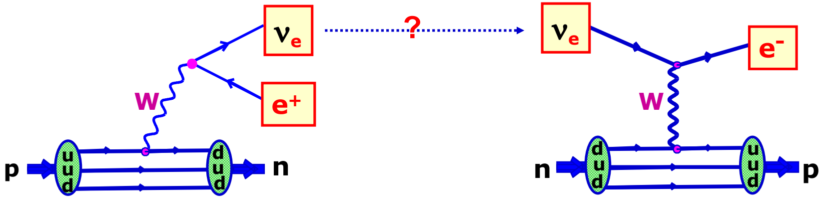
Neutrino flavors

- from the recent experiments: neutrinos have mass (very small)
- the flavor neutrino states, ν_e, ν_μ, ν_τ, are not the particles which propagate, these are ν_1, ν_2, ν_3
- concepts like "electron number" conservation do not hold
- we never directly observe neutrinos: can only detect them by their weak interactions. By definition ν_e is the neutrino state produced with e^+. Charged current weak interactions of the state ν_e produce e^-.

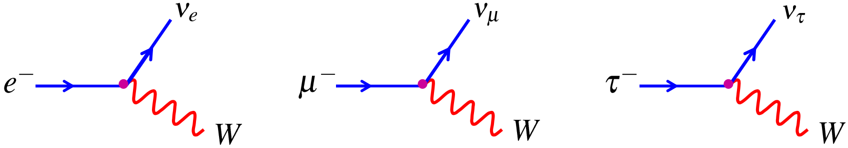
$\Longrightarrow$ ν_e, ν_μ, ν_τ are weak eigenstates



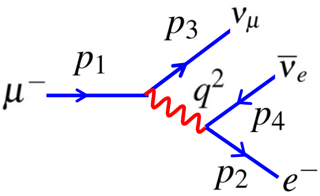
- unless dealing with very large distances: the neutrinos produced with e^+ will interact to produce e^-. For the discussion of the weak interaction continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

Muon decay and lepton universality

- the leptonic charged current ($W^{\pm}$) interaction vertices are:



- let's assume each of these vertices has its own coupling: G_F^e, G_F^μ, G_F^τ
- consider muon decay:



- it is straightforward to write down the matrix element (following Feynman rules and V-A interaction vertex)
- for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2 \implies$ working in a limit of Fermi theory
- and we will not do a calculation here

Muon decay and lepton universality

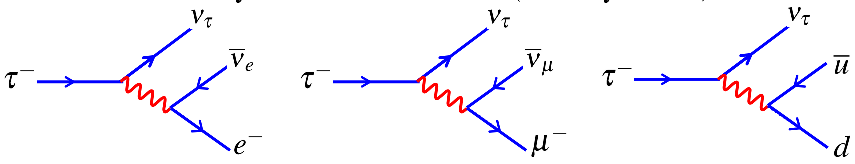
- the muon to electron rate can be computed to be:

$$\Gamma(\mu \to e\nu_e\nu_\mu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu} \text{ with } G_F = \frac{g_W^2}{4\sqrt{2}m_W^2} \tag{1}$$

- similarly for tau to electron rate:

$$\Gamma(\tau \to e\nu_e\nu_\tau) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3} \tag{2}$$

- but the tau can decay to various final states (not only to $e\nu\nu$):



- total particle width (or total transition rate) is the sum of all partial widths:

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau} \text{ (here } \tau \text{ denotes lifetime)} \tag{3}$$

- can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau \mathcal{B}(\tau \to e\nu\nu) = \mathcal{B}(\tau \to e\nu\nu)/\tau_\tau \tag{4}$$

- therefore predict:

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \qquad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5}\mathcal{B}(\tau \to e\nu\nu) \tag{5}$$

Muon decay and lepton universality

- muon and tau masses and lifetimes are precisely measured:

$$m_\mu = 0.1056583692(94)\ \text{GeV} \qquad \tau_\mu = 2.19703(4) \times 10^{-6}\ \text{s}$$
$$m_\tau = 1.77699(28)\ \text{GeV} \qquad \tau_\tau = 0.2906(10) \times 10^{-12}\ \text{s}$$
$$\mathcal{B}(\tau \to e\nu\nu) = 0.1784(5)$$

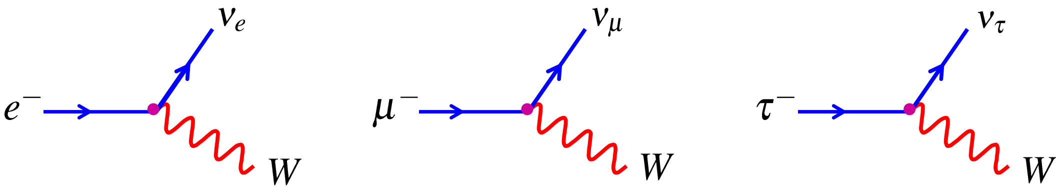
$$\Longrightarrow \quad \frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} \mathcal{B}(\tau \to e\nu\nu) = 1.0024 \pm 0.0033$$

- similarly by comparing $\mathcal{B}(\tau \to e\nu\nu)$ and $\mathcal{B}(\tau \to \mu\nu\nu)$:

$$\frac{G_F^e}{G_F^\mu} = 1.000 \pm 0.004$$

Muon decay and lepton universality

- demonstrates the weak charged current is the same for all leptonic vertices $\Longrightarrow$ Charged Current Lepton Universality



- lepton universality in charged current is an experimentally measured fact

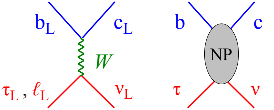
Hints of LFU violation in b decays

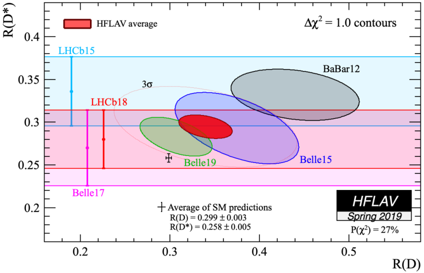
LFU tests in $b \to c$ transitions

Test of Lepton Flavor Universality in (charged current) $b \to c$ transitions [τ vs. light leptons (μ, e)]:

$$R(H_c) = \frac{\Gamma(B \to H_c\, \tau\nu)}{\Gamma(B \to H_c\, \ell\nu)}$$

H_c = D or D*





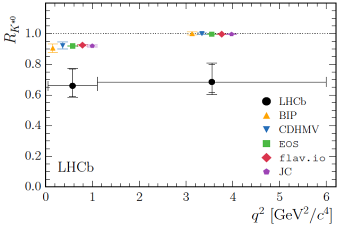
- SM prediction quite solid: hadronic uncertainties cancel (*to large extent*) in the ratio and deviations from 1 in $R(X)$ expected only from phase-space differences
- Consistent results by 3 different exps. → 3.1σ excess over SM ($D + D^*$)

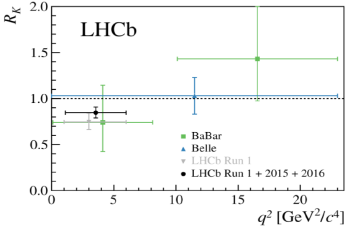
Hints of LFU violation in b decays

▶ *The $b \to s\ell\ell$ anomalies*

III. The "clean" LFU ratios:

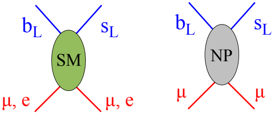
$$R_H = \frac{\int d\Gamma(B \to H\,\mu\mu)}{\int d\Gamma(B \to H\,ee)}$$





Deviations from the (*precise & reliable*) SM predictions ranging from 2.2σ to 2.5σ in each of the 3 bins measured by LHCb

What is particularly remarkable is that both these LFU breaking effects & the anomalies (I.+II.) are well described by the same set of Wilson coeff. assuming NP only in $b \to s\mu\mu$ and (& not in ee)



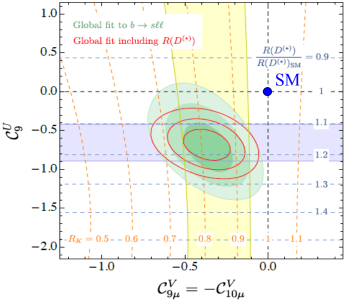
Slides by Gino Isidori

The $b \to s\ell\ell$ anomalies

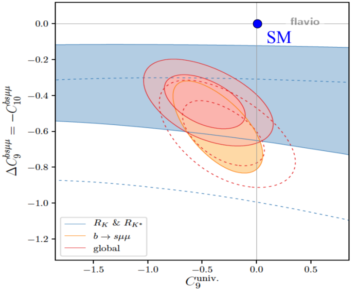
A very conservative analysis, taking into account only the observables III. & IV, with a single NP operator, leads to a pull of 3.2σ compared to the SM.

More sophisticated analyses, taking into account all observables, with state-of-the-art estimates of hadronic form factors + realistic (*but somehow model-dependent*) estimates of long-distance effects → pull exceeding 5σ:

Alguero *et al.* '19



Aebischer *et al.* '19



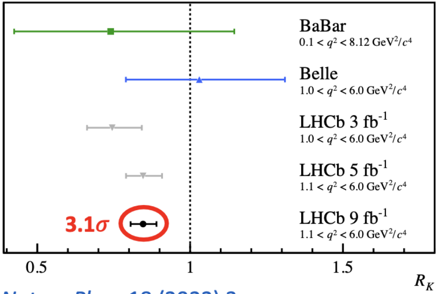
Lepton universality tests

- for theoretically precise observables, construct ratios:

$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to J/\psi(\to \mu^+\mu^-)K^+)} \Bigg/ \frac{\mathcal{B}(B^+ \to K^+e^+e^-)}{\mathcal{B}(B^+ \to J/\psi(\to e^+e^-)K^+)}$$

- R_K should be equal to 1 in the SM
- but decays to muons looked suppressed – a hint towards *lepton universality violation or possible new interaction*!

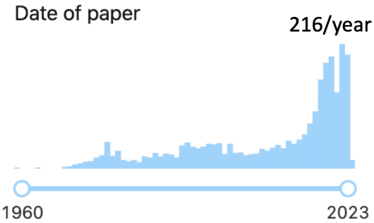
$B^{\pm}$ decays to $K^{\pm}\mu^+\mu^-$ look suppressed wrt $K^{\pm}e^+e^-$



Nature Phys. 18 (2022) 3

Tests of lepton universality

All “lepton universality” papers:



- over 2k papers in total
- almost 85k citations

LHCb papers ranked by citation number as of February 2023

2,599 results | cite all Citation Summary Most Cited

The LHCb Detector at the LHC #1
LHCb Collaboration • A.Augusto Alves, Jr. (Rio de Janeiro, CBPF) et al. (Aug 14, 2008)
Published in: *JINST* 3 (2008) S08005
DOI cite claim reference search 4,192 citations

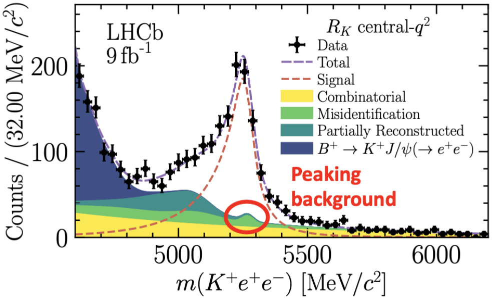
Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays #2
LHCb Collaboration • Roel Aaij (CERN) et al. (Jul 13, 2015)
Published in: *Phys.Rev.Lett.* 115 (2015) 072001 • e-Print: 1507.03414 [hep-ex]
pdf links DOI cite claim reference search 1,517 citations

Test of lepton universality using $B^+ \to K^+\ell^+\ell^-$ decays #3
LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Jun 25, 2014)
Published in: *Phys.Rev.Lett.* 113 (2014) 151601 • e-Print: 1406.6482 [hep-ex]
pdf DOI cite claim reference search 1,287 citations

Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays #4
LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)
Published in: *JHEP* 08 (2017) 055 • e-Print: 1705.05802 [hep-ex]
pdf links DOI cite datasets claim reference search 1,207 citations

Lepton universality restored in $b \to s\ell\ell$ ratios

Reconstructed B^+ mass in $K^+e^+e^-$ mode



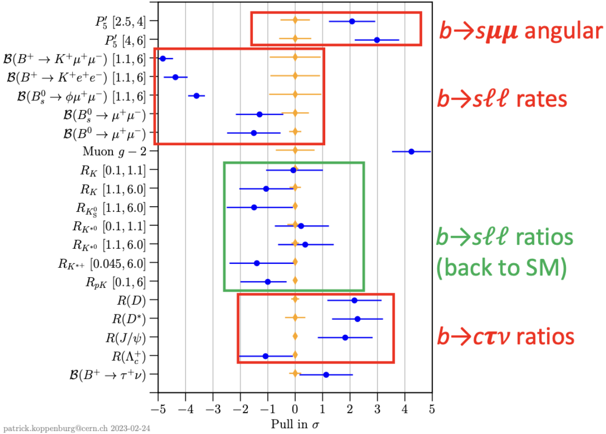
arXiv: 2212.09152, arXiv: 2212.09153 (subm. to PRL, PRD)

- new combined analysis finalized at the end of last year
- hadron to electron misidentification appeared to be important
- developed a dedicated method based on data to reliably estimate this background
- the new measurement is consistent with the SM within 0.2σ

https://actu.epfl.ch/news/lepton-universality-restored/

Lepton puzzles are not over

Orange: theory unc.; blue: experiment



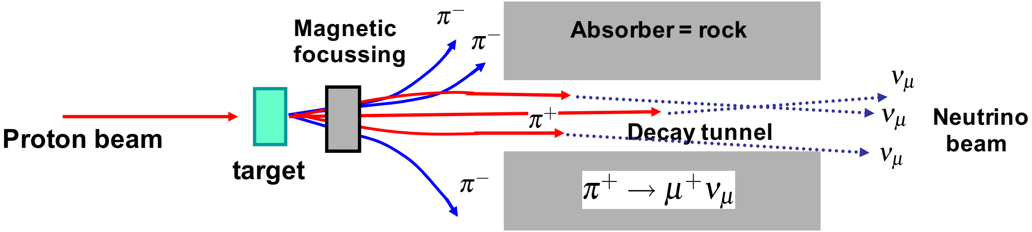
- other observables in the $b \to s\ell\ell$ transitions exhibit tensions with the SM
- some enhancement of $b \to c\tau\nu$ decays vs $b \to c\mu\nu$
- follow-up and complementary measurements are in the works!

Neutrino scattering

- last semester we looked into $e^- p$ deep inelastic scattering where a virtual photon is used to probe nucleon structure
- can also consider the weak interaction equivalent: neutrino deep inelastic scattering where a virtual W boson probes the structure of the nucleons
  - provides additional information about parton structure functions

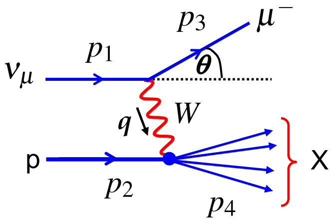
Neutrino beams:

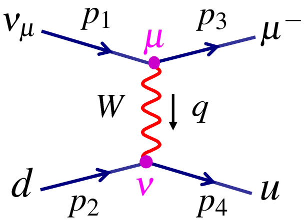
- smash high energy protons into a fixed target – get hadrons
- focus positive pions/kaons
- allow them to decay: $\pi^+ \to \mu^+\nu_\mu$, $K^+ \to \mu^+\nu_\mu$ ($\mathcal{B} \approx 64\%$)
- gives a beam of "collimated" ν_μ
- focus negative pions/kaons to get beam of $\bar{\nu}_\mu$



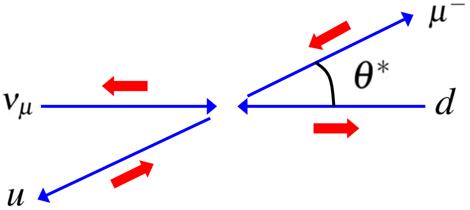
Neutrino-quark scattering

- for ν_μ-proton deep inelastic scattering, the underlying process is $\nu_\mu d \to \mu^- u$:





- let's do a bit better than making calculations directly, and use that:
  - in the limit of small momentum transfer $q^2 \ll m_W^2$, the W boson propagator is $g_{\mu\nu}/m_W^2$
  - in the relativistic limit can neglect muon and quark masses
  - in this limit only left-handed helicity particles participate in the weak interaction:



  - total spin of the system is 0 $\implies$ no preferred polar angle θ^* $\implies$ matrix element should be isotropic

- if we were to make calculations, we'd get our isotropic ME:

$$M_{fi} = \frac{g_W^2}{m_W^2}\hat{s}, \text{ where } \hat{s} = (2E)^2 \tag{6}$$

- this $(2E)^2$ would be acquired from the spinors normalization $\propto \sqrt{E}$, and 4 spinors participating in ME calculation

- to get correct number of factors of 2, need to sum over all possible spin states and average over all possible initial state spin states
- here, only one possible spin combination (LL→LL) and only 2 possible initial state combinations (the neutrino is always produced in a LH helicity state)

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2} \left| \frac{g_W^2}{m_W^2} \hat{s} \right|^2 \tag{7}$$

- the factor of a half arises because half of the time the quark will be in a RH state and won't participate in the charged current weak interaction

Neutrino-quark scattering

- to finalize this exercise, let's apply our known cross section expression for 2→2 body scattering in the extreme relativistic limit:

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |M_{fi}|^2 \rangle \tag{8}$$

- and let's use

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} \tag{9}$$

- we get:

$$\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s} \tag{10}$$

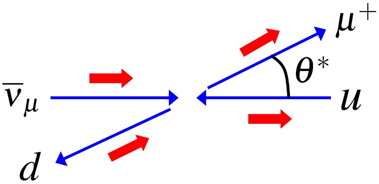
- integrating this isotropic distribution over $d\Omega^*$:

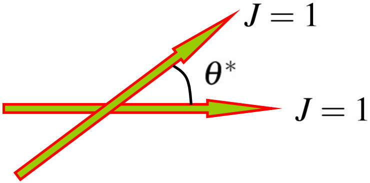
$$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi} \tag{11}$$

- cross section is a Lorentz invariant quantity so this is valid in any frame

Antineutrino-quark scattering

- for antineutrinos the things look different since we need right-handed helicity antiparticles (while before we had left-handed particles!)
- here, the interaction occurs in a total angular momentum 1 state (while before we had total spin 0!)





- because of this, we acquire angular dependence in the matrix element:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*} \times \frac{1}{4}(1 + \cos\theta^*)^2 \tag{12}$$

- this factor gives the overlap of the initial and final angular momentum wave-functions

Antineutrino-quark scattering

- for differential cross section we get:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s} \tag{13}$$

- for the full cross section:

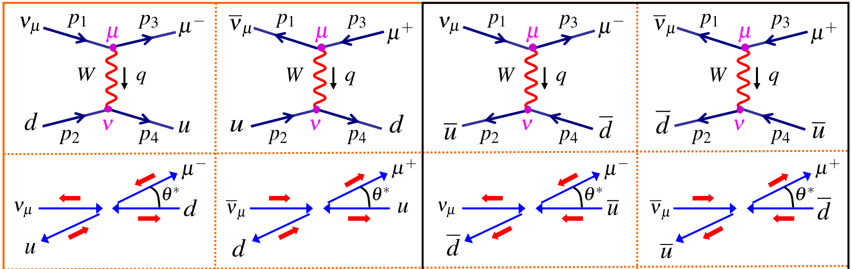
$$\sigma_{\bar{\nu}q} = \frac{G_F^2\hat{s}}{3\pi} \tag{14}$$

- which is a factor three smaller than the neutrino quark cross section:

$$\frac{\sigma_{\bar{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3} \tag{15}$$

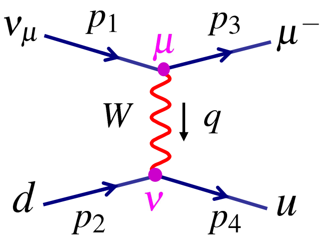
All combinations: (anti)neutrino-(anti)quark scattering

- non-zero antiquark component in the nucleon $\Longrightarrow$ also consider scattering from $\bar{q}$
- cross sections can be obtained immediately by comparing with quark scattering and remembering to only include LH particles and RH antiparticles



$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{d\sigma_{\bar{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\nu \bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\bar{\nu}\bar{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$	$\sigma_{\bar{\nu} q} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\nu \bar{q}} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\bar{\nu}\bar{q}} = \frac{G_F^2\hat{s}}{\pi}$

- to convert differential cross sections into Lorentz invariant form, replace an angle θ^* with a Lorentz invariant y:



- as previously for DIS, use $y \equiv \dfrac{p_2 \cdot q}{p_2 \cdot p_1}$
- it can be understood as a scattering angle, since in relativistic limit in C.o.M.:

$$y = \frac{1}{2}(1 - \cos\theta^*) \tag{16}$$

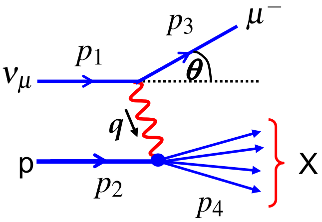
- in lab. frame:

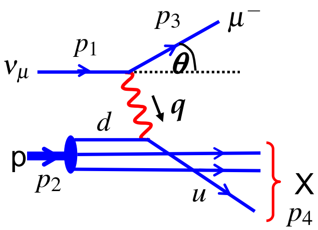
$$y = 1 - \frac{E_3}{E_1} \tag{17}$$

- using above relations between θ^* and y, can get:

$$\frac{\mathrm{d}\sigma_{\nu q}}{\mathrm{d}y} = \frac{\mathrm{d}\sigma_{\bar{\nu}\bar{q}}}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s} \quad \text{and} \quad \frac{\mathrm{d}\sigma_{\bar{\nu}q}}{\mathrm{d}y} = \frac{\mathrm{d}\sigma_{\nu\bar{q}}}{\mathrm{d}y} = \frac{G_F^2}{\pi}(1-y)^2\hat{s} \tag{18}$$

Parton model for neutrino deep inelastic scattering





Scattering from a proton with structure functions

Scattering from a point-like quark within the proton

- neutrino-proton scattering can occur via scattering from a **down quark** or from an **antiup quark**
- then can use this property to express scattering cross section through parton density functions of quarks of each flavor in a proton and a neutron

Parton model for neutrino deep inelastic scattering

- since ν cross sections are tiny, need massive detectors. Usually they are made of iron, and experimentally measure a combination of p and n scattering cross sections
- for an isoscalar target (i.e. equal numbers of protons and neutrons), the mean cross section per nucleon:

$$\frac{\mathrm{d}\sigma^{\nu N}}{\mathrm{d}y} = \frac{G_F^2}{2\pi} s \left[f_q + (1-y)^2 f_{\bar{q}} \right] \tag{19}$$

where f_q and $f_{\bar{q}}$ are the total momentum fractions carried by the quarks and by the antiquarks within a nucleon

$$f_q \equiv f_d + f_u = \int_0^1 x[u(x) + d(x)]\mathrm{d}x \tag{20}$$

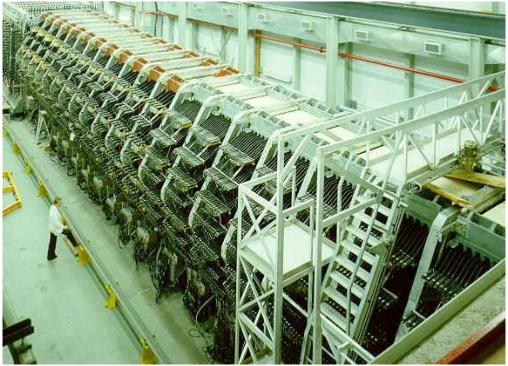
$$f_{\bar{q}} \equiv f_{\bar{d}} + f_{\bar{u}} = \int_0^1 x[\bar{u}(x) + \bar{d}(x)]\mathrm{d}x \tag{21}$$

- similarly:

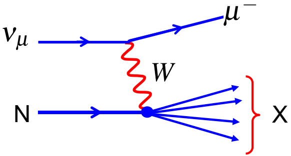
$$\frac{\mathrm{d}\sigma^{\bar{\nu} N}}{\mathrm{d}y} = \frac{G_F^2}{2\pi} s \left[(1-y)^2 f_q + f_{\bar{q}} \right] \tag{22}$$

CDHS Experiment (CERN 1976-1984)

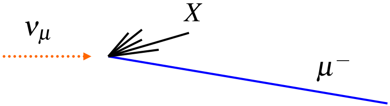
- 1250 tons
- Magnetized iron modules
- Separated by drift chambers



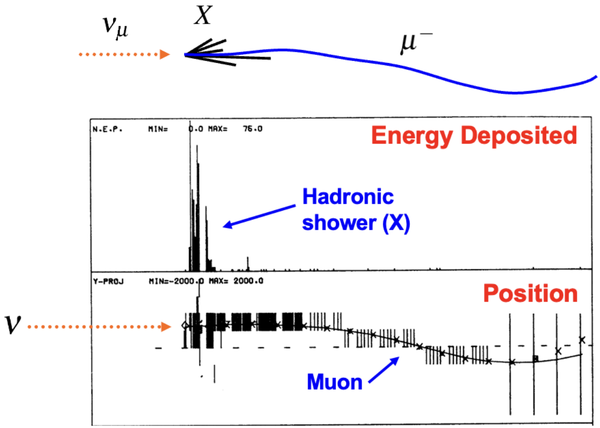
Study Neutrino Deep Inelastic Scattering



Experimental Signature:



CDHS Experiment (CERN 1976-1984)



- measure energy of X: E_X
- measure muon momentum from curvature in B-field: E_μ
- for each event can determine neutrino energy and y:

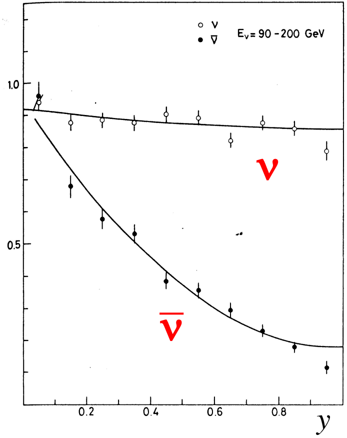
$$E_\nu = E_X + E_\mu$$

$$E_\mu = (1 - y)E_\nu$$

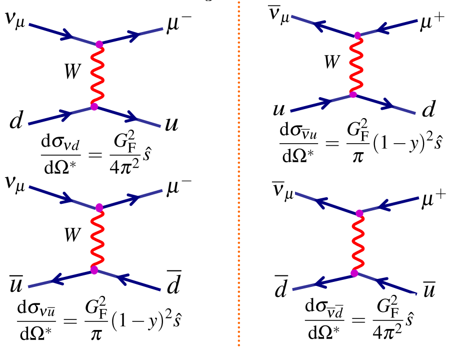
$$\implies y = \left(1 - \frac{E_\mu}{E_\nu}\right)$$

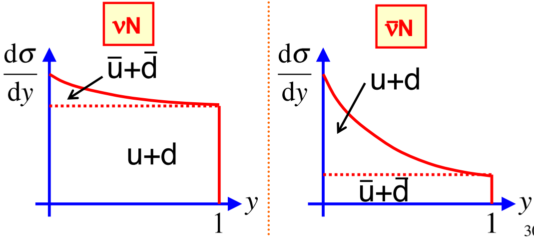
Measured y distributions

- CDHS measured y distribution
- shapes can be understood in terms of (anti)neutrino – (anti)quark scattering



J. de Groot et al., Z.Phys. C1 (1979) 143





Measured total cross sections

- integrating the expressions for $\frac{\mathrm{d}\sigma}{\mathrm{d}y}$:

$$\boxed{\sigma^{\nu N} = \frac{G_\mathrm{F}^2 s}{2\pi}\left[f_q + \frac{1}{3} f_{\overline{q}}\right]} \qquad \boxed{\sigma^{\overline{\nu} N} = \frac{G_\mathrm{F}^2 s}{2\pi}\left[\frac{1}{3} f_q + f_{\overline{q}}\right]}$$

$$\underset{(E_\nu, 0, 0, +E_\nu)}{\nu} \longrightarrow \underset{(m_p, 0, 0, 0)}{\bullet\, p} \qquad s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p^2 \approx 2E_\nu m_p$$

➡ **DIS cross section $\propto$ lab. frame neutrino energy**

- measured cross sections can be used to determine fraction of protons momentum carried by quarks, f_q, and fraction carried by antiquarks, $f_{\overline{q}}$

Measured total cross sections

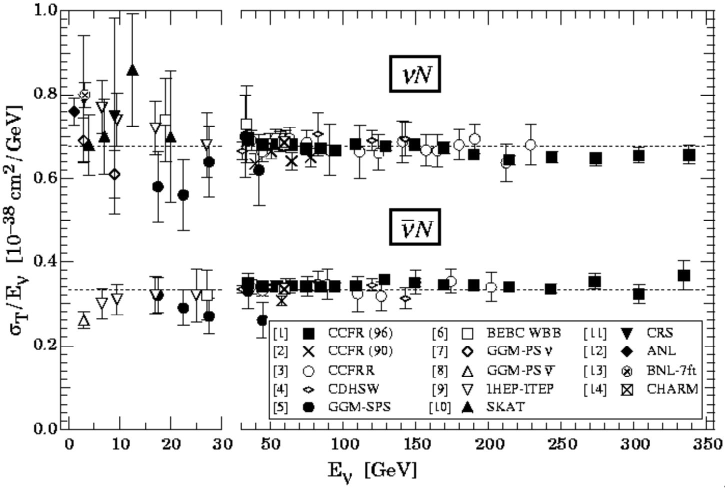
- find: $f_q \approx 0.41$, $f_{\bar{q}} \approx 0.08$
- $\sim 50\%$ of momentum carried by gluons (which do not interact with virtual W boson)

- if no antiquarks in nucleons, expect:

$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu}} N} = 3 \qquad (23)$$

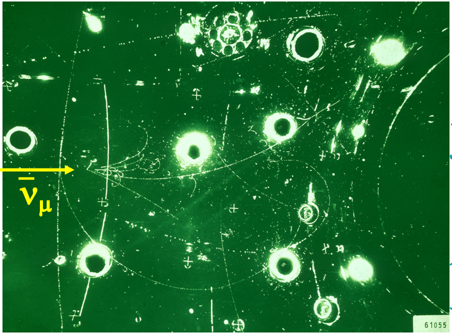
- including antiquarks:

$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu}} N} \approx 2 \qquad (24)$$



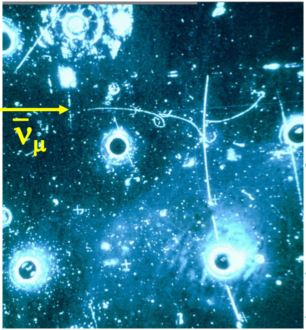
- neutrinos also interact via the neutral current
- first observed in the Gargamelle bubble chamber in 1973
- interaction of muon neutrinos produce a final state muon

$$\overline{\nu}_\mu + N \to \overline{\nu}_\mu + \text{hadrons}$$



F.J. Hasert et al., Phys. Lett. 46B (1973) 138

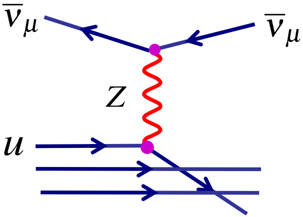
$$\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-$$

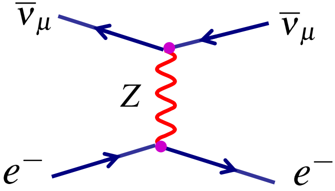


F.J. Hasert et al., Phys. Lett. 46B (1973) 121

Weak neutral current

- cannot be due to W exchange – first evidence for Z boson





Summary

- weak interaction is universal for all lepton flavors
- searching for deviations from this universality provides means to find new effects
- we looked at the neutrino/antineutrino – quark/antiquark weak charged current (CC) interaction cross sections
- neutrino - nucleon scattering yields extra information about parton distribution functions:
    - ν couples to d and $\bar{u}$; $\bar{\nu}$ couples to u and $\bar{d}$
        - $\Longrightarrow$ investigate flavor content of nucleon
    - can measure antiquark content of nucleon:
        - $\nu\bar{q}$ suppressed by factor $(1-y)^2$ compared to νq
        - $\bar{\nu}q$ suppressed by factor $(1-y)^2$ compared to $\bar{\nu}\bar{q}$
- finally, observe that neutrinos interact via weak neutral currents (NC)