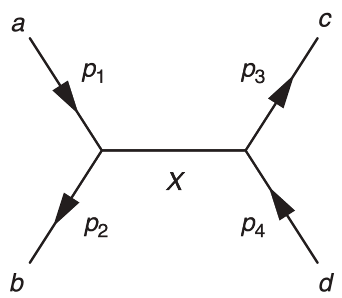
Particle Physics 1: Exercise 8

Exercise 1

Draw the two time-ordered diagrams for the s-channel process shown in the figure below. Show that the propagator has the same form as obtained for the t-channel process.

Hint: one of the time-ordered diagrams is non-intuitive. In the second-order perturbation theory the intermediate state does not conserve energy.



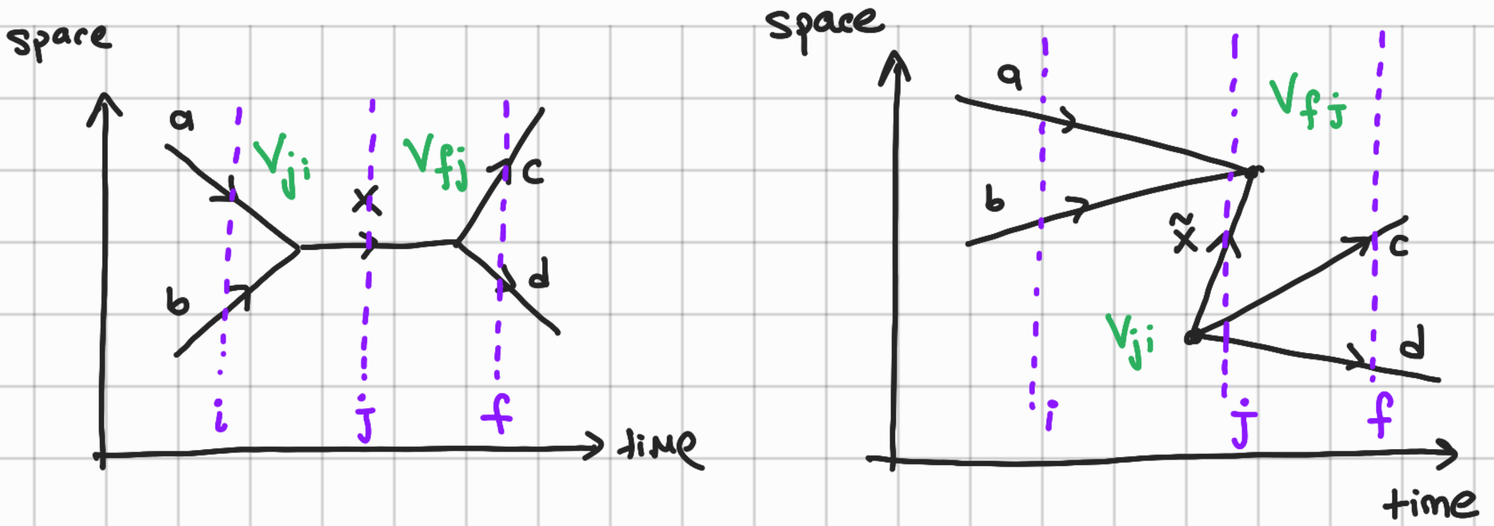
Exercise 2

Draw the lowest order Feynman diagrams for the Compton scattering

$$\gamma e^- \to \gamma e^-$$

Exercise 3

Draw the lowest order t-channel and u-channel Feynman diagrams for $e^+e^- \to \gamma\gamma$ and use the Feynman rules for QED to write down the corresponding matrix elements.



Matrix element given by second-order perturbation theory

Non-intuitive :)

c+d+$\tilde{x}$ pop out of the vacuum and subsequently a+b+$\tilde{x}$ annihilate into the vacuum

$$T_{fi}^{ab} = \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} = \frac{\langle c+d|V|x\rangle\langle x|V|a+b\rangle}{(E_a+E_b)-(E_x)}$$

Scalar interaction

$$V_{ji} = \mathcal{M}_{ij} \prod_k (2E_k)^{-1/2}$$

$$\mathcal{M}_{a+b\to x} = \mathcal{M}_{x\to c+d}$$

$$T_{fi}^{ab} = \frac{1}{2E_x} \frac{1}{(2E_a 2E_b 2E_c 2E_d)^{1/2}} \frac{g^2}{(E_a+E_b)-(E_x)}$$

$$\mathcal{M}_{fi}^{ab} = (2E_a 2E_b 2E_c 2E_d)^{1/2} T_{fi}^{ab} = \frac{1}{2E_x} \cdot \frac{g^2}{(E_a+E_b)-E_x}$$

For the 2^{nd} time-ordering

why?

vacuum state

$$T_{fi}^{cd} = \frac{\langle c+d+x|V|0\rangle\langle 0|V|a+b+x\rangle}{(\cancel{E_a}+\cancel{E_b})-(\cancel{E_a}+\cancel{E_b}+E_c+E_d+E_x)}$$

$$M_{fi}^{cd} = -\frac{1}{2E_x}\frac{g^2}{(E_c+E_d+E_x)} = -\frac{1}{2E_x}\frac{g^2}{(E_a+E_b)+E_x}$$

$$M = M_{fi}^{ab} + M_{fi}^{cd}$$

$$= \frac{g^2}{2E_x}\left[\frac{1}{(E_a+E_b)-E_x} - \frac{1}{(E_a+E_b)+E_x}\right]$$

$$(E_a+E_b)+E_x \qquad (E_a+E_b)-E_x$$

$$= \frac{g^2}{2E_x}\cdot\frac{2E_x}{(E_a+E_b)^2-E_x^2} = \frac{g^2}{(E_a+E_b)^2-E_x^2}$$

where $E_x^2 = (\vec{p_a}+\vec{p_b})^2 + m_x^2$

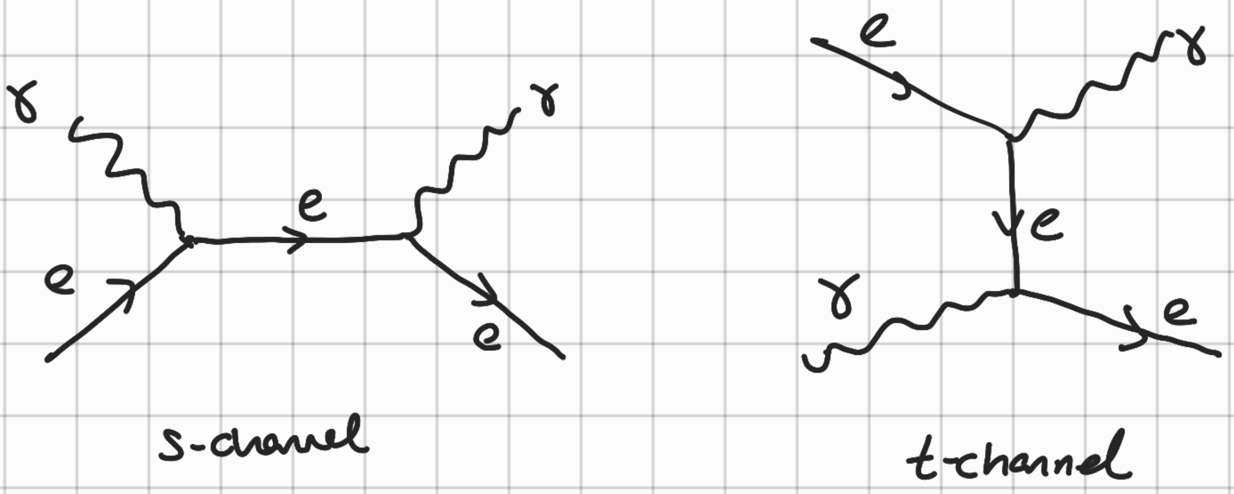
$$M = \frac{g^2}{\underbrace{(E_a+E_b)^2-(\vec{p_a}+\vec{p_b})^2}_{}-m_x^2} = \frac{g^2}{q^2-m_x^2}$$

$q^2 = (p_a+p_b)^2$

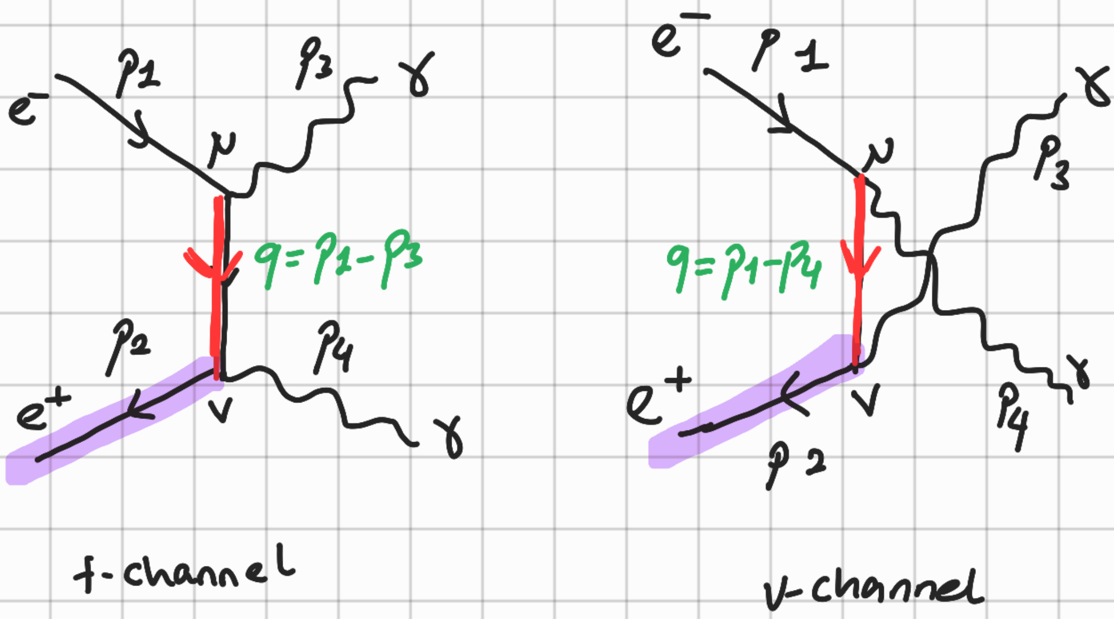
4 vector

same for the t-channel process

Exercise 2 $\gamma e^- \to \gamma e^-$ "Compton Scattering"



Exercise 3 $e^+ e^- \to \gamma\gamma$



$$-i\mathcal{M}_t = \left[\varepsilon^{\nu}(p_3)^* \; ie\gamma^{\nu} \; u_e(p_1) \right] \left[-\frac{i(\gamma^{\rho} q_{\rho} + m_e)}{q^2 - m_e^2} \right] \left[\bar{v}(p_2) \; ie\gamma^{v} \varepsilon_v(p_4)^* \right]$$

$\varepsilon^{\nu}(p_3)^* \downarrow$ outgoing photon

$$-i\mathcal{M}_u = \left[\varepsilon^{\nu}(p_4)^* \; ie\gamma^{\nu} u_e(p_1) \right] \left[\frac{-i(\gamma^{\rho} q_{\rho} + m_e)}{q^2 - m_e^2} \right] \left[\bar{v}(p_2) \; ie\gamma^{v} \varepsilon_v(p_3)^* \right]$$