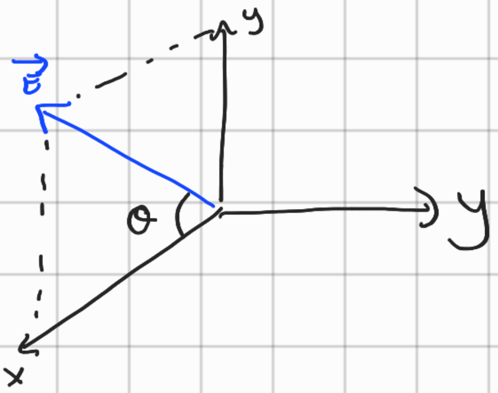
28/09/22

Polarization



$\vec{E} \perp \vec{B}$

$$\vec{E} = Re\left\{ E_0 \begin{pmatrix} \cos\theta \, e^{i\delta_x} \\ \sin\theta \, e^{i\delta_y} \\ 0 \end{pmatrix} e^{ikz} \, e^{-i\omega t} \right\}$$

Linear Polarization

$$\delta_x = \delta_y = 0 \Rightarrow \boxed{\vec{E} = E_0 \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \cos(kz - \omega t)}$$

$$k = \frac{2\pi}{\lambda} \quad , \quad \omega = 2\pi\nu$$

$$\lambda \nu = c$$

Circularly polarized case

$$\theta = \frac{\pi}{4} \qquad \cos\theta = \sin\theta = \frac{1}{\sqrt{2}} \quad , \quad \delta_x = 0 \, , \quad \delta_y = \pm\frac{\pi}{2}$$

$$\vec{E} = Re\left\{ \frac{E_0}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i(kz - \omega t)} \right\} = \frac{E_0}{\sqrt{2}} \begin{pmatrix} \cos(kz - \omega t) \\ \mp\sin(kz - \omega t) \\ 0 \end{pmatrix}$$

Quantum counterpart of a given mode of e.m which is polarized

$\downarrow$

Polarized photon

degree of freedom = polarization degree of freedom of a photon

$\equiv$ 1st example of qubit

Polarization qubit described by a vector $\underbrace{\begin{pmatrix} \cos\theta \, e^{i\delta_x} \\ \sin\theta \, e^{i\delta_y} \end{pmatrix} e^{ikz} e^{i\omega t}}_{\text{Wave function of photon}}$

$\psi_{\theta,\delta_x,\delta_y}(z,t)$

Vector $\in \left\{ \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \text{ with } \alpha \text{ and } \beta \text{ complex numbers st } |\alpha|^2 + |\beta|^2 = 1 \right\}$

Linearly polarized photon (qubit)

State vector $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = |\theta\rangle \rightarrow$ Dirac Notation (ket)

$(\cos\theta \quad \sin\theta) = \langle\theta| \rightarrow$ (bra)

Bracket = inner product in space of state vectors.

$$(\cos\theta_1 \,, \sin\theta_1) \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} = \langle\theta_1|\theta_2\rangle$$

ket-bra

$$|\theta_2\rangle\langle\theta_1| \qquad \underset{2\times 1}{\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix}} \underset{1\times 2}{(\cos\theta_1, \sin\theta_1)} = \underset{2\times 2}{\begin{pmatrix} & \\ & \end{pmatrix}}$$

States of circular polarization

$$\begin{pmatrix}1\\ \pm i\end{pmatrix} = \begin{cases} |\circlearrowleft\rangle \\ |\circlearrowright\rangle \end{cases} \quad \text{kets}$$

$$(1, \mp i) = \begin{cases} \langle\circlearrowleft| \\ \langle\circlearrowright| \end{cases} \quad \text{bras}$$

$$\underbrace{\langle\circlearrowright|\circlearrowleft\rangle}_{\text{orthogonal}} = \frac{1}{2}(1, +i)\begin{pmatrix}1\\ +i\end{pmatrix} = 1 + i^2 = 1 + (-1) = 0$$

$$\langle\circlearrowleft|\circlearrowleft\rangle = \frac{1}{2}(1, -i)\begin{pmatrix}1\\ +i\end{pmatrix} = \frac{1}{2}(1 - i^2) = (1-(-1)) \cdot \underbrace{\frac{1}{2}}_{\text{normalization}} = 1$$

Polarization state of photon (qubit)

State vector $\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = |\psi\rangle$

$$(\alpha^* \quad \beta^*) = \langle\psi|$$

$$|x\rangle = \begin{pmatrix}\gamma\\ \delta\end{pmatrix}$$

$$|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$\langle x|\psi\rangle = (\gamma^*, \delta^*)\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \gamma^*\alpha + \delta^*\beta \in \mathbb{C}$$

Properties of Brackets

① $\langle x|\psi\rangle = \langle \psi|x\rangle$

② $\langle x|\left(\lambda|\psi_1\rangle + \mu|\psi_2\rangle\right) = \lambda\langle x|\psi_1\rangle + \mu\langle x|\psi_2\rangle \rightarrow$ linearity

③ $\langle \psi|\psi\rangle \geq 0$

Question: how to extract information from $|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$

You can only extract 1 classical bit of information

BORN RULE

$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = |\psi\rangle \longrightarrow$ Measure pol. state of photon

linear pol.

a random projection on the basis of linear pol.

one example $\Leftarrow$ $\begin{pmatrix}1\\0\end{pmatrix} = |\theta=0\rangle$; $\begin{pmatrix}0\\1\end{pmatrix} = |\theta=\pi/2\rangle$

$\rightarrow |\theta=0\rangle \quad prob(\theta=0) = |\langle\theta=0|\psi\rangle|^2 = |\alpha|^2$

$\rightarrow |\theta=\pi/2\rangle \quad prob(\theta=\tfrac{\pi}{2}) = |\langle\theta=\pi/2|\psi\rangle|^2 = |\beta|^2$

$\boxed{\text{extracted 1 bit info}}$ $\quad \boxed{|\alpha|^2 + |\beta|^2 = 1}$

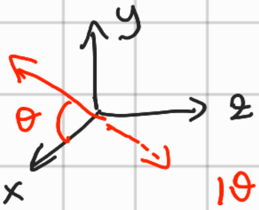
$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = |\psi\rangle \longrightarrow$ Measure CIRCULAR POL. Random Projection $|\circlearrowright\rangle$, $|\circlearrowleft\rangle$

$\rightarrow |\circlearrowright\rangle \quad prob(\circlearrowright) = |\langle\circlearrowright|\psi\rangle|^2$

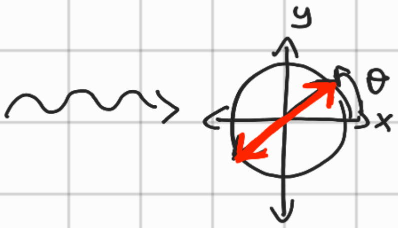
$\rightarrow |\circlearrowleft\rangle \quad prob(\circlearrowleft) = |\langle\circlearrowleft|\psi\rangle|^2$

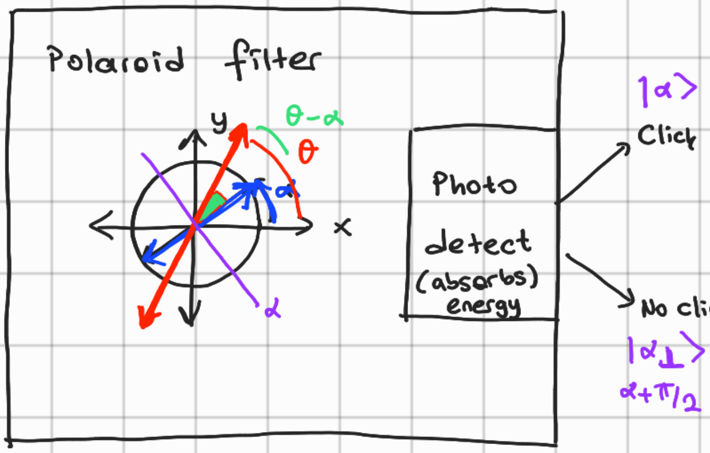
Thought Experiments: $\rightarrow$ Can all be done in lab?

Measurement of LIN POL.



$|\theta\rangle = \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$



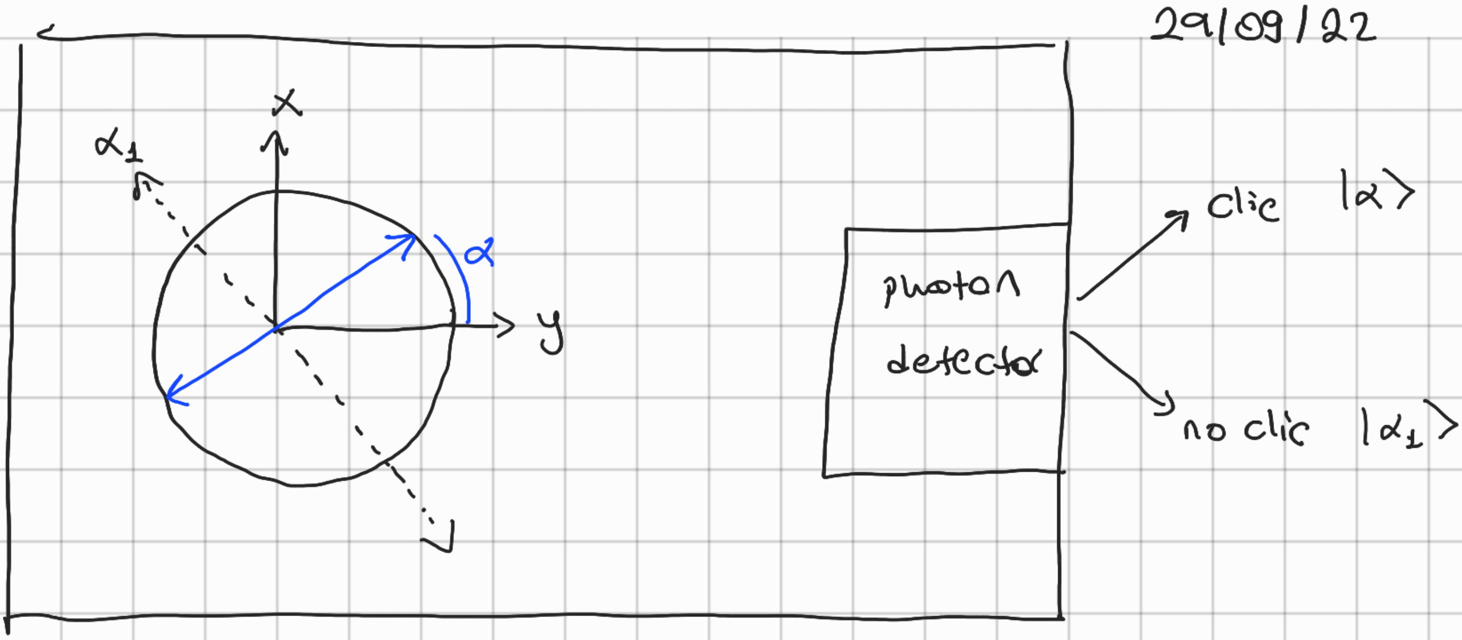


$$p(\text{click}) = |\langle \alpha | \theta \rangle|^2 = (\cos(\alpha-\theta))^2$$

$$p(\text{no click}) = |\langle \alpha_\perp | \theta \rangle|^2 = (\sin(\alpha-\theta))^2$$

Classically $\left\{ \begin{array}{l} (\cos(\theta-\alpha))^2 \\ (\sin(\theta-\alpha))^2 \end{array} \right\}$ intensity

29/09/22



Measurement device

Born Rule

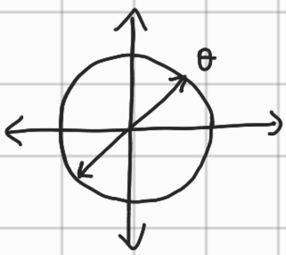
$$\text{prob}(\alpha) = |\langle \alpha | \theta \rangle|^2$$

$$\text{prob}(\alpha_1) = |\langle \alpha_1 | \theta \rangle|^2$$

what is the difference btw qubit & photons

1 1 0 0 - 1 . . . 1

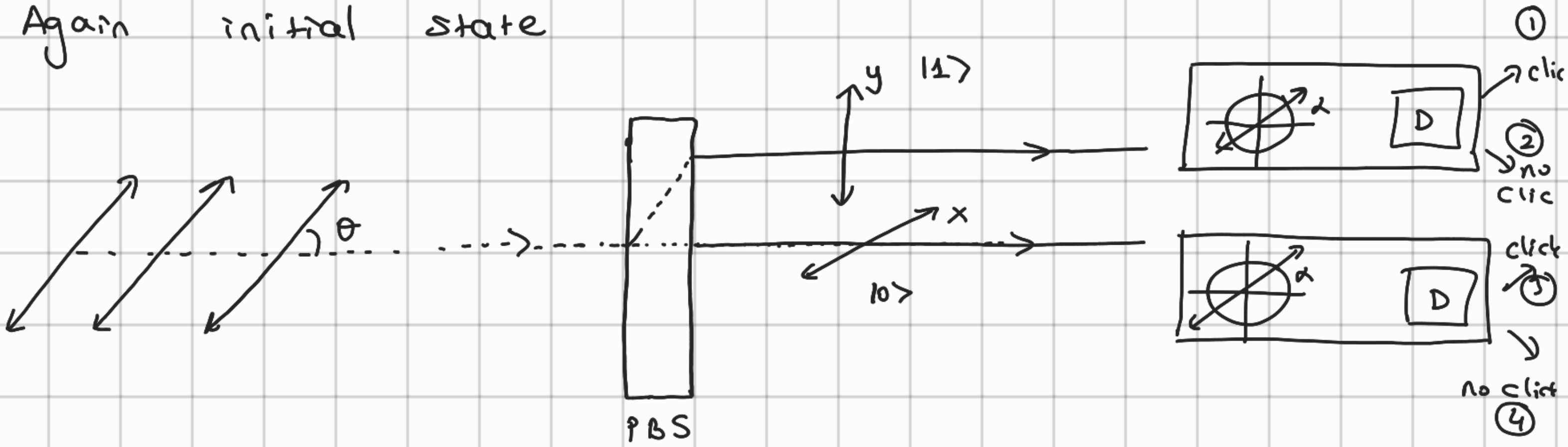
Initial state



⟶ direction of propagation

$$|\theta\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$

Again initial state



PBS
polarization beam splitter

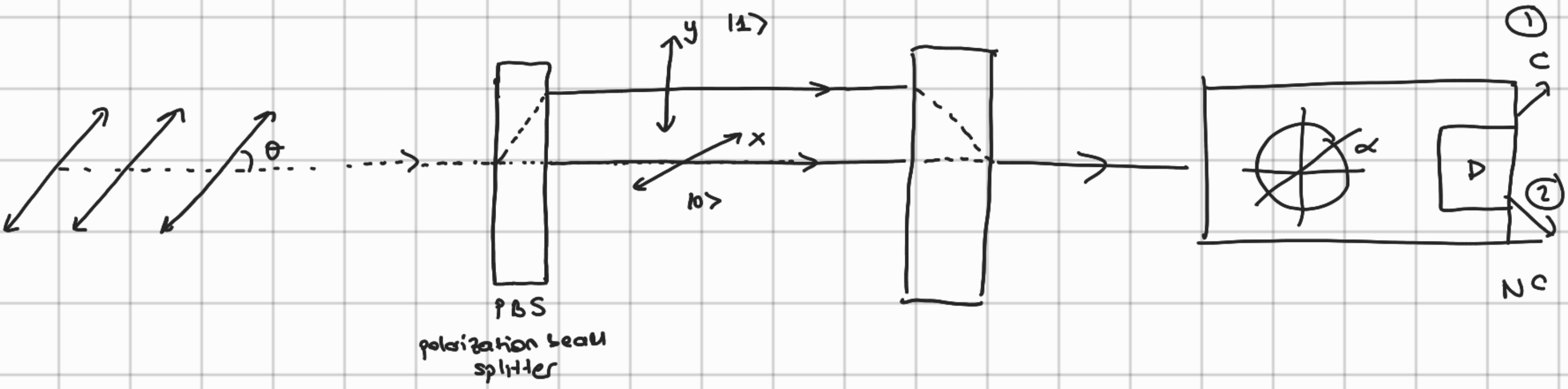
State of the photon has to be a superposition state

① $\text{prob}(\alpha) = |\langle \alpha | 1 \rangle|^2$

② $\text{prob}(\alpha_1) = |\langle \alpha_1 | 1 \rangle|^2$

③ $\text{prob}(\alpha) = |\langle \alpha | 0 \rangle|^2$

④ $\text{prob}(\alpha_1) = |\langle \alpha_1 | 0 \rangle|^2$

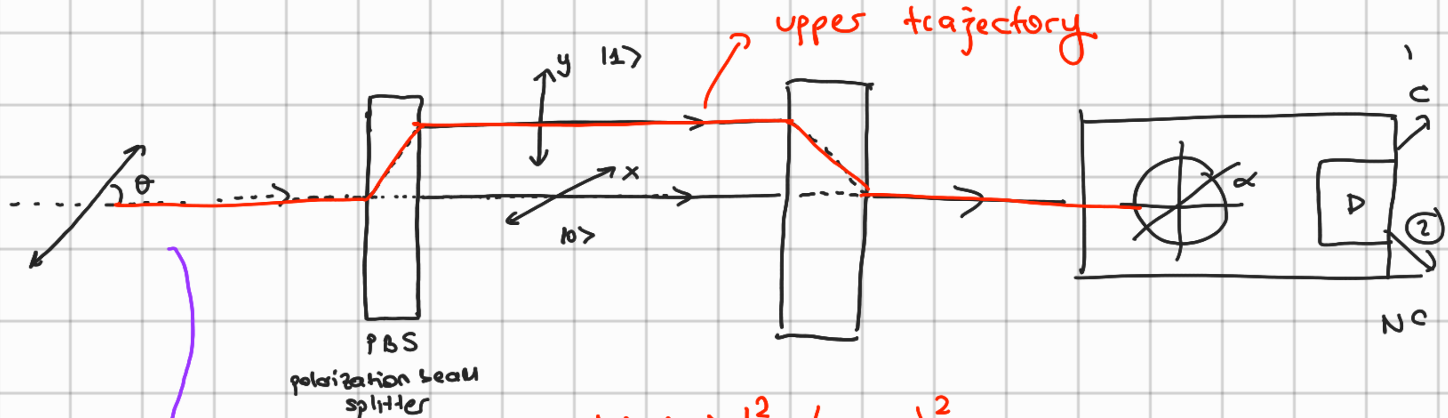


① $\text{prob}(\alpha) = |\langle \alpha | \theta \rangle|^2 = |\cos(\alpha - \theta)|^2$

② $\text{prob}(\alpha_1) = |\langle \alpha_1 | \theta \rangle|^2 = |\sin(\alpha_1 - \theta)|^2$

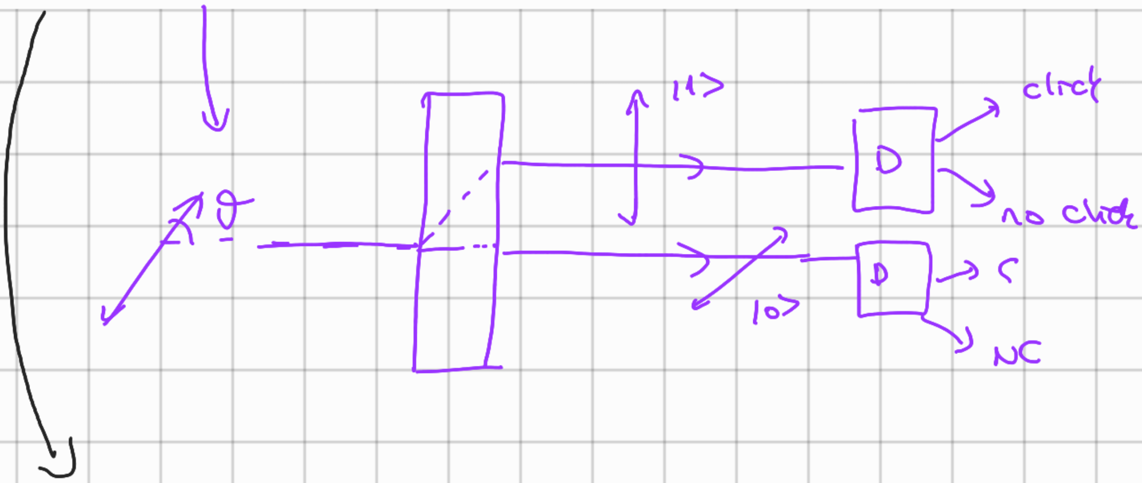
Calculation of prob (detection) or prob (no click) for classical "balls"

(click written above "detection")



①

$$\text{prob}(\text{click}) = \overbrace{\text{prob}(\text{upper tra})}^{(\sin\theta)^2} \cdot \overbrace{\text{prob}(\text{click} \mid \text{upper tra.})}^{|\langle \alpha | 1 \rangle|^2 = (\sin\alpha)^2} + \underbrace{\text{prob}(\text{down tra})}_{(\cos\theta)^2} \cdot \underbrace{\text{prob}(\text{click} \mid \text{down traj.})}_{\to |\langle \alpha | 0 \rangle|^2 = (\cos\alpha)^2}$$



$$|\langle 1|\theta\rangle|^2 = (\sin\theta)^2$$

$$|\langle 0|\theta\rangle|^2 = (\cos\theta)^2$$

① prob(click) $= (\sin\theta)^2 \cdot (\sin\alpha)^2 + (\cos\theta)^2 (\cos\alpha)^2$

$$\neq \cos(\theta-\alpha)^2 = (\sin\theta\sin\alpha - \cos\theta\cos\alpha)^2$$

CLASSICAL CALCULATION MISSES $\boxed{\text{DOUBLE PRODUCT}}$