4. WEAK INTERACTIONS OF LEPTONS

1) Muon decay and lepton universality

2) Lepton Universality Tests

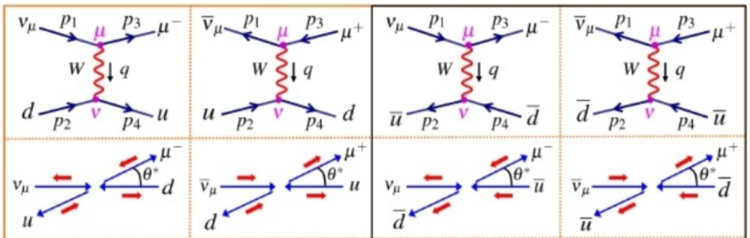
3) Neutrino-quark scattering

4) Anti-neutrino-quark scattering

$$\frac{\sigma_{\bar{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}$$

All combinations: (anti)neutrino-(anti)quark scattering

- non-zero antiquark component in the nucleon $\Longrightarrow$ also consider scattering from $\bar{q}$
- cross sections can be obtained immediately by comparing with quark scattering and remembering to only include LH particles and RH antiparticles



$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{d\sigma_{\bar{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\nu \bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\bar{\nu} \bar{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}$	$\sigma_{\bar{\nu} q} = \frac{G_F^2 \hat{s}}{3\pi}$	$\sigma_{\nu \bar{q}} = \frac{G_F^2 \hat{s}}{3\pi}$	$\sigma_{\bar{\nu} \bar{q}} = \frac{G_F^2 \hat{s}}{\pi}$

5. NEUTRINOS AND NEUTRINO OSCILLATIONS

1) Neutrino Flavours

$\nu_e, \nu_\mu, \nu_\tau \rightarrow$ weak eigenstates

Detected via their weak interactions

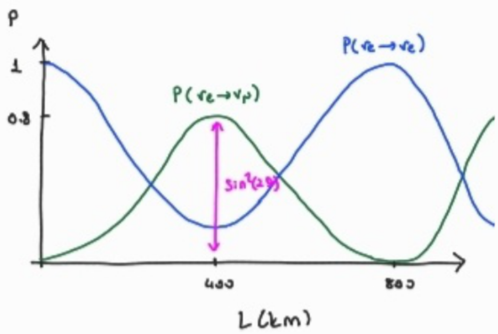
2) Mass and Weak Eigenstates

Mass eigenstates ν_1, ν_2

$$\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

2.1) Neutrino Oscillations for two flavours



$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta M_{21}^2 L}{4E}\right) \quad \Delta M_{21}^2 = M_2^2 - M_1^2$$

$$P(\nu_e \rightarrow \nu_e) = 1 - P(\nu_e \rightarrow \nu_\mu)$$

2.2) Neutrino Oscillations for 3 flavours

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{\text{Pontecorvo-Maki-Nakagawa-Sakata (PMNS)}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U^{-1} = U^{+}$$

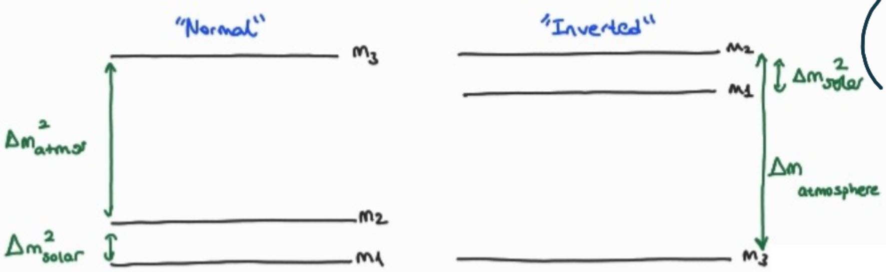
Hence, PMNS is unitary

$$P(\nu_e \rightarrow \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\Delta_{21} - 4|U_{e1}|^2|U_{e3}|^2\sin^2\Delta_{31} - 4|U_{e2}|^2|U_{e3}|^2\sin^2\Delta_{32}$$

2.2.1) CP and CPT in weak interaction

Unless the elements of the PMNS Matrix are real $P(\nu_e \rightarrow \nu_\mu) \neq P(\nu_\mu \rightarrow \nu_e)$

2.3) Neutrino mass hierarchy



Hence, we can approximate $\Delta m_{31}^2 \approx \Delta m_{32}^2$

$$\lambda_{21} = \frac{4\pi E}{\Delta m_{21}^2} \qquad \lambda_{32} = \frac{4\pi E}{\Delta m_{32}^2}$$

Long wavelength — SOLAR

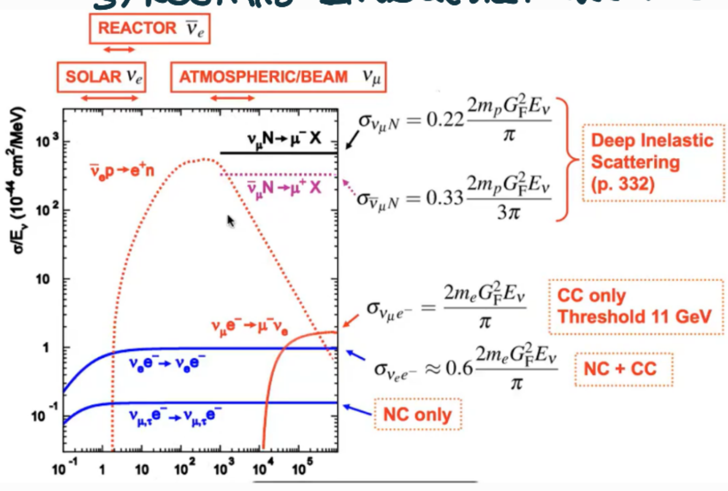
Short wavelength — ATMOSPHERIC

2.4) PMNS Matrix

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar}}$$

relates to CP violation

3) Neutrino Interaction Threshold



6. CKM MATRIX AND CP VIOLATION

1) CP violation in the Early Universe

- Matter dominates antimatter
- Asymmetry → 3 conditions
- Initially, CP violation observed in quark sector

2) Weak Interaction of Quarks

- Cabibbo Hypothesis (weak eigenstates are diff. from mass eigenstates.)
- GIM Mechanism (extra quark → c)

3) CKM Matrix

- Near Diagonal
- weak interaction ↑ with same generation
- off-diagonals are very small

4) Neutral Kaon System

- K^0, $\bar{K}^0$ are flavour states (eigenstates)
- $K^0 \leftrightarrow \bar{K}^0$ mixing
- Propagation States K_S and K_L

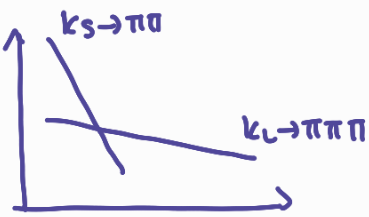
Without CP

$|K_S\rangle = |K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$ $\quad CP|K_1\rangle = +|K_1\rangle$

$|K_L\rangle = |K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$ $\quad CP|K_2\rangle = -|K_2\rangle$

Decay: $K_1 \to \pi\pi$ (CP even (+1))

$K_2 \to \pi\pi\pi$ (CP odd (−1))



5) Strangeness Oscillations

$M(K_L) \neq M(K_S) \to$ oscillation

$$P(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + 2e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t} \cos(\Delta m t)\right]$$

5.1) CPLEAR Experiment

$A_{\Delta m} \to$ Asymmetry

Δm is measured

6) CP violation in kaon System

K_S and K_L do not correspond exactly to CP eigenstates K_1 and K_2

$|\varepsilon| \sim 2\times10^{-3}$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right] \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

6.1) CP violation in semileptonic decay

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle\right]$$

($|K^0\rangle$ ↑ $\pi^+ e^- \bar{\nu}_e$; $|\bar{K}^0\rangle$ ↑ $\pi^- e^+ \nu_e$)

$$\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e) \propto |\langle\bar{K}^0|K_L\rangle|^2 \propto |1-\varepsilon|^2 \simeq 1 - 2\Re\{\varepsilon\}$$

$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto |\langle K^0|K_L\rangle|^2 \propto |1+\varepsilon|^2 \simeq 1 + 2\Re\{\varepsilon\}$$

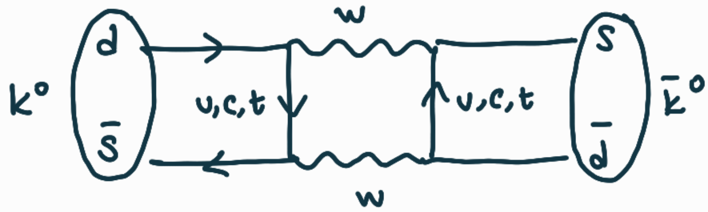
first evidence of diff. btw matter & antimatter

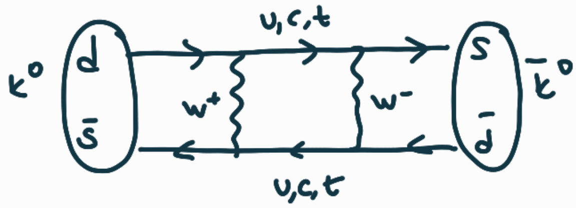
7) CP violation and the CKM Matrix

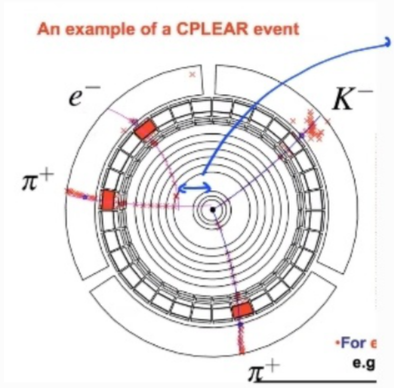
$$\Gamma(K^0 \to \bar{K}^0) - \Gamma(\bar{K}^0 \to K^0) \propto 2\,\Im\{M_{fi}\}$$

Rates can only be different if the CKM matrix has imaginary components

$$|\varepsilon| \propto \Im\{M_{fi}\}$$





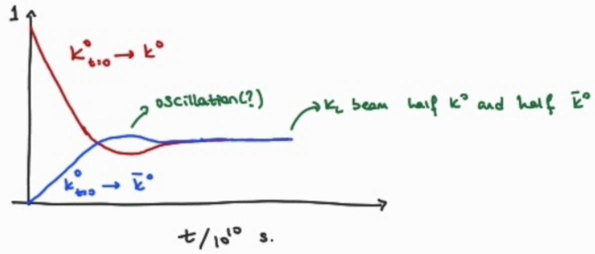


used to measure decay time

Production: $\bar{p}p \to K^- \pi^+ K^0$

Mixing

Decay : $\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$



7. CKM MATRIX PARAMETRIZATION AND B PHYSICS

1) Determination

- The CKM matrix is unitary: can be parameterized by 3 mixing angles and the CP-violating phase
- A standard choice is the same as was shown for the PMNS matrix:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Since experimentally $s_{13} \ll s_{23} \ll s_{12} \ll 1$, the matrix hierarchy is exhibited using the Wolfenstein parameterization (written in terms of λ, A, ρ, η):

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

MAINLY FROM LEPTONIC DECAYS

1) $|V_{ud}|$ from nuclear beta decay
2) $|V_{us}|$ semi leptonic π decay
3) $|V_{cd}|$ from neutrino scattering
4) $|V_{cs}|$ semi-leptonic D^+ decay
5) $|V_{cb}|$ semi-leptonic B hadron decay
6) $|V_{ub}|$ semi-leptonic B hadron decay
7) $|V_{td}|$ $|V_{ts}|$ → from strangeness oscillations
8) $|V_{tb}|$ from top quark decays

2) Particle- Anti-particle Mixing

↓

3) B- Physics

Oscillations have been also observed for heavy mesons:

$B^0(\bar{b}d) \leftrightarrow \bar{B}^0(b\bar{d})$

$B^0_s(\bar{b}s) \leftrightarrow \bar{B}^0_s(b\bar{s})$

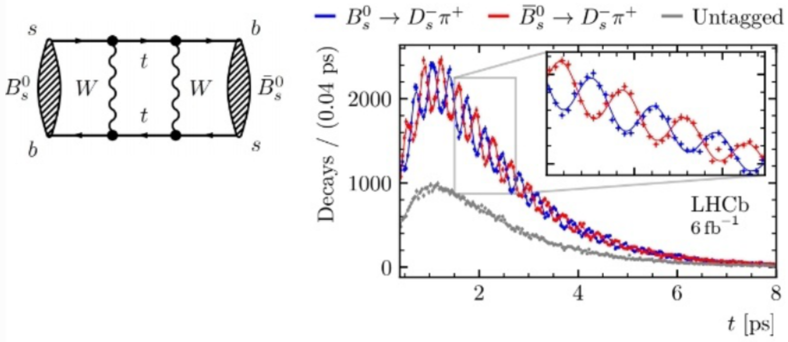
$D^0(\bar{c}u) \leftrightarrow \bar{D}^0(c\bar{u})$

→ Math descriptions are same for kaons

$$|B_L\rangle = \frac{1}{\sqrt{2}}\left[|B^0\rangle + e^{-i2\beta}|\bar{B}^0\rangle\right]$$

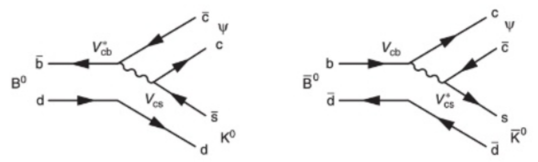
$$|B_H\rangle = \frac{1}{\sqrt{2}}\left[|B^0\rangle - e^{-i2\beta}|\bar{B}^0\rangle\right]$$

- The latest LHCb result: https://arxiv.org/abs/2104.04421



Δm_s=17.7656±0.0057ps^{-1}

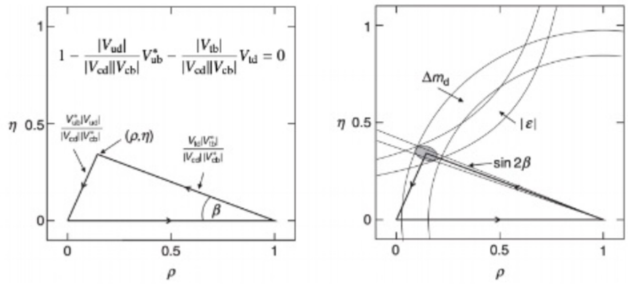
- Decay proceeds in two stages:
  - First, B^0 decays to flavour eigenstate: $B^0 \to J/\psi K^0$ and B^0-bar$\to J/\psi K^0$-bar
  - Then the neutral kaons system evolves as a linear combination of physical K_S and K_L states



- CP violation is measurable through the asymmetry

$$A_{CP}^{K_S} = \frac{\Gamma(\overline{B}^0_{t=0} \to \psi\, K_S) - \Gamma(B^0_{t=0} \to \psi\, K_S)}{\Gamma(\overline{B}^0_{t=0} \to \psi\, K_S) + \Gamma(B^0_{t=0} \to \psi\, K_S)} = \sin(\Delta m_d t)\ \sin(2\beta)$$

- Combination of various measurements:



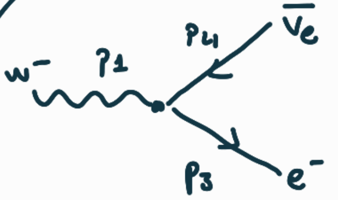
- CP violation in the weak interactions of hadrons is described by the single irreducible complex phase in the CKM matrix
- In the Wolfenstein parametrisation, CP violation is associated with the parameter η. To O(λ^4), the parameter η appears only in V_{ub} and V_{td}, with

$$V_{ub} \approx A\lambda^3(\rho - i\eta) \quad \text{and} \quad V_{td} \approx A\lambda^3(1 - \rho - i\eta)$$

- The measurements of non-zero values of |ε| and sin(2β) separately imply that η ≠ 0
- But ρ and η are determined from the measurements combinations

8) ELECTROWEAK UNIFICATION AND THE W and Z BOSONS

1) W boson decay

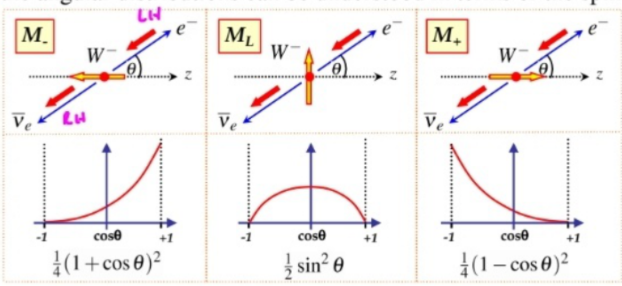


$j^{\mu} = \bar{u}(p_3)\gamma^{\mu}\frac{1}{2}(1-\gamma^5)v(p_4)$

$j^{\mu} = \bar{u}_{\downarrow}(p_3)\gamma^{\mu}v_{\uparrow}(p_4)$

In ultra-relativistic limit only LH PARTICLES and RH ANTI-PARTICLES participate in weak int.

- the angular distributions can be understood in terms of the spin of th



- the differential decay rate can be found using:

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$$

* Total W decay rate is independent of polarisation.

$\langle |M_{fi}|^2 \rangle = \frac{1}{3}g_W^2 m_W^2 \rightarrow$ no dependence on Direction

- the calculations for the other decay modes (neglecting final state particle masses) is the same. For quarks need to account for color and CKM matrix. No decays to t quark, since its mass (175 GeV) is greater than W boson mass (80 GeV)

$W^- \to e^-\bar{\nu}_e$	$W^- \to d\bar{u}$ $\times 3\vert V_{ud}\vert^2$	$W^- \to d\bar{c}$ $\times 3\vert V_{cd}\vert^2$
$W^- \to \mu^-\bar{\nu}_\mu$	$W^- \to s\bar{u}$ $\times 3\vert V_{us}\vert^2$	$W^- \to s\bar{c}$ $\times 3\vert V_{cs}\vert^2$
$W^- \to \tau^-\bar{\nu}_\tau$	$W^- \to b\bar{u}$ $\times 3\vert V_{ub}\vert^2$	$W^- \to b\bar{c}$ $\times 3\vert V_{cb}\vert^2$

- unitarity of CKM matrix gives $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- hence $\boxed{\mathcal{B}(W \to q\bar{q}') = 6\mathcal{B}(W \to e\nu)}$ and the total decay rate:

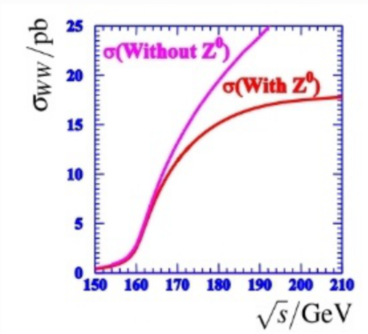
$$\boxed{\Gamma_W = 9\Gamma_{W\to e\nu} = \frac{3g_W^2 m_W}{16\pi} = 2.07\,\text{GeV}}$$

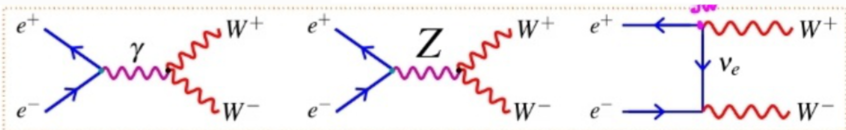
11%

Experiment: 2.14±0.04 GeV
(our calculation neglected a 3% QCD correction to decays to quarks)

No need to compute at all!

2) From W to Z





$$|M_{\gamma WW} + M_{ZWW} + M_{\nu WW}|^2 < |M_{\gamma WW} + M_{\nu WW}|^2$$

3) $SU(2)_L$: Weak Interaction

$\psi \to \psi' = \psi e^{i\vec{\alpha}(x)\cdot\frac{\vec{\sigma}}{2}}$ → generators of the SU(2)

⇓

3 Gauge Bosons → $W_1^{\mu}, W_2^{\mu}, W_3^{\mu}$

$\psi = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}$ ⇒ WEAK ISOSPIN

- for simplicity only consider $\chi_L = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L$
- the gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2) (note: including interaction strength in current):

$$j_\mu^1 = g_W\bar{\chi}_L\gamma^\mu\frac{1}{2}\sigma_1\chi_L; \quad (7)$$

$$j_\mu^2 = g_W\bar{\chi}_L\gamma^\mu\frac{1}{2}\sigma_2\chi_L; \quad (8)$$

$$j_\mu^3 = g_W\bar{\chi}_L\gamma^\mu\frac{1}{2}\sigma_3\chi_L; \quad (9)$$

- the charged current W^+/W^- interaction enters as a linear combinations of W_1, W_2:

$$\boxed{W^{\pm\mu} = \frac{1}{\sqrt{2}}(W_1^\mu \pm W_2^\mu)}$$

SU(2)$_L$: the weak interaction

- the $W^{\pm}$ interaction terms:

$$j^{\mu}_{\pm} = \frac{g_W}{\sqrt{2}}(j^{\mu}_1 \pm i j^{\mu}_2) = \frac{g_W}{\sqrt{2}}\bar{\chi}_L \gamma^{\mu}\frac{1}{2}(\sigma_1 \pm i\sigma_2)\chi_L$$

- express in terms of the weak isospin ladder operators: $\sigma_{\pm} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$

$$\boxed{j^{\mu}_{\pm} = \frac{g_W}{\sqrt{2}}\bar{\chi}_L\gamma^{\mu}\sigma_{\pm}\chi_L} \Big\} \text{ Origin of } \frac{1}{\sqrt{2}} \text{ in Weak CC}$$

W⁺  corresponds to $\boxed{j^{\mu}_{+} = \frac{g_W}{\sqrt{2}}\bar{\chi}_L\gamma^{\mu}\sigma_{+}\chi_L}$

which can be understood in terms of the weak isospin doublet

$$j^{\mu}_{+} = \frac{g_W}{\sqrt{2}}\bar{\chi}_L\gamma^{\mu}\sigma_{+}\chi_L = \frac{g_W}{\sqrt{2}}(\bar{\nu}_L \bar{e}_L)\gamma^{\mu}\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L$$
$$= \frac{g_W}{\sqrt{2}}\bar{\nu}_L\gamma^{\mu}e_L = \frac{g_W}{\sqrt{2}}\bar{\nu}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)e$$

- similarly:

W⁻  corresponds to $\boxed{j^{\mu}_{-} = \frac{g_W}{\sqrt{2}}\bar{\chi}_L\gamma^{\mu}\sigma_{-}\chi_L}$

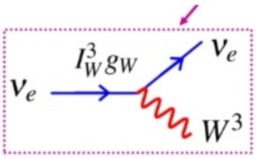
$$j^{\mu}_{-} = \frac{g_W}{\sqrt{2}}\bar{\chi}_L\gamma^{\mu}\sigma_{-}\chi_L$$
$$= \frac{g_W}{\sqrt{2}}(\bar{\nu}_L \quad \bar{e}_L)\gamma^{\mu}\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L$$
$$= \frac{g_W}{\sqrt{2}}\bar{e}_L\gamma^{\mu}\nu_L = \frac{g_W}{\sqrt{2}}\bar{e}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\nu$$

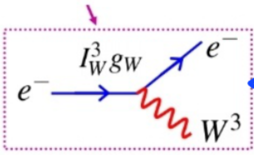
- however have an additional interaction due to W^3:

$$j^{\mu}_3 = g_W\bar{\chi}_L\gamma^{\mu}\frac{1}{2}\sigma_3\chi_L$$

- expanding this:

$$j^{\mu}_3 = g_W\frac{1}{2}(\bar{\nu}_L \quad \bar{e}_L)\gamma^{\mu}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L$$
$$= g_W\frac{1}{2}\bar{\nu}_L\gamma^{\mu}\nu_L - g_W\frac{1}{2}\bar{e}_L\gamma^{\mu}e_L$$





$\Longrightarrow$ **Neutral current interactions!**

★ It is tempting to identify W^3 as the Z-boson. YET W^3 is a MIXTURE of the (γ, Z)

$$A_{\mu} = B_{\mu}\cos\theta_W + W^3_{\mu}\sin\theta_W$$
$$Z_{\mu} = -B_{\mu}\sin\theta_W + W^3_{\mu}\cos\theta_W$$

θ_W is the weak mixing angle

TO UNIFY

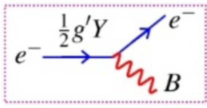
new spin-1 boson $B \rightarrow U(1)_Y$ ★★

- the charge of this symmetry is called **weak hypercharge Y**:

$$Y = 2Q - 2I^3_W,$$

Q is the EM charge of a particle,

I^3_W is the third component of weak isospin



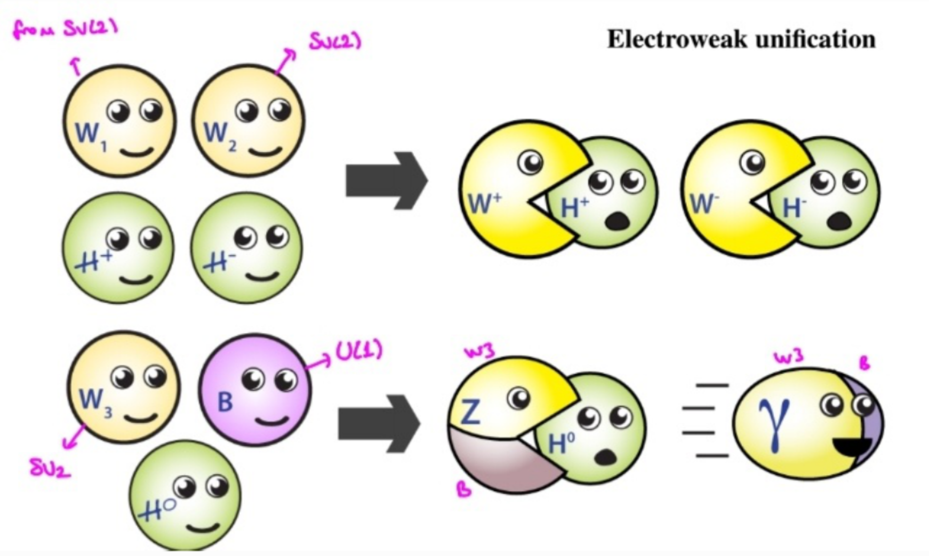
- by convention, the coupling to the B_{μ} is $\frac{1}{2}g'Y$:

$$e_L: Y = 2(-1) - 2(-\tfrac{1}{2}) = -1$$
$$e_R: Y = 2(-1) - 2(0) = -2$$
$$\nu_L: Y = +1$$
$$\nu_R: Y = 0$$

$$e = g_W\sin\theta_W = g'\cos\theta_W \quad \Leftarrow$$

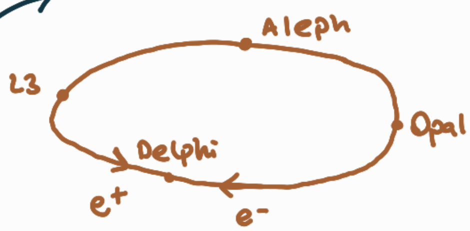
$$e = g_Z\cos\theta_W\sin\theta_W$$

$$g_Z = \frac{g_W}{\cos\theta_W}$$

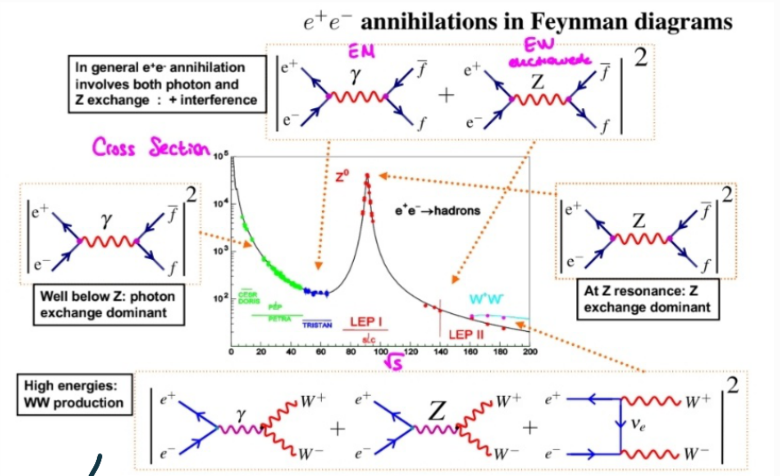


9. TESTS OF THE STANDARD MODEL

1) Electroweak Measurement at LEP

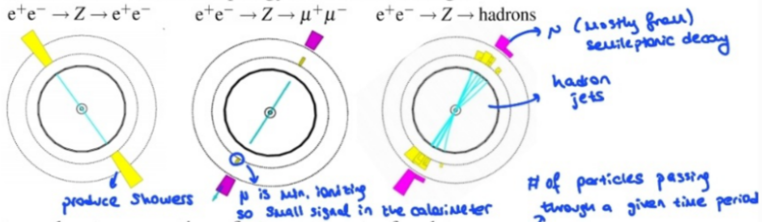


2) e^+e^- annihilations in Feynman Diagrams



3) Cross Section Measurements

- at Z resonance mainly observe four types of events: $e^+e^- \to Z \to e^+e^-$, $e^+e^- \to Z \to \mu^+\mu^-$, $e^+e^- \to Z \to \tau^+\tau^-$, $e^+e^- \to Z \to q\bar{q} \to$ hadrons
- each event has a distinct topology in the detectors, e.g.:

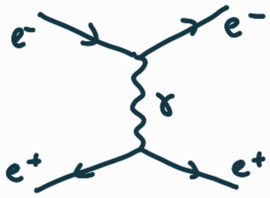


- to work out cross sections, first count events of each type
- then need to know "integrated luminosity" L of colliding beams, i.e. the relation between cross section σ and expected number of interactions N_{events}:

$$\boxed{N_{\text{events}} = \mathcal{L}\sigma}$$

\# of particles passing through a given time period

To calculate integrated luminosity need to know # of e^- and e^+s in the colliding beam



QED Bhabba Scattering

$$N_{\text{Bhabha}} = \mathcal{L}\, \sigma_{\text{Bhabha}} \checkmark$$

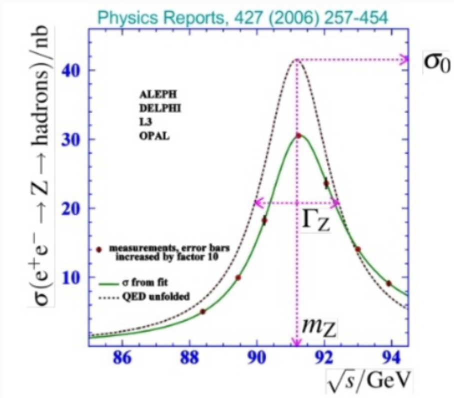
$$\sigma_i = \frac{N_i}{N_{\text{Bhabha}}}\, \sigma_{\text{Bhabha}}$$

★ Cross section measurements involve just EVENT COUNTING

4) Measurements of the Z-line shape

M_Z and Γ_Z measurement:

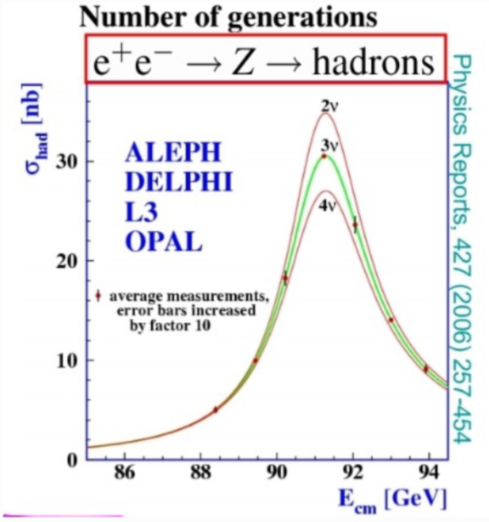
- .Run accelator at different energies
- .Measure cross sections
- .Account for ISR (Initial State) Radiation



4.1) Number of Generations

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + N_\nu \Gamma_{\nu\nu}$$

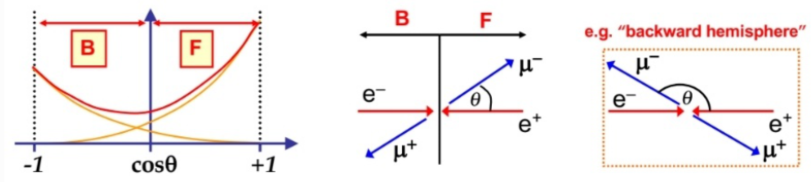
2.9840 ± 0.0082



- Took account beam-beam interaction effects

5) Forward - Backward Asymmetry

Couplings of the Z to both LH and RH particle are different ⇒ ASYMMETRY



6) Determination of the weak Mixing angle θ_W

$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q\sin^2\theta_W}{I_W^3} = 1 - 4|Q|\sin^2\theta_W$$

vector

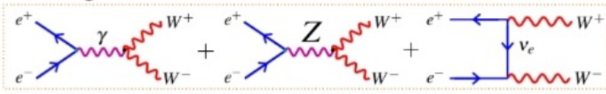
axial-vector

2 couplings

can be found using ASYMMETRY measurements

7) W^+W^- production

- in 1995–2000 LEP operated above the threshold for W-pair production
- three diagrams are involved:



- W bosons decay either to leptons or hadrons with branching fractions:

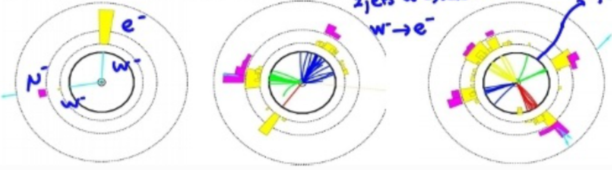
$$\mathcal{B}(W^- \to \text{hadrons}) \approx 0.67 \quad \mathcal{B}(W^- \to e^-\bar{\nu}_e) \approx 0.11$$
$$\mathcal{B}(W^- \to \mu^-\bar{\nu}_\mu) \approx 0.11 \quad \mathcal{B}(W^- \to \tau^-\bar{\nu}_\tau) \approx 0.11$$

- gives rise to three distinct topologies:

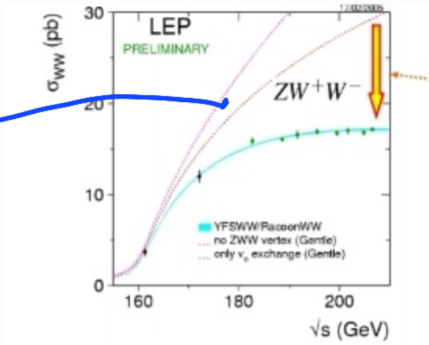
$W^- \to e^-$
$W^- \to \mu^-$

2jets $W^- \to$ hadron
$W^- \to e^-$

4jets $W^- \to$ hadron
$W^- \to$ hadron



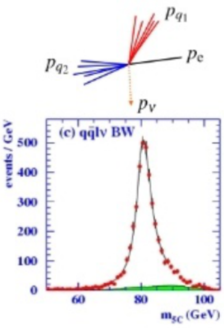
without Z, cross section violates unitarity



8) W-mass and W-width

Unlike $e^+e^- \to Z$, $e^+e^- \to W^+W^-$ is not a resonant process.

- measure energy and momenta of particles produced in the W boson decays, e.g. $W^+W^- \to q\bar{q}'e^-\bar{\nu}_e$



- ν 4-momentum from E-p conservation:
  $p_{q_1} + p_{q_2} + p_e + p_\nu = (\sqrt{s}, 0)$
- reconstruct masses of two W bosons:
  $M_+^2 = E^2 - \vec{p}^2 = (p_{q_1} + p_{q_2})^2$
  $M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$
- peak or reconstructed mass distribution gives:
  $m_W = 80.376 \pm 0.033$ GeV
- width of reconstructed mass distribution gives:
  $\Gamma_W = 2.196 \pm 0.083$ GeV (does not include Tevatron and LHC measurements)

9) Precision tests of the SM

(look at Ch. 10.2)

10) The Top Quark

$m_t^{meas} = 174.2 \pm 3.3$ GeV

$\Gamma_t = 1.5$ GeV

$\tau_t = 1/\Gamma_t \approx 5 \times 10^{-25}$ s $\to$ too short; thus, at the Tevatron or L

lifetime

top quark does not hadronize, DECAY BEFORE FORMING a bound state

10. HIGGS BOSON

1) The Higgs Mechanism

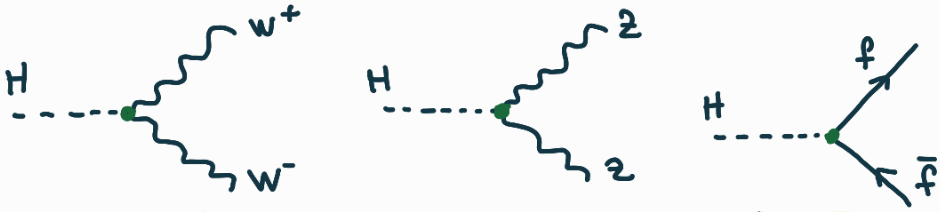
→ Gauge symmetry works only for massless gauge bosons

→ Higgs mechanism solves this issue

→ It gives mass to bosons and fermions

Analogy: Massless photons propagating through a plasma behave as massive particles

* Higgs boson is electrically neutral but carries WEAK HYPERCHARGE of 1/2
* Photons does not couple to the Higgs field and remains massless.



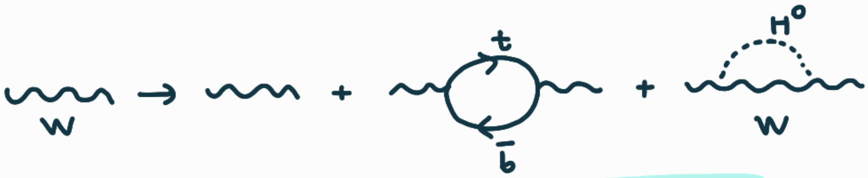
$ig_W m_W g^{\mu\nu}$ $\qquad$ $ig_Z m_Z g^{\mu\nu}$ $\qquad$ $-i\frac{g_W}{2m_W} m_f$

2) Precision tests of the standard model

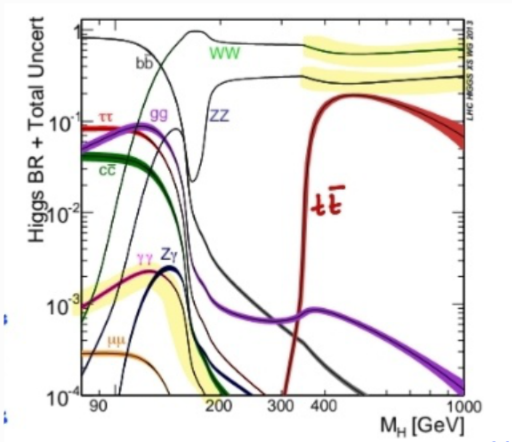
- LEP (Large Electron-Positron) collider
- Results are close but not quite right

↳ mass of W boson includes terms from VIRTUAL LOOPS

$m_W = \left(\frac{\pi \alpha_{em}}{\sqrt{2} G_F}\right)^{1/2} \frac{1}{\sin\vartheta_W}$, $\boxed{m_W = m_Z \cos\vartheta_W}$



$$\boxed{M_W = M_W^0 + a M_t^2 + b \ln\left(\frac{M_H}{M_W}\right)}$$



2.1) Hunting the Higgs boson at LEP

H decays predominantly to heaviest particles which are energetically allowed

$M_H < 2M_W$: $H \to b\bar{b}$, $\overbrace{H \to \tau^+\tau^-}^{10\%}$

$2m_W < M_H < 2M_t$: $H \to W^+W^-$, $H \to ZZ$

$M_H > 2M_t$: $H \to W^+W^-$, $H \to ZZ$, $H \to t\bar{t}$

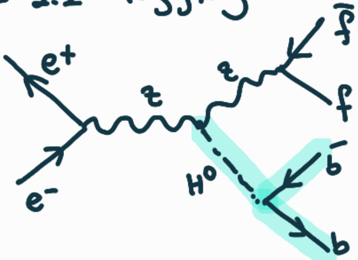
At LEP , $\sqrt{s}$ up to 207 GeV

↓

$M_H < 207\,\text{GeV} - M_Z$ ↙ 90

$M_H < 116\,\text{GeV}$

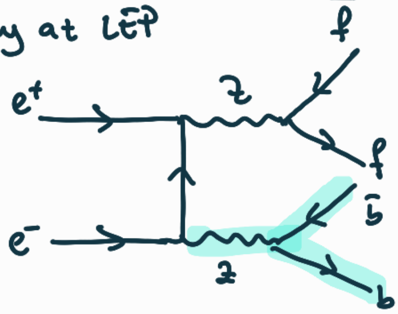
2.2 Tagging the H boson decay at LEP



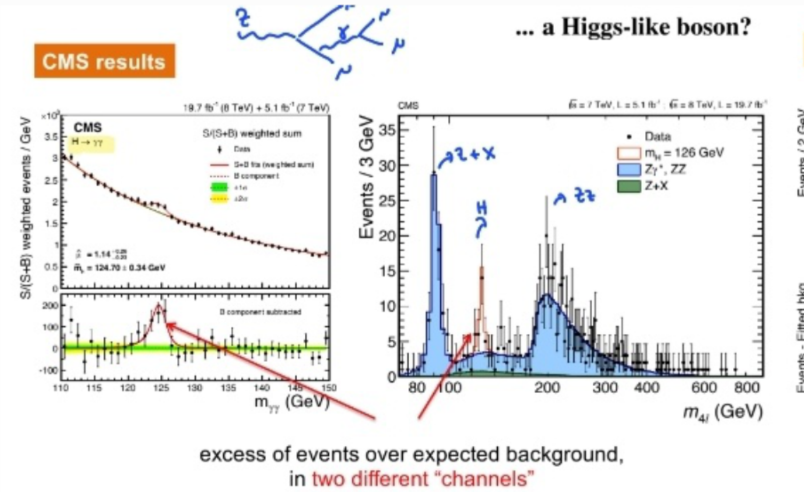
115 GeV

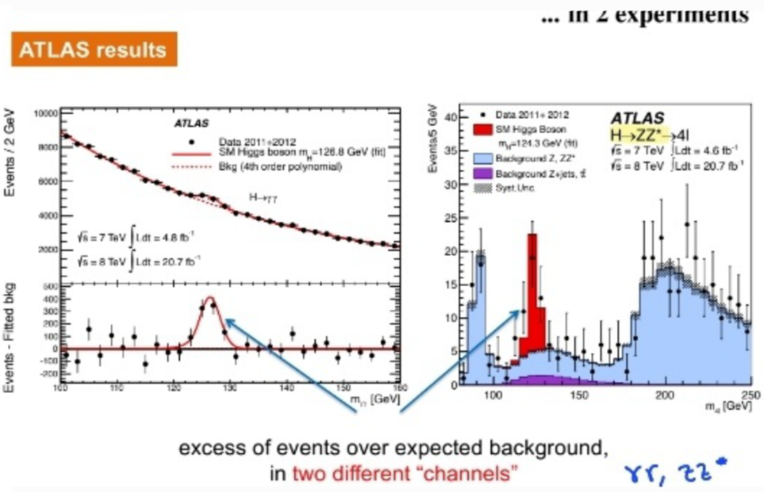
?

$M_{b\bar{b}} = M_H$



$M_{b\bar{b}} = m_Z$





11. HIGGS MECHANISM

1) Interacting Scalar fields

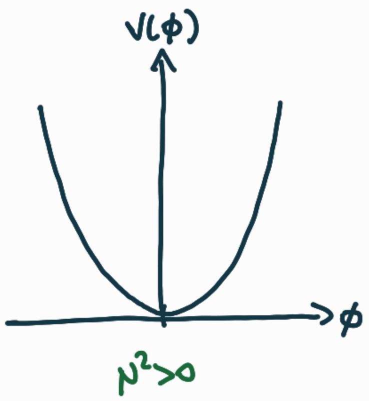
Free 1D scalar field

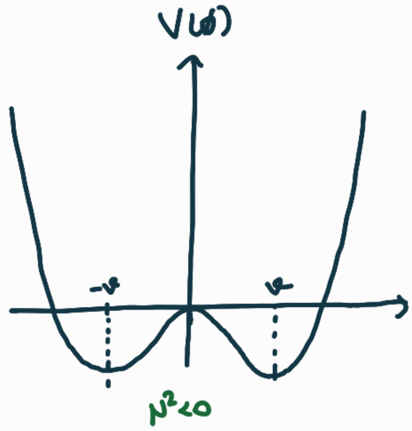
$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)$$

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi)}_{\text{kinetic energy}} - \underbrace{\frac{1}{2}\mu^2\phi^2}_{\text{mass of the particle}} - \underbrace{\frac{1}{4}\lambda\phi^4}_{\text{self-interaction of the scalar field}}$$





Vacuum state when $\phi = 0$

Scalar particle mass μ

Self interaction term ϕ^4

! Spontaneous symmetry breaking

To understand particle interactions

$\downarrow$

$$\phi(x) = v + \eta(x)$$

find excitations around its minimum

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \underbrace{\lambda v^2 \eta^2}_{\text{mass term}} - \underbrace{\lambda v \eta^3 - \frac{1}{4}\lambda\eta^4}_{\text{interaction vertices}} + \frac{1}{4}\lambda v^4$$

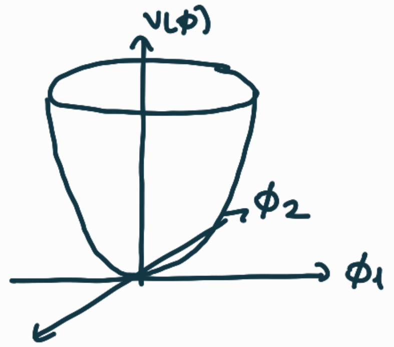
$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

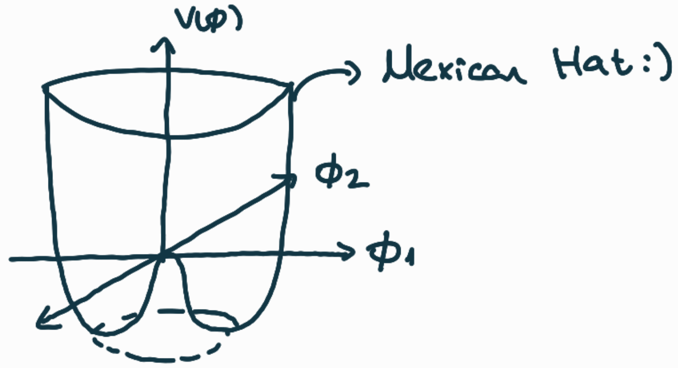
2) Symmetry breaking for a complex scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \frac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2)}_{\text{kinetic energy}} - \overbrace{\frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2)}^{\text{mass term}} - \overbrace{\frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2}^{\text{interaction}}$$

The Lagrangian is invariant under $\phi \to \phi' = e^{i\alpha}\phi$ (Global U(1) symmetry)





Again expand field around the vacuum state

$$\phi_1(x) = \eta(x) + v \quad \text{and} \quad \phi_2(x) = \xi(x) \Rightarrow \phi = \frac{1}{\sqrt{2}}(\eta + v + i\xi)$$

———— o ————

Local gauge symmetry $\phi(x) \to \phi'(x) = e^{ig\chi(x)}\phi(x)$

Fix, replace $\partial_\mu \to D_\mu = \partial_\mu + igB_\mu$

⋮

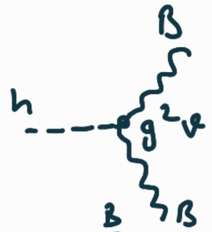
physical field in the unitary gauge

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) \equiv \frac{1}{\sqrt{2}}(v + h(x))$$

———— o ————

$$\mathcal{L} = \overbrace{\frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2}^{\text{massive } h \text{ scalar}} \overbrace{- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2v^2 B_\mu B^\mu}^{\text{massive gauge boson}}$$

$$\underbrace{+ g^2 v B_\mu B^\mu h + \frac{1}{2}g^2 B_\mu B^\mu h^2}_{h,B \text{ interaction}} \underbrace{- \lambda v h^3 - \frac{1}{4}\lambda h^4}_{h \text{ self interactions}}$$



Mass of gauge boson $m_B = gv$, $m_H = \sqrt{2\lambda}v$

12. HIGGS DECAY

1) Fermion Masses

Higgs also gives masses not only to W and Z boson but also to fermions.

$$-m\bar{\Psi}\Psi = -m(\bar{\Psi}_R\Psi_L + \bar{\Psi}_L\Psi_R)$$

It is not symmetric under $SU(2)_L \times U(1)_Y$ gauge symmetry

(can not be in the SM lagrangian) → WHY?

Thus fermion fields are introduced to the SM as massless states

2) Transformation of left doublet

LH chiral fermions → SU(2) doublets (L)

RH chiral fermions → SU(2) singlets (R)

$$\phi \rightarrow \phi' = (I + ig_W\vec{\varepsilon}(x)\cdot\vec{T})\phi$$

For $\bar{L} \rightarrow \bar{L}' = \bar{L}(I - ig_W\vec{\varepsilon}\cdot\vec{T})$

$\bar{L}\phi$ is invariant under $SU(2)_L$ gauge

* $\bar{L}\phi R$ ⇒ invariant under $SU(2)_L$ and $U(1)_Y$ gauge transformation

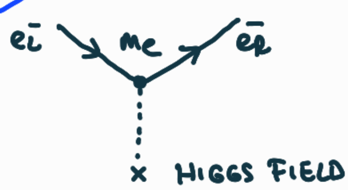
$$-g_f(\bar{L}\phi R + \bar{R}\phi^{+}L)$$

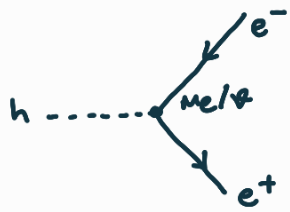
2.1) Electron Case

$$\mathcal{L}_e = -g_e\left[(\bar{\nu}_e\ \bar{e})_L\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}e_R + \bar{e}_R(\phi^{+*}\ \phi^{0*})\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right]$$

$$\mathcal{L}_e = -\frac{g_e}{\sqrt{2}}v(\bar{e}_L e_R + \bar{e}_R e_L) - \frac{g_e}{\sqrt{2}}h(\bar{e}_L e_R + \bar{e}_R e_L)$$

$$\mathcal{L}_e = -m_e\bar{e}e - \frac{m_e}{v}\bar{e}eh$$





2.2) Up-type fermions

$$\mathcal{L} = g_f\left[\bar{L}\phi_c R + \bar{R}\phi_c^{+}L\right]$$

$$\mathcal{L}_u = -m_u\bar{u}u - \frac{m_u}{v}\bar{u}uh$$

$$g_f = \sqrt{2}\frac{m_f}{v}$$

Yukawa Coupling

for top quark $g_f = 1$

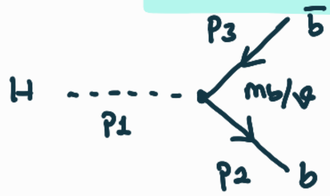
for neutrinos $g_f < 10^{-12}$

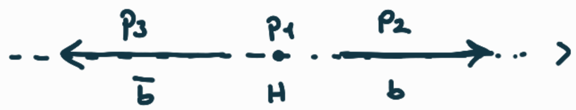
3) Higgs boson decays

Largest BF is to $b\bar{b}$: $B(H \rightarrow b\bar{b}) = 57.8\%$

* H is scalar → no polarization 4-vector is needed

$$\boxed{\mathcal{M} = \frac{m_b}{v}\bar{u}(p_2)v(p_3) = \frac{m_b}{v}u^{+}\gamma^0 v}$$





$p_2 \approx (E,0,0,E)$ and $p_3 \approx (E,0,0,-E)$

where $E = M_H/2$

$u_\uparrow(p_2), u_\downarrow(p_2)$ $\quad v_\downarrow(p_3), v_\uparrow(p_3)$

For the ultrarelativistic limit the spinors for two possible helicity state of the b

(

Only 2 of the 4 helicity combinations give non-0 matrix element $\Rightarrow \mathcal{M}_{\uparrow\uparrow} = -\mathcal{M}_{\downarrow\downarrow} = \frac{m_b}{v} 2E$

★ No angular dependence

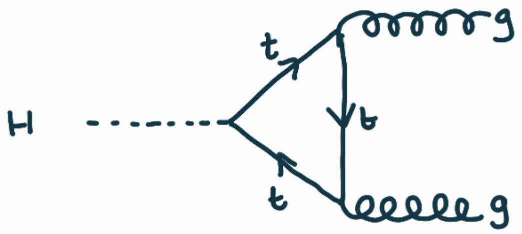
Spin-averaged ME: $\langle|\mathcal{M}|^2\rangle = |\mathcal{M}_{\uparrow\uparrow}|^2 + |\mathcal{M}_{\downarrow\downarrow}|^2 = \frac{m_b^2}{v^2} 8E^2 = \frac{2m_b^2 m_H^2}{v^2}$

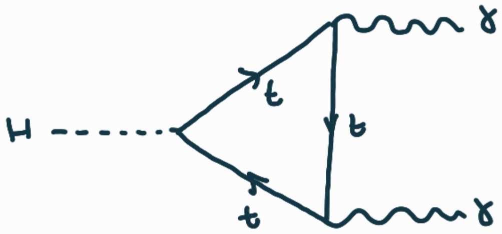
Partial decay width: $\Gamma(H \to b\bar{b}) = 3 \times \frac{m_b^2 m_H}{8\pi v^2}$

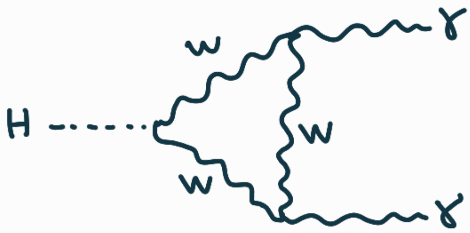
from 3 colors

$$\Gamma(H \to b\bar{b}) : \Gamma(H \to c\bar{c}) : \Gamma(H \to \tau^+\tau^-) \approx 3m_b^2 : 3m_c^2 : m_\tau^2$$

★ Decay to massless particle γ and g happen via loops :







13. DARK MATTER

1) Dark Matter Indications

- * Rotation speed of stars
- Gravational lensing
- Anisotropies in CMB (Cosmic Microwave Background)
- Bullet Cluster (not much depend on Newtonian gravity)

Non beyond SM

2) Non - BSM candidates

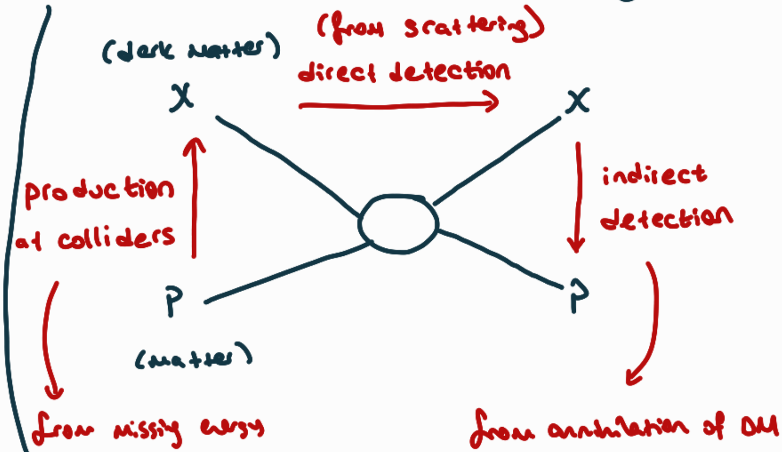
↳ 2.1) Modified Newtonian Dynamics (MOND)

↳ 2.2) Massive astrophysical compact halo objects (MACHOS)

↳ SM Neutrinos and Dark Matter

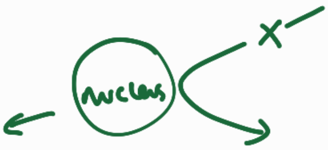
Cold DM

3) Different Detection Strategies



3.1) Direct Detection

Measure the nuclear recoil after "WIMP-nucleus" elastic scattering



$R \approx n \sigma \langle v \rangle / M_N$

- M_N → mass of target nucleus
- n → density of WIMPs (weakly interacting massive particle)
- σ → elastic-scattering cross-section
- $\langle v \rangle$ → speed of WIMPs