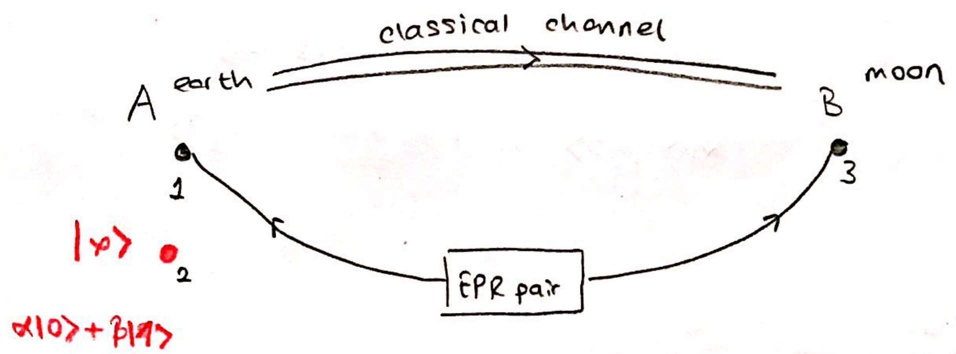
Teleportation Protocol



$$|B_{00}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$$

Goal: Teleport state $|\varphi\rangle$ into Bob's lab.

Using 2 classical bits of information.

Final situation:

$|B_{ij}\rangle_{12}$ for some (ij)

Global state initially is $\frac{1}{\sqrt{2}}|B_{00}\rangle_{13} \otimes |\varphi\rangle_2$

1) Alice does a measurement in the Bell basis.

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \qquad |B_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$|B_{10}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right) \qquad |B_{11}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

Outcome in her lab is some $|B_{ij}\rangle$ for some (ij) with prob. $\frac{1}{4}$

ket with Alice → projection

Let's compute global state: $\left(|B_{ij}\rangle_{12}\langle B_{ij}|_{12} \otimes \mathbb{I}_3\right)\frac{1}{\sqrt{2}}|B_{00}\rangle_{13} \otimes |\varphi\rangle_2$

Compute for $\underline{ij=00}$: $\frac{1}{2}\left(\langle 00|_{12} + \langle 11|_{12}\right)\left(|00\rangle_{13} + |11\rangle_{13}\right) \otimes \left(\alpha|0\rangle_2 + \beta|1\rangle_2\right)$

$$= \frac{1}{2}\left(\alpha|0\rangle_3 + \beta|1\rangle_3\right) = \frac{1}{2}|\varphi\rangle_3 \ ✓$$

$A \to$ 00 $\longrightarrow$ $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$

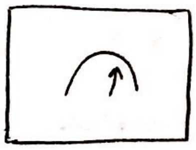
01 $\longrightarrow$ $\alpha|1\rangle - \beta|0\rangle$ $\leftarrow$ Bob applies X_3

11 $\longrightarrow$ $\alpha|0\rangle - \beta|1\rangle$ $\leftarrow$ Bob applies Z_3

10 $\longrightarrow$ $\alpha|1\rangle - \beta|0\rangle$ $\leftarrow$ Bob applies $Z_3 X_3$

Remark on Measurement Principle

$P_i = |\varphi_i\rangle\langle\varphi_i|$ projectors on some orthonormal basis $|\varphi_1\rangle, \ldots, |\varphi_k\rangle$

$|\psi\rangle \longrightarrow$  $\longrightarrow \dfrac{P_1|\psi\rangle}{\| P_1|\psi\rangle \|} = |\varphi_1\rangle, \ \ldots, \ \dfrac{P_k|\psi\rangle}{\| P_k|\psi\rangle \|}$

what is the probability of outcome?

$$\overbrace{|\varphi\rangle\langle\varphi|\varphi\rangle\langle\varphi|}^{1} \qquad \boxed{P^2 = P}$$

$$\left| \langle \varphi_i|\psi\rangle \right|^2 = \langle \varphi_i|\psi\rangle^* \langle \varphi_i|\psi\rangle$$

$$= \langle \psi|\varphi_i\rangle\langle\varphi_i|\psi\rangle = \langle\psi|P_i|\psi\rangle$$

$$= \langle\psi|P_i^2|\psi\rangle = (\langle\psi|P_i)(P_i|\psi\rangle)$$

$$= \big\| P_i|\psi\rangle \big\|^2$$

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