Example questions oral exam Solid State Systems for Quantum Information – Spring 2023

The exam will consist of a 30 min timeslot.
During the first 10-15 min of the exam, you will be provided with an exercise similar to the one solved during the course. This will be the starting point of the discussion that we will have in the remaining 15-20 min of the exam.
The questions will focus on one main topic that is randomly selected from a list of numbered questions. The main questions are all based on the material covered in the lectures. The lecture slides will be available, and additionally we provide background material to help you understand the concepts presented in the lectures. We will not ask questions about concepts covered in the background material which has not been covered in the lectures.
We may additionally ask broad questions (no detailed questions, you don't need to read those papers) on the papers presented in the student presentations. We will keep in mind how clear the student presentations were so that we won't questions you cannot reasonably be expected to answer. The student presentations will also be made available to you.

Here is the list of questions we will start with. If you answer well, we will probe your understanding or knowledge in more depth. For instance, when asked what are the limiting decoherence mechanisms for a specific qubit type, after you list the mechanisms, we may ask you to briefly explain the mechanisms. Or we may ask for realistic numbers, such as typical qubit energy splitting, or physical dimensions.

1) Introduction to quantum computing
a. How a quantum computer differs conceptually from a classical computer?
b. What are the reversible but gates for a classical computer? And for a quantum computer?
c. What is a universal gate set and cite one of it for a quantum computer?
d. Describe a single qubit state on the Block sphere and how the most common single qubit gate operations act on it.
e. Example of a single spin in a magnetic field. Larmor precession.
f. Two-qubit gates. Entangling gates. How they can be physically implemented?
g. Rabi oscillations.
h. Conceptually describe qubit dephasing, relaxation and how to measure them.

2) Superconducting qubits
a. Conceptually describe the transmon qubit, the role of its components and the relevant energy scales.
b. How are modern superconducting made, and what do they look like?
c. How is single-qubit control of transmon qubits achieved (three axis)?
d. What is the most straightforward way of coupling two superconducting qubits?
e. What is the Hamiltonian of a superconducting qubit plus resonator system? Which roles do microwave resonators play in superconducting qubit experiments, and which operating regimes do we distinguish?
f. Explain the concept of the vacuum Rabi splitting and vacuum Rabi oscillations.
g. Describe qubit readout in the dispersive regime. How is frequency multiplexing used to read out multiple qubits through a single feedline?
h. How can qubits be coupled together in the dispersive regime, and two-qubit gates implemented? What are the relevant energy scales?
i. How can additional energy levels be used to implement a CPhase gate?

j. What effect does charge noise have on the qubit properties and what steps have been taken to reduce those effects?
k. What our noise sources affect superconducting qubits?
l. Describe the scaling challenges.

3) Quantum dots
a. How can individual electrons be trapped in semiconductor quantum dots? What are some of the common materials platforms?
b. Explain how the state of a single electron spin in a quantum dot can be read out. What experimental conditions need to be met for the read-out to achieve high fidelity?
c. Explain two methods for the coherent control of a single electron spin in a quantum dot. Discuss the advantages and disadvantages of both. Bonus: explain a third method.
d. Explain two types of two-qubit gates between single-electron spin qubits in quantum dots. Discuss the advantages and disadvantages of both. Bonus: explain a third method.
e. What are the limiting decoherence mechanisms for single-spin qubits in quantum dots? How do they impact the fidelity of single-shot readout and of single- and two-qubit gates? How did the timescales and limiting mechanisms evolve over the years? To what extent can dynamical decoupling techniques extend the coherence times? What does this tell us about the decoherence mechanisms?
f. What are the limiting energy relaxation mechanisms for single-spin qubits in quantum dots? How do they impact the fidelity of single-shot readout and of single- and two-qubit gates? How did the timescales and limiting mechanisms evolve over the years?
g. What are the main challenges for scaling up spin qubits in quantum dots? What main ideas exist for overcoming these challenges?

4) Topological qubits
a. What ingredients (materials systems and their properties) are needed to obtain Majorana quasi-particles in a 1D semiconductor?
b. Intuitively, what is the reason for topological protection of Majorana quasiparticles? What makes it hard to achieve this protection in real devices?
c. How is the particle density of states probed in experiment, showing possible signatures of Majorana's? What are the signatures people have been looking for? What pitfalls exist in doing such measurements?
d. What other trends exist for realizing Majorana's besides 1D semiconductors?

1) Introduction to quantum computing

a. How a quantum computer differs conceptually from a classical computer?

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c. What is a universal gate set and cite one of it for a quantum computer?

d. Describe a single qubit state on the Block sphere and how the most common single qubit gate operations act on it.

e. Example of a single spin in a magnetic field. Larmor precession.

f. Two-qubit gates. Entangling gates. How they can be physically implemented?

g. Rabi oscillations.

h. Conceptually describe qubit dephasing, relaxation and how to measure them.

A)
- Bit → Qubit represent 0 and 1 simultaneously, speed↑
- Entanglement (qubit correlation)
- Similar to classical, there're gates
- More efficient such as Shor Algorithm (factoring large numbers), Grover's Search Algorithm
- No-cloning : used in encryption.

* Devise algorithm with O(poly(n))

B) Reversible Gate: UNIQUE INPUT associated with UNIQUE OUTPUT

Unitary

Classical | **Quantum**

Identity Gate (I)

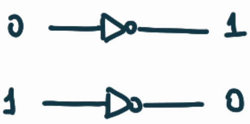
Free Evolution $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(Irreversible) Erase Gate

Relaxation

IN	OUT
0	0
1	0

(Reversible) NOT Gate



X Gate (Reversible)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(Irreversible)

AND Gate OR Gate

(Irreversible) XOR Gate 

00	0
01	1
10	1
11	0

CNOT Gate (REVERSIBLE)

00	00
01	01
10	11
11	10

Reversible Classical Gates:

Toffoli (CCNOT) : Takes 3 inputs, flips 3rd input only if 1st & 2nd input = 1

Fredkin (CSWAP) : Takes 3 input, replace 2nd & 3rd if 1st = 1

* Quantum gates can be irreversible such as measurement gates, otherwise rotational

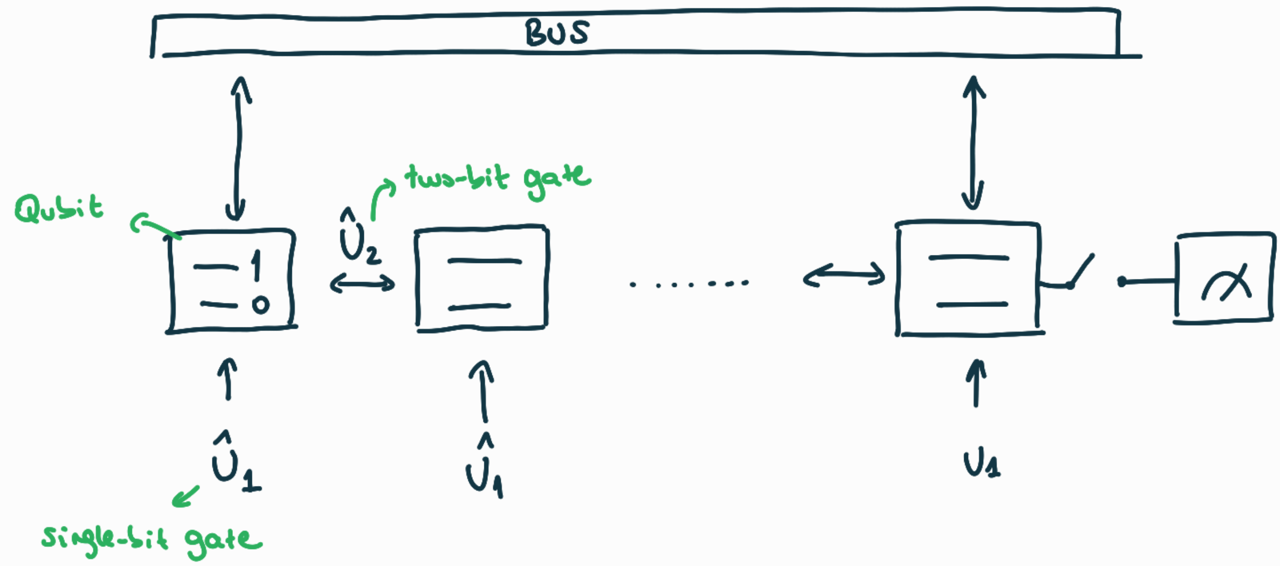
c) Universal Gate Set: {AND, NOT}, {OR, NOT}, {NAND} ★★

For Quantum (QC): $\{\underbrace{H, S, CNOT}_{\text{Clifford Group}}, T\}$

$\{R_x(\theta), R_y(\theta), R_z(\theta), CNOT\}$

Gottesman-Knill Theorem: Any quantum circuit made only of Clifford gates can be simulated efficiently on a classical computer.

A Generic Quantum Processor



1) Quantum bits [scalable]

2) Initialization of the qubit register

3) Coherence (>> gate time)

4) Set of universal gates

5) Read-out

Di Vincenzo Criteria

6) Interconvert stationary and flying qubits

7) Transmit flying qubits between specified locations

Networkability

D)

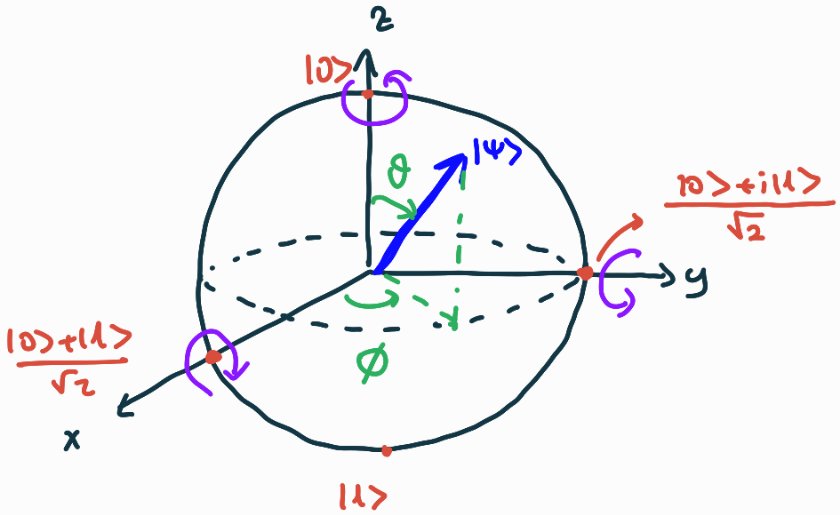
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$= e^{i\gamma}\left[\cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle\right]$$

γ: global phase

θ: polar angle

φ: azimuth angle



Rotation Operators

$$R_x(\theta) = e^{-i\theta X/2}$$

X → Pauli X : σ_x

$$R_y(\theta) = e^{-i\theta Y/2}$$

$$R_z(\theta) = e^{-i\theta Z/2}$$

Z-Y Decomposition

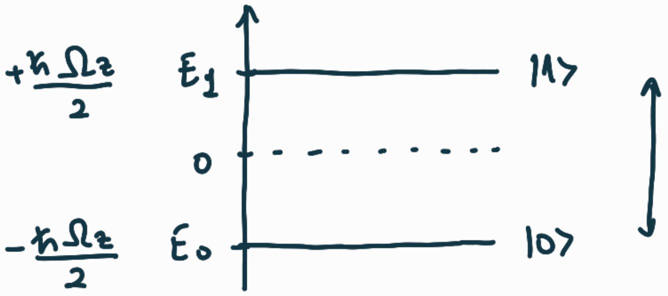
Any single qubit gate U can be rewritten as:

$$U = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$$

Ex. $R_x(\theta) = R_y\left(\frac{\pi}{2}\right) R_z(\theta) R_y\left(-\frac{\pi}{2}\right)$

E) Single electron spin in a magnetic B_z field.

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

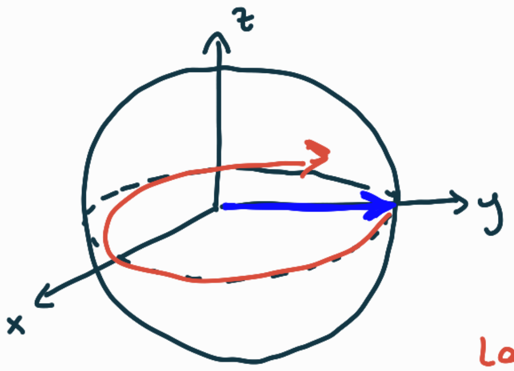


$\Delta E = \hbar\Omega_z = g\mu_B B_z$

$$\left(\vec{\mu} = \gamma\frac{\hbar}{2}\vec{S}, \quad E = -\vec{\mu}\cdot\vec{B}, \quad \Rightarrow \hat{H} = -\gamma\frac{\hbar}{2}\vec{S}\cdot\vec{B} = \underbrace{-\frac{\gamma\hbar}{2}B_z}_{-\frac{\hbar\Omega_z}{2}}\sigma_z \right)$$

intrinsic magnetic moment

If
$$|\psi(0)\rangle = |0\rangle \longrightarrow |\psi(t)\rangle = e^{+i\frac{\Omega_z t}{2}}|0\rangle$$
$$|\psi(0)\rangle = |1\rangle \longrightarrow |\psi(t)\rangle = e^{-i\frac{\Omega_z t}{2}}|1\rangle$$
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \longrightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}}e^{\frac{i\Omega_z t}{2}}\left(|0\rangle + e^{-i\Omega_z t}|1\rangle\right)$$



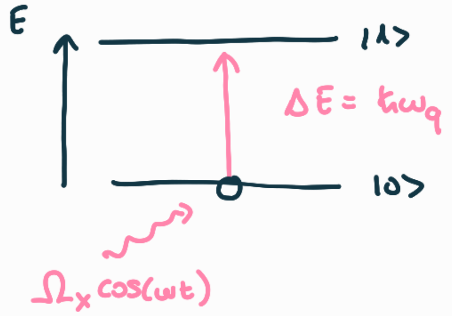
$$|\psi\rangle = e^{i\gamma}\left(\cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$$

where $\theta = \frac{\pi}{2}$ $\quad \varphi = -\Omega_z t$

Larmor Precession with frequency Ω_z

$g\mu_B B_z$

Control of single-qubit gates



Qubit hamiltonian with ac-drive:

$$H = \hbar\left[\frac{\omega_q}{2}\hat{\sigma}_z + \Omega_x\cos(\omega t)\hat{\sigma}_x + \Omega_y\cos(\omega t)\hat{\sigma}_y\right]$$

Rotating Wave Approximation (RWA)

driving field freq

$$H_{rot} = UHU^{+} - i\dot{U}U^{+} \quad \text{where} \quad U_{rot} = e^{+i\omega_d H_0 t}$$

$$H_{rot} \simeq \frac{\hbar}{2}\left[\Delta\hat{\sigma}_z + \Omega_x\hat{\sigma}_x + \Omega_y\hat{\sigma}_y\right]$$

drop fast rotating terms

Δ = detuning = $\omega - \omega_q$

* ac field control allows to realize arbitrary single qubit rotations

X_π pulse = $\Omega_x t = \pi$ $\qquad |0\rangle \rightarrow \boxed{X_\pi} \rightarrow |1\rangle$

Y_π pulse = $\Omega_y t = \pi$ $\qquad |0\rangle \rightarrow \boxed{Y_\pi} \rightarrow -i|1\rangle$

$X_{\frac{\pi}{2}}$ pulse = $\Omega_x t = \frac{\pi}{2}$ $\qquad |0\rangle \rightarrow \boxed{X_{\pi/2}} \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

F) Two-Qubit Gate

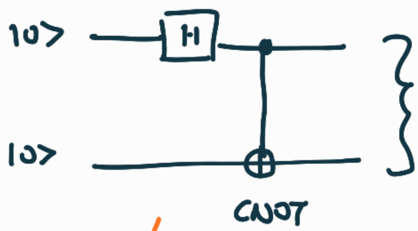
$\boxed{\text{CNOT}}$

$|00\rangle \rightarrow |00\rangle$

$|01\rangle \rightarrow |01\rangle$

$|10\rangle \rightarrow |11\rangle$

$|11\rangle \rightarrow |10\rangle$



$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$ → BELL STATE

to create entanglement

Physical Implementation of CNOT:

Using Ising Interaction: $H = -\sum_{ij} J_{ij} \hat{\sigma}_z^i \hat{\sigma}_z^j$ pairwise spin interaction

$$C(\gamma) = e^{-i \frac{\gamma}{2} \hat{\sigma}_{z1} \hat{\sigma}_{z2}}$$

↙

2-qubit unitary evolution

↓ Not a CNOT yet

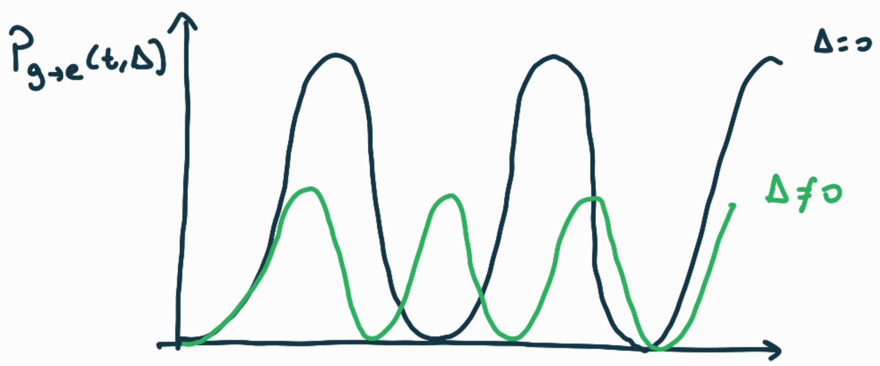
$$\mathrm{CNOT} = e^{-i\frac{3\pi}{4}} \underline{R_{x_2}\left(\tfrac{3\pi}{2}\right)}\, \underline{C\left(\tfrac{3\pi}{2}\right)}\, \underbrace{R_{z_2}\left(\tfrac{\pi}{2}\right) R_{x_2}\left(\tfrac{\pi}{2}\right) R_{z_2}\left(\tfrac{\pi}{2}\right) R_{z_1}\left(\tfrac{\pi}{2}\right)}\, \underline{C\left(\tfrac{3\pi}{2}\right)}$$

Other 2 qubit gates → C-Phase (CZ), SWAP

★ Any physical 2-qubit interaction that can generate entanglement can be turned into a universal 2-qubit gate (such as CNOT) when it's augmented by arbitrary single qubit operations.

G) Rabi Oscillations

Periodic exchange of energy between two quantum states under the influence of an oscillating electromagnetic field.

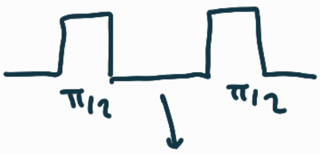


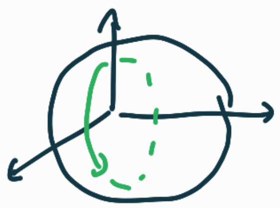
H) Qubit Dephasing, Relaxation, How to measure them

loss of coherence or decay of the quantum superposition

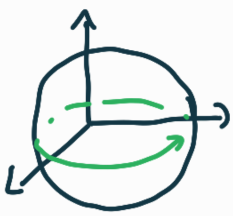
Relaxation, desire to be in the lower energy state

* Ramsey Interferometry
* Spin Echo
* Time-Resolved Measurements
* Quantum State Tomography





T1: longitudinal relax.



T2: transversal relaxation

2) Superconducting qubits

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d. What is the most straightforward way of coupling two superconducting qubits?

e. What is the Hamiltonian of a superconducting qubit plus resonator system? Which roles do microwave resonators play in superconducting qubit → cavity experiments, and which operating regimes do we distinguish? → resonance, dispersive

f. Explain the concept of the vacuum Rabi splitting and vacuum Rabi oscillations.

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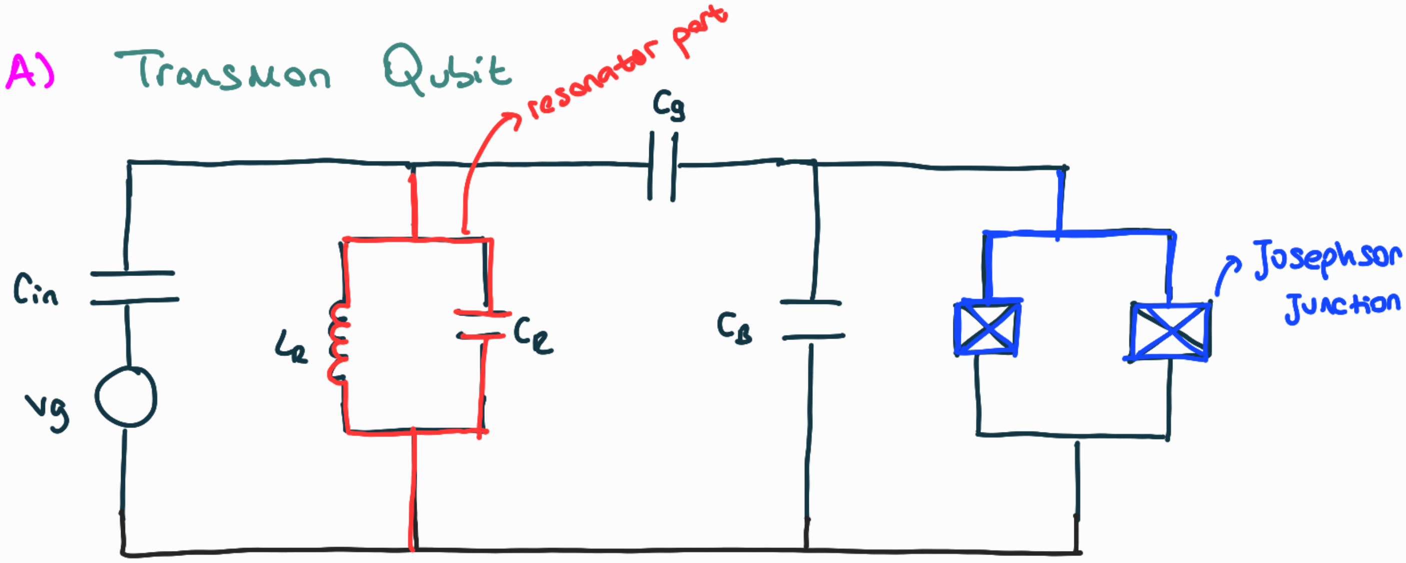
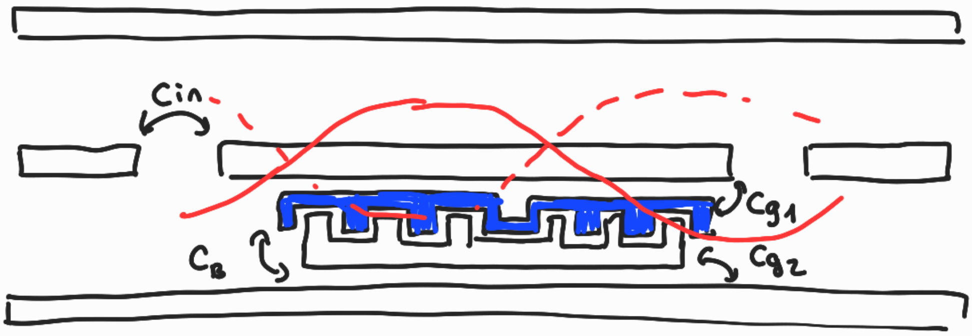
l. Describe the scaling challenges.

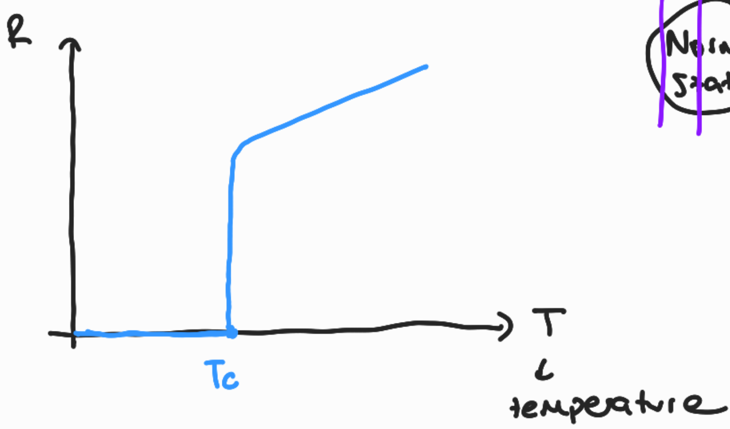
A) Transmon Qubit

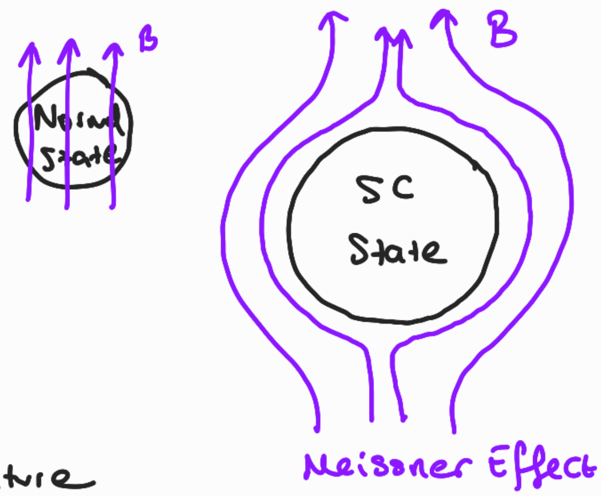




Before deep dive to the questions, initially review the basic concepts

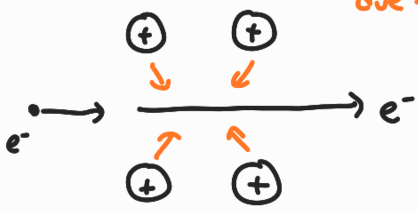
1) Superconductivity





1.1) BCS Theory

How can electrons attract each other?



due to electron-phonon interaction
↓
Lattice vibration

creates attractive interaction between 2 electrons

cooper pair with 2 e's in opposite spin and momentum

Total spin $S=0$

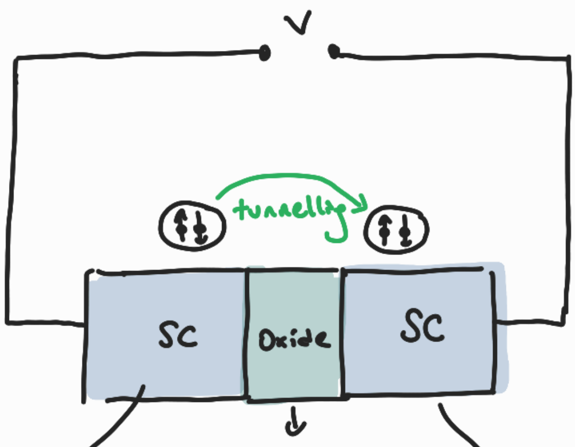
★ The superconducting state can be described by a single macroscopic wavefunction

$$\Psi = |\Psi| e^{i\phi}$$

$|\Psi|^2 = n_s$ (density of cooper pairs)

2. Josephson Junction

2.1) Josephson Effect



Gap

$$\Psi_1 = \sqrt{\frac{n_{s1}}{2}}\, e^{i\phi_1} \qquad \Psi_2 = \sqrt{\frac{n_{s2}}{2}}\, e^{i\phi_2}$$

$$J = J_c \sin(\overbrace{\Delta\phi}^{\phi_1 - \phi_2})$$

If voltage is applied $\quad (\dot{\Delta\phi}) = \dfrac{2e}{\hbar} V \quad \rightsquigarrow$ AC Josephson Effect

$$\frac{\partial I_s}{\partial t} = J_c \frac{\partial \phi}{\partial t} \cos(\phi) = J_c \frac{2e}{\hbar} V \cos(\phi)$$

$$V = \underbrace{\frac{\hbar}{2e I_c \cos\phi}}_{L} \frac{\partial I_s}{\partial t} = L \frac{\partial I}{\partial t}$$

(Josephson)
Inductance

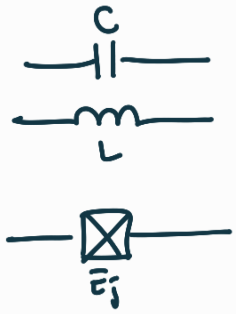
$$\Delta E = \int_{t_1}^{t_2} I(t) V(t)\, dt = \int_{t_1}^{t_2} I_c \sin\phi \left[\frac{\hbar}{2e} \dot{\phi}\right] dt$$

$$= -\frac{\Phi_0 I_c}{2\pi} \left[\cos\phi_2 - \cos\phi_1\right]$$

$$E = -E_j \cos\phi \qquad E_j = \frac{\Phi_0 I_c}{2\pi}$$

Josephson Energy

3) Quantum Electronic Circuit



Josephson Junction

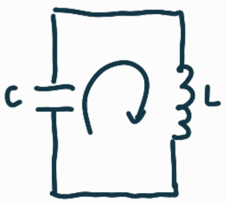
! No R (resistor)

Since it's dissipative

↓

Non-unitary evolution

LC Oscillator



$$\mathcal{E}_C = \frac{1}{2} C V_C^2 = \frac{1}{2} C \dot{\phi}^2$$

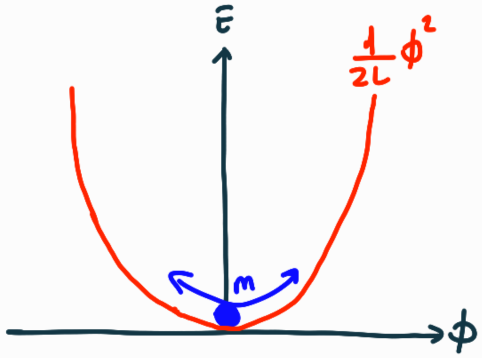
$$\mathcal{E}_L = \frac{1}{2} L I^2 = \frac{1}{2} \frac{\phi^2}{L}$$

$$E_{tot} = \underbrace{\frac{1}{2} C \dot{\phi}^2}_{E_{kin}} + \underbrace{\frac{1}{2L} \phi^2}_{E_{pot}}$$

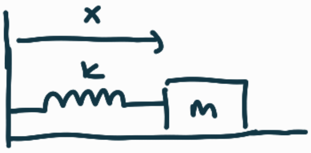
Kirchhoff Rule $V_C + V_L = 0$

$$V_L = -V_C$$

$$V_L = -L\dot{I} = -\dot{\phi} = -V_C \Rightarrow V_C = \dot{\phi}$$



similar to



$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$$

Lagrangian of the circuit

$$\mathcal{L} = K - U = \frac{1}{2} C \dot{\phi}^2 - \frac{1}{2L} \phi^2$$

Legendre Transformation $\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = C\dot{\phi} = CV = Q$

$$H = p\dot{\phi} - \mathcal{L} \Rightarrow H = Q\frac{Q}{C} - \left(\frac{1}{2} C\dot{\phi}^2 - \frac{1}{2L}\phi^2\right)$$

$$H = \frac{Q^2}{C} - \frac{1}{2} \cancel{C} \frac{Q^2}{C^{\cancel{2}}} + \frac{1}{2L}\phi^2$$

$$\boxed{\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{1}{2L}\hat{\phi}^2}$$

Quantum LC Circuit

$$\hat{H}(\hat{Q},\hat{\Phi}) = \frac{1}{2}\frac{\hat{Q}^2}{C} + \frac{1}{2}\frac{\hat{\Phi}^2}{L}$$

$$[\hat{\Phi},\hat{Q}] = i\hbar$$

$$\hat{Q} = -i\hbar\frac{\partial}{\partial\phi}$$

Mechanical Oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0\hat{x}^2$$

$$[\hat{x},\hat{p}] = i\hbar$$

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}$$

$$\boxed{\begin{aligned}\hat{Q} &= 2e\hat{n}\\ \hat{\Phi} &= \frac{\Phi_0}{2\pi}\hat{\varphi}\end{aligned}}$$

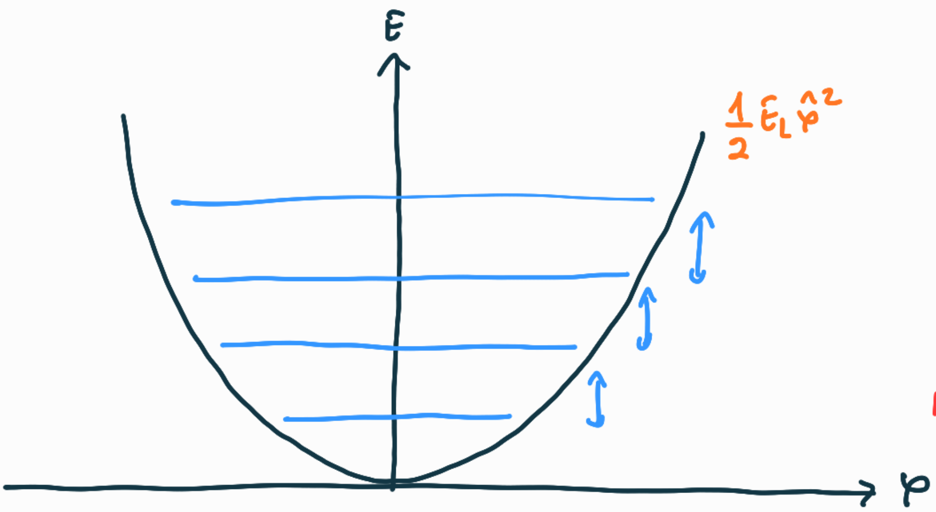
\# of cooper pairs

$\left(\Phi_0 = \frac{h}{2e}\right)$

Fourier transform

phase of the wavefunction

$$\hat{H} = \underbrace{4\frac{e^2}{2C}}_{E_C}\hat{n}^2 + \frac{1}{2}\underbrace{\frac{\Phi_0}{2\pi L}}_{E_L}\hat{\varphi}^2$$

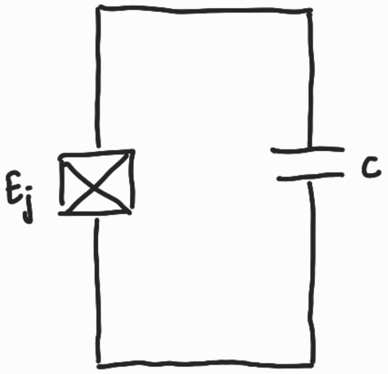


$$\omega_0 = \sqrt{8E_CE_L}/\hbar$$

$$E_n = \hbar\omega_0\left(n+\frac{1}{2}\right)$$

NOT A QUBIT YET!

Use non-linear inductance



$$L(\varphi) = \frac{\Phi_0}{2\pi}\frac{1}{I_c \cos\varphi} = L_0 \frac{1}{\cos\varphi}$$

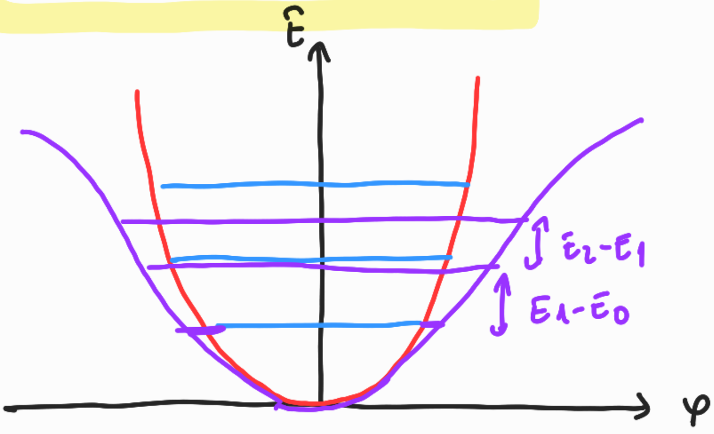
$$L_0 = \frac{\Phi_0}{2\pi I_c}$$

Energy of the Josephson Junction $= -E_j \cos\varphi$

$$= -\frac{\Phi_0 I_c}{2\pi}\left(1 - \frac{1}{2}\varphi^2 + \frac{1}{24}\varphi^4 + \dots\right) \approx \frac{1}{2}\frac{\Phi_0}{2\pi} I_c \varphi^2$$

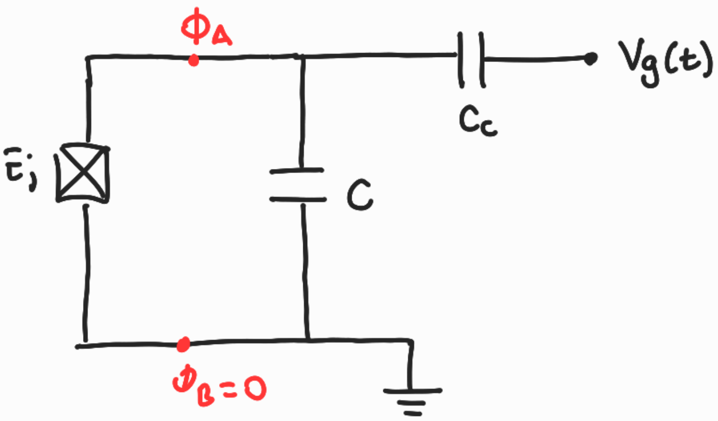
$$= \frac{1}{2}\left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_0}\varphi^2 = \frac{1}{2L_0}\left[\underbrace{\frac{\Phi_0}{2\pi}\varphi}_{\Phi_{eff}}\right]^2$$

$$\boxed{H = 4E_c n^2 - E_j \cos\phi}$$



$$E_2 - E_1 \neq E_1 - E_0$$

Then, the qubit is embedded into an environment



$$\mathcal{L} = \frac{1}{2} C \left[\dot{\phi}_A - \dot{\phi}_B\right]^2 + \frac{1}{2} C_c \left[\dot{\phi}_A - V_g\right]^2 - (-E_j \cos(\phi_A - \phi_B))$$

where $\phi_B = 0$, $\phi = \phi_A$

$$\mathcal{L} = \frac{1}{2}[C + C_c]\dot{\phi}^2 - C_c \dot{\phi} V_G + E_j \cos\phi$$

$$= \frac{1}{2}[C + C_c]\left[\dot{\phi} - \frac{C_c}{C + C_c} V_G\right]^2 + E_j \cos\phi$$

momentum

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = [C + C_c]\dot{\phi} - C_c V_G \Rightarrow \dot{\phi} = \frac{Q + C_c V_G}{C + C_c}$$

$$\mathcal{H} = Q\dot{\phi} - \mathcal{L} = Q\left[\frac{Q + C_c V_G}{C + C_c}\right]^2 - \frac{1}{2}[C + C_c]\left[\frac{Q}{C + C_c}\right]^2 - E_j \cos\phi$$

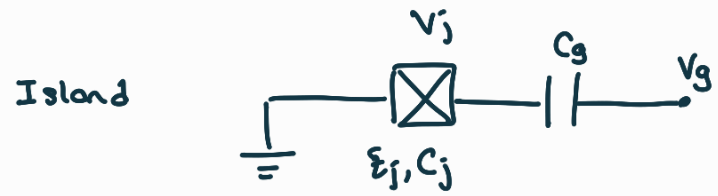
$$= \frac{1}{2}\frac{[Q - Q_g]^2}{C + C_c} - E_j \cos\phi$$

where $Q_g = -4 V_c V_G$

$Q = 2en$

$$\boxed{H = 4E_c(n - n_g)^2 - E_j \cos\phi}$$

4) Cooper Pair Box (Charged Qubit)



Smallness of the island : Coulomb blockade

Electrostatic Energy : $\dfrac{C_j V^2}{2} + \dfrac{C_g (V_g - V)^2}{2} \longrightarrow 4E_C (n - n_g)^2$

Total capacitance of the island: $C_\Sigma = C_J + C_G$

Characteristic Charging Energy : $E_C = \dfrac{e^2}{2C_\Sigma}$

$\dfrac{\hbar\dot{\varphi}}{(8E_C)}$

Number of the Cooper pair on island: $n = \dfrac{Q}{2e} = \dfrac{C_\Sigma V}{2e} = \dfrac{C_\Sigma \hbar \dot{\varphi}}{4e^2}$

? Dimensionless voltage on the gate electrode : $n_g = \dfrac{C_g V_g}{2e}$

System Lagrangian $\mathcal{L}(\varphi, \dot{\varphi}) = 4E_C\left(\dfrac{\hbar}{8E_C}\dot{\varphi} - n_g\right)^2 + E_j \cos\varphi$

Canonical momentum $p = \dfrac{\partial L}{\partial \dot{\varphi}} = \hbar(n - n_g)$

Then, Hamiltonian $\boxed{H(\varphi, p) = 4E_C (n - n_g)^2 - E_j \cos\varphi} = 4E_C \dfrac{p^2}{\hbar^2} - E_j \cos\varphi$

$\downarrow$

Quantize

Write this Hamiltonian in the charge basis: $\hat{n}|n\rangle = n|n\rangle$

Completeness Condition: $\boxed{\hat{n} = \sum_n n\,|n\rangle\langle n|}$ where $\sum_n |n\rangle\langle n| = \mathbb{I}$

$|\psi\rangle = \sum_N c_N |n\rangle$

$[\hat{\varphi}, \hat{n}] = i \rightarrow$ conjugate variable

$$\psi_{\vec{p}}(\vec{r}) = \langle \vec{r}|\vec{p}\rangle = e^{i\vec{r}\cdot\vec{p}/\hbar}$$

$|\varphi\rangle = e^{i\varphi}|1\rangle$

$$\langle \varphi|n\rangle = e^{in\varphi}$$

$$|\varphi\rangle = \sum_n |n\rangle\langle n|\varphi\rangle = \sum_n e^{in\varphi}|n\rangle$$

$\Big\downarrow$ Inverse Transformation

$$|n\rangle = \frac{1}{2\pi}\int d\varphi\, e^{in\varphi}|\varphi\rangle$$

$$|n+1\rangle = \frac{1}{2\pi}\int d\varphi\, e^{in\varphi}\cdot e^{i\varphi}|\varphi\rangle$$

$$|n+1\rangle = e^{i\varphi}|n\rangle$$

For $\cos\varphi$, one need to derive

$$|n\pm 1\rangle = e^{\pm i\varphi}|n\rangle$$

analogous to the finite-displacement operator

$\downarrow$

$$T_{\vec{a}} = \exp\left(\frac{i}{\hbar}\vec{a}\cdot\hat{\vec{p}}\right)$$

$$T_{\vec{a}}\,\psi(\vec{r}) = \psi(\vec{r}+\vec{a})$$

$$(e^{i\varphi} + e^{-i\varphi})|n\rangle = |n+1\rangle + |n-1\rangle$$

$$H = 4E_c(n-n_g)^2 - E_J\cos\varphi$$

$$(n-n_g)^2 = \left(\sum_n n\,|n\rangle\langle n| - n_g\right)^2$$

$$= \left(\sum_n n|n\rangle\langle n|\right)^2 - 2n_g\sum_n n|n\rangle\langle n| + n_g^2\overbrace{\sum_n |n\rangle\langle n|}^{\mathbb{I}}$$

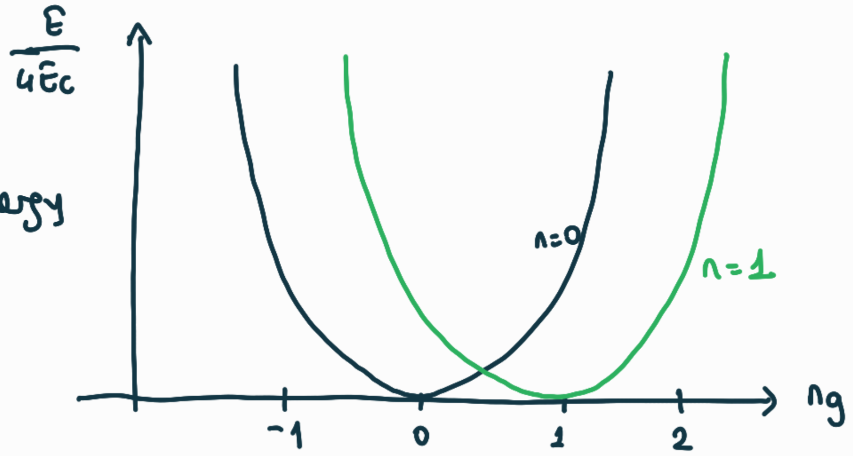
$$= \boxed{\sum_n (n-n_g)^2 |n\rangle\langle n|}$$

$$\cos\hat{\varphi} = \frac{1}{2}\left(\underbrace{e^{-i\varphi}}_{|n-1\rangle} + \underbrace{e^{+i\varphi}}_{|n+1\rangle}\right)\sum_n |n\rangle\langle n| = \boxed{\frac{1}{2}\sum_n \left(|n+1\rangle\langle n| + |n-1\rangle\langle n|\right)}$$

$$H = \sum_n \left\{ 4E_C (n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2}\left(|n+1\rangle\langle n| + \underbrace{|n-1\rangle\langle n|}_{|n\rangle\langle n+1|}\right)\right.$$

Charging Energy

$(E_J \ll E_C)$



In the 2-level approximation:

$$H = 4E_C \left\{ n_g^2 |0\rangle\langle 0| + (1-n_g)^2 |1\rangle\langle 1| \right\} - \frac{E_J}{2}\left\{ |0\rangle\langle 1| + |1\rangle\langle 0| \right\}$$

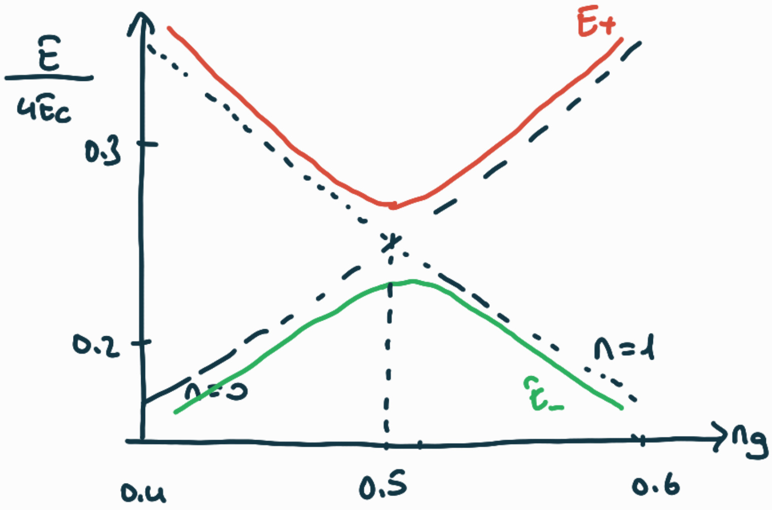
Using the completeness condition $|0\rangle\langle 0| + |1\rangle\langle 1| = \mathbb{I}$

$$H = -2E_C(1-2n_g)\{\underbrace{|0\rangle\langle 0| - |1\rangle\langle 1|}_{\sigma_z}\} - \frac{E_J}{2}\{\underbrace{|0\rangle\langle 1| + |1\rangle\langle 0|}_{\sigma_x}\}$$

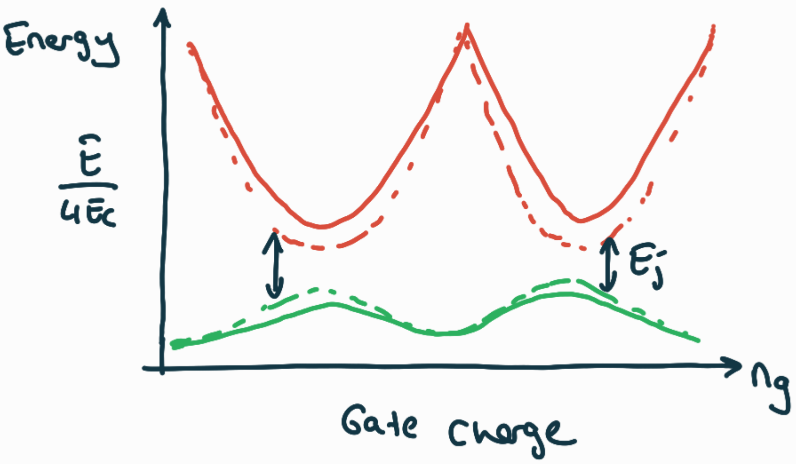
$$\boxed{H = -\frac{\Delta}{2}\sigma_x - \frac{\varepsilon}{2}\sigma_z} \quad ; \quad \Delta = E_J \quad \varepsilon = 4E_C(1-2n_g)$$

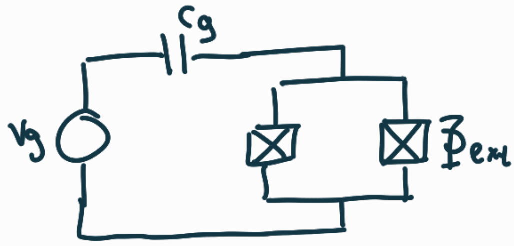
By diagonalizing this Hamiltonian

$$E_\pm = \pm\frac{1}{2}\sqrt{\Delta^2+\varepsilon^2} = \pm 2E_c\sqrt{(1-2n_g)^2+\left(\frac{E_j}{4E_c}\right)^2}$$



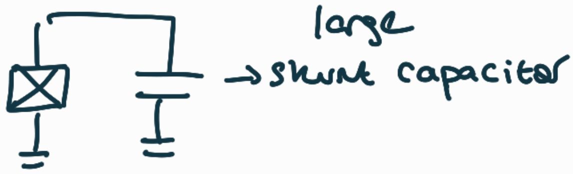
Tuning the Josephson Energy (additional junction)





$$E_j = E_{j,max}\cos\left(\pi\frac{\phi_{ext}}{\phi_0}\right)$$

5) Transmon Qubit



Previously where $E_J/E_C < 1$, n_g (gate charge) has a large impact on the transition frequency of the device.

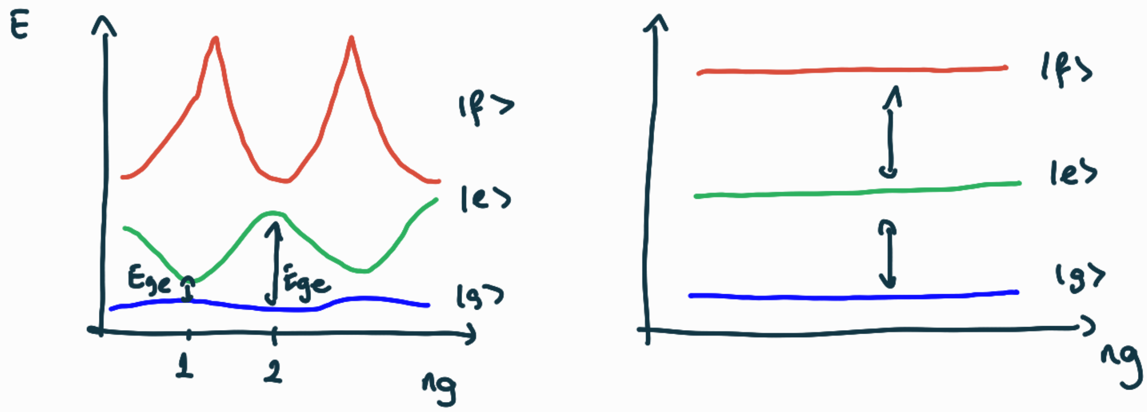
Unavoidable charge fluctuations yield dephasing.

To mitigate this problem, work in "transmon regime" where $E_J/E_C \sim 20-80$. Thus, the first energy levels of the device become essentially independent of gate charge.

* The charge dispersion (variation of the energy levels with gate charge) decreases exponentially with E_J/E_C in the transmon regime.

$$H_T = \underbrace{4E_C\hat{n}^2 + \frac{1}{2}E_J\hat{\varphi}^2}_{\text{LC harmonic oscillator}} - E_J\left(\cos\hat{\varphi} + \frac{1}{2}\hat{\varphi}^2\right)$$

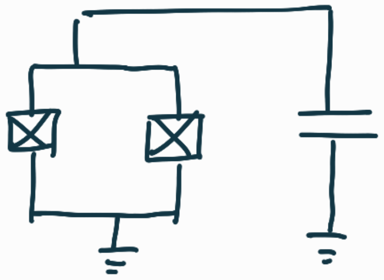
* A weakly anharmonic oscillator



Charge dispersion $\epsilon = E_g(n_g=1) - E_{ge}(n_g=2)$

5.1) Flux tunable Transmon

Use 2 JJ,



$$\hat{H}_T = 4E_C\hat{n}^2 - E_{J1}\cos\hat{\varphi}_1 - E_{J2}\cos\hat{\varphi}_2$$

$$\hat{H}_T = 4E_C\hat{n}^2 - E_J(\underset{}{\overset{\Phi_{ext}}{\Phi_x}})\cos(\hat{\varphi} - \hat{\varphi}_0)$$

where $E_j(\Phi_x) = E_{J\Sigma}\cos\left(\frac{\pi\Phi_x}{\Phi_0}\right)\sqrt{1+d^2\tan^2\left(\frac{\pi\Phi_x}{\Phi_0}\right)}$

$$\omega_q(\Phi_x) = \sqrt{8E_C E_J(\Phi_x)} - E_C$$

qubit transition frequency

Using an anharmonic oscillator as qubit

$$\hbar\omega = \sqrt{8E_j E_C}$$

$$H = \hbar\omega\hat{a}^+a - E_C\hat{a}^+\hat{a} - \frac{E_C}{2}\hat{a}^+\hat{a}^+\hat{a}\hat{a}$$

$$H = \hbar\omega_T\hat{a}^+\hat{a} - \frac{\hbar\alpha}{2}\hat{a}^+\hat{a}^+\hat{a}\hat{a}$$

$$\hbar\omega_T = \sqrt{8E_jE_c} - E_c$$

$E_1 - E_0 = \hbar\omega - E_c$ $\qquad$ $E_2 - E_1 = \hbar\omega - 2E_c$

$E_2 - E_1 \neq E_1 - E_0 \Rightarrow$ QUBIT

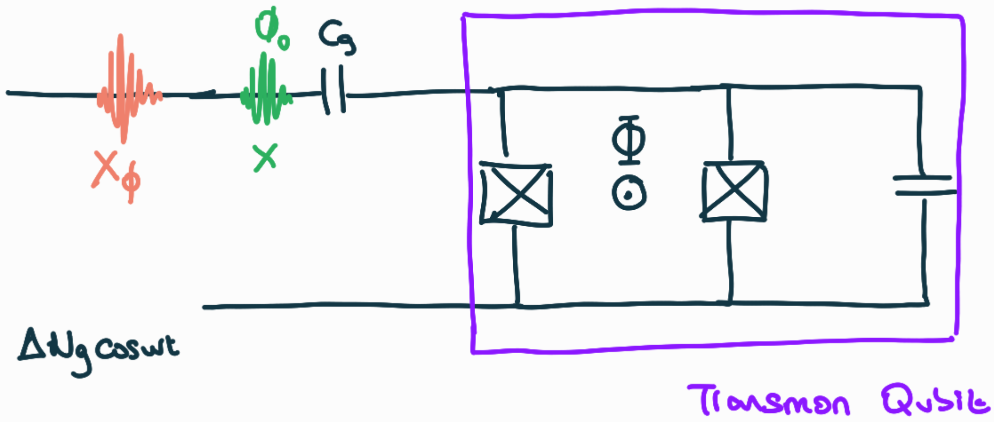
A) Transmon Qubit

Explained above.

Relevant energy scales: $\frac{E_C}{\hbar} \simeq 0.3$ GHz $\quad \frac{E_J}{\hbar} \simeq 30$ GHz

$\omega_q \sim$ several GHz

C) Single Qubit Drive

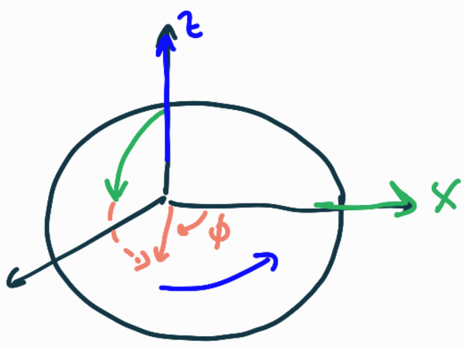


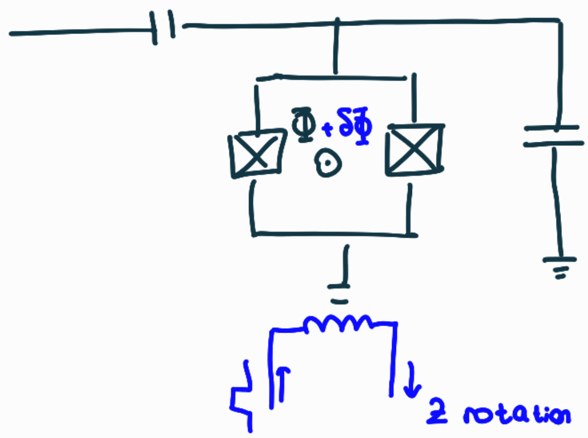
$N_g(t) = \Delta N_g \cos\omega t$

$$\hat{H} = E_C(\hat{N} - N_g(t))^2 - E_J \cos\hat{\theta}$$

$$\hat{H} = \underbrace{E_C(\hat{N}^2 - E_J \cos\hat{\theta})}_{\text{transmon}} - \underbrace{2E_C \Delta N_g \cos\omega t\, \hat{N}}_{\text{drive}}$$

$$\hat{H} = -\frac{\hbar\omega_{01}(\Phi)}{2}\hat{\sigma}_z + \hbar\Omega_R \cos(\omega t)\hat{\sigma}_x \rightarrow \text{"2 Level Approximation"}$$





6. Decoherence (Relaxation and Dephasing)

Relaxation Mechanism (T_1) → Ohmic losses

Dephasing mechanisms (T_2) → Magnetic flux noise, charge noise

Origin of noise:

1) ElectroMagnetic

Low frequency: Thermal Noise

High frequency: Spontaneous emission

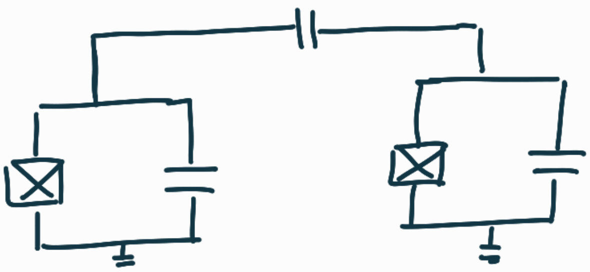
2) Microscopic

- Charge noise
- Flux noise

7) Two Qubit Gate

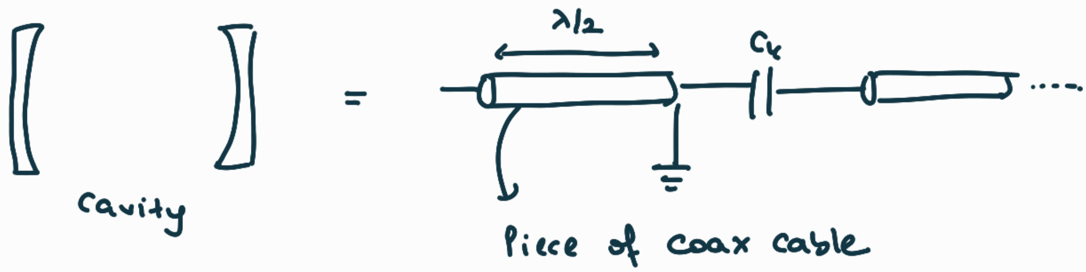
Coupling Strategies

1) Direct Capacitive Coupling

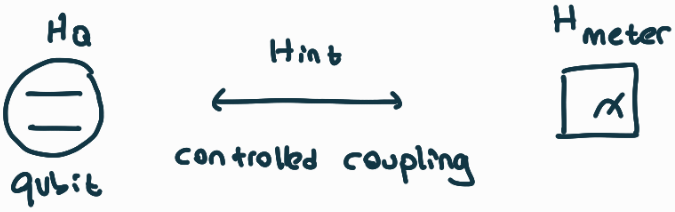


* iSWAP gate can be achieved

2) Cavity Mediated qubit qubit coupling (cQED)



8) Read-out



* Projective and Quantum non-demolition (QND)

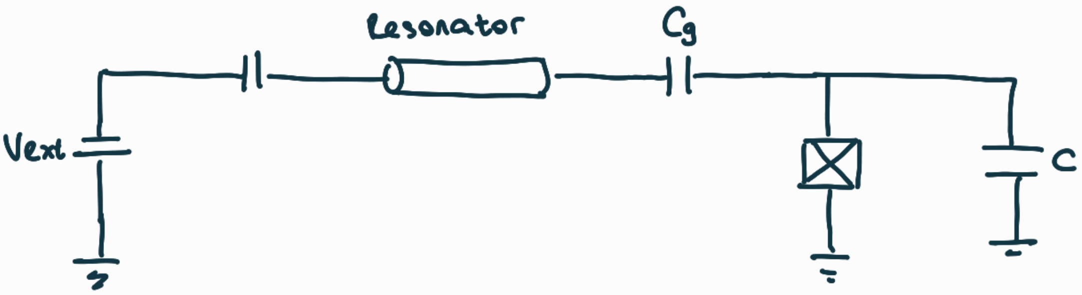
  - $[H_Q, H_{int}] = 0$
  - Repeated measurement yields the same outcome

* Good ON/OFF ratio

  - $[H_{int}, H_{meter}] = 0$ during "OFF"
  - $[H_{int}, H_{meter}] \neq 0$ during "ON"

Readout by a linear resonator

CPB coupled to a CPW Resonator



$$H_{tot} = -E_J \cos\hat{\theta} + 4E_C(n-n_g)^2 + \hbar\omega_c \hat{a}^\dagger \hat{a}$$

$$H_{tot} = \underbrace{-E_J\cos\hat{\theta} + 4E_C(\hat{n}-n_{g\,ext})^2}_{\hat{H}_{qubit}} \underbrace{+\hbar\omega_c \hat{a}^\dagger\hat{a}}_{\hat{H}_{cav}} + \underbrace{8(C_g\,\delta V_0 E_C/2e)\hat{n}(\hat{a}+\hat{a}^\dagger)}_{\hat{H}_{int}}$$

$\downarrow$ 2-level approximation + RWA

$$\boxed{H_{tot} \sim -\frac{\omega_{ge}}{2}\sigma_z + \omega_c\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + g(\sigma^+ a + \sigma^- a^\dagger)}$$

"Jaynes-Cummings Hamiltonian"

1-excitation manifold

$$H_{JC} = \hbar \begin{array}{c} |0_r,0_t\rangle \quad \underbrace{|0_r,1_t\rangle \quad |1_r,0_t\rangle}_{\text{1-excitation manifold}} \quad \ldots\ldots \\ \begin{bmatrix} 0 & & & \\ & \boxed{\begin{matrix} \omega_r & g \\ g & \omega_r \end{matrix}} & & \\ & & \boxed{\begin{matrix} 2\omega_r & \sqrt{2}g \\ \sqrt{2}g & \omega_r+\omega_q \end{matrix}} & \\ & & & \ddots \end{bmatrix} \end{array}$$

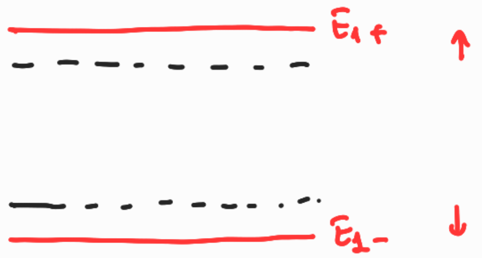
For 1-excitation → $|0_r, 1_t\rangle$ & $|1_r, 0_t\rangle$

↳ 1 photon in resonator

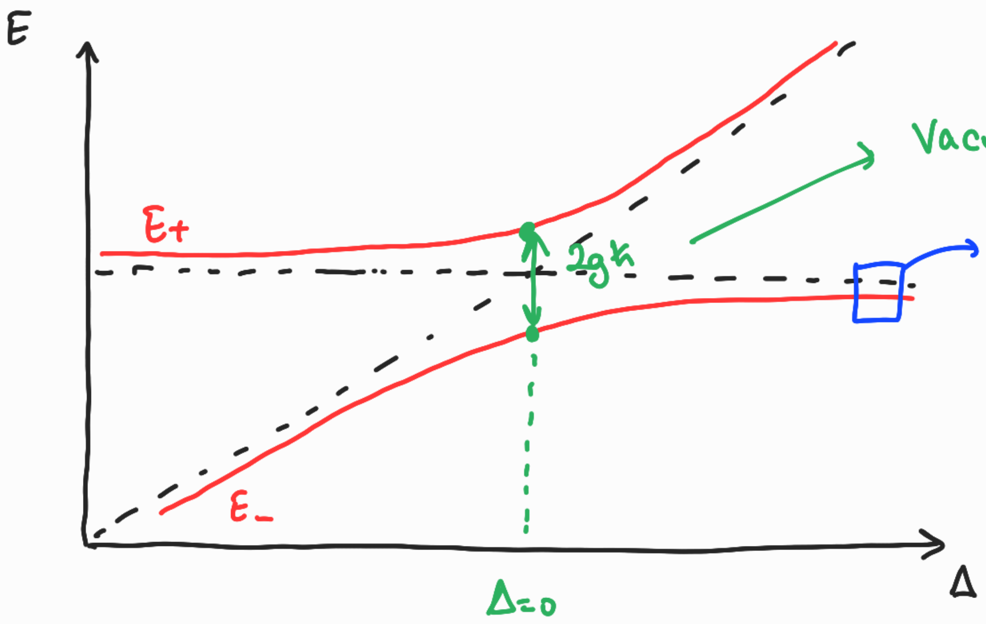
Transmon in ground state

Diagonalizing:

$$E_\pm = \hbar\left(\frac{\omega_{01}+\omega_r}{2} \pm \frac{1}{2}\sqrt{(\underbrace{\omega_{01}-\omega_r}_{\Delta})^2+4g^2}\right)$$



coupling g symmetrically repels the levels away from each other



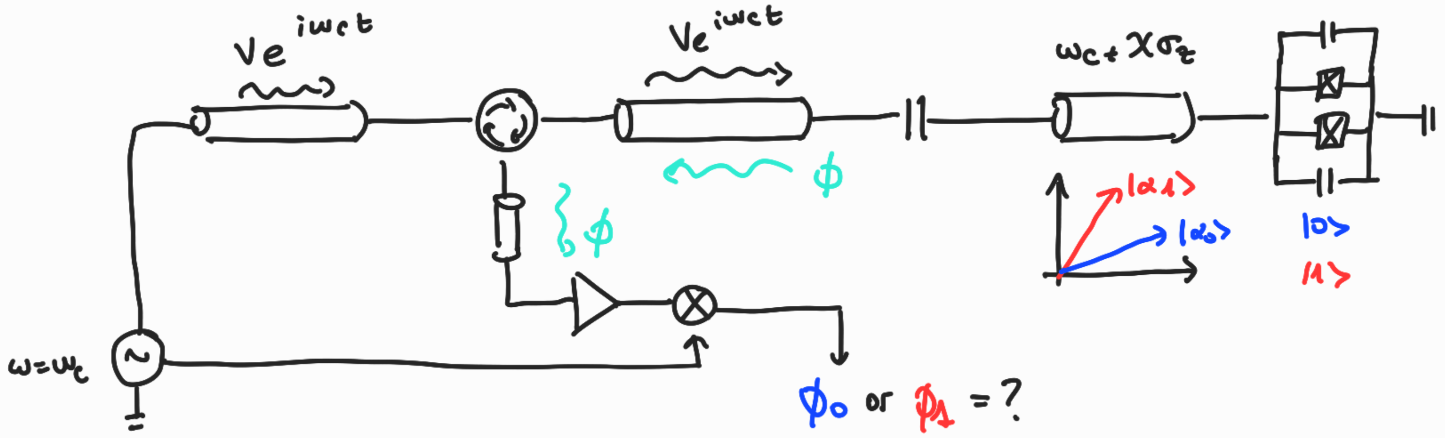
Vacuum Rabi splitting

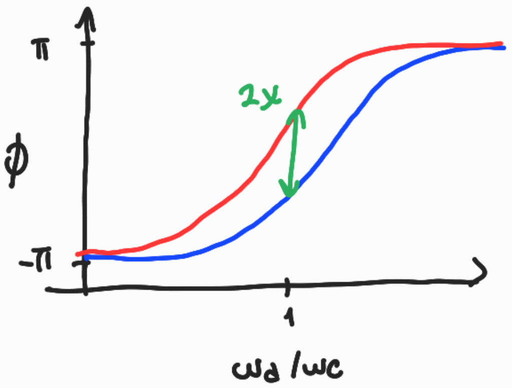
dispersive regime $|\Delta| \gg g$

$$H_{JC} \approx -\frac{\omega_{ge}+\chi}{2}\sigma_z + \underbrace{(\omega_c + \chi\sigma_z)}_{\text{shift of the cavity frequency}}\hat{a}^\dagger\hat{a}$$

($\hbar=1$)

6) - Dispersive Readout of a transmon





★ No discrimination in one shot

Frequency Multiplexing

↳ Read out multiple qubits through a single feedline.

1. Qubit Design: Each qubit is designed with unique resonance frequency

2. Use proper filtering and amplification

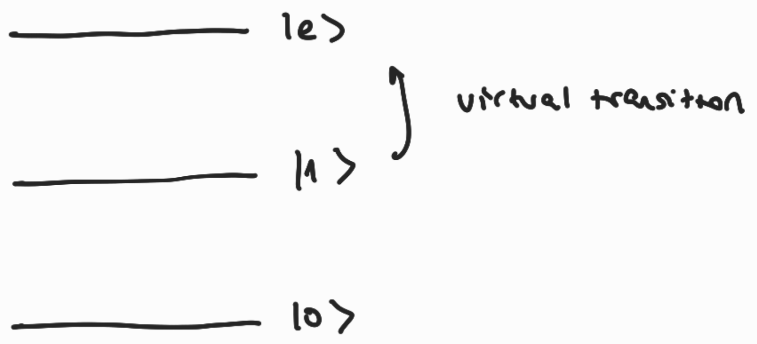
h. How can qubits be coupled together in the dispersive regime, and two-qubit gates implemented? What are the relevant energy scales?
i. How can additional energy levels be used to implement a CPhase gate?

j. What effect does charge noise have on the qubit properties and what steps have been taken to reduce those effects?
k. What our noise sources affect superconducting qubits?
l. Describe the scaling challenges.

H) Qubits should be in resonance with each other.

Qubit Energy levels
Qubit Anharmonicity
Qubit - Resonator Coupling Strength
Qubit - Environment Coupling Strength → T_1, T_2
Microwave frequency → GHz → THz
Gate Operation timescales → ns → μs

I)



|0⟩ → No phase accumulation

|1⟩ → Phase accumulation → change in phase

L) Scaling Challenges

→ Maintaining qubit coherence
→ Qubit Readout → Single-noise
→ Fabrication and Manifacturing