Qubit Disentanglement and decoherence via dephasing

I. Introduction

→ Time for decay of the qubit entanglement can be significantly shorter than time for local dephasing of the individual qubits.

→ Many degree of freedom
→ Decay of the off-diagonal elements of the density matrix
→ Local - Nonlocal Decoherence
→ Pure Dephasing

→ How does dephasing affect entanglement as well as local coherence?

→ How is the disentanglement rate related to those other coherence decay rates?

→ Coherence Decay
→ Quantum Map approach
→ Wootter's Concurrence

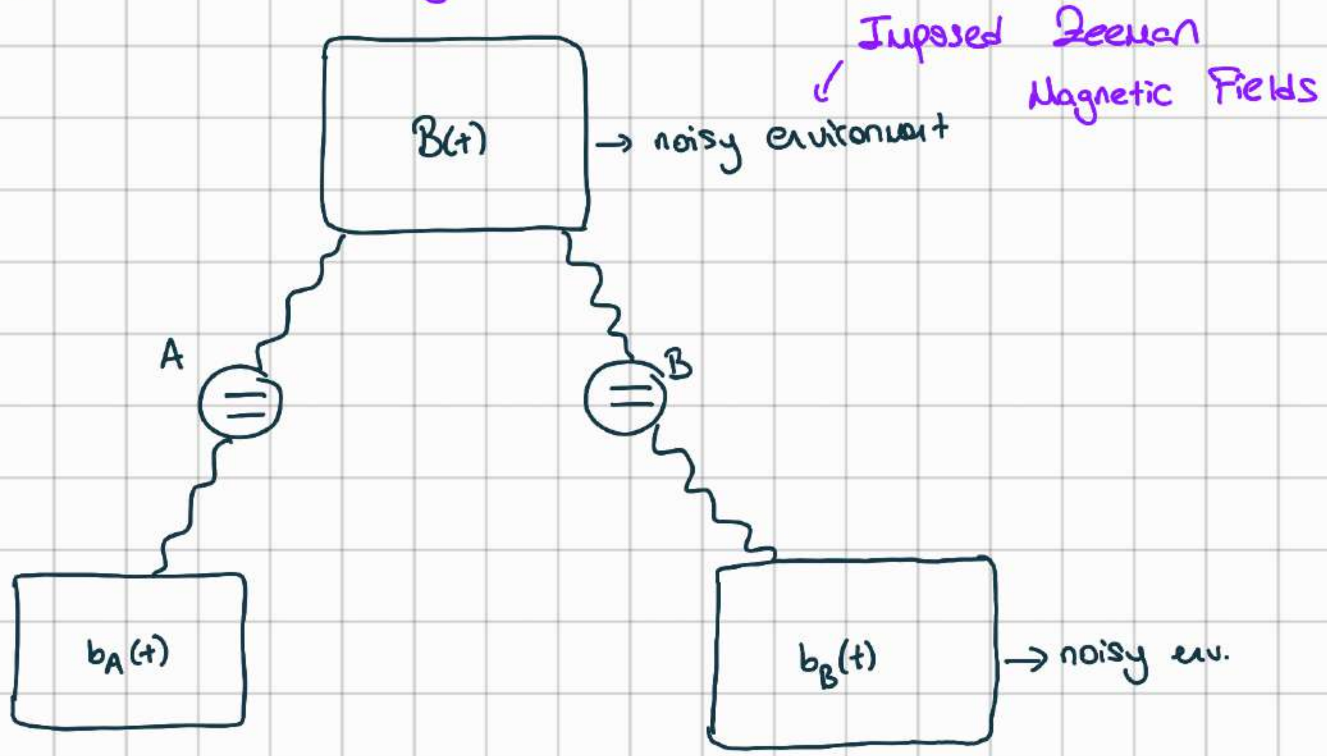
Quantum channel / quantum noisy channel

Dephasing Channel

Decoherence Rate

II. Two-Qubit System With Three Reservoirs

→ stochastic magnetic field



$$H(t) = -\frac{1}{2}\mu\left[B(t)\left(\sigma_z^A + \sigma_z^B\right) + b_A(t)\,\sigma_z^A + b_B(t)\,\sigma_z^B\right] \quad \text{where } \hbar = 1$$

<u>Two-qubit basis</u>

$|1\rangle_{AB} = |++\rangle_{AB}$

$|\pm\pm\rangle_{AB} = |\pm\rangle_A \otimes |\pm\rangle_B$

$|2\rangle_{AB} = |+-\rangle_{AB}$

$|3\rangle_{AB} = |-+\rangle_{AB}$

$\sigma_z^A \otimes \sigma_z^B$ with eigenvalues ± 1

$|4\rangle_{AB} = |--\rangle_{AB}$

Stochastic fields $B(t)$, $b_A(t)$, $b_B(t)$

$\downarrow$

Statistically independent Markov processes satisfying

ensemble average

$$\langle B(t) \rangle = 0$$

damping rate

$$\langle B(t) B(t') \rangle = \frac{\Gamma}{\mu^2} \delta(t-t')$$

$$\langle b_i(t) \rangle = 0$$

$$\langle b_i(t) b_i(t') \rangle = \frac{\Gamma_i}{\mu^2} \delta(t-t'), \quad i = A, B$$

$$\rho(t) = \langle\langle\langle \rho_{st}(t) \rangle\rangle\rangle \quad (?)$$

statistical density operator

$$\rho_{st}(t) = U(t) \rho(0) U^+(t)$$

$$U(t) = \exp\left[-i \int_0^t dt' H(t') \right]$$

III. Kraus Representation for Noisy Channels

Kraus Operators

$$\rho(t) = \mathcal{E}(\rho) = \sum_{\nu=1}^{N} K_\nu^{+}(t)\, \rho(0)\, K_\nu(t)$$

$$\sum_\nu K_\nu^{+} K_\nu = I$$

$$\rho(t) = \tilde{\mathcal{E}}(\rho(0)) = \sum_{i,j=1}^{2} \sum_{k=1}^{3} D_k^{+} E_j^{+} F_i^{+} \rho(0) F_i E_j D_k$$

Kraus operators describing interaction with the $b_A(t)$ & $b_B(t)$

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \gamma_A(t) \end{pmatrix} \otimes I \qquad E_2 = \begin{pmatrix} 0 & 0 \\ 0 & \omega_A(t) \end{pmatrix} \otimes I$$

$e^{-t/2T_2^A}$ $\qquad$ $\sqrt{1-e^{-t/T_2^A}}$

$$F_1 = I \otimes \begin{pmatrix} 1 & 0 \\ 0 & \gamma_B(t) \end{pmatrix} \qquad F_2 = I \otimes \begin{pmatrix} 0 & 0 \\ 0 & \omega_B(t) \end{pmatrix}$$

$e^{-t/2T_2^B}$ $\qquad$ $\sqrt{1-e^{-t/T_2^B}}$

Interaction with the environmental $B(t)$

$\sqrt{1-e^{-t/T_2}}$

$$D_1 = \begin{pmatrix} \gamma(t) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \gamma(t) \end{pmatrix} \qquad D_2 = \begin{pmatrix} \omega_1(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_2(t) \end{pmatrix}$$

$-e^{-t/T_2}\sqrt{1-e^{-t/T_2}}$

$e^{-t/2T_2}$

$T_2 = 1/\Gamma$

phase relaxation time

$$D_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_3(t) \end{pmatrix}$$

$\sqrt{(1-e^{-t/T_2})(1-e^{-2t/T_1})}$

IV. Explicit Solutions: Various Density Matrices

(i) Two-qubit local dephasing channel $\mathcal{E}_{AB}$ $\rightarrow B(t)=0$

$$M_1 = E_1 F_1$$

$$M_2 = E_1 F_2 \qquad \rho(t) = \mathcal{E}_{AB}(\rho(0)) = \sum_{\nu=1}^{4} M_\nu^{+} \rho(0) M_\nu$$

$$M_3 = E_2 F_1$$

$$M_4 = E_2 F_2$$

$$\rho(t) = \begin{pmatrix} \rho_{11} & \gamma_B \rho_{12} & \gamma_A \rho_{13} & \gamma_A \gamma_B \rho_{14} \\ \gamma_B \rho_{21} & \rho_{22} & \gamma_A \gamma_B \rho_{23} & \gamma_A \rho_{24} \\ \gamma_A \rho_{31} & \gamma_A \gamma_B \rho_{32} & \rho_{33} & \gamma_B \rho_{34} \\ \gamma_A \gamma_B \rho_{41} & \gamma_A \rho_{42} & \gamma_B \rho_{43} & \rho_{44} \end{pmatrix},$$

No effect on diagonal elements

(ii) One qubit local dephasing channels $\mathcal{E}_A$ and $\mathcal{E}_B$

$$\rho(t) = \mathcal{E}_A(\rho(0)) = \sum_{\nu=1}^{2} E_\nu^{+} \rho(0) E_\nu \qquad \text{for qubit } A$$

$$\rho(t) = \left(\begin{array}{cc|cc} \rho_{11} & \rho_{12} & \gamma_A \rho_{13} & \gamma_A \rho_{14} \\ \rho_{21} & \rho_{22} & \gamma_A \rho_{23} & \gamma_A \rho_{24} \\ \hline \gamma_A \rho_{31} & \gamma_A \rho_{32} & \rho_{33} & \rho_{34} \\ \gamma_A \rho_{41} & \gamma_A \rho_{42} & \rho_{43} & \rho_{44} \end{array}\right)$$

(iii) Collective Dephasing Channel $\mathcal{E}_D$

$$\rho(t) = \mathcal{E}_D(\rho(0)) = \sum_{\nu=1}^{3} D_\nu^\dagger(t)\,\rho(0)\,D_\nu(t)$$

$$\rho(t) = \begin{pmatrix} \rho_{11} & \gamma\rho_{12} & \gamma\rho_{13} & \gamma^4\rho_{14} \\ \gamma\rho_{21} & \rho_{22} & \rho_{23} & \gamma\rho_{24} \\ \gamma\rho_{31} & \rho_{32} & \rho_{33} & \gamma\rho_{34} \\ \gamma^4\rho_{41} & \gamma\rho_{42} & \gamma\rho_{43} & \rho_{44} \end{pmatrix}$$

V. LOCAL DEPHASING & MIXED DEPHASING

Reduced Density Matrix

$$S^A = \mathrm{Tr}_B\{\rho\} \qquad S^B = \mathrm{Tr}_A\{\rho\}$$

$$S^A(t) = \begin{pmatrix} \rho_{11}(t) + \rho_{22}(t) & \rho_{13}(t) + \rho_{24}(t) \\ \rho_{31}(t) + \rho_{42}(t) & \rho_{33}(t) + \rho_{44}(t) \end{pmatrix}$$

$$S^B(t) = \begin{pmatrix} \rho_{11}(t) + \rho_{33}(t) & \rho_{12}(t) + \rho_{34}(t) \\ \rho_{21}(t) + \rho_{43}(t) & \rho_{22}(t) + \rho_{44}(t) \end{pmatrix}$$

$$S_{12}^{A}(t) = \rho_{13}(t) + \rho_{24}(t) = \gamma_A S_{12}^{A}(0)$$

$$S_{12}^{B}(t) = \rho_{12}(t) + \rho_{34}(t) = \gamma_B S_{12}^{B}(0)$$

$$T_A = \frac{2}{\Gamma_A} \qquad T_B = \frac{2}{\Gamma_B}$$

Dephasing time   Dephasing time

$$\rho_{ij}(t) = e^{-\Gamma_{ij} t} \rho_{ij}(0)$$

$$T_{dec} = \frac{1}{\Gamma_{ij}}$$

$$\Gamma_{12} = \Gamma_{34} = \frac{\Gamma_B}{2}$$

$$\Gamma_{13} = \Gamma_{24} = \frac{\Gamma_A}{2}$$

$$\Gamma_{14} = \Gamma_{23} = \frac{(\Gamma_A + \Gamma_B)}{2}$$

VI. Entanglement Decay

$$C(\rho) = \max\left(0, \underbrace{\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}}\right)$$

concurrence

λ_i's are eigenvalues of Q

$$Q = \rho\left(\sigma_y^A \otimes \sigma_y^B\right) \rho^* \left(\sigma_y^A \otimes \sigma_y^B\right)$$

$$C(|\psi\rangle) = \left|\langle\psi| \sigma_y^A \otimes \sigma_y^B |\psi^*\rangle\right|$$

A. Entanglement decay under two-qubit dephasing channel

$$|\psi\rangle = a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle$$

$$C|\psi\rangle = 2\left|a_1 a_4 - a_2 a_3\right|$$