1. Find Ensembled Average Over 3 Noise Fields

2. Show How You found Krauss Operators

$$\rho(t) = \langle\langle\langle \rho_{st}(t) \rangle\rangle\rangle$$

$$\rho_{st}(t) = U(t)\,\rho(0)\,U^{\dagger}(t)$$

$$U(t) = \exp\left[-i\int_0^t dt' H(t')\right]$$

1.3

$$\langle O \rangle = \int DB\, Db_A\, Db_B\, \underbrace{P(B)\,P(b_A)\,P(b_B)}\, O$$

arbitrary operator

probability distribution of noise fields (noise fields are assumed to be uncorrelated, thus joint prob. distribution can be written as product of individual distributions)

Here, prob. distributions can be write as Gaussian distribut.

$$P(B) = \mathcal{N}(0, S_B)$$

$$P(b_A) = \mathcal{N}(0, S_{b_A})$$

$$P(b_B) = \mathcal{N}(0, S_{b_B})$$

wher S_B, S_{b_A}, S_{b_B} are power spectral densities

$$S_B = \frac{\Gamma}{N^2}$$

$$S_{b_A} = \frac{\Gamma_A}{N^2}$$

$$S_{b_B} = \frac{\Gamma_B}{N^2}$$

$$\langle\langle\langle \rho_{st}(t) \rangle\rangle\rangle = \iiint dB(t)\, db_A(t)\, d_{b_B}(t)\, P_{B(t)} P_{b_A}(t) P_{b_B}(t).$$

$$U(t)\rho(0)U^+(t)$$

Since noise fields are uncorrelated

$$\langle\langle\langle \rho_{st}(t) \rangle\rangle\rangle = \int dB(t)\, \underset{B(t)}{P_{B(t)}}\, U(t)\, \rho(0)\, U^+(t) \underset{b_A(t)}{\int d_{b_A(t)} P_{b_A(t)}} \underset{b_B(t)}{\int d_{B(t)} P_{b_B(t)}}$$

?

$$P_{B(t)} = \frac{1}{\sqrt{2\pi\Gamma}} \exp\left(\frac{-B_t^2}{2\Gamma}\right)$$

$$P_{b_A(t)} = \frac{1}{\sqrt{2\pi\Gamma_A}} \exp\left(\frac{-b_{A_t}^2}{2\Gamma_A}\right)$$

$$P_{b_B(t)} = \frac{1}{\sqrt{2\pi\Gamma_B}} \exp\left(\frac{-b_{B_t}^2}{2\Gamma_B}\right)$$

$$U(t) = \exp\left(-i\int_0^t t' H(t')\right)$$

$$\downarrow$$

$$\mathcal{T}\left\{ e^{\int_0^t A(t')dt'} \right\} \equiv \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^t \cdots\cdots \int_0^t \mathcal{T}\left\{ A(t_1')\ldots A(t_n') \right\}$$

$$\equiv \sum_{n=0}^{\infty} \int_0^t \cdots\cdots \int_0^{t_{n-1}'} A(t_1') \ldots\ldots A(t_n')$$

$$e^{\begin{pmatrix} -i\int B(t)\ldots dt & \\ & +i\int B(t) \end{pmatrix}_{A \atop 2\times 2}}$$

Path Integrals?

Magnus Series?

1.1

$$\langle O(t)\rangle = Tr\left[O(t)\rho_{AB}(t)\right] = Tr\left[O(t)\,U(t,0)\,\rho_{AB}(0)\,U^{\dagger}(t,0)\right]$$

$$= Tr\left[\underbrace{U^{\dagger}(t,0)\,O(t)\,U(t,0)}_{O_{eff}(t)}\,\rho_{AB}(0)\right] = Tr\left[O_{eff}(t)\,\rho_{AB}(0)\right]$$

$$\langle \rho_{AB}(t)\rangle = Tr\left[\rho_{AB}(t)\,\rho_{AB}(t)\right] = Tr\left[U\,\rho_{AB}(0)\,\underbrace{U^{\dagger}U}_{I}\,\rho_{AB}(0)\,U^{\dagger}\right]$$

$$\langle \rho_{AB}(t)\rangle = Tr\left[\rho_{AB}^{2}(0)\right]$$ $\rightsquigarrow$ It does not make sense!

1.2 $$H(t) = -\frac{1}{2}N\left[B(t)(\sigma_z^A+\sigma_z^B) + b_A(t)\sigma_z^A + b_B(t)\sigma_z^B\right]$$

time-ordering operator

$$U(t) = \mathcal{T}\exp\left[-i\int_0^t H(s)\,ds\right]$$

$$U(t) = \exp\left[-i\int_0^t H(s)\,ds\right] = U(t) = e^{ik(t)}\,e^{iA(t)}\,e^{iB(t)}$$

Using Baker-Campbell-Hausdorff

$$\rho_{st}(t) = U(t)\,\rho(0)\,U^{\dagger}(t)$$

Does it have to be an operator?

$$\langle\langle\langle \rho_{st}(t)\rangle\rangle\rangle = \cancel{Tr}_{\cancel{B(t),\,b_A(t),\,b_B(t)}}\left[\rho_{st}(t)\right]$$

2. To find Klauss Operators

$\langle\langle\langle \rho_{SH}(t) \rangle\rangle\rangle$

→ Diagonalize the ensembled averaged density operator

→ Select the <u>eigenvectors</u> corresponding to non-zero eigenvalues.

How to use these eigenvectors?