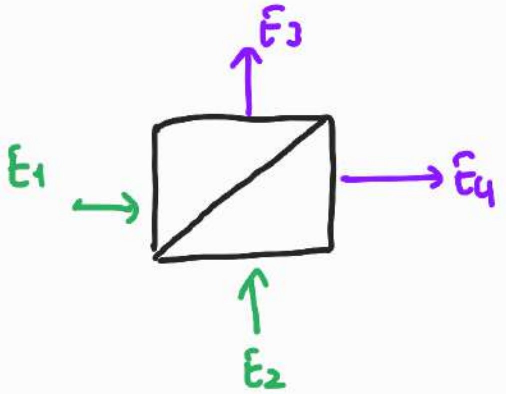
Immanuel Bloch Lectures

The Classical Beam Splitter



$$E_3 = R E_1 + T E_2$$

R, T: complex coefficients

$$E_4 = T' E_1 + R' E_2$$

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = \begin{pmatrix} R & T \\ T' & R' \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

Symmetric beamsplitter (simplified case)

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = \begin{pmatrix} R & T \\ T & R \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

Energy Conservation:

$$|E_1|^2 + |E_2|^2 = |E_3|^2 + |E_4|^2$$

$$|R|^2 + |T|^2 = 1$$

$$R T^* + T R^* = 0$$

$$R = |R| e^{i\phi_R} \qquad T = |T| e^{i\phi_T}$$

$$\cancel{|R|}\,\cancel{|T|}\, e^{i(\phi_R - \phi_T)} + \cancel{|R|}\,\cancel{|T|}\, e^{-i(\phi_R - \phi_T)} = 0$$

$$2\cos(\phi_R - \phi_T) = 0$$

$$\phi_R - \phi_T = \frac{\pi}{2} \qquad \text{Set } \phi_T = 0,\ \phi_R = \frac{\pi}{2}$$

$$R = |R| e^{i\phi_R} = |R| \underbrace{e^{i\pi/2}}_{i} = i|R|$$

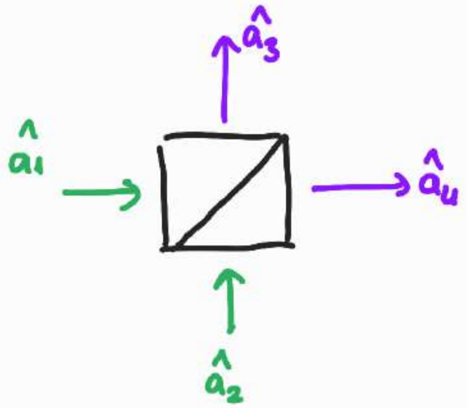
$$E_3 = i|R| E_1 + |T| E_2$$

$$E_4 = |T| E_1 + i|R| E_2$$

50/50 Beamsplitter Input-Output

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

The Quantum Beam Splitter



$$\begin{pmatrix} \hat{a}_3 \\ \hat{a}_4 \end{pmatrix} = \begin{pmatrix} R & T \\ T & R \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

$$\hat{a}_3 = R\hat{a}_1 + T\hat{a}_2$$

$$\hat{a}_4 = T\hat{a}_1 + R\hat{a}_2$$

$$R^* \hat{a}_3 = |R|^2 \hat{a}_1 + R^* T \hat{a}_2$$

$$T^* \hat{a}_4 = |T|^2 \hat{a}_1 + T^* R \hat{a}_2$$

$$\underbrace{|R|^2 \hat{a}_1 + |T|^2 \hat{a}_1}_{\hat{a}_1} + \underbrace{(R^* T + T^* R)}_{0} a_2 = R^* \hat{a}_3 + T^* \hat{a}_4$$

$$\hat{a}_1 = R^* \hat{a}_3 + T^* \hat{a}_4$$

$$\hat{a}_1 = R^* \hat{a}_3 + T^* \hat{a}_4 \qquad \hat{a}_1^\dagger = R \hat{a}_3^\dagger + T \hat{a}_4^\dagger$$

$$\hat{a}_2 = T^* \hat{a}_3 + R^* \hat{a}_4 \qquad \hat{a}_2^\dagger = T \hat{a}_3^\dagger + R \hat{a}_4^\dagger$$

$$\hat{a}_3 = R \hat{a}_1 + T \hat{a}_2 \qquad \hat{a}_3^\dagger = R^* \hat{a}_1^\dagger + T^* \hat{a}_2^\dagger$$

$$\hat{a}_4 = T \hat{a}_1 + R \hat{a}_2 \qquad \hat{a}_4^\dagger = T^* \hat{a}_1^\dagger + R^* \hat{a}_2^\dagger$$

Single Photon on BS

Input state $|1\rangle_1 |0\rangle_2 |0\rangle_3 |0\rangle_4 \rightarrow |1\rangle_1 |0\rangle_2$ (short hand notation)

$$|1\rangle_1 |0\rangle_2 = a_1^\dagger |0\rangle_1 |0\rangle_2$$

creation

$$\hat{a}_1^\dagger = R \hat{a}_3^\dagger + T \hat{a}_4^\dagger$$

$$\left(R \hat{a}_3^\dagger + T \hat{a}_4^\dagger \right) |0\rangle_3 |0\rangle_4 = R |1\rangle_3 |0\rangle_4 + T |0\rangle_3 |1\rangle_4$$

Entangled State of photon btw field modes

Single Photon on 50/50 BS

Average Output Photon Number

$$\langle \hat{n}_3 \rangle = \langle \hat{a}_3^\dagger \hat{a}_3 \rangle = \langle 0_2 1_1 | \hat{a}_3^\dagger \hat{a}_3 | 1_1 0_2 \rangle$$

$\hat{a}_3^\dagger = R^* \hat{a}_1^\dagger + T^* \hat{a}_2^\dagger$

$$= \langle 0_2 1_1 | \underbrace{(R^* \hat{a}_1^\dagger + T^* \hat{a}_2^\dagger)(R \hat{a}_1 + T \hat{a}_2)}_{|R|^2 \hat{a}_1^\dagger \hat{a}_1 + R^* T \hat{a}_1^\dagger \hat{a}_2 \ldots} | 1_1 0_2 \rangle$$

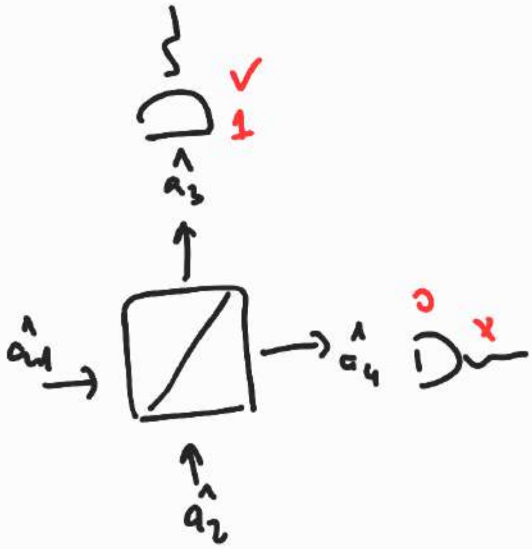
creates orthogonal states

$$= |R|^2 \underbrace{\langle 0_2 1_1 | 1_1 0_2 \rangle}_{1} = |R|^2 = \frac{1}{2} \text{ for } 50/50 \text{ BS}$$

$$\langle \hat{n}_4 \rangle = 1/2$$

Correlations

$$\langle 0_2 1_1 | \hat{n}_3 \hat{n}_4 | 1_1 0_2 \rangle = 0$$

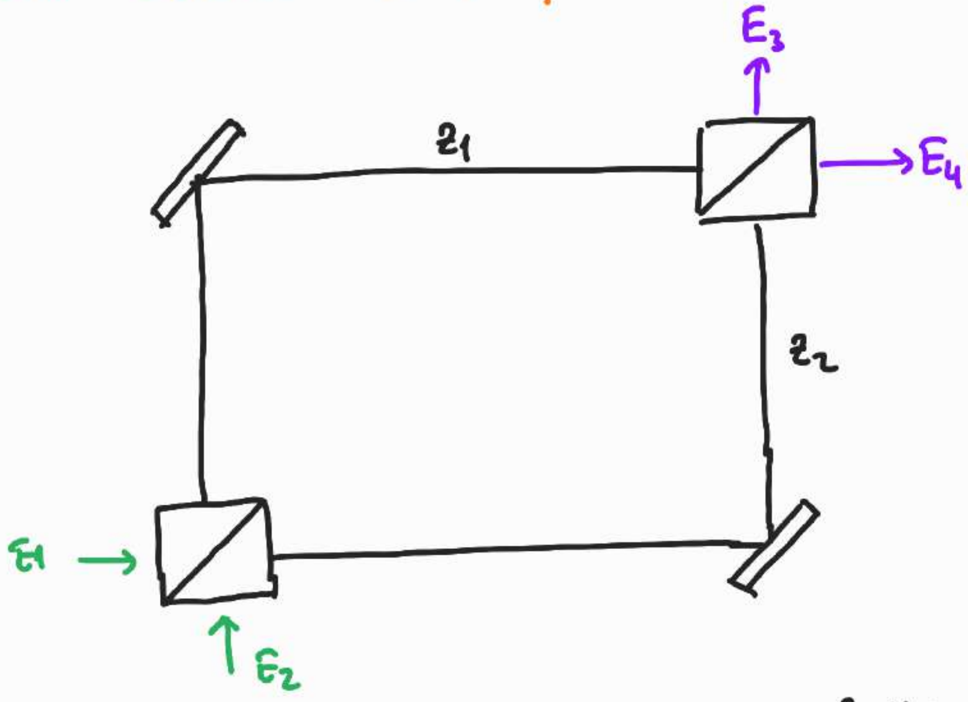


* Non-classical correlations

* Due to entangled states btw field modes

$$\frac{1}{\sqrt{2}} \left(i |1\rangle_3 |0\rangle_4 + |0\rangle_3 |1\rangle_4 \right)$$

Mach - Zender Interferometer



$$R_{MZ} = R^2 e^{ikz_1} + T^2 e^{ikz_2}$$

$$E_3 = R_{MZ} E_1 + T_{MZ} E_2 \qquad T_{MZ} = RT\left(e^{ikz_1} + e^{ikz_2}\right)$$

$$E_4 = T_{MZ} E_1 + R'_{MZ} E_2 \qquad R'_{MZ} = T^2 e^{ikz_1} + R^2 e^{ikz_2}$$

Output Intensities

Assume $E_2 = 0$

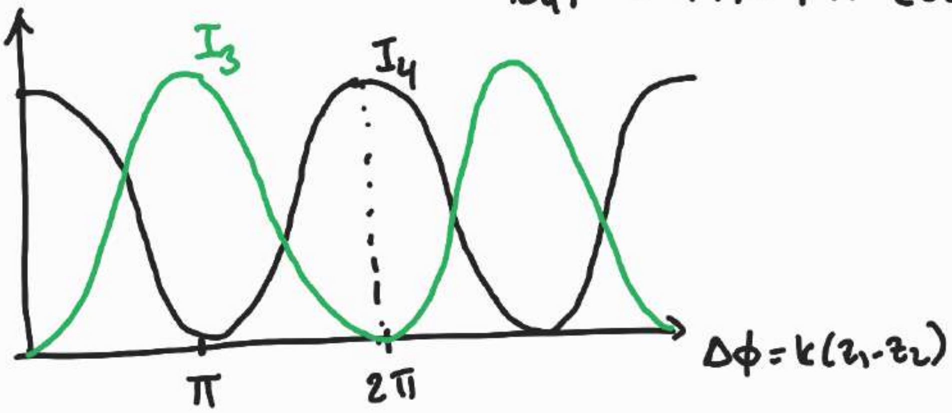
$$|E_3|^2 = |R_{MZ}|^2 |E_1|^2$$

$$|E_4|^2 = |T_{MZ}|^2 |E_1|^2$$

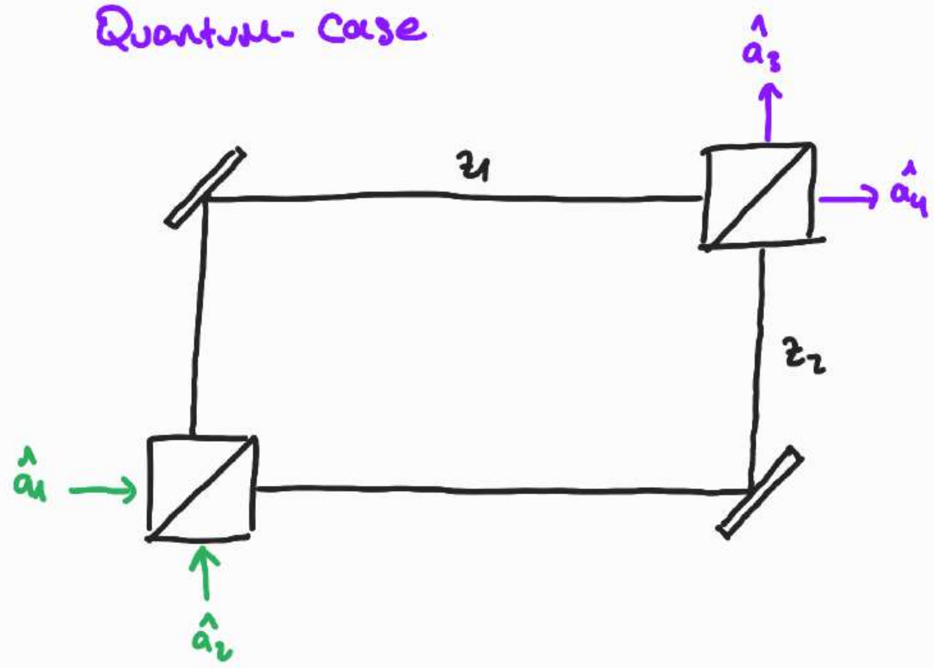
$$|E_4|^2 = |R|^2 |T|^2 \left|e^{ikz_1} + e^{ikz_2}\right|^2 |E_1|^2$$

50/50 BS

$$|E_4|^2 = 4|R|^2|T|^2 \cos^2\left(\frac{\overbrace{k(z_1 - z_2)}^{\Delta\phi}}{2}\right) |E_1|^2$$



Quantum-case



$$\hat{a}_3 = R_{MZ}\,\hat{a}_1 + T_{MZ}\,\hat{a}_2$$

$$\hat{a}_4 = T_{MZ}\,\hat{a}_1 + R'_{MZ}\,\hat{a}_2$$

Single photon input state $|1\rangle_1 |0\rangle_2$

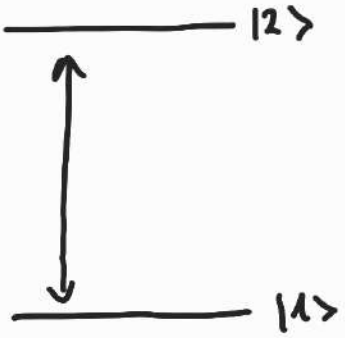
$$\langle \hat{n}_4 \rangle = \langle 1_1 0_2 |\, \hat{n}_4 \,| 1_1 0_2 \rangle$$

$$= \langle 10 |\, \hat{a}_4^{\dagger} \hat{a}_4 \,| 10 \rangle = \langle 10 |\, (T_{MZ}^{*}\hat{a}_1^{\dagger} + R'^{*}_{MZ}\hat{a}_2^{\dagger})(T_{MZ}\hat{a}_1 + R'_{MZ}\hat{a}_2) \,| 10 \rangle$$

$$= \langle 10 |\, |T_{MZ}|^2 \,| 10 \rangle = |T_{MZ}|^2 = 4\,|T|^2 |R|^2 \cos^2\left(\frac{k(z_1 - z_2)}{2}\right)$$

Collapse and Revival of Rabi Oscillations

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle \rightarrow \text{Light Field}$$



→ Atom (initially in excited state)

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} C_{2,n}(t)\, |2,n\rangle + C_{1,n}(t)\, |1,n\rangle$$

Combined Atom-Field State Amplitudes

$$C_{1,n}(0) = 0$$

$$|C_{2,n}(0)|^2 = e^{-\bar{n}} \frac{\bar{n}^n}{n!} \quad \text{where } \bar{n} = |\alpha|^2$$

↳ Poisson distribution

LOOK AT THEM

$$C_{2,n}(t) = \cos(g\sqrt{n+1}\, t)\, |C_{2,n}(0)|$$

$$C_{1,n}(t) = -i \sin(g\sqrt{n}\, t)\, |C_{2,n-1}(0)| \quad \forall n \geq 1$$

Inversion

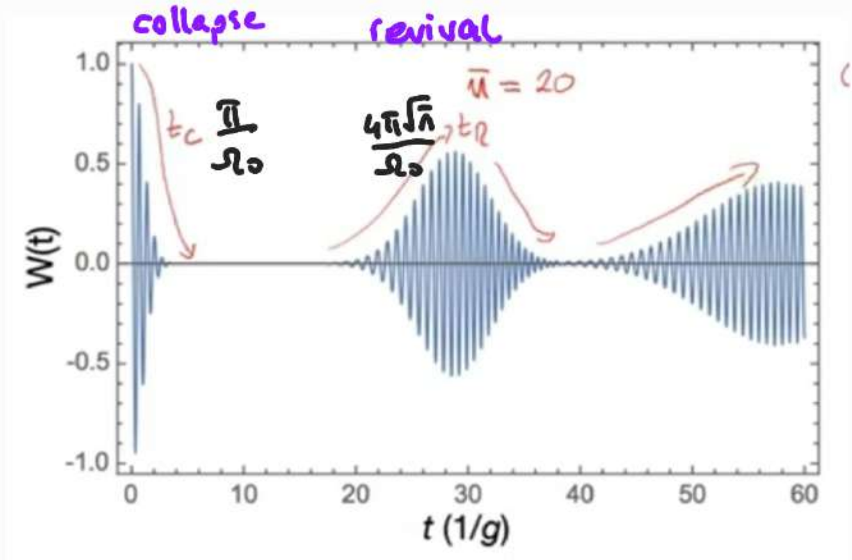
$$W(t) = P_2(t) - P_1(t)$$

$$= \underbrace{\sum_{n=0}^{\infty} |c_{2,n}(t)|^2}_{P_2(t)} - \underbrace{\sum_{n=0}^{\infty} |c_{1,n}(t)|^2}_{P_1(t)}$$

Rabi Osc.

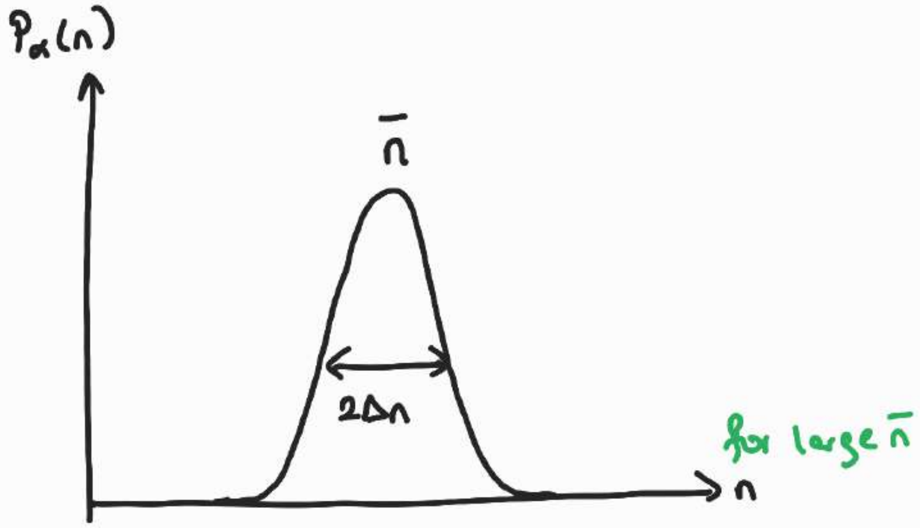
$$= \sum_{n=0}^{\infty} |c_{2,n}(0)|^2 \cos\left(2g\sqrt{n+1}\, t\right)$$

$$= e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \cos\left(2g\sqrt{n+1}\, t\right)$$



Collapse time

↳ Different Rabi Oscillations run out π phase



$\Delta n = \sqrt{\bar n}$

Rabi Oscillations

$$\Omega_{\bar n+\Delta n}\, t_c - \Omega_{\bar n-\Delta n}\, t_c = \pi$$

$$\left(2g\underbrace{\sqrt{\bar n+\sqrt{\bar n}}}_{\sqrt{\bar n}\left(\sqrt{1+\frac{1}{\sqrt{\bar n}}}\right)} - 2g\underbrace{\sqrt{\bar n-\sqrt{\bar n}}}_{\sqrt{\bar n}\left(1-\frac{1}{2}\frac{1}{\sqrt{\bar n}}\right)}\right) t_c \sim \pi$$

(Taylor Exp.) $1+\frac{1}{2}\frac{1}{\sqrt{\bar n}}$

$$\cancel{\sqrt{\bar n}} + \frac{1}{2} - \cancel{\sqrt{\bar n}} + \frac{1}{2} = 1$$

$$2g\,t_c \sim \pi$$

$$\boxed{t_c = \frac{\pi}{2g}} = \frac{\pi}{\Omega_0}$$

Ω_0 → vacuum Rabi freq.

Revival Time

$$\left(\Omega_{\bar{n}+1} - \Omega_{\bar{n}} \right) t_R = 2\pi$$

neighbour Rabi oscillations become in phase

$$\left(2g\sqrt{\bar{n}+1} - 2g\sqrt{\bar{n}} \right) t_R \sim 2\pi$$

$$\left(2g\sqrt{\bar{n}} \left(\sqrt{1+\frac{1}{\sqrt{\bar{n}}}} \right) - 2g\sqrt{\bar{n}} \right) t_R \sim 2\pi$$

$$\downarrow$$

$$1 + \frac{1}{2}\frac{1}{\bar{n}}$$

$$\left(\cancel{2} g \cancel{\sqrt{\bar{n}}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{\bar{n}}_{\sqrt{\bar{n}}}} \right) t_R \sim 2\pi$$

$$\frac{g}{\sqrt{\bar{n}}} t_R \sim 2\pi \longrightarrow t_R \sim \frac{2\pi\sqrt{\bar{n}}}{g} = \frac{4\pi\sqrt{\bar{n}}}{2g} = \frac{4\pi\sqrt{\bar{n}}}{\Omega_0}$$

Classical Limit

Quantized box $V \longrightarrow \infty$

$g \rightarrow 0$

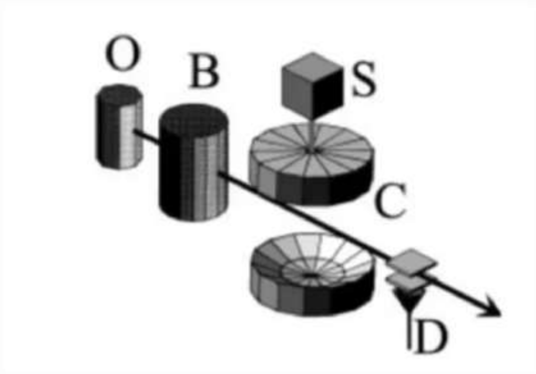
"Coupling constant b/w atom and the light field"

Keep Rabi Freq. Constant

$g \rightarrow 0 \quad \bar{n} \rightarrow \infty \qquad 2g\sqrt{\bar{n}} = \Omega$ = constant

$$t_c \sim \frac{\pi}{2g} \rightarrow \infty \quad \text{for} \quad g \rightarrow 0$$

Experiment



S: microwave signal generator

O: oven chamber (heat up atomic gas)

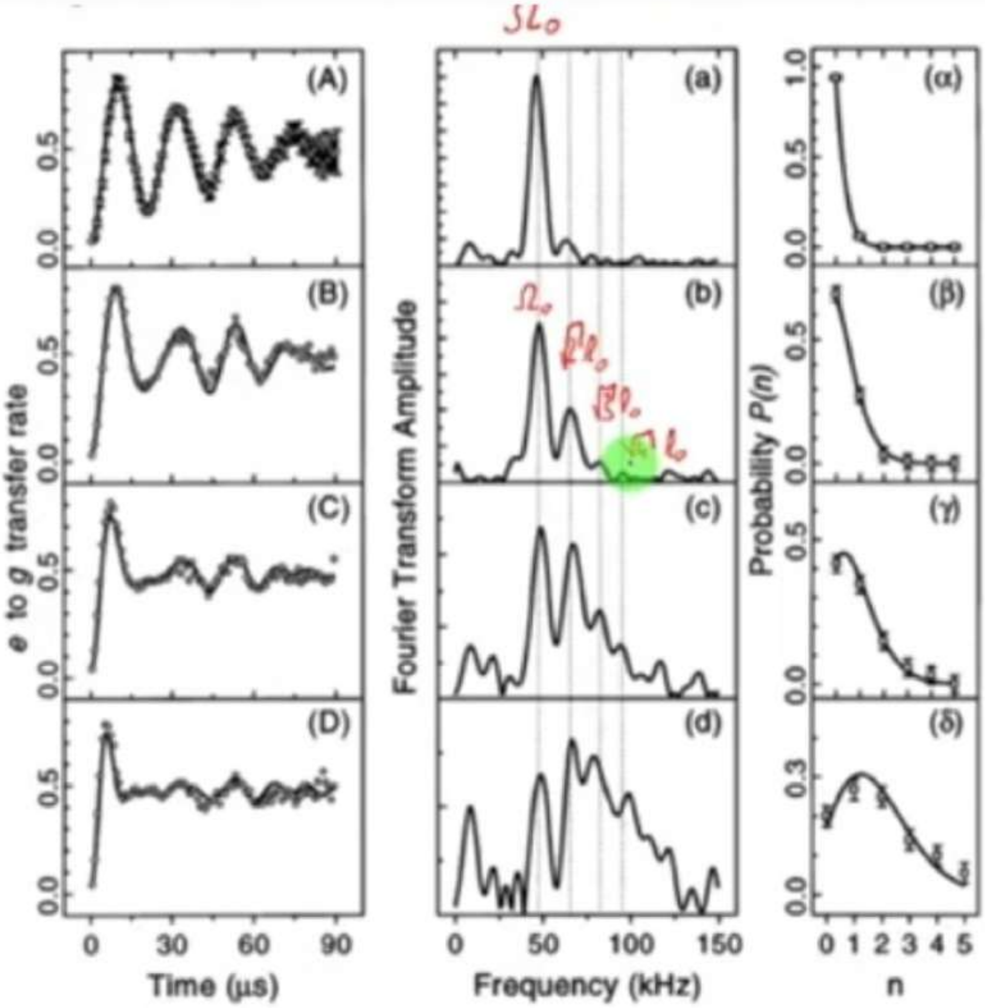
B: state preparation zone

Two level Atoms in ground state
&
Rydberg States

C: Two mirrors

Atom and light interaction

D: Detection zone (electric field plates)



Vacuum Field

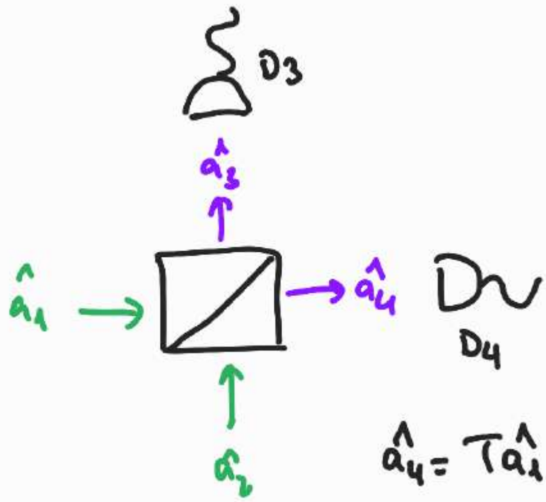
Coherent States

$\bar{n} = 0.4$

$\bar{n} = 0.85$

$\bar{n} = 1.77$

The Homodyne Detection of States



$\hat a_4 = T\hat a_1 + R\hat a_2$

50/50 BS

$T = \frac{1}{\sqrt{2}}$ $\quad R = \frac{1}{\sqrt{2}} i$

$$\hat n_4 = \hat a_4^\dagger \hat a_4$$

$$= \left(T^* \hat a_1^\dagger + R^* \hat a_2^\dagger\right)\left(T\hat a_1 + R\hat a_2\right)$$

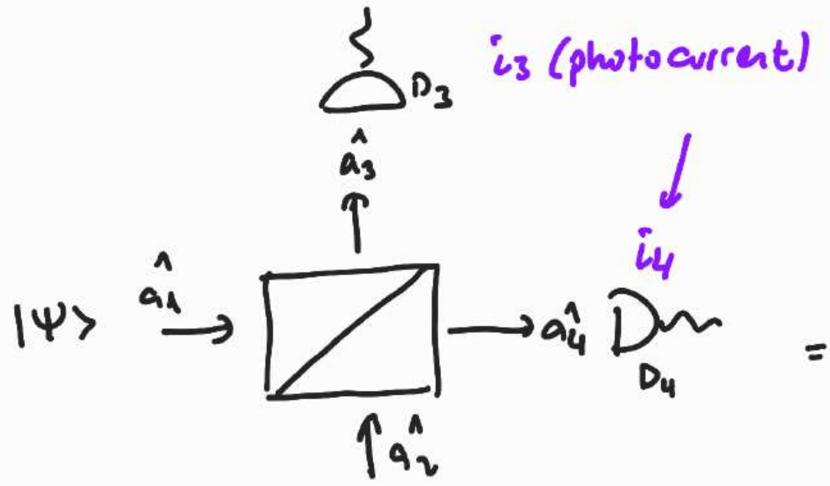
$$= \frac{1}{2}\left(\hat a_1^\dagger - i\hat a_2^\dagger\right)\left(\hat a_1 + i\hat a_2\right)$$

$$\hat n_4 = \frac{1}{2}\left(\hat a_1^\dagger \hat a_1 - i\hat a_2^\dagger \hat a_1 + i\hat a_1^\dagger \hat a_2 + \hat a_2^\dagger \hat a_2\right)$$

$$\hat n_3 = \hat a_3 \hat a_3 = \frac{1}{2}\left(\hat a_1^\dagger \hat a_1 - i\hat a_1^\dagger \hat a_2 + i\hat a_2^\dagger \hat a_1 + \hat a_2^\dagger \hat a_2\right)$$

$$\hat n_3 - \hat n_4 = \hat a_3^\dagger \hat a_3 - \hat a_4^\dagger \hat a_4$$

$$\hat n_3 - \hat n_4 = -2\left[\frac{1}{2i}\left(\hat a_2^\dagger \hat a_1 - \hat a_1^\dagger \hat a_2\right)\right]$$



$|\beta\rangle = |\beta| e^{i\varphi}$

coherent state

Input State: $|\psi\rangle_1 \, |\beta\rangle_2$

$$i_{34} = i_3 - i_4 \propto \langle in | \hat{n}_3 - \hat{n}_4 | in \rangle$$

$$= -2 \, \langle \psi_1 \beta_2 | \frac{1}{2i} \left(\hat{a}_2^{+} \hat{a}_1 - \hat{a}_1^{+} \hat{a}_2 \right) | \psi_1 \beta_2 \rangle$$

$$\hat{a}_2 |\beta\rangle = \beta |\beta\rangle$$

$$= -2 \, \langle \psi_1 | \frac{1}{2i} \left(\beta^* \hat{a}_1 - \hat{a}_1^{+} \beta \right) | \psi_1 \rangle$$

$$= -2\beta \, \langle \psi_1 | \underbrace{\frac{1}{2i} \left(\hat{a}_1 e^{-i\varphi} - \hat{a}_1^{+} e^{i\varphi} \right)}_{\hat{X}_1 \left(\varphi + \frac{\pi}{2} \right)} | \psi \rangle$$

quadrature operator