

WEEK 8 Interaction by Particle Exchange and QED

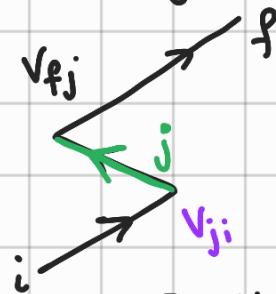
Lorentz Invariant Matrix Element

Interaction by particle exchange

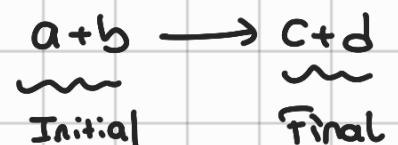
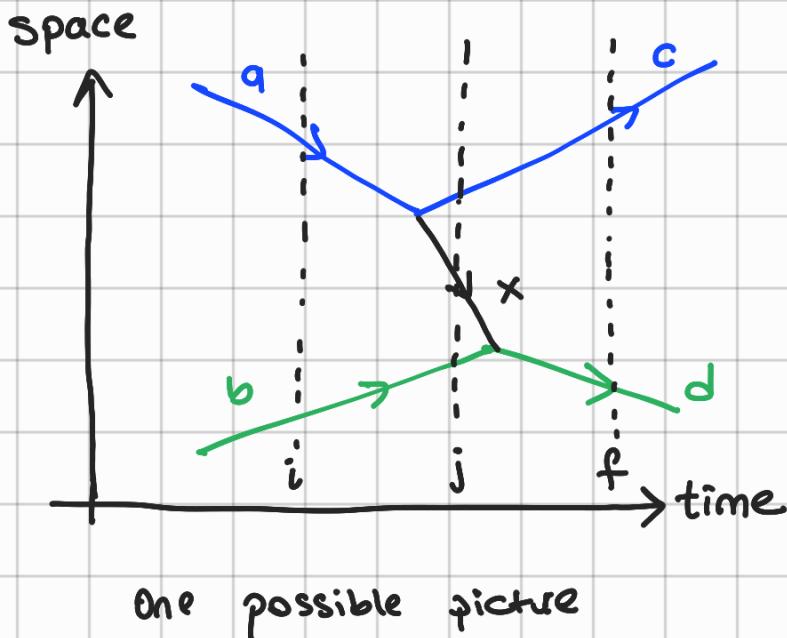
Fermi's Golden Rule

$$T_{fi} = 2\pi |\tau_{fi}|^2 \rho(E_f)$$

$$T_{fi} = \langle f | V | i \rangle + \sum_{j \neq i} \frac{\langle f | V | j \rangle \langle j | V | i \rangle}{E_i - E_j} + \dots$$

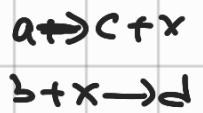


Scattering via an intermediate state



$j: c+b+x$
 b
 Intermediate state

a emits x before b absorbs it.



$$T_{fi} = \frac{\langle f | v | j \rangle \langle j | v | i \rangle}{\bar{E}_i - \bar{E}_j} \quad \text{and} \quad \bar{E}_i \neq \bar{E}_j, \text{ allowed by energy-time uncertainty relation}$$

a emits x before
b absorbs it

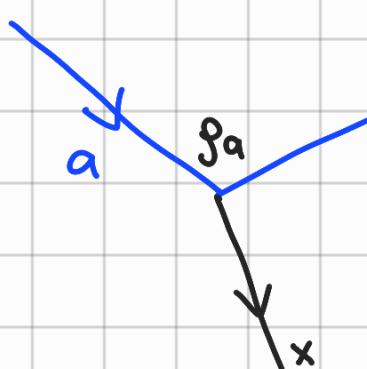
$$T_{fi}^{ab} = \frac{\langle d | v | x+b \rangle \langle c+x | v | a \rangle}{(\bar{E}_a + \bar{E}_b) - (\bar{E}_c + \bar{E}_x + \bar{E}_b)}$$

! $V_{ji} = \langle c+x | v | a \rangle$ (and $V_{fi} = \langle d | v | x+b \rangle$ are NON-INVARIANT.
Matrix elements

$$T_{fi} = \prod_k (2E_k)^{-1/2} M_{fi}$$

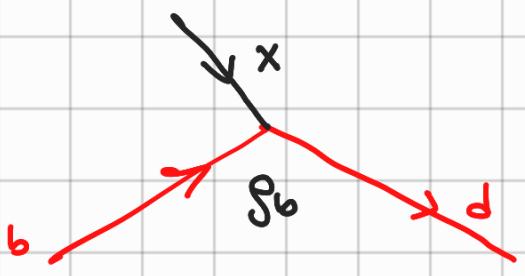
$$\langle c+x | v | a \rangle = \frac{M(a \rightarrow c+x)}{(2\bar{E}_a 2\bar{E}_c 2\bar{E}_x)^{1/2}}$$

Lorentz invariant



scalar L.I quantity g_a

$$\langle c+x | v | a \rangle = \frac{g_a}{(2\bar{E}_a 2\bar{E}_c 2\bar{E}_x)^{1/2}}$$



$$\langle d | V | b+x \rangle = \frac{g_b}{(2\bar{E}_a 2\bar{E}_d 2\bar{E}_x)^{1/2}}$$

$$T_{fi}^{ab} = \frac{\langle d | V | x+b \rangle \langle c+x | V | a \rangle}{(\bar{E}_a + \bar{E}_b) - (\bar{E}_c + \bar{E}_x + \bar{E}_b)}$$

↓

$$T_{fi}^{ab} = \frac{1}{2\bar{E}_x} \cdot \frac{1}{(2\bar{E}_a 2\bar{E}_b 2\bar{E}_c 2\bar{E}_d)^{1/2}} \cdot \frac{g_a g_b}{(\bar{E}_a - \bar{E}_c - \bar{E}_x)}$$

↗ Matrix element for the entire process

$$M_{fi}^{ab} = (2\bar{E}_a 2\bar{E}_b 2\bar{E}_c 2\bar{E}_d)^{1/2} T_{fi}^{ab} = \frac{1}{2\bar{E}_x} \cdot \frac{g_a g_b}{(\bar{E}_a - \bar{E}_c - \bar{E}_x)}$$

! M_{fi} is not Lorentz invariant, since order of event in time depends on frame.

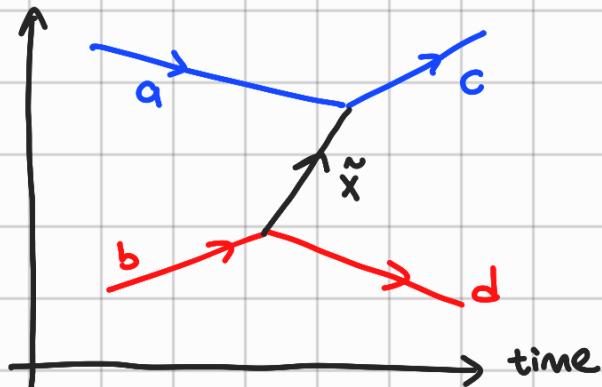
Note that:

Momentum is conserved at each interaction vertex but not energy.

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Let's consider the other
time orderings

space



$$M_{fi}^{ba} = \frac{1}{2\bar{E}_x} \cdot \frac{g_a g_b}{(E_b - E_d - \bar{E}_x)}$$

$$E_a + E_b = \bar{E}_c + \bar{E}_d$$

$$M_{fi} = M_{fi}^{ab} + M_{fi}^{ba} = \frac{g_a g_b}{2\bar{E}_x} \left(\frac{1}{\bar{E}_a - \bar{E}_c - \bar{E}_x} + \frac{1}{\underbrace{\bar{E}_b - \bar{E}_d - \bar{E}_x}_{-(\bar{E}_a - \bar{E}_c)}} \right)$$

$$= \frac{g_a g_b}{2\bar{E}_x} \left(\frac{1}{\bar{E}_a - \bar{E}_c - \bar{E}_x} - \frac{1}{\bar{E}_a - \bar{E}_c + \bar{E}_x} \right)$$

$$\cancel{\bar{E}_a - \bar{E}_c + \bar{E}_x} - (\bar{E}_a - \bar{E}_c - \bar{E}_x)$$

$$M_{fi} = \frac{g_a g_b}{2\bar{E}_x} \frac{2\bar{E}_x}{(\bar{E}_a - \bar{E}_c)^2 - \bar{E}_x^2} = \frac{g_a g_b}{(\bar{E}_a - \bar{E}_c)^2 - \bar{E}_x^2}$$

For both time orderings $\rightarrow \vec{p}_x = \vec{p}_b - \vec{p}_d = -(\vec{p}_a - \vec{p}_c)$

$$E_x^2 = p_x^2 + m_x^2 = (\vec{p}_a - \vec{p}_c)^2 + m_x^2$$

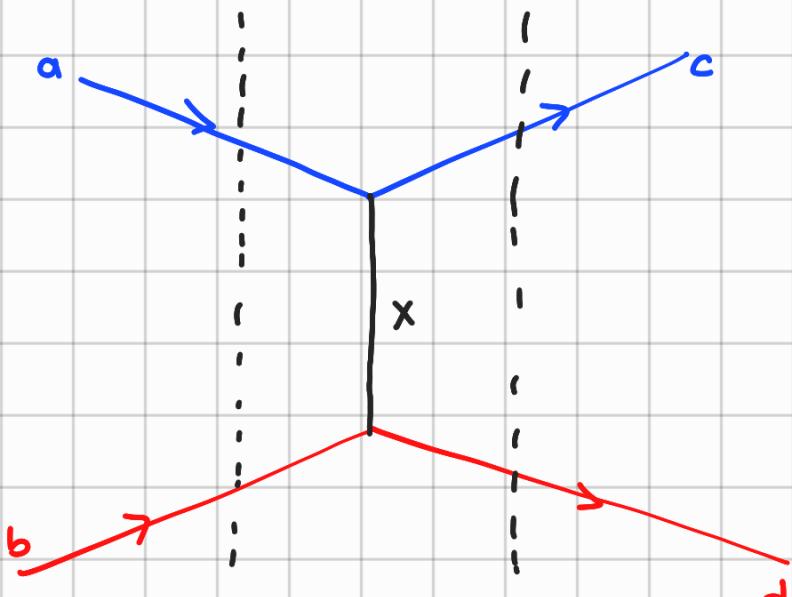
$$N_{fi} = \frac{g_a g_b}{(E_a - E_c)^2 - (\vec{p}_a - \vec{p}_c)^2 - Mx^2} = \frac{g_a g_b}{(p_a - p_c)^2 - Mx^2}$$

$$N_{fi} = \frac{g_a g_b}{q^2 - m^2}$$

$$\rightarrow q = p_a - p_c$$

\downarrow

4-momentum



Feynman Diagram

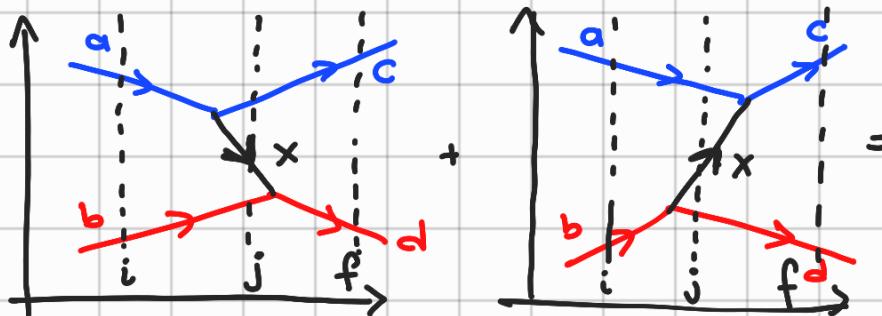
Note: Feynman Diagrams

→ Energy and Momentum are conserved at each int. vertex

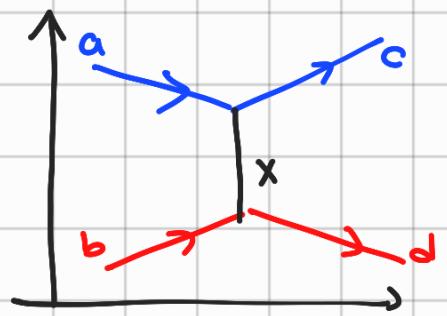
→ $\frac{1}{q^2 - M^2}$ is the propagator

Virtual Particles

Time-Ordered QM



Feynman Diagram



- Momentum conserved @ vertices
- energy NOT conserved @ vertices
- exchanged particle "on mass shell"

$$E_x^2 - |\vec{p}_x|^2 = m_x^2$$

- momentum & energy conserved @ vertices
- exchanged particle "off mass shell" virtual particle

$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

$$E_x^2 - |\vec{p}_x|^2 = q^2 \neq m_x^2$$

Quantum Electrodynamics - QED

Interaction of an electron and τ lepton by the exchange of a photon

$$\vec{p} \rightarrow \vec{p} - q\vec{A} \quad E \rightarrow E - q\phi \quad (q \rightarrow \text{charge})$$

in QM: $\vec{p} = -i\vec{\nabla}$ $E = i \frac{\partial}{\partial t}$

Therefore wave substitution $i\partial_\nu \rightarrow i\partial_\nu - qA_\nu$

where $A_\nu = (\phi, -\vec{A})$; $\partial_\nu = \left(\frac{\partial}{\partial t}, +\vec{\nabla} \right)$

Dirac Equation:

$$i\gamma^\mu \partial_\nu \Psi - m\Psi = 0 \Rightarrow i\gamma^\mu \partial_\nu \Psi - q\gamma^\mu A_\nu \Psi - m\Psi = 0$$

$$i\gamma^0 \frac{\partial \Psi}{\partial t} + i\vec{\gamma} \vec{\nabla} \Psi - q\gamma^\mu A_\mu \Psi - m\Psi = 0$$

$$i\gamma^0 \frac{\partial \Psi}{\partial t} = \gamma^0 \hat{H} \Psi = m\Psi - i\vec{\gamma} \vec{\nabla} \Psi + q\gamma^\mu A_\mu \Psi$$

$\times \gamma^0$ $\left(\gamma^0 \gamma^0 : I \right)$

$$\hat{H}\Psi = \underbrace{(\gamma_m - i\gamma^0 \vec{\gamma} \vec{\nabla}) \Psi}_{\text{Combined rest mass + } kE} + \underbrace{q\gamma^0 \gamma^\mu A_\mu \Psi}_{\text{potential energy}}$$

Combined rest mass + kE

potential
energy

$$\hat{V}_D = q \gamma^0 \gamma^N A_\mu$$

Note that: $q \gamma^0 \gamma^0 A_0 = q\phi$

$$A_\mu = \epsilon_\mu^{(\lambda)} e^{i(\vec{p} \cdot \vec{r} - Et)}$$

$$\epsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Two states of a real photon propagating
in \hat{z} direction

$$\epsilon_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}$$

$$\epsilon_+ = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}$$

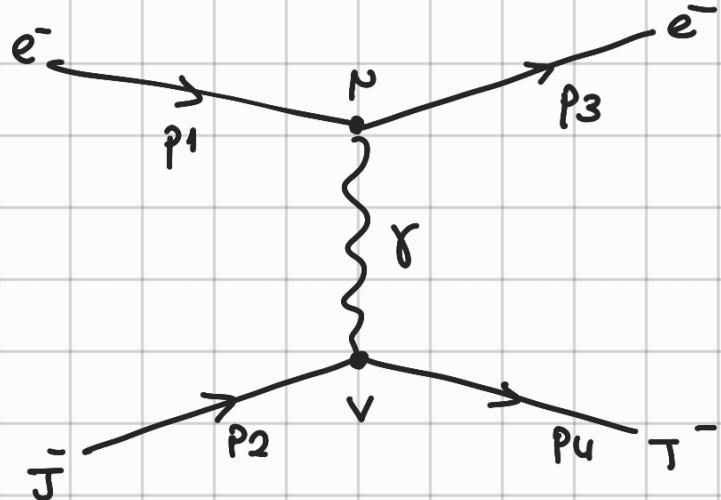
Circularly Polarized

Spinless Interaction:

$$H = \underbrace{\langle \Psi_C | V | \Psi_A \rangle}_{S_A} - \frac{1}{q^2 - \mu_x^2} \underbrace{\langle \Psi_D | V | \Psi_B \rangle}_{S_B}$$

$$M = \left[\bar{u}_e(p_3) q_e \gamma^0 \gamma^N u_e(p_1) \right] \sum_{\lambda} \epsilon_{\mu}^{\lambda} \frac{1}{q^2} \epsilon_{\nu}^{\lambda *} \left[\bar{u}_T(p_4) q_T \gamma^0 \gamma^V u_T(p_2) \right]$$

Interaction of \bar{e} with photon Sum over polarizations virtual photons interaction of \bar{J} with photon



$$\sum_{\lambda} \epsilon_{\mu}^{\lambda} (\epsilon_{\nu}^{\lambda})^* = -g_{\mu\nu}$$

Sum over the polarizations of VIRTUAL PHOTONS

$$\bar{\Psi} = \Psi^+ \gamma^0 \quad (\text{adjoint spinor})$$

$$M = \left[\bar{u}_e(p_3) q_e \gamma^0 \gamma^N u_e(p_1) \right] \frac{-g_{\mu\nu}}{q^2} \left[\bar{u}_T(p_4) q_T \gamma^0 \gamma^V u_T(p_2) \right]$$

4-vector

$$j_e^N = \bar{u}_e(p_3) \gamma^0 \gamma^N u_e(p_1)$$

$$j_T^V = \bar{u}_T(p_4) \gamma^0 \gamma^V u_T(p_2)$$

$$M = -q_e q_T \frac{\bar{j}_e \cdot \bar{j}_T}{q^2}$$

Basic Feynman Rules

- propagator factor for each internal line
"each virtual particle"
- Dirac spinor for each external line
"each real incoming or outgoing particle"
- vertex factor for each vertex

$-iM = \text{Product of all factors}$

external lines

spin $\frac{1}{2}$

{ incoming particle $\rightarrow \bullet$ $u(p)$

$\bullet \rightarrow$ $\bar{u}(p)$

{ outgoing particle

$\leftarrow \bullet$ $\bar{v}(p)$

{ incoming antiparticle

$\bullet \leftarrow$ $v(p)$

{ outgoing antiparticle

wavy $\epsilon^N(p)$

spin 1

{ incoming photon

wavy $\epsilon^N(p)^*$

{ outgoing photon

Internal lines:

spin 1

photon



$$\frac{-i\delta_{\mu\nu}}{q^2}$$

spin 1/2

fermion

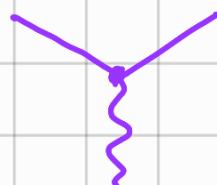


$$\frac{i(\gamma^N q_N + m)}{q^2 - m^2}$$

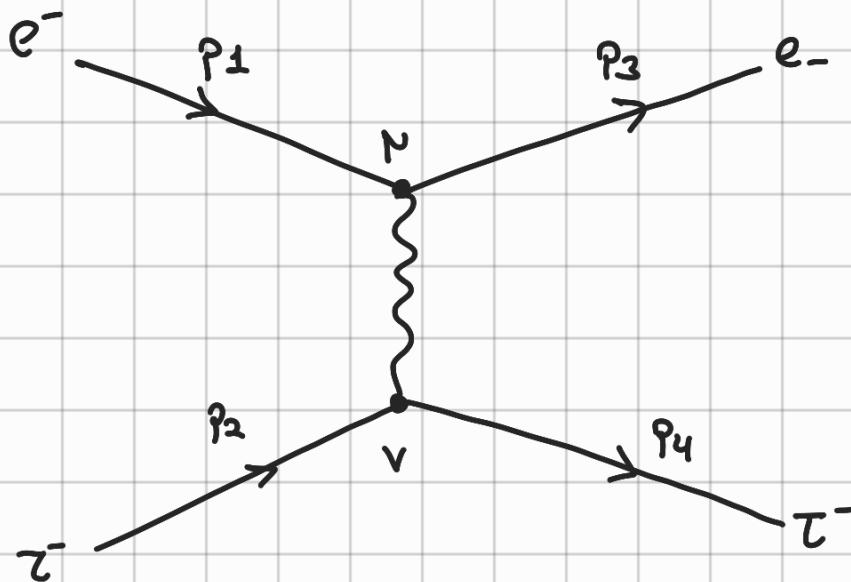
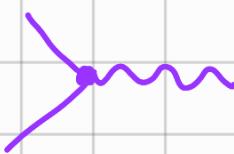
Vertex factors:

spin 1/2

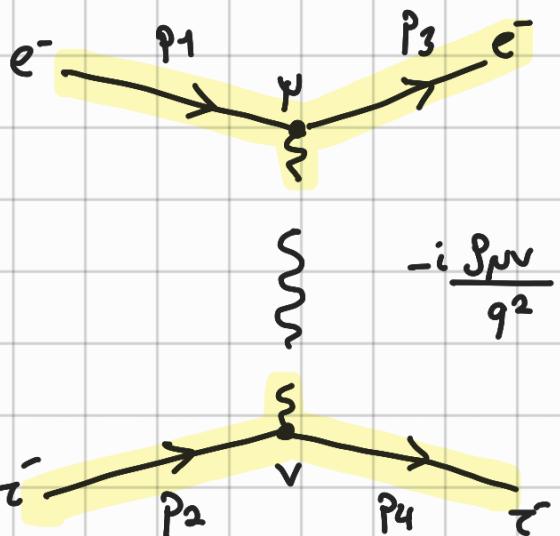
fermion (charge -|e|)



$$ie\gamma^N$$



$$\bar{v}_e(p_3) [ie\gamma^\mu] v_e(p_1)$$



$$\bar{v}_e(p_3) [ie\gamma^\nu] v_\tau(p_2)$$

$$-iM = [\bar{v}_e(p_3) ie\gamma^\mu v_e(p_1)] \frac{-i g_{\mu\nu}}{q^2} [\bar{v}_\tau(p_4) ie\gamma^\nu v_\tau(p_2)]$$