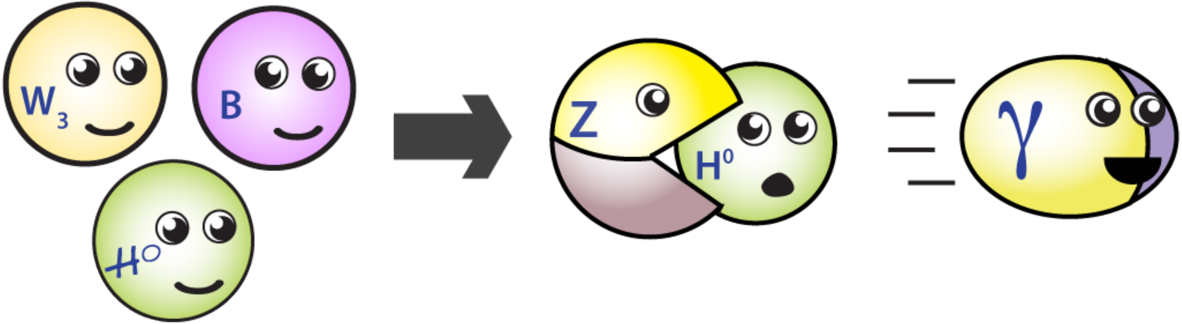
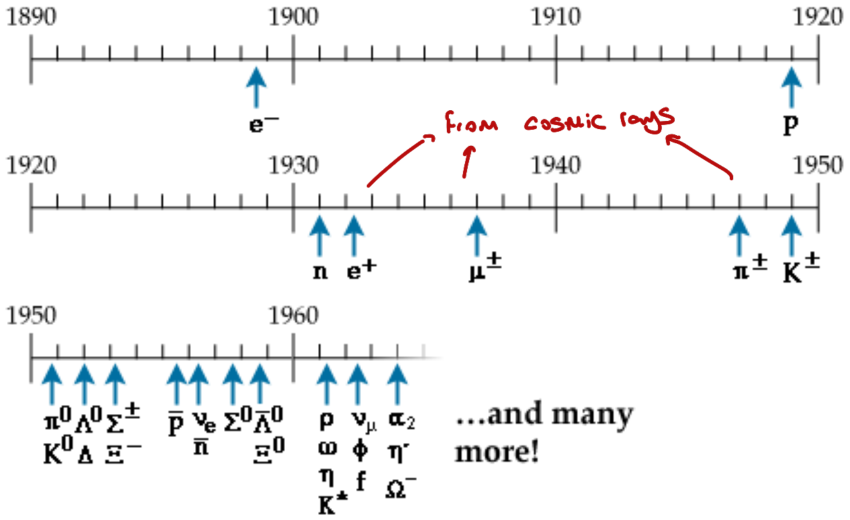


Particle Physics II
Lecture 8: Electroweak Unification and the W and Z Bosons

Lesya Shchutska

April 27, 2023

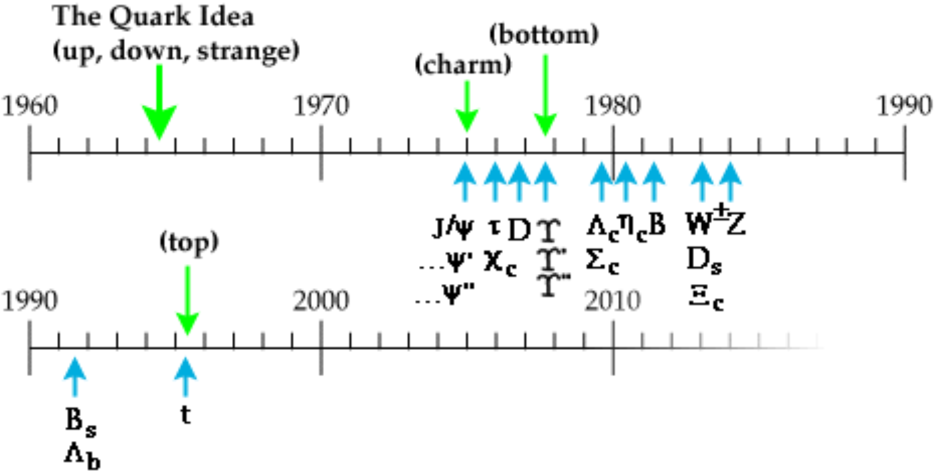
Step back: Discoveries in the past 100 years



Accidental discoveries: e.g. seeing new particles in the cosmic rays (e^+, $\mu^\pm$, $K^\pm$)

https://particleadventure.org/other/history/quantumt.html

Step back: Discoveries in the past 100 years



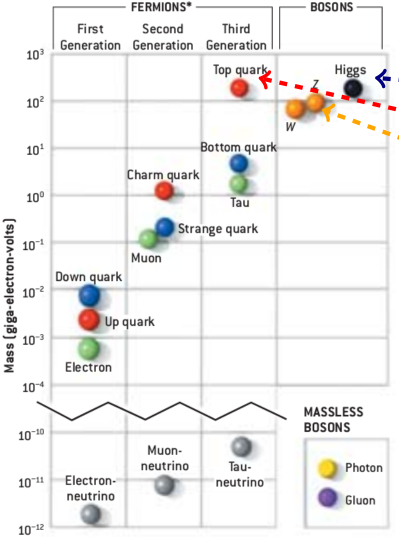
Theory-guided discoveries: e.g. J/ψ – needed a new quark to complete known u, d, s quarks, and have two complete generations

https://particleadventure.org/other/history/smt.html

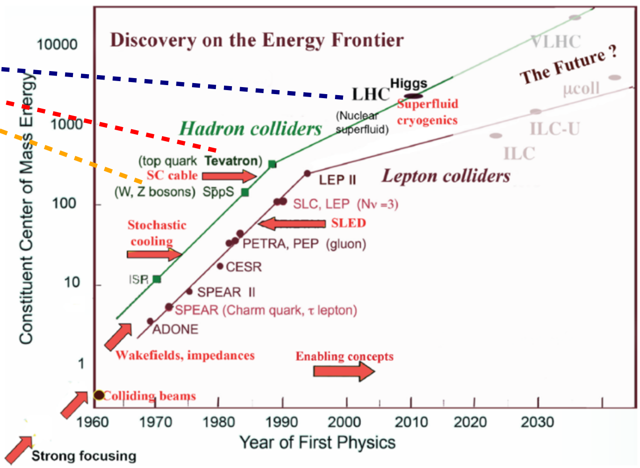
How to measure centre of mass energy?

The energy frontier for discoveries:

Known elementary particles



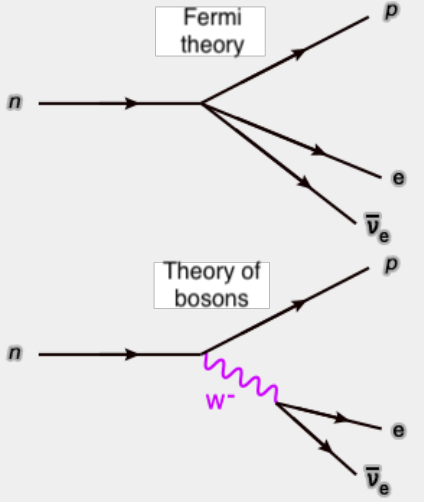
Tools to discover/study these particles



... guided by hints from other measurements!

Guidance for discoveries: W and Z

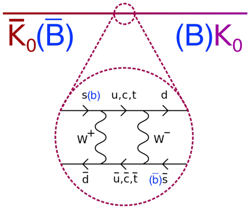
Starting with a β-decay discovery in 1896



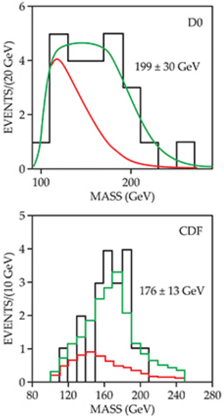
- description of β-decay (1934) suffered from divergences at high energies
- introduction of a heavy force carrier (1957) solved this problem
- first estimate of the "outrageous mass" $M_\mathrm{W} \approx 137 M_\mathrm{p}$ (1964)
- prediction and then evidence for neutral currents (1973)
- **dedicated construction:** upgrade of SPS (fixed target experiment) into SppS (collider) to be able to produce particles with masses of $\approx 100 M_\mathrm{p}$

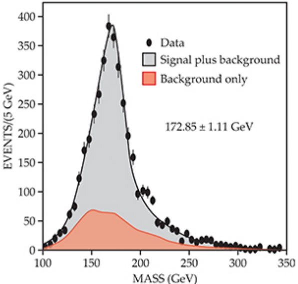
and leading to a discovery of W and Z bosons in 1983!

Guidance for discoveries: top quark



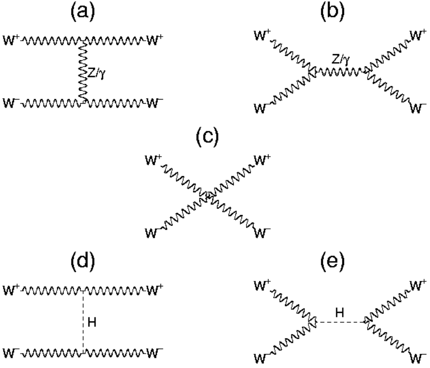
- $K_0 \leftrightarrow \bar{K}_0 \implies$ 3rd generation of quarks (b, t) existence (1973)
- $B_0 \leftrightarrow \bar{B}_0 \implies$ large top quark mass: $\gtrsim 50 \ldots 100$ GeV (1986/1987)
- precision measurements at LEP led to $m_{\text{top}} \approx 178 \pm 20$ GeV (1995)
- direct observation at the Tevatron (1995)





Guidance for discoveries: Higgs boson

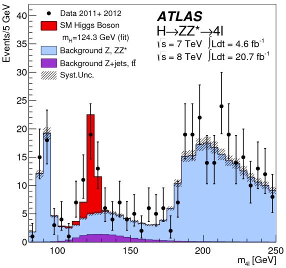
Theory developed and a new particle predicted in 1962-1964:

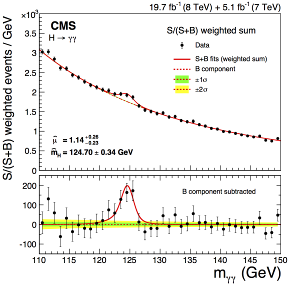


- something should happen below 1 TeV to avoid divergences in WW scattering
- $\Longrightarrow$ no-lose theorem for the LHC with $\sqrt{s} = 14$ TeV:
  - either a Higgs boson will be observed
  - or new phenomena appear below 1 TeV

Guidance for discoveries: Higgs boson

Higgs boson discovered in 2012!





W and Z bosons properties

- discovery of W and Z boson was a non-trivial task by itself
- was followed by the precise measurement of their properties at LEP and LEP2 accelerators
- these measurements were a key ingredient to the top quark and Higgs boson discoveries!
- and even now, precision W mass determination is one of few paramount experimental task
  - just because it is one of the fundamental SM parameters which allows to test SM consistency
- in this lecture we will consider in detail the decays of W and Z bosons

* Masses (80 GeV, 90 GeV)

* Their Decays

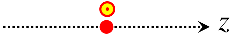
Boson Polarization States

- a real (i.e. not virtual) **massless** spin-1 boson can exist in two **transverse** polarization states
- a **massive** spin-1 boson can also be **longitudinally** polarized
- boson wave-functions are written in terms of the polarization four-vector ε^μ:

$$B^\mu = \varepsilon^\mu e^{-ip\cdot x} = \varepsilon^\mu e^{i(\vec{p}\cdot\vec{x}-Et)}$$

- for a spin-1 boson traveling along the z-axis, the polarization 4-vectors:

$$\varepsilon^\mu_- = \frac{1}{\sqrt{2}}(0,1,-i,0); \qquad \varepsilon_L = \frac{1}{m}(p_z,0,0,E) \qquad \varepsilon^\mu_+ = -\frac{1}{\sqrt{2}}(0,1,i,0)$$



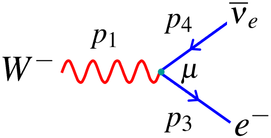
$\boxed{S_z = -1}$ $\qquad$ $\boxed{S_z = 0}$ $\qquad$ $\boxed{S_z = +1}$

transverse $\qquad$ **longitudinal** $\qquad$ **transverse**

Longitudinal polarization isn't present for on-shell massless particles, the photon can exist in two helicity states $h = \pm 1$ (LH and RH circularly polarized light)

W boson decay

- to calculate the W boson decay rate, first consider $W^- \to e^- \bar{\nu}_e$
- need matrix element for:



Incoming W-boson : $\varepsilon_\mu(p_1)$

Out-going electron : $\overline{u}(p_3)$

Out-going $\overline{\nu}_e$: $v(p_4)$

Vertex factor : $-i\frac{g_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)$

$$-iM_{fi} = \varepsilon_\mu(p_1) \cdot \bar{u}(p_3) \cdot -i\frac{g_W}{\sqrt{2}}\gamma^\mu \cdot \frac{1}{2}(1-\gamma^5) \cdot \nu(p_4)$$

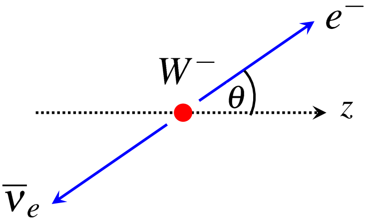
$$\Longrightarrow \; M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)\bar{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma^5)\nu(p_4)$$

- this can be written in terms of the 4-vector scalar product of the W boson polarization $\varepsilon_\mu(p_1)$ and the weak current j^μ:

  $M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1) \cdot j^\mu$ with $j^\mu = \bar{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma^5)\nu(p_4)$

W decay: the lepton current

- first consider the lepton current $j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)\nu(p_4)$
- work in the center-of-mass frame:



$$p_1 = (m_W, 0, 0, 0);$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta)$$
$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

with $E = \dfrac{m_W}{2}$

- in the ultra-relativistic limit only **LH particles** and **RH anti-particles** participate in the weak interaction, so:

$$j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)\nu(p_4) = \bar{u}_\downarrow(p_3)\gamma^\mu \nu_\uparrow(p_4)$$

Remember:

- $\frac{1}{2}(1-\gamma^5)\nu(p_4) = \nu_\uparrow(p_4)$ – chiral projection operator
- $\bar{u}(p_3)\gamma^\mu \nu_\uparrow(p_4) = \bar{u}_\downarrow(p_3)\gamma^\mu \nu_\uparrow(p_4)$ –"helicity conservation"

W decay: the lepton current

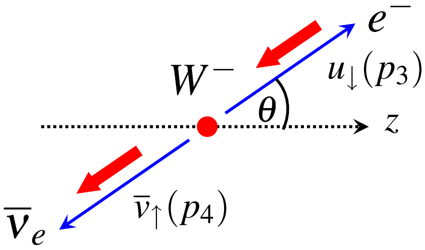
- we have already calculated the current

$$j^\mu = \bar{u}_\downarrow(p_3)\gamma^\mu \nu_\uparrow(p_4)$$

when considering $e^+e^- \to \mu^+\mu^-$

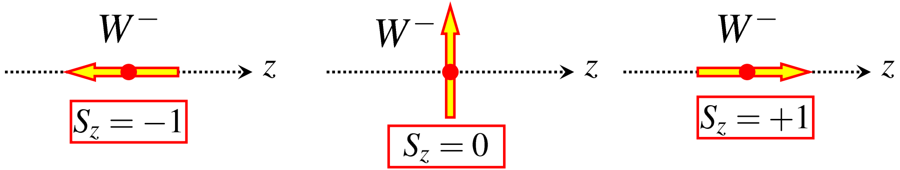
- we have for $\mu_L^-\mu_R^+$:

$$j^\mu_{\uparrow\downarrow} = 2E(0, -\cos\theta, -i, \sin\theta)$$



- for the charged current weak interaction we only have to consider this **single** combination of helicities:
  $j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)\nu(p_4) = \bar{u}_\downarrow(p_3)\gamma^\mu \nu_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$
  and the three possible W boson polarization states:

$$\varepsilon^\mu_- = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \quad \varepsilon_L = \frac{1}{m}(p_z, 0, 0, E); \quad \varepsilon^\mu_+ = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$



- for a W boson at rest these become:
  $\varepsilon_{-}^{\mu} = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$; $\varepsilon_{L} = (0, 0, 0, 1)$; $\varepsilon_{+}^{\mu} = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$
- the matrix elements for the different polarization states:
  $M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_{\mu}(p_1)j^{\mu}$ with $j^{\mu} = 2\frac{m_W}{2}(0, -\cos\theta, -i, \sin\theta)$

W decay

- giving

ε_-^μ: $M_- = \frac{g_W}{\sqrt{2}} \frac{1}{\sqrt{2}} (0, 1, -i, 0) \cdot m_W (0, -\cos\theta, -i, \sin\theta) = \frac{1}{2} g_W m_W (1 + \cos\theta)$

ε_L: $M_L = \frac{g_W}{\sqrt{2}} (0, 0, 0, 1) \cdot m_W (0, -\cos\theta, -i, \sin\theta) = -\frac{1}{\sqrt{2}} g_W m_W \sin\theta$

ε_+^μ: $M_+ = \frac{g_W}{\sqrt{2}} \frac{1}{\sqrt{2}} (0, 1, i, 0) \cdot m_W (0, -\cos\theta, -i, \sin\theta) = \frac{1}{2} g_W m_W (1 - \cos\theta)$

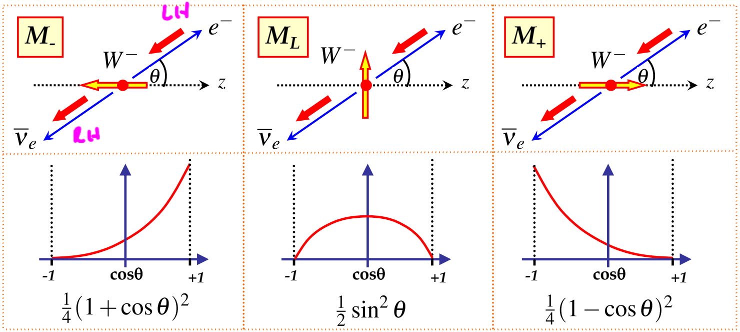
$$|M_-|^2 = g_W^2 m_W^2 \frac{1}{4} (1 + \cos\theta)^2 \tag{1}$$

$$|M_L|^2 = g_W^2 m_W^2 \frac{1}{2} \sin^2\theta \tag{2}$$

$$|M_+|^2 = g_W^2 m_W^2 \frac{1}{4} (1 - \cos\theta)^2 \tag{3}$$

W decay

- the angular distributions can be understood in terms of the spin of the particles:



$\theta = \pi$
$\cos\theta = -1$

- the differential decay rate can be found using:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$$

where p^* is the C.o.M. momentum of the final state particles, here $p^* = \frac{m_W}{2}$

- hence for the three different polarizations we obtain:

  $\frac{d\Gamma_-}{d\Omega} = \frac{g_W^2 m_W}{64\pi^2}\frac{1}{4}(1+\cos\theta)^2$; $\frac{d\Gamma_L}{d\Omega} = \frac{g_W^2 m_W}{64\pi^2}\frac{1}{2}\sin^2\theta$; $\frac{d\Gamma_+}{d\Omega} = \frac{g_W^2 m_W}{64\pi^2}\frac{1}{4}(1-\cos\theta)^2$;
- integrating over all angles using:

  $\int \frac{1}{4}(1\pm\cos\theta)^2\, d\phi\, d\cos\theta = \int \frac{1}{2}\sin^2\theta\, d\phi\, d\cos\theta = \frac{4\pi}{3}$
- gives: $\Gamma_- = \Gamma_L = \Gamma_+ = \frac{g_W^2 m_W}{48\pi}$
- total W decay rate is independent of polarization; this has to be the case as decay rate cannot depend on the arbitrary definition of the z-axis

- for a sample of unpolarized W bosons each polarization state is equally likely, for the average matrix element take sum over all possible matrix elements and average over the three initial polarization states:

$$
\begin{aligned}
\left\langle |M_{fi}|^2 \right\rangle &= \frac{1}{3}\left(|M_-|^2 + |M_L|^2 + |M_+|^2\right) && (4)\\
&= \frac{1}{3} g_W^2 m_W^2 \left[\frac{1}{4}(1+\cos\theta)^2 + \frac{1}{2}\sin^2\theta + \frac{1}{4}(1-\cos\theta)^2\right] && (5)\\
&= \frac{1}{3} g_W^2 m_W^2 && (6)
\end{aligned}
$$

- for a sample of unpolarized W, decays are isotropic (as expected)

no dependence on direction

- for this isotropic decay:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{|p^*|}{32\pi^2 m_W^2} \left\langle |M_{fi}|^2 \right\rangle \implies \Gamma = \frac{4\pi|p^*|}{32\pi^2 m_W^2} \left\langle |M_{fi}|^2 \right\rangle$$

$$\implies \boxed{\Gamma(W^- \to e^-\bar{\nu}) = \frac{g_W^2 m_W}{48\pi}}$$

W decay

- can not surpass Mass of W (then exclude t quark)
- width is proportional to $|M_{fi}|^2$
- "3" is coming from quarks

- the calculations for the other decay modes (neglecting final state particle masses) is the same. For quarks need to account for color and CKM matrix. No decays to t quark, since its mass (175 GeV) is greater than W boson mass (80 GeV)

$$
\begin{array}{lll}
W^- \to e^-\overline{\nu}_e & W^- \to d\overline{u} \quad \boxed{\times 3|V_{ud}|^2} & W^- \to d\overline{c} \quad \boxed{\times 3|V_{cd}|^2} \\
W^- \to \mu^-\overline{\nu}_\mu & W^- \to s\overline{u} \quad \boxed{\times 3|V_{us}|^2} & W^- \to s\overline{c} \quad \boxed{\times 3|V_{cs}|^2} \\
W^- \to \tau^-\overline{\nu}_\tau & W^- \to b\overline{u} \quad \boxed{\times 3|V_{ub}|^2} & W^- \to b\overline{c} \quad \boxed{\times 3|V_{cb}|^2}
\end{array}
$$

- unitarity of CKM matrix gives $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- hence $\boxed{\mathcal{B}(W \to q\bar{q}') = 6\mathcal{B}(W \to e\nu)}$ and the total decay rate:

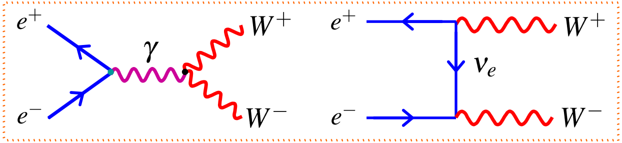
$$\boxed{\Gamma_W = 9\underbrace{\Gamma_{W\to e\nu}}_{11\%} = \frac{3g_W^2 m_W}{16\pi} = 2.07\,\text{GeV}}$$

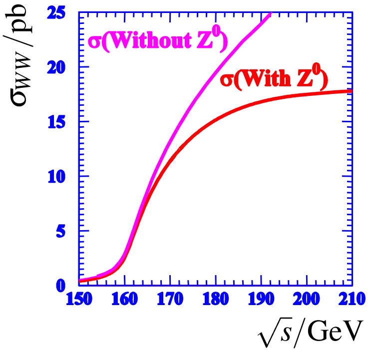
Experiment: 2.14±0.04 GeV
(our calculation neglected a 3% QCD correction to decays to quarks)

No need to compute at all!

- the $W^{\pm}$ bosons carry the EM charge: suggestive weak and EM forces are related

- W bosons can be produced in the $e^{+}e^{-}$ annihilation

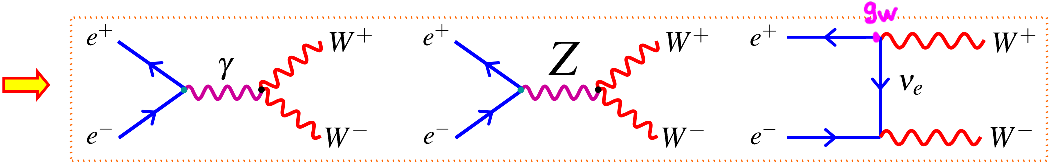




- with just these two diagrams there is a problem: the cross section increases with C.o.M. energy and at some points violates QM unitarity

UNITARITY VIOLATION: when QM calculation gives larger flux of W bosons than incoming flux of electrons/positrons

- problem can be "fixed" by introducing a new boson, the Z. The new diagram interferes negatively with the above two diagrams fixing the unitarity problem



$$|M_{\gamma WW} + M_{ZWW} + M_{\nu WW}|^2 < |M_{\gamma WW} + M_{\nu WW}|^2$$

- only works if Z, γ, W couplings are related: need **electroweak unification**

$\mathrm{SU(2)_L}$: the weak interaction

- the weak interaction arises from SU(2) local phase transformations:

$$\psi \to \psi' = \psi e^{i\vec{\alpha}(x)\cdot\frac{\vec{\sigma}}{2}}$$

where the $\vec{\sigma}$ are the generators of the SU(2) symmetry, i.e. the three Pauli spin matrices

$\Longrightarrow$ **3 Gauge bosons**: $W_1^\mu, W_2^\mu, W_3^\mu$

- the wave-functions have two components which, in analogy with isospin, are represented by "weak isospin"

- the fermions are placed in isospin doubles and the local phase transformation corresponds to:

$$\boxed{\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \to \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}' = e^{i\vec{\alpha}(x)\cdot\frac{\vec{\sigma}}{2}} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}}$$

SU(2)$_L$: the weak interaction

- weak interaction only couples to LH particles/RH antiparticles
- hence only place LH particles/RH antiparticles in weak isospin doubles: $I_W = \frac{1}{2}$
- RH particles/LH antiparticles placed in weak isospin singlets: $I_W = 0$

Weak Isospin

$$\boxed{I_W = \tfrac{1}{2}} \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad \begin{matrix} \leftarrow \boxed{I_W^3 = +\frac{1}{2}} \\ \leftarrow \boxed{I_W^3 = -\frac{1}{2}} \end{matrix}$$

$$\boxed{I_W = 0} \quad (\nu_e)_R,\ (e^-)_R, \ldots (u)_R,\ (d)_R, \ldots$$

Note: RH/LH refer to chiral states

singlets

flavour eigenstates
(not mass eigenstates)

$\mathbf{SU(2)_L}$: the weak interaction

- for simplicity only consider $\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$
- the gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2) (note: including interaction strength in current):

$$j^1_\mu = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2} \sigma_1 \chi_L; \tag{7}$$

$$j^2_\mu = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2} \sigma_2 \chi_L; \tag{8}$$

$$j^3_\mu = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2} \sigma_3 \chi_L; \tag{9}$$

- the charged current W^+/W^- interaction enters as a linear combinations of W_1, W_2:

$$\boxed{W^{\pm\mu} = \frac{1}{\sqrt{2}}(W_1^\mu \pm W_2^\mu)}$$

$\mathbf{SU(2)_L}$: the weak interaction

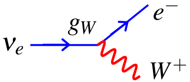
- the $W^{\pm}$ interaction terms:

$$j_{\pm}^{\mu} = \frac{g_W}{\sqrt{2}}(j_1^{\mu} \pm i j_2^{\mu}) = \frac{g_W}{\sqrt{2}}\bar{\chi}_L \gamma^{\mu} \frac{1}{2}(\sigma_1 \pm i\sigma_2)\chi_L$$

- express in terms of the weak isospin ladder operators: $\sigma_{\pm} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$

$$\boxed{j_{\pm}^{\mu} = \frac{g_W}{\sqrt{2}}\overline{\chi}_L \gamma^{\mu} \sigma_{\pm} \chi_L} \Big\} \text{ Origin of } \frac{1}{\sqrt{2}} \text{ in Weak CC}$$

W⁺



corresponds to $\boxed{j_{+}^{\mu} = \frac{g_W}{\sqrt{2}}\overline{\chi}_L \gamma^{\mu} \sigma_{+} \chi_L}$

which can be understood in terms of the weak isospin doublet

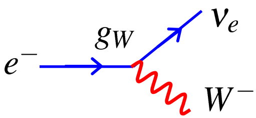
$$j_{+}^{\mu} = \frac{g_W}{\sqrt{2}}\bar{\chi}_L \gamma^{\mu} \sigma_{+} \chi_L = \frac{g_W}{\sqrt{2}}(\bar{\nu}_L \bar{e}_L)\gamma^{\mu} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad (10)$$

$$= \frac{g_W}{\sqrt{2}}\bar{\nu}_L \gamma^{\mu} e_L = \frac{g_W}{\sqrt{2}}\bar{\nu}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)e \quad (11)$$

$\mathrm{SU(2)_L}$: the weak interaction

- similarly:

W⁻



corresponds to $\boxed{j^\mu_- = \frac{g_W}{\sqrt{2}}\overline{\chi}_L \gamma^\mu \sigma_- \chi_L}$

$$j^\mu_- = \frac{g_W}{\sqrt{2}}\bar{\chi}_L \gamma^\mu \sigma_- \chi_L \tag{12}$$

$$= \frac{g_W}{\sqrt{2}}\begin{pmatrix}\bar{\nu}_L & \bar{e}_L\end{pmatrix}\gamma^\mu \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L \tag{13}$$

$$= \frac{g_W}{\sqrt{2}}\bar{e}_L \gamma^\mu \nu_L = \frac{g_W}{\sqrt{2}}\bar{e}\gamma^\mu \frac{1}{2}(1-\gamma^5)\nu \tag{14}$$

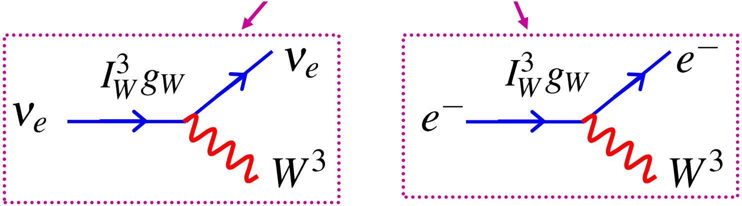
- however have an additional interaction due to W^3:

$$j_3^\mu = g_W \bar{\chi}_L \gamma^\mu \frac{1}{2}\sigma_3 \chi_L$$

- expanding this:

$$j_3^\mu = g_W \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L \tag{15}$$

$$= g_W \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - g_W \frac{1}{2} \bar{e}_L \gamma^\mu e_L \tag{16}$$



$\Longrightarrow$ **Neutral current interactions!**

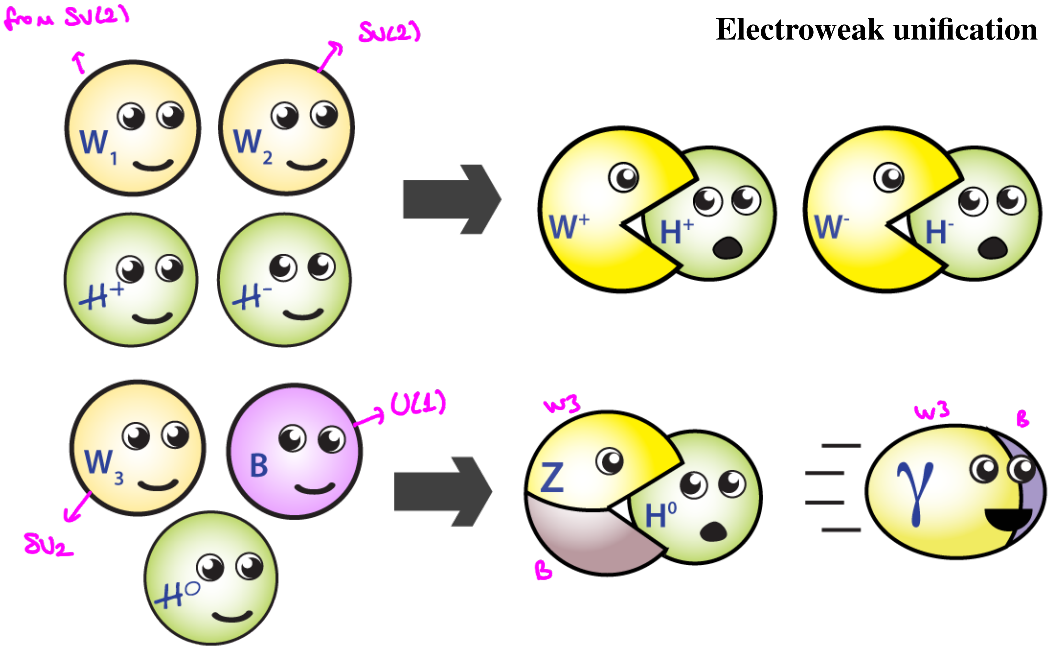
Electroweak unification

- it is tempting to identify W^3 as the Z boson
- however this is not the case: have two physical neutral spin-1 gauge bosons (γ, Z), and the W^3 is a mixture of the two
- equivalently, can write the photon and Z in terms of the W^3 and a new neutral spin-1 boson, the B
- the **physical** bosons (the Z and photon field, A) are:

$$\boxed{\begin{aligned} A_\mu &= B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W && (17)\\ Z_\mu &= -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W && (18)\\ &\theta_W \text{ is the weak mixing angle} && (19)\end{aligned}}$$

- the new boson is associated with a new gauge symmetry similar to that of electromagnetism: $U(1)_Y$
- the charge of this symmetry is called **weak hypercharge Y**

Electroweak unification



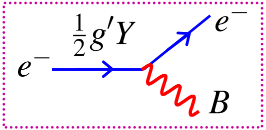
- the charge of this symmetry is called **weak hypercharge Y**:

$$Y = 2Q - 2I_W^3, \tag{20}$$

$$Q \text{ is the EM charge of a particle,} \tag{21}$$

$$I_W^3 \text{ is the third component of weak isospin} \tag{22}$$

- by convention, the coupling to the B_μ is $\frac{1}{2}g'Y$:



$$e_L : Y = 2(-1) - 2(-\frac{1}{2}) = -1 \tag{23}$$

$$e_R : Y = 2(-1) - 2(0) = -2 \tag{24}$$

$$\nu_L : Y = +1 \tag{25}$$

$$\nu_R : Y = 0 \tag{26}$$

- for this to work, the coupling constants of the W^3, B, and photon must be related
- consider contributions involving the neutral interactions of electrons:

$$\gamma : j_\mu^{em} = e\bar{\psi}Q_e\gamma_\mu\psi = e\bar{e}_L Q_e \gamma_\mu e_L + e\bar{e}_R Q_e \gamma_\mu e_R \tag{27}$$

$$W^3 : j_\mu^{W^3} = -\frac{g_W}{2}\bar{e}_L\gamma_\mu e_L \tag{28}$$

$$B : j_\mu^{Y} = \frac{g'}{2}\bar{\psi}Y_e\gamma_\mu\psi = \frac{g'}{2}\bar{e}_L Y_{e_L}\gamma_\mu e_L + \frac{g'}{2}\bar{e}_R Y_{e_R}\gamma_\mu e_R \tag{29}$$

- the relation $A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$ is equivalent to requiring

$$\boxed{j_\mu^{em} = j_\mu^{Y}\cos\theta_W + j_\mu^{W^3}\sin\theta_W}$$

- writing this in full:

$$e\bar{e}_L Q_e \gamma_\mu e_L + e\bar{e}_R Q_e \gamma_\mu e_R = \frac{1}{2} g' \cos\theta_W \left[\bar{e}_L Y_{e_L} \gamma_\mu e_L + \bar{e}_R Y_{e_R} \gamma_\mu e_R\right] \tag{30}$$

$$- \frac{1}{2} g_W \sin\theta_W \left[\bar{e}_L \gamma_\mu e_L\right] \tag{31}$$

$$-e\bar{e}_L \gamma_\mu e_L - e\bar{e}_R \gamma_\mu e_R = \frac{1}{2} g' \cos\theta_W \left[-\bar{e}_L \gamma_\mu e_L - 2\bar{e}_R \gamma_\mu e_R\right] \tag{32}$$

$$- \frac{1}{2} g_W \sin\theta_W \left[\bar{e}_L \gamma_\mu e_L\right] \tag{33}$$

EM   W boson   Z boson

which works if $e = g_W \sin\theta_W = g' \cos\theta_W$ (equate coefficients of L and R terms)

- couplings of electromagnetism, the weak interaction and the interaction of the $\mathrm{U}(1)_\mathrm{Y}$ symmetry are therefore related

The Z boson

- in this model we can now derive the couplings of the Z boson:

$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

$$j_\mu^Z = -\frac{1}{2} g' \sin\theta_W \left[\bar{e}_L Y_{e_L} \gamma_\mu e_L + \bar{e}_R Y_{e_R} \gamma_\mu e_R\right] - \frac{1}{2} g_W \cos\theta_W \left[\bar{e}_L \gamma_\mu e_L\right]$$

- writing this in terms of weak isospin and charge:

$$j_\mu^Z = -\frac{1}{2} g' \sin\theta_W \left[\bar{e}_L (2Q - 2I_W^3) \gamma_\mu e_L + \bar{e}_R (2Q) \gamma_\mu e_R\right] + I_W^3 g_W \cos\theta_W \left[\bar{e}_L \gamma_\mu e_L\right]$$

- gathering up the terms for LH and RH chiral states:

$$j_\mu^Z = \left[g' I_W^3 \sin\theta_W - g' Q \sin\theta_W + g_W I_W^3 \cos\theta_W\right] \bar{e}_L \gamma_\mu e_L - \left[g' Q \sin\theta_W\right] \bar{e}_R \gamma_\mu e_R$$

The Z boson

- finally, using $e = g_W \sin\theta_W = g' \cos\theta_W$ gives:

$$j_\mu^Z = \left[g' \frac{I_W^3 - Q\sin^2\theta_W}{\sin\theta_W} \right] \bar{e}_L \gamma_\mu e_L - \left[g' \frac{Q\sin^2\theta_W}{\sin\theta_W} \right] \bar{e}_R \gamma_\mu e_R$$

$$j_\mu^Z = g_Z (I_W^3 - Q\sin^2\theta_W) \left[\bar{e}_L \gamma_\mu e_L \right] - q_Z Q \sin^2\theta_W \left[\bar{e}_R \gamma_\mu e_R \right]$$

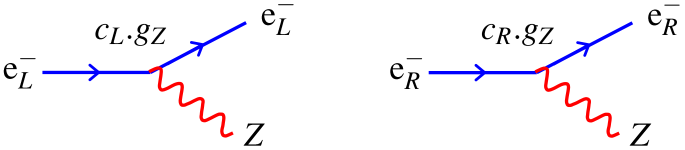
with $\boxed{e = g_Z \cos\theta_W \sin\theta_W \text{ or } g_Z = \frac{g_W}{\cos\theta_W}}$

The Z boson

- unlike for the charged current weak interaction (W), the Z boson couples to both LH and RH chiral components, but not equally:

$$j_\mu^Z = g_Z(I_W^3 - Q\sin^2\theta_W)\,[\bar{e}_L\gamma_\mu e_L] - g_Z Q\sin^2\theta_W\,[\bar{e}_R\gamma_\mu e_R] \quad (34)$$

$$= g_Z c_L\,[\bar{e}_L\gamma_\mu e_L] + g_Z c_R\,[\bar{e}_R\gamma_\mu e_R] \quad (35)$$



$\boxed{c_L = I_W^3 - Q\sin^2\theta_W}$ $\quad$ $\boxed{c_R = -Q\sin^2\theta_W}$

W^3 part of Z couples only to LH components (like $W^\pm$)

B_μ part of Z couples equally to LH and RH components

- use projection operators to obtain vector and axial couplings:
  $\bar{u}_L \gamma_\mu u_L = \bar{u}\gamma_\mu \frac{1}{2}(1-\gamma_5)u \quad \bar{u}_R \gamma_\mu u_R = \bar{u}\gamma_\mu \frac{1}{2}(1+\gamma_5)u$

$$j_\mu^Z = g_Z \bar{u}\gamma_\mu \left[c_L \frac{1}{2}(1-\gamma_5) + c_R \frac{1}{2}(1+\gamma_5) \right] u$$

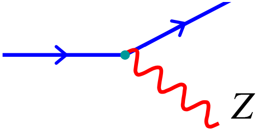
$$j_\mu^Z = \frac{g_Z}{2} \bar{u}\gamma_\mu \left[(c_L + c_R) + (c_R - c_L)\gamma_5 \right] u$$

- which in terms of V and A components gives:
  $j_\mu^Z = \frac{g_Z}{2} \bar{u}\gamma_\mu \left[c_V - c_A \gamma_5 \right] u$
  with
  $c_V = c_L + c_R = I_W^3 - 2Q\sin^2\theta_W$, $c_A = c_L - c_R = I_W^3$

The Z boson

- hence the vertex factor for the Z boson is:

$$\boxed{-ig_Z\frac{1}{2}\gamma_\mu\left[c_V - c_A\gamma_5\right]}$$



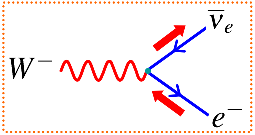
- using the experimentally determined value of the weak mixing angle:

$$\boxed{\sin^2\theta_W \approx 0.23}$$

Fermion	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

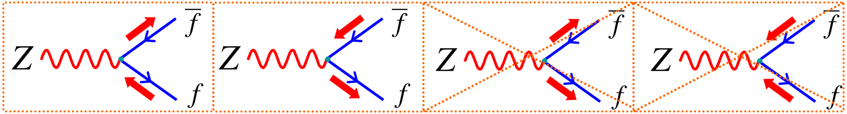
Z boson decay: Γ_Z

- in W decays only had to consider one helicity combination (assuming we can neglect final state masses: helicity states = chiral states)



W-boson couples:
to LH particles
and RH anti-particles

- but Z boson couples to LH and RH particles (with different strengths)
- need to consider only two helicity (or more correctly chiral) combinations:



- this can be seen by considering either of the combinations yielding 0:

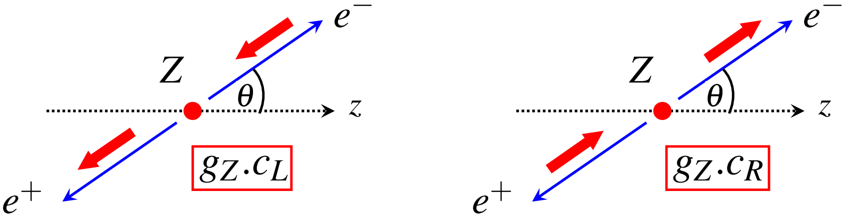
$$\bar{u}_R\gamma^\mu(c_V + c_A\gamma^5)\nu_R = u^\dagger\frac{1}{2}(1+\gamma^5)\gamma^0\gamma^\mu(c_V + c_A\gamma^5)\frac{1}{2}(1-\gamma^5)\nu \tag{36}$$

$$= \frac{1}{4}u^\dagger\gamma^0(1-\gamma^5)\gamma^\mu(1-\gamma^5)(c_V + c_A\gamma^5)\nu \tag{37}$$

$$= \frac{1}{4}\bar{u}\gamma^\mu(1+\gamma^5)(1-\gamma^5)(c_V + c_A\gamma^5)\nu = 0 \tag{38}$$

Z boson decay: Γ_Z

- in terms of left- and right-handed combinations need to calculate:



- for unpolarized Z bosons:

$$\left\langle |M_{fi}|^2 \right\rangle = \frac{1}{3}\left[2c_L^2 g_Z^2 m_Z^2 + 2c_R^2 g_Z^2 m_Z^2\right] = \frac{2}{3} g_Z^2 m_Z^2 (c_L^2 + c_R^2)$$

- using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$

$$\Longrightarrow \ \Gamma(Z \to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

Z branching ratios

- (neglecting fermion masses) obtain the same expression for the other decays:

$$\Gamma(Z \to f\bar{f}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

- using values for c_V and c_A listed previously, obtain:

$$\mathcal{B}(Z \to e^+e^-) = \mathcal{B}(Z \to \mu^+\mu^-) = \mathcal{B}(Z \to \tau^+\tau^-) \approx 3.5\% \tag{39}$$

$$\mathcal{B}(Z \to \nu_1\bar{\nu}_1) = \mathcal{B}(Z \to \nu_2\bar{\nu}_2) = \mathcal{B}(Z \to \nu_3\bar{\nu}_3) \approx 6.9\% \tag{40}$$

$$\mathcal{B}(Z \to d\bar{d}) = \mathcal{B}(Z \to s\bar{s}) = \mathcal{B}(Z \to b\bar{b}) \approx 15\% \tag{41}$$

$$\mathcal{B}(Z \to u\bar{u}) = \mathcal{B}(Z \to c\bar{c}) \approx 12\% \tag{42}$$

Z branching ratios

- the Z boson therefore predominantly decays to hadrons:
  $\mathcal{B}(Z \to \text{ hadrons}) \approx 69\%$ – mainly due to factor 3 from color
- also predict total decay rate (total width):

$$\Gamma_Z = \sum_i \Gamma_i = 2.5 \text{ GeV}$$

Experiment: $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$

Summary

- the standard model interactions are mediated by spin-1 gauge bosons
- the form of interactions are completely specified by assuming an underlying local phase transformation – **Gauge invariance**:

$U(1)_{em}$	⇒	QED
$SU(2)_L$	⇒	Charged Current Weak Interaction + W^3
$SU(3)_{col}$	⇒	QCD

- in order to "unify" the electromagnetic and weak interactions, introduced a new gauge symmetry: U(1) hypercharge

$U(1)_Y$	⇒	B_μ

Summary

- the physical Z boson and the photon are mixtures of the neutral W boson and B determined by the weak mixing angle:

$$\sin\theta_W \approx 0.23$$

- have we really unified the EM and weak interactions? Not really:
  - started with two independent theories with coupling constants g_W, e
  - ended up with coupling constants which are related but at the cost of introducing a new parameter to the standard model – θ_W
  - interactions not unified from any higher theoretical principle, but it works!