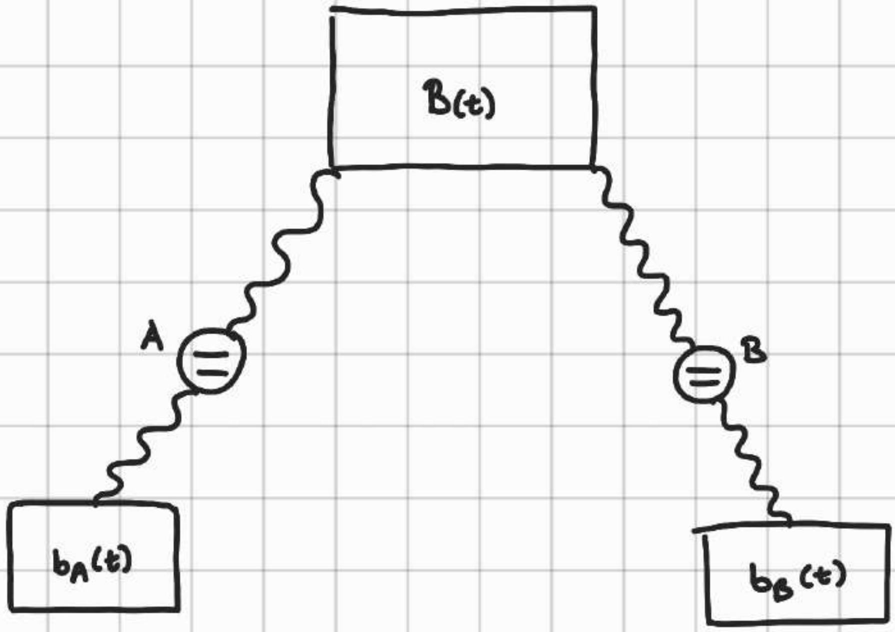
10/03/23

1- Find $\rho(t) = \langle\langle\langle \rho_{st}(t) \rangle\rangle\rangle$



Two qubits A and B that are coupled to a noisy environment both singly and collectively

$$H = -\mu \frac{\hbar}{2} \vec{S} \cdot \vec{B} \quad \text{where} \quad \vec{S} = (\sigma_x, \sigma_y, \sigma_z)$$

$$\vec{B} = (B_x, B_y, B_z)$$

$$H(t) = -\frac{1}{2}\mu \left[B(t)(\sigma_z^A + \sigma_z^B) + b_A(t)\sigma_z^A + b_B(t)\sigma_z^B \right]$$

For simplicity only z component is dealed

Also, $B(t)$, $b_A(t)$, $b_B(t)$ are statistically independent Markov processes satisfying:

(Gaussian)

$$\langle B(t) \rangle = 0 \qquad\qquad \langle b_i(t) \rangle = 0$$

$$\langle B(t)\, B(t') \rangle = \frac{\Gamma}{\mu^2}\delta(t-t') \qquad\qquad \langle b_i(t)\, b_i(t') \rangle = \frac{\Gamma_i}{\mu^2}\delta(t-t')$$

White Noise

$i = A, B$

$$\rho(t) = \langle\langle\langle \rho_{st}(t) \rangle\rangle\rangle$$

$$\rho_{st}(t) = U(t)\,\rho(0)\,U^{+}(t)$$

Statistical density operator

$$U(t) = \exp\left[-i \int_0^t dt' \, H(t')\right]$$

time-dependent Hamiltonian

$$\langle \omega \rangle = \mathrm{Tr}\left[\underbrace{U(t)\,\rho(0)\,U^{+}(t)}_{\rho_{st}(t)}\,\omega\right]$$

ensemble operator

$$\langle A \rangle = \mathrm{Tr}[\rho A]$$

Using cyclicity:

$$\langle \rho(t) \rangle = \mathrm{Tr}\left[U(t)\,\rho(0)\left(\underbrace{\omega\,U^{+}(t)}_{1}\right)\right]$$

$$\langle\langle\langle U(t) \rangle\rangle\rangle = \langle\langle\langle \exp\left(-i \int dt' \, H(t')\right) \rangle\rangle\rangle$$

$$= \exp\left(-i \int d(s)\, \cancel{b_A(s)}^{0}\, \sigma_z^A\right) \exp\left(-i \int d(s)\, \cancel{b_B(s)}^{0}\, \sigma_z^B\right)$$

$$\exp\left(-i \int d(s)\, \cancel{B(s)}^{0}\, (\sigma_z^A + \sigma_z^B)\right) = 1$$

$$\langle \rho(t) \rangle = \mathrm{Tr}\left[U(t)\, \rho(0) \right]$$

$$U(t) \to \exp\left[-i \int_0^t t'\, H(t')\right]$$

$$\rho(t) = \mathrm{Tr}\left(\begin{bmatrix} e^{-i\int_0^t t' H(t')} & 0 \\ 0 & e^{-i\int_0^t t' H(t')} \end{bmatrix} \begin{bmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{bmatrix} \right)$$

$$= \mathrm{Tr}\left(\begin{bmatrix} e^{-i\int_0^t t' H(t')} \rho_{11}(0) & \sim \\ \sim & e^{-i\int_0^t t' H(t')} \rho_{22}(0) \end{bmatrix} \right)$$

$$\rho(t) = \underbrace{e^{-i\int_0^t t' H(t')}}_{U(t)} \left(\rho_{11}(0) + \rho_{22}(0) \right)$$

→ time-ordered exponentials

Using <u>Baker - Campbell - Hausdorff</u> formula (BCH):

$$U(t) = \exp(itA) \cdot \exp(tB)$$

$$A = -\frac{1}{2} \nu \left[B(\sigma_z^A + \sigma_z^B) + b_A \sigma_z^A + b_B \sigma_z^B \right]$$

?

$$B = -\frac{1}{4} \nu^2 \left[\Gamma(\sigma_z^A + \sigma_z^B) + \Gamma_A \sigma_z^A + \Gamma_B \sigma_z^B \right]$$

$$\rho(t) = U(t)\left(\rho_{11}(0) + \rho_{22}(0)\right) = e^{itA} e^{tB}\left(\rho_{11}(0) + \rho_{22}(0)\right)$$

$$\langle \exp(A) \rangle = \langle \mathbb{1} + A + \frac{A^2}{2!} + \frac{A^3}{3!} \cdots \rangle$$

$$= \mathbb{1} + \langle A \rangle + \frac{1}{2!}\langle A^2 \rangle \neq \exp(\langle A \rangle)$$

$$\langle A^2 \rangle \neq \langle A \rangle \langle A \rangle$$

$$\int_0^t dt' \mathcal{H}(t') \quad , \quad \int_0^t ds' \mathcal{H}(s')$$

$$= \int_0^t \int_0^t dt' ds' \left(b_A(t') b_A(s') \right.$$

2. How to find Krauss Operators

$$\rho(t) = \mathcal{E}(\rho) = \sum_{\mu=1}^{N} K_\mu^+(t)\, \rho(0)\, K_\mu(t)$$

$$\sum_\mu K_\mu^+ K_\mu$$

Given $\rho(t)$

1. Calculate the eigenvalue decomposition of $\rho(t)$

$$\rho(t) = \sum_n \lambda_n |n\rangle\langle n|$$

2. Define set of operators E_k

$$E_k = \sum_{n,m} c_{k,n,m} |n\rangle\langle m|$$

3. Solve for the coefficients

$$E_k\, \rho(t)\, E_k^+ = \sum_{n,m} c_{k,n,m}\, c^*_{k,n',m'}\, \lambda_m |n\rangle\langle m|$$

$$= \sum_n \lambda_n E_k |n\rangle\langle n| E_k^+$$

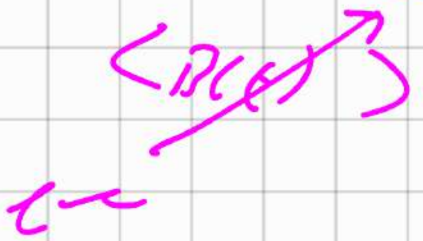
$$\sum_k c_{k,n,m}\, c^*_{k,n',m'} = \lambda_n\, \delta_{n,n'}\, \delta_{m m'}$$

This can be written in matrix form $CC^\dagger = \Lambda$

diagonal matrix with entries λ_n

Here, $\rho(t) = \tilde{\mathcal{E}}(\rho(0)) = \sum_{i,j=1}^{2} \sum_{k=1}^{3} D_k^\dagger E_j^\dagger F_i^\dagger \rho(0) F_i E_j D_k$

SVD $\rightsquigarrow$ Singular Value Decomposition ?



$$\langle A \rangle \equiv \bar{A} = \sum_i A(i)\, p(i) = \frac{\sum_i A(i)\, e^{-\frac{\mathcal{H}(i)}{k_B T}}}{\sum_i e^{-\frac{\mathcal{H}(i)}{k_B T}}}$$

$$\sum_i p_i = 1$$

$$= \frac{\int_0^\infty \rho(t)\, e^{-\frac{\mathcal{H}(t)}{k_B T}}}{\int_0^\infty e^{-\frac{\mathcal{H}(t)}{k_B T}}} \equiv \langle \rho(t) \rangle$$

$$\rho_{st}(t) = U(t)\, \rho_{st}(0)\, U^\dagger(t)$$

$$\langle A \rangle =$$

$$\rho(t) = \langle\langle\langle \rho_{st}(t) \rangle\rangle\rangle$$

$$\rho_{st}(t) = U(t)\,\rho(0)\,U^{+}(t)$$ How to create an operator using initial density matrix

$$U(t) = \exp\left[-i \int_0^t dt' H(t') \right]$$

$$H(t) = -\frac{1}{2}\mu \left[B(t)\,(\sigma_z^A + \sigma_z^B) + b_A(t)\,\sigma_z^A + b_B(t)\,\sigma_z^B \right]$$

✓ 1. What is the difference between $\rho(t)$ operator and density matrix? ⟷ Same

$$\rho_{st}(t) = e^{-i\int_0^t ds\, H(s)}\ \rho(0)\ e^{+i\int_0^t ds'\, H(s')}$$

2. How to make the system entangled?

3. Trotter - Suzuki Formula

4. Matrix Diagonalization-

5. Time Evolution by a numerical Method

6. Time- Ordered Hamiltonian Magnus Expansion

1. What's the definition to be an operator?

$$\rho_{st}(t) = U(t)\,\rho(0)\,U^{+}(t)$$

Let's define an ensemble Operator O (over three noise fields)

$$\langle \hat{O} \rangle = \mathrm{Tr}\left[\rho_{st}(t)\ \hat{O} \right]$$

$$= \mathrm{Tr}\left[U(t)\,\rho(0)\,U^{+}(t)\ \hat{O} \right]$$

$$= \mathrm{Tr}\left[U(t)\,\rho(0)\,\hat{O}\,U^{+}(t) \right]$$

$$\hat{O}\,U^{+}(t) = \langle\langle\langle U^{+}(t) \rangle\rangle\rangle$$

$$\langle\langle\langle U^{+}(t) \rangle\rangle\rangle = \langle\langle\langle e^{+i\int_0^t ds\, H(s)} \rangle\rangle\rangle$$

$$\langle \exp(A) \rangle = \langle \mathbb{I} + A + \frac{A^2}{2!} + \dots \rangle$$

Taylor Expansion

$$= \mathbb{I} + \langle A \rangle + \frac{\langle A^2 \rangle}{2} + \dots$$

$$= \mathbb{I} + \left\langle i\int_0^t ds\, H(s) \right\rangle + \left\langle \left(i\int_0^t ds\, H(s) \right)^2 \right\rangle + \dots$$