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Entanglement Generation for Two Coupled Multi-excitation Fields Interacting with Qubits

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Abstract We study the entanglement dynamics for two coupled multi-excitation cavity fields, each independently interacting with a two-level atom. We find that even when the atom-field system is initially not entangled, the fields can become heavily entangled through postselecting of the atomic state. The field entanglement entropy is dependent on the total excitation number but independent of the coherent photon hopping strength.

Keywords Entanglement entropy · Multi-excitation field · Coupled cavity · Postselection

1 Introduction

Dynamical behavior of entanglement has been extensively studied in various quantum systems and become a central issue in the quantum information processing (QIP) [1–4]. However, only the entanglement for two low-dimensional systems, such as two two-level atoms, spins and quantum dots, is well quantified and analytically easy to treat, and entanglement in the high dimensional Hilbert space that can be mathematically handy is few. Though the entanglement dynamics of high dimensional quantum states, such as coherent [5–7] and thermal states [8–10], has been investigated in different setups, in the context of the coupled cavity system which is an ideal candidate for implementing the distributed QIP, most researches deal with the interaction between qubits and the few-excitation fields which is confined in the low-dimensional Hilbert space [11–16]. In fact, previous efforts [5, 6] were devoted to a special setup where the separate high-dimension nodes do not interact with each other. Considering the recent experiment progresses in the arrays of interacting micro-cavities and their coupling to atoms [17–22], it is quite necessary to investigate the entanglement dynamics for qubits in coupled cavities with multi-excitation fields, which is believed to reveal the rudimentary physical mechanism of the interacting infinite-dimension systems. The system evolution between qubits and the coupled multi-excitation fields has not been extensively studied due to their complicated mutual-interaction process. There has

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been a previous work on the atomic entanglement induced by two thermal fields coupled to each other under the large detuning condition [23].

The entanglement dynamics of two maximally entangled two-level atoms in the coupled-cavity system with the initial vacuum field state has been discussed, in which both the photon loss and the atomic spontaneous emission are ignored [11]. Here, only the two-dimensional field states containing zero and one photon are involved. However, if there is one or more extra photons in the initial field state, how the entanglement dynamics of this system evolves differently? In this paper, we answer the above question by analytically and numerically studying the dynamics of the system with extra photons in the initial field state based on the similar setup in Ref. [11] via postselection. Considering the case that the atoms are both initially in the excited states or the ground states, we find that even when the initial states of the atoms and fields are not entangled, the field states can become maximally entangled via the Jaynes-Cummings interaction and the coherent photon hopping, and the maximal field entanglement is dependent on the total excitation number but independent of the coherent photon hopping strength. It is interesting to note that the entanglement of the multi-excitation fields can keep larger than one whenever the atoms leave the cavity in their ground states, and the maximal field entanglement becomes larger as the excitation number increases.

2 Dynamics for the Two-Excitation Fields

Two identical two-level atoms 1 and 2 are trapped in two directly coupled cavities, each containing a multi-excitation field. Taking the resonant interaction between the atoms and multi-excitation cavity fields into consideration, the interaction Hamiltonian under the rotating-wave approximation is ($\hbar = 1$):

$$H = g\sum_{i=1}^{2}\left(S_i^{+}a_i + S_i^{-}a_i^{\dagger}\right) + J\left(a_1^{\dagger}a_2 + a_1 a_2^{\dagger}\right), \tag{1}$$

where $S_i^{+} = |e_i\rangle\langle g_i|$ and $S_i^{-} = |g_i\rangle\langle e_i|$ with $|e_i\rangle$ and $|g_i\rangle$ being the excited state and ground state of the ith atom. $a_i^{\dagger}$ and a_i are the creation and annihilation operators for the ith cavity field, respectively. g is the coupling strength between the atom and the cavity field and J represents the coherent photon hopping strength between the two cavities. The total excitation number N is conserved during the system evolution since the excitation number operator $\hat{N} = \sum_{i=1}^{2}(|e_i\rangle\langle e_i| + a_i^{\dagger}a_i)$ commutes with the Hamiltonian H. For different initial excitation number, the whole system will evolve in different invariant subspace. When the initial excitation number $n \geq 3$, the analytical dynamics is so complex that is not given in the present paper. In order to explain the entanglement dynamics between the atoms and the interacting multi-excitation fields more explicitly, let us first consider the case that the total system evolves within the two-excitation subspace. The eigenenergies of the atom-cavity system are obtained as follows:

$$\begin{aligned}
\lambda_{1,2} &= \pm\sqrt{J^2 + 2g^2},\\
\lambda_{3,4} &= \pm\frac{1}{2}\sqrt{10J^2 + 12g^2 + 2\sqrt{9J^4 + 60g^2J^2 + 4g^4}},\\
\lambda_{5,6} &= \pm\frac{1}{2}\sqrt{10J^2 + 12g^2 - 2\sqrt{9J^4 + 60g^2J^2 + 4g^4}},\\
\lambda_7 &= \lambda_8 = 0.
\end{aligned} \tag{2}$$

The splitting of the energy-levels is caused by the Jaynes-Cummings interaction and the coherent photon hopping between the cavity fields. Suppose that the initial state of the whole system is $|e_1e_2\rangle|0\rangle_1|0\rangle_2$ and postselect the cavity fields conditioned on two atoms leaving the cavities both in their ground states, then the cavity fields finally collapse to the pure state $|\Phi(t)\rangle_{fee}$:

$$\big|\Phi(t)\big\rangle_{fee} = N_{fee}\big[U_5^{ee}(t)\big(|2\rangle_1|0\rangle_2 + |0\rangle_1|2\rangle_2\big) + U_6^{ee}(t)|1\rangle_1|1\rangle_2\big], \tag{3}$$

where $U_5^{ee}(t) = 2i(C_1 - A_1E_1)\sin(\lambda_5 t) - 2i(C_2 - A_2E_1)\sin(\lambda_3 t)$, $U_6^{ee}(t) = 2(C_2 - A_2E_1)[C_1\cos(\lambda_3 t) - A_1E_1] - 2(C_1 - A_1E_1)[C_2\cos(\lambda_5 t) - A_2E_1]$ and N_{fee} is the normalization factor. Here, $A_1 = A(\lambda_3)$, $A_2 = A(\lambda_5)$, $C_1 = C(\lambda_3)$, $C_2 = C(\lambda_5)$, $D_1 = D(\lambda_3)$, $D_2 = D(\lambda_5)$, and

$$\begin{aligned}
A(\lambda_k) &= \frac{\sqrt{2}\lambda_k}{6g(J^4+2g^4)}\big[8g^4 - g^2J^2 - 4J^4 + \lambda_k^2\big(J^2-g^2\big)\big],\\
B_1 &= \frac{\sqrt{2}}{6gJ}\left[-g^2 + \frac{3}{2}J^2 - \frac{1}{2}\sqrt{9J^4+60g^2J^2+4g^4}\right],\\
B_2 &= \frac{\sqrt{2}}{6gJ}\left[-g^2 + \frac{3}{2}J^2 + \frac{1}{2}\sqrt{9J^4+60g^2J^2+4g^4}\right],\\
C(\lambda_k) &= \frac{-\sqrt{2}\lambda_k}{6J(J^4+2g^4)}\big[2g^4 - g^2J^2 - 7J^4 + \lambda_k^2\big(J^2-g^2\big)\big],\\
D(\lambda_k) &= \frac{-\sqrt{2}\lambda_k}{6J(J^4+2g^4)}\big[2g^4 + 17g^2J^2 + 8J^4 - \lambda_k^2\big(2J^2+g^2\big)\big],\\
E_1 &= -\frac{g}{J}, \quad E_2 = -\frac{(g^2-J^2)}{gJ}.
\end{aligned} \tag{4}$$

If the initial state of the whole system is $|g_1g_2\rangle|1\rangle_1|1\rangle_2$, and we also postselect the cavity fields conditioned on two atoms leaving the cavities both in their ground states, then the cavity fields collapse to the pure state $|\Phi(t)\rangle_{fgg}$:

$$\big|\Phi(t)\big\rangle_{fgg} = N_{fgg}\big[U_5^{gg}(t)\big(|2\rangle_1|0\rangle_2 + |0\rangle_1|2\rangle_2\big) + U_6^{gg}(t)|1\rangle_1|1\rangle_2\big], \tag{5}$$

where $U_5^{gg}(t) = 2i(D_1 - A_1E_2)\sin(\lambda_5 t) - 2i(D_2 - A_2E_2)\sin(\lambda_3 t)$, $U_6^{gg}(t) = 2(D_2 - A_2E_2)[C_1\cos(\lambda_3 t) - A_1E_1] - 2(D_1 - A_1E_2)[C_2\cos(\lambda_5 t) - A_2E_1]$ and N_{fgg} is the normalization factor.

When $|U_6^{ee}(t)|$ or $|U_6^{gg}(t)|$ decreases to 0, the field component $|1\rangle_1|1\rangle_2$ vanishes and the two fields become a two-dimensional maximally entangled state. When $|U_5^{ee}(t)| = |U_6^{ee}(t)|$ or $|U_5^{gg}(t)| = |U_6^{gg}(t)|$, three field composite states $|2\rangle_1|0\rangle_2$, $|1\rangle_1|1\rangle_2$ and $|0\rangle_1|2\rangle_2$ are equally weighted in the superposition, which implies the fields are in a three-dimensional maximally entangled state. However, in the case that $|U_5^{ee}(t)| = 0$ or $|U_5^{gg}(t)| = 0$, only the field component $|1\rangle_1|1\rangle_2$ remains, meaning the fields are not entangled. We plot the von Neumann entropy of cavity 1, the probability of the atoms leaving the cavities both in the ground states, and the weighting factor of each field state component as functions of the interaction time for the initial states $|e_1e_2\rangle|0\rangle_1|0\rangle_2$ and $|g_1g_2\rangle|1\rangle_1|1\rangle_2$ in Figs. 1 and 2, respectively. It is interesting to see that even when the initial state of the whole system is not entangled, the entanglement entropy of the cavity fields can be larger than 1. The probabilities of obtaining the three-dimensional maximally entangled states for the fields are 37% and 18% in Figs. 1 and 2, respectively.

3 Generalization for the Multi-excitation Fields

Generalizing to the general multi-excitation fields, we assume that the whole system is initially in the state $|\Phi(0)\rangle_{ee} = |e_1e_2\rangle|k\rangle_1|k\rangle_2$ ($k = 0, 1, 2, 3, \ldots$). In Fig. 3, we plot the entropy

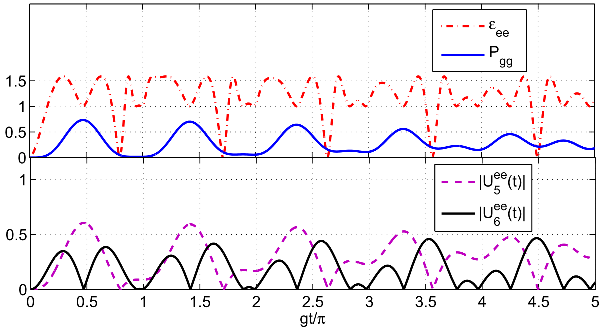

Fig. 1 The von Neumann entropy ϵ_{ee} of the cavity field one, the probability P_{gg} of the atoms leaving the cavities both in the ground states, and weighting factors ($|U_5^{ee}(t)|$ and $|U_6^{ee}(t)|$) versus the interaction time gt when $J = 1g$ and the initial state of the atom-field system is $|e_1e_2\rangle|0\rangle_1|0\rangle_2$ (Color figure online)

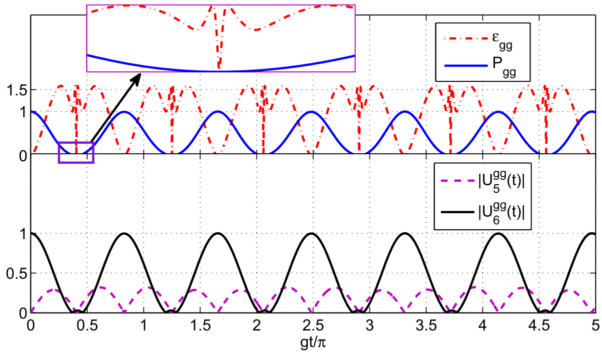

Fig. 2 The von Neumann entropy ϵ_{gg} of the cavity field one, the probability P_{gg} of the atoms leaving the cavities both in the ground states, and weighting factors ($|U_5^{gg}(t)|$ and $|U_6^{gg}(t)|$) versus the interaction time gt when $J = 0.5g$ and the initial state of the atom-field system is $|g_1g_2\rangle|1\rangle_1|1\rangle_2$ (Color figure online)

ϵ of field 1 against k and the interaction time gt (unit of π) under the condition that both the atoms are measured in the ground state after the interaction. The results show that the entanglement between the cavities increases with the initial photon number k. This is because that the number of the composite field state components increases with the increasing initial excitation number due to the photon hopping. For the case when the whole system is initially in the state $|\Phi(0)\rangle_{gg} = |g_1g_2\rangle|k\rangle_1|k\rangle_2$ ($k = 1, 2, 3, 4, \ldots$), the entanglement dynamics is similar to that in $|\Phi(0)\rangle_{ee}$, as shown in Fig. 4. The results of Figs. 3 and 4 show that the maximal field entanglement is dependent on the total excitation number but independent of the coherent photon hopping strength. It is interesting to note that the entanglement in the multi-excitation fields can keep larger than one whenever the atoms leave the cavity in their ground states. Comparing Figs. 3(f) and 4(f), we find that for $J \gg g$, when the atoms are initially in their ground states, the probability P_{gg} is apparently higher than that when they are both initially in the excited states. This is because that in this case the probability for the atoms exchanging energy with the fields in the coupled cavity system is small, due to the large detuning frequency shifts of the normal delocalized bosonic modes caused by the photon hopping.

4 Conclusion

In conclusion, we have investigated the entanglement dynamics between the atoms and the multi-excitation fields based on the coupled-cavity system. We first consider the situation that the atoms are initially both in the excited states or the ground states within the two-excitation subspace analytically. Then we generalize the situation to the multi-excitation

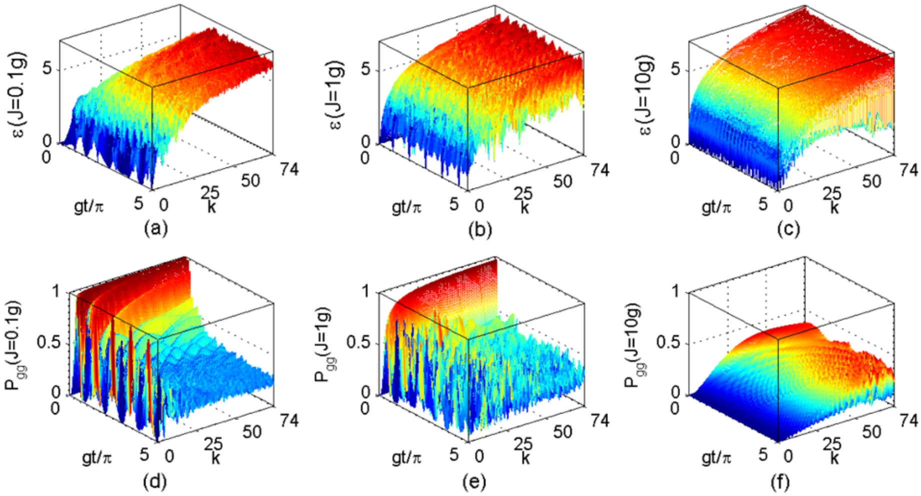

Fig. 3 Entanglement entropy ϵ for the fields versus the parameter k and the interaction time gt (unit of π) when the initial state is $|\Phi(0)\rangle_{ee}$: (**a**) $J = 0.1g$; (**b**) $J = 1g$; (**c**) $J = 10g$. Corresponding probabilities P_{gg} of the atoms leaving the cavities both in the ground states are: (**d**) $J = 0.1g$; (**e**) $J = 1g$; (**f**) $J = 10g$ (Color figure online)

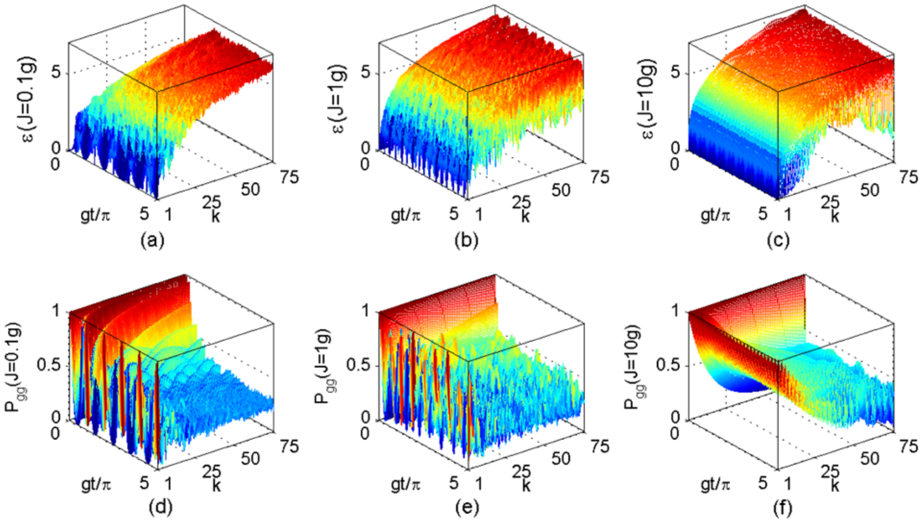

Fig. 4 Entanglement entropy ϵ for the fields versus the parameter k and the interaction time gt (unit of π) when the initial state is $|\Phi(0)\rangle_{gg}$: (**a**) $J = 0.1g$; (**b**) $J = 1g$; (**c**) $J = 10g$. Corresponding probabilities P_{gg} of the atoms leaving the cavities both in the ground states are: (**d**) $J = 0.1g$; (**e**) $J = 1g$; (**f**) $J = 10g$ (Color figure online)

field numerically and find that even when the atom-field system is initially not entangled, the field states can become maximally entangled, whose entropy is dependent on the total excitation number but independent of the coherent photon hopping strength. It is interesting to note that the entanglement of the multi-excitation fields can keep larger than one whenever the atoms leave the cavity in their ground states.

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