Particle Physics II
Lecture 11: The Higgs mechanism

Lesya Shchutska

May 11, 2023

- in this lecture we will cover more formal aspects of the Higgs mechanism so that you see where the main properties of the Higgs bosons are coming from
- the material can be split in three parts each giving an answer to a separate question:
  - appearance of mass terms for a scalar field (= Higgs field and Higgs boson mass)
  - appearance of mass term for a gauge boson from a broken U(1) local gauge symmetry
  - full Higgs mechanism by breaking the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ local gauge symmetry

Interacting scalar fields

- start with an example – lagrangian ($\mathcal{L} = T - V$) of QED:

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_e)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

  - here, the kinetic term for electron:

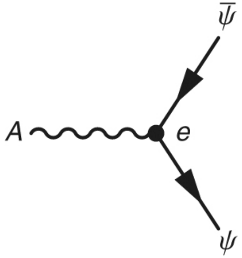
$$i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

  - the kinetic term for photon:

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

  - potential term for electron-photon interaction:

$$e\bar{\psi}\gamma^\mu\psi A_\mu$$



- in general, type of interactions and their strength are defined by the terms in lagrangian mixing the fields, like here $e\bar{\psi}\gamma^\mu\psi A_\mu$ defines the QED interaction vertex

Interacting scalar fields

- a free real (1D) scalar field has a lagrangian:

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

- for a scalar field ϕ with a potential:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

- the lagrangian will look like:

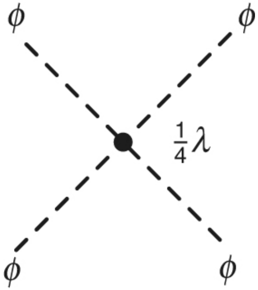
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \tag{1}$$

$$= \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 \tag{2}$$

- in this lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) \tag{3}$$

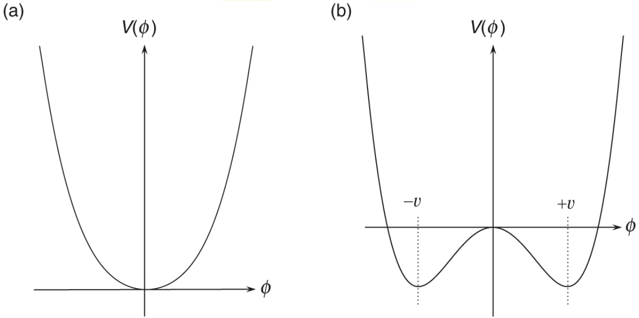
$$= \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 \tag{4}$$



- $\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)$ is the kinetic energy of the scalar particle
- $\frac{1}{2}\mu^2\phi^2$ represents the mass of the particle
- $\frac{1}{4}\lambda\phi^4$ is a self-interaction of the scalar field

Interacting scalar fields

- the vacuum state of the scalar field ϕ is its lowest energy state
- corresponds to the minimum of potential $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$
- for $V(\phi)$ to have a minimum, it is obligatory that $\lambda > 0$:



  - a) $\mu^2 > 0$: one minimum at $\phi = 0$,
  - b) $\mu^2 < 0$: two minima at $\phi = \pm v = \pm\left|\sqrt{\frac{-\mu^2}{\lambda}}\right|$

Interacting scalar fields

- a) $\mu^2 > 0$:
  - vacuum state is when $\phi = 0$
  - we have a scalar particle with mass μ
  - self-interaction term proportional to ϕ^4
- b) $\mu^2 < 0$:
  - lowest energy state when $\phi = +v$ or $\phi = -v$
  - choice of vacuum state breaks the symmetry of lagrangian – **spontaneous symmetry breaking**
  - to understand the particle interactions need to find excitations of the field around its minimum, e.g. for $\phi = +v$:

    $$\phi(x) = v + \eta(x),$$

    where $\eta(x)$ is the scalar field excitation

Interacting scalar fields

- we can expand the lagrangian with $\phi(x) = v + \eta(x)$ and $\partial_\mu \phi = \partial_\mu \eta$:
  $\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - V(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2}\mu^2(v+\eta)^2 - \frac{1}{4}\lambda(v+\eta)^4$
- now use the fact that $\mu^2 = -\lambda v^2$:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \frac{1}{4}\lambda v^4$$

some constant (ignore)

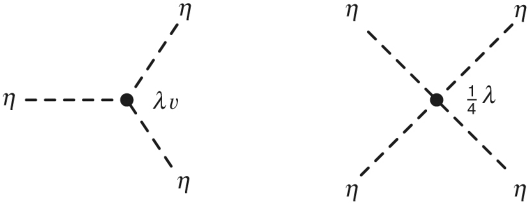
- term "$-\lambda v^2 \eta^2$" is equivalent to the mass term "$-\frac{1}{2}m^2\phi^2$" of $\mathcal{L}_S$

$$\Longrightarrow \quad m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

- it means that this lagrangian describes a massive scalar field η

Interacting scalar fields

- terms η^3 and η^4 represent triple (λv) and quartic ($\frac{1}{4}\lambda$) interaction vertices



- term $\frac{1}{4}\lambda v^4$ is a $const$ and does not have physical implications

Symmetry breaking for a complex scalar field

- can apply the same logic to a complex scalar field:

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

- the corresponding lagrangian is:

$$\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - V(\phi) \text{ with } V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$$

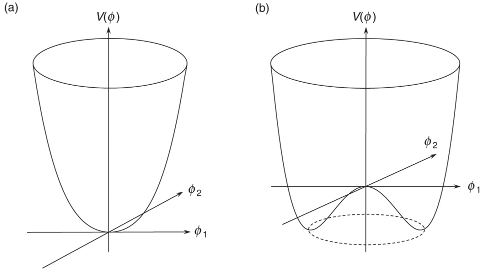
- can express the same in terms of two real scalar fields ϕ_1 and ϕ_2:

$$\mathcal{L} = \frac{1}{2}\underbrace{(\partial_\mu \phi_1)(\partial^\mu \phi_1)}_{\text{kin}} + \frac{1}{2}\underbrace{(\partial_\mu \phi_2)(\partial^\mu \phi_2)}_{\text{kin}} - \frac{1}{2}\underbrace{\mu^2(\phi_1^2 + \phi_2^2)}_{\text{mass term}} - \frac{1}{4}\underbrace{\lambda(\phi_1^2 + \phi_2^2)^2}_{\text{interaction}}$$

- again, for a potential to have a minimum, we need $\lambda > 0$

Symmetry breaking for a complex scalar field

- the lagrangian is invariant under $\phi \to \phi' = e^{i\alpha}\phi$, since $\phi'^* \phi' = \phi^* \phi$
  - $\Longrightarrow$ it has a global U(1) symmetry
- the shape of potential again depends on the sign of μ^2:
  - a) $\mu^2 > 0$: one minimum with $\phi_1 = \phi_2 = 0$
  - b) $\mu^2 < 0$: infinite set of minima with $\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} = v^2$



The physical vacuum state would be one point on this dashed circle breaking the global U(1) symmetry, e.g. $(\phi_1, \phi_2) = (v, 0)$

Symmetry breaking for a complex scalar field

- again, can expand the field around the vacuum state:

$$\phi_1(x) = \eta(x) + v \text{ and } \phi_2(x) = \xi(x)$$

$$\phi = \frac{1}{\sqrt{2}}(\eta + v + i\xi)$$

- need to rewrite lagrangian in terms of η and ξ:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)^*(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \xi)^*(\partial^\mu \xi) - V(\eta, \xi),$$

$$V(\eta, \xi) = \mu^2\phi^2 + \lambda\phi^4 \text{ with } \phi^2 = \phi\phi^* = \frac{1}{2}\left[(v+\eta)^2 + \xi^2\right]$$

- rewriting potential using $\mu^2 = -\lambda v^2$:

$$V(\eta, \xi) = \mu^2\phi^2 + \lambda\phi^4 \tag{5}$$

$$= -\frac{1}{2}\lambda v^2\left[(v+\eta)^2 + \xi^2\right] + \frac{1}{4}\left[(v+\eta)^2 + \xi^2\right]^2 \tag{6}$$

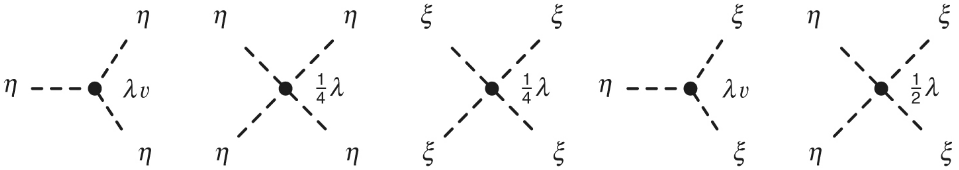
$$= -\frac{1}{4}\lambda v^4 + \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \frac{1}{2}\lambda\eta^2\xi^2 \tag{7}$$

Symmetry breaking for a complex scalar field

just number

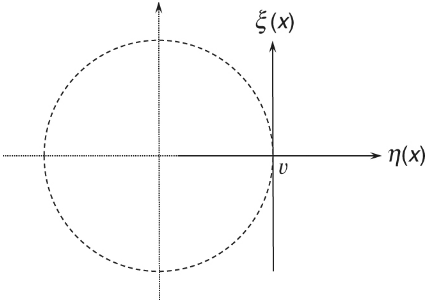
- $V(\eta,\xi) = -\frac{1}{4}\lambda v^4 + \lambda v^2\eta^2 + \overbrace{\lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \frac{1}{2}\lambda\eta^2\xi^2}^{\text{interaction terms}}$
  - term "$\lambda v^2\eta^2$" is a mass term for field η: $m_\eta = \sqrt{2\lambda v^2}$
  - terms "$\lambda v\eta^3$", "$\frac{1}{4}\lambda\eta^4$", "$\frac{1}{4}\lambda\xi^4$", "$\lambda v\eta\xi^2$", and "$\frac{1}{2}\lambda\eta^2\xi^2$" are three- and four-particle interaction terms
- lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)^*(\partial^\mu\eta) - \frac{1}{2}m_\eta^2\eta^2 + \frac{1}{2}(\partial_\mu\xi)^*(\partial^\mu\xi) - V_{int}(\eta,\xi)$$



Symmetry breaking for a complex scalar field

- this lagrangian contains two fields:
  - a massive scalar field η with mass $m_\eta = \sqrt{2\lambda v^2}$
  - a massless scalar field ξ
- excitations of the massive field η in the direction where the potential is quadratic
- excitations of ξ are in the direction of a constant potential – a Goldstone boson



The Higgs mechanism

- in the Higgs mechanism one more difference is that this spontaneous symmetry breaking happens in a theory with a local gauge symmetry
- local gauge transformation definition: $\phi(x) \to \phi'(x) = e^{ig\chi(x)}\phi(x)$
- $\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - V(\phi)$ is not invariant because of the derivatives
- this is fixed by replacing $\partial_\mu \to D_\mu = \partial_\mu + igB_\mu$
- $\mathcal{L} = (D_\mu \phi)^*(D^\mu \phi) - V(\phi)$ is gauge invariant if

$$B_\mu \to B'_\mu = B_\mu - \partial_\mu \chi(x)$$

- leading to the existence of a new gauge field B with gauge transformation properties:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu \phi)^*(D^\mu \phi) - \mu^2\phi^2 - \lambda\phi^4,$$

  where $F^{\mu\nu}F_{\mu\nu}$ is the kinetic term for the new field with

$$F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

- the field B is massless: the term $\frac{1}{2}m_B^2 B_\mu B^\mu$ breaks gauge invariance

- lagrangian will get additional terms when expanding “long” derivatives:

$$(D_\mu\phi)^*(D^\mu\phi) = (\partial_\mu - igB_\mu)\phi^*(\partial^\mu + igB^\mu)\phi \tag{8}$$

$$= (\partial_\mu\phi)^*(\partial^\mu\phi) - igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2B_\mu B^\mu\phi^*\phi \tag{9}$$

- the full lagrangian would be:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4 \tag{10}$$

$$-igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2B_\mu B^\mu\phi^*\phi \tag{11}$$

  - now need to repeat the same exercise of potential expansion around vacuum state taking into account additional terms in the lagrangian

- go directly to the case of $\mu^2 < 0$ and choose $\phi_1 + i\phi_2 = v$:

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$

- after all the transformations and algebra get:

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2}_{\text{massive } \eta} \quad (12)$$

$$+ \underbrace{\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)}_{\text{massless } \xi} \quad (13)$$

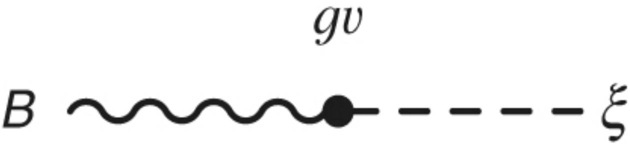
$$- \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2 v^2 B_\mu B^\mu}_{\text{massive gauge field}} \quad (14)$$

$$- V_{int} + gvB_\mu(\partial^\mu \xi) \quad (15)$$

where $V_{int}(\eta, \xi, B)$ contains 3- and 4-point interaction terms of the fields η, ξ and B

The Higgs mechanism

- we managed to provide a mass $m_B = gv$ to the field B, and retained local gauge invariance of the theory
- with doing this we acquired new particles: massive scalar field η and massless Goldstone boson ξ
- at the same time have two new issues:
  - **number of degrees of freedom**: had 4 (one of ϕ_1, one of ϕ_2, two polarizations of B), now have 5 (massive state B has one more polarization – longitudinal)?
  - **spin-1 to spin-0 particle transition**: term $gvB_\mu(\partial^\mu\xi)$ leads to such direct coupling



- these "problems" can be resolved by eliminating Goldstone field ξ with an appropriate gauge transformation:

$$\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + gvB_\mu(\partial^\mu \xi) + \frac{1}{2}g^2v^2 B_\mu B^\mu \tag{16}$$

$$= \frac{1}{2}g^2v^2 \left[B_\mu + \frac{1}{gv}(\partial_\mu \xi)\right]^2 \tag{17}$$

- can make gauge transformation:

$$B_\mu(x) \to B'_\mu(x) = B_\mu(x) + \frac{1}{gv}\partial_\mu \xi(x)$$

- then the lagrangian simplifies as:

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2}_{\text{massive } \eta} \tag{18}$$

$$- \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2 v^2 B'_\mu B^{\mu\prime}}_{\text{massive gauge field}} \tag{19}$$

$$- V_{int} \tag{20}$$

- the original lagrangian was invariant under local U(1) gauge transformations $\Longrightarrow$ physical predictions should be unchanged
- with the appropriate choice of gauge $\chi(x) = -\xi(x)/gv$ we do not have the Goldstone field ξ
- effect of this gauge on the scalar field ϕ:

$$\phi(x) \to \phi'(x) = e^{-ig\frac{\xi(x)}{gv}}\phi(x) = e^{-i\xi(x)/v}\phi(x)$$

- after the symmetry breaking we had:

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)) \approx \frac{1}{\sqrt{2}}\left[v + \eta(x)\right] e^{\frac{i\xi(x)}{v}}$$

- effect of the gauge transformation on this field:

$$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}}e^{-ig\frac{\xi(x)}{gv}}\left[v + \eta(x)\right] e^{\frac{i\xi(x)}{v}} = \frac{1}{\sqrt{2}}(v + \eta(x))$$

- so the effect of this gauge – **Unitary gauge** – is to choose the complex scalar field $\phi(x)$ to be real:
$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) \equiv \frac{1}{\sqrt{2}}(v + h(x))$$
- the field $\eta(x)$ is now denoted as the Higgs field $h(x)$ to show that it's the physical field in the unitary gauge
- unphysical term with $\xi(x)$ has disappeared
- extra degree of freedom disappeared with the Goldstone field $\xi(x)$: this boson was "eaten" by the massive gauge field B

- with all this the final lagrangian can be rewritten as (also ignoring a constant $\lambda v^4/4$):

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2}_{\text{massive } h \text{ scalar}} \tag{21}$$

$$- \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2 v^2 B_\mu B^\mu}_{\text{massive gauge boson}} \tag{22}$$

$$+ \underbrace{g^2 v B_\mu B^\mu h + \frac{1}{2}g^2 B_\mu B^\mu h^2}_{h, B \text{ interactions}} \tag{23}$$

$$- \underbrace{\lambda v h^3 - \frac{1}{4}\lambda h^4}_{h \text{ self-interactions}} \quad \rightarrow \text{not measured yet} \tag{24}$$

- mass of the gauge boson $m_B = gv$
- mass of the Higgs boson $m_{\mathrm{H}} = \sqrt{2\lambda}v$

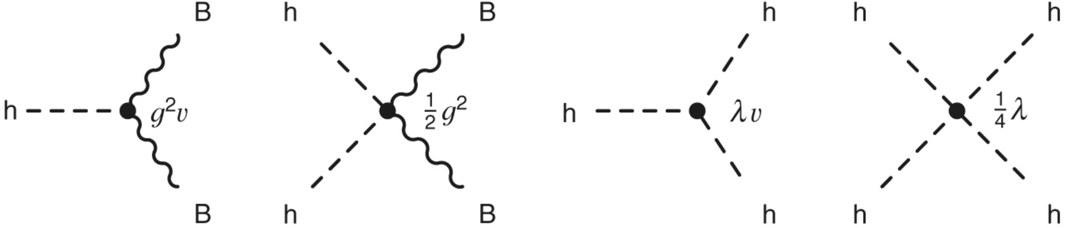
The Higgs mechanism

- interaction terms in the lagrangian:

$$+\underbrace{g^2 v B_\mu B^\mu h + \frac{1}{2} g^2 B_\mu B^\mu h^2}_{h,B \text{ interactions}} \tag{25}$$

$$-\underbrace{\lambda v h^3 - \frac{1}{4}\lambda h^4}_{h \text{ self-interactions}} \tag{26}$$

correspond to the following four diagrams:



The standard model Higgs boson

- the last step is to extend previous considerations from local gauge U(1) symmetry to local gauge $\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L$ symmetry
- **three** Goldstone bosons will be needed to provide longitudinal polarizations to W^+, W^- and Z bosons
- as before, after symmetry breaking there will be (at least) **one** massive scalar particle
- the simplest Higgs model with the necessary **four** degrees of freedom consists of two complex scalar fields
- to give masses to Z and $W^\pm$ one of the scalar fields must be neutral: ϕ^0; another one charged: ϕ^+ for W^+ and $(\phi^+)^* = \phi^-$ for W^-
- minimal Higgs model has two complex scalar fields in a weak isospin doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

(upper and lower components of doublet differ by one unit of charge)

- the lagrangian for this doublet is:

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi)$$

  with the Higgs potential $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$

- for $\mu^2 < 0$ the potential has an infinite set of minima with:

$$\phi^\dagger \phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}$$

- after symmetry breaking, the neutral photon remains massless $\implies$ minimum of the potential corresponds to a non-zero vacuum expectation value only for the neutral scalar field ϕ^0:

$$\langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

- the fields are expanded around this minimum:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{pmatrix}$$

- we will get three Goldstone bosons, which would be absorbed by $W^{\pm}$ and Z in the unitary gauge
- in this gauge Higgs doublet looks like:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- we need to identify masses of gauge bosons and interaction terms

The standard model Higgs boson

- for the SU(2)$_L\times$U(1)$_Y$ local gauge symmetry, the covariant derivatives would be:

$$\partial_\mu \to D_\mu = \partial_\mu + ig_W \vec{T}\cdot\vec{W}_\mu + ig'\frac{Y}{2}B_\mu,$$

where $\vec{T} = \frac{1}{2}\vec{\sigma}$ – the three generators of the SU(2) symmetry

- Higgs doublet hypercharge:

$$Y = 2(Q - I_W^3) = 2(0 + \frac{1}{2}) = 1$$

- hence for acting on the Higgs doublet ϕ the covariant derivative looks like:

$$D_\mu\phi = \frac{1}{2}\left[2\partial_\mu + \left(ig_W\vec{\sigma}\cdot\vec{W}_\mu + ig'B_\mu\right)\right]\phi$$

- the term in the lagrangian generating masses of the gauge bosons is $(D_\mu\phi)^\dagger(D^\mu\phi)$

- for determining the masses, need to expand the $(D_\mu\phi)^\dagger(D^\mu\phi)$ expression in the unitary gauge
- as a result, for terms quadratic in the gauge bosons fields will get:

$$\frac{1}{8}v^2 g_W^2\left(W_\mu^{(1)}W^{(1)\mu}+W_\mu^{(2)}W^{(2)\mu}\right) \tag{27}$$

$$+\frac{1}{8}v^2\left(g_W W_\mu^{(3)}-g'B_\mu\right)\left(g_W W^{(3)\mu}-g'B^\mu\right) \tag{28}$$

- the mass terms for $W^{(1)}$ and $W^{(2)}$ fields would be $\frac{1}{2}m_W^2 W_\mu^{(i)}W^{(i)\mu}$
- $\implies$ mass of the W boson is $m_W=\frac{1}{2}g_W v$

The standard model Higgs boson

- the terms containing neutral $W^{(3)}$ and B fields can be written as:
  $\frac{1}{8}v^2\left(g_W W^{(3)}_\mu - g' B_\mu\right)\left(g_W W^{(3)\mu} - g' B^\mu\right) = \frac{v^2}{8}\left(W^{(3)}_\mu B_\mu\right) M \begin{pmatrix} W^{(3)\mu} \\ B^\mu \end{pmatrix},$
  where M is the non-diagonal mass matrix:

$$M = \begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}$$

- off-diagonal matrix elements lead to mixing between $W^{(3)}$ and B
- to determine physical fields and their masses need to find M eigenvalues and eigenvectors leading to:

$$\frac{1}{8}v^2 (A_\mu \quad Z_\mu) \begin{pmatrix} 0 & 0 \\ 0 & g_W^2 + g'^2 \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}$$

with $m_A = 0$ and $m_Z = \frac{1}{2}v\sqrt{g_W^2 + g'^2}$

- we got the physical fields for a massless photon A_μ and for massive boson Z_μ:

$$A_\mu = \frac{g' W_\mu^{(3)} + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \text{ with } m_A = 0 \tag{29}$$

$$Z_\mu = \frac{g_W W_\mu^{(3)} - g' B_\mu}{\sqrt{g_W^2 + g'^2}} \text{ with } m_Z = \frac{1}{2} v \sqrt{g_W^2 + g'^2} \tag{30}$$

- by introducing

$$\frac{g'}{g_W} = \tan\theta_W$$

  we get:

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^{(3)}$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^{(3)}$$

- for a mass of a Z boson can also write:

$$m_Z = \frac{1}{2}\frac{g_W}{\cos\theta_W}v$$

- the relation between W and Z masses:

$$\frac{m_W}{m_Z} = \cos\theta_W$$

- resulting model of Glashow-Salam-Weinberg (GSW) is described by 4 parameters:

$$g_W, g', \mu, \lambda$$

- boson masses can be expressed through these parameters:

$$v^2 = \frac{-\mu^2}{\lambda} \text{ and } m_{\mathrm{H}}^2 = 2\lambda v^2$$

- from W mass and H mass measurements:

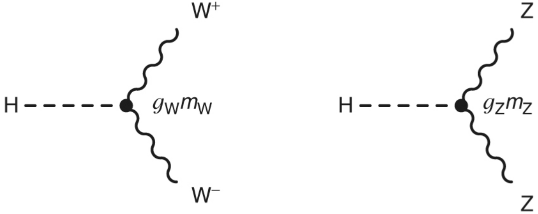
$$v = 246\ \text{GeV and } m_{\mathrm{H}} = 125\ \text{GeV}$$

The standard model Higgs boson

- the gauge bosons in the expansion of the $(D_\mu \phi)^\dagger (D^\mu \phi)$ appear as $VV(v+h)^2$
- VVv^2 terms lead to masses of gauge bosons
- the terms VVh and $VVhh$ lead to triple and quartic couplings between one or two H and gauge bosons
- by expanding all can find the following expression:

$$\frac{1}{4} g_W^2 v^2 W_\mu^- W^{+\mu} + \frac{1}{2} g_W^2 v W_\mu^- W^{+\mu} h + \frac{1}{4} g_W^2 W_\mu^- W^{+\mu} hh$$

- leading to $g_{\mathrm{HWW}} = \frac{1}{2} g_W^2 v \equiv g_W m_W$
- similarly, $g_{\mathrm{HZZ}} = g_Z m_Z$



Summary

- we have seen how a scalar field leads to the Higgs boson appearance
- derived Higgs boson mass and its interactions
- derived mass terms for the gauge bosons
- derived interactions of the Higgs boson with gauge bosons
- what is left: how fermions acquire their mass when interaction with the Higgs field