THE CKM MATRIX AND CP VIOLATION

1) CP violation in the Early Universe

Very early in the universe might expect equal numbers of baryons and anti-baryons

However, today the universe is matter dominated

$$\xi = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

For every baryon in the universe today there're 10^9 photons

How?

Early in the universe need to create a VERY SMALL ASYMMETRY between baryons and antibaryons.

For every 10^9 antibaryons there were $10^9 + 1$ baryons

Baryons and antibaryons annihilate

$\downarrow$

1 baryon + $\sim 10^9$ photons

To generate initial asymmetry, 3 conditions must be met:

* Baryon number violation
* C and CP violation
* Departure from thermal equilibrium

There're 2 places in the SM where CP violation enters:

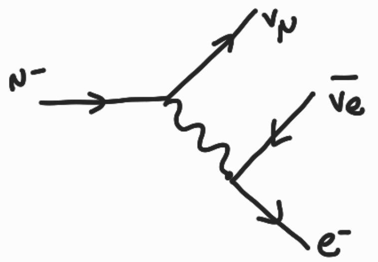
- PMNS matrix (neutrino sector)
- CKM matrix (quark sector)

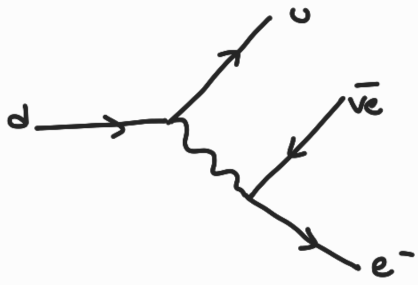
Initially, CP violation has been observed in the quark sector

Since we're dealing with quarks, which are only observed as BOUND States, this is a fairly complicated subject. It will be approached it in two steps:

1) Consider particle-antiparticle oscillations without CP violation
2) Then discuss the effects of CP violation.

2) Weak Interaction of Quarks

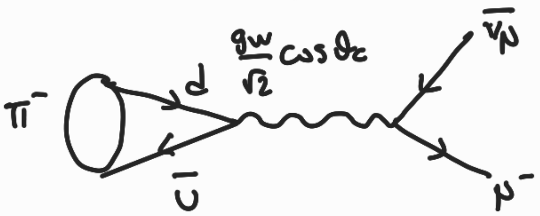


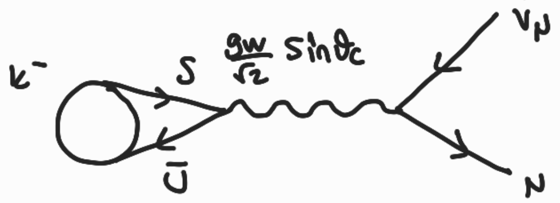


1) Slightly different values of G_F measured in μ decay and nuclear β decay.

$$G_F^{\mu} = (1.16632 \pm \dots) \times 10^{-5}\ \mathrm{GeV}^{-2} \qquad G_F^{(\beta)} = (1.1066 \pm \dots) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

2) Certain decay modes are observed to be suppressed:





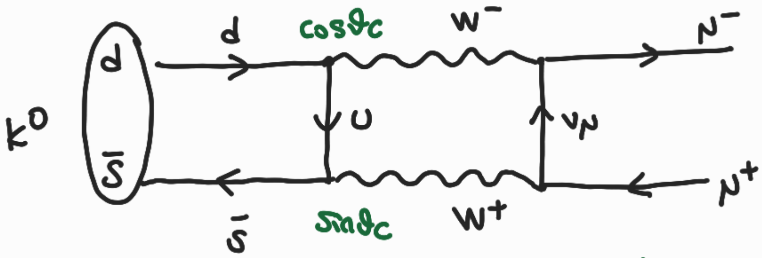
k decay rate is suppressed x20 compared to the expectation assuming a universal weak interaction of quarks.

Both observations explained by Cabbibo hypothesis: (1963)

☆ Weak eigenstates are different from mass eigenstates
weak interactions of quarks have same strength as for leptons but a u quark couples to a linear combination of s and d quarks:

↳ $K \to \mu^+\mu^-$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$



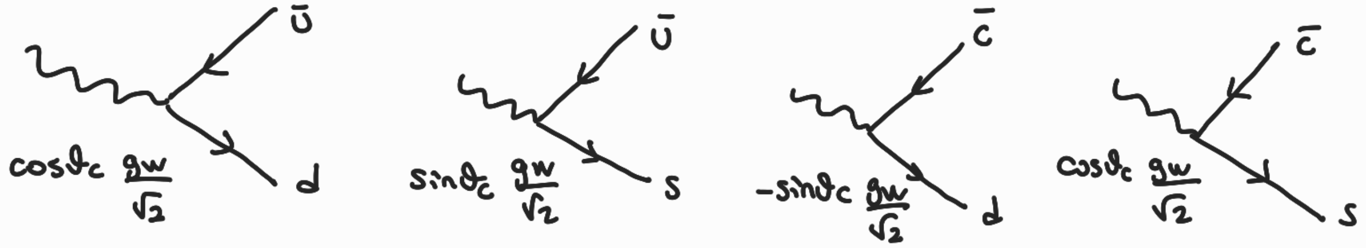
$M_1 \propto g_W{}^4 \cos\theta_c \sin\theta_c$

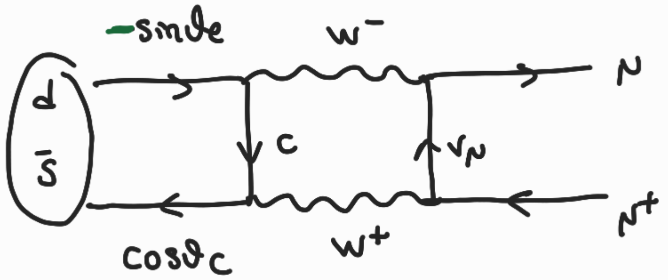
Since the Cabibbo mechanism allows for ud and us couplings, the flavour changing neutral-current (FCNC) decay of neutral kaon $K_L \to \mu^+\mu$ can occur via the exchange of a virtual up-quark.

However the observed branching ratio is MUCH SMALLER than expected.

★ Which leads Glashow, Illiopoulos, Maiani to postulate existence of an **EXTRA QUARK**

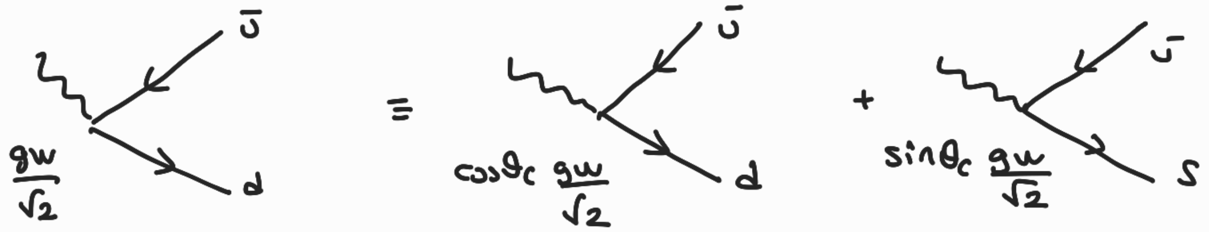
Before the discovery of charm quark (1974)

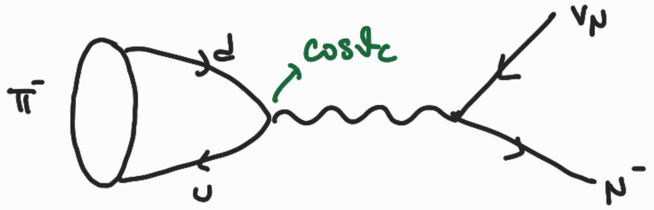


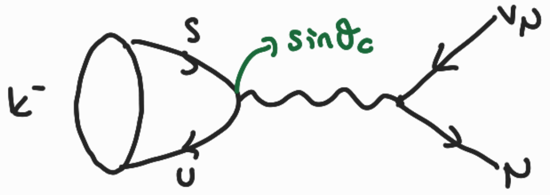


$$M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c$$

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$



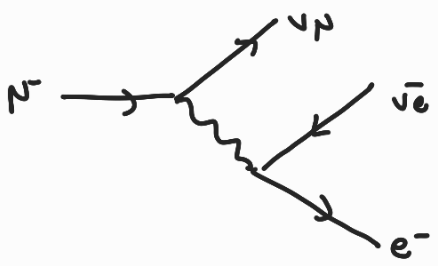




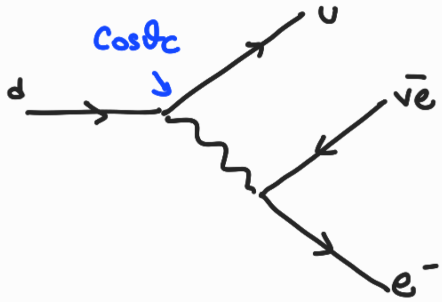
$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c$ $\qquad$ $\Gamma(K^- \to \mu^- \bar{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$

K decay suppressed by a factor of $\tan^2\theta_c \approx 0.05$ relative to π decay.

Thus expect $G_F^{\,B} = G_F^{\,N} \cos\theta_c$ (?)



$G_F G_F$



$G_F . (G_F \cos^2\theta_c)$

3) CKM Matrix

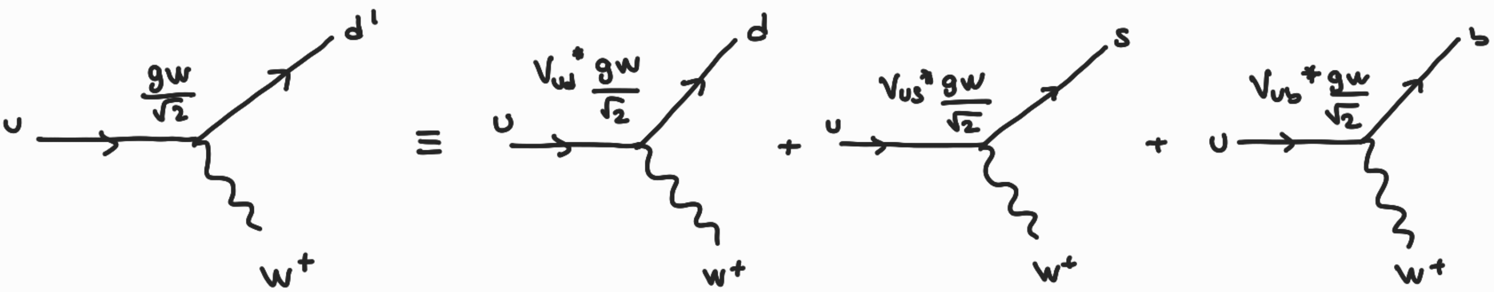
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — CKM matrix — mass eigenstates

Cabibbo, Kobayashi, Maskawa

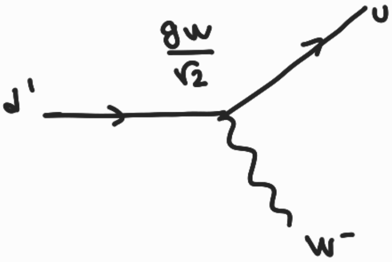
★ By convention CKM Matrix defined as acting on quarks with charge $-\frac{1}{3}e$

<u>Ex.</u> Weak eigenstate d' is produced in weak decay of an up quark



★ CKM matrix is unitary

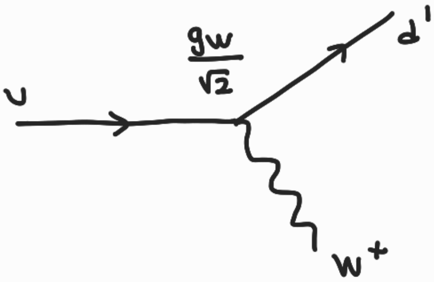
★ V_{ij} are not predicted by the SM, have to be determined from the experiment.



$$j_{d'u} = \underset{\text{adjoint spinor}}{\bar{u}}\left[-i\,\frac{g_W}{\sqrt{2}}\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma^5)\right] d'$$

$d' \to V_{ud}\,d + V_{us}\,s + V_{ub}\,b$

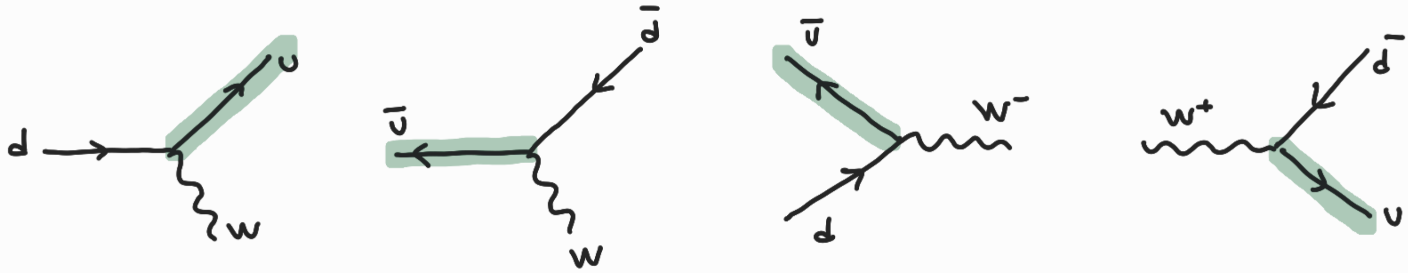
d → u weak current $\qquad j_{du} = \bar{u}\left[-i\,\frac{g_W}{\sqrt{2}}\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma^5)\right] V_{ud}\,d$



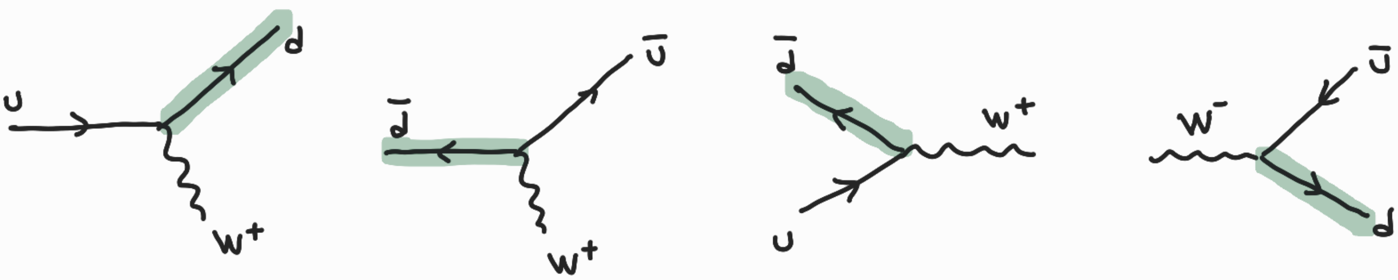
$$j_{ud'} = \bar{d'}\left[-i\,\frac{g_W}{\sqrt{2}}\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma^5)\right] u$$

$$\bar{d'} = d'^{\dagger}\gamma^0 = (V_{ud}\,d)^{\dagger}\gamma^0 = V_{ud}^{*}\,\underbrace{d^{\dagger}\gamma^0} = V_{ud}^{*}\,\bar{d}$$

u → d weak current $\qquad j_{ud} = \bar{d}\,V_{ud}^{*}\left[-i\,\frac{g_W}{\sqrt{2}}\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma^5)\right] u$



VERTEX FACTOR IS $\;-i\,\frac{g_W}{\sqrt{2}}\,V_{ud}\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma^5)$



VERTEX FACTOR IS $\;-i\,\frac{g_W}{\sqrt{2}}\,V_{ud}^{*}\,\gamma^\mu\,\frac{1}{2}\,(1-\gamma^5)$

Cabibbo Matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

* Near DIAGONAL

Weak Interaction in SM provides:

1) only way to CHANGE FLAVOUR

2) change from one generation of quarks or leptons to another

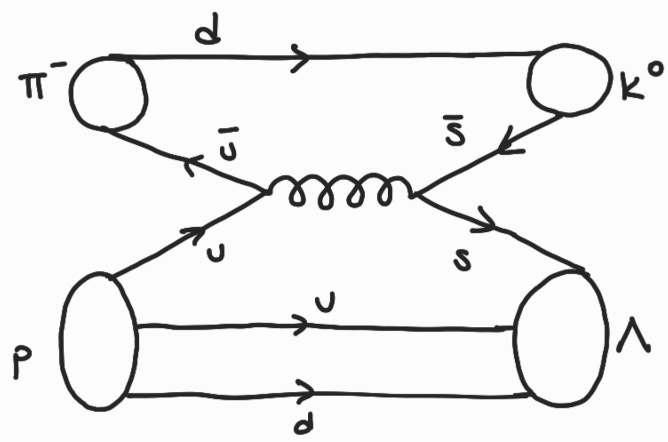
* The off-diagonal elements of the CKM matrix are relatively small:

* Weak interaction is LARGEST between quarks of the SAME generation.

* Coupling between 1^{st} and 3^{rd} generation quark is very SMALL

4) The Neutral Kaon System

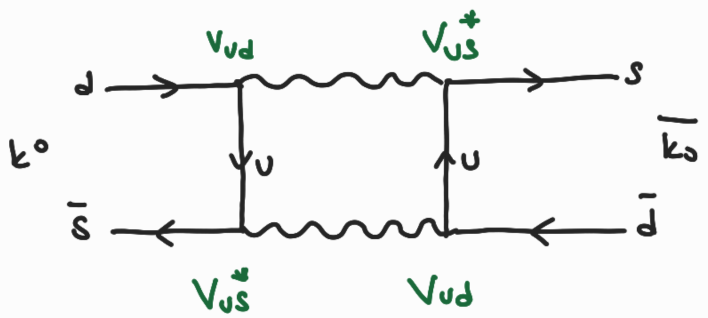
$$\pi^-(d\bar{u}) + p(uud) \longrightarrow \Lambda(uds) + K^0(d\bar{s})$$

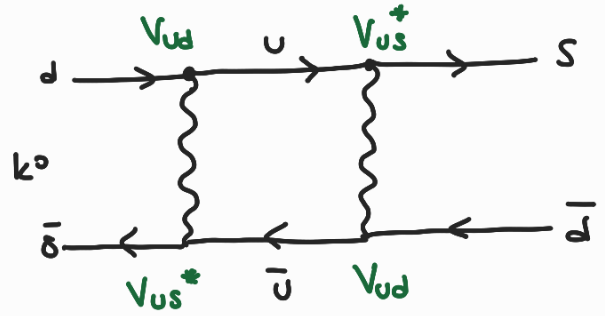


eigenstates of the strong interaction (flavour states)

$$\pi^+(u\bar{d}) + p(uud) \longrightarrow K^+(u\bar{s}) + \bar{K}^0(s\bar{d}) + p(uud)$$

- The neutral kaons decay via the weak interaction
- The weak interaction also allows mixing of neutral kaons via box diagrams





In quantum mechanics, the physical states are the eigenstates of the free-particle Hamiltonian. These are stationary states. Until now, independent stationary states have been used to describe each type of particle.

! However, here due to the $K^0 \leftrightarrow \bar{K}^0$ mixing process, a neutral kaon that is produced as a K^0 will develop a $\bar{K}^0$ component. Thus, the K^0–$\bar{K}^0$ system has to be considered as a whole.

The physical neutral kaon states are the stationary states of the combined Hamiltonian of the K^0–$\bar{K}^0$ system.

★ The neutral kaons propagate as linear combinations of K^0 and $\bar{K}^0$

These physical states are known as K-short (K_S) and K-long (K_L)

$$m(K_S) \approx m(K_L) \simeq 498\ \text{MeV}$$

! But very different lifetimes $\tau(K_S) = 0.9 \times 10^{-10}$ s
$\tau(K_L) = 0.5 \times 10^{-7}$ s

4.1) CP Eigenstates

K_S and K_L are closely related to CP-eigenstates

The strong eigenstates $K^0(d\bar{s})$ and $\bar{K}^0(s\bar{d})$ have $J^P = 0^-$

$$\hat{P}\,|K^0\rangle = -|K^0\rangle \qquad \hat{P}\,|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

$$\hat{C}\,|K^0\rangle = \hat{C}\,|d\bar{s}\rangle = +|s\bar{d}\rangle = |\bar{K}^0\rangle$$

$$\hat{C}\,|\bar{K}^0\rangle = |K^0\rangle$$

by convention

$$\hat{C}\hat{P}|K^0\rangle = -|\bar{K}^0\rangle$$
$$\hat{C}\hat{P}|\bar{K}^0\rangle = -|K^0\rangle$$

NOT EIGENSTATES of CP

$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$, $\quad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$

$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$, $\quad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$

4.2) Decays of CP Eigenstates

$u\bar{d} \to \pi^+ \quad d\bar{u} \to \pi^-$

Neutral kaons often decay to pions (the lightest hadrons)

Kaon masses are approximately 498 MeV ; whereas, pions ~ 140MeV

Hence neutral kaons can decay to either 2 or 3 pions.

4.2.1) Decay to Two Pions:

$$K^0 \to \pi^0\pi^0$$

Spins are all zero $\quad \overset{0}{J} = \overset{0}{L} + \overset{0}{S}$

$J^P: \; 0^- \to 0^- + 0^-$ → Conservation of angular momentum $\boxed{L=0}$

$$\hat{P}(\pi^0\pi^0) = (-1)(-1)(-1)^{2^0} = +1$$

each π^0 parity

→ $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ is an eigenstate of $\hat{C}$

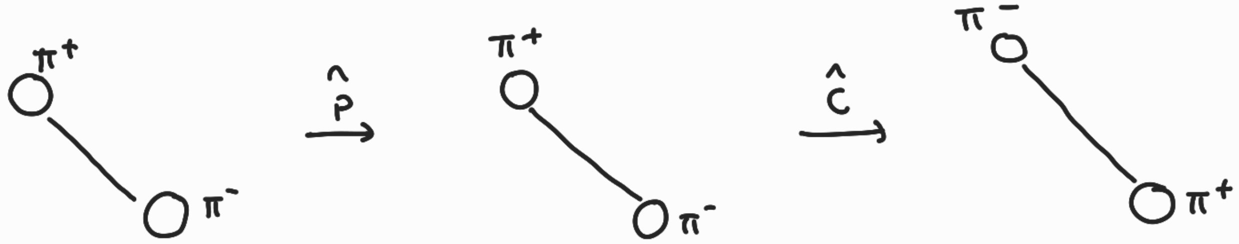
$$\hat{C}(\pi^0\pi^0) = C\pi^0 C\pi^0 = (+1)(+1) = +1$$

Thus $\boxed{\hat{C}\hat{P}(\pi^0\pi^0) = +1}$

$$\boxed{K^0 \rightarrow \pi^+\pi^-}$$

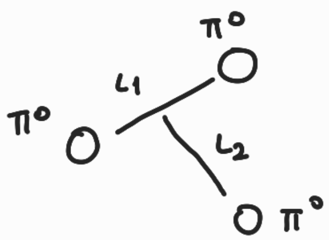
$$\hat{P}(\pi^+\pi^-) = 1 \qquad \boxed{\hat{C}\hat{P}(\pi^+\pi^-) = +1}$$

Here the C and P operations have identical effect



Neutral kaon decays to two pions occur in CP even (+1) eigenstates

4.2.2) Decay to Three Pions



$$K^0 \rightarrow \pi^0\pi^0\pi^0$$

$$J^P:\ 0^- \longrightarrow 0^- + 0^- + 0^-$$

WHY?

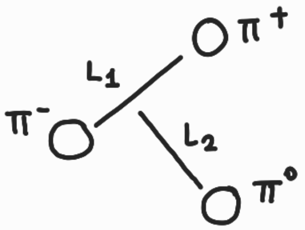
Conservation of angular momentum : $L_1 \oplus L_2 = 0 \quad \Rightarrow \quad L_1 = L_2$

$$P(\pi^0\pi^0\pi^0) = (-1)^3\,(-1)^{2l} = (-1)$$

$$C(\pi^0\pi^0\pi^0) = (+1)^3 = (+1)$$

$$\boxed{\hat{C}\hat{P}(\pi^0\pi^0\pi^0) = (-1)}$$

$K^0 \rightarrow \pi^+ \pi^- \pi^0$



$l_1 = l_2$

$$P(\pi^+\pi^-\pi^0) = (-1)^3 \, (-1)^{l_1} \, (-1)^{l_2} = (-1)$$

$$C(\pi^+\pi^-\pi^0) = (+1)\, C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^{L_1}$$

$$CP(\pi^+\pi^-\pi^0) = (-1)(-1)^{L_1}$$

can be taken '0'

The small amount of energy is available in the decay (~70MeV) meaning that the L>0 decays are strongly suppressed by the angular momentum barrier effects (recall QM tunnelling in alpha decay)

Neutral kaon decay to THREE pions occur in CP odd (-1) eigenstates

★ If CP were conserved in the weak decays of neutral kaons:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right), \quad \hat{C}\hat{P}|K_1\rangle = + |K_1\rangle \qquad K_1 \rightarrow \pi\pi : \text{CP-even}$$

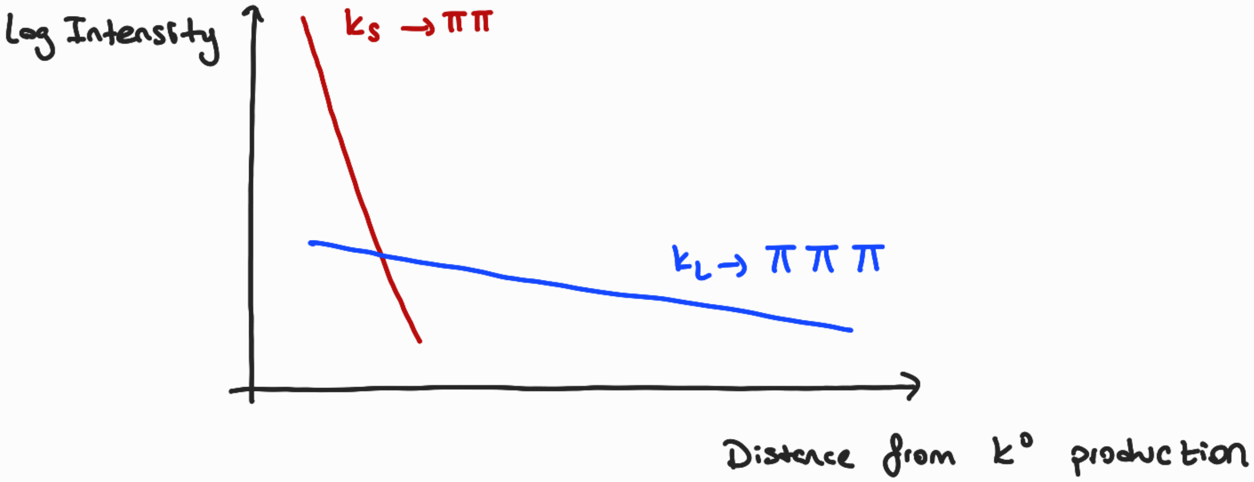
$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right), \quad \hat{C}\hat{P}|K_2\rangle = - |K_2\rangle \qquad K_2 \rightarrow \pi\pi\pi : \text{CP-odd}$$

$K_1 \rightarrow \pi\pi$ decay more rapid (more energy available ~220 MeV) K-short

$K_2 \rightarrow \pi\pi\pi$ decay slower (less energy available ~70MeV) K-long

if CP were EXACTLY conserved in the weak interaction,

$$K_S \equiv K_1 \qquad \& \qquad K_L \equiv K_2$$



Expected decay rate to pions from an initially K^0 beam, assuming no CP violation

At $t=0$ $\quad K^0 : |\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\left(|K_S\rangle + |K_L\rangle \right)$

$$|K_S(t)\rangle = |K_S\rangle e^{-im_S t - \Gamma_S t/2}$$

to ensure exponential decay

$$|\psi_S|^2 = \langle K_S | K_S \rangle = e^{-\Gamma_S t} = e^{-t/\tau_S}$$

Hence $\quad |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|K_S\rangle e^{-(im_S + \frac{\Gamma_S}{2})t} + |K_L\rangle e^{-(im_L + \frac{\Gamma_L}{2})t} \right]$

can be written as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle \right)$$

where $\quad \theta_S(t) = e^{-(im_S + \frac{\Gamma_S}{2}t)} \qquad \theta_L(t) = e^{-(im_L + \frac{\Gamma_L}{2}t)}$

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) \propto |\langle K_S | \psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_S t} = e^{-t/\tau_S}$$

Neutral kaons can also decay to leptons

$$\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e \qquad K_S \to \pi^- e^+ \nu_e$$
$$\bar{K}^0 \to \pi^+ \mu^- \bar{\nu}_\mu \qquad K_0 \to \pi^- \mu^+ \nu_\mu$$

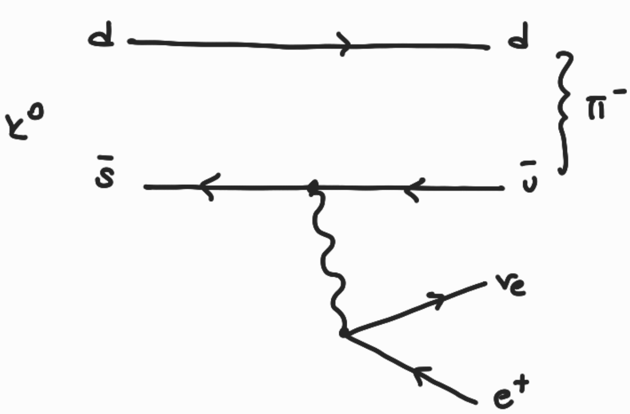
leptonic decays are more likely for K_L since the three-pion decay modes have a lower decay rate than the two pion modes of the K_S.

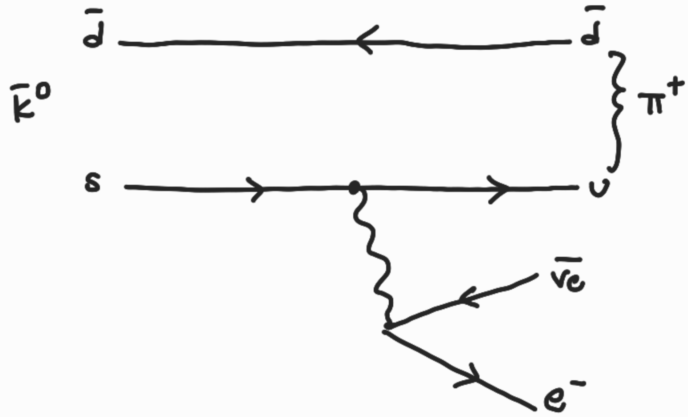
5) Strangeness Oscillations (neglecting CP violation)

In the previous chapter, neutrino oscillations arise because neutrinos are created and interact as weak eigenstates but propagate as mass eigenstates.

$K_0, \bar{K}_0$

Similarly, neutral kaons are produced and decay as flavour and/or CP eigenstates, but propagate as the K_S and K_L mass eigenstates.





$$K^0 \longrightarrow \pi^- e^+ \nu_e$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(\theta_S(t)\,|K_S\rangle + \theta_L(t)\,|K_L\rangle\right)$$

$$|\psi(t)\rangle = \frac{1}{2}\left[\theta_S(t)\left(|K^0\rangle - |\bar{K}^0\rangle\right) + \theta_L(t)\left(|K^0\rangle + |\bar{K}^0\rangle\right)\right]$$

$$|\psi(t)\rangle = \frac{1}{2}(\theta_S+\theta_L)\,|K^0\rangle + \frac{1}{2}(\theta_S-\theta_L)\,|\bar{K}^0\rangle$$

Becauses the masses of the K_S and K_L are slightly different $\theta_S \neq \theta_L$; thus, K^0 beam will develop a $\bar{K}^0$ component.

$$P(K^0_{t=0} \to K^0) = |\langle K^0|K(t)\rangle|^2 = \frac{1}{4}|\theta_S+\theta_L|^2$$

$$P(K^0_{t=0} \to \bar{K}^0) = |\langle \bar{K}^0|K(t)\rangle|^2 = \frac{1}{4}|\theta_S-\theta_L|^2$$

Can be simplified $|\theta_S \pm \theta_L|^2 = |\theta_S|^2 + |\theta_L|^2 \pm 2\,\mathrm{Re}(\theta_S\theta_L^*)$

$$|\theta_S(t) \pm \theta_L(t)|^2 = e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\,\mathrm{Re}\{e^{-im_S t}\,e^{-\frac{\Gamma_S t}{2}}\cdot e^{+im_L t}\,e^{-\frac{\Gamma_L t}{2}}$$

$$= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\,e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t}\,\mathrm{Re}\{e^{i\overbrace{(m_L-m_S)}^{\Delta m}t}\}$$

$$= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t}\cos(\Delta m t)$$

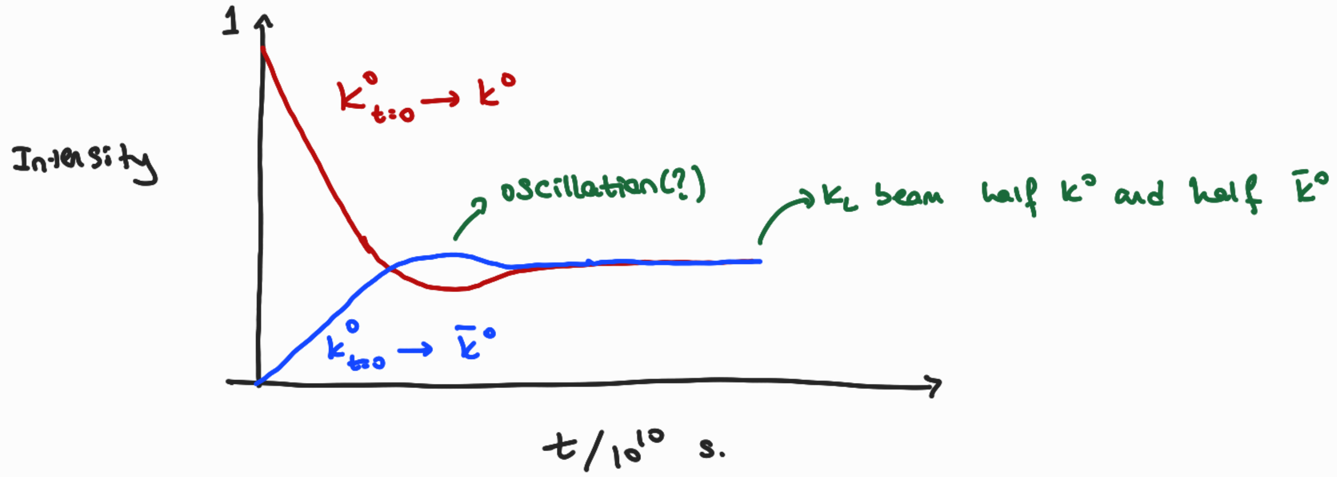
$\Delta m = m(K_L) - m(K_S)$

$$P(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t}\cos(\Delta m t)\right]$$

$$P(K^0_{t=0} \to \bar{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t}\cos(\Delta m t)\right]$$

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \simeq 1.2\times10^{-9}\,s \quad > \quad \tau(K_S) \simeq 0.9\times10^{-10}\,s$$

Consequently, after one oscillation period, the K_S and oscillatory components will decay away leaving an essentially pure K_L beam.



Experimentally $\Delta m = (3.506 \pm 0.006) \times 10^{-15}$ GeV

5.1) The CPLEAR Experiment (CP Large Exp. A and R)

Used a LOW ENERGY $\bar{p}$ beam

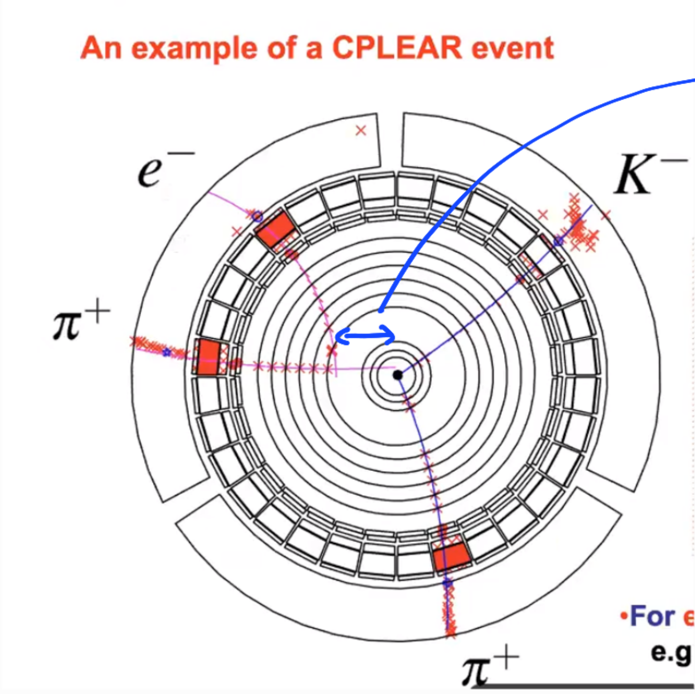
low-energy beams have a higher interaction rate with the experimental set-up

Neutral kaons produced:

$$\bar{p}p \longrightarrow K^- \pi^+ K^0$$

$$\bar{p}p \longrightarrow K^+ \pi^- \bar{K}^0$$

* Low energy, so particles produced almost at rest
* Observe production and decay in the same detector
* Charge of $K^\pm \pi^\mp$ in the production process tags the initial neutral kaon as either K^0 or $\bar{K}^0$.
* Direct probe of strangeness oscillations



Used to measure decay time

Production: $\bar{p}p \to K^- \pi^+ K^0$

Mixing

Decay : $\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$

Can measure decay rates as a function of time

$$R^+ = \Gamma(K^0_{t=0} \longrightarrow \pi^- e^+ \bar{\nu}_e) \propto \Gamma(K^0_{t=0} \to K^0)$$

Thus,

$$R_+ = \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) = N_{\pi e \nu} \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2} \cos \Delta m t\right]$$

$$R_- = \Gamma(K^0_{t=0} \to \pi^+ e^- \nu_e) = N_{\pi e \nu} \quad \ldots\ldots$$

overall normalization factor

$$\bar{R}_- = \Gamma(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e)$$

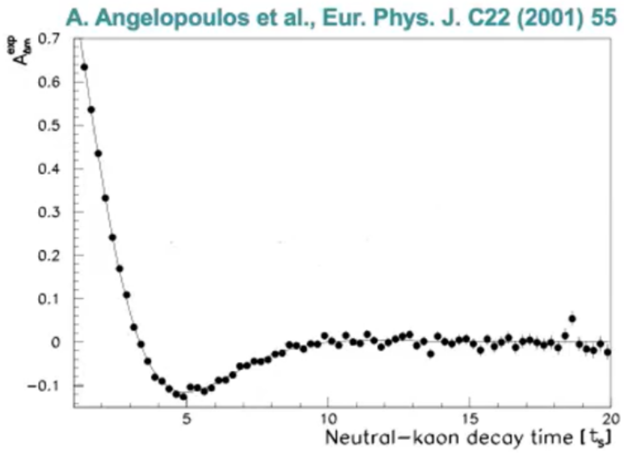
$$\bar{R}_+ = \Gamma(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e)$$

To remove dependence on $N_{\pi e \nu}$, express in terms of ASYMMETRY

$$A_{\Delta m} = \frac{(R_+ + \bar{R}_-) - (R_- + \bar{R}_+)}{(R_+ + \bar{R}_-) + (R_- + \bar{R}_+)}$$

For R_+ :

$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2} \cos \Delta m t}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$



★ Points show the data

★ The line shows the theoretical prediction for the value of Δm most consistent with the CPLEAR data:

$$\boxed{\Delta m = 3.485 \times 10^{-15}\,\text{GeV}}$$

6) CP Violation in the Kaon System

So far we ignored CP violation in the neutral kaon system

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad \text{decay}: \quad K_S \to \pi\pi \quad \boxed{CP = +1}$$

$$|K_L\rangle = |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \quad \text{decay}: \quad K_L \to \pi\pi\pi \quad \boxed{CP = -1}$$

At a long distance from the production point, a beam of neutral kaons will be 100% K_L

Hence, if CP conserved, would expect to see only 3π decays

In 1964, Fitch & Cronin (Nobel ,1980) observed 45 $K_L \to \pi^+\pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point. $\Rightarrow$ Weak Interaction Violates CP

At level of 2 parts in 1000

$K_L \longrightarrow$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$	$CP = -1$
$\longrightarrow$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$	$CP = -1$
$\longrightarrow$	$\pi^+\pi^-$	$BR = 0.2\%$	$CP = +1$
$\longrightarrow$	$\pi^0\pi^0$	$BR = 0.08\%$	$CP = +1$

Two possible explanations of CP violation in the kaon system:

i) The K_S and K_L do not correspond exactly to the CP eigenstates K_1 and K_2

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[\,|K_1\rangle + \varepsilon\,|K_2\rangle\,\right]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[\,|K_2\rangle + \varepsilon\,|K_1\rangle\,\right] \quad \text{with } |\varepsilon| \sim 2\times10^{-3}$$

In this case, the observation of $K_L \to \pi\pi$ is accounted for by:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[\,|K_2\rangle + \varepsilon\,|K_1\rangle\,\right]$$

$\varepsilon\,|K_1\rangle \to \pi\pi \quad CP = +1$

$|K_2\rangle \to \pi\pi\pi \quad CP = -1$

ii) and/or CP is violated in the decay:

$$|K_L\rangle = |K_2\rangle$$

$\to \pi\pi\pi \quad CP = -1$

$\to \pi\pi \quad CP = +1$

Experimentally i) dominates ii).

6.1) CP violation in semileptonic decays

If observe a neutral kaon beam a long time after production (at large distances) it will consist of a pure K_L component:

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\underset{\substack{\downarrow \\ \pi^+ e^- \bar{\nu}_e}}{|K^0\rangle} + (1-\varepsilon)\underset{\substack{\downarrow \\ \pi^- e^+ \nu_e}}{|\bar{K}^0\rangle}\right]$$

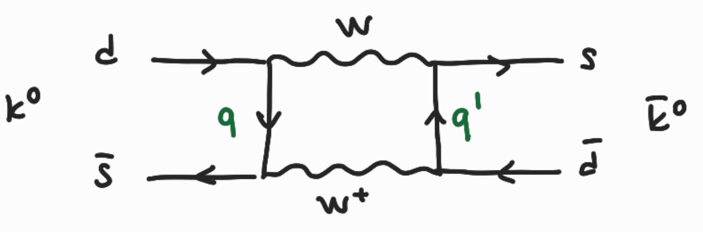
$$\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e) \propto |\langle \bar{K}^0|K_L\rangle|^2 \propto |1-\varepsilon|^2 \approx \boxed{1-2\Re\{\varepsilon\}}$$

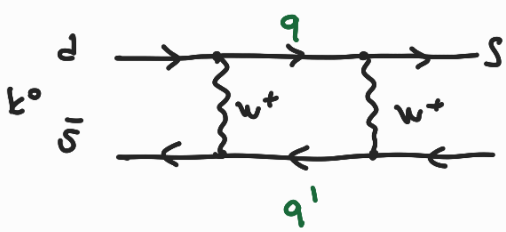
$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto |\langle K^0|K_L\rangle|^2 \propto |1+\varepsilon|^2 \approx \boxed{1+2\Re\{\varepsilon\}}$$

This difference provides the FIRST DIRECT EVIDENCE for an absolute difference between MATTER and ANTIMATTER

7) CP violation and CKM matrix

HOW CAN WE EXPLAIN $\Gamma(\bar{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \bar{K}^0)$ IN TERMS OF THE CKM matrix?

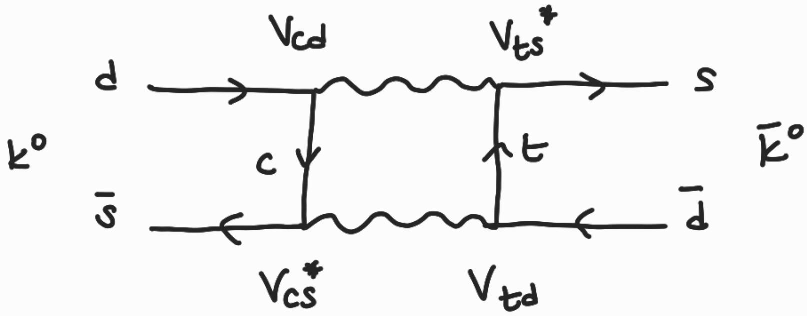




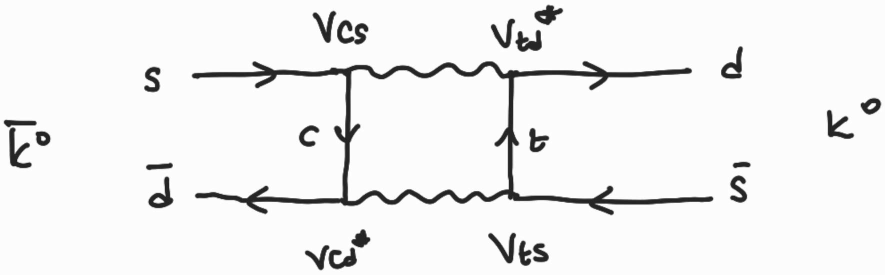
where $q = \{u, c, t\}$ $\quad q' = \{u, c, t\}$

Have to sum over all possible quark exchanges in the box.

For simplicity, consider just one diagram:



$$M_{fi} \propto \underset{\substack{\downarrow \\ \text{some constant}}}{A_{ct}} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$



$$M_{fi}' \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

Thus,

$$\Gamma(K^0 \to \bar{K}^0) - \Gamma(\bar{K}^0 \to \bar{K}^0) \propto M_{fi} - M_{fi}^* = 2I\{M_{fi}\}$$

* Hence the rates can only be different if the CKM matrix has imaginary components

$$|\varepsilon| \propto I\{M_{fi}\}$$

* SHOWS THAT CP VIOLATION IS RELATED TO THE IMAGINARY PARTS OF THE CKM MATRIX