<u>Review</u>

16/03/23

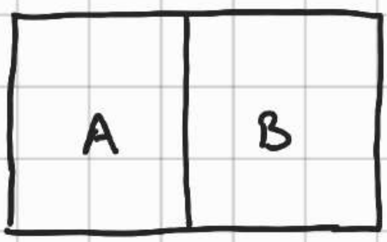
Tracing

Super-operator

- Unitary-evolution: $i\dot{\hat{\rho}} = [\hat{H}, \hat{\rho}]$

$$\hat{\rho}(t) = \hat{U}(t)\, \hat{\rho}\, \hat{U}^{\dagger}(t)$$

- Measurement:
  - Read result r: $\hat{\rho}' = \frac{1}{p(r)} \hat{M}_r \hat{\rho} \hat{M}_r^{\dagger} \quad \{r, M_r\}$
  - Don't read $\hat{\rho}' = \sum_r \hat{M}_r \rho \hat{M}_r^{\dagger}$
  - Unitary evolution + tracing:



$$\hat{\rho}_A \otimes |0\rangle\langle 0|_B$$

$$\hat{\rho}' = \hat{U}\, \overbrace{\hat{\rho}_A \otimes |0\rangle\langle 0|_B}\, \hat{U}^{\dagger}$$

$$\hat\rho_A' = \mathrm{Tr}_B\, \hat\rho' = \sum_\mu \langle \mu_B | \left(\hat U \hat\rho_A \otimes |0\rangle\langle 0|_B \hat U^\dagger \right) | \mu_B \rangle$$

$$= \sum_\mu \underbrace{\langle \mu_B | \hat U | 0_B \rangle}_{\hat M_\mu} \hat\rho_A \underbrace{\langle 0_B | \hat U^\dagger | \mu_B \rangle}_{\hat M_\mu^\dagger}$$

$\mathcal{S} : \hat\rho \longrightarrow \hat\rho'$ (channel)

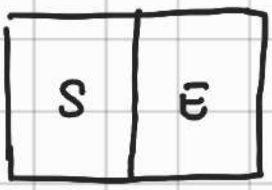
$\exists \{ M_\mu \}$ $\quad \mathcal{S}(\hat\rho) = \sum_\mu \hat M_\mu \hat\rho \hat M_\mu^\dagger$ Krauss Theorem

Not unique

$$\hat N_\nu = \sum_\mu U_{\mu\nu} \hat M_\mu$$

$\# \mu > 1 \Rightarrow \mathcal{S}$ can not be inverted

1) Amplitude Damping



Unitary Evolution:

$|0\rangle_S \otimes |0\rangle_E \longrightarrow |0_S\rangle \otimes |0_E\rangle$

$|1\rangle_S \otimes |0\rangle_E \longrightarrow \sqrt{p}\, |0_S\rangle \otimes |1\rangle_E + \sqrt{1-p}\, |1_S\rangle \otimes |0_E\rangle$

$$\hat{\rho}' = \mathrm{Tr}_E\left[\hat{U}\left(\hat{\rho}_S \otimes |0\rangle\langle 0|_E\right)\hat{U}^\dagger\right]$$

$$= \langle 0_E|\ \hat{U}\ \hat{\rho}_S \otimes |0\rangle\langle 0|_E\ \hat{U}^\dagger\ |0_E\rangle$$

$$+ \langle 1_E|\ \hat{U}\ \hat{\rho}_S \otimes |0\rangle\langle 0|_E\ U^\dagger\ |1_E\rangle$$

$$= \hat{M}_0\,\hat{\rho}_S\,\hat{M}_0^\dagger + \hat{M}_1\,\hat{\rho}_S\,\hat{M}_1^\dagger$$

$$\hat{M}_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \qquad \hat{M}_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

$$\hat{M}_0^\dagger \hat{M}_0 + \hat{M}_1^\dagger \hat{M}_1 = I$$

$$\left.\begin{array}{l} \overset{M_0^\dagger}{\begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}} \overset{M_0}{\begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}} = \begin{pmatrix} 1 & 0 \\ 0 & 1-p \end{pmatrix} \\[2ex] \underset{\hat{M}_1^\dagger}{\begin{pmatrix} 0 & 0 \\ \sqrt{p} & 0 \end{pmatrix}} \underset{\hat{M}_1}{\begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}} = \begin{pmatrix} 0 & 0 \\ 0 & p \end{pmatrix} \end{array}\right\} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Measurement

$$\hat{M}_1 \quad \text{"click"} \quad |\psi\rangle \longrightarrow \frac{\hat{M}_1|\psi\rangle}{\sqrt{p(1)}}$$

$$\hat{M}_1 = \sqrt{p}\ |0\rangle\langle 1|$$

$$\hat{E}_1 = M_1^{\dagger} M_1 = p\,|1\rangle\langle 1|$$

$$\langle\psi|\hat{E}_1^{\dagger}\hat{E}_1|\psi\rangle = p\,|\langle\psi|1\rangle|^2$$

<u>Heralded preparation</u>

$$\hat{M}_0 \quad \text{"no click"} \quad |\psi'\rangle = \frac{\hat{M}_0|\psi\rangle}{\sqrt{p(0)}}$$

$$\hat{M}_0 = |0\rangle\langle 0| + \sqrt{1-p}\ |1\rangle\langle 1|$$

$$\hat{E}_0 = |0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$$

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

$$|\psi\rangle = |\psi'\rangle$$

$$|\psi'\rangle = a|0\rangle + b\sqrt{1-p}\,|1\rangle$$

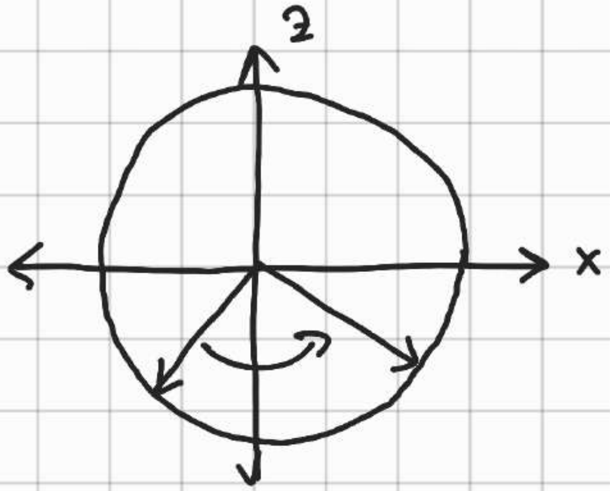
2) Phase Damping

$$|0\rangle \otimes |0\rangle = \sqrt{p}\,|0\rangle \otimes |0\rangle + \sqrt{1-p}\,|0\rangle \otimes |1\rangle$$

$$= |0\rangle \otimes \left(\sqrt{p}\,|0\rangle + \sqrt{1-p}\,|1\rangle \right)$$

$$|1\rangle \otimes |0\rangle = \sqrt{p}\,|1\rangle \otimes |0\rangle - \sqrt{1-p}\,|1\rangle \otimes |1\rangle$$

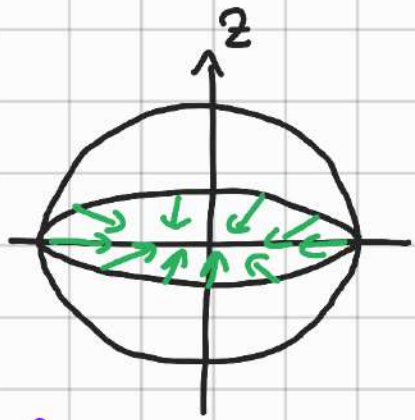
$$= |1\rangle \otimes \left(\sqrt{p}\,|0\rangle - \sqrt{1-p}\,|1\rangle \right)$$



$$\hat{M}_0 = \begin{pmatrix} \sqrt{p} & 0 \\ 0 & \sqrt{p} \end{pmatrix} \qquad \hat{M}_1 = \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & -\sqrt{1-p} \end{pmatrix}$$

$$\sqrt{p}\,\hat{\mathbb{I}} \qquad \sqrt{1-p}\,\hat{\sigma}_z$$

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \longrightarrow \begin{pmatrix} a_x\sqrt{1-p} \\ a_y\sqrt{1-p} \\ a_z \end{pmatrix}$$



Not reversible, inflation is not possible!