COHERENT STATES

A. Partially Entangled Bell State $|\Phi_{AB}\rangle$

$$|\Phi(0)\rangle = \underbrace{|\Phi_{AB}\rangle}_{\text{Atom}} \otimes \underbrace{|\alpha_A, \alpha_B\rangle}_{\text{Cavity}}$$

$$= \left(\cos\alpha\, |e_A, e_B\rangle + \sin\alpha\, |g_A, g_B\rangle\right) \otimes |\alpha_A, \alpha_B\rangle$$

$$= \cos\alpha\, |e_A, \alpha_A\rangle \otimes |e_B, \alpha_B\rangle + \sin\alpha\, |g_A, \alpha_A\rangle \otimes |g_B, \alpha_B\rangle$$

$$|e_A, \alpha_A\rangle = |e_A, \hat{D}(\alpha_A)|0\rangle\rangle = \hat{D}(\alpha_A)|e_A, 0\rangle$$

$$|\Phi(0)\rangle = \cos\alpha\left(\hat{D}(\alpha_A)|e_A, 0_a\rangle \otimes \hat{D}(\alpha_B)|e_B, 0_b\rangle\right)$$
$$+ \sin\alpha\left(\hat{D}(\alpha_A)|g_A, 0_a\rangle \otimes \hat{D}(\alpha_B)|g_B, 0_b\rangle\right)$$

$$|\Phi(0)\rangle = \cos\alpha\Big(\hat{D}(\alpha_A)\left(c|\Psi_1^+\rangle_A - s|\Psi_1^-\rangle_A\right) \otimes$$
$$\hat{D}(\alpha_B)\left(c|\Psi_1^+\rangle_B - s|\Psi_1^-\rangle_B\right)\Big) +$$
$$\sin\alpha\left(\hat{D}(\alpha_A)|\Psi^0\rangle_A \otimes \hat{D}(\alpha_B)|\Psi^0\rangle_B\right)$$

$$H_{JC}|\psi^{\pm}\rangle = \lambda^{\pm}|\psi^{\pm}\rangle$$

$$|\Phi^{\pm}(t)\rangle = e^{-i\lambda^{\pm}t}|\Phi^{\pm}(0)\rangle$$

$$|\Phi(t)\rangle = \cos\alpha\left(\hat{D}(\alpha_A)\left(c\,e^{-i\lambda^{+}t}|\psi_1^{+}\rangle_A - s\,e^{-i\lambda^{-}t}|\psi_1^{-}\rangle_A\right)\right. \otimes$$

$$\left.\hat{D}(\alpha_B)\left(c\,e^{-i\lambda^{+}t}|\psi_1^{+}\rangle_B - s\,e^{-i\lambda^{-}t}|\psi_1^{-}\rangle_B\right)\right) +$$

$$\sin\alpha\left(\hat{D}(\alpha_A)|\psi^0\rangle_A \otimes \hat{D}(\alpha_B)|\psi^0\rangle_B\right)$$

$\downarrow$ Revert back to basis

$$|\Phi(t)\rangle = \cos\alpha\left(\hat{D}(\alpha_A)\left(c\,e^{-i\lambda^{+}t}\left(c|e_A,0_a\rangle + s|g_A,1_a\rangle\right) -\right.\right.$$

$$\left. s\,e^{-i\lambda^{-}t}\left(-s|e_A,0_a\rangle + c|g_A,1_a\rangle\right)\right) \otimes$$

$$\left(\hat{D}(\alpha_B)\left(c\,e^{-i\lambda^{+}t}\left(c|e_B,0_b\rangle + s|g_B,1_b\rangle\right) -\right.\right.$$

$$\left.\left. s\,e^{-i\lambda^{-}t}\left(-s|e_B,0_b\rangle + c|g_B,1_b\rangle\right)\right)\right) +$$

$$+\sin\alpha\,\hat{D}(\alpha_A)|g_A,0_a\rangle \otimes \hat{D}(\alpha_B)|g_B,0_b\rangle$$

Let's write $\Phi(t)$ like below:

$$|\Phi(t)\rangle = x_1'|\uparrow\uparrow 00\rangle + x_2'|\downarrow\downarrow 11\rangle + x_3'|\uparrow\downarrow 01\rangle + x_4'|\downarrow\uparrow 10\rangle + x_5'|\downarrow\downarrow 00\rangle$$

$$x_1' = \hat{D}(\alpha_A) \otimes \hat{D}(\alpha_B)\, \underbrace{\cos\alpha \left(c^2 e^{-i\lambda^+ t} + s^2 e^{-i\lambda^- t}\right)^2}_{x_1}$$

$$x_2' = \hat{D}(\alpha_A) \otimes \hat{D}(\alpha_B)\, \underbrace{\cos\alpha \left(cs\, e^{-i\lambda^+ t} - sc\, e^{-i\lambda^- t}\right)^2}_{x_2}$$

$$x_3' = \hat{D}(\alpha_A) \otimes \hat{D}(\alpha_B)\, \underbrace{\left(c^2 e^{-i\lambda^+ t} + s^2 e^{-i\lambda^- t}\right)\left(cs e^{-i\lambda^+ t} - cs e^{-i\lambda^- t}\right)}_{x_3}$$

$$x_4' = x_3$$

$$x_5' = \hat{D}(\alpha_A) \otimes \hat{D}(\alpha_B)\, \underbrace{\sin\alpha}$$

$x_5 \rightarrow$ for one photon case

$$\hat{D}(\alpha_A)|0\rangle = |\alpha_A\rangle$$

$$\hat{D}(\alpha_A)|1\rangle = \hat{D}(\alpha_A).\hat{D}(1)|0\rangle = ✗\ \hat{D}(\alpha_A + 1)$$

$$\hat{D}(\alpha_A) = e^{\alpha_A \hat{a}^+}\, e^{-\alpha_A^* \hat{a}}\, e^{-|\alpha_A|^2/2}$$

$$\hat{D}(1) = e^{1.\hat{a}^+}\, e^{-1\hat{a}}\, e^{-1/2}$$

$$\hat{D}(\alpha_A)\hat{D}(1) = e^{\cancel{(\alpha_A+1)\hat{a}^+}}\, e^{\cancel{-(\alpha_A^*+1)}}\, e^{-|\alpha_A|^2/2}\, e^{-1/2}$$

$$\hat{D}(\alpha_{A+1}) = e^{\cancel{(\alpha_{A+1})\hat{a}^+}}\, e^{-\cancel{(\alpha_A^*+1)}\,\hat{a}}\, e^{-|\alpha_{A+1}|^2/2}$$

$$e^{-|\alpha_A|^2/2}\, e^{-1/2} = ✗\ e^{-(\alpha_A+1)(\alpha_A^*+1)/2}$$

$$e^{\cancel{-|\alpha_A|^2/2}}\, e^{\cancel{-1/2}} = ✗\ e^{\cancel{-|\alpha_A|^2/2}}\, e^{-\alpha_A/2}\, e^{-\alpha_A^*/2}\, e^{\cancel{-1/2}}$$

$$\swarrow$$

$$e^{(\alpha_A + \alpha_A^*)/2} = e^{\mathrm{Re}\{\alpha_A\}}$$

$$\begin{array}{r} \alpha_A = a + j\cancel{b} \\ + \ \alpha_A^* = a - j\cancel{b} \\ \hline 2a \end{array}$$

$$|\Phi(t)\rangle = x_1 |\uparrow\uparrow \alpha_A \alpha_B\rangle + x_2\, e^{Re\{\alpha_A\}+Re\{\alpha_B\}} |\downarrow\downarrow (\alpha_A+1)(\alpha_B+1)\rangle$$
$$+ x_3\, e^{Re\{\alpha_B\}} |\uparrow\downarrow \alpha_A (\alpha_B+1)\rangle + x_4\, e^{Re\{\alpha_A\}} |\downarrow\uparrow (\alpha_A+1)\alpha_B\rangle + x_5 |\downarrow\downarrow \alpha_A \alpha_B\rangle$$

A.1 $C_{AB}(t)$

Calculation Method:

$$\rho^{AB} = Tr_{ab}\left[|\Phi(t)\rangle\langle\Phi(t)| \right]$$

$$Tr_{\alpha_A\alpha_B} \Rightarrow |\tilde{x}_1|^2 |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\tilde{x}_5|^2 |\downarrow\downarrow\rangle\langle\downarrow\downarrow| + \tilde{x}_1 \tilde{x}_5^* |\uparrow\uparrow\rangle\langle\downarrow\downarrow| + \tilde{x}_1^* \tilde{x}_5 |\downarrow\downarrow\rangle\langle\downarrow\downarrow|$$

$$Tr_{\alpha_A(\alpha_B+1)} \Rightarrow |\tilde{x}_3|^2 |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$$

$$Tr_{(\alpha_A+1)\alpha_B} \Rightarrow |\tilde{x}_4|^2 |\downarrow\uparrow\rangle\langle\downarrow\uparrow|$$

$$Tr_{(\alpha_A+1)(\alpha_B+1)} \Rightarrow |\tilde{x}_2| \,|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$$

Similar to one-photon case:

$$C_{AB}(t) = 2\max\left[0, Q(t)\right]$$

$$C = 2\,|\tilde{x}_1||\tilde{x}_5| - 2|\tilde{x}_3||\tilde{x}_4|$$

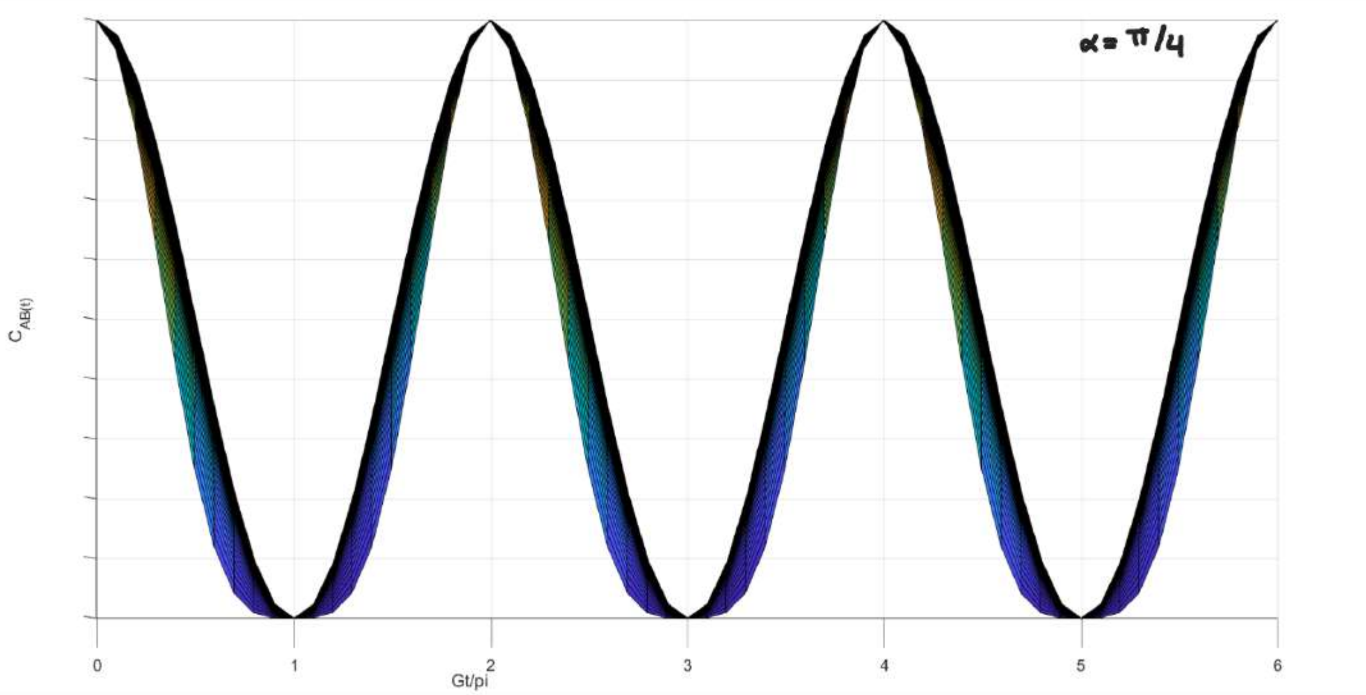
$$\left.\begin{aligned} |\tilde{x}_1| &= |x_1| = |\cos\alpha| \cos^2\left(\tfrac{Gt}{2}\right) \\ |\tilde{x}_5| &= |x_5| = |\sin\alpha| \end{aligned}\right\} 2|x_1||x_5| = \frac{1}{2}|\sin 2\alpha| \cos^2\left(\frac{Gt}{2}\right)$$

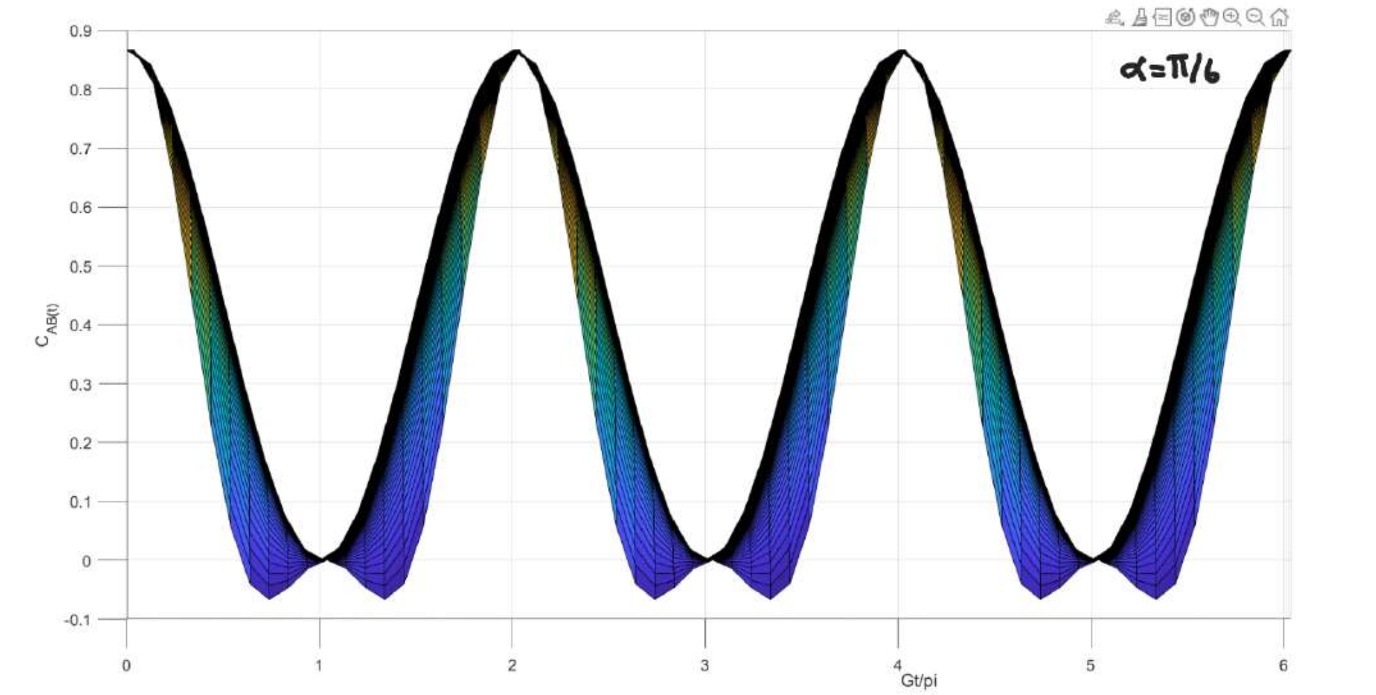
$$|\tilde{x}_3| = e^{Re\{\alpha_B\}} \cdot |\cos\alpha| \left|\cos\left(\tfrac{Gt}{2}\right)\right| \left|\sin\left(\tfrac{Gt}{2}\right)\right|$$

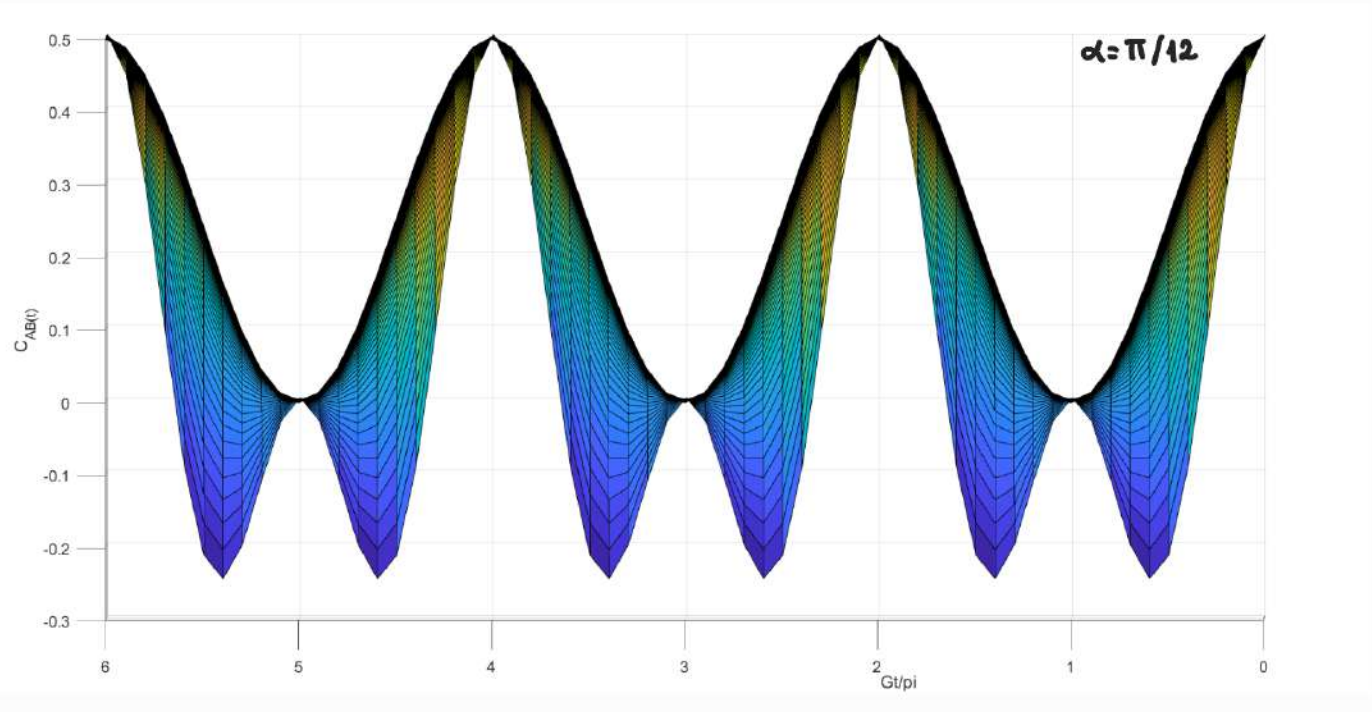
$$|\tilde{x}_4| = e^{Re\{\alpha_A\}} |\cos\alpha| \left|\cos\left(\tfrac{Gt}{2}\right)\right| \left|\sin\left(\tfrac{Gt}{2}\right)\right|$$

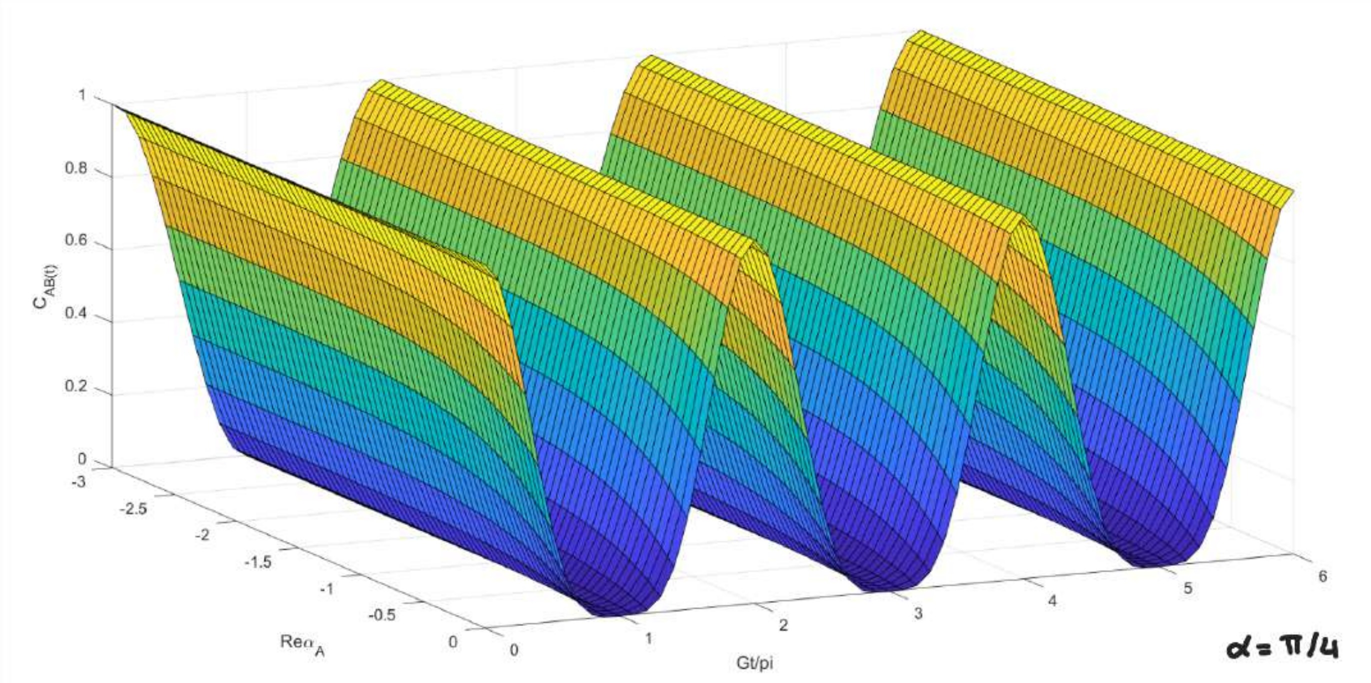
$$|\tilde{x}_3||\tilde{x}_4| = e^{Re\{\alpha_B\}+Re\{\alpha_A\}} \cos^2\alpha \cos^2\left(\tfrac{Gt}{2}\right) \sin^2\left(\tfrac{Gt}{2}\right)$$

$$C = 2|\tilde{x}_1||\tilde{x}_5| - 2|\tilde{x}_3||\tilde{x}_4| = \cos^2\left(\frac{Gt}{2}\right)\left[|\sin 2\alpha| - \cos^2\alpha \sin^2\left(\frac{Gt}{2}\right) e^{Re\{\alpha_B\}+Re\{\alpha_A\}} \right]$$

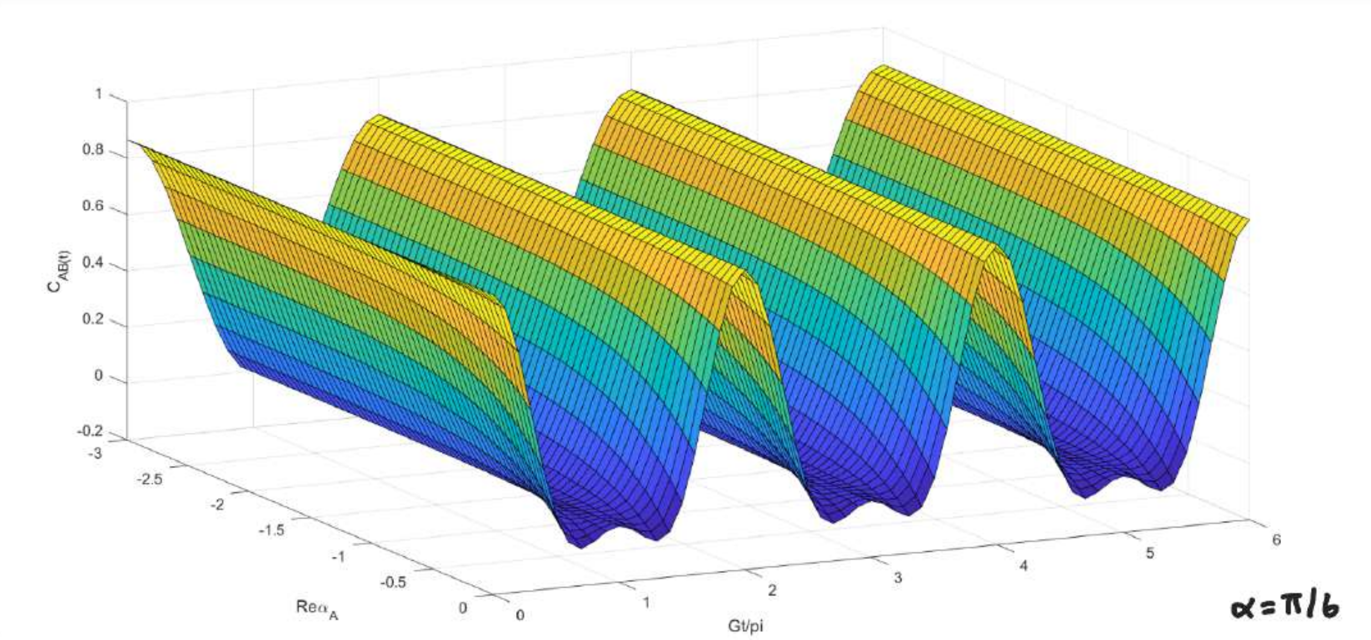

α=π/4
C_AB(t)
0
1
2
3
4
5
6
Gt/pi


α=π/6
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
C_AB(t)
0
1
2
3
4
5
6
Gt/pi

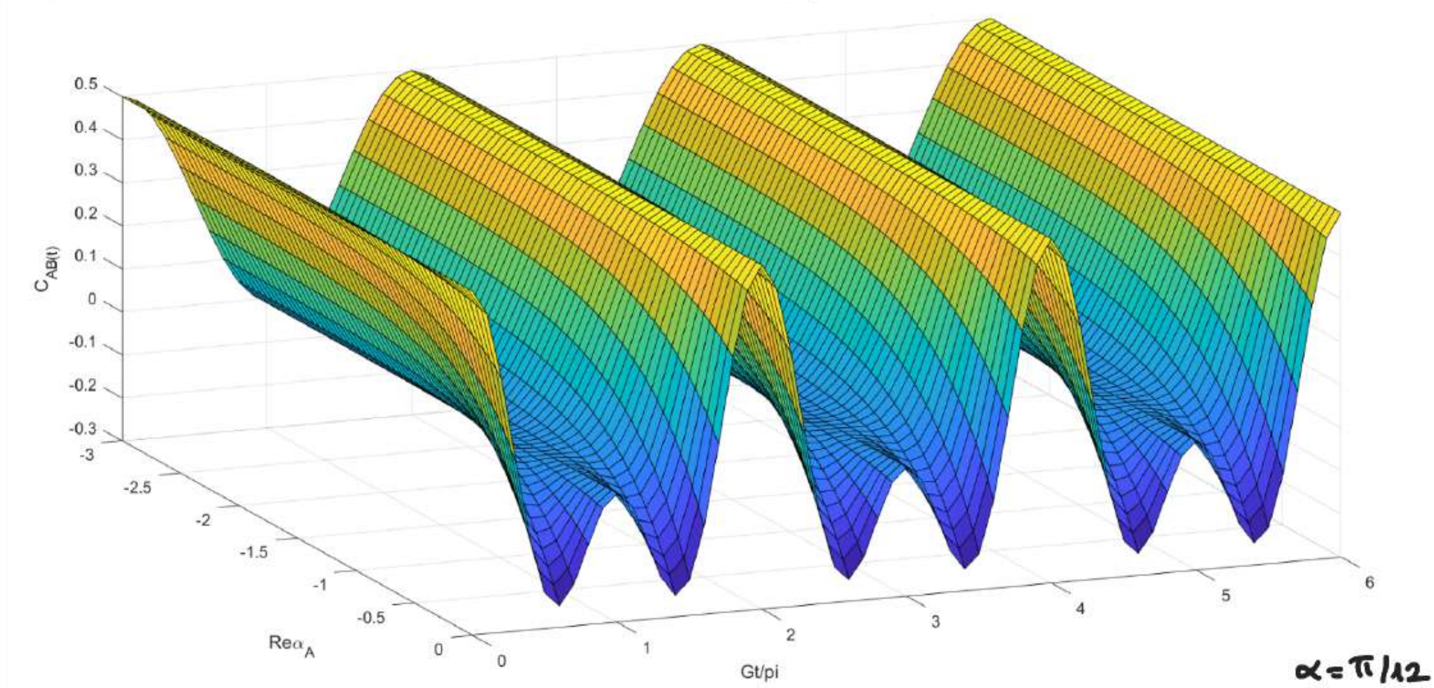

α=π/12
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
C_AB(t)
6
5
4
3
2
1
0
Gt/pi



$\alpha = \pi/4$



$\alpha = \pi/6$



$\alpha = \pi/12$

2. Cab(t)

$$C^{ab} = 2\,|\tilde{x}_2|\,|\tilde{x}_5| - 2\,|\tilde{x}_3|\,|\tilde{x}_4|$$

For $\Delta = 0$ Tuned Case:

$$|\tilde{x}_2| = |x_2|\, e^{Re\{\alpha_A\} + Re\{\alpha_B\}} = e^{Re\{\alpha_A\} + Re\{\alpha_B\}} \sin^2\left(\frac{6t}{2}\right) |\cos\alpha|$$

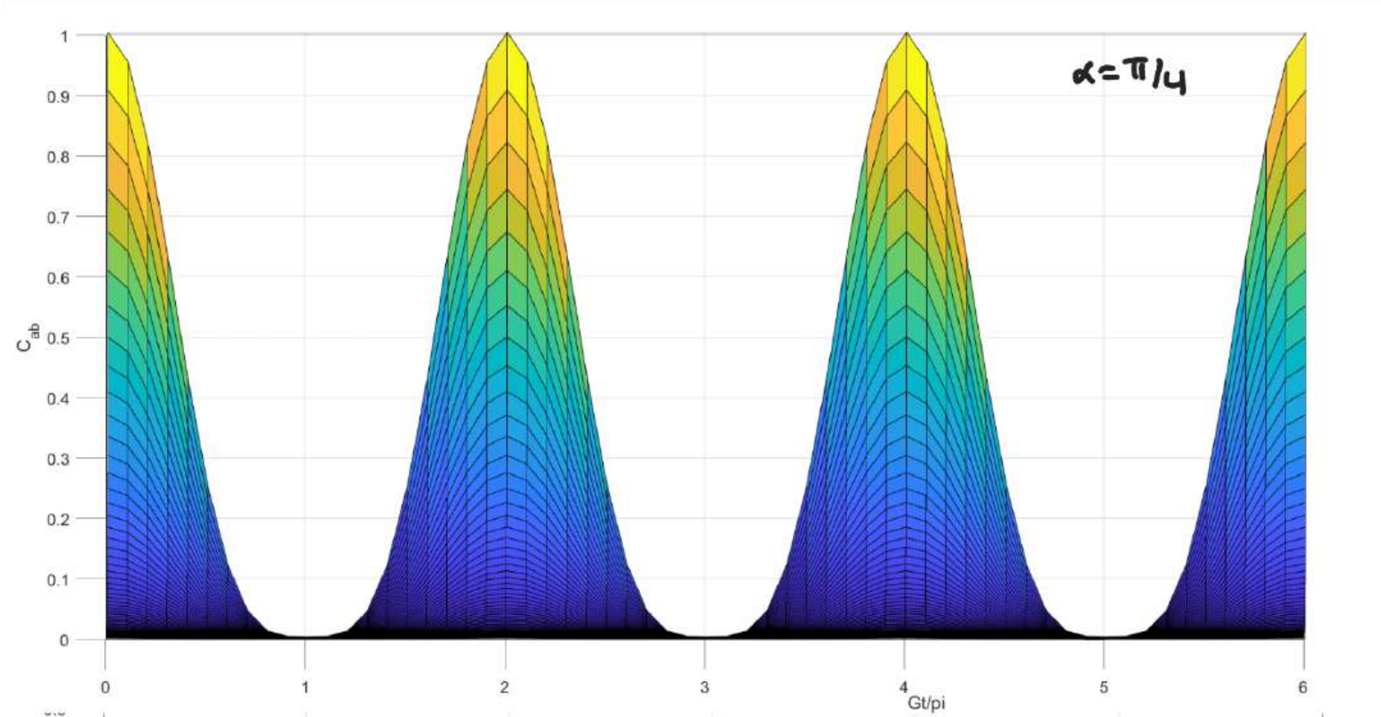
$$|\tilde{x}_5| = |\sin\alpha|$$

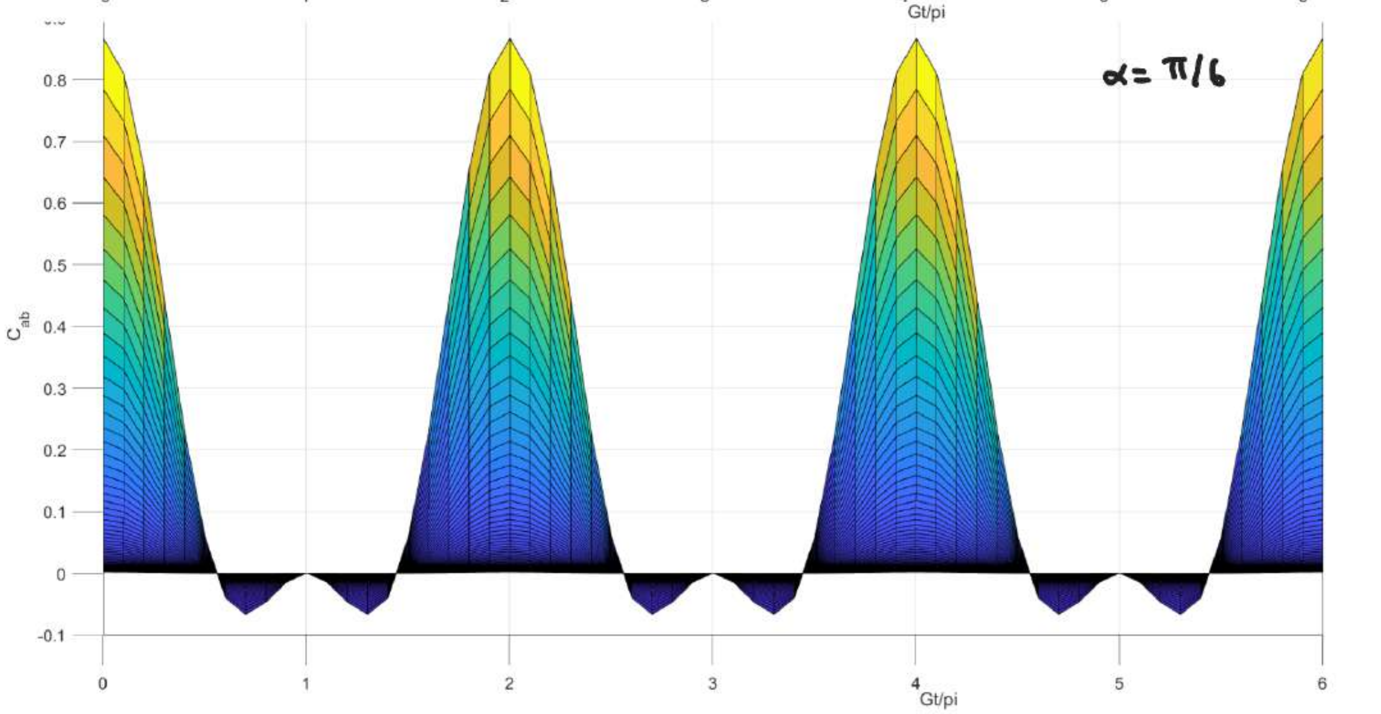
$$|\tilde{x}_3| = e^{Re\{\alpha_B\}} \cdot |\cos\alpha|\, \left|\cos\left(\frac{6t}{2}\right)\right| \left|\sin\left(\frac{6t}{2}\right)\right|$$

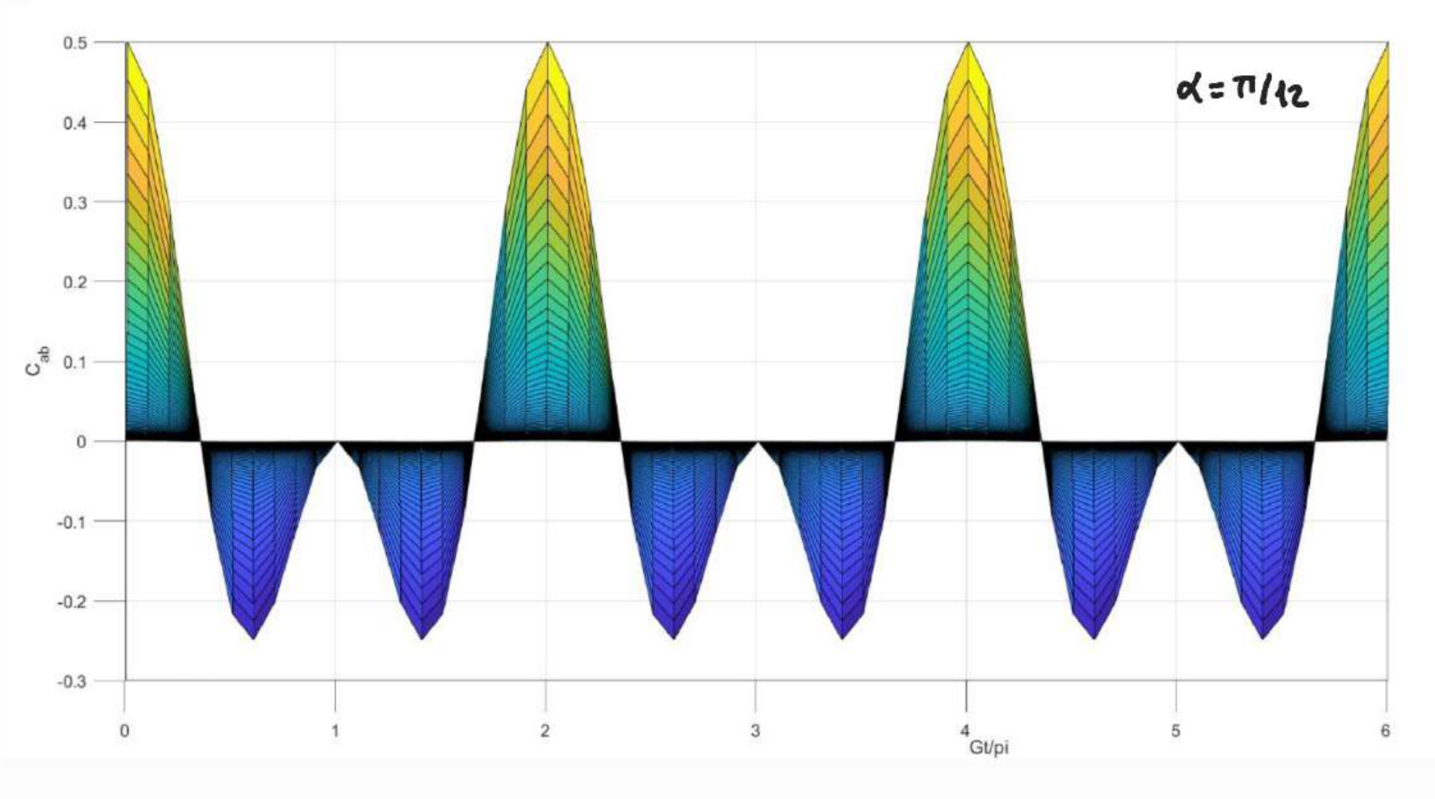
$$|\tilde{x}_4| = e^{Re\{\alpha_A\}}\, |\cos\alpha|\, \left|\cos\left(\frac{6t}{2}\right)\right| \left|\sin\left(\frac{6t}{2}\right)\right|$$

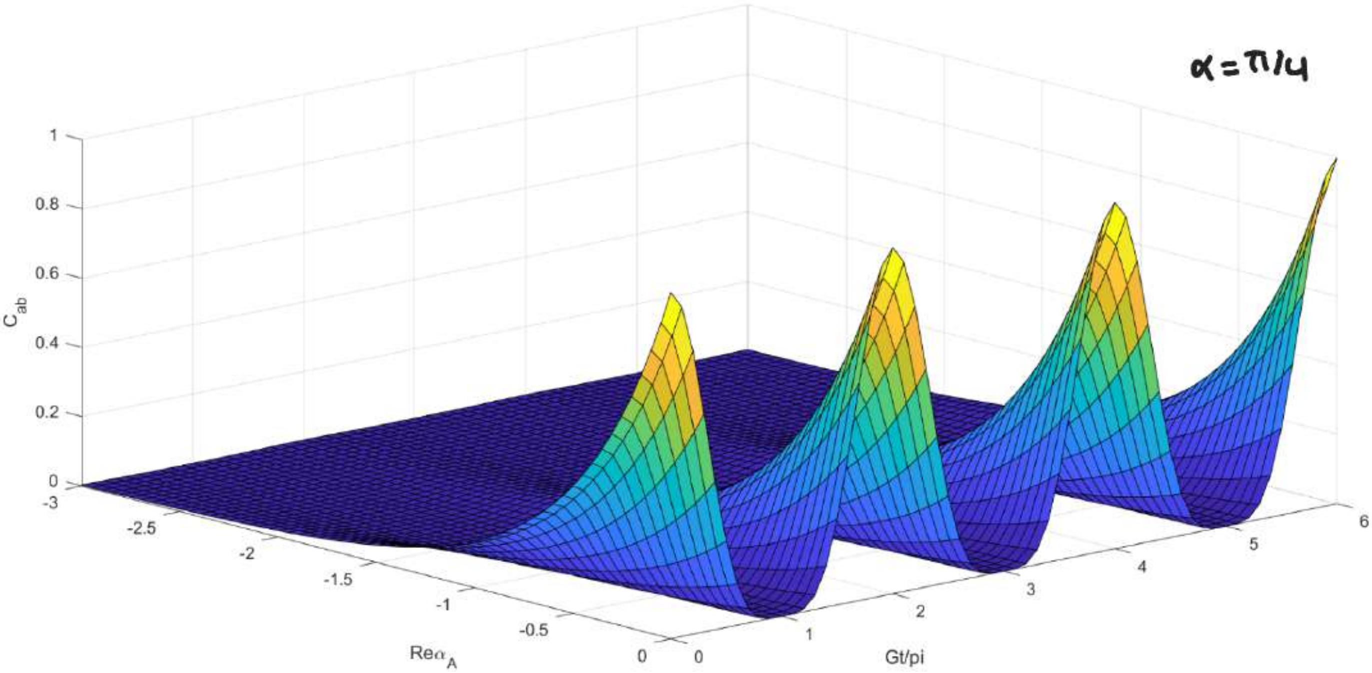
$$|\tilde{x}_3|\,|\tilde{x}_4| = e^{Re\{\alpha_B\} + Re\{\alpha_A\}} \cos^2\alpha\, \cos^2\left(\frac{6t}{2}\right) \sin^2\left(\frac{6t}{2}\right)$$

$$C^{ab} = \sin^2\left(\frac{6t}{2}\right) e^{Re\{\alpha_A\} + Re\{\alpha_B\}} \left[\, |\sin 2\alpha| - 2\cos^2\left(\frac{6t}{2}\right)\cos^2\alpha \,\right]$$

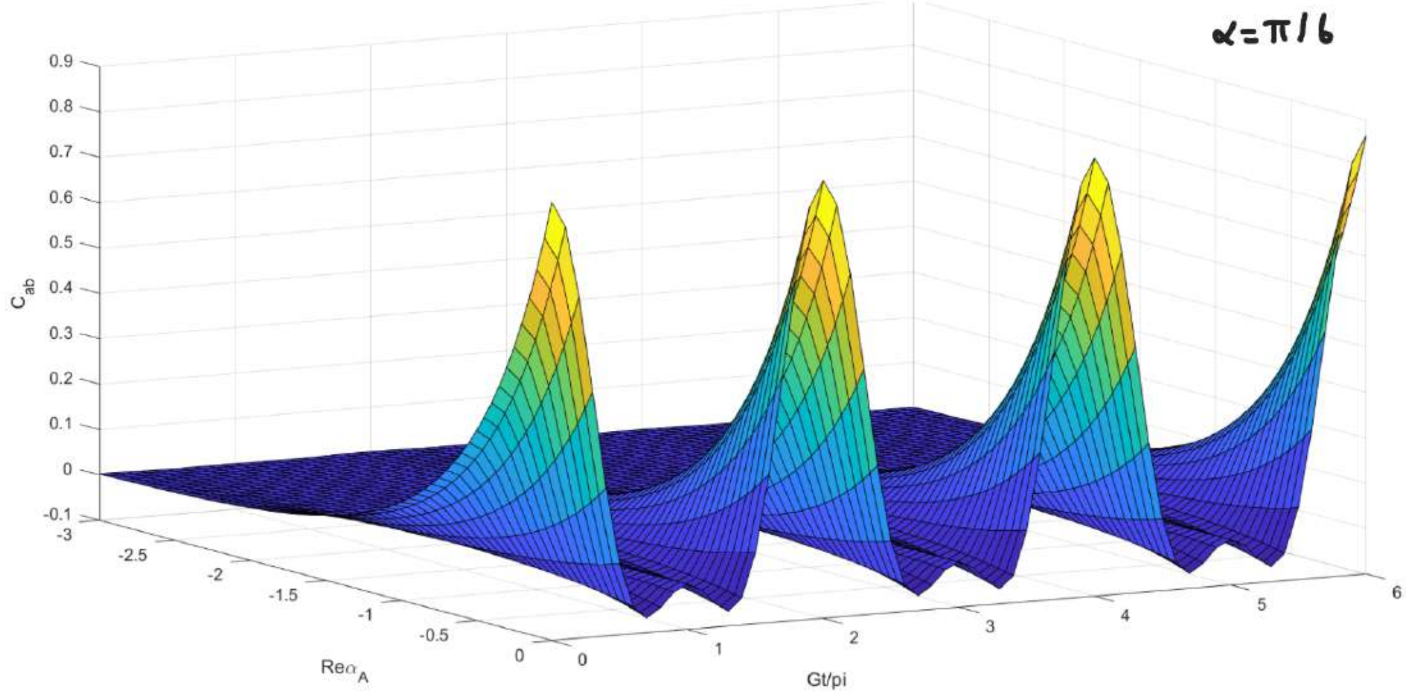

α=π/4
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
C_ab
0
1
2
3
4
5
6
Gt/pi


α=π/6
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
C_ab
0
1
2
3
4
5
6
Gt/pi

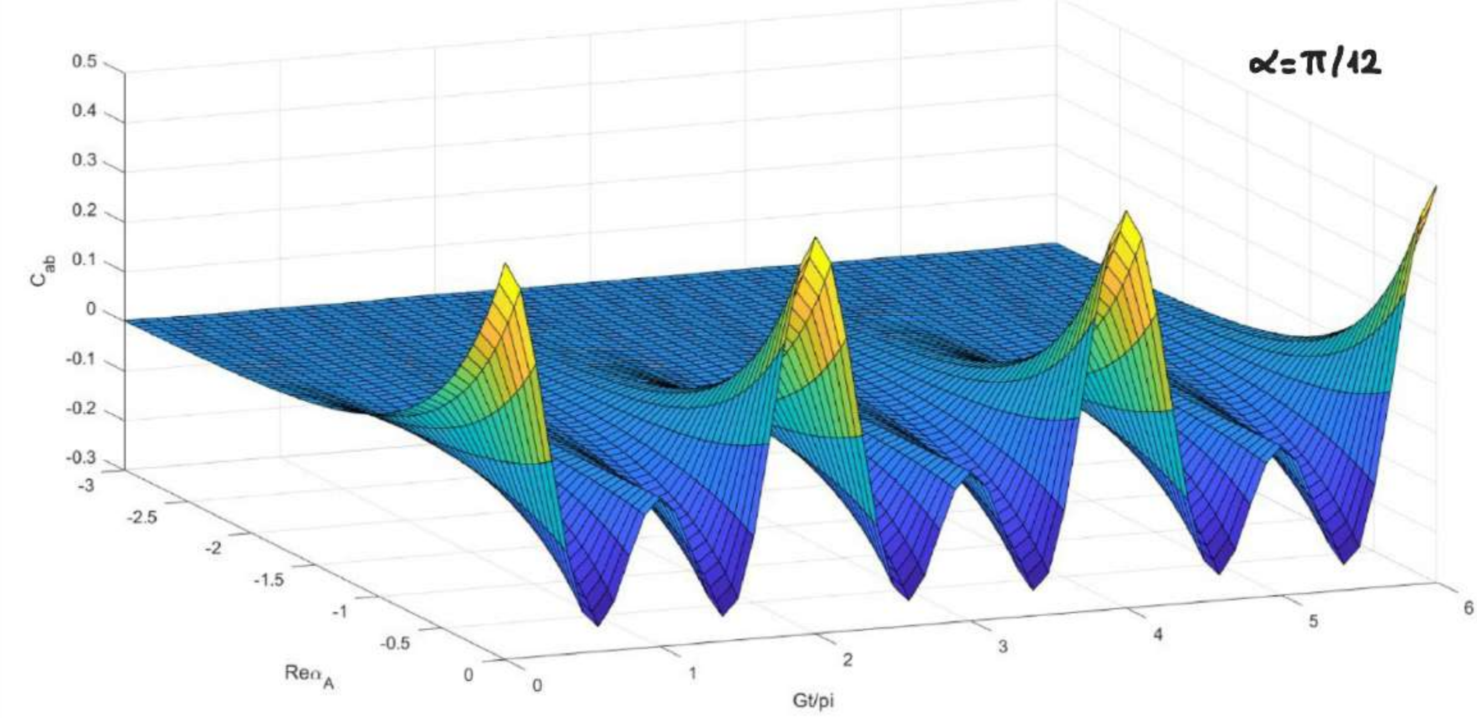

α=π/12
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
C_ab
0
1
2
3
4
5
6
Gt/pi



α=π/4
C_ab
1
0.8
0.6
0.4
0.2
0
-3
-2.5
-2
-1.5
-1
-0.5
0
Reα_A
0
1
2
3
4
5
6
Gt/pi



α=π/6
C_ab
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-3
-2.5
-2
-1.5
-1
-0.5
0
Reα_A
0
1
2
3
4
5
6
Gt/pi



α=π/12
C_ab
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-3
-2.5
-2
-1.5
-1
-0.5
0
Reα_A
0
1
2
3
4
5
6
Gt/pi

3. $C_{Ab}(t)$

$$C_{Ab}(t) = 2\,|\tilde{x}_3||\tilde{x}_5| - 2\,|\tilde{x}_1||\tilde{x}_2|$$

$$|\tilde{x}_3| = e^{Re\{\alpha_B\}} \cdot |\cos\alpha| \left|\cos\left(\frac{6t}{2}\right)\right| \left|\sin\left(\frac{6t}{2}\right)\right|$$

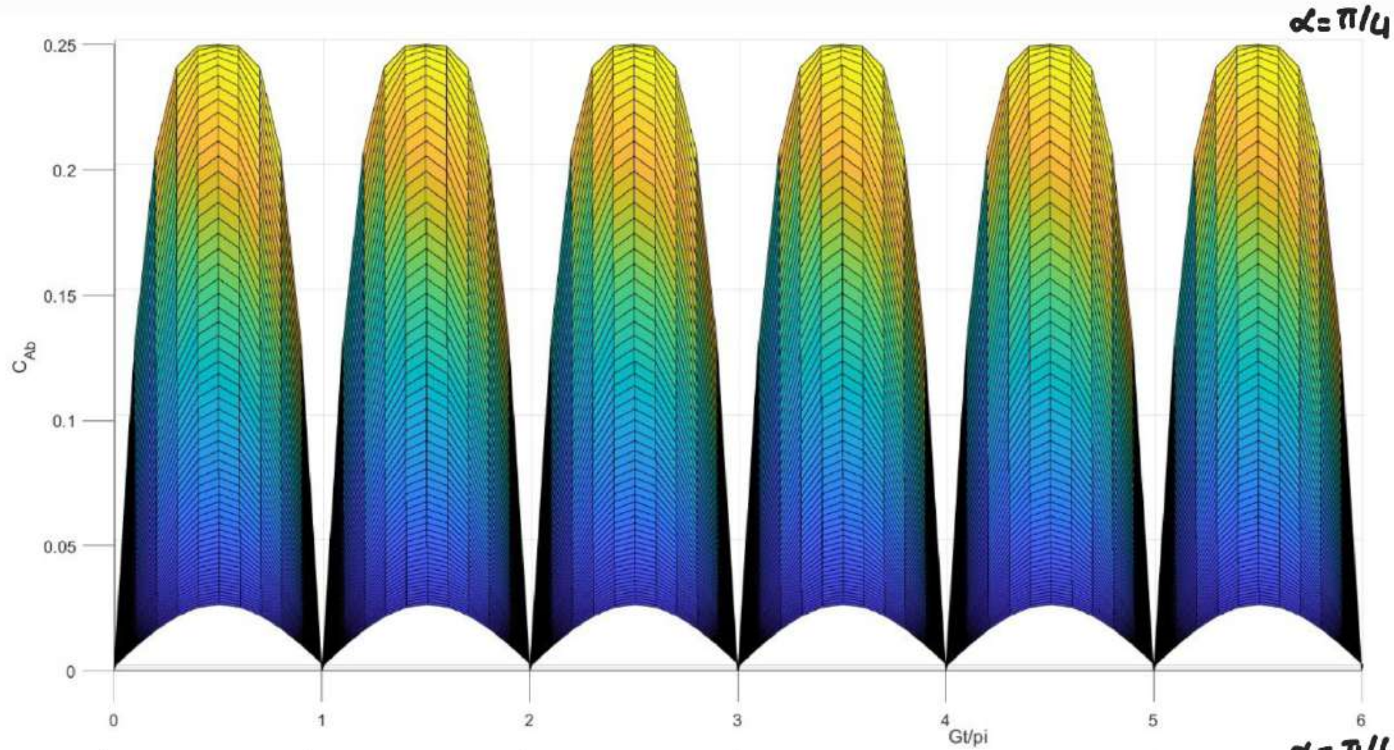
$$|\tilde{x}_5| = |x_5| = |\sin\alpha|$$

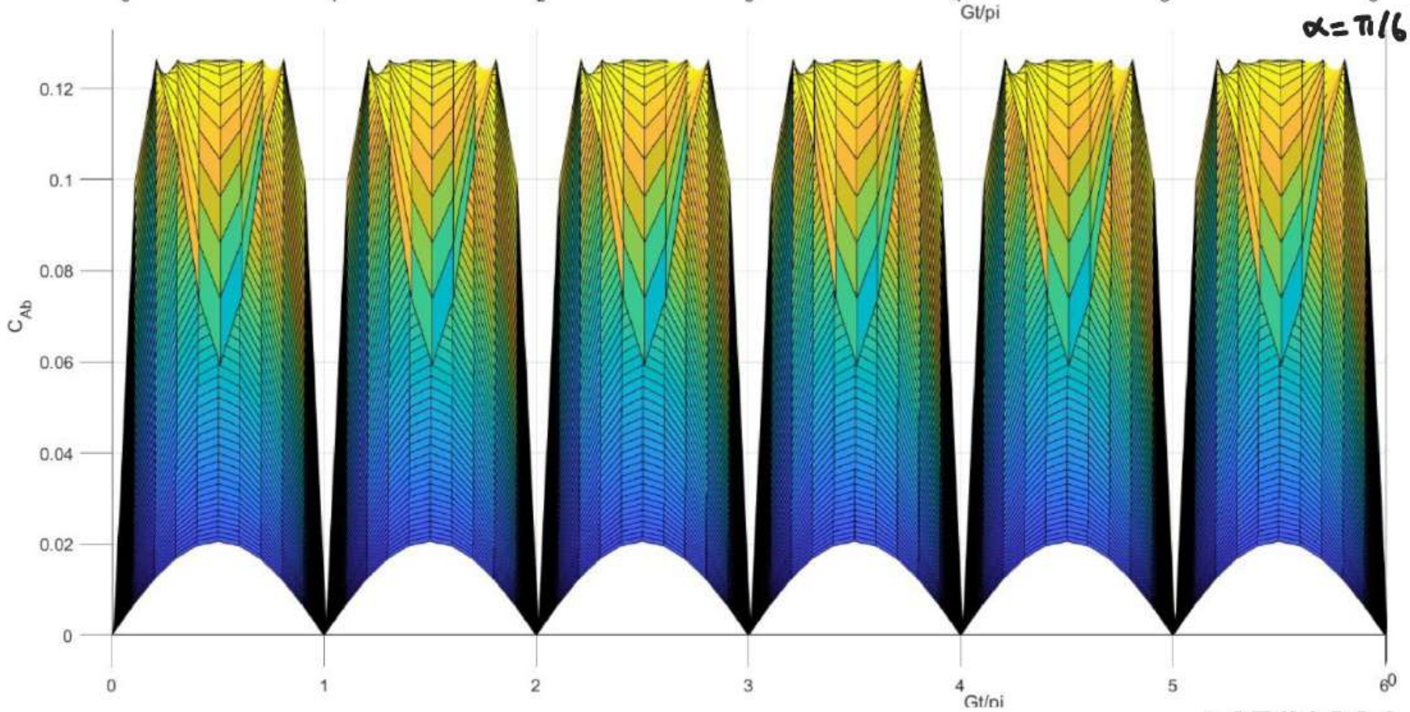
$$|\tilde{x}_1| = |x_1| = |\cos\alpha| \cos^2\left(\frac{6t}{2}\right)$$

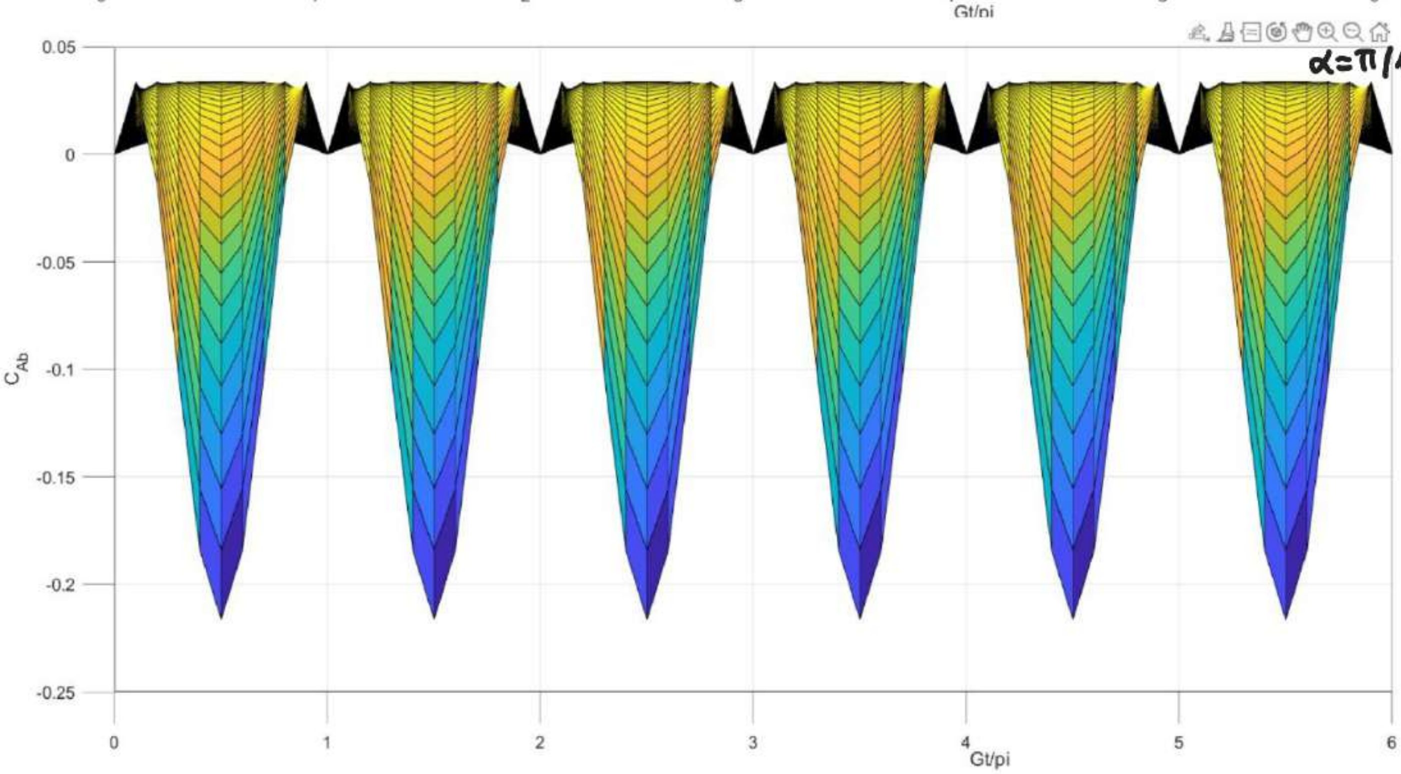
$$|\tilde{x}_2| = e^{Re\{\alpha_A\}+Re\{\alpha_B\}} \sin^2\left(\frac{6t}{2}\right)|\cos\alpha|$$

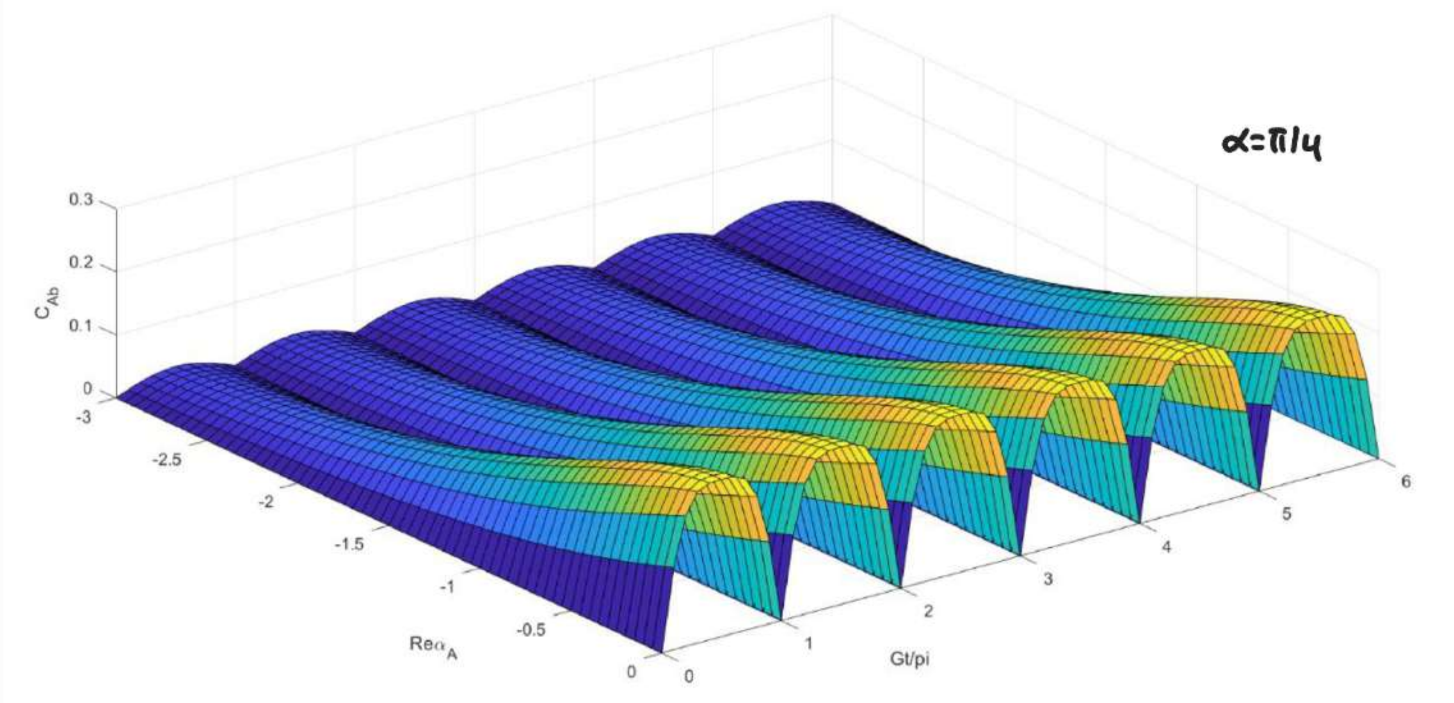
$$C_{Ab}(t) = e^{Re\{\alpha_B\}} |\sin 2\alpha| \frac{1}{2} |\sin(6t)| - e^{Re\{\alpha_A\}+Re\{\alpha_B\}} \cos^2\alpha \frac{1}{2} \sin^2(6t)$$

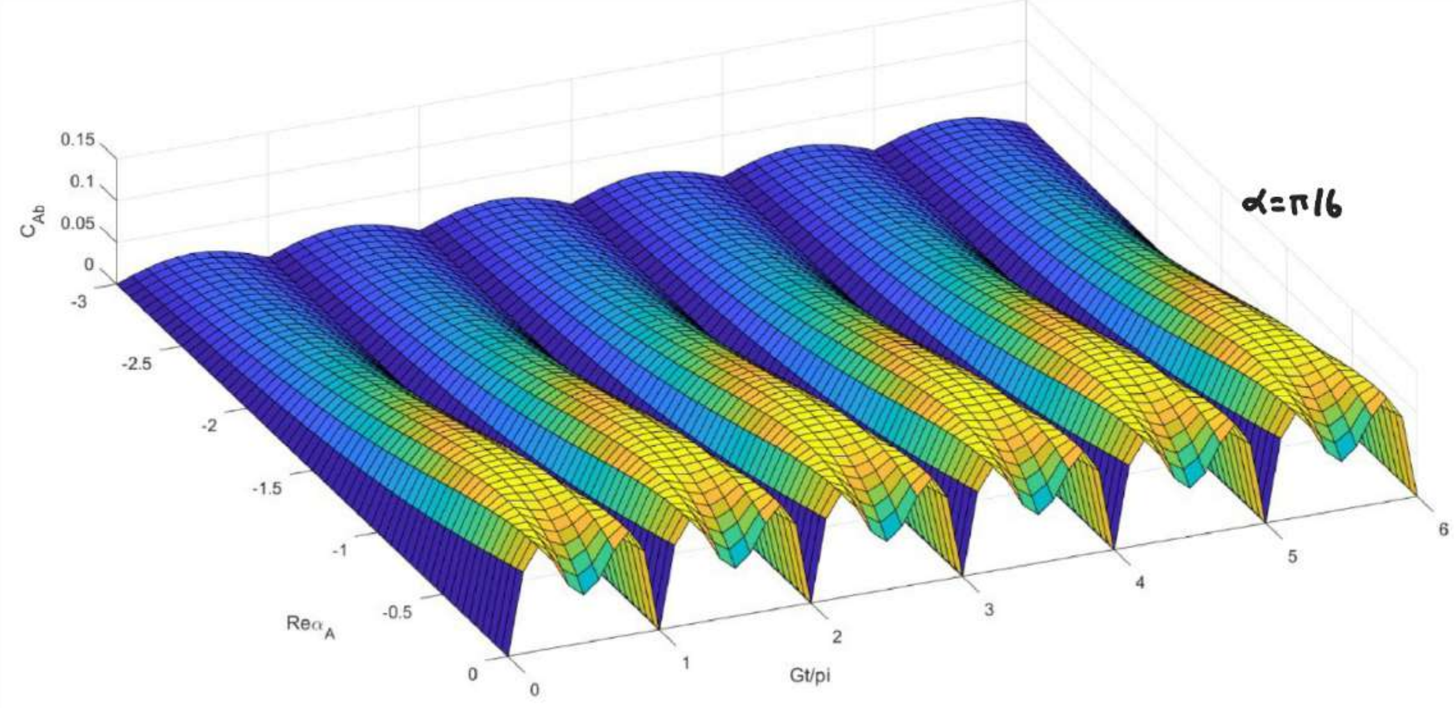
$$= \frac{1}{2} \cos^2\alpha\, |\sin(6t)|\, e^{Re\{\alpha_B\}} \left[2|\tan\alpha| - e^{Re\{\alpha_A\}} |\sin(6t)| \right]$$

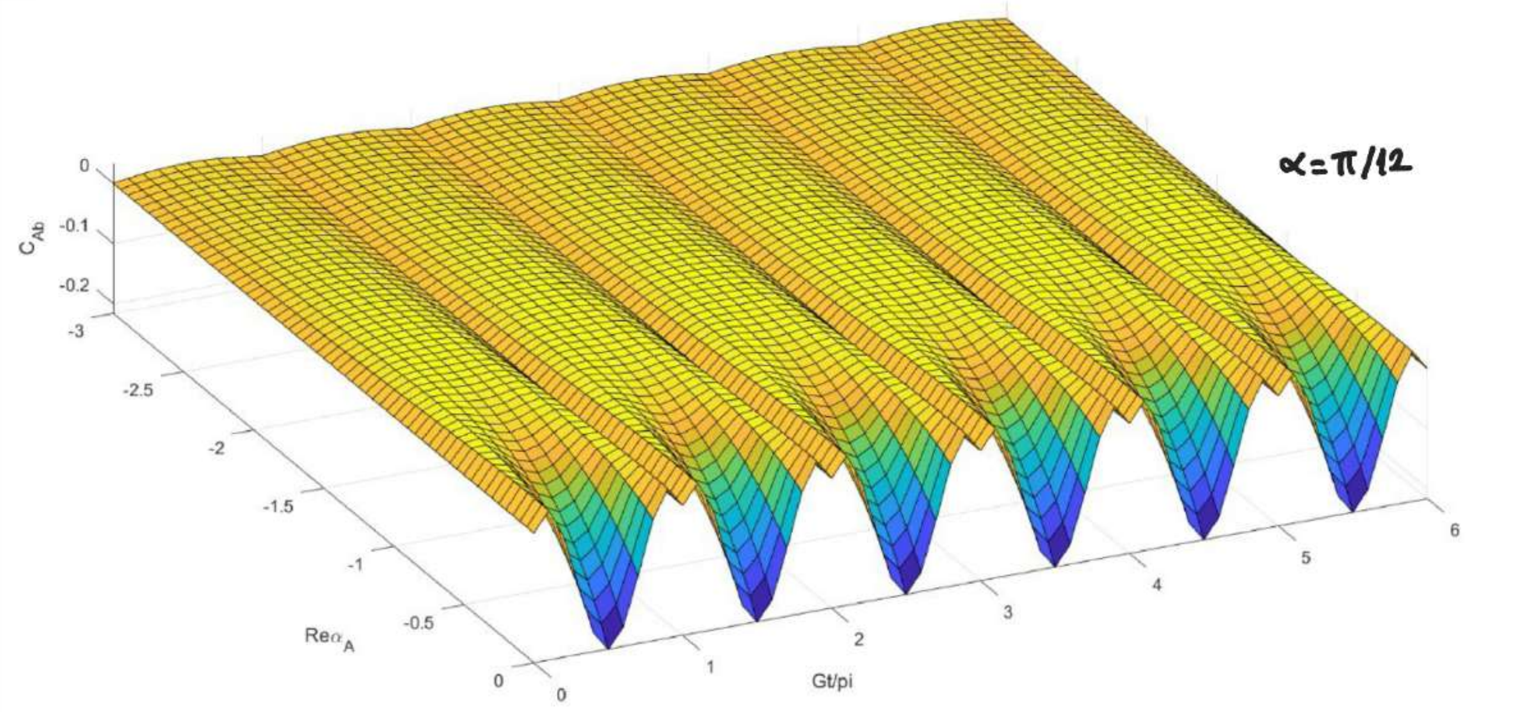

α=π/4
C_Ab
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
Gt/pi


α=π/6
C_Ab
0.12
0.1
0.08
0.06
0.04
0.02
0
0
1
2
3
4
5
6
Gt/pi


α=π/12
C_Ab
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
1
2
3
4
5
6
Gt/pi


α=π/4
C_Ab
0.3
0.2
0.1
0
-3
-2.5
-2
-1.5
-1
-0.5
0
Reα_A
0
1
2
3
4
5
6
Gt/pi


α=π/6
C_Ab
0.15
0.1
0.05
0
-3
-2.5
-2
-1.5
-1
-0.5
0
Reα_A
0
1
2
3
4
5
6
Gt/pi


α=π/12
C_Ab
0
-0.1
-0.2
-3
-2.5
-2
-1.5
-1
-0.5
0
Reα_A
0
1
2
3
4
5
6
Gt/pi

4. $C_{aB}(t)$

$$C_{aB}(t) = 2\,|\tilde{x}_4|\,|\tilde{x}_5| - 2\,|\tilde{x}_1|\,|\tilde{x}_2|$$

In the 1-photon case $C_{aB}(t) = C_{Ab}(t)$, since $|x_3| = |x_4|$

However for these case $|\tilde{x}_3| \neq |\tilde{x}_4|$ due to the exponential terms

Yet, in the numerical analysis, I assumed that $Re\{\alpha_A\} = Re\{\alpha_B\}$

thus plots are same with $C_{Ab}(t)$.

$$C_{aB}(t) = \frac{1}{2}\cos^2\alpha\,|\sin(6t)|\,e^{Re\{\alpha_A\}}\left[2|\tan\alpha| - e^{Re\{\alpha_B\}}|\sin(6t)|\right]$$

$$C_{Ab}(t) = \frac{1}{2}\cos^2\alpha\,|\sin(6t)|\,e^{Re\{\alpha_B\}}\left[2|\tan\alpha| - e^{Re\{\alpha_A\}}|\sin(6t)|\right]$$

5. $C_{Aa}(t)$

$$C_{Aa} = 2\left(|\tilde{x}_2 \tilde{x}_3 + \tilde{x}_4 \tilde{x}_1| \right)$$

$$\tilde{x}_2 = e^{Re\{\alpha_A\}+Re\{\alpha_B\}} \sin^2\left(\frac{6t}{2}\right) \cos\alpha$$

$$\tilde{x}_3 = e^{Re\{\alpha_B\}} \cdot \cos\alpha \overbrace{\cos\left(\frac{6t}{2}\right)\sin\left(\frac{6t}{2}\right)}^{\sin(6t)/2}$$

$$\tilde{x}_2 \tilde{x}_3 = e^{Re\{\alpha_A\}+2Re\{\alpha_B\}} \cos^2\alpha \sin^2\left(\frac{6t}{2}\right) \frac{1}{2}\sin(6t)$$

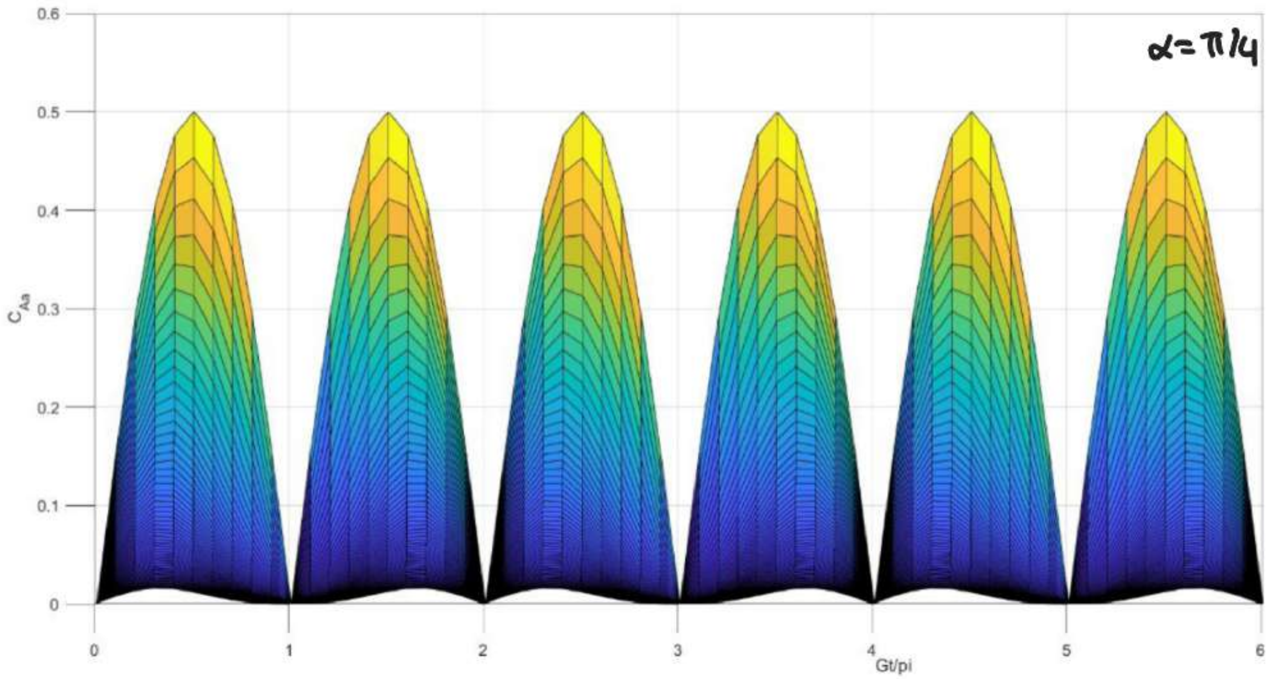
$$\tilde{x}_1 = \cos\alpha \cos^2\left(\frac{6t}{2}\right)$$

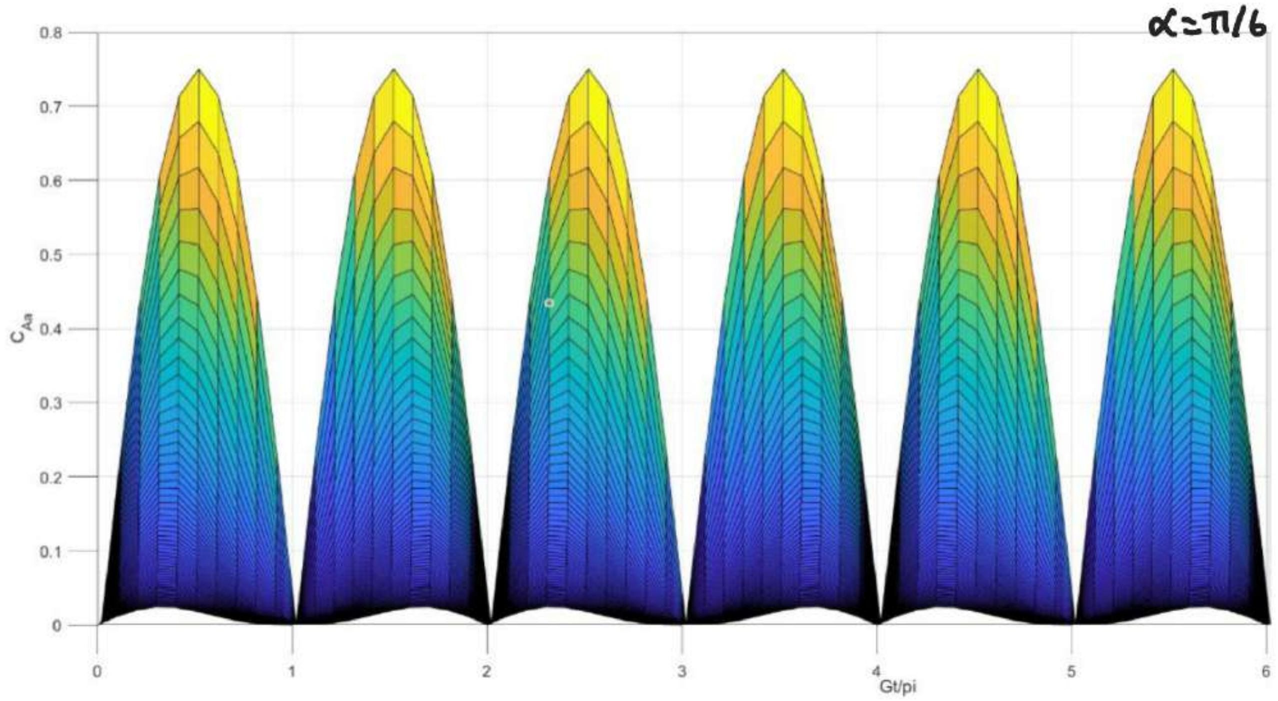
$$\tilde{x}_4 = e^{Re\{\alpha_A\}} \cos\alpha \overbrace{\cos\left(\frac{6t}{2}\right)\sin\left(\frac{6t}{2}\right)}^{\frac{1}{2}\sin(6t)}$$

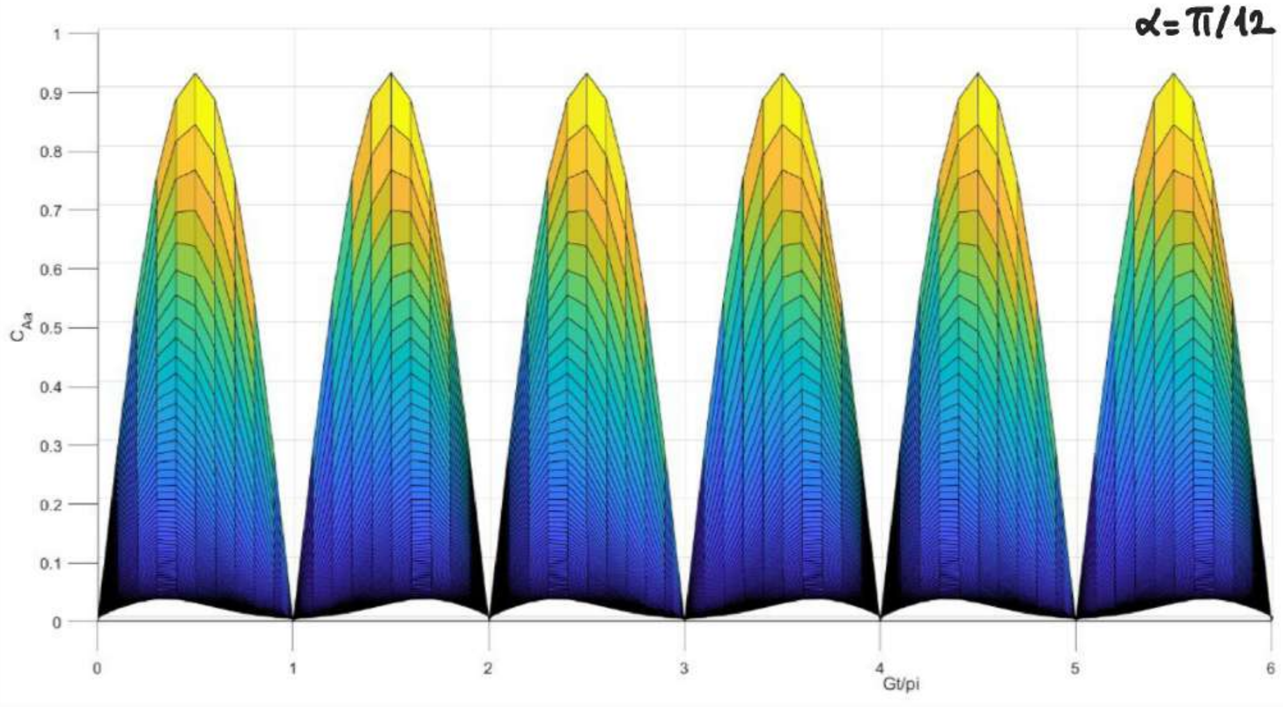
$$\tilde{x}_1 \tilde{x}_4 = e^{Re\{\alpha_A\}} \cos^2\alpha \cos^2\left(\frac{6t}{2}\right) \frac{1}{2}\sin(6t)$$

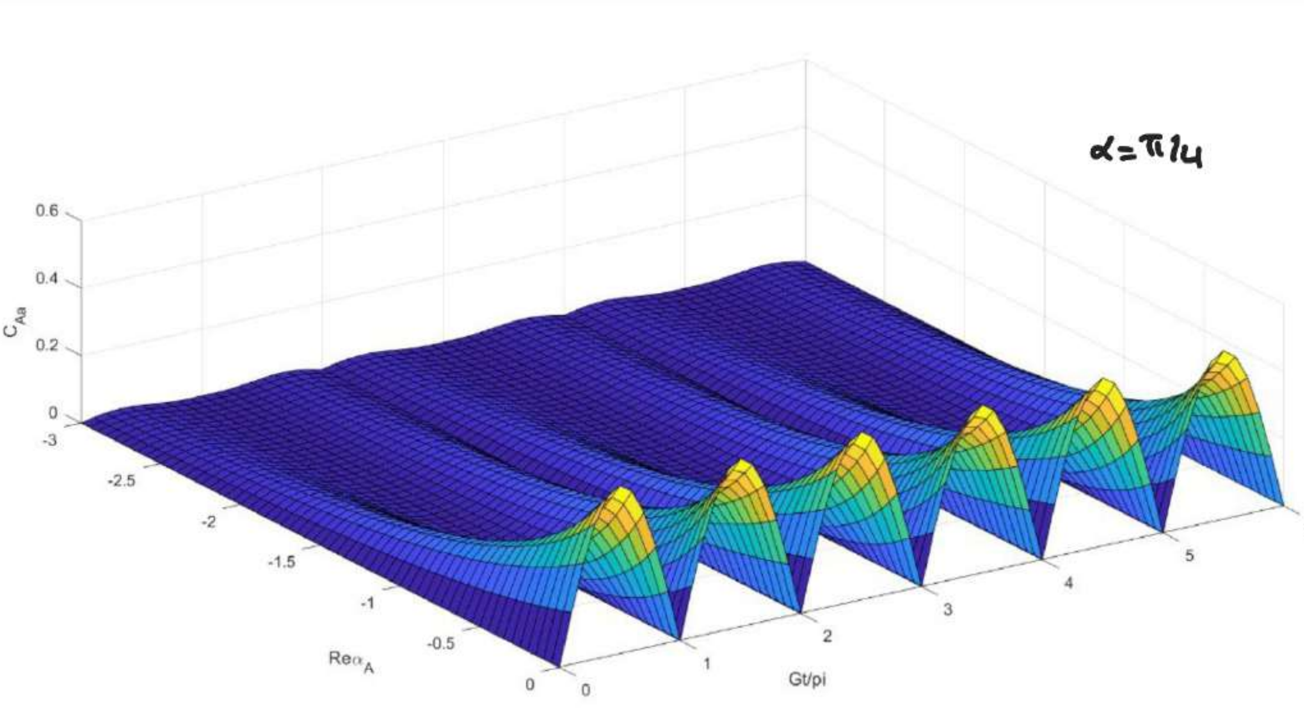
$$C_{Aa} = 2\left|\left(\cos^2\alpha \frac{1}{2}\sin(6t) e^{Re\{\alpha_A\}}\right)\left(e^{2Re\{\alpha_B\}}\sin^2\left(\frac{6t}{2}\right) + \cos^2\left(\frac{6t}{2}\right)\right)\right|$$

$$C_{Aa} = \cos^2\alpha\, |\sin(6t)|\, e^{Re\{\alpha_A\}} \left(e^{2Re\{\alpha_B\}}\sin^2\left(\frac{6t}{2}\right) + \cos^2\left(\frac{6t}{2}\right)\right)$$

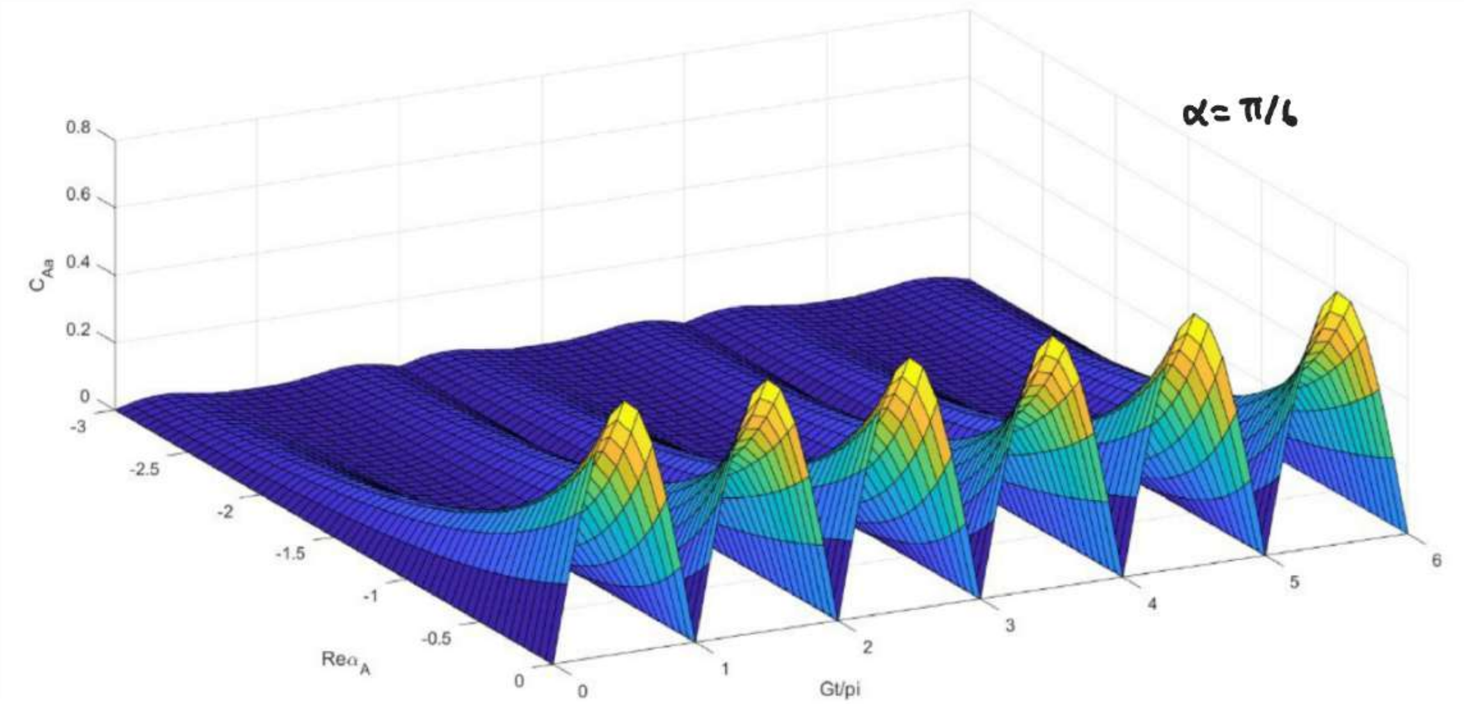

α=π/4
c_{Aa}
Gt/pi


α=π/6
c_{Aa}
Gt/pi

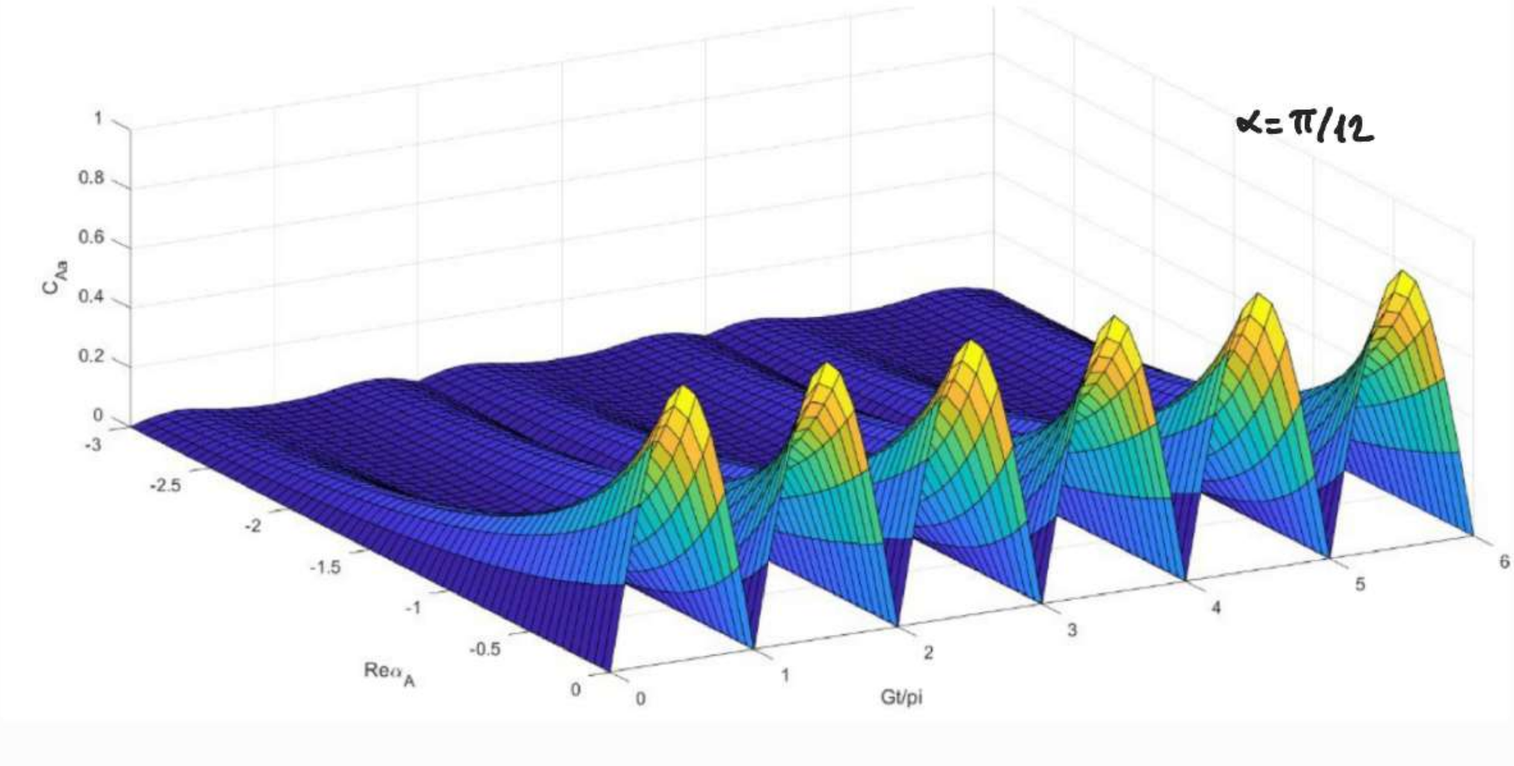

α=π/12
c_{Aa}
Gt/pi



α=π/4
C_Aa
Reα_A
Gt/pi



α=π/6
C_Aa
Reα_A
Gt/pi



α=π/12
C_Aa
Reα_A
Gt/pi

6. $C_{Bb}(t)$

For 1-photon case, we directly say $C_{Bb}(t) = C_{Aa}(t)$

But they're different for coherent states.

Yet, for numerical analysis I assumed $Re\{\alpha_A\} = Re\{\alpha_B\}$; therefore, plots are same with C_{Aa}.

$$C_{Bb}(t) = 2\left(|\tilde{x}_2 \tilde{x}_4 + \tilde{x}_3 \tilde{x}_1| \right)$$

$$\tilde{x}_2 = e^{Re\{\alpha_A\} + Re\{\alpha_B\}} \sin^2\left(\frac{Gt}{2}\right) \cos\alpha$$

$$\tilde{x}_4 = e^{Re\{\alpha_A\}} \cos\alpha \overbrace{\cos\left(\frac{Gt}{2}\right)\sin\left(\frac{Gt}{2}\right)}^{\frac{1}{2}\sin(Gt)}$$

$$\tilde{x}_2 \tilde{x}_4 = e^{Re\{\alpha_B\} + 2Re\{\alpha_A\}} \cos^2\alpha \, \frac{1}{2}\sin(Gt) \sin^2\left(\frac{Gt}{2}\right)$$

$$\tilde{x}_3 = e^{Re\{\alpha_B\}} \cdot \cos\alpha \overbrace{\cos\left(\frac{Gt}{2}\right)\sin\left(\frac{Gt}{2}\right)}^{\sin(Gt)/2}$$

$$\tilde{x}_1 = \cos\alpha \cos^2\left(\frac{Gt}{2}\right)$$

$$\tilde{x}_3 \tilde{x}_1 = e^{Re\{\alpha_B\}} \cos^2\alpha \, \frac{1}{2}\sin(Gt) \cos^2\left(\frac{Gt}{2}\right)$$

$$C_{Bb} = \cos^2\alpha \, |\sin(Gt)| \, e^{Re\{\alpha_B\}} \left(e^{2Re\{\alpha_A\}} \sin^2\left(\frac{Gt}{2}\right) + \cos^2\left(\frac{Gt}{2}\right) \right)$$

$$C_{Aa} = \cos^2\alpha \, |\sin(Gt)| \, e^{Re\{\alpha_A\}} \left(e^{2Re\{\alpha_B\}} \sin^2\left(\frac{Gt}{2}\right) + \cos^2\left(\frac{Gt}{2}\right) \right)$$

B. Partially Entangled Bell State $|\Psi_{AB}\rangle$

$$|\Psi(0)\rangle = |\Psi_{AB}\rangle \otimes |\alpha_A, \alpha_B\rangle$$

$$= \left(\cos\alpha\,|e_A, g_B\rangle + \sin\alpha\,|g_A, e_B\rangle\right) \otimes |\alpha_A, \alpha_B\rangle$$

Using the same procedure with Φ_{AB}, one can obtain:

$$|\Psi(t)\rangle = \tilde{x}_1\,|\uparrow\downarrow\alpha_A\alpha_B\rangle + \tilde{x}_2\,|\downarrow\uparrow\alpha_A\alpha_B\rangle + \tilde{x}_3\,|\downarrow\downarrow(\alpha_A+1)\alpha_B\rangle + \tilde{x}_4\,|\downarrow\downarrow\alpha_A(\alpha_B+1)\rangle$$

$$\tilde{x}_1 = x_1$$

$$\tilde{x}_2 = x_2$$

$$\tilde{x}_3 = x_3\,e^{\mathrm{Re}\{\alpha_A\}}$$

$$\tilde{x}_4 = x_4\,e^{\mathrm{Re}\{\alpha_B\}}$$

1. $C_{AB}(t)$

$$C_{AB}(t) = 2|\tilde{x}_1|\,|\tilde{x}_2|$$

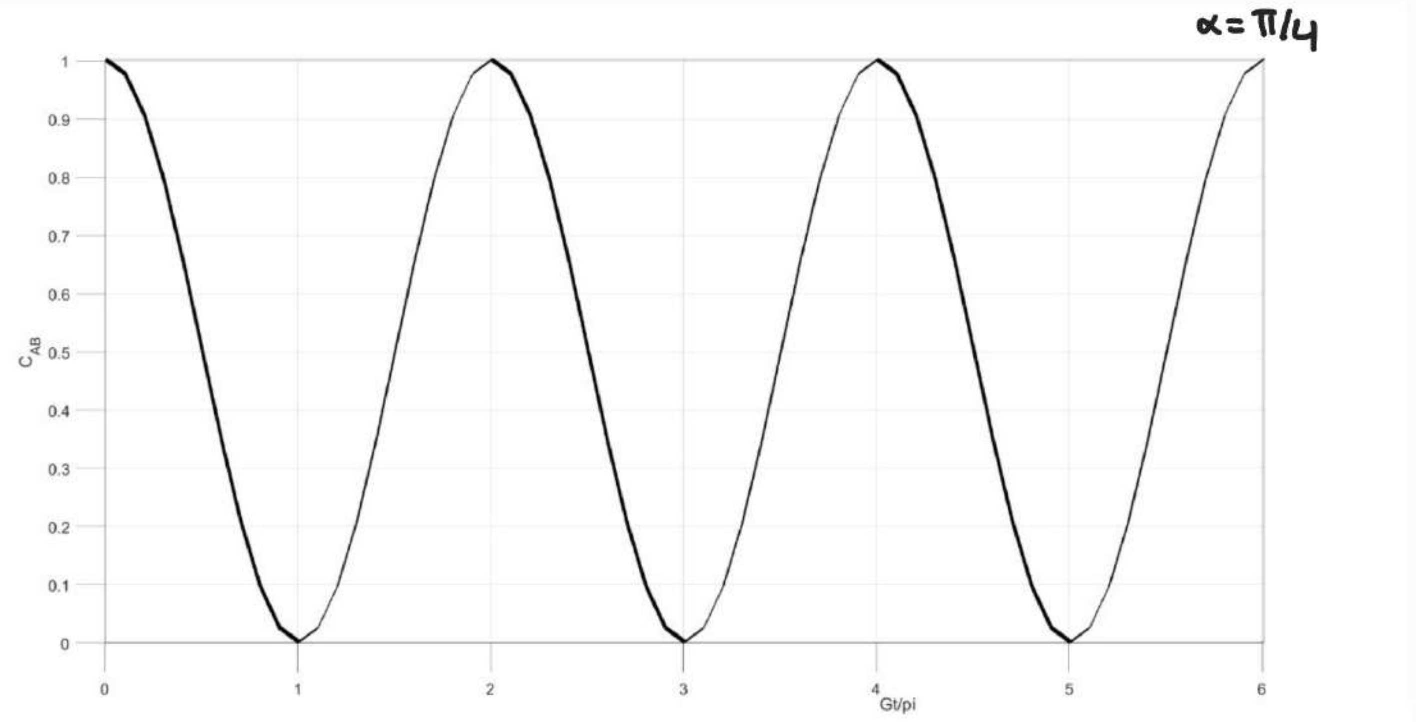
$$|\tilde{x}_1| = |\cos\alpha|\,\left|\cos\left(\frac{6t}{2}\right)\right|$$

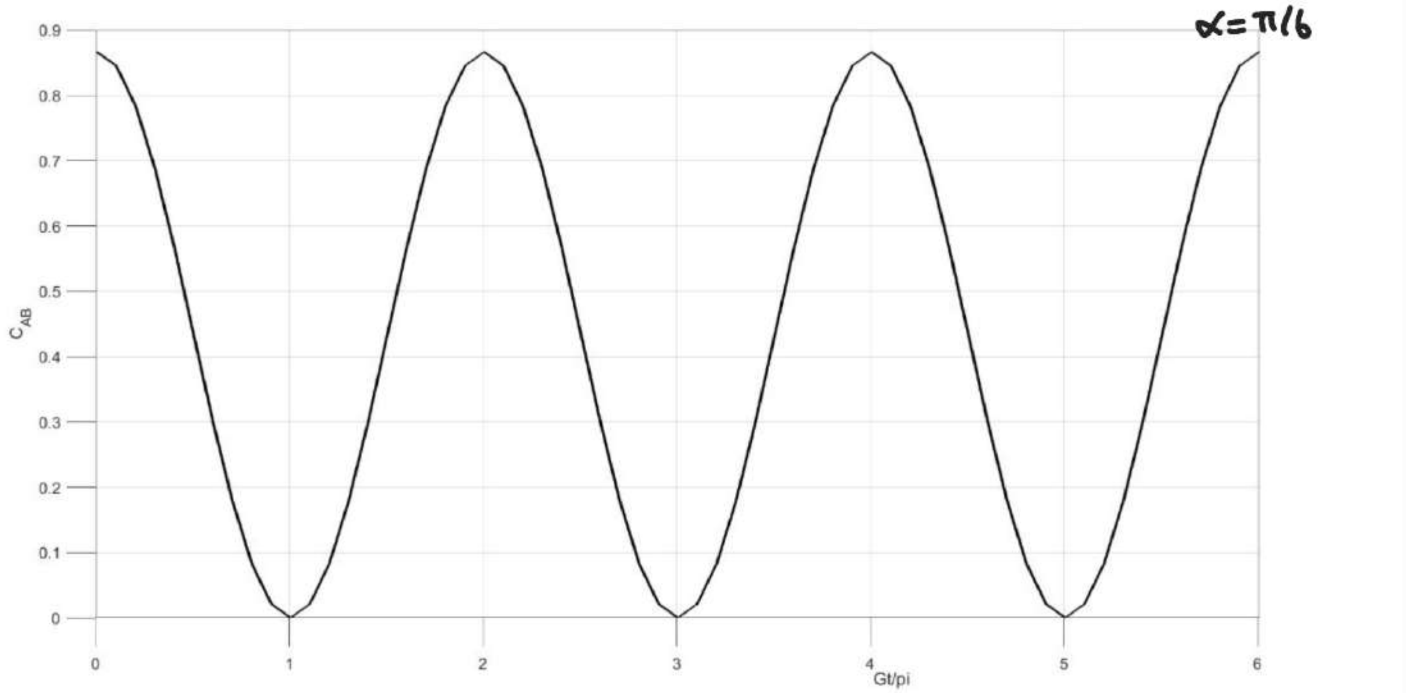
$$|\tilde{x}_2| = |\sin\alpha|\,\left|\cos\left(\frac{6t}{2}\right)\right|$$

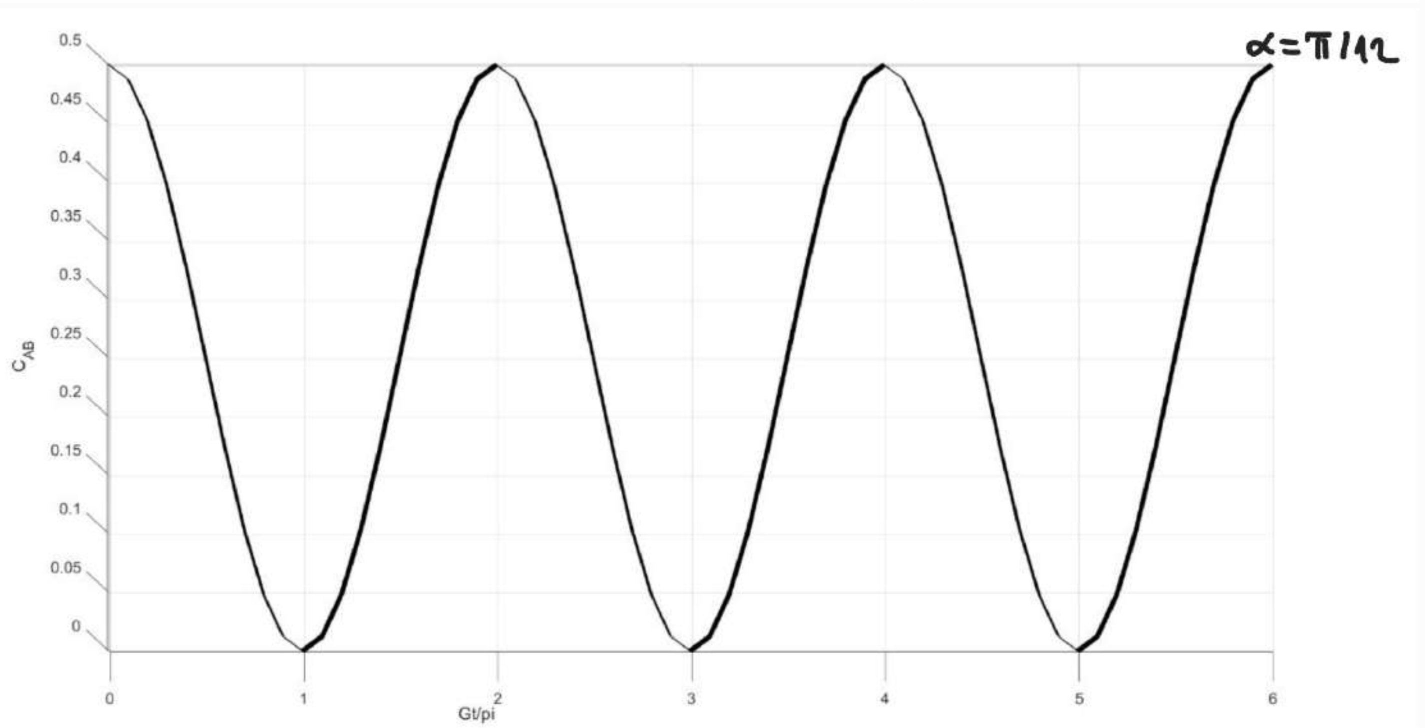
$$C_{AB}(t) = |\sin 2\alpha|\cos^2\left(\frac{6t}{2}\right)$$

This result is same as for the 1-photon case.

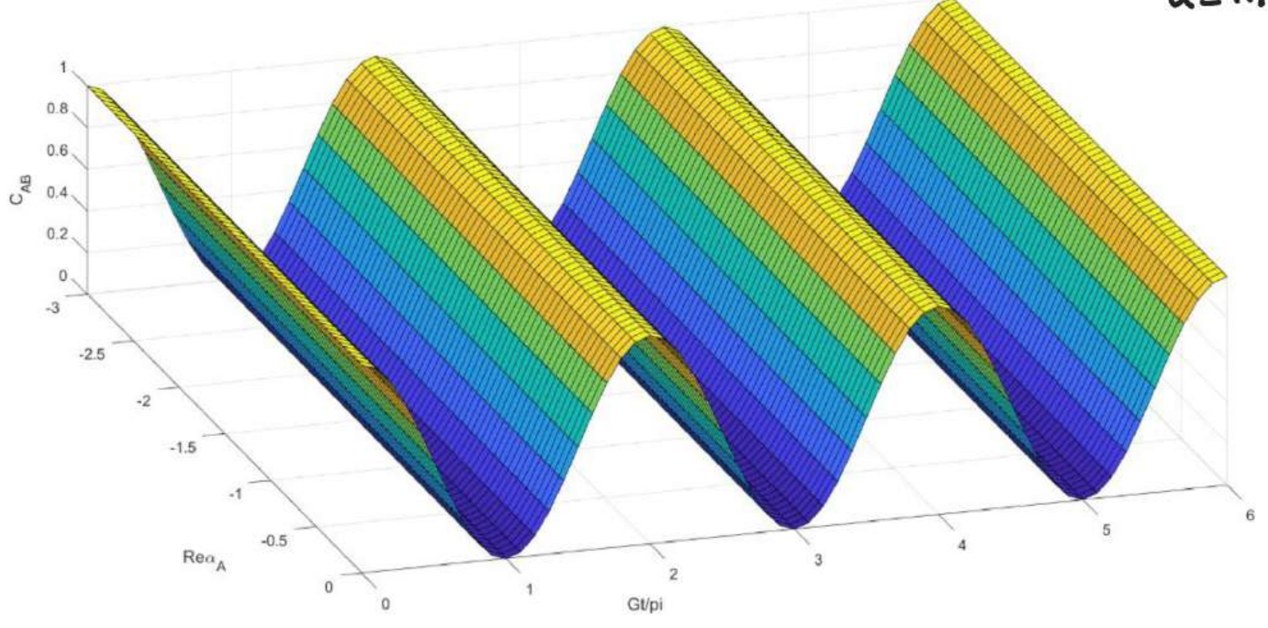
There's no dependence on $Re\{\alpha_A\}$ or $Re\{\alpha_B\}$


α=π/4
C_{AB}
Gt/pi

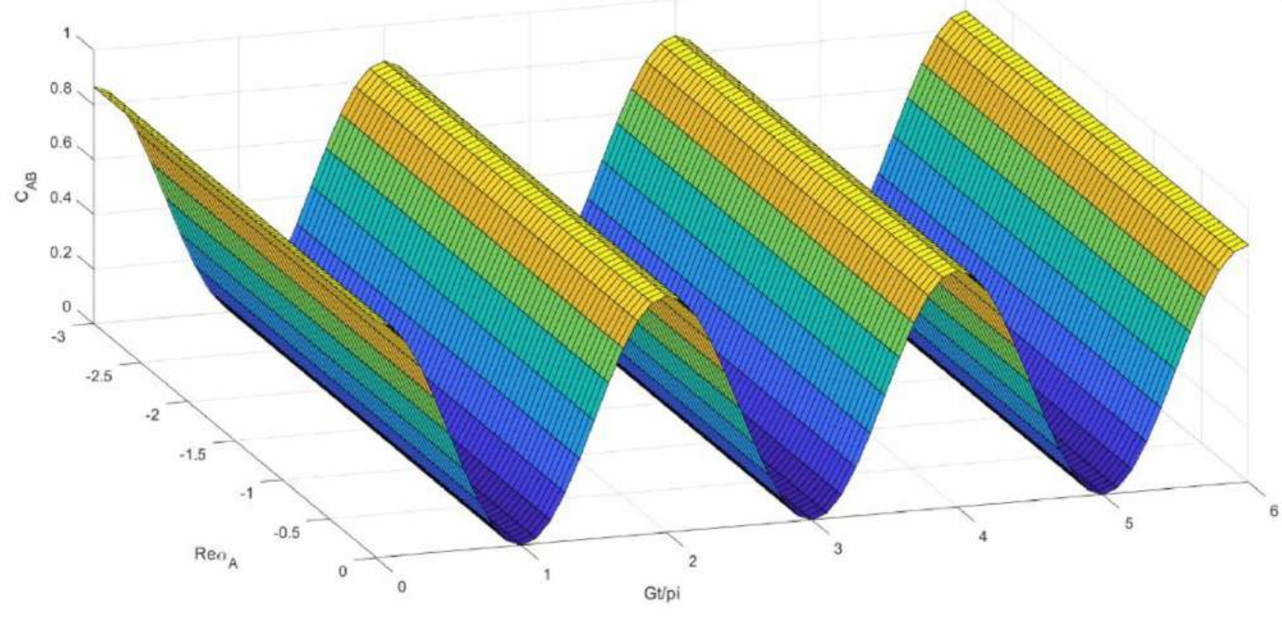

α=π/6
C_{AB}
Gt/pi


α=π/12
C_{AB}
Gt/pi

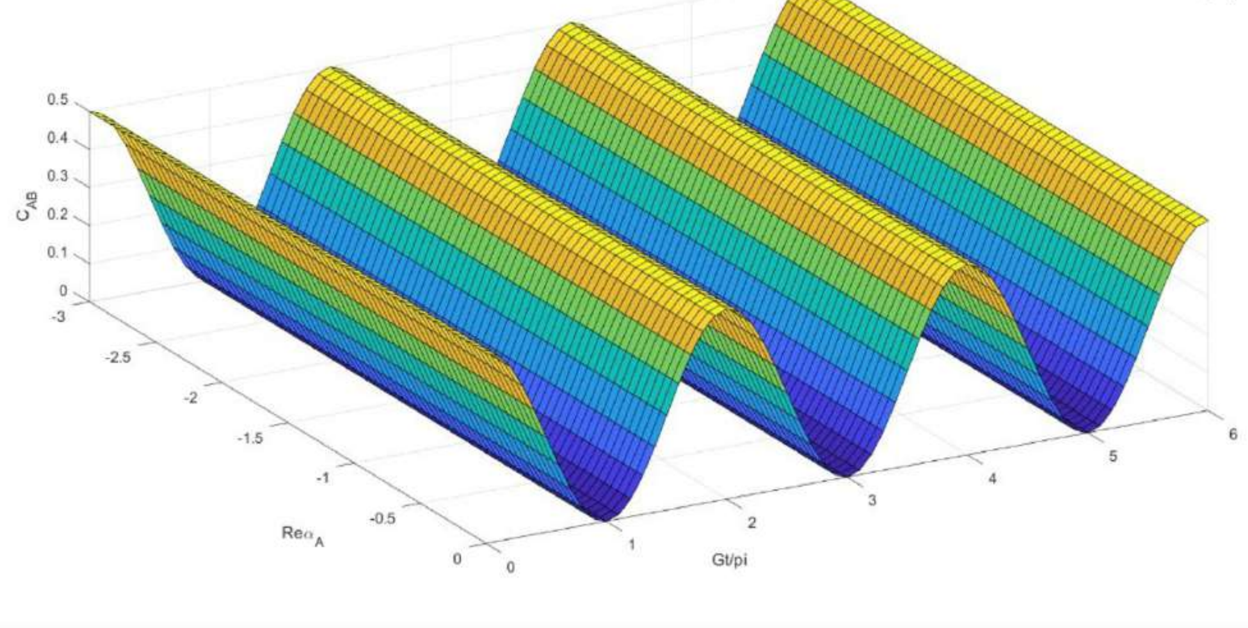
α=π/4



α=π/6



α=π/12



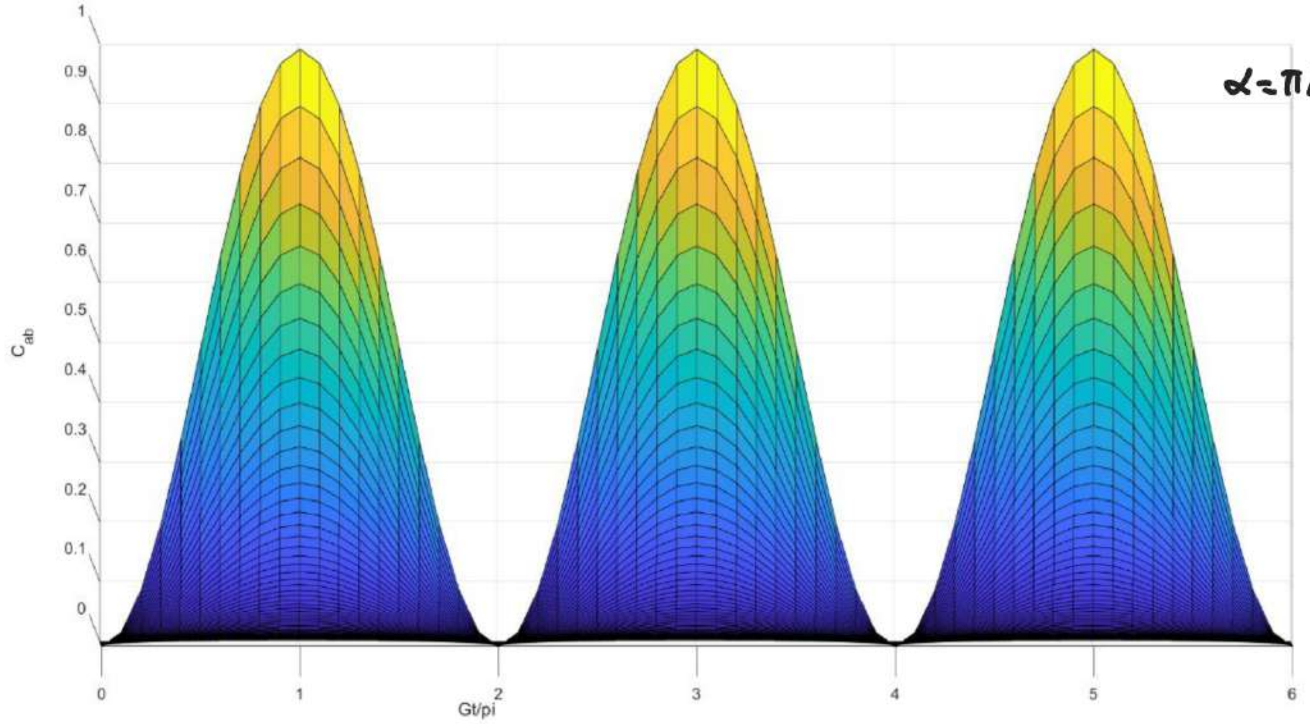
2. $C_{ab}(t)$

$$C_{ab}(t) = 2|\tilde{x}_3||\tilde{x}_4|$$

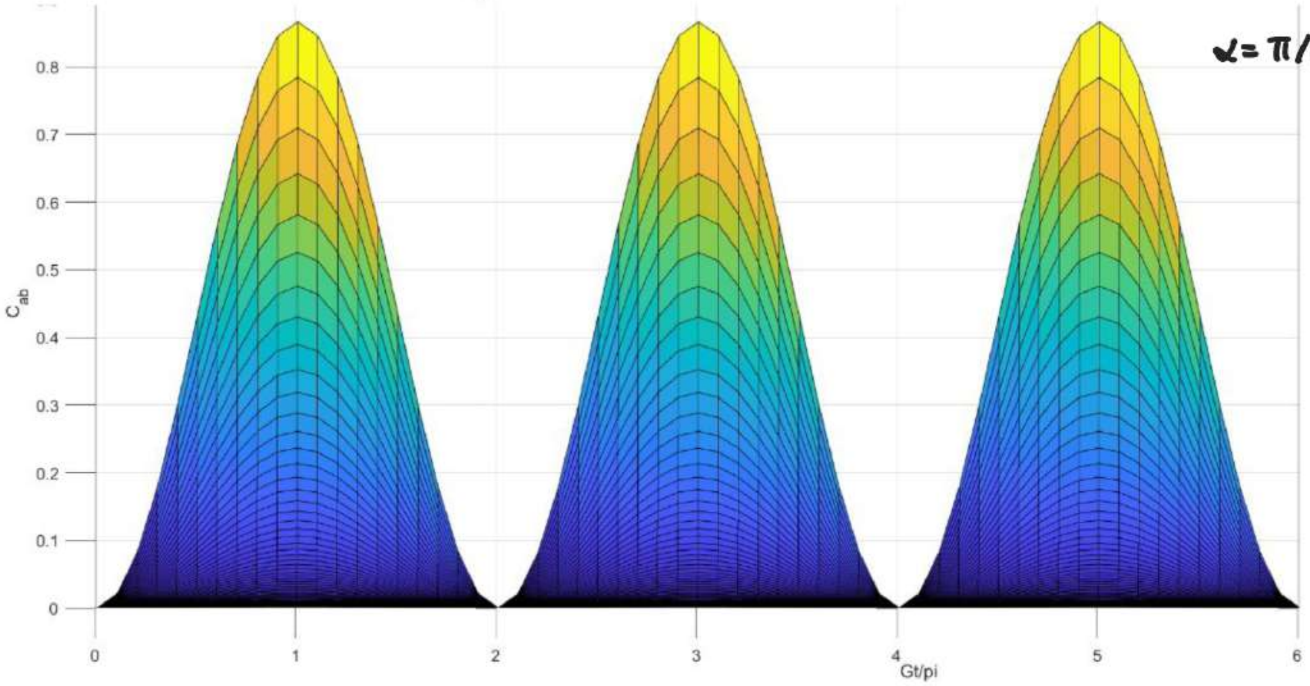
$$|\tilde{x}_3| = e^{Re\{\alpha_A\}} |\cos\alpha| \left|\sin\left(\frac{6t}{2}\right)\right|$$

$$|\tilde{x}_4| = e^{Re\{\alpha_B\}} |\sin\alpha| \left|\sin\left(\frac{6t}{2}\right)\right|$$

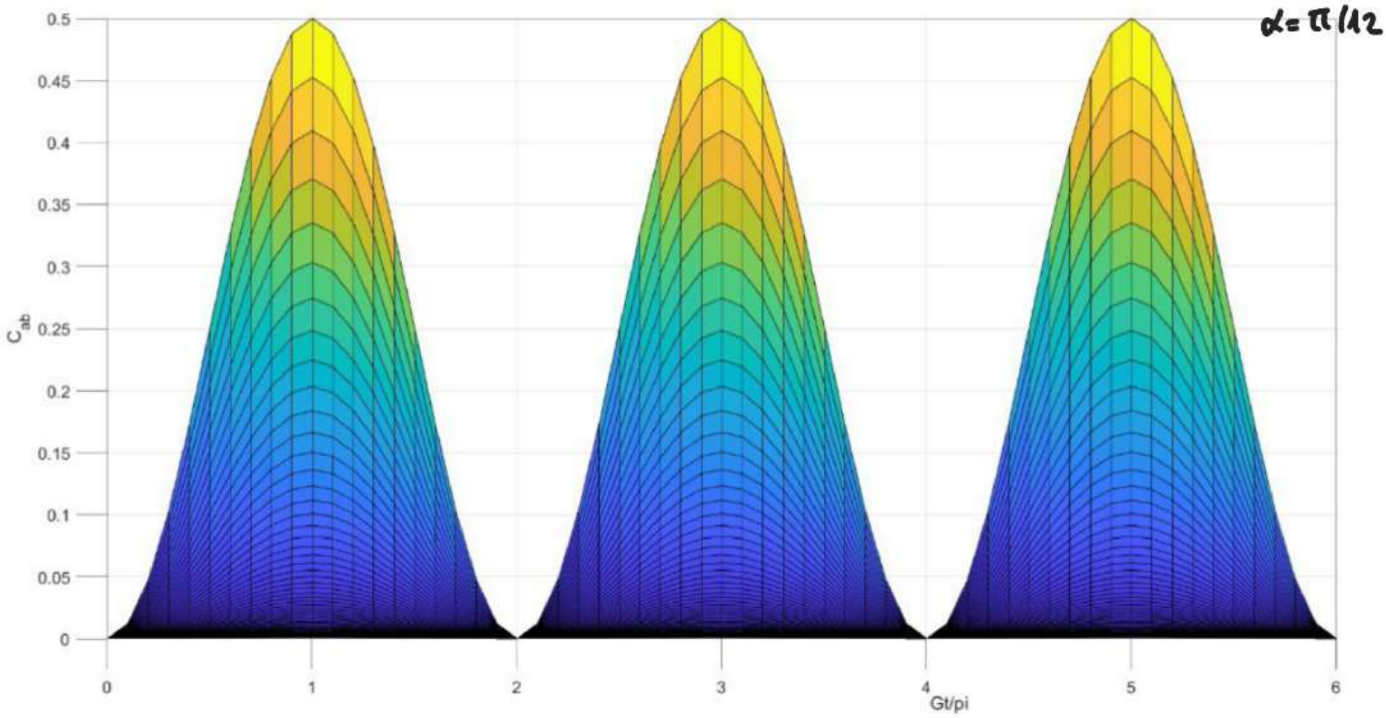
$$C_{ab}(t) = e^{Re\{\alpha_A\} + Re\{\alpha_B\}} |\sin 2\alpha| \sin^2\left(\frac{6t}{2}\right)$$



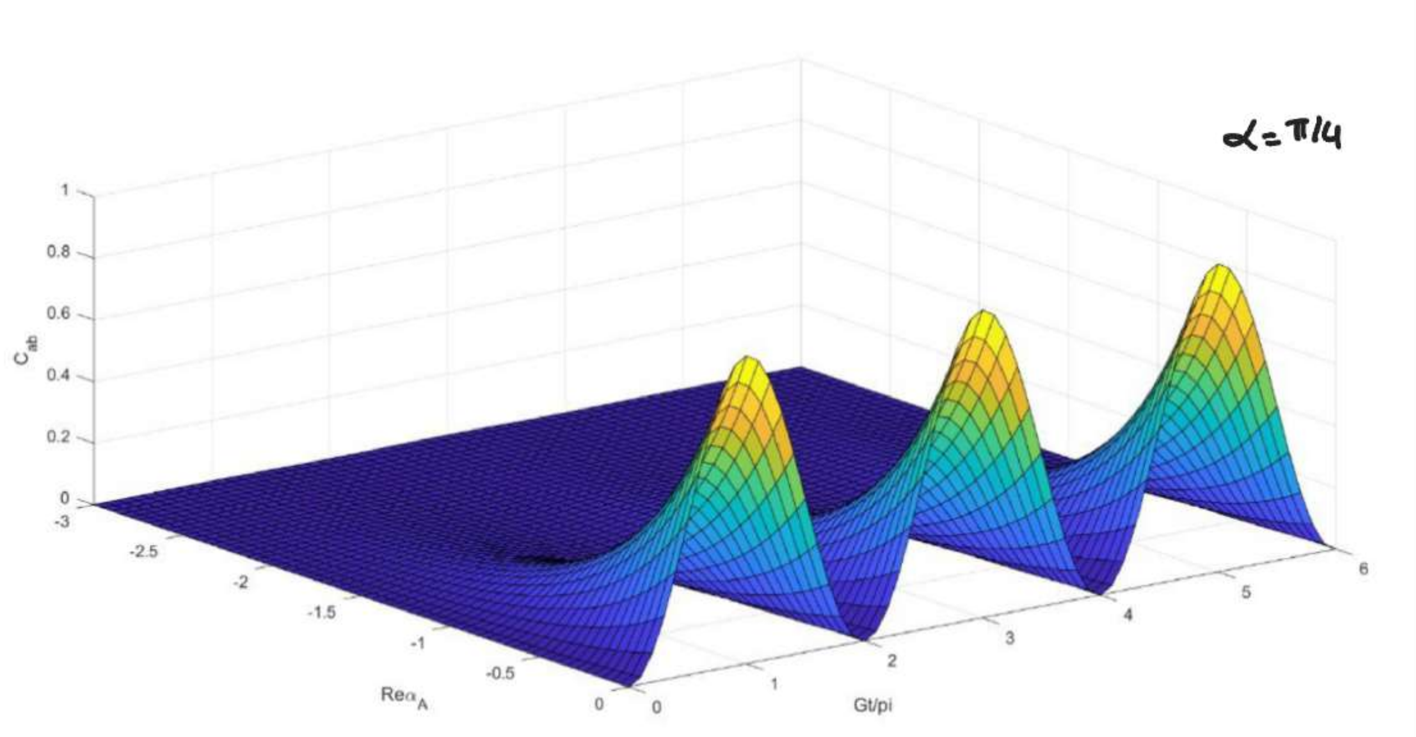
$\alpha = \pi/4$
C_{ab}
Gt/pi



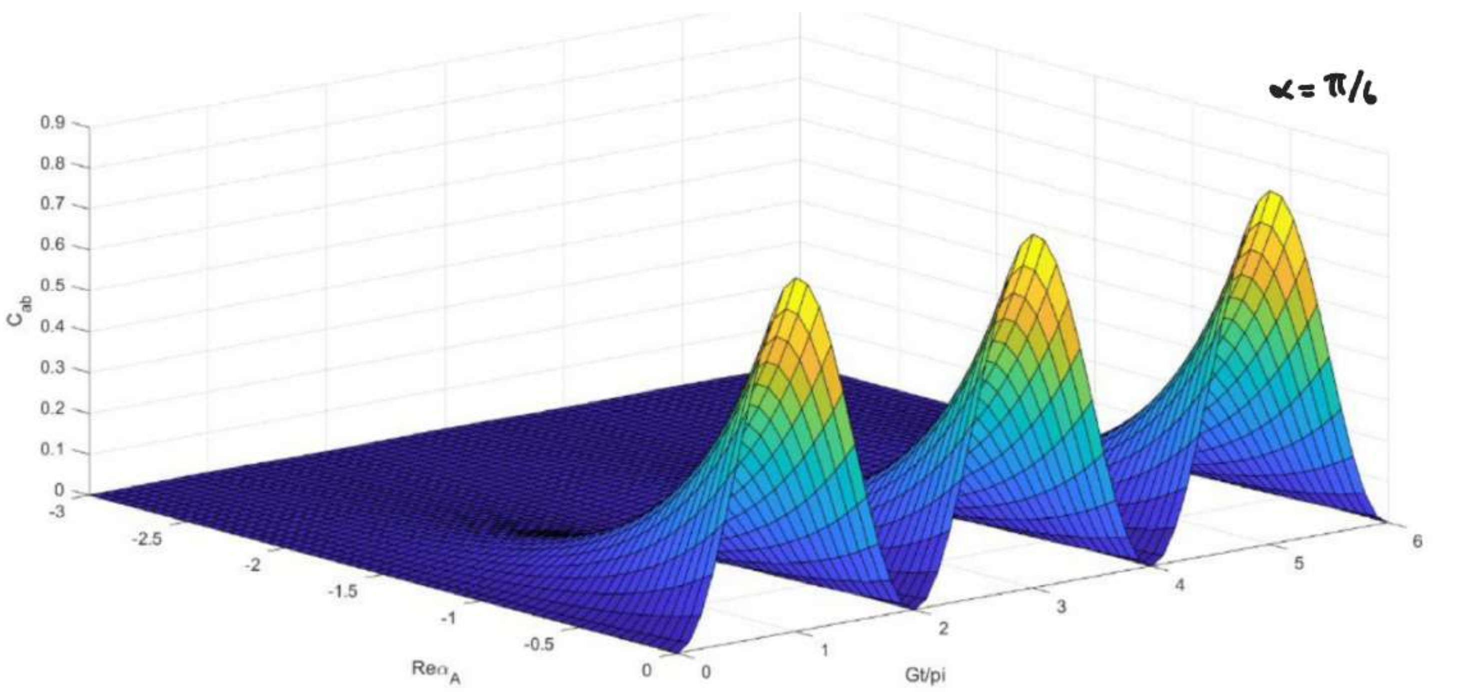
$\alpha = \pi/6$
C_{ab}
Gt/pi



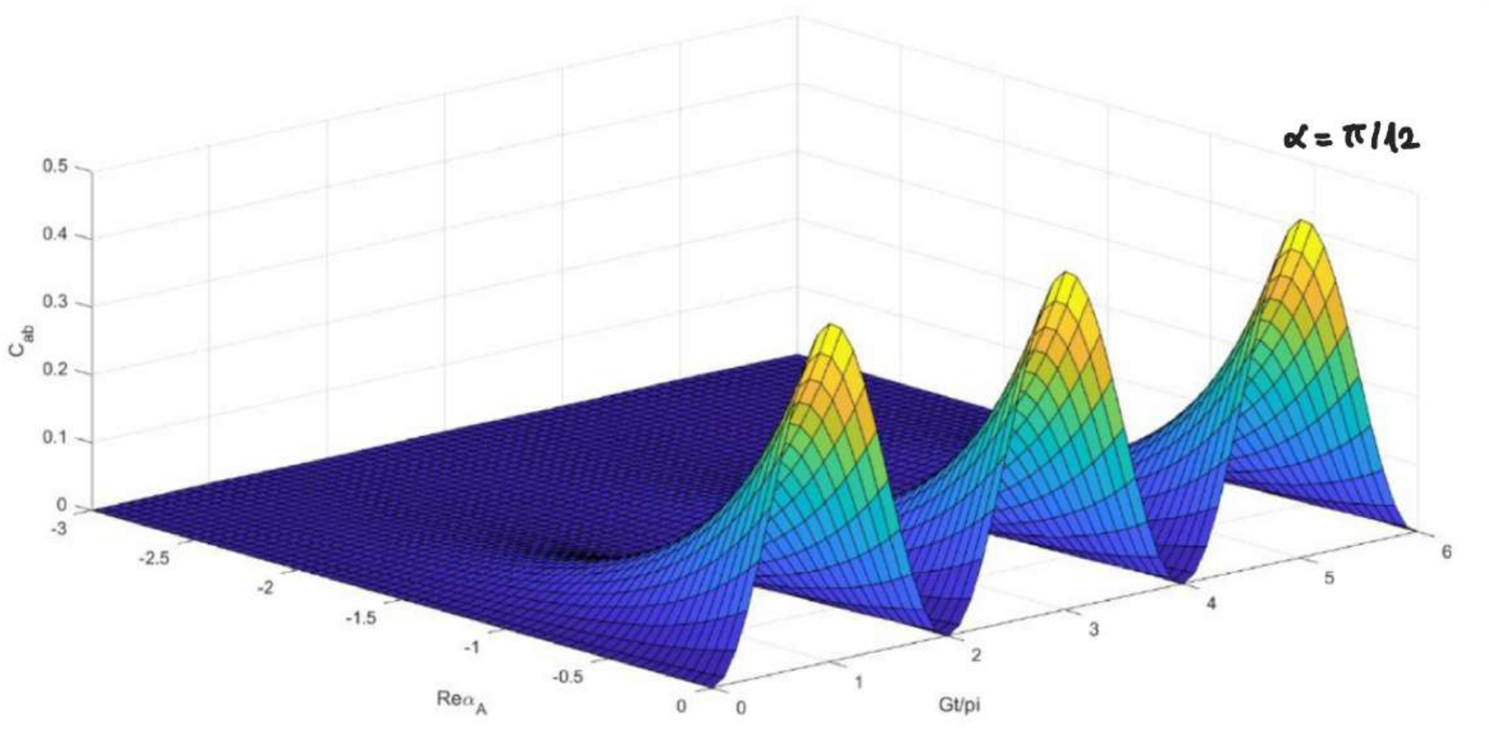
$\alpha = \pi/12$
C_{ab}
Gt/pi



α = π/4
C_ab
Reα_A
Gt/pi



α = π/6
C_ab
Reα_A
Gt/pi



α = π/12
C_ab
Reα_A
Gt/pi

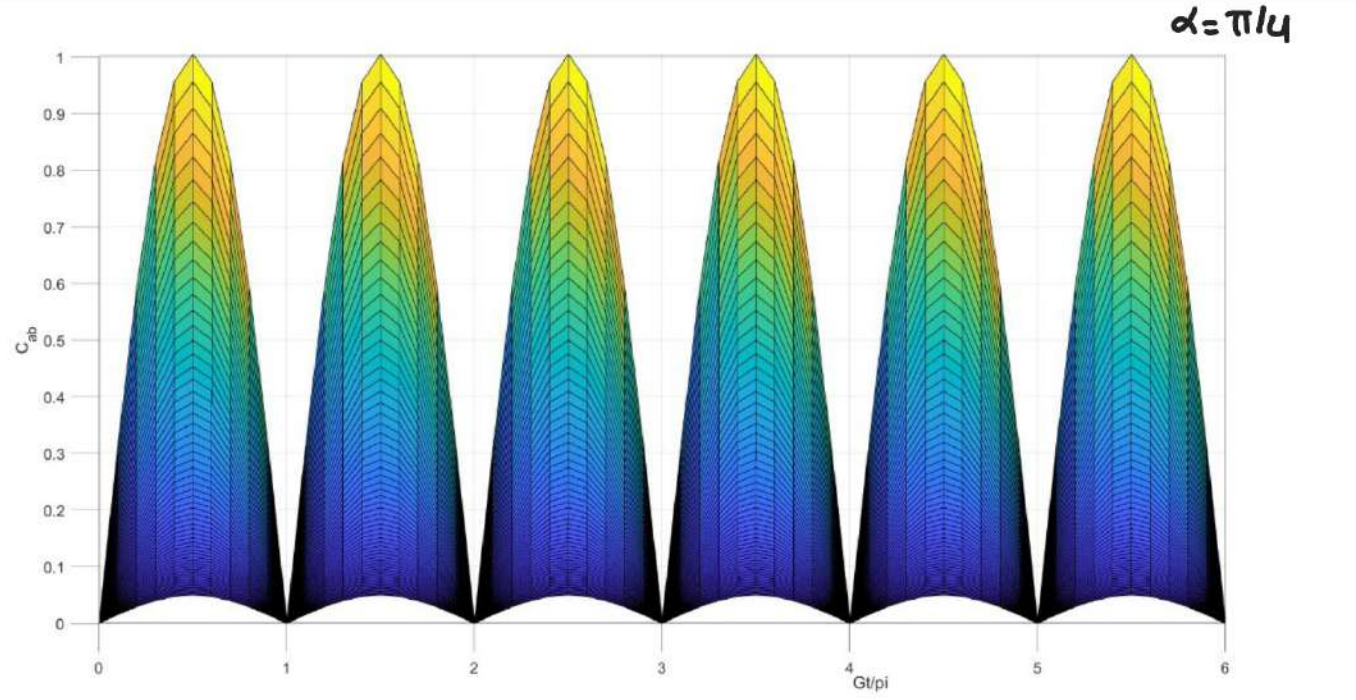
3. $C_{Ab}(t)$

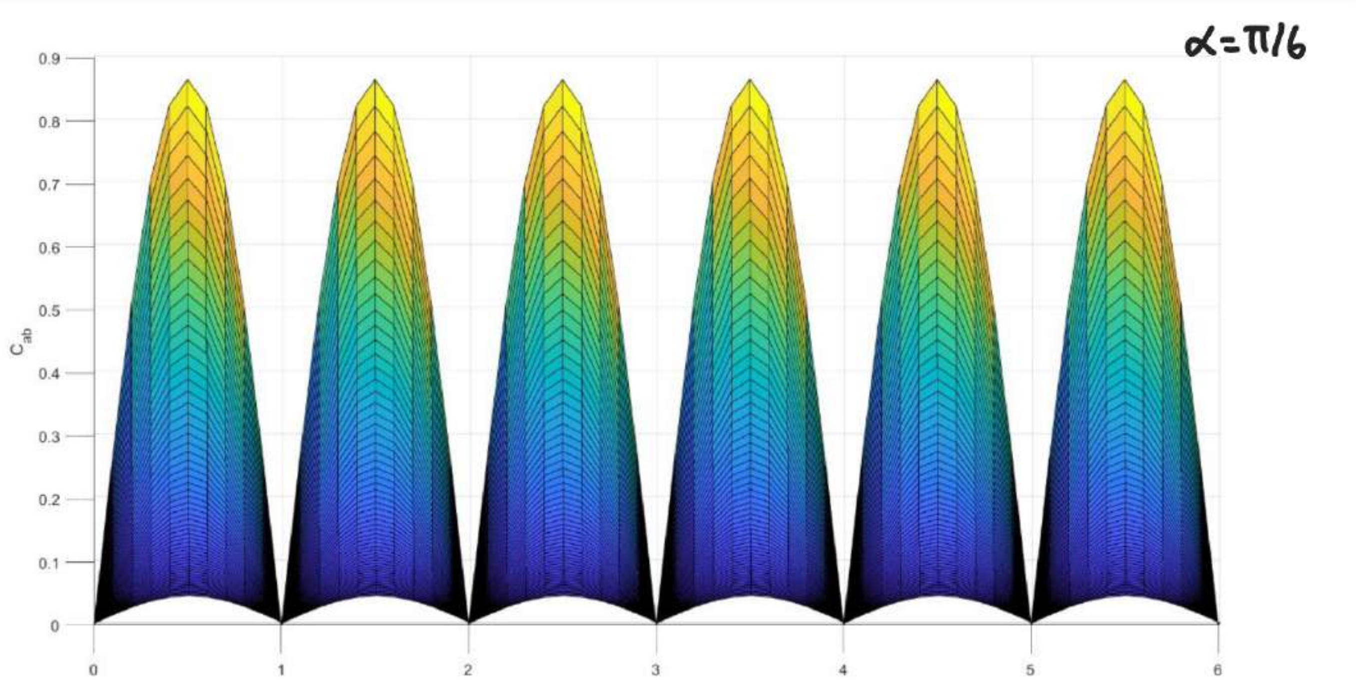
$$C_{Ab}(t) = 2|\tilde{x}_1||\tilde{x}_4|$$

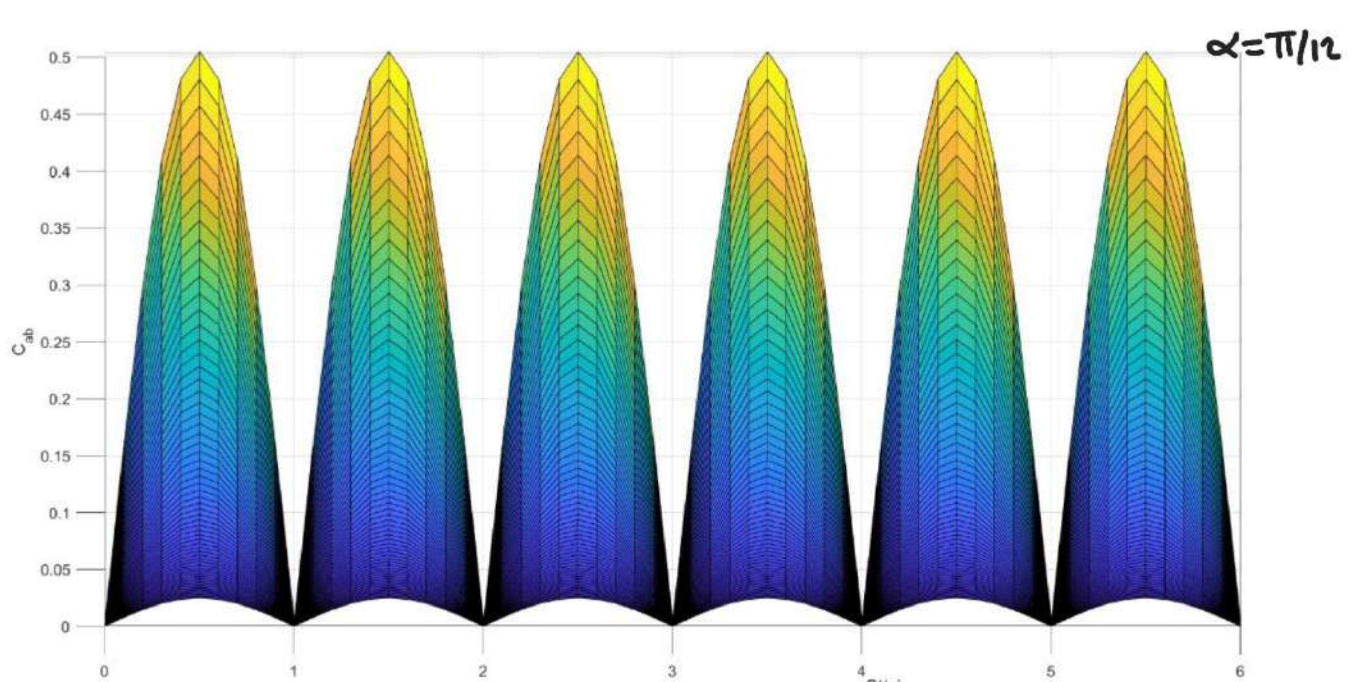
$$|\tilde{x}_1| = |\cos\alpha| \left|\cos\left(\frac{6t}{2}\right)\right|$$

$$|\tilde{x}_4| = e^{Re\{\alpha_B\}} |\sin\alpha| \left|\sin\left(\frac{6t}{2}\right)\right|$$

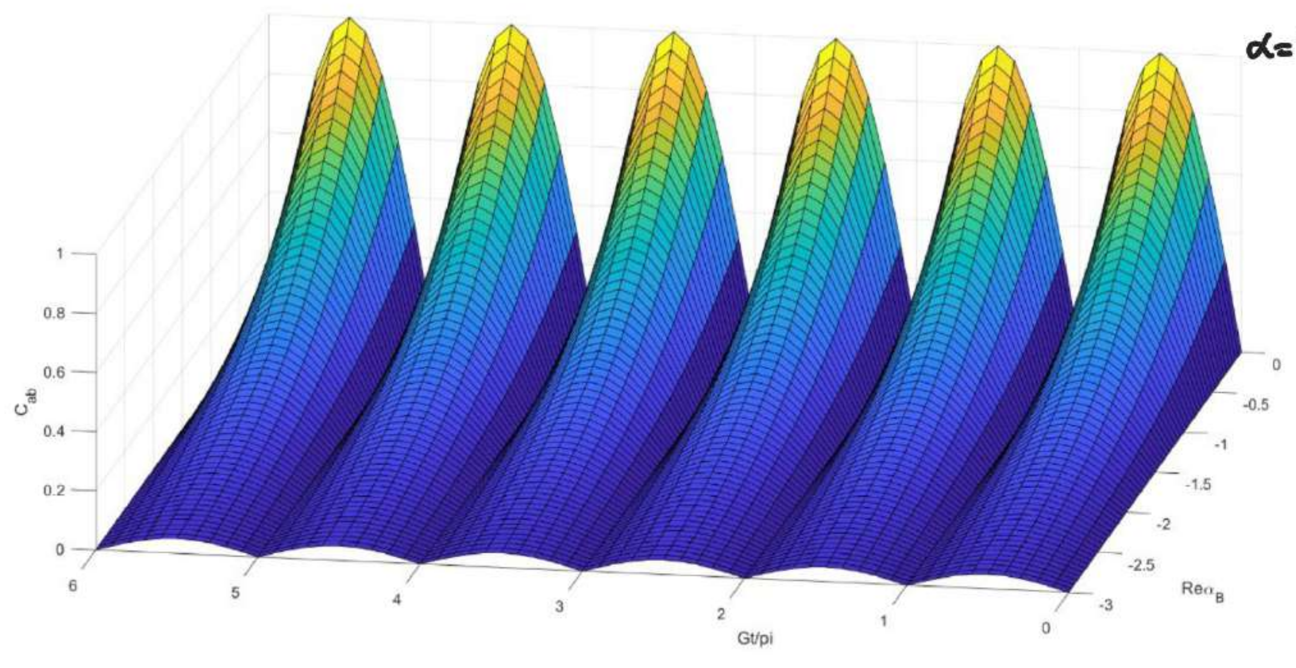
$$C_{Ab}(t) = \frac{1}{2} e^{Re\{\alpha_B\}} |\sin(2\alpha)| |\sin(6t)|$$


α=π/4
C_ab
Gt/pi

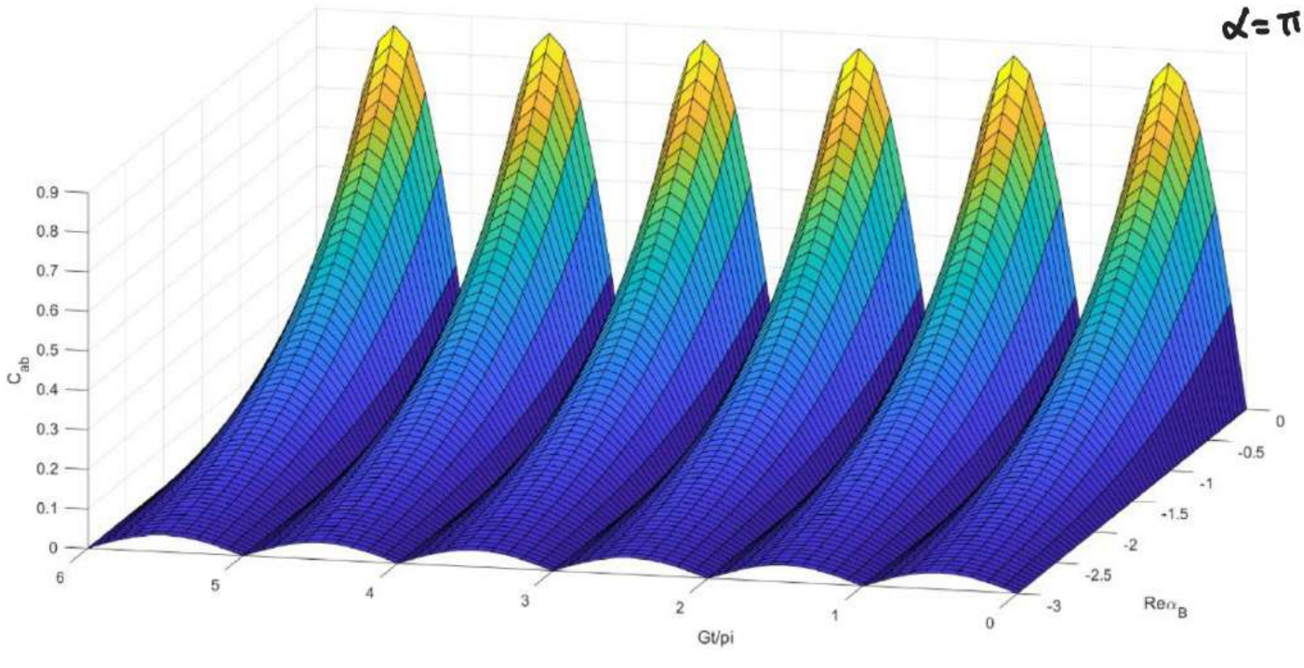

α=π/6
C_ab


α=π/12
C_ab
Gt/pi

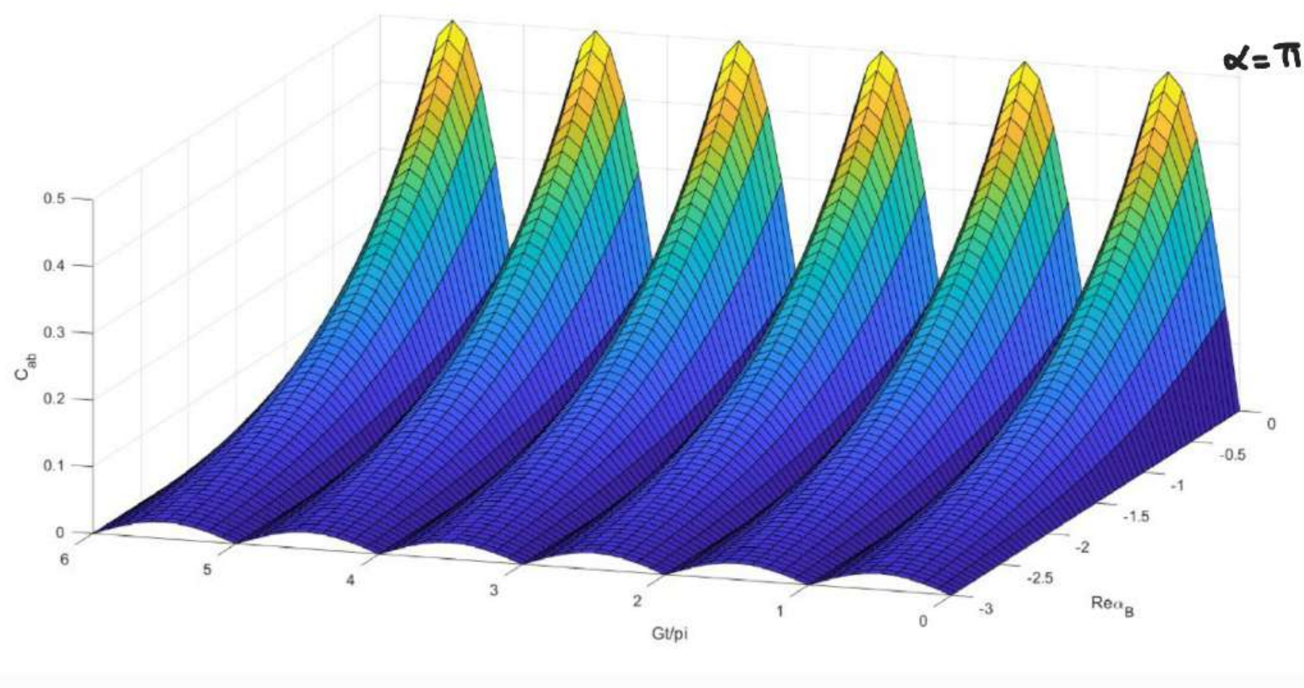
$\alpha = \pi/4$



$\alpha = \pi/6$



$\alpha = \pi/12$



4. $C_{Ba}(t)$

$$\rho^{Ba} = Tr_{Ab}\left[\, |\Psi(t)\rangle\langle\Psi(t)| \,\right]$$

A B a b

$$|\Psi(t)\rangle = \tilde{x}_1|\uparrow\downarrow\alpha_A\alpha_B\rangle + \tilde{x}_2|\downarrow\uparrow\alpha_A\alpha_B\rangle + \tilde{x}_3|\downarrow\downarrow(\alpha_A+1)\alpha_B\rangle + \tilde{x}_4|\downarrow\downarrow\alpha_A(\alpha_B+1)\rangle$$

$$Tr_{\uparrow\alpha_B} = |\tilde{x}_1|^2\,|\downarrow\alpha_A\rangle\langle\downarrow\alpha_A|$$

$$Tr_{\downarrow\alpha_B} = |\tilde{x}_2|^2\,|\uparrow\alpha_A\rangle\langle\uparrow\alpha_A| + |\tilde{x}_3|^2\,|\downarrow(\alpha_A+1)\rangle\langle\downarrow(\alpha_A+1)| + \tilde{x}_2\tilde{x}_3^*\,|\uparrow\alpha_A\rangle\langle\downarrow(\alpha_A+1)|$$
$$+\ \tilde{x}_2^*\tilde{x}_3\,|\downarrow(\alpha_A+1)\rangle\langle\uparrow\alpha_A|$$

$$Tr_{\downarrow(\alpha_B+1)} = |\tilde{x}_4|^2\,|\downarrow\alpha_A\rangle\langle\downarrow\alpha_A|$$

$$Tr_{\uparrow(\alpha_B+1)} = 0$$

$$\rho^{Ba} = \begin{array}{c} \\ |\downarrow\alpha_A\rangle \\ |\downarrow(\alpha_A+1)\rangle \\ |\uparrow\alpha_A\rangle \\ |\uparrow(\alpha_A+1)\rangle \end{array} \begin{array}{c} \begin{array}{cccc} \langle\downarrow\alpha_A| & \langle\downarrow(\alpha_A+1)| & \langle\uparrow\alpha_A| & \langle\uparrow(\alpha_A+1)| \end{array} \\ \begin{bmatrix} |\tilde{x}_1|^2+|\tilde{x}_4|^2 & & & \\ & |\tilde{x}_3|^2 & \tilde{x}_2^*\tilde{x}_3 & \\ & \tilde{x}_2\tilde{x}_3^* & |\tilde{x}_2|^2 & \\ & & & 0 \end{bmatrix} \end{array}$$

$$C_{AB}(t) = 2|\tilde{x}_2||\tilde{x}_3| = \frac{1}{2}|\sin(2\alpha)|\,|\sin(Gt)|\,e^{Re\{\alpha_A\}}$$

$$C_{Ab}(t) = \frac{1}{2}e^{Re\{\alpha_B\}}\,|\sin(2\alpha)|\,|\sin(Gt)|$$

For the numerical analysis, the plots are same

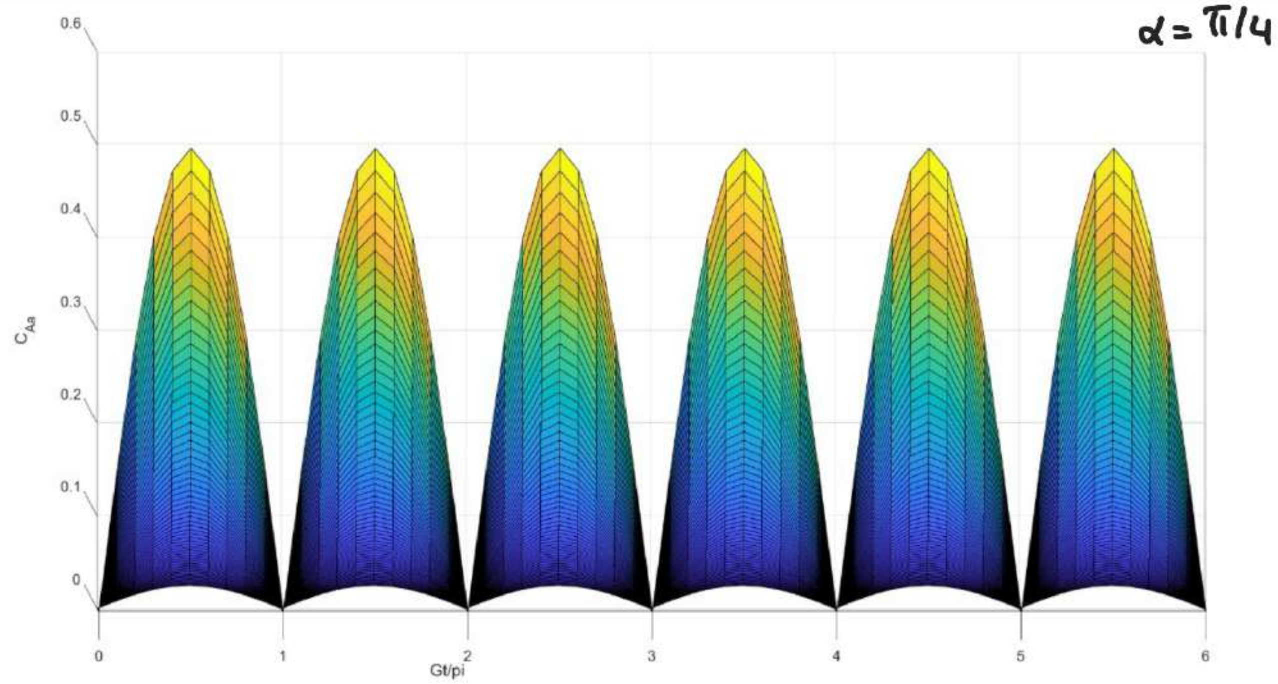
5. $C_{Aa}(t)$

$$C_{Aa}(t) = 2|\tilde{x}_1||\tilde{x}_3|$$

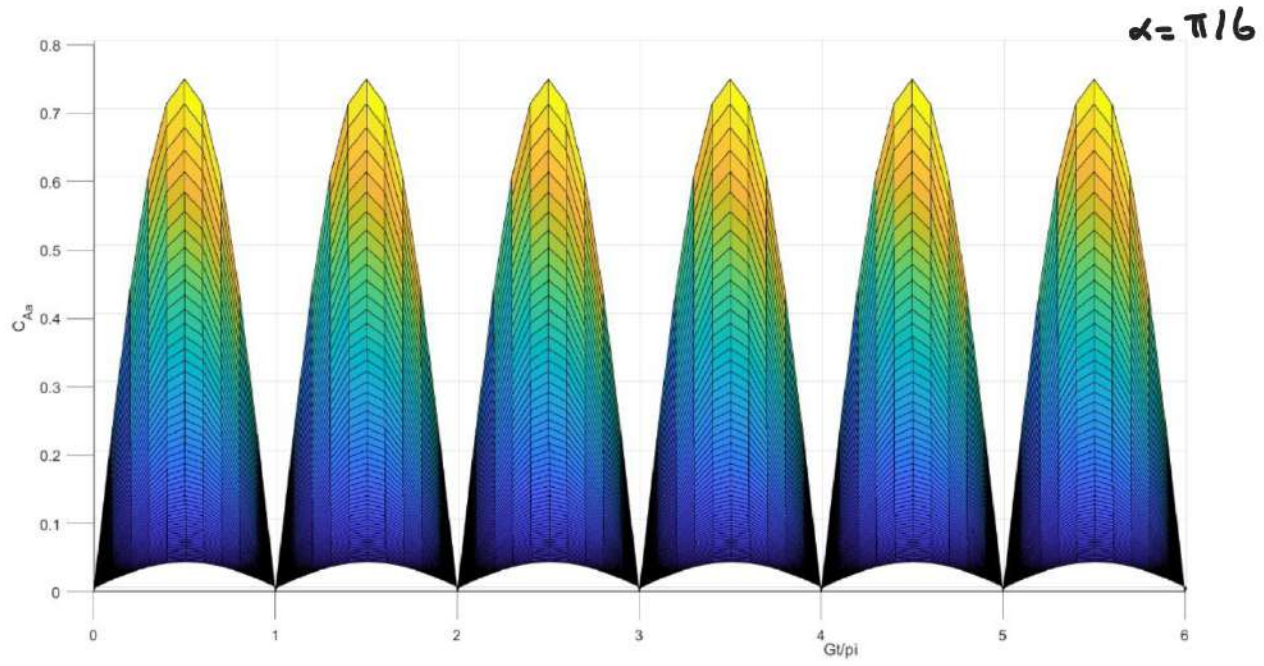
$$|\tilde{x}_1| = |\cos\alpha| \left|\cos\left(\frac{6t}{2}\right)\right|$$

$$|\tilde{x}_3| = e^{Re\{\alpha_A\}} |\cos\alpha| \left|\sin\left(\frac{6t}{2}\right)\right|$$

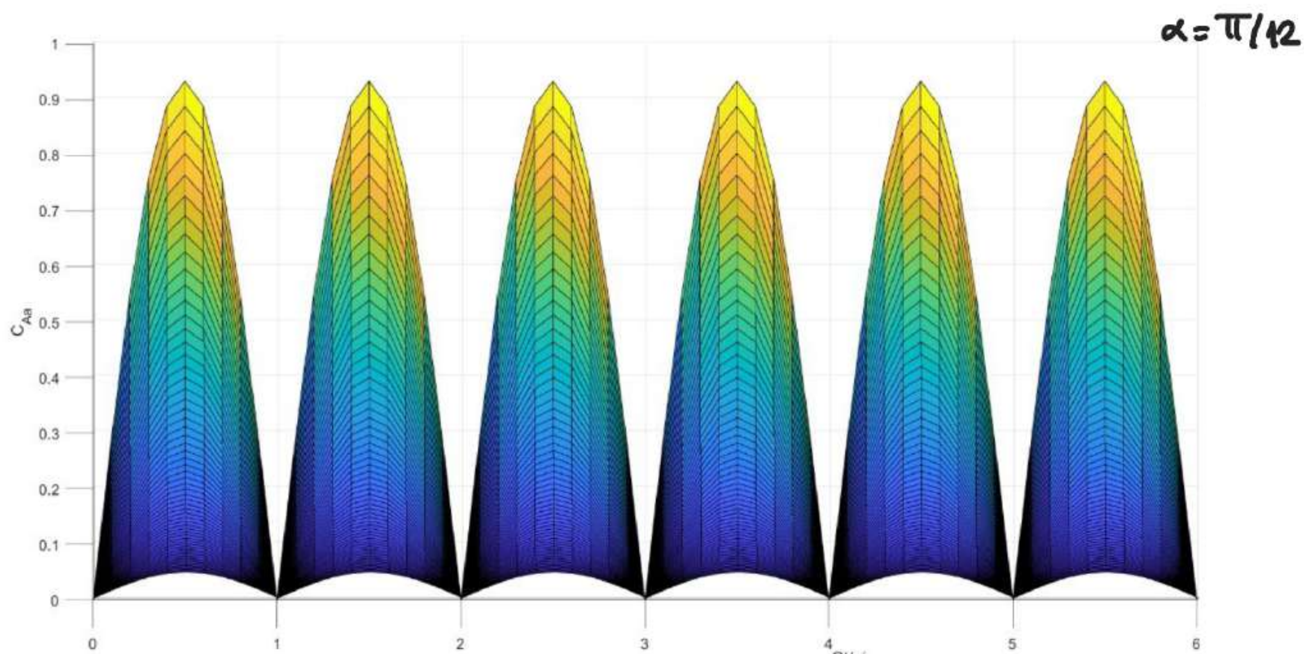
$$C_{Aa}(t) = e^{Re\{\alpha_A\}} \cos^2\alpha \,|\sin(6t)|$$



α=π/4
C_{Aa}
0
0.1
0.2
0.3
0.4
0.5
0.6
Gt/pi
0
1
2
3
4
5
6

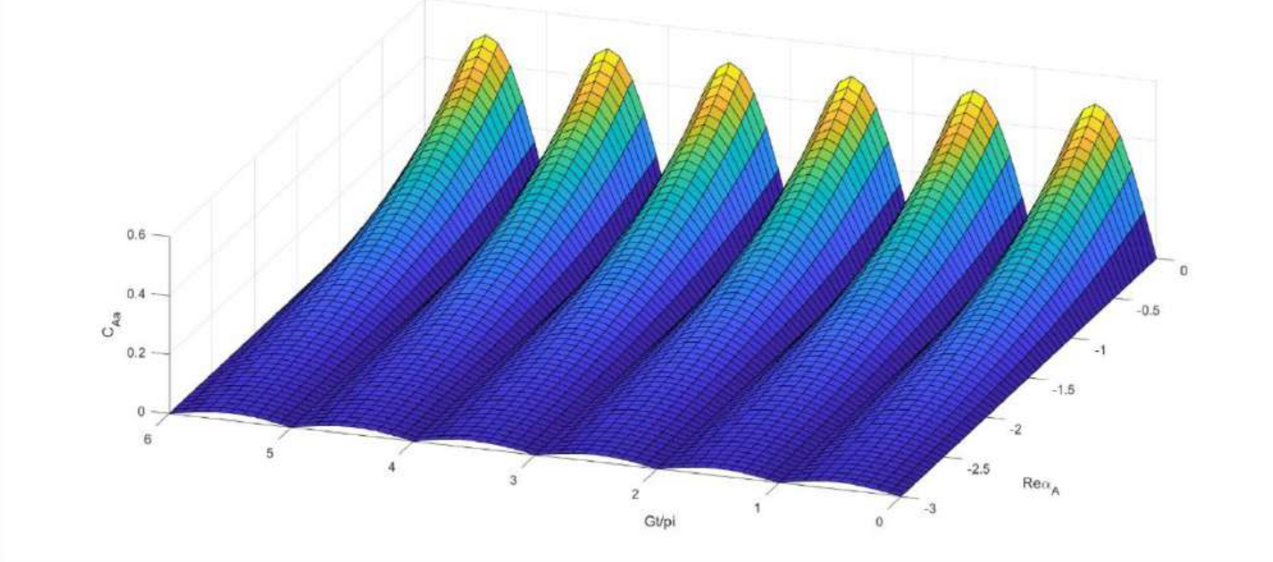


α=π/6
C_{Aa}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Gt/pi
0
1
2
3
4
5
6

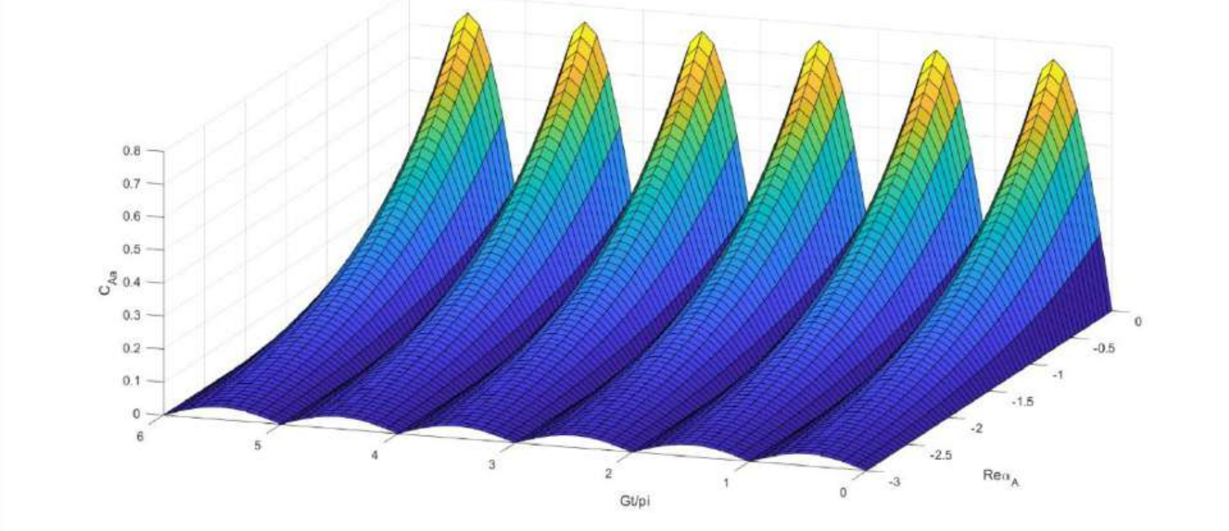


α=π/12
C_{Aa}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Gt/pi
0
1
2
3
4
5
6

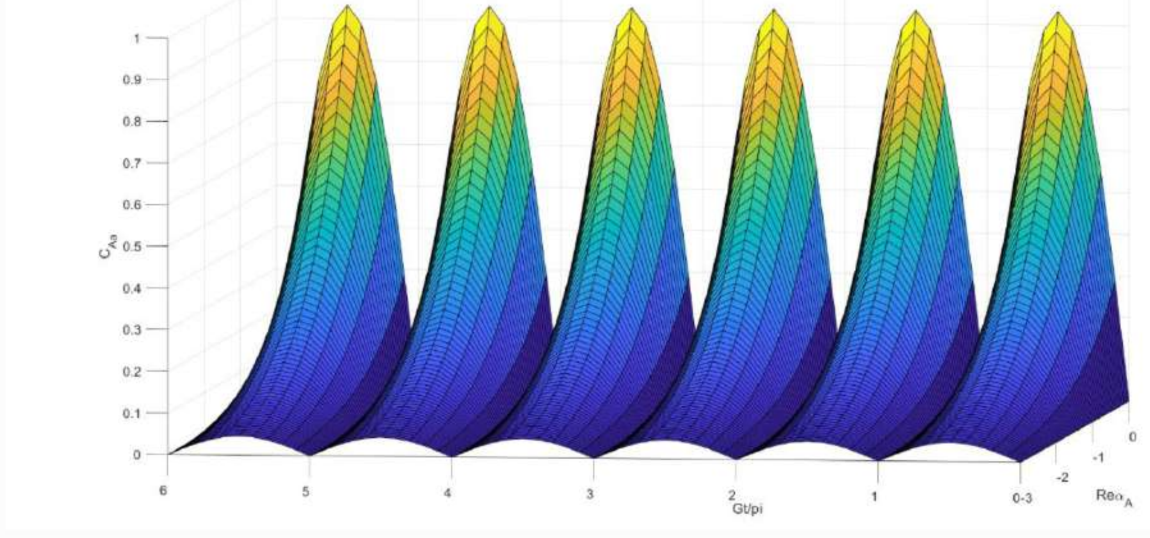
$\alpha = \pi/4$



$\alpha = \pi/6$



$\alpha = \pi/12$



6. $C_{Bb}(t)$

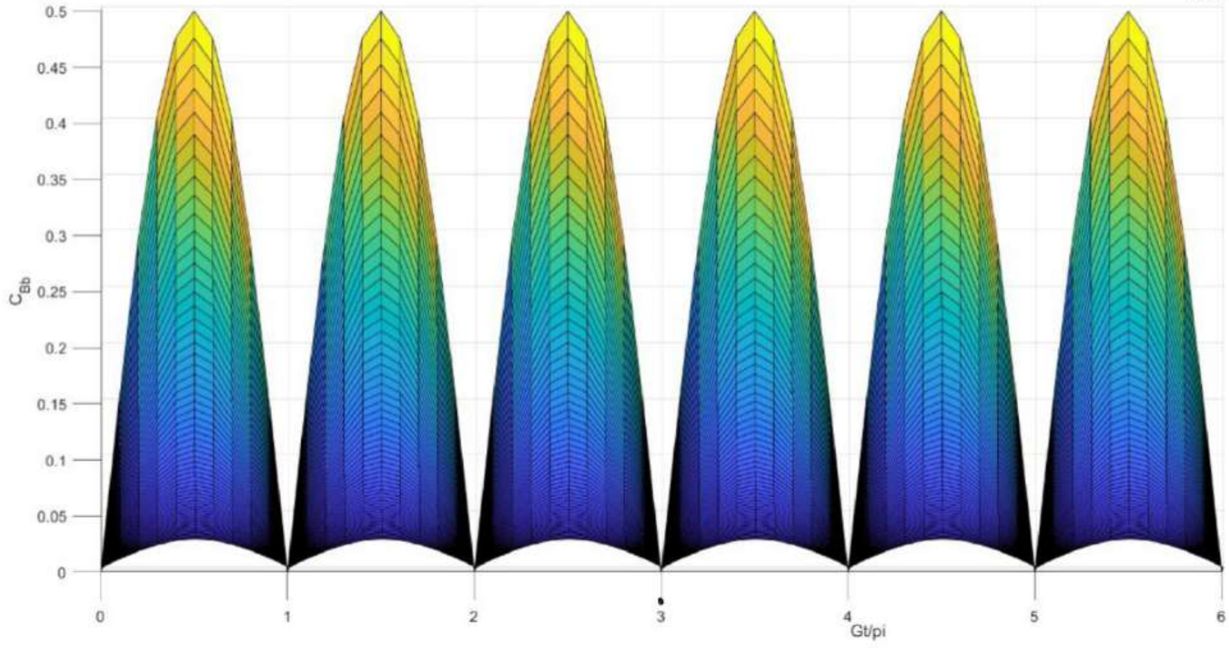
$$C_{Bb}(t) = 2|\tilde{x}_2||\tilde{x}_4|$$

$$|\tilde{x}_2| = |\sin\alpha|\left|\cos\left(\frac{6t}{2}\right)\right|$$

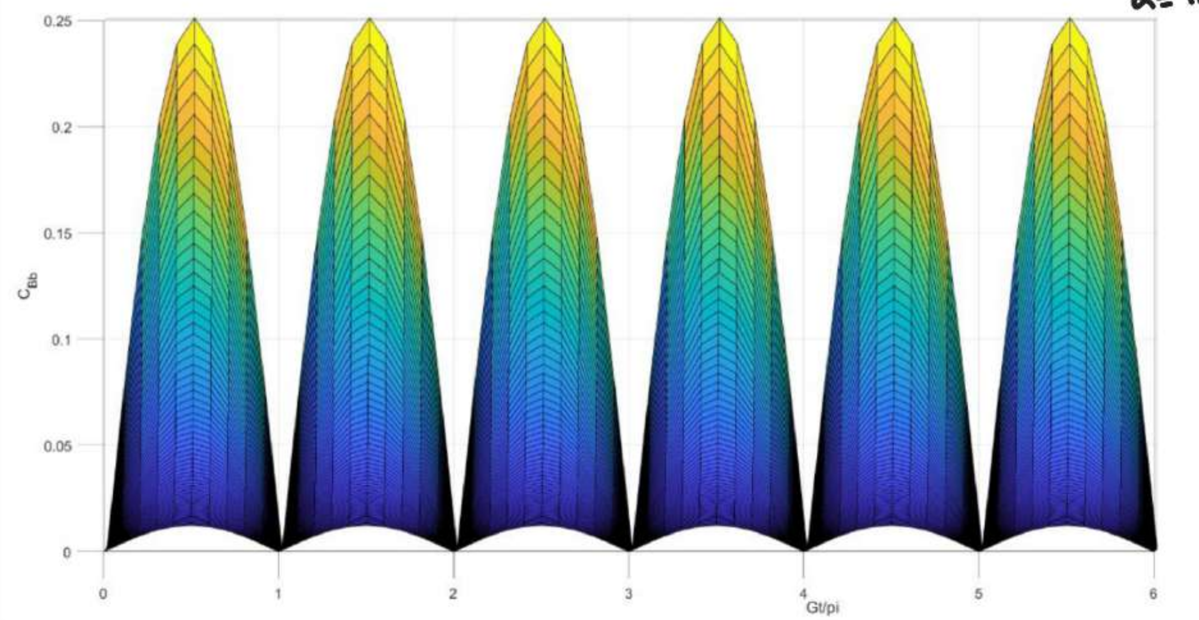
$$|\tilde{x}_4| = e^{Re\{\alpha_B\}}|\sin\alpha|\left|\sin\left(\frac{6t}{2}\right)\right|$$

$$C_{Bb}(t) = e^{Re\{\alpha_B\}}\sin^2\alpha\,|\sin(6t)|$$

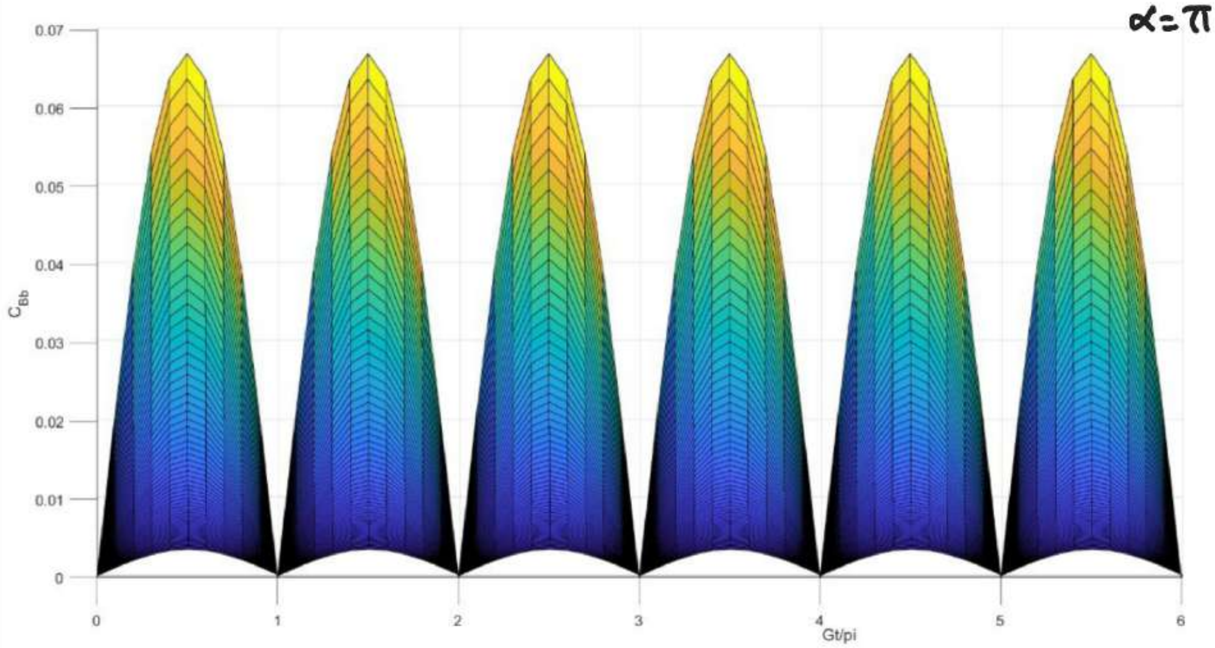
$\alpha = \pi/4$

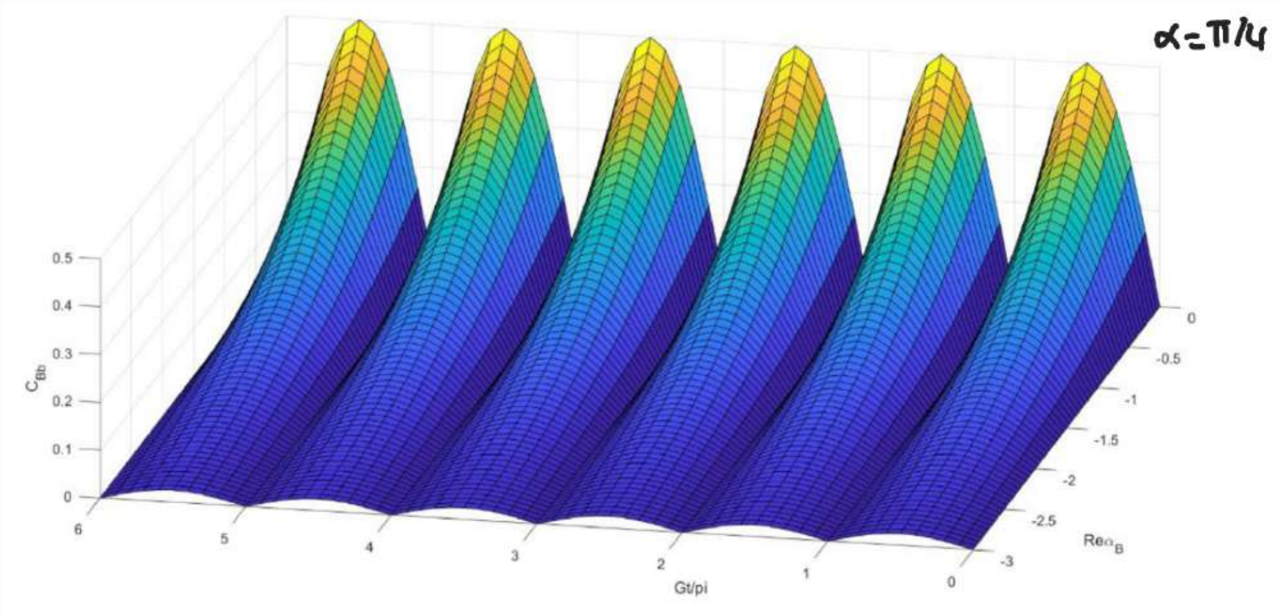


$\alpha = \pi/6$

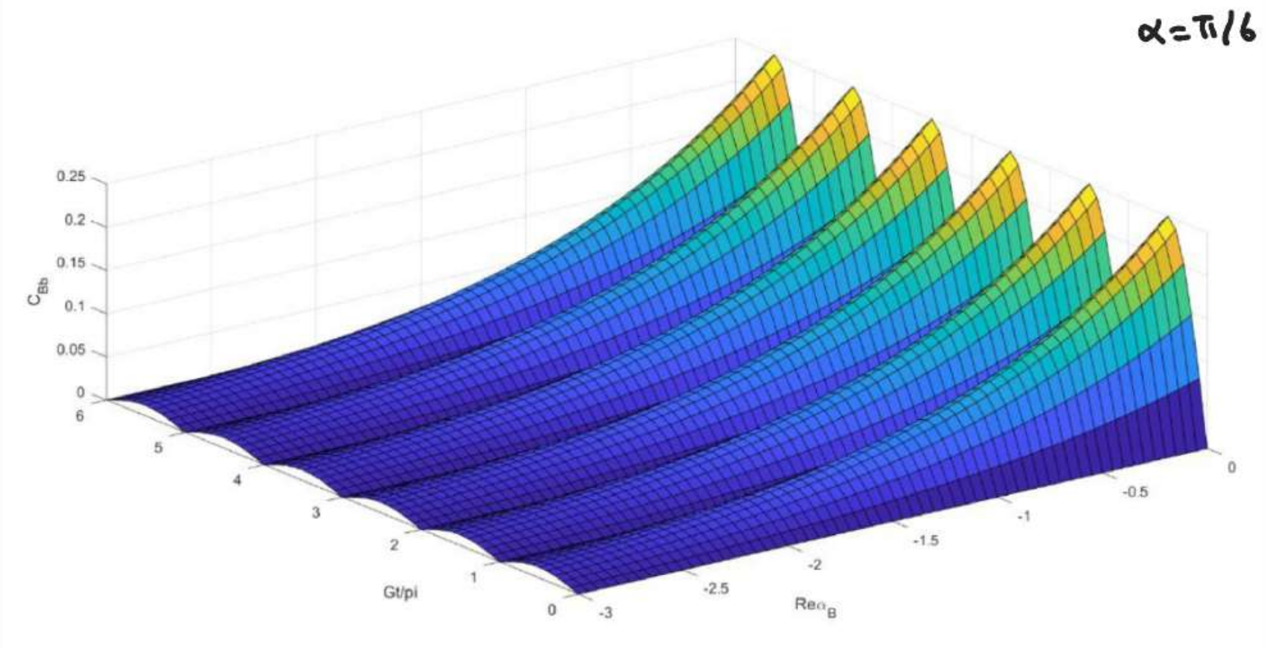


$\alpha = \pi/12$

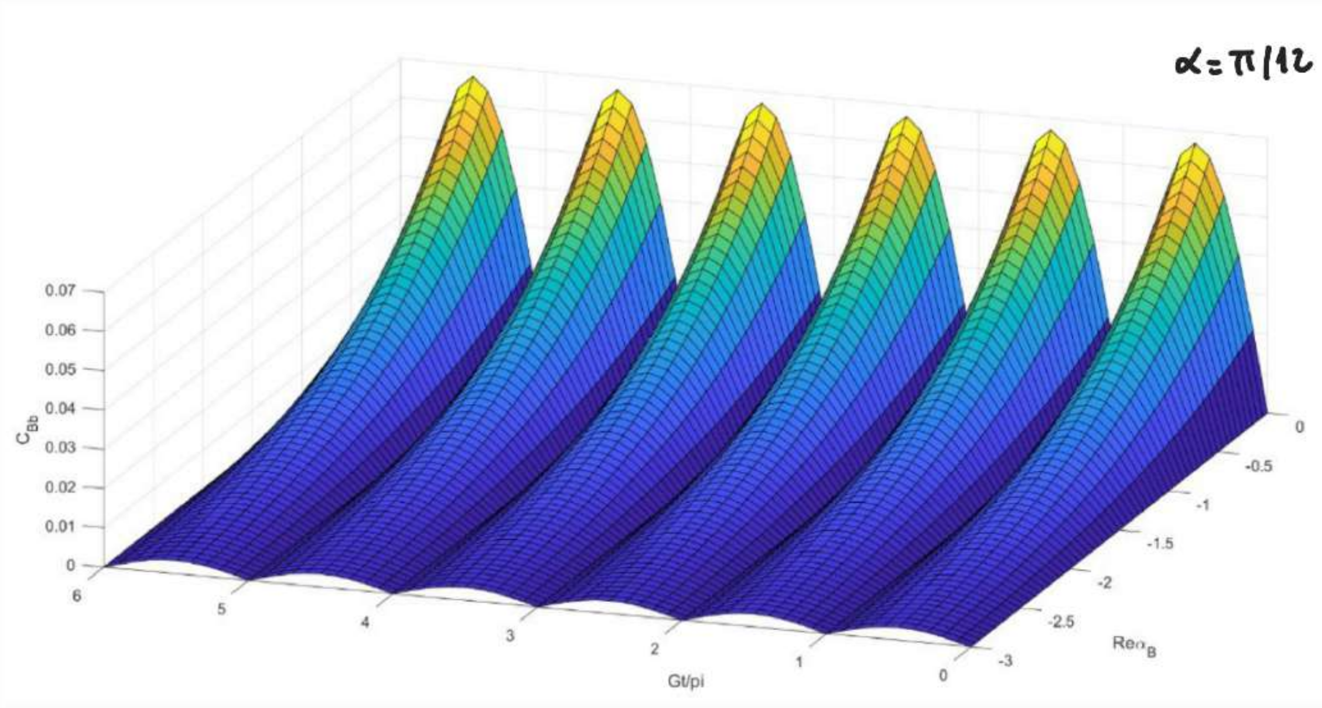




α=π/4
C_Bb
Gt/pi
Reα_B



α=π/6
C_Bb
Gt/pi
Reα_B



α=π/12
C_Bb
Gt/pi
Reα_B

CONCLUSION