09/11/22

Entanglement, Dense Coding, Teleportation, Entanglement Swapping

$$\mathcal{H}_A \simeq \mathcal{H}_B \in \mathbb{C}^2$$

Paradigmatic states that one "most entangled states" $\longrightarrow$ Bell States

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \qquad B_{01} = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$|B_{11}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right) \qquad B_{10} = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

Orthogonal basis of $\mathbb{C}^2 \otimes \mathbb{C}^2$

Thought Experiments with Measurements on the Bell States

A on the earth has a qubit — B on the moon has a qubit

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$$

Remark: $|\gamma\rangle = \cos\gamma|0\rangle + \sin\gamma|1\rangle$

$|\gamma_\perp\rangle = \sin\gamma|0\rangle - \cos\gamma|1\rangle$

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}\left\{ |\gamma\rangle_A \otimes |\gamma\rangle_B + |\gamma_\perp\rangle_A \otimes |\gamma_\perp\rangle_B \right\}$$

A & B local measurement

A chooses applatus $\rightarrow$ basis of $\mathbb{C}^2$ $\{|\alpha\rangle, |\alpha_\perp\rangle\}$

B " " $\rightarrow$ basis of $\mathbb{C}^2$ $\{|\beta\rangle, |\beta_\perp\rangle\}$

A measures state in her lab first

B " " " second

$$\{ |\alpha\rangle\langle\alpha| \otimes \mathbb{I}_B \; ; \; |\alpha_\perp\rangle\langle\alpha_\perp| \otimes \mathbb{I}_B \}$$

$|B_{00}\rangle \rightarrow$ Measurement of A

$\rightarrow |\alpha\rangle_A\langle\alpha|_A \otimes \mathbb{I}_B \; |B_{00}\rangle = |\alpha\rangle_A \otimes |\alpha\rangle_B$

$\rightarrow |\alpha_\perp\rangle\langle\alpha_\perp| \otimes \mathbb{I}_B \; |B_{00}\rangle = |\alpha_\perp\rangle_A \otimes |\alpha_\perp\rangle_B$

Bob measures

$|\beta\rangle$ prob with $\cos^2(\alpha-\beta)$

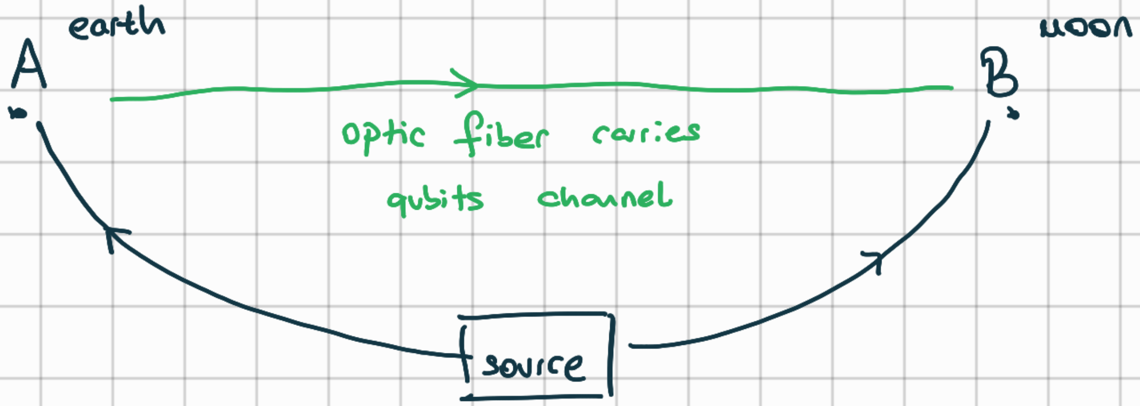
$|\beta_\perp\rangle$ prob with $\sin^2(\alpha-\beta)$

Bob measures

$|\beta\rangle$ prob with $\sin^2(\alpha-\beta)$

$|\beta_\perp\rangle$ prob with $\cos^2(\alpha-\beta)$

Dense Coding Protocol



$$|B_{00}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$$

EPR pairs

Goal: A wants to send 2 classical bits to Bob by using 1 EPR pair + sending physically one qubit

Messages of Alice : 00 , 01, 10, 11

- Suppose A wants to send 00 → she just sends her share of the EPR pair

Bob has the global $|B_{00}\rangle$ in his lab

- Suppose A wants to send 01 → she applies X_A →

the state becomes:

$$X_A \otimes I_B |B_{00}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle_A \otimes |0\rangle_B + |0\rangle_A \otimes |1\rangle_B\right) = |B_{01}\rangle$$

- Suppose A wants to send 11 $\rightarrow$ she applies $Z_A \otimes \mathbb{I}_B |B_{00}\rangle$

$= |B_{11}\rangle$

Alice sends $|B_{11}\rangle$, Bob gets $|B_{11}\rangle$

- Suppose A " " " 10 $\rightarrow$ $Z_A X_A \otimes I_B |B_{00}\rangle = |B_{10}\rangle$

Bob gets $|B_{10}\rangle$