

Homework #3

1.

At work, Alice, Bob and Chuck decide to prepare a GHZ state, where Alice keeps A, Bob keeps B and Chuck keeps C. Once they arrive at home, they are restricted to just classical communication, but Bob and Chuck want to communicate an arbitrary qubit to each other.

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C),$$

a) (15 points) Alice decides to apply a Hadamard transform to her qubit. Write down the resulting state.

b) (20 points) After applying the Hadamard transform, imagine now that Alice measures her qubit in the computational basis. Write down the state shared by Bob and Chuck for the case that Alice obtained measurement outcome $|0\rangle$ and for the case that she obtained outcome $|1\rangle$.

c) (25 points) Describe a protocol after which Bob and Chuck share the positive even parity Bell state $|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, where Alice is allowed to send one bit of classical communication to either Bob or Chuck.

2. (40 points) Using CNOTs, Toffoli gates and single qubit gates implement the circuit that results in the following unitary:

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Hint: Start from x_2 , keep in mind zero controlled Toffoli gate