

# 1 Drawing Feynman Diagrams

1. A fermion (quark, lepton, neutrino) is drawn by a straight line with an arrow pointing to the left:

$$f \longrightarrow f$$

2. An antifermion is drawn by a straight line with an arrow pointing to the right:

$$\overline{f} \longleftarrow \overline{f}$$

3. A photon or  $W^\pm$ ,  $Z^0$  boson is drawn by a wavy line:

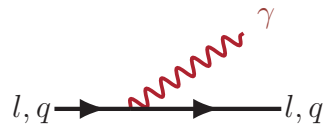
$$\gamma$$


$$W^\pm, Z^0$$

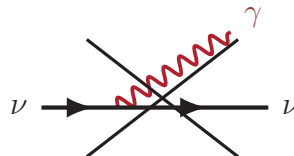

4. A gluon is drawn by a curled line:

$$g$$


5. The emission of a photon from a lepton or a quark doesn't change the fermion:

$$l, q \longrightarrow l, q$$


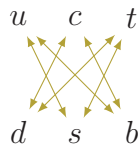
But a photon cannot be emitted from a neutrino:



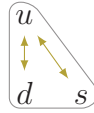
6. The emission of a  $W^\pm$  from a fermion changes the flavour of the fermion in the following way:

$Q = -1$	$e^-$	$\mu^-$	$\tau^-$		$u$	$c$	$t$	$Q = +\frac{2}{3}$
	$\updownarrow$	$\updownarrow$	$\updownarrow$		$\updownarrow$	$\updownarrow$	$\updownarrow$	
$Q = 0$	$\nu_e$	$\nu_\mu$	$\nu_\tau$		$d$	$s$	$b$	$Q = -\frac{1}{3}$

But for quarks, we have an additional mixing between families:



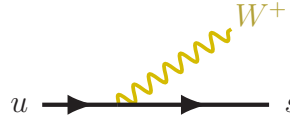
This means that when emitting a  $W^\pm$ , an  $u$  quark for example will mostly change into a  $d$  quark, but it has a small chance to change into a  $s$  quark instead, and an even smaller chance to change into a  $b$  quark. Similarly, a  $c$  will mostly change into a  $s$  quark, but has small chances of changing into an  $u$  or  $b$ . Note that there is no horizontal mixing, i.e. an  $u$  never changes into a  $c$  quark! In practice, we will limit ourselves to the light quarks ( $u, d, s$ ):



Some examples for diagrams emitting a  $W^\pm$ :

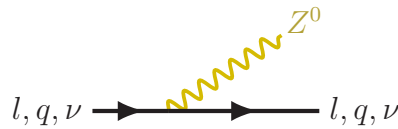


And using quark mixing:

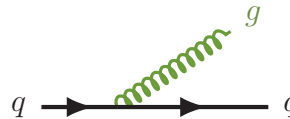


To know the sign of the  $W$ -boson, we use charge conservation: the sum of the charges at the left hand side must equal the sum of the charges at the right hand side.

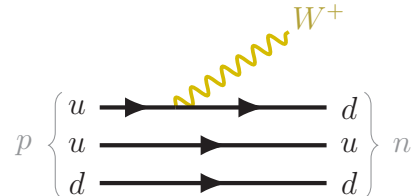
7. The emission of a  $Z^0$  boson doesn't change the fermion, as was the case for the emission of a photon, but a  $Z^0$  can also be emitted from a neutrino:



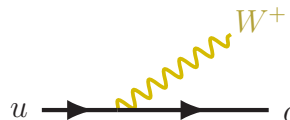
8. A gluon can only be emitted from quarks, and will change their *colour* charge. However, as we almost never draw the colour charge of a quark, in the diagram the quark doesn't change:



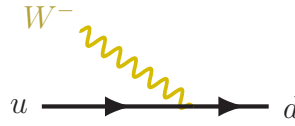
9. To make sure we never need to draw colour charge in our diagrams, we only allow quarks to combine into *colour neutral* particles, which consist of three quarks or a quark-antiquark pair. As an example we draw the emission of a  $W^+$  from a proton, changing it into a neutron:



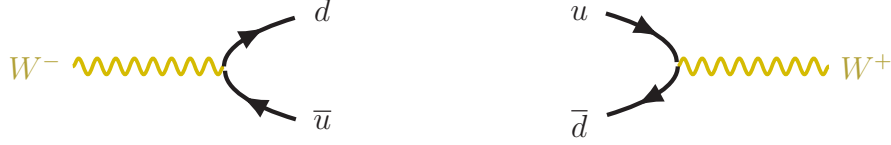
10. When moving a particle's placement in a Feynman diagram from right to left (or vice versa), we change a particle to its antiparticle. Using this rule, we can turn the emission of a  $W^+$



into the absorption of a  $W^-$  (because the antiparticle of a  $W^-$  is a  $W^+$ )

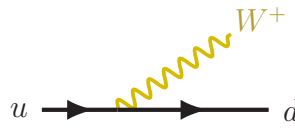


or into the creation or annihilation of a quark pair (note that when moving a fermion this way, the direction of the arrow stays the same relatively to the line)<sup>1</sup>

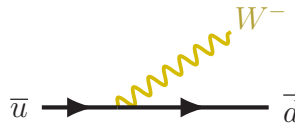


The same transformations can be applied on photon,  $Z^0$  and gluon emissions.

11. We also get a valid Feynman diagram by taking the antiparticle of every particle in the diagram. Using this rule, we can turn the emission of a  $W^+$

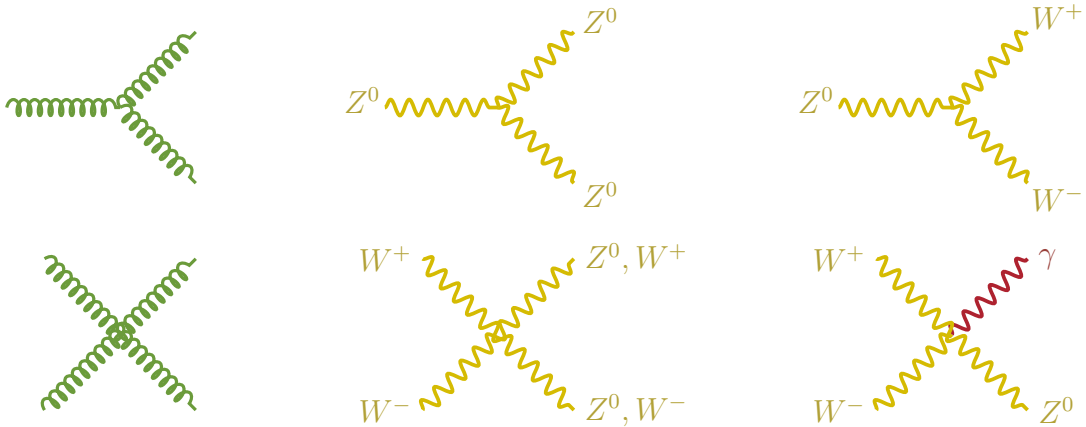


into the emission of a  $W^-$



Again, the same transformations can be applied on photon,  $Z^0$  and gluon emissions.

12. The  $W^\pm$ ,  $Z^0$  and the gluon also have three- and four-point interactions:



But in practice you will only need the gluon three-interaction, because the other three- and four-interactions are strongly suppressed.

13. To get an idea of the probability of a Feynman diagram, we count the number of *vertices* (another word for interaction points). The more, the less probable. Also, the strong interaction (gluons) is more probable than the electromagnetic interaction (photons), which in turn is more probable than the weak interaction ( $Z^0$  and  $W^\pm$  bosons).

<sup>1</sup>Or as a rule of thumb: an incoming particle has its arrow pointing inwards, an outgoing particle has its arrow pointing outwards; an incoming antiparticle has its arrow pointing outwards, an outgoing antiparticle has its arrow pointing inwards.

Hint: draw yourself a set of basic diagrams with the  $W^+$  and  $W^-$  bosons, in order to have a reference to know which particles couple to which. For the photon, the  $Z^0$  and the gluon this is straightforward (because then the particle doesn't change), but for the  $W^\pm$  this is a bit more elaborate.

## 2 Conservation Laws

Not every interaction satisfies all conservation laws. The **strong** force satisfies **all** conservation laws, the **electromagnetic** force conserves all but isospin, and the **weak** force satisfies most (not flavour nor isospin). There are quite a lot conservation laws, so we will limit ourselves to the easiest ones.

0. First a hint: any quantum number changes to its additive inverse when making an antiparticle from a particle. For example, a positron  $e^+$  has electron lepton number  $L_e = -1$  because an electron  $e^-$  has  $L_e = +1$ .
1. **Conservation of electric charge.** The sum of the charges of the initial particles should equal the sum of the final particles.
2. **Conservation of energy.** This is particularly important when the interaction is a decay (i.e. one particle going to several particles), because then in the rest frame of the initial particle we have that its energy (in other words its mass, as it is in rest) should equal the sum of the energies of the final particles. This implies

$$m_{\text{initial}} \geq \sum m_{\text{final}}$$

In case of a collision (two or more initial particles) energy conservation can always be satisfied by giving the initial particles enough energy. It is therefore only needed to check energy conservation in case of a decay (one initial particle).

3. **Conservation of lepton number.** This conservation is even true within the lepton families. For example, the decay

$$\begin{array}{rcll} \mu^- & \rightarrow & e^- + \bar{\nu}_e + \nu_\mu \\ L_e : & 0 & = 1 - 1 + 0 \\ L_\mu : & 1 & = 0 + 0 + 1 \\ L : & 1 & = 1 - 1 + 1 \end{array}$$

conserves electron lepton number  $L_e$ , muon lepton number  $L_\mu$  and total lepton number  $L$ . However, the decay

$$\begin{array}{rcll} \mu^- & \rightarrow & e^- + e^+ + e^- \\ L_e : & 0 & \neq 1 - 1 + 1 \\ L_\mu : & 1 & \neq 0 + 0 + 0 \\ L : & 1 & = 1 - 1 + 1 \end{array}$$

is not allowed, although total lepton number is conserved. But electron and muon number are not conserved separately, which is enough to prohibit the interaction.

4. **Conservation of baryon number.** This law tells us that the number of initial baryons (particles made from three quarks) should equal the number of final baryons. For example, the interactions

$$\begin{array}{ll} p \rightarrow n + e^- + \bar{\nu}_e & p + \bar{p} \rightarrow \pi^+ + \pi^- \\ B : \quad 1 = 1 + 0 + 0 & B : \quad 1 - 1 = 0 + 0 \end{array}$$

are allowed, but

$$\begin{array}{l} p \rightarrow \pi^+ + \pi^- + \pi^0 \\ B : \quad 1 \neq 0 + 0 + 0 \end{array}$$

is not. Keep in mind that there is no such rule for mesons!

5. **Conservation of flavour** (not conserved by the weak interaction). This law says that quark flavour should be conserved between initial and final states. As we are only considering light quarks, the only special flavour is that of a strange quark, which is called strangeness.<sup>2</sup> This implies for example that the interaction

$$\begin{array}{l} K^+ \rightarrow \pi^+ + \pi^0 \\ S : \quad 1 \neq 0 + 0 \end{array}$$

must take place using the weak interaction, as strangeness is not conserved (meaning it cannot take place using the strong nor the electromagnetic force, as those two do conserve flavour).

6. **Conservation of isospin** (not conserved by the weak interaction, nor by the electromagnetic).

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<sup>2</sup>To confuse you a bit, a particle with one  $s$  quark has strangeness  $S = -1$  and a particle with one anti- $s$  has strangeness  $S = +1$ . This is grown historically.