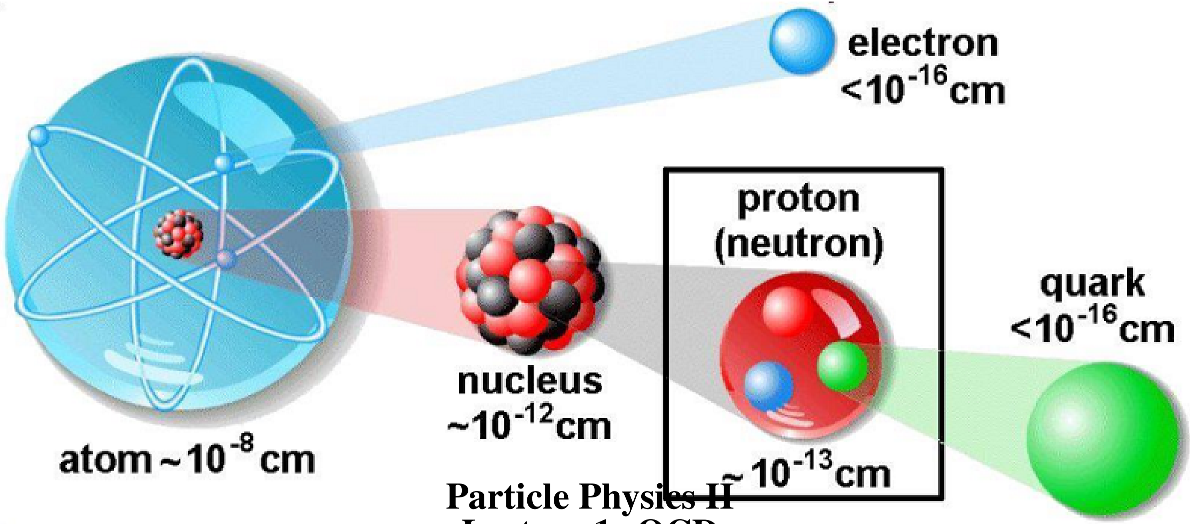

Particle Physics II
Lecture 1: QCD

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February 23, 2023

Practical information

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Course material:

moodle: https://moodle.epfl.ch/course/view.php?id=15032

will contain slides, pointers to video recordings, exercises, and solutions

books: Mark Thomson "Modern Particle Physics" (library)

Particle Data Group "The Review of Particle Physics" http://pdg.lbl.gov/

Course structure

1. Particle Physics I (fall semester)
   - Introduction, detectors, accelerators
   - The Klein-Gordon and Dirac equations and spin
   - Interaction by particle exchange
   - QED: quantum electrodynamics
   - Symmetries and quark model
2. Particle Physics II (spring semester)
   - QED: quantum electrodynamics (quick reminder)
   - QCD: quantum chromodynamics
   - The Weak Interaction
   - Electroweak unification and the W and Z bosons
   - The Higgs boson: theory and discovery
   - Beyond the standard model

Quantum Chromodynamics: main concepts

Theory:

- first principles: particle-wave function invariance under SU(3) local phase transformations (QED – from U(1) invariance)
- coupling constant $\alpha_s \sim 0.1 \ldots 1$ (QED – $\alpha \sim \frac{1}{137}$)
- 3 charges: 3 colors (QED – 1 electric charge)
- force carriers: 8 massless color-charged gluons (QED – 1 massless zero-electric charge photon)
- color confinement: only color-neutral states exist as free particles

Experiment:

- hadronization and jets
- evidence for color existence
- gluon discovery
- α_s measurements
- importance of understanding QCD at colliders

The local gauge principle

- all the interactions between fermions and spin-1 *gauge* bosons in the SM are specified by the principle of local *gauge* invariance:
  1. U(1) $\equiv$ local phase transformation of particle wave-functions $\Longrightarrow$ QED
  2. SU(2) $\equiv$ local SU(2) phase transformation $\Longrightarrow$ weak interaction
  3. SU(3) $\equiv$ local SU(3) phase transformation $\Longrightarrow$ QCD

The local gauge principle: how it works in QED

- require physics to be invariant under the local phase transformation of particle wave-functions:

$$\psi \rightarrow \psi' = \psi e^{iq\xi(x)} \tag{1}$$

where the change of phase $\xi(x) \equiv \xi(t, \vec{x})$ depends on the space-time coordinate

- under this transformation, the Dirac equation transforms as:

$$i\gamma^\mu \partial_\mu \psi - m\psi \rightarrow i\gamma^\mu(\partial_\mu + iq\partial_\mu \xi)\psi - m\psi = 0 \tag{2}$$

- to make "physics", i.e. the Dirac equation, invariant under this local phase transformation, we are **forced** to introduce a **massless gauge boson**, A_μ
- and we modify the Dirac equation by including this field:

$$i\gamma^\mu(\partial_\mu - qA_\mu)\psi - m\psi = 0 \tag{3}$$

The local gauge principle: how it works in QED

- the modified Dirac equation is invariant under local phase transformations if there is **gauge invariance**:

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \xi \tag{4}$$

- physics is unchanged $\equiv$ physical predictions are unchanged for this transformation
- this principle of invariance under local phase transformations completely specifies the interaction between a fermion and the gauge boson – photon:

$$i\gamma^\mu(\partial_\mu \psi - qA_\mu)\psi - m\psi = 0 \tag{5}$$

$\Longrightarrow$ interactions vertex is $i\gamma^\mu q A_\mu$ $\Longrightarrow$ **QED**

- the local phase transformation of QED is a unitary **U(1)** transformation:

$$\psi \rightarrow \psi' = \hat{U}\psi \text{ i.e. } \psi \rightarrow \psi' = \psi e^{iq\xi(x)} \text{ with } U^\dagger U = 1 \tag{6}$$

- suppose there is another fundamental symmetry of the universe, say

  "invariance under SU(3) local phase transformations"

  i.e. require invariance under $\psi \rightarrow \psi' = \psi e^{ig\vec{\lambda}\cdot\vec{\theta}(x)}$, where

  $\vec{\lambda}$ are the eight 3×3 Gell-Mann matrices

  $\vec{\theta}(x)$ are eight functions taking different values at each point in space-time

  $\Longrightarrow$ 8 spin-1 gauge bosons

  $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$ is a wave-function which is a vector in **color space**

  $\Longrightarrow$ QCD!

- QCD is fully specified by requiring invariance under SU(3) local phase transformations:

  $\Longrightarrow$ corresponds to rotating states in color space around an axis whose direction is different at every space-time point

  $\Longrightarrow$ interaction vertex: $-\frac{1}{2} i g_s \lambda^a \gamma^\mu$
- it predicts 8 massless gauge bosons – the gluons (one for each λ matrix)
- also predicts exact form for interactions between gluons, i.e. the 3- and 4-gluon vertices (which we will not consider here)

Color in Quantum Chromodynamics

The theory of the strong interaction, QCD is very similar to QED but with 3 conserved "color" charges:

In QED:

- the electron carries one unit of charge $-e$
- the antielectron carries one unit of anticharge $+e$
- the force is mediated by a massless "gauge boson" – the photon

In QCD:

- quarks carry color charge r, g, b
- antiquarks carry anticharge $\bar{r}, \bar{g}, \bar{b}$
- the force is mediated by massless gluons

In QCD, the strong interaction is invariant under rotations in color space:

$$r \leftrightarrow b; r \leftrightarrow g; b \leftrightarrow g$$

i.e. the same force for all three colors $\implies$ SU(3) color symmetry

This is an exact symmetry, unlike the approximate u-d-s flavor symmetry

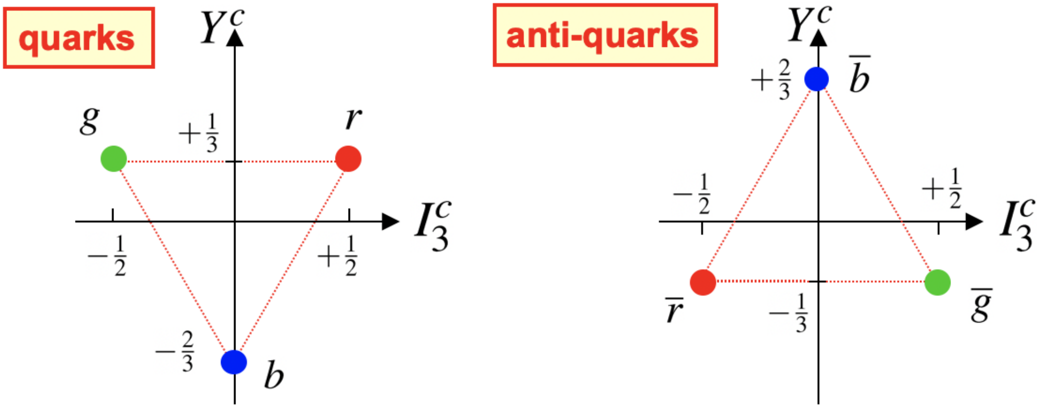
- represent r, g, b SU(3) color states by:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

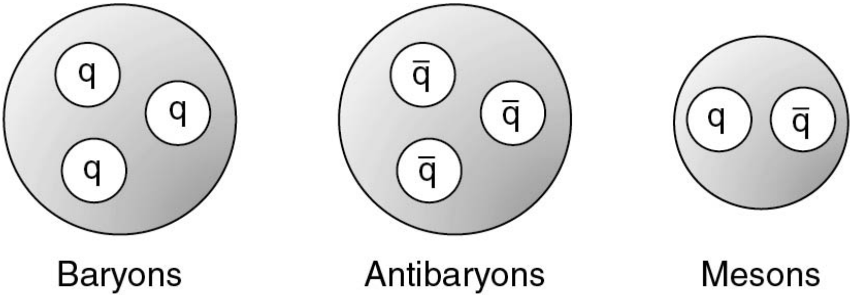
- color states can be labeled by two quantum numbers:
  - I_3^C color isospin
  - Y^C color hypercharge

Exactly analagous to labeling u,d,s flavor states by I_3 and Y

Each quark (antiquark) can have the following color quantum numbers:



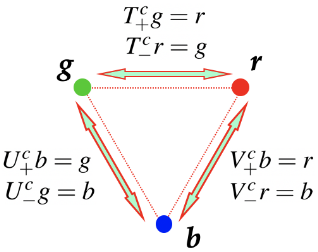
Question:
Why we have only mesons ($q\bar{q}'$), baryons ($qq'q''$), and antibaryons ($\bar{q}\bar{q}'\bar{q}''$) states in nature?



And no other quark combinations?

Color confinement

- it is believed (and not yet proven) that all observed free particles are **"colorless"**:
  - i.e. never observe a free quark (which would carry color charge)
  - quarks are always found in bound states – colorless hadrons
- **Color confinement hypothesis:** only **color singlet** states can exist as free particles
- all hadrons must be "colorless", i.e. color **singlets**
- to construct color wave-functions for hadrons can apply results for SU(3) flavor symmetry to SU(3) color with replacing quark flavors (u, d, s) by quark colors (r, g, b)



- in the same way can define color ladder operators:

- let's look what is meant by a singlet state
- as a reminder, consider spin states obtained with two spin-1/2 particles:
  - four spin combinations: $\uparrow\uparrow$, $\uparrow\downarrow$, $\downarrow\uparrow$, $\downarrow\downarrow$
  - gives four eigenstates of $\hat{S}^2$, $\hat{S}_z$ as $(2 \otimes 2 = 3 \oplus 1)$:

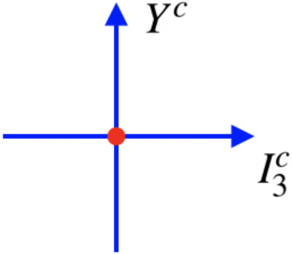
$$\left.\begin{array}{l} |1,+1\rangle = \uparrow\uparrow \\ |1,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ |1,-1\rangle = \downarrow\downarrow \end{array}\right. \quad \boxed{\text{spin-1 triplet}} \quad \oplus \quad |0,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \quad \boxed{\text{spin-0 singlet}}$$

- the singlet state is "spinless": it has 0 angular momentum, is invariant under SU(2) spin transformations and spin ladder operators yield 0:

$$S_\pm\,|0,0\rangle = 0$$

Color singlets

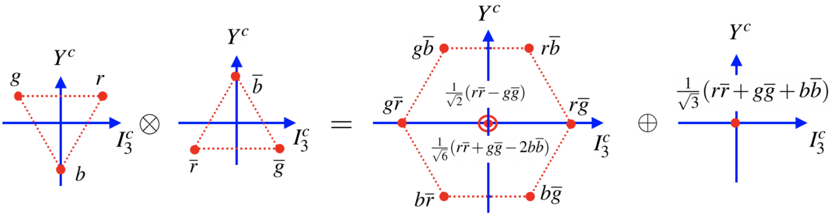
- in the same way color singlets are "colorless" combinations:
  - they have 0 color quantum numbers $I_3^C = 0$, $Y^C = 0$
  - invariant under SU(3) color transformations
  - ladder operators yield 0



- not sufficient to have $I_3^C = 0$, $Y^C = 0$: does not mean that such a state is a singlet

Meson color wave-function

- let's consider color wave-functions for $q\bar{q}$
- the combination of color with anticolor is mathematically identical to construction of meson wave-function with uds flavor symmetry:



$\Longrightarrow$ Colored octet and a colorless singlet

- color confinement implies that hadrons exist in color singlet states so the color wave-function for mesons is:

$$\psi_C^{q\bar{q}} = \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right) \tag{7}$$

Meson color wave-function

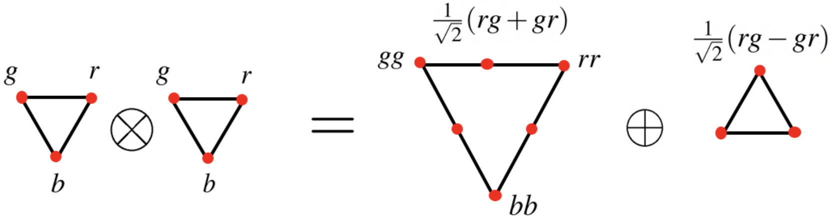
Question: can we have a $qq\bar{q}$ state?
I.e. by adding a quark to the above octet can we form a state with $Y^C = 0$, $I_3^C = 0$?

The answer is a clear no.

$\Longrightarrow$ $qq\bar{q}$ bound states do not exist in nature.

Baryon color wave-function

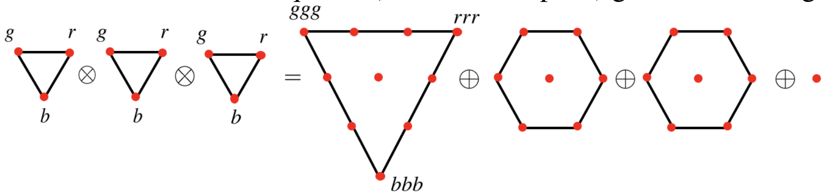
- do qq bound states exist? This is equivalent to asking whether it is possible to form a color singlet from two color triplets
- following the discussion of construction of baryon wave-functions in SU(3) flavor symmetry obtain:



- no qq color singlet state
- color confinement $\Longrightarrow$ bound states of qq do not exist

Baryon color wave-function

- but combination of three quarks (three color triplets) gives a color singlet state:



- the singlet color wave-function is:

$$\psi_C^{qqq} = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr) \tag{8}$$

$\Longrightarrow$ antisymmetric color wave-function (you can check yourselves if it's a color singlet: what are the conditions to check?)

- colorless singlet $\Longrightarrow$ qqq bound states exist!

Allowed hadrons, i.e. possible color singlet states:

$q\bar{q}$, qqq, $\bar{q}\bar{q}\bar{q}$: mesons, baryons and antibaryons

$q\bar{q}q\bar{q}$, $qqqq\bar{q}$: tetraquarks and pentaquarks

The exotic hadrons (pentaquarks and tetraquarks) were observed for the first time rather recently!

Pentaquark observation at LHCb in 2015 (#2)

All LHCb papers ranked by citation number as of February 2023 (link):

2,596 results | cite all — Citation Summary — Most Cited

The LHCb Detector at the LHC #1

LHCb Collaboration • A.Augusto Alves, Jr. (Rio de Janeiro, CBPF) et al. (Aug 14, 2008)

Published in: *JINST* 3 (2008) S08005

DOI | cite | claim | reference search | 4,191 citations

Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays #2

LHCb Collaboration • Roel Aaij (CERN) et al. (Jul 13, 2015)

Published in: *Phys.Rev.Lett.* 115 (2015) 072001 • e-Print: 1507.03414 [hep-ex]

pdf | links | DOI | cite | claim | reference search | 1,516 citations

Test of lepton universality using $B^+ \to K^+\ell^+\ell^-$ decays #3

LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Jun 25, 2014)

Published in: *Phys.Rev.Lett.* 113 (2014) 151601 • e-Print: 1406.6482 [hep-ex]

pdf | DOI | cite | claim | reference search | 1,287 citations

Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays #4

LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)

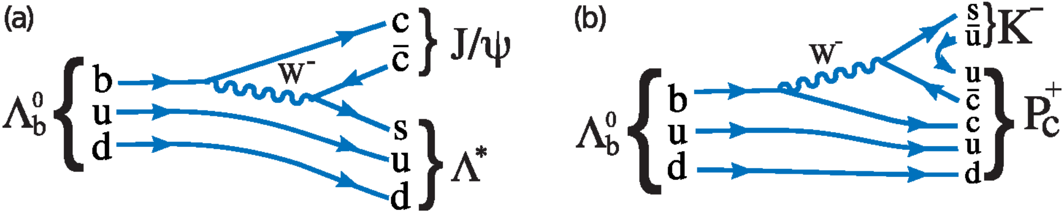
Published in: *JHEP* 08 (2017) 055 • e-Print: 1705.05802 [hep-ex]

pdf | links | DOI | cite | datasets | claim | reference search | 1,206 citations

Pentaquark observation at LHCb

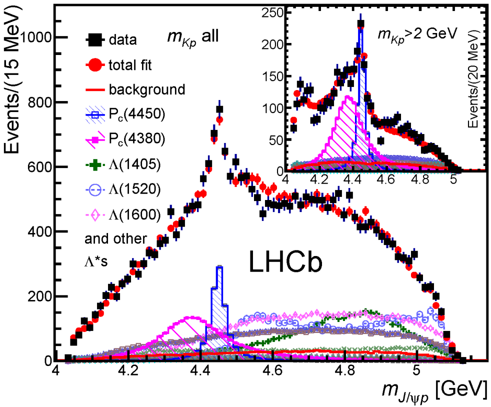
http://lhcb-public.web.cern.ch/lhcb-public/Welcome.html#Penta

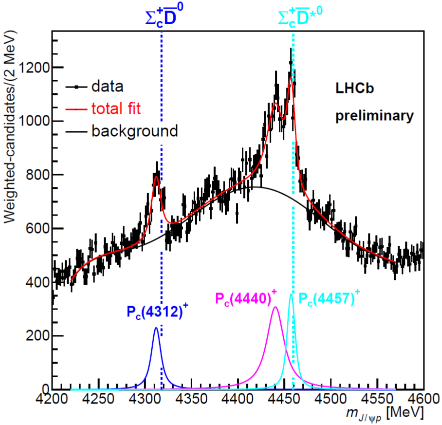
- analyzed a sample of about 26k $\Lambda_b^0 \to J/\psi p K^-$ decays:
  - a decay via known particles
  - a decay with a new pentaquark P_c^+ in the chain, $P_c^+ \to J/\psi p$



More information: https://arxiv.org/abs/1507.03414

Pentaquark observation at LHCb: 2015 and 2019

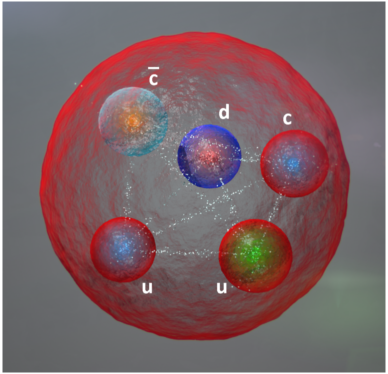


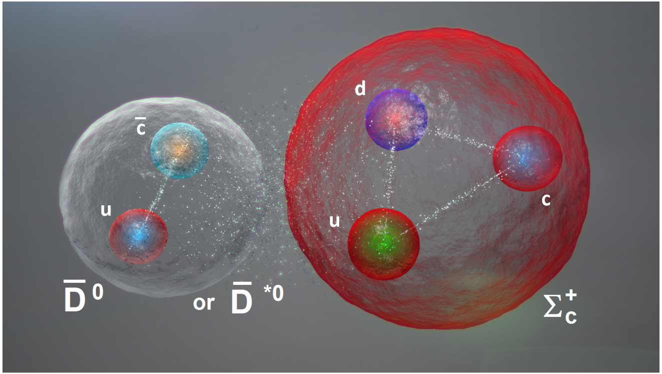


- $P_c(4450)^+$ state seen in 2015 (left) is zoomed into two peaks, $P_c(4440)^+$ and $P_c(4457)^+$ in 2019 (right)

More information: https://arxiv.org/abs/1904.03947

Pentaquark possible models



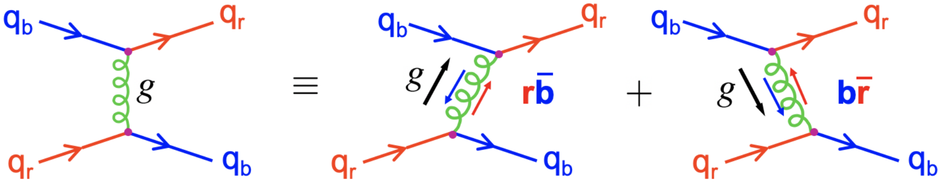


The color of the central part of each quark is related to the strong interaction color charge, while the external part shows its electric charge. The quarks could be tightly bound, or they could also be loosely bound in meson-baryon molecule, in which color-neutral meson and baryon feel a residual strong force similar to the one that binds nucleons together within nuclei.

To be studied by the LHCb in the future.

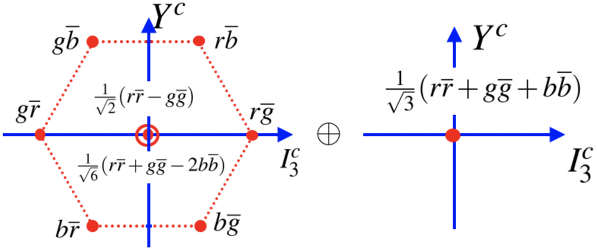
Gluons

- in QCD quarks interact by exchanging virtual massless gluons, e.g.:



- gluons carry color and anticolor, e.g.:

- gluon color wave-functions (color + anticolor) are the same as those obtained for mesons (also color + anticolor) $\Longrightarrow$ we get an octet + "colorless" singlet



- so we might expect 9 physical gluons:

octet: $r\bar{g}, r\bar{b}, g\bar{b}, b\bar{r}, b\bar{g}, \dfrac{1}{\sqrt{2}}(r\bar{r} - g\bar{g}), \dfrac{1}{\sqrt{6}}\left(r\bar{r} + g\bar{g} - 2b\bar{b}\right)$

singlet: $\dfrac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$

- but remember color confinement hypothesis:

only colour singlet states can exist as free particles $\Rightarrow$ **Colour singlet gluon would be unconfined. It would behave like a strongly interacting photon $\rightarrow$ infinite range Strong force.**

- empirically, the strong force is short range and therefore we know that physical gluons are confined $\Longrightarrow$ the color singlet state does not exist in nature!
- **note:** this is not entirely ad hoc. In the context of gauge field theory the strong interaction arises from a fundamental SU(3) symmetry. The gluons arise from the generators of the symmetry group (the Gell-Mann λ matrices). There are 8 such matrices $\Longrightarrow$ 8 gluons. Had nature "chosen" a U(3) symmetry, would have 9 gluons, the additional gluon would be the colour singlet state and QCD would be an unconfined long-range force.
- **note:** the "gauge symmetry" determines the exact nature of the interaction $\Longrightarrow$ Feynman rules

Feynman rules for QCD

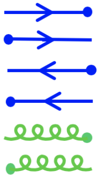
- **External Lines**

spin 1/2:

incoming quark	$u(p)$	
outgoing quark	$\bar{u}(p)$	
incoming anti-quark	$\bar{v}(p)$	
outgoing anti-quark	$v(p)$	

spin 1:

incoming gluon	$\varepsilon^{\mu}(p)$	
outgoing gluon	$\varepsilon^{\mu}(p)^{*}$	



- **Internal Lines (propagators)**

spin 1 gluon

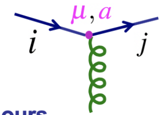
$$\boxed{\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}}$$

a, b = 1,2,…,8 are gluon colour indices

- **Vertex Factors**

spin 1/2 quark

$$\boxed{-ig_s\tfrac{1}{2}\lambda^{a}_{ji}\gamma^{\mu}}$$



i, j = 1,2,3 are quark colours,

λ^{a} a = 1,2,..8 are the Gell-Mann SU(3) matrices

- **+ 3 gluon and 4 gluon interaction vertices**
- **Matrix Element** $-iM$ = product of all factors

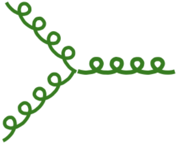
Gluon-gluon interaction

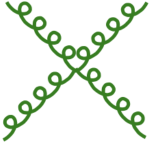
- in QED, photon does not carry EM interaction charge: photons are electrically neutral
- in contrast, in QCD, gluons do carry color charge

$$\Longrightarrow \textbf{gluon self-interactions}$$

- two new vertices appear (no QED analogues):

triple-gluon vertex

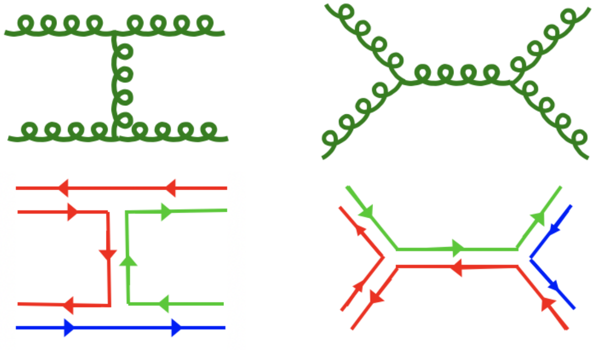




quartic-gluon vertex

Gluon-gluon interaction

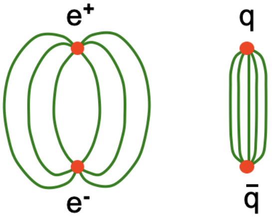
- in addition to quark-quark scattering, can have gluon-gluon scattering:



a possible color-flow is shown in the bottom

Gluon self-interaction and confinement

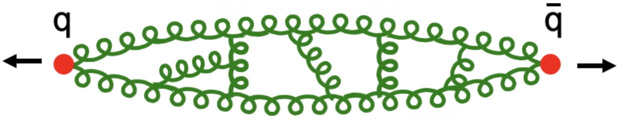
- gluon self-interactions are believed to give rise to color confinement
- qualitative picture:



  - compare QED with QCD:
  - in QCD, “gluon self-interactions squeeze lines of force into a flux tube”

Gluon self-interaction and confinement

What happens when try to separate two colored objects, e.g. $q\bar{q}$:

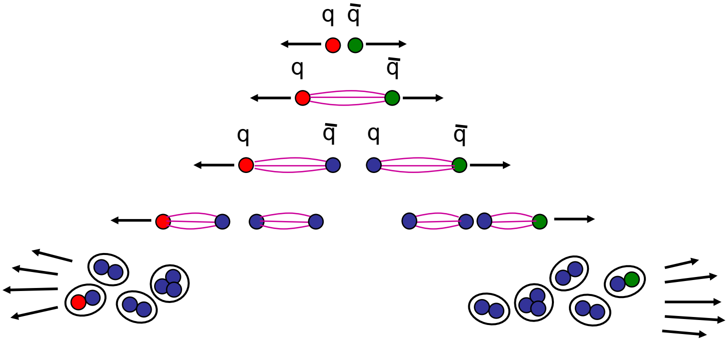


- form a flux tube if interacting gluons of approximately constant energy density ~ 1 GeV/fm $\implies V(r) \sim \lambda r$
- require infinite energy to separate colored objects to infinity
- colored quarks and gluons are always confined within colorless states
- in this way QCD provides a plausible explanation of confinement – but not yet proven

Hadronization and Jets

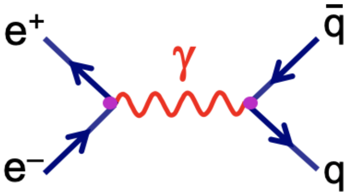
For example, take electron positron annihilation $e^+e^- \to q\bar{q}$:

- quarks separate at high velocity
- color flux tube forms between quarks
- energy stored in the flux tube sufficient to produce $q\bar{q}$ pairs
- continue until quarks pair up into jets of colorless hadrons



- this process is called hadronization: it is not (yet) calculable
- the main consequence is that at collider experiments quarks and gluons observed as jets of particles

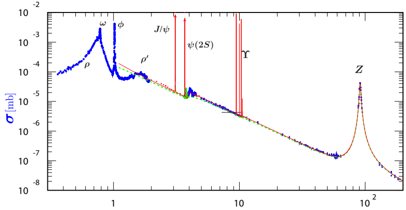
e^+e^- colliders are an excellent place to study QCD:



Well-defined production of quarks

- QED process is well-understood
- no need to know parton structure functions
- experimentally very clean – no proton remnants

Quark spin in e^+e^-



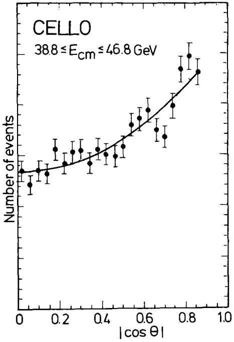


Fig. 1. Angular distribution of the corrected sphericity axis. The solid line corresponds to a fit of $1 + a\cos^2\theta$ with $a = 1.00 \pm 0.01$.

From PPI, for $e^+e^- \to \mu^+\mu^-$ cross section:

$$\sigma = \frac{4\pi\alpha^2}{3s}, \qquad \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta) \tag{9}$$

- produce all quark flavors for which $\sqrt{s} > 2m_q$
- in general, unless producing a $q\bar{q}$ bound state, produce jets of hadrons
- can't tell which jet comes from quark and which from anti-quark
- measured angular distribution of jets $\propto (1 + \cos^2\theta) \implies$ quarks are spin $\frac{1}{2}$

CELLO at PETRA in DESY: "Determination of α_s and $\sin^2\theta_W$ from measurements of the total hadronic cross section in e^+e^- annihilation", Phys. Lett. B 183 (1987) 400

- color is conserved, $q\bar{q}$ produced with three color/anti-color variants
- for a single quark flavor and single color:

$$\sigma(e^+e^- \to q_i\bar{q}_i) = \frac{4\pi\alpha^2}{3s} Q_q^2 \qquad (10)$$

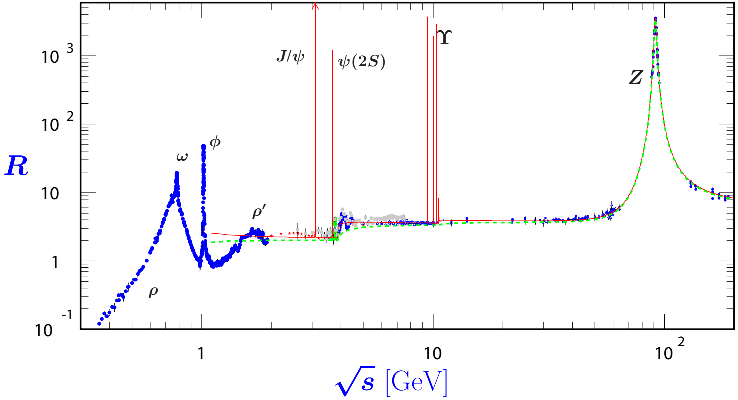
- experimentally observe jets of hadrons of all flavors (and 3 colors):

$$\sigma(e^+e^- \to \text{hadrons}) = 3 \sum_{u,d,s,\ldots} \frac{4\pi\alpha^2}{3s} Q^2_{u,d,s,\ldots} \qquad (11)$$

- usually use a ratio R wrt $\sigma(e^+e^- \to \mu^+\mu^-)$:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \sum_{u,d,s,\ldots} Q^2_{u,d,s,\ldots} \qquad (12)$$

Quark color in e^+e^-: measurements



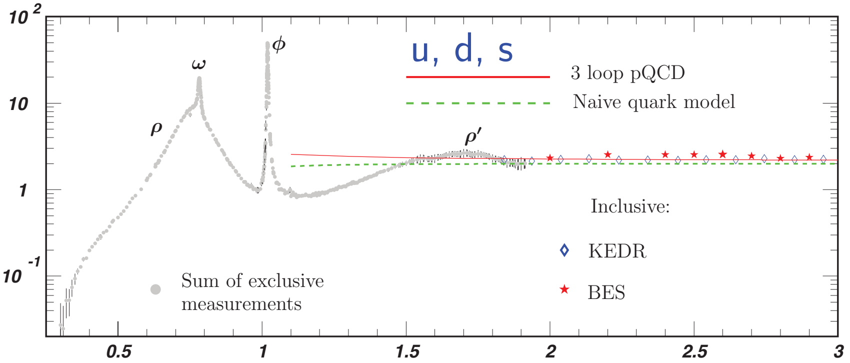
u, d, s: $R = 3 \times (\frac{1}{9} + \frac{4}{9} + \frac{1}{9}) = 2$

u, d, s, c: $R = 3 \times (\frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9}) = \frac{10}{3}$

u, d, s, c, b: $R = 3 \times (\frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} +) = \frac{11}{3}$

Data are consistent with factor 3 from color!

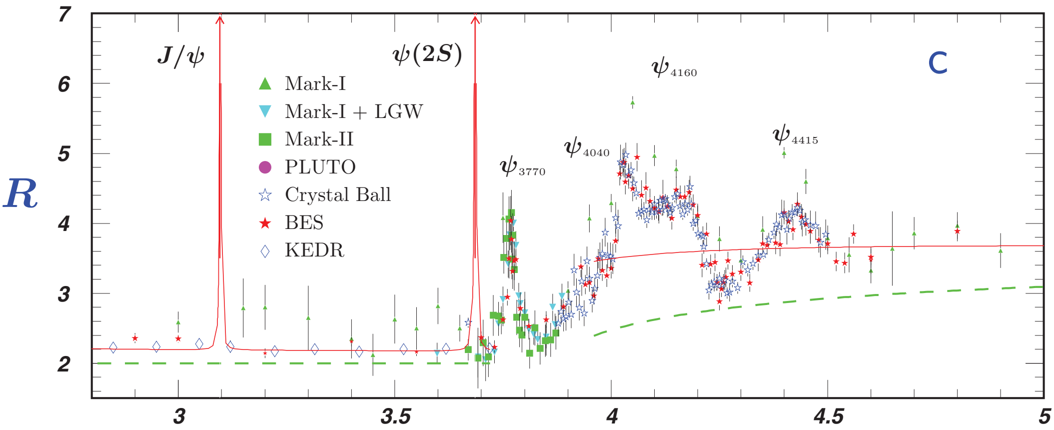
Quark color in e^+e^-: zoomed-in measurements



u, d, s: $R = 3 \times (\frac{1}{9} + \frac{4}{9} + \frac{1}{9}) = 2$

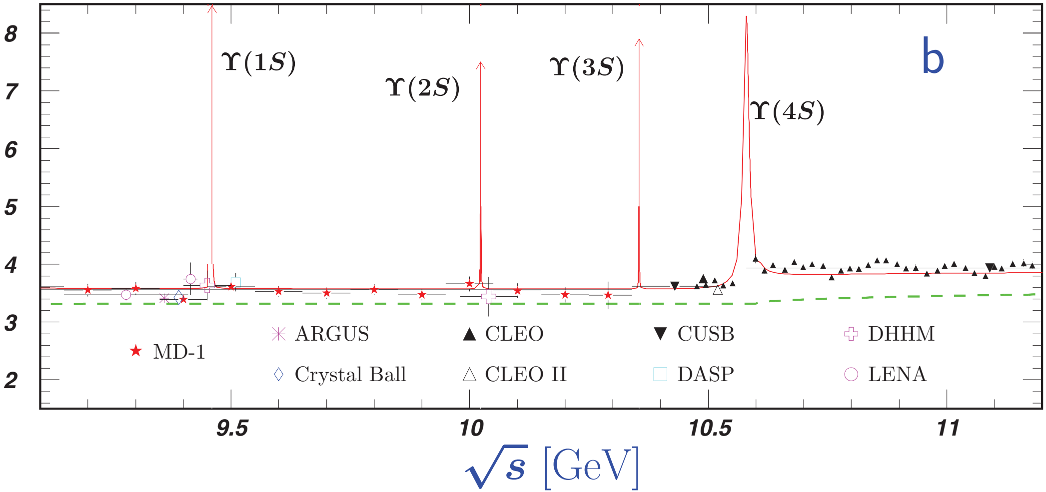
Green dashed line is the lowest order prediction assuming three colors (as shown here). The solid red lines are computed with 3-loop perturbative QCD prediction, which includes the next order contributions as e.g. $e^+e^- \to \mu^+\mu^-\gamma$, $e^+e^- \to q\bar{q}\gamma$ or $e^+e^- \to q\bar{q}g$, summing up to about 10% correction dominated by the last diagram and α_s value.

Quark color in e^+e^-: zoomed-in measurements



u, d, s, c: $R = 3 \times \left(\frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{10}{3}$

Quark color in e^+e^-: zoomed-in measurements



u, d, s, c, b: $R = 3 \times \left(\frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} +\right) = \frac{11}{3}$

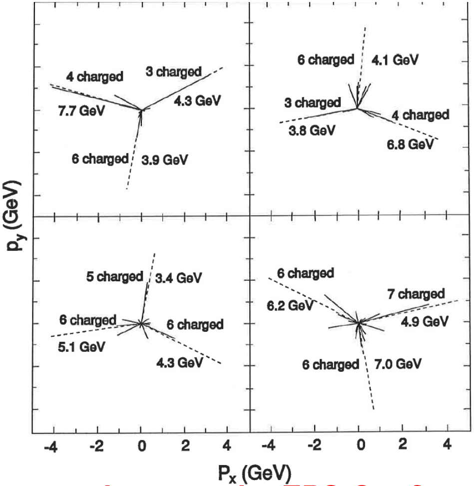
The Discovery of the Gluon

From John Ellis in https://arxiv.org/abs/1409.4232:

Abstract: *"Soon after the postulation of quarks, it was suggested that they interact via gluons, but direct experimental evidence was lacking for over a decade. In 1976, Mary Gaillard, Graham Ross and the author suggested searching for the gluon via 3-jet events due to gluon bremsstrahlung in* e^+e^- *collisions. Following our suggestion, the gluon was discovered at DESY in 1979 by TASSO and the other experiments at the PETRA collider."*

p.4: *This was the context in 1976 when I was walking over the bridge from the CERN cafeteria back to my office one day. Turning the corner by the library, the thought occurred that the simplest experimental situation to search directly for the gluon would be through production via bremsstrahlung in electron-positron annihilation:* $e^+e^- \to q\bar{q}g$*. What could be simpler?*

The Discovery of the Gluon

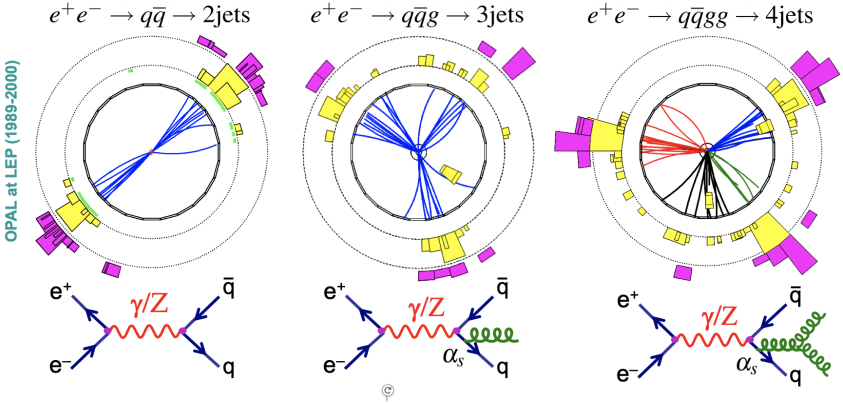


TASSO

Three-jet events shown at the EPS Conference in Geneva, June 27-July 4, 1979, by Paul Söding of TASSO.

https://indico.cern.ch/event/704471/contributions/3012502/attachments/1670841/2680256/Wu.pdf

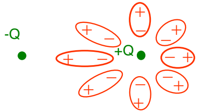
Four decades of gluons celebrated in 2019



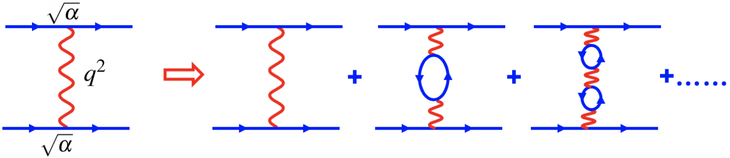
- three jet rate $\Longrightarrow$ measurement of α_s
- angular distributions $\Longrightarrow$ gluons are spin-1
- four jet rate and distributions $\Longrightarrow$ QCD has an underlying SU(3) symmetry

https://home.cern/news/news/physics/four-decades-gluons

Running coupling constants: QED



- "bare" electron charge is screened by virtual e^+e^- pairs
- behaves like a polarizable dielectric
- in terms of Feynman diagrams:

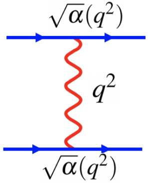


  - same final state $\Longrightarrow$ add amplitudes: $M = M_1 + M_2 + M_3 + \ldots$
  - sum is equivalent to a single diagram with "running" coupling constant:

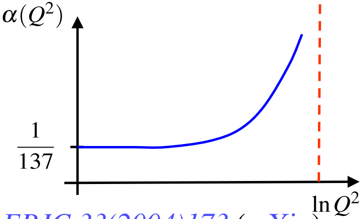
$$\alpha(Q^2) = \alpha(Q_0^2) \Big/ \left[1 - \frac{\alpha(Q_0^2)}{3\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right]$$

$$Q^2 \gg Q_0^2$$

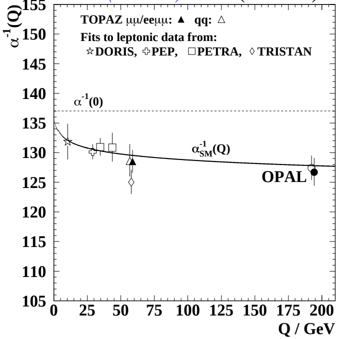
Note sign



Running coupling constants: QED



EPJC 33(2004)173 (arXiv)



EM coupling becomes infinite at $\ln\left(\frac{Q^2}{Q_0^2}\right) = \frac{3\pi}{1/137}$, i.e. at $Q \sim 10^{26}$ GeV

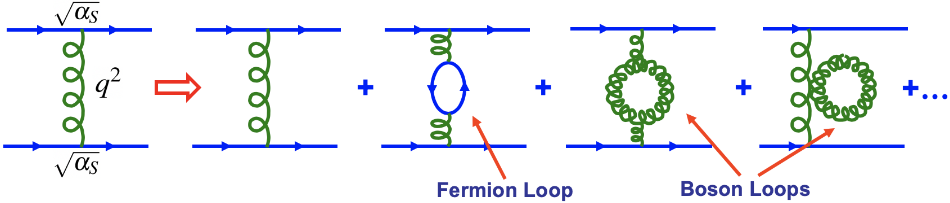
- but quantum gravity effects would come in way below this energy and it is highly unlikely that QED "as is" would be valid in this regime

In QED, α increases very slowly:

- atomic physics $Q^2 \sim 0$: $1/\alpha = 137.03599976(50)$
- high energy physics: $1/\alpha(193\text{ GeV}) = 127.4 \pm 2.1$

Running coupling constants: QCD

Similar to QED, but also have gluon loops:



- adding amplitudes: bosonic loops interfere negatively

$$\alpha_S(Q^2) = \alpha_S(Q_0^2) \Big/ \left[1 + B\alpha_S(Q_0^2)\ln\left(\frac{Q^2}{Q_0^2}\right)\right]$$

with $B = \dfrac{11N_c - 2N_f}{12\pi}$ $\left\{ \begin{array}{l} N_c \text{ = no. of colours} \\ N_f \text{ = no. of quark flavours} \end{array} \right.$

$N_c = 3;\ N_f = 6$ ➡ $B > 0$

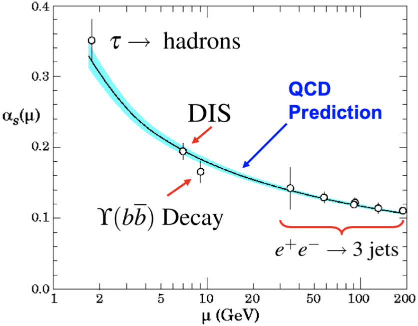
➡ $\boxed{\alpha_S \text{ decreases with } Q^2}$

Nobel Prize for Physics, 2004
(Gross, Politzer, Wilczek)

Running coupling constants: QCD

★ **Measure α_s in many ways:**
- jet rates
- DIS
- tau decays
- bottomonium decays
- +...

✶ **As predicted by QCD, α_s decreases with Q^2**



$\mu \approx Q$

At low Q^2: α_s is large, e.g. at $Q^2 = 1\ \text{GeV}^2$ find $\alpha_s \sim 1$

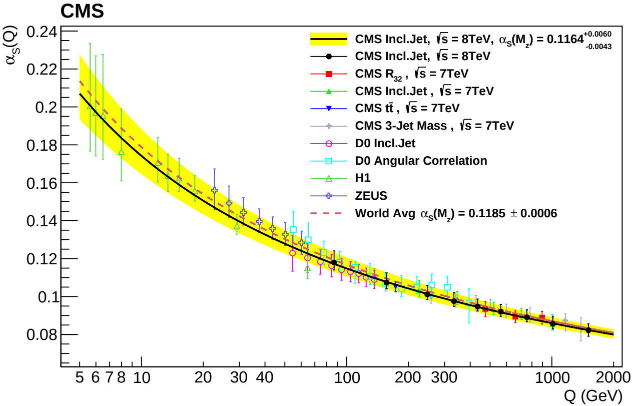
- can't use perturbation theory! This is the reason why QCD calculations at low energies are so difficult, e.g. properties of hadrons, hadronization of quarks to jets, ...

At high Q^2: α_s is rather small, e.g. at $Q^2 = M_Z^2$ find $\alpha_s \sim 0.12$

- **asymptotic freedom**: can use perturbation theory and this is the reason that in DIS at high Q^2 quarks behave as if they are quasi-free (i.e. only weakly bound within hadrons)

EPJC 64(2009)689 arxiv:0908.1135

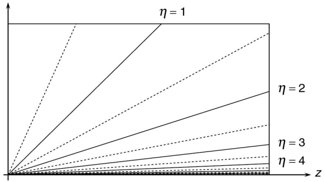
Running coupling constants: QCD at the LHC



The strong coupling $\alpha_s(Q)$ (solid line) and its total uncertainty (band) as determined in JHEP 03(2017)156 analysis using a 2-loop solution to the renormalization group equation as a function of the momentum transfer $Q = p_{\rm T}$. The extractions of $\alpha_s(Q)$ in nine separate ranges of Q are shown together with results from the H1, ZEUS, and D0 experiments at the HERA and Tevatron colliders. Recent other CMS measurements are displayed as well: $\rm R_{32}$: EPJC 73(2013)2604, $t\bar{t}$ cross section: Phys. Lett. B 738 (2014) 526, 3-jet mass: EPJC 75(2015)186 and inclusive jets at 7 TeV EPJC 75(2015)288

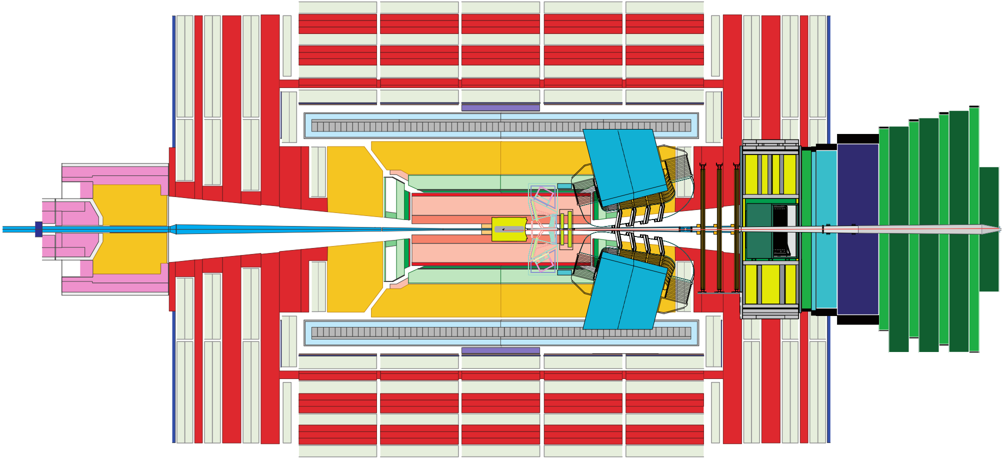
Hadron–hadron collisions

- in e^+e^- collisions one angle θ is enough to parameterize event
- in hh collisions there are more unknowns: x_1, x_2 of colliding partons
- for hh need three variables to describe event, e.g. for $pp \to jj + X$ two angles between the jets and beam axis, and jet p_T
  - $p_\mathrm{T} = \sqrt{p_x^2 + p_y^2}$
  - rapidity $y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$ – Lorentz-invariant wrt the boost along z
  - for massless particles rapidity is the same as pseudorapidity $\eta \equiv - \ln \left(\tan \frac{\theta}{2} \right)$ – depends only on the angle between the particle momentum and beam axis



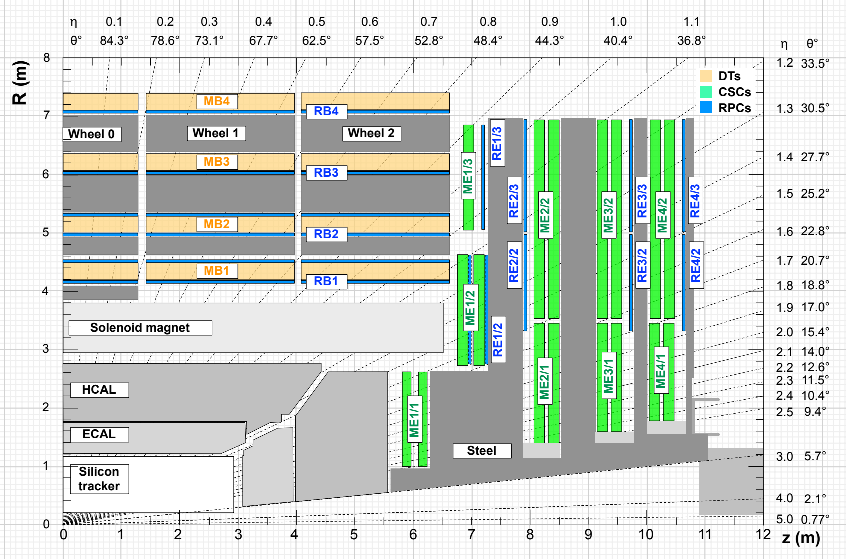
Pseudorapidity coverage in CMS vs. LHCb

Broadly speaking, the differential cross sections for jet production in hadron–hadron collisions are approximately constant in pseudorapidity, implying that roughly equal numbers of jets are observed in each interval of pseudorapidity, reflecting the forward nature of jet production in pp and $p\bar{p}$ collisions.



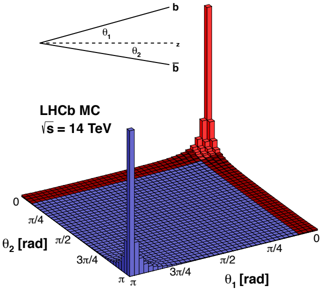
Pseudorapidity coverage in CMS

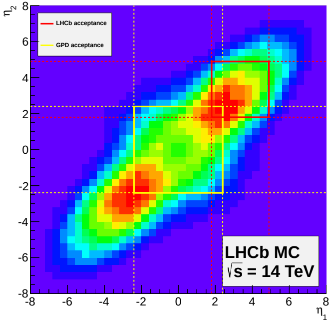
CMS is basically instrumented up to $|\eta| \approx 2.5$



Pseudorapidity coverage in LHCb

LHCb is instrumented in a region $2 < \eta < 5$:



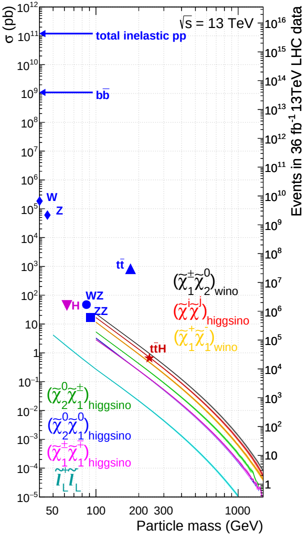


Geometry chosen to get forwardly produced $b\bar{b}$ in acceptance:

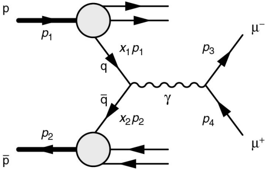
- LHCb intercepts 27% of b or $\bar{b}$ quarks; 24% of $b\bar{b}$ quark pairs
- CMS intercepts 49% of b or $\bar{b}$ quarks; 41% of $b\bar{b}$ quark pairs

https://lhcb.web.cern.ch/lhcb/speakersbureau/html/bb_ProductionAngles.html

QCD cross sections not sustainable

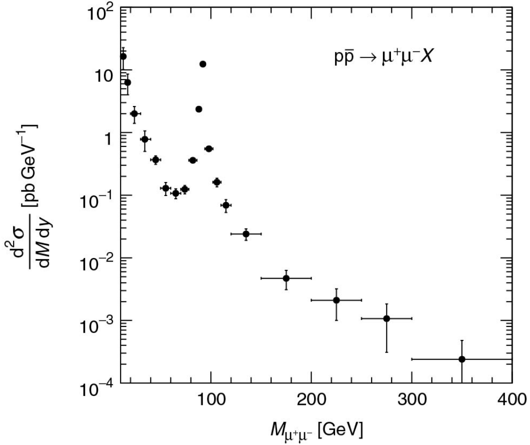


- inelastic pp cross section is 6 orders of magnitude larger than those of W or Z bosons production
- $b\bar{b}$ cross section is such that a B meson is produced in every collision at LHCb
- dedicated online selection strategies – triggers – are in place to record only interesting events
- more rare process – Drell–Yan process: QED lepton pair production



$$\sigma(q\bar{q} \to \mu^+\mu^-) = \frac{1}{N_c} Q_q^2 \frac{4\pi\alpha^2}{3\hat{s}},$$ where Q_q is the quark/antiquark charge and $\hat{s}$ is the centre-of-mass energy of the colliding $q\bar{q}$ system

DY production at CDF: $p\bar{p} \to \mu^+\mu^- + X$



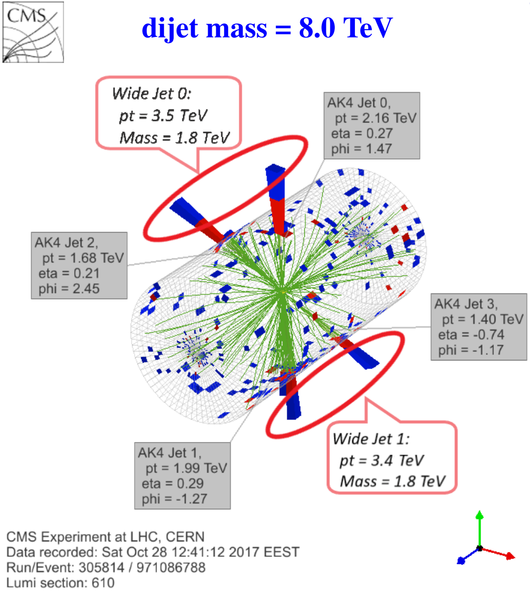
- to obtain full cross section, need to integrate over parton distribution functions (PDF) of quarks and atniquarks in a proton and an anti-proton
- using invariant mass of muon system $M^2 = x_1 x_2 s$, and its rapidity y can obtain:

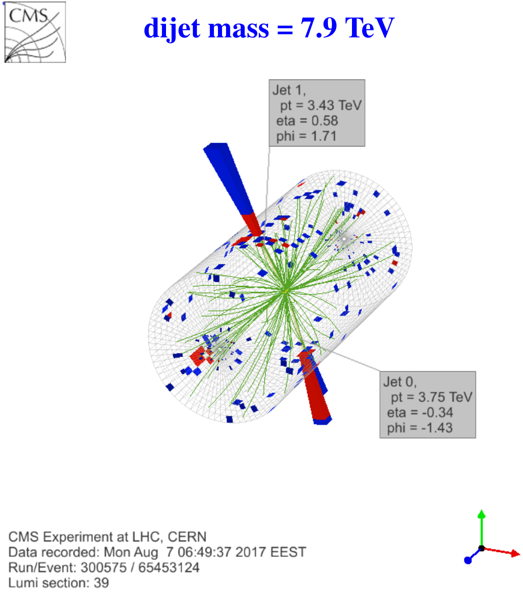
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}y^2} M = \frac{8\pi\alpha^2}{9Ms} f(x_1, x_2),$$

$$f(x_1, x_2) = \left[\frac{4}{9}\left(u(x_1)u(x_2) + \bar{u}(x_1)\bar{u}(x_2)\right) + \frac{1}{9}\left(d(x_1)d(x_2) + \bar{d}(x_1)\bar{d}(x_2)\right)\right]$$

The cross section can grow with the collider energy as it is impacted by the PDFs of colliding partons.

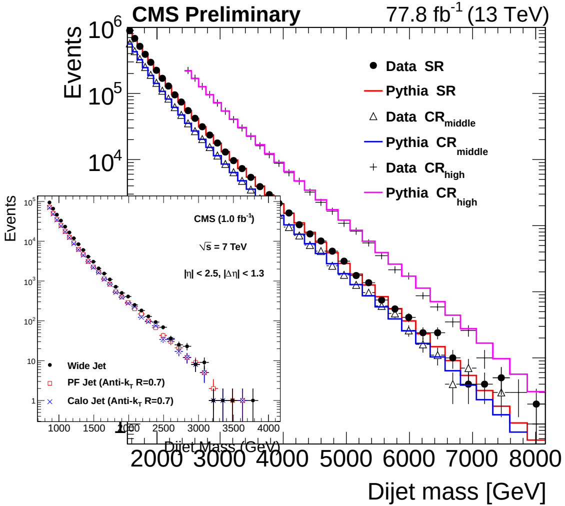
Dijet production at CMS

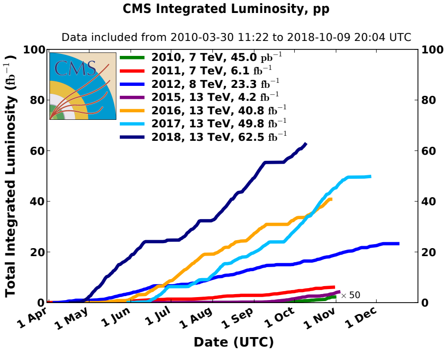




$7 \to 8 \to 13$ TeV

Dijet invariant mass shape is well described by MC generator up to highest values:



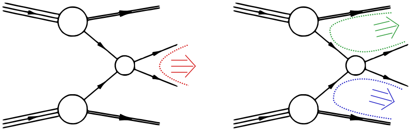


7 TeV $\to$ 13 TeV
1.0 fb^{-1} $\to$ $\sim$100 fb^{-1}
in m(X): 3.5 TeV $\to$ 8 TeV!

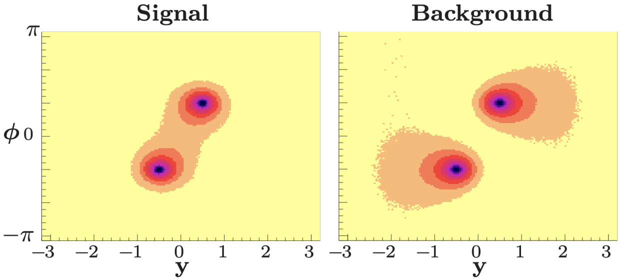
QCD is the most important process to understand and control at hadron colliders!

Seeing color flow in detectors

Color singlet ($pp \to H \to b\bar{b}$) vs. colored particle ($pp \to g \to b\bar{b}$) hadronization:



Distributions seen in the detector:



Many machine learning applications are developed to consider energy patterns in the jets in the detector, e.g. papers citing this result: list

PRL 105(2010)022001 arXiv:1001.5027

Summary

- superficially QCD is very similar to QED
- but gluon self-interactions are believed to result in colour confinement
- all hadrons are colour singlets which explains why only observe mesons or baryons
- at low energies $\alpha_S \sim 1$

$\Longrightarrow$ cannot use perturbation theory
$\Longrightarrow$ non-perturbative regime

- coupling constant runs, smaller coupling at higher energy scales $\alpha_S(100\text{ GeV}) \sim 0.1$

$\Longrightarrow$ can use perturbation theory
$\Longrightarrow$ asymptotic freedom

- where calculations can be performed, QCD provides a good description of relevant experimental data