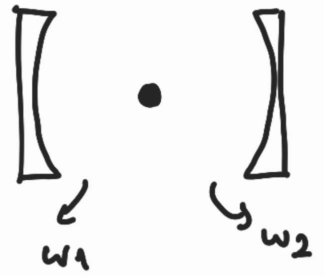
Atom In 2-Modes Cavity



$$H_{at} = \frac{1}{2}\omega_A \sigma_z^A$$

$$H_{cav} = \omega_1 \hat{a}_1^{\dagger} \hat{a}_1 + \omega_2 \hat{a}_2^{\dagger} \hat{a}_2$$

$$H_{int} = g_1 (\sigma_-^A \hat{a}_1^{\dagger} + \sigma_+^A \hat{a}_1) + g_2 (\sigma_-^A \hat{a}_2^{\dagger} + \sigma_+^A \hat{a}_2)$$

let's define $\hat{N}_{exc} = \hat{a}_1^{\dagger}\hat{a}_1 + \hat{a}_2^{\dagger}\hat{a}_2 + \hat{\sigma}_+ \hat{\sigma}_-$

Eigenstates	Eigenvalue
$\lvert g, n_1, n_2\rangle$	
$\lvert e, n_1, n_2-1\rangle$	n_1+n_2
$\lvert e, n_1-1, n_2\rangle$	

$[\hat{H}, \hat{N}_{exc}] = 0$ Thus, they've common eigenstates

$$\hat{H} = \begin{bmatrix} \lvert g,0,0\rangle & & \\ & \ddots & \\ & & \begin{array}{ccc} \lvert g,n_1,n_2\rangle & \lvert e,n_1-1,n_2\rangle & \lvert e,n_1,n_2-1\rangle \\ A & X & Y \\ X & B & 0 \\ Y & 0 & C \end{array} \\ & & & \ddots \end{bmatrix}$$

$$\hat{H} = \bigoplus_{n_1+n_2=0}^{\infty} H^{(n_1+n_2)}$$

$$\hat{H} = \begin{pmatrix} H^{(0)} & & & 0 \\ & \boxed{H^{(1)}} & & \\ & & \boxed{H^{(2)}} & \\ 0 & & & \ddots \end{pmatrix}$$

$$A = \langle g, n_1, n_2 | \hat{H} | g, n_1, n_2 \rangle = -\frac{1}{2}\omega_A + \omega_1(n_1) + \omega_2(n_2)$$

$$B = \langle e, n_1-1, n_2 | \hat{H} | e, n_1-1, n_2 \rangle = +\frac{1}{2}\omega_A + \omega_1(n_1-1) + \omega_2 n_2$$

$$C = \langle e, n_1, n_2-1 | \hat{H} | e, n_1, n_2-1 \rangle = +\frac{1}{2}\omega_A + \omega_1(n_1) + \omega_2(n_2-1)$$

$$X = \langle g, n_1, n_2 | \underbrace{\hat{H}}_{g_1(\hat{\sigma}_- \hat{a}_1^+)} | e, n_1-1, n_2 \rangle = g_1\sqrt{n_1}$$

$$Y = \langle g, n_1, n_2 | \underbrace{\hat{H}}_{g_2(\hat{\sigma}_- \hat{a}_2^+)} | e, n_1, n_2-1 \rangle = g_2\sqrt{n_2}$$

$$\hat{H}^{(n_1+n_2)} = \begin{bmatrix} A & X & Y \\ X & B & 0 \\ Y & 0 & C \end{bmatrix} \longrightarrow \det \begin{bmatrix} A-\lambda & X & Y \\ X & B-\lambda & 0 \\ Y & 0 & C-\lambda \end{bmatrix} = 0$$

$$\begin{matrix} + & - & + \end{matrix}$$

$$y \begin{vmatrix} x & y \\ B-\lambda & 0 \end{vmatrix} + (C-\lambda) \begin{vmatrix} A-\lambda & x \\ x & B-\lambda \end{vmatrix} = 0$$

$$-y^2 \cdot (B-\lambda) + (C-\lambda)\left[(A-\lambda)(B-\lambda) - x^2 \right] = 0$$

$$-y^2(B-\lambda) + (A-\lambda)(B-\lambda)(C-\lambda) - (C-\lambda)x^2 = 0$$

$$(B-\lambda)\Big[\underbrace{-y^2 + (A-\lambda)(C-\lambda)}_{-y^2 + AC - \lambda(A+C) + \lambda^2}\Big] - (C-\lambda)x^2 = 0$$

$$-By^2 + BAC - \lambda B(A+C) + B\lambda^2$$

$$+\lambda y^2 \quad -\lambda AC \quad + \lambda^2(A+C) \quad - \lambda^3$$

$$-Cx^2 \quad +\lambda x^2$$

$$+$$

$$+\lambda^3 - \lambda^2(A+B+C) + \lambda(AB+BC+AC-y^2-x^2) - (By^2+Cx^2-ABC) = 0$$

General cubic formula

$$ax^3+bx^2+cx+d=0$$

$$\Delta_0 = b^2-3ac$$

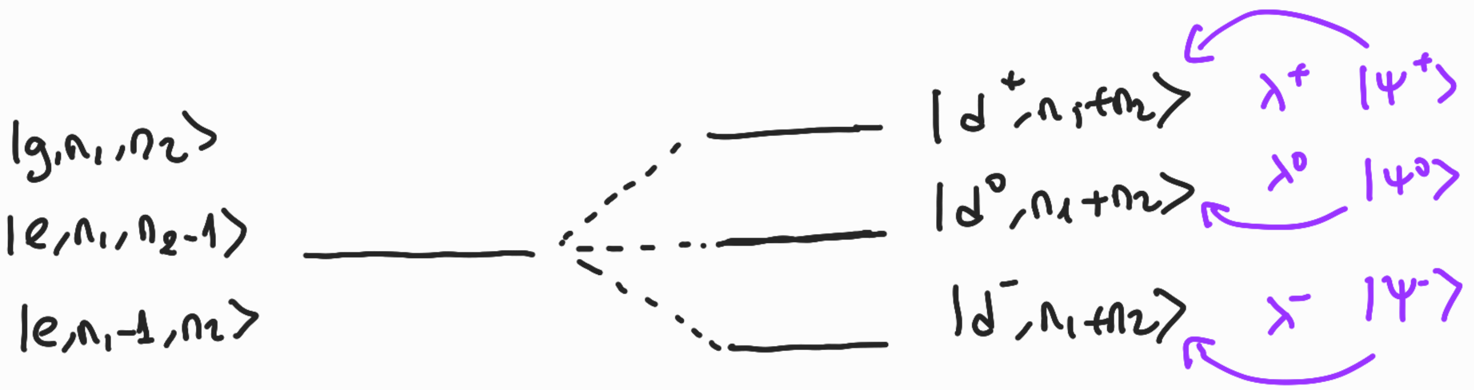
$$\Delta_1 = 2b^3-9abc+27a^2d$$

$$\frac{1}{4}\cdot(-1+\sqrt{3}i)(-1+\sqrt{3}i)$$

$$\xi^2 \quad +1-2\sqrt{3}i-3 = \frac{-2\sqrt{3}i-2}{4} = \frac{-1-\sqrt{3}i}{2}$$

$$C=\sqrt[3]{\frac{\Delta_1 \pm \sqrt{\Delta_1^2-4\Delta_0^3}}{2}}$$

$$x_k = -\frac{1}{3a}\left(b+\xi^k C+\frac{\Delta_0}{\xi^k C}\right), \quad k\in\{0,1,2\}, \quad \xi=\left(\frac{-1+\sqrt{3}i}{2}\right)$$



3 Assume initially 2 photon in the cavity $(\underset{1}{n_1}+\underset{1}{n_2}=2)$

$$|\psi_2^+\rangle = a_1|g,\overset{1}{n_1},\overset{1}{n_2}\rangle + a_2|e,\overset{1}{n_1},\overset{0}{n_2-1}\rangle + a_3|e,\overset{0}{n_1-1},\overset{1}{n_2}\rangle$$

$$|\psi_2^0\rangle = b_1|g,\overset{1}{n_1},\overset{1}{n_2}\rangle + b_2|e,\overset{1}{n_1},\overset{0}{n_2-1}\rangle + b_3|e,\overset{0}{n_1-1},\overset{1}{n_2}\rangle$$

$$|\psi_2^-\rangle = c_1|g,\overset{1}{n_1},\overset{1}{n_2}\rangle + c_2|e,\overset{1}{n_1},\overset{0}{n_2-1}\rangle + c_3|e,\overset{0}{n_1-1},\overset{1}{n_2}\rangle$$

$(a_1^2+a_2^2+a_3^2)=1$ (due to normalization)

for one-photon case $(a_1^2+a_2^2=1)$

Write down $|e,0,0\rangle$ in terms of $|\psi_1^+\rangle, |\psi_1^0\rangle, |\psi_1^-\rangle$

$|g,0,1\rangle$

$$|e,0,0\rangle = x_1|\psi_1^+\rangle + \cancel{x_2|\psi_1^0\rangle} + x_3|\psi_1^-\rangle$$

2x2 matrix

Hamiltonian becomes 2x2

Partially Entangled Bell State Φ_{AB}

$$|\Phi_{AB}\rangle = \cos(\alpha)|e_A,e_B\rangle + \sin(\alpha)|g_A,g_B\rangle$$

$$|\Phi(0)\rangle = |\Phi_{AB}\rangle \otimes |0_{a_1}, 1_{a_2}, 0_{b_1}, 1_{b_2}\rangle$$

$$|\Phi(0)\rangle = \cos\alpha\, |e_A, 0_{a_1}, 1_{a_2}\rangle \otimes |e_B, 0_{b_1}, 1_{b_2}\rangle$$

$$+ \sin\alpha\, |g_A, 0_{a_1}, 1_{a_2}\rangle \otimes |g_B, 0_{a_1}, 1_{a_2}\rangle$$

→ can be found from the 2x2 Hamilton

$$|\Phi(0)\rangle = \cos\alpha \left(x_1|\psi_1^+\rangle_A + x_2|\psi_1^0\rangle_A + x_3|\psi_1^-\rangle_A \right) \otimes \left(x_1|\psi_1^+\rangle_B + x_2|\psi_1^0\rangle_B + x_3|\psi_1^-\rangle_B \right)$$

$$+ \sin\alpha\, |g_A, 0_{a_1}, 0_{a_2}\rangle \otimes |g_B, 0_{b_1}, 0_{b_2}\rangle$$

$d_1|g,0,1\rangle + d_2|e,0,0\rangle$ $\quad e_1\ e_2$ $\quad f_1\ f_2$

$$|\Phi(t)\rangle = \cos\alpha \left(x_1 e^{-i\lambda^+ t}|\psi_2^+\rangle_A + x_2 e^{-i\lambda^0 t}|\psi_2^0\rangle_A + x_3 e^{-i\lambda^- t}|\psi_2^-\rangle_A \right) \otimes$$

$$\left(x_1 e^{-i\lambda^+ t}|\psi_2^+\rangle_B + x_2 e^{-i\lambda^0 t}|\psi_2^0\rangle_B + x_3 e^{-i\lambda^- t}|\psi_2^-\rangle_B \right)$$

$d_1\ d_2$ $\quad e_1\ e_2$ $\quad f_1\ f_2$

$$+ \sin\alpha\, |g_A, 0_{a_1}, 1_{a_2}\rangle \otimes |g_B, 0_{b_1}, 1_{b_2}\rangle$$

$$\left(s\, e^{-i\lambda^+ t}|\psi_1^+\rangle_A + c\, e^{-i\lambda^- t}|\psi_1^-\rangle_A \right) \rightarrow$$ B version of ?

$$
\begin{aligned}
|\Phi(t)\rangle = \cos\alpha \Big[& \Big(\overbrace{x_1 e^{-i\lambda^+ t} d_1 + x_2 e^{-i\lambda^0 t} e_1 + x_3 e^{-i\lambda^- t}}^{W} f_1 \Big) |g, 0, 1\rangle_{A\, a_1\, a_2} + \\
& \Big(\underbrace{x_1 e^{-i\lambda^+ t} d_2 + x_2 e^{-i\lambda^0 t} e_2 + x_3 e^{-i\lambda^- t}}_{T} f_2 \Big) |e, 0, 0\rangle_{A\, a_1\, a_2} \Big] \otimes \\
\Big[& \Big(\underbrace{x_1 e^{-i\lambda^+ t} d_1 + x_2 e^{-i\lambda^0 t} e_1 + x_3 e^{-i\lambda^- t}}_{W} f_1 \Big) |g_B, 0_{b_1}, 1_{b_2}\rangle \\
& \Big(\underbrace{x_1 e^{-i\lambda^+ t} d_2 + x_2 e^{-i\lambda^0 t} e_2 + x_3 e^{-i\lambda^- t}}_{T} f_2 \Big) |e, 0, 0\rangle_{B\, b_1\, b_2} \Big] +
\end{aligned}
$$

$$
\sin\alpha \; |g_A, 0_{a_1}, 0_{a_2}\rangle \otimes |g_B, 0_{b_1}, 0_{b_2}\rangle
$$

$$
\begin{aligned}
|\Phi(t)\rangle = \; & \overbrace{k_1}^{\cos\alpha\, T^2} |e_A, e_B, 0_{a_1}, 0_{a_2}, 0_{b_1}, 0_{b_2}\rangle + \overbrace{k_2}^{\cos\alpha\, W^2} |g_A, g_B, 0_{a_1}, 1_{a_2}, 0_{b_1}, 1_{b_2}\rangle \\
& + \overbrace{k_3}^{\cos\alpha\, TW} |e_A, g_B, 0_{a_1}, 0_{a_2}, 0_{b_1}, 1_{b_2}\rangle + \overbrace{k_4}^{\cos\alpha\, TW} |g_A, e_B, 0_{a_1}, 1_{a_2}, 0_{b_1}, 0_{b_2}\rangle \\
& + \underbrace{k_5}_{\sin\alpha} |g_A, g_B, 0_{a_1}, 0_{a_2}, 0_{b_1}, 0_{b_2}\rangle
\end{aligned}
$$

$C_{AB}(t)$

$$\rho^{AB} = \mathrm{Tr}_{a_1,a_2,b_1,b_2}\left[\,|\Phi(t)\rangle\langle\Phi(t)|\,\right]$$

Tr

$$Q(t) = 2|k_1||k_5| - 2|k_3||k_4|$$

HIGHLIGHT DIFFERENCE BTW 2 photo freq

[HOW CAN WE SHOW THE ~~ESD~~ TIME INCREASE]

CONTINUE EXPAND THE

0

0 1

$$|\psi_1^+\rangle = a_1|g,\overset{0}{n_1},\overset{1}{n_2}\rangle + a_2|e,\overset{0}{n_1},\overset{0}{n_2-1}\rangle + a_3|e,\overset{-1}{\cancel{n_1-1}},n_2\rangle$$

$$|\psi_1^0\rangle = b_1|g,\overset{0}{n_1},\overset{1}{n_2}\rangle + b_2|e,\overset{0}{n_1},\overset{0}{n_2-1}\rangle + b_3|e,\overset{-1}{\cancel{n_1-1}},n_2\rangle$$

$$|\psi_1^-\rangle = c_1|g,\overset{0}{n_1},\overset{1}{n_2}\rangle + c_2|e,\overset{0}{n_1},\overset{0}{n_2-1}\rangle + c_3|e,\cancel{n_1-1},n_2\rangle$$

$(a_1^2+a_2^2+a_3^2)=1$ (due to normalization)

for one-photon case $(a_1^2+a_2^2=1)$

Write down $|e,0,0\rangle$ in terms of $|\psi_1^+\rangle, |\psi_1^0\rangle, |\psi_1^-\rangle$

$$\boxed{|e,0,0\rangle = x_1|\psi_1^+\rangle + \cancel{x_2|\psi_1^0\rangle} + x_3|\psi_1^-\rangle}$$

0

$$|\psi_1^+\rangle \quad c/ \quad \overset{c}{a_1}x + \overset{s}{a_2}y = d_1$$

$$\cancel{b_1x + b_2y = d_2}$$

$$|\psi_1^-\rangle \quad -s/ \quad \underset{-s}{c_1}x + \underset{c}{c_2}y = d_3$$

$[\quad]$

$$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \underset{\sim}{\begin{bmatrix} x \\ y \end{bmatrix}} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

determine $\underline{\underline{\theta}}$

$$x_k = -\frac{1}{3a}\left(b + \xi^k C + \frac{\Delta_0}{\xi^k C}\right), \quad k \in \{0,1,2\}, \quad \xi = \left(\frac{-1+\sqrt{3}i}{2}\right)$$

$$x_0 = -\frac{1}{3a}\left(b + C + \frac{\Delta_0}{C}\right)$$

$$x_1 = -\frac{1}{3a}\left(b + \xi C + \frac{\Delta_0(\xi)}{\xi^2 C}\right)$$

$$x_2 = -\frac{1}{3a}\left(b + \xi^2 C + \frac{\Delta_0}{\xi^2 C}\right)$$

$$\xi^2 = \frac{(-1+\sqrt{3}i)^2}{4}$$

$$\xi^2 = \frac{1+(-3)-2\sqrt{3}i}{4} = \frac{-2-2\sqrt{3}i}{4}$$

$$\xi^2 = \frac{-1-\sqrt{3}i}{2}$$

$$x_1 = -\frac{1}{3a}\left(b + \left(\frac{-1}{2}\right)C\left(+\frac{\sqrt{3}i}{2}\right)C + \frac{\Delta_0}{\left(\frac{-1}{2}\right)C + \left(\frac{\sqrt{3}i}{2}\right)C}\right)$$

Choose another representation:

$$\lambda_n^{\pm} = K(n_1, n_2) \pm Q(n_1, n_2)$$

$$Q(n_1,n_2) = \frac{\sqrt{3}i}{2}\left(\frac{\beta}{\sqrt[3]{\Delta}} + \sqrt[3]{\Delta}\right)$$

$$\frac{\sqrt{3}i}{2\sqrt[3]{\Delta}}\left(\beta + \sqrt[6]{\Delta}\right)$$

$$e^{-i\lambda_1 t}$$

$$e^{-i(-i\phi)} A + e^{-i(+i\phi)} . A$$

$$A(e^{\phi} + e$$

for the case:

$$\hat{H} = \begin{array}{c|ccc} & |g,n_1,n_2\rangle & |e,n_1-1,n_2\rangle & |e,n_1,n_2-1\rangle \\ \hline |g,0,0\rangle & & & \\ & A & X & Y \\ & X & B & 0 \\ & Y & 0 & C \end{array}$$

let's investigate the case $n_1+n_2=1$

Assume $n_1=0$ & $n_2=1$ $\rightarrow \boxed{\{|g,0,1\rangle, |e,0,0\rangle\}}$

$$\begin{array}{c|cc} & |g,0,1\rangle & |e,0,0\rangle \\ \hline \langle g,0,1| & A & X \\ \langle e,0,0| & X & B \end{array}$$

$$H_{at} = \frac{1}{2}\omega_A \sigma_z^A$$

$$H_{cav} = \omega_1 \hat{a}_1^+ \hat{a}_1 + \omega_2 \hat{a}_2^+ \hat{a}_2$$

$$H_{int} = g_1 (\sigma_-^A \hat{a}_1^+ + \sigma_+^A \hat{a}_1)$$
$$+ g_2 (\sigma_-^A \hat{a}_2^+ + \sigma_+^A \hat{a}_2)$$

$$A = \langle g,0,1 | \hat{H} | g,0,1 \rangle = -\frac{1}{2}\omega_A + \cancel{0 \cdot \omega_1} + \omega_2$$

$$B = \langle e,0,0 | \hat{H} | e,0,0 \rangle = +\frac{1}{2}\omega_A + \cancel{0\omega_1} + \cancel{0\omega_2} = \boxed{\frac{1}{2}\omega_A}$$

$$X = \langle g,0,1 | \hat{H} | e,0,0 \rangle = g_2 \cdot (1)$$

$$(\cancel{g_1 \hat{\sigma}_- \hat{a}_1^+}_{0} + g_2 \hat{\sigma}_- \hat{a}_2) | e,0,0 \rangle$$

$$g_2 \sqrt{1} \cdot | g\ 0,1 \rangle$$

for $n_1 = 1 \quad n_2 = 0$

$$\boxed{\{ | g,1,0 \rangle , | e,0 0 \rangle \}}\ ?$$

$$\underset{(n_1=0,\, n_2=1)}{\hat{H}} = \begin{bmatrix} -\frac{1}{2}\omega_A + \omega_2 - \lambda & g_2\sqrt{1} \\ g_2\sqrt{1} & +\frac{1}{2}\omega_A - \lambda \end{bmatrix} \text{ for } n_1 = 0,\ n_2 = 1$$

$$\left(-\frac{1}{2}\omega_A+\omega_2-\lambda\right)\left(\frac{1}{2}\omega_A-\lambda\right)-g_2^2=0$$

$$-\frac{1}{4}\omega_A^2+\frac{1}{2}\omega_A\omega_2-\cancel{\frac{1}{2}\omega_A\lambda}+\cancel{\frac{1}{2}\omega_A\lambda}-\lambda\omega_2+\lambda^2-g_2^2=0$$

$$\lambda^2-\lambda\omega_2+\underbrace{\left(-\frac{1}{4}\omega_A^2+\frac{1}{2}\omega_A\omega_2-g_2^2\right)}_{c}=0$$

$a=1 \qquad b=-\omega_2$

$$\lambda_2^{\pm}=\frac{b\pm\sqrt{\Delta}}{2a}=\frac{-\omega_2\pm\sqrt{\omega_2^2-4\left(-\frac{1}{4}\omega_A^2+\frac{1}{2}\omega_A\omega_2-g_2^2\right)}}{2}$$

$$=-\frac{\omega_2}{2}\pm\sqrt{\underbrace{\omega_2^2+\omega_A^2-2\omega_A\omega_2}_{(\omega_2-\omega_A)^2}+\underbrace{4g_2^2}_{(G_1)^2}}$$

$\omega_2-\omega_A$: δ (detuning)

$$\boxed{G=2g\sqrt{n}}$$

for $\delta=0$ the previous case

$$\boxed{\lambda_1^{\pm}=-\frac{\omega_1}{2}\pm\sqrt{\delta^2+(2g_1)^2}}$$

$$\boxed{\lambda_2^{\pm}=-\frac{\omega_2}{2}\pm\sqrt{\delta^2+(2g_2)^2}}$$

} Two same results occurs with the Section III.