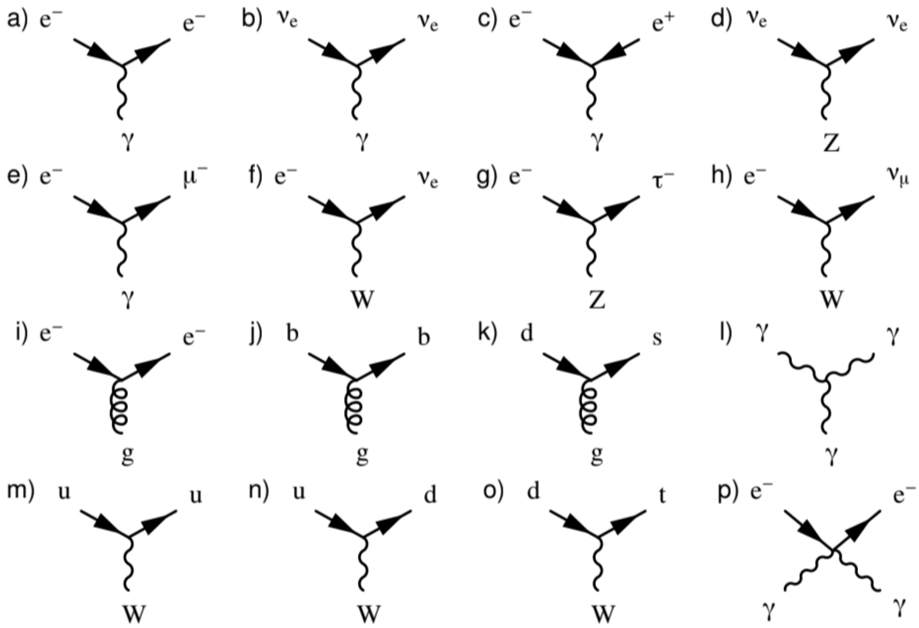
Particle Physics 1: Exercise 1

Exercise 1 - Feynman diagrams



1) State and explain your reasoning whether each of the diagrams below represents a valid Standard Model vertex
2) Draw the Feynman diagram for $\tau^- \to \pi^- \nu_\tau$ (the π^- is the lightest of $u\bar{d}$ meson)
3) Draw the Feynman diagrams for the decays:
    a) $\Delta(uud) \to n(udd)\pi^+(u\bar{d})$,
    b) $\Sigma^0(uds) \to \Lambda(uds)\gamma$,
    c) $\pi^+(u\bar{d}) \to \mu^+\nu_\mu$

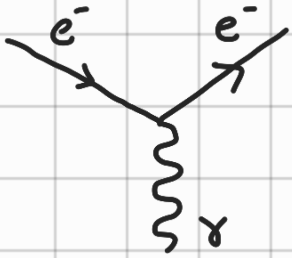
    and place them in order of increasing lifetime

Exercise 2 - Lorentz transformation

1) In a collider experiment, Λ baryons can be identified from the decay $\Lambda \to \pi^- p$ that gives rise to a displaced vertex in a tracking detector. In a particular decay, the momenta of the π^- and p are measured to be 0.75 GeV and 4.25 GeV respectively, and the opening angle between the tracks is 9°. The masses of the pion and proton are 139.6 MeV and 938.3 MeV.
    a) Calculate the mass of the Λ baryon
    b) On average, Λ baryon of this energy are observed to decay at a distance of 0.35 m from the point of the production. Calculate the lifetime of the Λ
2) Find the minimum opening angle between the photons produced in the decay $\pi^0 \to \gamma\gamma$, if the energy of the pion is 10 GeV, given that m_π^0 = 135 MeV

Exercise 1

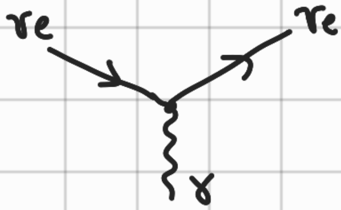
a)



✓ Charged particles

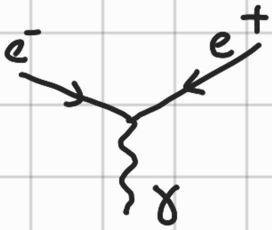
EM → no change of flavour

b)



× Neutrino ν_e has no charge

c)

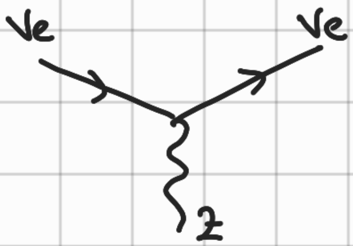


✓ Charged particles

No flavour change

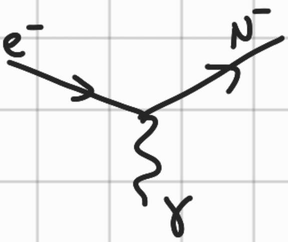
X No charge conservation

d)



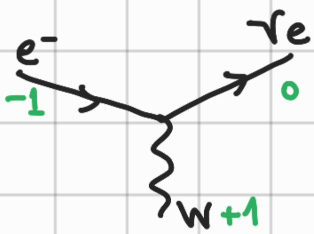
✓ No flavour change

e)



× Flavour has changed (No change)

f)



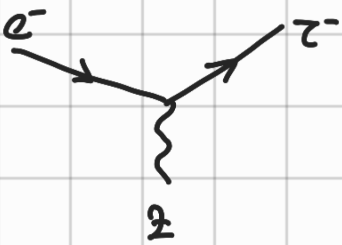
✓ Flavour changed

All fermions

(?)

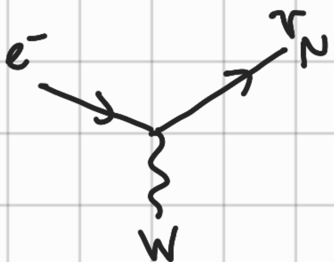
flavour (?)

g)



× No flavour change ($e^- \cancel{\to} \tau^-$)

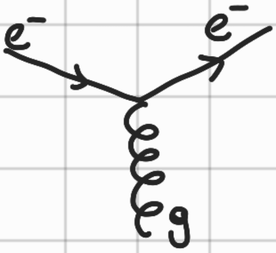
h)



✓ Flavour change !

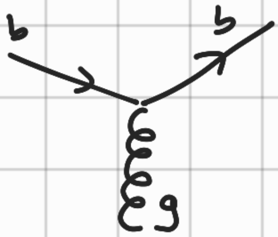
All fermion

X Flavour change only for leptons with corresponding neutrino

i) 

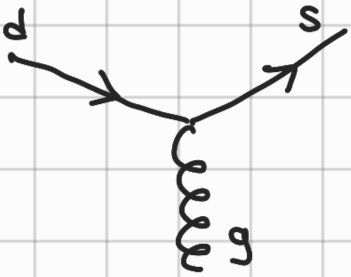
× Only quarks

e^- does not carry colour charge

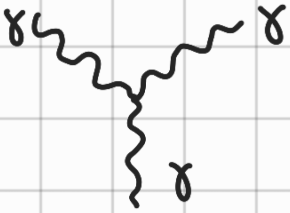
j) 

✓ quarks

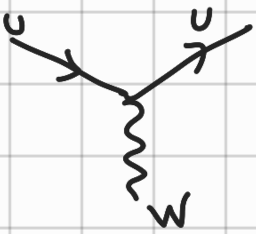
no flavour change

k) 

× No flavour change

l) 

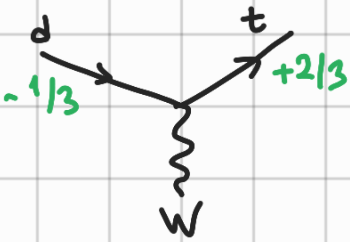
× Forbidden vertices

m) 

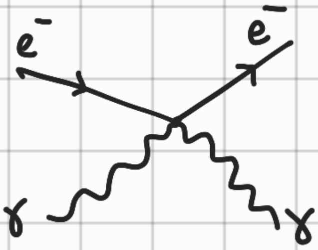
× Flavour must change

n) 

✓ Flavour change

o) 

✓ Flavour change

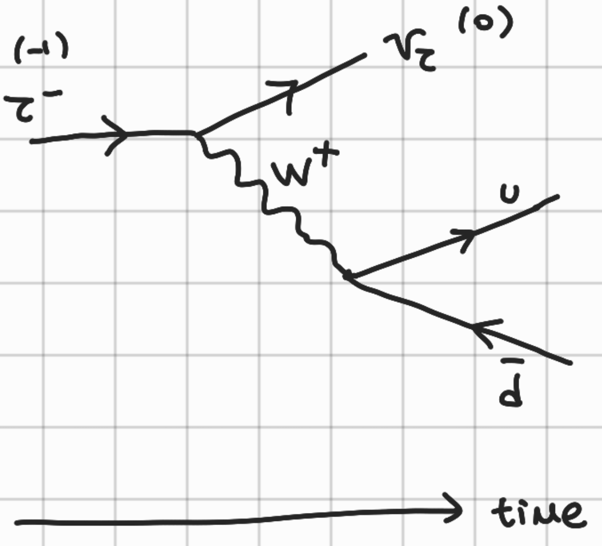
p) 

× No 4 vertices for leptons(?)

All fermion vertices involve a coupling to a single gauge boson.

2) $\tau^- \longrightarrow \pi^- \nu_\tau$ where $\pi^- = u\bar{d}$ (meson)

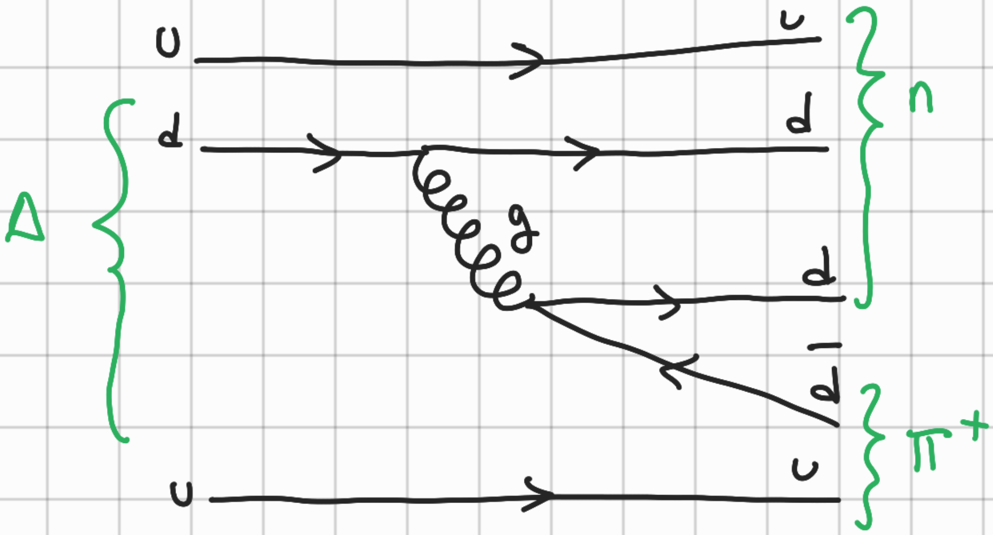
$\tau^- \longrightarrow u + \bar{d} + \nu_\tau$



★ Never have a vertex connecting a lepton to a quark.

3) a) $\Delta(uud) \rightarrow n(udd)\pi^+(u\bar{d})$,

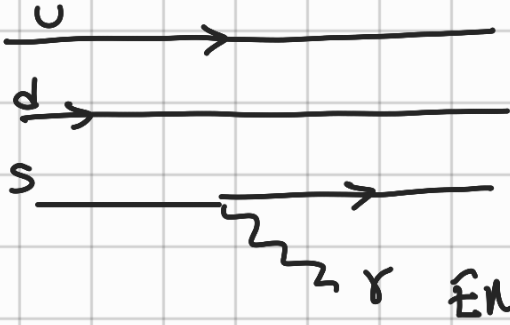
$uud \longrightarrow udd \quad u\bar{d}$



high probability
↑
Happens quickly
due to strong force

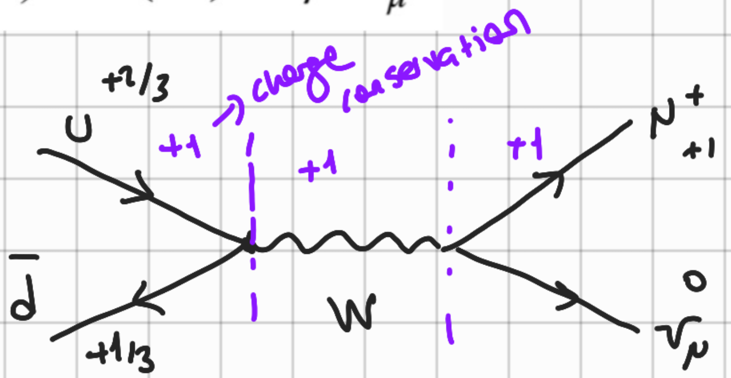
Strong force

b) $\Sigma^0(uds) \to \Lambda(uds)\gamma$,



EM

c) $\pi^+(u\bar{d}) \to \mu^+\nu_\mu$



(Lifetime is high)

Weak

Increasing life time

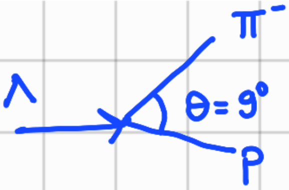
A → B → C

1) In a collider experiment, Λ baryons can be identified from the decay $\Lambda \rightarrow \pi^- p$ that gives rise to a displaced vertex in a tracking detector. In a particular decay, the momenta of the π^- and p are measured to be 0.75 GeV and 4.25 GeV respectively, and the opening angle between the tracks is 9°. The masses of the pion and proton are 139.6 MeV and 938.3 MeV.

a) Calculate the mass of the Λ baryon

b) On average, Λ baryon of this energy are observed to decay at a distance of 0.35 m from the point of the production. Calculate the lifetime of the Λ

$$E^2 = p^2 + m^2$$



$\boxed{E\beta = p}$ corresponding velocities

$\beta_\pi = \frac{E_\pi}{p_\pi}$, $\beta_p = \frac{E_p}{p_p}$

$$E_\pi = \sqrt{p_\pi^2 + m_\pi^2} = 0.763 \text{ GeV}$$

$$E_p = \sqrt{p_p^2 + m_p^2} = 4.352 \text{ GeV}$$

$E(\Lambda) = E(p) + E(\pi)$ Energy Conservation

$p(\Lambda) = p(p) + p(\pi)$ Momentum Conservation

$$m^2(\Lambda) = E(\Lambda)^2 - p(\Lambda)^2$$

$$m_\Lambda^2 = (E_p + E_\pi)^2 - (p_p + p_\pi)^2$$

$\langle p_p p_\pi \rangle = p_p p_\pi \cos\theta$

$$m_\Lambda^2 = E_p^2 + 2E_pE_\pi + E_\pi^2 - (p_p^2 + p_\pi^2 - 2\underbrace{p_p \cdot p_\pi})$$

$$m_\Lambda^2 = m_p^2 + m_\pi^2 + 2E_pE_\pi - 2p_1p_2\cos\theta$$

$$m_\Lambda^2 = m_p^2 + m_\pi^2 + 2E_pE_\pi(1 - \beta_\pi\beta_p\cos\theta) = 1.244 \text{ GeV}^2$$

$$m_\Lambda = 1.115 \text{ GeV}$$

b) Relativistic time dilation

$$d = vt = v\,\underbrace{\gamma\tau} = \beta c \cdot \gamma\tau$$

$p = \gamma m \beta$

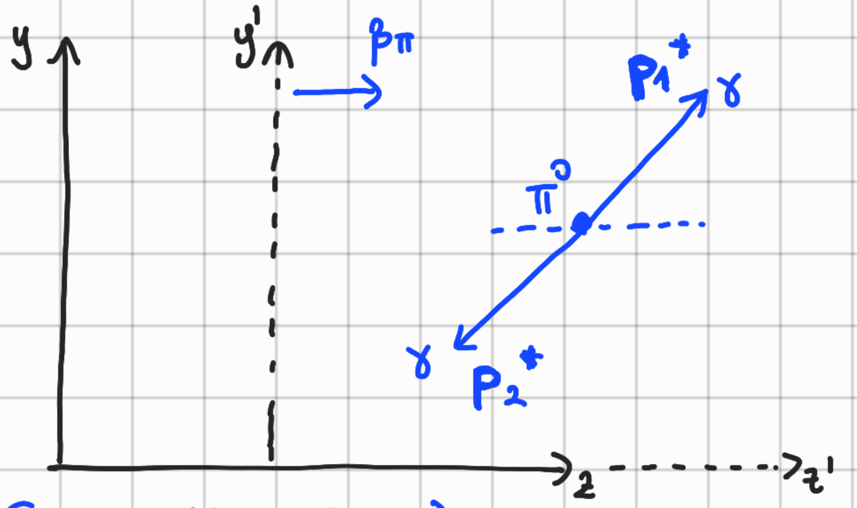
$\gamma\beta = \frac{p}{m}$

$p = \gamma m v$

$p = \gamma m \beta . c \;\; (c=1)$

$$d = \underbrace{\gamma\beta}_{\frac{p}{m}} c\tau \longrightarrow \tau = \frac{d^{\to m}}{\frac{p}{m} c_{\to m/s}} = \dots \text{ s}$$

2) Find the minimum opening angle between the photons produced in the decay $\pi^0 \to \gamma\gamma$, if the energy of the pion is 10 GeV, given that $m_\pi^0 = 135$ MeV



CM Frame (Rest Frame)

How did we determine this?

$$p_1^* = (\underset{E_1^*}{E},\ \underset{p_{x_1}^*}{0},\ \underset{p_{y_1}^*}{E\sin\theta^*},\ \underset{p_{z_1}^*}{E\cos\theta^*})$$

$$p_2^* = (E, 0, -E\sin\theta^*, -E\cos\theta^*)$$

$$E = \frac{m_\pi}{2}$$

To find the Lab Frame ⤳ Exploit Inverse Lorentz Transformation

$$\underbrace{\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}}_{p_1} = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix}$$

$t \to E_1$, $x \to p_x = 0$, $y \to p_y$, $z \to p_z$

$t' \to E = E_1^*$, $x' \to 0 = p_x^*$, $y' \to E\sin\theta^* = p_{y_1}^*$, $z' \to E\cos\theta^* = p_{z_1}^*$

$$E_1 = \gamma E_1^* + \gamma\beta p_{z_1}^* = \gamma E + \gamma\beta E\cos\theta^*$$
$$= \gamma E(1 + \beta\cos\theta^*)$$

$$E_2 = \gamma E_2^* + \gamma\beta p_{z_2}^* = \gamma E(1 - \beta\cos\theta^*)$$

$$p_x = p_x^* \quad , \quad p_y = p_y^*$$

$$p_{z_1} = \gamma\beta \underbrace{E}_{E_1^*} + \gamma \underbrace{E\cos\theta^*}_{p_{z_1}^*} = \gamma E(\beta + \cos\theta^*)$$

$$p_{z_2} = \gamma\beta E_2^* + \gamma p_{z_2}^* = \gamma E(\beta - \cos\theta^*)$$

$E_1 \sin\theta^*$

$E_1 \cos\theta^*$

$\sqrt{E_1^2(s^2+c^2)} = E_1$

$$\cos\theta = \frac{\mathbf{p_1} \cdot \mathbf{p_2}}{p_1 p_2} = \frac{p_{x1}p_{x2} + p_{y1}p_{y2} + p_{z1}p_{z2}}{E_1 E_2}$$

Magnitude

$$= \frac{0 - \cancel{E^2}\sin^2\theta^* + \gamma^2\cancel{E^2}(\beta^2 - \cos^2\theta^*)}{\gamma^2\cancel{E^2}(1-\beta^2\cos^2\theta^*)}$$

$$\left(\gamma = (1-\beta^2)^{-1/2}\right)$$

$$\frac{1}{\gamma^2} = (1-\beta^2)$$

$$= \frac{-\sin^2\theta^*/\gamma^2 + \beta^2 - \cos^2\theta^*}{1-\beta^2\cos^2\theta^*}$$

$$= \frac{-\sin^2\theta^*(1-\beta^2) + \beta^2 - \cos^2\theta^*}{1-\beta^2\cos^2\theta^*}$$

$$\cos\theta = \frac{\beta^2(1+\sin^2\theta^*) - 1}{1-\beta^2\cos^2\theta^*}$$

$$\cos\theta_{min} = \frac{\beta^2(1+1)-1}{1-0} = 2\beta^2 - 1$$

$\cos\theta^* = 0$

$\sin\theta^* = 1$

$$\cos\theta_{min} = \frac{\beta^2(1+0)-1}{1-\beta^2 1} = \frac{\beta^2-1}{1-\beta^2} = -1 \rightsquigarrow \theta = \pi$$

$\cos\theta^* = 1$

$\sin\theta^* = 0$

$\cos\theta_{min} = 2\beta^2 - 1$ where $\beta^2 = 1 - \frac{1}{\gamma^2}$ & $E_\pi = \gamma m_\pi$

10 GeV

135 MeV

74.1

0.9963

$\theta_{min} = 0.027$