05/04/23

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PAIRWISE CONCURRENCE DYNAMICS: A FOUR-QUBIT MODEL

1. JC Hamiltonian

$$H_{tot} = \frac{\omega_0}{2}\sigma_z^A + g\left(a^\dagger\sigma_-^A + \sigma_+^A a\right) + \omega a^\dagger a +$$

$$+ \frac{\omega_0}{2}\sigma_z^B + g\left(b^\dagger\sigma_-^B + \sigma_+^B b\right) + \omega b^\dagger b \quad \text{where } \hbar = 1$$

ω_0: freq. of atom
ω : freq. of cavity

$$H_{JC}\,|\psi_n^\pm\rangle = \lambda_n^\pm\,|\psi_n^\pm\rangle$$

eigenstates (dressed states)

$$\lambda_n^\pm = n\omega + \frac{1}{2}\left(\Delta \pm \sqrt{\Delta^2 + G_n^2}\right)$$

n : photon number

$\Delta = \omega - \omega_0$ (detuning)

$$G_n = \frac{2g}{\sqrt{n}}$$

Dressed states:

$$|\Psi_0\rangle = |g,0\rangle$$

$$\left.\begin{aligned} |\Psi_n^+\rangle &= c_n |e, n-1\rangle + s_n |g,n\rangle \\ |\Psi_n^-\rangle &= -s_n |e, n-1\rangle + c_n |g,n\rangle \end{aligned}\right\} n>0$$

$$c_n = \cos\left(\frac{\theta_n}{2}\right) \qquad , \quad s_n = \sin\left(\frac{\theta_n}{2}\right)$$

where $\cos(\theta_n) = \dfrac{\Delta}{\sqrt{\Delta^2 + G_n^2}}$ $\qquad \sin(\theta_n) = \dfrac{G_n}{\sqrt{\Delta^2 + G_n^2}}$

For this article, we're only dealing with 1 photon

So, dressed states become:

$$|\Psi_0\rangle = |g,0\rangle$$

$$|\Psi_1^+\rangle = \cos\left(\frac{\theta}{2}\right)|e,0\rangle + \sin\left(\frac{\theta}{2}\right)|g,1\rangle$$

$$|\Psi_1^-\rangle = -\sin\left(\frac{\theta}{2}\right)|e,0\rangle + \cos\left(\frac{\theta}{2}\right)|g,1\rangle$$

Two group of pure initial states:

↳ superpositions of Bell states

$$|\Phi_{AB}\rangle = \cos\alpha\,|e_A e_B\rangle + \sin\alpha\,|g_A g_B\rangle$$

$$|\Psi_{AB}\rangle = \cos\alpha\,|e_A g_B\rangle + \sin\alpha\,|g_A e_B\rangle$$

2. Partially Entangled Bell States $|\Phi_{AB}\rangle$

$$|\Phi(0)\rangle = |\Phi_{AB}\rangle \otimes |0_a, 0_b\rangle$$

$$|\Phi(0)\rangle = \left(\cos\alpha\,|e_A, e_B\rangle + \sin\alpha\,|g_A, g_B\rangle\right) \otimes |0_a, 0_b\rangle$$

write them in terms of eigenstates

$$c\,|\Psi_1^+\rangle = c\,|e,0\rangle + s\,|g,1\rangle \qquad s\,|\Psi_1^+\rangle = c\,|e,0\rangle + s\,|g,1\rangle$$

$$-s\,|\Psi_1^-\rangle = -s\,|e,0\rangle + c\,|g,1\rangle \qquad c\,|\Psi_1^-\rangle = -s\,|e,0\rangle + c\,|g,1\rangle$$

$$+ \qquad\qquad\qquad\qquad +$$

$$c|\Psi_1^+\rangle = c^2\,|e,0\rangle + cs\,\cancel{|g,1\rangle} \qquad s|\Psi_1^+\rangle = cs\,\cancel{|e,0\rangle} + s^2\,|g,1\rangle$$

$$-s\,|\Psi_1^-\rangle = +s^2\,|e,0\rangle - cs\,\cancel{|g,1\rangle} \qquad c|\Psi_1^-\rangle = -cs\,\cancel{|e,0\rangle} + c^2\,|g,1\rangle$$

$$+ \qquad\qquad\qquad\qquad +$$

$$|e,0\rangle = c|\Psi_1^+\rangle - s|\Psi_1^-\rangle \qquad |g,1\rangle = s|\Psi_1^+\rangle + c|\Psi_1^-\rangle$$

$$|g_A, 0_a\rangle = |\Psi_0\rangle$$

Thus initial atom-atom entangled state is:

$$|\Phi(0)\rangle = \cos\alpha\, |e_A, 0_a\rangle \otimes |e_B, 0_b\rangle + \sin\alpha\, |g_A, 0_a\rangle \otimes |g_B, 0_b\rangle$$

$$|\Phi(0)\rangle = \cos\alpha \left(c|\psi_1^+\rangle_A - s|\psi_1^-\rangle_A \right) \otimes \left(c|\psi_1^+\rangle_B - s|\psi_1^-\rangle_B \right)$$

$$+ \sin\alpha\, |\psi_0\rangle_A \otimes |\psi_0\rangle_B$$

$$|\psi^\pm(t)\rangle = e^{-i\lambda_\pm t} |\psi^\pm(0)\rangle$$

$$|\Phi(t)\rangle = \cos\alpha \left(e^{-i\lambda_+ t} |\psi_1^+\rangle_A - e^{-i\lambda_- t} |\psi_1^-\rangle_A \right)$$

$$\otimes \left(e^{-i\lambda_+ t} |\psi_1^+\rangle_B - e^{i\lambda_- t} |\psi_1^-\rangle_B \right) + \sin\alpha\, |\psi_0\rangle_A \otimes |\psi_0\rangle_B$$

Here, $|\psi^\pm\rangle$ states refer to the states at $t=0$

Also c_0, s_0 refer to $c(t=0)$ and $s(t=0)$

Now to take partial trace over individual atoms or cavities, one need to REVERT to the BARE bases

$|e_A, 0_A\rangle$, $|g_A, 1_a\rangle$

$|e_B, 0_b\rangle$, $|g_B, 1_b\rangle$

$|g_A, 0_a\rangle$, $|g_B, 0_b\rangle$

$$
\begin{aligned}
|\Phi(t)\rangle = \cos\alpha \Big(& e^{-i\lambda_+ t}\left(c_0 |e_A, 0_a\rangle + s_0 |g_A, 1_a\rangle\right) \\
& - e^{-i\lambda_- t}\left(-s_0 |e_A, 0_a\rangle + c_0 |g_A, 1_a\rangle\right)\Big) \otimes \\
& \Big(e^{-i\lambda_+ t}\left(c_0 |e_B, 0_b\rangle + s_0 |g_B, 0_b\rangle\right) \\
& - e^{-i\lambda_- t}\left(-s_0 |e_B, 0_b\rangle + c_0 |g_B, 1_b\rangle\right)\Big) \\
& + \sin\alpha \, |g_A, 0_a\rangle \otimes |g_B, 0_b\rangle
\end{aligned}
$$

$$
\begin{aligned}
|\Phi(t)\rangle = \cos\alpha \Big(& \left(e^{-i\lambda_+ t} c_0 + e^{-i\lambda_- t} s_0\right) \overset{\uparrow\ 0}{|e_A, 0_a\rangle} + \left(e^{-i\lambda_+ t} s_0 - e^{-i\lambda_- t} c_0\right) \overset{\downarrow\ 1}{|g_A, 1_a\rangle}\Big) \\
\otimes \Big(& \left(e^{-i\lambda_+ t} c_0 + e^{-i\lambda_- t} s_0\right) \overset{\uparrow\ 0}{|e_B, 0_b\rangle} + \left(e^{-i\lambda_+ t} s_0 - e^{-i\lambda_- t} c_0\right) \overset{\downarrow\ 1}{|g_B, 1_b\rangle}\Big) \\
& + \sin\alpha \, \underset{\downarrow\ 0}{|g_A, 0_a\rangle} \otimes \underset{\downarrow\ 0}{|g_B, 0_b\rangle}
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
|\Phi(t)\rangle = \; & x_1 |\uparrow\uparrow 00\rangle + x_2 |\downarrow\downarrow 11\rangle + x_3 |\uparrow\downarrow 01\rangle + x_4 |\downarrow\uparrow 10\rangle \\
& + x_5 |\downarrow\downarrow 00\rangle
\end{aligned}
}
$$

$$x_1 = \left(\overset{L}{c_0}\, e^{-i\lambda_+ t} + \overset{M}{s_0}\, e^{-i\lambda_- t}\right)^2 \cos\alpha$$

$$x_2 = \left(\overset{\sqrt{LM}}{s_0}\, e^{-i\lambda_+ t} - \overset{\sqrt{LM}}{c_0}\, e^{-i\lambda_- t}\right)^2 \cos\alpha$$

$$\left.\begin{aligned} x_3 &= \left(\overset{L}{c_0}\, e^{-i\lambda_+ t} + \overset{M}{s_0}\, e^{-i\lambda_- t}\right)\left(e^{-i\lambda_+ t} \underset{N}{s_0} - e^{-i\lambda_- t} \underset{N}{c_0}\right) \cos\alpha \\ x_4 &= \left(\overset{L}{c_0}\, e^{-i\lambda_+ t} + \overset{M}{s_0}\, e^{-i\lambda_- t}\right)\left(e^{-i\lambda_+ t} \underset{N}{s_0} - e^{-i\lambda_- t} \underset{N}{c_0}\right) \cos\alpha \end{aligned}\right\} x_3 = x_4$$

$$x_5 = \sin\alpha$$

Question 1: why didn't we use $\cos\left(\frac{\theta}{2}\right)$

$$L = \frac{1}{2}\left(1 + \frac{\Delta}{\sqrt{\Delta^2 + G^2}}\right) = \frac{1}{2}\left(1 + \cos(\theta)\right) = \cos^2\left(\frac{\theta}{2}\right)$$

$$M = \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2 + G^2}}\right) = \frac{1}{2}\left(1 - \cos(\theta)\right) = \sin^2\left(\frac{\theta}{2}\right)$$

$$N = \frac{1}{2}\,\frac{G}{\sqrt{\Delta^2 + G^2}} = \frac{1}{2}\sin(\theta) = \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \sqrt{LM}$$

2.1 $C_{AB}(t)$

$$\rho^{AB} = \mathrm{Tr}_{ab}\left[\,|\Phi(t)\rangle\langle\Phi(t)|\,\right]$$

$$\mathrm{Tr}_{0_a0_b} \Rightarrow |x_1|^2\,|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |x_5|^2\,|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$$
$$x_1 x_5^*\,|\uparrow\uparrow\rangle\langle\downarrow\downarrow| + x_1^* x_5\,|\downarrow\downarrow\rangle\langle\uparrow\uparrow|$$

$$\mathrm{Tr}_{0_a1_b} \Rightarrow |x_3|^2\,|\uparrow\downarrow\rangle\langle\uparrow\downarrow|$$

$$\mathrm{Tr}_{1_a0_b} \Rightarrow |x_4|^2\,|\downarrow\uparrow\rangle\langle\downarrow\uparrow|$$

$$\mathrm{Tr}_{1_a1_b} \Rightarrow |x_2|^2\,|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$$

In the form of X

$\langle\uparrow\uparrow| \rightarrow (1\ 0\ 0\ 0)$

$\langle\uparrow\downarrow| \rightarrow (0\ 1\ 0\ 0)$

$\langle\downarrow\uparrow| \rightarrow (0\ 0\ 1\ 0)$

$\langle\downarrow\downarrow| \rightarrow (0\ 0\ 0\ 1)$

$$\begin{array}{c c} & \begin{array}{cccc} \langle\uparrow\uparrow| & \langle\uparrow\downarrow| & \langle\downarrow\uparrow| & \langle\downarrow\downarrow| \end{array} \\ \begin{array}{c} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{array} & \begin{bmatrix} |x_1|^2 & & & x_1 x_5^* \\ & |x_3|^2 & & \\ & & |x_4|^2 & \\ x_1 x_5^* & & & |x_2|^2 + |x_5|^2 \end{bmatrix} \end{array}$$

For X Matrices : (with only diagonal or anti-diagonal) elements

$$\rho^{AB} = \begin{bmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{bmatrix}$$

From Supporting Material

$$C(\rho) = 2\max[0, Q(t)] = 2\max[0, |z| - \sqrt{ad}, |w| - \sqrt{bc}]$$

Thus $\boxed{C^{AB}(t) = 2\,|x_1||x_5| - 2\,|x_3||x_4|}$

$$|x_1| = \left|\left(\overset{L}{c_0}\, e^{-i\lambda_+ t} + \overset{M}{s_0}\, e^{-i\lambda_- t}\right)^2 \cos\alpha\right| \qquad |x_5| = |\sin\alpha|$$

$$|x_3| = \left|\left(\overset{L}{c_0}\, e^{-i\lambda_+ t} + \overset{M}{s_0}\, e^{-i\lambda_- t}\right)\left(e^{-i\lambda_+ t}\, \underset{N}{s_0} - e^{-i\lambda_- t}\, \underset{N}{c_0}\right) \cos\alpha\right|$$

$$|x_3| = |x_4|$$

For the case $\Delta = 0$: "TUNED CASE"

$$\left.\begin{aligned} L &= \frac{1}{2} \\ M &= \frac{1}{2} \\ N &= \frac{G}{2\sqrt{0+G^2}} = \frac{1}{2} \end{aligned}\right\} \quad \begin{aligned} &L = M = N = \frac{1}{2} \\ &\delta = \lambda^+ - \lambda^- = \sqrt{\cancel{\Delta^2} + G^2} = G \end{aligned}$$

$$\lambda^{\pm} = \overset{\text{global factor}}{\nu} + \cancel{\frac{\Delta}{2}} \pm \frac{\sqrt{\cancel{\Delta^2} + G^2}}{2} = \nu \pm \frac{G}{2}$$

$$|x_1| = \left| \left(\frac{1}{2} e^{-i\lambda_+ t} + \frac{1}{2} e^{-i\lambda_- t} \right)^2 \cos\alpha \right|$$

$$= \left| \frac{1}{4} \left(e^{-i\lambda_+ t} + e^{-i\lambda_- t} \right)^2 \cos\alpha \right|$$

$$= \frac{1}{4} |\cos\alpha| \left| \left(e^{-i\lambda_+ t} + e^{-i\lambda_- t} \right) \right|^2$$

$$= \frac{1}{4} |\cos\alpha| \left| \left(\cancel{e^{-i\nu t}} e^{-i\frac{G}{2}t} + \cancel{e^{-i\nu t}} e^{+i\frac{G}{2}t} \right) \right|^2$$

$$= \frac{1}{4} |\cos\alpha| \left(2\cos\left(\frac{G}{2}t\right) \right)^2 = |\cos\alpha| \cos^2\left(\frac{Gt}{2}\right)$$

$$|x_1||x_5| = \cancel{\frac{1}{4}} \underbrace{|\cos\alpha||\sin\alpha|}_{\frac{1}{2}|\sin 2\alpha|} \cancel{4} \cos^2\left(\frac{Gt}{2}\right) = \frac{1}{2} |\sin 2\alpha| \cos^2\left(\frac{Gt}{2}\right)$$

$$|x_3| = \left| \left(\overset{L}{c_0} e^{-i\lambda_+ t} + \overset{M}{s_0} e^{-i\lambda_- t} \right) \left(e^{-i\lambda_+ t} \underset{N}{s_0} - e^{-i\lambda_- t} \underset{N}{c_0} \right) \cos\alpha \right|$$

$$|x_3| = \cancel{\frac{1}{4}} \, \underbrace{\left| e^{-i\lambda_+ t} + e^{-i\lambda_- t} \right|}_{\left|\cancel{2}\cos\left(\frac{\delta t}{2}\right)\right|} \, \underbrace{\left| e^{-i\lambda_+ t} - e^{-i\lambda_- t} \right|}_{\left|\cancel{-2i}\sin\left(\frac{\delta t}{2}\right)\right|} \, |\cos\alpha|$$

$$|x_3| = \left|\cos\left(\frac{\delta t}{2}\right)\right| \left|\sin\left(\frac{\delta t}{2}\right)\right| |\cos\alpha| = |x_4|$$

$$|x_3||x_4| = \cos^2\left(\frac{\delta t}{2}\right) \sin^2\left(\frac{\delta t}{2}\right) \cos^2\alpha$$

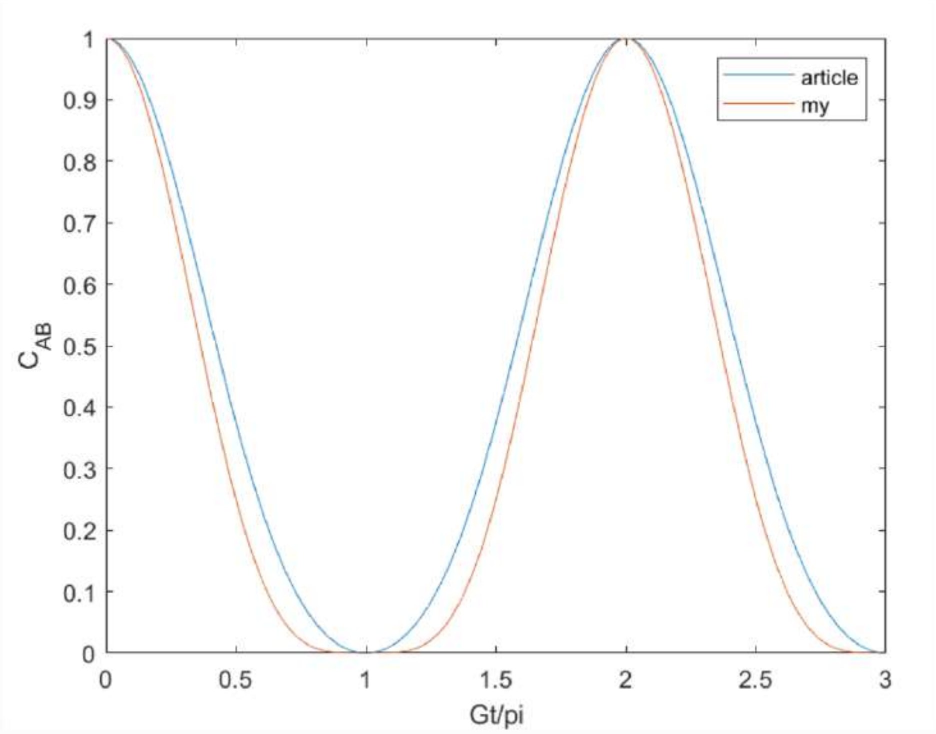
$$Q(t) = 2\left[|x_1||x_5| - |x_3||x_4| \right]$$

$$= |\sin 2\alpha| \cos^2\left(\frac{\delta t}{2}\right) - 2\cos^2\left(\frac{\delta t}{2}\right) \sin^2\left(\frac{\delta t}{2}\right) \cos^2\alpha$$

$$= \cos^2\left(\frac{\delta t}{2}\right) \left[|\sin 2\alpha| - \boxed{2} \sin^2\left(\frac{\delta t}{2}\right) \cos^2\alpha \right]$$

I have extra "2" here!

MATLAB Results:

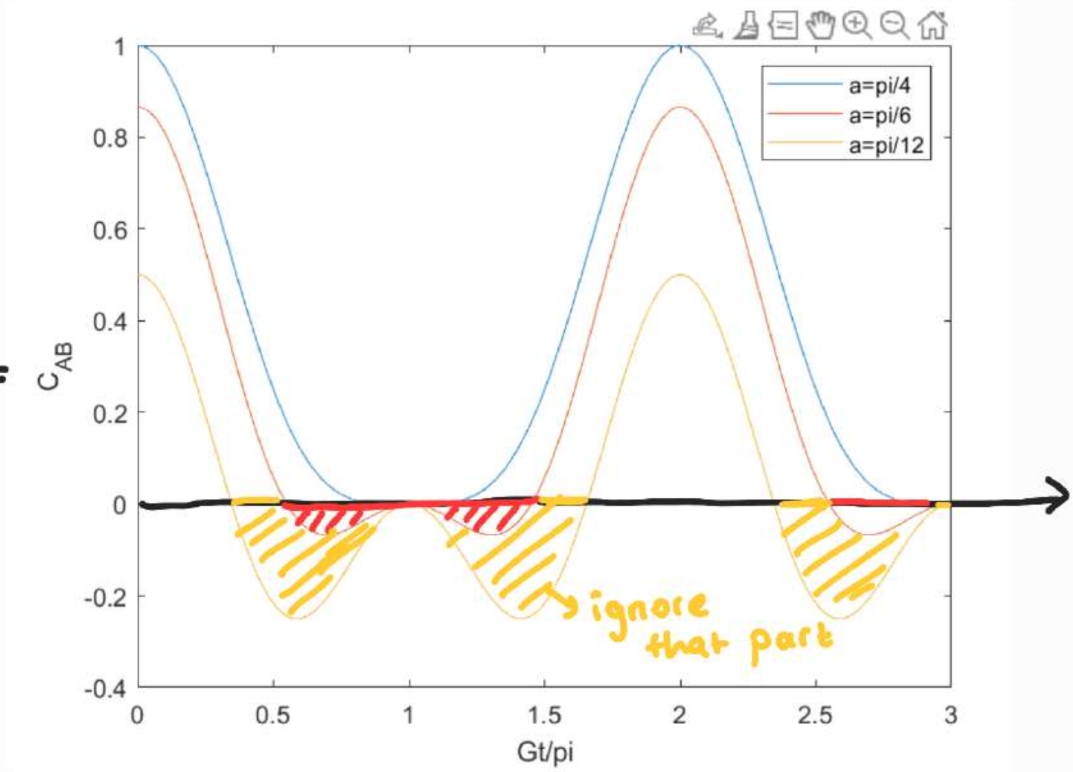


$\Delta = 0$

$\alpha = \frac{\pi}{4}$

From this result, there should have been "2" in the article.

$Q(t) =$



2.2 $C_{ab}(t)$

$$\rho^{ab} = \mathrm{Tr}_{AB}\left[\,|\Phi(t)\rangle\langle\Phi(t)|\,\right]$$

$$\Phi(t) = x_1\,|\uparrow\uparrow 00\rangle + x_2\,|\downarrow\downarrow 11\rangle + x_3\,|\uparrow\downarrow 01\rangle + x_4\,|\downarrow\uparrow 10\rangle$$

$$+\,x_5\,|\downarrow\downarrow 00\rangle$$

$$\mathrm{Tr}_{\uparrow_A\uparrow_B} \rightarrow |x_1|^2\,|00\rangle\langle 00|$$

$$\mathrm{Tr}_{\uparrow_A\downarrow_B} \rightarrow |x_3|^2\,|01\rangle\langle 01|$$

$$\mathrm{Tr}_{\downarrow_A\uparrow_B} \rightarrow |x_4|^2\,|10\rangle\langle 10|$$

$$\mathrm{Tr}_{\downarrow_A\downarrow_B} \rightarrow |x_2|^2\,|11\rangle\langle 11| + |x_5|^2\,|00\rangle\langle 00|$$

$$+\,x_2 x_5^*\,|11\rangle\langle 00| + x_5 x_2^*\,|00\rangle\langle 11|$$

$$\rho^{ab} = \begin{array}{c} \\ |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array}\begin{array}{c} \begin{array}{cccc} \langle 00| & \langle 01| & \langle 10| & \langle 11| \end{array} \\ \begin{bmatrix} |x_1|^2+|x_5|^2 & & & x_2^* x_5 \\ & |x_3|^2 & & \\ & & |x_4|^2 & \\ x_2 x_5^* & & & |x_2|^2 \end{bmatrix} \end{array}$$

$$C^{ab} = 2\max\{0,\ Q(t)\}$$

$$Q(t) = |w| - \sqrt{bc} \quad = |x_2||x_5| - |x_3||x_4|$$

$$\boxed{C^{ab} = 2|x_2||x_5| - 2|x_3||x_4|}$$

For $\Delta = 0$ TUNED CASE

$$|x_2| = \left|\frac{1}{2}\cdot\frac{1}{2}\left(e^{-i\lambda^+ t} - e^{-i\lambda^- t}\right)^2\right| |\cos\alpha|$$

$$= \cancel{\frac{1}{4}}\left|\cancel{-2i}\sin\left(\frac{6t}{2}\right)\right|^2 |\cos\alpha| = \sin^2\left(\frac{6t}{2}\right)|\cos\alpha|$$

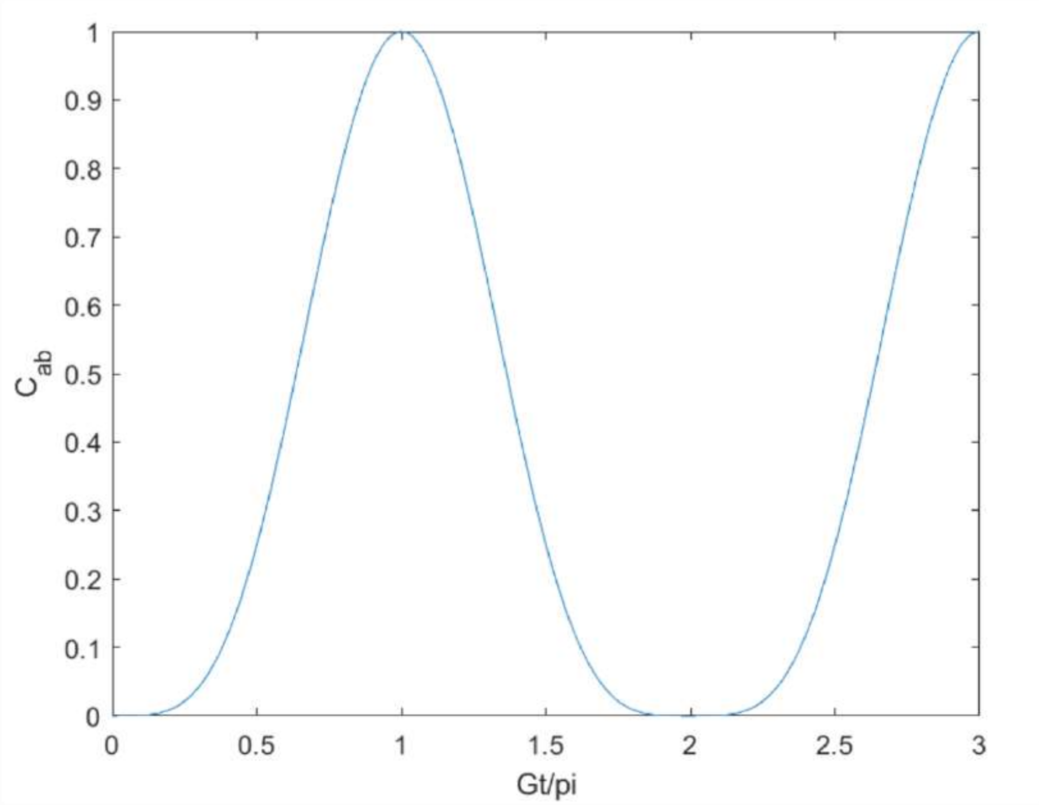
$$|x_5| = |\sin\alpha|$$

$$|x_3| = \left|\cos\left(\frac{6t}{2}\right)\right| \left|\sin\left(\frac{6t}{2}\right)\right| |\cos\alpha| = |x_4|$$

$$|x_3||x_4| = \cos^2\left(\frac{6t}{2}\right)\sin^2\left(\frac{6t}{2}\right)\cos^2\alpha$$

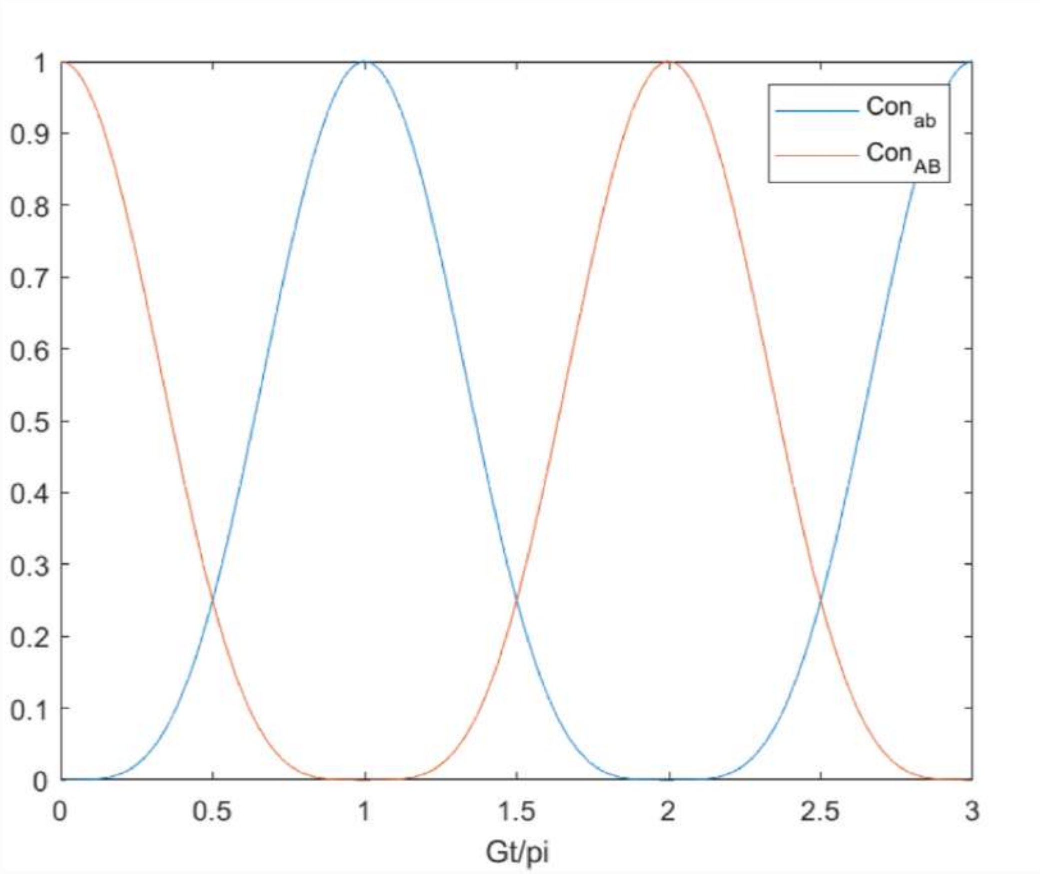
$$C^{ab} = \cancel{2}\sin^2\left(\frac{6t}{2}\right)\underbrace{|\sin\alpha||\cos\alpha|}_{|\sin 2\alpha|} - 2\sin^2\left(\frac{6t}{2}\right)\cos^2\left(\frac{6t}{2}\right)\cos^2\alpha$$

$$C^{ab} = \sin^2\left(\frac{6t}{2}\right)\left[|\sin 2\alpha| - 2\cos^2\left(\frac{6t}{2}\right)\cos^2\alpha\right]$$



$\Delta = 0$

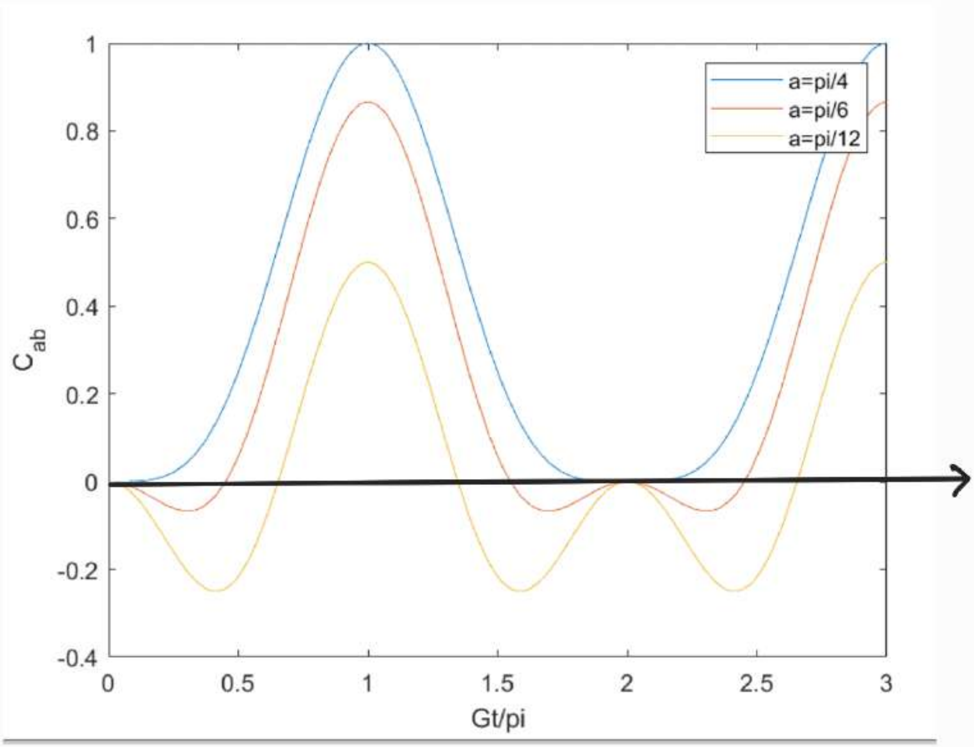
$\alpha = \frac{\pi}{4}$



$\Delta = 0$

$\alpha = \frac{\pi}{4}$

Loss of atom-atom entanglement is compensated by concurrence gain in the photon-photon space. This loss and gain may be interpreted as entanglement transfer between atomic and photonic variables.



$$\text{If } Q(t) < 0 \rightsquigarrow C_{ab} = 0$$

2.3 $C_{Ab}(t)$

$$\rho^{Ab} = \mathrm{Tr}_{aB}\left[|\Phi(t)\rangle\langle\Phi(t)| \right]$$

$$\Phi(t) = x_1 |\uparrow\uparrow 00\rangle + x_2 |\downarrow\downarrow 11\rangle + x_3 |\uparrow\downarrow 01\rangle + x_4 |\downarrow\uparrow 10\rangle$$
$$+ x_5 |\downarrow\downarrow 00\rangle$$

$$\mathrm{Tr}_{0\uparrow} \rightarrow |x_1|^2 |\uparrow 0\rangle\langle\uparrow 0|$$

$$\mathrm{Tr}_{0\downarrow} \rightarrow |x_5|^2 |\downarrow 0\rangle\langle\downarrow 0| + |x_3|^2 |\uparrow 1\rangle\langle\uparrow 1|$$
$$+ x_3 x_5^* |\uparrow 1\rangle\langle\downarrow 0| + x_3^* x_5 |\downarrow 0\rangle\langle\uparrow 1|$$

$$\mathrm{Tr}_{1\uparrow} \rightarrow |x_4|^2 \, |\downarrow 0 \rangle\langle \downarrow 0|$$ ✓

$$\mathrm{Tr}_{1\downarrow} \rightarrow |x_2|^2 \, |\downarrow 1 \rangle\langle \downarrow 1|$$

$$\rho^{Ab} = \begin{array}{c|cccc} & \langle\uparrow 0| & \langle\uparrow 1| & \langle\downarrow 0| & \langle\downarrow 1| \\ \hline |\uparrow 0\rangle & |x_1|^2 & & & \\ |\uparrow 1\rangle & & |x_3|^2 & x_3 x_5^* & \\ |\downarrow 0\rangle & & x_3^* x_5 & |x_5|^2 + |x_4|^2 & \\ |\downarrow 1\rangle & & & & |x_2|^2 \end{array}$$

$$C_{Ab}(t) = 2\left(|z| - \sqrt{ad}\right)$$

$$C_{Ab}(t) = 2\,|x_3||x_5| - 2|x_1||x_2|$$

$$|x_1| = |\cos\alpha| \cos^2\left(\frac{6t}{2}\right) \qquad |x_3| = \left|\cos\left(\frac{6t}{2}\right)\right| \left|\sin\left(\frac{6t}{2}\right)\right| |\cos\alpha|$$

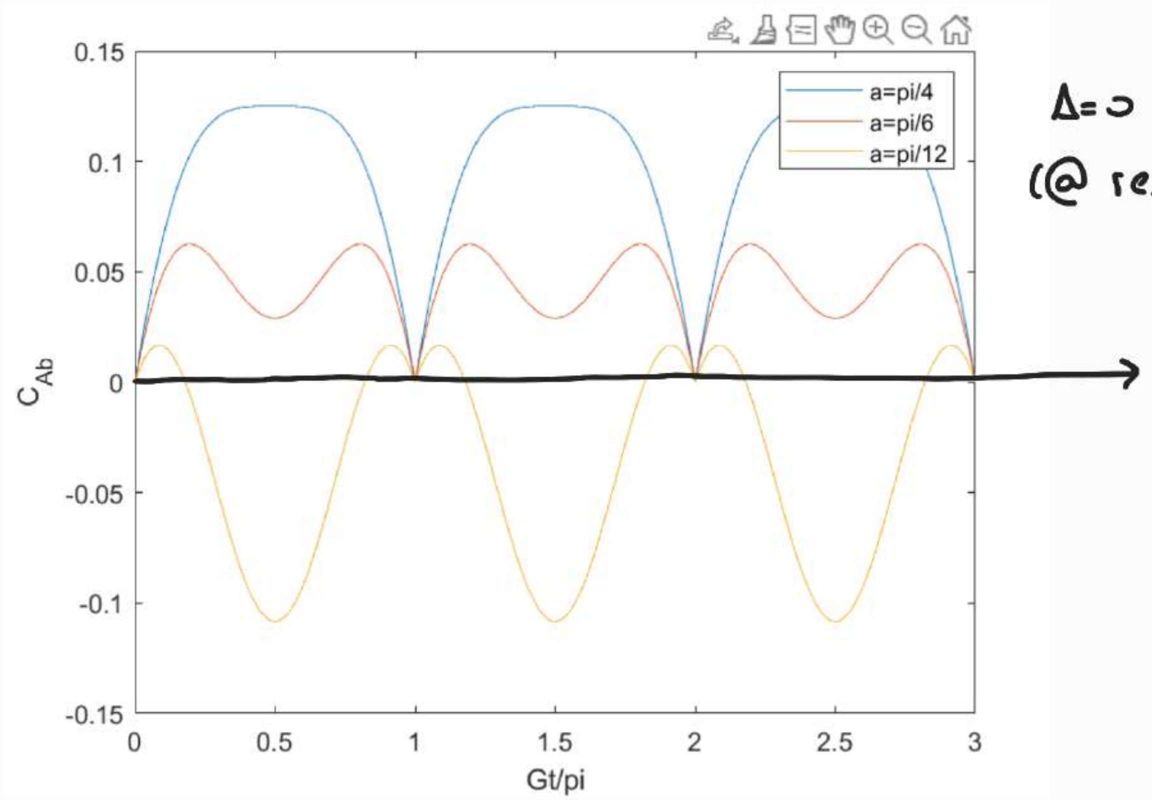
$$|x_2| = \sin^2\left(\frac{6t}{2}\right)|\cos\alpha| \qquad |x_5| = |\sin\alpha|$$

$$C_{Ab}(t) = 2\cdot\frac{1}{2}\,|\sin(Gt)|\,|\cos\alpha|\,|\sin\alpha| - 2\cos^2\alpha\,\cos^2\left(\frac{Gt}{2}\right)\sin^2\left(\frac{Gt}{2}\right)$$

$$C_{Ab}(t) = \frac{1}{2}|\sin(Gt)|\,\sin(2\alpha) - \frac{1}{2}\cos^2\alpha\,\sin^2(Gt)$$

$$= \frac{1}{2}|\sin(Gt)|\left[\sin(2\alpha) - \cos^2\alpha\,|\sin(Gt)|\right]$$

$$= \frac{1}{4}\cos^2\alpha\,|\sin(Gt)|\left(2|\tan\alpha| - |\sin Gt|\right)$$



$\Delta = 0$
(@ resonance)

C_{Ab} is always less than 1 so the parts A and b cannot be maximally entangled unlike parts A and B (or a and b)

2.4 $C_{aB}(t)$

$$\rho^{aB} = \mathrm{Tr}_{Ab}\left[\, |\Phi(t)\rangle\langle\Phi(t)| \,\right]$$

$$\Phi(t) = x_1 |\uparrow\uparrow 00\rangle + x_2 |\downarrow\downarrow 11\rangle + x_3 |\uparrow\downarrow 01\rangle + x_4 |\downarrow\uparrow 10\rangle + x_5 |\downarrow\downarrow 00\rangle$$

$$\mathrm{Tr}_{\uparrow 0} \rightarrow |x_1|^2 \, |\underset{B\ a}{\uparrow 0}\rangle\langle\uparrow 0|$$

$$\mathrm{Tr}_{\uparrow 1} \rightarrow |x_3|^2 \, |\downarrow 0\rangle\langle\downarrow 0|$$

$$\mathrm{Tr}_{\downarrow 0} \rightarrow |x_4|^2 \, |\uparrow 1\rangle\langle\uparrow 1| + |x_5|^2 \, |\downarrow 0\rangle\langle\downarrow 0| + x_4 x_5^* \, |\uparrow 1\rangle\langle\downarrow 0| + x_5 x_4^* \, |\downarrow 0\rangle\langle\uparrow 1|$$

$$\mathrm{Tr}_{\downarrow 1} \rightarrow |x_2|^2 \, |\downarrow 1\rangle\langle\downarrow 1|$$

$$\begin{array}{c|cccc}
 & \downarrow 0 & \downarrow 1 & \uparrow 0 & \uparrow 1 \\
\hline
\downarrow 0 & |x_3|^2 + |x_5|^2 & & & x_5 x_4^* \\
\downarrow 1 & & |x_2|^2 & & \\
\uparrow 0 & & & |x_1|^2 & \\
\uparrow 1 & x_4 x_5^* & & & |x_4|^2
\end{array}$$

$$C_{aB}(t) = 2\max\left\{0,\ |w| - \sqrt{bc},\ |\tilde{\mathring{z}}| - \cancel{\sqrt{ad}}\right\}$$

$$C_{aB}(t) = 2\left(|x_4||x_5| - |x_1||x_2|\right)$$

From the previous result of C_{Ab}

$$\boxed{C_{Ab}(t) = C_{aB}(t)}$$

Since the initial state $|\Phi(0)\rangle = |\underbrace{\Phi_{AB}}_{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}\rangle \otimes |0_a, 0_b\rangle$

$A \leftrightarrow B$, $a \leftrightarrow b$ the state remains unchanged.

2.5 $C_{Aa}(t)$

$$\rho^{Aa} = \mathrm{Tr}_{Bb}\left[\,|\Phi(t)\rangle\langle\Phi(t)|\,\right]$$

$$\Phi(t) = x_1\,|\uparrow\uparrow 00\rangle + x_2\,|\downarrow\downarrow 11\rangle + x_3\,|\uparrow\downarrow 01\rangle + x_4\,|\downarrow\uparrow 10\rangle$$

$$+ x_5\,|\downarrow\downarrow 00\rangle$$

$$\mathrm{Tr}_{\downarrow 0} \rightarrow |x_5|^2 \, |\uparrow 0\rangle\langle \uparrow 0|$$

$$\mathrm{Tr}_{\downarrow 1} \rightarrow |x_2|^2 \, |\downarrow 1\rangle\langle \downarrow 1| + |x_3|^2 \, |\uparrow 0\rangle\langle \uparrow 0| + x_2 x_3^* \, |\downarrow 1\rangle\langle \uparrow 0| + x_2^* x_3 \, |\uparrow 0\rangle\langle \downarrow 1|$$

$$\mathrm{Tr}_{\uparrow 0} \rightarrow |x_4|^2 \, |\downarrow 1\rangle\langle \downarrow 1| + |x_1|^2 \, |\uparrow 0\rangle\langle \uparrow 0| + x_4 x_1^* \, |\downarrow 1\rangle\langle \uparrow 0| + x_1 x_4^* \, |\uparrow 0\rangle\langle \downarrow 1|$$

$$\mathrm{Tr}_{\uparrow 1} \rightarrow 0$$

$$\begin{array}{c c}
 & \begin{array}{cccc} \langle \downarrow 0| & \langle \downarrow 1| & \langle \uparrow 0| & \langle \uparrow 1| \end{array} \\
\begin{array}{c} |\downarrow 0\rangle \\ |\downarrow 1\rangle \\ |\uparrow 0\rangle \\ |\uparrow 1\rangle \end{array} &
\begin{bmatrix}
|x_5|^2 & & & \\
 & |x_2|^2 + |x_4|^2 & x_2 x_3^* + x_4 x_1^* & \\
 & x_2^* x_3 + x_4^* x_1 & |x_3|^2 + |x_1|^2 & \\
 & & & 0
\end{bmatrix}
\end{array}$$

no imaginary parts

$$C_{Aa}(t) = 2\left(|z| - \sqrt{ad}\right)$$

$$= 2\left(|x_2 x_3 + x_4 x_1| - \cancel{|x_5| \cdot 0}\right)$$

$$= 2\,|x_3|\left(|x_2 + x_1|\right)$$

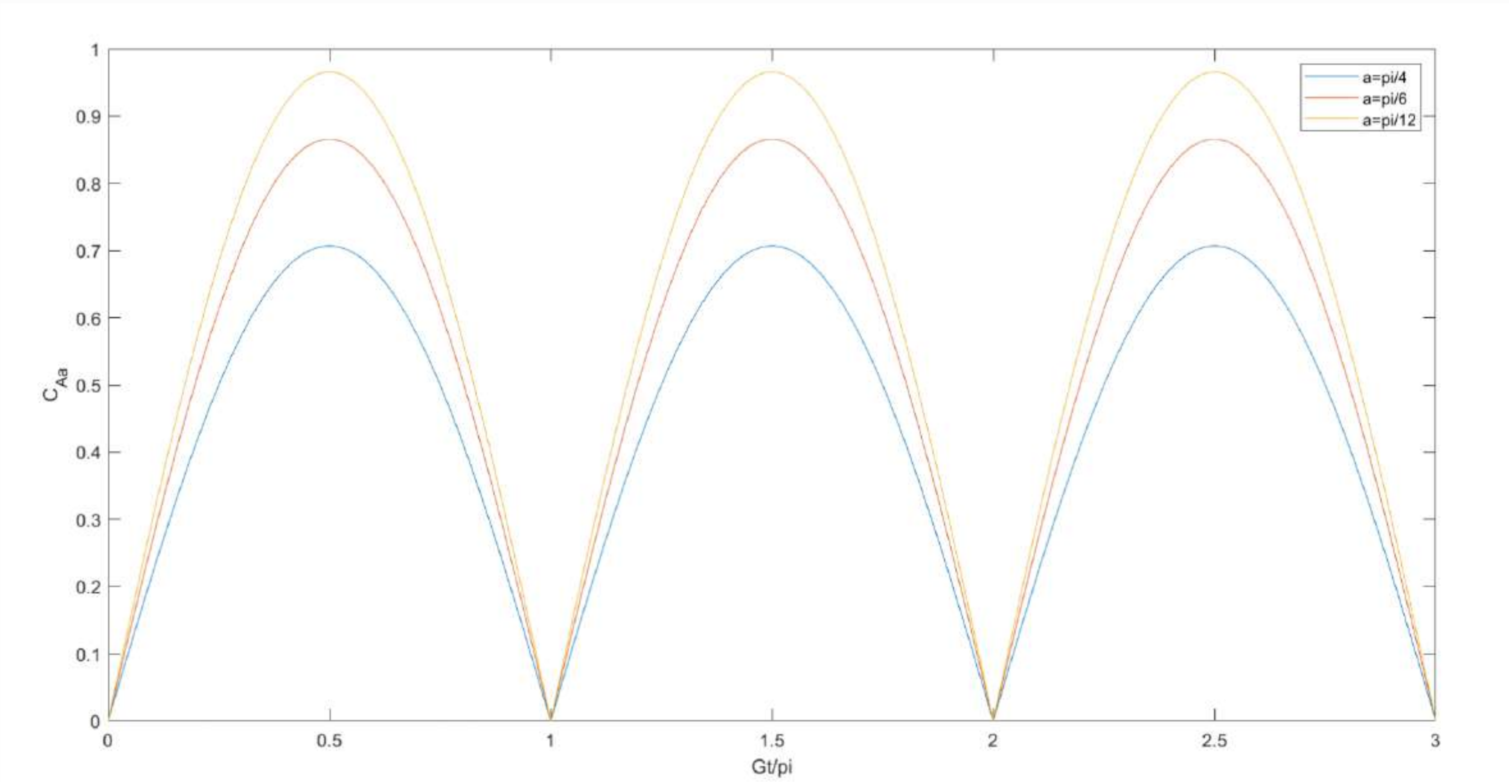
$$x_1 = \cos\alpha \cos^2\left(\frac{Gt}{2}\right) \qquad x_3 = \cos\left(\frac{Gt}{2}\right)\sin\left(\frac{Gt}{2}\right)\cos\alpha = x_4$$

$$x_2 = \sin^2\left(\frac{Gt}{2}\right)\cos\alpha$$

$$+$$

$$x_1 + x_2 = \cos\alpha$$

$$C_{Aa}(t) = \cancel{2}\cdot\frac{1}{\cancel{2}}\left|\sin(Gt)\right|\cos^2\alpha$$



! Here, decreasing alpha yields higher concurrence

2.6 $C_{Bb}(t)$

One can directly say that $C_{Bb}(t) = C_{Aa}(t)$

3. Partially Entangled Bell States $|\Psi_{AB}\rangle$

$$|\psi(0)\rangle = |\Psi_{AB}\rangle \otimes |0_a, 0_b\rangle$$

$$= \left(\cos\alpha\, |e_A, g_B\rangle + \sin\alpha\, |g_A, e_B\rangle\right) \otimes |0_a, 0_b\rangle$$

Following the methods from Part 2 $|\Phi_{AB}\rangle$

$$|e_A, 0_a\rangle = c\,|\psi_1^+\rangle - s\,|\psi_1^-\rangle$$

$$|g_A, 1_a\rangle = s\,|\psi_1^+\rangle + c\,|\psi_1^-\rangle$$

$$|g_A, 0_a\rangle = |\psi_0\rangle$$

$$|\psi(0)\rangle = \cos\alpha\, |e_A, 0_a\rangle \otimes |g_B, 0_b\rangle + \sin\alpha\, |g_A, 0_a\rangle \otimes |e_B, 0_b\rangle$$

$$= \cos\alpha \left(c\,|\psi_1^+\rangle - s\,|\psi_1^-\rangle \right) \otimes |\psi_0\rangle_B$$

$$+ \sin\alpha\, |\psi_0\rangle_A \otimes \left(c|\psi_1^+\rangle - s\,|\psi_1^-\rangle \right)$$

$$|\psi^{\pm}(t)\rangle = e^{-i\lambda^{\pm} t}\, |\psi^{\pm}(0)\rangle$$

$$|\psi(t)\rangle = \cos\alpha \left(e^{-i\lambda^+ t} |\psi_1^+\rangle_A - e^{-i\lambda^- t} |\psi_1^-\rangle_A \right) \otimes |\psi_0\rangle_B$$

$$+ \sin\alpha\, |\psi_0\rangle_A \otimes \left(e^{-i\lambda^+ t} |\psi_1^+\rangle_B - e^{-i\lambda^- t} |\psi_1^-\rangle_B \right)$$

$$|\Psi(t)\rangle = \cos\alpha\left(e^{-i\lambda^+ t}\left(c_0\,|\overset{\uparrow\,0}{e_A,0_a}\rangle + s_0\,|\overset{\downarrow\,1}{g_A,1_a}\rangle\right) - e^{-i\lambda^- t}\left(-s_0\,|\overset{\uparrow\,0}{e_A,0_a}\rangle + c_0\,|\overset{\downarrow\,1}{g_A,1_a}\rangle\right)\right) \otimes |\overset{\downarrow\,0}{g_B,0_b}\rangle$$

$$+ \sin\alpha\,|\overset{\downarrow\,0}{g_A,0_a}\rangle \otimes \left(e^{-i\lambda^+ t}\left(c_0\,|\overset{\uparrow\,0}{e_B,0_b}\rangle + s_0\,|\overset{\downarrow\,1}{g_B,1_b}\rangle\right) - e^{-i\lambda^- t}\cdot\left(-s_0\,|\underset{\uparrow\,0}{e_B,0_b}\rangle + c_0\,|\underset{\downarrow\,1}{g_B,1_b}\rangle\right)\right)$$

$$|\Psi(t)\rangle = x_1\,|\uparrow\downarrow 00\rangle + x_2\,|\downarrow\uparrow 00\rangle + x_3\,|\downarrow\downarrow 10\rangle + x_4\,|\downarrow\downarrow 01\rangle$$

$$x_1 = \cos\alpha\left(\overset{L}{c_0} e^{-i\lambda^+ t} + \overset{M}{s_0} e^{-i\lambda^- t}\right) \qquad L = \frac{1}{2}\left(1 + \overbrace{\frac{\Delta}{\sqrt{\Delta^2+G^2}}}^{\cos(\theta)}\right) = \cos^2\left(\frac{\theta}{2}\right)$$

$$x_2 = \sin\alpha\left(\overset{L}{c_0} e^{-i\lambda^+ t} + \overset{M}{s_0} e^{-i\lambda^- t}\right) \qquad M = \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2+G^2}}\right) = \sin^2\left(\frac{\theta}{2}\right)$$

$$x_3 = \cos\alpha\left(\overset{N}{s_0} e^{-i\lambda^+ t} - \overset{N}{c_0} e^{-i\lambda^- t}\right)$$

$$x_4 = \sin\alpha\left(\underset{N}{c_0} e^{-i\lambda^- t} - \underset{N}{s_0} e^{-i\lambda^- t}\right) \qquad N = \frac{G}{2\sqrt{\Delta^2+G^2}} = \frac{1}{2}\sin(\theta) = \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

Why $c_0 \neq \cos\left(\frac{\theta}{2}\right)$ for this cases?

Initially for the dressed states $c_0 = \sqrt{L}$, $s_0 = \sqrt{M}$

How are these values determined?

3.1 $C_{AB}(t)$

$$\rho^{AB} = \mathrm{Tr}_{ab} |\psi(t)\rangle\langle\psi(t)|$$

$$|\psi(t)\rangle = x_1 |\overset{ABab}{\uparrow\downarrow\underset{\sim}{00}}\rangle + x_2 |\downarrow\uparrow\underset{\sim}{00}\rangle + x_3 |\downarrow\downarrow\underset{\sim}{10}\rangle + x_4 |\downarrow\downarrow\underset{\sim}{01}\rangle$$

$$\mathrm{Tr}_{00} = |x_1|^2 |\uparrow\downarrow\rangle\langle\uparrow\downarrow| \checkmark + |x_2|^2 |\downarrow\uparrow\rangle\langle\downarrow\uparrow| \checkmark + x_1 x_2^* |\uparrow\downarrow\rangle\langle\downarrow\uparrow|$$
$$+ x_2 x_1^* |\downarrow\uparrow\rangle\langle\uparrow\downarrow|$$

$$\mathrm{Tr}_{01} = |x_4|^2 |\downarrow\downarrow\rangle\langle\downarrow\downarrow| \checkmark$$

$$T_{10} = |x_3|^2 |\downarrow\downarrow\rangle\langle\downarrow\downarrow| \checkmark$$

$$\mathrm{Tr}_{11} = 0$$

$$\rho^{AB} = \begin{array}{c} \\ |\downarrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \\ |\uparrow\uparrow\rangle \end{array} \begin{array}{c} \begin{array}{cccc} |\downarrow\downarrow\rangle & |\downarrow\uparrow\rangle & |\uparrow\downarrow\rangle & |\uparrow\uparrow\rangle \end{array} \\ \begin{bmatrix} |x_3|^2+|x_4|^2 & & & \\ & |x_2|^2 & x_1 x_2^* & \\ & x_1 x_2^* & |x_1|^2 & \\ & & & 0 \end{bmatrix} \end{array}$$

$$C_{AB}(t) = 2\left(|z| - \sqrt{ad}\right) = 2\left(|x_1||x_2| - 0\right)$$

$$C_{AB}(t) = 2\,|x_1||x_2|$$

For the resonant case: $(\Delta = 0)$

$$L = \frac{1}{2}\left(1 + \overbrace{\frac{\Delta}{\sqrt{\Delta^2+G^2}}}^{\cos(\theta)}\right) = \frac{1}{2}$$

$$\lambda^{\pm} = \nu + \cancel{\frac{\Delta}{2}} \pm \frac{\sqrt{\cancel{\Delta^2} + G^2}}{2} = \nu \pm \frac{G}{2}$$

global factor

$$M = \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2+G^2}}\right) = \frac{1}{2}$$

$$x_1 = \cos\alpha\left(\overset{L}{C_0}\, e^{-i\lambda^+ t} + \overset{M}{S_0}\, e^{-i\lambda^- t}\right)$$

$$|x_1| = \left|\frac{1}{2}\cos\alpha\left(\cancel{e^{-i\nu}}\cdot e^{-iG/2\,t} + \cancel{e^{-i\nu}}\cdot e^{+iG/2\,t}\right)\right|$$

$$|x_1| = \frac{1}{\cancel{2}}\,|\cos\alpha|\;\cancel{2}\cdot\left|\cos\left(\frac{G}{2}t\right)\right| = |\cos\alpha|\,\left|\cos\left(\frac{Gt}{2}\right)\right|$$

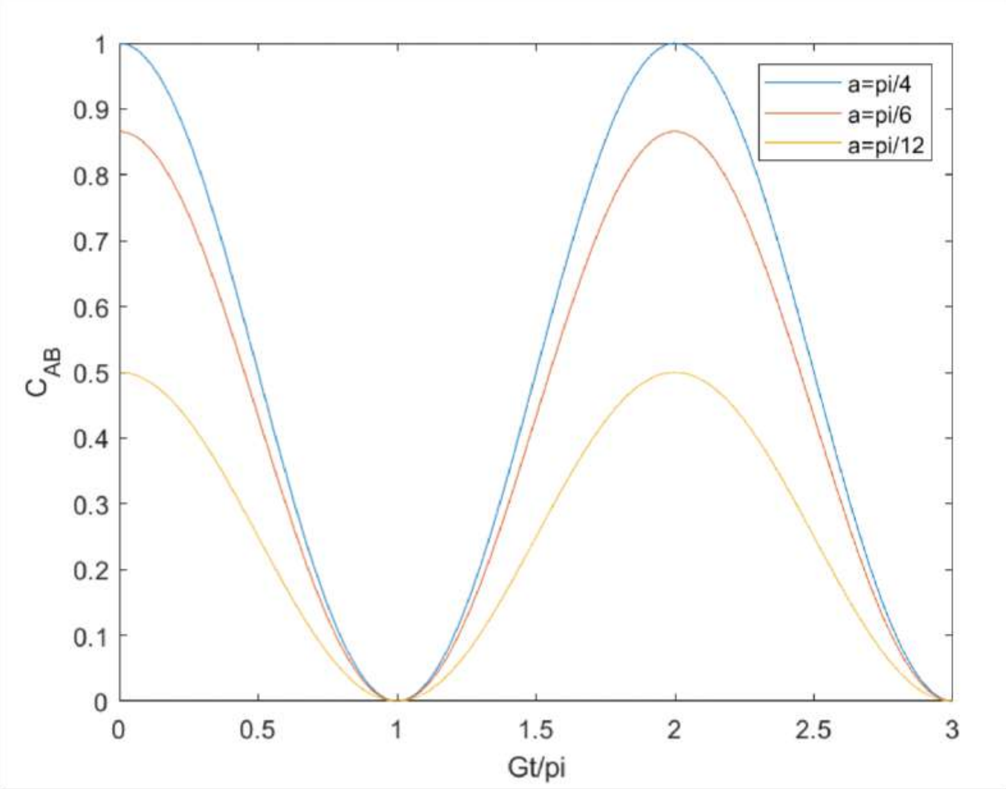
$$x_2 = \sin\alpha\left(\overset{L}{C_0}\, e^{-i\lambda^+ t} + \overset{M}{S_0}\, e^{-i\lambda^- t}\right)$$

$$x_2 = \sin\alpha\,\frac{1}{2}\left(e^{-i\lambda^+ t} + e^{-i\lambda^- t}\right)$$

$$|x_2| = \frac{1}{\cancel{2}}\,|\sin\alpha|\;\cancel{2}\cdot\left|\cos\left(\frac{G}{2}t\right)\right|$$

$$C_{AB}(t) = 2\,|\sin\alpha|\,|\cos\alpha|\,\cos^2\left(\frac{Gt}{2}\right) = |\sin 2\alpha|\,\cos^2\left(\frac{Gt}{2}\right)$$

MATLAB Plot:



3.2 $C_{ab}(t)$

$$\rho^{ab} = Tr_{AB} |\psi(t)\rangle\langle\psi(t)|$$

ABab

$$|\psi(t)\rangle = x_1 |\uparrow\downarrow 00\rangle + x_2 |\downarrow\uparrow 00\rangle + x_3 |\downarrow\downarrow 10\rangle + x_4 |\downarrow\downarrow 01\rangle$$

$$Tr_{\downarrow\downarrow} \rightarrow |x_3|^2 |10\rangle\langle 10| + |x_4|^2 |01\rangle\langle 01| + x_3 x_4^* |10\rangle\langle 01| + x_3^* x_4 |01\rangle\langle 10|$$

$$Tr_{\downarrow\uparrow} \rightarrow |x_2|^2 |00\rangle\langle 00|$$

$$Tr_{\uparrow\downarrow} \rightarrow |x_1|^2 |00\rangle\langle 00|$$

$$Tr_{\uparrow\uparrow} \rightarrow 0$$

$$
\rho^{ab} = \begin{array}{c} \\ |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array}
\begin{array}{c}
\begin{array}{cccc} \langle 00| & \langle 01| & \langle 10| & \langle 11| \end{array} \\
\begin{bmatrix} |x_1|^2+|x_2|^2 & & & \\ & |x_4|^2 & x_3^* x_4 & \\ & x_3 x_4^* & |x_3|^2 & \\ & & & 0 \end{bmatrix}
\end{array}
$$

$$
C_{ab}(t) = 2\left(|z| - \sqrt{ad}\right) = 2|z| = 2|x_3||x_4|
$$

$$
x_3 = \cos\alpha \left(\underbrace{s_0}_{N} e^{-i\lambda^+ t} - \underbrace{c_0}_{N} e^{-i\lambda^- t} \right) \quad \text{where} \quad N = \frac{1}{2}
$$

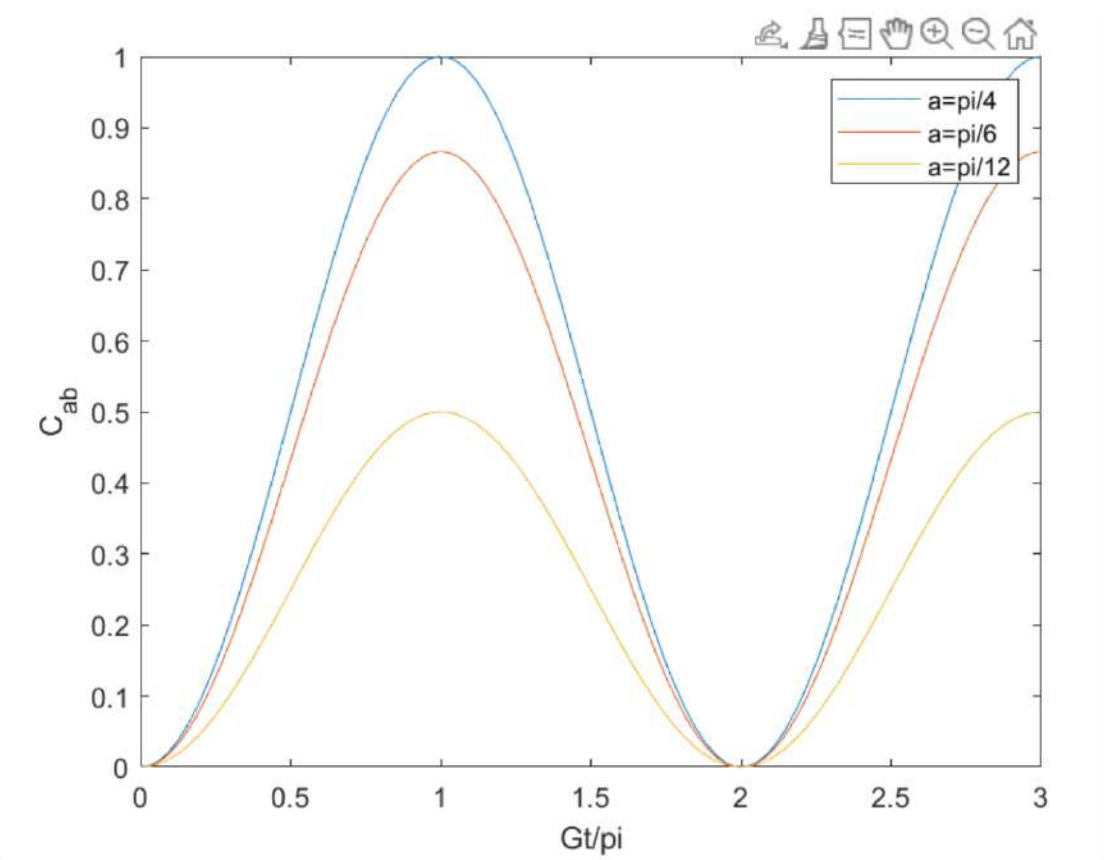
$$
|x_3| = \left| \frac{1}{\cancel{2}} \cos\alpha \left(\underbrace{e^{-i\lambda^+ t} - e^{-i\lambda^- t}}_{-\cancel{2}i \sin\left(\frac{6t}{2}\right)} \right) \right| = |\cos\alpha| \cdot \left|\sin\left(\frac{6t}{2}\right)\right|
$$

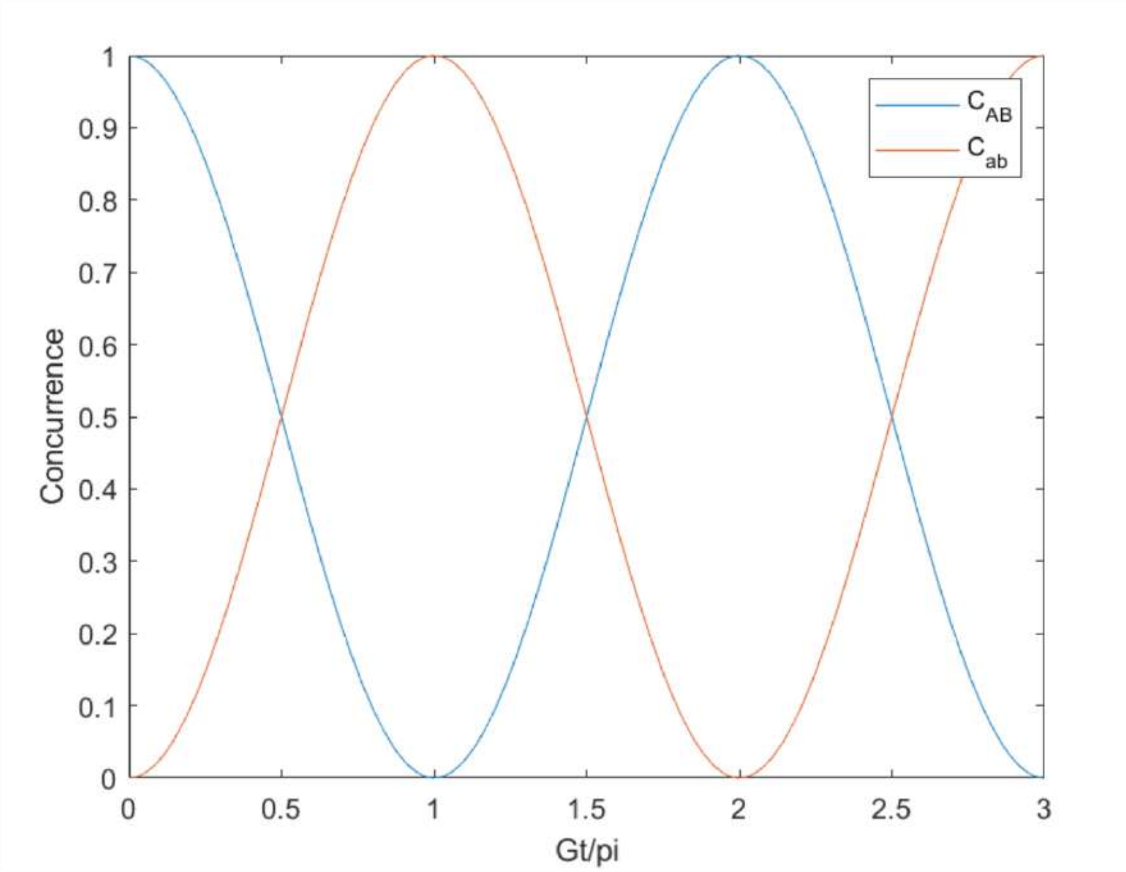
$$
x_4 = \sin\alpha \left(\underbrace{c_0}_{N} e^{-i\lambda^- t} - \underbrace{s_0}_{N} e^{-i\lambda^- t} \right)
$$

$$
|x_4| = |\sin\alpha| \left| \sin\left(\frac{6t}{2}\right) \right|
$$

$$
C_{ab}(t) = 2|\cos\alpha||\sin\alpha| \sin^2\left(\frac{6t}{2}\right) = |\sin(2\alpha)| \sin^2\left(\frac{6t}{2}\right)
$$

MATLAB Plot:





3.3 $C_{Ab}(t)$

$$\rho^{Ab} = Tr_{Ba} |\psi(t)\rangle\langle\psi(t)|$$

$$|\psi(t)\rangle = x_1 \overset{ABab}{|\uparrow\underbrace{\downarrow 0}0\rangle} + x_2 |\downarrow\underbrace{\uparrow 0}0\rangle + x_3 |\downarrow\underbrace{\downarrow 1}0\rangle + x_4 |\downarrow\underbrace{\downarrow 0}1\rangle$$

$$Tr_{\downarrow 0} \rightarrow |x_1|^2 |\uparrow 0\rangle\langle\uparrow 0| \checkmark + |x_4|^2 |\downarrow 1\rangle\langle\downarrow 1| \checkmark + x_1 x_4^* |\uparrow 0\rangle\langle\downarrow 1| + x_1^* x_4 |\downarrow 1\rangle\langle\uparrow 0|$$

$$Tr_{\downarrow 1} \rightarrow |x_3|^2 |\downarrow 0\rangle\langle\downarrow 0| \checkmark$$

$$Tr_{\uparrow 0} \rightarrow |x_2|^2 |\downarrow 0\rangle\langle\downarrow 0| \checkmark$$

$$Tr_{\uparrow 1} \rightarrow 0$$

$$\rho^{Ab} \quad \begin{array}{c|cccc} & \langle\downarrow 0| & \langle\downarrow 1| & \langle\uparrow 0| & \langle\uparrow 1| \\ \hline |\downarrow 0\rangle & |x_3|^2+|x_2|^2 & & & \\ |\downarrow 1\rangle & & |x_4|^2 & x_1^* x_4 & \\ |\uparrow 0\rangle & & x_1 x_4^* & |x_1|^2 & \\ |\uparrow 1\rangle & & & & 0 \end{array}$$

$$C_{Ab}(t) = 2\left(|z| - \sqrt{ad}\right) = 2|z| = 2|x_1||x_4|$$

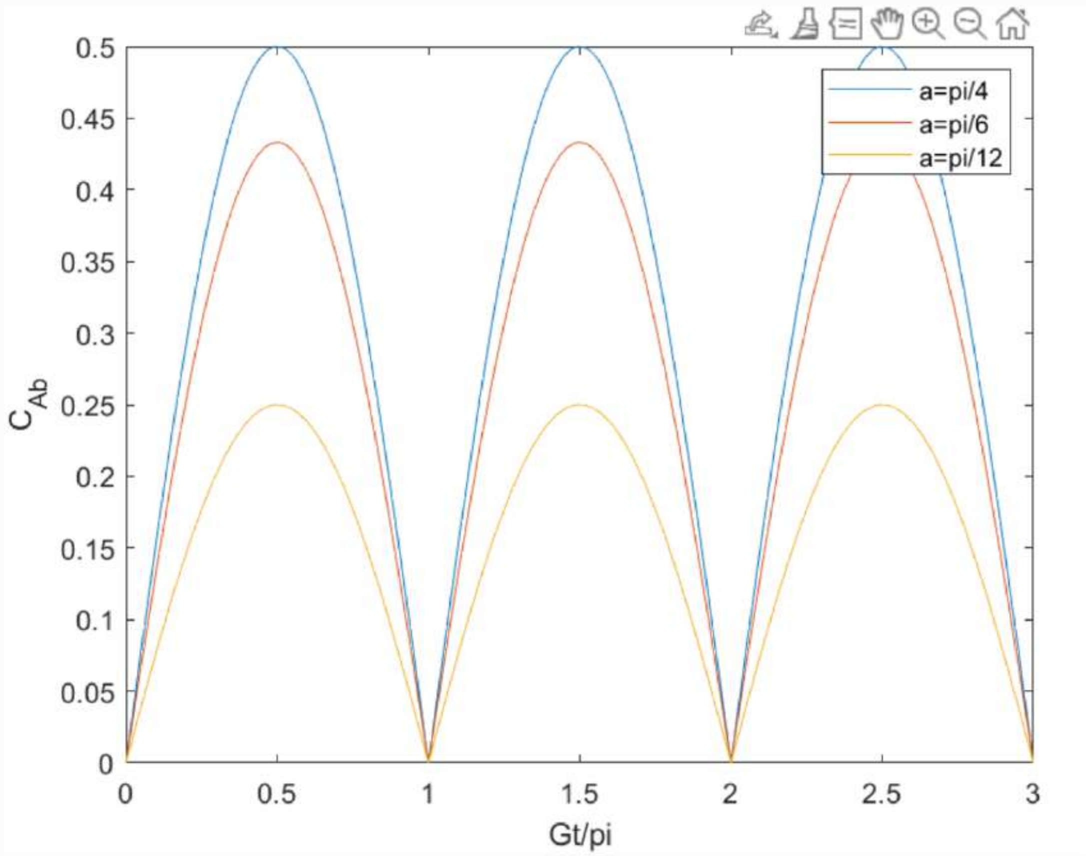
$$|x_1| = |\cos\alpha|\left|\cos\left(\frac{Gt}{2}\right)\right|$$

$$|x_4| = |\sin\alpha|\left|\sin\left(\frac{Gt}{2}\right)\right|$$

$$C_{Ab}(t) = 2|\sin\alpha||\cos\alpha|\left|\cos\left(\frac{Gt}{2}\right)\right|\left|\sin\left(\frac{Gt}{2}\right)\right|$$

$$= \frac{1}{2}|\sin 2\alpha||\sin(Gt)|$$

MATLAB plot:



3.4 $C_{Ba}(t)$

Under the transformation $A \leftrightarrow B$, $a \leftrightarrow b$

and $\cos\alpha \leftrightarrow \sin\alpha$ the state remains unchanged.

If we apply $\cos\alpha \leftrightarrow \sin\alpha$, but this transformation

won't change the equation. Thus $C_{Ab}(t) = C_{Ba}(t)$

$$|\psi(0)\rangle = \Big(\underset{\sin\alpha\,|\downarrow\uparrow 00\rangle}{\cos\alpha\,|\overset{AB\,ab}{\uparrow\downarrow 00}\rangle} + \underset{\cos\alpha\,|\uparrow\downarrow 00\rangle}{\sin\alpha\,|\overset{AB\,ab}{\downarrow\uparrow 00}\rangle}\Big)$$

3.5 $C_{Aa}(t)$

$$\rho^{Aa} = Tr_{Bb} |\psi(t)\rangle\langle\psi(t)|$$

$$|\psi(t)\rangle = x_1 |\overset{ABab}{\uparrow\downarrow 00}\rangle + x_2 |\downarrow\uparrow 00\rangle + x_3 |\downarrow\downarrow 10\rangle + x_4 |\downarrow\downarrow 01\rangle$$

$Tr_{\downarrow 0} \rightarrow |x_1|^2 |\uparrow 0\rangle\langle\uparrow 0| + |x_3|^2 |\downarrow 1\rangle\langle\downarrow 1| + x_1 x_3^* |\uparrow 0\rangle\langle\downarrow 1|$ ✓

$+ x_1^* x_3 |\downarrow 1\rangle\langle\uparrow 0|$

$Tr_{\downarrow 1} \rightarrow |x_4|^2 |\downarrow 0\rangle\langle\downarrow 0|$ ✓

$Tr_{\uparrow 0} \rightarrow |x_2|^2 |\downarrow 0\rangle\langle\downarrow 0|$ ✓

$Tr_{\uparrow 1} \rightarrow 0$

$$
\rho^{Aa} = \begin{array}{c} \\ |\downarrow 0\rangle \\ |\downarrow 1\rangle \\ |\uparrow 0\rangle \\ |\uparrow 1\rangle \end{array}
\begin{array}{c} \begin{array}{cccc} \langle\downarrow 0| & \langle\downarrow 1| & \langle\uparrow 0| & \langle\uparrow 1| \end{array} \\
\begin{bmatrix} |x_2|^2 + |x_4|^2 & & & \\ & |x_3|^2 & x_1^* x_3 & \\ & x_1 x_3^* & |x_1|^2 & \\ & & & 0 \end{bmatrix} \end{array}
$$

$$C_{Aa}(t) = 2\left(|z| - \underset{0}{\cancel{\sqrt{ga}}}\right) = 2|z| = 2|x_1||x_3|$$

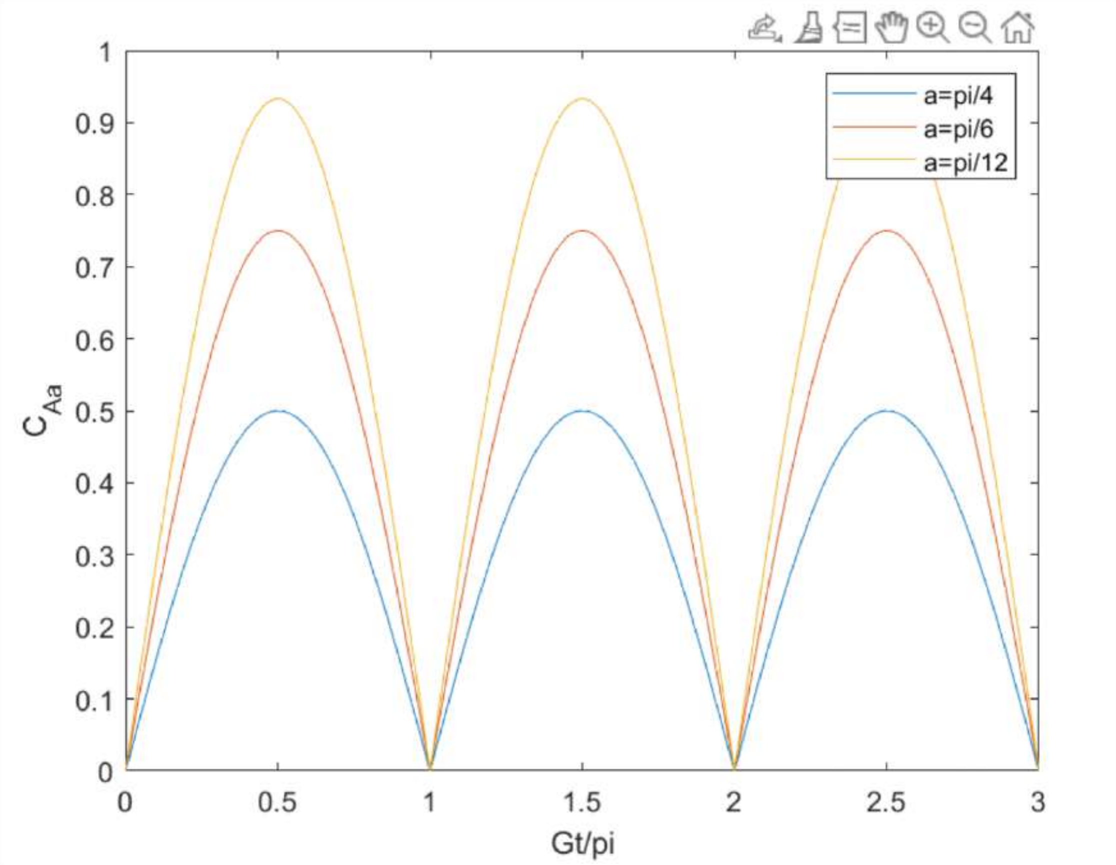
$$|x_1| = |\cos\alpha| \left|\cos\left(\frac{Gt}{2}\right)\right|$$

$$|x_3| = |\cos\alpha| . \left|\sin\left(\frac{Gt}{2}\right)\right|$$

$$C_{Aa}(t) = 2|x_1||x_3| = \underbrace{2. \left|\sin\left(\frac{Gt}{2}\right)\right| \left|\cos\left(\frac{Gt}{2}\right)\right|} \cos^2\alpha$$

$$C_{Aa}(t) = |\sin(Gt)| \cos^2\alpha$$

MATLAB Plot:



3,6 $C_{Bb}(t)$

Using transformation $A \leftrightarrow B$

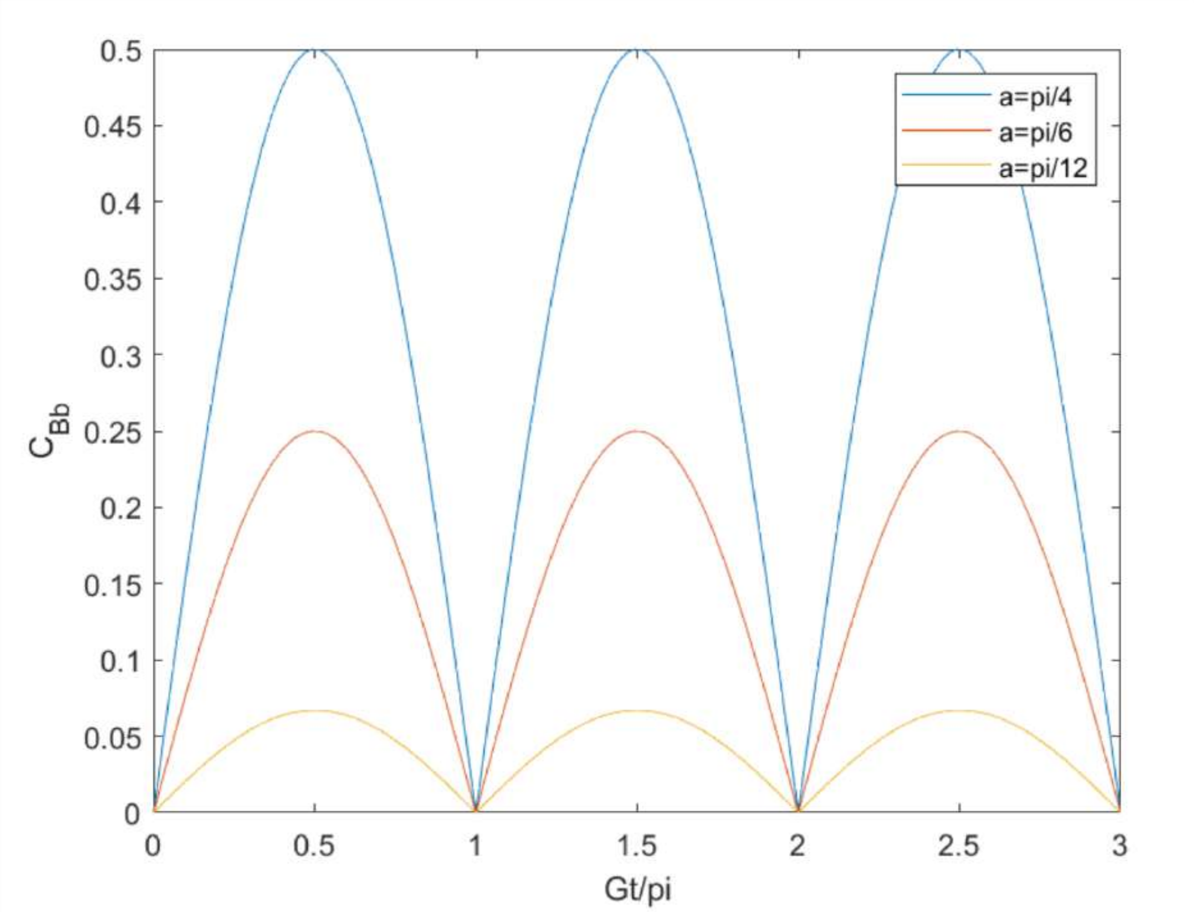
$a \leftrightarrow b$

$\cos\alpha \leftrightarrow \sin\alpha$

Thus, previously we found $C_{Aa}(t) = \cos^2\alpha \left|\sin(Gt)\right|$

$$C_{Bb}(t) = \sin^2\alpha \left|\sin(Gt)\right|$$

MATLAB Ploti



4. CONCLUSION

$$|\Phi(0)\rangle = \left(\cos\alpha\, |\overset{\uparrow}{e_A}, \overset{\uparrow}{e_B}\rangle + \sin\alpha\, |\overset{\downarrow}{g_A}, \overset{\downarrow}{g_B}\rangle\right) \otimes |0_a 0_b\rangle$$

$$|\Phi(t)\rangle = x_1 |\uparrow\uparrow 00\rangle + x_2 |\downarrow\downarrow 11\rangle + x_3 |\uparrow\downarrow 01\rangle + x_4 |\downarrow\uparrow 10\rangle + x_5 |\downarrow\downarrow 00\rangle$$

$$|\Psi(0)\rangle = \left(\cos\alpha\, |\overset{\uparrow}{e_A}, \overset{\downarrow}{g_B}\rangle + \sin\alpha\, |\overset{\downarrow}{g_A}, \overset{\uparrow}{e_B}\rangle\right) \otimes |0_a 0_b\rangle$$

$$|\Psi(t)\rangle = x_1' |\uparrow\downarrow 00\rangle + x_2' |\downarrow\uparrow 00\rangle + x_3' |\downarrow\downarrow 10\rangle + x_4' |\downarrow\downarrow 01\rangle$$

* Existence of the additional term of $|\Phi(t)\rangle$ yields negative term in the $Q(t)$; thus, "0" concurrence.
  Where $C(t) = 2\max\{0, Q(t)\}$

  In Φ_{AB} 2 photons may later be present at the same time, whereas there can never be more than one photon present in the $|\Psi_{AB}\rangle$ case.

* When ESD does not occur, the sum of the entanglements of the atomic and photonic parts gives the INITIAL ENTANGLEMENT.

$$C^{AB} + C^{ab} = |\sin 2\alpha| = \text{CONSTANT}$$

* For this model, there are no couplings between the two atoms or the two cavitys. That's evolution of entanglement is a pure information-exchange evolution. (Not a conventional decoherence process.)

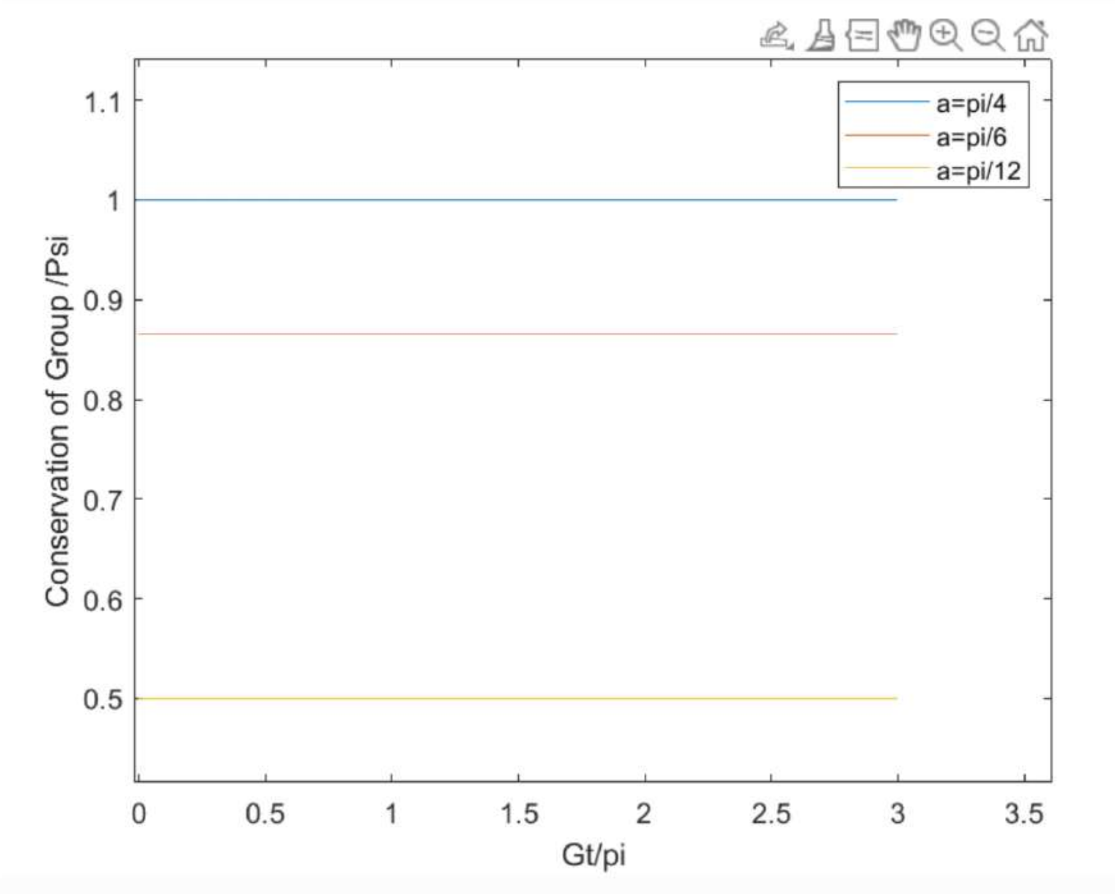
★ The local atom-cavity couplings not only cause the creation of entanglement between atom and its cavity, but also generate non-local entanglement btw the cavities, if the atom-atom subsystem is initially prepared in an entangled state.

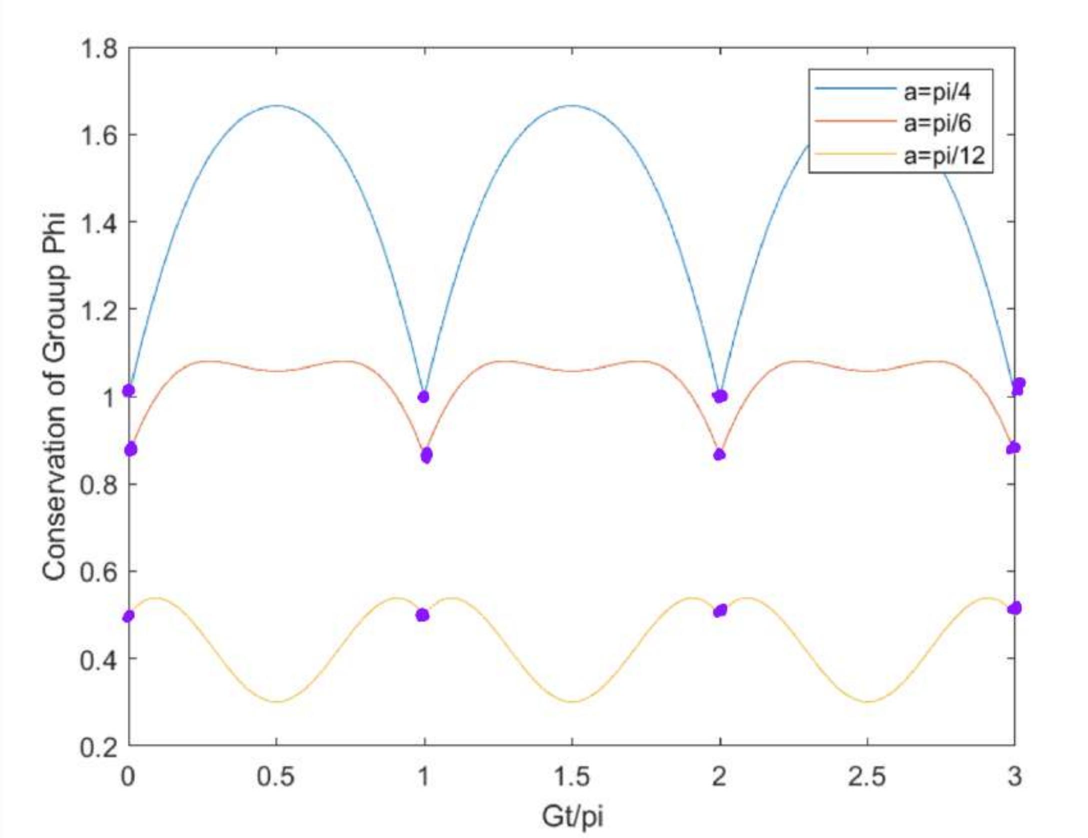
★ Entanglement conservation is an open issue since we can not expect conservation of entanglement is not defined as an observable or represented by a Hermitian operator.

★ The following equation holds for both $|\Psi_{AB}\rangle$ and $|\Phi_{AB}\rangle$

$$Q^{AB} + Q^{ab} + 2Q^{Aa}|\tan\alpha| - 2Q^{Ab} = |\sin 2\alpha|$$

NOT CORRECT for $|\Phi_{AB}\rangle$





! Equation only holds for the purple points for $|\Phi_{AB}\rangle$