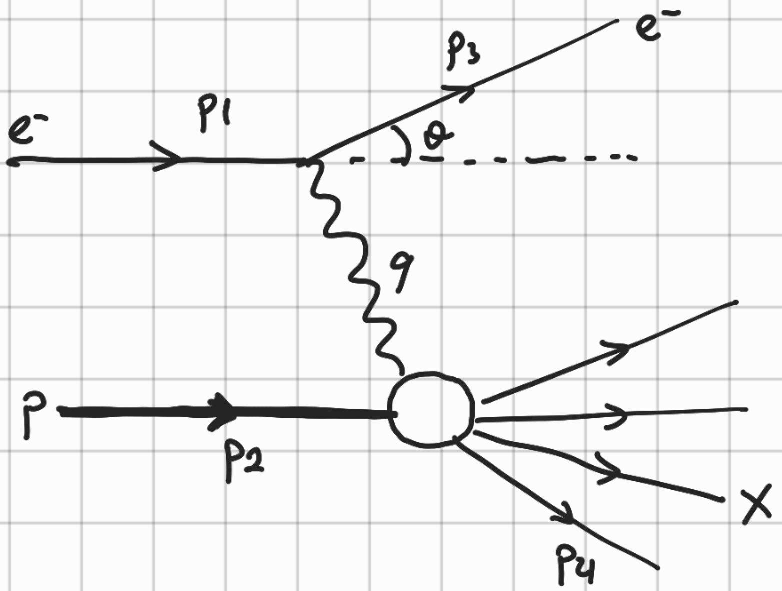
Week 11 Deep inelastic Scattering

Electron-proton inelastic Scattering



Kinematic variables for inelastic scattering

$$Q^2 = -q^2$$

$$Q^2 = -(p_1 - p_3)^2 = -2m_e^2 + 2p_1 \cdot p_3 = -2m_e^2 + (2E_1E_3 - 2p_1p_3\cos\theta)$$

Neglect the electron mass and energies are high:

$$Q^2 \approx 2E_1E_3(1-\cos\theta) = 4E_1E_3\sin^2\frac{\theta}{2} > 0$$

Bjorken $x = \dfrac{Q^2}{2p_2 \cdot q}$

$$W^2 \equiv p_4^2 = (q+p_2)^2 = q^2 + 2p_2 \cdot q + p_2^2$$

$$W^2 \underbrace{-q^2}_{+Q^2} - \underbrace{p_2^2}_{M_p^2} = 2p_2 \cdot q$$

$$W^2 + Q^2 - M_p^2 = 2p_2 \cdot q$$

$$x = \frac{Q^2}{W^2 + Q^2 - M_p^2}$$

$$W^2 \equiv p_4^2 \geq M_p^2 \quad , \quad Q^2 \geq 0$$

$$\boxed{0 \leq x \leq 1}$$

elasticity

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$$

$$y = \frac{m_p (E_1 - E_3)}{M_p E_1} = 1 - \frac{E_3}{E_1}$$

$$\boxed{0 \leq y \leq 1}$$

$$p_1 = (E_1, 0, 0, E_1)$$

$$p_2 = (M_p, 0, 0, 0)$$

$$p_3 = (E_3, E_3 \sin\theta, 0, E_3 \cos\theta)$$

$$q = (E_1 - E_3, \vec{p_1} - \vec{p_3})$$

$$\nu = \frac{p_2 \cdot q}{M_p} = E_1 - E_3$$

$\rightarrow$ energy lost by the election

Relationships between kinematic variables

$$\boxed{x = \frac{Q^2}{2M_p\nu}}$$

$$S = (p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = 2p_1 \cdot p_2 + M_p^2 + \underbrace{\cancel{M_e^2}}_{\approx 0}$$

$$2p_1 \cdot p_2 \simeq S - M_p^2$$

$$\boxed{y = \left(\frac{2M_p}{S - M_p^2}\right)\nu}$$

$$\boxed{Q^2 = (S - M_p^2)\, x y}$$

Deep Inelastic Scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

$\searrow$

$e^- p \rightarrow e^- p$ elastic scattering cross section

"Rosenbluth formula"

In terms of Q^2 and y

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\underbrace{\frac{G_E^2+\tau G_M^2}{(1+\tau)}}_{f_2(Q^2)}\left(1-y-\frac{m_p^2y^2}{Q^2}\right)+\frac{1}{2}y^2\underbrace{G_M^2}_{f_1(Q^2)}\right]$$

elastic scattering

Structure Functions

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\cancel{\frac{M_p^2y^2}{Q^2}}\right)\frac{F_2(x,Q^2)}{x}+y^2F_1(x,Q^2)\right]$$

$Q^2 \gg M_p^2y^2$

$$\frac{d^2\sigma}{dx\,dQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x}+y^2F_1(x,Q)\right]$$

Bjorken Scaling and the Callan-Gross Relation

Exp. data revealed two striking features:

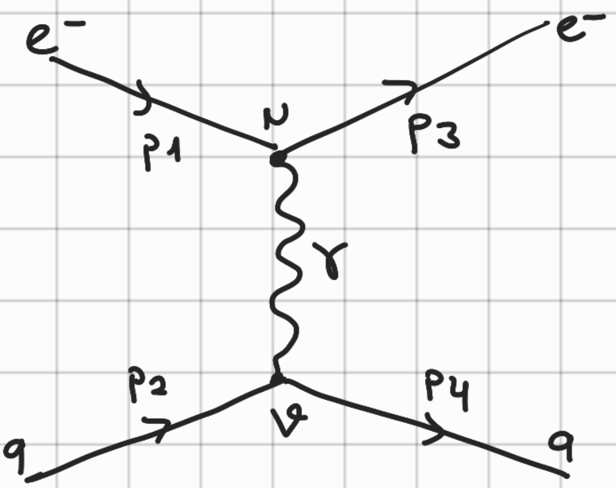
$$\left.\begin{aligned}F_1(x,Q^2) &\longrightarrow F_1(x)\\ F_2(x,Q^2) &\longrightarrow F_2(x)\end{aligned}\right\}$$

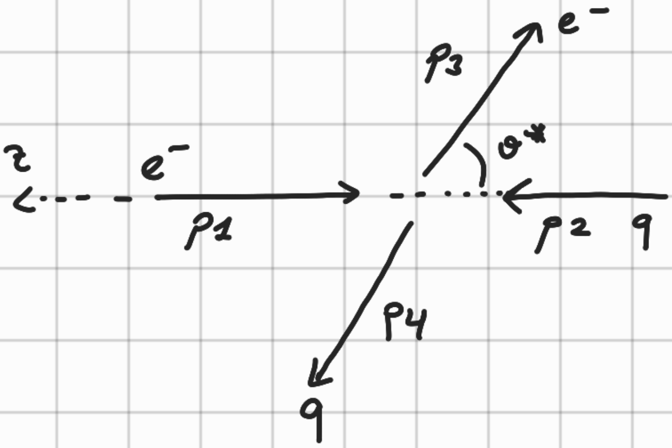
scattering from point-like constituents within the proton

For $Q^2 >$ a few GeV^2 $\quad F_2(x) = 2xF_1(x)$

"Callan-Gross Relation"

Electron-quark Scattering





CoM frame

$$E_e = E = \sqrt{s}/2$$

m_e and m_q are neglected

$$p_1 = (E, 0, 0, +E)$$

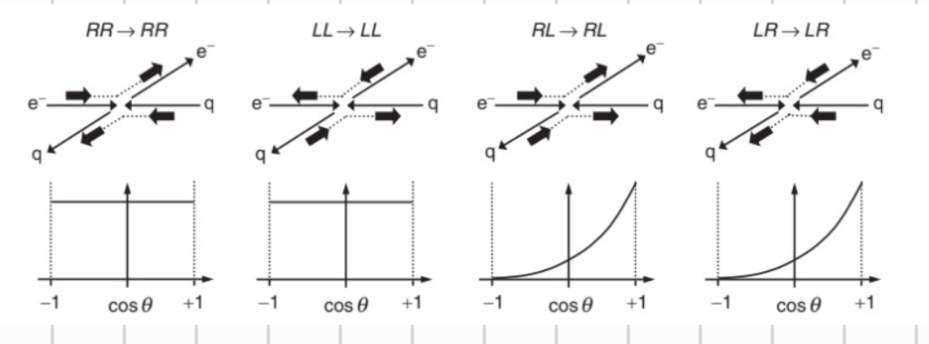
$$p_2 = (E, 0, 0, -E) \qquad p_1 \cdot p_2 = 2E^2$$

$$p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*) \qquad p_1 \cdot p_3 = E^2(1-\cos\theta)$$

$$p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*) \qquad p_1 \cdot p_4 = E^2(1+\cos\theta)$$

$$\langle |M_{fi}|^2 \rangle = \frac{2 Q_q^2 e^4 \; 4E^4 + E^4(1+\cos\theta^*)^2}{E^4(1-\cos\theta^*)^2}$$

$$\frac{d\sigma}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s} \; \frac{\left[1 + \frac{1}{4}(1+\cos\theta^*)^2\right]}{(1-\cos\theta^*)^2}$$



$$\frac{d\sigma}{dq^2} = \frac{1}{64\pi s\, p_i^{*2}} \langle |M_{fi}|^2 \rangle = \frac{Q_q^2 e^4}{32\pi s\, p_i^{*2}} \left(\frac{s^2 + u^2}{t^2} \right)$$

Since $p_i^* = \sqrt{s}/2$, $t = q^2$

$$\frac{d\sigma}{dq^2} = \frac{Q_q^2 e^4}{8\pi q^4} \left(\frac{s^2 + u^2}{s^2} \right)$$ where $u \approx -s - t = -s - q^2$

$\Big\downarrow$ In terms of s and q^2

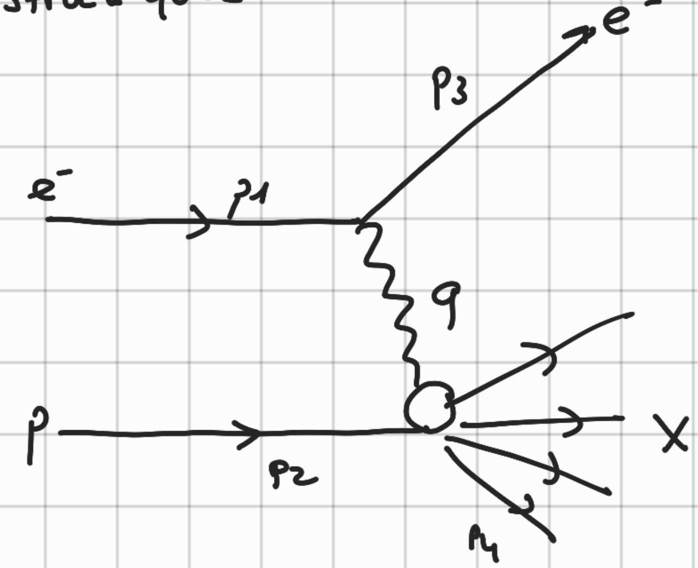
$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4} \left[1 + \left(1 + \frac{q^2}{s} \right)^2 \right]$$

The quark-parton Model

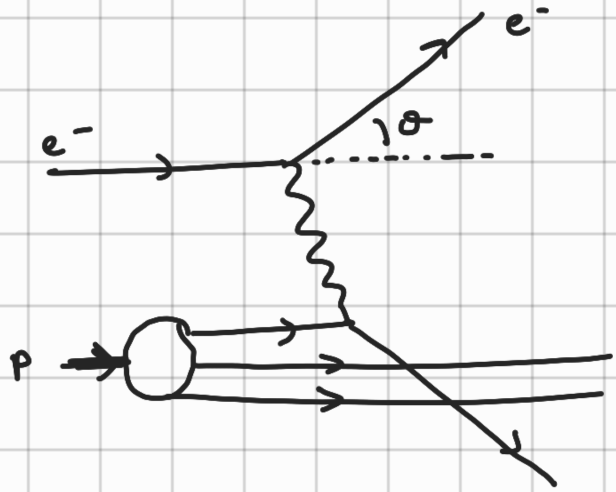
$p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2)$

$\downarrow$

struck quark

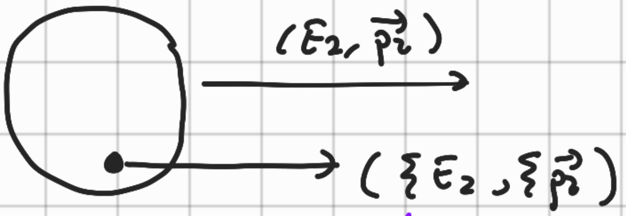


CoM frame

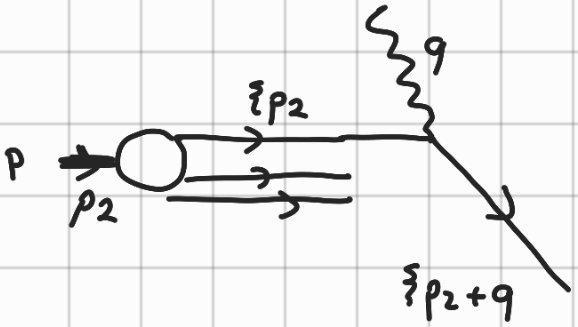


Feynman Digram of lowest order

$p_2 = (E_2, 0, 0, E_2)$



fraction of momentum of proton carried by the quark



$$(\xi p_2 + q)^2 = \overbrace{\cancel{\xi^2 p_2^2}}^{M_q^2} + 2\xi p_2 \cdot q + q^2 = \cancel{m_q^2}$$

$$\xi p_2 = (\xi E_2, 0, 0, \xi E_2) \rightarrow \xi^2 p_2^2 = m_q^2$$

$$q^2 + 2\xi p_2 \cdot q = 0$$

$$\xi = \frac{-q^2}{2p_2 \cdot q} = \frac{Q^2}{2p_2 \cdot q} \equiv x$$

Bjorken x

Neglecting M_e and M_p

$e^- p$ Initial State: $s = (p_1 + p_2)^2 \simeq 2p_1 \cdot p_2$

$$p_q = x p_2$$

4-momentum of struck quark

CoM energy of the initial $e^- q$:

$$s_q = (p_1 + x p_2)^2 \approx 2x p_1 \cdot p_2 = xs$$

The kinematic variables in terms of p_2 → 4-momentum of the proton

$$y = \frac{p_2 \cdot q}{p_2 \cdot p_1} \qquad x = \frac{Q^2}{2p_2 \cdot q}$$

Similarly $e^- q$ system:

$$y_q = \frac{p_q \cdot q}{p_q \cdot p_1} = \frac{x p_2 \cdot q}{x p_2 \cdot p_1} = y$$

$$\boxed{S_q = xS \qquad y_q = y \qquad x_q = 1}$$

electron-quark elastic scattering

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

$$\boxed{q^2 = -Q^2 = -(s_q - M_q^2)\, x_q\, y_q}$$

(M_q is neglected) $\quad \frac{q^2}{s_q} = -x_q y_q = -y$

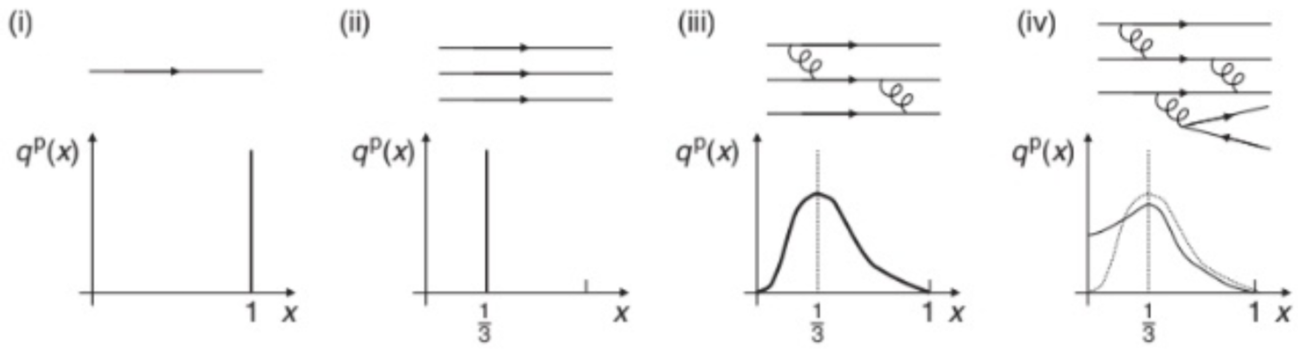
$$\Downarrow$$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + (1-y)^2\right]$$

$\Downarrow$ using $q^2 = -Q^2$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

Parton Distribution Functions



Four possible forms of the quark PDFs within a proton: (i) a single point-like particle; (ii) three static quarks each sharing 1/3 of the momentum of the proton; (iii) three interacting quarks which can exchange momentum; and (iv) interacting quarks including higher-order diagrams. After Halzen and Martin (1984).

The cross section for elastic scattering from a particular flavour of quark i with charge Q_i, momentum fraction in the range $x \to x + \delta x$

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right] \times Q_i^2 \, q_i^p(x) \, \delta x$$

PDF for that flavour of quark

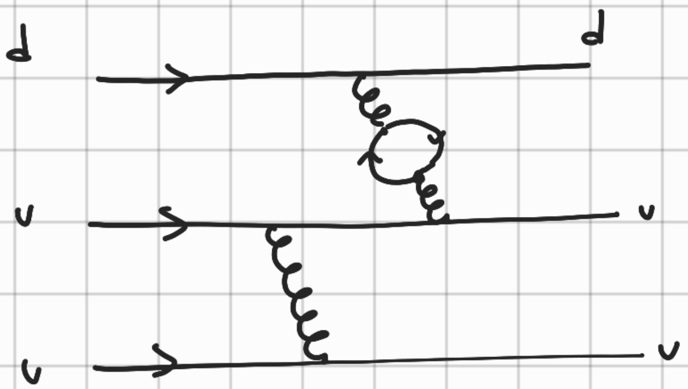
$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right] \sum_i Q_i^2 \, q_i^p(x)$$

Before, we found

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

$$\underbrace{F_2(x,Q^2)}_{F_2(x)} = 2x \underbrace{F_1(x,Q^2)}_{F_1(x)} = x \sum_i Q_i^2 \, q_i^p(x)$$

Determination of the parton distribution functions



(electron-proton)

$$F_2^{ep}(x) = x \sum_i Q_i^2 q_i^p(x) \approx x\left(\frac{4}{9}u^p(x) + \frac{1}{9}d^p(x) + \frac{4}{9}\underset{\text{anti-up}}{\bar{u}^p(x)} + \frac{1}{9}\underset{\text{anti-down}}{\bar{d}^p(x)}\right)$$

$$F_2^{en}(x) = x \sum_i Q_i^n q_i^n(x) \simeq x\left(\frac{4}{9}u^n(x) + \frac{1}{9}d^n(x) + \frac{4}{9}\bar{u}^n(x) + \frac{1}{9}\bar{d}^n(x)\right)$$

Due to isospin symmetry: $\rightsquigarrow$ $d^n(x) = u^p(x)$

$u^n(x) = d^p(x)$

In order to simplify the notation:

Proton PDFs $\longrightarrow$ $u(x), d(x), \bar{u}(x), \bar{d}(x)$

Neutron PDFs $\longrightarrow$ $d^n(x) = u^p(x) = u(x)$

$u^n(x) = d^p(x) = d(x)$

$\bar{d}^n(x) = \bar{u}^p(x) = \bar{u}(x)$

$\bar{u}^n(x) = \bar{d}^p(x) = \bar{d}(x)$

Rewrite $F_2^{ep}(x)$ & $F_2^{en}(x)$

$$F_2^{ep}(x) = 2x F_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right)$$

$$F_e^{en}(x) = 2x F_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)\right)$$

$$\int_0^1 F_2^{ep}(x)dx = \frac{4}{9}f_U + \frac{1}{9}f_D$$

$$f_U = \int_0^1 [x u(x) + x\bar{u}(x)]dx$$

$$\int_0^1 F_2^{en}(x)\,dx = \frac{4}{9}f_D + \frac{1}{9}f_U$$

$$f_D = \int_0^1 [x d(x) + x\bar{d}(x)]dx$$

f_U and f_D can be obtained directly from the experiment

Valence and Sea quarks

$$u(x) = u_v(x) + u_s(x)$$

u_v: valence quark

u_s: from the sea of up quarks (produced from virtual gluons)

$$d(x) = d_v(x) + d_s(x)$$

d_v: valence

d_s: sea quark

$$\bar{u}(x) = \bar{u}_s(x) \qquad \bar{d}(x) = \bar{d}_s(x)$$

$p = uud$ (proton) $\rightarrow$

$$\int_0^1 u_v(x)\,dx = 2$$

$$\int_0^1 d_v(x)\,dx = 1$$

$$u_S(x) = \bar{u}_S(x) \simeq d_S(x) = \bar{d}_S(x) \simeq S(x)$$

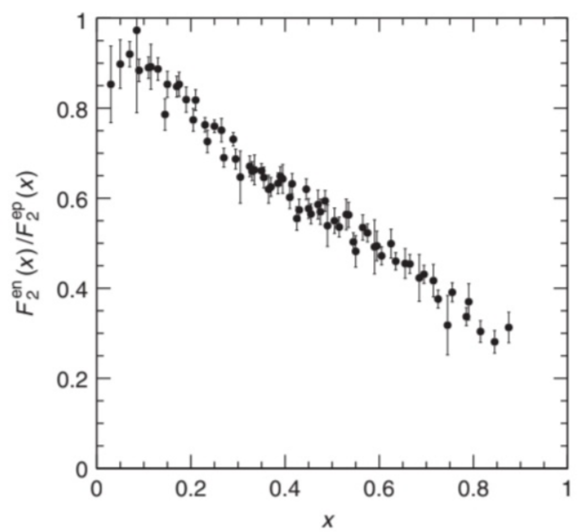
since $Mass_{up} \simeq Mass_{down}$

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_v(x) + \frac{1}{9}d_v(x) + \frac{10}{9}S(x)\right)$$

$$F_2^{en}(x) = x\left(\frac{4}{9}d_v(x) + \frac{1}{9}u_v(x) + \frac{10}{9}S(x)\right)$$

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d_v(x) + u_v(x) + 10S(x)}{4u_v(x) + d_v(x) + 10S(x)}$$

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \longrightarrow 1 \quad \text{as} \quad x \to 0$$



The ratio of $F_2^{en}(x)/F_2^{ep}(x)$ obtained from electron–deuterium and electron–proton deep inelastic scattering measurements at SLAC. Data from Bodek *et al.* (1979).

Due to $1/q^2$ gluon propagator, which will supress the production of sea quarks at high x, it might be expected the high-x PDFs will be dominated by the valence quarks.

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \rightarrow \frac{4d_v(x) + u_v(x)}{4u_v(x) + d_v(x)} \quad \text{as} \quad x \rightarrow 1$$

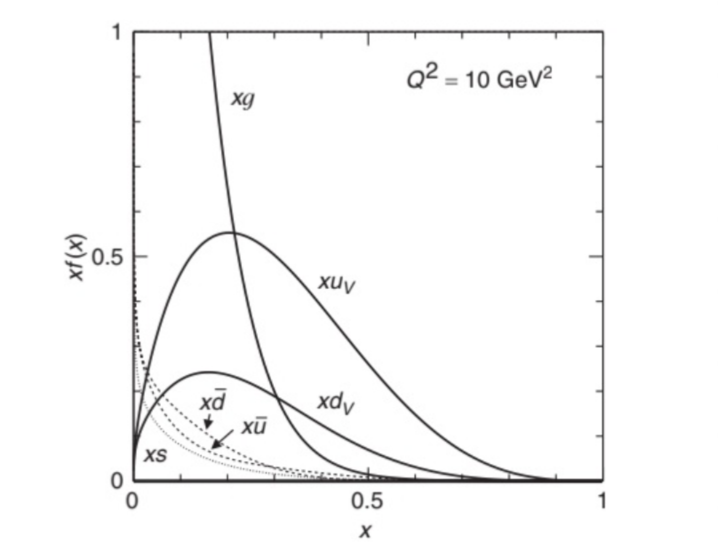
Electron - proton Scattering at the HERA collider

Q^2 and x are determined from the energy and scattering angle of the electron

For $0.01 < x < 0.5 \rightarrow Q^2 = 2 \times 10^4 \ GeV^2$

$\rightarrow$ Quarks appear to be point-like particles at scales up to $Q^2 \approx 2\times10^4 \ GeV^2$

$\rightarrow r_q < 10^{-18} m$



$\rightarrow$ Apart from high x values, $u_v(x) \simeq 2\ d_v(x)$

$\rightarrow$ Although $\bar{d}(x) > \bar{u}(x)$, $\bar{d}(x) > \bar{u}(x)$

Maybe $\rightarrow$ relative suppression of the $g \rightarrow u\bar{u}$ process due to the exc. principle

Summary

In this chapter the process of deep inelastic scattering has been described in terms of the quark–parton model, where the underlying process is the elastic scattering of the electron from the quasi-free spin-half constituent quarks. The kinematics of inelastic scattering were described in terms of two of the kinematic variables defined below

$$Q^2 \equiv -q^2, \quad x \equiv \frac{Q^2}{2p_2 \cdot q}, \quad y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1} \quad \text{and} \quad \nu \equiv \frac{p_2 \cdot q}{m_\mathrm{p}}.$$

In the quark–parton model, x is identified as the fraction of the momentum of the proton carried by the struck quark in the underlying $\mathrm{e^-q \to e^-q}$ elastic scattering process. The quark–parton model naturally describes the experimentally observed phenomena of Bjorken scaling, $F(x, Q^2) \to F(x)$, and the Callan–Gross relation, $F_2(x) = 2xF_1(x)$.

In the quark–parton model, cross sections can be described in terms of parton distribution functions (PDFs) which represent the momentum distributions of quarks and antiquarks within the proton. The PDFs can not yet be calculated from first principles but are determined from a wide range of experimental data. The resulting PDF measurements reveal the proton to be much more complex than a static bound state of three quarks (uud); it is a dynamic object consisting of three valence quarks and a sea of virtual quarks, antiquarks and gluons, with almost 50% of the momentum of the proton carried by the gluons. The precise knowledge of the PDFs is an essential ingredient to the calculations of cross sections for all high-energy processes involving protons, such as proton–proton collisions at the LHC.