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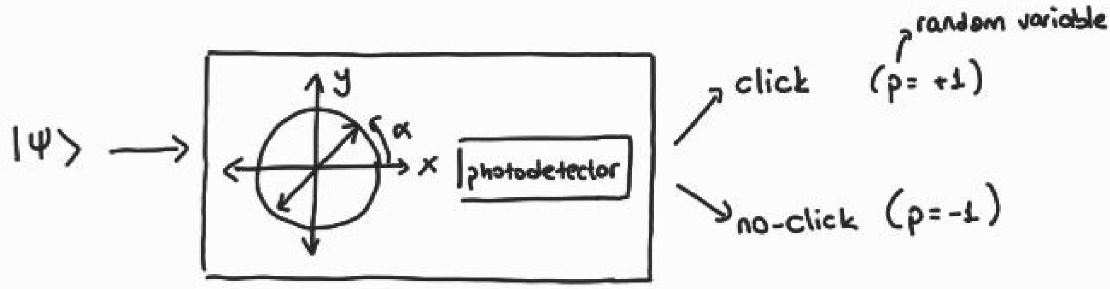
COM-309 QUANTUM INFORMATION PROCESSING

Mini Project

Measurement

1.

$$\left.\begin{aligned} \Pi_a &= |a\rangle\langle a| \\ \Pi_{a_\perp} &= |a_\perp\rangle\langle a_\perp| \end{aligned}\right\} \quad A = (+1)\,|a\rangle\langle a| + (-1)\,|a^\perp\rangle\langle a^\perp|$$



$$|a\rangle = \cos(\alpha)\,|0\rangle + \sin(\alpha)\,|1\rangle \quad \& \quad |a_\perp\rangle = \left|\alpha + \frac{\pi}{2}\right\rangle$$

Here, measurement is done on the $|a\rangle$ and $|a_\perp\rangle$ basis not on the computational basis.

$$P(|\psi\rangle \rightarrow |a\rangle) = P(p=+1) = |\langle\psi|a\rangle|^2 = \langle\psi|\Pi_a|\psi\rangle$$

$$= \langle\psi|a\rangle\langle a|\psi\rangle = \cos\alpha\,\underbrace{\langle 0|0\rangle}_{1} \cdot \cos\alpha\,\underbrace{\langle 0|0\rangle}_{1}$$

$$= \cos^2\alpha$$

Therefore, one can find directly

$$P(|\psi\rangle \rightarrow |a_\perp\rangle) = P(p=-1) = 1 - \cos^2\alpha = \sin^2\alpha$$

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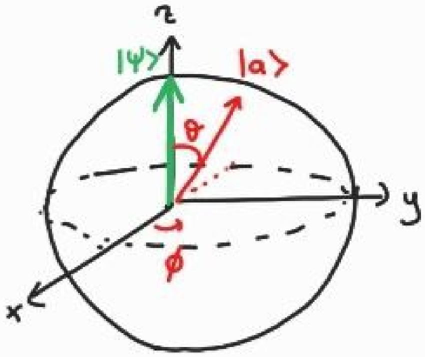
$R_y(\theta) = e^{-i\frac{\theta}{2}\sigma_y}$ is a rotation matrix of angle θ around Y axis on the Bloch sphere.

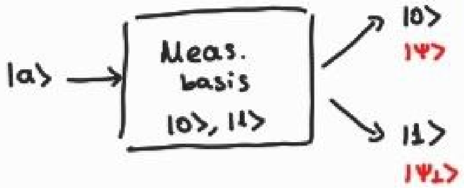
$|a\rangle = \cos(\alpha)|0\rangle + \sin(\alpha)|1\rangle$

$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$

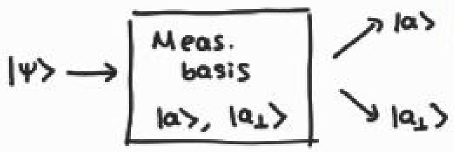


Here $\theta = 2\alpha$ & $\phi = 0$

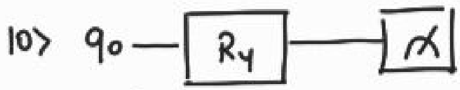
In NISQ devices measurements are done in computational basis. The problem turns into finding $P(|a\rangle \to |\psi\rangle)$



with $\text{Prob}(|a\rangle \to |\psi\rangle) = |\langle a|\psi\rangle|^2$



with $\text{Prob}(|\psi\rangle \to |a\rangle) = |\langle\psi|a\rangle|^2$



(Initial state)

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2.

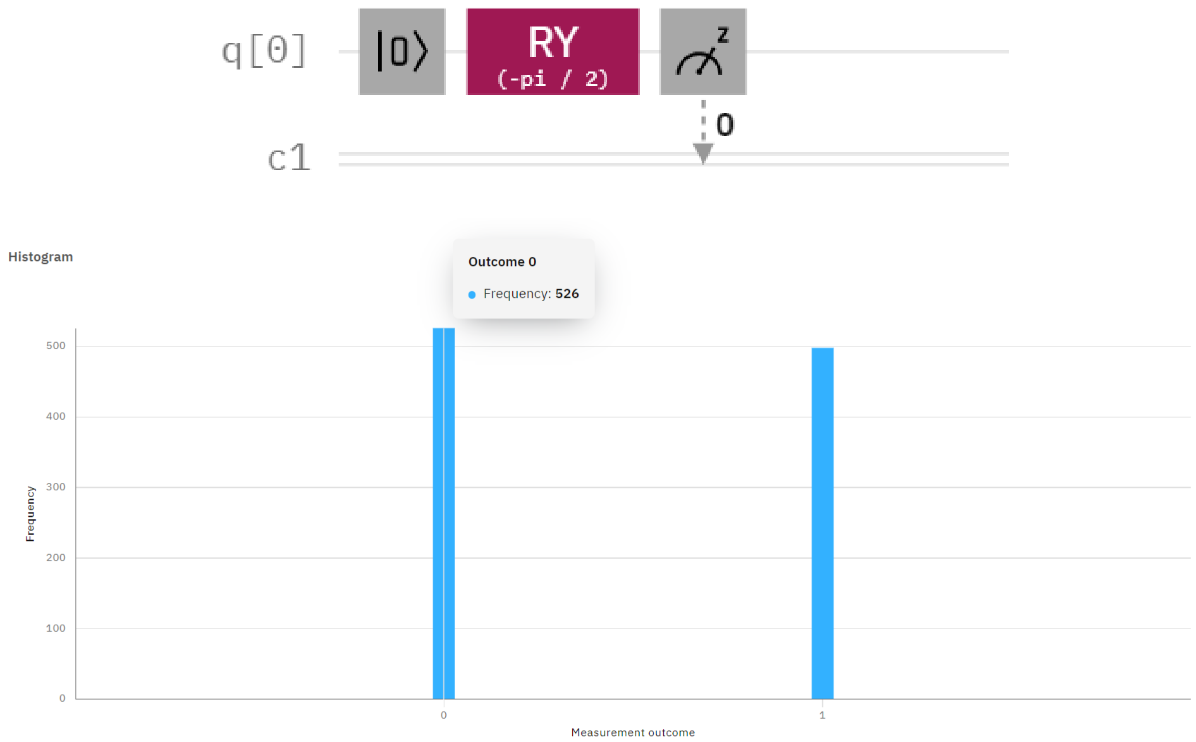

Figure 1. Histogram of the result of α=-π/4 on basis α run on IBMQ Manila

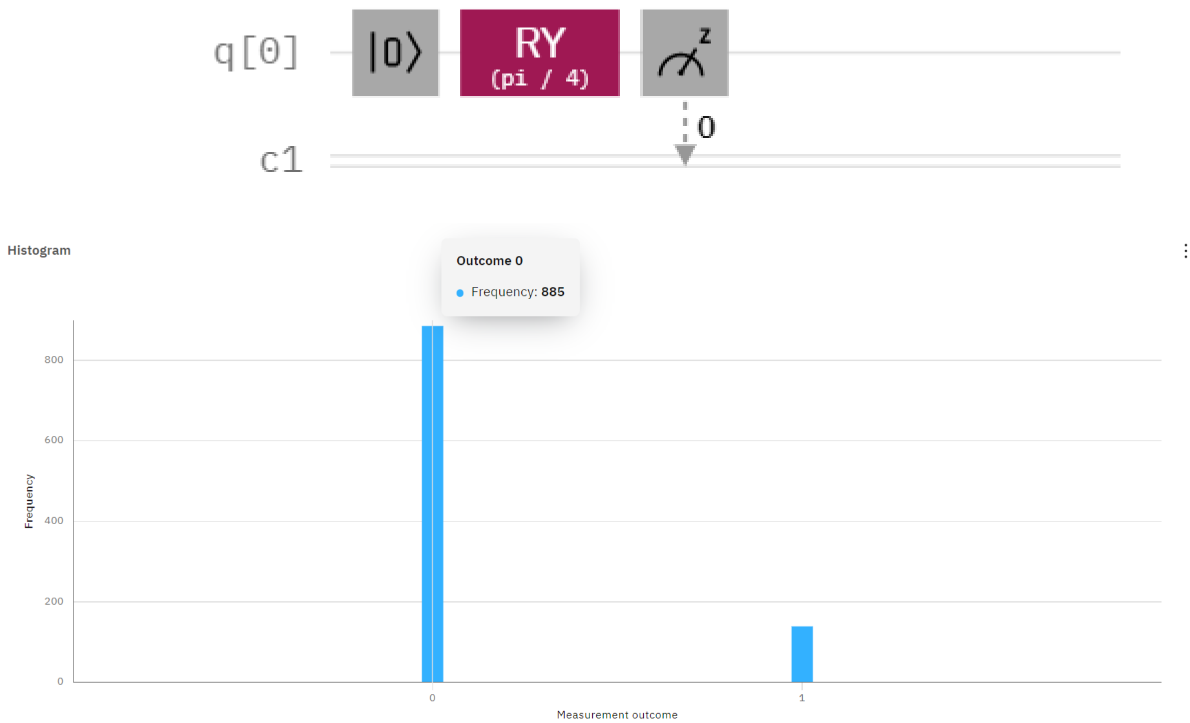

Figure 2. Histogram of the result of α=π/8 on basis α, run on IBMQ Manila

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3.

$$A = (+1)|a\rangle\langle a| + (-1)|a_\perp\rangle\langle a_\perp|$$

$$A = (+1)\,\Pi_a + (-1)\,\Pi_{a_\perp}$$

$$\langle\Psi|A|\Psi\rangle = (+1)\underbrace{\langle\Psi|\Pi_a|\Psi\rangle}_{P(p=+1)} + (-1)\underbrace{\langle\Psi|\Pi_{a_\perp}|\Psi\rangle}_{Prob(p=-1)} = E(p)$$

(Expectation value)

$$\langle\Psi|A|\Psi\rangle = (+1)\cos^2\alpha + (-1)\sin^2\alpha = \cos(2\alpha)$$

$$E(p) \text{ for } \alpha = -\frac{\pi}{4} \longrightarrow \cos\left(2\cdot\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$$

$$E(p) \text{ for } \alpha = +\frac{\pi}{8} \longrightarrow \cos\left(2\cdot\left(\frac{\pi}{8}\right)\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \simeq 0.71$$

Let's check the experimental expected values:

For $\alpha = -\frac{\pi}{4}$

$$P(p=+1) = P(|\Psi\rangle \rightarrow |a\rangle) = P(|a\rangle \rightarrow |\Psi\rangle) = \frac{526}{1024}$$

$$P(p=-1) = \frac{1024-526}{1024} = \frac{498}{1024}$$

$$E(p) = +1.\frac{526}{1024} - 1.\frac{498}{1024} = \frac{28}{1024} = 0.027 \simeq 0 \checkmark$$

For $\alpha = \frac{\pi}{8}$

$$P(p=+1) = \frac{885}{1024}$$

$$P(p=-1) = \frac{1024-885}{1024} = \frac{139}{1024}$$

$$E(p) = \frac{885}{1024} - \frac{139}{1024} = \frac{746}{1024} \simeq 0.72 \checkmark$$

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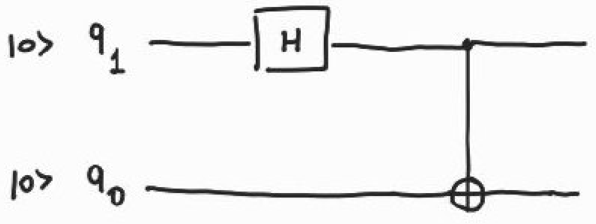
State Preparation

1.

a)

$$|\psi_0\rangle = |B_{00}\rangle = \frac{1}{\sqrt{2}}\left(\underset{q_1 q_0}{|00\rangle} + \underset{q_1 q_0}{|11\rangle}\right)$$

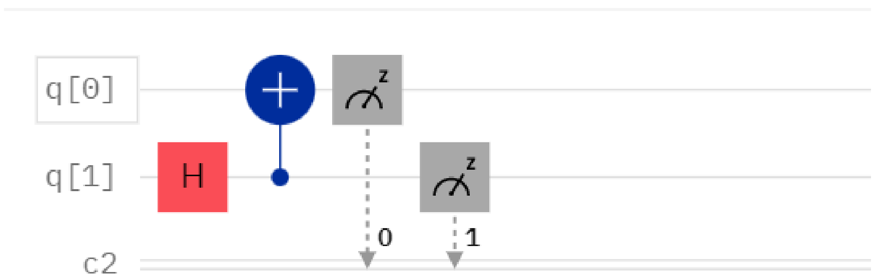


$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$\text{CNOT } H|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

! In Qiskit, qubit order is little-endian. The most significant bit (MSB) is on the left, LSB is on the right

b)



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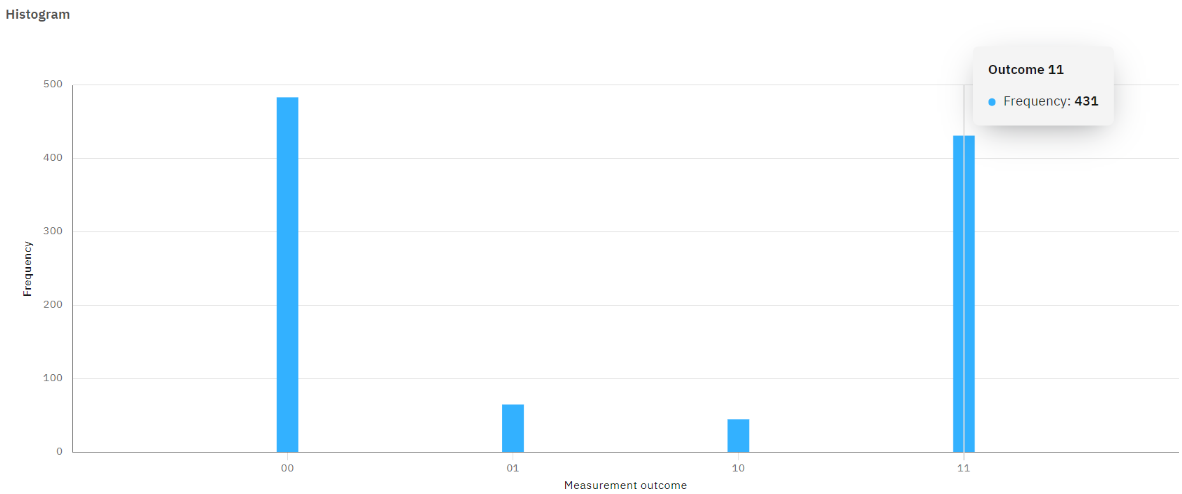


Figure 4. Histogram result of the Bell state, run on IBMQ Quito

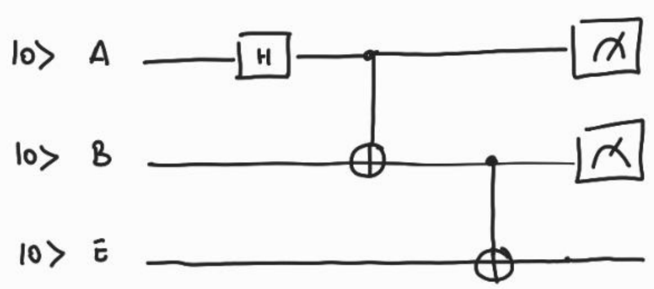
$$P(00) = \frac{483}{1024} = 47.17\% \qquad P(10) = \frac{45}{1024} = 4.39\%$$

$$P(01) = \frac{65}{1024} = 6.35\% \qquad P(11) = \frac{431}{1024} = 42.09\%$$

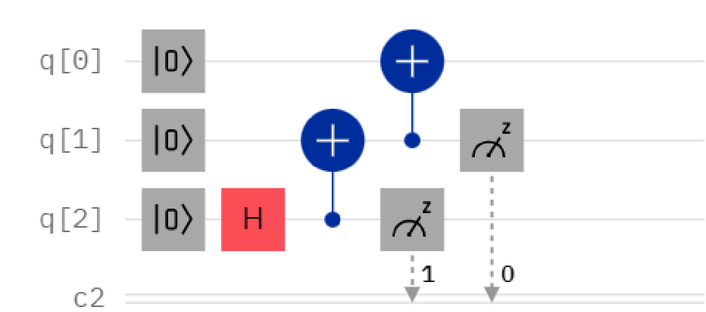
2.

a)

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle_{ABE} + |111\rangle_{ABE}\right)$$



b)



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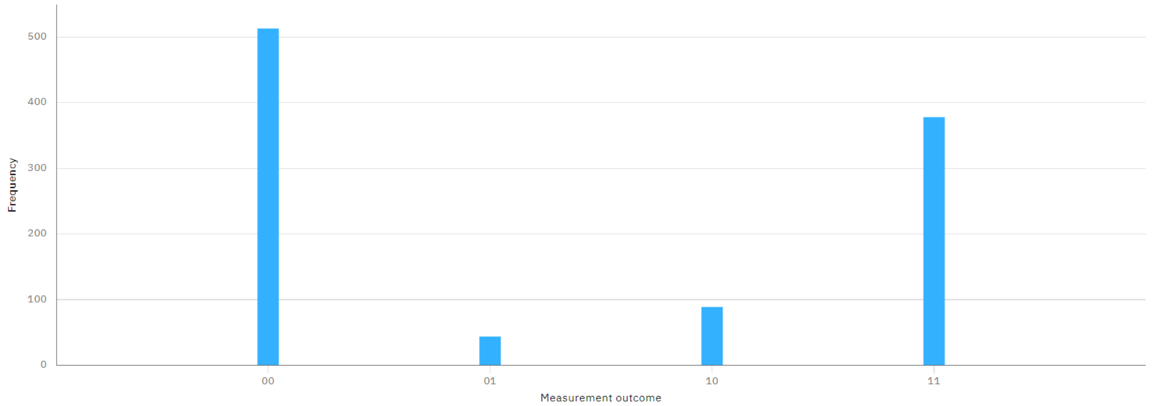

Figure 5. Histogram result of the GHZ state, run on IBMQ Quito

$$P(00) = \frac{513}{1024} \simeq 50\,\% \qquad P(10) = \frac{89}{1024} = 8.7\,\%$$

$$P(01) = \frac{44}{1024} = 4.3\,\% \qquad P(11) = \frac{378}{1024} = 36.9\,\%$$

3.

From experiment results, no difference can be observed. However, their density Matrixs are different.

Bell State :

$$\rho_{AB,\,bell} = |B_{00}\rangle\langle B_{00}| = \frac{1}{2}\left(|00\rangle + |11\rangle\right)\left(\langle 00| + \langle 11|\right)$$

$$= \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$$

Reduced Density Matrix of GHZ state:

$\langle 0|1\rangle = 0$ $\qquad$ $\langle 1|0\rangle = 0$

$$\rho_{GHZ} = |GHZ\rangle\langle GHZ| = \frac{1}{2}\left\{ |000\rangle\langle 000| + |000\rangle\langle 111| + |111\rangle\langle 000| + |111\rangle\langle 111| \right\}$$

$$\rho_{AB} = Tr_C\, \rho_{GHZ} = \frac{1}{2}\left\{ |00\rangle\langle 00| + |11\rangle\langle 11| \right\}$$

Bell State $\neq$ Statistical Mixture of $|00\rangle$ and $|11\rangle$

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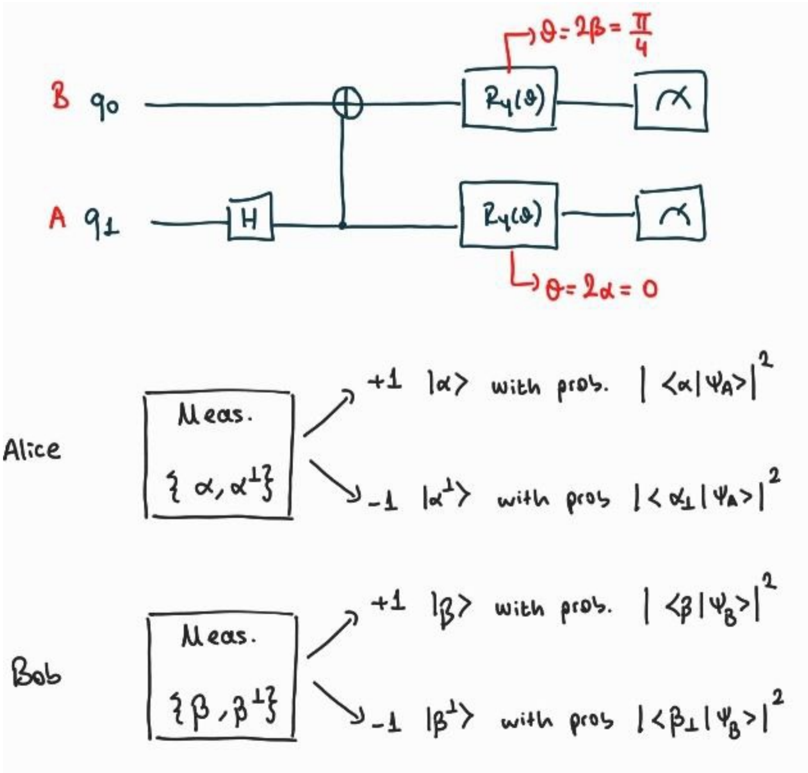
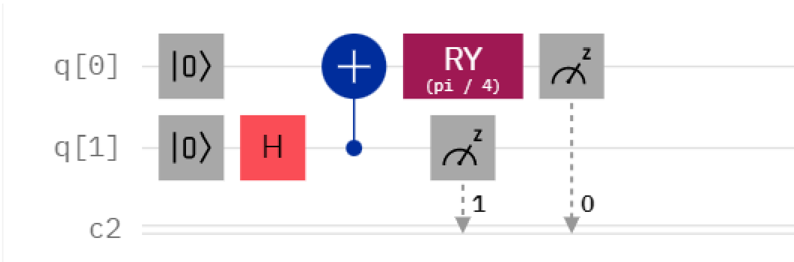
CHSH Operator

1.

a)



b)



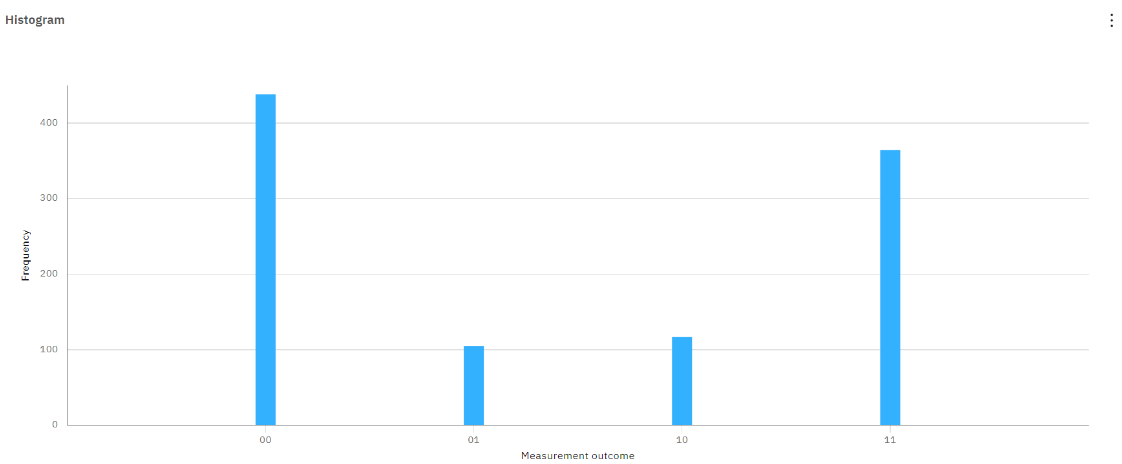


Figure 6. Histogram Result for AXB observable on IBMQ Belem

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$\rightarrow P(11) - P(10) - P(01) + P(11)$

From experimental results: $\rightarrow$ [IBMQ. Belem]

$$\frac{364-117-105+438}{1024} = \frac{580}{1024} \simeq 57\% \rightsquigarrow \text{Noisy}$$

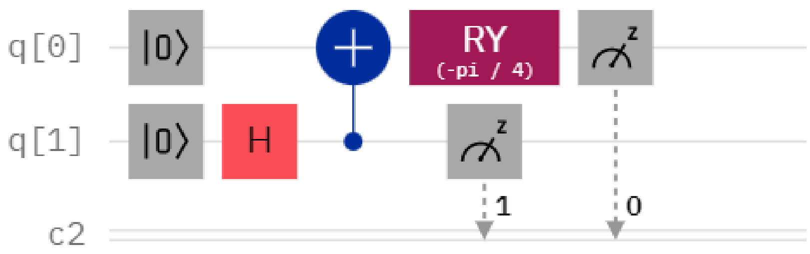
IBMQ. Qasm. Simulator Result:

$$\frac{423-84-77+440}{1024} = \frac{702}{1024} \simeq 69\% \rightsquigarrow \text{Closer Result}$$

From Lectures: $\langle\Psi| A\otimes B |\Psi\rangle = \cos(2(\alpha-\beta)) = \cos\left(2\left(0-\frac{\pi}{8}\right)\right)$

$$= \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \simeq 0.7$$

c)



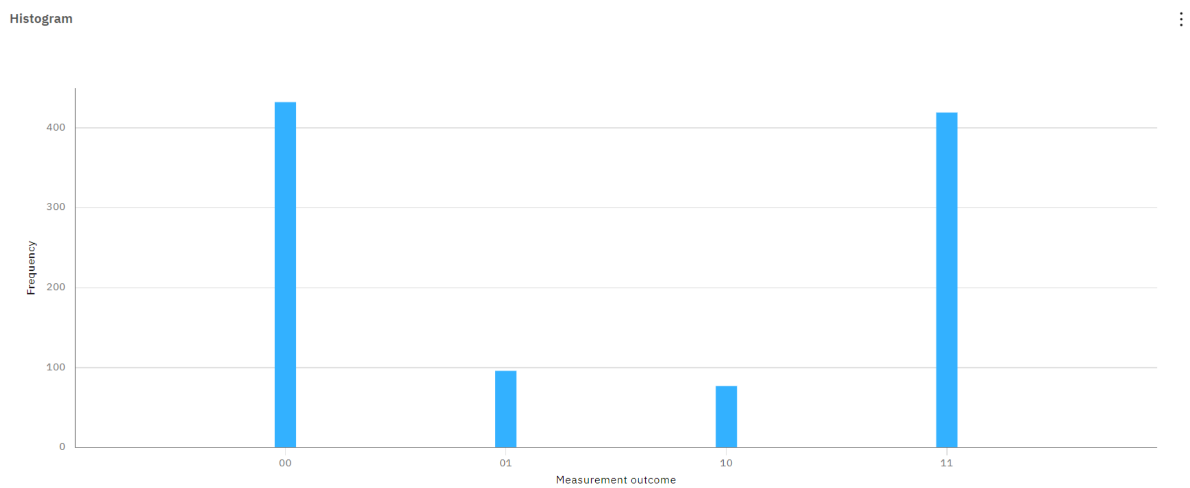


Figure 7. Histogram Result for AXB' observable on IBMQ Belem

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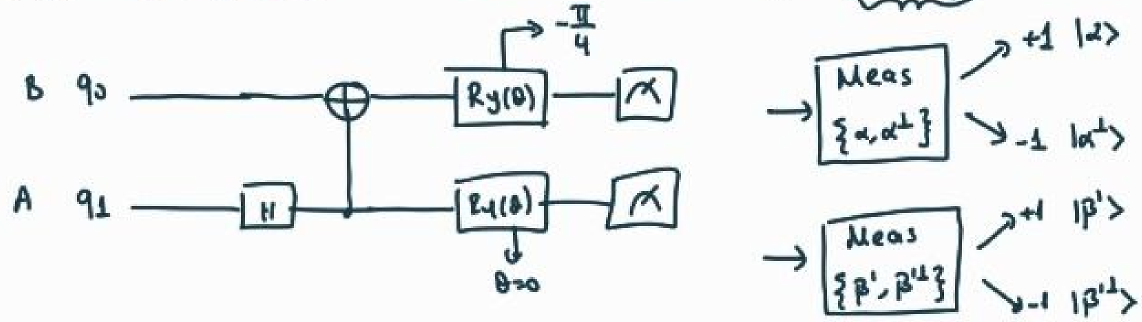
$\langle \Psi_0 | A \otimes B' | \Psi_0 \rangle$ $\quad \alpha = 0,\ \alpha' = -\frac{\pi}{4} \quad \beta = \frac{\pi}{8} \quad \beta' = -\frac{\pi}{8}$



$$\langle \Psi | A \otimes B' | \Psi_0 \rangle = \cos(2(\alpha - \beta')) = \cos\left(2\left(0 + \frac{\pi}{8}\right)\right) = \cos\left(+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

≈ 0.7

Experimental mean value:

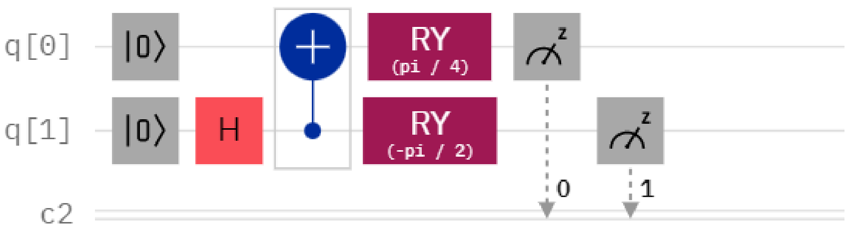
$$P(11) - P(10) - P(01) + P(00)$$

From IBMQ-Belem Results:

$$\frac{419 - 77 - 96 + 432}{1024} = \frac{678}{1024} = 66\%$$

From IBMQ-Qasm_Simulator Results:

$$\frac{462 - 63 - 76 + 423}{1024} = \frac{745}{1024} = 73\%$$



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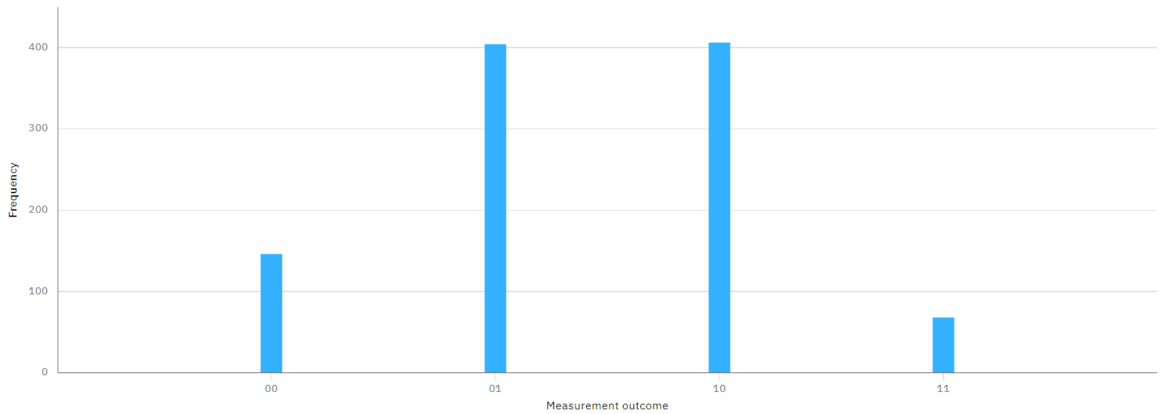
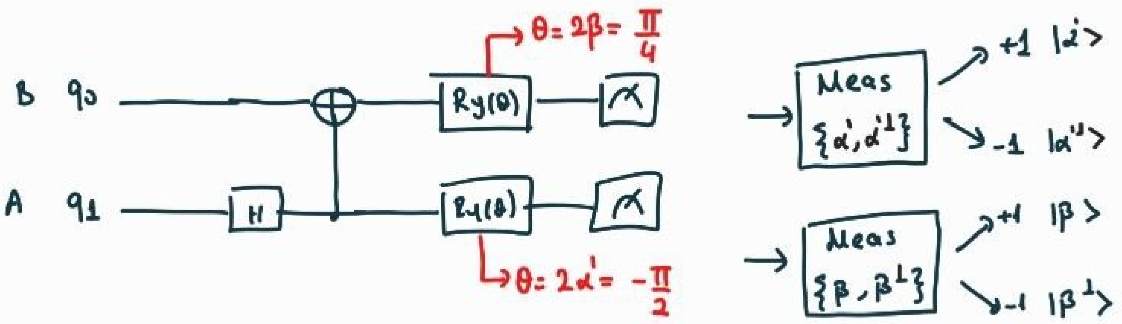


Figure 8. Histogram Result for A'XB observable on IBMQ Belem

$$\langle \Psi_0 | A' \otimes B | \Psi_0 \rangle = \cos(2(\alpha' - \beta)) = \cos\left(2.\left(-\frac{\pi}{4} - \frac{\pi}{8}\right)\right)$$

$$= \cos\left(2.\left(\frac{-3\pi}{8}\right)\right) = \cos\left(-\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -0.7$$



Experimental Mean value: $P(11) - P(10) - P(01) + P(00)$

From IBMQ. Belem Results:

$$\frac{68 - 404 - 406 + 146}{1024} = -58.2\ \%$$

From IBMQ. Qasm. Simulator Results:

$$\frac{75 - 419 - 448 + 82}{1024} = -69.3\%$$ → Closer to the ideal result

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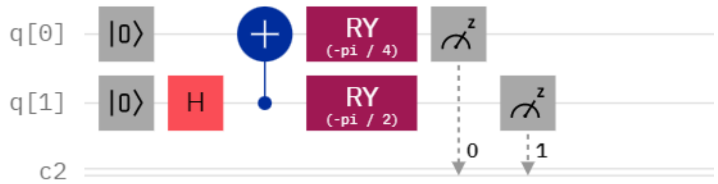


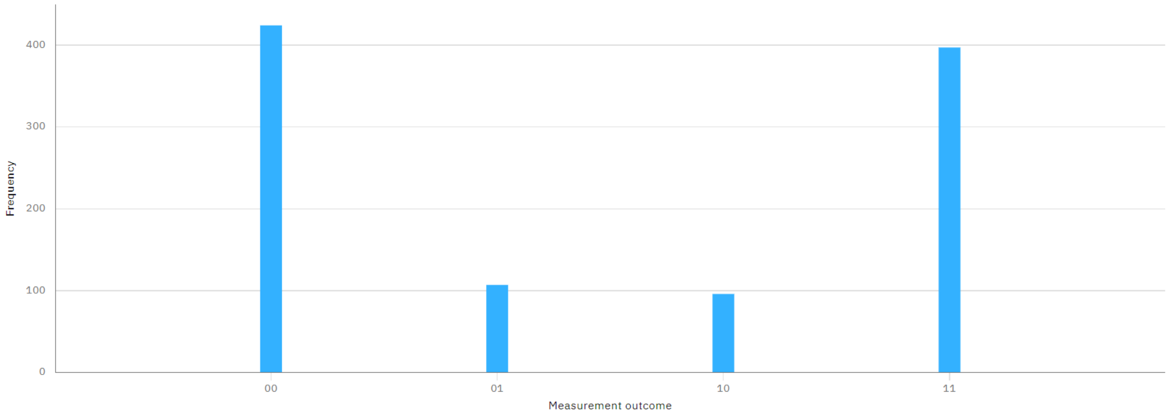
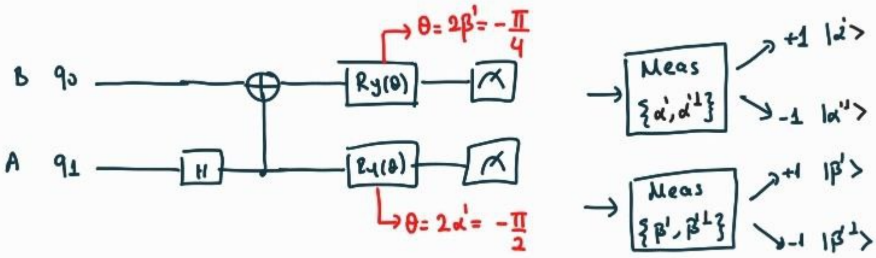


Figure 9. Histogram Result for A'XB' observable on IBMQ Belem

$$\langle \Psi_0 | A' \otimes B' | \Psi_0 \rangle = \cos(2(\alpha' - \beta')) = \cos\left(2.\left(-\frac{\pi}{4} + \frac{\pi}{8}\right)\right)$$

$$= \cos\left(\cancel{2}.\left(\frac{-\pi}{\cancel{8}_4}\right)\right) = \cos\left(\frac{\pi}{4}\right) = +\frac{1}{\sqrt{2}} = +0.7$$



Experimental Mean value: $P(11) - P(10) - P(01) + P(00)$

From IBMQ. Belem Results:

$$\frac{397 - 96 - 107 + 424}{1024} = 60.3\%$$

From IBMQ. Qasm. Simulator Results:

$$\frac{447 - 74 - 78 + 425}{1024} = 70.3\%$$ ⇝ Closer to the ideal result

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d)

$$\mathcal{B} = A \otimes B - A' \otimes B + A \otimes B' + A' \otimes B'$$

$$\langle \Psi_0 | \mathcal{B} | \Psi_0 \rangle = \langle \Psi_0 | A \otimes B | \Psi_0 \rangle - \langle \Psi_0 | A' \otimes B | \Psi_0 \rangle + \langle \Psi_0 | A \otimes B' | \Psi_0 \rangle + \langle \Psi_0 | A' \otimes B' | \Psi_0 \rangle$$

IBMQ. Belem Results $\rightsquigarrow$ Noisy

$$\langle \Psi_0 | \mathcal{B} | \Psi_0 \rangle = 57\% - (-58.2\%) + 66\% + 60.3\%$$

$$\langle \Psi_0 | \mathcal{B} | \Psi_0 \rangle = 241.5\% > 2$$

IBMQ. Qasm. Simulator Results

$$\langle \Psi_0 | \mathcal{B} | \Psi_0 \rangle = 69\% - (-69.3\%) + 73\% + 70.3\%$$

$$\langle \Psi_0 | \mathcal{B} | \Psi_0 \rangle = 281.6\% > 2$$

↳ which is the closest value to $2\sqrt{2}$ (more ideal)

Since B_{00} is an entangled, we expect inequality meaning larger value ($2\sqrt{2}$) than 2. From experiment results, it can be said that CHSH inequality is observed ✓

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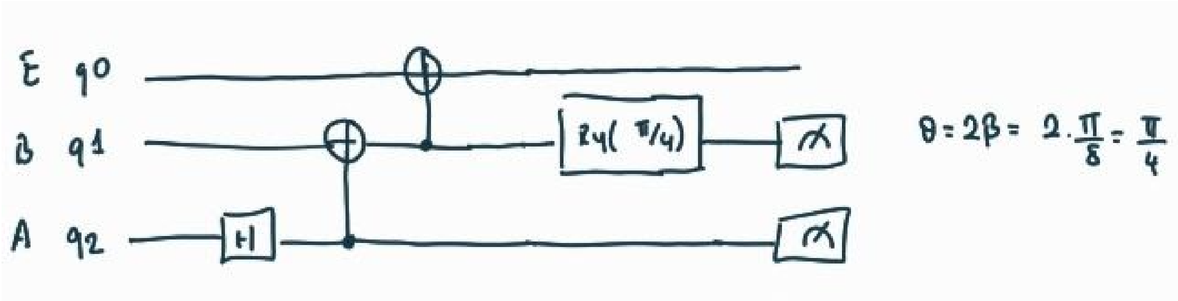
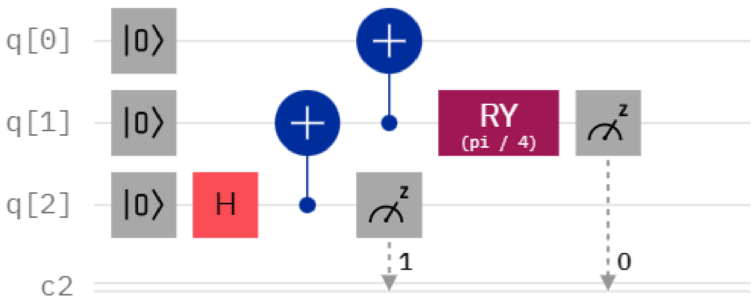
2)

a)



b)



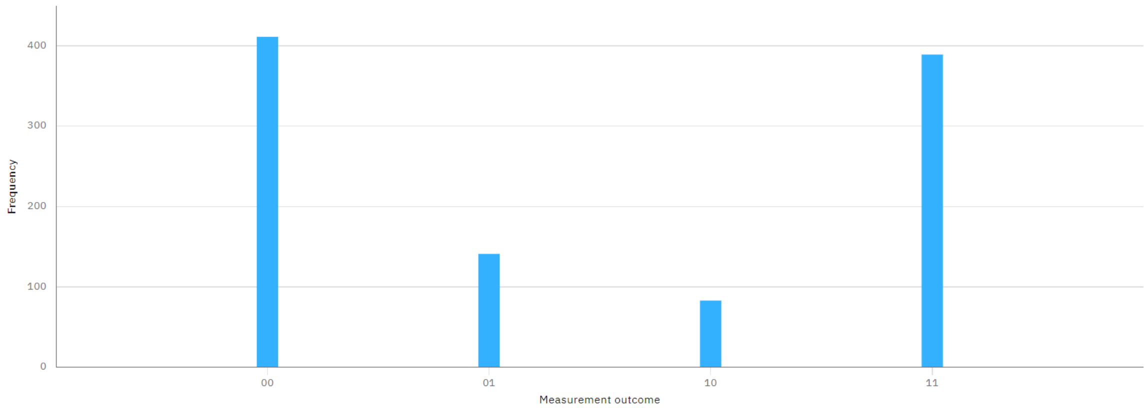


Figure 10. Histogram Result for AXB observable of Psi1 on IBMQ Belem

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$$\langle\psi_1|A\otimes B|\psi_1\rangle = Tr(A\otimes B\,\rho_{AB}) = \frac{1}{2}\langle 00|A\otimes B|00\rangle + \frac{1}{2}\langle 11|A\otimes B|11\rangle$$

$$= \frac{1}{2}\langle 0|A|0\rangle\langle 0|B|0\rangle + \frac{1}{2}\langle 1|A|1\rangle\langle 1|B|1\rangle$$

$$= (\cos^2\alpha - \sin^2\alpha)(\cos^2\beta - \sin^2\beta) = \cos 2\alpha \cos 2\beta = \cos 0 . \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = 0.7$$

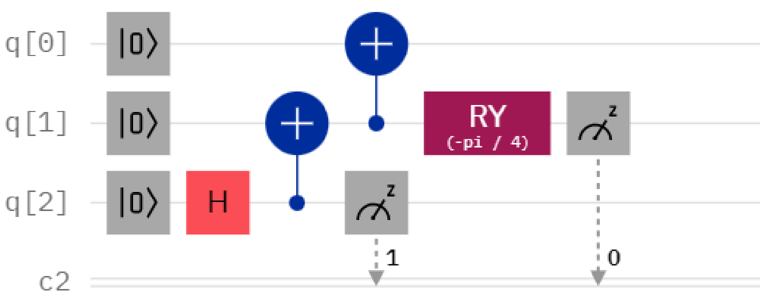
From Experimental Results IBMQ-Belem:

$$P(11) - P(10) - P(01) + P(00) = \frac{389 - 83 - 141 + 411}{1024} = 56.25\%$$

From Experimental Results IBMQ-Qasm:

$$P(11) - P(10) - P(01) + P(00) = \frac{382 - 76 - 84 + 482}{1024} = 68.75\%$$

c)



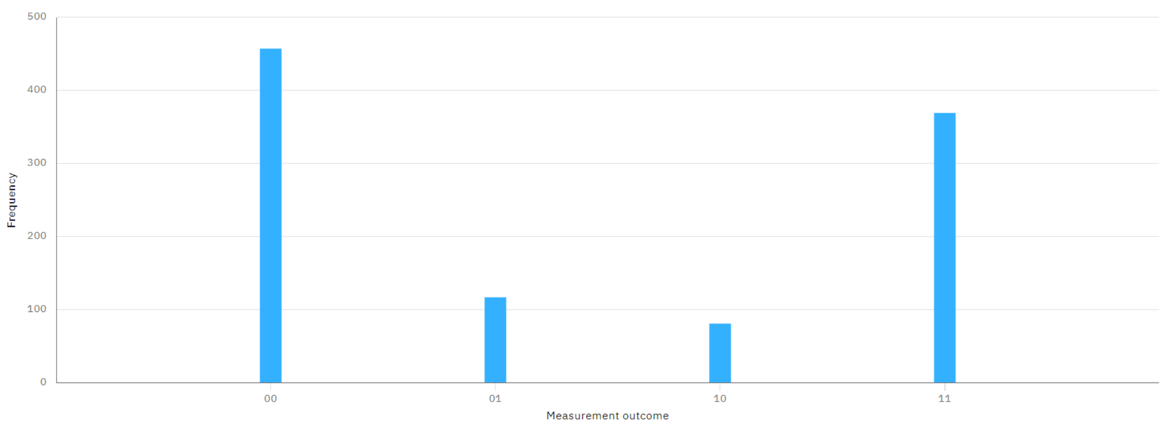


Figure 11. Histogram Result for AXB' observable of Psi1 on IBMQ Belem

$$\langle\psi_1|A\otimes B'|\psi_1\rangle = \cos 2\alpha \cos 2\beta' = \cos 0 . \cos(-\tfrac{\pi}{4}) = \frac{1}{\sqrt{2}} = 0.7$$

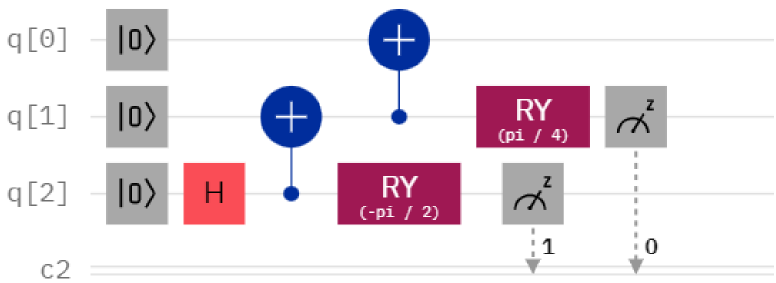
From Experimental Results IBMQ-Belem:

$$+P(11) - P(10) - P(01) + P(00) = \frac{+369 - 81 - 117 + 457}{1024} = 61.3\%$$

From Experimental Results IBMQ-Qasm:

$$+P(11) - P(10) - P(01) + P(00) = \frac{+458 - 85 - 58 + 423}{1024} = 72\%$$

d)



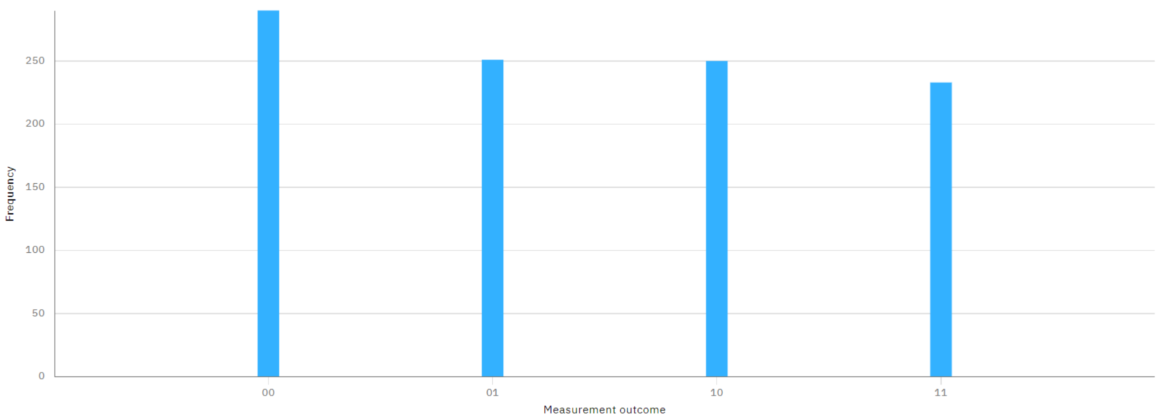


Figure 12. Histogram Result for A’xB observable of Psi1 on IBMQ Belem

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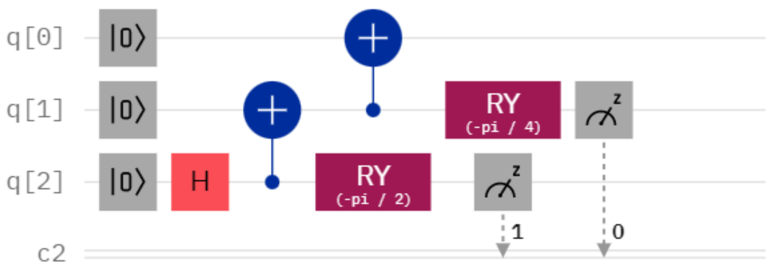
$$\langle \Psi_1 | A' \otimes B | \Psi_1 \rangle = \cos(2\alpha')\cos(2\beta) = \underbrace{\cos\left(2\cdot\left(-\frac{\pi}{4}\right)\right)}_{0} \cdot \cos\left(2\cdot\frac{\pi}{8}\right) = 0$$

From Experimental Results IBMQ-Belem:

$$+P(11) - P(10) - P(01) + P(00) = \frac{233 - 250 - 251 + 290}{1024} = 2.1\,\%$$

From Experimental results IBMQ-Qasm:

$$+P(11) - P(10) - P(01) + P(00) = \frac{247 - 264 - 262 + 251}{1024} = -2.7\,\%$$



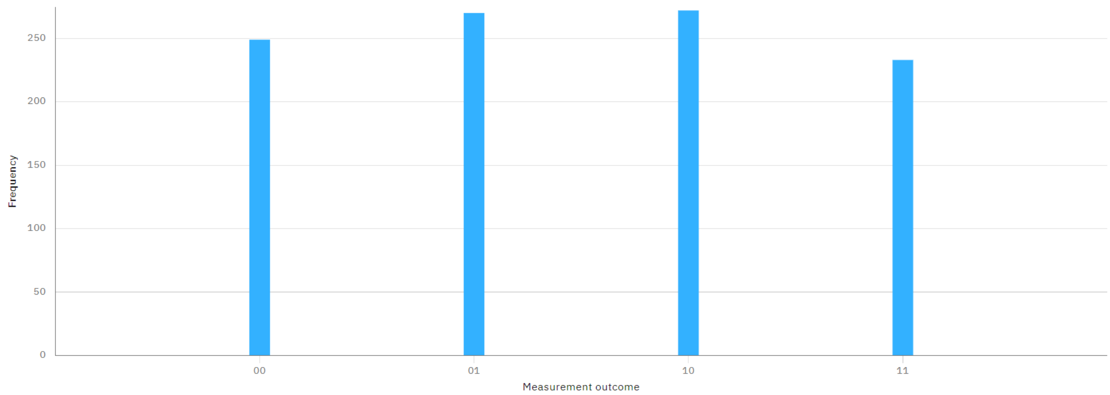


Figure 12. Histogram Result for A’xB’ observable of Psi1 on IBMQ Belem

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$$\langle \Psi_1 | A' \otimes B' | \Psi_1 \rangle = \cos(2\alpha')\cos(2\beta') = \underbrace{\cos\left(2\cdot\left(-\frac{\pi}{4}\right)\right)}_{0} \cdot \cos\left(2\left(\frac{-\pi}{8}\right)\right) = 0$$

From Experimental Results IBMQ-Belem:

$$+P(11) - P(10) - P(01) + P(00) = \frac{233 - 272 - 270 + 249}{1024} = -5.8\%$$

From Experimental Results IBMQ-Qasm:

$$+P(11) - P(10) - P(01) + P(00) = \frac{255 - 240 - 240 + 289}{1024} = 6.2\%$$

d)

$$\langle \Psi_1 | \mathcal{B} | \Psi_1 \rangle = \langle \Psi_1 | A \otimes B | \Psi_1 \rangle - \langle \Psi_1 | A' \otimes B | \Psi_1 \rangle + \langle \Psi_1 | A \otimes B' | \Psi_1 \rangle + \langle \Psi_1 | A' \otimes B' | \Psi_1 \rangle$$

Theoritically: $= \frac{1}{\sqrt{2}} - 0 + \frac{1}{\sqrt{2}} + 0 = \underbrace{\sqrt{2}}_{1.41} < 2$

Experiment Results from IBMQ. Belem:

$$\langle \Psi_1 | \mathcal{B} | \Psi_1 \rangle = 56.25\% - 2.1\% + 61.3\% + (-5.8\%) = 109.65\% < 2$$

Experiment Results from IBMQ. Qasm:

$$\langle \Psi_1 | \mathcal{B} | \Psi_1 \rangle = 68.75\% - (-2.7\%) + 72\% + 6.2\% = 149.65\% < 2$$

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Bonus

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In this part, 9 random angle samples and their CHSH expectation for both Bell and GHZ states are obtained.

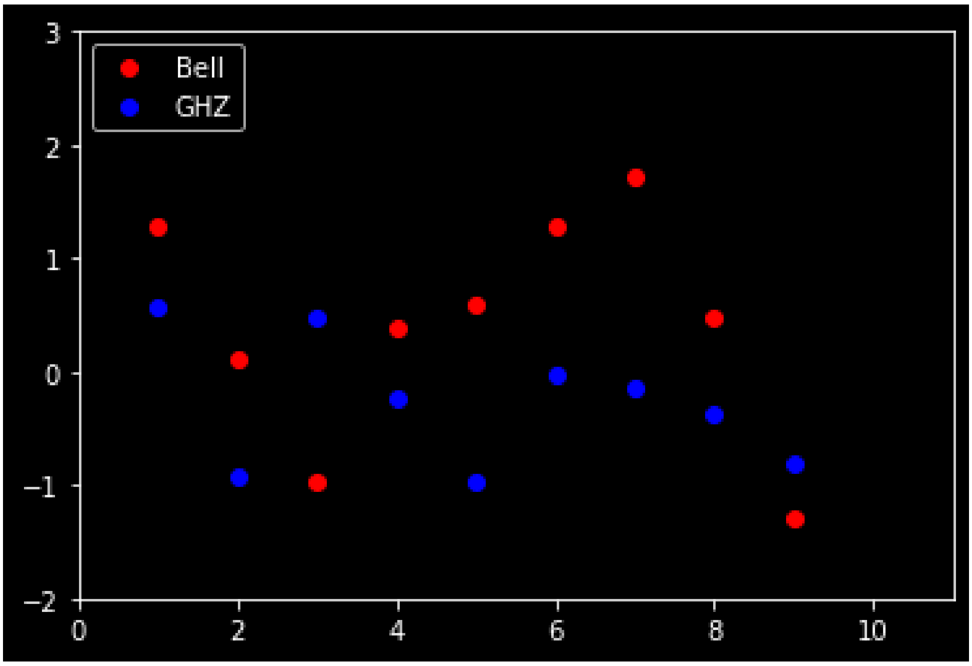

Figure 13. Plot for the CHSH expectations of Bell and GHZ states for 9 different angle samples

Appendix

CHSH Operator-1

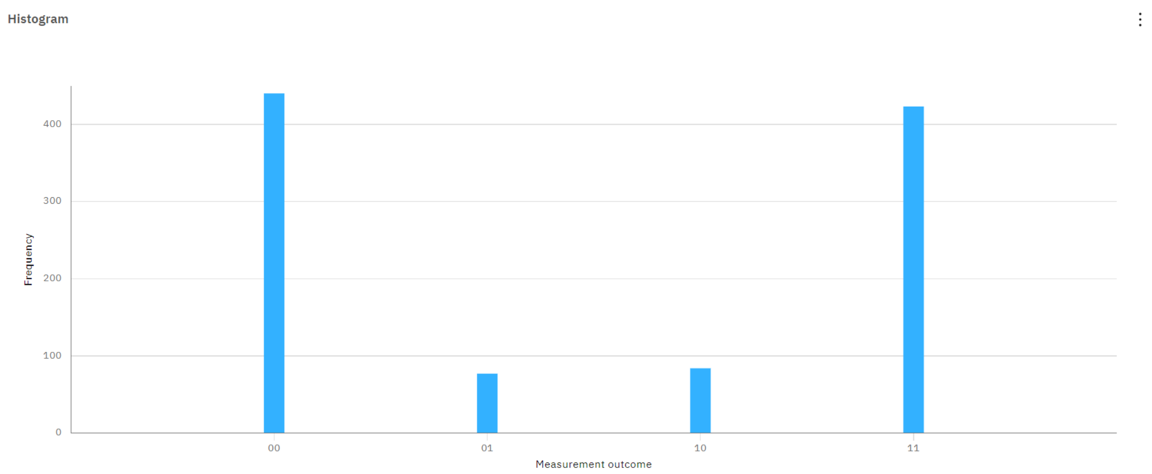

Figure 14. Histogram Result for AXB observable on IBMQ_QASM Simulator

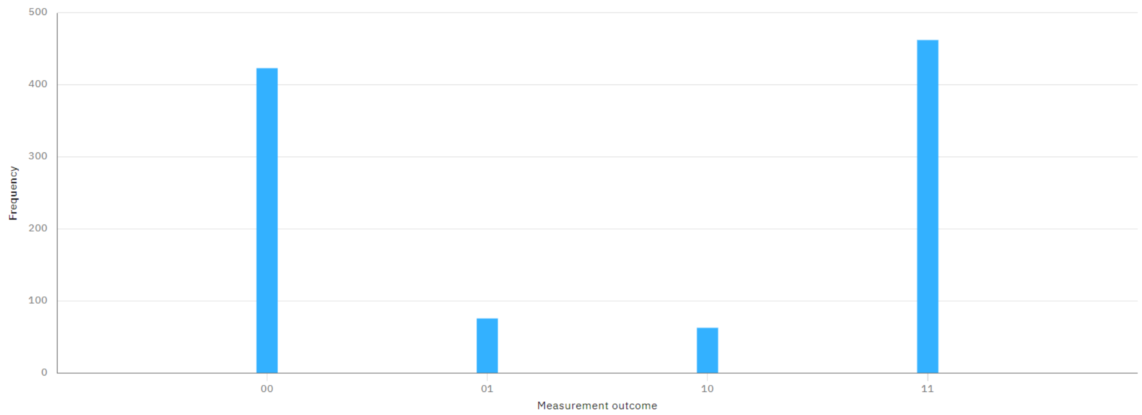

Figure 15. Histogram Result for AXB’ observable on IBMQ_QASM Simulator

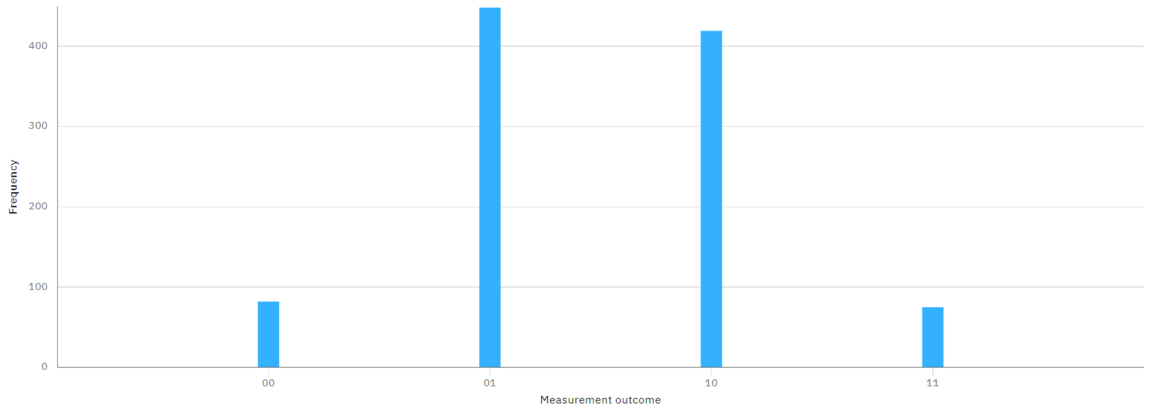

Figure 16. Histogram Result for A’XB observable on IBMQ_QASM Simulator

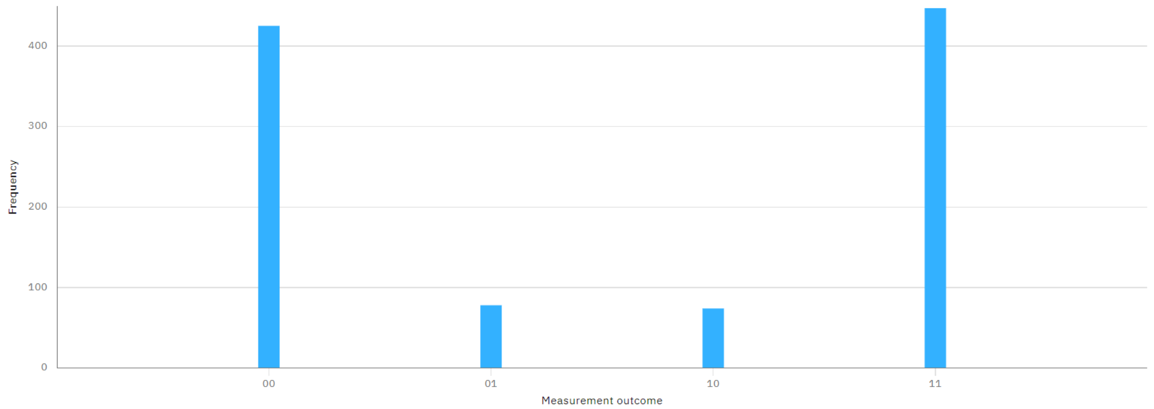

Figure 17. Histogram Result for A’XB’ observable on IBMQ_QASM Simulator

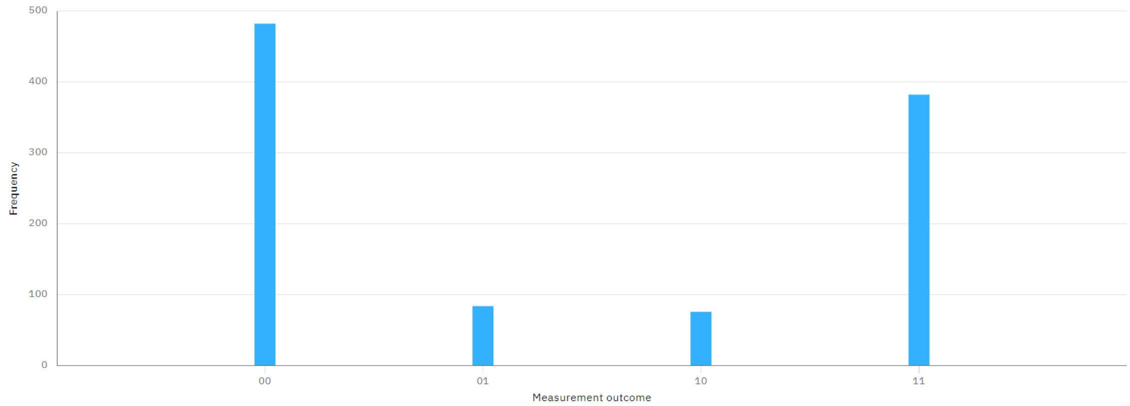

Figure 18. Histogram Result for AXB observable of Psi1 on IBMQ_QASM Simulator

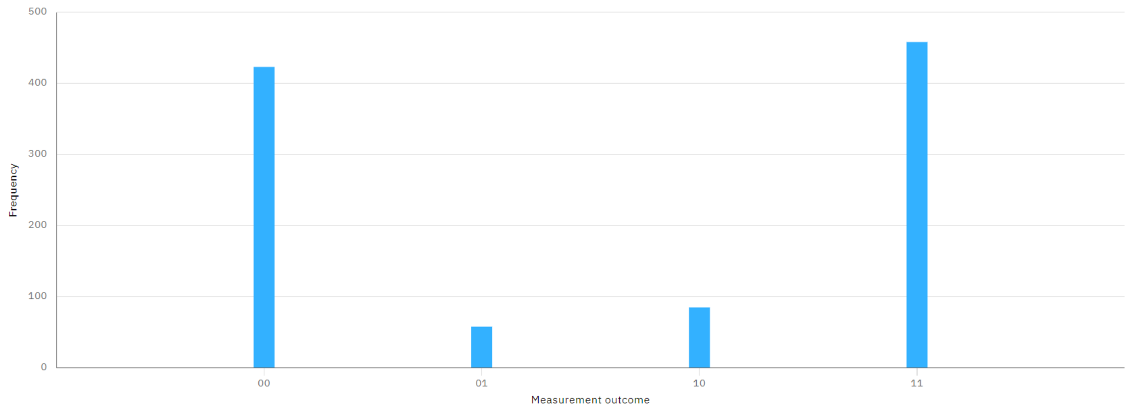

Figure 19. Histogram Result for AXB' observable of Psi1 on IBMQ_QASM Simulator

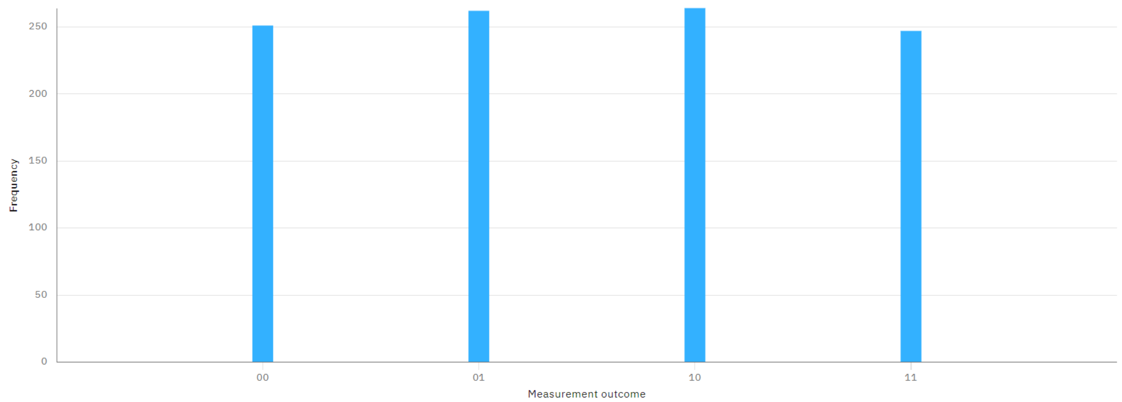

Figure 20. Histogram Result for A'XB observable of Psi1 on IBMQ_QASM Simulator

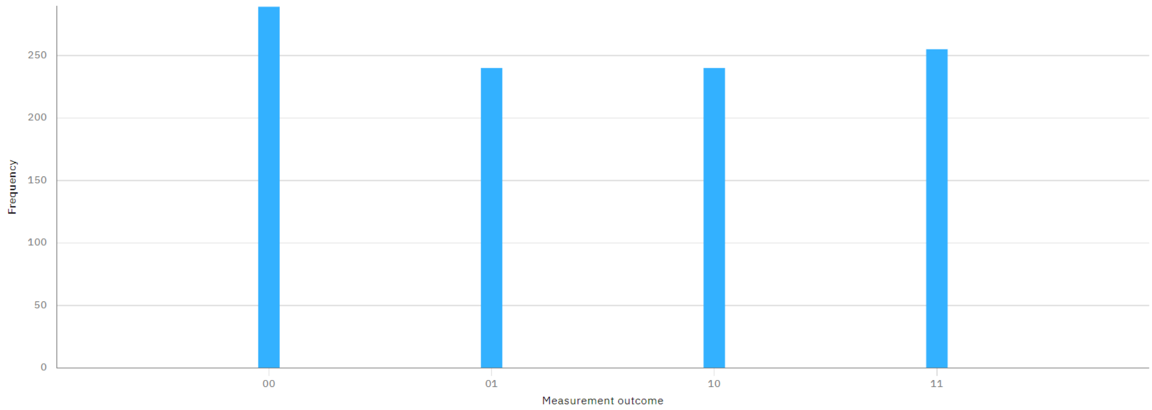

Figure 21. Histogram Result for A'XB' observable of Psi1 on IBMQ_QASM Simulator

Bonus Part Code Snippets:

```
[1]: import numpy as np
     import random
     import matplotlib.pyplot as plt

     # Importing standard Qiskit libraries
     from qiskit import QuantumCircuit, transpile, Aer, IBMQ
     from qiskit.tools.jupyter import *
     from qiskit.visualization import *
     from ibm_quantum_widgets import *
     from qiskit.providers.aer import QasmSimulator

     # Loading your IBM Quantum account(s)
     provider = IBMQ.load_account()

     from qiskit import QuantumRegister, ClassicalRegister, execute

     from qiskit.quantum_info import Statevector
```

```
[2]: #Register and Circuit Creation
     q=QuantumRegister(2,'q')
     c=ClassicalRegister(2,'c')

     q_ghz=QuantumRegister(3,'q_ghz')
     c_ghz=ClassicalRegister(2,'c_ghz')

     Bell=QuantumCircuit(q,c)
     GHZ=QuantumCircuit(q_ghz,c_ghz)

     Exp_meas_bell_col=[]
     Exp_meas_ghz_col=[]
     Exp_meas_bell=[] #becomes 2-d array
     Exp_meas_ghz=[] #later becomes 2-d array
     Exp_CHSH_bell=[]
     Exp_CHSH_ghz=[]

     Angle_array=[]

     shots=1024 #number of samples
     backend=provider.get_backend('ibmq_qasm_simulator')
```

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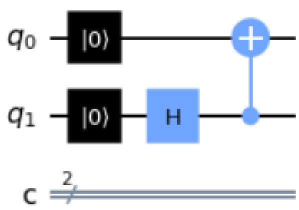
[3]:
```
#state preparation for Bell
Bell.reset(q[1])
Bell.reset(q[0])

Bell.h(q[1])
Bell.cx(q[1],q[0])

Bell.draw('mpl')
```

[3]:

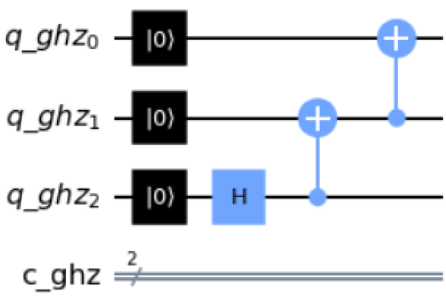


[4]:
```
#state preparation for GHZ
GHZ.reset(q_ghz[2])
GHZ.reset(q_ghz[1])
GHZ.reset(q_ghz[0])

GHZ.h(q_ghz[2])
GHZ.cx(q_ghz[2],q_ghz[1])
GHZ.cx(q_ghz[1],q_ghz[0])

GHZ.draw('mpl')
```

[4]:



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```
[*]: for i in range(9):
#Angle Initialization
    alpha=random.randrange(-90,90,2)*np.pi/180
    beta=random.randrange(-90,90,2)*np.pi/180
    alpha_=random.randrange(-90,90,2)*np.pi/180
    beta_=random.randrange(-90,90,2)*np.pi/180

    Angle_array=[alpha, beta, alpha_, beta_]
    print("Angle_array=", Angle_array)

    #Measurement in AxB basis-alpha,beta
    theta0ab=2*beta
    theta1ab=2*alpha
    measureAB=QuantumCircuit(2,2)
    measureAB.ry(theta0ab,0)
    measureAB.ry(theta1ab,1)

    #Measurement in A'xB basis-alpha',beta
    theta0a_b=2*beta
    theta1a_b=2*alpha_
    measureA_B=QuantumCircuit(2,2)
    measureA_B.ry(theta0a_b,0)
    measureA_B.ry(theta1a_b,1)
```

```
    #Measurement in AxB' basis-alpha,beta
    theta0ab_=2*beta_
    theta1ab_=2*alpha
    measureAB_=QuantumCircuit(2,2)
    measureAB_.ry(theta0ab_,0)
    measureAB_.ry(theta1ab_,1)

    #Measurement in A'xB' basis-alpha,beta
    theta0a_b_=2*beta_
    theta1a_b_=2*alpha_
    measureA_B_=QuantumCircuit(2,2)
    measureA_B_.ry(theta0a_b_,0)
    measureA_B_.ry(theta1a_b_,1)
```

```
#Compose both Bell State and Different Measurement and calculate probabilities
for state_init in [Bell, GHZ]:
    for measure_circuit in [measureAB, measureA_B, measureAB_, measureA_B_]:

        #run the circuit with the selected measurement and the counts from the simulation
        if state_init == Bell:
            qc=state_init.compose(measure_circuit)
            qc.measure(q[1],c[1])
            qc.measure(q[0],c[0])
        else:
            qc=state_init.compose(measure_circuit, qubits=[2,1])
            qc.measure(q_ghz[2],c_ghz[1])
            qc.measure(q_ghz[1],c_ghz[0])

        job=execute(qc, backend, shots=shots)
        counts=job.result().get_counts()

        #calculate the probabilities from counts results
        probs={}
        for output in ['00','01','10','11']:
            if output in counts:
                probs[output]=counts[output]/shots
            else:
                probs[output]=0
```

```
            #Expection Value=P(11)+P(00)-P(10)-P(01)
            if state_init == Bell:
                Exp_meas_bell.append(probs['11']+probs['00']-probs['10']-probs['01'])

            else:
                Exp_meas_ghz.append(probs['11']+probs['00']-probs['10']-probs['01'])


        if state_init == Bell:
            Exp_CHSH_bell.append(Exp_meas_bell[4*i+0]-Exp_meas_bell[4*i+1]+Exp_meas_bell[4*i+2]+Exp_meas_bell[4*i+3])
            print("Exp_meas_bell=", Exp_meas_bell)
        else:
            Exp_CHSH_ghz.append(Exp_meas_ghz[4*i+0]-Exp_meas_ghz[4*i+1]+Exp_meas_ghz[4*i+2]+Exp_meas_ghz[4*i+3])
            print("Exp_meas_ghz=", Exp_meas_ghz)



    print("Exp_CHSH_bell[%d]=" %i, Exp_CHSH_bell[i])
    print("Exp_CHSH_ghz[%d]="%i, Exp_CHSH_ghz[i])


print("Exp_CHSH_bell=", Exp_CHSH_bell)
print("Exp_CHSH_ghz=", Exp_CHSH_ghz)
```

```
Angle_array= [-0.9075712110370514, -1.1519173063162575,
-1.117010721276371, -1.43116998663535]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875]
Exp_CHSH_bell[0]= 1.279296875
Exp_CHSH_ghz[0]= 0.5703125

Angle_array= [1.3264502315156903, 0.5235987755982988,
-0.3141592653589793, 1.2566370614359172]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125]
Exp_CHSH_bell[1]= 0.115234375
Exp_CHSH_ghz[1]= -0.923828125

Angle_array= [0.9075712110370514, -1.2566370614359172,
0.4886921905584123, -1.1868238913561442]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0, -0.3984375,
-0.939453125, -0.52734375, -0.98046875]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125, 0.224609375,
-0.4609375, 0.224609375, -0.42578125]
Exp_CHSH_bell[2]= -0.966796875
Exp_CHSH_ghz[2]= 0.484375

Angle_array= [-1.43116998663535, -0.5934119456780721,
-0.8377580409572781, -0.8377580409572781]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0, -0.3984375,
-0.939453125, -0.52734375, -0.98046875, -0.08203125, 0.900390625,
```

```
0.37109375, 1.0]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125, 0.224609375,
-0.4609375, 0.224609375, -0.42578125, -0.3671875, -0.02734375,
0.111328125, 0.005859375]
Exp_CHSH_bell[3]= 0.388671875
Exp_CHSH_ghz[3]= -0.22265625

Angle_array= [-0.8377580409572781, -1.3962634015954636,
-1.3962634015954636, -0.6981317007977318]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0, -0.3984375,
-0.939453125, -0.52734375, -0.98046875, -0.08203125, 0.900390625,
0.37109375, 1.0, 0.45703125, 1.0, 0.97265625, 0.169921875]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125, 0.224609375,
-0.4609375, 0.224609375, -0.42578125, -0.3671875, -0.02734375,
0.111328125, 0.005859375, 0.107421875, 0.857421875, -0.0546875,
-0.158203125]
Exp_CHSH_bell[4]= 0.599609375
Exp_CHSH_ghz[4]= -0.962890625

Angle_array= [0.767944870877505, 0.3839724354387525,
-0.9773843811168246, -0.10471975511965977]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0, -0.3984375,
-0.939453125, -0.52734375, -0.98046875, -0.08203125, 0.900390625,
0.37109375, 1.0, 0.45703125, 1.0, 0.97265625, 0.169921875,
0.73046875, -0.89453125, -0.1875, -0.15625]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125, 0.224609375,
-0.4609375, 0.224609375, -0.42578125, -0.3671875, -0.02734375,
0.111328125, 0.005859375, 0.107421875, 0.857421875, -0.0546875,
-0.158203125, 0.03125, -0.298828125, 0.0390625, -0.390625]
Exp_CHSH_bell[5]= 1.28125
Exp_CHSH_ghz[5]= -0.021484375

Angle_array= [-0.9424777960769379, -1.1519173063162575,
-1.0471975511965976, -0.767944870877505]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0, -0.3984375,
-0.939453125, -0.52734375, -0.98046875, -0.08203125, 0.900390625,
0.37109375, 1.0, 0.45703125, 1.0, 0.97265625, 0.169921875,
0.73046875, -0.89453125, -0.1875, -0.15625, 0.921875, 0.984375,
0.9453125, 0.830078125]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125, 0.224609375,
-0.4609375, 0.224609375, -0.42578125, -0.3671875, -0.02734375,
0.111328125, 0.005859375, 0.107421875, 0.857421875, -0.0546875,
-0.158203125, 0.03125, -0.298828125, 0.0390625, -0.390625,
0.2578125, 0.34765625, 0.021484375, -0.078125]
Exp_CHSH_bell[6]= 1.712890625
Exp_CHSH_ghz[6]= -0.146484375
```

```
Angle_array= [0.8726646259971648, 0.3490658503988659,
1.2217304763960306, 0.20943951023931953]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0, -0.3984375,
-0.939453125, -0.52734375, -0.98046875, -0.08203125, 0.900390625,
0.37109375, 1.0, 0.45703125, 1.0, 0.97265625, 0.169921875,
0.73046875, -0.89453125, -0.1875, -0.15625, 0.921875, 0.984375,
0.9453125, 0.830078125, 0.51171875, -0.15234375, 0.271484375,
-0.462890625]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125, 0.224609375,
-0.4609375, 0.224609375, -0.42578125, -0.3671875, -0.02734375,
0.111328125, 0.005859375, 0.107421875, 0.857421875, -0.0546875,
-0.158203125, 0.03125, -0.298828125, 0.0390625, -0.390625,
0.2578125, 0.34765625, 0.021484375, -0.078125, -0.07421875,
-0.576171875, -0.17578125, -0.70703125]
Exp_CHSH_bell[7]= 0.47265625
Exp_CHSH_ghz[7]= -0.380859375

Angle_array= [-1.53588974175501, 0.8028514559173915,
0.767944870877505, 0.3141592653589793]
Exp_meas_bell= [0.9140625, 0.99609375, 0.552734375, 0.80859375,
-0.025390625, -0.150390625, 0.990234375, -1.0, -0.3984375,
-0.939453125, -0.52734375, -0.98046875, -0.08203125, 0.900390625,
0.37109375, 1.0, 0.45703125, 1.0, 0.97265625, 0.169921875,
0.73046875, -0.89453125, -0.1875, -0.15625, 0.921875, 0.984375,
0.9453125, 0.830078125, 0.51171875, -0.15234375, 0.271484375,
-0.462890625, -0.064453125, 0.99609375, -0.841796875, 0.609375]
Exp_meas_ghz= [0.091796875, 0.37890625, 0.220703125, 0.63671875,
-0.451171875, 0.486328125, 0.689453125, -0.67578125, 0.224609375,
-0.4609375, 0.224609375, -0.42578125, -0.3671875, -0.02734375,
0.111328125, 0.005859375, 0.107421875, 0.857421875, -0.0546875,
-0.158203125, 0.03125, -0.298828125, 0.0390625, -0.390625,
0.2578125, 0.34765625, 0.021484375, -0.078125, -0.07421875,
-0.576171875, -0.17578125, -0.70703125, 0.048828125, 0.009765625,
-0.822265625, -0.021484375]
Exp_CHSH_bell[8]= -1.29296875
Exp_CHSH_ghz[8]= -0.8046875


Exp_CHSH_bell= [1.279296875, 0.115234375, -0.966796875,
0.388671875, 0.599609375, 1.28125, 1.712890625, 0.47265625,
-1.29296875]

Exp_CHSH_ghz= [0.5703125, -0.923828125, 0.484375, -0.22265625,
-0.962890625, -0.021484375, -0.146484375, -0.380859375,
-0.8046875]
```