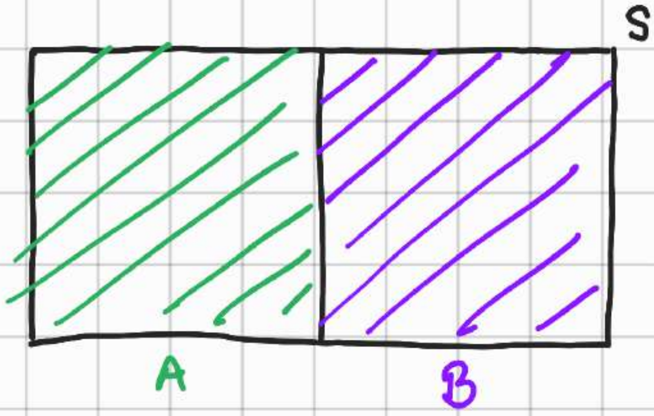
Bipartite Systems - Entanglement

I. Density Matrix



1) Bipartite System

$$\mathcal{H}_S = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$|\psi_{AB}\rangle \in \mathcal{H}_S$$

$\hat{M}_A$: observable on system A

$\hat{M} = \hat{M}_A \otimes \hat{I}$: observeble on S (acts on A only)

$$\langle \hat{M} \rangle = \langle \psi_{AB} | \hat{M} | \psi_{AB} \rangle$$

now with $|\Psi_{AB}\rangle = \sum_{i\mu} \alpha_{i\mu} |i_A\rangle \otimes |\mu_B\rangle$

$$\langle \hat{M} \rangle = \sum_{\substack{i\mu \\ j\nu}} \langle i_A | \hat{M}_A | j_A \rangle \langle \mu_B | \hat{\mathbb{I}} | \nu_B \rangle \alpha^*_{i\mu} \alpha_{j\nu}$$

$$\langle \hat{M} \rangle = \sum_{ij\mu} \alpha_{i\mu}^* \alpha_{j\mu} \langle i | \hat{M}_A | j \rangle$$

$$\hat{\rho}_A = \sum_{\mu, j, i} \alpha_{i\mu}^* \alpha_{j\mu} |j\rangle\langle i|$$ density matrix A

$$\langle \hat{M} \rangle = \mathrm{Tr}\left(\hat{\rho}_A \hat{M}_A \right)$$

$\rightarrow$ $\hat{\rho}_A$ is obtained from $|\Psi_{AB}\rangle$:

Partial trace $\hat{\rho}_A = \mathrm{Tr}_B \left[|\Psi\rangle_{AB} \langle \Psi|_{AB} \right]$

2. States

- Pure state on A: $|\psi_A\rangle \in \mathcal{H}_A$ such that $\hat{\rho}_A = |\psi_A\rangle\langle\psi_A|$

- Product state of S: $|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ for some $|\psi_A\rangle$ and $|\psi_B\rangle$
(Seperate)

- A state which is not a product state is <u>entangled.</u>

$\Leftrightarrow$ A is entangled with B when S is in state $|\psi_{AB}\rangle$

3. Discussion

- Here we defined entanglement for bipartite systems

- $\hat{\rho}_A$ can be diagonalized $\hat{\rho}_A = \sum_i \underset{\text{prob.}}{p_i} |i\rangle\langle i|$

$\hat{\rho}_A$ represents an ensemble of states

$\langle \hat{M}_A \rangle = \mathrm{Tr}(\hat{\rho}_A \hat{M}_A) \longrightarrow$ ① quantum mech. expectation value for each state of the ensemble

$\langle i | \hat{M}_A | i \rangle$

$\downarrow$

② Classical average over all possible states $|i\rangle$

- Purity:

$$\hat{\rho}_A^2 = \sum_i p_i^2 |i\rangle\langle i|$$

So $\mathrm{Tr}(\hat{\rho}_A^2) = \sum_i p_i^2 \leq \sum_i p_i = 1$