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Decoherence, Disentanglement and Foundations of Quantum Mechanics

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Abstract. Decoherence and disentanglement are phenomena central to quantum mechanics. Here, we consider the relative rates of decoherence and disentanglement in two-qubit, three-qubit, and two-qutrit systems when subject to pure dephasing noise alone, and a very recent result for $d \times d$ systems. Of particular interest is the specific counterintuitive effect related to the nonadditivity of such weak noises, known as *Entanglement Sudden Death (ESD)*, in which the entanglement of a composite quantum system goes abruptly to zero in finite time, coherence only exponentially decaying. We discuss these results in the context of the foundations of quantum mechanics.

Keywords: Decoherence, Disentanglement, Dephasing, Damping, Entanglement Sudden Death, Foundations of Quantum Mechanics
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INTRODUCTION

When considering fundamental issues of quantum theory, the phenomena of quantum decoherence and disentanglement are of great significance [1, 2]. Quantum coherence is the property of a quantum system permitting superposition states to persist. However, in natural environments for most physical systems, this property is relatively delicate, being easily degraded by interaction of a system with its environment [3]. The resulting decoherence of the quantum system typically increases state mixedness and causes uniquely quantum behavior to disappear. In particular, the closely related property of quantum entanglement, which is responsible for nonclassical features, is lost. This is referred to as "disentanglement," and "entanglement death" when entanglement is entirely lost [4–8]. To study the behavior of a quantum physical system interacting with its environment, the environment may be modeled as a noise source. In studying these uniquely quantum phenomena, it is important to characterize the relationship between decoherence and disentanglement. Here, we consider recent work examining this relationship in few-qubit open quantum systems with their environments modeled as white Markov noise [4–10].

Intuitively, one might expect that weak noise influences, for example, acting locally on the components of a composite quantum system to be additive, that is, for the combined effect of these weak noises to be the sum of the various local effects as separately manifested. Surprisingly, however, this has been found not to be the case. Historically, this was first found in a very simple continuous system, a pair of gaussian states of position [11], and the simplest possible discrete quantum system, a two-qubit system [5]. Here, we focus on systems described by finite-dimensional Hilbert spaces. It has been shown that when such systems are subjected to two different sorts of multi-local noise, dephasing and damping, each individual source serves to reduce component subsystem coherence as well as joint-system coherence and entanglement, but only asymptotically. However, when these two noise sources act simultaneously on a system appropriately prepared, entanglement abruptly reaches zero, that is, it does so in finite time, whereas the quantum coherence decays only exponentially in time, a qualitatively significant effect termed *Entanglement Sudden Death (ESD)*. Here, we critically comment on both the models used and interpret the results in a broader context. Theoretical results such as those of the Żurek group, *e.g.* [12, 13], as well as experimental results have been produced with the hope that decoherence might hold the key to ultimate foundational questions in quantum mechanics. Here, we suggest rather that finite-time disentanglement, that is ESD, rather than decoherence may be key to some important questions in quantum foundations, such as the measurement problem. The confirmation of ESD in photonic [14] and atomic systems [15], showing that this effect indeed occurs in practice, lends greater importance to the exploration of this possibility. Moreover, we explain that local dephasing noise alone is sufficient to induce ESD.

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DISENTANGLEMENT AND DECOHERENCE

The simplest model in which to study quantitatively the relationship between decoherence and disentanglement is a bipartite two-level (qubit) system. Qubits may be represented by a variety of quantum physical systems, such as electron spins, photon polarization states, and atomic energy levels. Yu and Eberly [9] first considered a two-qubit system as an open quantum system subjected to multi-local and collective pure dephasing Markovian noise using the operator-sum decomposition within the general approach we discuss here. We later extended these results by examined a three-qubit system subjected to dephasing noise in three ways, differentiated by scope: multi-locally in which the noise acting on each qubit is independent, collectively in which all qubits experience the same noise, and sub-collective noise (in which some subset of two qubits experiences the same noise and the remaining qubit may experience a different sort of noise) [10]. We then further extended this further to a pair of three-level systems subjected to multi-local dephasing noise [8]. Let us comment on these models and summarize the problems and solutions to each in turn.

Two qubits

The first finite-dimensional case considered was that of the two-qubit system, coupled to an environment giving rise to white Markovian *dephasing noise* alone, both local and collective in nature [9]. The interaction Hamiltonian was taken to be

$$H(t) = -\frac{1}{2}\mu\left[B(t)\left(\sigma_z^{\mathrm{A}}+\sigma_z^{\mathrm{B}}\right)+b_{\mathrm{A}}(t)\,\sigma_z^{\mathrm{A}}+b_{\mathrm{B}}(t)\,\sigma_z^{\mathrm{B}}\right] , \tag{1}$$

where μ is, for example, the gyromagnetic ratio of the particle, $\hbar = 1$, σ_z is the well-known Pauli matrix, and $b_{\mathrm{A}}(t)$, $b_{\mathrm{B}}(t)$, and $B(t)$ represent Markovian dephasing noise acting locally on qubit A, qubit B, and collectively on both, respectively. Although this Hamiltonian was constructed for spin qubits, the model is readily generalized to any physical system consisting of distinct component subsystems subjected to multi-local and collective noise. The noise terms satisfy the following relations.

$$\langle B(t)\rangle = 0\,, \tag{2}$$

$$\langle B(t)B(t')\rangle = \frac{\Gamma_{\mathrm{AB}}}{\mu^2}\delta\left(t-t'\right) , \tag{3}$$

$$\langle b_X(t)\rangle = 0\,, \tag{4}$$

$$\langle b_X(t)\,b_X(t')\rangle = \frac{\Gamma_X}{\mu^2}\delta\left(t-t'\right) , \tag{5}$$

with $X = \mathrm{A,B}$. Eqs. 2 and 4 impose the condition that the ensemble average $\langle\cdot\rangle$ of the noise fields are zero; the relations Eqs. 3 and 5 require the temporal evolution to be Markovian. For this model and all subsequent similar models involving dephasing noise fields discussed, these two conditions will continue to hold, although the associated relations will not be explicitly exhibited.

Note that there exists two distinct noise terms here, multilocal noises $b_{\mathrm{A}}(t)$ and $b_{\mathrm{B}}(t)$ acting independently at the level of each individual qubit and collective noise $B(t)$ acting on both qubits in the same way. These situations may be physically interpreted as qubits in spatially separated environments or a single global environment, respectively. Markovian evolution assumes there is no back-action of the system to its environment and that there is no memory of interaction after the effective timescale of interaction. Furthermore, the nature of this noise is such that the relaxation timescales of either the qubit or its environment are much shorter than the timescales of the qubit-environment interaction between them, allowing one to disregard the internal dynamics of both the system and environment. In constructing a model for dephasing, the relative timescales and effective dynamics must be specified. We have constructed our model as one of an open quantum system. This involves what corresponds to the tracing out of environmental degrees of freedom under a closed system construction, discarding fine-grained features in favor of coarse-grained ones assumed to be Markovian in character. That is, one can think of any quantum system as the reduced subsystem of a larger system. After purifying this system by a unitarily embedding into a higher-dimensional system, with the support of the Stinespring dilation theorem [16], one can unitary time-evolve the larger system, over which one traces to get the system state as a reduced state. Although the dynamics of the composite system is still unitary, the reduced subsystem dynamics may not be. This naturally lends itself to the use of completely positive trace-preserving (CPTP) maps of statistical operators, described below.

To represent states in the density matrix formalism, one must consider a specific base in which to represent matrices. In the present case, it is convenient to use the basis

$$|1\rangle_{\mathrm{AB}} = |++\rangle_{\mathrm{AB}}\ ,\ |2\rangle_{\mathrm{AB}} = |+-\rangle_{\mathrm{AB}}\ ,\ |3\rangle_{\mathrm{AB}} = |-+\rangle_{\mathrm{AB}}\ ,\ |4\rangle_{\mathrm{AB}} = |--\rangle_{\mathrm{AB}}\ . \tag{6}$$

It is important also to note that quantum coherence has typically been studied in a basis-dependent fashion in the literature on decoherence, to which the studies considered here belong. Any discussion of the rate of decoherence requires a specification of the basis whenever this is done. Similarly, although entanglement is basis *independent*, there are a number of entanglement measures used for its quantification; different measures may give different numerical values of entanglement, although all good measures must be monotones of one another. Thus, one must take care to specify both the basis and entanglement measure used in any such analysis. In determining the time-evolved density matrix, one must take the ensemble averages over all three noise fields,

$$\rho(t) = \langle\langle\langle \rho_{\mathrm{st}}(t)\rangle\rangle\rangle \quad , \text{ where } \quad \rho_{\mathrm{st}}(t) = U(t)\rho(0)U^{\dagger}(t) \quad \text{and} \quad U(t) = \exp\left[-i\int_0^t dt' H(t')\right] \tag{7}$$

are the standard unitary evolution with the unitary operator given by the solution of the Schrodinger equation. We have used the very practical operator-sum decomposition to find the time-evolved state, in which one expresses dynamical equations via completely positive trace preserving (CPTP) maps. In particular, operators satisfying the completeness relation, $\sum_\mu \overline{K}_\mu^{\dagger}\overline{K}_\mu = I$ and the trace-preserving condition, $\sum_\mu \overline{K}_\mu\overline{K}_\mu^{\dagger} = I$, in which in general $\left[\overline{K}_\mu, \overline{K}_\mu^{\dagger}\right] \neq 0$, may be used to find time-evolved the density matrix. The time-evolved density matrix takes the form of $\rho(t) = \mathscr{E}(\rho(0)) = \sum_{\mu=1}^{N}\overline{E}_\mu^{\dagger}(t)\rho(0)\overline{E}_\mu(t)$, where $\mathscr{E}$ is a superoperator representing the CPTP map, which is an operator acting on operators, the $\overline{E}_\mu$ represent the non-unique Kraus operators, and N is not necessarily fixed, because the $\overline{E}_\mu$ are typically nonunique. In the two-qubit case under the conditions specified above, the temporally evolved state and operators are

$$\rho(t) = \sum_{i,j=1}^{2}\sum_{k=1}^{3}\left(D_k^{\dagger}E_j^{\mathrm{B}\dagger}E_i^{\mathrm{A}\dagger}\right)\rho(0)\left(E_i^{\mathrm{A}}E_j^{\mathrm{B}}D_k\right)\ , \tag{8}$$

$$E_1^{\mathrm{A}} = \begin{pmatrix} 1 & 0 \\ 0 & \gamma_{\mathrm{A}}(t) \end{pmatrix}\otimes I,\ E_2^{\mathrm{A}} = \begin{pmatrix} 0 & 0 \\ 0 & \omega_{\mathrm{A}}(t) \end{pmatrix}\otimes I,\ E_1^{\mathrm{B}} = I\otimes\begin{pmatrix} 1 & 0 \\ 0 & \gamma_{\mathrm{B}}(t) \end{pmatrix},\ E_2^{\mathrm{B}} = I\otimes\begin{pmatrix} 0 & 0 \\ 0 & \omega_{\mathrm{B}}(t) \end{pmatrix}, \tag{9}$$

$$D_1 = \begin{pmatrix} \gamma(t) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \gamma(t) \end{pmatrix},\ D_2 = \begin{pmatrix} \omega_1(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_2(t) \end{pmatrix},\ D_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_3(t) \end{pmatrix}, \tag{10}$$

and the elements in the matrices are given by

$$\gamma_{\mathrm{A}}(t) = e^{-t/2T_{\mathrm{A}}},\ \gamma_{\mathrm{B}}(t) = e^{-t/2T_{\mathrm{B}}},\ \omega_{\mathrm{A}}(t) = \sqrt{1-\gamma_{\mathrm{A}}^2},\ \omega_{\mathrm{B}}(t) = \sqrt{1-\gamma_{\mathrm{B}}^2}, \tag{11}$$

$$\gamma(t) = e^{-t/2T_{\mathrm{AB}}},\ \omega_1(t) = \sqrt{1-\gamma^2},\ \omega_2(t) = -\gamma^2\sqrt{1-\gamma^2},\ \omega_3(t) = \sqrt{(1-\gamma^2)(1-\gamma^4)}, \tag{12}$$

where $T_X = \frac{1}{\Gamma_X}$ $(X = \mathrm{A}, \mathrm{B}, \mathrm{AB})$. In a similar way that Eqs. 2, 3, 4, and 5 specifying the white noise and Markovian conditions are assumed for subsequent discussions, Eqs. 11 and 12 defining the terms in the matrices will be assumed when defining subsequent dephasing operators. For tractability of notation, time does not explicitly appear as an argument for these quantities from hereon, but rather is implied. These operators act to dephase the system, both at the joint-system and the individual subsystem level, in the sense that the off-diagonal elements in the respective density matrix (at least asymptotically) decay to zero.

The timescales in which density matrix elements decay define the pertinent decoherence timescales, which we compare to disentanglement timescales. For example, these timescales may represent the transverse spin-spin dephasing timescales T_2, differing from the longitudinal spin-lattice damping timescales T_2. In general, $T_1 = 2T_2$. This and subsequent similar analyses comparing decoherence and disentanglement timescales focus only on the T_2 timescales and make no mention of T_1 timescales. The reason for this is to keep the analysis as simple as possible, retaining only the uniquely quantum dephasing aspect of noise and thereby avoiding complications associated with amplitude damping, such as such as energy exchange between system and environment and modification of relative state populations. This simplicity is not merely a matter of convenience; as shown below during the examination of the effects of two different sorts of weak noises, there is not necessarily the expected additivity.

As evidenced by the ever-expanding literature concerning entanglement measures, there are many complications. Nonetheless, the entanglement of a two-qubit system *is* easily quantified by the concurrence measure, as defined by Wootters [17], as an intermediate quantity, from which one can calculate a value for this system the entanglement of formation E_f. The entanglement of formation is the canonical entanglement measure; it corresponds to the minimum entanglement of the weighted average of possible ensembles that may form a given two-qubit mixed state. Since concurrence is a monotonic function of the entanglement of formation, it is a valid entanglement measure in its own right and is typically used as such. The concurrence may be written

$$C(\rho_{AB}) = \max\left[0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right] , \tag{13}$$

where λ_i are the eigenvalues of the matrix

$$\rho_{AB}\tilde{\rho}_{AB} \equiv \rho_{AB}\left(\sigma_y^{\mathrm{A}} \otimes \sigma_y^{\mathrm{B}}\right)\rho_{AB}^{*}\left(\sigma_y^{\mathrm{A}} \otimes \sigma_y^{\mathrm{B}}\right), \tag{14}$$

ordered from largest to smallest, with ρ^* the component-wise complex-conjugation of the density matrix, and σ_y is the well-known Pauli matrix. Entanglement of formation is thus defined as

$$E_f(\rho) = h\left(\frac{1+\sqrt{1-C^2(\rho)}}{2}\right), \tag{15}$$

where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is the binary Shannon entropy. Note that, formally, this suggests that the concurrence serves as a probability. Under this interpretation, which to our knowledge we introduce for the first time here, the concurrence is the probability that a density matrix under some abstract conjugation operation is indistinguishable from its original unconjugated state [18]: a large overlap implies a value close to unity indicating high indistinguishability and thus high entanglement, whereas low overlap implies a value close to zero indicating distinguishability and thus low entanglement. Although the conjugation used to define concurrence may be seen as a time-reversal operation, it may be taken to be any operation which brings a density matrix defined on some Hilbert space into another subspace whose distinguishability defines the degree of entanglement.

The state of a generic two-qubit system in the computational basis is

$$|\Psi\rangle = a|1\rangle_{\mathrm{AB}} + b|2\rangle_{\mathrm{AB}} + c|3\rangle_{\mathrm{AB}} + d|4\rangle_{\mathrm{AB}}, \tag{16}$$

with the usual normalization condition $a^2+b^2+c^2+d^2 = 1$. Although this general state may be analyzed explicitly, it is vastly more illuminating and computationally tractable to define two classes of states, *fragile* and *robust*, according to their behavior when subjected to collective dephasing noise. The fragile states, are defined as those which lose coherence asympototically and contain the Bell states $|\Phi^{\pm}\rangle$. The robust states, in contrast, maintain coherence for all time and contain $|\Psi^{\pm}\rangle$.

(i) The *fragile* class has the forms

$$|\phi_1\rangle = a|1\rangle_{\mathrm{AB}} + b|2\rangle_{\mathrm{AB}} + d|4\rangle_{\mathrm{AB}} \quad \text{and} \quad |\phi_2\rangle = a|1\rangle_{\mathrm{AB}} + c|3\rangle_{\mathrm{AB}} + d|4\rangle_{\mathrm{AB}} . \tag{17}$$

(ii) The *robust* class has the forms

$$|\psi_1\rangle = a|1\rangle_{\mathrm{AB}} + b|2\rangle_{\mathrm{AB}} + c|3\rangle_{\mathrm{AB}} \quad \text{and} \quad |\psi_2\rangle = b|2\rangle_{\mathrm{AB}} + c|3\rangle_{\mathrm{AB}} + d|4\rangle_{\mathrm{AB}} . \tag{18}$$

The main features of this model are as follows: operator-sum decomposition representing Markovian dephasing noise acting at the local and collective level, dephasing and damping timescales, the definition of concurrence used to compute the amount of entanglement, and finally the classes of states used. For multi-local and collective Markov dephasing noise, when disentanglement does occur, as quantified by the decay of the concurrence, it always proceeds at least as fast as decoherence as quantified by the decay of the off-diagonal elements of the state density matrix.

$$t_{\mathrm{dis}} \leq t_{\mathrm{dec}} \tag{19}$$

This is important because it shows that the rate at which the pair of systems loses correlation may be faster than the rate at which each individually loses quantum coherence. It is valuable to relate this result to properties of decoherence-free subspaces (DFS) in order to qualify the phrase "when disentanglement occurs" in the previous paragraph. DFS are protected subspaces in the entire Hilbert space due to certain symmetries of the state. It effectively makes the state immune to the effects of the environment. As a specific example, let us examine the singlet state $|\Psi^-\rangle$ and let the environmental fields be magnetic fields. Since there is no net spin for such a field to act upon, this state does not decohere and thus protected from a magnetic environment. Such situations are a core component of the "decoherence program," because the coupling of environment to the specified degrees of freedom of the system, effectively chooses a basis in which to decohere the system. This is discussed further in the final section.

Three qubits

More complex systems than qubit pairs are those containing a greater number of component subsystems and/or greater dimensionality in each subsystem. In general, there is no simple way to quantify the entanglement of multipartite systems of arbitrary dimensions. One reason is that entanglement is contingent upon the definition of a subsystem, which in general is not uniquely defined; the entanglement between different partitions of different subsystems cannot be described by a single scalar, effectively making it a vector quantity [19, 20]. Interestingly, when the number of dimensions of each subsystem is infinite as in continuous-variable systems, much is known about the entanglement for a special class of states, the Gaussian states. Although an analogous situation doesn't occur when the number of component subsystems is increased, we may consider instead a large number of *copies* of a state, from which asympototic entanglement measures may be defined.

The system immediately more complex than the above simple system of two-qubits involves three qubits. After examining this case, in which we increase the number of subsystems, we will then examine the case in which the dimension of each system increases, that of a two three-level (qutrit) system. The Hamiltonian describing environmental dephasing noise acting on the three-qubit system is given by

$$\begin{aligned} H(t) = -\tfrac{1}{2}\mu\Big[& B_{\mathrm{A}}^{(1)}(t)\,\sigma_z^{\mathrm{A}} + B_{\mathrm{B}}^{(1)}(t)\,\sigma_z^{\mathrm{B}} + B_{\mathrm{C}}^{(1)}(t)\,\sigma_z^{\mathrm{C}} \\ & + B_{\mathrm{AB}}^{(2)}(t)\left(\sigma_z^{\mathrm{A}}+\sigma_z^{\mathrm{B}}\right) + B_{\mathrm{BC}}^{(2)}(t)\left(\sigma_z^{\mathrm{B}}+\sigma_z^{\mathrm{C}}\right) + B_{\mathrm{AC}}^{(2)}(t)\left(\sigma_z^{\mathrm{A}}+\sigma_z^{\mathrm{C}}\right) \\ & + B^{(3)}(t)\left(\sigma_z^{\mathrm{A}}+\sigma_z^{\mathrm{B}}+\sigma_z^{\mathrm{C}}\right)\Big], \end{aligned} \tag{20}$$

where μ is, for example, the gyromagnetic ratio of the particle, $\hbar = 1$, σ_z is the well-known Pauli matrix, and $B^{(1)}(t)$, $B^{(2)}(t)$, and $B^{(3)}(t)$ represent noise fields acting locally on a single qubit, a subset of two qubits, and collectively on all three, respectively, exhausting all subsets of qubits that noise may act on. As stated earlier, all dephasing noise fields satisfy the analogous relations given in Eqs. 2, 3, 4, and 5 to ensure Markov time evolution. The time evolution for this system given in the operator-sum decomposition is

$$\rho(t) = \sum_{i,j,k=1}^{2}\ \sum_{l,m,n,p=1}^{3} \left(F_p^{\mathrm{ABC}\dagger}E_n^{\mathrm{AC}\dagger}E_m^{\mathrm{BC}\dagger}E_l^{\mathrm{AB}\dagger}D_k^{\mathrm{C}\dagger}D_j^{\mathrm{B}\dagger}D_i^{\mathrm{A}\dagger}\right)\rho(0)\left(D_i^{\mathrm{A}}D_j^{\mathrm{B}}D_k^{\mathrm{C}}E_l^{\mathrm{AB}}E_m^{\mathrm{BC}}E_n^{\mathrm{AC}}F_p^{\mathrm{ABC}}\right), \tag{21}$$

where the D_i are multi-local operators acting on the level of a single qubit defined similarly to Eq. 9, E_j acts collectively on a subset of two qubits defined similar to Eq. 10, and F_k acts collectively on the entire qubit system and is given by

$$F_1(t) = \mathrm{diag}(\gamma(t),1,1,1,1,1,1,\gamma(t)), \tag{22}$$

$$F_2(t) = \mathrm{diag}(\omega_1(t),0,0,0,0,0,0,\omega_2(t)), \tag{23}$$

$$F_3(t) = \mathrm{diag}(0,0,0,0,0,0,0,\omega_3(t)) \tag{24}$$

with the elements in the density matrix defined as in Eq. 12. These operators act to decay the off-diagonal elements in the respective density matrices to zero.

The basis used to describe the joint system is again the "computational basis," which is the one naturally selected by the environment, say for electron spins in a magnetic environment.

$$\begin{aligned} &|1\rangle_{\mathrm{ABC}} = |000\rangle_{\mathrm{ABC}}, |2\rangle_{\mathrm{ABC}} = |001\rangle_{\mathrm{ABC}}, |3\rangle_{\mathrm{ABC}} = |010\rangle_{\mathrm{ABC}}, |4\rangle_{\mathrm{ABC}} = |011\rangle_{\mathrm{ABC}}, \\ &|5\rangle_{\mathrm{ABC}} = |100\rangle_{\mathrm{ABC}}, |6\rangle_{\mathrm{ABC}} = |101\rangle_{\mathrm{ABC}}, |7\rangle_{\mathrm{ABC}} = |110\rangle_{\mathrm{ABC}}, |8\rangle_{\mathrm{ABC}} = |111\rangle_{\mathrm{ABC}}, \end{aligned} \tag{25}$$

For a system of three qubits, the analogous robust and fragile states are the W and GHZ states, respectively. The W-state has the property that it has no genuine tripartite entanglement [21] according to the tangle (squared concurrence) measure, yet becomes a maximally entangled two-qubit state under the loss of one qubit. The GHZ-state, in contrast, has maximal tripartite entanglement according to the tangle measure, yet has zero bipartite entanglement at the two-qubit level. Let us note that these simple classifications according to robustness and fragility do not in general exist either in systems of higher numbers of qubits or higher dimensions. For example, it was shown by [22, 23] that there are *nine* inequivalent classes under stochastic local operations with classical communication. There is much evidence supporting the view that the structure of entanglement is of a complex nature. The tripartite classes are given by

$$|\mathrm{W^g}\rangle = \bar{a}_1|001\rangle + \bar{a}_2|010\rangle + \bar{a}_4|100\rangle \quad \text{and} \quad |\mathrm{GHZ^g}\rangle = \bar{a}_0|000\rangle + \bar{a}_7|111\rangle, \tag{26}$$

Since there is no entanglement measure for mixed three-qubit states, the pairwise entanglement content was examined using the concurrence. In a similar fashion to the two-qubit analysis, the rate of decay of off-diagonal elements of the joint-system and component subsystem density matrices were used to quantify decoherence. In all cases, it was shown that disentanglement, when it occurs between any two qubits of the system subjected to dephasing noise on some subset of qubits, takes place at a faster rate than decoherence, in agreement with results obtained for the two-qubit system [9].

Two qutrits

The next step in this line of investigation is the analysis of a bipartite three-level (qutrit) system which may represent, for example, two three-level atoms in the "V" configuration. This extends the two-qubit system by increasing the dimension of each subsystem instead of increasing the number of component subsystems. Here, the Hamiltonian is given by

$$H(t) = -\frac{\mu}{2}\left[b_{\mathrm{A}}^{(1)}(t)\,\sigma_{\mathrm{z}}^{\mathrm{A}} + b_{\mathrm{B}}^{(1)}(t)\,\sigma_{\mathrm{z}}^{\mathrm{B}} + b_{\mathrm{AB}}^{(2)}(t)\left(\sigma_{\mathrm{z}}^{\mathrm{A}} + \sigma_{\mathrm{z}}^{\mathrm{B}}\right)\right], \tag{27}$$

where $\hbar = 1$, μ is the gyromagnetic moment of the atom, and $\sigma_{\mathrm{z}} = \mathrm{diag}(1, e^{\frac{i2\pi}{3}}, e^{\frac{i4\pi}{3}})$ is the dephasing operator for three-level systems with subscripts denoting qutrits A, B, or both. σ_z represents the raising operator for spin-one systems, since we are now dealing with $SU(3)$ operators instead of those contained in $SU(2)$ as in the previous two-level systems. As before, the noise fields $b_X^{(i)}(i = 1,2)$ satisfy conditions for white noise and Markov evolution.

The basis eigenstates for each qutrit are given by $\{|0\rangle, |+1\rangle, |-1\rangle\}$, representing the ground state, first-excited state, and second-excited state of the atom, respectively. Since we are assuming the "V" configuration the states $|+1\rangle, |-1\rangle$ couple to the ground state but not to each other. The basis eigenstates for the two-qutrit system is given by

$$\begin{aligned}&\{|1\rangle, |2\rangle, |3\rangle, |4\rangle, |5\rangle, |6\rangle, |7\rangle, |8\rangle, |9\rangle\}\\ &\quad \doteq \{|00\rangle, |0,+1\rangle, |0,-1\rangle, |+1,0\rangle, |+1,+1\rangle, |+1,-1\rangle, |-1,0\rangle, |-1,+1\rangle, |-1,-1\rangle\}.\end{aligned} \tag{28}$$

The temporal evolution of the density matrix $\rho(t)$ is again given via a completely positive trace preserving (CPTP) linear map,

$$\rho(t) = \sum_{i,j=1}^{2}\sum_{k=1}^{3}\left(D_k^{\mathrm{AB}\dagger}E_j^{\mathrm{B}\dagger}E_i^{\mathrm{A}\dagger}\right)\rho(0)\left(E_i^{\mathrm{A}}E_j^{\mathrm{B}}D_k^{\mathrm{AB}}\right), \tag{29}$$

where the operators of the operator-sum decomposition are

$$E_1^{\mathrm{A}} = \mathrm{diag}(1, \gamma_{\mathrm{A}}(t), \gamma_{\mathrm{A}}(t)) \otimes I_3\,,\ E_2^{\mathrm{A}} = \mathrm{diag}(0, \omega_{\mathrm{A}}(t)\,\omega_{\mathrm{A}}(t)) \otimes I_3\,, \tag{30}$$

$$E_1^{\mathrm{B}} = I_3 \otimes \mathrm{diag}(1, \gamma_{\mathrm{B}}(t)\ ,\ \gamma_{\mathrm{B}}(t))\,,\ E_2^{\mathrm{B}} = I_3 \otimes \mathrm{diag}(0, \omega_{\mathrm{B}}(t), \omega_{\mathrm{B}}(t))\,, \tag{31}$$

$$D_1^{\mathrm{AB}} = \mathrm{diag}(\gamma_{\mathrm{AB}}(t), 1, 1, 1, \gamma_{\mathrm{AB}}(t), 1, 1, 1, \gamma_{\mathrm{AB}}(t))\,, \tag{32}$$

$$D_2^{\mathrm{AB}} = \mathrm{diag}(\omega_{\mathrm{AB1}}(t), 0, 0, 0, \omega_{\mathrm{AB2}}(t), 0, 0, 0, \omega_{\mathrm{AB2}}(t))\,, \tag{33}$$

$$D_3^{\mathrm{AB}} = \mathrm{diag}(0, 0, 0, 0, 0, 0, 0, 0, \omega_{\mathrm{AB3}}(t))\,, \tag{34}$$

where the time-dependent parameters are as specified in Eq. 11 and Eq. 12. Although decoherence is quantified as in the previous case, now we must use a different entanglement measure, since the Wootters concurrence is no longer applicable. However, the negativity measure, which is the sum of the negative eigenvalues of the partially transposed density matrix is applicable: $\mathcal{N}(\rho) = \left(\left\|\rho^{\mathrm{T_A}}\right\|_1 - 1\right)/2$, where $\rho^{\mathrm{T_A}}$ is the partial transpose of the density matrix with respect to qutrit A and $\|\cdot\|_1$ denotes the trace norm [24]. Mathematically, the negativity is the degree to which a density matrix fails to be positive under a positive map but not completely positive map, the positive map in this case being the partial transpose with respect to a particular subsystem.

The general state of a bipartite three-level system is $|\Psi\rangle_{\mathrm{AB}} = \bar{a}_1|1\rangle + \bar{a}_2|2\rangle + \bar{a}_3|3\rangle + \bar{a}_4|4\rangle + \bar{a}_5|5\rangle + \bar{a}_6|6\rangle + \bar{a}_7|7\rangle + \bar{a}_8|8\rangle + \bar{a}_9|9\rangle$. In a similar fashion to the previous analyses, we define two classes of states characterized by their asymptotic behavior in the presence of collective dephasing noise.

(i) The *fragile* class $|\phi\rangle = \bar{a}_1|1\rangle + \bar{a}_5|5\rangle + \bar{a}_9|9\rangle$, in which $\bar{a}_1, \bar{a}_5,$and $\bar{a}_9$ may be non-zero and all other terms $\bar{a}_i = 0$, has the forms

$$|\phi_1\rangle = \bar{a}_1|1\rangle + \bar{a}_5|5\rangle \quad , \quad |\phi_2\rangle = \bar{a}_1|1\rangle + \bar{a}_9|9\rangle \quad , \quad \text{and} \quad |\phi_3\rangle = \bar{a}_5|5\rangle + \bar{a}_9|9\rangle \ . \tag{35}$$

(ii) The *robust* class $|\psi\rangle = \bar{a}_2|2\rangle + \bar{a}_3|3\rangle + \bar{a}_4|4\rangle + \bar{a}_6|6\rangle + \bar{a}_7|7\rangle + \bar{a}_8|8\rangle$, in which all $\bar{a}_i$ listed may be non-zero and $\bar{a}_1 = \bar{a}_9 = 0$, has the forms

$$|\psi_1\rangle = \bar{a}_2|2\rangle + \bar{a}_4|4\rangle \quad , \quad |\psi_2\rangle = \bar{a}_3|3\rangle + \bar{a}_7|7\rangle \quad , \quad \text{and} \quad |\psi_3\rangle = \bar{a}_6|6\rangle + \bar{a}_8|8\rangle \ . \tag{36}$$

In this system–environmental-noise model, as in previous models [9, 10], we found that under pure dephasing noise, the timescale of disentanglement is *never* slower than that of decoherence.

In principle, we could extend this method of analysis to an even greater number of constituent subsystems and to higher dimensions, taking multi-local and collective noise to act on subsets of N $d-$level subsystems from 1 to N. However, this has the drawback that in physical systems of higher dimensions, either due to a greater number of constituent subsystems or due to each subsystem being of high dimension, there exist many different entanglement classes, which are not amenable to the simple fragile and robust classifications considered. Furthermore, there are no widely used mixed-state entanglement measures for multipartite systems and none that are rigorously defined, to our knowledge. Although the simplest two-qubit states are, experimentally, merely difficult to produce, protecting them from degrading to classical states in light of other types of decoherence is extremely difficult. Furthermore, identifying components and reconstructing the entire $d_{\mathrm{A}}d_{\mathrm{B}} \times d_{\mathrm{A}}d_{\mathrm{B}}$ density matrix is an extraordinarily difficult resource-intensive procedure. Any further analysis along this path must be carefully considered in light of conditions that simplify the situation, such as symmetries of the state and the noise.

ENTANGLEMENT SUDDEN DEATH

Let us now consider the phenomenon of the complete loss of entanglement in finite time. Intuitively, the combined effect of weak noises would be expected, on the basis of classical physical experience, to be additive. However, this is not always the case in quantum systems, even subjected to dephasing noise which can affect classical as well as quantum systems. Yu and Eberly [4] first found that in finite-dimensional systems that when a two-qubit system is subjected to two different types of multi-local noises, dephasing and damping, disentanglement occurred in finite time, despite only asymptotically occurring when only individual noise sources were present. They quantified disentanglement by the reduction of concurrence and decoherence by the decay of off-diagonal elements in the two-qubit system joint density matrix. They termed this phenomenon *Entanglement Sudden Death (ESD)*. After its initial discovery in a bipartite qubit system, we demonstrated its existence in a qubit-qutrit system [7], the only other finite-dimensional bipartite quantum system in which the only fully general mixed state entanglement measure, negativity, is applicable. The negativity which is based on the Peres-Horodečki criterion [24–27].

Two-qubit

To understand the evolution of this direction of investigation, we begin by summarizing the two-qubit density matrix ansatz considered by Yu and Eberly [4].

$$\rho_\lambda^{AB} = \frac{1}{9}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & \lambda & 0 \\ 0 & \lambda & 4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \tag{37}$$

The concurrence of an arbitrary two-qubit density matrix is given by

$$C^{AB} = 2\max\{0, \sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}, \sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}\}. \tag{38}$$

For this class, there is an initial concurrence of $C_\lambda(0) = 2\lambda/9$. The relevant decomposition operators for dephasing noise are those of Eq. 9, whereas the amplitude damping noise that serve to modify the population of the so that all states end up in the ground state is described by

$$E_1'^{\mathrm{A}} = \begin{pmatrix} 1 & 0 \\ 0 & \gamma_{\mathrm{A}}'(t) \end{pmatrix} \otimes I,\ E_2'^{\mathrm{A}} = \begin{pmatrix} 0 & \omega_{\mathrm{A}}'(t) \\ 0 & 0 \end{pmatrix} \otimes I,\ F_1'^{\mathrm{B}} = I \otimes \begin{pmatrix} 1 & 0 \\ 0 & \gamma_{\mathrm{B}}'(t) \end{pmatrix},\ F_2'^{\mathrm{B}} = I \otimes \begin{pmatrix} 0 & \omega_{\mathrm{B}}'(t) \\ 0 & 0 \end{pmatrix}, \tag{39}$$

with $\gamma_A'(t) = \gamma_B'(t) = \exp[-\Gamma_1 t] \equiv \gamma'$ and $\omega_A'(t) = \omega_B'(t) = \sqrt{1-\gamma'^2(t)} \equiv \omega'$, in contrast to the Γ_2 terms corresponding to the elements in the dephasing matrices.

Let us now review the effects of each of the dephasing, damping, and both combined in turn, as found by Yu and Eberly. When only dephasing exists, the diagonal elements are unaffected $\{a(0),\ b(0),\ c(0),\ d(0)\} = \{a(t),\ b(t),\ c(t),\ d(t)\} = \{1/9,\ 4/9,\ 4/9,\ 0\}]$ and only the off-diagonal elements change according to $z(0) = \frac{\lambda}{9} \rightarrow z(t) = \frac{\lambda}{9}\exp[-\Gamma_2 t]$ when plugging into Eq. 38 yields $C_\lambda^{ph.} = (2\lambda/9)\exp[-\Gamma_2 t]$. Note that this expression approaches zero only in the limit $t \rightarrow \infty$, so there always remains some amount of bipartite entanglement under dephasing noise. When only amplitude damping exists, the elements of the density matrix are affected in the following way for the range $3 \le \lambda \le 4$: $\{a(0),\ d(0),\ z(0)\} = \{\frac{1}{9},\ 0,\ \frac{\lambda}{9}\} \rightarrow \{a(t),\ d(t),\ z(t)\} = \{\frac{1}{9}\exp[-2\Gamma_1 t], \frac{1}{9}\omega'^4 + \frac{8}{9}\omega'^2, \frac{\lambda}{9}\exp[-\Gamma_1 t]\}$, with the values of b and c unaffected. These values yield a concurrence of $C_\lambda^{amp.} = (2/9)[\lambda - \sqrt{\omega'^4 + 8\omega'^2}]\exp[-\Gamma_1 t]$. Note that this also approaches zero only asymptotically in time under this damping noise. When both dephasing and damping are combined, an interesting qualitatively new effect occurs. $C_\lambda^{ph.+amp.} = 2\max\left\{0,\ \lambda e^{-\Gamma_2 t} - \sqrt{\omega'^4 + 8\omega'^2}\right\}$ since the off-diagonal element $z = \lambda/9$ now falls off as a result of both the effects of dephasing and amplitude damping noise. Thus, one sees that one does not have an additivity of noise as expected, because each of the dephasing and damping rates go smoothly to zero separately, whereas they cause the combined expression to go to zero abruptly in finite time.

Qubit-qutrit

After this result, we were interested to know whether ESD phenomenon is specific to a two-qubit system or a generic phenomenon. The two-qubit density matrix was chosen so that the analysis would be as simple as possible, yet still be sufficiently complex to demonstrate the desired effect. However, it is possible that complex effects may occur in higher dimensions that prevent the existence of ESD. Let us remark that our focus in on finite dimensional systems. For infinite-dimensional continuous-variable bipartite systems, ESD was shown to exist [11] for Gaussian systems, though these are qualitatively different from finite-dimensional systems. The original analysis of bipartite ESD was extended to a four-qubit model [5], however, that system was only analyzed according to bipartite dynamics since there are no multipartite entanglement measures for arbitrary mixed four-qubit states. Thus, the first extension to higher finite-dimensional systems was ours for a qubit-qutrit system in [7]. Incidently, this is the only other bipartite finite-dimensional case in which an entanglement measure existed for arbitrary mixed state systems, namely, the negativity. In order to show the existence of ESD clearly in a simple ansatz similar to [4], we derive an ansatz in the same way as [28] which has the feature that it only looks at terms responsible for joint-system coherence by setting those solely responsible for individual subsystem coherence to zero.

The joint qubit-qutrit system is described by the density matix $\rho_{\mathrm{AB}} = [\rho_{ij}]$, where $\rho_{ji}^* = \rho_{ij}$, $\sum_i \rho_{ii} = 1$, with $i, j = 1, \ldots, 6$. We wanted to focus on the terms that had only density matrix terms containing joint system coherence. Thus, the general density matrix of the qubit, by tracing out the qutrit.

$$\rho_{\mathrm{A}} = \begin{pmatrix} (\rho_{11}+\rho_{22}+\rho_{33}) & (\rho_{14}+\rho_{25}+\rho_{36}) \\ (\rho_{41}+\rho_{52}+\rho_{63}) & (\rho_{44}+\rho_{55}+\rho_{66}) \end{pmatrix}. \tag{40}$$

Now, when there is no coherence at all at the reduced density matrix level, all the off-diagonal elements can be set to zero and thus gives: $\rho_{\mathrm{A}}' = \mathrm{diag}\big((\rho_{11}+\rho_{22}+\rho_{33}), (\rho_{44}+\rho_{55}+\rho_{66})\big)$. Similarly, the qutrit reduced density matrix is

$$\rho_{\mathrm{B}} = \begin{pmatrix} (\rho_{11}+\rho_{44}) & (\rho_{12}+\rho_{45}) & (\rho_{13}+\rho_{46}) \\ (\rho_{21}+\rho_{54}) & (\rho_{22}+\rho_{55}) & (\rho_{23}+\rho_{56}) \\ (\rho_{31}+\rho_{64}) & (\rho_{32}+\rho_{65}) & (\rho_{33}+\rho_{66}) \end{pmatrix}, \tag{41}$$

Again, when there is no subsystem coherence, we have $\rho'_{\rm B} = \mathrm{diag}\big((\rho_{11}+\rho_{44}),(\rho_{22}+\rho_{55}),(\rho_{33}+\rho_{66})\big)$. The full joint-system density matrix corresponding to those reduced density matrices are given by the following.

$$\rho'_{\rm AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 & \rho_{15} & \rho_{16} \\ 0 & \rho_{22} & 0 & \rho_{24} & 0 & \rho_{26} \\ 0 & 0 & \rho_{33} & \rho_{34} & \rho_{35} & 0 \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} & 0 & 0 \\ \rho_{51} & 0 & \rho_{53} & 0 & \rho_{55} & 0 \\ \rho_{61} & \rho_{62} & 0 & 0 & 0 & \rho_{66} \end{pmatrix} , \tag{42}$$

The task now is to show when this generic joint-system density matrix exhibits ESD. In order to achieve this, we make use of the negativity measure consisting of the sum of the negative eigenvalues of the joint system density matrix as defined in the qutrit analysis. To simplify the analysis even further, we choose the following ansatz,

$$\rho'_{AB}(\bar{x}) = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \bar{x} \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ \bar{x} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} , \tag{43}$$

where $0 \le \bar{x} \le \frac{1}{4}$ in order to satisfy the positivity requirement. The time-evolution is obtained again, much like the previous analyses, according to the operator sum decomposition. $\rho\,(t) = \mathscr{E}\,(\rho\,(0)) = \sum_\mu K_\mu\,(t)\,\rho\,(0)\,K_\mu^\dagger\,(t)$. For local and multi-local dephasing environments, the $K_\mu(t)$ are of the form $K_\mu(t) = F_j(t)E_i(t)$, so that

$$\begin{aligned}
\rho_{\rm AB}\,(t) &= \mathscr{E}\,(\rho\,(0)) = \sum_{i=1}^{2}\sum_{j=1}^{3} F_j\,(t)\,E_i\,(t)\,\rho_{\rm AB}\,(0)\,E_i^\dagger\,(t)\,F_j^\dagger\,(t)\ , && (44)\\
E_1(t) &= \mathrm{diag}(1,\gamma_{\rm A})\otimes\mathrm{diag}(1,1,1) = \mathrm{diag}(1,1,1,\gamma_{\rm A},\gamma_{\rm A},\gamma_{\rm A})\ , && (45)\\
E_2(t) &= \mathrm{diag}(0,\omega_{\rm A})\otimes\mathrm{diag}(1,1,1) = \mathrm{diag}(0,0,0,\omega_{\rm A},\omega_{\rm A},\omega_{\rm A})\ , && (46)\\
F_1(t) &= \mathrm{diag}(1,1)\otimes\mathrm{diag}(1,\gamma_{\rm B},\gamma_{\rm B}) = \mathrm{diag}(1,\gamma_{\rm B},\gamma_{\rm B},1,\gamma_{\rm B},\gamma_{\rm B})\ , && (47)\\
F_2(t) &= \mathrm{diag}(1,1)\otimes\mathrm{diag}(0,\omega_{\rm B},0) = \mathrm{diag}(0,\omega_{\rm B},0,0,\omega_{\rm B},0)\ , && (48)\\
F_3(t) &= \mathrm{diag}(1,1)\otimes\mathrm{diag}(0,0,\omega_{\rm B}) = \mathrm{diag}(0,0,\omega_{\rm B},0,0,\omega_{\rm B})\ , && (49)
\end{aligned}$$

with the factors in the matrices defined as before and the $E_i(t)$ and $F_j(t)$ operators again satisfy the CPTP relations.

For a specific example, let us analyze the case in which there exists local dephasing noise on both the qubit and the qutrit, with the qualification that dephasing noise on either subsystem individually have the same consequences. The time-evolved matrix in this case then given by

$$\rho'_{\rm AB}(x,t) = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & x\gamma_{\rm A}\gamma_{\rm B} \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ x\gamma_{\rm A}\gamma_{\rm B} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} , \tag{50}$$

for which $\bar{x} = x\gamma_{\rm A}\gamma_{\rm B}$. The eigenvalues of this matrix are given by $\{\lambda_k(x,t)\} = \{\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{4}(1+4x\gamma_{\rm A}\gamma_{\rm B}),\frac{1}{4}(1-4x\gamma_{\rm A}\gamma_{\rm A})\}$, of which only the last one can possibly be negative. According to the positivity condition, this is a valid density matrix for values of x such that all eigenvalues are positive, $0 \le x \le \frac{1}{4}$, since $\gamma_{\rm A},\gamma_{\rm B} \le 1$. The eigenvalues $\left\{\lambda_k^{\rm T_A}(x,t)\right\}$ of the partial transpose with respect to qubit A are $\{\frac{1}{4},\frac{1}{4},\frac{1}{8},\frac{1}{8},\frac{1}{8}(1+8x\gamma_{\rm A}\gamma_{\rm B}),\frac{1}{8}(1-8x\gamma_{\rm A}\gamma_{\rm B})\}$. As before, the only eigenvalue of interest is the last one, which takes on a negative eigenvalue when $\frac{1}{8} < x \le \frac{1}{4}$. Thus, we find

$$\mathscr{N}\,(\rho'_{\rm AB}(x,t)) = \max\{0, x\gamma_{\rm A}(t)\gamma_{\rm B}(t) - 1/8\}\ , \tag{51}$$

may reach zero in finite time as the first term decays according to the γ_A and γ_B factors. Thus, we showed that ESD can take place in the next largest system finite-dimensional bipartite system, supporting the conjecture that it is not specific to the simplest quantum system and may be a generic effect. Most recently, we have extended this result to pairs of d-dimensional systems for all d in the case of initially isotropic entangled systems, exploiting their symmetry [29].

RELATION TO FOUNDATIONS OF QUANTUM MECHANICS

An attempt to solve the quantum measurement problem was made 25 years ago by Żurek, invoking a process termed environmentally induced superselection or *einselection* [30]. The idea is that the environment of the system under measurement naturally chooses the basis in which state is measured by the measuring apparatus, for the class of interactions that include measurements. Although this approach has encountered a number of difficulties in solving the measurement problem, such as the failure of *coherence* to be entirely lost in finite times under local noise, the Żurek group has continued to pursue the study of decoherence and to produce increasingly subtle treatments of system–apparatus–environment interactions to better understand its effects. We have been following a related direction of investigation that focuses not on decoherence, but rather on *disentanglement* which, in comparison with decoherence, can relatively easily occur and can even do so in finite time under local noise, as we have discussed here. Because the measurement process requires the entanglement between measured system and measurement apparatus and occurs in a larger noisy environment, we are suggesting that if a sufficient formal description of physical conditions for a successful completed measurement process is characterized primarily by disentanglement then environmental noise may play the key role in measurement and its study may allow for the solution the quantum measurement problem, through its ability to induced disentanglement between measuring apparatus and measured system.

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