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MICRO 435 HW2

a) $$\left.\begin{aligned} |\psi_1\rangle &= \alpha_1|0\rangle + \beta_1|1\rangle \\ |\psi_2\rangle &= \alpha_2|0\rangle + \beta_2|1\rangle \end{aligned}\right\} \quad U\left(|\psi_1\rangle \otimes |\psi_2\rangle\right) = |\psi_2\rangle \otimes |\psi_1\rangle$$

$$|\psi_1\rangle = \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} \qquad |\psi_2\rangle = \begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix}$$

$$|\psi_1\rangle \otimes |\psi_2\rangle = \begin{pmatrix}\alpha_1\alpha_2\\ \alpha_1\beta_2\\ \beta_1\alpha_2\\ \beta_1\beta_2\end{pmatrix} \qquad |\psi_2\rangle \otimes |\psi_1\rangle = \begin{pmatrix}\alpha_2\alpha_1\\ \alpha_2\beta_1\\ \beta_2\alpha_1\\ \beta_2\beta_1\end{pmatrix}$$

$$\underbrace{\underset{4\times4}{\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}}}_{U}\underset{4\times1}{\begin{pmatrix}\alpha_1\alpha_2\\ \alpha_1\beta_2\\ \beta_1\alpha_2\\ \beta_1\beta_2\end{pmatrix}} = \underset{4\times1}{\begin{pmatrix}\alpha_2\alpha_1\\ \alpha_2\beta_1\\ \beta_2\alpha_1\\ \beta_2\beta_1\end{pmatrix}}$$

b) Write down the unitary ZCSWAP

$$ZCSWAP\left(|0\rangle \otimes |\psi_1\rangle \otimes |\psi_2\rangle\right) = |0\rangle \otimes |\psi_2\rangle \otimes |\psi_1\rangle$$

$$ZCSWAP\left(|1\rangle \otimes |\psi_1\rangle \otimes |\psi_2\rangle\right) = |1\rangle \otimes |\psi_1\rangle \otimes |\psi_2\rangle$$

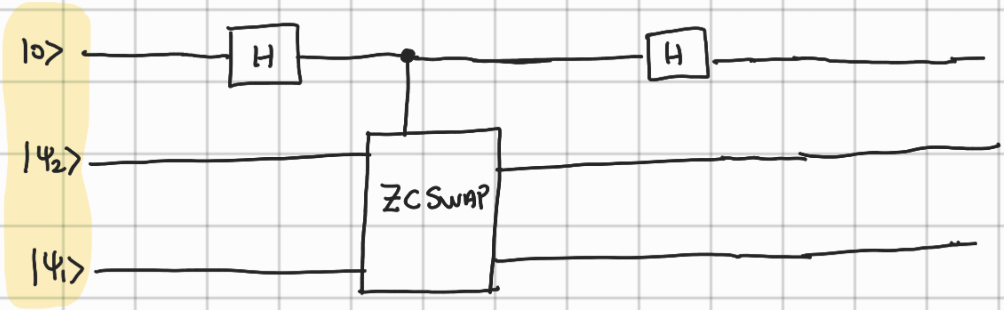
$$ZCSWAP = \left[\begin{array}{cccc|cccc} 1&0&0&0 & & & & \\ 0&0&1&0 & & 0 & & \\ 0&1&0&0 & & & & \\ 0&0&0&1 & & & & \\ \hline & & & & 1&0&0&0 \\ & 0 & & & 0&1&0&0 \\ & & & & 0&0&1&0 \\ & & & & 0&0&0&1 \end{array}\right]$$

The upper-left block is U.

H

8x8 matrix

c)



d) Output $|\phi\rangle$

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$ZCSWAP\left(H|0\rangle \otimes |\psi_1\rangle \otimes |\psi_2\rangle\right)$$

$$\frac{1}{\sqrt{2}} ZCSWAP\left(|0\rangle \otimes |\psi_1\rangle \otimes |\psi_2\rangle\right) + \frac{1}{\sqrt{2}} ZCSWAP\left(|1\rangle \otimes |\psi_1\rangle \otimes |\psi_2\rangle\right)$$

changed

$$= \frac{1}{\sqrt{2}}|0\,\psi_2\,\psi_1\rangle + \frac{1}{\sqrt{2}}|1\,\psi_1\,\psi_2\rangle$$

Apply Hadamard

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|\psi_1\psi_2\rangle + \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)|\psi_2\psi_1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\psi_1\psi_2\rangle + \frac{1}{\sqrt{2}}|1\psi_1\psi_2\rangle + \frac{1}{\sqrt{2}}|0\psi_2\psi_1\rangle - \frac{1}{\sqrt{2}}|1\psi_2\psi_1\rangle\right)$$

$$= \frac{1}{2}|0\psi_1\psi_2\rangle + \frac{1}{2}|1\psi_1\psi_2\rangle + \frac{1}{2}|0\psi_2\psi_1\rangle - \frac{1}{2}|1\psi_2\psi_1\rangle)$$

e)

$$p_0 = \langle\phi|0\rangle\langle 0| \otimes I \otimes I|\phi\rangle$$

$$p_1 = \langle\phi|1\rangle\langle 1| \otimes I \otimes I|\phi\rangle$$

$$p_0 = \langle \phi | \, |0\rangle\langle 0| \otimes I \otimes I \, |\phi\rangle$$

$$\left(\tfrac{1}{2}|0\psi_1\psi_2\rangle + \tfrac{1}{2}|0\psi_2\psi_1\rangle\right) \qquad \left(\tfrac{1}{2}|0\psi_1\psi_2\rangle + \tfrac{1}{2}|0\psi_2\psi_1\rangle\right)$$

$$\frac{1}{4}\langle \psi_2\psi_1 0| \, |0\rangle\langle 0| \otimes I \otimes I \, |0\psi_1\psi_2\rangle + \frac{1}{4}\langle \psi_2\psi_1 0| \, |0\rangle\langle 0| \otimes I \otimes I \, |0\psi_2\psi_1\rangle$$

$$+\frac{1}{4}\langle \psi_1\psi_2 0| \, |0\rangle\langle 0| \otimes I \otimes I \, |0\psi_1\psi_2\rangle + \frac{1}{4}\langle \psi_1\psi_2 0| \, |0\rangle\langle 0| \otimes I \otimes I \, |0\psi_2\psi_1\rangle$$

$$= \frac{1}{4}\underbrace{\langle \psi_2\psi_1 \overbrace{0|0}^{1} \psi_1\psi_2\rangle}_{1} + \frac{1}{4}\overbrace{\langle \psi_2\psi_1 | \psi_2\psi_1\rangle}^{|\langle\psi_1|\psi_2\rangle|^2}$$

$$+\frac{1}{4}\underbrace{\langle \psi_1\psi_2 | \psi_1\psi_2\rangle}_{|\langle\psi_2|\psi_1\rangle|^2} + \frac{1}{4}\underbrace{\langle \psi_1\psi_2 | \psi_2\psi_1\rangle}_{1}$$

$$= \frac{1}{4}\times 2 + \frac{1}{4}\times 2 \, |\langle\psi_1|\psi_2\rangle|^2 = \boxed{\frac{1}{2} + \frac{1}{2}|\langle\psi_1|\psi_2\rangle|^2}$$

0

$$p_1 = \langle \phi | \, |1\rangle\langle 1| \otimes I \otimes I \, |\phi\rangle$$

$$\left(\tfrac{1}{2}|1\psi_1\psi_2\rangle - \tfrac{1}{2}|1\psi_2\psi_1\rangle\right) \qquad \left(\tfrac{1}{2}|1\psi_1\psi_2\rangle - \tfrac{1}{2}|1\psi_2\psi_1\rangle\right)$$

$$p_1 = \frac{1}{4}\underbrace{\langle \psi_2\psi_1 1 | 1\psi_1\psi_2\rangle}_{1} - \frac{1}{4}\langle \psi_2\psi_1 1 | 1\psi_2\psi_1\rangle$$

$$-\frac{1}{4}\underbrace{\langle \psi_1\psi_2 1 | 1\psi_1\psi_2\rangle}_{|\langle\psi_2|\psi_1\rangle|^2} + \frac{1}{4}\underbrace{\langle \psi_1\psi_2 1 | 1\psi_2\psi_1\rangle}_{1}$$

$$p_1 = \frac{1}{4}\times 2 - \frac{1}{4}\times 2 \, |\langle\psi_2|\psi_1\rangle|^2 = \boxed{\frac{1}{2} - \frac{|\langle\psi_2|\psi_1\rangle|^2}{2}}$$

f) When $|\psi_1\rangle = |\psi_2\rangle$

$$p_0 = \frac{1}{2} + \frac{|\overbrace{\langle\psi_1|\psi_1\rangle}^{1}|^2}{2} = 1 \;_{//}$$

$$p_1 = \frac{1}{2} - \frac{|\overbrace{\langle\psi_1|\psi_1\rangle}^{1}|^2}{2} = 0 \;_{//}$$

Thus, only "0" result could appear.

To be sure many ($\sim 1000 \uparrow$) measurements should be done.