COHERENT STATES ENTANGLEMENT CONSERVATION

A. Partially Entangled Bell State $|\Psi_{AB}\rangle$

$$|\Psi(0)\rangle = |\Psi_{AB}\rangle \otimes |\alpha_A, \alpha_B\rangle$$

$$|\Psi(0)\rangle = \left(\cos\alpha\,|e_A, g_B\rangle + \sin\alpha\,|g_A, e_B\rangle\right) \otimes |\alpha_A, \alpha_B\rangle$$

$$Q_{AB}(t) = |\sin 2\alpha|\,\cos^2\left(\frac{Gt}{2}\right)$$

$$Q_{ab}(t) = e^{Re\{\alpha_A\}+Re\{\alpha_B\}}\,|\sin 2\alpha|\,\sin^2\left(\frac{Gt}{2}\right)$$

$$Q_{Ab}(t) = \frac{1}{2}\,e^{Re\{\alpha_B\}}\,|\sin 2\alpha|\,|\sin(Gt)|$$

$$Q_{Ba}(t) = \frac{1}{2}\,e^{Re\{\alpha_A\}}\,|\sin 2\alpha|\,|\sin(Gt)|$$

$$Q_{Aa}(t) = e^{Re\{\alpha_A\}}\,\cos^2\alpha\,|\sin(Gt)|$$

$$Q_{Bb}(t) = e^{Re\{\alpha_B\}}\,\sin^2\alpha\,|\sin(Gt)|$$

$$Q_{AB}(t) = |\sin 2\alpha| \cos^2\left(\frac{6t}{2}\right) \cdot e^{Re\{\alpha_A\}+Re\{\alpha_B\}}$$

$$Q_{ab}(t) = e^{Re\{\alpha_A\}+Re\{\alpha_B\}} |\sin 2\alpha| \sin^2\left(\frac{6t}{2}\right)$$

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$$Q_{Ab}(t) = \frac{1}{2} e^{Re\{\alpha_B\}} |\sin 2\alpha| |\sin(6t)| \times e^{Re\{\alpha_A\}}$$

$$-\ /\ Q_{Ba}(t) = \frac{1}{2} e^{Re\{\alpha_A\}} |\sin 2\alpha| |\sin(6t)| \times e^{Re\{\alpha_B\}}$$

$$Q_{Aa}(t) = e^{Re\{\alpha_A\}} \cos^2\alpha\, |\sin(6t)| \cdot e^{Re\{\alpha_B\}} \sin^2\alpha$$

$$Q_{Bb}(t) = e^{Re\{\alpha_B\}} \sin^2\alpha\, |\sin(6t)| \cdot e^{Re\{\alpha_A\}} \cos^2\alpha$$

$-$

$+$

$$e^{Re\{\alpha_A\}+Re\{\alpha_B\}} Q_{AB} + Q_{ab} = |\sin(2\alpha)|\, e^{Re\{\alpha_A\}+Re\{\alpha_B\}}$$

$$e^{Re\{\alpha_A\}} Q_{Ab} - e^{Re\{\alpha_B\}} Q_{Ba} = 0$$

$$e^{Re\{\alpha_B\}} \sin^2\alpha\, Q_{Aa} - e^{Re\{\alpha_A\}} \cos^2\alpha\, Q_{Bb} = 0$$

$+$

$$\boxed{e^{Re\{\alpha_A\}+Re\{\alpha_B\}} Q_{AB} + Q_{ab} + \underbrace{e^{Re\{\alpha_A\}}}_{x} Q_{Ab} - \underbrace{e^{Re\{\alpha_B\}}}_{y} Q_{Ba} + e^{Re\{\alpha_B\}} \sin^2\alpha\, Q_{Aa} - e^{Re\{\alpha_A\}} \cos^2\alpha\, Q_{Bb} = |\sin(2\alpha)|\, e^{\underbrace{Re\{\alpha_A\}}_{x}+\underbrace{Re\{\alpha_B\}}_{y}}}$$

For the case $|\Psi_{AB}\rangle \otimes |0_a 0_b\rangle$, we had:

$$\left(Q_{AB} + Q_{ab}\right) + \left(Q_{Ab} - Q_{Ba}\right) + \left(\sin^2\alpha\, Q_{Aa} - \cos^2\alpha\, Q_{Bb}\right)$$

$\big\downarrow$ For the coherent state case:

$$\left(XY\, Q_{AB} + Q_{ab}\right) + \left(X\, Q_{Ab} - Y\, Q_{Ba}\right) + \left(Y \sin^2\alpha\, Q_{Aa} - X \cos^2\alpha\, Q_{Bb}\right)$$

B. Partially Entangled Bell State $|\Phi_{AB}\rangle$

$$|\Phi(0)\rangle = |\Phi_{AB}\rangle \otimes |\alpha_A, \alpha_B\rangle$$

$$\downarrow$$

$$\cos\alpha\, |e_A, e_B\rangle + \sin\alpha\, |g_A, g_B\rangle$$

$$Q_{AB}(t, \alpha_A : \alpha_B) = \cos^2\left(\frac{Gt}{2}\right)\left[\,|\sin 2\alpha| - 2\cos^2\alpha \sin^2\left(\frac{Gt}{2}\right) e^{Re\{\alpha_B\} + Re\{\alpha_A\}}\right]$$

$$Q_{ab}(t, \alpha_A : \alpha_B) = \sin^2\left(\frac{Gt}{2}\right) e^{Re\{\alpha_A\} + Re\{\alpha_B\}}\left[\,|\sin 2\alpha| - 2\cos^2\left(\frac{Gt}{2}\right)\cos^2\alpha\right]$$

$$Q_{Ab}(t, \alpha_A : \alpha_B) = \frac{1}{2}\cos^2\alpha\, |\sin(Gt)|\, e^{Re\{\alpha_B\}}\left[\,2|\tan\alpha| - e^{Re\{\alpha_A\}}|\sin(Gt)|\right]$$

$$Q_{aB}(t, \alpha_A : \alpha_B) = \frac{1}{2}\cos^2\alpha\, |\sin(Gt)|\, e^{Re\{\alpha_A\}}\left[\,2|\tan\alpha| - e^{Re\{\alpha_B\}}|\sin(Gt)|\right]$$

$$Q_{Aa}(t, \alpha_A : \alpha_B) = \cos^2\alpha\, |\sin(Gt)|\, e^{Re\{\alpha_A\}}\left(e^{2Re\{\alpha_B\}}\sin^2\left(\frac{Gt}{2}\right) + \cos^2\left(\frac{Gt}{2}\right)\right)$$

$$Q_{Bb}(t, \alpha_A : \alpha_B) = \cos^2\alpha\, |\sin(Gt)|\, e^{Re\{\alpha_B\}}\left(e^{2Re\{\alpha_A\}}\sin^2\left(\frac{Gt}{2}\right) + \cos^2\left(\frac{Gt}{2}\right)\right)$$

$$Q_{AB} = \cos^2\left(\frac{6t}{2}\right)|\sin 2\alpha| - \frac{1}{2}\cos^2\alpha \sin^2(6t)\, XY$$

$$Q_{ab} = \sin^2\left(\frac{6t}{2}\right)|\sin 2\alpha|\, XY - \frac{1}{2}\cos^2\alpha \sin^2(6t)\, XY$$

$$Q_{Ab} = \cos^2\alpha\, |\sin(6t)|\,|\tan\alpha|\, Y - \frac{1}{2}\cos^2\alpha \sin^2(6t)\, XY$$

$$Q_{Ba} = \cos^2\alpha\, |\sin(6t)|\,|\tan\alpha|\, X - \frac{1}{2}\cos^2\alpha \sin^2(6t)\, XY$$

$$Q_{Aa} = \cos^2\alpha\, |\sin(6t)| \sin^2\left(\frac{6t}{2}\right) XY^2 + \cos^2\alpha\, |\sin(6t)| \cos^2\left(\frac{6t}{2}\right) X$$

$$Q_{Bb} = \cos^2\alpha\, |\sin(6t)| \sin^2\left(\frac{6t}{2}\right) X^2Y + \cos^2\alpha\, |\sin(6t)| \cos^2\left(\frac{6t}{2}\right) Y$$

For the case $|\Phi_{AB}\rangle \otimes |0_a 0_b\rangle$ we had:

$$Q_{AB} + Q_{ab} - 2Q_{Ab} + 2|\tan\alpha|\, Q_{Aa} = |\sin 2\alpha|$$

$$(XY\, Q_{AB} + Q_{ab}) - (XY\, Q_{Ab} + Q_{Ba}) + |\tan\alpha|\,(?\, Q_{Aa} + ?\, Q_{Bb})$$

$$(XY\, Q_{AB} + Q_{ab}) = XY|\sin 2\alpha| \boxed{- \frac{1}{2}\cos^2\alpha \sin^2(6t)\, XY(XY+1)}$$

$$\begin{aligned} -(XY\, Q_{Ab} + Q_{Ba}) = {} & \cos^2\alpha\, |\sin(6t)|\,|\tan\alpha|\, XY^2 - \frac{1}{2}\cos^2\alpha \sin^2(6t)\, X^2Y^2 \\ & + \cos^2\alpha\, |\sin(6t)|\,|\tan\alpha|\, X - \frac{1}{2}\cos^2\alpha \sin^2(6t)\, XY \end{aligned}$$

$$+$$

$$- \cos^2\alpha\, |\sin(6t)|\,|\tan\alpha|\, X(Y^2+1)$$

$$|\tan\alpha|\left(A\, Q_{Aa} + B\, Q_{Bb}\right) =$$

$$|\tan\alpha| \cos^2\alpha\, |\sin(6t)|\, \sin^2\left(\frac{6t}{2}\right) XY^2 + |\tan\alpha| \cos^2\alpha\, |\sin(6t)|\, \cos^2\left(\frac{6t}{2}\right) X$$

$$|\tan\alpha| \cos^2\alpha\, |\sin(6t)|\, \sin^2\left(\frac{6t}{2}\right) X^2 Y + |\tan\alpha| \cos^2\alpha\, |\sin(6t)|\, \cos^2\left(\frac{6t}{2}\right) Y$$

Are there and coefficient A, B such that

$$X(Y^2+1) = A \sin^2\left(\frac{6t}{2}\right) XY^2 + A \cos^2\left(\frac{6t}{2}\right) X + B \sin^2\left(\frac{6t}{2}\right) X^2 Y + B \cos^2\left(\frac{6t}{2}\right) Y$$