Particle Physics 1: Exercise 5

Exercise 1

The total $e^+e^- \to \gamma \to \mu^+\mu^-$ annihilation cross section is $\sigma = 4\pi\alpha^2/(3s)$, where $\alpha \sim 1/137$. Calculate the cross section at $\sqrt{s} = 50\,\mathrm{GeV}$, expressing your answer in both natural units and barns ($1\,\mathrm{b} = 10^{-28}\,\mathrm{m}^2$). Compare this to the total pp cross section at $\sqrt{s} = 50\,\mathrm{GeV}$ which is approximately 40 mb and comment on the result.

Exercise 2

A 1 GeV muon neutrino is fired at a 1 m thick block of iron ($^{56}_{26}\mathrm{Fe}$) with density $\rho = 7.874 \times 10^3$ Kg m^{-3}. If the average neutrino-nucleon interaction cross section is $\sigma = 8 \times 10^{-39}\,\mathrm{cm}^2$, calculate the (small) probability that the neutrino interacts in the block.

Exercise 3

For the process $a + b \to 1 + 2$ the Lorentz-invariant flux term is

$$F = 4\left[(p_a \cdot p_b)^2 - m_a^2 m_b^2\right]^{\frac{1}{2}}.$$

In the non-relativistic limit $\beta_\alpha \ll 1$ and $\beta_b \ll 1$, show that

$$F \approx 4 m_a m_b \,|\,\overrightarrow{v_a} - \overrightarrow{v_b}\,|$$

where $\overrightarrow{v_a}$ and $\overrightarrow{v_b}$ are the (non-relativistic) velocities of the two particles.

Exercise 4

At the LEP e^+e^- collider, which had a circumference of 27 km, the electron and positron beams consisted of four equally spaced bunches in the accelerator. Each bunch corresponded to a beam current of 1.0 mA. The beams collided head-on at the interaction point, where the beam spot had an rms profile of $\sigma_\mathrm{x} \approx 250\,\mu\mathrm{m}$ and $\sigma_\mathrm{y} \approx 4\,\mu\mathrm{m}$, giving an effective area of $1.0 \times 10^3\,\mu\mathrm{m}^2$. Calculate the instantaneous luminosity and estimate the event rate for the process $e^+e^- \to Z$, which has a cross section of about 40 nb.

Exercise 5

At a future e^+e^- linear collider operating as a Higgs factory at a centre-of-mass energy of $\sqrt{s} = 250\,\mathrm{GeV}$, the cross section for the process $e^+e^- \to HZ$ is 250 fb. If the collider has an instantaneous luminosity of $2 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ and is operational for 50 % of the time, how many Higgs bosons will be produced in five years of running?

Exercise 1

The total $e^+e^- \to \gamma \to \mu^+\mu^-$ annihilation cross section is $\sigma = 4\pi\alpha^2/(3s)$, where $\alpha \sim 1/137$. Calculate the cross section at $\sqrt{s} = 50\,\text{GeV}$, expressing your answer in both natural units and barns ($1\,\text{b} = 10^{-28}\,\text{m}^2$). Compare this to the total pp cross section at $\sqrt{s} = 50\,\text{GeV}$ which is approximately 40 mb and comment on the result.

In natural units: $\sigma = \dfrac{4\pi\alpha^2}{3s} = \dfrac{4\pi \cdot \left(\frac{1}{137}\right)^2}{3 \cdot 50^2\ \text{GeV}^2}$

$$\sigma = 8.9 \times 10^{-8}\ \text{GeV}^{-2}$$

$$\sigma = 8.9 \times 10^{-8} \times (\hbar c)^2$$

$\downarrow$

$\hbar c = 0.197\ \text{GeV fm}$

$1\,\text{fm} = 10^{-15}\,\text{m}$

$1\,\text{fm}^2 = 10^{-30}\,\text{m}^2 = 10^{-2} \times \overset{10^{-28}\,\text{m}^2}{\overbrace{1\,\text{b}}}$

$$\sigma = 8.9 \times 10^{-8}\ \text{GeV}^{-2} \times 0.197^2 \times 0.01\,\text{b} = 34\ \underset{10^{-12}}{\text{p}}\text{b}$$

This is a factor of approximately 10^9 smaller than the strong interaction pp cross section.

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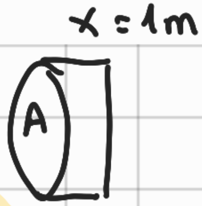

$$\text{Rate} = \phi \, \sigma \, N_t$$

$$N_{\text{interaction}} = \sigma N_t \underbrace{\int \phi \, dt}_{\frac{1}{A}} \;?$$

$$\int \frac{dProb}{dt} dt = Prob$$

$$\overset{\sim x}{n_a v . t} = \frac{1}{A}$$

$$\underset{1}{n_a} . V = 1$$

$$N_t = A x \, n$$ → # density of target nuclei

$$N_{int} = \sigma A x n . \frac{1}{A} = \sigma x n$$

$$n = \frac{\rho}{Y \, m_n} = \frac{7874 \text{ kg/m}^3}{56 \times 1.67 \times 10^{-27} \text{kg}} = 8.4 \times 10^{28} \text{ m}^{-3}$$

Y → Nucleus number, m_n → Average mass of nucleon

10^{24} cm^{-2}

$$N_{int} = \sigma n x = 8 \times 10^{-39} \text{ cm}^2 \; 8.4 \times 10^{28} \underbrace{\text{m}^{-3} (1\text{m})}_{\text{m}^{-2}}$$

$$N_{int} = 7 \times 10^{-14} < 10^{-13}$$

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In the non-relativistic limit $\beta_a \ll 1$ and $\beta_b \ll 1$, show that

$$F \approx 4 m_a m_b |\vec{v_a} - \vec{v_b}|$$

where $\vec{v_a}$ and $\vec{v_b}$ are the (non-relativistic) velocities of the two particles.

$$p_a p_b = E_a E_b - \vec{p_a} \vec{p_b}$$

In Natural Units:

$$E = \gamma m = m\left(1-\beta^2\right)^{-1/2} \approx m\left(1 - \left(-\tfrac{1}{2}\right)\beta^2\right)$$

$$E \simeq m + \frac{m}{2}\beta^2$$

$v = \beta c = \beta$ $(c=1)$

$$\vec{p} = \gamma m \vec{\beta} = m\left(1-\beta^2\right)^{-1/2}\beta \simeq m\vec{\beta}$$

$$p_a p_b = E_a E_b - \vec{p_a}\,\vec{p_b}$$

$$\simeq m_a m_b\left(1+\tfrac{1}{2}\beta_a^2\right)\left(1+\tfrac{1}{2}\beta_b^2\right) - m_a m_b \vec{\beta_a}\cdot\vec{\beta_b}$$

$$= m_a m_b\left[1 + \frac{1}{2}\left(\beta_a^2+\beta_b^2\right) + \frac{1}{4}\beta_a^2\beta_b^2 - \vec{\beta_a}\cdot\vec{\beta_b}\right]$$

$$\simeq m_a m_b\left[1+\frac{1}{2}\left(\vec{\beta_a}-\vec{\beta_b}\right)^2 + \frac{1}{4}\cancel{\beta_a^2\beta_b^2}\right]$$

$\to 0$ (too small)

$$\simeq m_a m_b\left[1+\frac{1}{2}\left(\vec{\beta_a}-\vec{\beta_b}\right)^2\right]$$

$$(p_a p_b)^2 \approx \underbrace{m_a^2 m_b^2\left[1+\left(\vec{\beta_a}-\vec{\beta_b}\right)^2\right.} + \left.\frac{1}{4}\cancel{\left(\vec{\beta_a}-\vec{\beta_b}\right)^4}\right]$$

$\to 0$

$$F = 4\left[(p_a p_b)^2 - m_a^2 m_b^2\right]^{1/2}$$

$v_a/c \ll 1$

$$F \simeq 4\left[m_a^2 m_b^2\left(\vec{\beta_a}-\vec{\beta_b}\right)^2\right]^{1/2} \simeq 4 m_a m_b\left|\vec{\beta_a}-\vec{\beta_b}\right|$$

In SI units: $4 m_a m_b \left|\vec{v_a}-\vec{v_b}\right|$

Exercise 4

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(1.0 mA) beam current

Instantaneous luminosity

$$\mathcal{L} = f \frac{n_1 n_2}{4\pi \sigma_x \sigma_y}$$

$\sigma_x \sigma_y \to 10^{-5}\,\text{cm}^{-2}$

# of electrons/positrons in each bunch = n_e

Each bunch circulates ring at frequency $f_b = \dfrac{c}{(27 \times 10^3)}$

$f_b = 11.1$ kHz

Bunch current $I = f_b \cdot n_e \cdot e$

$$\frac{1\,\text{mA}}{f_b\, e} = n_e = 5.6 \times 10^{11} = n_1 = n_2$$

4 Bunches per beam $\to f = 4 \times f_b = 44.4$ kHz

$$\mathcal{L} = \underset{(1/s)}{44.4 \times 10^3} \frac{(5.1 \times 10^{11})^2}{4\pi \times 10^{-5}\,\text{cm}^2} = 1.1 \times 10^{32}\,\text{cm}^{-2}\text{s}^{-1}$$

Event Rate $= \sigma \mathcal{L} = \underbrace{40\,\text{nb}}_{4 \times 10^{-32}\,\text{cm}^{-2}\text{s}^{-1}} \times 1.1 \times 10^{32} \simeq 4\,\text{s}^{-1}$

Exercise 5

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Event Rate

$$N = \int \overbrace{\sigma \mathcal{L}} \, dt$$

$$\text{Rate} = \sigma \mathcal{L} = (250 \times \underbrace{10^{-15}}_{f} \times \underbrace{10^{-24}}_{\text{barn}} \text{cm}^2) \times (2 \times 10^{34} \text{cm}^{-2} \text{s}^{-1})$$

$$\text{Rate} = 0.005 \, \text{s}^{-1}$$

50% of the time

$$\text{In five years} = (0.005 \, \text{s}^{-1}) \cdot \frac{1}{2} \cdot 365 \cdot 86400 \, \text{s} \simeq 394000$$