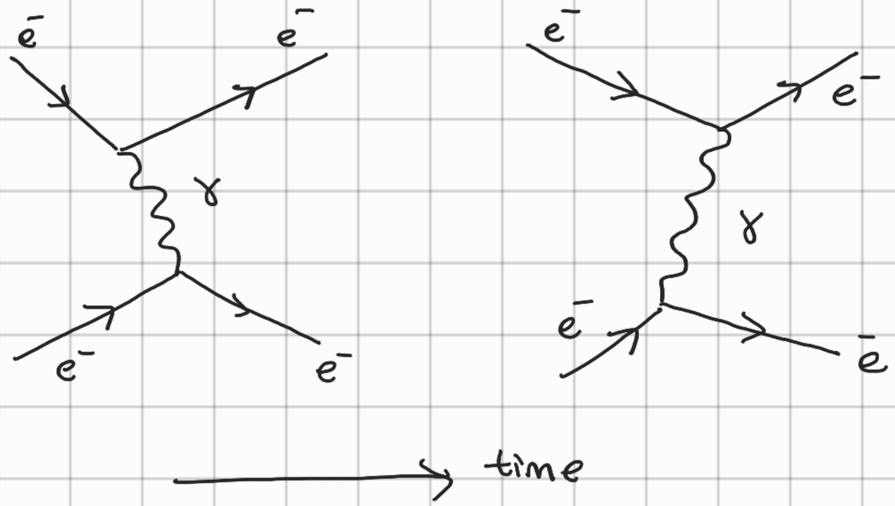
Week 1 Summary

1^{st} generation	ν_e (0)	e^- (-1)	d ($-1/3$)	u ($+2/3$)
2^{nd} generation	ν_μ (0)	μ^- (-1)	s ($-1/3$)	c ($+2/3$)
3^{rd} generation	ν_τ (0)	τ^- (-1)	b ($-1/3$)	t ($+2/3$)

				Strong	EM	Weak
Quarks	d	s	b	✓	✓	✓
	u	c	t			
Leptons	e^-	μ^-	τ^-		✓	✓
	ν_e	ν_μ	ν_τ			✓

★ Quarks are never observed as free particles, always confined to bound states called hadrons (such as proton, neutron)



In classical EM, scalar potential is not enough to explain transfer of momentum

★ In QED interactions btw. charged particles are mediated by the exchange of virtual photons
?
(later)

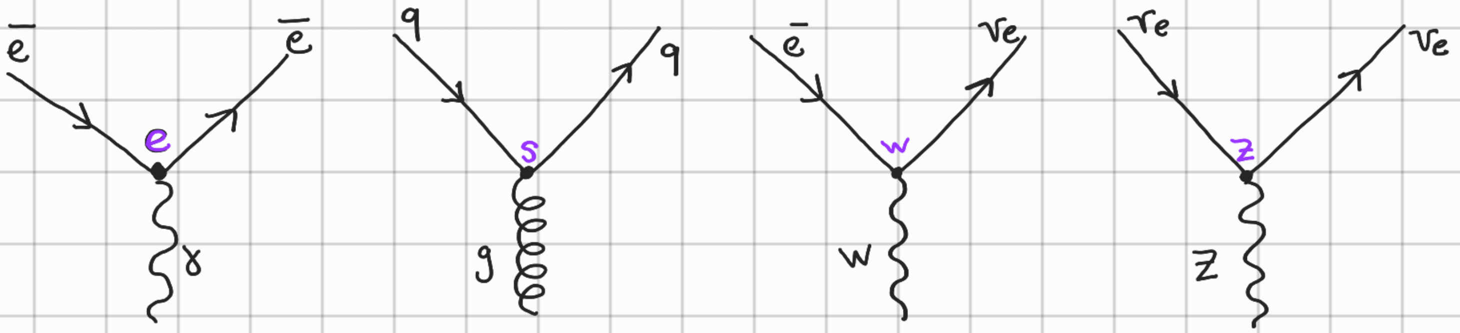
Forces	Strength	Boson		Spin	Mass /GeV
Strong	1	Gluon	g	1	0
EM	10^{-3}	Photon	γ	1	0
Weak	10^{-8}	W bozon	$W^{\pm}$	1	80.4
		Z bozon	Z	1	91.2
Gravity	10^{-37}	Graviton	G	2	0

☆ Higgs Boson → spin=0, $m_H \simeq 125$ GeV

→ only fundamental scalar in SM.

→ Excitation of the Higgs field

SM Interaction Vertices

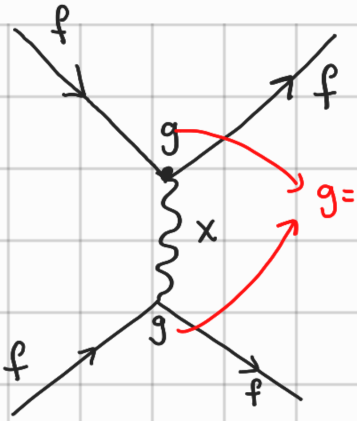


Electromagnetism	Strong	Weak	Weak
• All charged particle	• only quarks	• All fermions	• All fermions
• Never changes flavour	• Never changes fla.	• Always changes flavour	• Never changes flavour
$\alpha = 1/137$	$\alpha_S = 1$		$\alpha_{W/Z} = 1/30$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$ → weak interaction couples

up-type quarks $(+\frac{2}{3})$ → (u, c, t)

down-type quarks $(-\frac{1}{3})$ → (d, s, b)



g = coupling constant → shows strength

$M \propto g^2$

matrix element

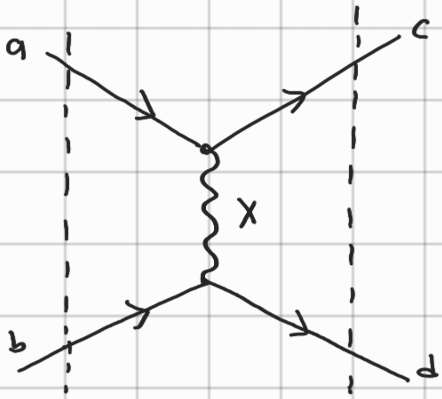
Interaction prob $= |M|^2 \propto g^4$

Instead of using g, α is mostly used. $(\alpha \propto g^2)$

Where in EM → fine structure constant $= \alpha = \dfrac{e^2}{4\pi \epsilon_0 \hbar c} = \dfrac{1}{137}$ (has no dimension)

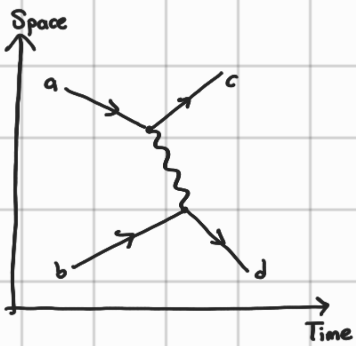
Feynman Diagrams

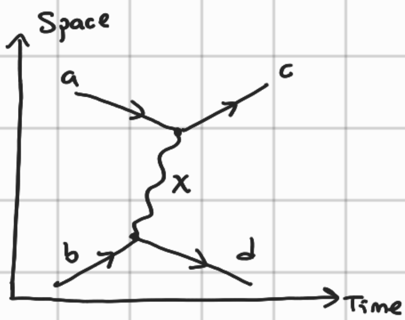
$a + b \rightarrow c + d$ (Infinite possible representation)



initial state

final state

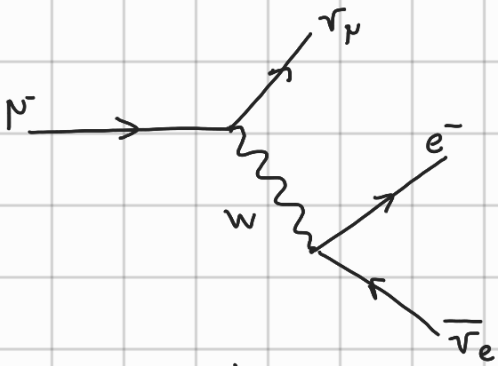




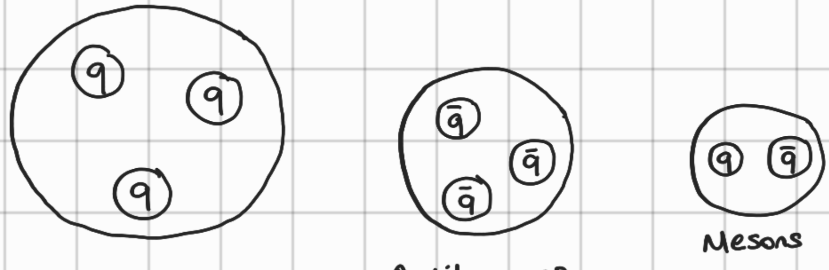
2 examples

Particle decays

* weak interactions
* → smaller masses
* e is stable, no decay occurs for e

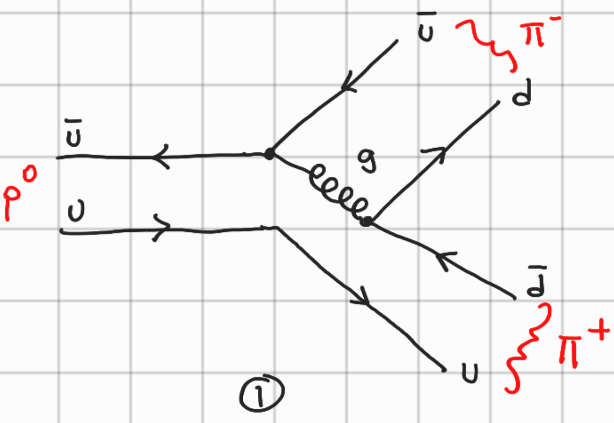


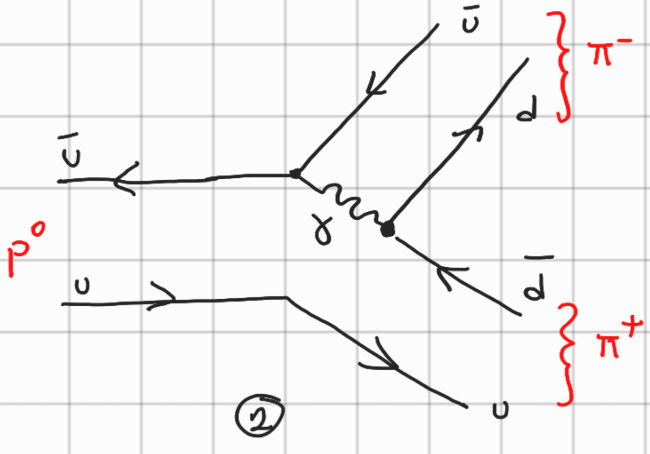
Muon decay



Baryons   Antibaryons   Mesons

★ The only stable hadron is proton





2 possible diagrams

① $|M_g|^2 \propto \alpha_s^2$

② $|M_\gamma|^2 \propto \alpha^2$

$\alpha_s^2 > \alpha^2$

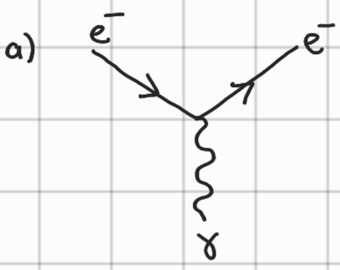
Strong interaction dominates EM or weak decays.

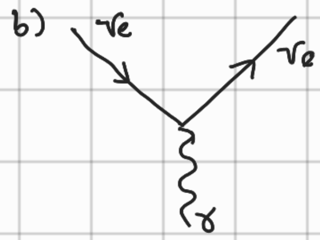
★ Particles where only weak decay processes are possible are relatively long-lived.

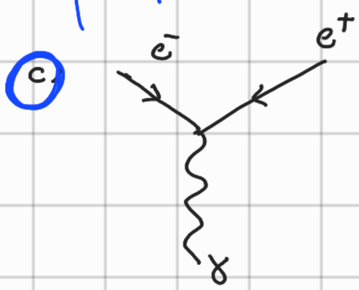
Exercises - Feynman Diagrams

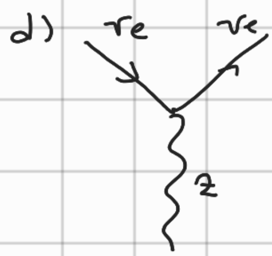
1) State correct/incorrect acc. to SM.

No charge conversion particle to antiparticle (?)







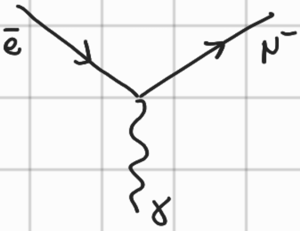


a) EM
✓ All charged particles
no change of flavour

b) EM
✗ ν_e has no charge

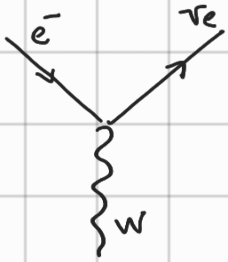
c) EM
✗ No change of ~~flavour~~
Flavour is same

d) ✓ Weak
• No change

e)


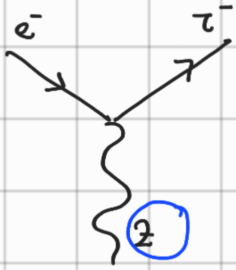
EM

× No change of flavour

f)


Weak

✓ Change of flavour

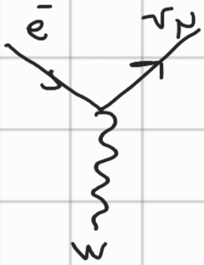
g)


Weak

✓ ✗ e = -1, τ = -1, Z = 0

~~change of flavour~~

no change of flavour

h)


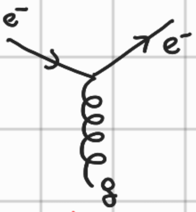
Weak

✗ e = -1, ν_μ = 0

W = -1

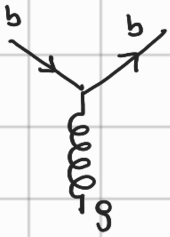
no ~~change of flav.~~

~~W~~ only couple together leptons with the corresponding neutrino

i)


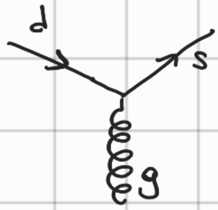
Strong

× Only quarks

j)


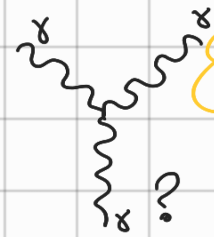
Strong

✓ Quarks, no change of flav.

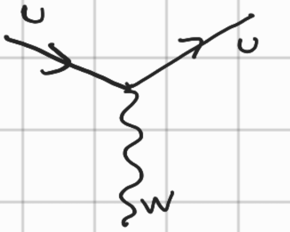
k)


Strong

× No change of flavours

l)


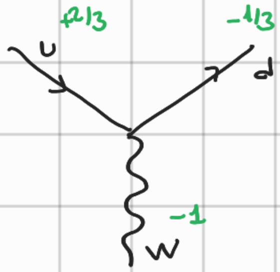
No three photon vertex

m)


Weak

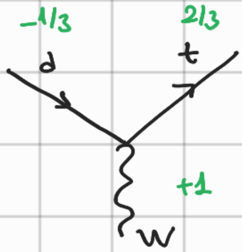
× ~~Quarks don't interact weak int.~~

W is charged boson, hence it change flavour.

n)


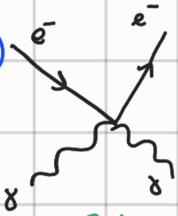
Weak

✓ All fermions, change of flavours

o)


Weak

✓ Change of flav.

p)


EM

✗

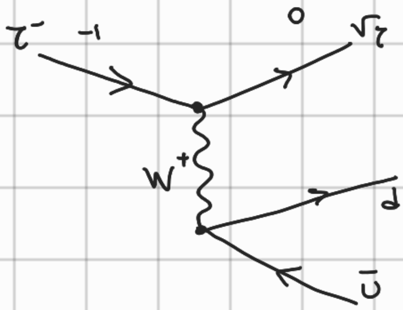
✓ All charged part. no change of fla.

All fermion vertices involve a coupling to a single gauge boson

2) Draw the Feynman diagram for $\tau^- \longrightarrow \pi^- \nu_\tau$ where $\pi^- = \bar{u}d$ meson

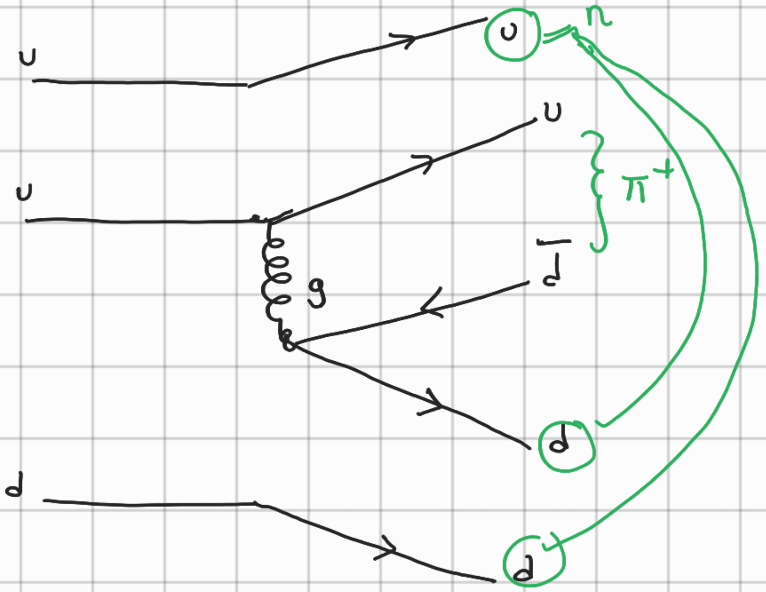
$\pi^- = \bar{u}d \longrightarrow \bar{u} = -2/3 \quad d = 1/3 \qquad \bar{u}d = -2/3 - 1/3 = -1$

$\tau^- \longrightarrow \pi^- \nu_\tau$



3) a) $\Delta \longrightarrow n\,\pi^+$ $\quad +2/3 \; +1/3 = +\underline{\underline{1}}$

$(uud) \qquad (udd) \; (u\bar{d})$



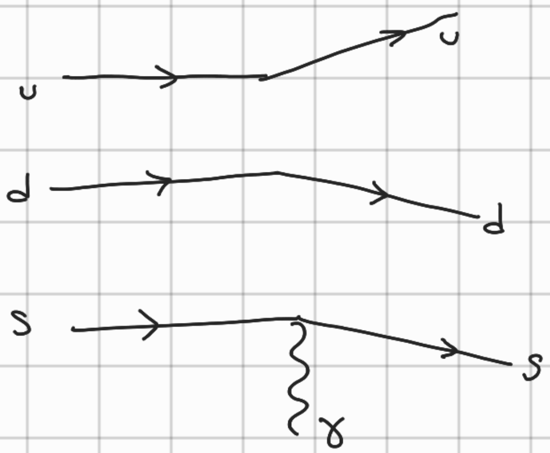
Domination:

Strong > EM > Weak

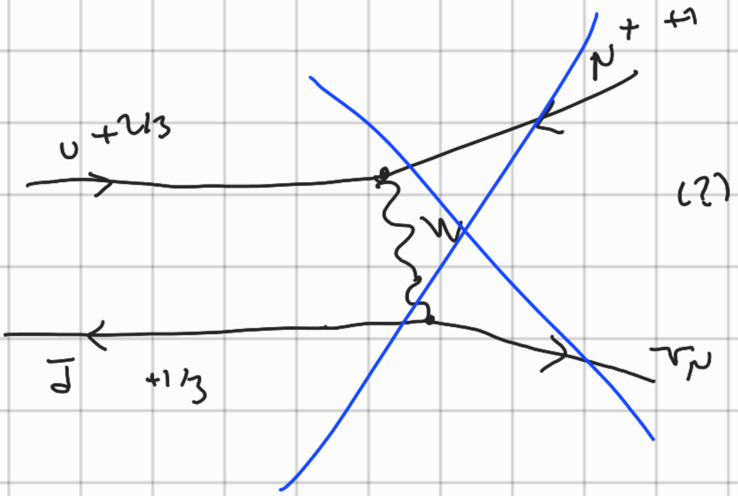
Order in increasing life time

Strong → EM → Weak

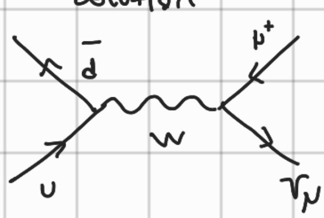
b) $\Sigma^0 (uds) \longrightarrow \Lambda (uds)\ \gamma$

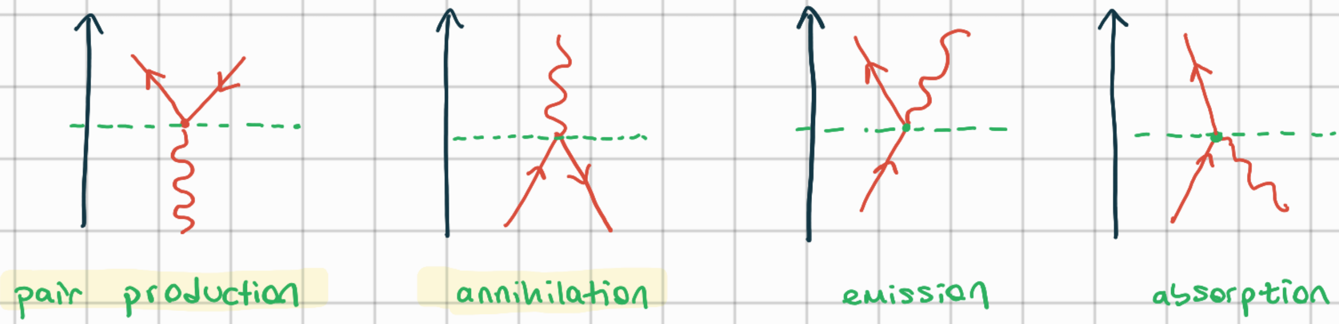


c) $\pi^+ (u\bar{d}) \longrightarrow \mu^+ \nu_\mu$



Solution

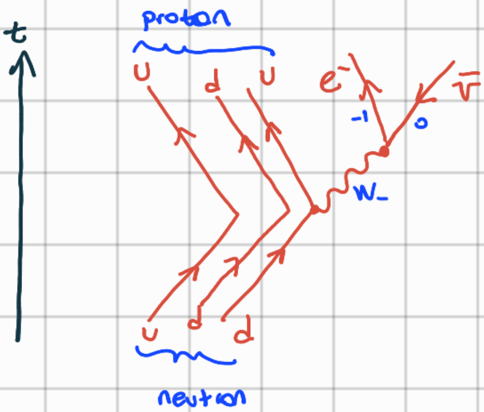




β decay:

$${}^{1}_{0}n \rightarrow {}^{1}_{+1}p + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$$

udd → uud



Hadrons

(composite particles from quarks and/or antiquarks)

- Mesons $(q\bar{q})$
- Baryons $(qqq$ or $\bar{q}\bar{q}\bar{q})$
- Tetraquark
- Pentaquark

1) K meson = $\bar{u}s$ → Charge: $\bar{u} = -\frac{2}{3}e - \frac{1}{3}e = -e$ → elementary charge

Strangeness : $(-1) = S$

Quark # : $(-1) + (+1) = 0$

$E = mc^2$

2) $^{14}_{6}C \longrightarrow ^{14}_{7}N + ^{0}_{-1}e + ^{0}_{0}\bar{\nu} + E$

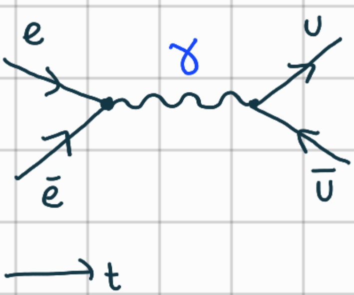
why the mass of a C-14 nucleus and the mass of a N-14 nucleus are slightly different even though they have the same nucleon number.

3) $e^- + e^+ \longrightarrow \nu + \bar{\nu}$

This process involves the electromagnetic interaction $\rightarrow \gamma$

a) what is a virtual particle?

An exchange particle that mediates one of the 4 fundamental interactions



4) Some particles are identical to their antiparticles. Discuss whether the neutron and the antineutron are identical.

$\cancel{\bar{n}^0 = n^0}$?

They have zero electric charge in total

$n = udd$, $\bar{n} = \bar{u}\bar{d}\bar{d}$

5) $K^- \to \pi^+ + \pi^- + \pi^-$

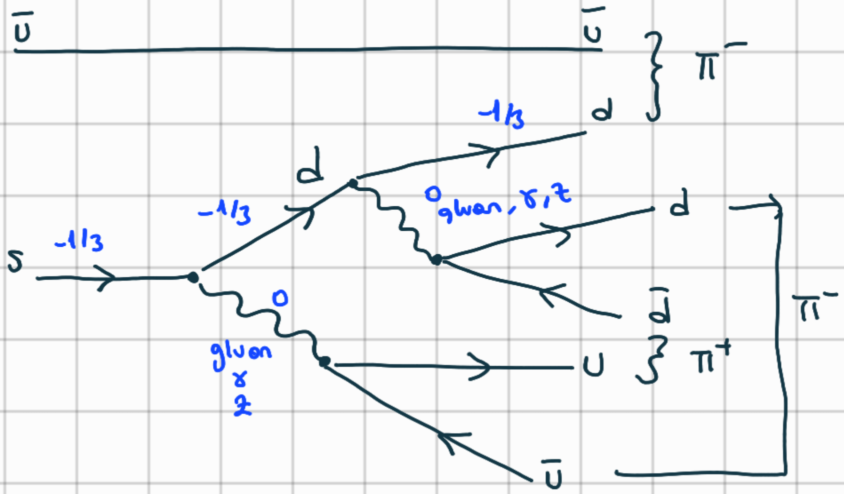
$\pi^- = \bar{u}d$

$\pi^+ = u\bar{d}$

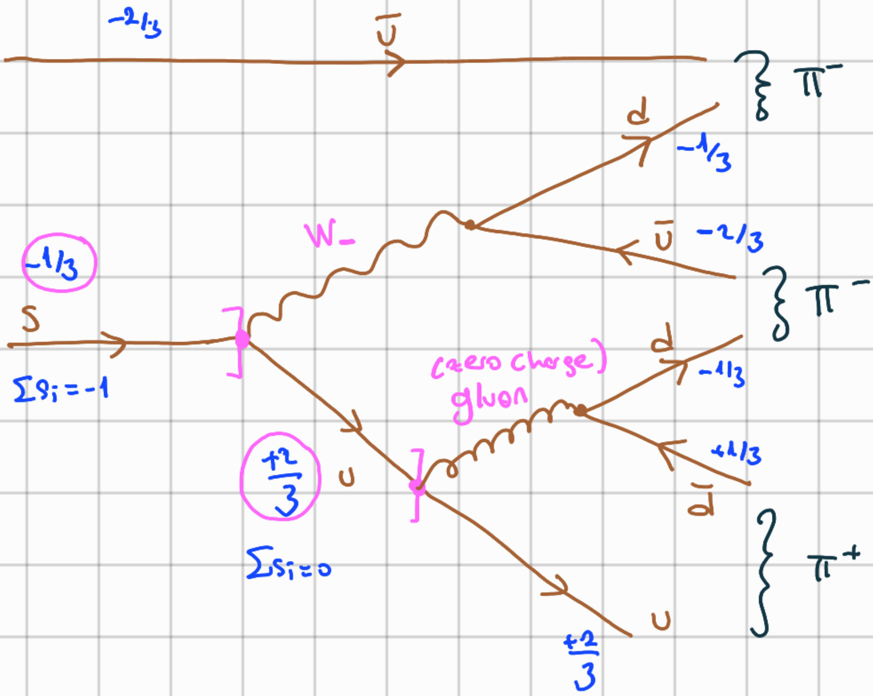
$K^- = \bar{u}s$

$\bar{u}s \to \bar{u}d + \bar{u}d + u\bar{d}$

$s \to d + \bar{u} + d + u + \bar{d}$



Solution :

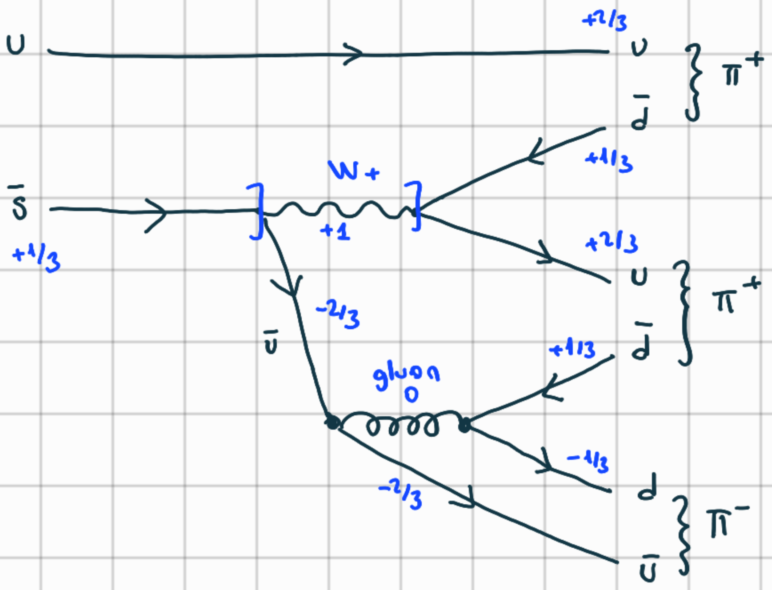


$\sum s_i = \sum s_f$

unless weak interaction

6) $K^+ \longrightarrow \pi^+ + \pi^+ + \pi^-$

$u\bar{s} \qquad u\bar{d} \qquad u\bar{d} \qquad \bar{u}d$



7) $n \longrightarrow K^+ + e^-$! Can not happen

$n \longrightarrow u + \bar{s} + \bar{e}$

$\underbrace{u+d+d} \rightarrow \underbrace{u + \bar{s} + \bar{e}}$

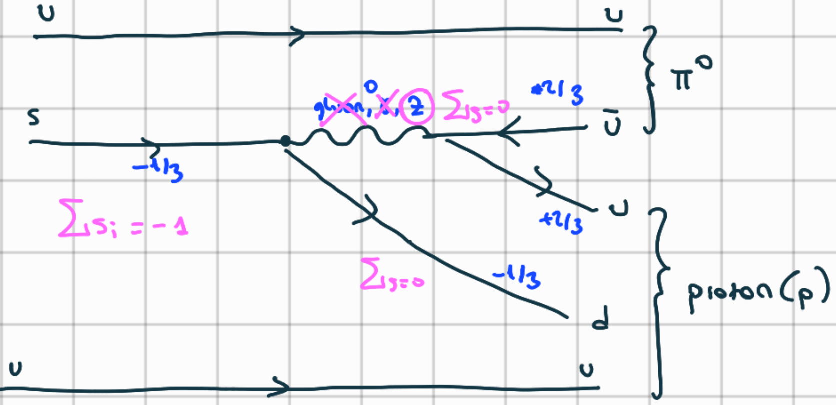
$\sum q_i = 3 \qquad \sum q_i = +1 + (-1) = 0 \quad \times$

lepton number $\leftarrow \sum L_e = 0 \qquad \sum L_e = 1 \quad \times$

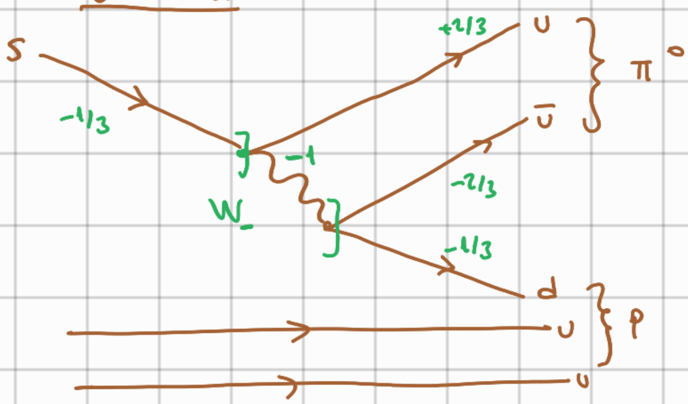
! Conservation Rule

8) $\Sigma^+ \longrightarrow \pi^0 + p$

$uus \qquad u\bar{u} \qquad uud$

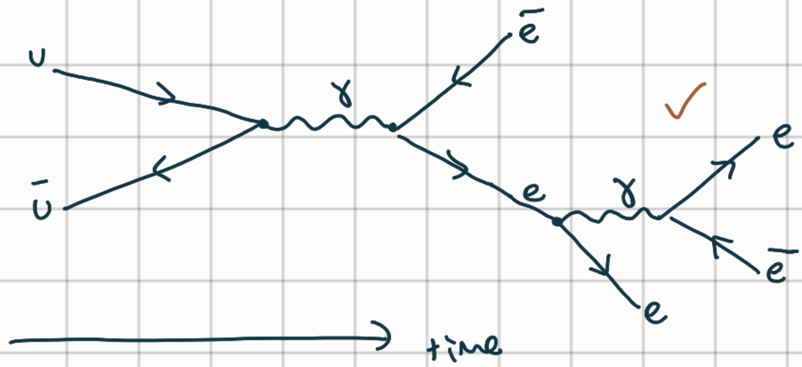


Solution:

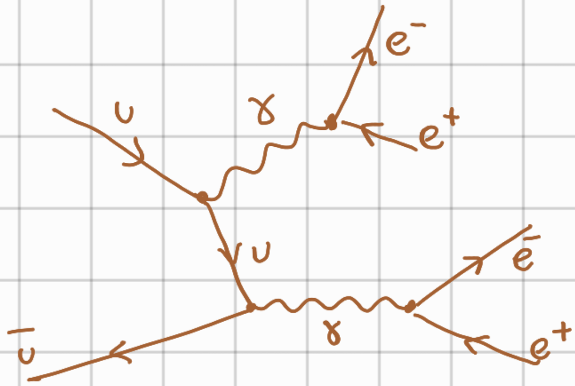


9) π^0 → emission of two γ gamma rays, each one subsequently produce an electron and a positron. ⇒ pair production

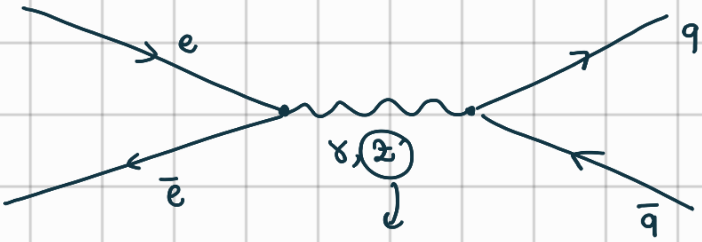
$\nu\bar{\nu} \to e + \bar{e} + e + \bar{e} + \gamma + \gamma$



Solution



10)



question says weak interaction; thus, it should be Z.

11) Neutrino → particle is same as its own antiparticle?

$$\bar{\nu}^0 = \nu^0$$

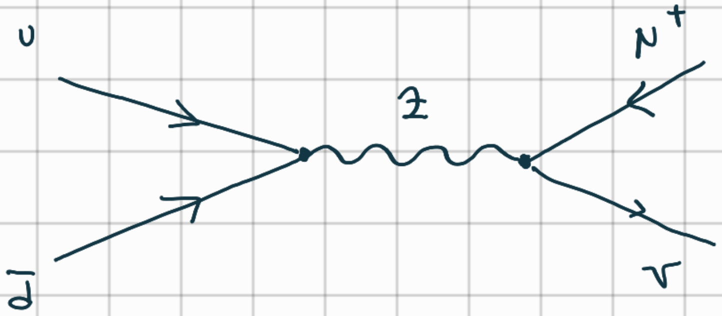
12) $\pi^+ \longrightarrow \nu + \mu^+$

pion — neutrino — antimuon

$Q=1$ $\quad$ $Q=0$ $\quad$ $Q=1$

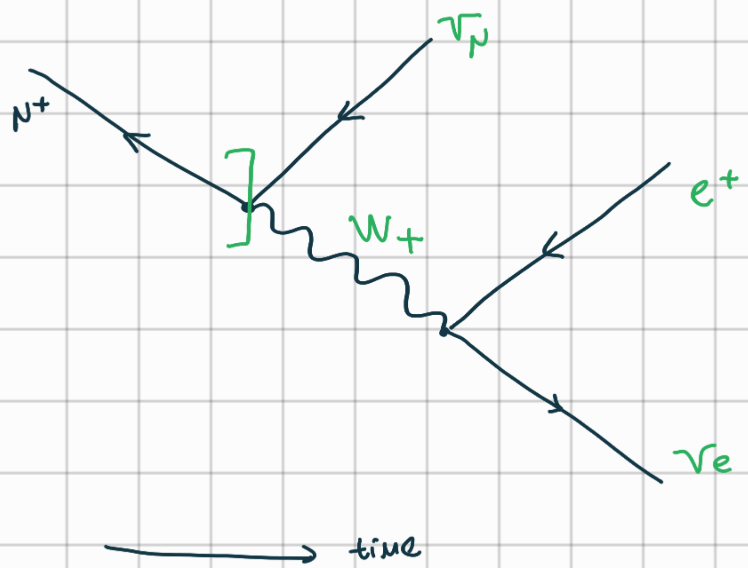
$u\bar{d} \longrightarrow \nu + \mu^+$

$+\frac{2}{3} + \frac{1}{3} = +1$

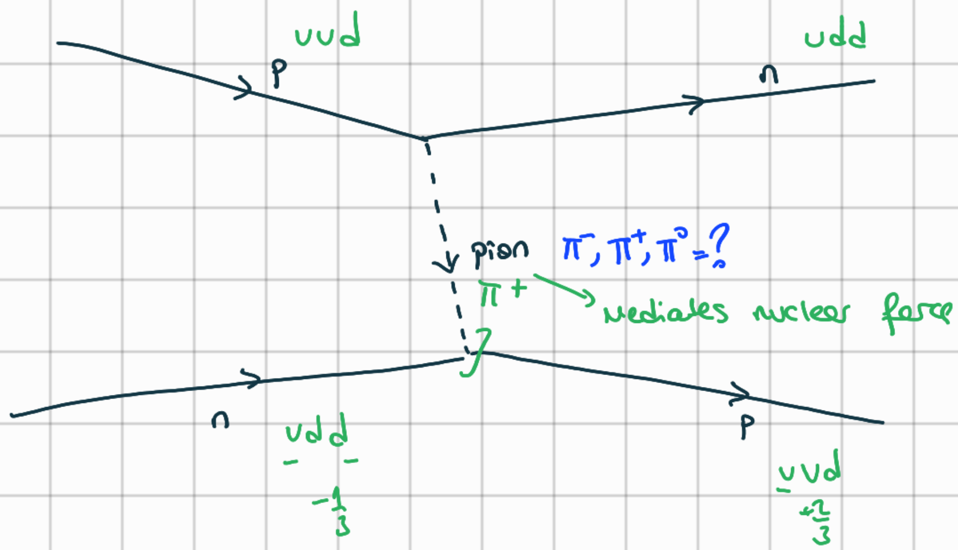


13) Antimuon Decay

$\mu^+ \longrightarrow e^+ + \nu_e + \nu_\mu$

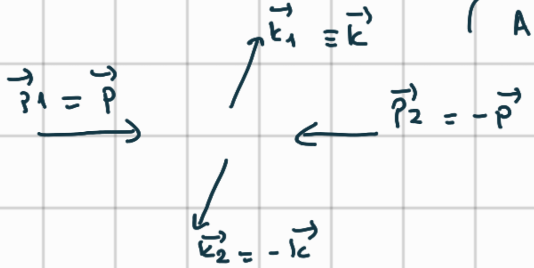


14)



Mandelstam Variables

From Altuğ Özpineci's Video



$$s = (p_1 + p_2)^2$$

$$t = (p_1 - k_1)^2$$

$$u = (p_1 - k_2)^2$$

$$p_1 = \left(\frac{E}{2}, \vec{p}\right)$$

$$p_2 = \left(\frac{E}{2}, -\vec{p}\right)$$

$$k_1 = \left(\frac{E}{2}, \vec{k}\right)$$

$$k_2 = \left(\frac{E}{2}, -\vec{k}\right)$$

$$s = (p_1 + p_2)^2 = (E, 0)^2 = E^2 \Rightarrow E = \sqrt{s}$$

$$t = (p_1 - k_1)^2 = (0, \vec{p} - \vec{k})^2 = -(\vec{p} - \vec{k})^2 = -\vec{p}^2 - \vec{k}^2 + 2\,\vec{p}\cdot\vec{k}$$

$$= 2p^2(\cos\theta - 1)$$

$$|\vec{p}| = |\vec{k}| = p$$

$$\cos\theta = 1 + \frac{t}{2p^2} = 1 + \frac{t}{2\left(\frac{E^2}{4} - m^2\right)} = 1 + \frac{2t}{E^2 - 4m^2}$$

$$1 + 2 \longrightarrow 3 + 4$$

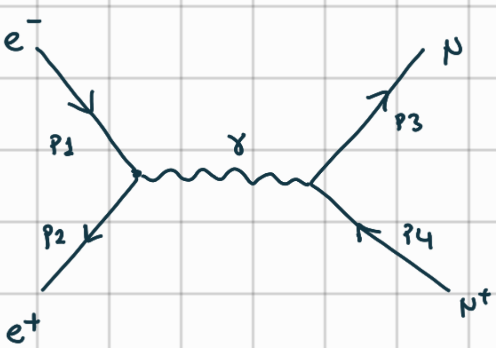
$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

Lorentz Invariant

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$



$$s = (p_1 + p_2)^2$$

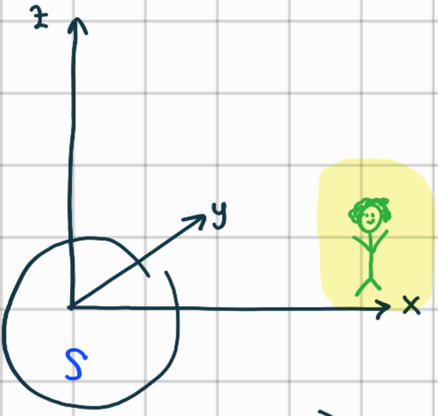
Since Lorentz Invariant (LI) can be evaluated any frame

In the center of mass (CM)

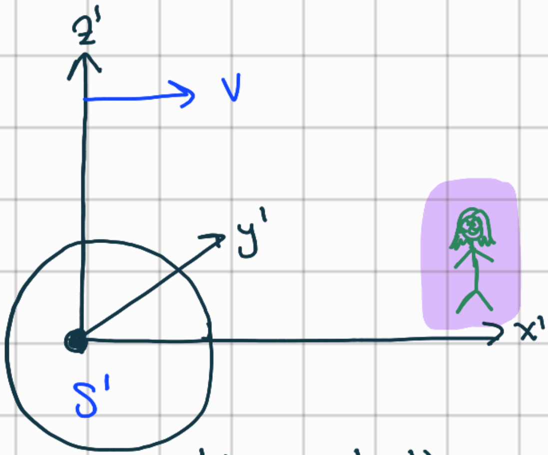
$p_1 = (E_1, \vec{p})$, $p_2 = (E_2, -\vec{p})$

$$s = (E_1 + E_2)^2 \rightarrow \sqrt{s}$$ is the total energy of collision

Lorentz Transformation



$S(x, y, z, t)$



$S'(x', y', z', t')$

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\}$$

Galilean Transformation Equations

✗ not compatible with the reality

$\boxed{\text{LINEAR}}$

→ one-to-one relationship

→ no acceleration arising

* Physical Event Happens At the Origin of S' :

$x' = ax + bt \quad (1)$ → a,b are constants

$x = vt$

$x' = 0$

Using Equation (1)

$x' = ax + bt$

$0 = a(vt) + bt$

$avt = -bt \longrightarrow b = -av \quad (2)$

Substitude Eq 2 in Eq 1

$x' = a(x - vt)$ (3) $(S \to S')$

$x = a(x' + vt')$ (4) $\to$ Reverse, from other ref. frame $(S' \to S)$

* Pulse of light goes off at the origin at time $t=0$

S: $x = ct$ 5.a

S': $x' = ct'$ 5.b

Substitute 5.a and 5.b in 3

$x' = a(x - vt)$	$x = a(x' + vt')$
$ct' = a(ct - vt)$	$ct = a(ct' + vt')$
$ct' = a(c-v)t$	$ct = a(c+v)t'$
$\frac{t}{t'} = \frac{c}{a(c-v)}$ (6)	$\frac{t}{t'} = \frac{a(c+v)}{c}$ (7)

6 = 7

$$\frac{c}{a(c-v)} = \frac{a(c+v)}{c}$$

$$c^2 = a^2 (c+v)(c-v)$$

$$c^2 = a^2 (c^2 - v^2) \longrightarrow a = \sqrt{\frac{c^2}{c^2 - v^2}} = \boxed{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} = \gamma \quad (8)$$

$$x' = a(x - vt) \qquad\qquad x = a(x' + vt')$$

$$\Downarrow$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad\qquad x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \gamma(x - vt) \qquad\qquad x = \gamma(x' + vt')$$

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x\sqrt{1 - \frac{v^2}{c^2}} = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} + vt'$$

$$x\sqrt{1 - \frac{v^2}{c^2}} = \frac{x - vt + vt'\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x\left(1 - \frac{v^2}{c^2}\right) = x - vt + vt'\sqrt{1 - \frac{v^2}{c^2}}$$

$$\cancel{x} - \frac{x v^{\cancel{2}}}{c^2} = \cancel{x} - \cancel{v}t + \cancel{v}t'\sqrt{1 - \frac{v^2}{c^2}}$$

$$t - \frac{xv}{c^2} = t'\sqrt{1 - \frac{v^2}{c^2}}$$

$$\boxed{t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

To sum up:

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$y' = y$$
$$z' = z$$

$$x = \gamma(x' + vt')$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$
$$y = y'$$
$$z = z'$$

Rewrite Equations

$$ct' = \gamma(ct) - \gamma\beta x \qquad \text{where} \quad \beta = \frac{v}{c}$$

$$x' = \gamma x - \gamma\beta(ct)$$

$$y' = y$$

$$z' = z$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \longrightarrow \text{For motion along } x$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\gamma\beta & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

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Motion Along y-axis

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

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Motion along z-axis