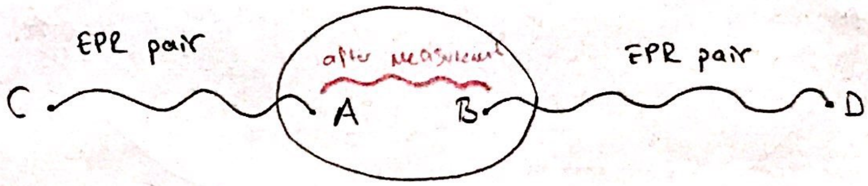
Entanglement Swapping



Initial state: $|B_{00}\rangle_{AC} \otimes |B_{00}\rangle_{BD} = |\Psi\rangle$

Idea: Local Measurement in AB-lab using Bell-basis.

Projected state: $|B_{00}\rangle_{AB}\langle B_{00}|_{AB} \otimes \mathbb{I}_C \otimes \mathbb{I}_D |\Psi\rangle$

$|B_{01}\rangle_{AB}\langle B_{01}|_{AB} \otimes \mathbb{I}_C \otimes \mathbb{I}_D |\Psi\rangle$

$|B_{10}\rangle_{AB}\langle B_{10}|_{AB} \otimes \mathbb{I}_C \otimes \mathbb{I}_D |\Psi\rangle$

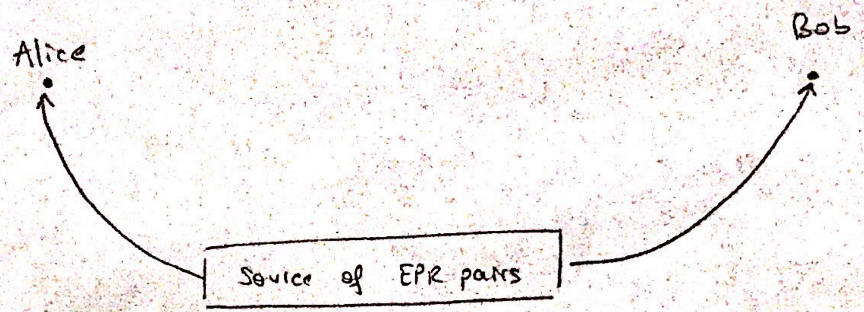
Final Possible State in AB-lab $|B_{00}\rangle_{AB}$

$$\frac{1}{\sqrt{2}}\left(\langle 00|_{AB} + \langle 11|_{AB}\right) \frac{1}{\sqrt{2}}\left(|00\rangle_{AC} + |11\rangle_{AC}\right) \otimes \frac{1}{\sqrt{2}}\left(|00\rangle_{BD} + |11\rangle_{BD}\right)$$

$\langle 0|0\rangle_A = 1$ $\qquad$ $\langle 0|0\rangle_B = 1$

$$= \frac{1}{\sqrt{2}}\left(|00\rangle_{CD} + |11\rangle_{CD}\right)$$ A* C & D becomes entangled

Bell Inequalities (CSHS)



$i=1,\dots,N$

$|B_{00}\rangle$

$\mathcal{H} = \mathbb{C}^2_A \otimes \mathbb{C}^2_B$

Local Measurement of A : $\{|\alpha\rangle, |\alpha_\perp\rangle\} \to a = \{\pm 1$ (random variable)

$\{|\alpha'\rangle, |\alpha'_\perp\rangle\} \to a' = \{^{+1}_{-1}$

A choices at random one of these basis

Local Measurement of B with element at random $\{|\beta\rangle, |\beta_\perp\rangle\} \to b = \{^{+1}_{-1}$

$\{|\beta'\rangle, |\beta'_\perp\rangle\} \to b' = \{^{+1}_{-1}$

After getting all experiment results, A & B get together and compute correlation coefficient

$$\boxed{X_{exp} = \frac{1}{N_1}\sum_{\underset{(\alpha,\beta)}{n_1}} a_{n_1} b_{n_1} + \frac{1}{N_2}\sum_{\underset{(\alpha,\beta')}{n_2}} a_{n_2} b'_{n_2} - \frac{1}{N_3}\sum_{\underset{(\alpha',\beta)}{n_3}} a'_{n_3} b_{n_3} + \frac{1}{N_4}\sum_{n_4} a'_{n_4} b'_{n_4}}$$

total time instances (N_1)

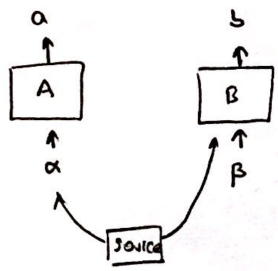
time instance (n_1)

Imagine that A & B are a 'classical' theory to describe the experiment

$\{$ Impose some inequality on X_{clth} (classical theory)

CHSH inequality

Local Hidden Variables Theories



Pairs are described by some hidden variables $\{\lambda\}$

$\lambda \sim q(\lambda)$ pdf

• Assume there exists prob. distribution $P(a,b|\alpha,\beta)$

$$P(a,b|\alpha,\beta) = \int d\lambda\, q(\lambda)\, p(a,b|\alpha,\beta;\underbrace{\lambda}_{\text{hidden varible}})$$

$\boxed{\text{Locality}}$ $p(a,b|\alpha,\beta;\lambda) = p(\alpha|\alpha,\lambda)\, p(b|\beta,\lambda)$

Then under the assumptions $|X_{LHV}| \leq 2$ CSHS

(X_{LHV}: local hidden variable th.)

Here $X_{LHV} = \underbrace{E_1(ab)}_{\text{expectation}} + E_2(ab') - E_3(a'b) + E_4(a'b')$

$$\sum_{a,b=\pm 1} ab\, p(a,b|\alpha,\beta) \qquad \ldots \qquad \ldots \qquad \sum_{a',b'=\pm 1} a'b'\, p(a',b'|\alpha',\beta')$$

Proof of CSHS

$$X_{LHV} = \sum_{a,b} ab\, p(a,b|\alpha,\beta) + \sum_{a,b'} ab'\, p(a,b'|\alpha,\beta') - \sum_{a',b} a'b\, p(a',b|\alpha',\beta) + \sum_{a,b} ab\, prob(a',b'|\alpha',\beta')$$

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$$X_{LHV} = \sum_{a,b} ab \int d\lambda\, q(\lambda)\, p(a|\alpha,\lambda)\, p(b|\beta,\lambda) + \sum_{a,b'} ab' \int d\lambda\, q(\lambda)\, p(a|\alpha,\lambda)\, p(b'|\beta',\lambda)$$

$$- \sum_{a',b} a'b \int d\lambda\, q(\lambda)\, p(a'|\alpha',\lambda)\, p(b|\beta,\lambda) + \sum_{a',b'} a'b' \int d\lambda\, q(\lambda)\, p(a'|\alpha',\lambda)\, p(b'|\beta',\lambda)$$

$$= \int d\lambda\, q(\lambda) \Big\{ \sum_{a,b} ab\, p(a|\alpha,\lambda)\, p(b|\beta,\lambda) + \sum_{a,b'} ab'\, p(a|\alpha,\lambda)\, p(b'|\beta',\lambda) +$$

$$\sum_{a',b} a'b\, p(a'|\alpha',\lambda)\, p(b|\beta,\lambda) + \sum_{a,b} a'b'\, p(a'|\alpha',\lambda)\, p(b'|\beta',\lambda)$$

$$\underbrace{p(a|\alpha,\lambda)\, p(b|\beta,\lambda)}_{\text{MARGINAL}} = \sum_{a',b'} \underbrace{p(a|\alpha,\lambda)\, p(b|\beta,\lambda)\, \underbrace{p(a'|\alpha'\lambda)}_{1}\, \underbrace{p(b'|\beta'\lambda)}_{1}}_{\text{Distribution over } a,\, a',\, b,\, b'}$$

let's put this into the equation

$$= \int d\lambda\, q\lambda \Big\{ \sum_{a,b,a',b'} ab\, p(a|\alpha,\lambda)\, p(b|\beta,\lambda)\, p(a'|\alpha',\lambda)\, p(b'|\beta',\lambda)$$

$$+ \sum_{a,b',a',b} ab'\, p(a|\alpha,\lambda)\, p(b'|\beta'\lambda)\, p(a'|\alpha',\lambda)\, p(b|\beta,\lambda)$$

$$- \sum_{a',b,a,b'} a'b\, p(a'|\alpha',\lambda)\, p(b|\beta,\lambda)\, p(a|\alpha,\lambda)\, p(b'|\beta',\lambda)$$

$$+ \sum_{a',b',a,b} a'b'\, p(a'|\alpha',\lambda)\, p(b'|\beta',\lambda)\, p(a|\alpha,\lambda)\, p(b|\beta,\lambda) \Big\}$$

$$= \int d\lambda\, q(\lambda) \sum_{a,b,a',b'} \underbrace{\{ ab + a'b - ab' + a'b' \}}_{b(a+a') - b'(a-a')} p(a|\alpha,\lambda)\, p(b|\beta,\lambda)\, p(a'|\alpha',\lambda)\, p(b'|\beta',\lambda)$$

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$$= E_{a,b,a',b'} \left\{ \underbrace{(a+a')b - (a-a')b'}_{\pm 2} \right\} \rightarrow \boxed{|X_{LHV}| \leq 2}$$

$a = a' \longrightarrow \pm 2$

$a \neq a' \longrightarrow \pm 2$

Experiment $\alpha = 0$, $\beta = \frac{\pi}{8}$, $\alpha' = \frac{\pi}{4}$, $\beta' = -\frac{\pi}{8}$ $\rightarrow X_{exp} = 2\sqrt{2}$ (max)

the most ideal

Quantum Prediction $\rightarrow$ same!

$$\boxed{X_{QM} = \cos 2(\alpha-\beta) + \cos 2(\alpha'-\beta) + \cos 2(\alpha'-\beta) - \cos 2(\alpha-\beta') + \cos 2(\alpha'-\beta')}$$

$|\varphi_A\rangle \otimes |\varphi_B\rangle$: $|X_{QM}^{\text{Product State}}| \leq 2$?

17/11/22

QM Prediction

$$X_{QM} = \underbrace{\cos 2(\alpha-\beta)} + \cos 2(\alpha'-\beta) - \cos 2(\alpha-\beta') + \cos 2(\alpha'-\beta')$$

$E_1(a,b) \rightarrow$ quantum mechanical expectation

A & B choose α & β

$\rightsquigarrow$ Observable that A measures is

$$\boxed{\begin{aligned} A &= (+1)|\alpha\rangle\langle\alpha| + (-1)|\alpha_\perp\rangle\langle\alpha_\perp| \\ B &= (+1)|\beta\rangle\langle\beta| + (-1)|\beta_\perp\rangle\langle\beta_\perp| \end{aligned}}$$

$A|\alpha\rangle = (+1)|\alpha\rangle$

$A|\alpha_\perp\rangle = (-1)|\alpha_\perp\rangle$

$$\langle\Psi|A \otimes B|\Psi\rangle = \langle\Psi|\left(|\alpha\rangle_A\langle\alpha|_A - |\alpha_\perp\rangle_A\langle\alpha_\perp|_A\right) \otimes B \,\Big|\, \frac{1}{\sqrt{2}}\left(|\alpha\rangle_A \otimes |\alpha\rangle_B + |\alpha_\perp\rangle_A \otimes |\alpha_\perp\rangle_B\right)$$

Here $|\Psi\rangle = |B_{00}\rangle$

$$= \frac{1}{\sqrt{2}}\left(|\alpha\rangle_A \otimes |\alpha\rangle_B + |\alpha_\perp\rangle_A \otimes |\alpha_\perp\rangle_B\right)$$

$$= \frac{1}{\sqrt{2}} \langle\Psi| \, |\alpha\rangle_A \otimes B|\alpha\rangle_B - |\alpha_\perp\rangle_A \otimes B|\alpha_\perp\rangle_B)$$

$\downarrow$

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$$= \frac{1}{\sqrt{2}} \langle \Psi | \; |\alpha\rangle_A \otimes |\beta\rangle_B \overbrace{\langle \beta|\alpha\rangle_B}^{\cos(\beta-\alpha)} - |\alpha_\perp\rangle_A \otimes |\beta\rangle_B \langle \beta|\alpha_\perp\rangle$$

$$- \frac{1}{\sqrt{2}} \langle \Psi| \; |\alpha\rangle_A \otimes |\beta_\perp\rangle_B \langle \beta_\perp|\alpha\rangle_B - |\alpha_\perp\rangle_A \otimes |\beta_\perp\rangle_B \langle \beta_\perp|\alpha_\perp\rangle$$

$$= \frac{1}{\sqrt{2}} \Big\{ \langle \Psi|\alpha\beta\rangle \cos(\beta-\alpha) - \langle \Psi|\alpha_\perp\beta\rangle \cos(\beta-\alpha_\perp) - \langle \Psi|\alpha\beta_\perp\rangle \cos(\beta_\perp-\alpha)$$

$$+ \langle \Psi|\alpha_\perp \beta_\perp\rangle \cos(\beta_\perp - \alpha_\perp)$$

$$\langle \Psi| = \frac{1}{\sqrt{2}} \left(\langle \alpha\,\alpha| + \langle \alpha_\perp \alpha_\perp| \right)$$

$$= \frac{1}{2} \Big\{ \cos^2(\beta-\alpha) - \underbrace{\cos^2(\beta-\alpha_\perp)}_{} - \cos^2(\beta_\perp-\alpha) + \cos^2\underbrace{(\beta_\perp - \alpha_\perp)}_{} \Big\}$$

$$\langle\alpha\,\alpha|\alpha\beta\rangle = \langle\alpha|\beta\rangle\cos(\beta-\alpha)$$

$$\sin^2(\beta-\alpha) \qquad \cos^2(\beta-\alpha)$$

$$\cos^2\left(\beta-\alpha-\frac{\pi}{2}\right) = \sin^2(\beta-\alpha)$$

$$= \frac{1}{2} \Big\{ \cos(2(\beta-\alpha)) + \cos(2(\beta-\alpha)) \Big\} = \boxed{\cos(2(\beta-\alpha))}$$

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