MECHANICAL EFFECTS OF LIGHT

Idea:
- Photon have momentum.
- "Effective" theory

motion of atom → center-of-mass → "effective" action

motion of e^- → internal degrees of freedom ← addressed by laser

I. FORMULATION

1) Hilbert Space

$$\mathcal{H} = \mathcal{H}_{at} \otimes \mathcal{H}_{field} = \mathcal{H}_{C.O.M} \otimes \mathcal{H}_{internal} \otimes \mathcal{H}_{\substack{laser\\ mode}} \otimes \mathcal{H}_{vac}$$

$\mathcal{H}_{c.o.m} = Sp\{\,|\vec{p}\rangle\,,\ \vec{p} \in \mathbb{R}^3\}$ atom in free space

($\vec{p}$ → momentum)

$\mathcal{H}_{internal} = Sp\{\,|nLJm_J\ldots\rangle \ldots\}$ energy levels of the atom

$\mathcal{H}_{internal} = Sp\{\,|n\rangle,\ n \in \mathbb{N}\}$ one particular mode addressed by the laser

$\mathcal{H}_{vac} = \mathcal{F}$ Fock space

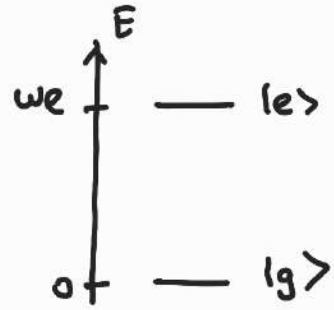
2) Simplification

- Reduce $\mathcal{H}_{int}$ to a 2 dim. Hilbert space $= Sp\{|g\rangle, |e\rangle\}$

  → cycling transition

- Laser mode is in a coherent state → classical field
- Vacuum as a reservoir (Markov approximation) → Lindblad Equation
- C.o.m motion is slow compared with internal dynamics.

3) Hamiltonian



Acts on $\mathcal{H}_{c.o.m} \otimes \mathcal{H}_{int}$

$$\hat{H} = \frac{\vec{p}^2}{2m} \otimes \hat{\mathbb{I}} + \hat{\mathbb{1}} \otimes w_e |e\rangle\langle e| + \frac{\Omega(\hat{\vec{r}})}{2}\left[e^{i(w_0 t - \vec{k}_L \hat{\vec{r}})} \otimes |g\rangle\langle e| + hc \right]$$

key point: Ω and $\phi = w_0 t - \vec{k}_L \hat{\vec{r}}$ are operators on $\mathcal{H}_{com}$

within dipole approximation : "size" of the atom $<< \lambda$

the atom "feels" Ω and ϕ at the position of its c.o.m.

$\Omega(\hat{\vec{r}}) = \langle \Psi_{com} | \Omega(\hat{\vec{r}}) | \Psi_{com} \rangle$ for c.o.m in state $|\Psi_{com}\rangle$

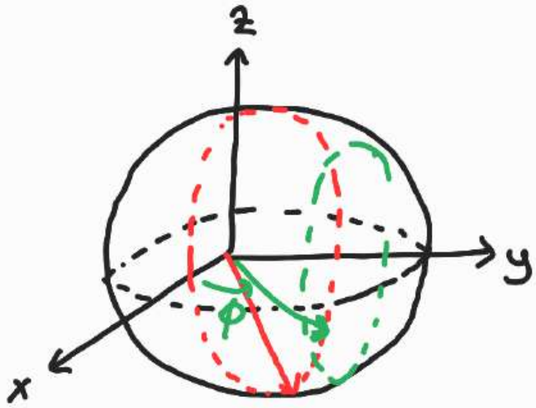
$\Omega(\hat{\vec{r}}) = -\vec{E}(\hat{\vec{r}})\, d_0$ → how e^- and atom bound together

$\phi(\vec{r}) = w_0 t - \vec{k}_L \cdot \hat{\vec{r}}$

II. Recoil and Doppler Effects

1) Phase Imprinting

Remainder: $\hat{H} = \frac{\Omega}{2}\left(e^{-i\phi}|e\rangle\langle g| + hc\right) + \frac{|\Omega|}{2}\vec{n}\vec{\sigma}$



Here $\phi(\vec{r})$

Consider $|\Psi_{at}\rangle = |\vec{p}\rangle \otimes |g\rangle$

$$\hat{H}_{int}|\Psi_{at}\rangle = \frac{\Omega}{2}\underbrace{\left(e^{-i\phi(\hat{\vec{r}})}|\vec{p}\rangle\right)}\otimes|e\rangle$$

$$\int d^3\vec{R}\, |\vec{R}\rangle\langle\vec{R}|\, e^{-i\phi(\hat{\vec{r}})}|\vec{p}\rangle$$

$$= \int d^3R\, e^{-i\phi(\vec{R})}\, |\vec{R}\rangle \underbrace{\langle\vec{R}|\vec{p}\rangle}_{e^{i\vec{p}.\vec{R}}}$$

$$= \int d^3R\, e^{-i\phi(\vec{R}) + i\vec{p}.\vec{R}}\, |\vec{R}\rangle$$

Plane wave: $\Phi(\vec{R}) = \omega_0 t - \vec{k}_L \cdot \vec{R}$ then:

$$\hat{H}_{int}|\Psi_{at}\rangle = \frac{\Omega}{2} e^{i\omega_0 t} \underbrace{\int d^3R\, e^{i(\vec{k}_L \cdot \vec{R} + \vec{p}\cdot\vec{R})} |\vec{R}\rangle}_{|\vec{p}+\vec{k}_L\rangle}$$

So $\hat{H}_{int} = \frac{\Omega}{2} \sum_{\vec{p}} e^{i\omega_0 t} |\vec{p}\, g\rangle\langle \vec{p}+\vec{k}_L, e| + hc$

2) Shifts

Move to the rest frame of the atom (from the laboratory frame where momentum $\vec{p}$)

Unitary transformation:

$$\hat{U}(t) = e^{\frac{i\hat{\vec{p}}^2 t}{2m}} \qquad \hat{H} \to (i\partial_t \hat{U})\hat{U}^+ + \hat{U}\hat{H}\hat{U}^+$$

In this frame:

$$\hat{H} = \frac{\Omega}{2} \sum_{\vec{p}} e^{i\omega_0 t} \underbrace{\hat{U}|\vec{p}\, g\rangle\langle\vec{p}+\vec{k}_L, e|\hat{U}^+}_{e^{\frac{i\hat{p}^2 t}{2m}} |\vec{p}, g\rangle\langle \vec{p}+\vec{k}_L, e| e^{-i\frac{(\vec{p}+\vec{k}_L)^2}{2m} t}} + h.c$$

$$= \frac{\Omega}{2}\sum_{\vec{p}} e^{i\left(\omega_0 - \frac{\vec{p}\vec{k_i}}{m} - \frac{\vec{k_i}^2}{2m}\right)t} \, |\vec{p}, g\rangle\langle \vec{p}+\vec{k_i}, e| + hc$$

$$\omega_0 \longrightarrow \omega_0 - \frac{\vec{p}\vec{k_i}}{m} - \frac{\vec{k_i}^2}{2m}$$

$^6Li \sim 0.1\ ms^{-1}$

$\uparrow$

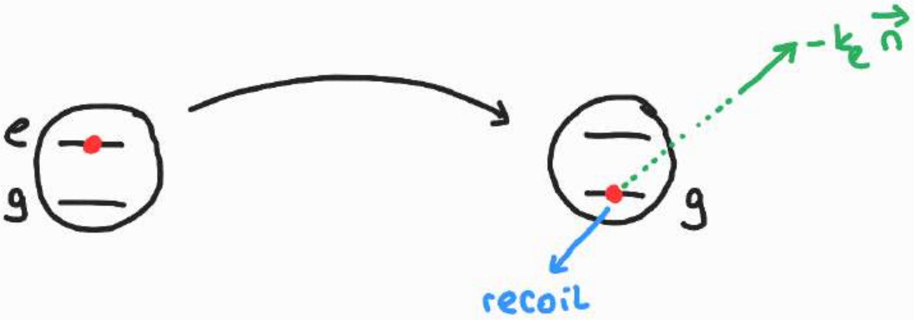
Doepler Shift: $\dfrac{\vec{p}\cdot\vec{k_i}}{m} = \vec{p}\,\vec{v}_{rec}$, $\vec{v}_{rec} = \dfrac{\vec{k_i}}{m}$

$\hookrightarrow$ Ex: ^{6}Li at 300K : 1GHz

Recoil Shift: $E_R = \dfrac{\hbar^2 k_i^2}{2m} = h \cdot 77kHz$ for ^{6}Li

$\hookrightarrow$ negligible cmp with doppler shift

3) Jump Operators



$k_e = \dfrac{\omega_e}{c}$

Family of jump operators: $\hat{L}(\vec{n}) = \sum_{\vec{p}} |\vec{p}+k_e\vec{n}, g\rangle\langle \vec{p}, e|$

$$= e^{ik_e\vec{n}\cdot\vec{r}} \otimes |g\rangle\langle e|$$

Rate $\Gamma_{\vec{n}}$ where $\underbrace{\hat{M}}_{\text{measurement operator}}(\vec{n}) = \sqrt{\Gamma_{\vec{n}}}\,\hat{L}(\vec{n})$

4) Lindblad Equation

$$\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] + \sum_{\vec{n}} \Gamma_{\vec{n}} \left(\hat{L}(n)\hat{\rho}\hat{L}(\vec{n})^{\dagger} - \frac{1}{2}\hat{L}^{\dagger}\hat{L}(\vec{n})\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{L}^{\dagger}\hat{L}(\vec{n}) \right)$$

Note that: $\hat{L}^{\dagger}(\vec{n})\hat{L}(\vec{n}) = |e\rangle\langle e|$

$$\cancel{e^{-i\cdot}}|e\rangle\langle g|\,|g\rangle\langle e|\cancel{e^{+i\cdot}}$$

ρ_{ee} ρ_{ee}

Introduce $\Gamma = \sum_{\vec{n}} \Gamma_{\vec{n}}$

$$\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] \underbrace{- \frac{\Gamma}{2}\left(|e\rangle\langle e|\hat{\rho} + \hat{\rho}|e\rangle\langle e|\right)}_{\text{Non-Hermitian "decay" of } |e\rangle} + \underbrace{\sum_{\vec{n}} \hat{L}(\vec{n})\hat{\rho}\hat{L}^{\dagger}(\vec{n})}_{\text{kicks upon spontaneous emission}}$$

Last term $\hat{\rho} = |e,\vec{p}\rangle\langle\vec{p},e|$

Then $\hat{L}(\vec{n})\hat{\rho}\hat{L}^{\dagger}(\vec{n}) = |g,\vec{p}+k_e\vec{n}\rangle\langle g,\vec{p}+k_e\vec{n}|$

III. Semi-classical Forces

1) Further simplification

. The momentum "spread" of the atom is large $\Delta p \geq k_e, k_L$ ...

$\Leftrightarrow$ position spread is small $\Delta R \ll \lambda$

. k_e is small (momentum exchanges with em field are continous)

. internal dynamics is much faster than motion of c.o.m

Steady state description of the internal state for each position.

$$\Gamma, \Omega \gg \underbrace{E_R}_{\to \text{recoil}}$$

-> Ansatz for the density matrix: $\hat{\rho} = \hat{\rho}_{com} \otimes \hat{\rho}_{int}^{\overset{\text{steady state}}{\uparrow}\, st}$

$$\hat{\rho}_{int}^{st}(\vec{R}(t))$$

2) Sponteneous Emission

$$\sum_{\vec{n}} \Gamma_{\vec{n}}\, e^{ik_e \vec{n}\cdot\hat{\vec{r}}} |g\rangle\langle e| \hat{\rho} |e\rangle\langle g| e^{-ik_e \vec{n}\cdot\vec{r}}$$

since k_e is small: $e^{ik_e\vec{n}\cdot\hat{\vec{r}}} \sim 1 + ik_e\vec{n}\cdot\vec{r} - \frac{k_e^2}{2}(\vec{n}\cdot\hat{\vec{r}})^2 + \dots$

So: $\underbrace{\sum_{n} \Gamma_{\vec{n}}}_{\Gamma} |g\rangle\langle e|\hat{\rho}|e\rangle\langle g| + \underbrace{\sum_{n} ik_e\vec{n}\cdot\vec{r}\, |g\rangle\langle e|\hat{\rho}|e\rangle\langle g|}_{0} + \underbrace{\sum_{n} |g\rangle\langle e|\hat{\rho}|e\rangle\langle g| (-ik_e\vec{n}\cdot\vec{r})}_{0}$

$+ \cancel{O(k_e^2)}$

Up to $O(k_e^2)$: no net effect of spont. emission of the center of motion!

Now, up to $O(k_e^2)$:

$$\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] + \Gamma |g\rangle\langle e|\hat{\rho}|e\rangle\langle g| - \frac{\Gamma}{2}\left(|e\rangle\langle e|\hat{\rho} + \hat{\rho}|e\rangle\langle e|\right)$$

the dissipative part of the Lindblad Eq. is only affecting the internal state!!

3) Forces

$$\vec{F} = \frac{d\langle \vec{p} \rangle}{dt} = \frac{d}{dt}\, \mathrm{Tr}(\hat{\rho}\,\vec{p}) = \mathrm{Tr}(\dot{\hat{\rho}}\,, \vec{p})$$

using cyclicity

$$= -i\, \mathrm{Tr}([\hat{H}, \hat{\rho}] . \hat{\vec{p}})$$

$$= -i\, \mathrm{Tr}(\hat{\rho} \cdot [\vec{p}, \hat{H}])$$

$$\vec{F} = -i \langle [\hat{\vec{p}}, \hat{H}] \rangle$$

$\hat{H}$ depends on $\hat{\vec{r}}$: move to position representation (via $\Omega(\vec{r})$, $\phi(\vec{r})$...)

$$\vec{p} = -i\, \vec{\nabla}_{\vec{r}}$$

then $\langle [\vec{p}, \hat{H}] \rangle = -i \langle \vec{\nabla}_r \hat{H} \rangle$

Proof: for any state $|\alpha\rangle$:

$$\langle \alpha | \vec{\nabla}_r (\hat{H} | \alpha \rangle) = \langle \alpha | [\vec{\nabla}_r \hat{H}] | \alpha \rangle + \langle \alpha | \hat{H} (\vec{\nabla}_r | \alpha \rangle$$

So, $\boxed{\vec{F} = -\langle \vec{\nabla}_r \hat{H} \rangle}$

for an atom at rest:

$$\hat{H} = \omega_e |e\rangle\langle e| + \frac{\Omega(\vec{r})}{2}\left(e^{i(\omega_0 t - \vec{k}_L \cdot \vec{r})} |g\rangle\langle e| + h.c \right)$$

In a frame rotating at ω_0 (for internal degrees of freedom)

$$\hat{H} = \Delta |e\rangle\langle e| + \frac{\Omega(\vec{r})}{2}\left(e^{-i k_L \cdot \vec{r}} |g\rangle\langle e| + h.c \right)$$

$$\langle -\vec{\nabla}_r \hat{H} \rangle = - \frac{\nabla\Omega(\vec{r})}{2}\left(\underbrace{\langle e^{-i k_L \cdot \vec{r}} |g\rangle\langle e| + h.c\rangle}_{} \right)$$

internal d.o.f

↓

OBE

optical bloch equation

$$+ i k_L \frac{\Omega}{2}\left(\langle e^{-i k_L \cdot \vec{r}} |g\rangle\langle e| + \dots \right)$$

$$\boxed{\vec{F} = \underbrace{\frac{\vec{\nabla}\Omega}{\Omega} \Delta . \frac{s(\vec{r})}{1+s(\vec{r})}}_{\text{dipole force}} + \underbrace{k_L \frac{\Gamma}{2} \frac{s(\vec{r})}{1+s(\vec{r})}}_{\text{radiation pressure}}}$$

$$s(\vec{r}) = \frac{\Omega^2(\vec{r})/2}{\Delta^2 + \Gamma^2/4}$$

dipole force

$\vec{F}_{dip}$

↓

$$\frac{\Delta}{2} \frac{\nabla \vec{s}}{1+s}$$

radiation pressure

$\vec{F}_{rad}$

↓

$$\vec{k}_L . \Gamma . \langle \rho_{ee} \rangle_{st}$$

→ steady state

IV Interpretation

1) Dipole force

$$\vec{F}_{dip} = -\vec{\nabla}\left(-\frac{\Delta}{2}\,Ln\left(1+s(\vec{r})\right)\right)$$

dipole force is conservative and $\overset{\rightarrow \text{potential}}{U_{dip}} = -\frac{\Delta}{2}Ln(1+s)$

Limiting case : $\Delta >> \Omega, \Gamma$

$$s << 1 \Rightarrow \langle \rho_{ee} \rangle << 1$$

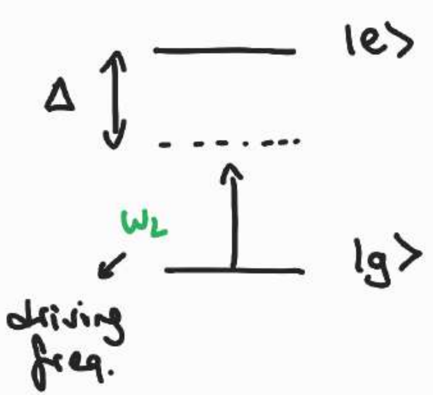
Then $U_{dip} = -\frac{s}{2}\Delta = -\frac{\Omega^2(\vec{r})}{4\Delta}$

- $\Omega \propto \mathcal{E}$ E-field amplitude

  $U_{dip} \propto I$ light intensity

- U_{dip} is also independent of Γ!

Recovering the dipole potential: 2-level atom in external field.

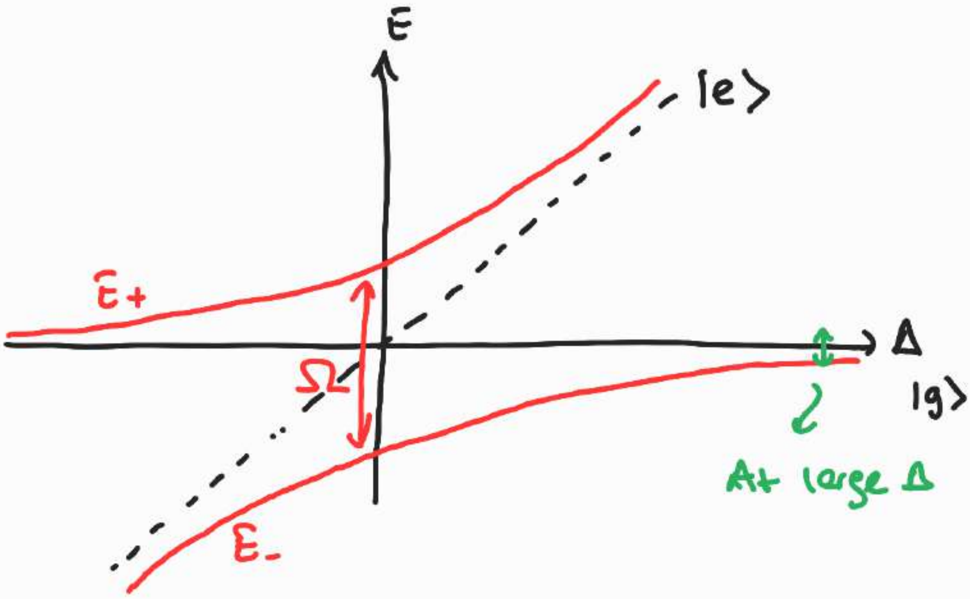


$$\hat{H} = \omega_e |e\rangle\langle e| + \frac{\Omega}{2}\left(e^{-i\omega_L t}|e\rangle\langle g| + hc\right)$$

In a rot. frame:

$$\hat{H} = \Delta |e\rangle\langle e| + \frac{\Omega}{2}\left(|e\rangle\langle g| + hc\right)$$

Energy spectrum:



Large Δ $\quad E_\pm = \frac{1}{2}\left(\Delta \pm \sqrt{\Delta^2+\Omega^2}\right) \qquad E_- \to \frac{-\Omega^2}{4\Delta}$

and $|-\rangle \to |g\rangle$

Now if $\Omega(\bar{r}) \longrightarrow E_-(\bar{r})$

Remarks

* $U_{dip} \Leftrightarrow$ AC- sterk shift

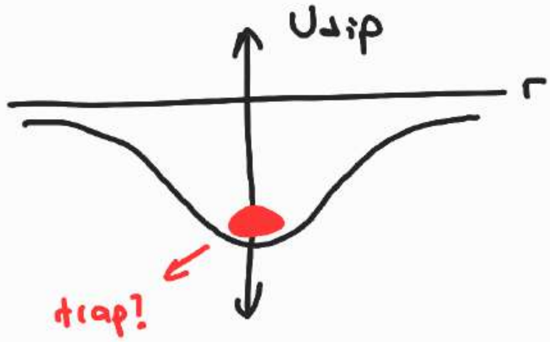
(can also be obtained directly using 2nd order perturbation theory.

* Also possible for DC fields: $\Delta \to \omega_e$

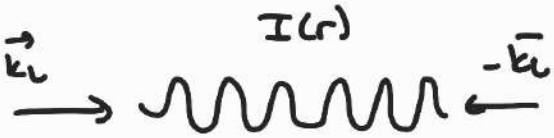
* Optical dipole trap:

for $\Delta \geq 0$: $(\omega_e \geq \omega_L)$

"optical tweezer"



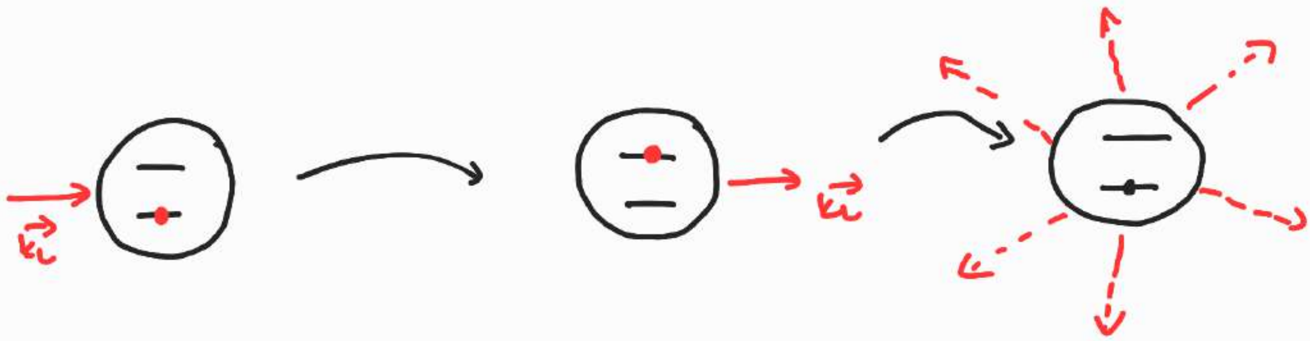
Optical lattices



$$U_{dip} = U_0 \cos^2(k_L r) \Rightarrow \text{band structure}$$

2) Radiation pressure

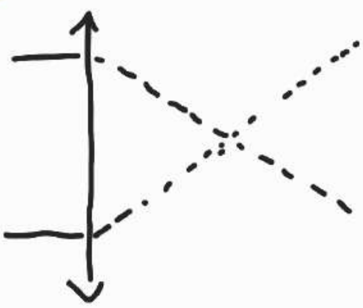
$$\vec{F} = \Gamma \langle \rho_{ee} \rangle^{st} \vec{k_L}$$



Remarks: Intrinsically related to spontaneous emission

$\Leftrightarrow$ incoherent

* Dipole force: stimulated emission



- Radiation pressure is related to photon absorption:

3) Doppler cooling

An atom moving with momentum $\vec{p}$

$$\Delta_0 \longrightarrow \Delta_0 - \vec{p}\cdot\vec{v}_{rec} - \cancel{E_{rec}}$$

negligible

$$\vec{F}_{rad} = \vec{k}_L \frac{\Gamma\Omega^2}{4} \frac{1}{\Delta_0^2 + \Gamma'^2} \Bigg\} \; \Gamma' = \text{power broadened linewidth}$$

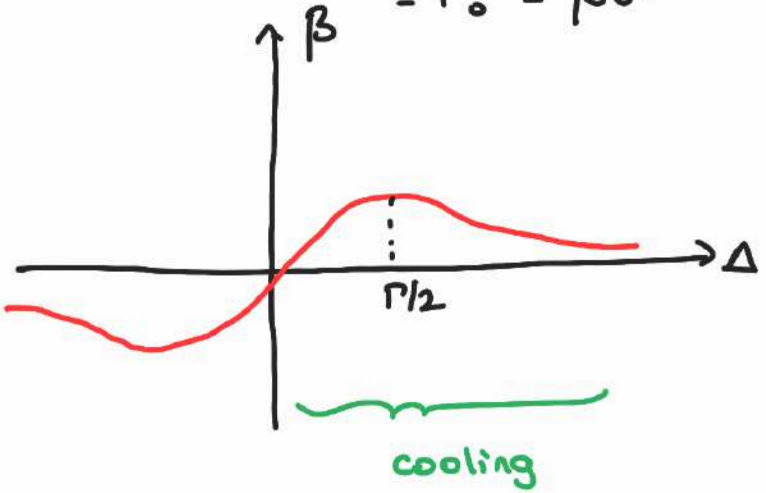
$$= \vec{k}_L \frac{\Gamma\Omega^2}{4} \frac{1}{(\Delta_0 + \vec{p}\cdot\vec{v}_{rec})^2 + \frac{\Gamma'^2}{2}}$$

First order expansion for Doppler shift:

$$\vec{F}_{rad} = \vec{k}_L \frac{\Gamma\Omega^2}{4} \left(\frac{1}{\Delta_0^2 + \Gamma'^2} - \vec{p}\cdot\vec{v}_{rec} \frac{2\Delta_0}{(\Delta_0^2 + \Gamma'^2)^2} + \dots \right)$$

$$= F_0 - \beta\vec{v}$$

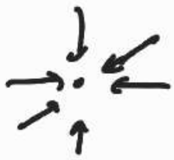
β: friction coefficient



Interpretation: Doppler shift bring the atom closer to resonance when it moves against the beam.

Remarks: to cool in all directions, you need 6 laser beams

"optical molasses"



4) Limitations and Extensions

Limit to Doppler cooling: terms $O(k_e^2)$ in the Lindblad equation.

- Random walk in k (momentum) space $\langle \Delta p^2 \rangle \sim 2m\hbar\Gamma$

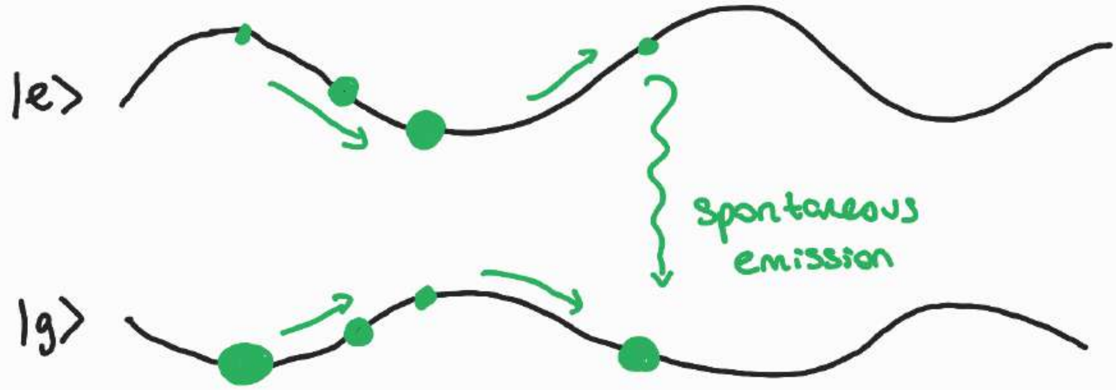
- Limit temperature: $kT \sim \hbar\Gamma$ $\left(\begin{array}{l} {}^{6}Li = 280\ \mu K \\ {}^{88}Sr = 400\ nK \end{array} \right.$

the finite k_e $\Rightarrow$ can not reach $\vec{v} = 0$ in free space.

E_{rec} is the hard limit.

For trapped atoms $\rightarrow$ reaching the ground state is possible.

Other mechanism to cool in free space: Sisyphus cooling



|e>
|g>
spontaneous
emission