Coplanar Waveguide Resonators for circuit quantum electrodynamics

I. INTRO

II. Device Geometry, Fabrication, Measurement Technique

III. Basic Resonator Properties

Questions

→ what's coplanar waveguide?

→ what's resonators? /why do we use it?

→ why is it used in quantum electro dynamics?

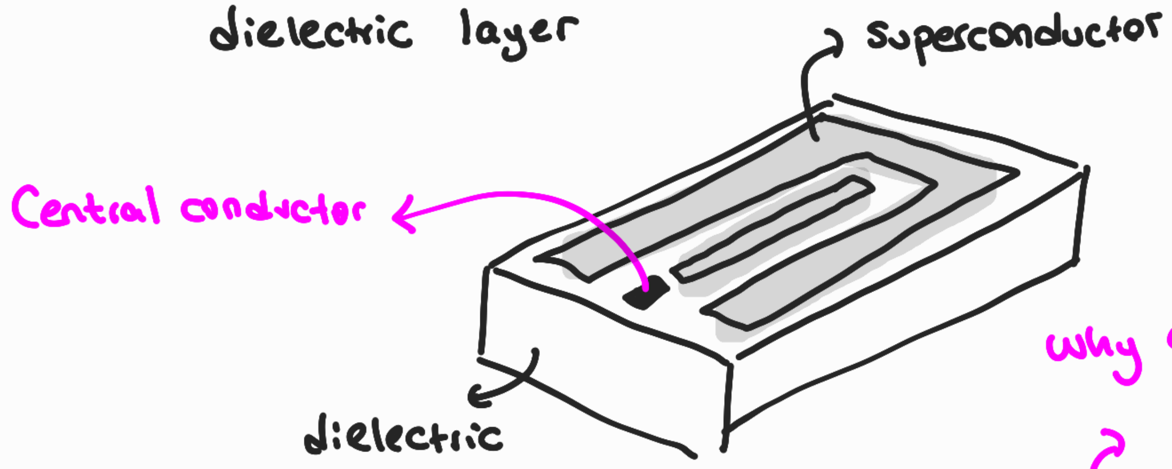
→ Quality factors

→ Insertion losses

→ whatare the gap and finger capacitors?

1. CPWs are used as the transmission line to connect the qubits to the readout and control electronics

- Microwave transmission line
- Central conductor, two ground planes, dielectric layer



why do we need cavitys?

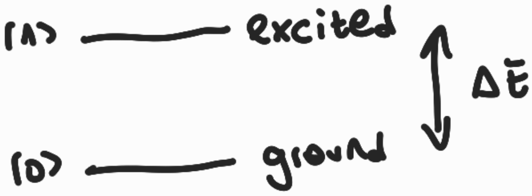
CPW resonators are often used to create cavitys.

2. Resonator / Resonant frequency ⇝ typically btw 4-8 GHz

- Resonant frequency determines at which the qubit and cavity interact mostly.

- The interaction strength btw the qubit and the cavity is crucial for the operation of the qubit.

Qubit



Cavity

- zero photon
- One photon

- When the cavity resonant frequency matches the ΔE of qubit, they become strongly coupled. Energy can be exchange btw them. This interaction is the basis for many quantum operations.

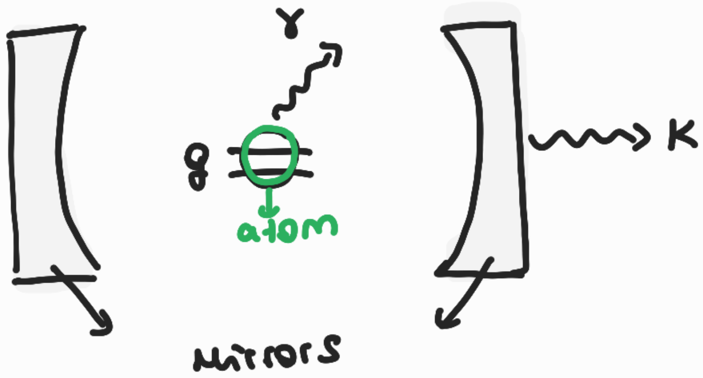
How?

such as gates

- Resonant freq. is also important for the detecting the qubit state. When a photon is added to the cavity, it can change the resonant freq. of the cavity. It can be detected by measuring the transmission or reflection of the probe signal through the cavity.

How?

3. From Cavity QED to Circuit QED



κ: transmission losses of cavity

γ: Spontaneous emission of atom into free space

g: coherent coupling atom btw 1 mode radiation field

provide a single resonant frequency

- When the atom interacts with the single mode radiation field, it can emit or absorb a single photon at a time.

4. How is transmission (S_{21}) measured?

How does one experimentally extract the relevant parameters from the measurement?

4.1 What is S matrix

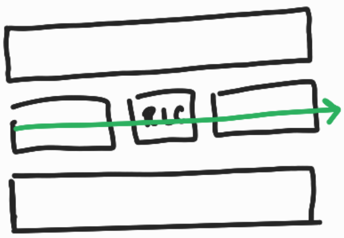
√1. Mention why do we need resonators?

√2. What's S matrix, why do we use S_{21} (transmission) for the frequency response?

3. Why do we use Q (quality factors

** 4. What's transmitted here? → Microwave Control Signal
√

S_{21} Forward Transmission



Network Analyzer

High Quality Factor :

Resonator is able to store energy for a long time.

$$Q = \frac{f_0}{\delta f}$$

* It's important for storing and manipulating quantum information

5. From the Article:

for larger coupling

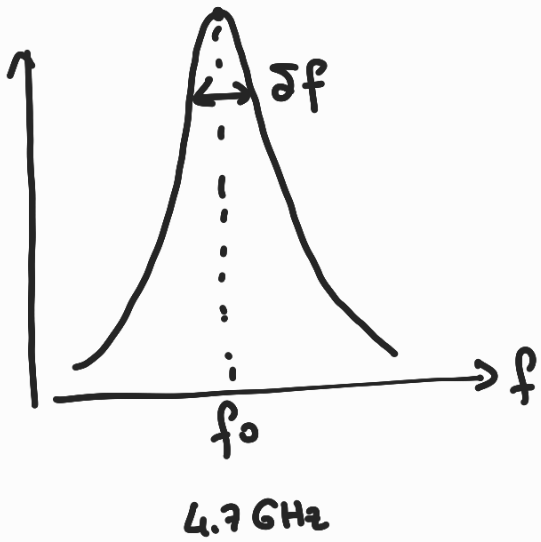
· CPW with finger capacitors? ↘ what're the

· CPW with gap capacitors? ↗ differences

for small coupling capacitances

· Using a 40 GHz vector network analyzer, S_{21} transmission measurements

III. Basic Resonator Properties



$$f_0 = \frac{c}{\sqrt{\epsilon_{eff}}} \frac{1}{2\ell}$$

phase velocity

λ_0 wavelength for the fundamental resonator mode

ϵ_{eff} : effective permittivity

$$v_{ph} = \frac{1}{\sqrt{L_\ell C_\ell}}$$

$$L_\ell = \frac{\mu_0}{4} \frac{K(k_0')}{K(k_0)}$$

complete elliptic integral

$$C_\ell = 4\epsilon_0\epsilon_{eff} \frac{K(k_0)}{K(k_0')}$$

$$Z_0 = \sqrt{\frac{L_\ell}{C_\ell}}$$

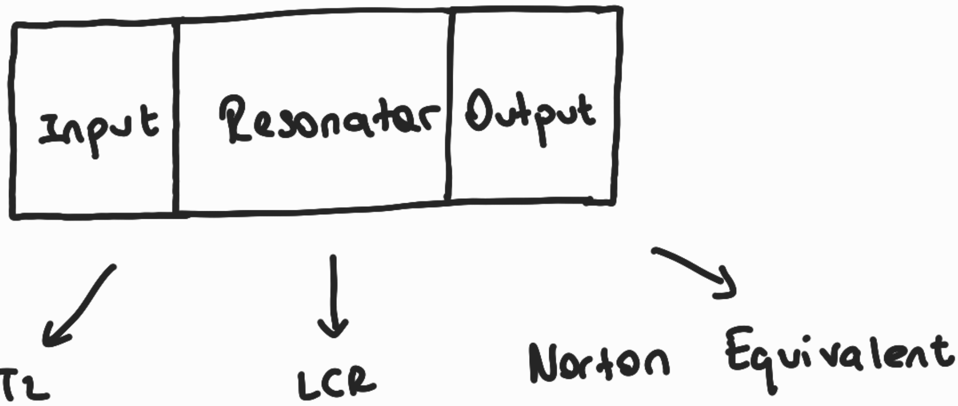
$L_\ell = L_\ell^m + L_\ell^k$ → kinetic (temp. dependent)

magnetic (temp. independent)

IV. Input/Output Coupling

→ Explain how transmission S_{21} is obtained

→ First explain S matrix



$$Z_{TL} = Z_0 \frac{1 + i\tan(\beta l)\tan(\alpha l)}{\tanh(\alpha l) + i\tan(\beta l)} \simeq \frac{Z_0}{\alpha l + i\frac{\pi}{\omega_0}(\omega - \omega_n)}$$

α: attenuation constant

β: $\frac{\omega_n}{v_{ph}}$ phase constant of the TL

it holds when

$\alpha l \ll 1$

& ω close to ω_n

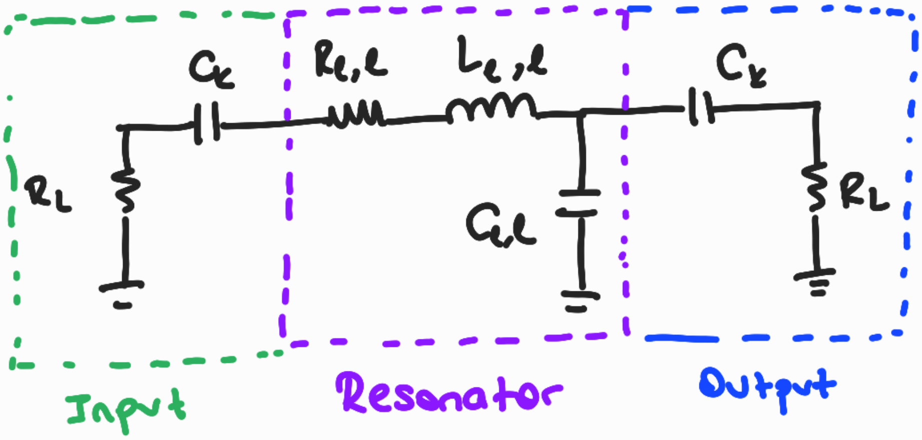
$$\omega_n = n\omega_0 = \frac{1}{\sqrt{L_n C}}$$

angular frequency of nth mode

resonance mode #

$$Z_{LCR} = \left(\frac{1}{i\omega L_n} + i\omega C + \frac{1}{R}\right)^{-1} \simeq \frac{R}{1 + 2iRC(\omega - \omega_n)}$$

$\omega \simeq \omega_n$



R_ℓ: Resistance per unit length

L_ℓ: Inductance per unit length

C_ℓ: Capacitance per unit length

$$Z_{TL} = Z_0 \frac{1 + i\tan(\beta\ell)\tanh(\alpha\ell)}{\tanh(\alpha\ell) + i\tan\beta\ell} \simeq \frac{Z_0}{\alpha\ell + i\frac{\pi}{\omega_0}(\omega-\omega_n)}$$

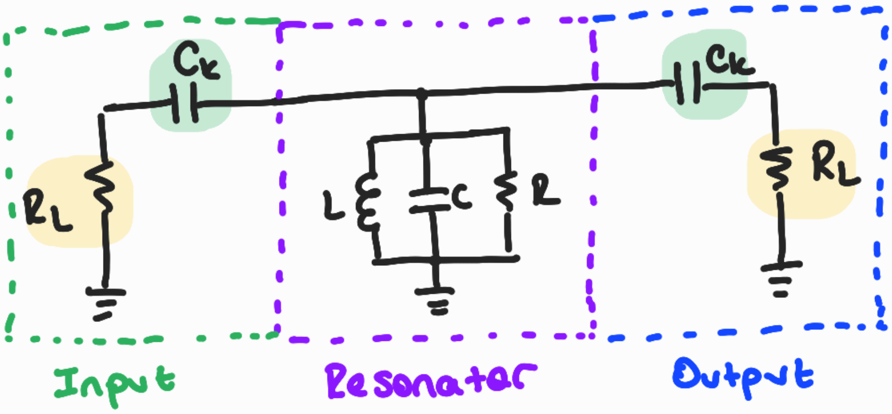
For $\alpha\ell \leq 1$ (small losses)

$\omega \simeq \omega_n$

α: attenuation constant

$\beta = \frac{\omega_n}{v_{ph}}$: phase constant

$\omega_n = n\omega_0 = \frac{1}{\sqrt{L_n C}}$: angular frequency of the nth mode.



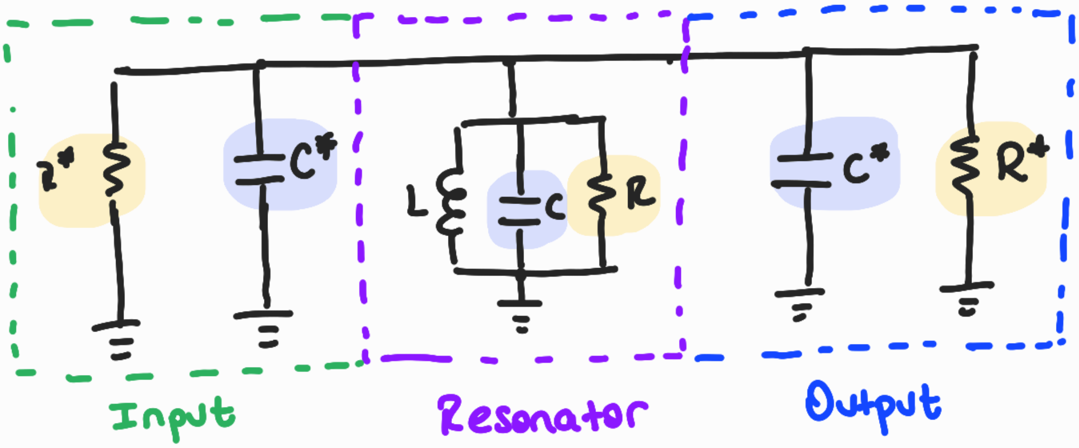
$$L_n = \frac{2L_\ell \ell}{n^2\pi^2}$$

$$C = \frac{C_\ell \ell}{2}$$

$$R = \frac{Z_0}{\alpha\ell} \quad (\text{since } \omega = \omega_n)$$

$$Q_{int} = R\sqrt{\frac{C}{L_n}} = \omega_n RC$$

- Q_L is reduced due to the resistive loading.
- The frequency is shifted due to the C_k.



$$R^* = \frac{1+\omega_n^2 C_k^2 R_L^2}{\omega_n^2 C_k^2 R_L^2}$$

$$C^* = \frac{C_k}{1+\omega_n^2 C_k^2 R_L^2}$$

The small capacitor C_k transforms $R_L = 50\,\Omega$ into large impedance

$$R^* = \frac{R_L}{k^2} \quad \text{where} \quad k = \omega_n C_k R_L \ll 1$$

$$Q_L = \omega_n^* \frac{C + 2C^*}{\frac{1}{R} + \frac{2}{R^*}}$$

$$\omega_n^* = \frac{1}{\sqrt{L_n (C + 2C^*)}} \approx \omega_n$$

$$C + 2C^* \approx C$$

$$Q_L \approx \omega_n \frac{C}{1/R + 2/R^*}$$

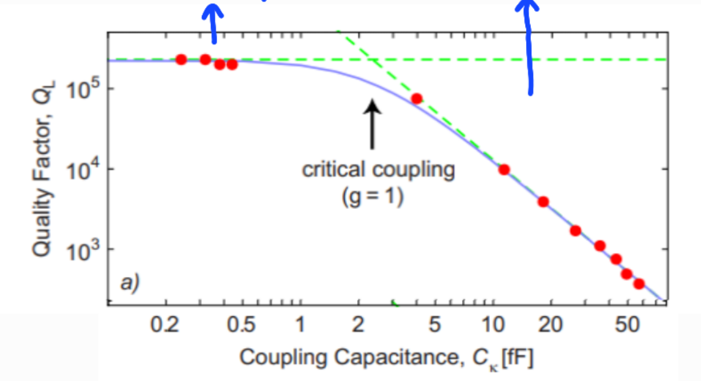
$$\frac{1}{Q_L} = \frac{1}{Q_{int}} + \frac{1}{Q_{ext}}$$

$$Q_{int} = \omega_n R C = \frac{n\pi}{2\alpha l}$$

$$Q_{ext} = \frac{\omega_n R^* C}{2}$$

Undercoupled

Overcoupled



- For small C_k's, Q_L is constant
- For large C_k's, Q_L decreases

Coupling coefficient $g = \dfrac{Q_{int}}{Q_{ext}}$

Overcoupled Regime

$Q_{int} \geq Q_{ext}$

$$\frac{1}{Q_L} = \cancel{\frac{1}{Q_{int}}}^{\,0} + \frac{1}{Q_{ext}}$$

$$Q_L \simeq Q_{ext}$$

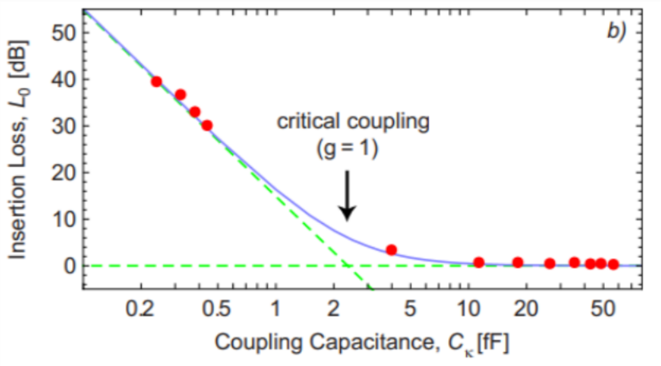
$$Q_L = \frac{\omega_n R^* C}{2} = \frac{C}{2\omega_n R_L C_k^2}$$

$$Q_L \propto \frac{1}{C_k^2}$$

Undercoupled Regime

$Q_{ext} \gg Q_{int}$

- Q_L saturates at the internal quality factor
- $Q_{int} \simeq 2.3 \times 10^5$
- Determined by the intrinsic losses of the resonator.



Insertion Loss:

Deviation of peak transmission from unity.

The measured values of L_0 are extracted from Fig.4.

For $g>1$ (Large C_k)

- Overcoupled Regime
- $L_0 = 0 \rightarrow$ Unity transmission

For $g<1$ (Small C_k)

- Undercoupled
- $L_0 \simeq -20 \log\left(\dfrac{2\omega_n Q_{int} R_L C_k^2}{C}\right)$

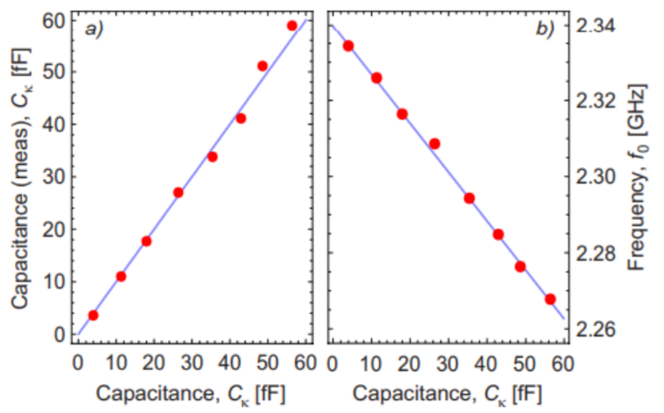


FIG. 7. (Color online) (a) Comparison of C_κ values extracted from the measured quality factors using Eqs. (14), (19), and (21) to the EM-simulated values for C_κ (red points) for devices A–H. The blue curve is a line through origin with slope one. (b) Dependence of f_0 on C_κ. The mapped *LCR* model prediction given by Eqs. (15) and (18) is shown (blue line) for resonators coupled via finger capacitors together with the measured values for f_0 (red points).

For the overcoupled devices A-H :

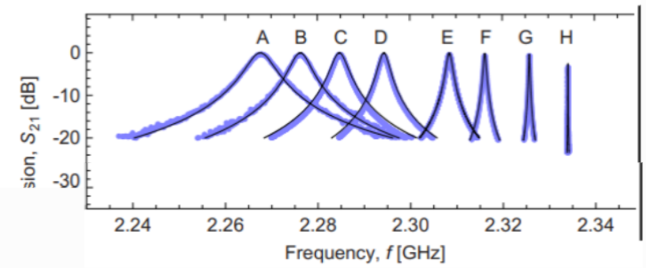
The experimental values C_k are in good agreement with calculations.

$$\omega_n^* = \frac{1}{\sqrt{L_n(C+2C^*)}}$$

$$C^* = \frac{C_k}{1+\omega_n^2 C_k^2 R_L^2}$$

For $C^* \approx C_k$ and $C \gg C_k$

$$\omega_n^* \simeq \omega_n \left(1 - \frac{C_k}{C}\right)$$



Relative Resonator Frequency Shift:

$$\frac{(\omega_n^* - \omega_n)}{\omega_n} = -\frac{C_k}{C}$$

Another Method : Transmission Matrix (ABCD)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & Z_{in} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} 1 & Z_{out} \\ 0 & 1 \end{pmatrix}$$

$$Z_{in} = Z_{out} = \frac{1}{i\omega C_k}$$

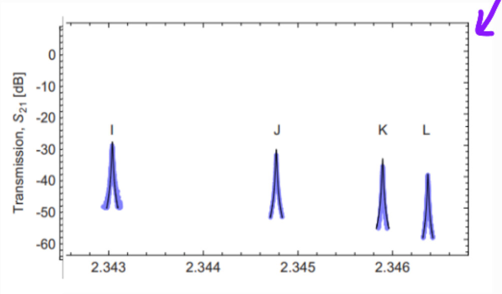
$$\left.\begin{aligned} t_{11} &= \cosh(\gamma l) \\ t_{12} &= Z_0 \sinh(\gamma l) \\ t_{21} &= \frac{1}{Z_0} \sinh(\gamma l) \\ t_{22} &= \cosh(\gamma l) \end{aligned}\right\} \quad \gamma = \boxed{\alpha} + i\boxed{\beta}$$

determined by Q_{int}

determined by ϵ_{eff}

TL wave propagation constant

$$S_{21} = \frac{2}{A + \frac{B}{R_L} + CR_L + D}$$



* For gap capacitors, the measured data fit very well to the transmission spectrum calculated with ABCD matrix

* For finger capacitors, each finger length should be added to the length of the bare resonators

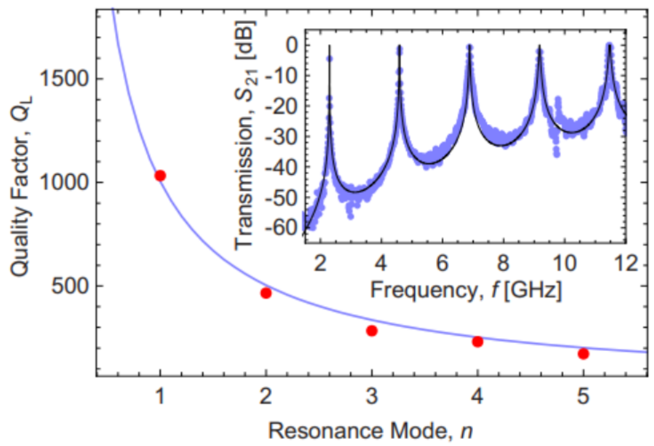


FIG. 8. (Color online) Measured quality factors for the overcoupled resonator D vs mode number n (red points) together with the prediction of the mapped *LCR* model given by Eqs. (18) and (21) (solid blue line). The inset shows the S_{21} transmission spectrum of resonator D with fundamental mode and harmonics. The measured data (blue) are compared to the S_{21} spectrum (black) obtained by the ABCD matrix method.

$$\frac{1}{Q_L} = \frac{1}{Q_{int}} + \frac{1}{Q_{ext}}$$

$$Q_{ext} = \frac{\omega_n R^* C}{2}$$

D is overcoupled $Q_{int} \gg Q_{ext}$

$$Q_L \simeq Q_{ext} = \frac{\omega_n C}{2}\left(\frac{1}{\omega_n^2 C_k^2 R_L}\right)$$

$$Q_L = \frac{C}{2\omega_n C_k^2 R_L} = \frac{C}{2 n \omega_0 R_L C_k^2}$$

① • From now on, we continue with input/output coupling.

- Here, we can see the symmetrically coupled transmission line
- We denote R, L, C with sub-ℓ since in Transmission Line voltages and currents differ over its LENGTH.
- You see the important parameters.

★ Around resonance, the properties of TL can be approximated to parallel LCR oscillator.

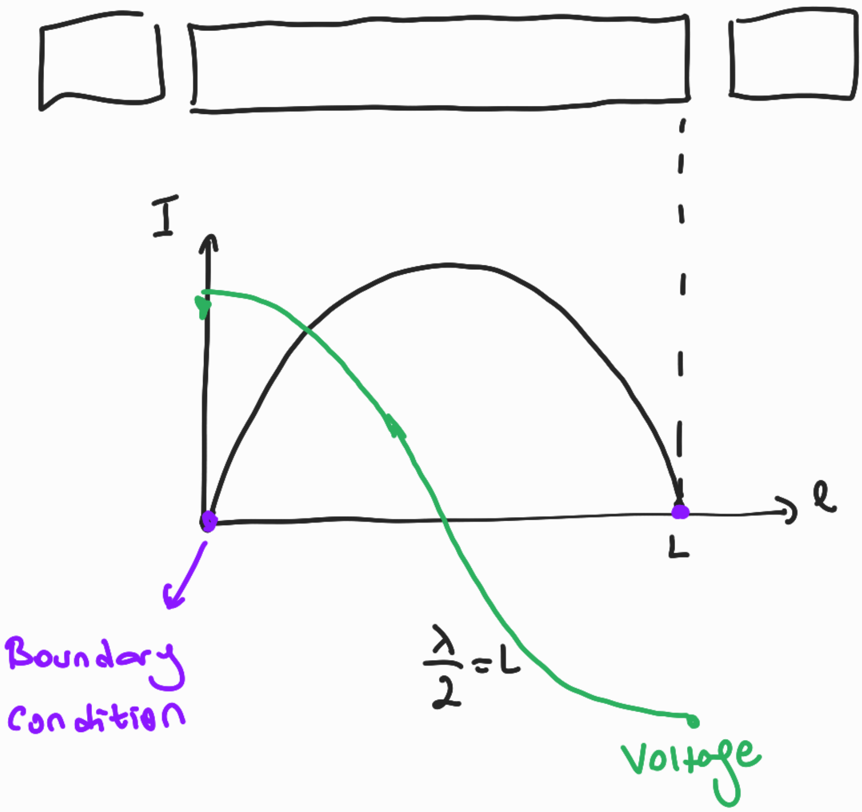
② There are 2 points that should be noted

→ Q_L
→ C_k

To understand these effects series connection of C_k and R_L can be transformed into a Norton equivalent

Stray Capacitor

Inside the Inductor for high frequencies



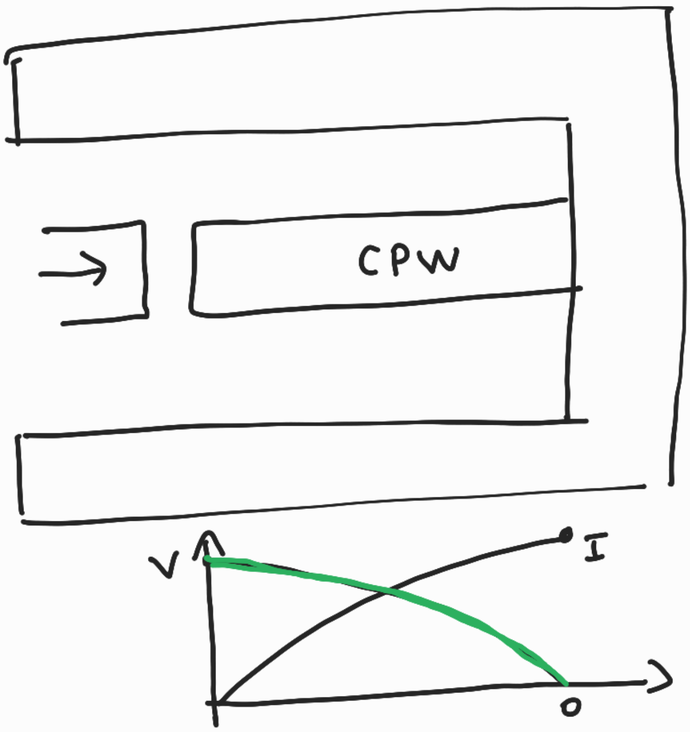
$$\boxed{f = \frac{c}{\sqrt{\varepsilon_{eff}}} \cdot \frac{1}{2l}}$$

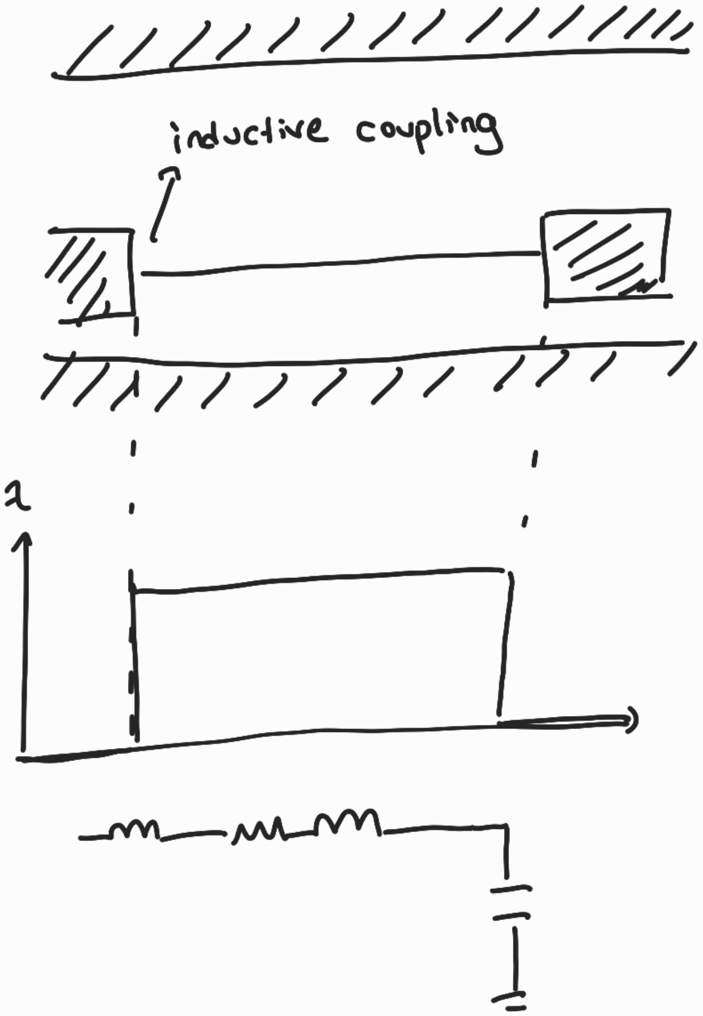
<u>Coupling</u> (put the qubit to the end of the reson)

Capacitive → Max Voltage

Inductive → Max Current

(put the qubit to into the middle of resonator)




inductive coupling
x