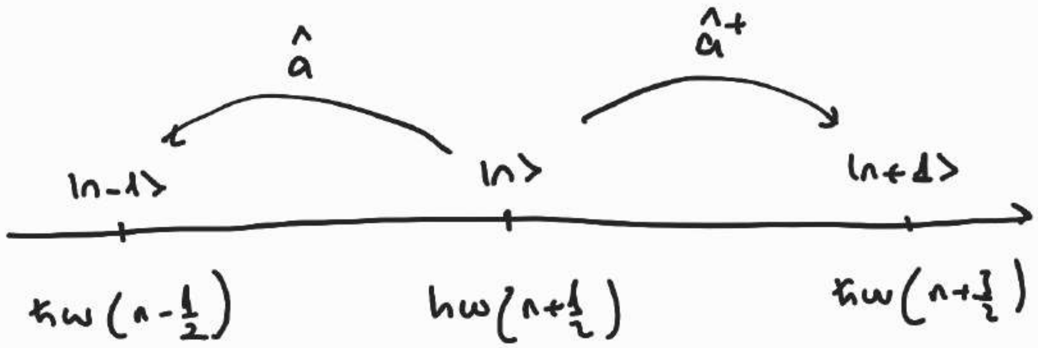
COHERENT STATE WAVE FUNCTION

Quantum Harmonic Oscillator:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 = \hbar\omega\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$$

$$\hat{H}|n\rangle = E_n|n\rangle$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad \rightarrow n = 0, 1, 2, \ldots$$



Coherent States

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad \alpha \in \mathbb{C}$$

→ In the energy basis: $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

→ With displacement operator: $\hat{D}(\alpha) = e^{\alpha\hat{a}^+ - \alpha^*\hat{a}}$

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad |\alpha\rangle = \hat{D}(\alpha)|0\rangle$$

Position Representation: $\hat{x}|x\rangle = x|x\rangle$

$$|\psi\rangle = \int dx\, \psi(x)|x\rangle \qquad \psi(x) = \langle x|\psi\rangle$$

"wave function"

Coherent state wave function:

$$\psi_\alpha(x) = \langle x|\alpha\rangle = \langle x|\hat{D}(\alpha)|0\rangle$$

$$\hat{D}(\alpha) = e^{\alpha\hat{a}^+ - \alpha^*\hat{a}}$$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} + i\frac{1}{\sqrt{2m\hbar\omega}}\hat{p}$$

$$\hat{a}^+ = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} - i\frac{1}{\sqrt{2m\hbar\omega}}\hat{p}$$

$$\alpha\hat{a}^+ - \alpha^*\hat{a} = \alpha\left(\sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} - i\frac{1}{\sqrt{2m\hbar\omega}}\hat{p}\right)$$

$$-\alpha^*\left(\sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} + i\frac{1}{\sqrt{2m\hbar\omega}}\hat{p}\right)$$

$$\alpha\hat{a}^{+} - \alpha^{*}\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^{*})\hat{x} - \frac{i}{\sqrt{2m\hbar\omega}}(\alpha+\alpha^{*})\hat{p}$$

$$\hat{D}(\alpha) = \exp\left[\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^{*})\boxed{\hat{x}} - \frac{i}{\sqrt{2m\hbar\omega}}(\alpha+\alpha^{*})\boxed{\hat{p}}\right]$$

$$[\hat{A},[\hat{A},\hat{B}]] = 0,\ [\hat{B},[\hat{A},\hat{B}]] = 0 : \quad e^{\hat{A}+\hat{B}} = \underbrace{e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}[\hat{A},\hat{B}]}}_{\checkmark \text{ satisfied}}$$

$$[\hat{x},\hat{p}] = i\hbar \quad \rightarrow \quad [\hat{x},\underbrace{[\hat{x},\hat{p}]}_{i\hbar}] = 0$$

$$[\hat{p},\underbrace{[\hat{x},\hat{p}]}_{i\hbar}] = 0$$

$$\hat{D}(\alpha) = \exp\left(\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^{*})\hat{x}\right)\exp\left(-\frac{i}{\sqrt{2m\hbar\omega}}(\alpha+\alpha^{*})\hat{p}\right)$$

$$\times\exp\left(-\frac{1}{2}\underbrace{\left[\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^{*})\hat{x},\ \frac{-i}{\sqrt{2m\hbar\omega}}(\alpha+\alpha^{*})\hat{p}\right]}\right)$$

$$\sqrt{\frac{\cancel{m\omega}}{2\hbar}}\left(-\frac{i}{\sqrt{2\cancel{m}\hbar\cancel{\omega}}}\right)\underbrace{(\alpha-\alpha^{*})(\alpha+\alpha^{*})}_{\alpha^{2}-(\alpha^{*})^{2}}\underbrace{[\hat{x},\hat{p}]}_{i\hbar}$$

$$-\frac{i}{2\cancel{\hbar}}(\alpha^{2}-(\alpha^{*})^{2})\,i\cancel{\hbar} = \frac{1}{2}(\alpha^{2}-(\alpha^{*})^{2})$$

$$\hat{D}(\alpha) = \exp\left(\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^*)\hat{x}\right)\exp\left(-\frac{i}{\sqrt{2m\hbar\omega}}(\alpha+\alpha^*)\hat{p}\right)$$

$$\times \exp\left(-\frac{1}{4}\left(\alpha^2-(\alpha^*)^2\right)\right)$$

$$\Psi_\alpha(x) = \langle x|\hat{D}(\alpha)|0\rangle$$

$$\Psi_\alpha(x) = \exp\left(-\frac{1}{4}\left(\alpha^2-(\alpha^*)^2\right)\right)\overbrace{\langle x|\exp\left(\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^*)\hat{x}\right)}^{\langle x|\exp\left(\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^*)\overset{\text{scalar}}{x}\right)}$$

$$\exp\left(-\frac{i}{\sqrt{2m\hbar\omega}}(\alpha+\alpha^*)\hat{p}\right)|0\rangle$$

$$\Psi_\alpha(x) = \exp\left(-\frac{1}{4}\left(\alpha^2-(\alpha^*)^2\right)\right)\exp\left(\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^*)x\right)$$

$$\langle x|\underbrace{\exp\left(-\frac{i}{\sqrt{2m\omega\hbar}}(\alpha+\alpha^*)\hat{p}\right)}_{\hat{T}\left(\sqrt{\frac{\hbar}{2m\omega}}(\alpha+\alpha^*)\right)}|0\rangle$$

Translation Operator: $\hat{T}(\lambda) = e^{-i\lambda\hat{p}/\hbar}$

$$\hat{T}(\lambda)|x\rangle = |x+\lambda\rangle \longleftrightarrow \langle x|\hat{T}(\lambda) = \langle x-\lambda|$$

$$\langle x | \hat{T}\left(\sqrt{\frac{\hbar}{2m\omega}}(\alpha+\alpha^*)\right) = \langle x - \sqrt{\frac{\hbar}{2m\omega}}(\alpha+\alpha^*) |$$

$$\psi_\alpha(x) = \exp\left[-\frac{1}{4}\left(\alpha^2 - (\alpha^*)^2\right)\right] \exp\left[\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^*)\,x\right]$$

$$\underbrace{\langle x - \sqrt{\frac{\hbar}{2m\omega}}(\alpha+\alpha^*) | 0 \rangle}$$

ground state wavefunction
the harmonic oscillator

$$\psi_0\left(x - \sqrt{\frac{\hbar}{2m\omega}}(\alpha+\alpha^*)\right)$$

$$\psi_\alpha(x) = \exp\left[-\frac{1}{4}\left(\alpha^2 - (\alpha^*)^2\right)\right] \exp\left[\sqrt{\frac{m\omega}{2\hbar}}(\alpha-\alpha^*)\,x\right]$$

$$\psi_0\left(x - \sqrt{\frac{\hbar}{2m\omega}}(\alpha+\alpha^*)\right)$$

$$\langle \hat{x} \rangle_\alpha = \sqrt{\frac{\hbar}{2m\omega}}(\alpha+\alpha^*) \qquad \langle \hat{p} \rangle_\alpha = -i\sqrt{\frac{m\hbar\omega}{2}}(\alpha-\alpha^*)$$

$$\boxed{\psi_\alpha(x) = \exp\left[-\frac{1}{4}\left(\alpha^2 - (\alpha^*)^2\right)\right] \exp\left[\frac{i}{\hbar}\langle p \rangle_\alpha\, x\right] \psi_0(x - \langle x \rangle_\alpha)}$$

$$-\frac{1}{4}\left(\alpha^2-(\alpha^*)^2\right) = -i\,\mathrm{Re}(\alpha)\,\mathrm{Im}(\alpha) = i\vartheta(\alpha)$$

$$(\alpha)^2 = (a+ib)^2 = \cancel{a^2} + 2iab - \cancel{b^2}$$

$$-\quad (\alpha^*)^2 = (a-ib)^2 = \cancel{a^2} - 2iab - \cancel{b^2}$$

$$\left(-\frac{1}{4}\right)\left((\alpha)^2-(\alpha^*)^2\right) = (4iab)\left(-\frac{1}{4}\right) = -iab$$

$$\theta(\alpha) = -\mathrm{Re}(\alpha)\,\mathrm{Im}(\alpha) \rightarrow \vartheta(\alpha) \in \mathbb{R}$$

$$\boxed{\Psi_\alpha(x) = e^{i\vartheta_\alpha}\, e^{i\langle\hat{p}\rangle_\alpha x/\hbar}\, \Psi_0\left(x-\langle\hat{x}\rangle_\alpha\right)}$$

$$\Psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

$$|\psi_\alpha(x)|^2 = |\psi_0(x - \langle \hat{x} \rangle_\alpha)|^2$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

$$|\psi_\alpha(x)|^2 = \sqrt{\frac{m\omega}{\pi\hbar}} \exp\left(-\frac{m\omega}{\hbar}\left(x - \langle \hat{x} \rangle_\alpha\right)^2\right)$$

$$\Delta\hat{x}_\alpha = \sqrt{\frac{\hbar}{2m\omega}} \Rightarrow \frac{m\omega}{\hbar} = \frac{1}{2(\Delta\hat{x}_\alpha)^2}$$

QUASI- CLASSICAL STATES

DOES THE QUANTUM HARMONIC OSCILLATOR ACTUALLY OSCILLATE ?

Classical Harmonic Oscillator:

$$x(t) = x_0 \cos(\omega t - \varphi)$$

$$p(t) = m\,\dot{x}(t) = -m\omega x_0 \sin(\omega t - \varphi)$$

$$E(t) = \frac{1}{2m}[p(t)]^2 + \frac{1}{2}m\omega^2 [x(t)]^2$$

$$E(t) = \frac{1}{2\cancel{m}} m^{\cancel{2}}\omega^2 x_0^2 \sin^2(\omega t - \varphi) + \frac{1}{2} m\omega^2 x_0^2 \cos^2(\omega t - \varphi)$$

$$= \frac{1}{2} m\omega^2 x_0^2 \Big[\underbrace{\sin^2(\omega t - \varphi) + \cos^2(\omega t - \varphi)}_{1} \Big] = \frac{1}{2} m\omega^2 x_0^2$$

$\leftrightarrow$

<u>Quantum harmonic oscillator:</u>

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 = \hbar\omega\left(\hat{a}^+ \hat{a} + \frac{1}{2}\right) = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$$

$$\hat{H}|n\rangle = E_n|n\rangle \qquad E_n = \hbar\omega\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots$$

How to connect classical and quantum oscillator?

position operator

$$\langle \hat{x} \rangle_n = \langle n | \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^+) | n \rangle = 0$$

$$\langle n | \underbrace{\hat{a} | n \rangle}_{\sqrt{n}|n-1\rangle} = \sqrt{n} \langle n | n-1 \rangle = 0$$

$$\langle n | \underbrace{\hat{a}^+ | n \rangle}_{\sqrt{n+1}|n+1\rangle} = \sqrt{n+1} \langle n | n+1 \rangle = 0$$

$$\langle \hat{p} \rangle_n = 0 \qquad \langle \hat{H} \rangle_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

Classical

$|n\rangle$

$x(t) = x_0 \cos(\omega t - \varphi)$

$p(t) = -m\omega x_0 \sin(\omega t - \varphi)$

$E(t) = \frac{1}{2} m\omega^2 x_0^2$

no resemble of oscillation

$\langle \hat{x} \rangle_n (t) = 0$

$\langle \hat{p} \rangle_n (t) = 0$

$\langle \hat{H} \rangle_n (t) = \hbar\omega\left(n + \frac{1}{2}\right)$

THEREFORE WE NEED COHERENT STATE

Coherent States: (Quasi-classical States)

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \quad , \quad \alpha \in \mathbb{C}$$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \xleftrightarrow{\text{Dual space}} \langle\alpha|\hat{a}^{+} = \langle\alpha|\alpha^*$$

$$\langle \hat{a} \rangle_\alpha = \langle \alpha | \underbrace{\hat{a} | \alpha \rangle}_{\alpha|\alpha\rangle} = \alpha\langle\alpha|\alpha\rangle = \alpha$$

$$\langle \hat{a}^n \rangle_\alpha = \langle\alpha| \hat{a}^n |\alpha\rangle = \alpha^n$$

$$\langle \hat{a}^+ \rangle_\alpha = \underbrace{\langle \alpha | \hat{a}^+}_{\langle \alpha | \alpha^*} | \alpha \rangle = \alpha^*$$

$$\langle \hat{a}^{+n} \rangle_\alpha = (\alpha^*)^n$$

$$\underbrace{\langle \alpha | \hat{a}^+}_{\langle \alpha | \alpha^*} \underbrace{\hat{a} | \alpha \rangle}_{\alpha | \alpha \rangle} = |\alpha|^2$$

$$\langle \alpha | \hat{a} \hat{a}^+ | \alpha \rangle = \overbrace{\langle \alpha | a^+ a | \alpha \rangle}^{|\alpha|^2} + 1 = |\alpha|^2 + 1$$

$$[\hat{a}, \hat{a}^+] = 1 \Rightarrow a a^+ = a^+ a + 1$$

$$\langle \hat{x} \rangle_\alpha = \langle \alpha | \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^+) | \alpha \rangle$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \left(\underbrace{\langle \alpha | \hat{a} | \alpha \rangle}_{\alpha} + \underbrace{\langle \alpha | \hat{a}^+ | \alpha \rangle}_{\alpha^*} \right)$$

$$\boxed{\langle \hat{x} \rangle_\alpha = \sqrt{\frac{\hbar}{2m\omega}} (\alpha + \alpha^*) = \sqrt{\frac{2\hbar}{m\omega}} \operatorname{Re}(\alpha)}$$

$$\langle \hat{x}^2 \rangle_\alpha = \langle \alpha | \frac{\hbar}{2m\omega} (\hat{a} + \hat{a}^+)^2 | \alpha \rangle$$

$$= \frac{\hbar}{2m\omega} \left(\overbrace{\langle \alpha | \hat{a}^2 | \alpha \rangle}^{\alpha^2} + \overbrace{\langle \alpha | \hat{a}^{+2} | \alpha \rangle}^{(\alpha^*)^2} + \overbrace{\langle \alpha | \hat{a} \hat{a}^+ | \alpha \rangle}^{1 + \alpha \alpha^*} + \underbrace{\langle \alpha | \hat{a}^+ \hat{a} | \alpha \rangle}_{\alpha \alpha^*} \right)$$

$$\langle \hat{x}^2 \rangle_\alpha = \frac{\hbar}{2m\omega}\left(\alpha^2 + (\alpha^*)^2 + 2\alpha\alpha^* + 1 \right) = \frac{\hbar}{2m\omega}\left((\alpha+\alpha^*)^2 + 1 \right)$$

$$\Delta \hat{x}_\alpha = \sqrt{\langle \hat{x}^2 \rangle_\alpha - \langle \hat{x} \rangle_\alpha^2}$$

$$\Delta \hat{x}_\alpha = \sqrt{\frac{\hbar}{2m\omega}\left(\cancel{(\alpha+\alpha^*)^2} + 1 \right) - \frac{2\hbar}{m\omega}\cancel{(\alpha+\alpha^*)^2}}$$

$$\Delta \hat{x}_\alpha = \sqrt{\frac{\hbar}{2m\omega}}$$ does not depend on α

Assume:

$$|\Psi(0)\rangle = |\alpha_0\rangle \qquad \alpha_0 = |\alpha_0| e^{i\varphi}$$

$$|\Psi(t)\rangle = e^{-i\omega t/2} |\alpha\rangle, \qquad \alpha = \alpha_0 e^{-i\omega t} = |\alpha_0| e^{-i(\omega t - \varphi)}$$

$$\langle x \rangle_\alpha (t) = \sqrt{\frac{\hbar}{2m\omega}} \left(|\alpha_0| e^{-i(\omega t - \varphi)} + |\alpha_0| e^{i(\omega t - \varphi)} \right)$$

$$\alpha(t) + \alpha^*(t)$$

$$e^{i\delta} + e^{-i\delta} = 2\cos(\delta)$$

$$\boxed{\langle x \rangle_\alpha (t) = \sqrt{\frac{2\hbar}{m\omega}} |\alpha_0| \cos(\omega t - \varphi)}$$

$$\langle \hat{p} \rangle_\alpha = -i\sqrt{\frac{m\hbar\omega}{2}}\,(\alpha-\alpha^*) = \sqrt{2m\hbar\omega}\;\mathrm{Im}(\alpha)$$

$$\langle \hat{p}^2 \rangle_\alpha = \frac{m\hbar\omega}{2}\left[1-(\alpha-\alpha^*)^2\right]$$

$$\Delta\hat{p}_\alpha = \sqrt{\frac{m\hbar\omega}{2}}$$

$$|\psi(0)\rangle = |\alpha_0\rangle \longrightarrow |\psi(t)\rangle = e^{-i\omega t/2}\,|\alpha\rangle$$

$$\alpha = |\alpha_0| e^{-i(\omega t-\varphi)}$$

$$\boxed{\langle \hat{p} \rangle_\alpha(t) = -\sqrt{2m\hbar\omega}\,|\alpha_0|\,\sin(\omega t-\varphi)}$$

$$\langle \hat{H} \rangle_\alpha = \hbar\omega\left(|\alpha|^2+\frac{1}{2}\right)$$

$$\langle \hat{H}^2 \rangle_\alpha = \hbar^2\omega^2\left(|\alpha|^4+2|\alpha|^2+\frac{1}{4}\right)$$

$$\Delta\hat{H}_\alpha = \hbar\omega|\alpha|$$

$$|\psi(0)\rangle = |\alpha_0\rangle \longrightarrow |\psi(t)\rangle = e^{-i\omega t/2}\,|\alpha\rangle \quad \text{for } \alpha = \alpha_0 e^{-i\omega t}$$

$$\boxed{\langle \hat{H} \rangle_\alpha(t) = \hbar\omega\left(|\alpha_0|^2+\frac{1}{2}\right) \rightsquigarrow \text{TIME INDEPENDENT}}$$

QUASI-CLASSICAL STATES

Classical

$|\alpha\rangle$

$x(t) = x_0 \cos(\omega t - \varphi)$

$\langle \hat{x} \rangle_\alpha(t) = \sqrt{\frac{2\hbar}{m\omega}}\, |\alpha_0| \cos(\omega t - \varphi)$

$p(t) = -m\omega x_0 \sin(\omega t - \varphi)$

$\langle \hat{p} \rangle_\alpha(t) = -\sqrt{2m\hbar\omega}\, |\alpha_0| \sin(\omega t - \varphi)$

$E(t) = \frac{1}{2} m\omega^2 x_0$

$\langle \hat{H} \rangle_\alpha(t) = \hbar\omega \left(|\alpha_0|^2 + \frac{1}{2} \right)$

$x_0 = \sqrt{\frac{2\hbar}{m\omega}}\, |\alpha_0| \Rightarrow \quad x(t) = \langle \hat{x} \rangle_\alpha(t)$

$p(t) = \langle \hat{p} \rangle_\alpha(t)$

$\frac{1}{\cancel{2}} \cancel{m}\omega^{\cancel{2}} \left(\frac{\cancel{2}\hbar}{\cancel{m}\cancel{\omega}} \right) |\alpha_0|^2$

$\hbar\omega |\alpha_0|^2$

$E(t) = \langle \hat{H}_\alpha(t) \rangle - \frac{1}{2}\hbar\omega$

zero-point energy of quantum harmonic oscillator

"PURELY QUANTUM" PHENOMENON

for classical $\rightarrow |\alpha_0| \gg \frac{1}{2}\hbar\omega$

RETURN TO $|\psi_\alpha(x)|^2$

$$|\psi_\alpha(x)|^2 = |\psi_0(x - \langle \hat{x} \rangle_\alpha)|^2$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

$$|\psi_\alpha(x)|^2 = \sqrt{\frac{m\omega}{\pi\hbar}}\, \exp\left(-\frac{m\omega}{\hbar}(x - \langle \hat{x} \rangle_\alpha)^2\right)$$

$$\Delta\hat{x}_\alpha = \sqrt{\frac{\hbar}{2m\omega}} \Rightarrow \frac{m\omega}{\hbar} = \frac{1}{2(\Delta\hat{x}_\alpha)^2}$$

$$\boxed{|\psi_\alpha(x)|^2 = \frac{1}{\sqrt{2\pi}\,\Delta\hat{x}_\alpha} \exp\left(-\frac{1}{2}\left(\frac{x - \langle \hat{x} \rangle_\alpha}{\Delta\hat{x}_\alpha}\right)^2\right)}$$

"Gaussian"

$$\Delta\hat{x}_\alpha = \sqrt{\frac{\hbar}{2m\omega}} \qquad \Delta\hat{p}_\alpha = \sqrt{\frac{m\hbar\omega}{2}}$$

$$\Delta\hat{x}_\alpha\, \Delta\hat{p}_\alpha = \sqrt{\frac{\hbar}{2\cancel{m\omega}}}\sqrt{\frac{\cancel{m}\hbar\cancel{\omega}}{2}} = \frac{\hbar}{2}$$

Heisenberg Uncertainity Principle:

$$\Delta \hat{x} \, \Delta \hat{p} \geq \frac{\hbar}{2}$$

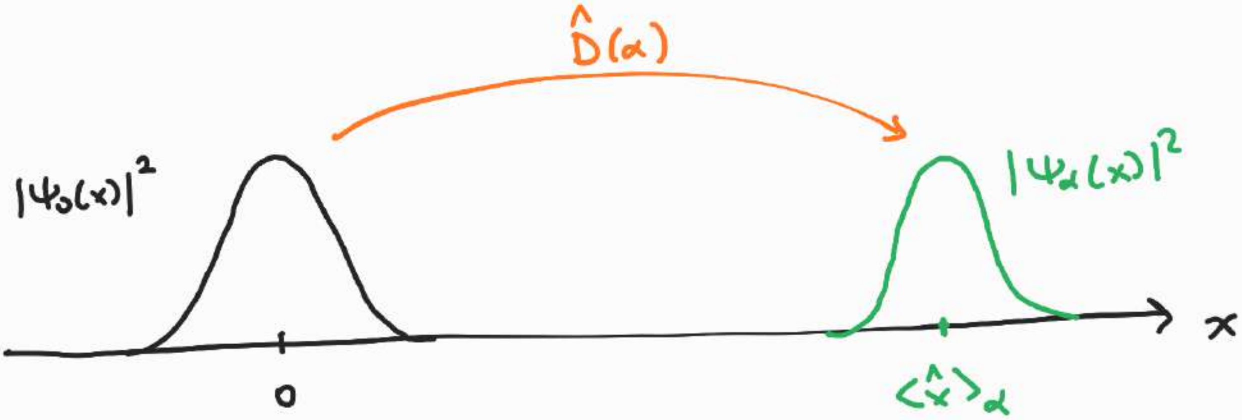
For coherent states $\Delta \hat{x}_\alpha \, \Delta \hat{p}_\alpha = \frac{\hbar}{2}$

"MINIMUM UNCERTINITY STATES"

$\updownarrow$

GAUSSIAN ?

$$|\alpha\rangle = \hat{D}(\alpha) \, |0\rangle$$



Time Evolution:

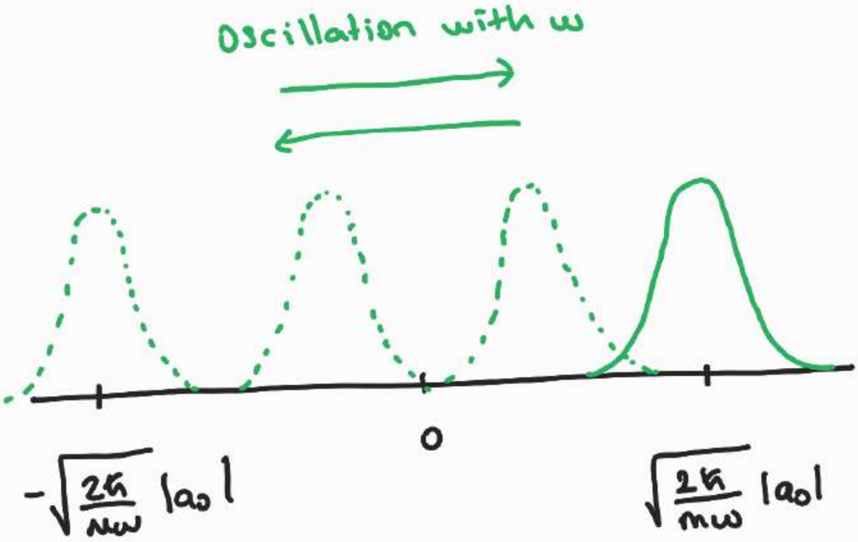
$$|\Psi(0)\rangle = |\alpha_0\rangle \Rightarrow |\Psi(t)\rangle = e^{-i\omega t/2}|\alpha\rangle, \quad \alpha = \alpha_0 e^{-i\omega t}$$

$$|\Psi_\alpha(x,t)|^2 = |\Psi_0(x - \langle\hat{x}\rangle_\alpha(t))|^2$$

$$|\Psi_\alpha(x,t)|^2 = \frac{1}{\sqrt{2\pi}\,\Delta\hat{x}_\alpha} \exp\left(-\frac{1}{2}\left(\frac{x - \langle\hat{x}\rangle_\alpha(t)}{\Delta\hat{x}_\alpha}\right)^2\right)$$

$$\langle\hat{x}\rangle_\alpha(t) = \sqrt{\frac{2\hbar}{m\omega}}\,|\alpha_0|\cos(\omega t - \varphi)$$

$$\Delta\hat{x}_\alpha = \sqrt{\frac{\hbar}{2m\omega}}$$ → Time Independent. Thus Gaussian does not spread over time.



Where do we use coherent states?

How entanglement is achieved?

→ In Quantum Optics: Laser beam

. well defined phase and amplitude

→ In QFT

→ In Quantum-Information: creating entangled states (SPDC)