Particle Physics II
Lecture 12: The Higgs boson

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- Higgs mechanism gives masses not only to W and Z bosons but also to fermions
- due to different transformation of LH and RH chiral states the mass term of the Dirac lagrangian

$$-m\bar{\psi}\psi = -m\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right)$$

  is not symmetric under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry

  $\Longrightarrow$ cannot be in the SM lagrangian
- therefore the fermion fields are introduced to the SM as massless states and they acquire their mass thanks to the interaction with Higgs field

Transformations of left doublet

- in the SM, LH chiral fermions are placed in SU(2) doublets (L)
- RH chiral fermions – in SU(2) singlets (R)
- since two complex scalar Higgs fields are in an SU(2) doublet $\phi(x)$, a local gauge transformation affects it as:

$$\phi \to \phi' = (I + ig_W \vec{\epsilon}(x) \cdot \vec{T})\phi$$

- the same local gauge transformation applies to the LH fermion doublet L
- for $\bar{L} \equiv L^\dagger \gamma^0$:

$$\bar{L} \to \bar{L}' = \bar{L}(I - ig_W \vec{\epsilon}(x) \cdot \vec{T})$$

- $\implies$ $\bar{L}\phi$ is invariant under $\mathrm{SU}(2)_L$ gauge transformations

Invariant form

- combined with a RH singlet: $\bar{L}\phi R \implies$ invariant under $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge transformations
- its Hermitian conjugate is invariant too:

$$\left(\bar{L}\phi R\right)^\dagger = \bar{R}\phi^\dagger L$$

- $\implies$ such term satisfies the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry:

$$\boxed{-g_f\left(\bar{L}\phi R + \bar{R}\phi^\dagger L\right)}$$

Electron case

- lagrangian part for electrons would look like:

$$\mathcal{L}_e = -g_e \left[\begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e}_R \begin{pmatrix} \phi^{+*} & \phi^{0*} \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right]$$

- g_e is a constant – Yukawa coupling of the electron to the Higgs field
- after spontaneous symmetry breaking the Higgs doublet in the unitary gauge is:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- electron term in the lagrangian becomes:

$$\mathcal{L}_e = -\frac{g_e}{\sqrt{2}} v(\bar{e}_L e_R + \bar{e}_R e_L) - \frac{g_e}{\sqrt{2}} h(\bar{e}_L e_R + \bar{e}_R e_L)$$

- first term there has exactly the form needed for the fermion masses

Electron case

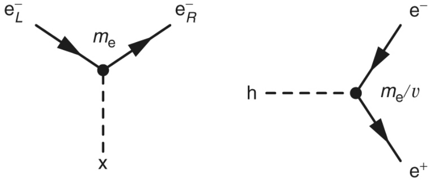
- the Yukawa coupling g_e is not predicted by the Higgs mechanism but can be chosen to be consistent with the observed electron mass:

$$g_e = \sqrt{2}\frac{m_e}{v}$$

- then electron mass lagrangian becomes:

$$\mathcal{L}_e = -m_e\bar{e}e - \frac{m_e}{v}\bar{e}eh$$

  - first term (giving the mass to electron) – is a coupling of electron to the **Higgs field** through its non-zero v
  - second term – is a coupling between the electron and the **Higgs boson**



Up-type fermions

- term $\bar{L}\phi R + \bar{R}\phi^\dagger L$ can generate masses only for down-type fermions:
  - this happens since non-zero vacuum expectation value v exists only in the lower (neutral) component of the Higgs doublet
- to have masses for up-type fermions, use the conjugate doublet ϕ_c formed as:

$$\phi_c = -i\sigma_2\phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\phi_3 + i\phi_4 \\ \phi_1 - i\phi_2 \end{pmatrix}$$

- the conjugate doublet ϕ_c transforms in the same way as ϕ under SU(2)
- this is similar to the representation of up- and down-quarks and anti-up and anti-down quarks in SU(2) isospin symmetry

Up-type fermions

- a gauge-invariant mass term for up-type quarks comes from the term:

$$\boxed{\bar{L}\phi_c R + \bar{R}\phi_c^\dagger L}$$

- as an example, for u quark:

$$\mathcal{L}_u = g_u \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} u_R + H.C.$$

- after symmetry breaking it becomes:

$$\mathcal{L}_u = -\frac{g_u}{\sqrt{2}} v(\bar{u}_L u_R + \bar{u}_R u_L) - \frac{g_u}{\sqrt{2}} h(\bar{u}_L u_R + \bar{u}_R u_L)$$

- with Yukawa coupling $g_u = \sqrt{2} m_u / v$ leading to:

$$\boxed{\mathcal{L}_u = -m_u \bar{u}u - \frac{m_u}{v} \bar{u}uh}$$

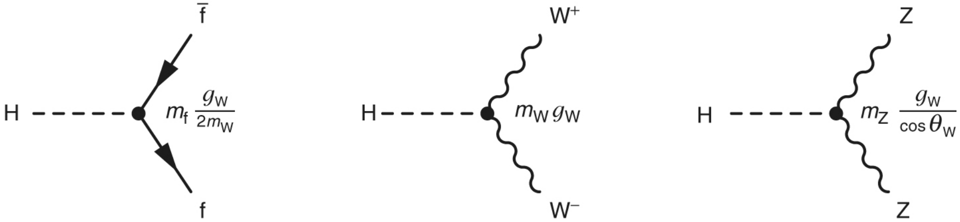
- so for all Dirac fermions gauge invariant mass terms can be constructed from:
$$\mathcal{L} = -g_f \left[\bar{L}\phi R + \left(\bar{L}\phi R\right)^\dagger\right] \text{ or } \mathcal{L} = g_f \left[\bar{L}\phi_c R + \left(\bar{L}\phi_c R\right)^\dagger\right]$$
- these terms produce masses of the fermions and the interactions between the Higgs boson and the fermion
- the Yukawa couplings of the fermions to the Higgs field are given by:
$$g_f = \sqrt{2}\frac{m_f}{v},$$
where $v = 246$ GeV
- e.g. for the top quark with $m_t \approx 173.1 \pm 0.9$ GeV, the Yukawa coupling is almost exactly 1, resonating with a notion that maybe Yukawa couplings of the fermions should be $\mathcal{O}(1)$
- a special case is neutrinos where Yukawa couplings are $< 10^{-12}$: other mechanisms are suggested for neutrino masses

Properties of the Higgs boson

- the SM Higgs boson H is a neutral scalar particle
- its mass is a free parameter of the SM $m_H = 2\lambda v^2$
- H couples to all fermions with a coupling strength proportional to the fermion mass, vertex factor is:

$$-i\frac{m_f}{v} \equiv -i\frac{m_f}{2m_W}g_W$$

- H decays as $H \rightarrow f\bar{f}$ to all kinematically allowed decay modes $m_H > 2m_f$
- for vector bosons, coupling is also proportional to boson mass:

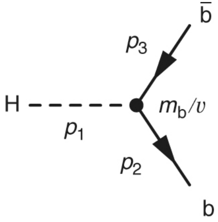


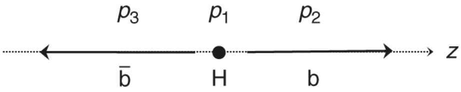
- for a discovered H boson with $m = 125$ GeV the largest BF is to $b\bar{b}$:

$$\mathcal{B}(H \to b\bar{b}) = 57.8\%$$

- we can compute partial decay width:
  - vertex is known
  - H is scalar $\implies$ no polarization 4-vector is needed
  - matrix element is simply:

$$\mathcal{M} = \frac{m_b}{v}\bar{u}(p_2)v(p_3) = \frac{m_b}{v}u^\dagger\gamma^0 v$$





- can take b-quark momentum along the z-axis
- $m_H \gg m_b$: can neglect b mass and take

$$p_2 \approx (E, 0, 0, E) \text{ and } p_3 \approx (E, 0, 0, -E), \text{ where } E = m_H/2$$

- in the ultra-relativistic limit the spinors for two possible helicity state for each of the b $(\theta = 0, \phi = 0)$ and the $\overline{b}$ $(\theta = \phi, \phi = \pi)$ are:

$$u_\uparrow(p_2) = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}, u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix},$$

$$v_\uparrow(p_3) = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}, v_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}0\\-1\\0\\-1\end{pmatrix}$$

Higgs boson decays

- due to $u^\dagger \gamma^0 v$ form of the ME only two of the possible four helicity combinations give non-0 matrix element:

$$\mathcal{M}_{\uparrow\uparrow} = -\mathcal{M}_{\downarrow\downarrow} = \frac{m_b}{v} 2E$$

- these correspond to spin configurations where $b\bar{b}$ are produced in a spin-0 state
- since H is a spin-0 scalar, it decays isotropically and ME does not have angular dependence
- and since there is only one spin state for H, the spin-averaged ME is:

$$\left\langle |\mathcal{M}|^2 \right\rangle = |\mathcal{M}_{\uparrow\uparrow}|^2 + |\mathcal{M}_{\downarrow\downarrow}|^2 = \frac{m_b^2}{v^2} 8E^2 = \frac{2m_b^2 m_H^2}{v^2}$$

- the partial decay width:

$$\Gamma(H \to b\bar{b}) = 3 \times \frac{m_b^2 m_H}{8\pi v^2},$$

  where factor 3 accounts for three colors of $b\bar{b}$ pair
- for SM H: $\Gamma(H \to b\bar{b}) \approx 2$ MeV

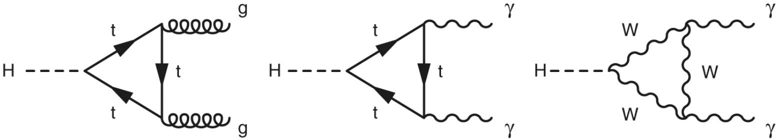
Higgs boson decays

- partial H decay rate to fermions is $\propto m_f^2$:

$$\Gamma(H \to b\bar{b}) : \Gamma(H \to c\bar{c}) : \Gamma(H \to \tau^+\tau^-) \approx 3m_b^2 : 3m_c^2 : m_\tau^2$$

- m_q run with q^2 similarly to α_S: masses should be taken at $q^2 = m_H^2$?
- there $m_c(m_H^2) \approx 0.6$ GeV and $m_b(m_H^2) \approx 3.0$ GeV
- decay to massless particles γ and g happen via loops:

Decay mode	Branching ratio
$H \to b\bar{b}$	57.8%
$H \to WW^*$	21.6%
$H \to \tau^+\tau^-$	6.4%
$H \to gg$	8.6%
$H \to c\bar{c}$	2.9%
$H \to ZZ^*$	2.7%
$H \to \gamma\gamma$	0.2%



Higgs boson decays measurements

- example of measurements by the ATLAS experiment, testing H couplings to all particles: