Neutrinos and Neutrino Oscillations

1) Neutrino Flavours

2) Mass and Weak Eigenstates

  2.1) Neutrino Oscillations for 2 Flavours

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    2.2.1) CP and CPT in the weak interaction
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  3.1) Neutrino Interaction Thresholds

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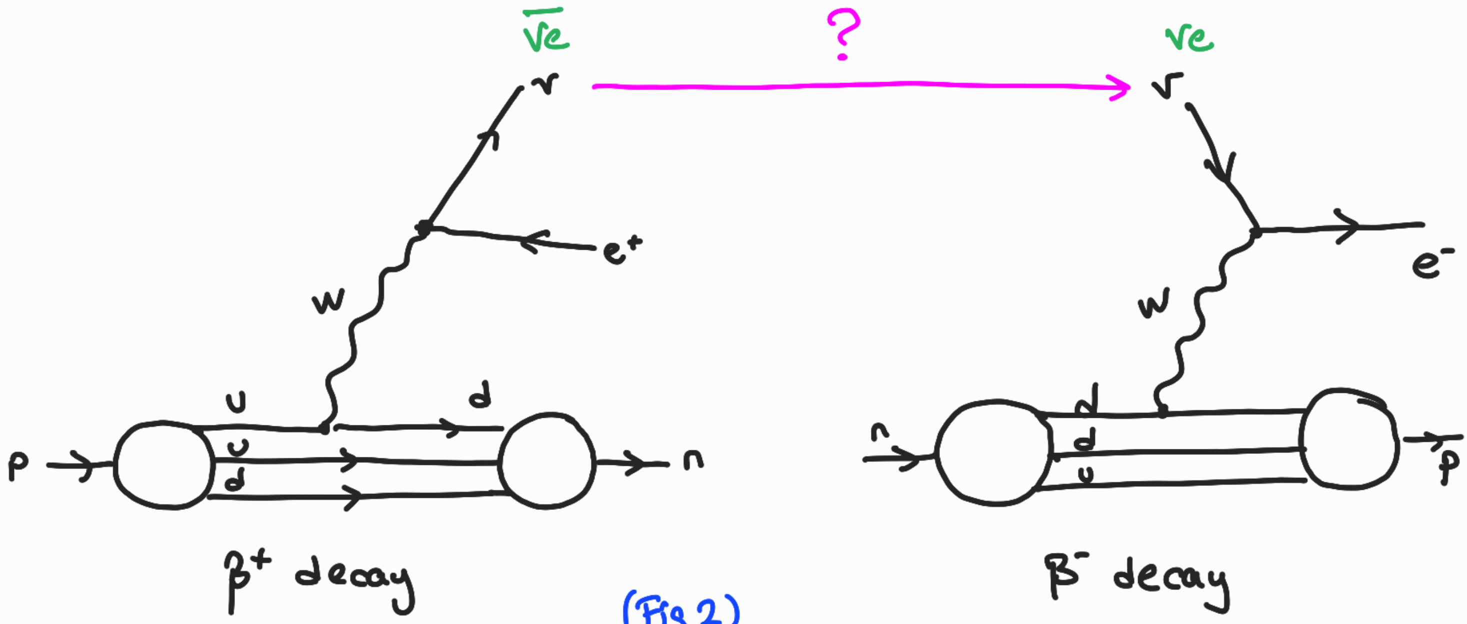
1) Neutrino Flavors

ν_e, ν_μ, ν_τ are distinct neutrinos. → weak eigenstates

Non-observation of the decay $\mu^- \to e^- \gamma$ can be an evidence

(Fig1)



(Fig 2)

* ν produced along with e^+ always lead to e^- in CC weak interactions.

2) Mass and weak eigenstates

Mass eigenstates are the stationary states of the free-particle Hamiltonian

$$\hat{H}\Psi = i\frac{\partial \Psi}{\partial t} = E\Psi$$

$$\Psi(x,t) = \phi(x)\, e^{-iEt}$$

2.1) Neutrino Oscillations for 2 flavors

When considering Fig.2,

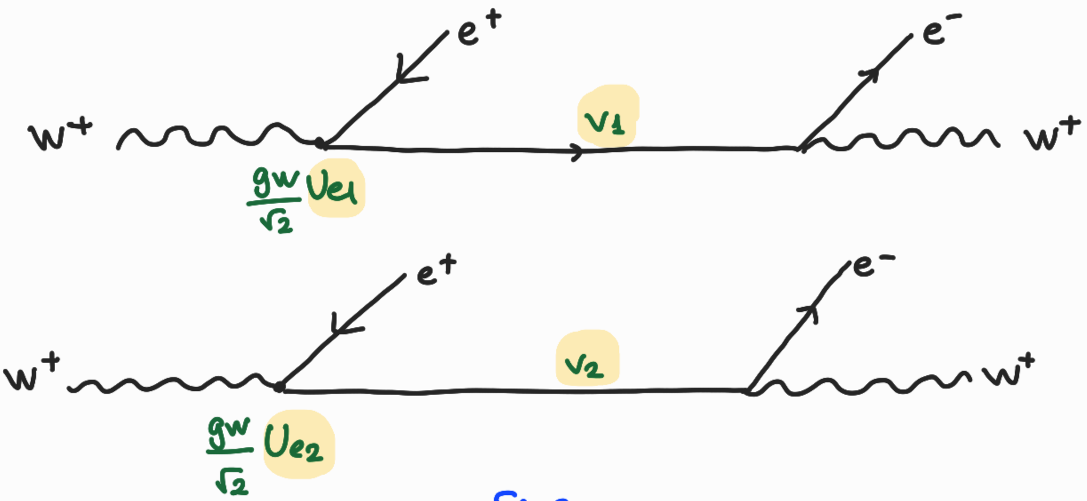


Fig3.

we can't know which mass eigenstates (ν_1, ν_2) was involved

$$\boxed{\text{Coherent state } |\psi\rangle = \nu_e = U_{e1}\nu_1 + U_{e2}\nu_2}$$

weak eigenstate

mass eigenstates

free particle solutions

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{i(\vec{p}_1\cdot\vec{x} - E_1 t)}$$

$$|\nu_2(t)\rangle = |\nu_2\rangle e^{i(\vec{p}_2\cdot\vec{x} - E_2 t)}$$

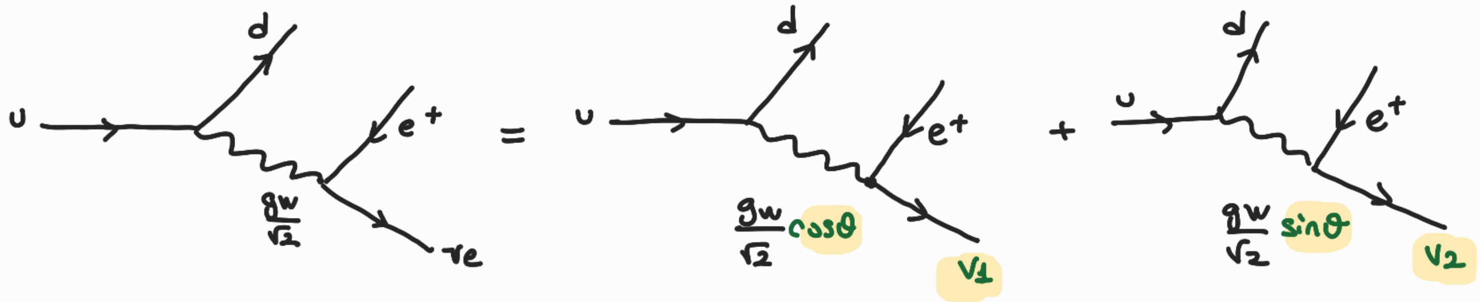


Fig4.

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad (*)$$

At time $t=0$, a neutrino is produced in a pure ν_e state by decay $u \rightarrow d e^+ \nu_e$ (fig.4)

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\psi(t)\rangle = \cos\theta\,|\nu_1\rangle\, e^{-i p_1 . x} + \sin\theta\,|\nu_2\rangle\, e^{-i p_2 . x}$$

where $\underbrace{p_i}_{\text{4-momenta}} . x = E_i t - \vec{p_i} . \vec{x} = E_i t - |\vec{p_i}|\, z$ in z direction

Neutrino interacts in a detector at a distance L and a time T:

$$\phi_i = p_i x = E_i T - |\vec{p_i}| L$$

$$|\psi(L,T)\rangle = \cos\theta\,|\nu_1\rangle\, e^{-i\phi_1} + \sin\theta\,|\nu_2\rangle\, e^{-i\phi_2}$$

Inverse of the above matrix $(*)$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$|\Psi(L,T)\rangle = \cos\theta\left(\cos\theta\,|\nu_e\rangle - \sin\theta\,|\nu_\mu\rangle\right)e^{-i\phi_1} + \sin\theta\left(\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle\right)e^{-i\phi_2}$$

$$= \left(e^{-i\phi_1}\cos^2\theta + e^{-i\phi_2}\sin^2\theta\right)|\nu_e\rangle - \left(e^{-i\phi_1}\cos\theta\sin\theta + e^{-i\phi_2}\cos\theta\sin\theta\right)|\nu_\mu\rangle$$

$$= e^{-i\phi_1}\left[\left(\cos^2\theta + e^{i\Delta\phi_{12}}\sin^2\theta\right)|\nu_e\rangle - \left(1 - e^{i\Delta\phi_{12}}\right)\cos\theta\sin\theta\,|\nu_\mu\rangle\right]$$

$$\Delta\phi_{12} = \phi_1 - \phi_2 = (E_1 - E_2)T - (|p_1| - |p_2|)L$$

If $\Delta\phi_{12} = 0$ → neutrino remains in a pure electron neutrino

If $\Delta\phi_{12} \neq 0$ → There's also muon neutrino component.

$$|\Psi(L,T)\rangle = c_e|\nu_e\rangle + c_\mu|\nu_\mu\rangle$$

$$P(\nu_e \to \nu_\mu) = c_\mu c_\mu^* = \left(1 - e^{i\Delta\phi_{12}}\right)\left(1 - e^{-i\Delta\phi_{12}}\right)\cos^2\theta\sin^2\theta$$

$$= \left(2 - 2\cos(\Delta\phi_{12})\right)\frac{\sin^2(2\theta)}{4}$$

$$= \boxed{\sin^2\left(\frac{\Delta\phi_{12}}{2}\right)\frac{\sin^2(2\theta)}{4}}$$

One can assume $|p_1| = |p_2| = p$:

$$\Delta\phi_{12} = (E_1 - E_2)T - \left[(p^2 + m_1^2)^{1/2} - (p^2 + m_2^2)^{1/2}\right]L$$

$p^\mu = (E, \vec{p})$

$p^\mu p_\mu = \underbrace{E^2 - p^2}_{} = m^2$

Lorentz-invariant quantity

$T \approx L$ since $\beta \approx 1$

$(L = cT)$

$c = 1$ (natural units)

$\beta = \frac{v}{c} \approx 1$

$$\boxed{p = E\beta}$$

$$\Delta\Phi_{12} = p\left[\left(1+\frac{m_1^2}{p^2}\right)^{1/2} - \left(1+\frac{m_2^2}{p^2}\right)^{1/2}\right]L$$

Since $m \ll E$ → squareroots can be approximated:

$$\left(1+\frac{m^2}{p^2}\right)^{1/2} \approx 1 + \frac{m^2}{2p^2}$$

$$\boxed{\Delta\Phi_{12} = \frac{m_1^2 - m_2^2}{2p} L = \frac{\Delta m^2}{2E} L}$$

★ Here, it is neglected that for the same momentum, different mass eigenstates propagate at different velocities, thus observed at different times.

$$\Delta\Phi_{12} = (E_1 - E_2)T - \left(\frac{|p_1|^2 - |p_2|^2}{|p_1| + |p_2|}\right)L$$

$$= (E_1 - E_2)T - \left(\frac{E_1^2 - m_1^2 - E_2^2 - m_2^2}{p_1 + p_2}\right)L$$

$$= (E_1 - E_2)\left[T - \left(\frac{E_1 + E_2}{p_1 + p_2}\right)L\right] + \frac{m_1^2 - m_2^2}{p_1 + p_2}$$

If $E_1 = E_2$ or $\beta_1 = \beta_2$ $\Rightarrow$ $\Delta\phi = \dfrac{m_1^2 - m_2^2}{p_1 + p_2}$ → same as previous $\Delta\phi$

$$P(\nu_e \to \nu_\mu) = \sin^2(2\vartheta)\,\sin^2\left(\frac{(m_1^2 - m_2^2)L}{4E_\nu}\right)$$

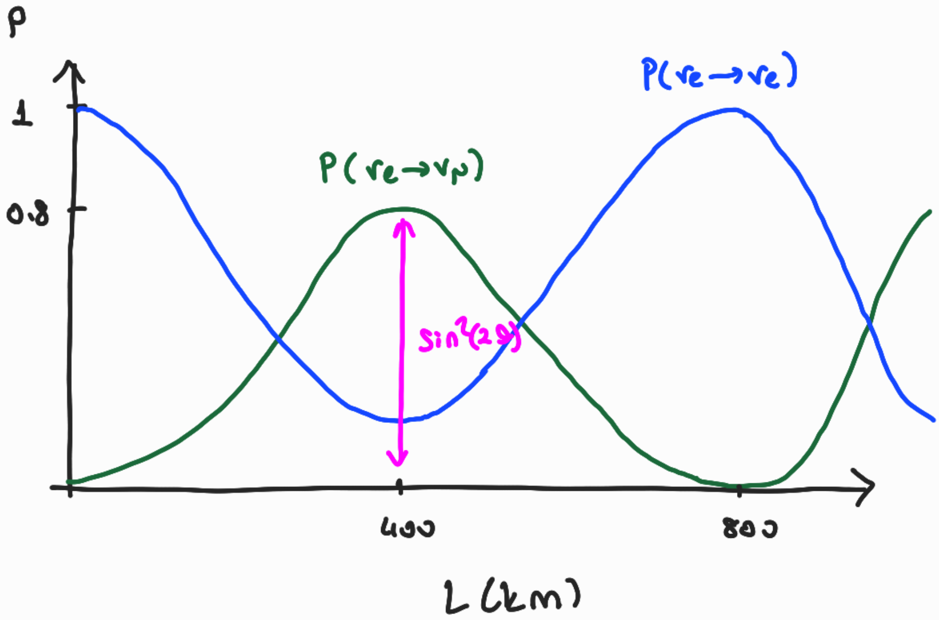
$$P(\nu_e \to \nu_e) = 1 - P(\nu_e - \nu_\mu)$$

$E_\nu = 1\ \text{GeV}$

$\Delta m^2 = 0.003\ eV^2$

$\Delta L \rightarrow \text{km}$

$\lambda_{osc} = \frac{\pi\, 4E}{\Delta m^2}$



2.2) Neutrino Oscillations for 3 flavours

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{\text{Pontecorvo-Maki-Nakagawa-Sakata (PMNS)}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

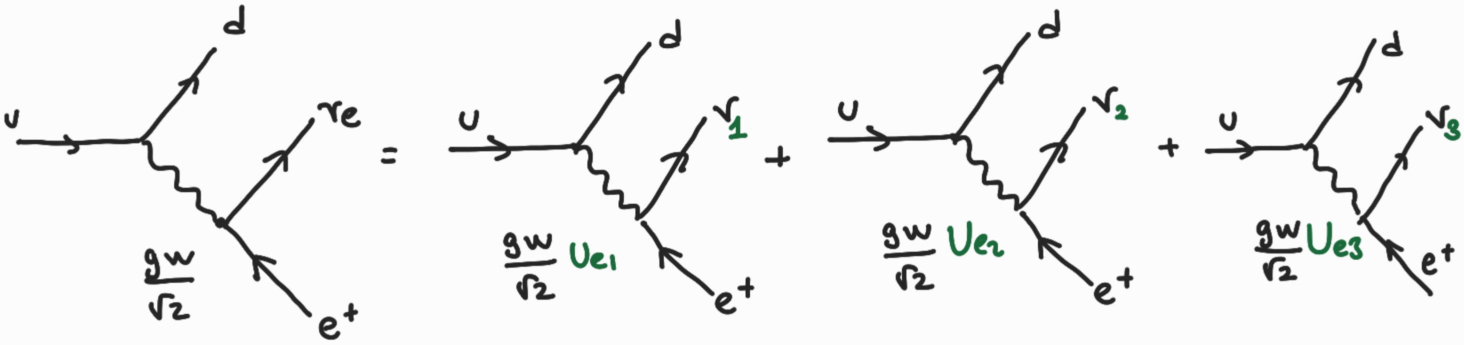
$$U^{-1} = U^{+}$$

Hence, PNMS is unitary

$$U_{e1} U^*_{e1} + U_{e2} U^*_{e2} + U_{e3} U^*_{e3} = 1$$
$$U_{\mu 1} U^*_{\mu 1} + \dots = 1$$
$$U_{\tau 1} U^*_{\tau 1} + \dots = 1$$

otherwise 0

$$U_{e1} U^*_{\mu 1} + U_{e2} U^*_{\mu 2} + U_{e3} U^*_{\mu 3} = 0$$



$$|\Psi(t=0)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

$$|\Psi(t)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle e^{-ip_1\cdot x} + U_{e2}|\nu_2\rangle e^{-ip_2 x} + U_{e3}|\nu_3\rangle e^{-ip_3\cdot x}$$

$$p_i\cdot x = E_i t - \vec{p_i}\cdot\vec{x} = E_i t - |\vec{p}|z$$ axis of propagation

$$\boxed{|\Psi(L)\rangle = U_{e1}|\nu_1\rangle e^{-i\phi_1} + U_{e2}|\nu_2\rangle e^{-i\phi_2} + U_{e3}|\nu_3\rangle e^{-i\phi_3}}$$

$$\phi_1 = p_i\cdot x = E_i t - |p|L = (E_i - |p|_i)L$$

we can again approximate $\phi_i \approx \frac{m_i^2}{2E_i}L$

$$\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \begin{pmatrix}U_{e1}^* & U_{\mu1}^* & U_{\tau1}^*\\ U_{e2}^* & U_{\mu2}^* & U_{\tau2}^*\\ U_{e3}^* & U_{\mu3}^* & U_{\tau3}^*\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$|\Psi(L)\rangle = U_{e1}\left[U_{e1}^*|\nu_e\rangle + U_{\mu1}^*|\nu_\mu\rangle + U_{\tau1}^*|\nu_\tau\rangle\right]e^{-i\phi_1}$$

$$+ U_{e2}\left[U_{e2}^*|\nu_e\rangle + U_{\mu2}^*|\nu_\mu\rangle + U_{\tau2}^*|\nu_\tau\rangle\right]e^{-i\phi_2}$$

$$+ U_{e3}\left[U_{e3}^*|\nu_e\rangle + U_{\mu3}^*|\nu_\mu\rangle + U_{\tau3}^*|\nu_\tau\rangle\right]e^{-i\phi_3}$$

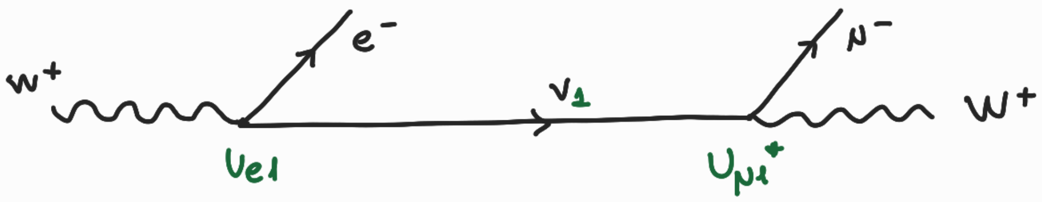
$$|\psi(L)\rangle = \left[U_{e1}U_{e1}^* e^{-i\phi_1} + U_{e2}U_{e2}^* e^{-i\phi_2} + U_{e3}U_{e3}^* e^{-i\phi_3} \right] |\nu_e\rangle$$

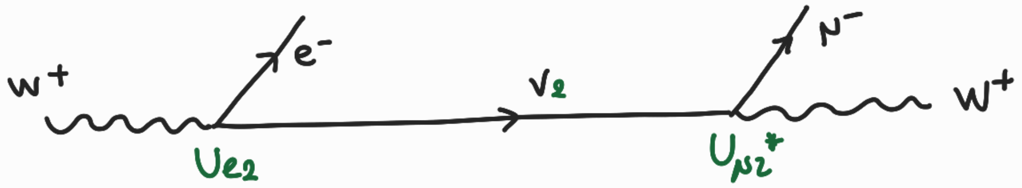
$$+ \left[U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3} \right] |\nu_\mu\rangle$$

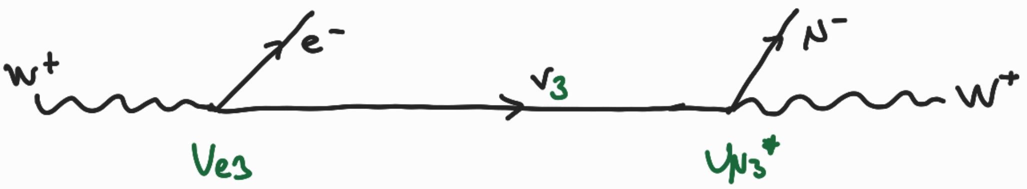
$$+ \left[U_{e1}U_{\tau 1}^* e^{-i\phi} + U_{e2}U_{\tau 2}^* e^{-i\phi_2} + U_{e3}U_{\tau 3}^* e^{-i\phi_1} \right] |\nu_\tau\rangle$$

$$P(\nu_e \to \nu_\mu) = \left| \langle \nu_\mu | \psi(L) \rangle \right|^2$$

$$= \left| \underbrace{U_{e1}U_{\mu 1}^* e^{-i\phi_1}}_{z_1} + \underbrace{U_{e2}U_{\mu 2}^* e^{-i\phi_2}}_{z_2} + \underbrace{U_{e3}U_{\mu 3}^* e^{-i\phi_3}}_{z_3} \right|^2$$







If the phases were all the same $P(\nu_e \to \nu_\mu)$ becomes 0.

Thus neutrino oscillations only occur if the neutrinos have mass and the masses are not the same.

$P(\nu_e \to \nu_\mu)$ can be written as:

$$|z_1+z_2+z_3|^2 \equiv |z_1|^2+|z_2|^2+|z_3|^2 + 2\mathbb{R}(z_1z_2^* + z_1z_3^* + z_2z_3^*)$$

Using the property:

$$|U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^*|^2 = 0$$

$$P(\nu_e \to \nu_\mu) = 2R\{U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2}[e^{-i(\phi_1-\phi_2)}-1]\}$$
$$+2R\{U_{e1}U_{\mu 1}^* U_{e3}^* U_{\mu 3}[e^{-i(\phi_1-\phi_3)}-1]\}$$
$$+2R\{U_{e2}U_{\mu 2}^* U_{e3}^* U_{\mu 3}[e^{-i(\phi_3-\phi_2)}-1]\}$$

$P(\nu_e \to \nu_e) =$ (using $U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^* = 1$)

$$= 1 + 2|U_{e1}|^2|U_{e2}|^2 R\{e^{-i(\phi_1-\phi_2)}-1\}$$
$$+ 2|U_{e1}|^2|U_{e3}|^2 R\{e^{-i(\phi_1-\phi_3)}-1\}$$
$$+ 2|U_{e2}|^2|U_{e3}|^2 \mathbb{R}\{e^{-i(\phi_2-\phi_3)}-1\}$$

where $R\{e^{-i(\phi_1-\phi_2)}-1\} = \cos(\phi_2-\phi_1) - 1 = -2\sin^2\left(\frac{\phi_2-\phi_1}{2}\right)$

$$= -2\sin^2\left(\frac{(m_2^2-m_1^2)L}{4E}\right)$$

Define $\Delta_{21} = \frac{(m_2^2-m_1^2)L}{4E} = \frac{\phi_2-\phi_1}{2}$

$$\boxed{\begin{aligned} P(\nu_e \to \nu_e) = 1 &- 4\,|U_{e1}|^2\,|U_{e2}|^2 \sin^2\Delta_{21} \\ &- 4\,|U_{e1}|^2\,|U_{e3}|^2 \sin^2\Delta_{31} \\ &- 4\,|U_{e2}|^2\,|U_{e3}|^2 \sin^2\Delta_{32} \end{aligned}}$$

* Since we have only three neutrino generations there're only 2 independent mass-squared differences:

$$m_3^2 - m_1^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2)$$

Only 2 of the Δ_{ij} are independent

Converting to practical units:

$$\Delta_{21} = (1.27)\,\frac{\Delta m_{21}^2 (eV^2)\; L(km)}{E\,(GeV)}$$

$$\lambda_{osc}\,(km) = (2.47)\,\frac{E\;(GeV)}{\Delta m^2\,(eV^2)}$$

2.2.1) CP and CPT in the weak interaction

There're 3 important discrete symmetries:

Parity: $\hat{P}$: $\vec{r} \longrightarrow -\vec{r}$

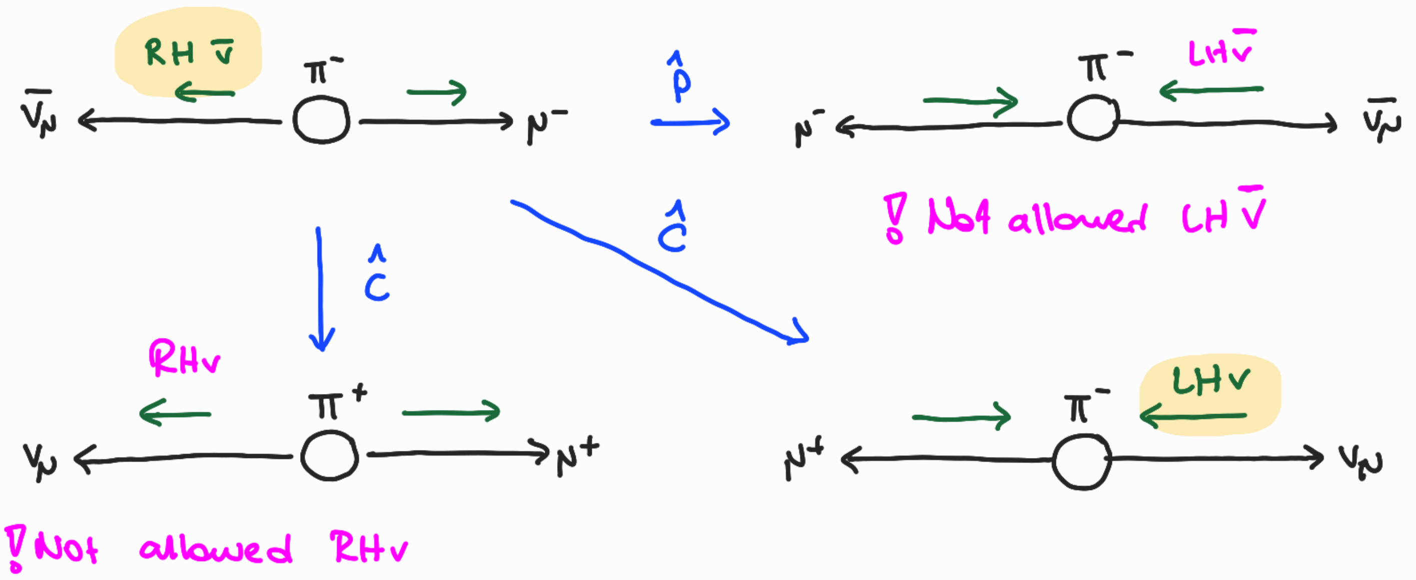
Time Reversal: $\hat{T}$: $t \longrightarrow -t$

Charge Conjugation: $\hat{C}$: Particle $\rightarrow$ Antiparticle

★ The weak interaction violates parity conservation P, but also C.

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

★ Neutrino is ultra-relativistic. And only left handed ν and right handed $\bar{\nu}$ particle.



Hence weak interaction also violates charge conjugation symmetry but appears to be invariant under combined effect of C and P

$\hat{C}\hat{P}$ transforms:

RH particles $\leftrightarrow$ LH antiparticles

LH particles $\leftrightarrow$ RH antiparticles

If the weak interaction were invariant under CP, expect:

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu) = \Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$$ BUT

All local Lorentz-invariant Quantum Field Theories can be shown to be invariant under the combined operation of C, P and T. One consequence of this CPT symmetry:

"Particles and anti-particles have identical masses, magnetic moments, lifetime etc"

However in account for the small excess of matter over antimatter that must have existed early in the universe, require CP violation in particle physics

CP violation can arise in the weak interaction.

CP and T violation in neutrino oscillations

$$P(\nu_e \to \nu_\mu) = 2\mathbb{R}\{U_{e1} U_{\mu 1}^* U_{e2}^* U_{\mu 2} [e^{-i(\Phi_1-\Phi_2)} - 1]\}$$

$$+ 2\mathbb{R}\{U_{e1} U_{\mu 1}^* U_{e3}^* U_{\mu 3} [e^{-i(\Phi_1-\Phi_3)} - 1]\}$$

$$+ 2\mathbb{R}\{U_{e2} U_{\mu 2}^* U_{e3}^* U_{\mu 3} [e^{-i(\Phi_2-\Phi_3)} - 1]\}$$

The oscillation probability for $\nu_\mu \to \nu_e$ can be obtained in the same manner or by simply exchanging the labels $(e) \leftrightarrow (\mu)$:

$$P(\nu_\mu \to \nu_e) = 2\mathbb{R}\{U_{\mu 1} U_{e1}^* U_{\mu 2}^* U_{e2} [e^{-i(\Phi_1-\Phi_2)} - 1]\}$$

$$+ 2\mathbb{R}\{U_{\mu 1} U_{e1}^* U_{\mu 3}^* U_{e3} [e^{-i(\Phi_1-\Phi_3)} - 1]\}$$

$$+ 2\mathbb{R}\{U_{\mu 2} U_{e2}^* U_{\mu 3}^* U_{e3} [e^{-i(\Phi_2-\Phi_3)} - 1]\}$$

* Unless the elements of the PMNS matrix REAL

$$P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$$

$$\nu_e \to \nu_\mu \xrightarrow{\hat{T}} \nu_\mu \to \nu_e$$

$$\nu_e \to \nu_\mu \xrightarrow{\hat{C}\hat{P}} \bar{\nu}_e \to \bar{\nu}_\mu$$

$$\nu_e \to \nu_\mu \xrightarrow{CPT} \bar{\nu}_\mu \to \bar{\nu}_e$$

If the weak interactions are inverient under CPT:

$$P(\nu_e \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_e)$$

$$P(\nu_\mu \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_\mu)$$

If the PMNS matrix is not purely real then

$$P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e) \quad \to T \text{ violation}$$

$$P(\nu_e \to \nu_\mu) \neq P(\bar{\nu}_e \to \bar{\nu}_\mu) \quad \to CP \text{ violation}$$

CP is violated in neutrino oscillations!!

2.3) Neutrino mass hierarchy

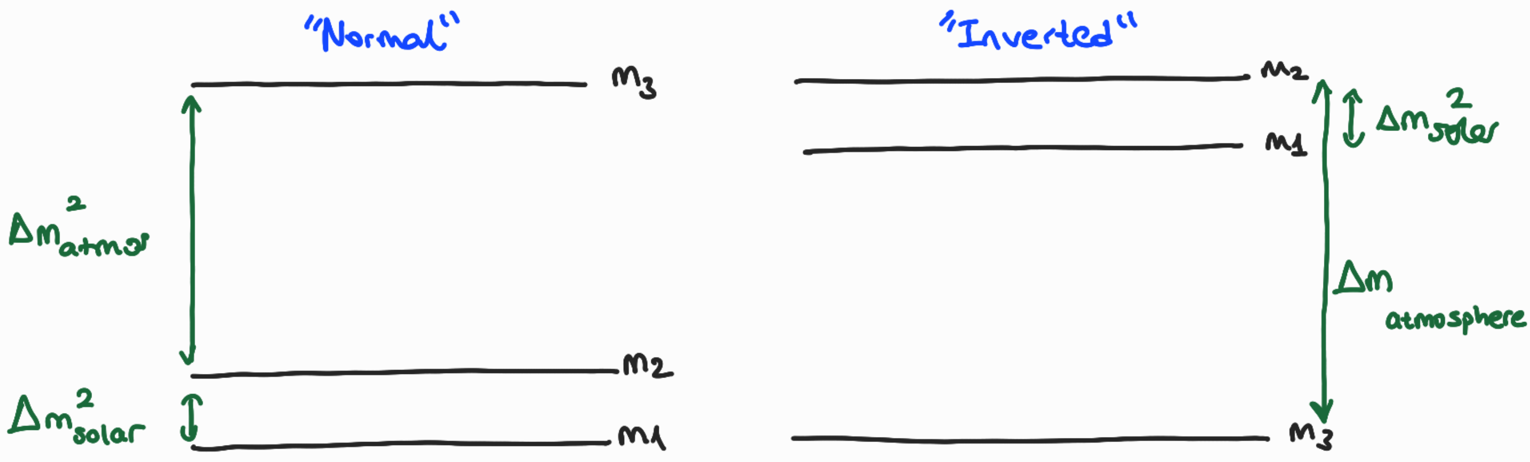
Results on neutrino oscillations only determine:

$$|\Delta m_{ij}^2| = |m_j^2 - m_i^2|$$

There're 2 distinct and very distinct mass scales:

* Atmospheric neutrino oscillations: $|\Delta m^2|_{atmosph.} \sim 2.5 \times 10^{-3}\, eV^2$
* Solar Neutrino Oscillations: $|\Delta m^2|_{solar} \sim 8 \times 10^{-5}\, eV^2$

Two possible assignments of mass hierarchy:



Hence, we can approximate $\Delta m_{31}^2 \approx \Delta m_{32}^2$

3- Flavor Oscillations neglecting CP violation

Neglecting CP violation,

$$P(\nu_e \to \nu_\mu) = -4\, U_{e1} U_{\mu1} U_{e2} U_{\mu2} \sin^2 \Delta_{21}$$

$$-4\, U_{e1} U_{\mu1} U_{e3} U_{\mu3} \sin^2 \Delta_{31}$$

$$-4\, U_{e2} U_{\mu2} U_{e3} U_{\mu3} \sin^2 \Delta_{32}$$

where $\Delta_{ij} = \dfrac{(m_j^2 - m_i^2) L}{4E} = \dfrac{\Delta m_{ij}^2 \, L}{4E}$

Using $\Delta_{31} \approx \Delta_{32}$

$$P(\nu_e \to \nu_\mu) = -4 U_{e1} U_{\mu1} U_{e2} U_{\mu2} \sin^2 \Delta_{21}$$

$$-4 \overbrace{(U_{e1} U_{\mu1} + U_{e2} U_{\mu2})}^{-U_{e3} U_{\mu3}} U_e U_{\mu3} \sin^2 \Delta_{32}$$

using $U_{e1} U_{\mu1}^* + U_{e2} U_{\mu2}^* + U_{e3} U_{\mu3}^* = 0$

$$P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu1}U_{e2}U_{\mu2}\sin^2\Delta_{21} + 4U_{e3}^2U_{\mu3}^2\sin^2\Delta_{32}$$

For ν_e survival, again apply $\Delta_{31} \approx \Delta_{32}$:

$$P(\nu_e \to \nu_e) = 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4U_{e1}^2U_{e3}^2\sin^2\Delta_{31} - 4U_{e2}^2U_{e3}^2\sin^2\Delta_{32}$$

$$\approx 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4(U_{e1}^2+U_{e2}^2)U_{e3}^2\sin^2\Delta_{32}$$

$$P(\nu_e \to \nu_e) \approx 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4(1-U_{e3}^2)U_{e3}^2\sin^2\Delta_{32}$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4U_{\mu1}^2U_{\mu2}^2\sin^2\Delta_{21} - 4(1-U_{\mu3}^2)U_{\mu3}^2\sin^2\Delta_{32}$$

$$P(\nu_\tau \to \nu_\tau) \approx 1 - 4U_{\tau1}^2U_{\tau2}^2\sin^2\Delta_{21} - 4(1-U_{\tau3}^2)U_{\tau3}^2\sin^2\Delta_{32}$$

$$P(\nu_e \to \nu_\mu) = P(\nu_\mu \to \nu_e) \approx -4U_{e1}U_{\mu1}U_{e2}U_{\mu2}\sin^2\Delta_{21} + 4U_{e3}^2U_{\mu3}^2\sin^2\Delta_{32}$$

$$P(\nu_e \to \nu_\tau) = P(\nu_\tau \to \nu_\mu) \approx -4U_{\mu1}U_{\tau1}U_{\mu2}U_{\tau2}\sin^2\Delta_{21} + 4U_{\mu3}^2U_{\tau3}^2\sin^2\Delta_{32}$$

$$\boxed{\lambda_{21} = \frac{4\pi E}{\Delta m_{21}^2}} \qquad \boxed{\lambda_{32} = \frac{4\pi E}{\Delta m_{32}^2}}$$

$\downarrow$ Long Wavelength — SOLAR

$\downarrow$ Short-Wavelength — ATMOSPHERIC

PMNS matrix

PMNS matrix is expressed in terms of 3 rotation angles θ_{12}, θ_{23}, θ_{13} and a complex phase δ, using the notation $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$:

relates to CP violation

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar}}$$

There're 6 SM parameters that can be measured in ν oscillation experiments

$|\Delta m_{21}|^2 = |m_2^2 - m_1^2|$ → Solar & reactor neutrino exp.

$|\Delta m_{32}|^2 = |m_3^2 - m_2^2|$ → Atmos. & beam neutrino exp.

θ_{12} → Solar & reac.

θ_{23} → Atmos & beam

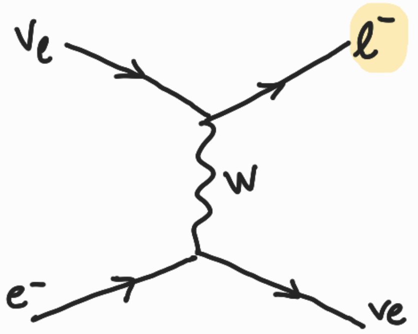
θ_{13} → Reactor neutrino exp + future beam

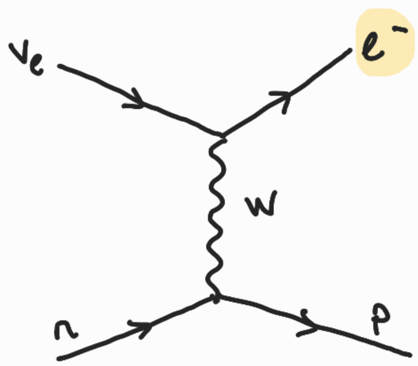
δ → future beam

* Two processes
  - charged current (CC) interactions (via a W boson) → Charged lepton
  - neutral current (NC) interactions (via a Z boson)

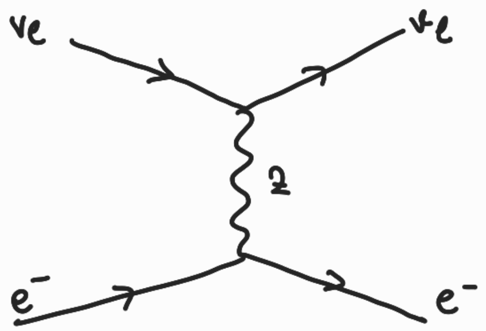
* Two possible targets
  - atomic electrons
  - nucleons within the nucleus

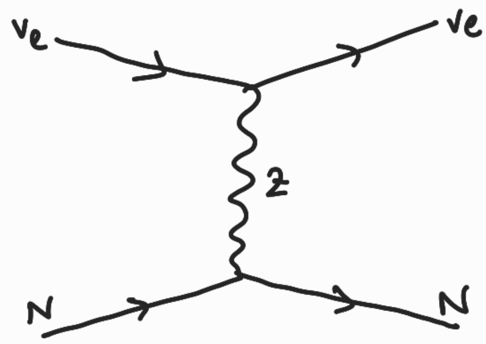
Charged Current





Neutral Current





Neutrino detection method depends on the neutrino energy and (weak) flavour

- From the sun and nuclear reactions $E_\nu \sim 1\,MeV$
- Atmospheric neutrinos have $E_\nu \sim 1\,GeV$
  → From cosmic rays $p \to \pi \to \mu \to \nu_\mu$ (mostly ν_μ in atmosphere)

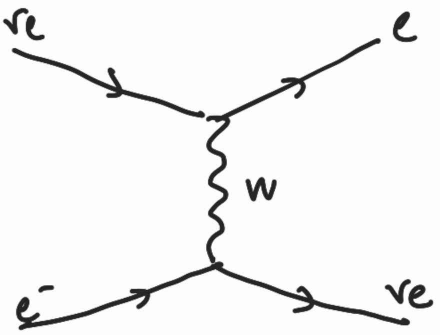
These energies are relatively low, not all interactions are allowed. There's a threshold energy

Neutrino interaction thresholds

1) Charged Current interactions on atomic electrons (lab frame)

$p_\nu = (E_\nu, 0, 0, E_\nu)$

$p_e = (m_e, 0, 0, 0)$



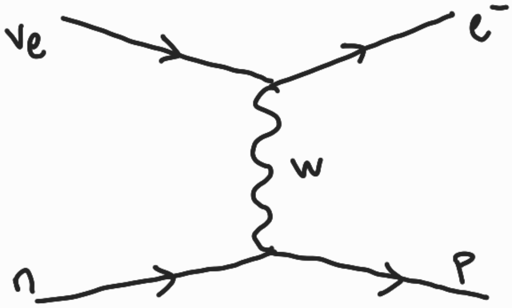
$s = (p_\nu + p_e)^2 = (E_\nu + m_e)^2 - E_\nu^2$

Require $s > M_e^2$

$$E_\nu > \left[\left(\frac{M_e}{m_e}\right)^2 - 1\right]\frac{M_e}{2}$$

Thus $E_{\nu_e} > 0$, $E_{\nu_\mu} > 11\,\text{GeV}$, $E_{\nu_\tau} > 3090\,\text{GeV}$ → too high

2) CC interaction on nucleons (lab frame)



$s = (p_\nu + p_n)^2 = (E_\nu + m_n)^2 - E_\nu^2$

Require $s > (M_e + M_p)^2$

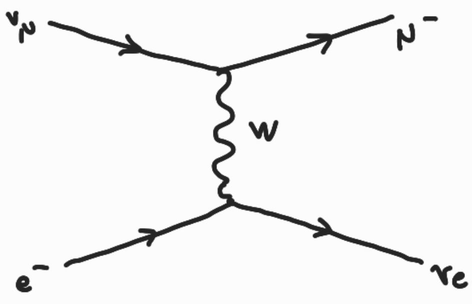
$$E_\nu > \frac{(m_p^2 - m_n^2) + M_e^2 + 2M_p M_e}{2m_n}$$

$E_{\nu_e} > 0$, $E_{\nu_\mu} > 110\,\text{MeV}$, $E_{\nu_\tau} > 3.5\,\text{GeV}$

ν_e from the sun and nuclear reactors ($E_\nu \sim 1\,\text{MeV}$) which oscillate into ν_μ and ν_τ can NOT interact via charged current interactions (DISAPPEAR)

Atmospheric ν_μ ($E_\nu \sim 1\,\text{GeV}$) which oscillates into ν_τ can NOT interact via charged current interactions (DISAPPEAR)

Signature for neutrino oscillation is DEFICIT of neutrino interactions



$$\sigma_{\nu_\mu, e^-} = \frac{G_F^2 s}{\pi}$$

from previous chapter

with $s = (E_\nu + M_e)^2 - E_\nu^2 \approx 2 M_e E_\nu$

$$\boxed{\sigma_{\nu_\mu, e^-} = \frac{2 m_e G_F^2 E_\nu}{\pi}}$$

cross section increases linearly with lab frame neutrino energy