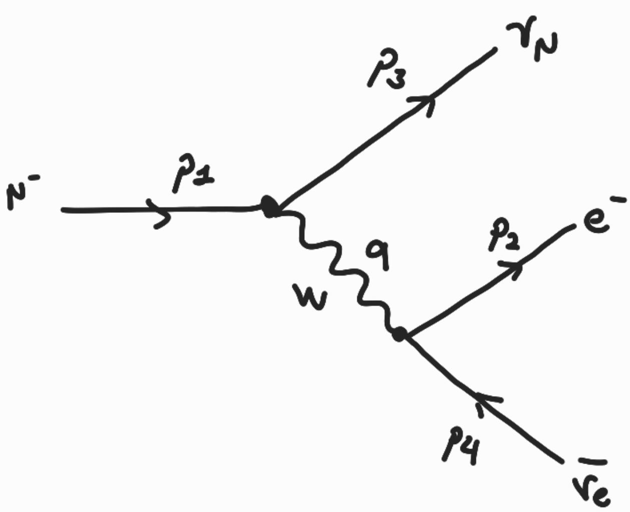
THE WEAK INTERACTIONS OF LEPTONS

1. Lepton Universality

2. Neutrino Scattering

1. Lepton Universality

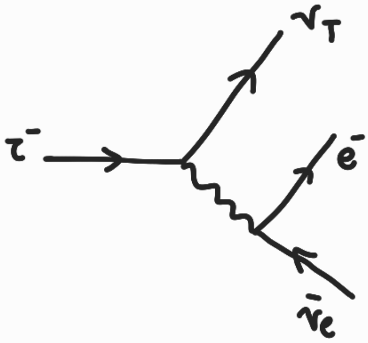


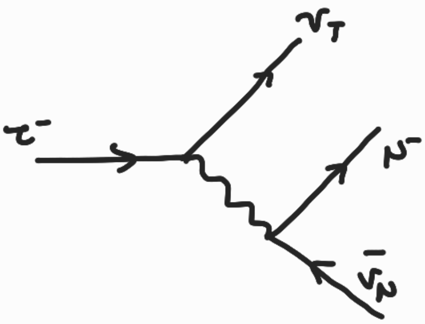
Muon decay rate

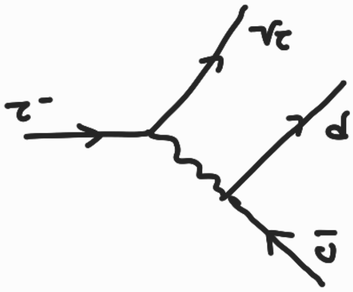
$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{1}{\tau_\mu}$$

$$= \frac{G_F^{(e)} G_F^{(\mu)} m_\mu^5}{192\pi^3}$$

from previous chapter







$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \frac{G_F^{(e)} G_F^{(\tau)} m_\tau^5}{192\pi^3}$$

Tau lepton is sufficiently massive that it can also decay into a muon or to mesons. Thus tau life-time needs to expressed in terms of the total decay rate.

$$\frac{1}{\tau_\tau} = \Gamma = \sum_i \Gamma_i$$

$$Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \frac{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\Gamma} = \Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \times \tau_\tau$$

branching ratio

$$\tau_\tau = \frac{192\pi^3}{G_F^{(e)} G_F^{(\tau)} m_\tau^5} Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$$

0.1783

$$\frac{G_F^{(\tau)}}{G_F^{(\mu)}} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$$

Measured mass & lifetimes

$m_\mu = 0.105..$ GeV $\qquad \tau_\mu = 2.19.... \times 10^{-6}$ s

$m_\tau = 1.77682$ GeV $\qquad \tau_\tau = 0.29...\times 10^{-12}$ s

Using these measurements

$$\frac{G_F^{(\tau)}}{G_F^{(\mu)}} = 1.0023 \pm 0.0..3$$

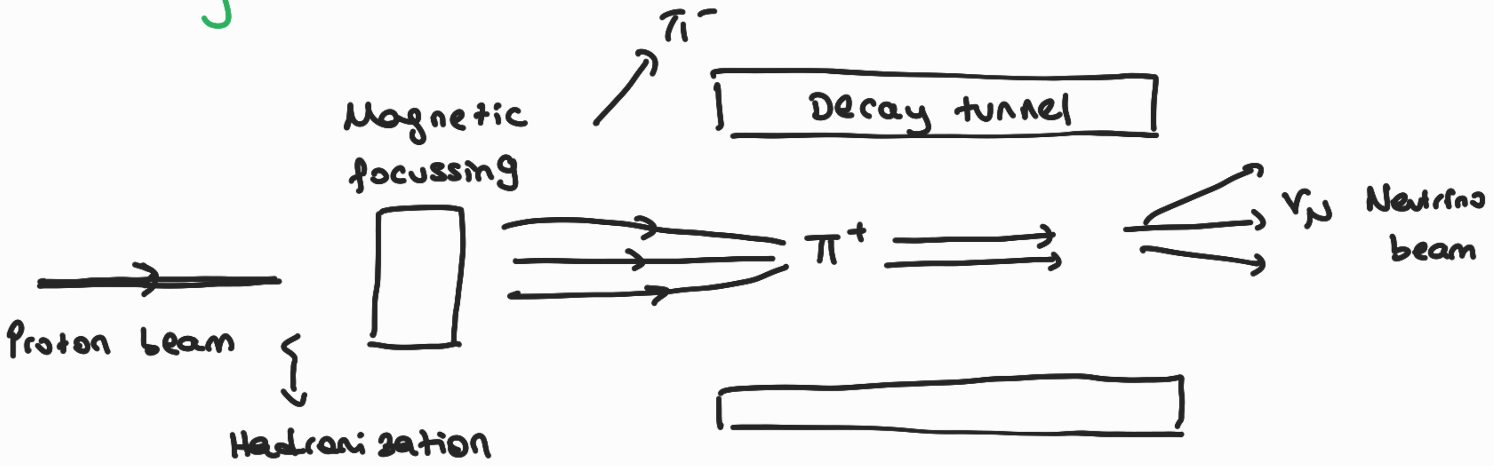
Similarly, $$\frac{G_F^{(e)}}{G_F^{(\mu)}} = 1.000 \pm 0.004$$

★ Thus, within the accuracy of the experimental measurements, it can be concluded that $G_F^{(e)} = G_F^{(\mu)} = G_F^{(\tau)}$

There's a universal coupling strength at the $We\nu_e$, $W\mu\nu_\mu$, $W\tau\nu_\tau$ interaction vertices.

2. Neutrino Scattering

Producing neutrino

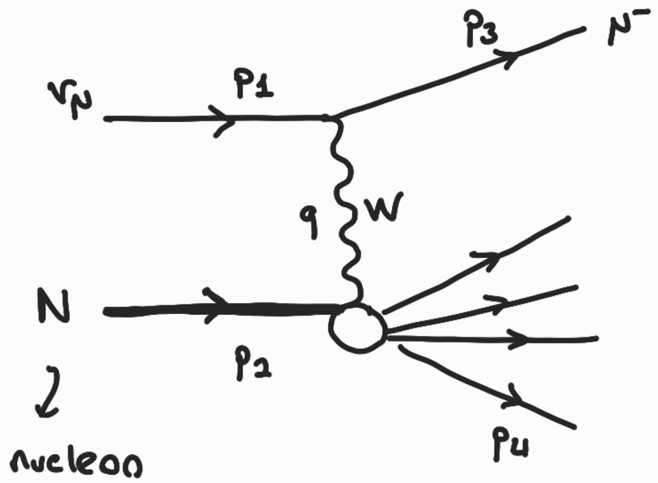


$$\pi^+ \longrightarrow \mu^+ \nu_\mu$$

$$\pi^- \longrightarrow \mu^- \bar{\nu}_\mu$$

to obtain $\bar{\nu}_\mu$ beam magnetic focus is arranged so π^- is focused.

The neutrino energy is sufficiently high that "only" the deep inelastic



For a neutrino interacting with a nucleon at rest,

$$s = (p_1 + p_2)^2 = (E_\nu + m_N)^2 - E_\nu^2 = 2M_N E_\nu + M_N^2$$

$$p_1^\mu p_{1\mu} + p_2^\mu p_{2\mu} + 2p_1^\mu p_{2\mu} = \cancel{E_\nu^2} - \cancel{E_\nu^2} + M_N^2 + 2M_N E_\nu$$

Neutrino

$p_1 = (E_\nu, 0, 0, E_\nu)$

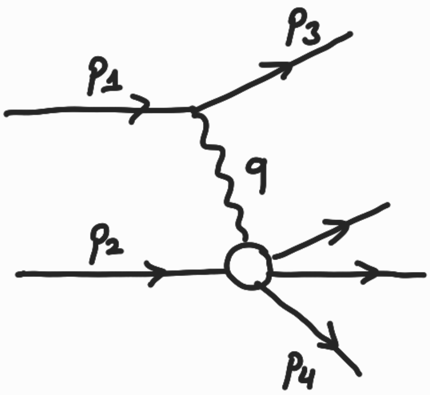
$p_2 = (M_N, 0, 0, 0)$

Nucleon

found in Ch.8

$$Q^2 = -q^2 = (s - m_N^2)\,xy = 2m_N E_\nu xy$$

Mini Review



$$(p_4)^2 = (q + p_2)^2 = q^2 + 2p_2 \cdot q + p_2^2$$

$$(p_4)^2 \overbrace{-q^2}^{Q^2} - \overbrace{p_2^2}^{m_p^2} = 2p_2 \cdot q$$

$$(p_4)^2 + Q^2 - m_p^2 = 2p_2 \cdot q$$

$$0 \leq y = \frac{p_2 \cdot q}{p_2 \cdot p_1} \leq 1$$

where Bjorken (Elasticity) $x = \dfrac{Q^2}{2p_2 \cdot q}$

$$0 \leq x = \frac{Q^2}{(p_4)^2 + Q^2 - M_p^2} \leq 1$$

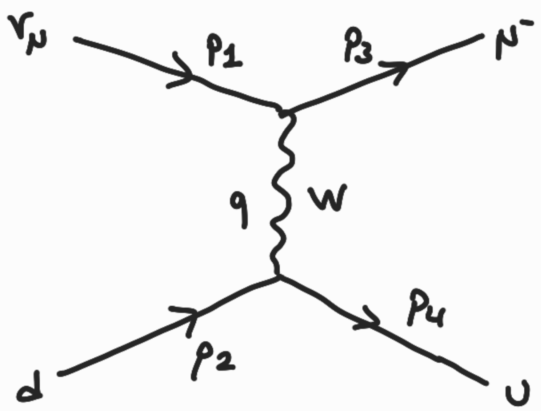
$$\boxed{Q^2 = (s - m_N^2)\,xy}$$

$$Q^2 = (s - m_N^2)\, xy = 2 m_N E_\nu\, xy$$

$$Q^2 \leq 2 m_N E_\nu$$

The underlying interactions in neutrino-nucleon scattering are the parton-level processes:

$$\nu_\mu d \longrightarrow \mu^- u \qquad \& \qquad \nu_\mu \bar{u} \longrightarrow \mu^- \bar{d}$$



2.1 Neutrino-Quark Scattering

$$-i\mathcal{M}_{fi} = \left[-i \frac{g_W}{\sqrt{2}} \bar{u}(p_3) \gamma^\mu \frac{1}{2}(1-\gamma^5) u(p_1) \right] \frac{i g_{\mu\nu}}{m_W^2} \left[-i \frac{g_W}{\sqrt{2}} \bar{u}(p_4) \gamma^\nu \frac{1}{2}(1-\gamma^5) u(p_2) \right]$$

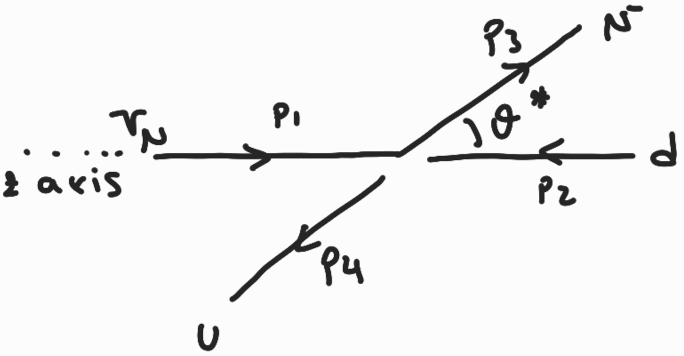
$$\mathcal{M}_{fi} = \frac{g_W^2}{2 m_W^2} g_{\mu\nu} \left[\bar{u}(p_3) \gamma^\mu \frac{1}{2}(1-\gamma^5) u(p_1) \right] \left[\bar{u}(p_4) \gamma^\nu \frac{1}{2}(1-\gamma^5) u(p_2) \right]$$

* For high-energy neutrino scattering, both the masses of the neutrinos and quarks are sufficiently small that LH chiral state = LH helicity state.

$$\mathcal{M}_{fi} = \frac{g_W^2}{2m_W^2}\, g_{\mu\nu} \underbrace{\left[\bar{u}_{\downarrow}(p_3)\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)u_{\downarrow}(p_1)\right]}_{j_\ell^{\mu}} \underbrace{\left[\bar{u}_{\downarrow}(p_4)\gamma^{\nu}\tfrac{1}{2}(1-\gamma^5)u_{\downarrow}(p_2)\right]}_{j_q^{\nu}}$$

(lepton -current) (quark current)

let's evaluate the matrix element in the center-of-mass-frame

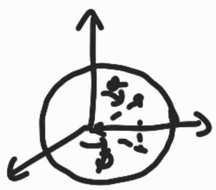


$(\theta_1, \phi_1) = (0,0)$

$(\theta_2, \phi_2) = (\pi, \pi)$

$(\theta_3, \phi_3) = (\theta^*, 0)$

$(\theta_4, \phi_4) = (\pi-\theta^*, \pi)$



$$u_{\downarrow}(p) = \sqrt{E+m}\begin{pmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2}e^{i\phi} \\ \frac{p}{E+m}\sin\frac{\theta}{2} \\ -\frac{p}{E+m}\cos\frac{\theta}{2}e^{i\phi}\end{pmatrix}$$

$$u_{\downarrow}(p_1) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} \quad u_{\downarrow}(p_2) = \sqrt{E}\begin{pmatrix}-1\\0\\1\\0\end{pmatrix} \quad u_{\downarrow}(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix} \quad u_{\downarrow}(p_4) = \sqrt{E}\begin{pmatrix}-c\\-s\\c\\s\end{pmatrix}$$

$E \gg m \quad E \approx p$

from Ch.6 $\curvearrowright \bar{\psi}\gamma^{\mu}\phi = \psi^{\dagger}\gamma^{0}\gamma^{\mu}\phi$

$$j_\ell^{\mu} = \bar{u}_{\downarrow}(p_3)\gamma^{\mu}u_{\downarrow}(p_1) = 2E(c, s, -is, c)$$

$$j_q^{\nu} = \bar{u}_{\downarrow}(p_4)\gamma^{\nu}u_{\downarrow}(p_2) = 2E(c, -s, -is, -c)$$

$$\downarrow$$

$$\mathcal{M}_{fi} = \frac{g_w^2}{2m_w^2} j_e \cdot j_q = \frac{g_w^2}{\cancel{2}m_w^2} 4E^2(\underbrace{c^2 + s^2 + s^2 + c^2}_{\cancel{2}})$$

$$\mathcal{M}_{fi} = \frac{g_w^2}{m_w^2} 4E^2 = \frac{g_w^2}{m_w^2} \hat{s}$$

centre-of-mass-energy of $\nu_\mu d$

In the limit where particle masses can be neglected, the centre of mass frame differential cross section is given by

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |\mathcal{M}_{fi}|^2 \rangle$$

spin-averaged matrix element

Previously, the average over the spins of the two initial-state fermions gave rise to a factor 1/4 in $\langle \mathcal{M}_{fi} \rangle^2$

$$|\langle \mathcal{M}_{fi} \rangle|^2 = \frac{1}{4} \left(|\mathcal{M}_{RR}|^2 + |\mathcal{M}_{RL}|^2 + |\mathcal{M}_{LR}|^2 + |\mathcal{M}_{LL}|^2 \right)$$

☆ The neutrino will always be left-handed since it is produced in a weak decay, it is enough to average over the two spin states of the quark.

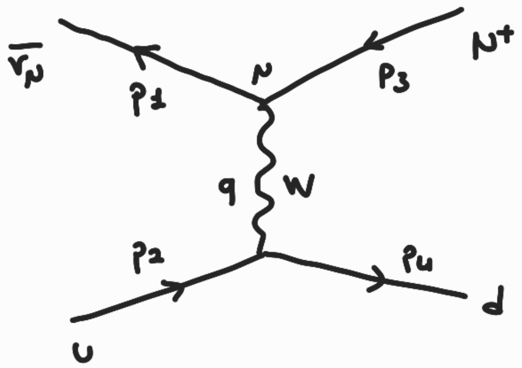
$$\langle |\mathcal{M}_{fi}| \rangle^2 = \frac{1}{2} \left(\frac{g_w^2}{m_w^2} \hat{s} \right)^2$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |\mathcal{M}_{fi}|^2 \rangle = \left(\frac{g_w^2}{8\sqrt{2}\, m_w^2} \right)^2 \hat{s} = \frac{G_F^2}{4\pi^2} \hat{s}$$

Total cross section, by integrating over $d\Omega^*$

$$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}$$

2.2 Antineutrino-quark scattering



$$\bar{\nu}_\mu u \longrightarrow \mu^+ d$$

$$\mathcal{M}_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{v}(p_1) \gamma^\mu \frac{1}{2}(1-\gamma^5) v(p_3) \right] \left[\bar{u}(p_4) \gamma^\nu \frac{1}{2}(1-\gamma^5) u(p_2) \right]$$

In the high-energy limit, only LH helicity particles and RH helicity antiparticles participate in the charged-current weak interaction. Thus only non-zero Matrix element is:

$$\mathcal{M}_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{v}_\uparrow (p_1) \gamma^\mu v_\uparrow(p_3) \right] \left[\bar{u}_\downarrow(p_4) \gamma^\nu u_\downarrow(p_2) \right]$$

$$M_{\bar{\nu}q} = \frac{1}{2}(1+\cos\theta^*) \frac{g_W^2}{m_W^2} \hat{s}$$

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{1}{4}(1+\cos\theta^*)^2 \frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2 \hat{s}$$

where $$\int (1+\cos\theta^*)^2 d\Omega^* = \int_0^{2\pi} d\phi^* \int_{-1}^{+1} (1+x)^2 dx = \frac{16\pi}{3}$$

$x = \cos\theta^*$

Thus $$\sigma_{\bar{\nu}q} = \frac{\hat{G}_F^2}{3\pi}$$

total antineutrino-quark cross section

$$\frac{\sigma_{\bar{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}$$