Particle Physics II
Lecture 9: Tests of the Standard Model

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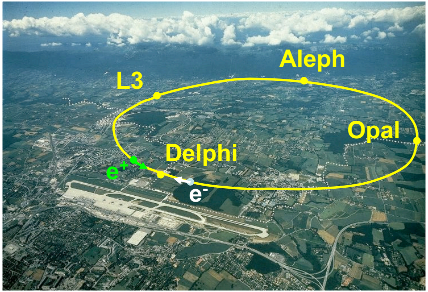
Benchmark tests of the SM consistency

- Z line shape measurement
- Z, W and WW production cross sections at lepton colliders
- number of light neutrino generation from Z decays
- forward-backward asymmetries for Z decays
- W mass and width measurements
- weak angle θ_W measurement
- mass of the top quark
- Higgs boson mass (next lectures)

Electroweak Measurements at LEP

- the Large Electron Positron (LEP) Collider at CERN (1989–2000) was designed to make precise measurements of the properties of the Z and W bosons



- 26 km circumference accelerator over French/Swiss border
- e^+ and e^- collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):

  ALEPH, DELPHI, L3, OPAL

On-shell ?

Basically a large Z and W factory:

- 1989 – 1995: e^+e^- collisions at $\sqrt{s} = 91.2$ GeV
  17M Z bosons detected
- 1996 – 2000: e^+e^- collisions at $\sqrt{s} = 161 - 208$ GeV
  30k W^+W^- events detected

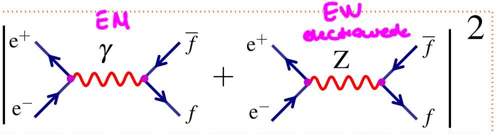
2 × mass(W)

Z + H

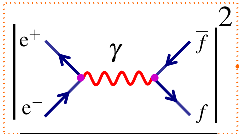
91 125 ≈ 216 GeV

e^+e^- annihilations in Feynman diagrams

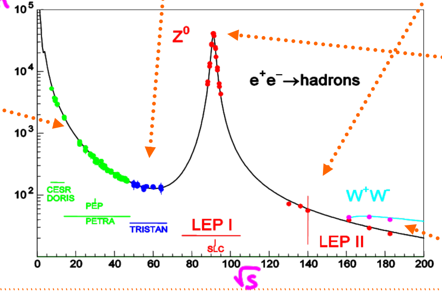
In general e^+e^- annihilation involves both photon and Z exchange : + interference

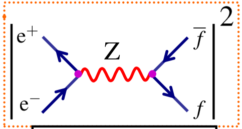


Cross Section



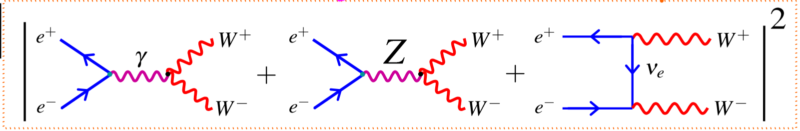
Well below Z: photon exchange dominant





At Z resonance: Z exchange dominant

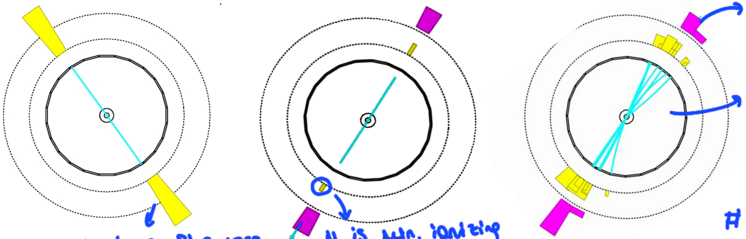
High energies: WW production



Cross section measurements

- at Z resonance mainly observe four types of events: $e^+e^- \to Z \to e^+e^-$, $e^+e^- \to Z \to \mu^+\mu^-$, $e^+e^- \to Z \to \tau^+\tau^-$, $e^+e^- \to Z \to q\bar{q} \to$ hadrons
- each event has a distinct topology in the detectors, e.g.:

$\mathrm{e^+e^-} \to \mathrm{Z} \to \mathrm{e^+e^-}$ $\qquad$ $\mathrm{e^+e^-} \to \mathrm{Z} \to \mu^+\mu^-$ $\qquad$ $\mathrm{e^+e^-} \to \mathrm{Z} \to$ hadrons



# of particles passing
through a given time period

- to work out cross sections, first count events of each type
- then need to know "integrated luminosity" L of colliding beams, i.e. the relation between cross section σ and expected number of interactions N_{events}:

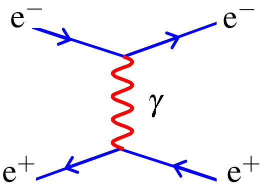
$$\boxed{N_{\text{events}} = \mathcal{L}\sigma}$$

Cross section measurements

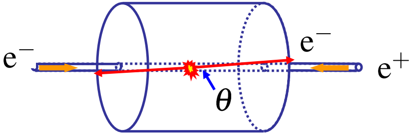
- to calculate the integrated luminosity need to know numbers of electrons and positrons in the colliding beams and the exact beam profile:

  - very difficult to achieve with precision of better than 10%

- instead “normalize” using another type of events:



- use the QED Bhabha scattering process
- QED: cross section can be calculated very precisely
- very large cross section (large N): small statistical uncertainty
- reaction is very forward peaked, i.e. the electron tends not to get deflected much



$$\frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \propto \frac{1}{\sin^4 \theta/2} \Rightarrow \boxed{\frac{d\sigma}{d\theta} \propto \frac{1}{\theta^3}}$$

Photon propagator

Cross section measurements

- count events where the electron is scattered in the very forward direction:

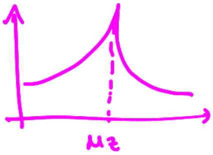
$$N_{\text{Bhabha}} = \mathcal{L}\sigma_{\text{Bhabha}} \implies \mathcal{L}, \sigma_{\text{Bhabha}} \text{ known from QED calculations}$$

- hence all other cross sections can be expressed as:

$$\boxed{\sigma_i = \frac{N_i}{N_{\text{Bhabha}}}\sigma_{\text{Bhabha}}}$$

Cross section measurements involve just event counting!

Measurements of the Z line shape



- measurements of the Z resonance line shape determine:
  - m_Z: peak position of the resonance
  - Γ_Z: FWHM of resonance
  - Γ_f: partial decay widths (to given fermions f)
  - N_ν: number of light neutrino generations (no signal ?)
- measure cross sections to different final states versus C.o.M. energy $\sqrt{s}$

indirect measurement

- starting from (derivation is in the *Bonus slides* at the end):

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2} \frac{s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \Gamma_{ee}\Gamma_{ff}$$

maximum cross section occurs at $\sqrt{s} = m_Z$ with peak cross section equal to:

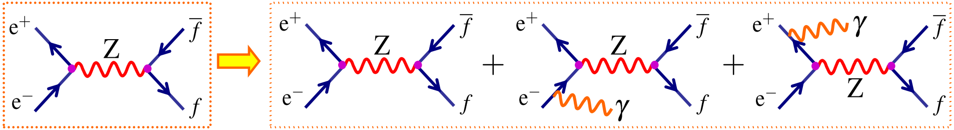
$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

- cross section falls to half peak value at $\sqrt{s}_Z \pm \frac{\Gamma_Z}{2}$
- hence $\Gamma_Z = \frac{\hbar}{\tau_Z}$ = FWHM of resonance

full width half maximum

Measurements of the Z line shape: ISR

- in practice, it is not that simple: QED corrections distort the measured line shape
- one particularly important correction: initial state radiation (ISR)



- ISR reduces the center-of-mass energy of the e^+e^- collision:

$$\mathrm{e}^+ \xrightarrow{E} \quad \xleftarrow{E} \mathrm{e}^- \qquad \sqrt{s} = 2E$$

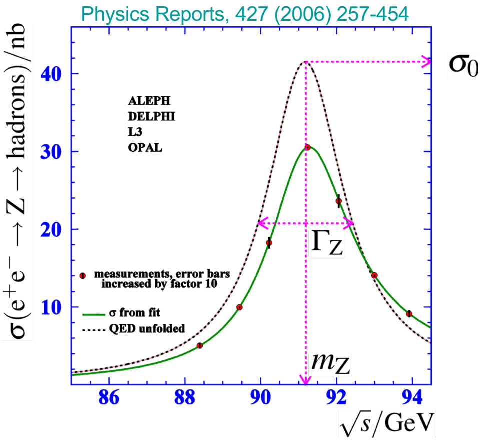
becomes

$$\xrightarrow{E} \quad \xleftarrow{E - E_\gamma} \qquad \sqrt{s'} \approx 2E\left(1 - \frac{E_\gamma}{2E}\right)$$

Measurements of the Z line shape: ISR

- measured cross section can be written:
  $\sigma_{\text{meas}}(E) = \int \sigma(E')f(E', E)dE'$

  f: probability of e^+e^- colliding with C.o.M. energy E' when C.o.M. energy before ISR is E
- fortunately, can calculate $f(E', E)$ very precisely, just QED, and then obtain Z line shape from measured cross section



Measurements of the Z line shape

- in principle the measurement of m_Z and Γ_Z is rather simple: run accelerator at different energies, measure cross sections, account for ISR, then find peak and FWHM:

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV} \qquad \Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

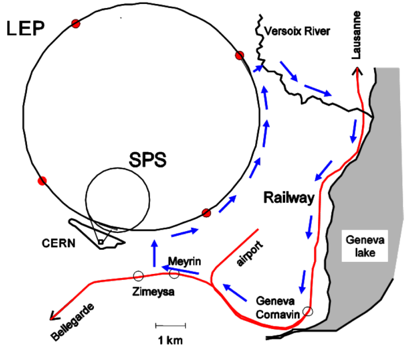
- 0.002% measurement of m_Z!
- to achieve this level of precision need to know energy of the colliding beams to better than 0.002%: sensitive to unusual systematic effects

Measurements of the Z line shape: Moon

* as the moon orbits the Earth it distorts the rock in Geneva area very slightly
* the nominal radius of the accelerator of 4.3 km varies by ± 0.15 mm
* changes beam energy by ~ 10 MeV: need to correct for tidal effects!

Measurements of the Z line shape: Trains

- leakage currents from the TGV railway line return to Earth following the path of least resistance
- traveling via the Versoix river and using the LEP ring as a conductor
- each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changed by $\sim$10 MeV



Number of generations

- total decay width measured from Z line shape: $\Gamma_Z = 2.4952 \pm 0.0023$ GeV
- if there were an additional 4$^{\text{th}}$ generation would expect $Z \to \nu_4 \bar{\nu}_4$ decays even if the charged leptons and fermions were too heavy (i.e. $> m_Z/2$)
- total decay width is the sum of the partial widths:

$$\boxed{\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?}$$

- although don't observe neutrinos, $Z \to \nu\bar{\nu}$ decays affect the Z resonance shape for **all** final states

Number of generations

- for all other final states can determine partial decay widths from peak cross sections:

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

- assuming lepton universality:

$$\boxed{\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + N_\nu \Gamma_{\nu\nu}}$$

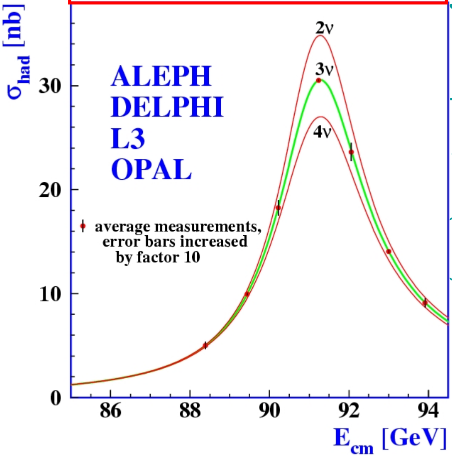
Γ_Z: measured from Z line shape

$\Gamma_{\ell\ell}, \Gamma_{\text{hadrons}}$: measured from peak cross sections

$\Gamma_{\nu\nu}$: calculated from the SM

$$\Longrightarrow \boxed{N_\nu = 2.9840 \pm 0.0082}$$

$e^+e^- \rightarrow Z \rightarrow$ hadrons



Physics Reports, 427 (2006) 257-454

Only 3 generations: unless a new 4th generation neutrino has a very large mass

Number of generations: 30 years later

- a very recent talk at one of the most important conferences in the field: 55th Rencontres de Moriond
- the slides are at the link: Light neutrinos families update
- based on two papers: arXiv:1908.01704 and arXiv:1912.02067 about two main ingredients of the N_ν measurement

Beam-beam effects on the luminosity measurement at LEP and the number of light neutrino species

Marking the 30th anniversary of the first Z detected at LEP on 13 August 1989

and

Improved Bhabha cross section at LEP and the number of light neutrino species

Marking the 30th anniversary of the first N_ν determination at LEP on 13 October 1989

Number of generations: What was the question?

$$N_\nu = 2.9840 \pm 0.0082$$ why not 3?

Why the 2σ deficit?

Number of light neutrino families is 3

But we observe a 2σ deficit

- Is it a simply a statistics fluke?
- Or is it due to some new physics?
- Result invited some theoretical speculation, e.g.
  - Heavy neutrinos (Phys. Rev. D 67,073012 (2003))
  - Extra dimensions (Nucl. Phys. B 623, 395 (2002))

Number of generations: What has changed?

- better estimation of the integrated luminosity $\mathcal{L}$:
  - enters in $\sigma^0_{f\bar{f}} = N_{f\bar{f}}/\mathcal{L}$ and impacts Γ_{hadrons}, $\Gamma_{\ell\ell}$
  - took into account beam-beam interaction effects
  - $\implies$ increased integrated $\mathcal{L}$
- better estimation of the Bhabha cross section (σ_{Bhabha}):
  - higher order correction for the Z exchange contribution (used only tree level at LEP times)
  - vacuum polarisation (corrections to the photon propagator)
  - full inclusion of light fermion pairs production (i.e. $e^+e^- \to e^+e^- f\bar{f}$, where f are undetected)
  - lower $\sigma_{\text{Bhabha}} \implies$ increased integrated $\mathcal{L}$
- two effects shift the measurement in the same direction

Conclusions

After including the corrections due to the beam-beam effects and the update theoretical calculation of the Bhabha cross section

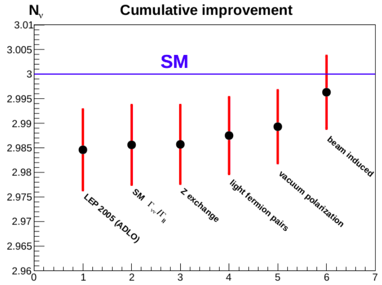
N_ν : from $N_\nu = 2.9840 \pm 0.0082$ to $N_\nu = 2.9963 \pm 0.0074$

- The 2σ deficit is gone

σ^0_{had}: from $\sigma^0_{had} = 41.540 \pm 0.037$ to $\sigma^0_{had} = 41.4737 \pm 0.0326$

Γ_z: from $\Gamma_z = 2.4952 \pm 0.0023$ GeV to $\Gamma_z = 2.4955 \pm 0.0023$ GeV

Theoretical uncertainty in L measurement decreased from 0.061% → 0.037%



No other EW observables were found to be affected

Forward-backward asymmetry

- in the *Bonus slides* there is a derivation of the expression for the differential cross section for $e^+e^- \to Z \to \mu^+\mu^-$:

$$\left\langle |M_{fi}|^2 \right\rangle \propto \left[(c_R^e)^2 + (c_L^e)^2\right] \left[\left(c_R^\mu\right)^2 + \left(c_L^\mu\right)^2\right] \left(1 + \cos^2\theta\right) \tag{1}$$

$$+ 2\left[(c_R^e)^2 - (c_L^e)^2\right] \left[\left(c_R^\mu\right)^2 - \left(c_L^\mu\right)^2\right] \cos\theta \tag{2}$$

where c_R^e is a number giving a coupling of the Z to the RH electron etc (see Lecture 8)

- the differential cross section is therefore of the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \kappa \times \left[A\left(1 + \cos^2\theta\right) + 2B\cos\theta\right] \tag{3}$$

$$A = \left[(c_R^e)^2 + (c_L^e)^2\right] \left[\left(c_R^\mu\right)^2 + \left(c_L^\mu\right)^2\right] \tag{4}$$

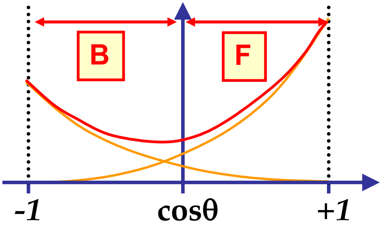
$$B = \left[(c_R^e)^2 - (c_L^e)^2\right] \left[\left(c_R^\mu\right)^2 - \left(c_L^\mu\right)^2\right] \tag{5}$$

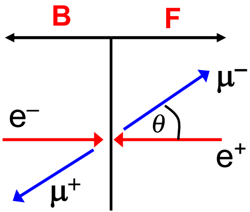
R and L are different

Forward-backward asymmetry

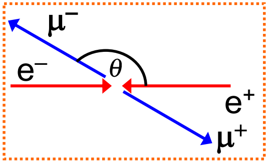
- define the forward and backward cross sections in terms of angle between incoming electron and outgoing particle:

$$\sigma_F \equiv \int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\,\mathrm{d}\cos\theta \qquad \sigma_B \equiv \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\,\mathrm{d}\cos\theta$$

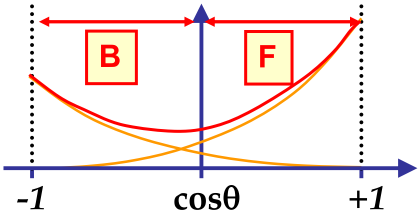




e.g. “backward hemisphere”



Forward-backward asymmetry



- the level of asymmetry about $\cos\theta = 0$ is expressed in terms of the forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

- integrating Eq. 3:

$$\sigma_F = \kappa \int_0^1 \left[A(1+\cos^2\theta) + 2B\cos\theta\right] \mathrm{d}\cos\theta = \kappa\left(\frac{4}{3}A + B\right)$$

$$\sigma_B = \kappa \int_{-1}^0 \left[A(1+\cos^2\theta) + 2B\cos\theta\right] \mathrm{d}\cos\theta = \kappa\left(\frac{4}{3}A - B\right)$$

- which gives:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3B}{4A} = \frac{3}{4}\left[\frac{\left(c_L^e\right)^2 - \left(c_R^e\right)^2}{\left(c_L^e\right)^2 + \left(c_R^e\right)^2}\right]\left[\frac{\left(c_L^\mu\right)^2 - \left(c_R^\mu\right)^2}{\left(c_L^\mu\right)^2 + \left(c_R^\mu\right)^2}\right]$$

Forward-backward asymmetry

- this can be written as:

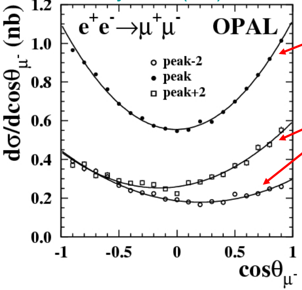
$$A_{FB} = \frac{3}{4} A_e A_\mu \text{ with } A_f = \frac{\left(c_L^f\right)^2 - \left(c_R^f\right)^2}{\left(c_L^f\right)^2 + \left(c_R^f\right)^2} = \frac{2c_V^f c_A^f}{\left(c_V^f\right)^2 + \left(c_A^f\right)^2}$$

- observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB (forward-backward) symmetric

Measured forward-backward asymmetries

- forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g $e^+e^- \to Z \to \mu^+\mu^-$

OPAL Collaboration,
Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_W \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \to \gamma \to \mu^+\mu^-$ leads to a larger asymmetry

★LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$

$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$

$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

In principal, they should be same

- to relate these measurements to the coupling use $A_{FB} = \frac{3}{4}A_e A_\mu$
- in all cases asymmetries depend on A_e
- to obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4}A_e^2$

Determination of the weak mixing angle θ_W

- from LEP: $A_{FB}^{0,f} = \frac{3}{4} A_e A_f$
- from SLC: $A_{LR} = A_e$
- putting everything together (including results from other measurements):

  $A_e = 0.1514 \pm 0.0019$

  $A_\mu = 0.1456 \pm 0.0091$

  $A_\tau = 0.1449 \pm 0.0040$

  with $A_f \equiv \dfrac{2 c_V^f c_A^f}{\left(c_V^f\right)^2 + \left(c_A^f\right)^2} = 2 \dfrac{c_V / c_A}{1 + (c_V / c_A)^2}$

Determination of the weak mixing angle θ_W

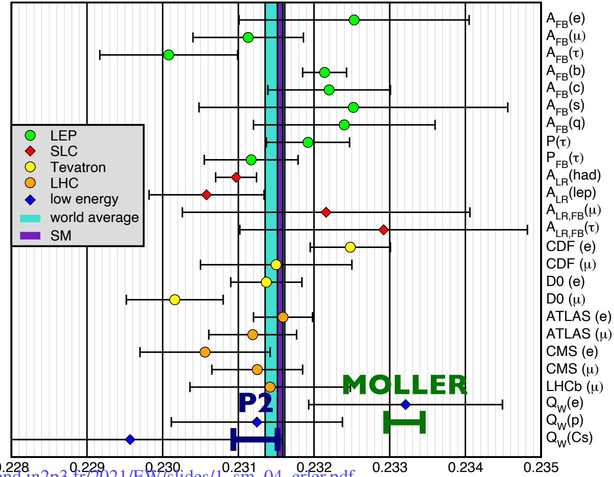
- measured asymmetries give ratio of vector (c_V) to axial-vector (c_A) Z couplings
- in SM these are related to the weak mixing angle:

$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q\sin^2\theta_W}{I_W^3} = 1 - \frac{2Q}{I_W^3}\sin^2\theta_W = 1 - 4|Q|\sin^2\theta_W$$

- asymmetry measurements give precise determination of $\sin^2\theta_W$:

$$\sin^2\theta_W = 0.23154 \pm 0.00016$$

Weak mixing angle measurements



LEP & SLC:

0.23151 ± 0.00016

Tevatron:

0.23148 ± 0.00033

LHC:

0.23129 ± 0.00033

average direct

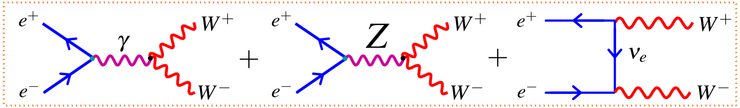
0.23148 ± 0.00013

global fit

0.23153 ± 0.00004

http://moriond.in2p3.fr/2021/EW/slides/1_sm_04_erler.pdf

- in 1995–2000 LEP operated above the threshold for W-pair production
- three diagrams are involved:

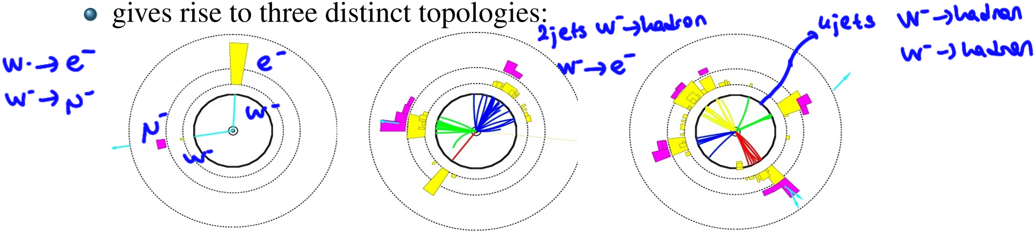


- W bosons decay either to leptons or hadrons with branching fractions:

$$\mathcal{B}(W^- \to \text{hadrons}) \approx 0.67 \quad \mathcal{B}(W^- \to e^-\bar{\nu}_e) \approx 0.11 \tag{6}$$

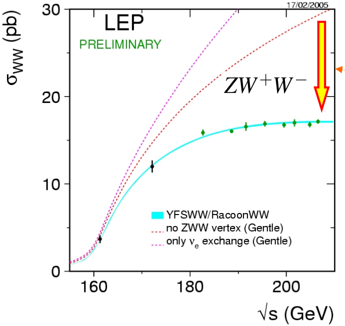
$$\mathcal{B}(W^- \to \mu^-\bar{\nu}_\mu) \approx 0.11 \quad \mathcal{B}(W^- \to \tau^-\bar{\nu}_\tau) \approx 0.11 \tag{7}$$

- gives rise to three distinct topologies:

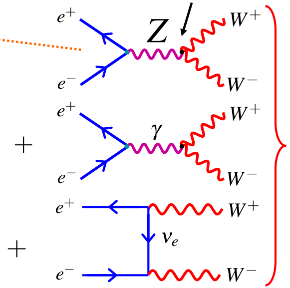


$e^+e^- \to W^+W^-$ **cross section**

- measure cross sections by counting events and normalizing to low angle Bhabha scattering events
- data are consistent with SM expectation
- provides a direct test of ZW^+W^- vertex



Thus, we can achieve convergence



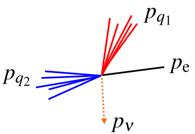
- recall that without the Z diagram the cross section violates unitarity
- presence of Z fixes this problem

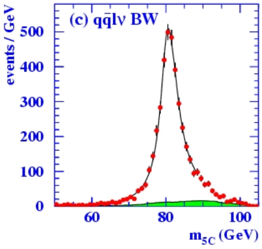
W-mass and W-width

- unlike $e^+e^- \to Z$, the process $e^+e^- \to W^+W^-$ is not a resonant process

  $\Longrightarrow$ different method to measure W boson mass

- measure energy and momenta of particles produced in the W boson decays, e.g. $W^+W^- \to q\bar{q}'e^-\bar{\nu}_e$





- ν 4-momentum from E-p conservation:

  $$p_{q_1} + p_{q_2} + p_e + p_\nu = (\sqrt{s}, 0)$$

- reconstruct masses of two W bosons:

  $$M_+^2 = E^2 - \vec{p}^2 = (p_{q_1} + p_{q_2})^2$$
  $$M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$$

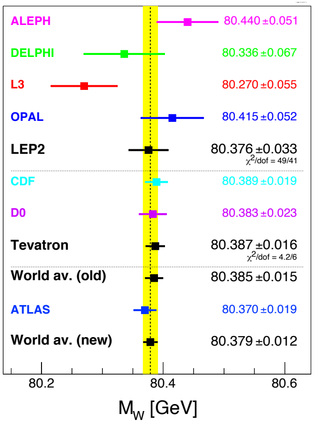
- peak or reconstructed mass distribution gives:

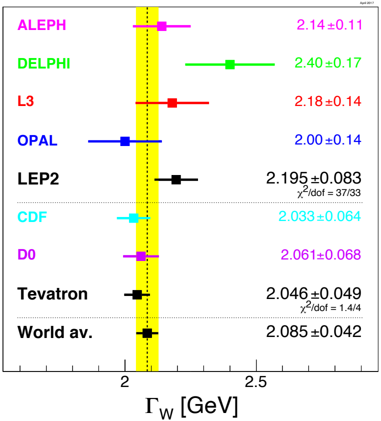
  $$m_W = 80.376 \pm 0.033 \text{ GeV}$$

- width of reconstructed mass distribution gives:

  $\Gamma_W = 2.196 \pm 0.083$ GeV (does not include Tevatron and LHC measurements)

Summary of all measurements and their combination:

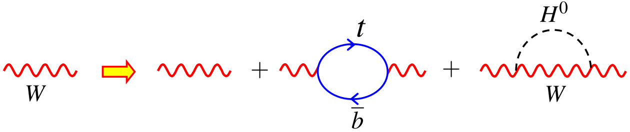




Uncertainties are $\times 2$ smaller than after LEP2!

Precision tests of the standard model

- from LEP and elsewhere have precise measurements: can test predictions of the standard model:
  - e.g. predict $m_W = m_Z \cos\theta_W$ (derivation in Higgs boson lecture in the future!)
  - measure $m_Z = 91.1875 \pm 0.0021$ GeV, $\sin^2\theta_W = 0.23154 \pm 0.00016$
  - therefore expect: $m_W = 79.946 \pm 0.008$ GeV
  - but measure $m_W = 80.376 \pm 0.033$ GeV
- close but not quite right: we only considered lowest order diagrams
- mass of W boson also includes terms from virtual loops

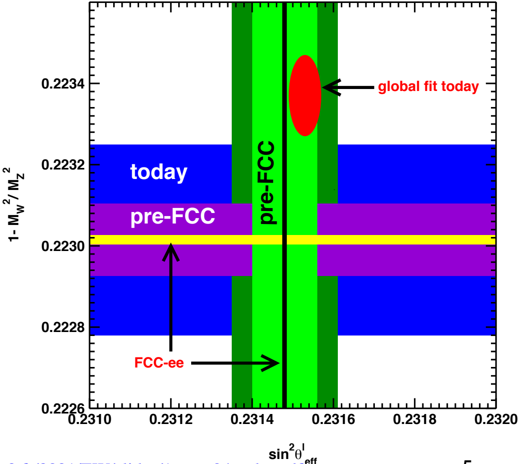


$$\boxed{m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)}$$

- above "discrepancy" due to these virtual loops: making very high precision measurements become sensitive to the masses of particles inside the virtual loops

Another determination of the weak mixing angle θ_W

on-shell *vs.* effective weak mixing angle



ΔM_W (LHC)

$\approx 3.8_{stat} + 3.8_{syst} + 3.8_{PDF}$ MeV

$\approx (5/3)^{1/2} \times 3.8$ MeV ≈ 5 MeV

(for 3 detectors) based on

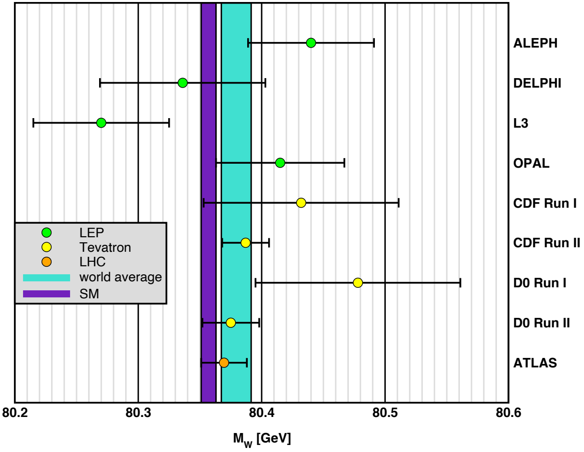
Azzi et al.
arXiv:1902.04070

$\Delta\sin^2\theta_W$ (LHC) $\approx 10^{-4}$

http://moriond.in2p3.fr/2021/EW/slides/1_sm_04_erler.pdf

W mass: persisting tension

W boson mass



average direct

80.379 ± 0.012 GeV

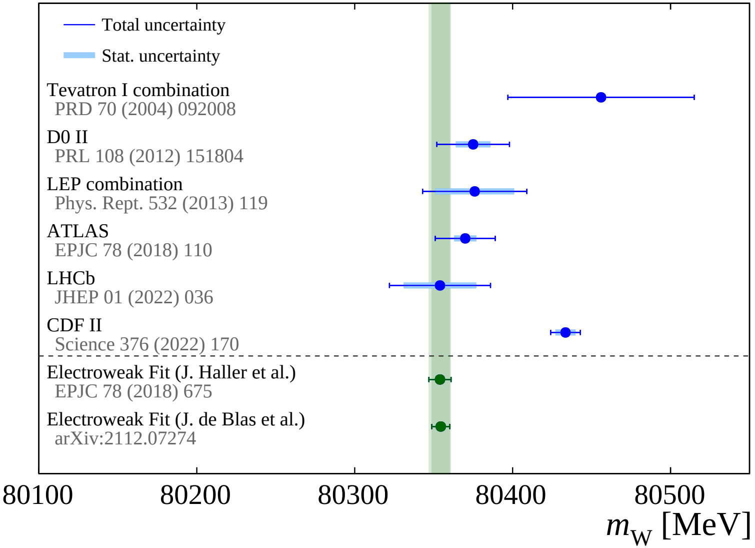
indirect

80.357 ± 0.006 GeV

(1.7 σ low)

http://moriond.in2p3.fr/2021/EW/slides/1_sm_04_erler.pdf

W mass: fresh news, 7σ discrepancy with the SM EW fit



W mass: CDF seminar recording

- detailed seminar about the measurement (recording available): https://indico.cern.ch/event/1150962/

New CDF Result (8.8 fb^{-1})
Combined Fit Systematic Uncertainties

Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

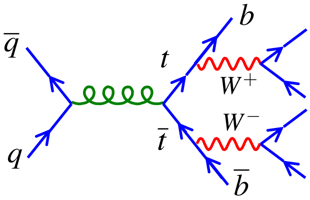
Table 2

The Top quark

- from virtual loop corrections and precise LEP data can predict the top quark mass:

$$m_t^{\text{loop}} = 173 \pm 11 \text{ GeV}$$

- in 1994 top quark observed at the Tevatron $p\bar{p}$ collider at Fermilab - with the predicted mass



- the top quark almost exclusively decays to a bottom quark since

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$$

- complicated final state topologies:

$$t\bar{t} \to b\bar{b}q\bar{q}q\bar{q} \to 6 \text{ jets}$$
$$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu \to 4 \text{ jets} + \ell + \nu$$
$$t\bar{t} \to b\bar{b}\ell\nu\ell\nu \to 2 \text{ jets} + 2\ell + 2\nu$$

- mass determined by direct reconstruction:

$$m_t^{\text{meas}} = 174.2 \pm 3.3 \text{ GeV}$$

- top quark width:

$$\Gamma(t \to bW^+) = \frac{G_F m_t^3}{8\sqrt{2}\pi}\left(1 - \frac{m_W^2}{m_t^2}\right)^2\left(1 + \frac{2m_W^2}{m_t^2}\right)$$

- for the measured values of $m_t = 173$ GeV, $m_W = 80.4$ GeV, $G_F = 1.166 \times 10^{-5}$ GeV^{-2}

$$\Gamma_t = 1.5 \text{ GeV}$$

- hence the lifetime: $\tau_t = 1/\Gamma_t \approx 5 \times 10^{-25}$ s: too short, so that at the Tevatron or LHC top quark does not hadronize, and top quarks decay before forming a bound state

The most recent top quark mass measurements:

Table 67.1: Measurements of top-quark mass from Tevatron and LHC. $\int \mathcal{L}dt$ is given in fb^{-1}. The results are a selection of both published and preliminary (not yet submitted for publication as of September 2019) measurements. For a complete set of published results see the Listings. Statistical uncertainties are listed first, followed by systematic uncertainties.

m_t (GeV/c^2)	Source	$\int \mathcal{L}dt$	Ref.	Channel
$172.08 \pm 0.25 \pm 0.41$	ATLAS	20.2	[165]	ℓ+jets+$\ell\ell$+All jets
$172.44 \pm 0.13 \pm 0.47$	CMS	19.7	[166]	ℓ+jets+$\ell\ell$+All jets
$172.35 \pm 0.16 \pm 0.48$	CMS	19.7	[166]	ℓ+jets
$172.34 \pm 0.20 \pm 0.70$	CMS	35.9	[179]	$\ell\ell$
$173.72 \pm 0.55 \pm 1.01$	ATLAS	20.2	[180]	All jets
$172.25 \pm 0.08 \pm 0.62$	CMS	35.9	[167]	ℓ+jets
$174.30 \pm 0.35 \pm 0.54$	CDF,DØ (I+II)	$\leq$9.7	[181]	publ. or prelim.
$173.34 \pm 0.27 \pm 0.71$	Tevatron+LHC	$\leq$8.7+$\leq$4.9	[3]	publ. or prelim.

World average: $m_t = 172.9 \pm 0.4$ GeV

Summary

- Z boson measured with very high (unbeatable) precision at LEP:

$$m_Z = 91.1875 \pm 0.0021\ \text{GeV}$$

- observed couplings of the Z boson are consistent with the SM expectations with

$$\sin^2\theta_W = 0.23146 \pm 0.00012$$

- measurements of Γ_Z and the $e^+e^- \to Z \to f\bar{f}$ cross sections show that there are only 3 light neutrino generations $\Longrightarrow$ have only 3 fundamental fermion generations
- W mass and top quark masses are measured precisely at LEP, Tevatron and the LHC $\Longrightarrow$ sensitive to quantum loop effects, in particular to Higgs boson
- SM precision measurements in the electroweak sector is one of the main motivations for the next generation of colliders!

Electroweak precision physics

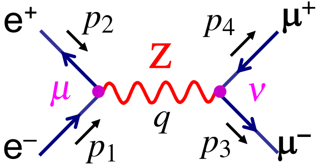
* The electroweak (EW) precision program started about 50 years ago
* M_W, M_Z, m_t, M_H (and m_c) have all been successfully predicted before their discoveries
* 2012 the Standard Model (SM) was completed …

  … and it is as successful as it is unsatisfactory (dark matter, naturalness, …)
* no new states discovered at the LHC yet, so perhaps they show up in EW physics first
* General remark: the higher the precision, the more physics issues will enter in the interpretation of precision measurements
* this is an obstacle when looking at single observables but is a feature in global analyses (across different observables and subfields of particle, nuclear and atomic physics)
* some tensions in g_μ–2, M_W, and the first row CKM matrix unitarity constraint

http://moriond.in2p3.fr/2021/EW/slides/1_sm_04_erler.pdf

Bonus slides

- want to calculate the cross section for $e^+e^- \to Z \to \mu^+\mu^-$
  - Feynman rules for the relevant diagram give:



$$e^+e^- \text{ vertex: } \bar{\nu}(p_2) \cdot -ig_Z\gamma^\mu \frac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1) \tag{8}$$

$$\text{Z propagator: } \frac{-ig_{\mu\nu}}{q^2 - m_Z^2} \tag{9}$$

$$\mu^+\mu^- \text{ vertex: } \bar{u}(p_3) \cdot -ig_Z\gamma^\nu \frac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot \nu(p_4) \tag{10}$$

$$\implies -iM_{fi} =$$

$$\left[\bar{\nu}(p_2) \cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)\right] \cdot \tfrac{-ig_{\mu\nu}}{q^2-m_Z^2} \cdot \left[\bar{u}(p_3) \cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot \nu(p_4)\right]$$

$$\implies M_{fi} = -\tfrac{g_Z^2}{q^2-m_Z^2} g_{\mu\nu} \left[\bar{\nu}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5) \cdot u(p_1)\right] \cdot \left[\bar{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5) \cdot \nu(p_4)\right]$$

- convenient to work in terms of helicity states by explicitly using the Z coupling to LH and RH chiral states (ultra-relativistic limit so helicity = chirality)

$$\frac{1}{2}(c_V - c_A\gamma^5) = c_L\frac{1}{2}(1-\gamma^5) + c_R\frac{1}{2}(1+\gamma^5)$$

hence $c_V = (c_L + c_R)$, $c_A = (c_L - c_R)$, and
$\frac{1}{2}(c_V - c_A\gamma^5) = \frac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) = c_L\frac{1}{2}(1-\gamma^5) + c_R\frac{1}{2}(1+\gamma^5)$
with $c_L = \frac{1}{2}(c_V + c_A)$, $c_R = \frac{1}{2}(c_V - c_A)$

- rewriting the matrix element in terms of LH and RH couplings:

  $M_{fi} = -\frac{g_Z^2}{q^2-m_Z^2} g_{\mu\nu} \left[c_L^e \bar{\nu}(p_2)\gamma^\mu \frac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \bar{\nu}(p_2)\gamma^\mu \frac{1}{2}(1+\gamma^5)u(p_1)\right] \times$
  $\left[c_L^\mu \bar{u}(p_3)\gamma^\nu \frac{1}{2}(1-\gamma^5)\nu(p_4) + c_R^\mu \bar{u}(p_3)\gamma^\nu \frac{1}{2}(1+\gamma^5)\nu(p_4)\right]$
- apply projection operators in the ultra-relativistic limit:

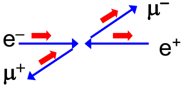
  $\frac{1}{2}(1-\gamma^5)u = u_\downarrow$; $\frac{1}{2}(1+\gamma^5)u = u_\uparrow$; $\frac{1}{2}(1-\gamma^5)\nu = \nu_\uparrow$; $\frac{1}{2}(1+\gamma^5)\nu = \nu_\downarrow$

  $\implies M_{fi} = -\frac{g_Z^2}{q^2-m_Z^2} g_{\mu\nu} \left[c_L^e \bar{\nu}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \bar{\nu}(p_2)\gamma^\mu u_\uparrow(p_1)\right] \times$
  $\left[c_L^\mu \bar{u}(p_3)\gamma^\nu \nu_\uparrow(p_4) + c_R^\mu \bar{u}(p_3)\gamma^\nu \nu_\downarrow(p_4)\right]$
- for a combination of V and A currents, $\bar{u}_\uparrow\gamma^\mu\nu_\uparrow = 0$ etc, gives four orthogonal contributions:

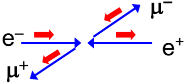
  $\implies -\frac{g_Z^2}{q^2-m_Z^2} g_{\mu\nu} \left[c_L^e \bar{\nu}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \bar{\nu}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)\right] \times$
  $\left[c_L^\mu \bar{u}_\downarrow(p_3)\gamma^\nu \nu_\uparrow(p_4) + c_R^\mu \bar{u}_\uparrow(p_3)\gamma^\nu \nu_\downarrow(p_4)\right]$

- sum of 4 terms:

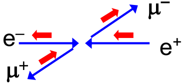
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



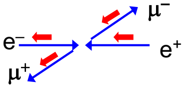
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Here L/R refer to the helicities if the initial/final state particles

- fortunately we have obtained these terms before when considering $e^+e^- \to \gamma \to \mu^+\mu^-$ giving

$$[\bar{\nu}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]\,[\bar{u}_\uparrow(p_3)\gamma^\nu \nu_\downarrow(p_4)] = s(1+\cos\theta)$$

etc

The Z resonance

- applying the QED results to the Z exchange with $\frac{e^2}{q^2} \to \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$:

$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

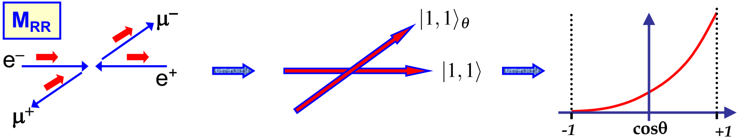
$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

where $q^2 = s = 4E_e^2$

- as before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles, e.g.:



The Breit-Wigner resonance

- need to consider carefully the propagator term $1/(s - m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z
- to do this need to account for the fact that the Z is an unstable particle:
  - for a stable particle at rest the time development of the wave function is
    $$\psi \sim e^{-imt}$$
  - for an unstable particle this must be modified to
    $$\psi \sim e^{-imt} e^{-\Gamma t/2}$$
    so that the particle probability decays away exponentially:
    $$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \text{ with } \tau = 1/\Gamma$$
  - equivalent to making the replacement:
    $$m \to m - i\Gamma/2$$
- in the Z boson propagator make the substitution:
  $$m_Z \to m_Z - i\Gamma_Z/2$$

The Breit-Wigner resonance

- which gives (assuming that $\Gamma_Z \ll m_Z$)

$$(s - m_Z^2) \to \left[s - (m_Z - i\Gamma_Z/2)^2\right] = s - m_Z^2 + i m_Z \Gamma_Z + \frac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + i m_Z \Gamma_Z$$

- which gives

$$\left|\frac{1}{s - m_Z^2}\right|^2 \to \left|\frac{1}{s - m_Z^2 + i m_Z \Gamma_Z}\right|^2 = \frac{1}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

- the matrix element becomes:

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

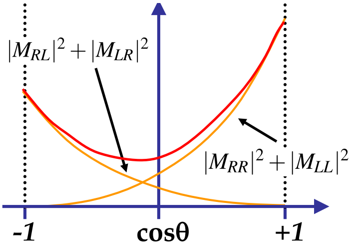
- in the limit where initial and final state particle masses can be neglected:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s} |M_{fi}|^2$$

The Breit-Wigner resonance

- giving:

$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$
$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$
$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$
$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$



- because $|M_{LL}|^2+|M_{RR}|^2 \neq |M_{LR}|^2+|M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation is present (although not maximal as was the case for the W boson)

Cross section with unpolarized beams

- to calculate the total cross section, need to sum over all matrix elements and average over the initial spin states. Here, assuming unpolarized beams (i.e. both e^+ and both e^- states equally likely) there are four combinations of initial electron/positron spins, so:

$$\left\langle |M_{fi}|^2 \right\rangle = \frac{1}{2} \cdot \frac{1}{2} \cdot \left(|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 \right) \tag{11}$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{g_Z^4 s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \tag{12}$$

$$\times \left\{ \left[(c_R^e)^2 (c_R^\mu)^2 + (c_L^e)^2 (c_L^\mu)^2 \right] (1 + \cos\theta)^2 \right. \tag{13}$$

$$\left. + \left[(c_L^e)^2 (c_R^\mu)^2 + (c_R^e)^2 (c_L^\mu)^2 \right] (1 - \cos\theta)^2 \right\} \tag{14}$$

Cross section with unpolarized beams

- the part of the expression in $\Big\{\dots\Big\}$ can be rearranged:

$$\Big\{\dots\Big\} = \left[(c_R^e)^2 + (c_L^e)^2\right]\left[(c_R^\mu)^2 + (c_L^\mu)^2\right]\left(1+\cos^2\theta\right) \tag{15}$$

$$+\,2\left[(c_R^e)^2 - (c_L^e)^2\right]\left[(c_R^\mu)^2 - (c_L^\mu)^2\right]\cos\theta \tag{16}$$

and using $c_V^2 + c_A^2 = 2\left(c_L^2 + c_R^2\right)$ and $c_V c_A = c_L^2 - c_R^2$:

$$\Big\{\dots\Big\} = \frac{1}{4}\left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\mu)^2 + (c_A^\mu)^2\right]\left(1+\cos^2\theta\right) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta$$

Cross section with unpolarized beams

- hence the complete expression for the unpolarized differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\left\langle |M_{fi}|^2 \right\rangle \tag{17}$$

$$= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times \tag{18}$$

$$\left\{\frac{1}{4}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]\left(1+\cos^2\theta\right)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta\right\} \tag{19}$$

- integrating over solid angle $d\Omega = d\phi\, d\cos\theta = 2\pi\, d\cos\theta$

$\int_{-1}^{+1}\left(1+\cos^2\theta\right) d\cos\theta = \int_{-1}^{+1}(1+x^2)\, dx = \frac{8}{3}$ and $\int_{-1}^{+1}\cos\theta\, d\cos\theta = 0$

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

- note the total cross section is proportional to the sums of the squares of the vector and axial-vector couplings of the initial and final state fermions: $\left(c_V^f\right)^2+\left(c_A^f\right)^2$

Connection to the Breit-Wigner Formula

- can write the total cross section:

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

in terms of the Z boson decay rates (partial widths) from the previous lecture:
$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2+(c_A^e)^2\right]$ and $\Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$

$$\implies \sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

- writing the partial widths as $\Gamma_{ee}=\Gamma(Z\to e^+e^-)$ etc, the total cross section can be written:

$$\sigma(e^+e^-\to Z\to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff}$$

where f is the final state fermion flavor