Circuit Quantum Electrodynamics

Superconducting platform

(10th Lecture)

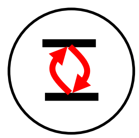
Covering: basic concepts, measurement techniques, implementations, qubit approaches, current trends

With figures and slides borrowed from

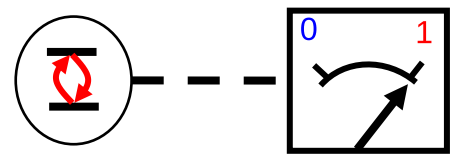
P. Bertet (CEA Saclay)

Requirements for QC

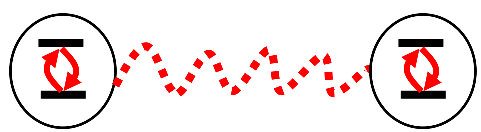
High-Fidelity
Single Qubit Operations



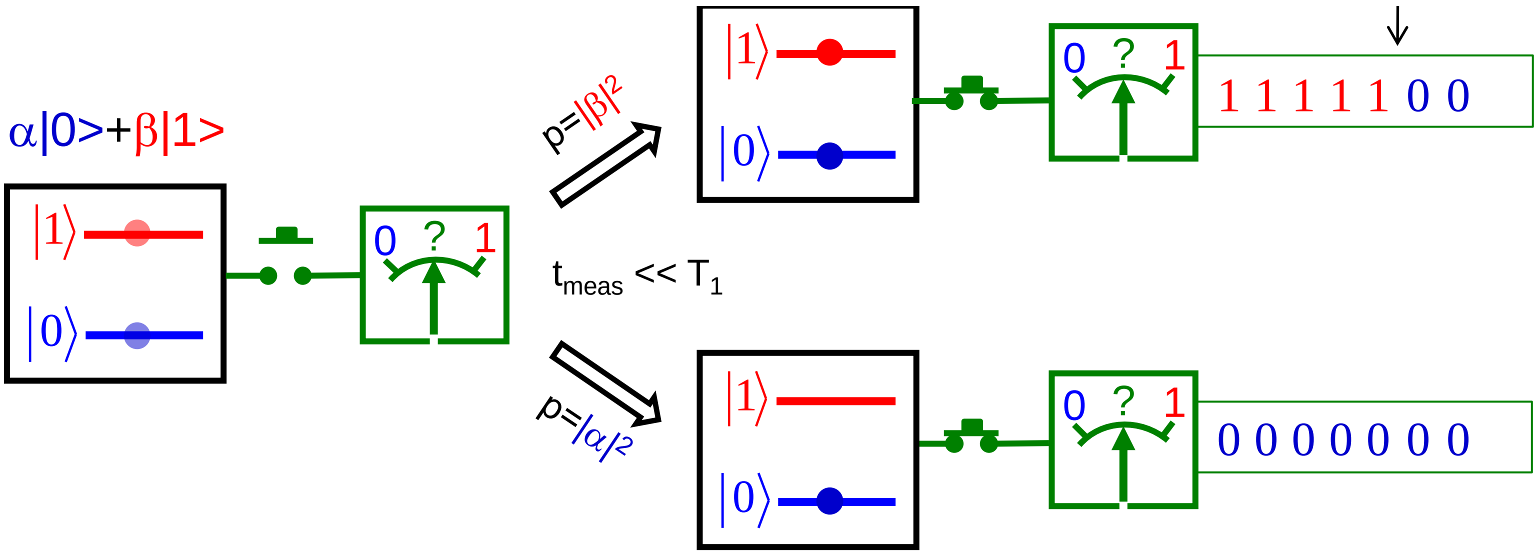
High-Fidelity Readout
of Individual Qubits



Deterministic, On-Demand
Entanglement between Qubits



The ideal qubit readout



Hi-Fi

Quantum Non Demolishing (QND)

BUT ….HOW ???

SURPRISING DIFFICULT AND INTERESTING QUESTION FOR SUPERCONDUCTING QUBITS

courtesy CEA Saclay

The readout problem

1) Readout should be **FAST** :

$t_{meas} \ll T_1 \sim \text{from } 1\mu s \; to \; 100\mu s$ for high fidelity $(F \leq 1 - t_{meas} / T_1)$

Ideally, $t_{meas} \sim 10ns$

2) Readout should be **NON-INVASIVE**

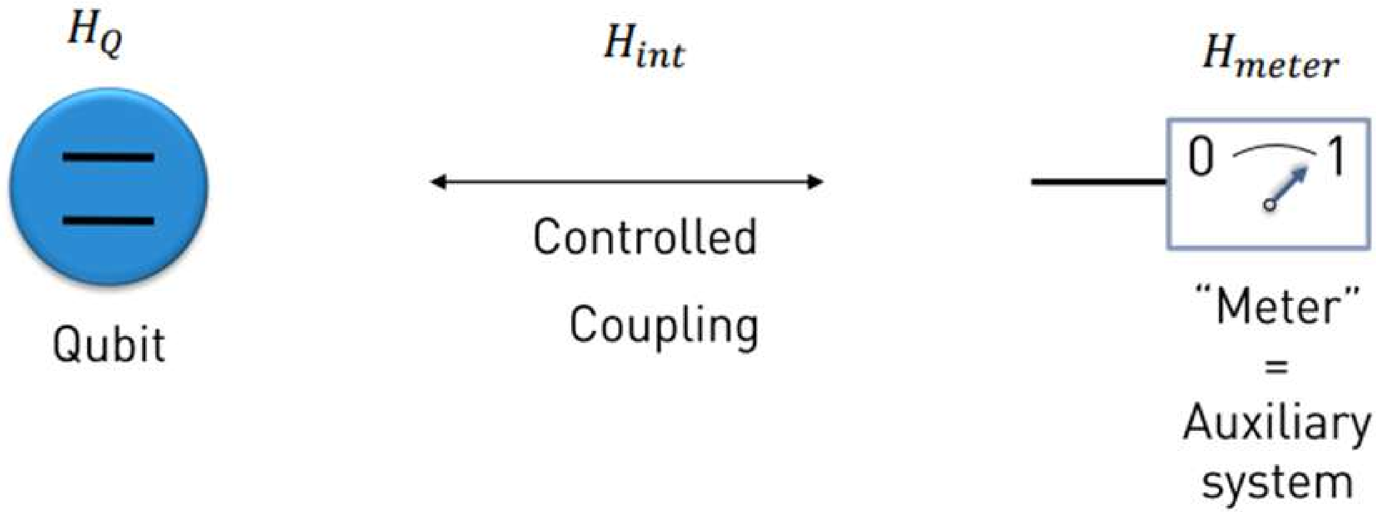
Unwanted transition caused by readout process $\Longrightarrow$ **errors**
(but full dephasing can't be avoided !!!)

3) Readout should be **COMPLETELY OFF** during quantum state preparation

(avoid backaction)

courtesy CEA Saclay

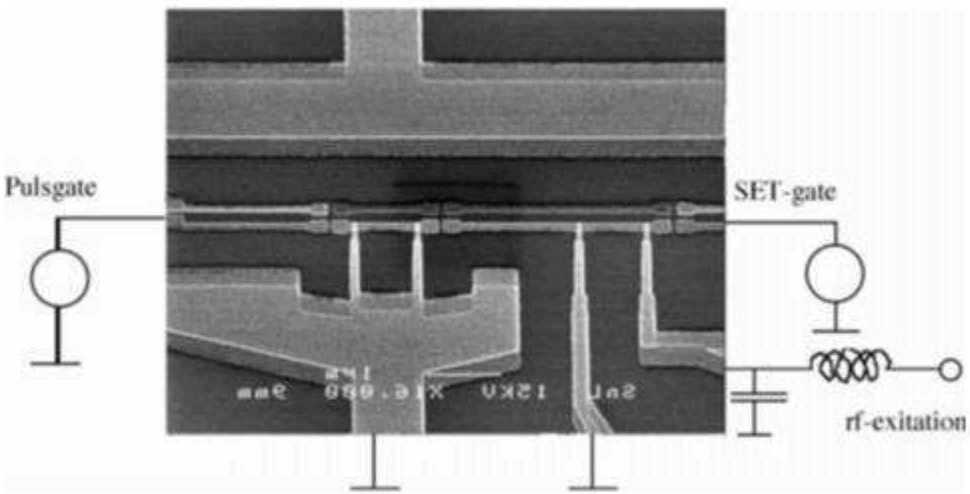
General properties of quantum measurements

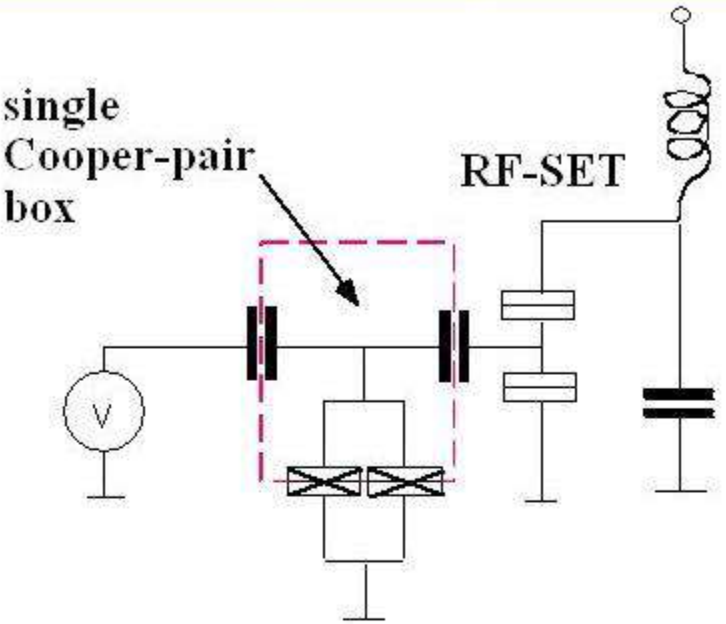


Desirable properties:

- Projective and Quantum non-demolition (QND)
  - Coupling to the meter does not change the state of the qubit $[H_Q, H_{int}] = 0.$
  - Repeated measurement yields the same outcome.
- Good ON/OFF ratio
  - $[H_{int}, H_{meter}] = 0$ during "OFF"
  - $[H_{int}, H_{meter}] \neq 0$ during "ON"
- No spontaneous decay/excitation due to measurement apparatus
- Fast and high fidelity

Example: A single CPB integrated with an RF-SET Read-out system





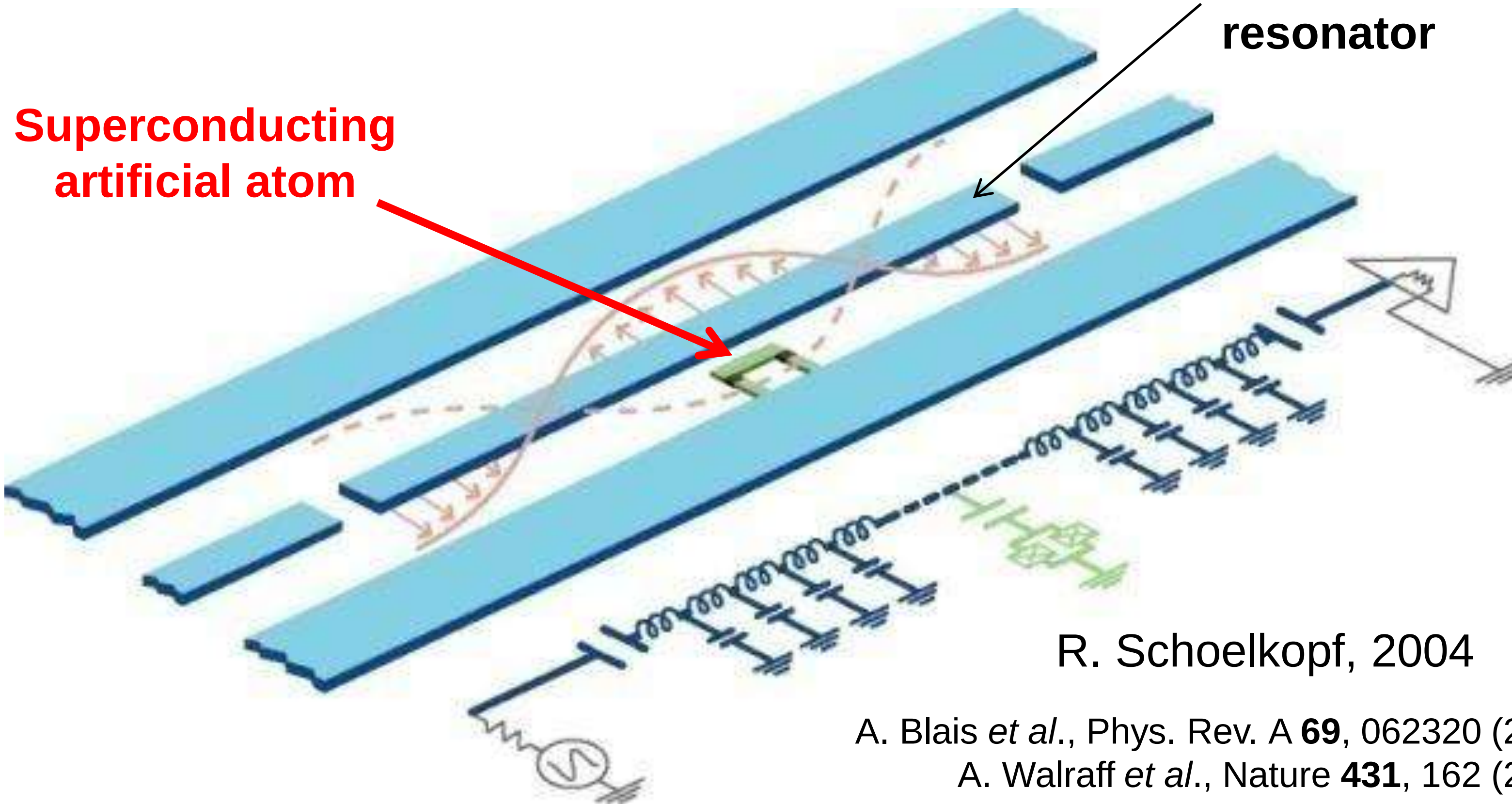
A two level systen based on the charge states
|1> = One extra Cooper-pair in the box
|0> = No extra Cooper-pair in the box

Δ	>>	E_C	>>	$E_J(B)$	>>	T
2.5K		0.5-1.5K		0.05-1K		20mK

Büttiker, PRB (86)
Bouchiat et al. Physica Scripta (99)
Nakamura et al., Nature (99)
Makhlin et al. Rev. Mod. Phys. (01)
Aassime, PD et al., PRL (01)
Vion et al. Nature (02)

courtesy P. Delsing (Chalmers)

Readout by a linear resonator



R. Schoelkopf, 2004

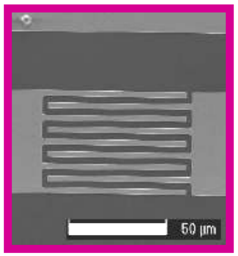
A. Blais *et al*., Phys. Rev. A **69**, 062320 (2004)
A. Walraff *et al*., Nature **431**, 162 (2004)
I. Chiorescu et al., Nature **431**, 159 (2004)

Modern readout methods by coupling to a resonator

(CIRCUIT QUANTUM ELECTRODYNAMICS)

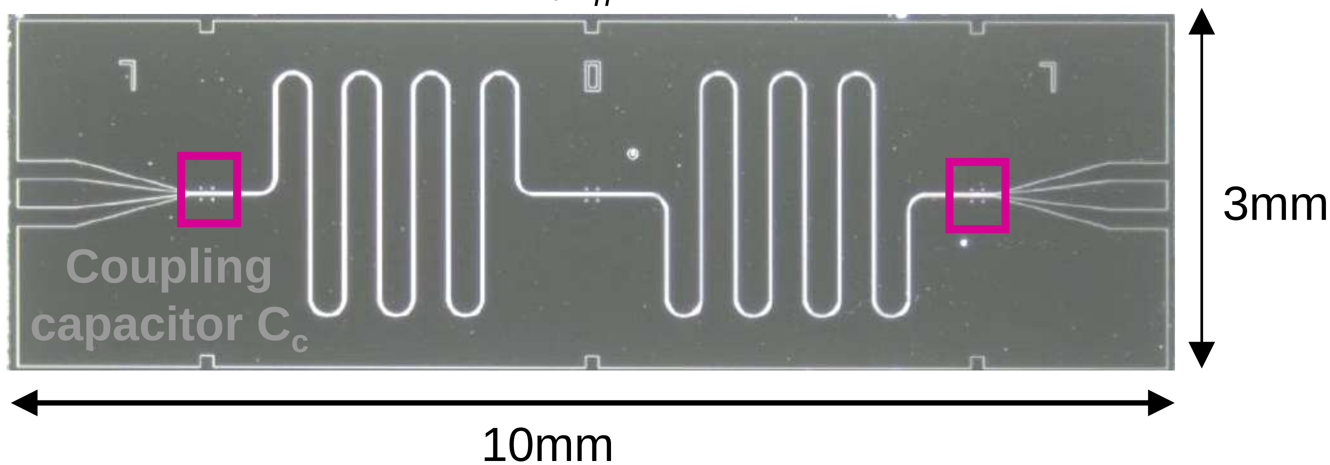
courtesy CEA Saclay

Readout by a linear resonator

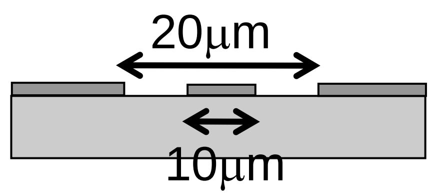


50µm

L=3.2cm, f_n=n 1.8GHz



Typical lateral dimensions :

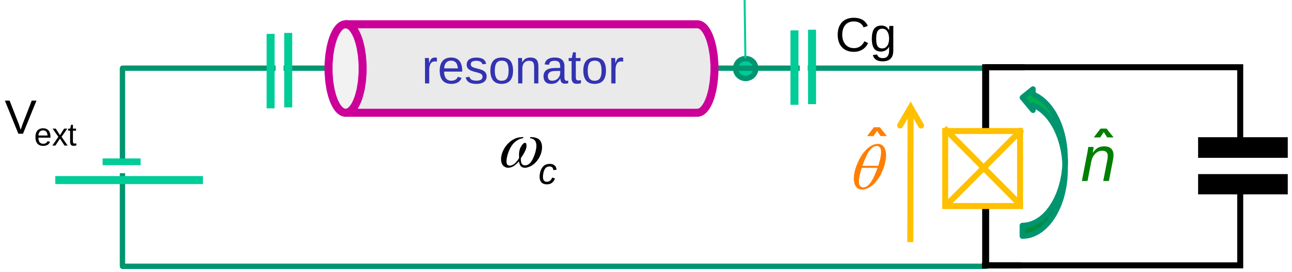


- **1-dimensional** mode
- Very **confined :** $V_{cav} \approx 10^{-5} \lambda^3$
- Large voltage quantum fluctuations $\delta V_0 \sim uV$
- Quality factor easily tuned by designing C_c

courtesy CEA Saclay

CPB coupled to a CPW resonator

$$\hat{V}_g = \delta V_0\left(\hat{a}+\hat{a}^+\right)+V_{gext}$$



photon operators $\left(\hat{a}\ ,\hat{a}^+\right)$

$\delta V_0 = \sqrt{\hbar\omega_c/2C_c},$

$C_c\delta V_0 = \sqrt{\hbar\omega_c C_c/2}$

res.
- $\hat{V}_g = \delta V_0\left(\hat{a}+\hat{a}^+\right)$
- $\hat{Q}_c = C_c\delta V_0\left(\hat{a}+\hat{a}^+\right)$

$$\hat{H}_{tot} = -E_J\cos\hat{\theta}+4E_C\left(\hat{n}-\hat{n}_g\right)^2+\hbar\omega_c\hat{a}^+\hat{a}$$

$$\hat{H}_{tot} = \underbrace{-E_J\cos\hat{\theta}+4E_C\left(\hat{n}-n_{gext}\right)^2}_{\hat{H}_q}+\underbrace{\hbar\omega_c\hat{a}^+\hat{a}}_{\hat{H}_{cav}}+\underbrace{8(C_g\delta V_0E_C/2e)\hat{n}(a+a^+)}_{\hat{H}_{\mathrm{int}}}$$

2-level approximation + Rotating Wave Approximation

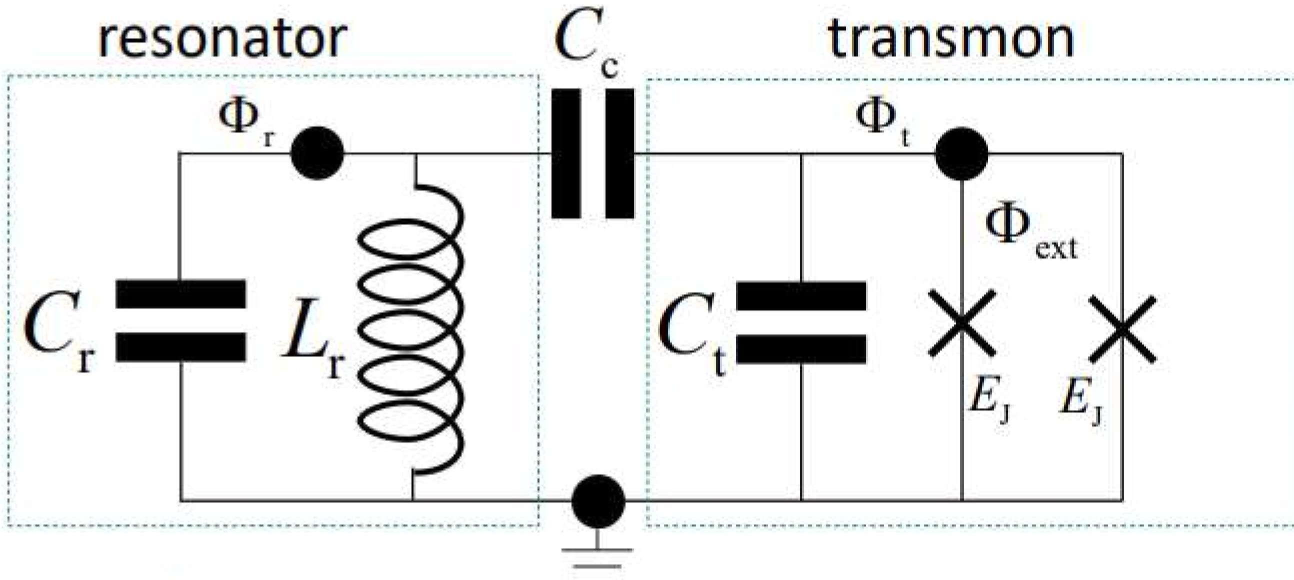
$$\boxed{H_{tot}\sim-\frac{\omega_{ge}}{2}\sigma_z+\omega_c(a^+a+1/2)+g(\sigma^+a+\sigma^-a^+)}$$

Jaynes-Cummings Hamiltonian

A. Blais *et al.*, PRA **69**, 062320 (2004)

courtesy CEA Saclay

Coupling a Resonator to a Transmon



Calculate the classical Lagrangian:

$$\mathfrak{L} = E_e - E_m$$

$$E_e = \frac{1}{2}C_r\dot{\Phi}_r^{\,2} + \frac{1}{2}C_t\dot{\Phi}_t^{\,2} + \frac{1}{2}C_c\left(\dot{\Phi}_r - \dot{\Phi}_t\right)^2 \qquad E_m = \frac{\Phi_r^{\,2}}{2L_r} - E_J(\Phi_{ext})\cos\left(2\pi\frac{\Phi_t}{\Phi_0} + \varphi(\Phi_{ext})\right)$$

Find the variables conjugate to the fluxes:

$$Q_r = \frac{d\mathfrak{L}}{d\dot{\Phi}_r} = C_r\dot{\Phi}_r + C_c\left(\dot{\Phi}_r - \dot{\Phi}_t\right) = \left(C_r + C_c\right)\dot{\Phi}_r - C_c\dot{\Phi}_t$$

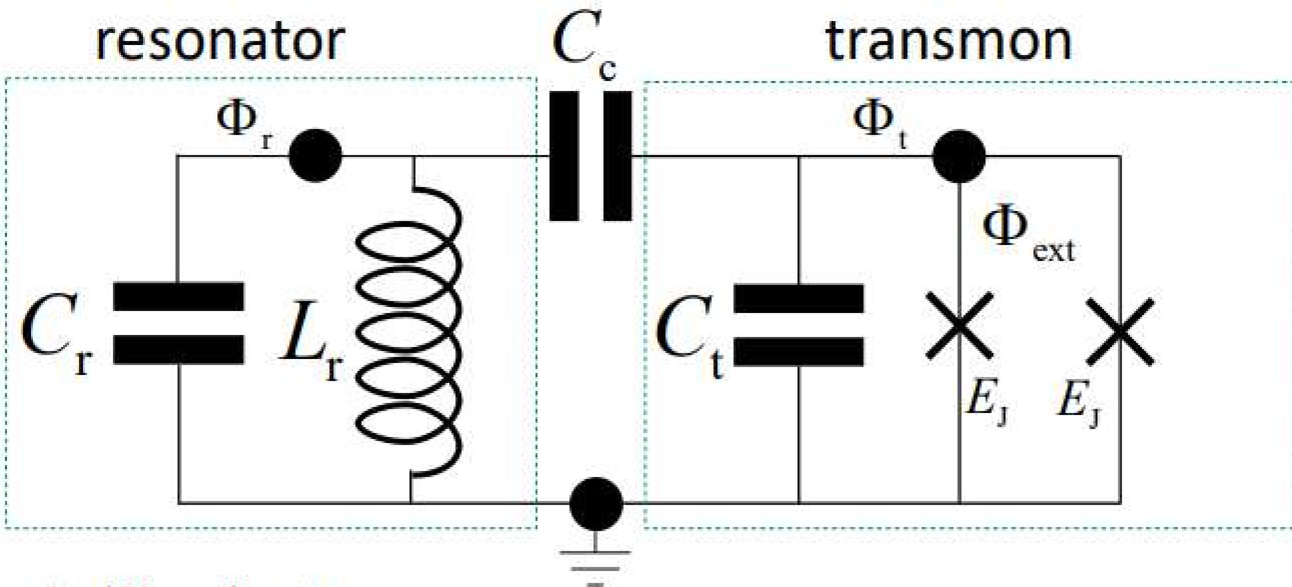
$$Q_t = \frac{d\mathfrak{L}}{d\dot{\Phi}_t} = C_t\dot{\Phi}_t + C_c\left(\dot{\Phi}_t - \dot{\Phi}_r\right) = \left(C_t + C_c\right)\dot{\Phi}_t - C_c\dot{\Phi}_r$$

In matrix form:

$$\begin{pmatrix} Q_r \\ Q_t \end{pmatrix} = \begin{pmatrix} C_r + C_c & -C_c \\ -C_c & C_t + C_c \end{pmatrix}\begin{pmatrix} \dot{\Phi}_r \\ \dot{\Phi}_t \end{pmatrix} = M^{-1}\begin{pmatrix} \dot{\Phi}_r \\ \dot{\Phi}_t \end{pmatrix}$$

courtesy of L. DiCarlo (TU Delft)

Coupling a Resonator to a Transmon



Calculate the classical Hamiltonian:

$$H(\Phi_r,\Phi_t,Q_r,Q_t)=\dot{\Phi}_r Q_r+\dot{\Phi}_t Q_t-\mathfrak{L}=E_m+\frac{1}{2}Q^T\times M\times Q$$

$$=\frac{\Phi_r^{\;2}}{2L_r}+\frac{Q_r^{\;2}}{2C_{\Sigma r}}\quad+\quad-E_J(\Phi_{ext})\cos\left(2\pi\frac{\Phi_t}{\Phi_0}+\varphi(\Phi_{ext})\right)+\frac{Q_t^{\;2}}{2C_{\Sigma t}}\quad+\quad\beta Q_r\otimes Q_t$$

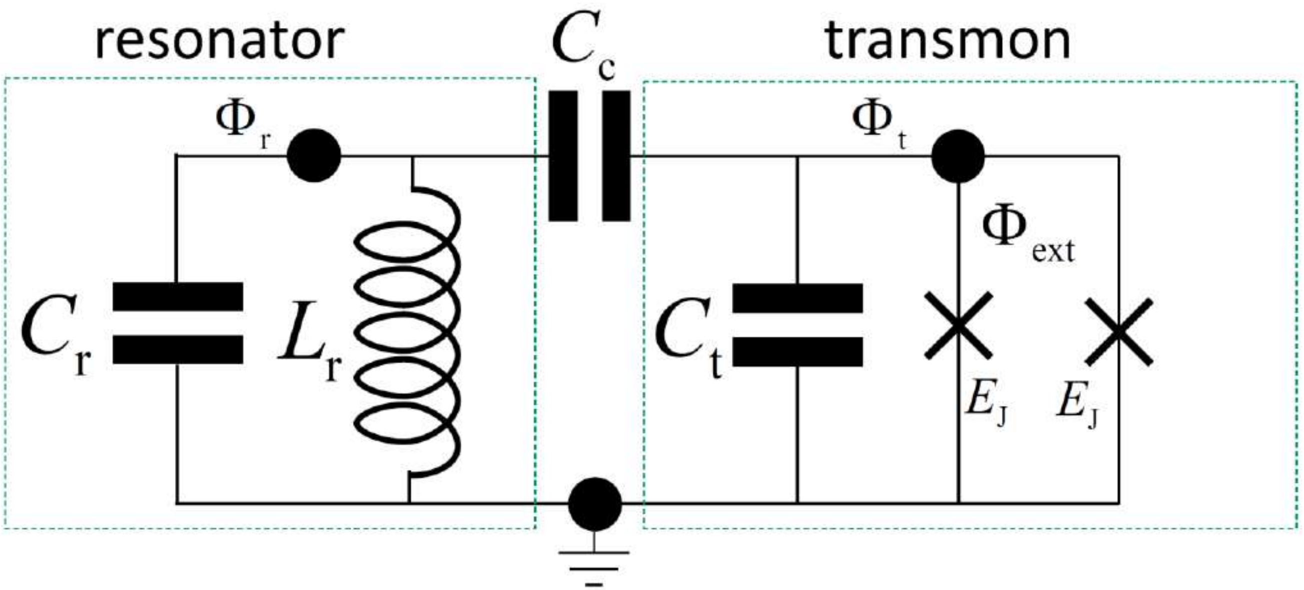
where $C_{\Sigma r}\equiv C_r+C_c\,||\,C_t$ $\qquad \beta\equiv\frac{1}{C_{\Sigma r}}\left(\frac{C_c}{C_c+C_t}\right)$

$C_{\Sigma t}\equiv C_t+C_c\,||\,C_r$

Definition: $a\,||\,b\equiv\frac{ab}{a+b}$

courtesy of L. DiCarlo (TU Delft)

Coupling a Resonator to a Transmon



Calculate the quantum Hamiltonian:

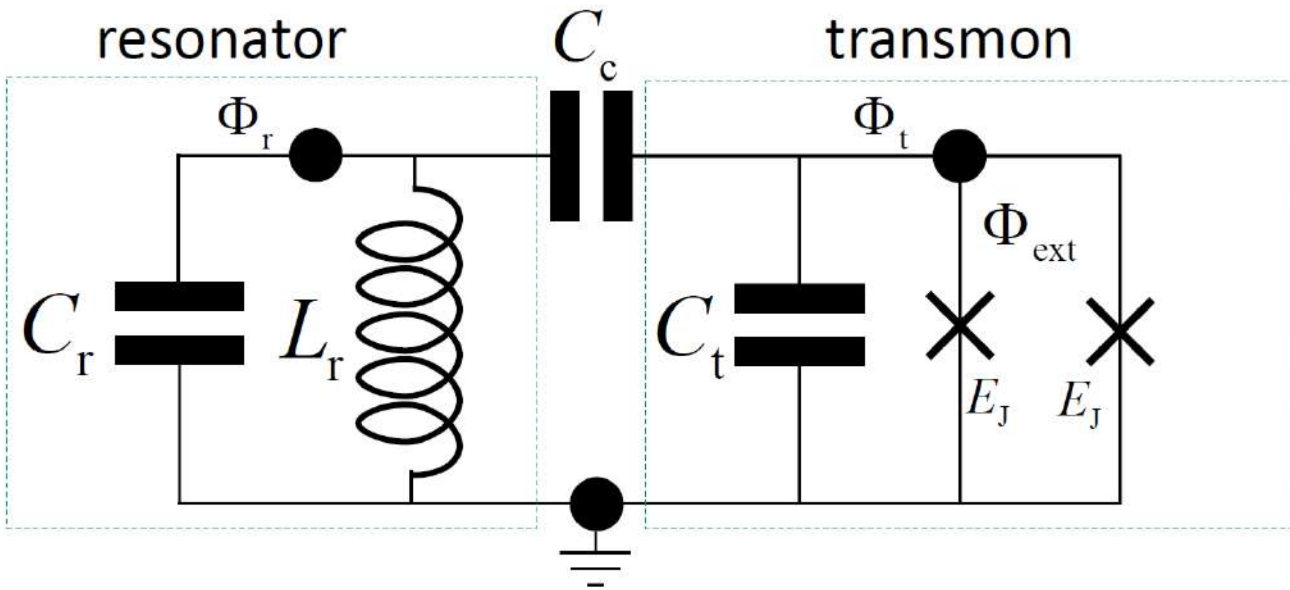
$$\hat{H} = \underbrace{\frac{\hat{\Phi}_r^2}{2L_r} + \frac{\hat{Q}_r^2}{2C_{\Sigma r}}}_{\hat{H}_{\text{resonator}}} + \underbrace{-E_J(\Phi_{ext})\cos\left(2\pi\frac{\hat{\Phi}_t}{\Phi_0} + \varphi(\Phi_{ext})\right) + \frac{\hat{Q}_t^2}{2C_{\Sigma t}}}_{\hat{H}_{\text{transmon}}} + \underbrace{\beta \hat{Q}_r \otimes \hat{Q}_t}_{\hat{H}_{\text{coupling}}}$$

$$\left[\hat{\Phi}_r, \hat{Q}_r\right] = i\hbar \quad \left[\hat{\Phi}_r, \hat{\Phi}_t\right] = 0 \quad \left[\hat{Q}_r, \hat{\Phi}_t\right] = 0$$

$$\left[\hat{\Phi}_t, \hat{Q}_t\right] = i\hbar \quad \left[\hat{\Phi}_r, \hat{Q}_t\right] = 0 \quad \left[\hat{Q}_r, \hat{Q}_t\right] = 0$$

courtesy of L. DiCarlo (TU Delft)

Coupling a Resonator to a Transmon



Let's take a close look at the coupling term:

$$\hat{H}_{\text{coupling}} = \beta \hat{Q}_r \otimes \hat{Q}_t$$

$$= i\beta Q_{\text{zpf}}\left(\hat{a}_r^\dagger - \hat{a}_r\right) \otimes \hat{Q}_t$$

$$= i\beta Q_{\text{zpf}}\left(\hat{a}_r^\dagger - \hat{a}_r\right) \otimes \left(\underbrace{\left(\sum_k |k_t\rangle\langle k_t|\right)}_{\hat{I}_t} \hat{Q}_t \underbrace{\left(\sum_l |l_t\rangle\langle l_t|\right)}_{\hat{I}_t}\right)$$

$$= i\beta Q_{\text{zpf}}\left(\hat{a}_r^\dagger - \hat{a}_r\right) \otimes \left(\sum_{k,l} \langle k_t|\hat{Q}_t|l_t\rangle |k_t\rangle\langle l_t|\right)$$

Recall: $\hat{Q}_r = iQ_{\text{zpf}}\left(\hat{a}_r^\dagger - \hat{a}_r\right)$

$$Q_{\text{zpf}} \equiv \sqrt{\frac{\hbar}{2Z_r}}$$

$$Z_r = \sqrt{\frac{L_r}{C_{\Sigma r}}}$$

courtesy of L. DiCarlo (TU Delft)

The Transmon Dipole Moment

$$D_{kl} = \langle k_t | \hat{Q}_t | l_t \rangle = -2e \langle k_t | \hat{N}_t | l_t \rangle$$

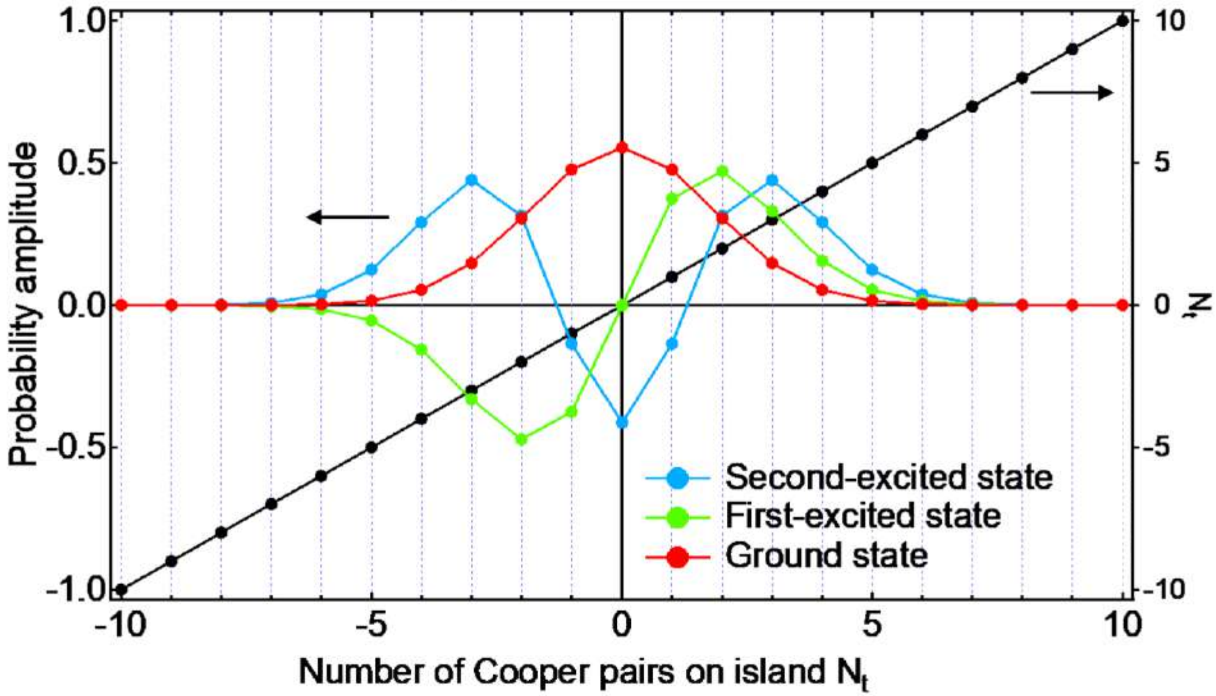
Is the *dipole moment* between transmon levels k_t and l_t

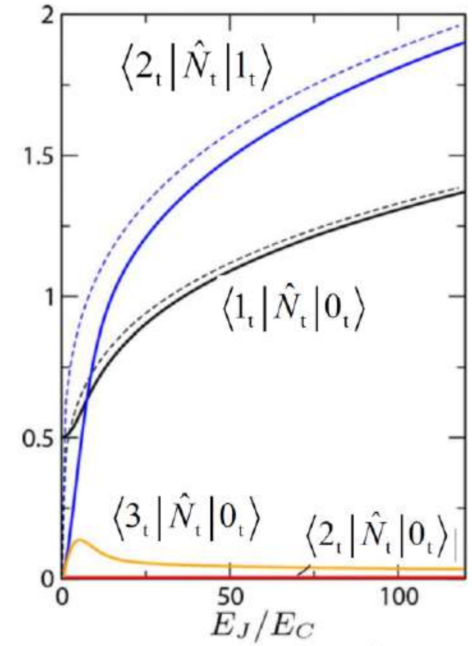
$$= -2e \left(\sum_{N_t=-\infty}^{\infty} c^*_{k,N_t} \langle N = N_t | \right) \hat{N}_t \left(\sum_{M_t=-\infty}^{\infty} c_{l,M_t} | N = M_t \rangle \right)$$

$$= -2e \sum_{N_t=-\infty}^{\infty} c^*_{k,N_t} N_t c_{l,N_t}$$

since $\langle N = N_t | \hat{N}_t | N = M_t \rangle = \delta_{N_t,M_t}$

The coupling between neighbouring transmon states is the only relevant coupling in the transmon limit





Koch et al., PRA (2007)

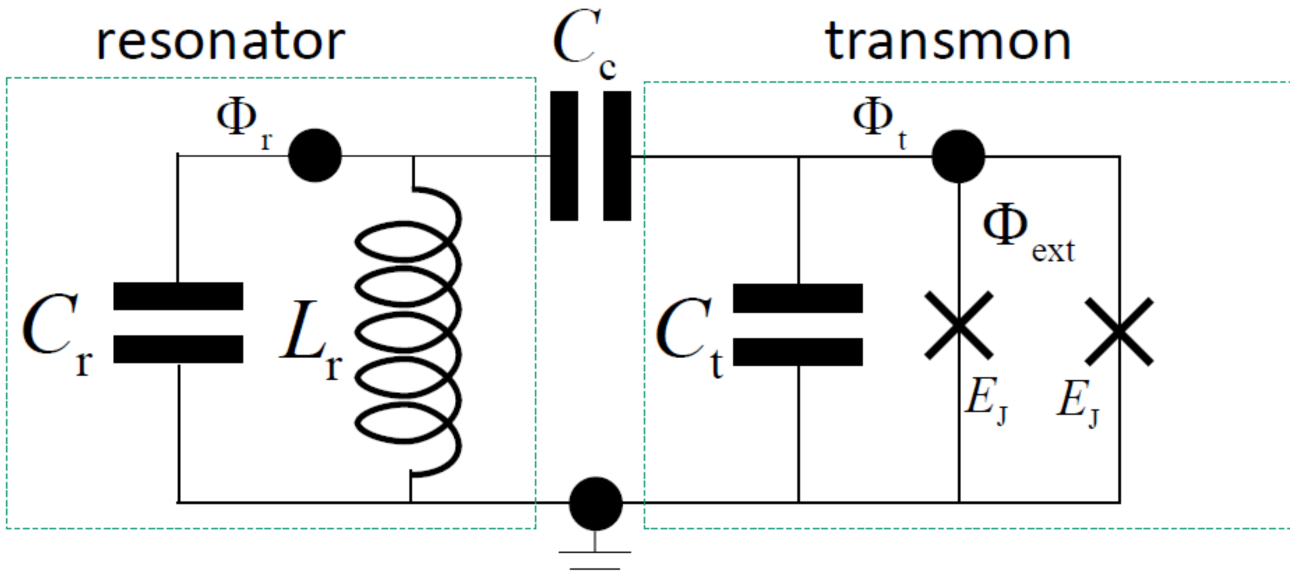
Due to the symmetry properties of the transmon eigenstate wavefunctions:

$\langle k_t | \hat{Q}_t | l_t \rangle = 0$ for all levels where k_t *and* l_t differ by an even number.

$\langle k_t | \hat{Q}_t | l_t \rangle \neq 0$ for all levels where k_t *and* l_t differ by an odd number.

courtesy of L. DiCarlo (TU Delft)

Coupling a Resonator to a Transmon



Let's take a close look at the coupling term:

$$\hat{H}_{\text{coupling}} = i\beta Q_{\text{zpf}}\left(\hat{a}_{\text{r}}^{\dagger} - \hat{a}_{\text{r}}\right) \otimes \left(\sum_{k,l} \langle k_{\text{t}} | \hat{Q}_{\text{t}} | l_{\text{t}} \rangle |k_{\text{t}}\rangle\langle l_{\text{t}}| \right)$$

$$= i\beta Q_{\text{zpf}}\left(\hat{a}_{\text{r}}^{\dagger} - \hat{a}_{\text{r}}\right) \otimes \left(\sum_{|k-l| \text{ odd}} \langle k_{\text{t}} | \hat{Q}_{\text{t}} | l_{\text{t}} \rangle |k_{\text{t}}\rangle\langle l_{\text{t}}| \right)$$

Keeping only terms with non-zero dipole moment

$$= i\beta Q_{\text{zpf}}\left(\hat{a}_{\text{r}}^{\dagger} - \hat{a}_{\text{r}}\right) \otimes \left(\sum_{k>l,\ \text{odd } |k-l|} D_{kl}\sigma_{kl}^{+} + D_{kl}^{*}\sigma_{kl}^{-} \right)$$

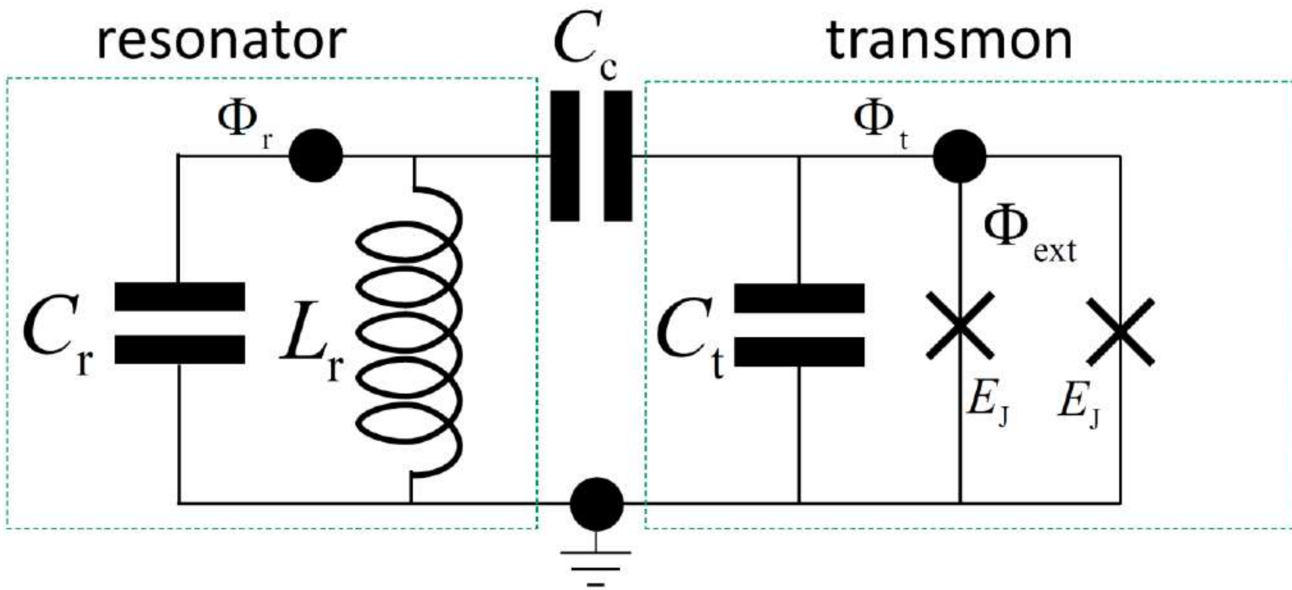
$$= i\beta Q_{\text{zpf}}\left(\sum_{k>l, |k-l| \text{ odd}} D_{kl}\hat{a}_{\text{r}}^{\dagger} \otimes \sigma_{kl}^{+} - D_{kl}\hat{a}_{\text{r}} \otimes \sigma_{kl}^{+} + D_{kl}^{*}\hat{a}_{\text{r}}^{\dagger} \otimes \sigma_{kl}^{-} - D_{kl}^{*}\hat{a}_{\text{r}} \otimes \sigma_{kl}^{-} \right)$$

Ladder operators for the transmon:

$$\sigma_{kl}^{+} \equiv |k_{\text{t}}\rangle\langle l_{\text{t}}|$$

$$\sigma_{kl}^{-} \equiv \sigma_{kl}^{+\dagger} = |l_{\text{t}}\rangle\langle k_{\text{t}}|$$

Coupling a Resonator to a Transmon



Next, we will make two approximations:

Approximation 1: Rotating-wave approximation (RWA)
we neglect the terms $\hat{a}_r^\dagger \sigma_{kl}^+$ and $\hat{a}_r \sigma_{kl}^-$, which do not conserve total excitation number in the resonator + transmon system:

$$\hat{H}_{\text{coupling, RWA}} = i\beta Q_{\text{zpf}} \left(\sum_{k>l,\ |k-l|\ \text{odd}} D_{kl}^* \hat{a}_r^\dagger \otimes \sigma_{kl}^- - D_{kl} \hat{a}_r \otimes \sigma_{kl}^+ \right)$$

courtesy of L. DiCarlo (TU Delft)

Coupling a Resonator to a Transmon

Approximation 2: We truncate the transmon to the qubit subspace (lowest two energy levels).

$$\hat{H}_{\text{transmon}} = -E_{\text{J}}(\Phi_{\text{ext}})\cos\left(2\pi\frac{\hat{\Phi}_{\text{t}}}{\Phi_0} + \varphi(\Phi_{\text{ext}})\right) + \frac{\hat{Q}_{\text{t}}^2}{2C_{\Sigma\text{t}}}$$

$$= \sum_k E_k |k_{\text{t}}\rangle\langle k_{\text{t}}|$$

$$\hat{H}_{\text{qubit}} = E_0|0_{\text{t}}\rangle\langle 0_{\text{t}}| + E_1|1_{\text{t}}\rangle\langle 1_{\text{t}}|$$

$$= \left(-\frac{E_1 - E_0}{2} + \frac{E_1 + E_0}{2}\right)(|0_{\text{t}}\rangle\langle 0_{\text{t}}|) + \left(\frac{E_1 - E_0}{2} + \frac{E_1 + E_0}{2}\right)|1_{\text{t}}\rangle\langle 1_{\text{t}}|$$

$$= -\frac{E_1 - E_0}{2}(|0_{\text{t}}\rangle\langle 0_{\text{t}}| - |1_{\text{t}}\rangle\langle 1_{\text{t}}|) + \frac{E_1 + E_0}{2}(|0_{\text{t}}\rangle\langle 0_{\text{t}}| + |1_{\text{t}}\rangle\langle 1_{\text{t}}|)$$

$$= -\frac{\omega_{01}}{2}(|0_{\text{t}}\rangle\langle 0_{\text{t}}| - |1_{\text{t}}\rangle\langle 1_{\text{t}}|) + \frac{E_1 + E_0}{2}(|0_{\text{t}}\rangle\langle 0_{\text{t}}| + |1_{\text{t}}\rangle\langle 1_{\text{t}}|)$$

$$= -\frac{\omega_{01}}{2}\hat{\sigma}_z + \text{const}$$

$$\hat{\sigma}_z \equiv |0_{\text{t}}\rangle\langle 0_{\text{t}}| - |1_{\text{t}}\rangle\langle 1_{\text{t}}|$$

Pauli *Z* operator

courtesy of L. DiCarlo (TU Delft)

The Jaynes-Cummings Hamiltonian

With these two approximations, we arrive at the **Jaynes-Cummings (JC) Hamiltonian!**

$$\hat{H}_{\mathrm{JC}} = \hbar\omega_{\mathrm{r}}\left(\hat{a}_{\mathrm{r}}^{\dagger}\hat{a}_{\mathrm{r}} + \frac{1}{2}\right) - \frac{\hbar\omega_{01}}{2}\hat{\sigma}_z + \hbar g\hat{a}_{\mathrm{r}}^{\dagger}\hat{\sigma}_- + \hbar g^*\hat{a}_{\mathrm{r}}\hat{\sigma}_+$$

$$\hat{\sigma}_+ \equiv |1_{\mathrm{t}}\rangle\langle 0_{\mathrm{t}}|$$

$$\hat{\sigma}_- \equiv |0_{\mathrm{t}}\rangle\langle 1_{\mathrm{t}}|$$

$$D \equiv D_{10}$$

where $g \equiv \dfrac{i\beta Q_{\mathrm{zpf}} D^*}{\hbar}$ is called the vacuum-Rabi coupling constant

Note: most textbooks present the JC Hamiltonian as

$$\hat{H}_{\mathrm{JC}} = \hbar\omega_{\mathrm{r}}\left(\hat{a}_{\mathrm{r}}^{\dagger}\hat{a}_{\mathrm{r}} + \frac{1}{2}\right) - \frac{\hbar\omega_{01}}{2}\hat{\sigma}_z + \hbar g(\hat{a}_{\mathrm{r}}^{\dagger}\hat{\sigma}_- + \hat{a}_{\mathrm{r}}\hat{\sigma}_+)$$

with real-valued, positive g. This can be done by absorbing the phase of $g = |g|e^{i\theta_g}$ into the definition of $|1_{\mathrm{t}}\rangle$: $|1_{\mathrm{t}}\rangle \to e^{-i\theta_g}|1_{\mathrm{t}}\rangle$

courtesy of L. DiCarlo (TU Delft)

The Jaynes-Cummings Hamiltonian

$$\hat{H}_{JC} = \hbar\omega_r\left(\hat{a}_r^\dagger\hat{a}_r + \frac{1}{2}\right) - \frac{\hbar\omega_{01}}{2}\hat{\sigma}_z + \hbar g(\hat{a}_r^\dagger\hat{\sigma}_- + \hat{a}_r\hat{\sigma}_+)$$

e.g., 2 photons in resonator and qubit in ground state

In the composite basis of resonator and transmon eigenstates:

$$|0_r0_t\rangle, \underbrace{|0_r1_t\rangle, |1_r0_t\rangle}_{\text{1-excitation manifold}}, \underbrace{|2_r0_t\rangle, |1_r1_t\rangle}_{\text{2-excitation manifold}}, \underbrace{|3_r0_t\rangle, |2_r1_t\rangle}_{\text{3-excitation manifold}}, \ldots$$

$$H_{JC} \doteq \hbar \begin{pmatrix} \boxed{0} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \omega_r & g & 0 & 0 & 0 & 0 & \\ 0 & g & \omega_{01} & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 2\omega_r & \sqrt{2}g & 0 & 0 & \\ 0 & 0 & 0 & \sqrt{2}g & \omega_r + \omega_{01} & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 3\omega_r & \sqrt{3}g & \\ 0 & 0 & 0 & 0 & 0 & \sqrt{3}g & 2\omega_r + \omega_{01} & \\ \vdots & & & & & & & \ddots \end{pmatrix} + \text{constant}$$

$$= \frac{1}{2}\hbar(\omega_r - \omega_{01})$$

$$= -\frac{1}{2}\hbar\Delta$$

$$\Delta \equiv \omega_{01} - \omega_r$$

We can will ignore this energy offset moving forward, referencing energies w.r.t. ground state

The block diagonal form makes it easy to find the eigenstates and their energies!
We only need to work with 2x2 matrices!

courtesy of L. DiCarlo (TU Delft)

JC Hamiltonian: 1-excitation manifold

$$H_{\mathrm{JC},1} \doteq \hbar \begin{pmatrix} \omega_{\mathrm{r}} & g \\ g & \omega_{01} \end{pmatrix}$$

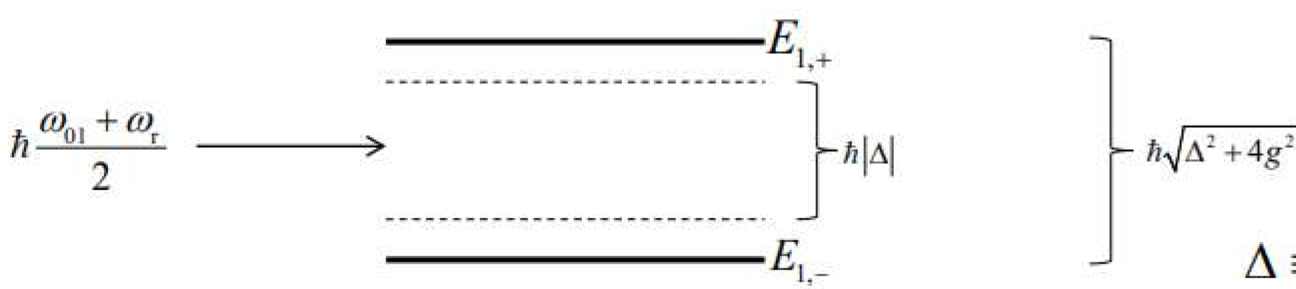
Finding the energy:

$$\det\left(H_{\mathrm{JC},1} - E_1 I\right) = \det\left(\begin{pmatrix} \hbar\omega_{\mathrm{r}} - E_1 & \hbar g \\ \hbar g & \hbar\omega_{01} - E_1 \end{pmatrix}\right) = E_1^{\,2} - \hbar\left(\omega_{\mathrm{r}} + \omega_{01}\right)E_1 + \hbar^2\left(\omega_{\mathrm{r}}\omega_{01} - g^2\right) = 0$$

$$\Leftrightarrow E_{1,\pm} = \hbar\left(\frac{\omega_{01} + \omega_{\mathrm{r}}}{2} \pm \frac{1}{2}\sqrt{\left(\omega_{01} - \omega_{\mathrm{r}}\right)^2 + 4g^2}\right)$$

Recall quadratic formula:

$$ax^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



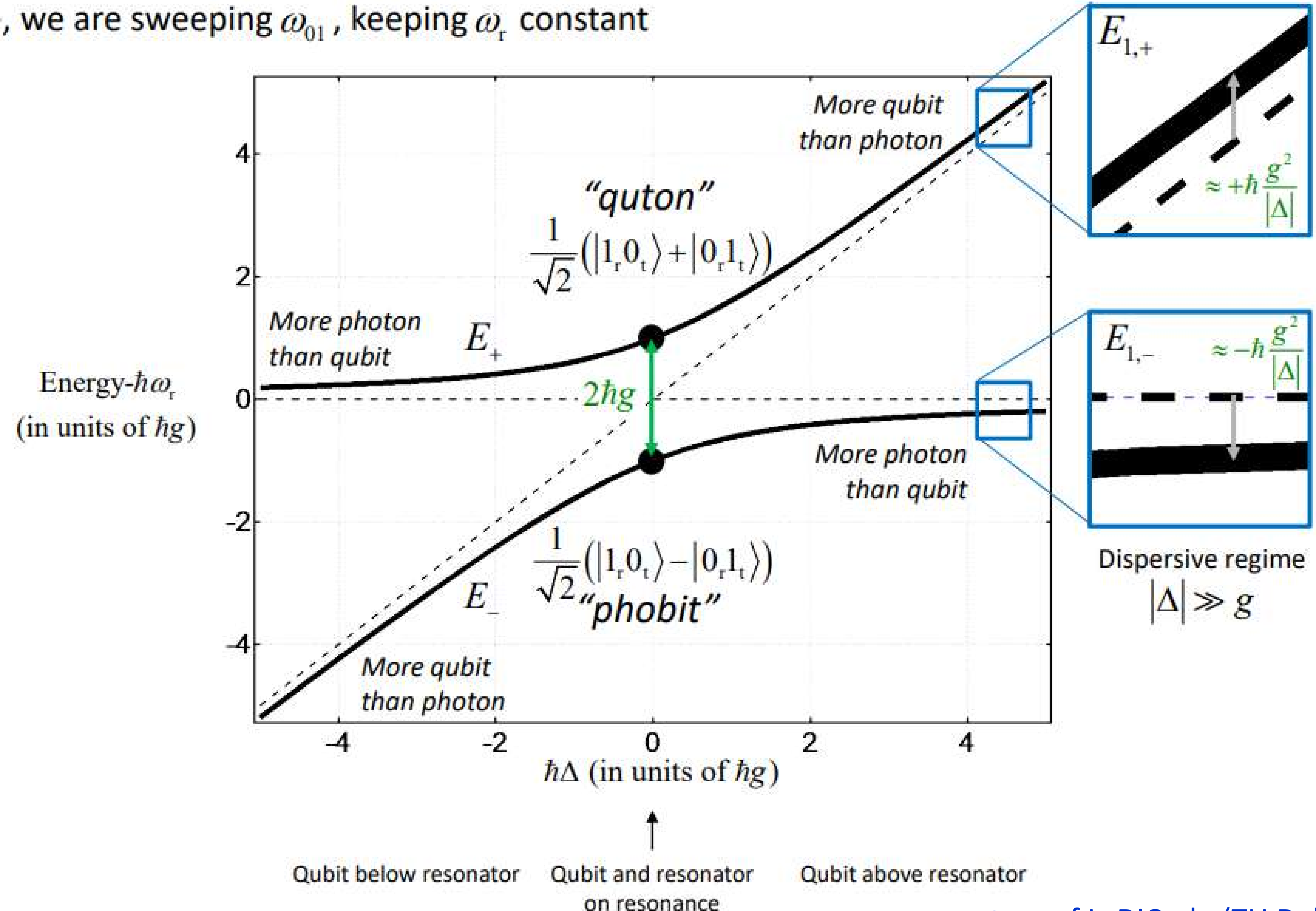
$$\Delta \equiv \omega_{01} - \omega_{\mathrm{r}}$$

The coupling *g* symmetrically *repels* the levels away from each other

courtesy of L. DiCarlo (TU Delft)

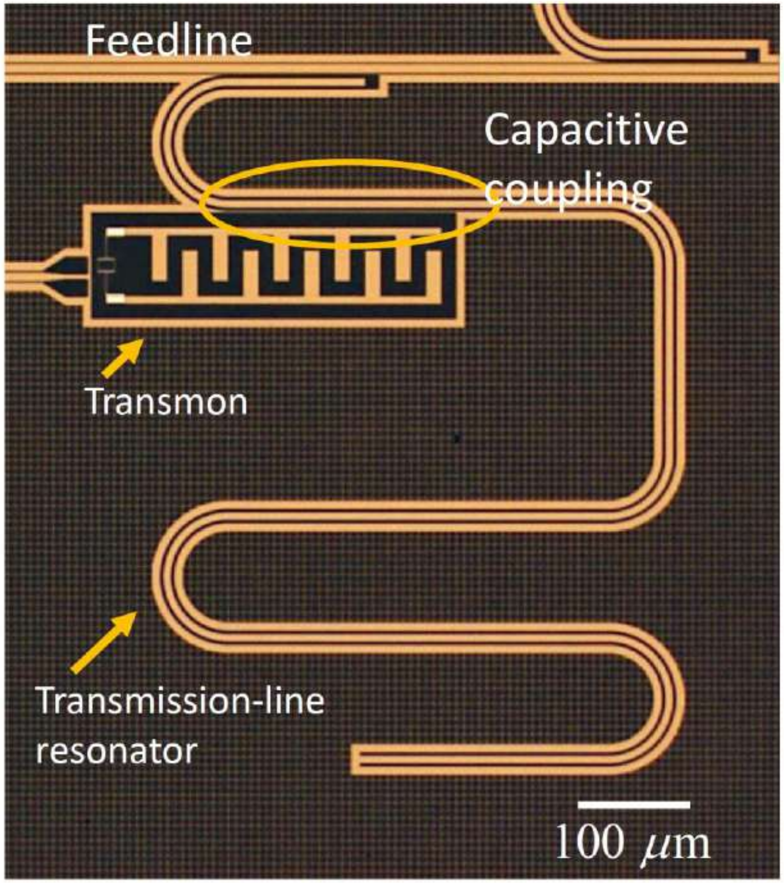
JC Hamiltonian: 1-excitation manifold

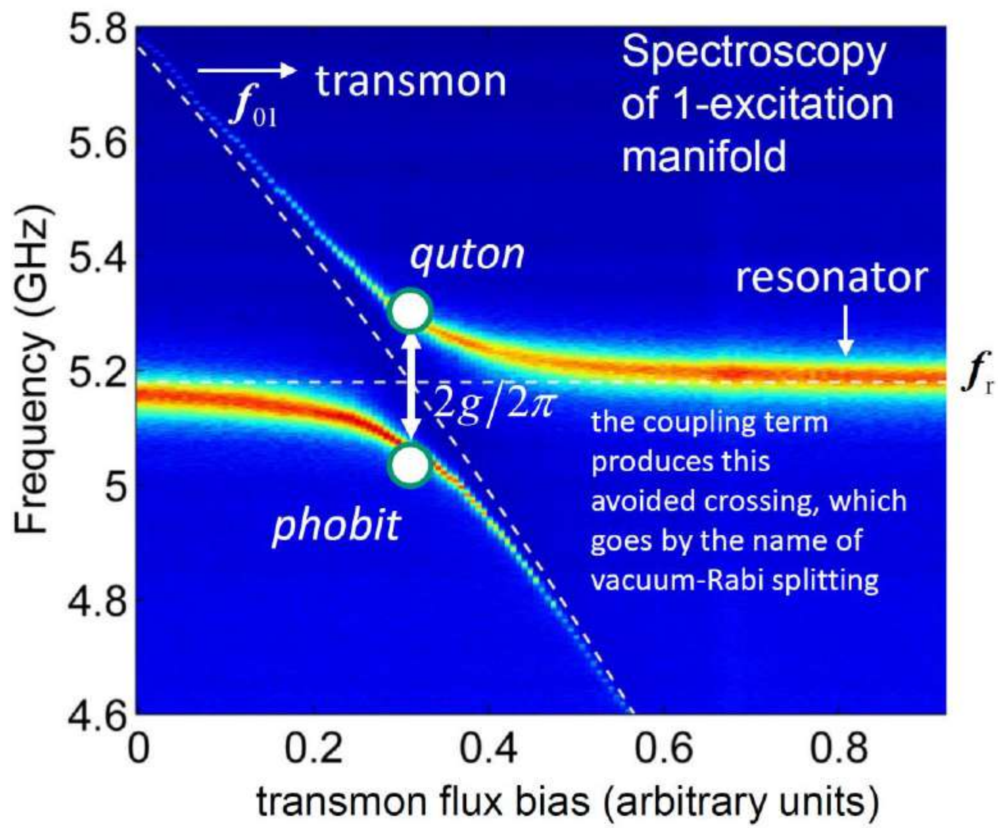
Here, we are sweeping ω_{01}, keeping ω_r constant



courtesy of L. DiCarlo (TU Delft)

Observation of the vacuum Rabi splitting with electrical circuits





"quton" $\frac{1}{\sqrt{2}}\left(|1_r 0_t\rangle + |0_r 1_t\rangle\right)$

"phobit" $\frac{1}{\sqrt{2}}\left(|1_r 0_t\rangle - |0_r 1_t\rangle\right)$

Data: A.A. Houck, D. I. Schuster et al., Nature (2007)

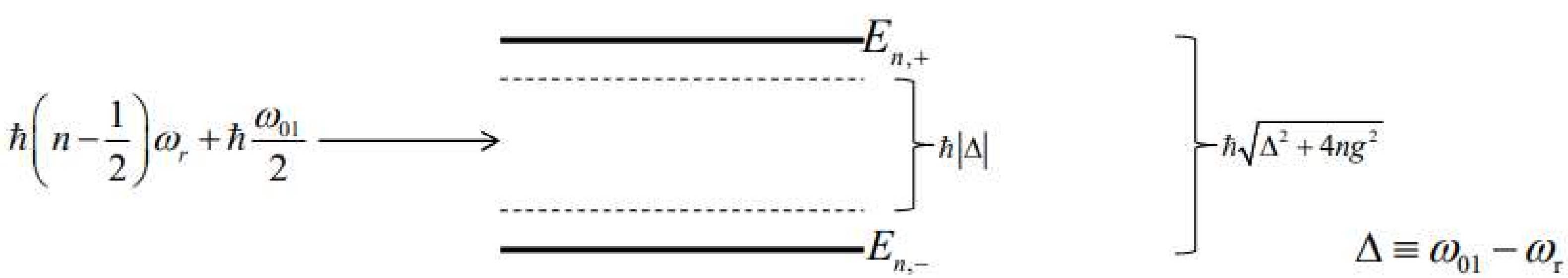
JC Hamiltonian: *n*-excitation manifold

$$H_{\mathrm{JC},n} \doteq \hbar \begin{pmatrix} n\omega_{\mathrm{r}} & \sqrt{n}g \\ \sqrt{n}g & (n-1)\omega_{\mathrm{r}} + \omega_{01} \end{pmatrix}$$

Finding the energy:

$$\det\left(H_{\mathrm{JC},n} - E_n I\right) = 0$$

$$\Leftrightarrow E_{n,\pm} = \hbar\left(\left(n-\frac{1}{2}\right)\omega_r + \frac{\omega_{01}}{2} \pm \frac{1}{2}\sqrt{\left(\omega_{01}-\omega_{\mathrm{r}}\right)^2 + 4ng^2}\right)$$

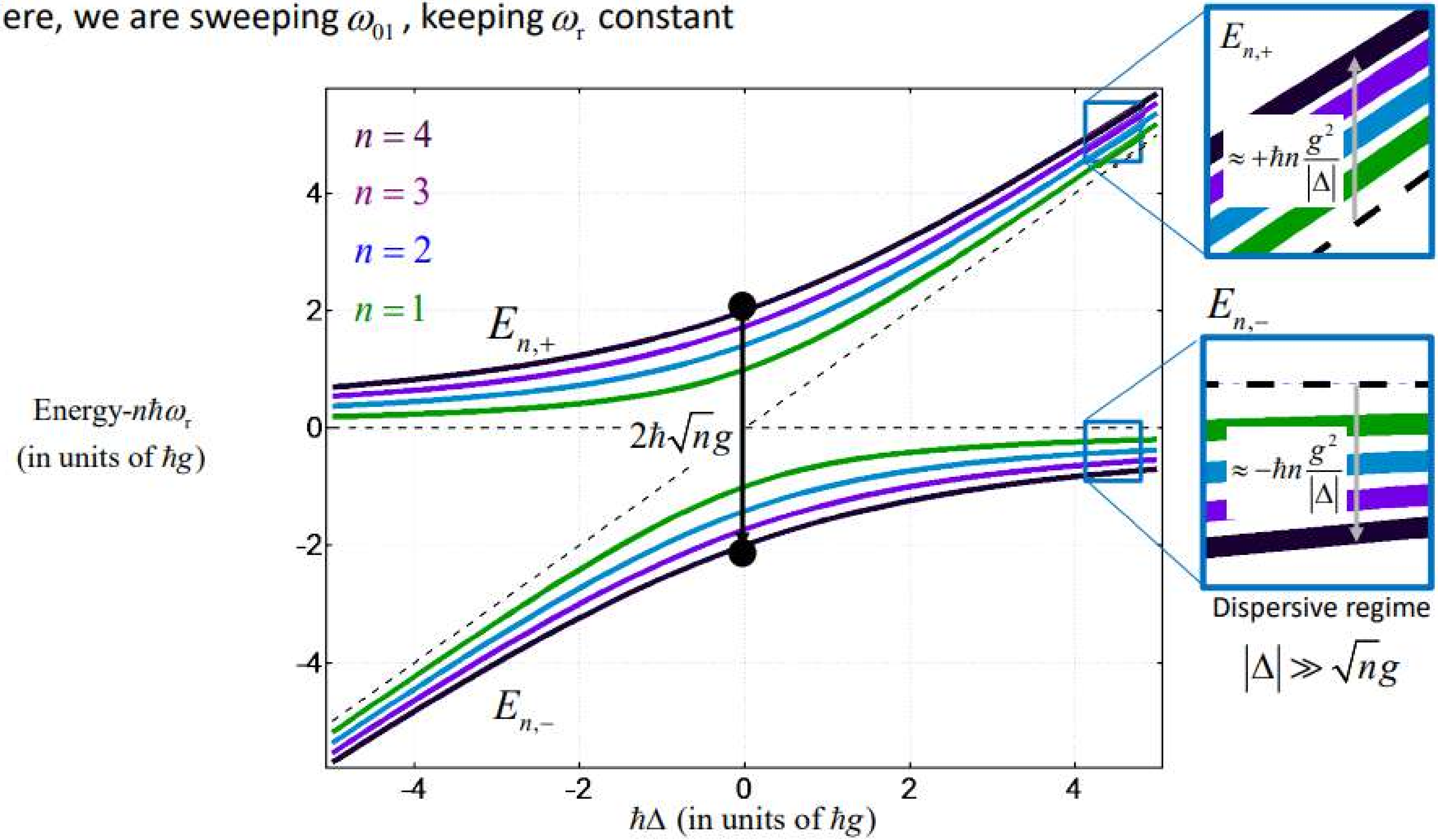


The coupling *g* symmetrically *repels* the levels away from each other

courtesy of L. DiCarlo (TU Delft)

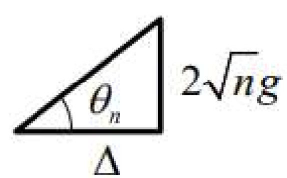
JC Hamiltonian: *n*-excitation manifold

Here, we are sweeping ω_{01} , keeping ω_r constant



$$|\psi_{n,+}\rangle = \sin\left(\frac{\theta_n}{2}\right)|n_r 0_t\rangle + \cos\left(\frac{\theta_n}{2}\right)|(n-1)_r 1_t\rangle$$

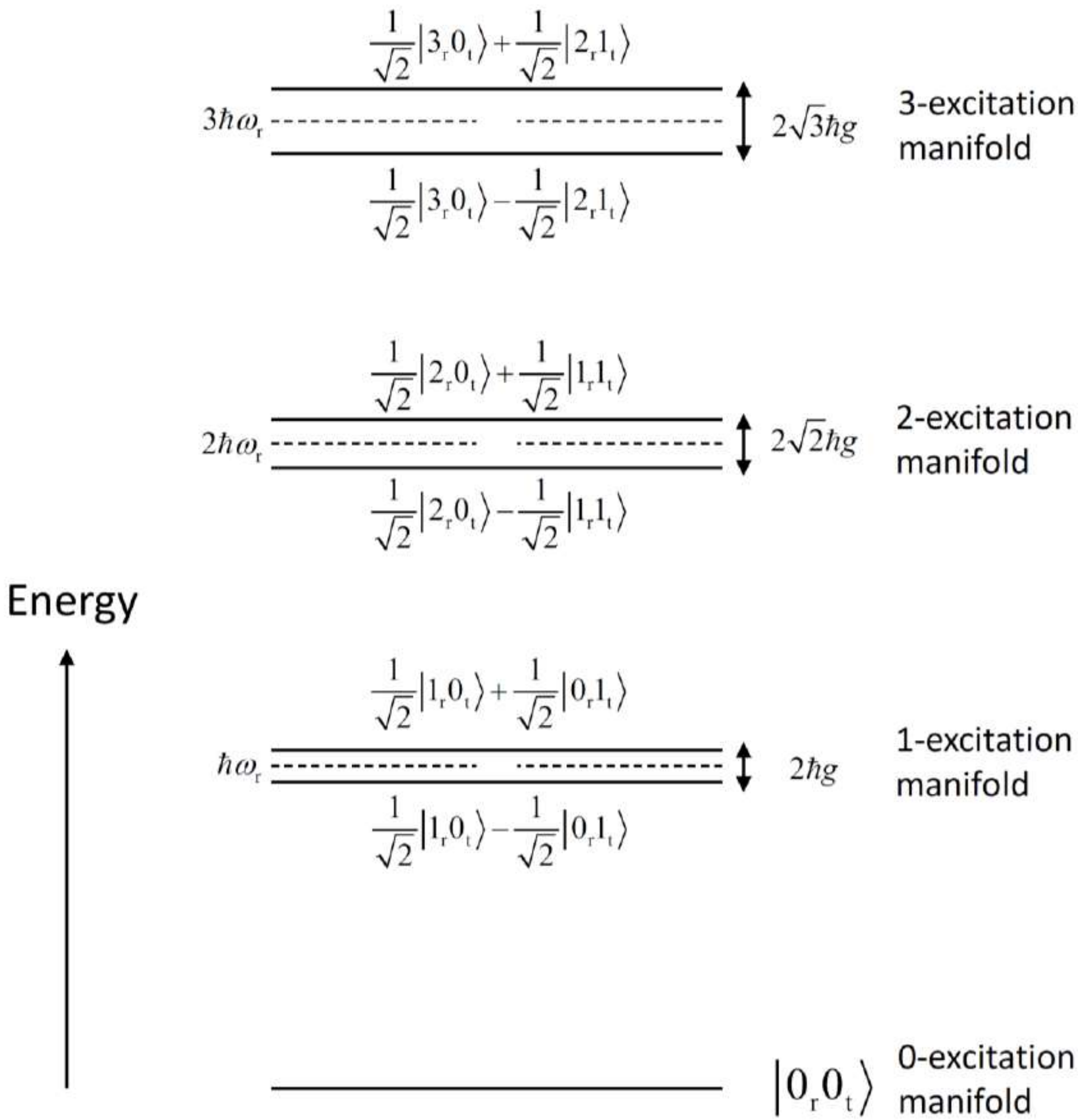
$$|\psi_{n,-}\rangle = \cos\left(\frac{\theta_n}{2}\right)|n_r 0_t\rangle - \sin\left(\frac{\theta_n}{2}\right)|n-1_r 1_t\rangle$$



courtesy of L. DiCarlo (TU Delft)

Spectrum on the JC Hamiltonian on resonance

When the resonator and qubit are on resonance: $\Delta = \omega_{01} - \omega_{\mathrm{r}} = 0$

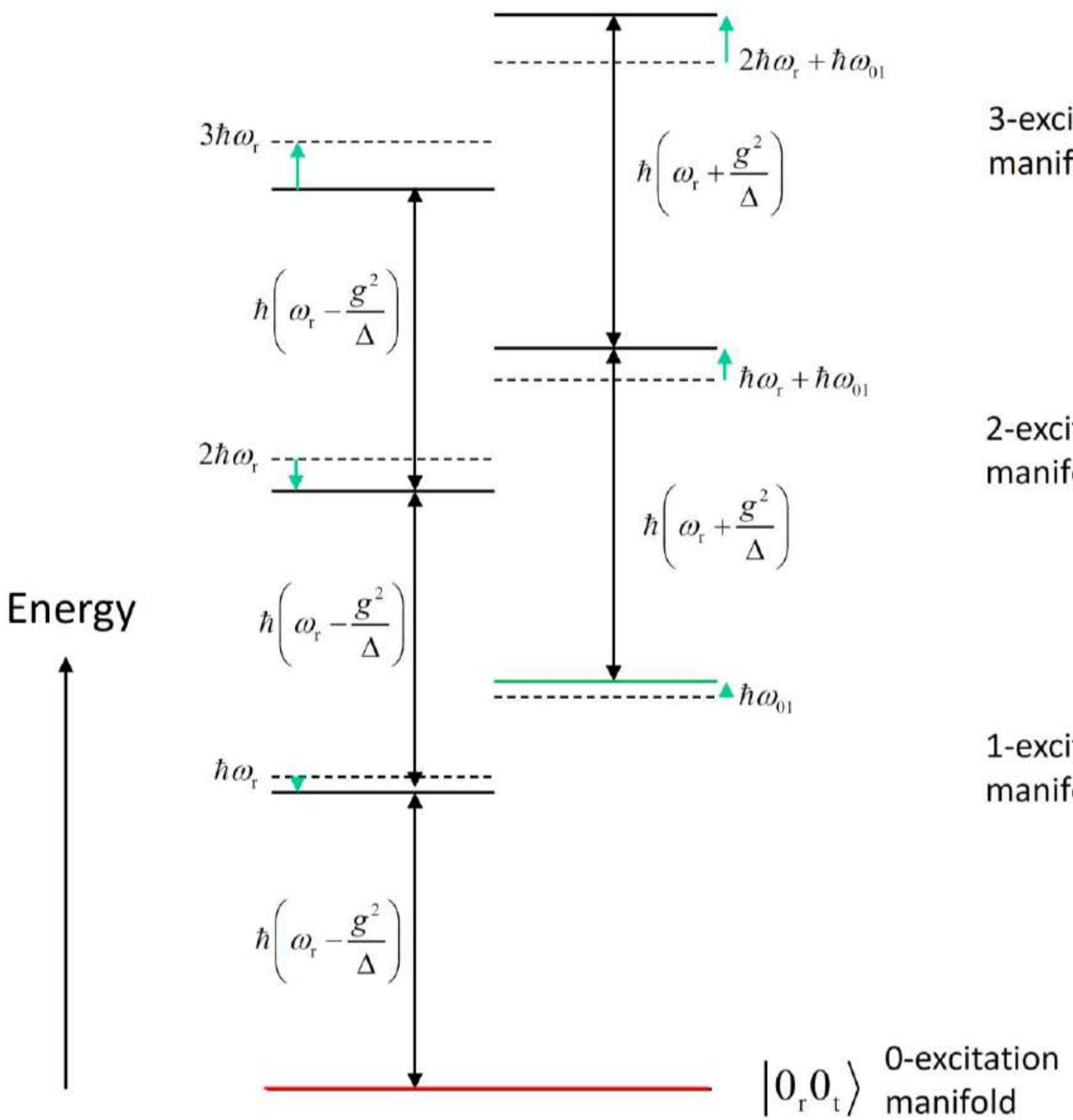


- The interaction term fully hybridizes the two levels in each manifold.
- The eigenstates in the *n*-th excitation manifold consist of the symmetric and antisymmetric superpositions of $|n_{\mathrm{r}}0_{\mathrm{t}}\rangle$ and $|(n-1)_{\mathrm{r}}1_{\mathrm{t}}\rangle$.

courtesy of L. DiCarlo (TU Delft)

Spectrum on the JC Hamiltonian in dispersive regime

In the dispersive regime: $|\Delta| \gg g$



- In the dispersive regime, there are two (approximately) harmonic ladders.
- The transition frequencies in these ladders are

$$\omega_r \pm \frac{g^2}{\Delta}$$

Strong coupling regime with superconducting qubits

$$g = 2e\delta V_0 (C_g / C)\langle 0|\hat{n}|1\rangle$$

GEOMETRICAL dependence of g

→ Easily tuned by circuit design
Can be made very large !

(Typically : 0 – 200MHz)

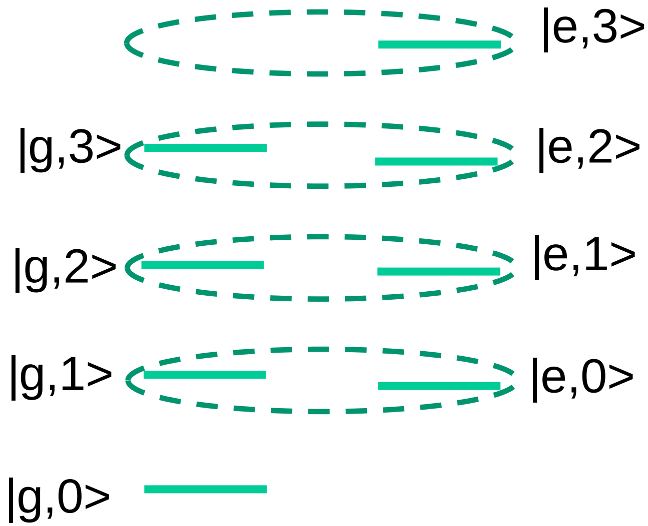
$$\boxed{g \approx 200MHz >> \gamma, \kappa \approx 10 - 500\ kHz}$$

Strong coupling condition naturally fulfilled with superconducting circuits

(Q=100 enough for strong coupling !!)

courtesy CEA Saclay

The Jaynes-Cummings model



$$\boxed{H_{J-C} = -\frac{\omega_{ge}}{2}\sigma_z + \omega_c(a^+a + 1/2) + g(\sigma^+ a + \sigma^- a^+)}$$

H_{J-C} couples only level doublets

$\{|g,n+1>, |e,n>\}$

Exact diagonalization possible

Restriction of $H_{J\text{-}C}$ to $\{|g,n+1>, |e,n>\}$

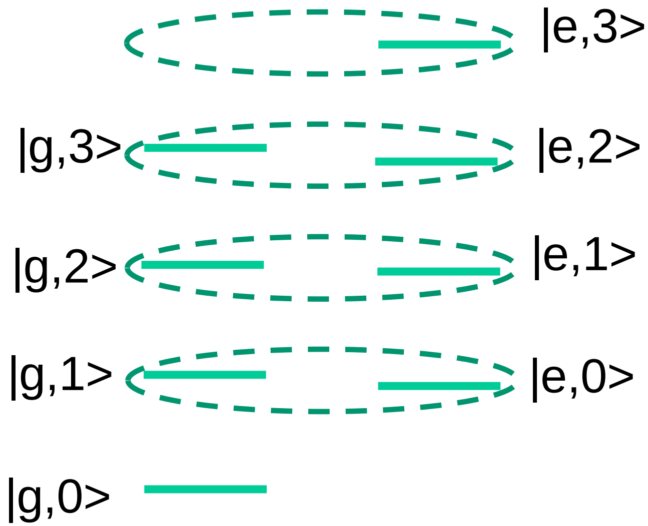
$$\left(\delta = \omega_{ge} - \omega_c\right)$$

$$\begin{array}{ccc} & |g,n+1\rangle & |e,n\rangle \\ \begin{array}{c}|g,n+1\rangle \\ |e,n\rangle\end{array} & \multicolumn{2}{c}{\begin{pmatrix} (n+1)\omega_c - \delta/2 & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1)\omega_c + \delta/2 \end{pmatrix}} \end{array}$$

Note : |g,0> state is left unchanged by $H_{J\text{-}C}$ with $E_{g,0}=-\delta/2$

courtesy CEA Saclay

The Jaynes-Cummings model



$$\boxed{H_{J-C} = -\frac{\omega_{ge}}{2}\sigma_z + \omega_c(a^+a + 1/2) + g(\sigma^+ a + \sigma^- a^+)}$$

H_{J-C} couples only level doublets

$$\{|g,n+1\rangle , |e,n\rangle\}$$

⟹ Exact diagonalization possible

$$\left(\delta = \omega_{ge} - \omega_c\right)$$

Coupled states

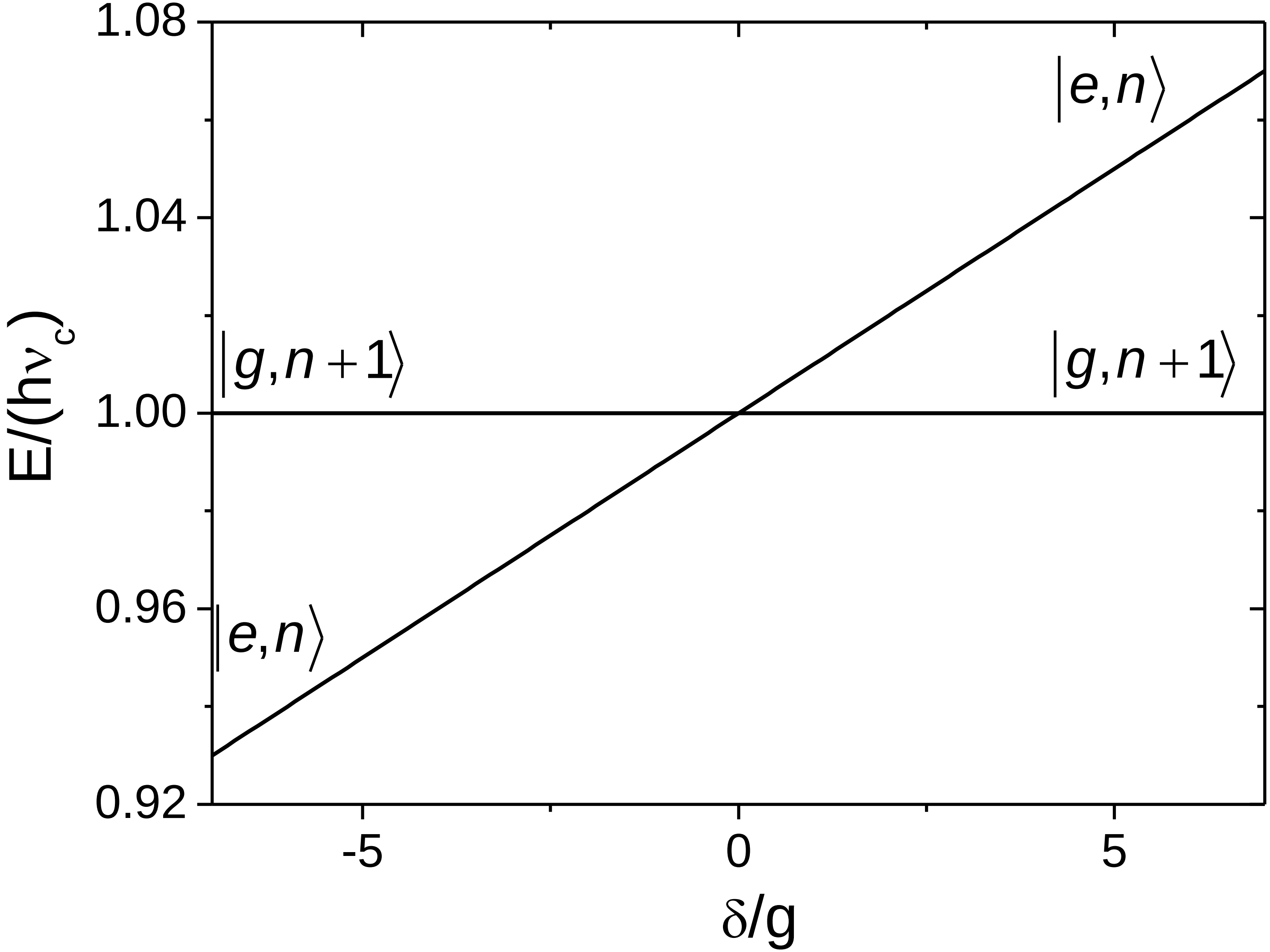
$$|+,n\rangle = \cos\theta_n |e,n\rangle + \sin\theta_n |g,n+1\rangle \qquad E_{+,n} = (n+1)\hbar\omega_c + \frac{\hbar}{2}\sqrt{4g^2(n+1)+\delta^2}$$

$$|-,n\rangle = -\sin\theta_n |e,n\rangle + \cos\theta_n |g,n+1\rangle \qquad E_{-,n} = (n+1)\hbar\omega_c - \frac{\hbar}{2}\sqrt{4g^2(n+1)+\delta^2}$$

$$\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\delta}\right)$$

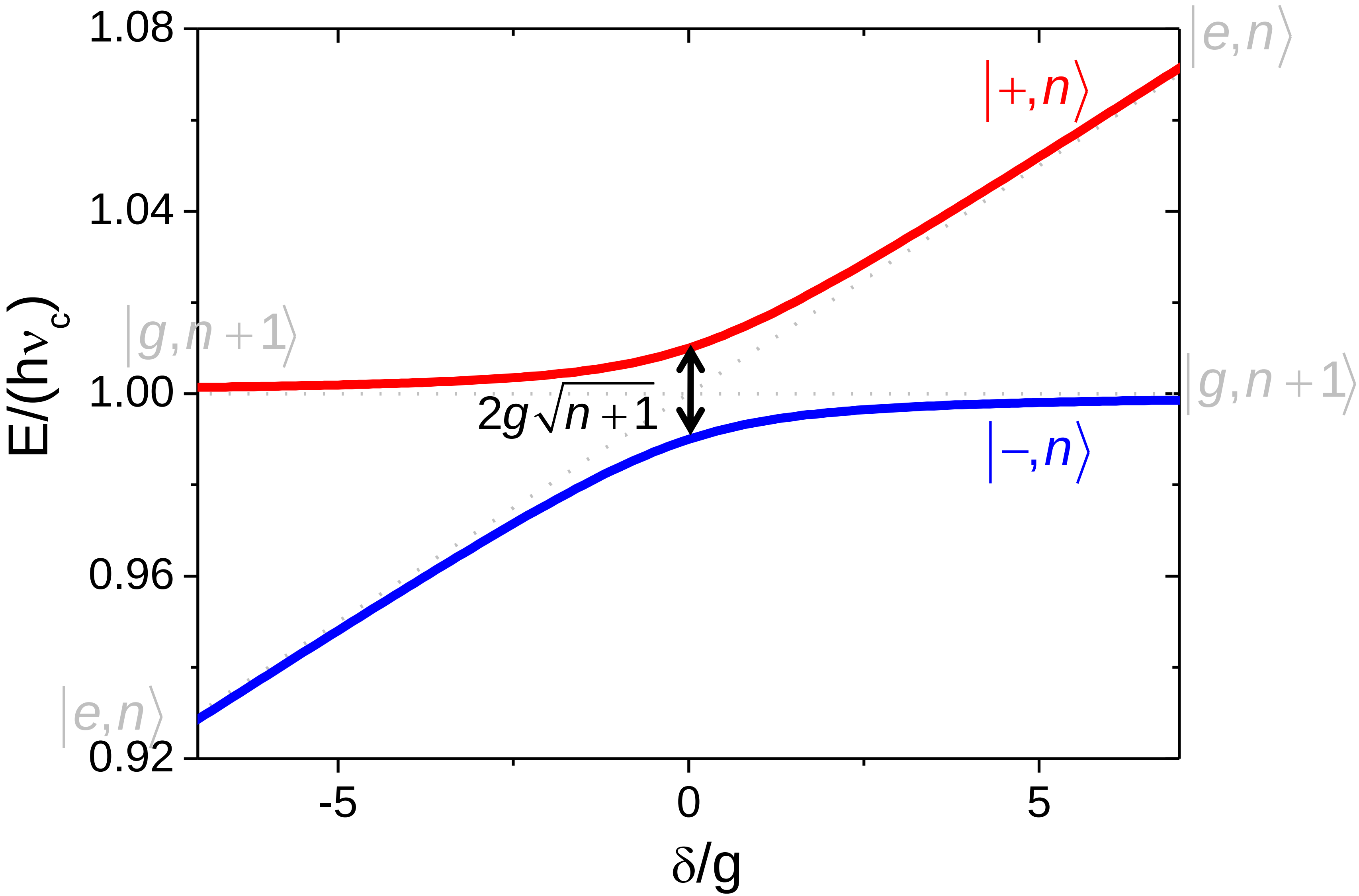
courtesy CEA Saclay

The Jaynes-Cummings model



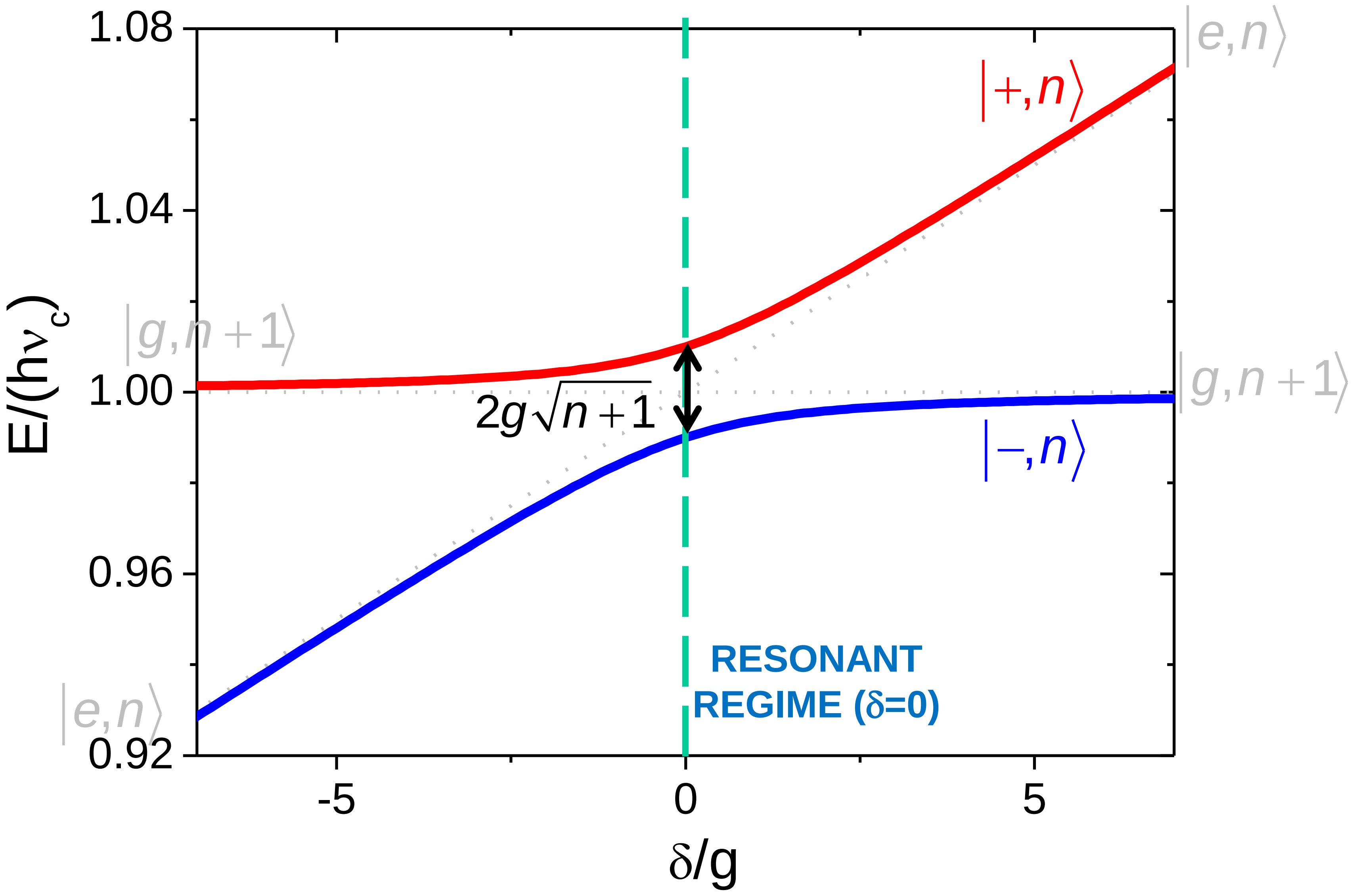
courtesy CEA Saclay

The Jaynes-Cummings model



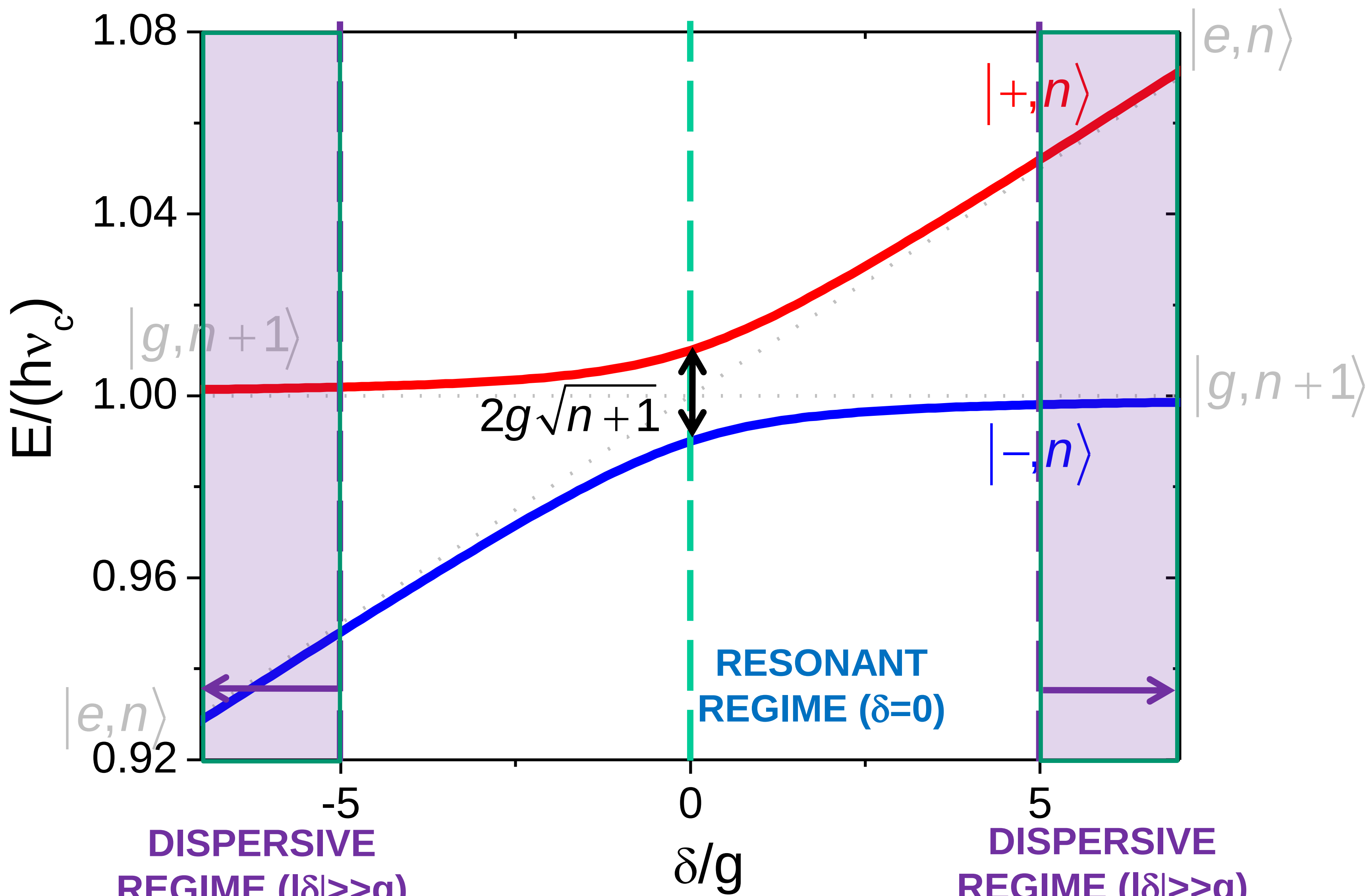
courtesy CEA Saclay

Two interesting limits



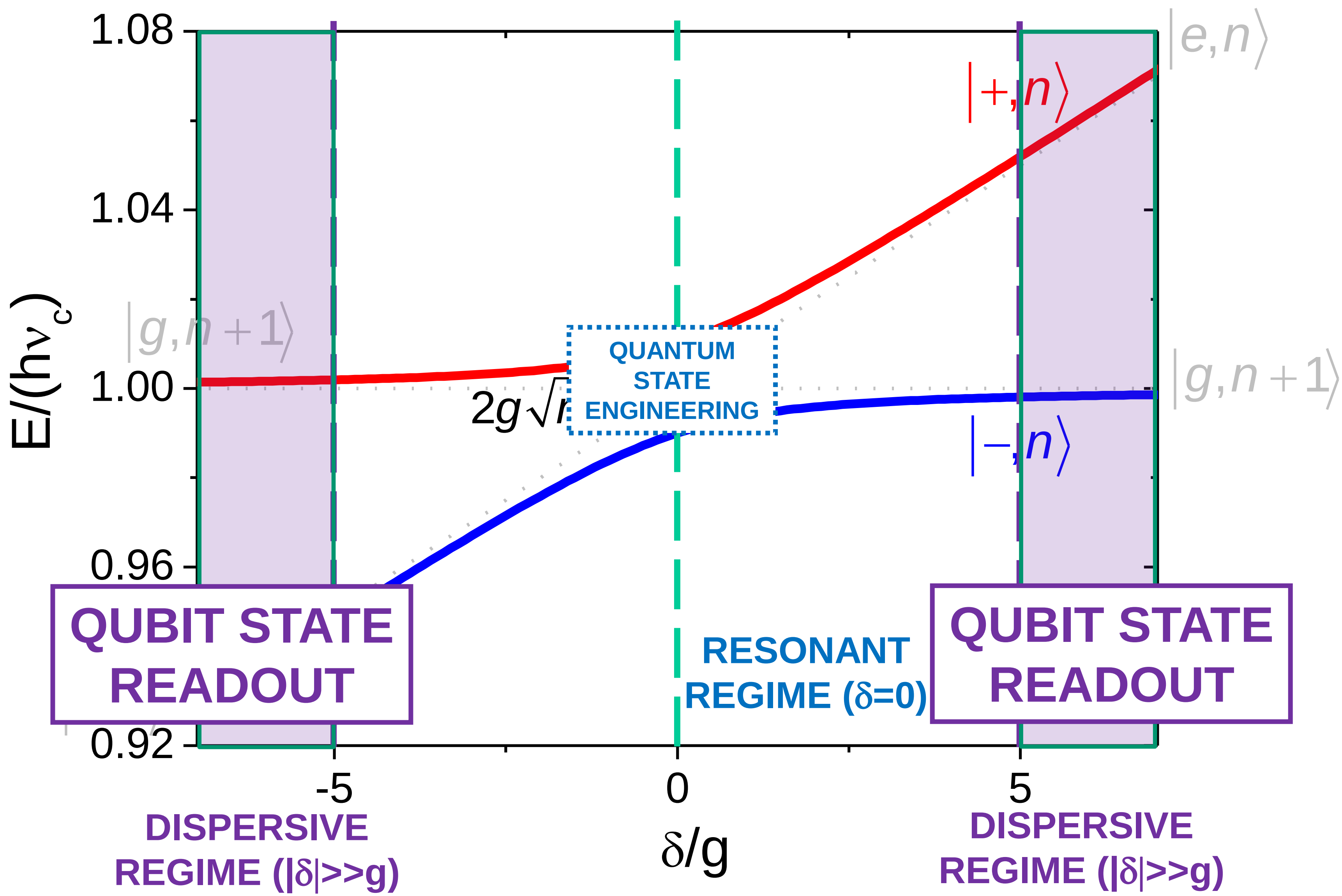
courtesy CEA Saclay

Two interesting limits



courtesy CEA Saclay

Two interesting limits



courtesy CEA Saclay

The Jaynes-Cummings model : dispersive interaction $\delta >> g$

$$H_{J-C}/\hbar \approx -\frac{\omega_{ge}+\chi}{2}\sigma_z + \boxed{(\omega_c+\chi\sigma_z)}a^+a = \boxed{-\frac{\omega_{ge}+2\chi(a^+a+1/2)}{2}}\sigma_z + \omega_c a^+a$$

with $\chi = \frac{g^2}{\delta}$ the dispersive coupling constant

1) Qubit state-dependent **shift of the cavity frequency**

$$\boxed{\tilde{\omega}_c = \omega_c + \chi\sigma_z}$$

⟹ **Cavity can probe the qubit state non-destructively**

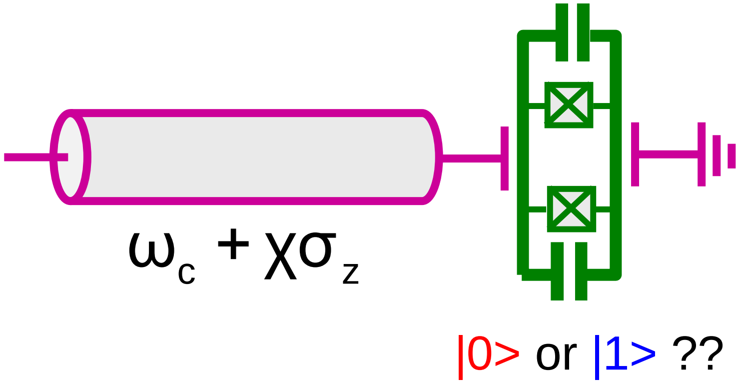
2) **Light shift** of the qubit transition in the presence of n photons

$$\boxed{\delta\omega_{ge} = -2\chi n}$$

⟹ **Field in the resonator causes qubit frequency shift and decoherence**

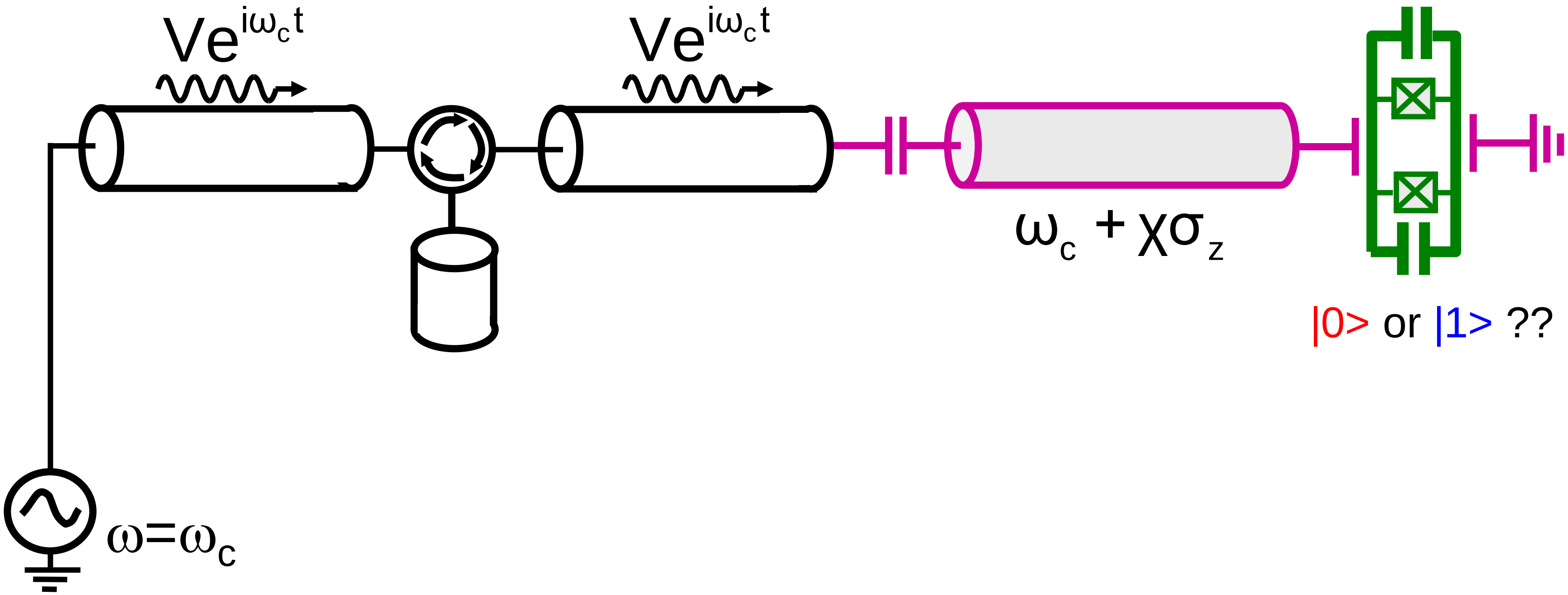
courtesy CEA Saclay

Dispersive readout of a transmon: principle



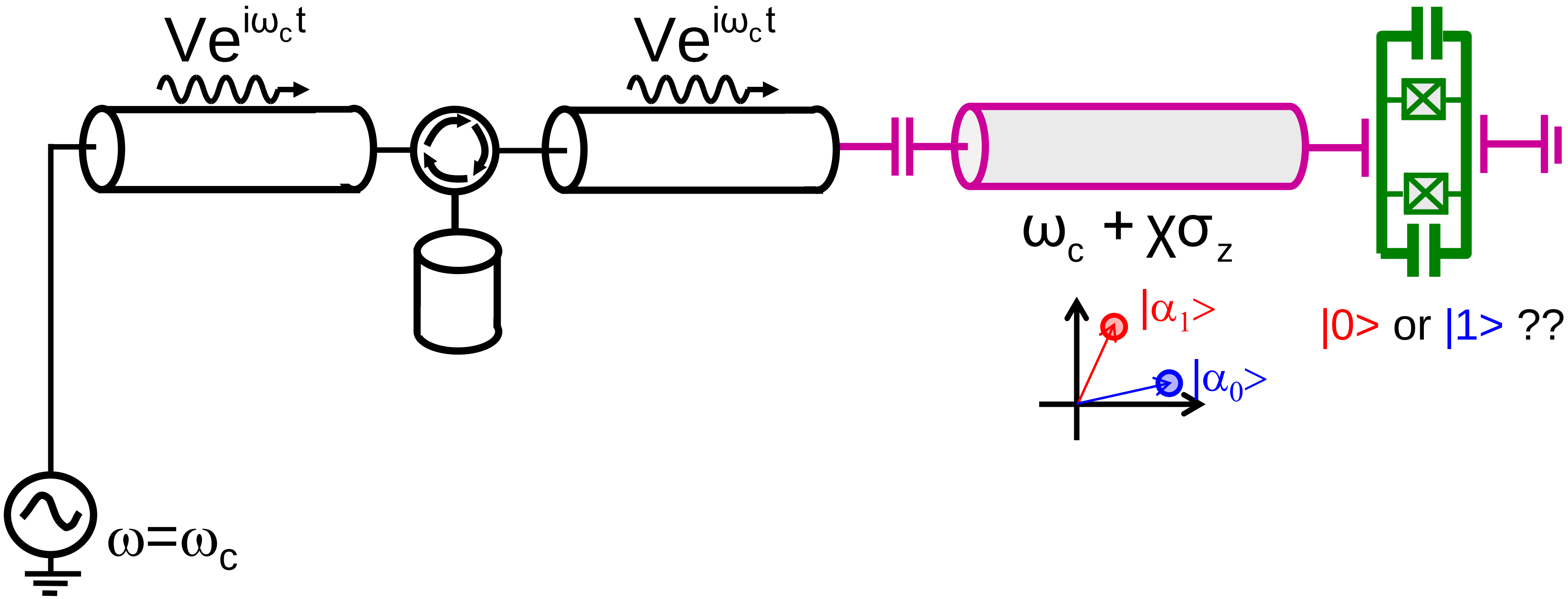
courtesy CEA Saclay

Dispersive readout of a transmon: principle



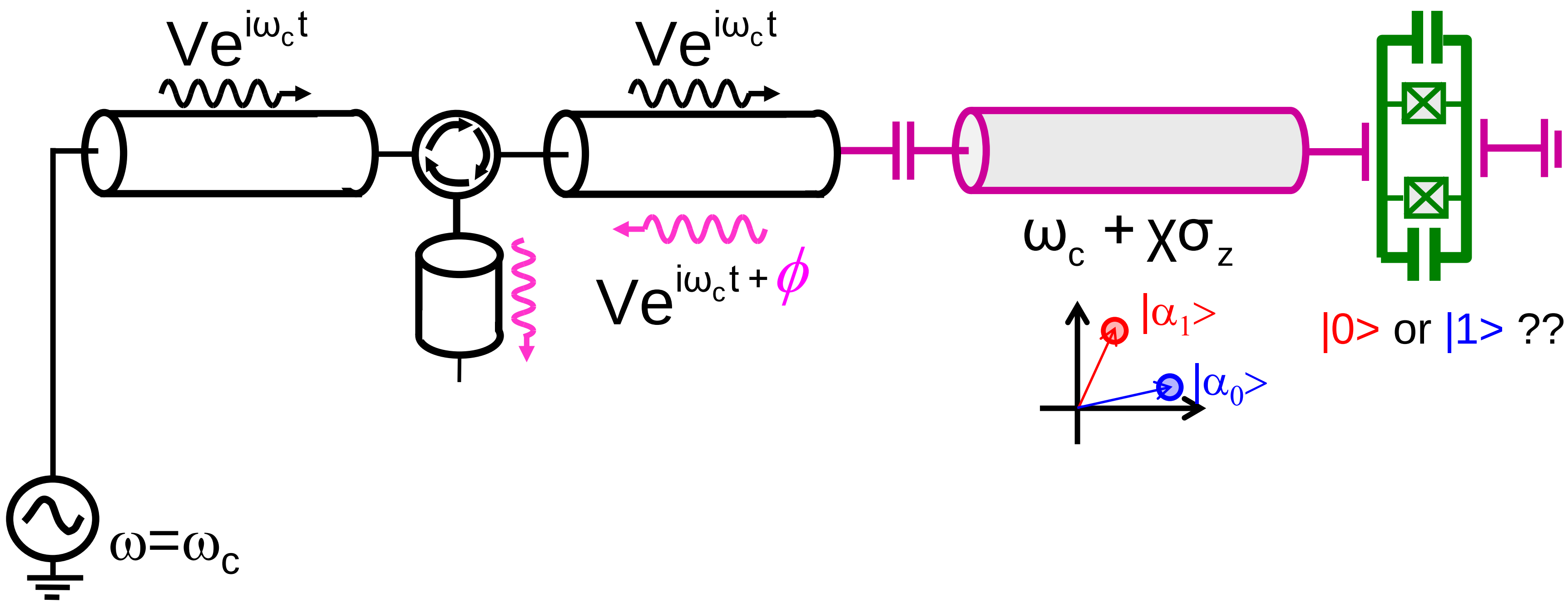
courtesy CEA Saclay

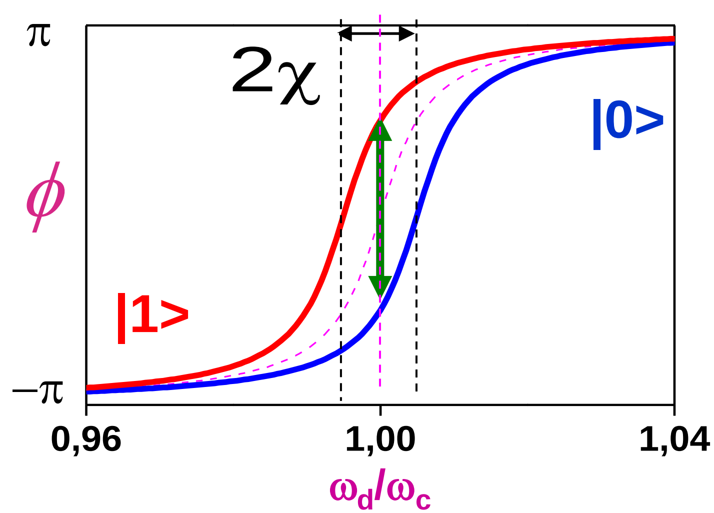
Dispersive readout of a transmon: principle



courtesy CEA Saclay

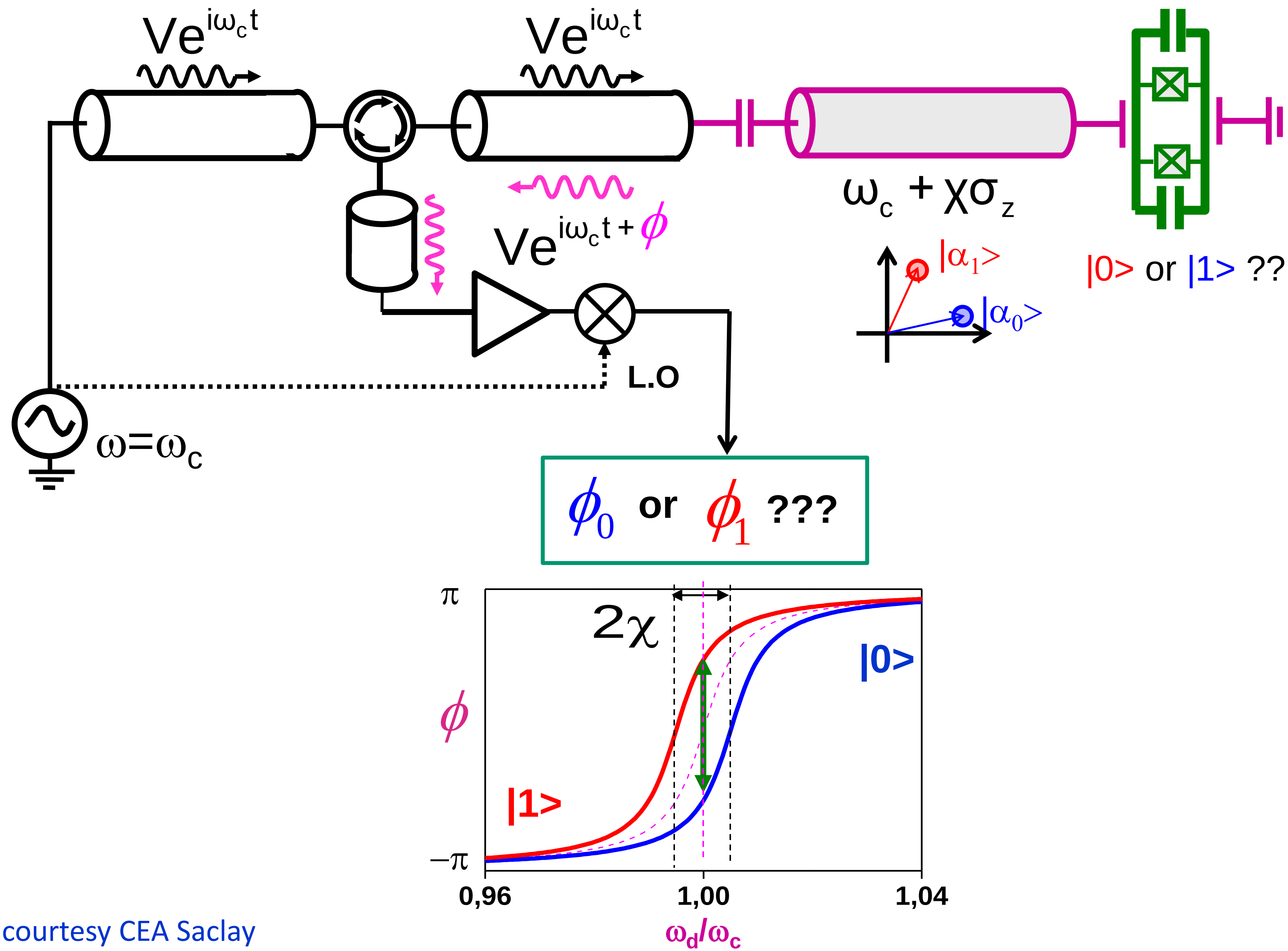
Dispersive readout of a transmon: principle





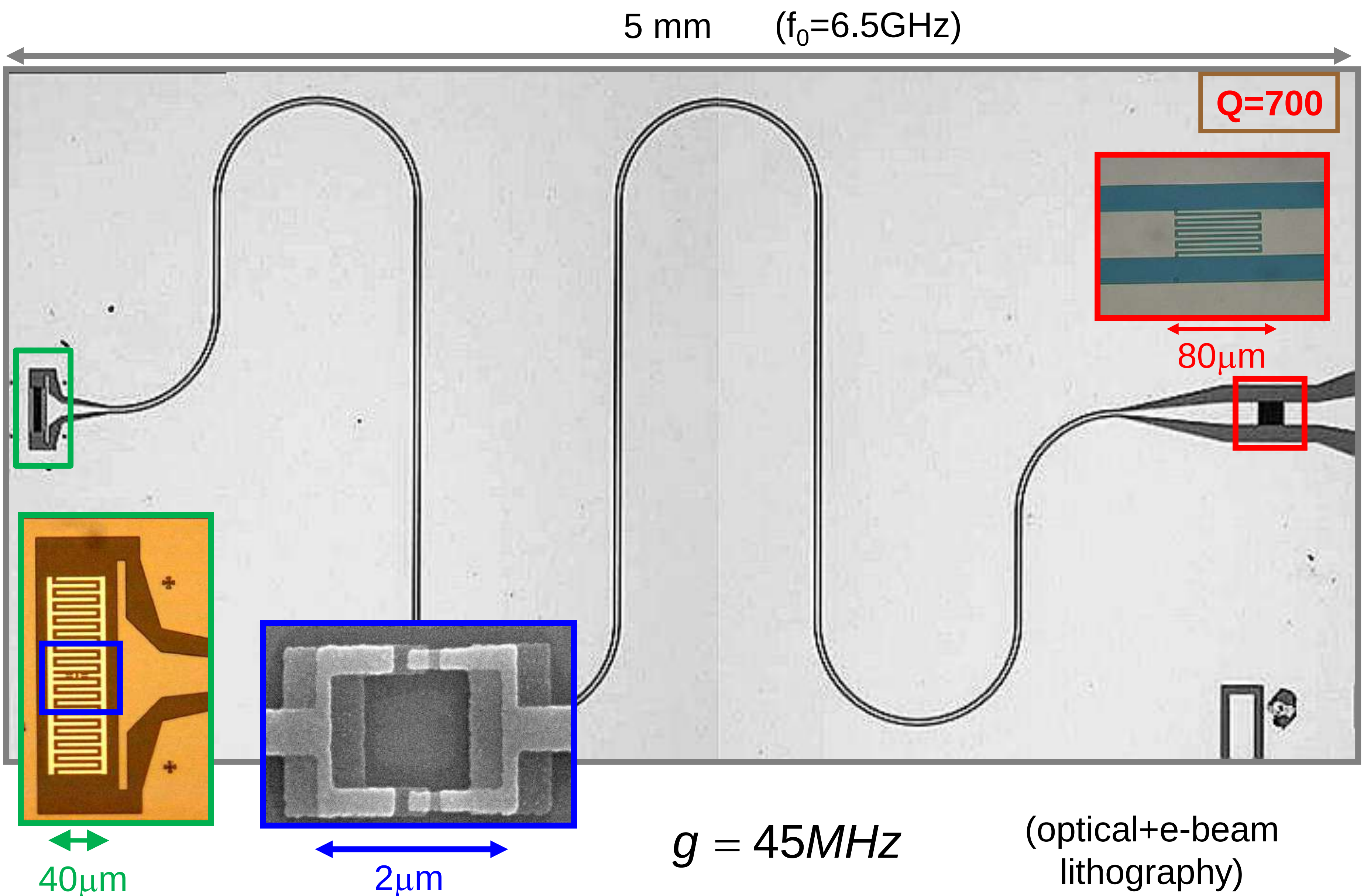
courtesy CEA Saclay

Dispersive readout of a transmon: principle



courtesy CEA Saclay

Typical implementation (Saclay)

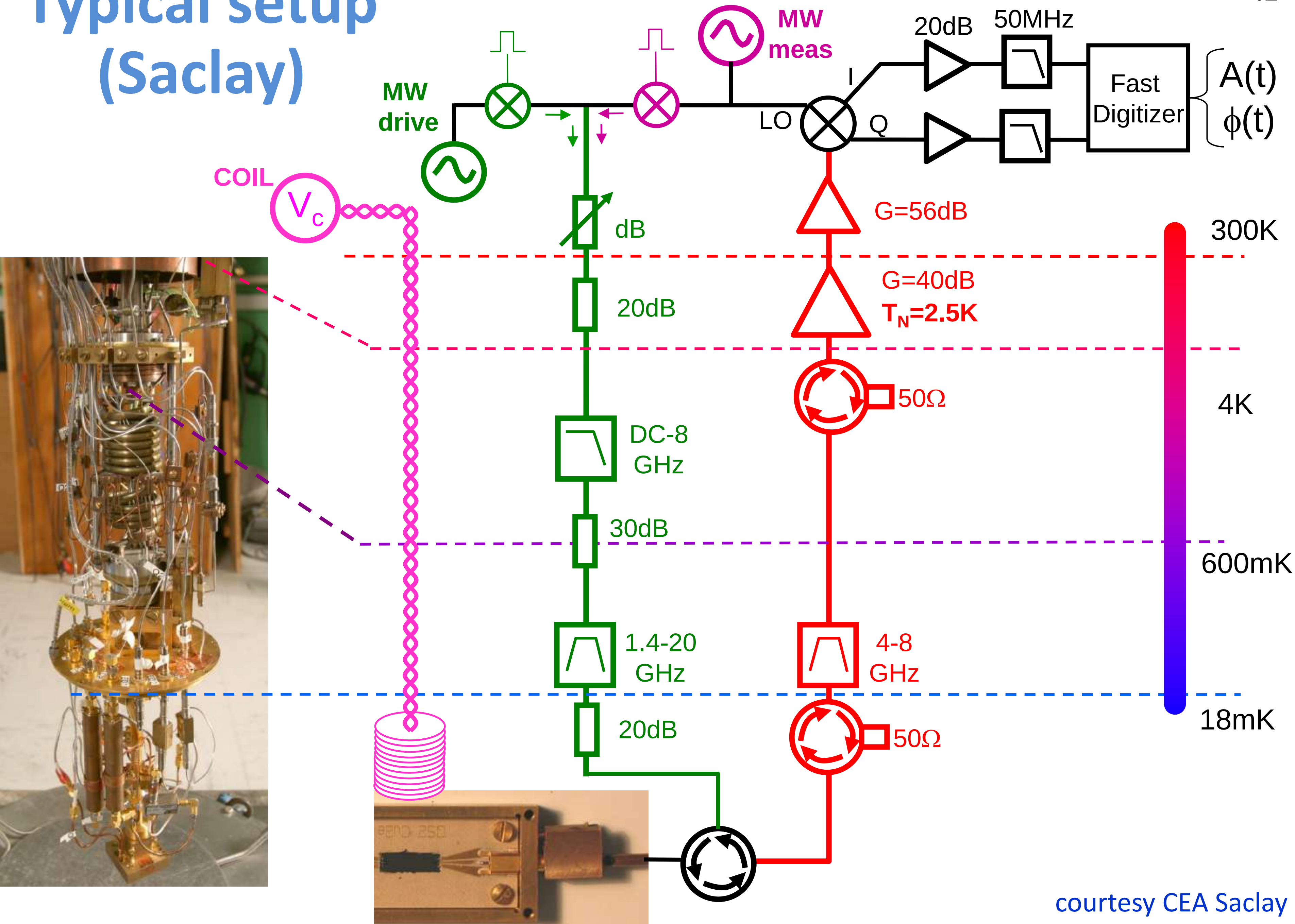


$$g = 45MHz$$

(optical+e-beam lithography)

courtesy CEA Saclay

Typical setup (Saclay)



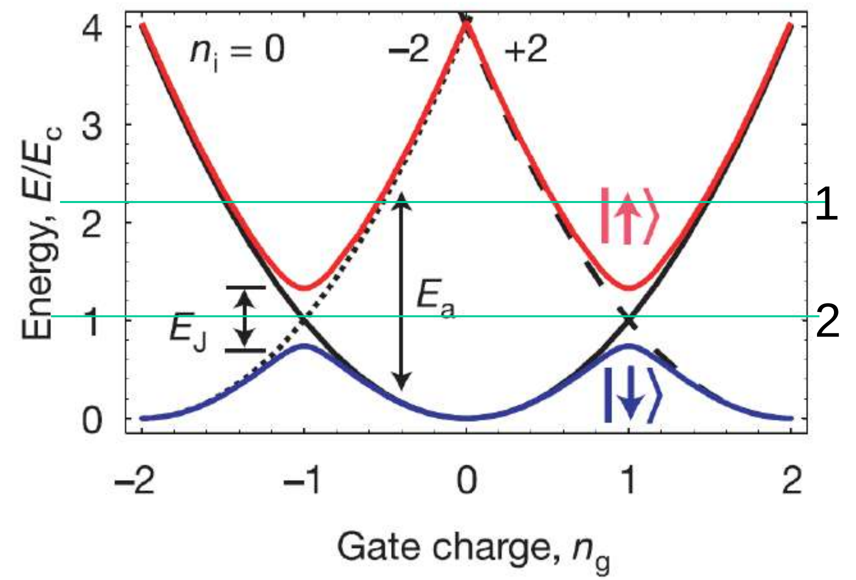
courtesy CEA Saclay

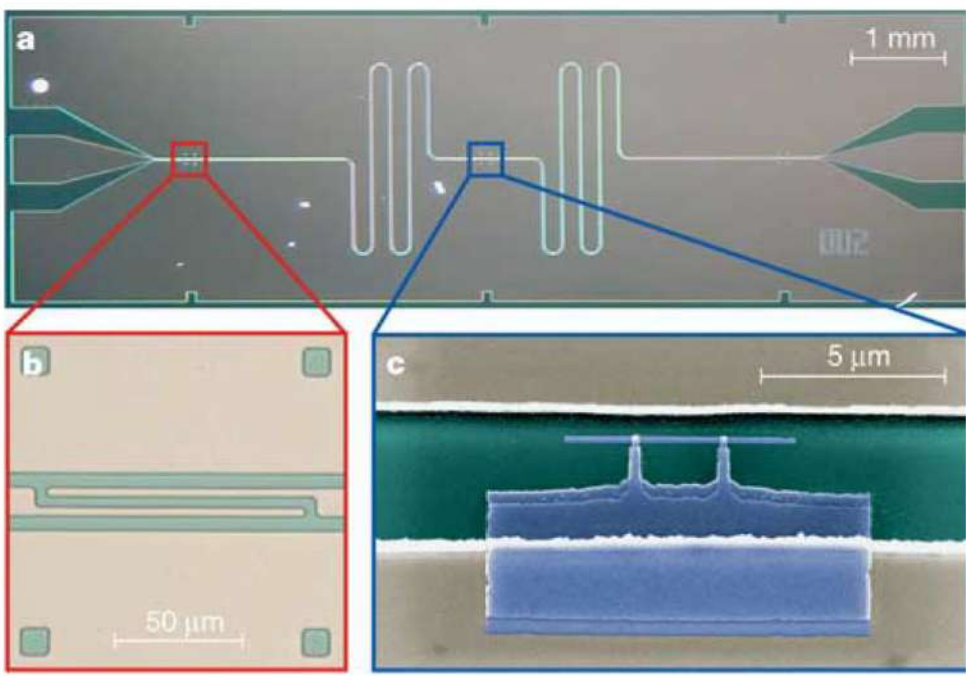
A. Wallraff et al., Nature **431**, 162 (2004)

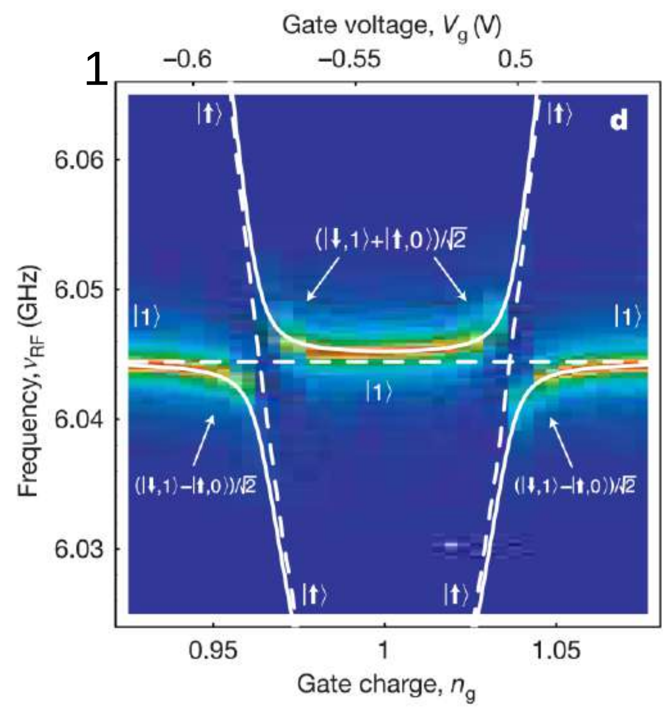
Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics

A. Wallraff[1], D. I. Schuster[1], A. Blais[1], L. Frunzio[1], R.- S. Huang[1,2], J. Majer[1], S. Kumar[1], S. M. Girvin[1] & R. J. Schoelkopf[1]

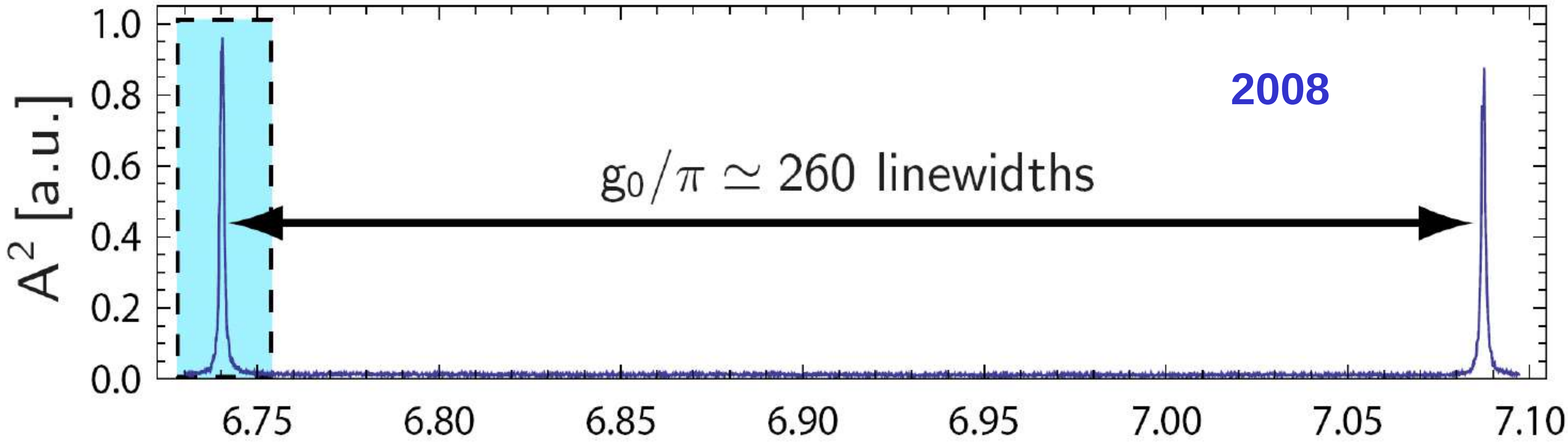
[1]*Departments of Applied Physics and Physics, Yale University, New Haven, Connecticut 06520, USA*
[2]*Department of Physics, Indiana University, Bloomington, Indiana 47405, USA*







Observation of the vacuum Rabi splitting with electrical circuits



cavity QED

R.J. Thompson et al., PRL **68**, 1132 (1992)

I. Schuster et al. Nature Physics **4,** 382-385 (2008)

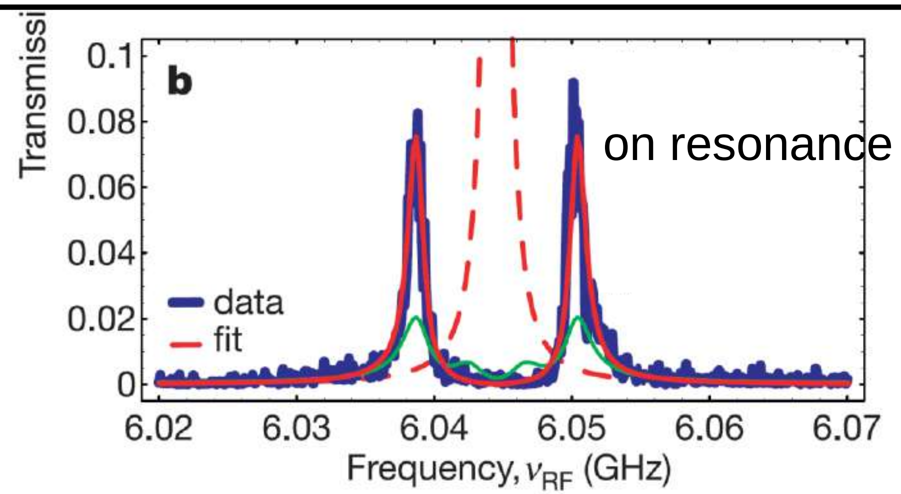
circuit QED

A. Wallraff et al., Nature **431**, 162 (2004)

quantum dot systems

J.P. Reithmaier et al., Nature **432**, 197 (2004)

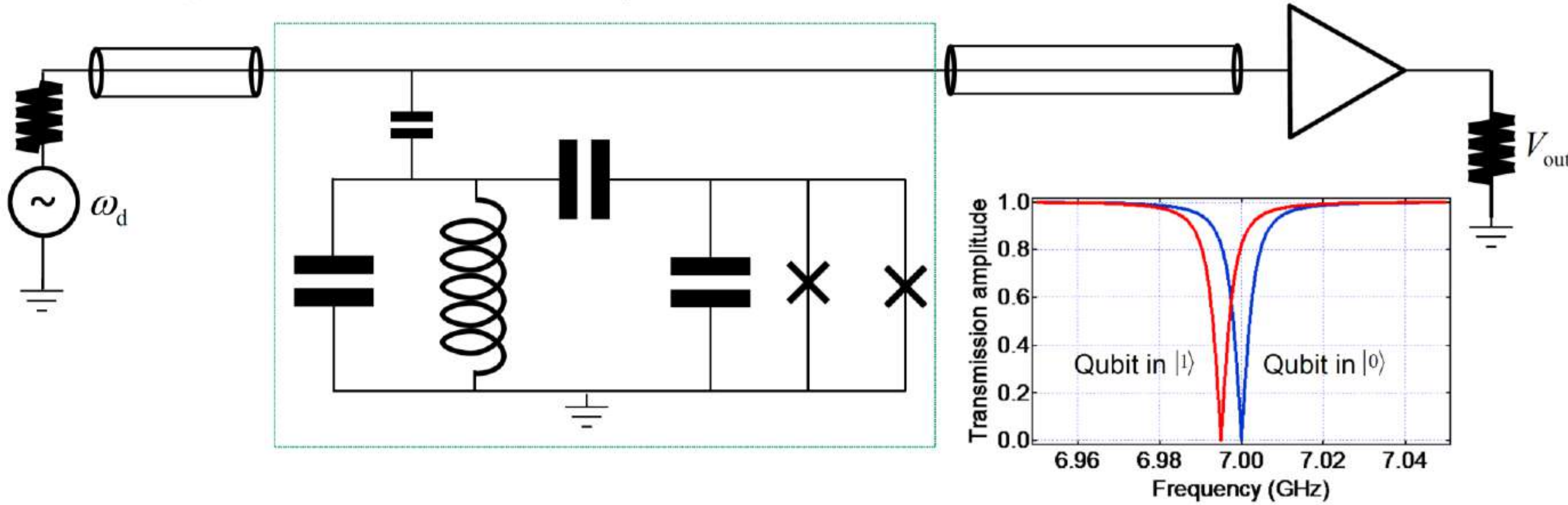
T. Yoshie et al., Nature **432**, 200 (2004)



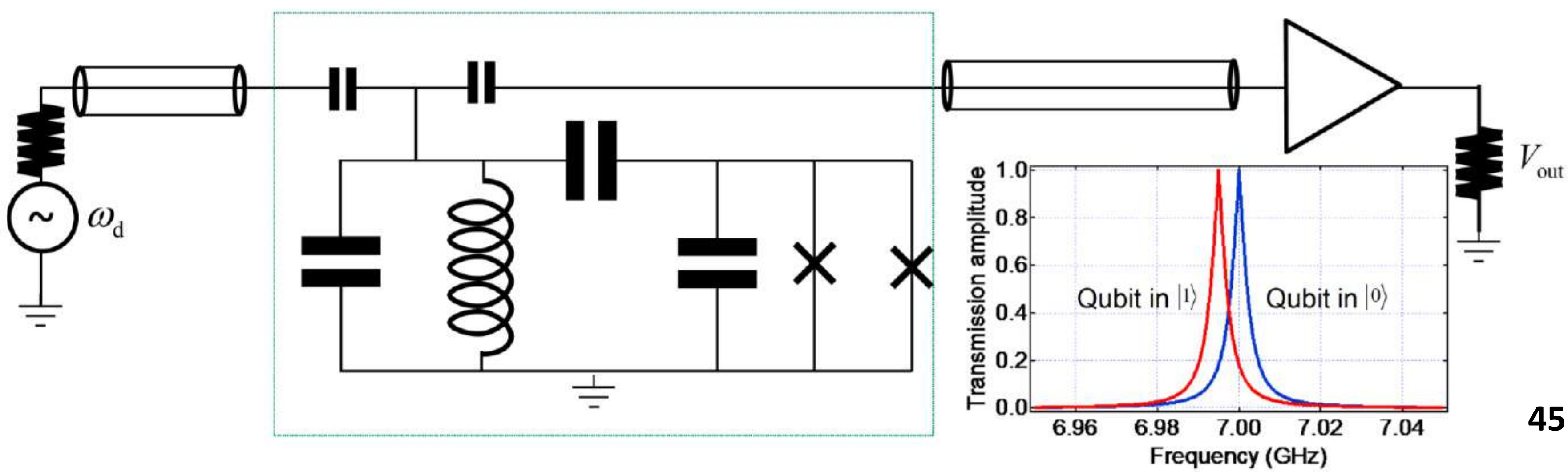
A. Wallraff et al., Nature **431**, 162 (2004)

Different ways to measure a qubit

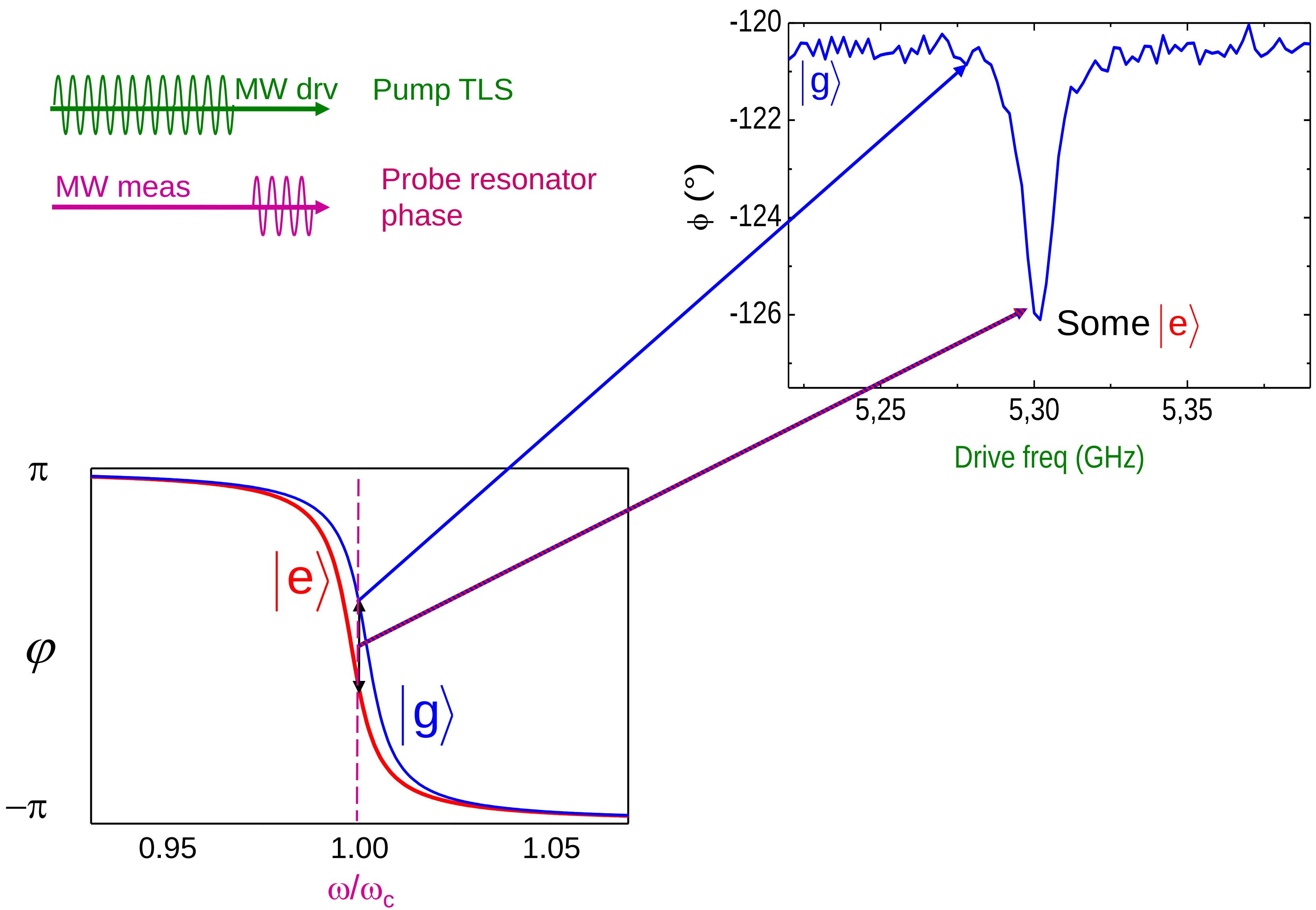
Shunt configuration: a transmission dip on resonance



Series configuration: a transmission peak on resonance

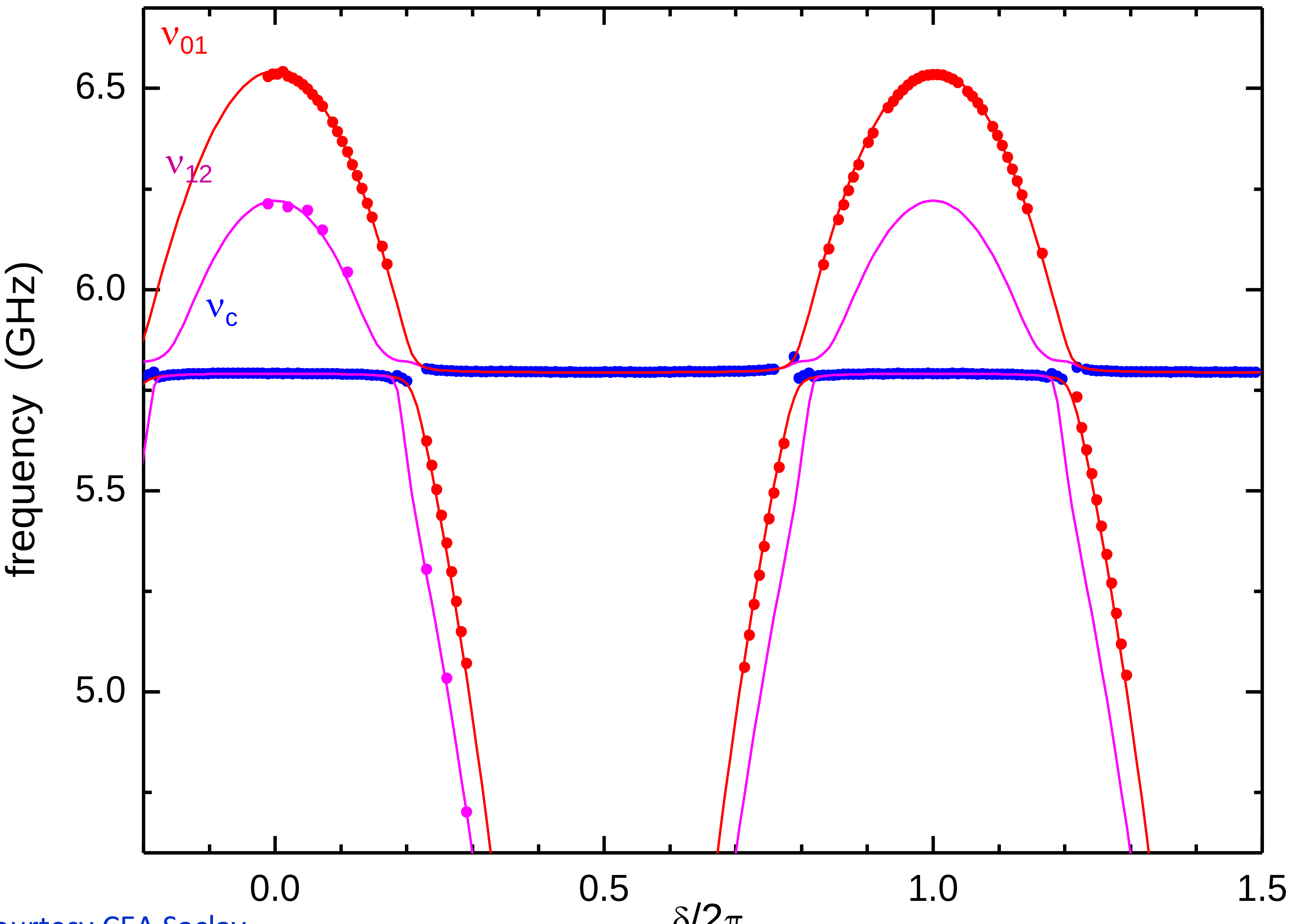


Qubit spectroscopy with dispersive readout



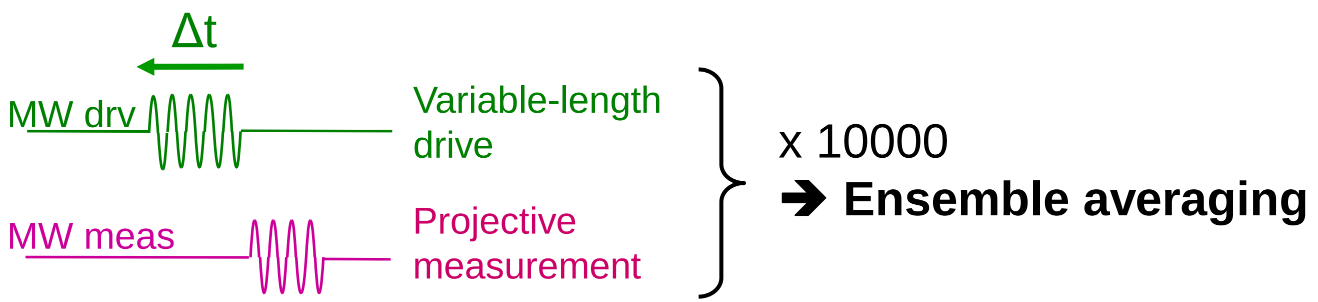
courtesy CEA Saclay

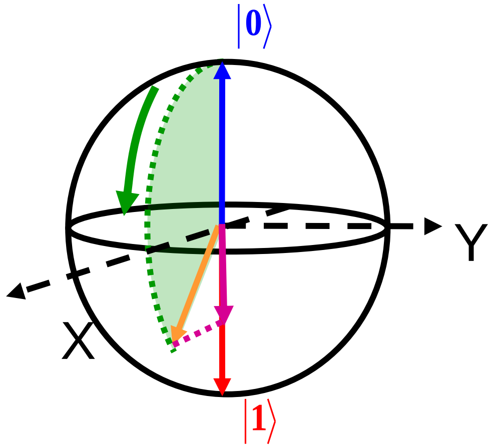
Typical spectroscopy of a transmon + cavity circuit

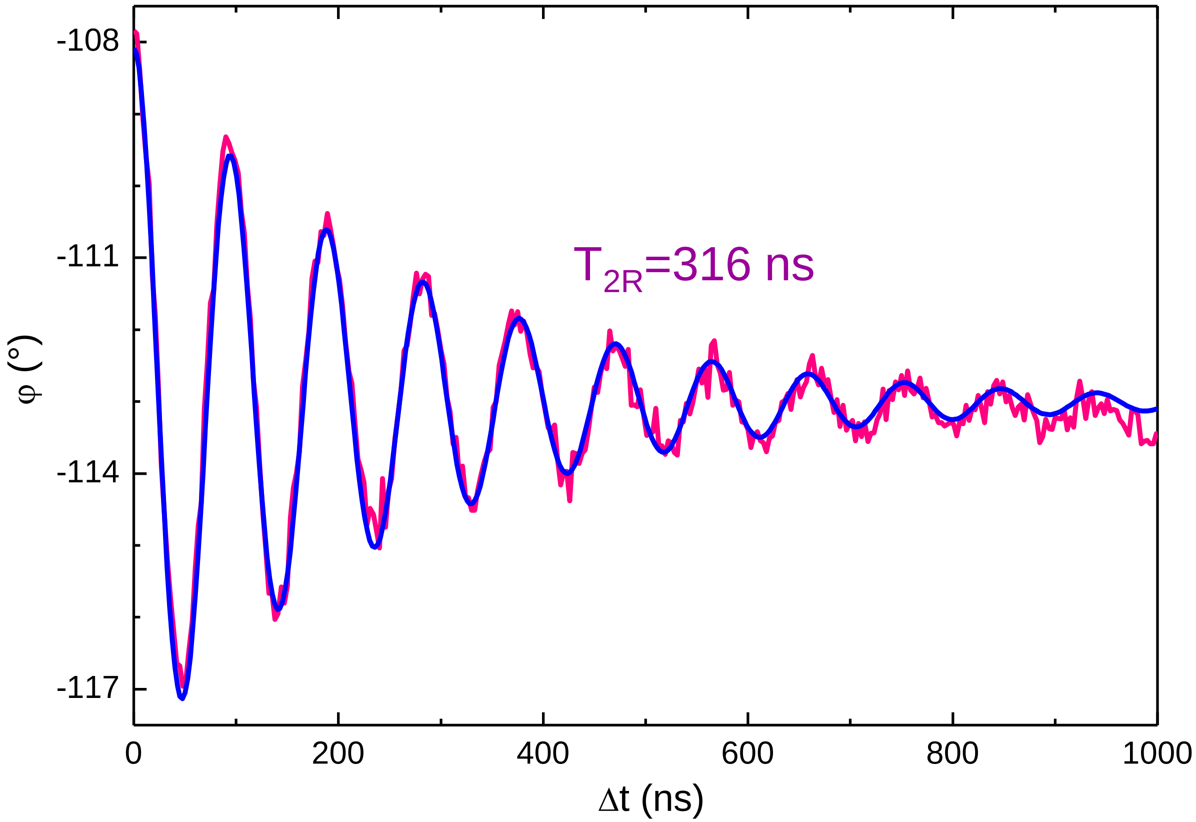


courtesy CEA Saclay

Rabi oscillations measured with dispersive readout

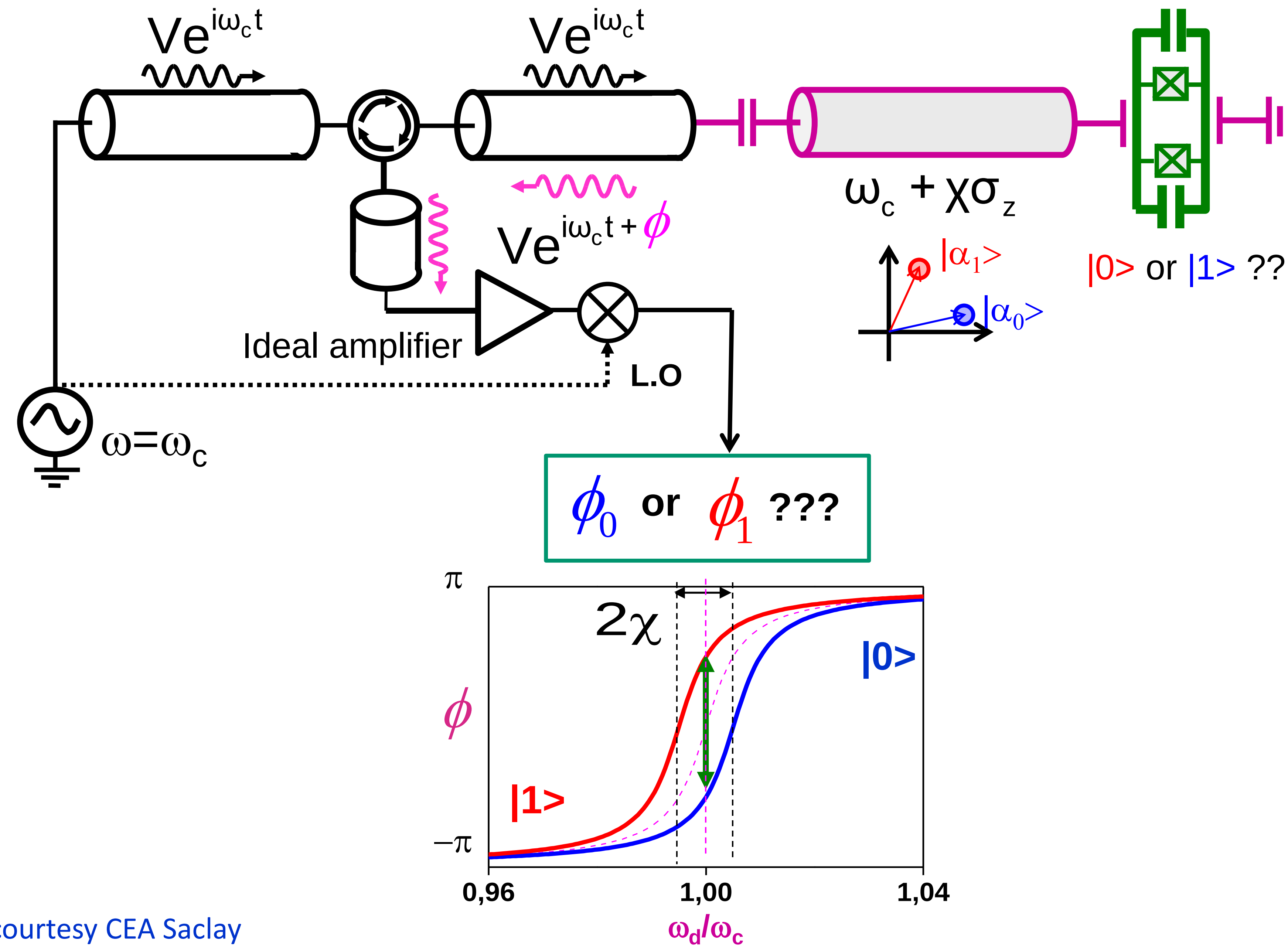






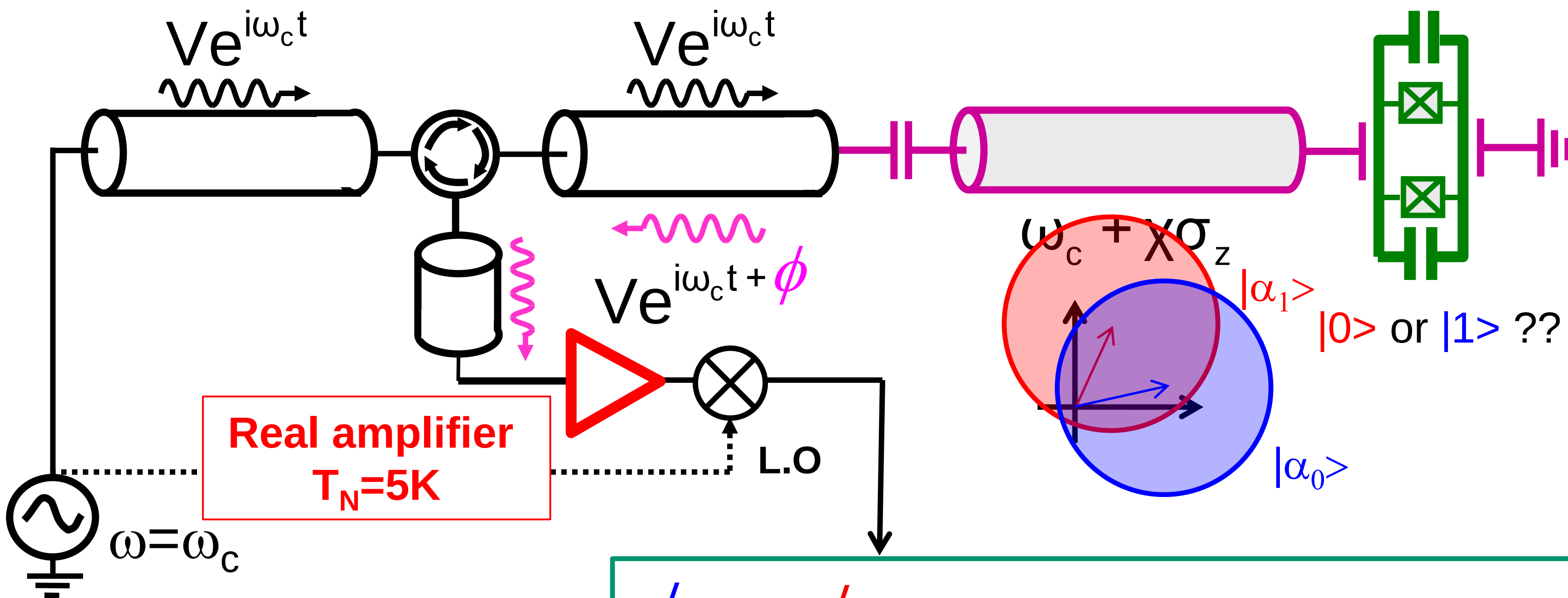
Quantum Limited Amplifier

Dispersive readout : the signal-to-noise issue

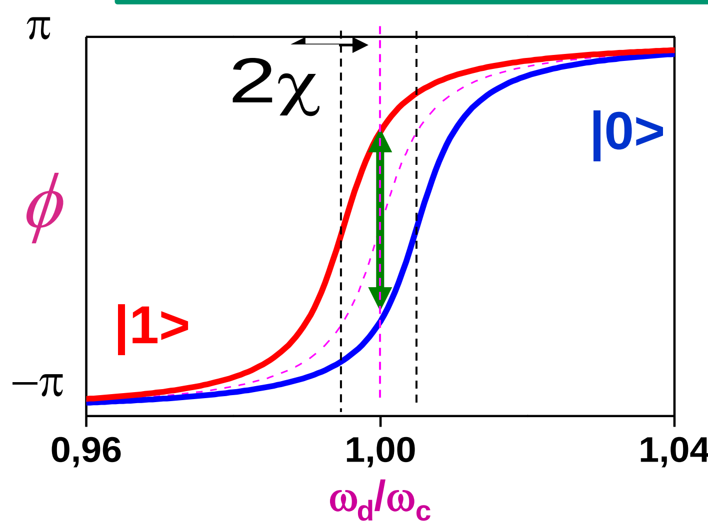


courtesy CEA Saclay

Dispersive readout : the signal-to-noise issue

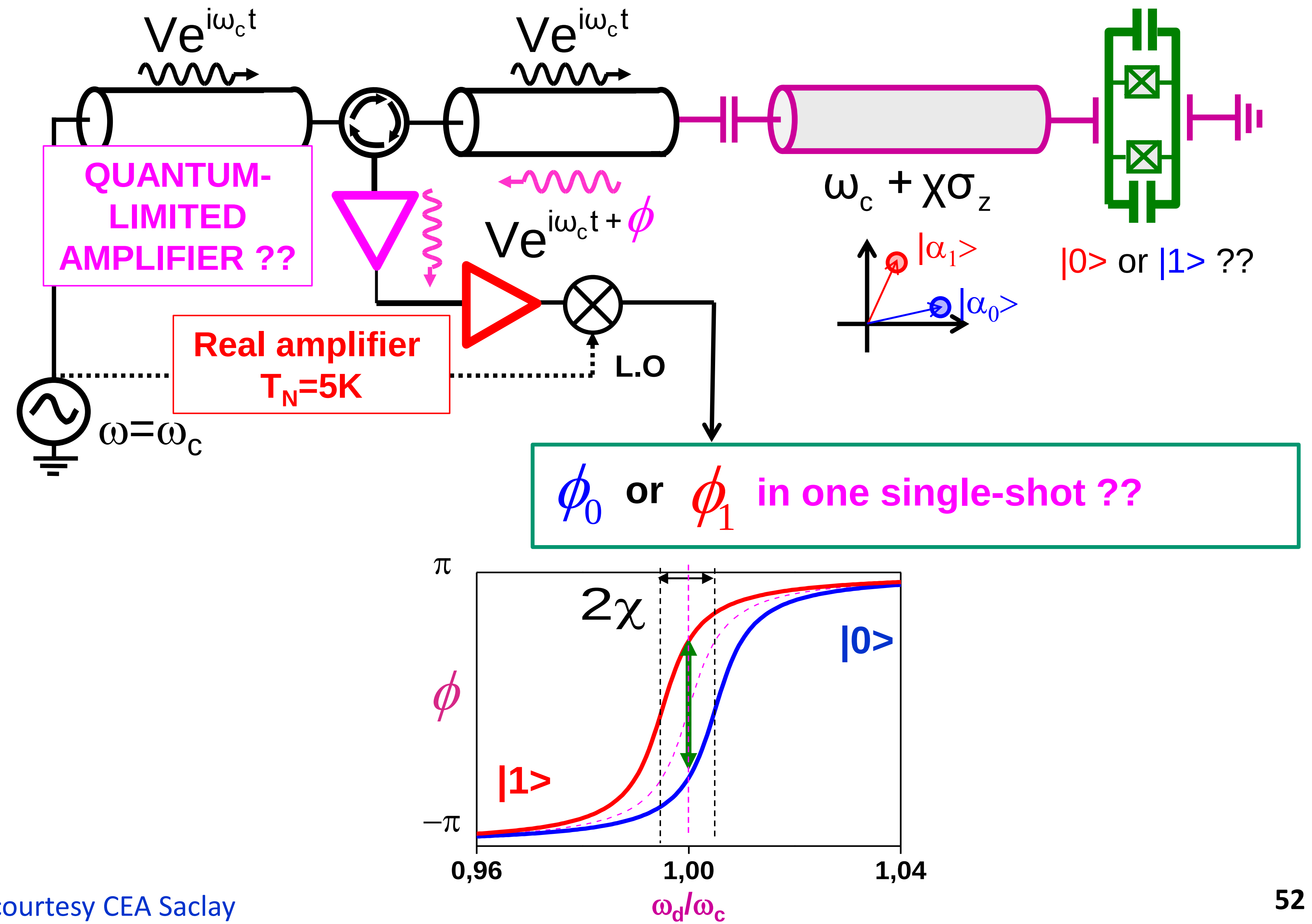


ϕ_0 or ϕ_1 ??? No discrimination in 1 shot



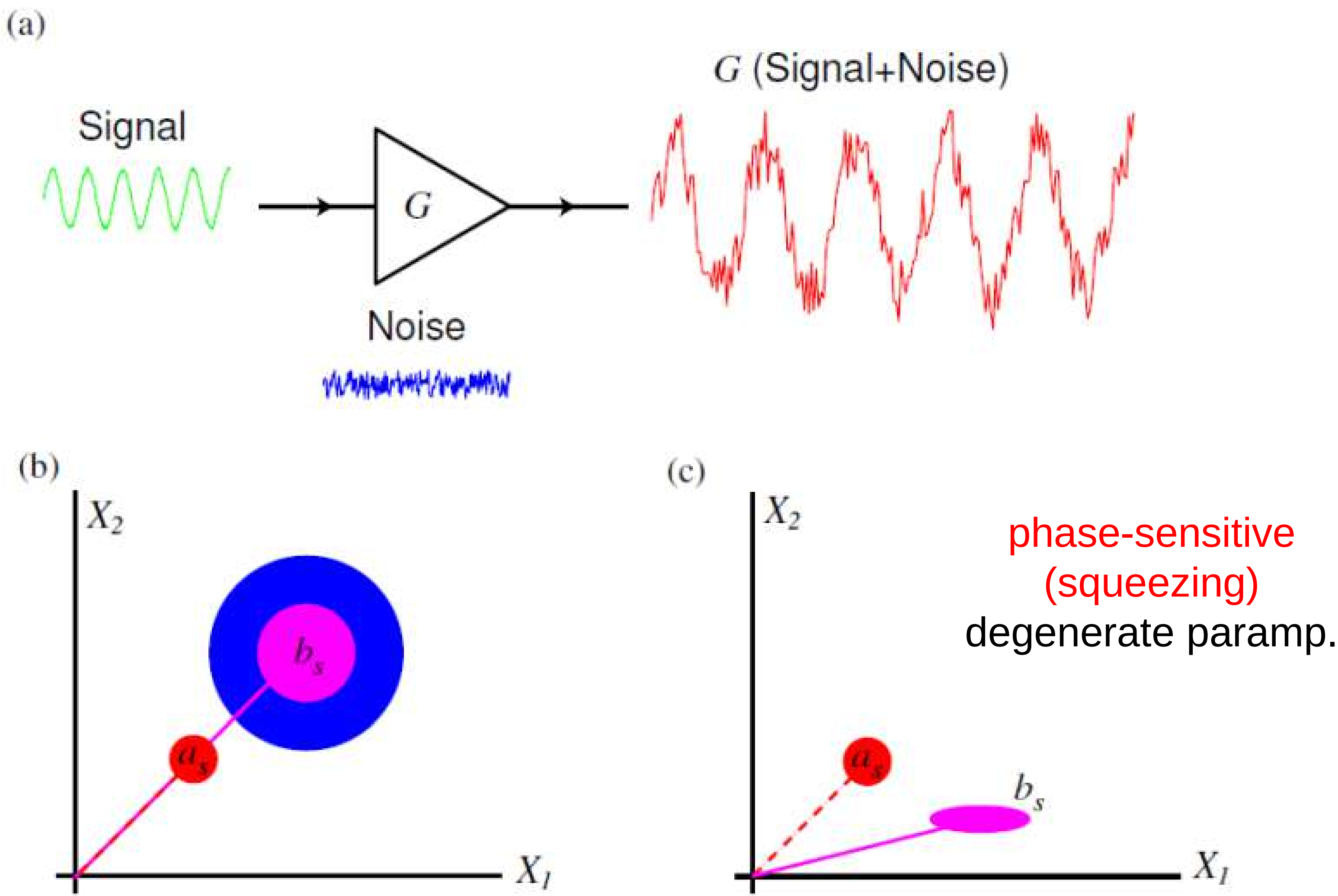
courtesy CEA Saclay

Dispersive readout : the signal-to-noise issue



courtesy CEA Saclay

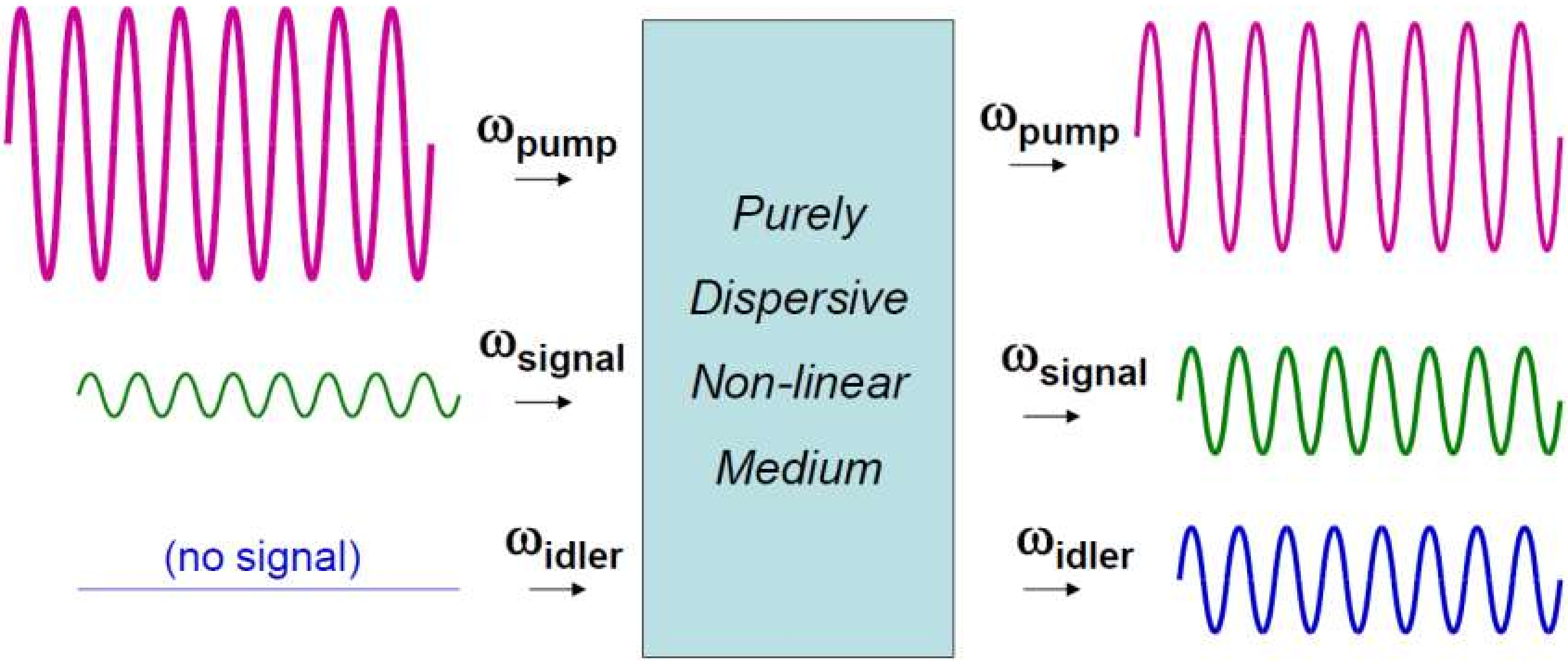
Parametric Amplification



phase-preserving
non-degenerate paramp.

Castellanos, PhD thesis (2010)

Parametric Amplification



$$\omega_{signal} + \omega_{idler} = \omega_{pump}$$ "3-wave process"

$$\omega_{signal} + \omega_{idler} = 2\omega_{pump}$$ "4-wave process"

(Slide by Michel Devoret)

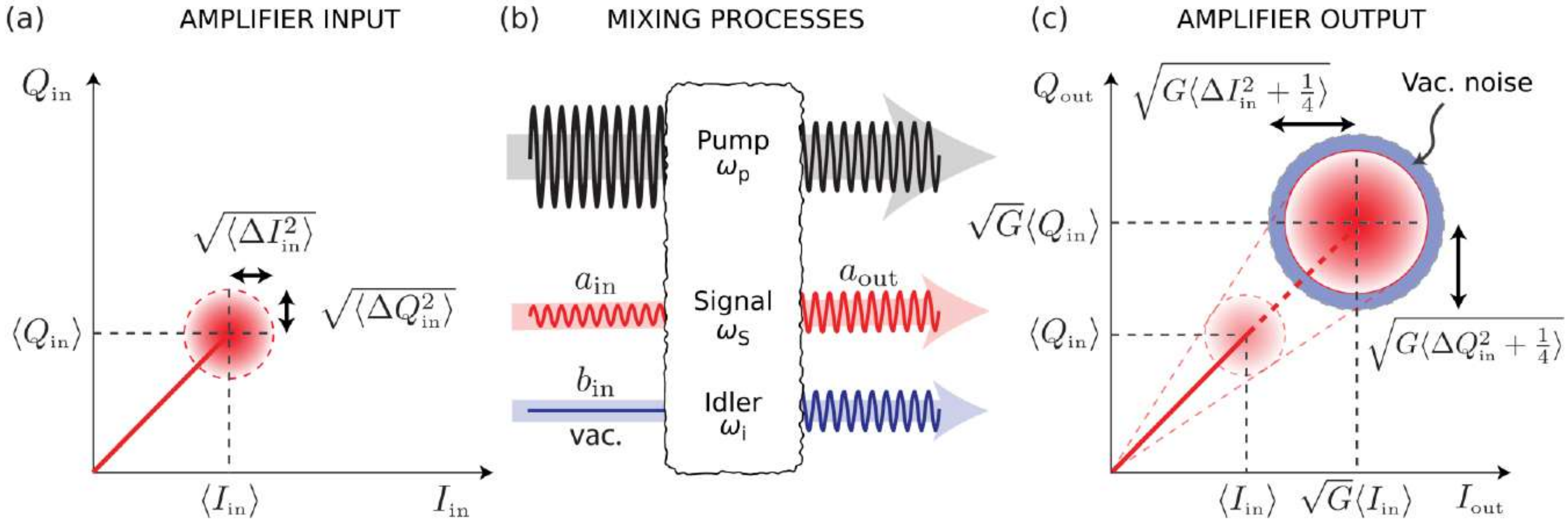
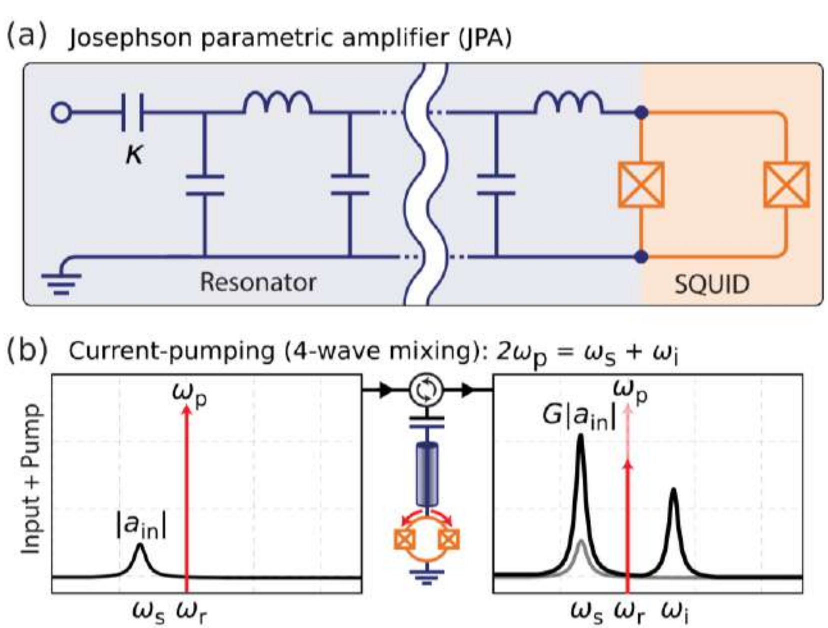


FIG. 25. Schematic illustration of a quantum-limited, phase-preserving parametric amplification process of a coherent input state, $a_{in} = I_{in} + iQ_{in}$. (a) The state is centered at $(\langle I_{in} \rangle, \langle Q_{in} \rangle)$ and has a noise represented by the radii of the circles along the real and imaginary axes, respectively. (b) Scattering representation of parametric mixing, where the signal and pump photons are interacting via a purely dispersive nonlinear medium. (c) In the case of phase-preserving amplification, both quadratures get amplified by a factor $\sqrt{G}$, while (in the ideal case) half a photon of noise gets added to the output distribution (blue). Image inspired by Flurin.[332]



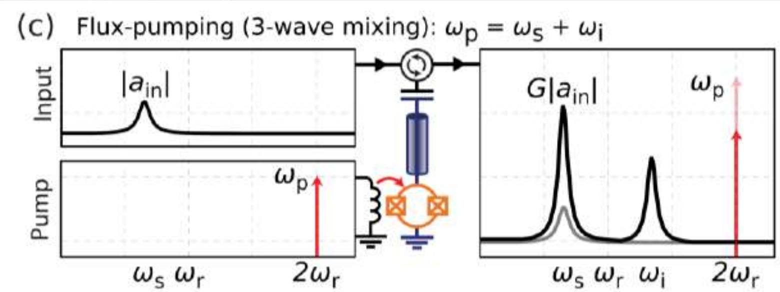
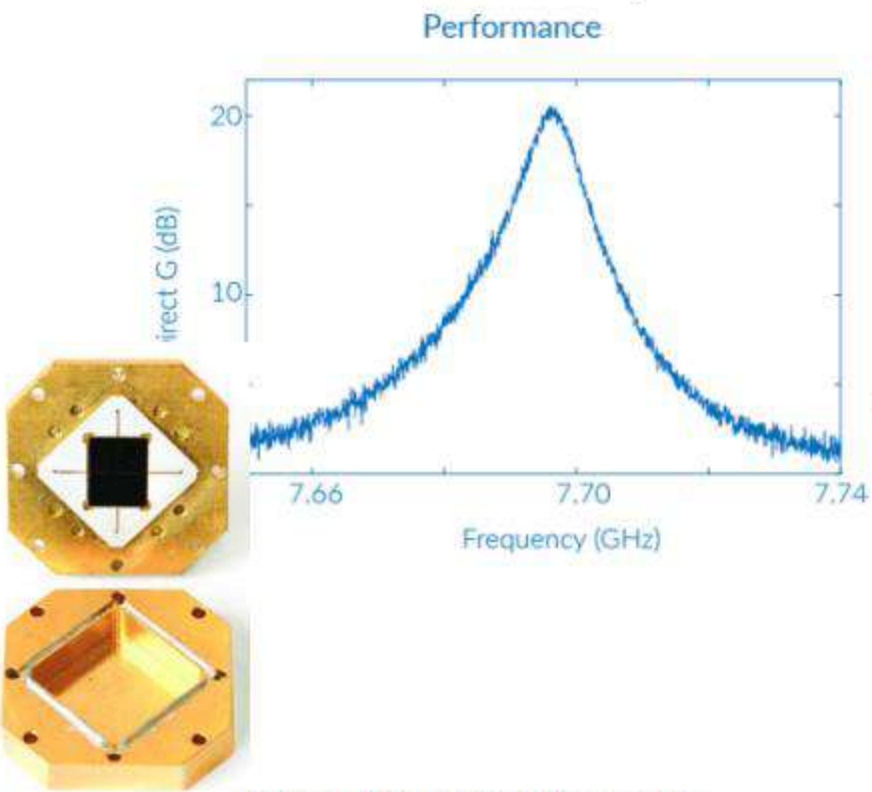


FIG. 27. Circuit schematics and pump schemes of a Josephson parametric amplifier. (a) The device consists of a quarter-wavelength resonator (blue), represented as lumped elements, shorted to ground via a Kerr-nonlinearity consisting of two parallel Josephson junctions (orange) forming a SQUID. The pump (red) can be applied in two ways; (b) either by modulating the current through the junctions (four-wave mixing) at the resonant frequency, $\omega_p \approx \omega_r$, or (c) by modulating the ac-flux Φ_{ac} around a static DC-flux point Φ_{dc} using a separate fast-flux line (three-wave mixing). The flux pump is applied at twice the resonance frequency, $\omega_p \approx 2\omega_r$.

Parametric Amplification

JPA / JPC

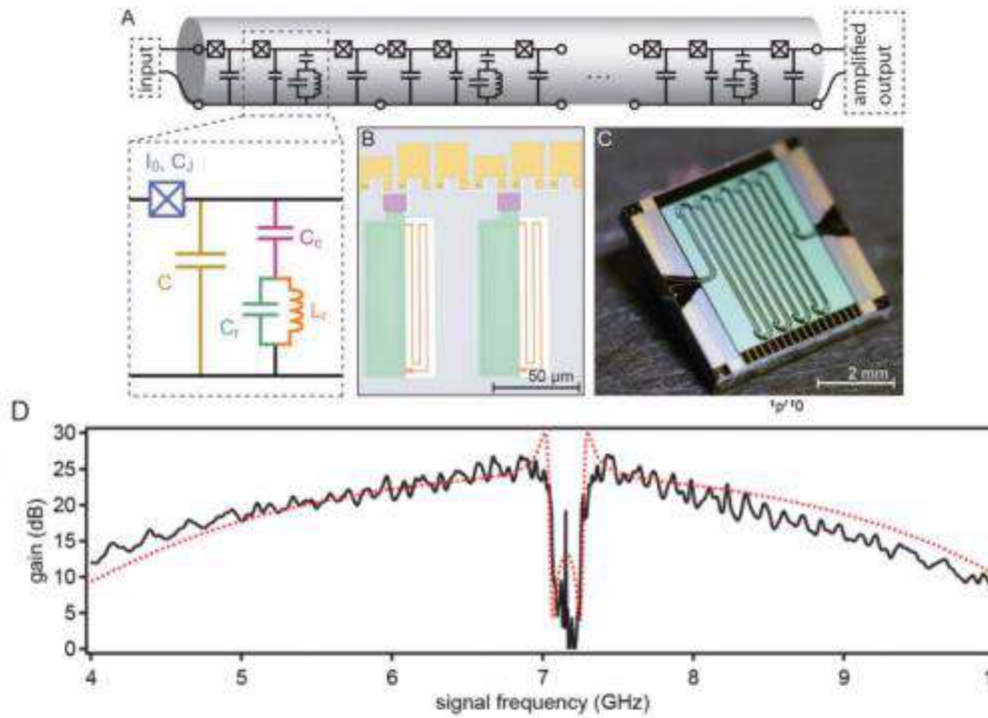
+ Very simple to make
- Narrow band
+ Ultimate efficiency



Quantumcircuits.com

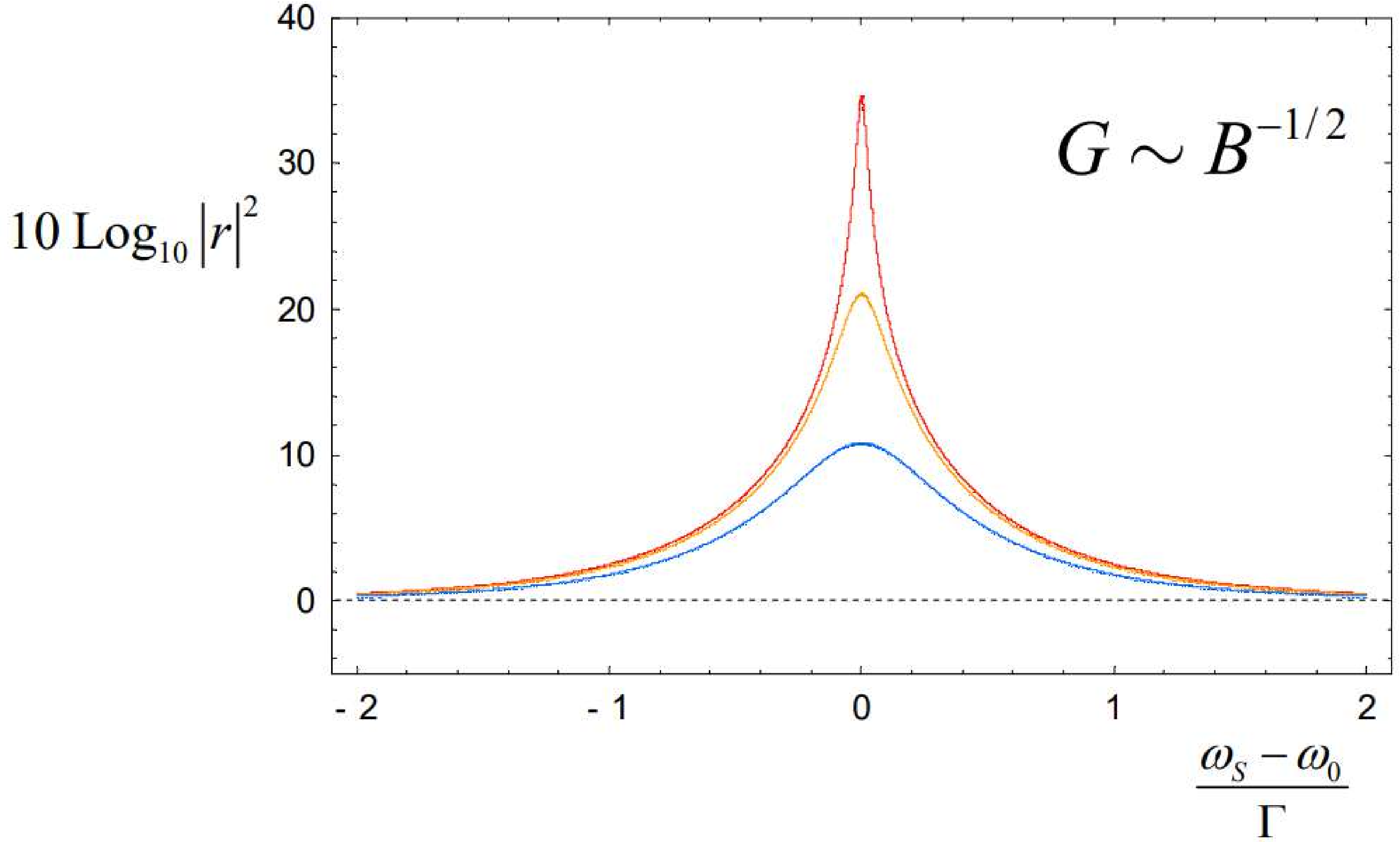
TWPA

- Complicated:1000s JJ
- Slightly less efficient now
+ Broadband, directional

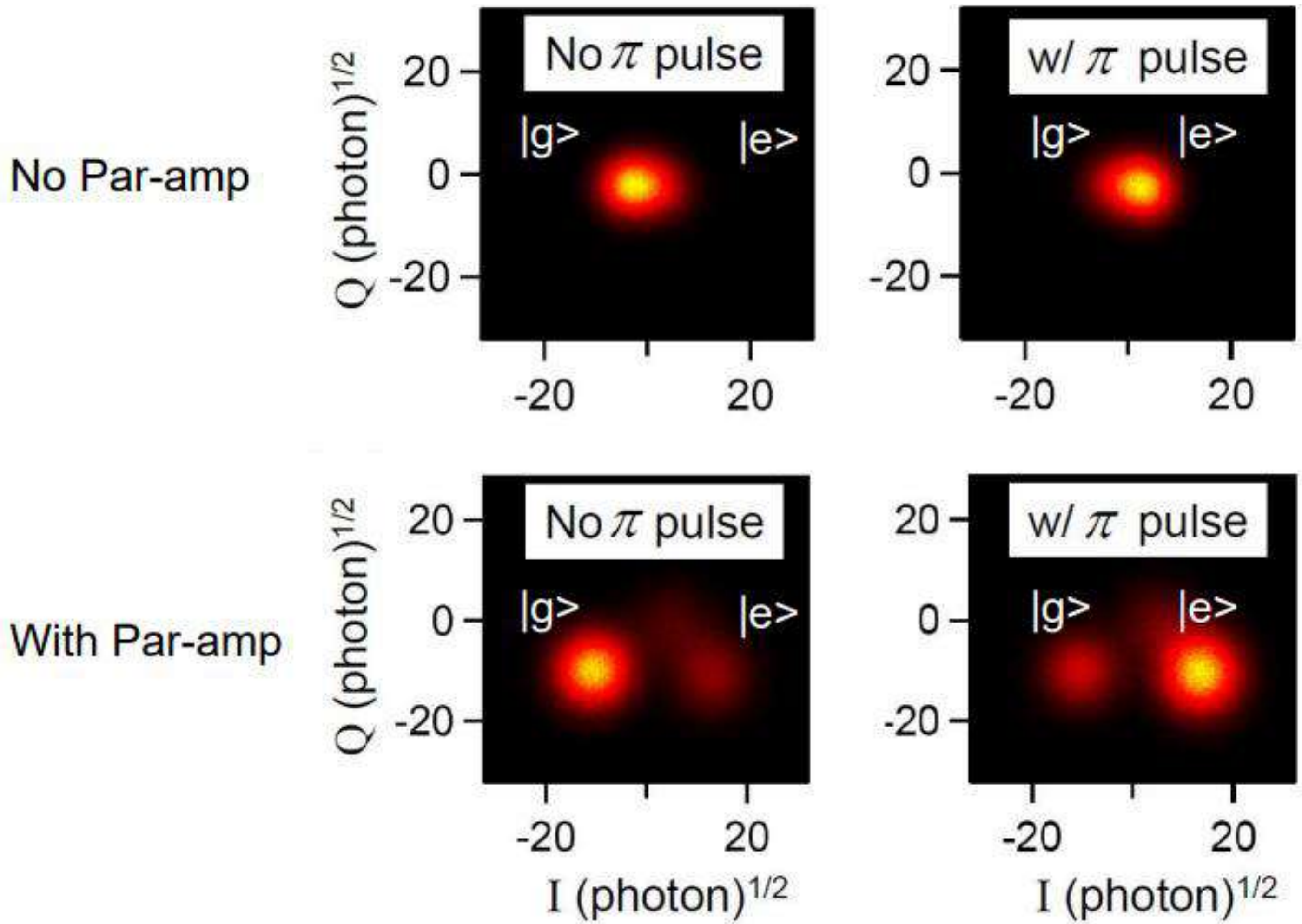


Macklin et al., Science (2015)

Gain-Bandwidth Compromise
(for resonator based amplifiers)



Single-Shot Histograms



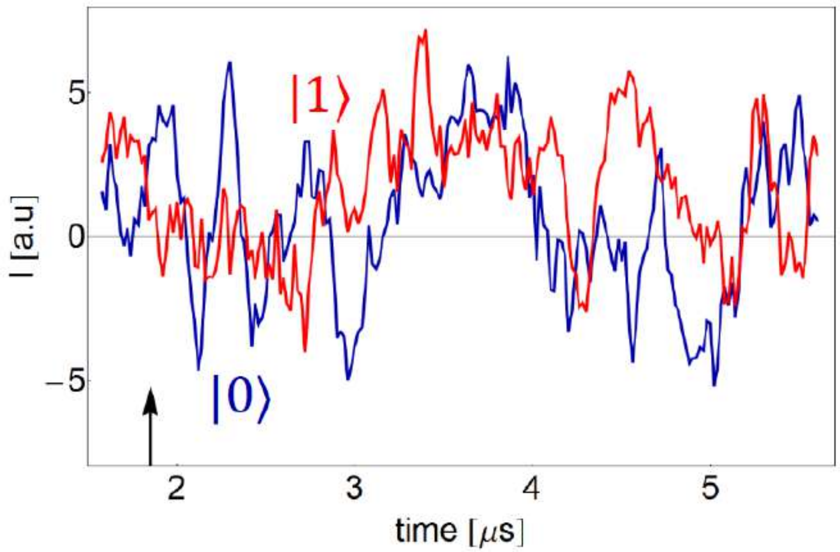
→ Single-shot discrimination of qubit state

Abdo *et al.*, PRL (2014)

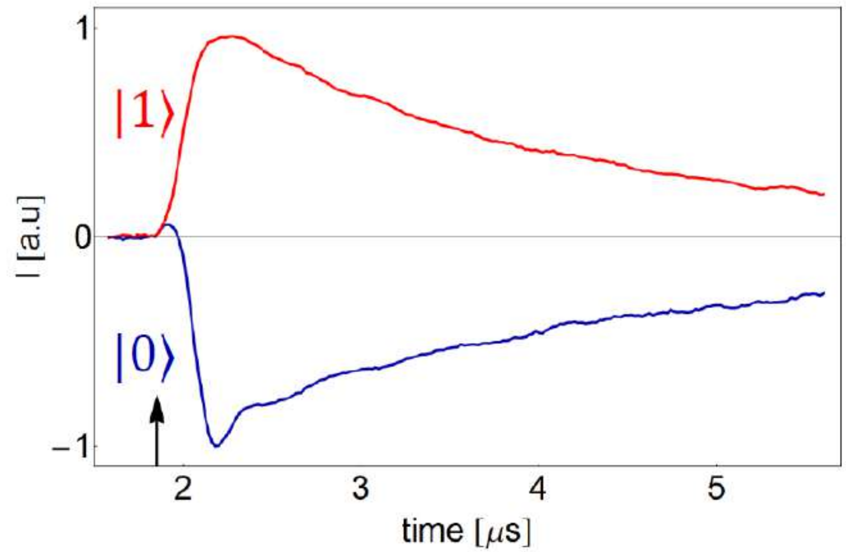
Single-shot qubit readout

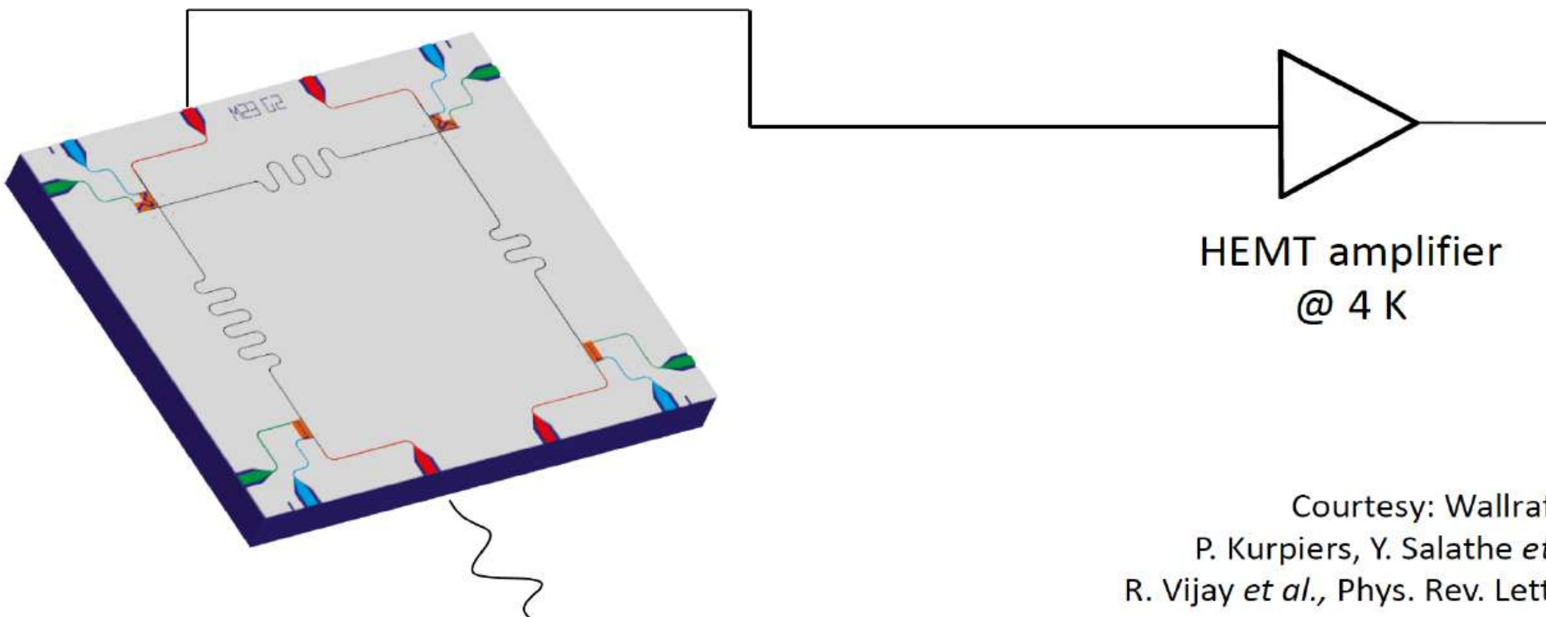
Conventional HEMT

single-shot measurements:



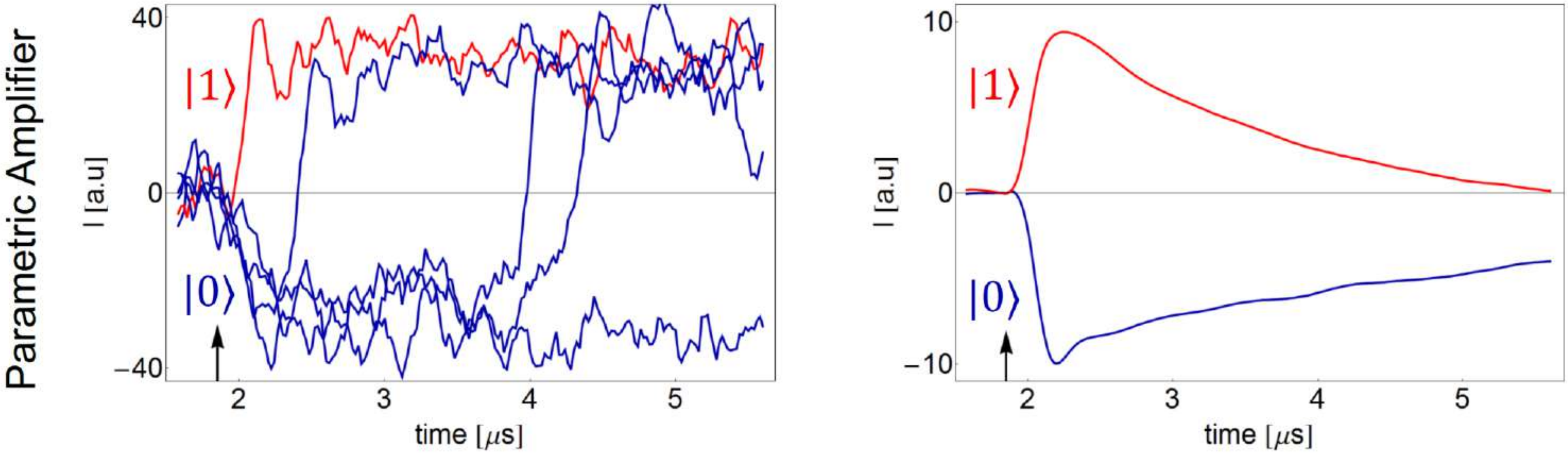
averaged measurements (8 10⁴):

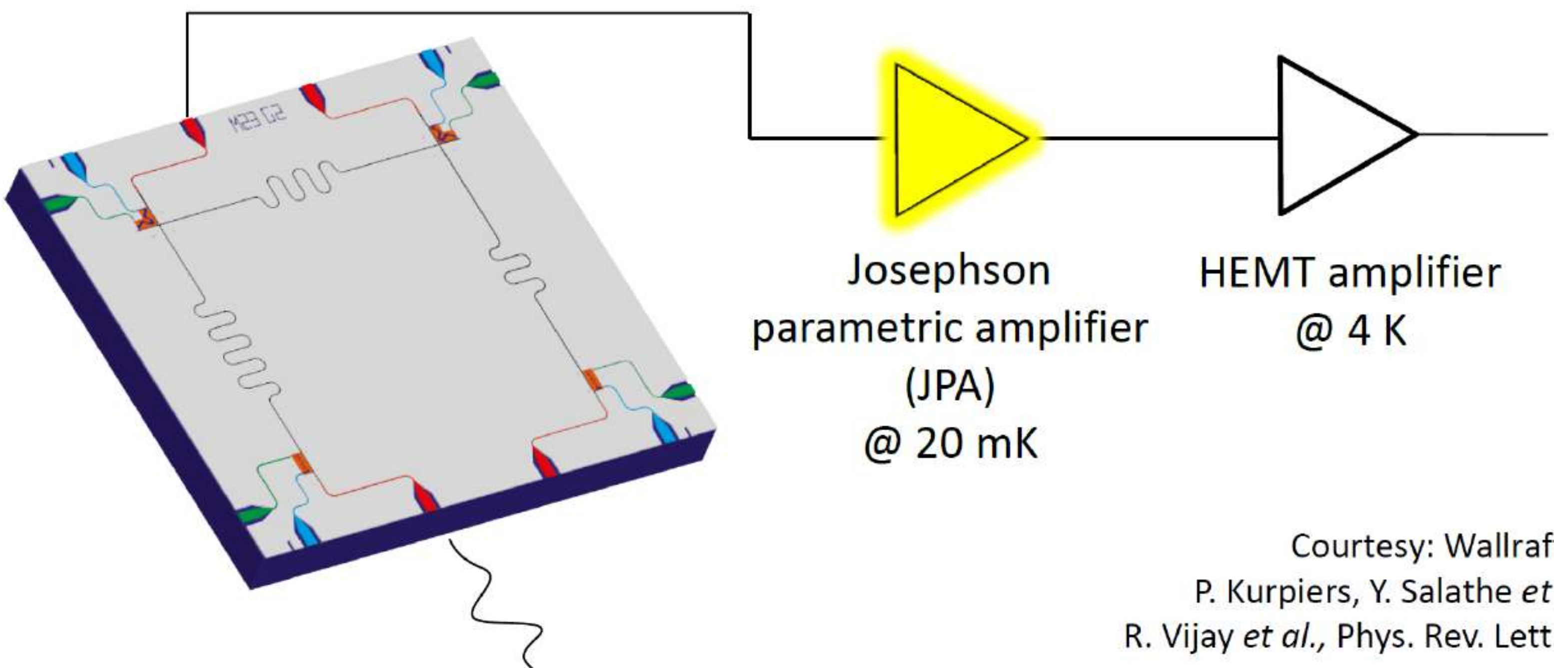




Courtesy: Wallraff group @ ETH Zurich
P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, Phys. Rev. Lett. **106**, 110502 (2011)

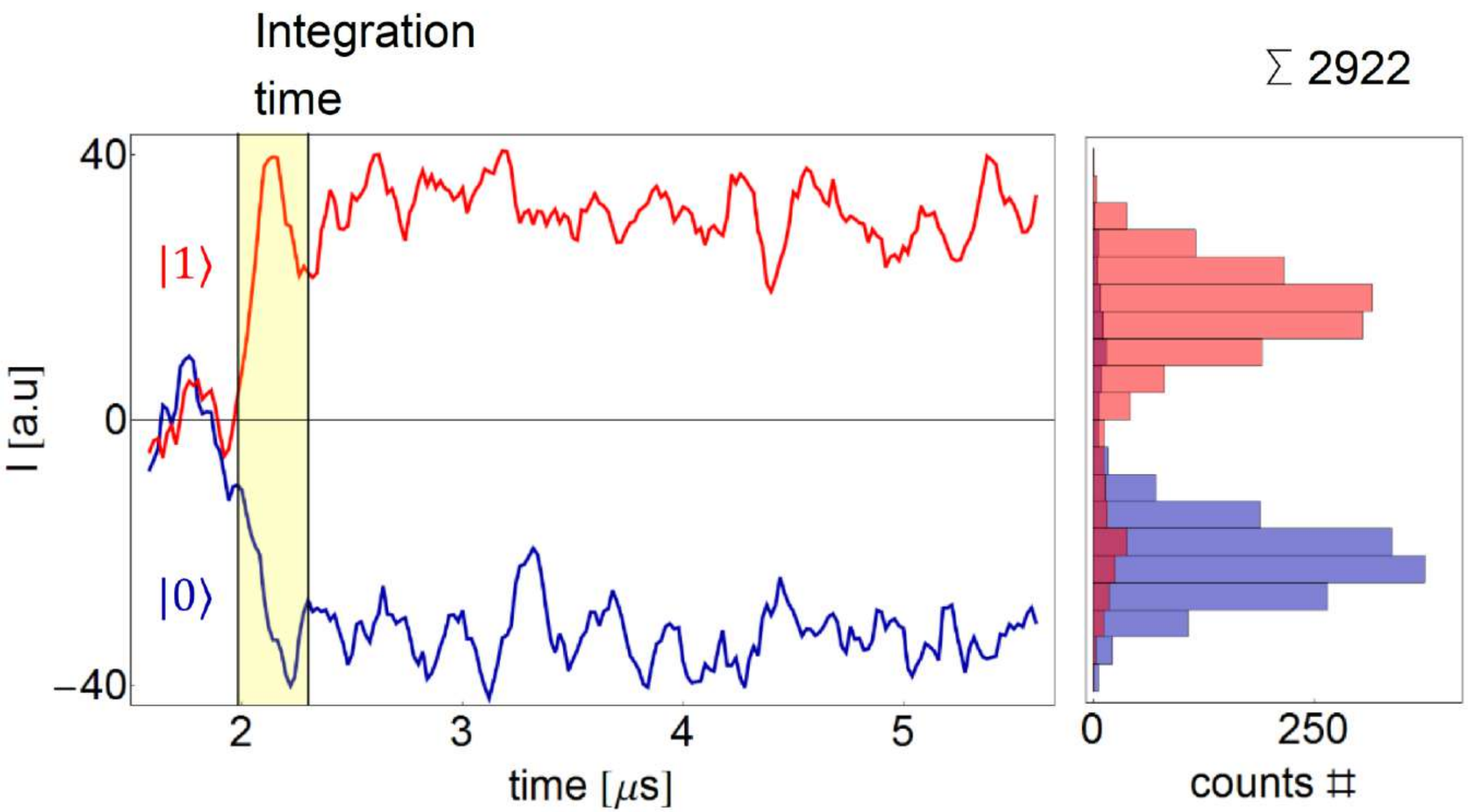
Single-shot qubit readout



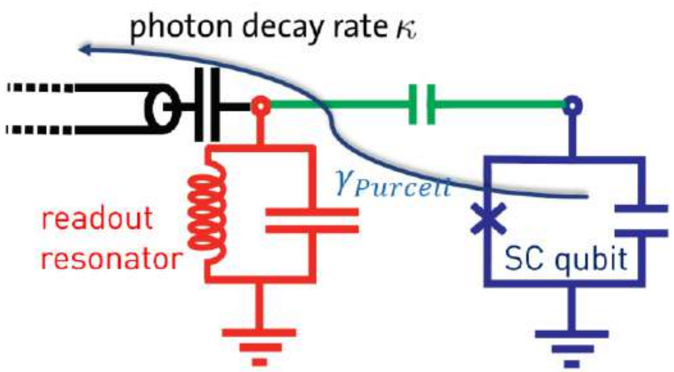


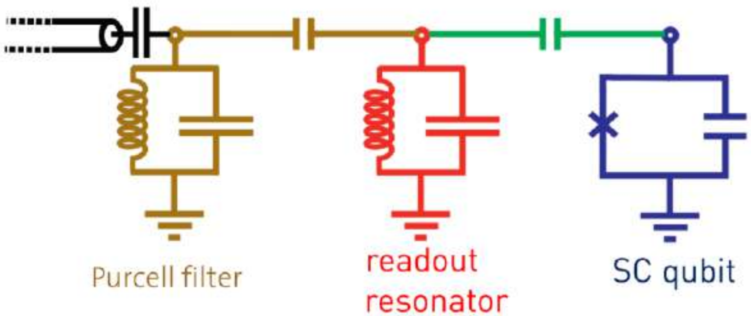
Courtesy: Wallraff group @ ETH Zurich
P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, Phys. Rev. Lett. **106**, 110502 (2011)

Statistics of Integrated Single-shot qubit readout



Purcell decay and protection





Walter et al., *Phys. Rev. Applied* **7**, 054020 (2017)

What about decay of the qubit into the measurement line via the resonator?

- In the limit of large detuning we find
  $$\gamma_{Purcell} \approx \kappa \frac{g^2}{\Delta^2} \approx \kappa \frac{|\chi|}{\alpha}$$
- BUT: Fast readout requires large κ and $|\chi|$.
- Solution: Include an additional filter, called "Purcell filter" to suppress qubit decay while allowing for large κ and $|\chi|$.
- Purcell filter can be realized e.g. as an an additional LC- resonator (see schematic).
- In this case
  $$\gamma_{Purcell} \propto 1/\Delta^4$$
  is strongly suppressed.

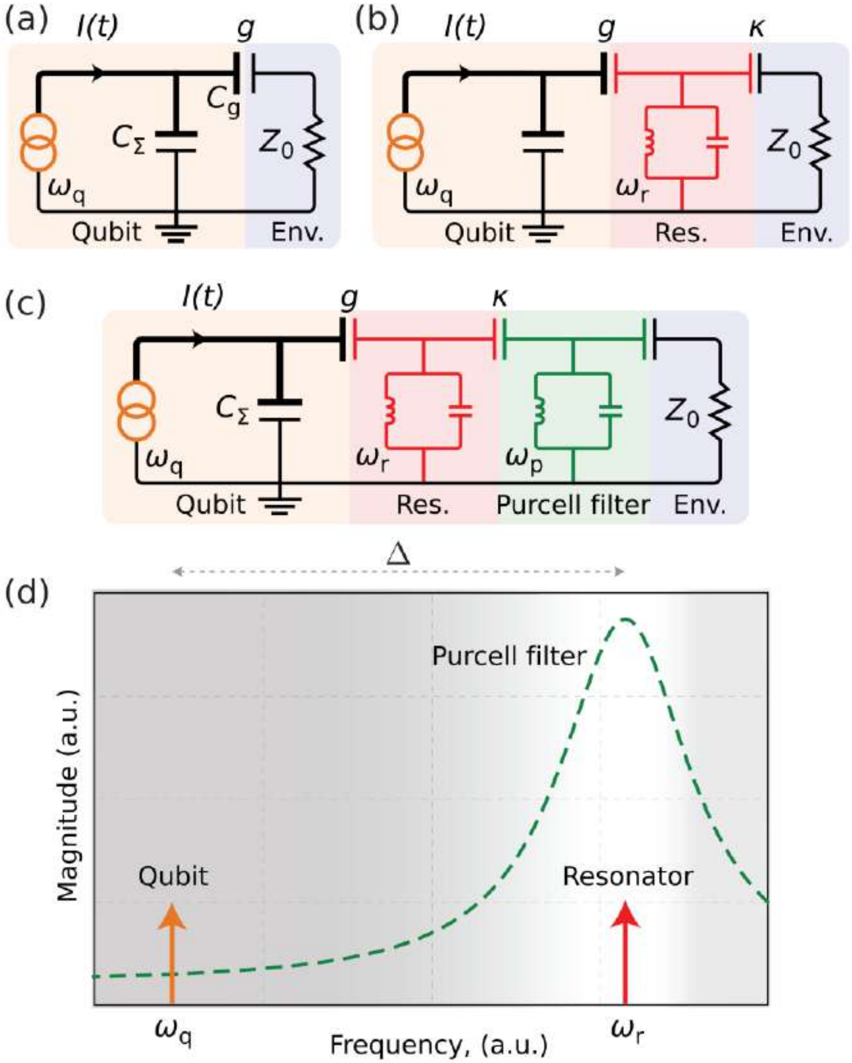
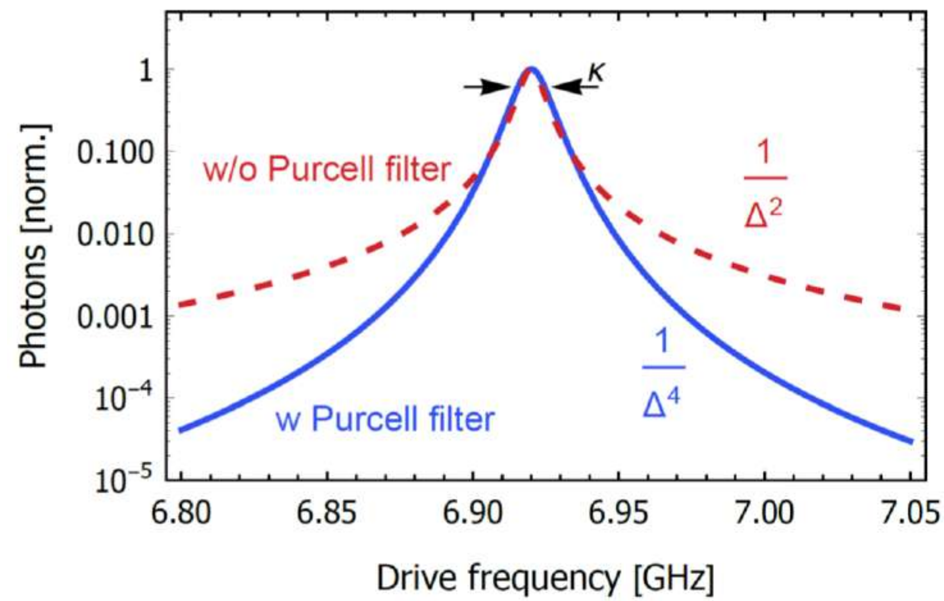


FIG. 24. (a) Circuit representation of the qubit (orange) coupled to an environment (blue) with a load resistor, Z_0, via a capacitor C_g. To study the decay rate, the Josephson junction has been replaced with a current source, $I(t)$. (b) By adding a resonator (red) with frequency ω_r in-between the qubit and the 50 Ω environment, we get the case found in a regular dispersive readout. (c) A Purcell-filter (green) is added to the circuit, providing protection for the qubit, while allowing the resonator field to decay fast in the environment. (d) Transmission spectrum of a Purcell filter (dashed green), centered around the resonator frequency (red arrow), whereas the qubit frequency (orange arrow) is far detuned.



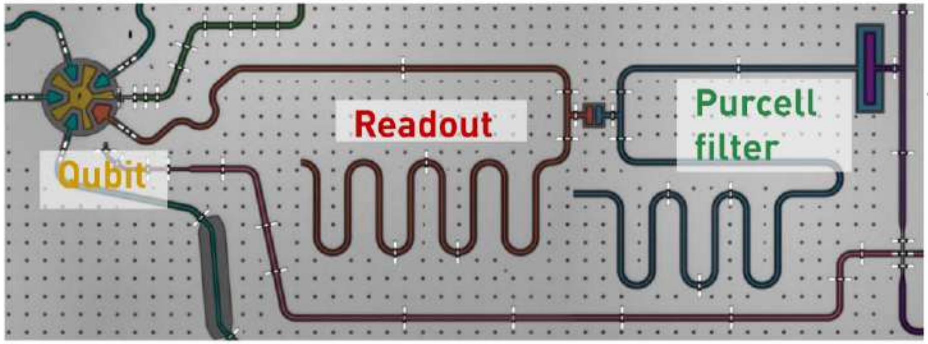
(a) $$\gamma_{\mathrm{env}}^{\mathrm{Purcell}} = \frac{1}{T_1} = \frac{P}{\hbar\omega} = \frac{(\beta e \omega)^2 Z_0}{\hbar\omega} = \frac{g^2}{\omega}.$$

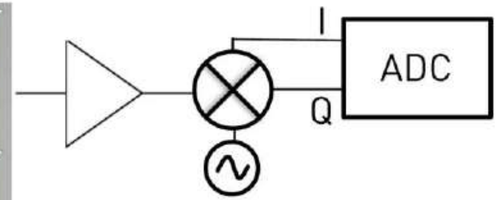
(b) $$\gamma_{\mathrm{res-env}}^{\mathrm{Purcell}} = \frac{g^2}{\omega_r}\frac{\mathrm{Re}[Z_r]}{Z_0} \underset{\Delta \gg g,\kappa}{=} \frac{g^2}{\omega_r} Q \left(\frac{\kappa}{\Delta}\right)^2 = \left(\frac{g}{\Delta}\right)^2 \kappa$$

(c) $$\gamma_{\mathrm{res-filter-env}}^{\mathrm{Purcell}} = \kappa \left(\frac{g}{\Delta}\right)^2 \left(\frac{\omega_q}{\omega_r}\right) \left(\frac{\omega_r}{2 Q_F \Delta}\right)$$

Applied Physics Reviews **6**, 021318 (2019)

Principle of Dispersive Qubit Measurement





$$A\, e^{i\phi} = I + i\, Q$$

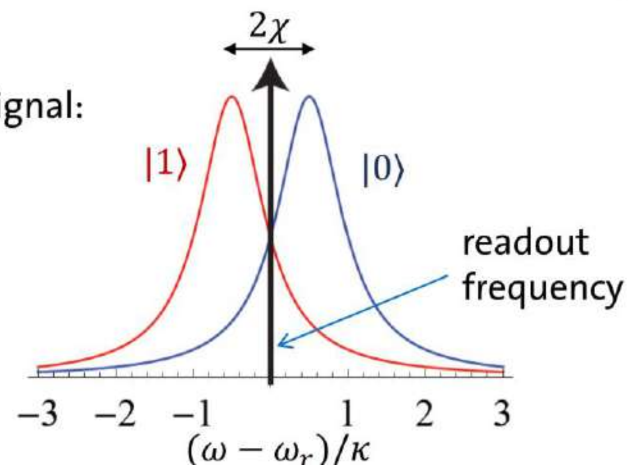
signal amplitude

Phase

In-phase and quadrature components

$$H/\hbar \approx (\omega_r + \chi\sigma_z)a^\dagger a$$,with $\chi \approx -\alpha \frac{g^2}{\Delta(\Delta-\alpha)}$

Amplitude of transmitted signal:



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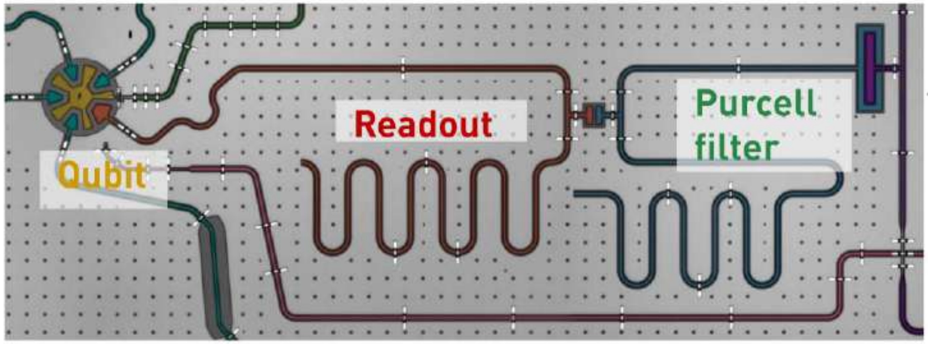
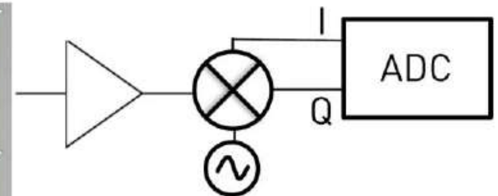
Principle of Dispersive Qubit Measurement





$$A\, e^{i\phi} = I + i\, Q$$

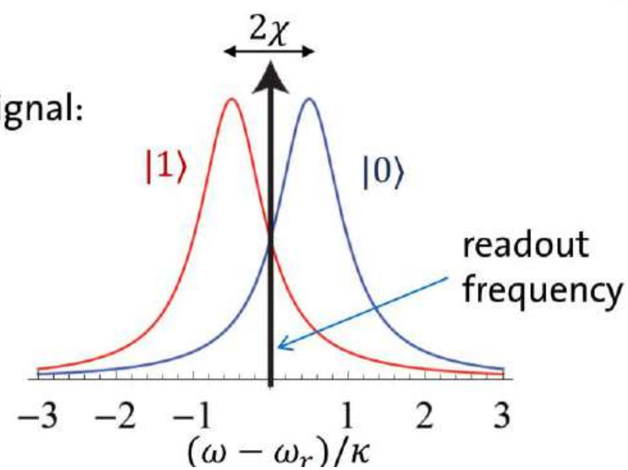
signal amplitude

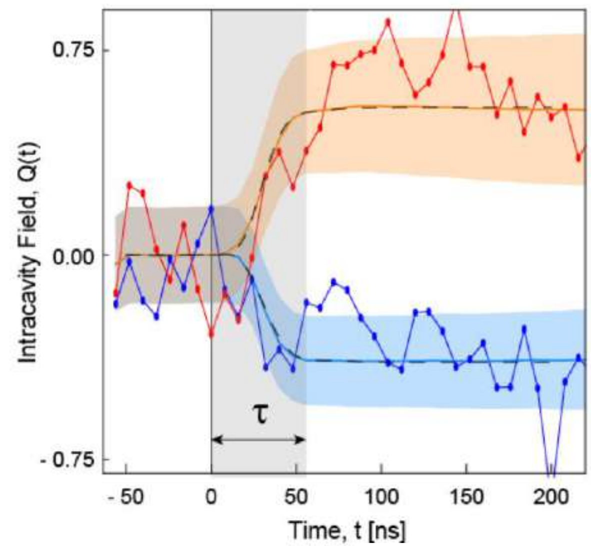
Phase

In-phase and quadrature components

$$H/\hbar \approx (\omega_r + \chi\sigma_z)a^\dagger a \quad \text{,with } \chi \approx -\alpha\frac{g^2}{\Delta(\Delta-\alpha)}$$

Amplitude of transmitted signal:





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- Integration time τ

Observations:

- Fast rise of measurement signal (< 50 ns) due large χ (and κ)
- Small decay of average excited state trace due to Purcell protected T_1
- Little increase of average ground state trace due to measurement induced mixing