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Sudden death of entanglement of two Jaynes–Cummings atoms

Muhammed Yönaç, Ting Yu and J H Eberly

Rochester Theory Center for Optical Science and Engineering and Department of Physics and Astronomy, University of Rochester, New York 14627, USA

E-mail: ting@pas.rochester.edu

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Abstract

We investigate entanglement dynamics of two isolated atoms, each in its own Jaynes–Cummings cavity. We show analytically that initial entanglement has an interesting subsequent time evolution, including the so-called sudden death effect.

Entanglement is a defining feature of quantum mechanics that makes fundamental distinctions between quantum and classical physics. As an unambiguous and quantifiable property of sufficiently simple multi-party quantum systems, entanglement has a definite time evolution that has recently begun to be studied in several contexts [1–8].

Entanglement in a quantum system may deteriorate due to interaction with background noise or with other systems usually called environments. Interest was originally concerned with the consequences for quantum measurement and the quantum-classical transition [9–11]. More recently, entanglement decoherence has been studied in connection with obstacles to realizing various quantum information processing schemes. Particularly, we have shown that entanglement can decay to zero abruptly, in a finite time, a phenomenon termed entanglement sudden death [4, 5].

The purpose of this paper is to examine two interesting time-evolving quantum systems that have no route for mutual interaction, but whose mutual entanglement nevertheless evolves in an unusual way. We have chosen the 'double Jaynes–Cummings (JC)' model consisting of two two-level atoms (see figure 1). Each is in a perfect one-mode near-resonant cavity and interacts with its initially unexcited cavity mode, but each is completely isolated from the other atom and cavity.

By tracing over the cavity modes at time t we are left with a mixed state of atoms A and B similar to that treated previously by us [4], but in this case the underlying intra-cavity dynamics are quite different since the cavities are treated here as lossless rather than as perfect reservoirs. The evolution of the entanglement of the non-interacting atoms here shows striking new features.

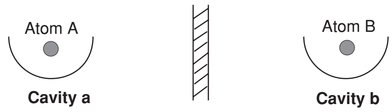

Figure 1. A schematic diagram of the double JC model used in this paper. There is no communication between the cavities.

By tracing cavity modes we are forcibly creating a two-qubit scenario, and various measures of entanglement are available. For a pair of qubits, all of them are equivalent, in the sense that when any of them indicates no entanglement (separable states), the others[1] also indicate no entanglement. Throughout the paper we will use Wootters' [12] concurrence $C(\rho)$ as the conveniently normalized entanglement measure ($1 \geqslant C \geqslant 0$).

The double JC Hamiltonian for our system may be written as

$$H_{\text{tot}} = \omega\sigma_z^A + \omega\sigma_z^B + g\left(a^\dagger\sigma_-^A + a\sigma_+^A\right) + g\left(b^\dagger\sigma_-^B + b\sigma_+^B\right) + \nu a^\dagger a + \nu b^\dagger b. \tag{1}$$

Clearly there will be no interaction between atom A and atom B or between cavity a and cavity b. The eigenstates of this Hamiltonian are products of the dressed states of the separate JC systems, which are well known [13].

For greatest simplicity, we assume that both cavities are prepared initially in the vacuum state $|0_a\rangle \otimes |0_b\rangle$ and the two atoms are in a pure entangled state specified below. Under these assumptions, there is never more than one photon in each cavity, so the cavity mode is essentially equivalent to a two-level system. This allows a uniform measure of quantum entanglement—concurrence—for both atoms and the cavity modes.

In that connection we note that there are, in principle, six different concurrences that provide information about the overall entanglements that may arise. With an obvious notation we can denote these as C^{AB}, C^{ab}, C^{Aa}, C^{Bb}, C^{Ab}, C^{Ba}. Symmetry considerations can provide natural relations among these, which we will report elsewhere [14]. Here we confine our attention to C^{AB}.

For a partially entangled atomic pure state that is a combination of the Bell states usually denoted as $|\Psi^\pm\rangle$, we have

$$|\Psi_{\text{atom}}\rangle = \cos\alpha|\uparrow\downarrow\rangle + \sin\alpha|\downarrow\uparrow\rangle \tag{2}$$

with the first index denoting the state of atom A and the second denoting the state of atom B ($\uparrow =$ excited state $\downarrow =$ ground state); the initial state for the total system (1) is given by

$$\begin{aligned}|\Psi_0\rangle &= (\cos\alpha|\uparrow\downarrow\rangle + \sin\alpha|\downarrow\uparrow\rangle) \otimes |00\rangle \\ &= \cos\alpha|\uparrow\downarrow 00\rangle + \sin\alpha|\downarrow\uparrow 00\rangle.\end{aligned} \tag{3}$$

Then the solution of the model in terms of the standard basis can be written as

$$|\Psi(t)\rangle = x_1|\uparrow\downarrow 00\rangle + x_2|\downarrow\uparrow 00\rangle + x_3|\downarrow\downarrow 10\rangle + x_4|\downarrow\downarrow 01\rangle, \tag{4}$$

where the coefficients stand for the following time-dependent formulae:

$$\begin{aligned}x_1 &= (L\,\mathrm{e}^{-\mathrm{i}\lambda^+ t} + M\,\mathrm{e}^{-\mathrm{i}\lambda^- t})\cos\alpha \\ x_2 &= (L\,\mathrm{e}^{-\mathrm{i}\lambda^+ t} + M\,\mathrm{e}^{-\mathrm{i}\lambda^- t})\sin\alpha \\ x_3 &= N(\mathrm{e}^{-\mathrm{i}\lambda^+ t} - \mathrm{e}^{-\mathrm{i}\lambda^- t})\cos\alpha \\ x_4 &= N(\mathrm{e}^{-\mathrm{i}\lambda^+ t} - \mathrm{e}^{-\mathrm{i}\lambda^- t})\sin\alpha.\end{aligned} \tag{5}$$

[1] For a two-qubit state entanglement a set of alternative measures of entanglement equivalent to concurrence in the sense mentioned in the text includes entropy of formation, tangle, Schmidt number and negativity.

Note here that λ^- and λ^+ are given by

$$\lambda^{\pm} = \nu + \frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2 + G^2}}{2}, \tag{6}$$

where $\Delta = \omega - \nu$ is the detuning and $G = 2g$ represents the strength of interaction between the atoms and cavities. The constant coefficients L, M and N are given by

$$L = \frac{1}{2}\left(1 + \frac{\Delta}{\sqrt{\Delta^2 + G^2}}\right) \tag{7}$$

$$M = \frac{1}{2}\left(1 - \frac{\Delta}{\sqrt{\Delta^2 + G^2}}\right) \tag{8}$$

$$N = \frac{G}{2\sqrt{\Delta^2 + G^2}}. \tag{9}$$

The information about the entanglement of two atoms is contained in the reduced density matrix ρ^{AB} for the two atoms which can be obtained from (4) by tracing out the photonic parts of the total pure state. The explicit 4×4 matrix written in the basis $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ is given by

$$\rho^{AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_1 x_2^* & 0 \\ 0 & x_1^* x_2 & |x_2|^2 & 0 \\ 0 & 0 & 0 & |x_3|^2 + |x_4|^2 \end{pmatrix}, \tag{10}$$

which is in the 'normal' or 'standard' form of the two-qubit mixed state we have noted previously [15]. The time-dependent matrix elements are given by (5).

It can be shown that the concurrence of the density matrix (10) is given by

$$C^{AB}(t) = |\sin 2\alpha|[1 - 4N^2 \sin^2(\delta t/2)], \tag{11}$$

where

$$\delta = \lambda^+ - \lambda^- = \sqrt{\Delta^2 + G^2}.$$

A particularly interesting example is the case of zero detuning ($\Delta = 0$) in which the concurrence becomes $C^{AB}(t) = |\sin 2\alpha| \cos^2(Gt/2)$, which is shown in figure 2.

Alternatively, the initial state for the total system may be based on a combination of the other two Bell states $|\Phi^{\pm}\rangle$:

$$|\Phi_0\rangle = \cos\alpha|\uparrow\uparrow 00\rangle + \sin\alpha|\downarrow\downarrow 00\rangle, \tag{12}$$

in which case the state of the total system at time t can be expressed in the standard basis:

$$|\Phi(t)\rangle = x_1|\uparrow\uparrow 00\rangle + x_2|\downarrow\downarrow 11\rangle + x_3|\uparrow\downarrow 01\rangle + x_4|\downarrow\uparrow 10\rangle + x_5|\downarrow\downarrow 00\rangle \tag{13}$$

where the coefficients are now given by

$$\begin{aligned} x_1 &= (L\,\mathrm{e}^{-\mathrm{i}\lambda^+ t} + M\,\mathrm{e}^{-\mathrm{i}\lambda^- t})^2 \cos\alpha \\ x_2 &= LM(\mathrm{e}^{-\mathrm{i}\lambda^+ t} - \mathrm{e}^{-\mathrm{i}\lambda^- t})^2 \cos\alpha \\ x_3 &= N(\mathrm{e}^{-\mathrm{i}\lambda^+ t} - \mathrm{e}^{-\mathrm{i}\lambda^- t})(L\,\mathrm{e}^{-\mathrm{i}\lambda^+ t} + M\,\mathrm{e}^{-\mathrm{i}\lambda^- t}) \cos\alpha \\ x_4 &= N(\mathrm{e}^{-\mathrm{i}\lambda^+ t} - \mathrm{e}^{-\mathrm{i}\lambda^- t})(L\,\mathrm{e}^{-\mathrm{i}\lambda^+ t} + M\,\mathrm{e}^{-\mathrm{i}\lambda^- t}) \cos\alpha \\ x_5 &= \sin\alpha. \end{aligned} \tag{14}$$

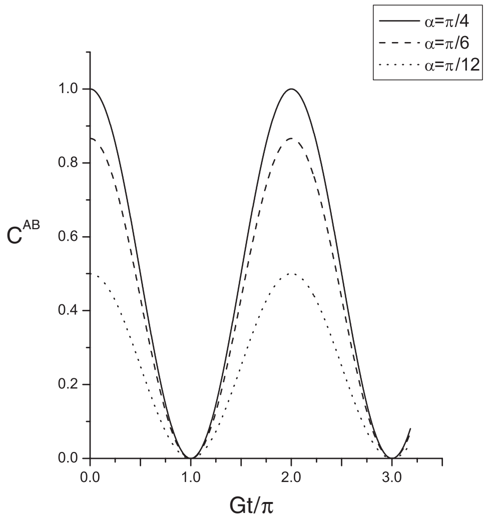

Figure 2. The concurrence for atom–atom entanglement with the initial atomic state $|\Psi_{\text{atom}}\rangle = \cos\alpha|\uparrow\downarrow\rangle + \sin\alpha|\downarrow\uparrow\rangle$ for zero detuning $\Delta = 0$.

In the basis of $|\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle$ the reduced density matrix ρ^{AB} is now found to be another example of the 'standard' two-qubit mixed state:

$$\rho^{AB} = \begin{pmatrix} |x_3|^2 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_1 x_5 & 0 \\ 0 & x_1^* x_5 & |x_2| + |x_5|^2 & 0 \\ 0 & 0 & 0 & |x_4|^2 \end{pmatrix} \tag{15}$$

and the concurrence for this matrix is given by $C^{AB}(t) = \max\{0, f(t)\}$ where

$$\begin{aligned} f(t) &= 2|x_1||x_5| - 2|x_3||x_4| \\ &= (1 - 4N^2\sin^2(\delta t/2))(|\sin 2\alpha| - 8N^2\sin^2(\delta t/2)\cos^2\alpha). \end{aligned} \tag{16}$$

For $\Delta = 0$, it becomes

$$f(t) = \cos^2(Gt/2)(|\sin 2\alpha| - \sin^2(Gt/2)\cos^2\alpha). \tag{17}$$

Unlike the previous case, figure 3 shows that entanglement can fall abruptly to zero (the two lower curves in the figure), and will remain zero for a period of time before entanglement recovers. The length of the time interval for the zero entanglement is dependent on the degree of entanglement of the initial state. The smaller the initial degree of entanglement, the longer the state will stay in the disentangled separable state.

In summary, let us emphasize again that there are no communications or interactions between the two atoms. The main result is the appearance of entanglement sudden death in a new environment, the first instance of sudden death without decoherence in the traditional sort. That is, because the cavities in our double Jaynes–Cummings model are lossless, they are as far from being standard decoherence reservoirs as possible. Nevertheless, we have shown that the non-interacting and non-communicating atoms A and B can abruptly lose their entanglement with each other. Given the lossless nature of the evolution, one can expect the resurrection of the original entanglement value to occur in a periodic way following each sudden death event, as is evident in figure 3. Finally, we point out that the onset of disappearance of two-atom entanglement is due to the information loss of atomic dynamics to the cavity modes

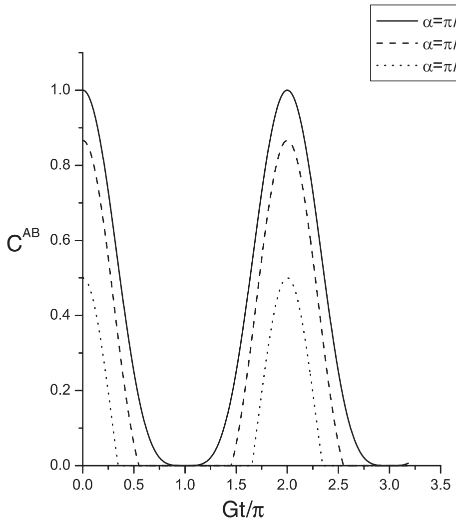

Figure 3. The concurrence for atom–atom entanglement with the initial atomic state $|\Phi_{\mathrm{atom}}\rangle = \cos\alpha|\uparrow\uparrow\rangle + \sin\alpha|\downarrow\downarrow\rangle$ for zero detuning $\Delta = 0$.

manifested by the operation of tracing over the cavity variables. On the other hand, it is the ‘small’ numbers of the cavity modes that lead to the entanglement resurrection. Namely, the lost information will come back to the atomic systems in finite times—a memory effect that is often associated with the Rabi frequency in JC cavities. Clearly, the entanglement of the two atoms will remain constant without their interactions with the local cavities.

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