

Determination of the CKM Matrix elements

- In total, there are 9 matrix elements to determine:

$$V_{\text{CKM}} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM Matrix parameterization

- The CKM matrix is unitary: can be parameterized by 3 mixing angles and the CP-violating phase
- A standard choice is the same as was shown for the PMNS matrix:

$$\begin{aligned} V_{\text{CKM}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned}$$

- Since experimentally $s_{13} \ll s_{23} \ll s_{12} \ll 1$, the matrix hierarchy is exhibited using the Wolfenstein parameterization (written in terms of λ, A, ρ, η):

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Determination of the CKM Matrix elements

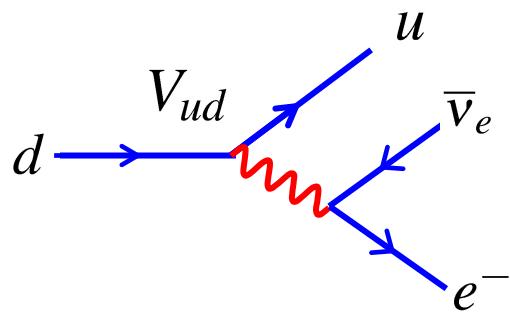
- The experimental determination of the **CKM matrix** elements comes mainly from measurements of leptonic decays (the leptonic part is well understood).
- It is easy to produce/observe meson decays, however theoretical uncertainties associated with the decays of bound states often limits the precision
- Contrast this with the measurements of the **PMNS matrix**, where there are few theoretical uncertainties and the experimental difficulties in dealing with neutrinos limits the precision.

1

$|V_{ud}|$

from nuclear beta decay

$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$



Super-allowed $0^+ \rightarrow 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

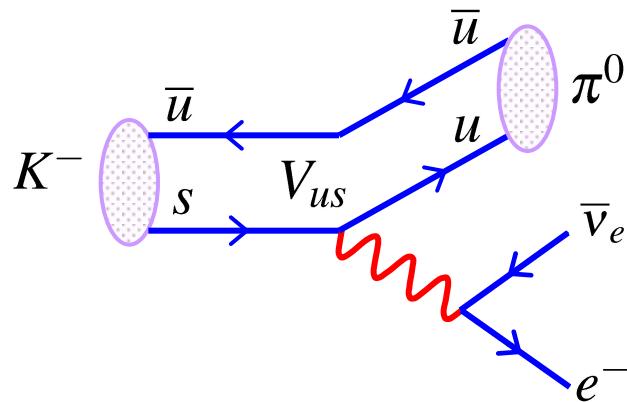
$$|V_{ud}| = 0.97370 \pm 0.00014$$

$$(\approx \cos \theta_c)$$

2

 $|V_{us}|$

from semi-leptonic kaon decays



$$\Gamma \propto |V_{us}|^2$$

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$|V_{us}| = 0.2245 \pm 0.0008$$

$$(\approx \sin \theta_c)$$

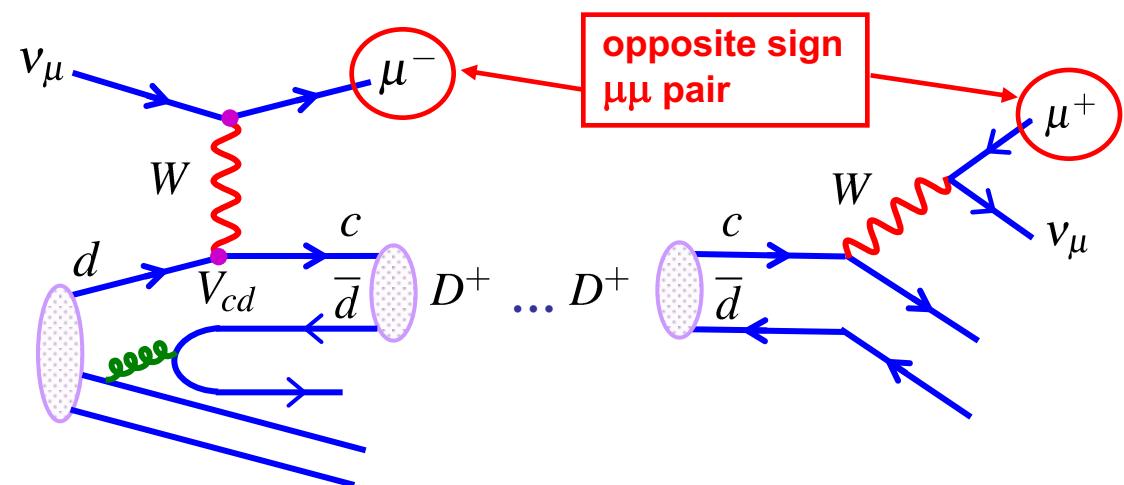
3

 $|V_{cd}|$ from neutrino scattering;
(semi-)leptonic D decays

$$\nu_\mu + N \rightarrow \mu^+ \mu^- X$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson



$$\text{Rate} \propto |V_{cd}|^2 \text{Br}(D^+ \rightarrow X \mu^+ \nu_\mu)$$

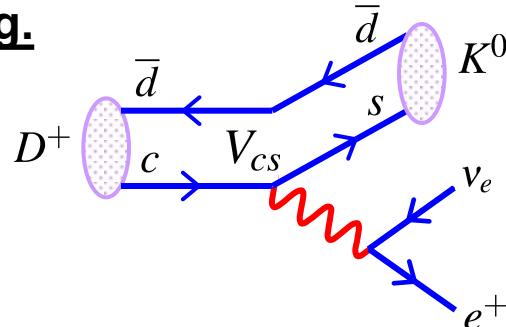
Measured in various
collider experiments

$$|V_{cd}| = 0.221 \pm 0.004$$

4

 $|V_{cs}|$

from semi-leptonic charmed meson decays

e.g.

$$\Gamma \propto |V_{cs}|^2$$

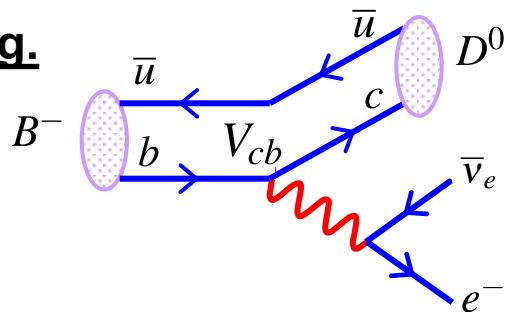
$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$|V_{cs}| = 0.987 \pm 0.011$$

5

 $|V_{cb}|$

from semi-leptonic B hadron decays

e.g.

$$\Gamma \propto |V_{cb}|^2$$

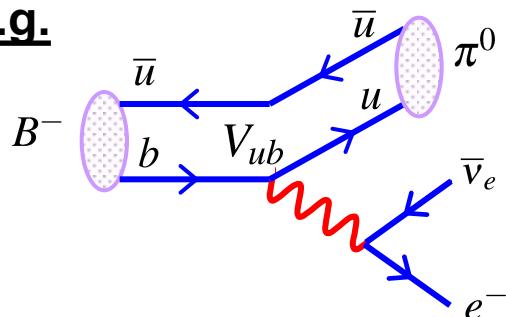
$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$|V_{cb}| = (41.0 \pm 1.4) \times 10^{-3}$$

6

 $|V_{ub}|$

from semi-leptonic B hadron decays

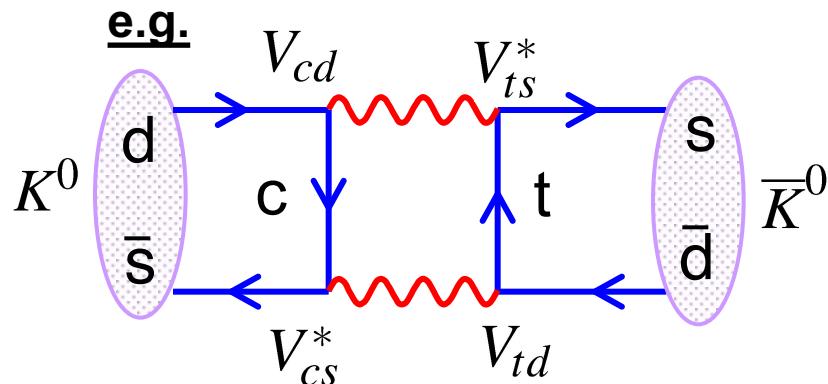
e.g.

$$\Gamma \propto |V_{ub}|^2$$

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$|V_{ub}| = (3.82 \pm 0.24) \times 10^{-3}$$

7/8 $|V_{td}|$ and $|V_{ts}|$ from K or B oscillations measurements



- E.g. ratio $|V_{td}/V_{ts}|$ determined from $\Delta m_d/\Delta m_s$ (mass difference in B^0 and in B_s systems)
- In ratios many uncertainties cancel out
- Precision limited by theoretical uncertainties

$$|V_{td}/V_{ts}| = 0.205 \pm 0.001 \pm 0.006$$

experimental error

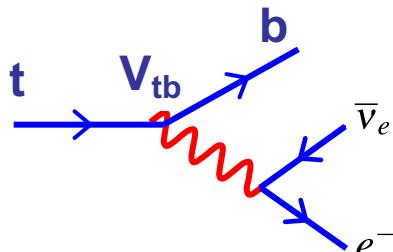
theory uncertainty

- Compare with uncertainties here:

$$|V_{td}| = (8.0 \pm 0.3) \times 10^{-3}, \quad |V_{ts}| = (38.8 \pm 1.1) \times 10^{-3}$$

9 $|V_{tb}|$ from top quark decays

e.g.



- Use ratio of BF: $B(t \rightarrow Wb)/B(t \rightarrow Wq) \propto |V_{tb}|^2$
- Also use single top quark production

$$|V_{tb}| = 1.013 \pm 0.030$$

- Experimental uncertainties dominate

Particle – Anti-Particle Mixing

- The wave-function for a single particle with lifetime $\tau = 1/\Gamma$ evolves with time as:

$$\psi(t) = Ne^{-\Gamma t/2} e^{-iMt}$$

which gives the appropriate exponential decay of

$$\langle \psi(t) | \psi(t) \rangle = \langle \psi(0) | \psi(0) \rangle e^{-t/\tau}$$

- The wave-function satisfies the time-dependent wave equation:

$$\hat{H}|\psi(t)\rangle = (M - \frac{1}{2}i\Gamma)|\psi(t)\rangle = i\frac{\partial}{\partial t}|\psi(t)\rangle \quad (\text{A1})$$

- For a bound state such as a K^0 the mass term includes the “mass” from the weak interaction “potential” \hat{H}_{weak}

$$M = m_{K^0} + \langle K^0 | \hat{H}_{\text{weak}} | K^0 \rangle + \sum_j \frac{|\langle K^0 | \hat{H}_{\text{weak}} | j \rangle|^2}{m_{K^0} - E_j}$$

Sum over intermediate states j

The third term is the 2nd order term in the perturbation expansion corresponding to box diagrams resulting in $K^0 \rightarrow K^0$

- The total decay rate is the sum over all possible decays $K^0 \rightarrow f$

$$\Gamma = 2\pi \sum_f |\langle f | \hat{H}_{weak} | K^0 \rangle|^2 \rho_F$$

Density of final states

- Because there are also diagrams which allow $K^0 \leftrightarrow \bar{K}^0$ mixing need to consider the time evolution of a mixed state

$$\psi(t) = a(t)K^0 + b(t)\bar{K}^0 \quad (\text{A2})$$

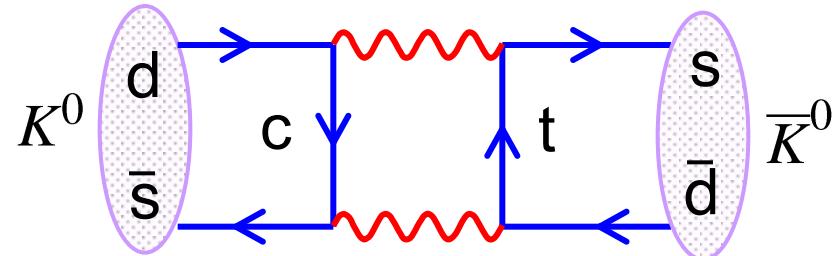
- The time dependent wave-equation of (A1) becomes

$$\begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix} \begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} = i \frac{\partial}{\partial t} \begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} \quad (\text{A3})$$

the diagonal terms are as before, and the off-diagonal terms are due to mixing.

$$M_{11} = m_{K^0} + \langle K^0 | \hat{H}_{weak} | K^0 \rangle + \sum_n \frac{|\langle K^0 | \hat{H}_{weak} | K^0 \rangle|^2}{m_{K^0} - E_n}$$

$$M_{12} = \sum_j \frac{\langle K^0 | \hat{H}_{weak} | j \rangle^* \langle j | \hat{H}_{weak} | \bar{K}^0 \rangle}{m_{K^0} - E_j}$$



- The off-diagonal decay terms include the effects of interference between decays to a common final state

$$\Gamma_{12} = 2\pi \sum_f \langle f | \hat{H}_{weak} | K^0 \rangle^* \langle f | \hat{H}_{weak} | \bar{K}^0 \rangle \rho_F$$

- In terms of the time dependent coefficients for the kaon states, (A3) becomes

$$[\mathbf{M} - i\frac{1}{2}\boldsymbol{\Gamma}] \begin{pmatrix} a \\ b \end{pmatrix} = i \frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix}$$

where the Hamiltonian can be written:

$$\mathbf{H} = \mathbf{M} - i\frac{1}{2}\boldsymbol{\Gamma} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{1}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

- Both the mass and decay matrices represent observable quantities and are Hermitian

$$M_{11} = M_{11}^*, \quad M_{22} = M_{22}^*, \quad M_{12} = M_{21}^*$$

$$\Gamma_{11} = \Gamma_{11}^*, \quad \Gamma_{22} = \Gamma_{22}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

- Furthermore, if CPT is conserved then the masses and decay rates of the K^0 and \bar{K}^0 are identical:

$$M_{11} = M_{22} = M; \quad \Gamma_{11} = \Gamma_{22} = \Gamma$$

- Hence the time evolution of the system can be written:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i \frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix} \quad (\text{A4})$$

- To solve the coupled differential equations for $a(t)$ and $b(t)$, first find the eigenstates of the Hamiltonian (the K_L and K_S) and then transform into this basis. The eigenvalue equation is:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (\text{A5})$$

- Which has non-trivial solutions for

$$|\mathbf{H} - \lambda I| = 0$$

$$\Rightarrow (M - \frac{1}{2}i\Gamma - \lambda)^2 - (M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12}) = 0$$

with eigenvalues

$$\lambda = M - \frac{1}{2}i\Gamma \pm \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})}$$

- The eigenstates can be obtained by substituting back into (A5)

$$(M - \frac{1}{2}i\Gamma)x_1 + (M_{12} - \frac{1}{2}i\Gamma_{12}) = (M - \frac{1}{2}i\Gamma \pm \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})})x_1$$



$$\frac{x_2}{x_1} = \pm \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

★ Define

$$\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

★ Hence the normalised eigenstates are

$$|K_{\pm}\rangle = \frac{1}{\sqrt{1+|\eta|^2}} \begin{pmatrix} 1 \\ \pm\eta \end{pmatrix} = \frac{1}{\sqrt{1+|\eta|^2}} (|K^0\rangle \pm \eta |\bar{K}^0\rangle)$$

★ Note, in the limit where M_{12}, Γ_{12} are real, the eigenstates correspond to the CP eigenstates K_1 and K_2 . Hence we can identify the general eigenstates as as the long and short lived neutral kaons:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}} (|K^0\rangle + \eta |\bar{K}^0\rangle)$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}} (|K^0\rangle - \eta |\bar{K}^0\rangle)$$

★ Substituting these states back into (A2):

$$\begin{aligned}
 |\psi(t)\rangle &= a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle \\
 &= \sqrt{1+|\eta|^2} \left[\frac{a(t)}{2}(K_L + K_S) + \frac{b(t)}{2\eta}(K_L - K_S) \right] \\
 &= \sqrt{1+|\eta|^2} \left[\left(\frac{a(t)}{2} + \frac{b(t)}{2\eta} \right) K_L + \left(\frac{a(t)}{2} - \frac{b(t)}{2\eta} \right) K_S \right] \\
 &= \frac{\sqrt{1+|\eta|^2}}{2} [a_L(t)K_L + a_S(t)K_S]
 \end{aligned}$$

with

$$a_L(t) \equiv a(t) + \frac{b(t)}{\eta}$$

$$a_S(t) \equiv a(t) - \frac{b(t)}{\eta}$$

★ Now consider the time evolution of $a_L(t)$

$$i \frac{\partial a_L}{\partial t} = i \frac{\partial a}{\partial t} + \frac{i}{\eta} \frac{\partial b}{\partial t}$$

★ Which can be evaluated using (A4) for the time evolution of $a(t)$ and $b(t)$:

$$\begin{aligned}
i \frac{\partial a_L}{\partial t} &= [(M - \frac{1}{2}i\Gamma_{12})a + (M_{12} - \frac{1}{2}i\Gamma_{12})b] + \frac{1}{\eta} \left[(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)a + (M - \frac{1}{2}i\Gamma)b \right] \\
&= (M - \frac{1}{2}i\Gamma) \left(a + \frac{b}{\eta} \right) + (M_{12} - \frac{1}{2}i\Gamma_{12})b + \frac{1}{\eta} (M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)a \\
&= (M - \frac{1}{2}i\Gamma)a_L + (M_{12} - \frac{1}{2}i\Gamma_{12})b + \left(\sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})} \right) a \\
&= (M - \frac{1}{2}i\Gamma)a_L + \left(\sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})} \right) \left(a + \frac{b}{\eta} \right) \\
&= (M - \frac{1}{2}i\Gamma)a_L + \left(\sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})} \right) a_L \\
&= (m_L - \frac{1}{2}i\Gamma_L)a_L
\end{aligned}$$

★ Hence:

$$i \frac{\partial a_L}{\partial t} = (m_L - \frac{1}{2}i\Gamma_L)a_L$$

with $m_L = M + \Re \left\{ \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})} \right\}$

and $\Gamma_L = \Gamma - 2\Im \left\{ \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})} \right\}$

★ Following the same procedure obtain:

$$i \frac{\partial a_S}{\partial t} = (m_S - \frac{1}{2}i\Gamma_S) a_S$$

with $m_S = M - \Re \left\{ \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})} \right\}$

and $\Gamma_S = \Gamma + 2\Im \left\{ \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})} \right\}$

★ In matrix notation we have

$$\begin{pmatrix} M_L - \frac{1}{2}i\Gamma_L & 0 \\ 0 & M_S - \frac{1}{2}i\Gamma_S \end{pmatrix} \begin{pmatrix} a_L \\ a_S \end{pmatrix} = i \frac{\partial}{\partial t} \begin{pmatrix} a_L \\ a_S \end{pmatrix}$$

★ Solving we obtain

$$a_L(t) \propto e^{-im_L t - \Gamma_L t/2} \quad a_S(t) \propto e^{-im_S t - \Gamma_S t/2}$$

★ Hence in terms of the K_L and K_S basis the states propagate as independent particles with definite masses and lifetimes (the mass eigenstates). The time evolution of the neutral kaon system can be written

$$|\psi(t)\rangle = A_L e^{-im_L t - \Gamma_L t/2} |K_L\rangle + A_S e^{-im_S t - \Gamma_S t/2} |K_S\rangle$$

where A_L and A_S are constants

CP Violation : $\pi\pi$ decays

- ★ Consider the development of the $K^0 - \bar{K}^0$ system now including CP violation
- ★ Repeat previous derivation using

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_1\rangle + \varepsilon |K_2\rangle] \quad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_2\rangle + \varepsilon |K_1\rangle]$$

- Writing the CP eigenstates in terms of K^0, \bar{K}^0

$$|K_L\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon) |K_0\rangle + (1-\varepsilon) |\bar{K}^0\rangle \right]$$

$$|K_S\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon) |K_0\rangle - (1-\varepsilon) |\bar{K}^0\rangle \right]$$

- Inverting these expressions obtain

$$|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} (|K_L\rangle + |K_S\rangle)$$

$$|\bar{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1-\varepsilon} (|K_L\rangle - |K_S\rangle)$$

- Hence a state that was produced as a K^0 evolves with time as:

$$|\psi(t)\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} (\theta_L(t) |K_L\rangle + \theta_S(t) |K_S\rangle)$$

where as before $\theta_S(t) = e^{-(im_S + \frac{\Gamma_S}{2})t}$ and $\theta_L(t) = e^{-(im_L + \frac{\Gamma_L}{2})t}$

- If we are considering the decay rate to $\pi\pi$ need to express the wave-function in terms of the CP eigenstates (remember we are neglecting CP violation in the decay)

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{\sqrt{2}} \frac{1}{1+\varepsilon} [(|K_2\rangle + \varepsilon|K_1\rangle)\theta_L(t) + (|K_1\rangle + \varepsilon|K_2\rangle)\theta_S(t)] \\ &= \frac{1}{\sqrt{2}} \frac{1}{1+\varepsilon} [(\theta_S + \varepsilon\theta_L)|K_1\rangle + (\theta_L + \varepsilon\theta_S)|K_2\rangle] \end{aligned}$$

CP Eigenstates

- Two pion decays occur with $CP = +1$ and therefore arise from decay of the $CP = +1$ kaon eigenstate, i.e. K_1

$$\Gamma(K_{t=0}^0 \rightarrow \pi\pi) \propto |\langle K_1 | \psi(t) \rangle|^2 = \frac{1}{2} \left| \frac{1}{1+\varepsilon} \right|^2 |\theta_S + \varepsilon\theta_L|^2$$

- Since $|\varepsilon| \ll 1$

$$\left| \frac{1}{1+\varepsilon} \right|^2 = \frac{1}{(1+\varepsilon^*)(1+\varepsilon)} \approx \frac{1}{1+2\Re\{\varepsilon\}} \approx 1 - 2\Re\{\varepsilon\}$$

- Now evaluate the $|\theta_S + \varepsilon\theta_L|^2$ term again using

$$|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re(z_1 z_2^*)$$

$$\begin{aligned}
|\theta_S + \varepsilon \theta_L|^2 &= |e^{-im_S t - \frac{\Gamma_S}{2} t} + \varepsilon e^{-im_L t - \frac{\Gamma_L}{2} t}|^2 \\
&= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2\Re\{e^{-im_S t - \frac{\Gamma_S}{2} t} \cdot \varepsilon^* e^{+im_L t - \frac{\Gamma_L}{2} t}\}
\end{aligned}$$

• Writing $\varepsilon = |\varepsilon| e^{i\phi}$

$$\begin{aligned}
|\theta_S + \varepsilon \theta_L|^2 &= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S + \Gamma_L)t/2} \Re\{e^{i(m_L - m_S)t - \phi}\} \\
&= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m.t - \phi)
\end{aligned}$$

• Putting this together we obtain:

$$\Gamma(K_{t=0}^0 \rightarrow \pi\pi) = \frac{1}{2}(1 - 2\Re\{\varepsilon\})N_{\pi\pi} \left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m.t - \phi) \right]$$

Short lifetime component
 $K_S \rightarrow \pi\pi$

CP violating long lifetime component
 $K_L \rightarrow \pi\pi$

Interference term

• In exactly the same manner obtain for a beam which was produced as \bar{K}^0

$$\Gamma(\bar{K}_{t=0}^0 \rightarrow \pi\pi) = \frac{1}{2}(1 + 2\Re\{\varepsilon\})N_{\pi\pi} \left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m.t - \phi) \right]$$

Interference term changes sign

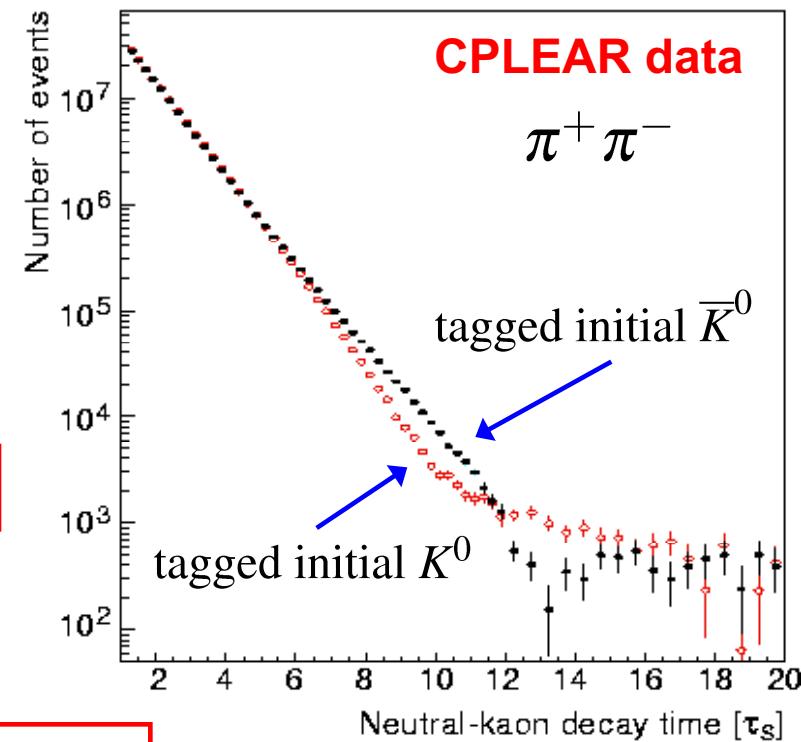
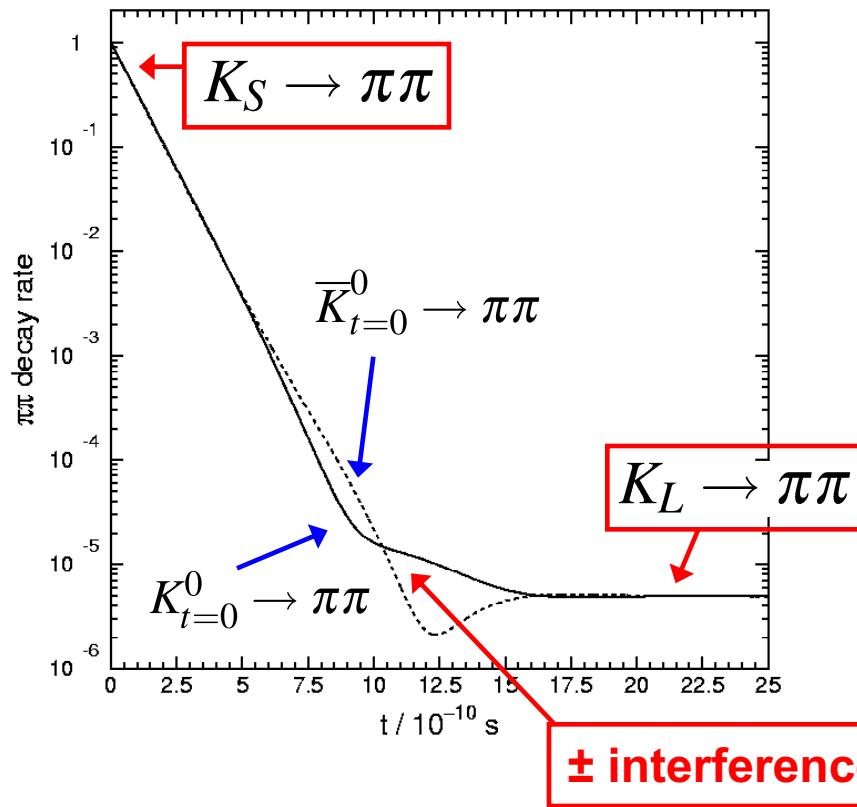
★ At large proper times only the long lifetime component remains :

$$\Gamma(K_{t=0}^0 \rightarrow \pi\pi) \rightarrow \frac{1}{2}(1 - 2\Re\{\varepsilon\})N_{\pi\pi}|\varepsilon|^2 e^{-\Gamma_L t}$$

i.e. CP violating $K_L \rightarrow \pi\pi$ decays

★ Since CPLEAR can identify whether a K^0 or \bar{K}^0 was produced, able to measure $\Gamma(K_{t=0}^0 \rightarrow \pi\pi)$ and $\Gamma(\bar{K}_{t=0}^0 \rightarrow \pi\pi)$

Prediction with CP violation



★ The CPLEAR data shown previously can be used to measure $\varepsilon = |\varepsilon|e^{i\phi}$

- Define the asymmetry:

$$A_{+-} = \frac{\Gamma(\bar{K}_{t=0}^0 \rightarrow \pi\pi) - \Gamma(K_{t=0}^0 \rightarrow \pi\pi)}{\Gamma(\bar{K}_{t=0}^0 \rightarrow \pi\pi) + \Gamma(K_{t=0}^0 \rightarrow \pi\pi)}$$

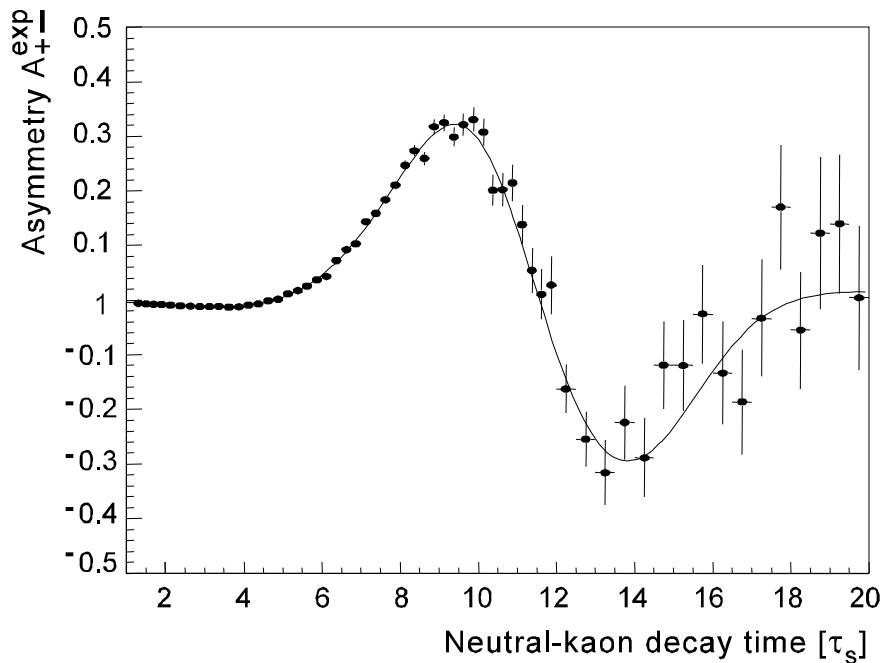
- Using previous expressions:

$$A_{+-} = \frac{4\Re\{\varepsilon\} [e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}] - 4|\varepsilon|e^{-(\Gamma_L + \Gamma_S)t/2} \cos(\Delta m.t - \phi)}{2[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}] - 8\Re\{\varepsilon\}|\varepsilon|e^{-(\Gamma_L + \Gamma_S)t/2} \cos(\Delta m.t - \phi)}$$

$\propto |\varepsilon|\Re\{\varepsilon\}$ i.e. two small quantities and
can safely be neglected

$$\begin{aligned} A_{+-} &\approx \frac{2\Re\{\varepsilon\} [e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}] - 2|\varepsilon|e^{-(\Gamma_L + \Gamma_S)t/2} \cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}} \\ &= 2\Re\{\varepsilon\} - \frac{2|\varepsilon|e^{-(\Gamma_L + \Gamma_S)t/2} \cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}} \\ &= 2\Re\{\varepsilon\} - \frac{2|\varepsilon|e^{(\Gamma_S - \Gamma_L)t/2} \cos(\Delta m.t - \phi)}{1 + |\varepsilon|^2 e^{(\Gamma_S - \Gamma_L)t}} \end{aligned}$$

A.Apostolakis et al., Eur. Phys. J. C18 (2000) 41



Best fit to the data:

$$|\epsilon| = (2.264 \pm 0.035) \times 10^{-3}$$
$$\phi = (43.19 \pm 0.73)^\circ$$

CP Violation via Mixing

- ★ A full description of the SM origin of CP violation in the kaon system is beyond the level of this course, nevertheless, the relation to the box diagrams is illustrated below
- ★ The K-long and K-short wave-functions depend on η

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle + \eta|\bar{K}^0\rangle)$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}(|K^0\rangle - \eta|\bar{K}^0\rangle)$$

with

$$\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

- ★ If $M_{12}^* = M_{12}$; $\Gamma_{12}^* = \Gamma_{12}$ then the K-long and K-short correspond to the CP eigenstates K_1 and K_2

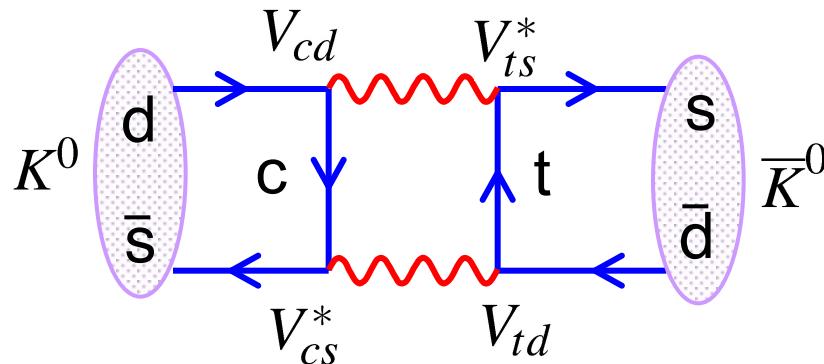
- CP violation is therefore associated with imaginary off-diagonal mass and decay elements for the neutral kaon system

- Experimentally, CP violation is small and $\eta \approx 1$

• Define: $\epsilon = \frac{1-\eta}{1+\eta}$ $\Rightarrow \eta = \frac{1-\epsilon}{1+\epsilon}$

- Consider the mixing term M_{12} which arises from the sum over all possible intermediate states in the mixing box diagrams

e.g.



$$M_{12} = A_{ct} V_{cd} V_{cs}^* V_{ts}^* V_{td} + \dots$$

- Therefore it can be seen that, in the Standard Model, CP violation is associated with the imaginary components of the CKM matrix
- It can be shown that mixing leads to CP violation with

$$|\varepsilon| \propto \Im\{M_{12}\}$$

- The differences in masses of the mass eigenstates can be shown to be:

$$\Delta m_K = m_{K_L} - m_{K_S} \approx \sum_{q,q'} \frac{G_F^2}{3\pi^2} f_K^2 m_K |V_{qd} V_{qs}^* V_{q'd} V_{q's}^*| m_q m_{q'}$$

where q and q' are the quarks in the loops and f_K is a constant

• In terms of the small parameter ε

$$|K_L\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle \right]$$

$$|K_S\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}} \left[(1-\varepsilon)|K^0\rangle + (1+\varepsilon)|\bar{K}^0\rangle \right]$$

★ If epsilon is non-zero we have CP violation in the neutral kaon system

Writing $\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}} = \sqrt{\frac{z^*}{z}}$ and $z = ae^{i\phi}$

gives $\eta = e^{-i\phi}$

★ From which we can find an expression for ε

$$\varepsilon \cdot \varepsilon^* = \frac{1 - e^{-i\phi}}{1 + e^{-i\phi}} \cdot \frac{1 - e^{+i\phi}}{1 + e^{i\phi}} = \frac{2 - \cos \phi}{2 + \cos \phi} = \tan^2 \frac{\phi}{2}$$

$$|\varepsilon| = |\tan \frac{\phi}{2}|$$

★ Experimentally we know ε is small, hence ϕ is small

$$|\varepsilon| \approx \frac{1}{2}\phi = \frac{1}{2}\arg z \approx \frac{1}{2} \frac{\Im\{M_{12} - \frac{1}{2}i\Gamma_{12}\}}{|M_{12} - \frac{1}{2}i\Gamma_{12}|}$$

Time Reversal Violation

- Previously obtained expressions for strangeness oscillations in the absence of CP violation, e.g.

$$\Gamma(K_{t=0}^0 \rightarrow K^0) = \frac{1}{4} \left[e^{-\Gamma_{St}} + e^{-\Gamma_{Lt}} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

- This analysis can be extended to include the effects of CP violation to give the following rates:

$$\Gamma(K_{t=0}^0 \rightarrow K^0) \propto \frac{1}{4} \left[e^{-\Gamma_{St}} + e^{-\Gamma_{Lt}} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\Gamma(\bar{K}_{t=0}^0 \rightarrow \bar{K}^0) \propto \frac{1}{4} \left[e^{-\Gamma_{St}} + e^{-\Gamma_{Lt}} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\Gamma(\bar{K}_{t=0}^0 \rightarrow K^0) \propto \frac{1}{4} (1 + 4\Re\{\varepsilon\}) \left[e^{-\Gamma_{St}} + e^{-\Gamma_{Lt}} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

$$\Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) \propto \frac{1}{4} (1 - 4\Re\{\varepsilon\}) \left[e^{-\Gamma_{St}} + e^{-\Gamma_{Lt}} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right]$$

★ Including the effects of CP violation find that

$$\Gamma(\bar{K}_{t=0}^0 \rightarrow K^0) \neq \Gamma(K_{t=0}^0 \rightarrow \bar{K}^0)$$

Violation of time reversal symmetry !

★ No surprise, as CPT is conserved, CP violation implies T violation

B-physics

- The oscillation of neutral mesons have been observed for heavy mesons as well:

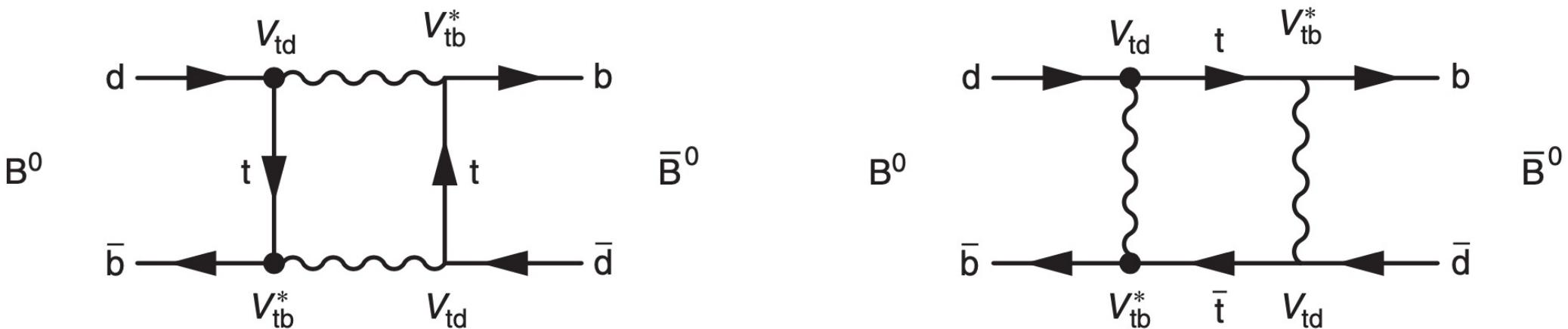
$$B^0(\bar{b}d) \leftrightarrow \bar{B}^0(\bar{b}\bar{d}), \quad B_s^0(\bar{b}s) \leftrightarrow \bar{B}_s^0(b\bar{s}) \quad \text{and} \quad D^0(\bar{c}u) \leftrightarrow \bar{D}^0(c\bar{u})$$

- The studies of B^0 mesons by BaBar, Belle and LHCb experiments, and of B_s mesons by the LHCb provided crucial information on the CKM matrix and CP violation
- Mathematical description of B-oscillations is the same as for kaons
- Major difference:
 - as B-mesons are heavy they have many decay modes (few hundreds)
 - Typically the decays for B^0 and \bar{B}^0 are different
 - Hence the interference between the decays is small
 - Consequence: oscillations can be described by a single angle β

$$|B_L\rangle = \frac{1}{\sqrt{2}} \left[|B^0\rangle + e^{-i2\beta} |\bar{B}^0\rangle \right] \quad \text{and} \quad |B_H\rangle = \frac{1}{\sqrt{2}} \left[|B^0\rangle - e^{-i2\beta} |\bar{B}^0\rangle \right]$$

B_L and B_H are a lighter and a heavier states with almost identical lifetimes

B⁰ mixing



- In kaon mixing, the contributions from all quarks to a box diagram were similar
- Here, since $|V_{tb}| \gg |V_{ts}| > |V_{td}|$, only box diagrams with top quark matter
- Off-diagonal mass element $M_{12} \propto (V_{td} V_{tb}^*)^2$
- Mass difference $\Delta m_d = m(B_H) - m(B_L) = 2|M_{12}| \propto |(V_{td} V_{tb}^*)^2|$
- Since $|V_{tb}| \approx 1$, Δm_d allows to measure V_{td}

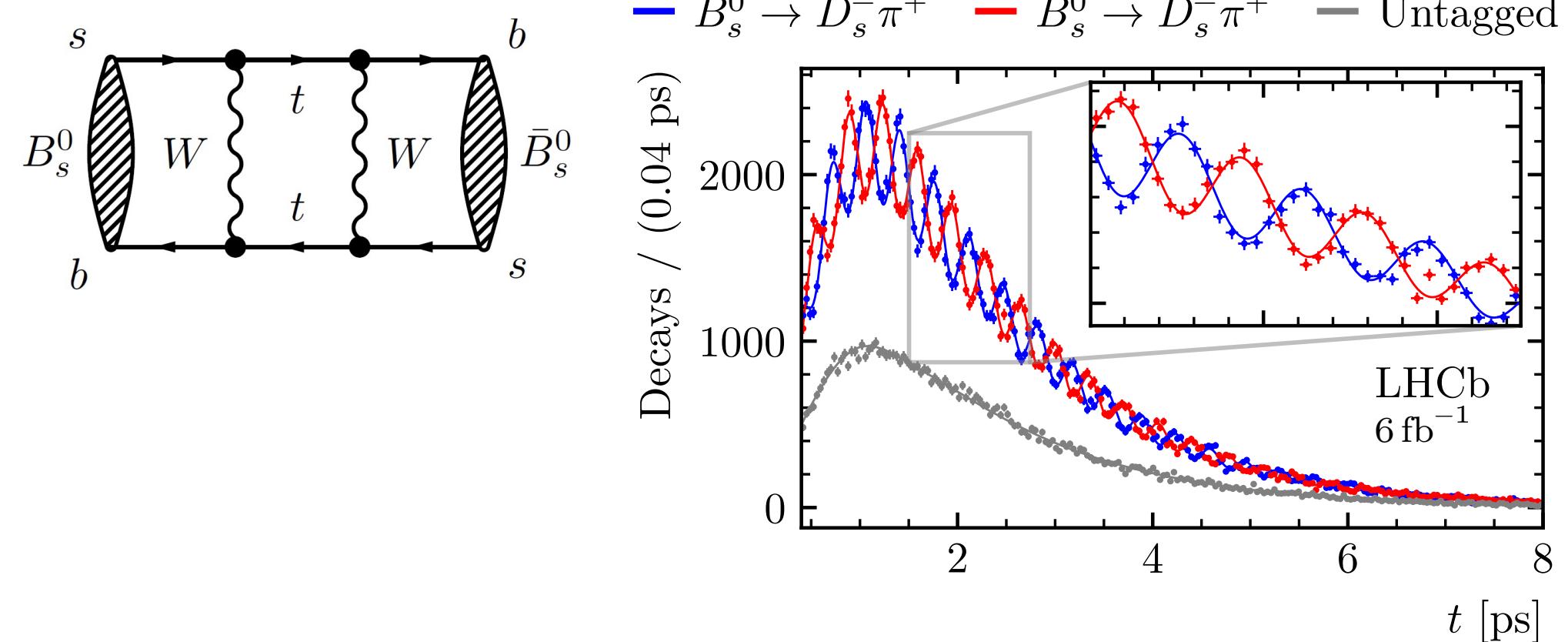
$$\Delta m_d = (0.507 \pm 0.005) \text{ ps}^{-1} \equiv (3.34 \pm 0.03) \times 10^{-13} \text{ GeV}$$

- Similarly, V_{ts} is derived from Δm_s in B_s oscillations

$$\Delta m_s = 17.7656 \pm 0.0057 \text{ ps}^{-1}$$

B_s mixing

- The latest LHCb result: <https://arxiv.org/abs/2104.04421>



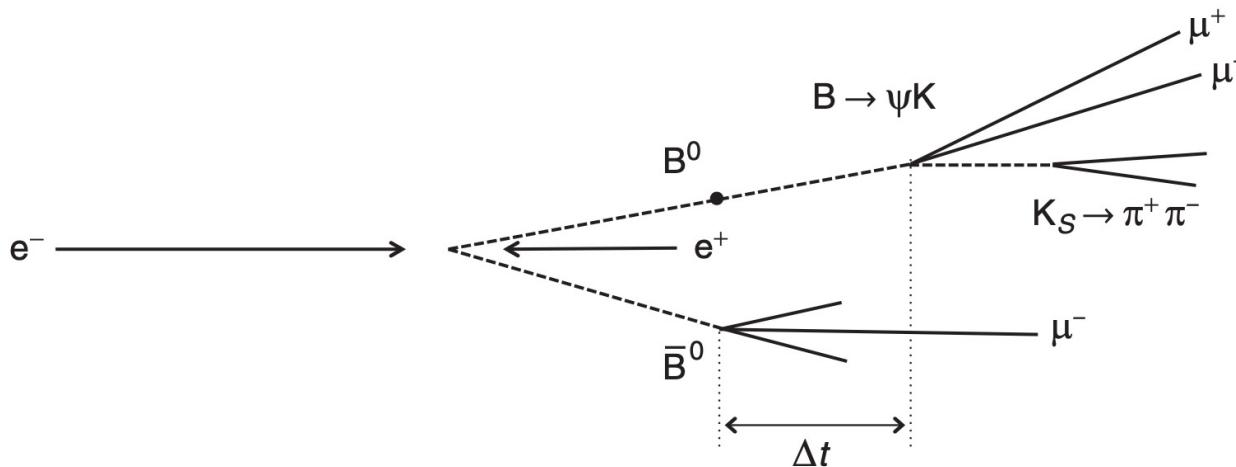
$$\Delta m_s = 17.7656 \pm 0.0057 \text{ ps}^{-1}$$

CP violation in B-meson system

- CP violation is observed as three effects:
 - Direct CP violation in decay: $\Gamma(A \rightarrow X) \neq \Gamma(\bar{A} \rightarrow \bar{X})$, as parameterised by ϵ' in the neutral kaon system;
 - CP violation in the mixing of neutral mesons as parameterised by ϵ in the kaon system;
 - CP violation in the interference between decays to a common final state f with and without mixing, for example $B^0 \rightarrow f$ and $B^0 \rightarrow B^0\bar{B} \rightarrow f$.
- In the SM, the CP violation in B^0 mixing is small
- CP-violating effects in the interference between decays $B^0 \rightarrow f$ and $B^0 \rightarrow B^0\bar{B} \rightarrow f$ can be relatively large and have been studied extensively by the BaBar, Belle and LHCb experiments

CP violation in the interference: $B^0 \rightarrow J/\psi K_S$

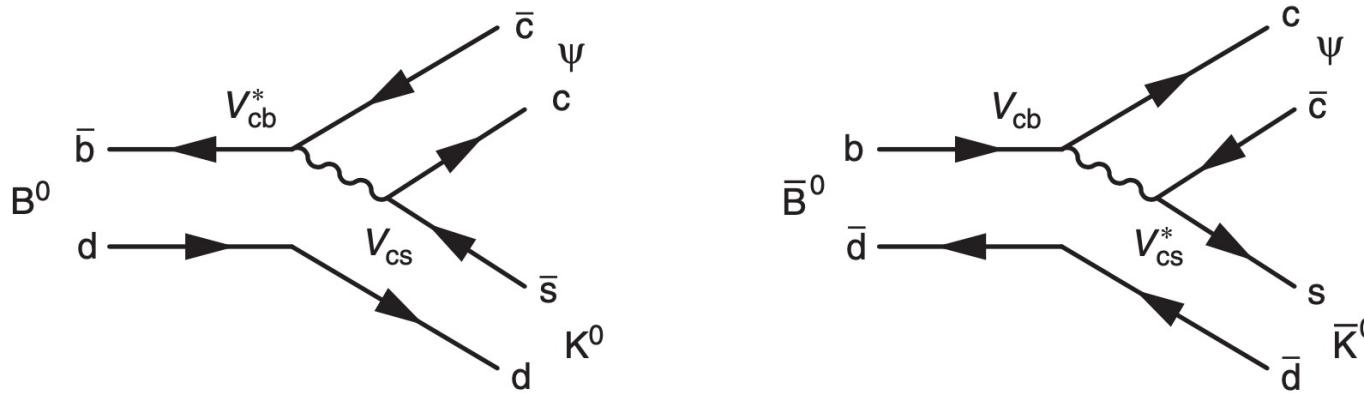
- J/ψ : $J^P = 1^-$, CP = +1
- K_S : $J^P = 0^-$, CP = +1
- B^0 are spin-0 $\Rightarrow B^0 \rightarrow J/\psi K_S$ has L=1 orbital angular momentum
- $CP(J/\psi K_S) = CP(J/\psi)CP(K_S)(-1)^L = (+1)(+1)(-1) = -1$
- Similarly, $B^0 \rightarrow J/\psi K_L$ occurs in the CP-even state



- Muon charge from $B^0\text{-bar} \rightarrow D^+\mu^-\nu_\mu$ -bar tags a flavour of B^0 -bar, and a second B^0 at the moment $t=0$
- Decay to $J/\psi K_S$ happens either directly or after mixing

CP violation in the interference: $B^0 \rightarrow J/\psi K_S$

- Decay proceeds in two stages:
 - First, B^0 decays to flavour eigenstate: $B^0 \rightarrow J/\psi K^0$ and $\bar{B}^0 \rightarrow J/\psi \bar{K}^0$
 - Then the neutral kaons system evolves as a linear combination of physical K_S and K_L states

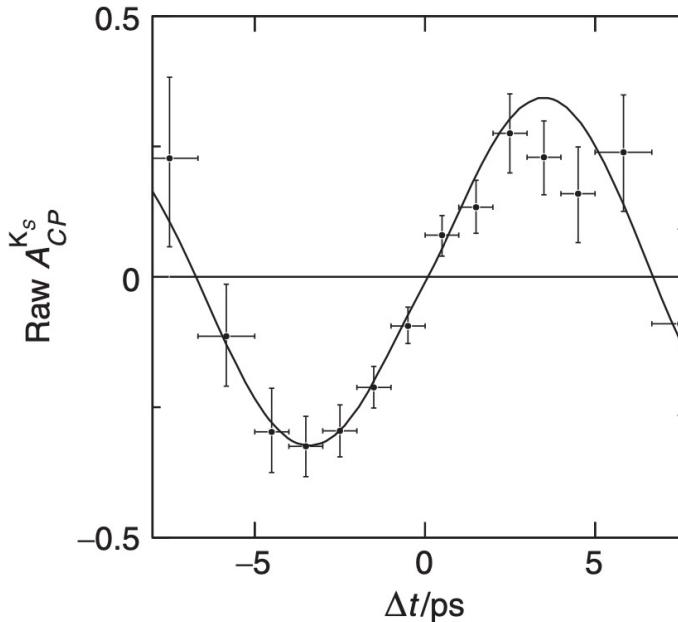
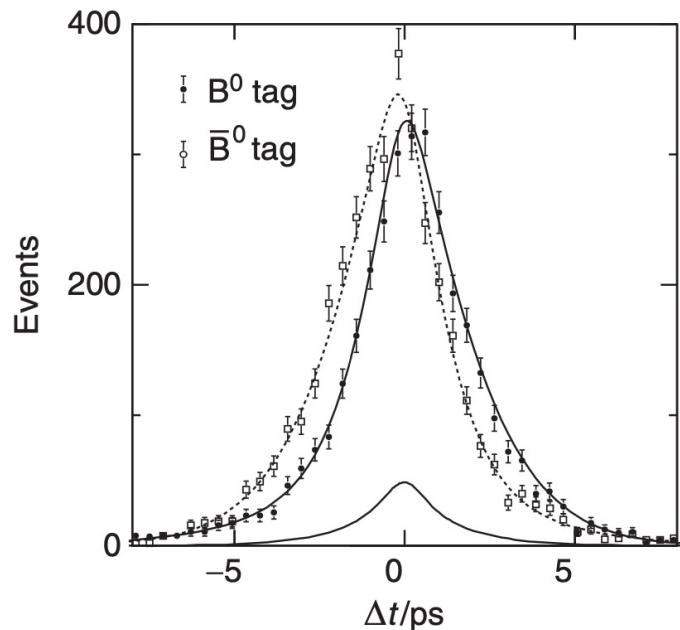


- CP violation is measurable through the asymmetry

$$A_{CP}^{K_S} = \frac{\Gamma(\bar{B}_{t=0}^0 \rightarrow \psi K_S) - \Gamma(B_{t=0}^0 \rightarrow \psi \bar{K}_S)}{\Gamma(\bar{B}_{t=0}^0 \rightarrow \psi K_S) + \Gamma(B_{t=0}^0 \rightarrow \psi \bar{K}_S)} = \sin(\Delta m_d t) \sin(2\beta)$$

CP violation in the interference: $B^0 \rightarrow J/\psi K_s$

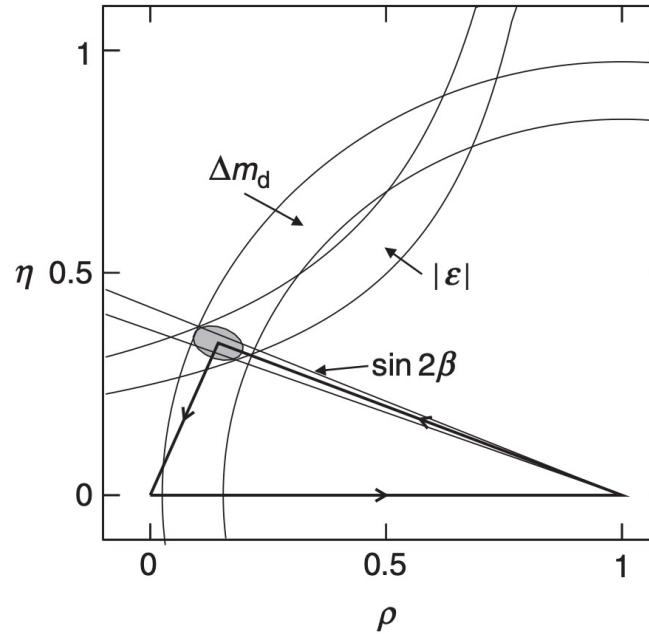
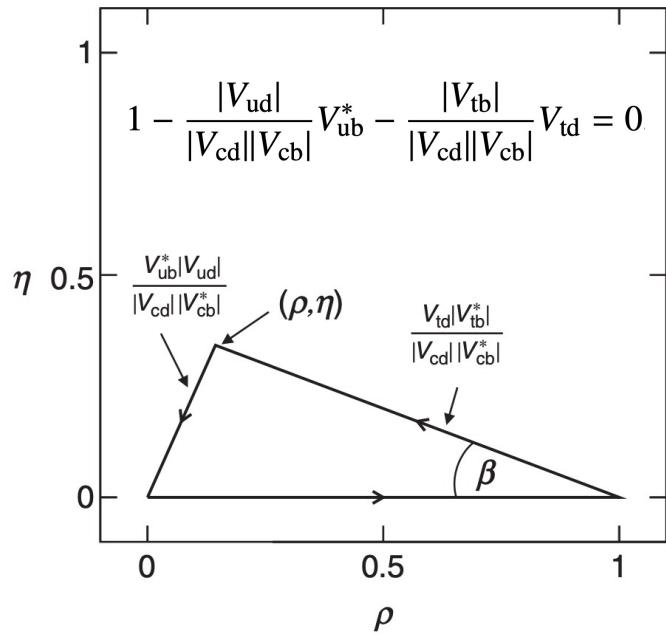
- Results from Belle:



- the observation of a non-zero value of $\sin(2\beta)$ is a direct manifestation of CP violation in the B-meson system
- Measured value: $\sin(2\beta) = 0.685 \pm 0.032$

CP violation in the quark sector

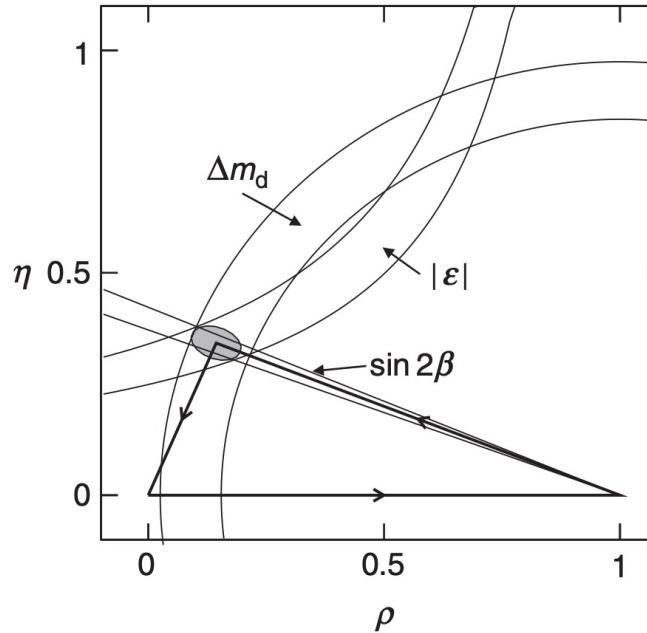
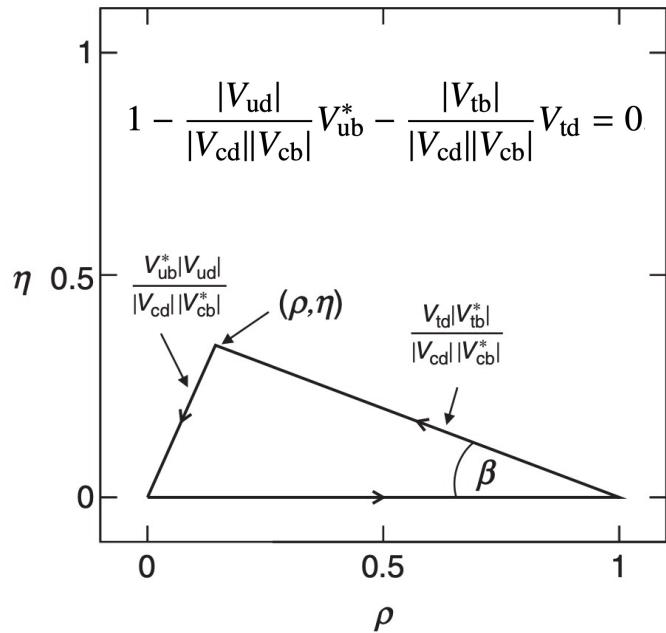
- Combination of various measurements:



- CP violation in the weak interactions of hadrons is described by the single irreducible complex phase in the CKM matrix
 - In the Wolfenstein parametrisation, CP violation is associated with the parameter η . To $O(\lambda^4)$, the parameter η appears only in V_{ub} and V_{td} , with
- $$V_{ub} \approx A\lambda^3(\rho - i\eta) \quad \text{and} \quad V_{td} \approx A\lambda^3(1 - \rho - i\eta)$$
- The measurements of non-zero values of $|\epsilon|$ and $\sin(2\beta)$ separately imply that $\eta \neq 0$
 - But ρ and η are determined from the measurements combinations

CP violation in the quark sector

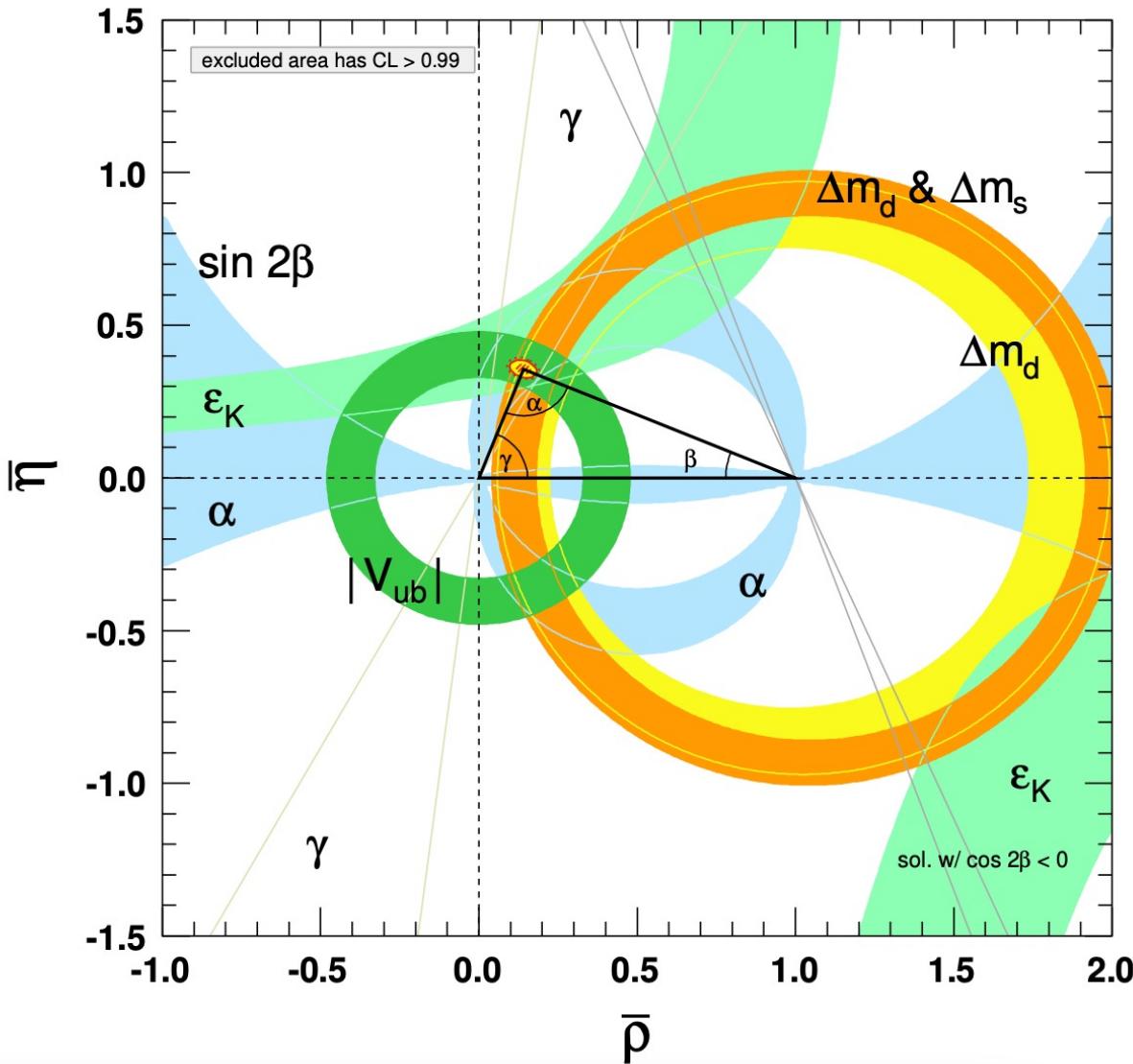
- Combination of various measurements:



$$\lambda = 0.2253 \pm 0.0007, A = 0.811^{+0.022}_{-0.012}, \rho = 0.13 \pm 0.02, \eta = 0.345 \pm 0.014.$$

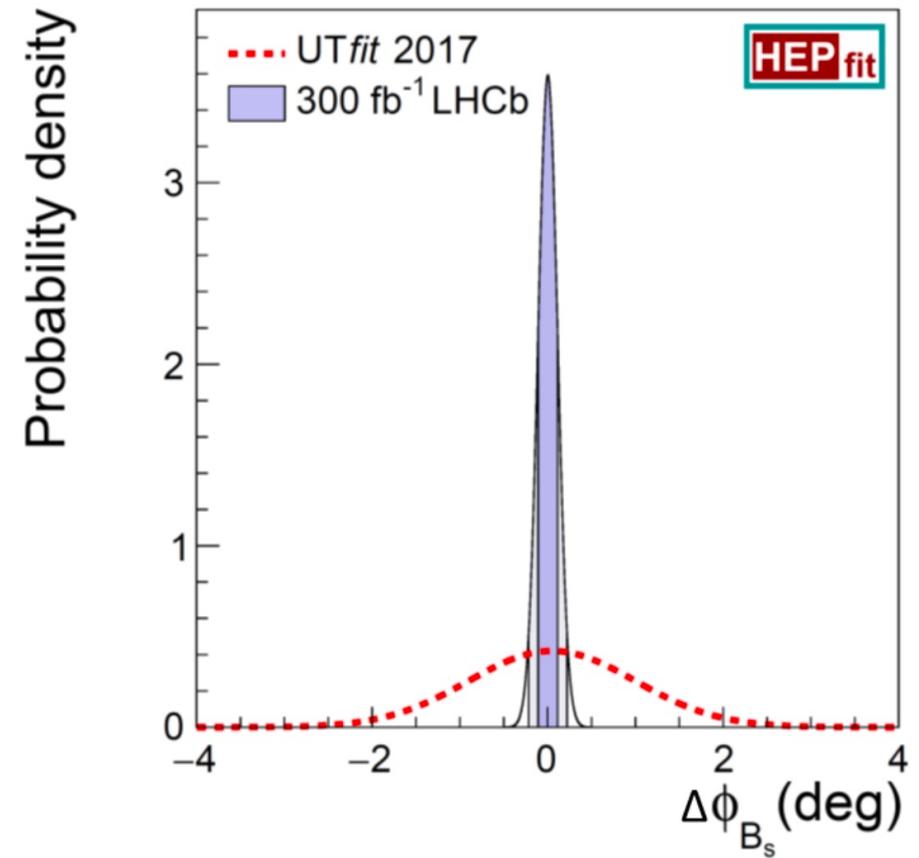
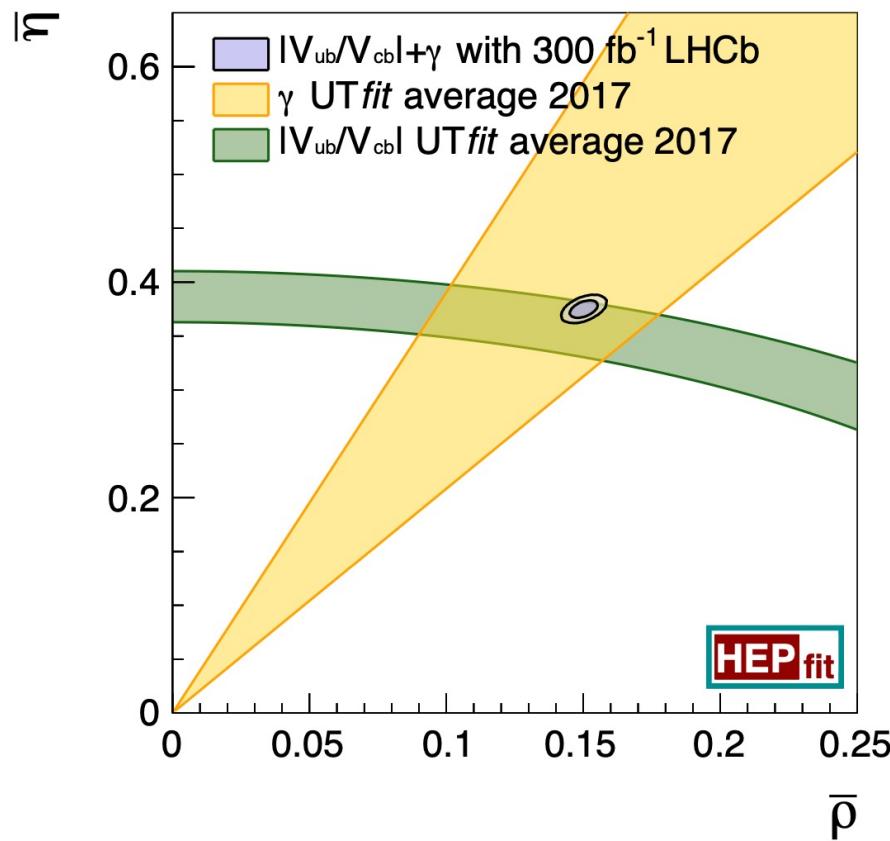
- With all the measurements, the unitary triangle is closed
- A deviation from this could indicate a presence of a new physics
- Future experiments, LHCb in particular, will improve many measurements by orders of magnitude

CKM triangle now



LHCb in the HL-LHC phase

- In case the LHCb upgrade in 2030 happens:



- CKM triangle measurements will come on a new level