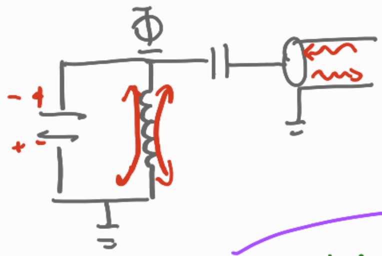
Superconducting Qubits

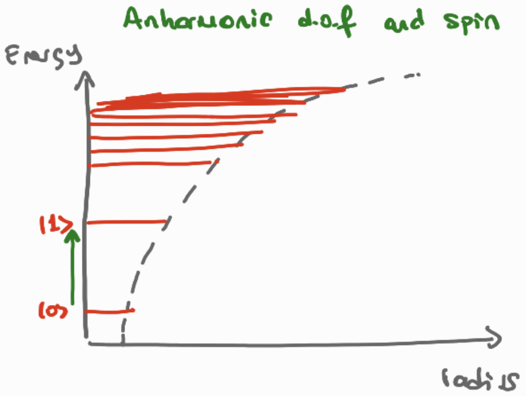
FIOMI 2020 Qiskit Summer School - Zlatko Minev

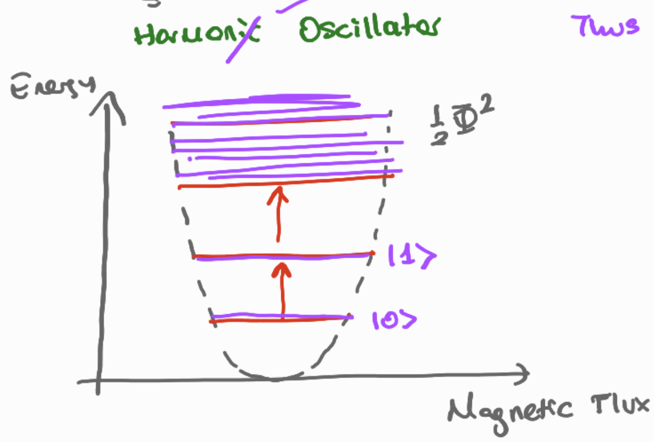
Making Your First Qubit from an Oscillator

Look at References page

Artificial Atoms





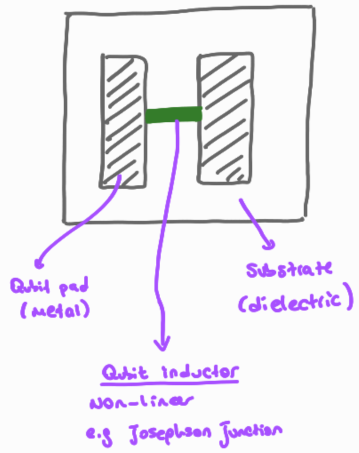


→ It should be anharmonic

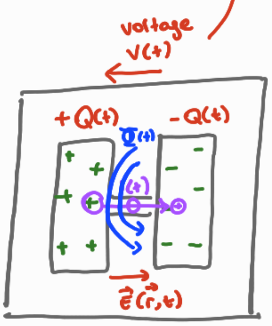
Thus inductor should be nonlinear

Circuit Quantum Electro Dynamics (cQED)

Transmon Qubit



$v(t) = \int_{\vec{r}_a}^{\vec{r}_b} E(\vec{r},t) \cdot d(\vec{r})$



charge

$Q(t) = C\, v(t)$

$\frac{dQ(t)}{dt} = i(t)$

$\Phi(t) = \int_{-\infty}^{t} v(\tau)\, d\tau \rightarrow \frac{d}{dt}\Phi(t) = v(t)$

magnetic flux

$\Phi(t) = L\, i(t)$

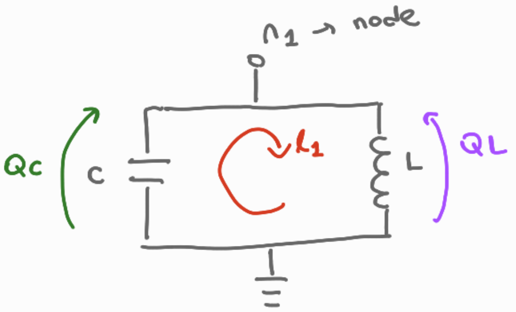
Energy

$\frac{d\mathcal{E}(t)}{dt} = p(t) = v(t)\, I(t)$

instantaneous power

$\mathcal{E}_{cap}(\dot{\Phi}) = \frac{1}{2} C \dot{\Phi}^2 = \frac{Q^2}{2C}$

$\mathcal{E}_{ind}(\Phi) = \frac{\Phi^2}{2L} = \frac{1}{2} L \dot{Q}^2$



$n_1: \quad \dot{Q}_C + \dot{Q}_L = 0$

$l_1: \quad \dot{\Phi}_C - \dot{\Phi}_L = 0 \rightarrow \boxed{\Phi_C = \Phi_L}$

$\sum_{\text{nodes}} \pm \dot{Q}_b(t) = 0$

Kirchoff's Current Law

$\sum_{\text{loop}} \pm \dot{\Phi}_b(t) = 0$

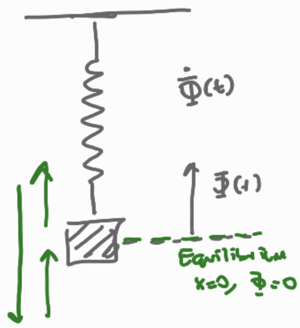
$Q = CV = C\dot{\Phi} \qquad \Phi = Li$

$\dot{Q} = C\ddot{\Phi} \qquad \Phi = L\dot{Q}_L \rightarrow \dot{Q}_L = L^{-1}\Phi$

$n_1: \quad \dot{Q}_c + \dot{Q}_L = \boxed{C\ddot{\Phi} + L^{-1}\Phi = 0}$ ☆

$$C\ddot{\Phi} + L^{-1}\Phi = 0$$

$$\ddot{\Phi} = -\omega_0^2 \Phi \quad \text{where} \quad \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow \Phi = \Phi_0 e^{-i\omega_0 t}$$



Lagrangian and Hamiltonian

position, velocity

$\mathcal{L}(\Phi, \dot{\Phi}) = \mathcal{E}_{cap}(\dot{\Phi}) - \mathcal{E}_{ind}(\Phi) \rightarrow$ kinetic Energy − Potential Energy

$= \frac{1}{2}C\dot{\Phi}^2 - \frac{\Phi^2}{2L}$

momentum

$Q = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\dot{\Phi} \longrightarrow$ canonically conjugate variable (momentum, charge)

$\boxed{C\ddot{\Phi} + L^{-1}\Phi = 0} \leftarrow \underbrace{\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\Phi}}}_{i(t)} = \underbrace{\frac{\partial \mathcal{L}}{\partial \Phi}}_{i(t)} \longrightarrow$ Euler-Lagrange equations, $F = ma$

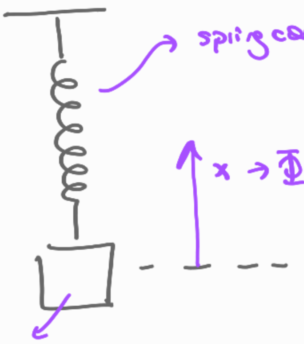
$i_C(t)$ $\quad i_L(t)$

-KCL-

$H(\Phi, Q) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \longrightarrow$ Kinetic Energy + Potential Energy

position $\quad$ momentum

Analogy - LC classical harmonic oscillator



Mass $m \to C$

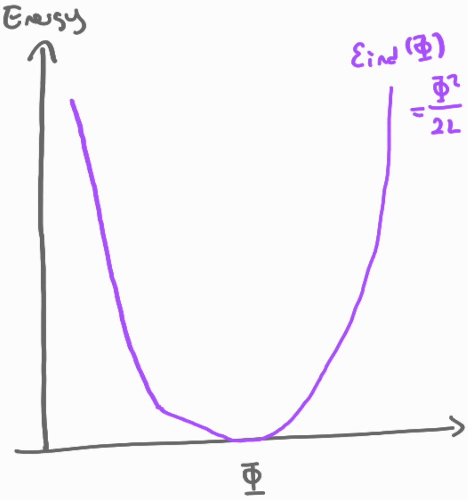
Momentum $p \to Q$

$$H(\Phi, Q) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$z_0^2 = \frac{L}{C}$$

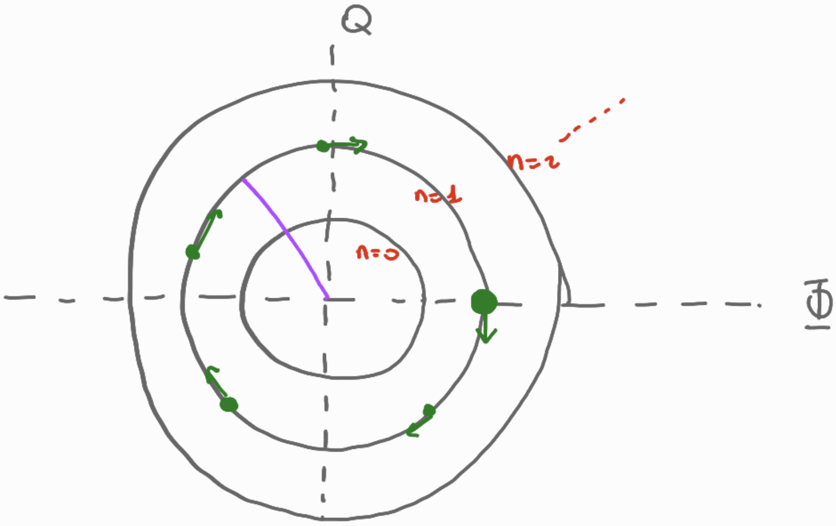
impedance



Hamiltonian dynamics and phase space

$$H(\Phi, Q) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \qquad E = \hbar\omega_0\left(n + \frac{1}{2}\right) = \frac{1}{2}\left(Q^2 + \Phi^2\right)$$

with $L = C = 1$



complex action-angle variable

$$\alpha(t) = \alpha(0)\, e^{-i\omega_0 t}$$

$\to$ to describe the point of the phase space

trajectory of motion (position)

$$\alpha(t) = \sqrt{\frac{1}{2\hbar z}}\left[\Phi(t) + i z\, Q(t)\right]$$

$$H(\alpha, \alpha^*) = \frac{1}{2}\hbar\omega_0\left(\alpha^*\alpha + \alpha\alpha^*\right)$$

Unveiling the Quantum

Classical	Quantum
$\Phi(t)$	$\hat{\Phi}$
$Q(t)$	$\hat{Q}$
$H(\Phi, Q)$	$\hat{H}(\hat{\Phi}, \hat{Q})$
poisson bracket $\{\Phi, Q\} = 1$	$[\hat{\Phi}, \hat{Q}] = i\hbar \hat{1}$
$\{\alpha, \alpha^*\} = \frac{1}{(i\hbar)}$	$[\hat{a}, \hat{a}^\dagger] = \hat{1}$

Hamiltonian

$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$= \frac{1}{2}\hbar\omega_0 (\alpha^*\alpha + \alpha\alpha^*)$$

$$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}$$

$$= \hbar\omega_0 (\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger)$$

$$\alpha(t) = \sqrt{\frac{1}{2\hbar Z}}\,[\Phi(t) + iZQ(t)]$$

$$\boxed{\hat{a} = \sqrt{\frac{1}{2\hbar Z}}\,(\hat{\Phi} + iZ\hat{Q})}$$

Phase Space

$$\alpha(t) = \alpha(0)e^{-i\omega_0 t}$$

$$\hat{a}(t) = \hat{a}(0)\, e^{-i\omega_0 t}$$

$$\Phi(t) = \sqrt{\frac{\hbar Z}{2}}\,(\alpha^*(t) + \alpha(t))$$

$$\hat{\Phi} = \Phi_{ZPF}\,(\hat{a}^\dagger + \hat{a})$$

$$Q(t) = i\sqrt{\frac{\hbar}{2Z}}\,(\alpha^*(t) - \alpha(t))$$

$$\hat{Q} = iQ_{ZPF}\,(\hat{a}^\dagger - \hat{a})$$

Ladder Operators

$$\hat{a}|0\rangle = 0$$

$$\hat{a}|1\rangle = \sqrt{1}|0\rangle$$

annihilation

$$\hat{a}^\dagger|0\rangle = \sqrt{1}|1\rangle$$

$$\hat{a}^\dagger|1\rangle = \sqrt{2}|2\rangle$$

creation

$$\boxed{\begin{aligned}\hat{a}|n\rangle &= \sqrt{n}\,|n-1\rangle \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}\,|n+1\rangle\end{aligned}}$$

$$\hat{a} = \begin{array}{c|cccc} & \langle 0| & \langle 1| & \langle 2| & \langle 3| \\ \hline |0\rangle & 0 & 1 & 0 & 0 \\ |1\rangle & 0 & 0 & \sqrt{2} & 0 \\ |2\rangle & 0 & 0 & 0 & \sqrt{3} \\ |3\rangle & 0 & 0 & 0 & \ddots \end{array}$$

$$\hat{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

$$\hat{a}^\dagger\hat{a} = \begin{pmatrix} 1 & & 0 \\ & 2 & \\ 0 & & 3 \\ & & & \ddots \end{pmatrix}$$

$$\hat{H} = \frac{\hbar\omega_0}{2}\left(a^\dagger a + a a^\dagger\right)$$

$$a^\dagger a\,|n\rangle = n\,|n\rangle$$

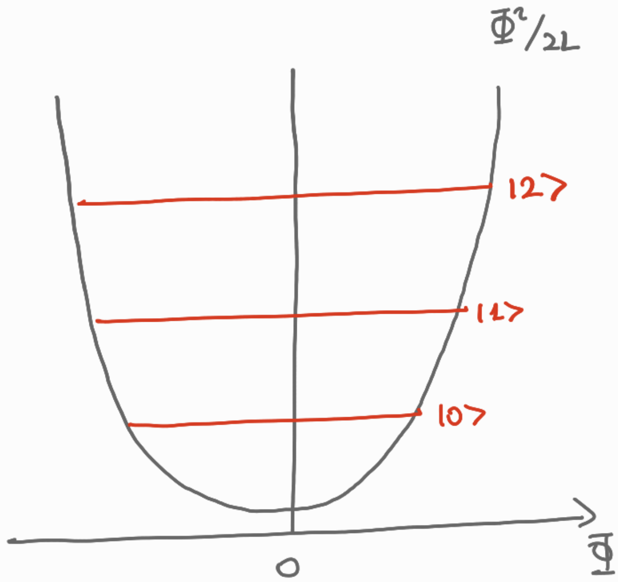
$$a^\dagger \underbrace{(a|n\rangle)}_{\sqrt{n}|n-1\rangle} = \sqrt{n}\,\underbrace{\hat{a}^\dagger|n-1\rangle}_{\sqrt{n}|n\rangle} = (\sqrt{n})^2\,|n\rangle = n|n\rangle$$

$$[a, a^\dagger] = 1$$

$$a a^\dagger - a^\dagger a = 1$$

$$a a^\dagger = 1 + a^\dagger a$$

$\hat{a}^\dagger a =$ Photon number n



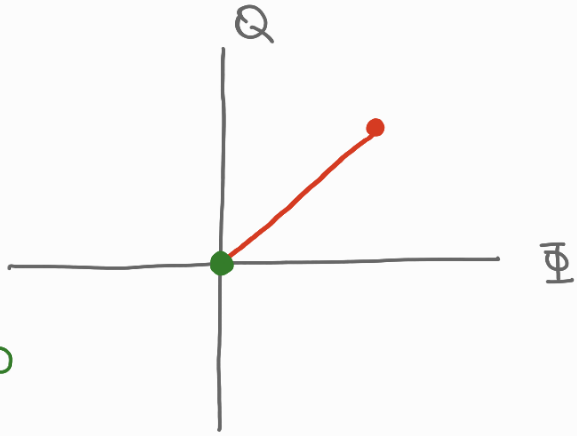
$$\hat{H} = \frac{\hbar\omega_0}{2}\left(a^\dagger a + \overbrace{a a^\dagger}^{1 + a^\dagger a}\right)$$

$$= \frac{\hbar\omega_0}{2}\left(2a^\dagger a + 1\right)$$

$$\boxed{\hat{H} = \hbar\omega_0\left(\underbrace{a^\dagger a}_{n} + \underbrace{1/2}_{\text{zero-point energy}}\right)}$$

$$\hat{H}|n\rangle = E_n|n\rangle$$

$$E_n = \hbar\omega_0\left(n + \tfrac{1}{2}\right) \qquad n = 0, 1, 2, \ldots$$



expectation value of pos.

$$\langle\hat{\Phi}\rangle = \langle n|\, \Phi_{ZPF}\,(\hat{a}^\dagger + \hat{a})\,|n\rangle = 0 \qquad \Phi_{ZPF} = \sqrt{\frac{\hbar Z}{2}}$$

$$\langle n|a|n\rangle = \sqrt{n}\,\langle n|n-1\rangle = 0$$

$$\langle n|a^\dagger|n\rangle = \sqrt{n+1}\,\langle n|n+1\rangle = 0$$

$$\langle\Phi\rangle = \Phi_{ZPF}\left(\langle\hat{a}\rangle + \langle\hat{a}^\dagger\rangle\right) = 0$$

$$\langle Q\rangle = \langle n|\,{-i}\,Q_{ZPF}\,(a^\dagger - a)\,|n\rangle = 0 \qquad Q_{ZPF} = \sqrt{\frac{\hbar}{2Z}}$$

$$\mathrm{Var}(\Phi) = \langle \Phi^2 \rangle - \cancel{\langle \Phi \rangle^2}_{\to 0}$$

$$= \langle \Phi^2 \rangle = \langle n | (a^\dagger + a)^2 | n \rangle \, \Phi_{ZPF}^2$$

$$= \langle n | \, a^{\dagger 2} + a^2 + a^\dagger a + a a^\dagger \, | n \rangle \, \Phi_{ZPF}^2$$

$$= \left(\cancel{\langle n | a^{\dagger 2} | n \rangle}_{\to 0} + \cancel{\langle a^2 \rangle}_{\to 0} + \langle a^\dagger a + a a^\dagger \rangle \right) \Phi_{ZPF}^2$$

$$= \Phi_{ZPF}^2 \langle a^\dagger a + a a^\dagger \rangle$$

$$= \Phi_{ZPF}^2 \langle a^\dagger a + a^\dagger a + 1 \rangle$$

$$\mathrm{Var}(\hat{\Phi}) = \langle \hat{\Phi}^2 \rangle = (2n+1)\, \Phi_{ZPF}^2$$

For $|n\rangle = 0 \longrightarrow \langle \hat{\Phi}^2 \rangle = \Phi_{ZPF}^2$

i.e RMS fluctuations at $\hat{\Phi}$ in ground state

$$\boxed{\sigma_{\Phi} = \Phi_{ZPF}}$$

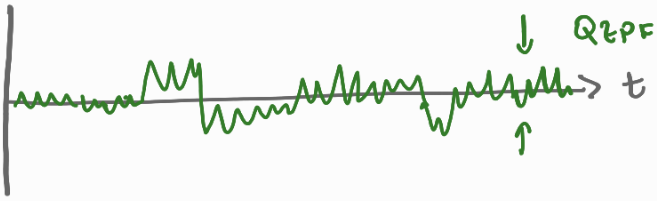
Measured $\Phi(t)$

mean = 0

std = Φ_{ZPF}

standard deviation

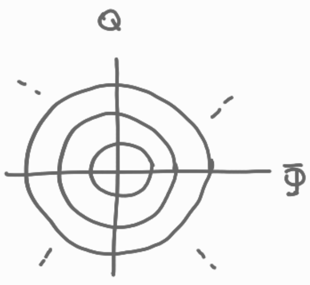
$Q(t)$

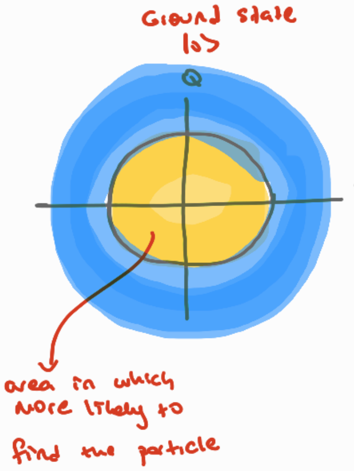


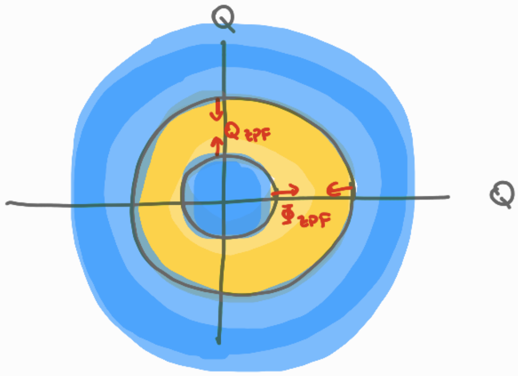
mean = 0

std = Q_{ZPF}

Husimi- Q Function







The flux and charge operators are Hermitian observables

How can some expectations be imaginary, or negative?

Example: $\langle 0 | \hat{\Phi} \hat{Q} | 0 \rangle = \frac{1}{4} i$

?