Zeynepnur ŞAHİNEL
2305399

PHYS 455 FINAL

Question 1

$$|\psi\rangle_C = a|0\rangle_c + b|1\rangle_c$$

$$|\psi\rangle_{AB} = \frac{|00\rangle + i|11\rangle}{\sqrt{2}}$$

Teleportation Protocol

1) Alice does a measurement on CA in Bell-Basis

Before the measurement, state of CAB:

$$|\psi_{1}\rangle = |\psi_c\rangle \otimes |\psi\rangle_{AB}$$

$$= (a|0\rangle + b|1\rangle)_c \otimes \left(\frac{|00\rangle + i|11\rangle}{\sqrt{2}} \right)_{AB}$$

$$|\psi_{1}\rangle = \frac{a|000\rangle + b|100\rangle + ia|011\rangle + ib|111\rangle}{\sqrt{2}}$$

$$|\psi_{1}\rangle = \frac{a}{\sqrt{2}} \left(\frac{|\Phi_+\rangle + |\Phi_-\rangle}{\sqrt{2}} \right)_{CA} \otimes |0\rangle_B + \frac{b}{\sqrt{2}} \left(\frac{|\Psi_+\rangle - |\Psi_-\rangle}{\sqrt{2}} \right) \otimes |0\rangle +$$

$$\frac{ia}{\sqrt{2}} \left(\frac{|\Psi_+\rangle + |\Psi_-\rangle}{\sqrt{2}} \right) \otimes |1\rangle + \frac{ib}{\sqrt{2}} \left(\frac{|\Phi_+\rangle - |\Phi_-\rangle}{\sqrt{2}} \right) \otimes |1\rangle$$

$$|\psi_{1}\rangle = \frac{1}{2} \Big(|\Phi_+\rangle_{CA} \otimes \boxed{(a|0\rangle + ib|1\rangle)_B} + |\Phi_-\rangle \otimes \boxed{(a|0\rangle - ib|1\rangle)} +$$

$$|\Psi_+\rangle \otimes \boxed{(b|0\rangle + ia|1\rangle)} + |\Psi_-\rangle \otimes \boxed{(ia|1\rangle - b|0\rangle)} \Big)$$

After Measurement, Alice could achieve 4 different states with $\frac{1}{4}$ prob.

2) Alice encodes these states with 2-bit classical info.
later she sends this info through classical infor. channel. to Bob.

$a|0\rangle + ib|1\rangle \longrightarrow 00$

$a|0\rangle - ib|1\rangle \longrightarrow 01$

$b|0\rangle + ia|1\rangle \longrightarrow 10$

$ia|1\rangle - b|0\rangle \longrightarrow 11$

3) Bob does a special unitary transformation on his particle B

	State of B after meas.	Unitary Bob should apply	Final state of B after Bob's unitary
1)	$a|0\rangle + ib|1\rangle$	$SZ = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$
2)	$a|0\rangle - ib|1\rangle$	$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$
3)	$b|0\rangle + ia|1\rangle$	$SZY = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$
4)	$ia|1\rangle - b|0\rangle$	$SY = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$

Here S gate (phase gate) represents a 90' degree rotation around the z-axis

For the first case:

1) $SZ = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$

$SZ|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

$SZ|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -i \end{bmatrix} = -i|1\rangle$

$SZ(a|0\rangle + ib|1\rangle) = a\,SZ|0\rangle + ib\,SZ|1\rangle = a|0\rangle + ib(-i)|1\rangle = a|0\rangle + b|1\rangle$

2) $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

$S|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

$S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$

$S(a|0\rangle - ib|1\rangle) = a\,S|0\rangle - ib\,S|1\rangle = a|0\rangle - ibi|1\rangle = a|0\rangle + b|1\rangle$

3) $SZY = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix}$

$SZY\,|0\rangle = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$

$SZY\,|1\rangle = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$

$SZY\left(b|0\rangle + ia|1\rangle\right) = b\,\underbrace{SZY|0\rangle} + ia\,SZY|1\rangle$

$= b|1\rangle + ia(-i)|0\rangle = b|1\rangle + a|0\rangle$ //

4) $SY = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix}$

$SY|0\rangle = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$

$SY|1\rangle = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$

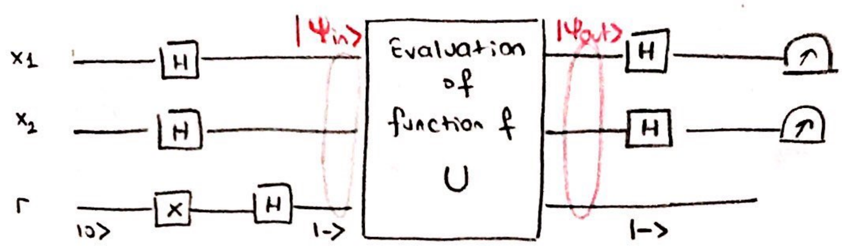
$SY\left(ia|1\rangle - b|0\rangle\right) = ia\,SY|1\rangle - b\,SY|0\rangle$

$= ia(-i)|0\rangle - b(-1)|1\rangle$

$= +a|0\rangle + b|1\rangle$ //

According to given information over the telefone (whether 00, 01, 10, 11) Bob should apply a proper unitary which is shown in the above table.

$\boxed{\text{Question 2}}$



$$|\Psi_{in}\rangle = |x_1, x_2, r\rangle \longrightarrow U|x_1, x_2, r\rangle = |x_1, x_2, r \oplus f(x_1, x_2)\rangle$$

$$|\Psi_{in}\rangle = \frac{1}{2^{n/2}}\left(\sum_x |x\rangle_I\right) \otimes |-\rangle_R$$

where $n=2$

$$|\Psi_{out}\rangle = U|\Psi_{in}\rangle = \frac{1}{2^{n/2}} U \sum_x |x\rangle_I \otimes |-\rangle_R = \left(\frac{1}{2^{n/2}} \sum_x (-)^{f(x)} |x\rangle\right) \otimes |-\rangle_R$$

$$(H^{\otimes n} \otimes 1_R)\,|\Psi_{out}\rangle = \frac{1}{2^{n/2}}\left(\sum_x (-)^{f(x)} H^{\otimes n}|x\rangle\right) \otimes |-\rangle_R$$

$$= \left[\sum_z |z\rangle \left(\frac{\sum_x (-)^{f(x) + \langle x|z\rangle}}{2^n}\right)\right] \otimes |-\rangle_R$$

a_z: amplitude of z
where z is the output state (x_1', x_2')

Part a)

Table 1

$x_1' x_2'$	$P(x_1'x_2' \mid f_1)$	$P(x_1'x_2' \mid \overline{f_1})$	$P(x_1'x_2' \mid f_2)$	$P(x_1'x_2' \mid \overline{f_2})$	$P(x_1'x_2' \mid f_3)$	$P(x_1'x_2' \mid \overline{f_3})$	$P(x_1'x_2' \mid f_4)$	$P(x_1'x_2' \mid \overline{f_4})$
00	1	1	0	0	0	0	0	0
01	0	0	0	0	1	1	0	0
10	0	0	1	1	0	0	0	0
11	0	0	0	0	0	0	1	1

a.1) $x_1'x_2' = 00$, f_1

$$a_{00} = \frac{1}{2^2}\left(\sum_{x\in\{0,1\}^2} (-1)^{\overset{0}{f_1} + \underbrace{\overset{0}{x_1x_1'} + \overset{0}{x_2x_2'}}_{0}}\right) = \frac{1}{4}(4) = 1$$

(it is expected, since f_1 is constant which is shown lecture notes for $|00\rangle$ case)

$$P(00|f_1) = |a_{00}|^2 = 1$$

a.2) $x_1'x_2' = 00$, $\overline{f_1}$

$a_{00} = -1$; since f_1 is constant $\rightarrow$ $P(00|\overline{f_1}) = 1$

a.3) $x_1'x_2' = 00$, f_2 or $\overline{f_2}$, $f_3, \overline{f_3}$, $f_4, \overline{f_4}$

$P(00|f_2) = P(00|\overline{f_2})$ ..... $= P(00|\overline{f_4}) = 0$ since these functions are balanced, can be seen in lecture notes

b.1) $x_1'x_2' = 01$, f_1

$$a_{01} = \frac{1}{4}\left(\sum_{x\in\{0,1\}^2} (-1)^{\overset{0}{f_1} + \langle x|z\rangle}\right) = \frac{1}{4}\left(\overset{x=00}{(-1)^0} + \overset{x=01}{(-1)^1} + \overset{x=10}{(-1)^0} + \overset{x=11}{(-1)^1}\right) = 0$$

$P(01|f_1) = 0$

b.2) $x_1'x_2' = 01$, $\overline{f_1}$

$a_{01} = \frac{1}{4}\left((-1)^1 + (-1)^{0} + (-1)^1 + (-1)^0\right) = 0 \rightarrow P(01|\overline{f_1}) = 0$

b.3) $x_1'x_2' = 01$, f_2

$a_{01} = \frac{1}{4}\left((-1)^0 + (-1)^1 + (-1)^1 + (-1)^0\right) = 0$

b.4) $x_1'x_2' = 01$, $\overline{f_2}$

$a_{01} = \frac{1}{4}\left((-1)^{1+0} + (-1)^{1+1} + (-1)^{0+0} + (-1)^{0+1}\right) = 0$

b.5) $x_1'x_2' = 01$, f_3

$a_{01} = \frac{1}{4}\left((-1)^{0+0} + (-1)^{1+1} + (-1)^{0+0} + (-1)^{1+1}\right) = \frac{1}{4}\cdot(4) = 1$ $\rightarrow$ $P(01|f_3) = 1$

b.6) $x_1'x_2' = 01$, $\overline{f_3}$

$a_{01} = \frac{1}{4}\left((-1)^{1+0} + (-1)^{0+1} + (-1)^{1+0}, (-1)^{0+1}\right) = \frac{1}{4}(-4) = -1 \rightarrow P(01|\overline{f_3}) = 1$

b.7) f_4: $a_{01} = \frac{1}{4}\left((-1)^{0+0} + (-1)^{1+1} + (-1)^{1+0} + (-1)^{0+1}\right) = 0$

b.8) $\overline{f_4}$: a_{01} $\frac{1}{4}\left((-1)^{1+0} + (-1)^{0+1} + (-1)^{0+0} + (-1)^{1+1}\right) = 0$

Similar to $x_1'x_2' = 01$ case, $x_1'x_2' = 10$ and $x_1'x_2' = 11$ are calculated.

And for these possible cases:

$$P(10 \mid f_2) = P(10 \mid \bar{f}_2) = 1 \;, \quad \text{others} = 0$$

$$P(11 \mid f_4) = P(11 \mid \bar{f}_4) = 1 \;, \quad \text{others} = 0$$

For each possible outcome $x_1'x_2'$, list all possible functions:

$x_1'x_2'$	Possible functions might be computed
00	$f_1, \bar{f}_1$
01	$f_3, \bar{f}_3$
10	$f_2, \bar{f}_2$
11	$f_4, \bar{f}_4$

Part b)

x_1x_2	g_H
00	1
01	1
10	1
11	0

$\rightarrow$

$x_1'x_2'$	$P(x_1'x_2' \mid g)$
00	1/4
01	1/4
10	1/4
11	1/4

$$a_{00}|g = \frac{1}{4}\Big(\underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+0}}_{\cancel{-1}} + \underbrace{(-1)^{0+0}}_{\cancel{1}}\Big) = -\frac{1}{2} \rightarrow P = |a_{00}|^2 = \frac{1}{4}$$

$$a_{01}|g = \frac{1}{4}\Big(\underbrace{(-1)^{1+0}}_{\cancel{-1}} + \underbrace{(-1)^{1+1}}_{\cancel{1}} + \underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{0+1}}_{-1}\Big) = -\frac{1}{2} \rightarrow P = |a_{01}|^2 = \frac{1}{4}$$

$$a_{10}|g = \frac{1}{4}\Big(\underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+1}}_{\cancel{1}} + \underbrace{(-1)^{0+1}}_{\cancel{-1}}\Big) = -\frac{1}{2} \rightarrow P = |a_{10}|^2 = \frac{1}{4}$$

$$a_{11}|g = \frac{1}{4}\Big(\underbrace{(-1)^{1+0}}_{\cancel{-1}} + \underbrace{(-1)^{1+1}}_{\cancel{1}} + \underbrace{(-1)^{1+1}}_{1} + \underbrace{(-1)^{0+2}}_{1}\Big) = \frac{1}{2} \rightarrow P = |a_{11}|^2 = \frac{1}{4}$$

Adding g_H, we can not distinguish between functions based on the measurement outcome.
Since g_H is possible for **all** measurements, one can **not** understand at **once** whether function g_H or f_n $(1,2,3,4)$

Question 3

Part a) $|\langle \phi_{b0} | \alpha_0 \rangle|^2 \overset{?}{=} |\langle \phi_{b1} | \alpha_1 \rangle|^2$ ✓

$\underline{b=0}$

$$\bullet\ |\langle \phi_{00} | \alpha_0 \rangle|^2 = \left| \cos\frac{\theta}{2} \underbrace{\langle 0|0\rangle}_{1} + \sin\frac{\theta}{2} \cancel{\langle 0|1\rangle}_{0} \right|^2 = \cos^2\frac{\theta}{2}$$

$$\bullet\ |\langle \phi_{01} | \alpha_1 \rangle|^2 = \left| -\sin\frac{\theta}{2} \cancel{\langle 1|0\rangle}_{0} + \cos\frac{\theta}{2} \underbrace{\langle 1|1\rangle}_{1} \right|^2 = \cos^2\frac{\theta}{2}$$

✓

$\underline{b=1}$

$$\bullet\ |\langle \phi_{10} | \alpha_0 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\cos\frac{\theta}{2} \underbrace{\langle 0|0\rangle}_{1} + \sin\frac{\theta}{2} \underbrace{\langle 1|1\rangle}_{1} \right) \right|^2 = \frac{1}{2}(1+\sin\theta)$$

$$\frac{1}{2}(1+\sin\theta) = \frac{1}{2}(1+\cos\phi) = \frac{1}{\cancel{2}}\left(\cancel{1} + \cancel{2}\cos^2\frac{\phi}{2} - \cancel{1}\right) = \cos^2\frac{\phi}{2}$$

$$\bullet\ |\langle \phi_{11} | \alpha_1 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(-\sin\frac{\theta}{2} \langle 0|0\rangle - \cos\frac{\theta}{2} \langle 1|1\rangle \right) \right|^2 = \frac{1}{2}(1+\sin\theta) = \cos^2\frac{\phi}{2}$$

✓

$$|\langle \phi_{b0} | \alpha_1 \rangle|^2 = |\langle \phi_{b1} | \alpha_0 \rangle|^2$$

$\underline{b=0}$

$$\bullet\ |\langle \phi_{00} | \alpha_1 \rangle|^2 = \left| -\sin\frac{\theta}{2} \langle 0|0\rangle \right|^2 = \sin^2\frac{\theta}{2}$$

$$\bullet\ |\langle \phi_{01} | \alpha_0 \rangle|^2 = \left| \sin\frac{\theta}{2} \langle 1|1\rangle \right|^2 = \sin^2\frac{\theta}{2}$$

✓

$\underline{b=1}$

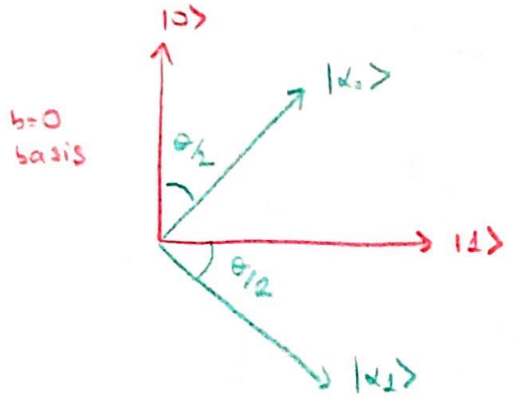
$$\bullet\ |\langle \phi_{10} | \alpha_1 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(-\sin\frac{\theta}{2} \underbrace{\langle 0|0\rangle}_{0} + \cos\frac{\theta}{2} \underbrace{\langle 1|1\rangle}_{1} \right) \right|^2 = \frac{1}{2}(1-\sin\theta)$$

$$\frac{1}{2}(1-\sin\theta) = \frac{1}{2}(1-\cos\phi) = \frac{1}{\cancel{2}}\left(\cancel{1} - \left(\cancel{1} - \cancel{2}\sin^2\frac{\phi}{2}\right)\right) = \sin^2\frac{\phi}{2}$$

$$\bullet\ |\langle \phi_{11} | \alpha_0 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\cos\frac{\theta}{2} \langle 0|0\rangle - \sin\frac{\theta}{2} \langle 1|1\rangle \right) \right|^2 = \frac{1}{2}(1-\sin\theta) = \sin^2\frac{\phi}{2}$$

✓

Part b)



1) $A=0$ $E=0$ $B=0$ $\longrightarrow$ given $E=0$

$\quad\downarrow \qquad\quad \downarrow$

$\quad \frac{1}{2}$

$$P(B=0 \mid E=0) = |\langle \phi_{00} | \alpha_0 \rangle|^2$$

given $A=0 \rightarrow |\phi_{00}\rangle$

$$P(E=0 \mid A=0) = |\langle \alpha_0 | \phi_{00} \rangle|^2 = \cos^2\frac{\theta}{2}$$

$$P(A=0, E=0, B=0) = \frac{1}{2}\cos^2\frac{\theta}{2} \cdot \cos^2\frac{\theta}{2} = \frac{1}{2}\cos^4\left(\frac{\theta}{2}\right) //$$

2) $A=0$ $E=0$ $B=1$

$\frac{1}{2}$ $|\langle \phi_{00} | \alpha_0 \rangle|^2$ $|\langle \phi_{01} | \alpha_0 \rangle|^2$

$$P(A=0, E=0, B=1) = \frac{1}{2}\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2} //$$

3) $A=0$ $E=1$ $B=0$

$\frac{1}{2}$ $|\langle \phi_{00} | \alpha_1 \rangle|^2$ $|\langle \phi_{00} | \alpha_1 \rangle|^2$

$$P(010) = \frac{1}{2}\sin^4\left(\frac{\theta}{2}\right) //$$

4) $A=0$ $E=1$ $B=1$

$\frac{1}{2}$ $|\langle \phi_{00} | \alpha_1 \rangle|^2$ $|\langle \phi_{01} | \alpha_1 \rangle|^2$

$$P(011) = \frac{1}{2}\sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right) //$$

5) $A=1$ $E=0$ $B=0$

$\frac{1}{2}$ $|\langle \phi_{01} | \alpha_0 \rangle|^2$ $|\langle \phi_{00} | \alpha_0 \rangle|^2$

$$P(100) = \frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} //$$

6) $A=1 \rightarrow \frac{1}{2}$ $\quad E=0 \rightarrow |\langle \phi_{01}|\alpha_0\rangle|^2$ $\quad B=1 \rightarrow |\langle \phi_{01}|\alpha_0\rangle|^2$

$$P(101) = \frac{1}{2}\sin^2\frac{\theta}{2}\sin^2\frac{\theta}{2} = \frac{1}{2}\sin^4\frac{\theta}{2}$$

7) $A=1 \rightarrow \frac{1}{2}$ $\quad E=1 \rightarrow |\langle \phi_{01}|\alpha_1\rangle|^2$ $\quad B=0 \rightarrow |\langle \phi_{00}|\alpha_1\rangle|^2$

$$P(110) = \frac{1}{2}\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}$$

8) $A=1 \rightarrow \frac{1}{2}$ $\quad E=1 \rightarrow |\langle \phi_{01}|\alpha_1\rangle|^2$ $\quad B=1 \rightarrow |\langle \phi_{01}|\alpha_1\rangle|^2$

$$P(111) = \frac{1}{2}\cos^2\frac{\theta}{2}\cos^2\frac{\theta}{2} = \frac{1}{2}\cos^4\frac{\theta}{2}$$

Table 2

$\boxed{\text{For } b=0}$				$\boxed{\text{For } b=1}$
A	E	B	Prob $(b=0)$	Prob $(b=1)$
0	0	0	$\frac{1}{2}\cos^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\cos^4\left(\frac{\phi}{2}\right)$
0	0	1	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} = \frac{\sin^2\theta}{8}$	$\frac{1}{8}\sin^2\phi$
0	1	0	$\frac{1}{2}\sin^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\sin^4\left(\frac{\phi}{2}\right)$
0	1	1	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}$	$\frac{1}{8}\sin^2\phi$
1	0	0	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}$	$\frac{1}{8}\sin^2\phi$
1	0	1	$\frac{1}{2}\sin^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\sin^4\left(\frac{1}{2}\right)$
1	1	0	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}$	$\frac{1}{8}\sin^2\phi$
1	1	1	$\frac{1}{2}\cos^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\cos^4\left(\frac{\phi}{2}\right)$

Part c)

Disagreement between A and E

$A=0 \quad E=1 \rightarrow 2$ case

$A=1 \quad E=0 \rightarrow 2$ case

$$D_0^{AE} = \cancel{2}\times\frac{1}{\cancel{2}}\sin^4\left(\frac{\theta}{2}\right) + \frac{1}{\cancel{2}}\times\cancel{2}\left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\right)$$

$$= \sin^2\left(\frac{\theta}{2}\right)\Big[\underbrace{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}_{1}\Big] = \sin^2\frac{\theta}{2} \;//$$

Disagreement between A and B

$A=0 \quad B=1 \rightarrow 2$ cases

$A=1 \quad B=0 \rightarrow 2$ cases

$$D_0^{AB} = 4\times\frac{\sin^2\theta}{8} = \frac{\sin^2\theta}{2}$$

Part d)

$$D_1^{AE} = \cancel{2}\times\frac{1}{\cancel{2}}\sin^4\left(\frac{\phi}{2}\right) + \cancel{2}\times\frac{1}{\cancel{8}_4}\sin^2\phi$$

↓ in terms of θ

$$\frac{1}{4}\cos^2\theta + \frac{1}{4}(1-\sin\theta)^2 = \frac{1}{4}\Big(\underbrace{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}_{}\Big)$$

$$= \frac{1}{4}\,2(1-\sin\theta) = \frac{1}{2}(1-\sin\theta) = \sin^2\frac{\phi}{2} \;//$$

$$D_1^{AB} = \cancel{4}\,\frac{1}{\cancel{8}_2}\cos^2\theta = \frac{\cos^2\theta}{2} = \frac{\sin^2\phi}{2} \;//$$

Part e) $2 D_0^{AB} + 2 D_1^{AB} = 1$

$$\cancel{2}\times \frac{\sin^2\theta}{\cancel{2}} + \cancel{2}\times \frac{\cos^2\theta}{\cancel{2}} = 1 \quad ✓$$

Part f) $\sqrt{2 D_0^{AB}} + 2 D_1^{AE} = 1$

$$\sqrt{\cancel{2}\times \frac{\sin^2\theta}{\cancel{2}}} + \cancel{2}\times \frac{1}{\cancel{2}}(1-\sin\theta)$$

$$= \sin\cancel{\theta} + 1 - \sin\cancel{\theta} = 1 \quad ✓$$

Part g)

	$\phi = \pi/2$, $\theta = 0$	$\phi = 0$, $\theta = \frac{\pi}{2}$
$D_0^{AE} = (\sin^2\frac{\theta}{2})$	0	$\frac{1}{2}$
$D_0^{AB} = \frac{1}{2}\sin^2\theta$	0	$\frac{1}{2}$
$D_1^{AE} = \sin^2\frac{\phi}{2}$	$\frac{1}{2}$	0
$D_1^{AB} = \frac{\sin^2\phi}{2}$	$\frac{1}{2}$	0