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PHYS 455   FINAL

Question 1

$$|\psi\rangle_C = a|0\rangle_c + b|1\rangle_c$$

$$|\psi\rangle_{AB} = \frac{|00\rangle + i|11\rangle}{\sqrt{2}}$$

Teleportation Protocol

1) Alice does a measurement on CA in Bell-Basis

Before the measurement, state of CAB:

$$|\psi_{\textcircled{1}}\rangle = |\psi_c\rangle \otimes |\psi\rangle_{AB}$$

$$= (a|0\rangle + b|1\rangle)_c \otimes \left(\frac{|00\rangle + i|11\rangle}{\sqrt{2}}\right)_{AB}$$

$$|\psi_{\textcircled{1}}\rangle = \frac{a|\widehat{000}\rangle + b|\widehat{100}\rangle + ia|\widehat{011}\rangle + ib|\widehat{111}\rangle}{\sqrt{2}}$$

$$|\psi_{\textcircled{1}}\rangle = \frac{a}{\sqrt{2}}\left(\frac{|\Phi_+\rangle + |\Phi_-\rangle}{\sqrt{2}}\right)_{CA} \otimes |0\rangle_B + \frac{b}{\sqrt{2}}\left(\frac{|\Psi_+\rangle - |\Psi_-\rangle}{\sqrt{2}}\right) \otimes |0\rangle +$$

$$\frac{ia}{\sqrt{2}}\left(\frac{|\Psi_+\rangle + |\Psi_-\rangle}{\sqrt{2}}\right) \otimes |1\rangle + \frac{ib}{\sqrt{2}}\left(\frac{|\Phi_+\rangle - |\Phi_-\rangle}{\sqrt{2}}\right) \otimes |1\rangle$$

$$|\psi_{\textcircled{1}}\rangle = \frac{1}{2}\left(|\Phi_+\rangle_{CA} \otimes \boxed{(a|0\rangle + ib|1\rangle)_B} + |\Phi_-\rangle \otimes \boxed{(a|0\rangle - ib|1\rangle)} + \right.$$
$$\left. |\Psi_+\rangle \otimes \boxed{(b|0\rangle + ia|1\rangle)} + |\Psi_-\rangle \otimes \boxed{(ia|1\rangle - b|0\rangle)} \right)$$

①

After Measurement, Alice could achieve 4 different states with $\frac{1}{4}$ prob.

2) Alice encodes these states with 2-bit classical info. later she sends this info through classical infor. channel to Bob.

$$a|0\rangle + ib|1\rangle \longrightarrow 00$$
$$a|0\rangle - ib|1\rangle \longrightarrow 01$$
$$b|0\rangle + ia|1\rangle \longrightarrow 10$$
$$ia|1\rangle - b|0\rangle \longrightarrow 11$$

3) Bob does a special unitary transformation on his particle B

State of B after meas.	Unitary Bob should apply	Final state of B after Bob's unitary
1) $a|0\rangle + ib|1\rangle$	$SZ = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$
2) $a|0\rangle - ib|1\rangle$	$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$
3) $b|0\rangle + ia|1\rangle$	$SZY = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$
4) $ia|1\rangle - b|0\rangle$	$SY = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix}$	$a|0\rangle + b|1\rangle = |\psi\rangle$

Here S gate (phase gate) represents a 90° degree rotation around the z-axis

for the first case:

1) $SZ = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$

$SZ|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

$SZ|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -i \end{bmatrix} = -i|1\rangle$

$SZ(a|0\rangle + ib|1\rangle) = a \, SZ|0\rangle + ib \, SZ|1\rangle = a|0\rangle + ib(-i)|1\rangle = a|0\rangle + b|1\rangle$

2) $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

$S|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

$S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$

$S(a|0\rangle - ib|1\rangle) = a \, S|0\rangle - ib \, S|1\rangle = a|0\rangle - ib \cdot i|1\rangle = a|0\rangle + b|1\rangle$

②

3) $SZY = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix}$

$SZY |0\rangle = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$

$SZY |1\rangle = \begin{bmatrix} 0 & -i \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$

$SZY (b|0\rangle + ia|1\rangle) = b \, SZY|0\rangle + ia \, SZY|1\rangle$

$= b|1\rangle + ia(-i)|0\rangle = b|1\rangle + a|0\rangle$ ✓

4) $SY = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix}$

$SY|0\rangle = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$
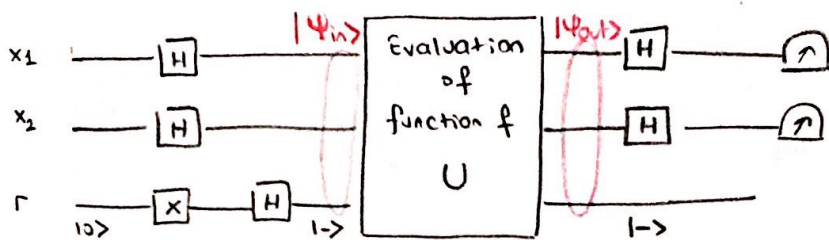
$SY|1\rangle = \begin{bmatrix} 0 & -i \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$

$SY (ia|1\rangle - b|0\rangle) = ia \, SY|1\rangle - b \, SY|0\rangle$

$= ia(-i)|0\rangle - b(-1)|1\rangle$

$= +a|0\rangle + b|1\rangle$ ✓

According to given information over the telefone (whether 00,01,10,11) Bob should apply a proper unitary which is shown in the above table.

$$|\Psi_{in}\rangle = |x_1, x_2, r\rangle \longrightarrow U|x_1, x_2, r\rangle = |x_1, x_2, r \oplus f(x_1, x_2)\rangle$$

$$|\Psi_{in}\rangle = \frac{1}{2^{n/2}} \left(\sum_x |x\rangle_I \right) \otimes |-\rangle_R$$

where $n = 2$

$$|\Psi_{out}\rangle = U|\Psi_{in}\rangle = \frac{1}{2^{n/2}} U \sum_x |x\rangle_I \otimes |-\rangle_R = \left(\frac{1}{2^{n/2}} \sum_x (-)^{f(x)} |x\rangle \right) \otimes |-\rangle_R$$

$$(H^{\otimes n} \otimes \mathbb{1}_R) |\Psi_{out}\rangle = \frac{1}{2^{n/2}} \left(\sum_x (-)^{f(x)} H^{\otimes n} |x\rangle \right) \otimes |-\rangle_R$$

$$= \left[\sum_z |z\rangle \left(\frac{\sum_x (-)^{f(x) + \langle x|z\rangle}}{2^n} \right) \right] \otimes |-\rangle_R$$

a_z : amplitude of z

where z is the output state (x_1', x_2')

Part a)

Table 1

$x_1' x_2'$	$P(x_1' x_2' | f_1)$	$P(x_1' x_2' | \bar{f_1})$	$P(x_1' x_2' | f_2)$	$P(x_1' x_2' | \bar{f_2})$	$P(x_1' x_2' | f_3)$	$P(x_1' x_2' | \bar{f_3})$	$P(x_1' x_2' | f_4)$	$P(x_1' x_2' | f_4)$
00	1	1	0	0	0	0	0	0
01	0	0	0	0	1	1	0	0
10	0	0	1	1	0	0	0	0
11	0	0	0	0	0	0	1	1

a.1) $x_1' x_2' = 00$, f_1

$$a_{00} = \frac{1}{2^2} \left(\sum_{x \in \{0,1\}^2} (-1)^{\overset{0}{\cancel{f_1}} + \underbrace{x_1 \overset{0}{x_1'} + x_2 \overset{0}{x_2'}}_{0}} \right) = \frac{1}{4}(4) = 1$$

(it is expected, since f_1 is constant which is shown lecture notes for $|00\rangle$ case)

$$P(00|f_1) = |a_{00}|^2 = 1$$

a.2) $x_1' x_2' = 00$, $\bar{f_1}$

$a_{00} = -1$; since f_1 is constant $\rightarrow$ $P(00|\bar{f_1}) = 1$

a.3) $x_1' x_2' = 00$, f_2 or $\bar{f_2}$, f_3, $\bar{f_3}$, f_4, $\bar{f_4}$

$P(00|f_2) = P(00|\bar{f_2}) \cdots = P(00|\bar{f_4}) = 0$ since these functions are balanced, can be seen in lecture notes

b.1) $x_1' x_2' = 01$, f_1

$$a_{01} = \frac{1}{4} \left(\sum_{x \in \{0,1\}^2} (-1)^{\overset{0}{\cancel{f_1}} + \langle x | z \rangle} \right) = \frac{1}{4} \left(\overset{x=00}{(-1)^{\cancel{0}}} + \overset{x=01}{(-1)^{\cancel{1}}} + \overset{x=10}{(-1)^{\cancel{0}}} + \overset{x=11}{(-1)^{\cancel{1}}} \right) = 0$$

$P(01|f_1) = 0$

b.2) $x_1' x_2' = 01$, $\bar{f_1}$

$$a_{01} = \frac{1}{4} \left((-1)^1 + (-1)^{\overset{0}{x}} + (-1)^1 + (-1)^0 \right) = 0 \rightarrow P(01|\bar{f_1}) = 0$$

b.3) $x_1' x_2' = 01$, f_2

$$a_{01} = \frac{1}{4} \left((-1)^0 + (-1)^1 + (-1)^1 + (-1)^0 \right) = 0$$

b.4) $x_1' x_2' = 01$, $\bar{f_2}$

$$a_{01} = \frac{1}{4} \left((-1)^{1+0} + (-1)^{1+1} + (-1)^{0+0} + (-1)^{0+1} \right) = 0$$

b.5) $x_1' x_2' = 01$, f_3

$$a_{01} = \frac{1}{4} \left((-1)^{0+0} + (-1)^{1+1} + (-1)^{0+0} + (-1)^{1+1} \right) = \frac{1}{4} \cdot (4) = 1 ,, \rightarrow P(01|f_3) = 1$$

b.6) $x_1' x_2' = 01$, $\bar{f_3}$

$$a_{01} = \frac{1}{4} \left((-1)^{1+0} + (-1)^{0+1} + (-1)^{1+0} , (-1)^{0+1} \right) = \frac{1}{4}(-4) = -1 \rightarrow P(01|\bar{f_3}) = 1$$

b.7) f_4: $a_{01} = \frac{1}{4} \left((-1)^{0+0} + (-1)^{1+1} + (-1)^{1+0} , (-1)^{0+1} \right) = 0$

b.8) $\bar{f_4}$: $a_{01} \ \frac{1}{4} \left((-1)^{1+0} + (-1)^{0+1} + (-1)^{0+0} + (-1)^{1+1} \right) = 0$

⑤

Similar to $x_1'x_2' = 01$ case, $x_1'x_2' = 10$ and $x_1'x_2' = 11$ are calculated.
And for these possible cases:

$$P(10 \mid f_2) = P(10 \mid \bar{f}_2) = 1 \quad , \quad \text{others} = 0$$

$$P(11 \mid f_4) = P(11 \mid \bar{f}_4) = 1 \quad , \quad \text{others} = 0$$

For each possible outcome x_1x_2', list all possible functions:

$x_1'x_2'$	Possible functions might be computed
00	$f_1, \bar{f}_1$
01	$f_3, \bar{f}_3$
10	$f_2, \bar{f}_2$
11	$f_4, \bar{f}_4$

Part b)

$x_1 x_2$	g_H
00	1
01	1
10	1
11	0

$\rightarrow$

$x_1'x_2'$	$P(x_1'x_2' \mid g)$
00	$1/4$
01	$1/4$
10	$1/4$
11	$1/4$

$$a_{00}|g = \frac{1}{4}\left(\underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{0+0}}_{1} \right) = -\frac{1}{2} \rightarrow P = |a_{00}|^2 = \frac{1}{4}$$

$$a_{01}|g = \frac{1}{4}\left(\underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+1}}_{1} + \underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{0+1}}_{-1} \right) = -\frac{1}{2} \rightarrow P = |a_{01}|^2 = \frac{1}{4}$$

$$a_{10}|g = \frac{1}{4}\left(\underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+1}}_{1} + \underbrace{(-1)^{0+1}}_{-1} \right) = -\frac{1}{2} \rightarrow P = |a_{10}|^2 = \frac{1}{4}$$

$$a_{11}|g = \frac{1}{4}\left(\underbrace{(-1)^{1+0}}_{-1} + \underbrace{(-1)^{1+1}}_{1} + \underbrace{(-1)^{1+1}}_{1} + \underbrace{(-1)^{0+2}}_{1} \right) = \frac{1}{2} \rightarrow P = |a_{11}|^2 = \frac{1}{4}$$

★ Adding g_H, we can not distinguish between functions based on the measurement outcome.
Since g_H is possible for all measurements, one can not understand at once whether function g_H or f_n (1,2,3,4)

Part a) $\qquad |\langle \phi_{b0} | \alpha_0 \rangle|^2 \overset{?}{=} |\langle \phi_{b\perp} | \alpha_1 \rangle|^2 \checkmark$

$\underline{b=0}$

$\cdot \; |\langle \phi_{00} | \alpha_0 \rangle|^2 = \left| \cos\frac{\theta}{2} \underbrace{\langle 0 | 0 \rangle}_{1} + \sin\frac{\theta}{2} \underbrace{\langle 0 | 1 \rangle}_{0} \right|^2 = \cos^2\frac{\theta}{2} \; \Big\} \checkmark$

$\cdot \; |\langle \phi_{01} | \alpha_1 \rangle|^2 = \left| -\sin\frac{\theta}{2} \underbrace{\langle 1 | 0 \rangle}_{0} + \cos\frac{\theta}{2} \underbrace{\langle 1 | 1 \rangle}_{1} \right|^2 = \cos^2\frac{\theta}{2}$

$\underline{b=1}$

$\cdot \; |\langle \phi_{10} | \alpha_0 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\cos\frac{\theta}{2} \underbrace{\langle 0 | 0 \rangle}_{1} + \sin\frac{\theta}{2} \underbrace{\langle 1 | 1 \rangle}_{1} \right) \right|^2 = \frac{1}{2}(1 + \sin\theta)$

$\qquad \frac{1}{2}(1+\sin\theta) = \frac{1}{2}(1+\cos\phi) = \frac{1}{2}(\cancel{1} + \cancel{2}\cos^2\frac{\phi}{2} - \cancel{1}) = \cos^2\frac{\phi}{2} \; \Big\} \checkmark$

$\cdot \; |\langle \phi_{11} | \alpha_1 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(-\sin\frac{\theta}{2} \langle 0 | 0 \rangle - \cos\frac{\theta}{2} \langle 1 | 1 \rangle \right) \right|^2 = \frac{1}{2}(1+\sin\theta) = \cos^2\frac{\theta}{2}$

$\qquad\qquad |\langle \phi_{b0} | \alpha_1 \rangle|^2 = |\langle \phi_{b\perp} | \alpha_0 \rangle|^2$

$\underline{b=0}$

$\qquad \langle 0|$
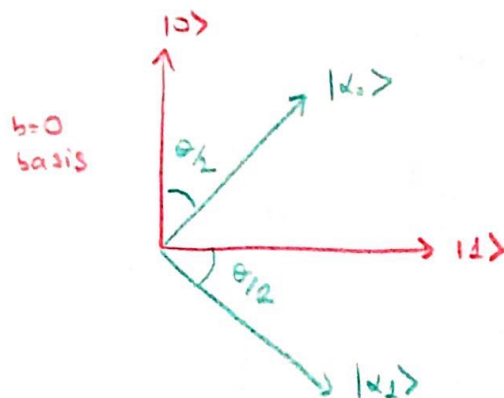$\cdot \; |\langle \phi_{00} | \alpha_1 \rangle|^2 = \left| -\sin\frac{\theta}{2} \langle 0 | 0 \rangle \right|^2 = \sin^2\frac{\theta}{2} \; \Big\} \checkmark$

$\cdot \; |\langle \phi_{01} | \alpha_0 \rangle|^2 = \left| \sin\frac{\theta}{2} \langle 1 | 1 \rangle \right|^2 = \sin^2\frac{\theta}{2}$
$\qquad \langle 1|$

$\underline{b=1}$

$\cdot \; |\langle \phi_{10} | \alpha_1 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(-\sin\frac{\theta}{2} \underbrace{\langle 0 | 0 \rangle}_{0} + \cos\frac{\theta}{2} \underbrace{\langle 1 | 1 \rangle}_{1} \right) \right|^2 = \frac{1}{2}(1-\sin\theta)$

$\qquad \frac{1}{2}(1-\sin\theta) = \frac{1}{2}(1-\cos\phi) = \frac{1}{2}\left(\cancel{1} - (\cancel{1} - 2\sin^2\frac{\phi}{2}) \right) = \sin^2\frac{\phi}{2}$

$\cdot \; |\langle \phi_{11} | \alpha_0 \rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\cos\frac{\theta}{2} \langle 0 | 0 \rangle - \sin\frac{\theta}{2} \langle 1 | 1 \rangle \right) \right|^2 = \frac{1}{2}(1-\sin\theta)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \sin^2\frac{\phi}{2} \; \Big\} \checkmark$

⑦

Part b)

$b = 0$ basis

1)
$$A = 0 \qquad E = 0 \qquad B = 0 \qquad \longrightarrow \qquad \text{given } E = 0$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad\qquad\qquad\qquad P(B=0 \mid E=0) = |\langle \phi_{00} | \alpha_0 \rangle|^2$$
$$\tfrac{1}{2} \qquad \text{given } A = 0 \rightarrow |\phi_{00}\rangle$$

$$P(E=0 \mid A=0) = |\langle \alpha_0 | \psi_{00} \rangle|^2 = \cos^2 \tfrac{\theta}{2}$$

$$P(A=0, E=0, B=0) = \tfrac{1}{2} \cos^2 \tfrac{\theta}{2} \cdot \cos^2 \tfrac{\theta}{2} = \tfrac{1}{2} \cos^4 \left(\tfrac{\theta}{2} \right) \; //$$

2)
$$A = 0 \qquad E = 0 \qquad B = 1$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \searrow$$
$$\tfrac{1}{2} \qquad |\langle \phi_{00} | \alpha_0 \rangle|^2 \qquad |\langle \phi_{0\perp} | \alpha_0 \rangle|^2$$

$$P(A=0, E=0, B=1) = \tfrac{1}{2} \cos^2 \tfrac{\theta}{2} \sin^2 \tfrac{\theta}{2} \; //$$

3)
$$A = 0 \qquad E = 1 \qquad B = 0$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \searrow$$
$$\tfrac{1}{2} \qquad |\langle \phi_{00} | \alpha_\perp \rangle|^2 \qquad |\langle \phi_{00} | \alpha_\perp \rangle|^2$$

$$P(010) = \tfrac{1}{2} \sin^4 \left(\tfrac{\theta}{2} \right) \; //$$

4)
$$A = 0 \qquad E = 1 \qquad B = 1$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \searrow$$
$$\tfrac{1}{2} \qquad |\langle \phi_{00} | \alpha_\perp \rangle|^2 \qquad |\langle \phi_{01} | \alpha_\perp \rangle|^2$$

$$P(011) = \tfrac{1}{2} \sin^2 \left(\tfrac{\theta}{2} \right) \cos^2 \left(\tfrac{\theta}{2} \right) \; //$$

5)
$$A = 1 \qquad E = 0 \qquad B = 0$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \searrow$$
$$\tfrac{1}{2} \qquad |\langle \phi_{01} | \alpha_0 \rangle|^2 \qquad |\langle \phi_{00} | \alpha_0 \rangle|^2$$

$$P(100) = \tfrac{1}{2} \sin^2 \tfrac{\theta}{2} \cos^2 \tfrac{\theta}{2} \; //$$

6) $A = 1$ $E = 0$ $B = 1$

$\quad\quad\downarrow$ $\quad\quad\downarrow$ $\quad\quad\downarrow$

$\quad\quad\frac{1}{2}$ $|\langle\phi_{01}|\alpha_0\rangle|^2$ $|\langle\phi_{01}|\alpha_0\rangle|^2$

$P(101) = \frac{1}{2}\sin^2\frac{\theta}{2}\sin^2\frac{\theta}{2} = \frac{1}{2}\sin^4\frac{\theta}{2}\;//$

7) $A = 1$ $E = 1$ $B = 0$

$\quad\quad\downarrow$ $\quad\quad\downarrow$ $\quad\quad\downarrow$

$\quad\quad\frac{1}{2}$ $|\langle\phi_{01}|\alpha_1\rangle|^2$ $|\langle\phi_{00}|\alpha_1\rangle|^2$

$P(110) = \frac{1}{2}\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}\;//$

8) $A = 1$ $E = 1$ $B = 1$

$\quad\quad\downarrow$ $\quad\quad\downarrow$ $\quad\quad\downarrow$

$\quad\quad\frac{1}{2}$ $|\langle\phi_{01}|\alpha_1\rangle|^2$ $|\langle\phi_{01}|\alpha_1\rangle|^2$

$P(111) = \frac{1}{2}\cos^2\frac{\theta}{2}\cos^2\frac{\theta}{2} = \frac{1}{2}\cos^4\frac{\theta}{2}$

Table 2

For b=0				For b=1
A	E	B	Prob (b=0)	Prob (b=1)
0	0	0	$\frac{1}{2}\cos^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\cos^4\left(\frac{\phi}{2}\right)$
0	0	1	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} = \frac{\sin^2\theta}{8}$	$\frac{1}{8}\sin^2\phi$
0	1	0	$\frac{1}{2}\sin^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\sin^4\left(\frac{\phi}{2}\right)$
0	1	1	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}$	$\frac{1}{8}\sin^2\phi$
1	0	0	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}$	$\frac{1}{8}\sin^2\phi$
1	0	1	$\frac{1}{2}\sin^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\sin^4\left(\frac{\phi}{2}\right)$
1	1	0	$\frac{1}{2}\sin^2\frac{\theta}{2}\cos^4\frac{\theta}{2}$	$\frac{1}{8}\sin^2\phi$
1	1	1	$\frac{1}{2}\cos^4\left(\frac{\theta}{2}\right)$	$\frac{1}{2}\cos^4\left(\frac{\phi}{2}\right)$

Part c)

Disagreement between A and E

$$A = 0 \quad E = 1 \quad \rightarrow \quad 2 \text{ case}$$

$$A = 1 \quad E = 0 \quad \rightarrow \quad 2 \text{ case}$$

$$D_0^{AE} = 2 \times \frac{1}{2} \sin^4\left(\frac{\theta}{2}\right) + \frac{1}{2} \times 2 \left(\sin^2\frac{\theta}{2} \cos^2\frac{\theta}{2} \right)$$

$$= \sin^2\left(\frac{\theta}{2}\right) \left[\underbrace{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}_{1} \right] = \sin^2\frac{\theta}{2} \; //$$

Disagreement between A and B

$$A = 0 \quad B = 1 \quad \rightarrow \quad 2 \text{ cases}$$

$$A = 1 \quad B = 0 \quad \rightarrow \quad 2 \text{ cases}$$

$$D_0^{AB} = 4 \times \frac{\sin^2\theta}{8} = \frac{\sin^2\theta}{2}$$

Part d)

$$D_1^{AE} = 2 \times \frac{1}{2} \sin^4\left(\frac{\phi}{2}\right) + 2 \times \frac{1}{8} \sin^2\phi$$

in terms of θ

$$\frac{1}{4}\cos^2\theta + \frac{1}{4}(1-\sin\theta)^2 = \frac{1}{4}\left(\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta \right)$$

$$= \frac{1}{4} \; 2(1-\sin\theta) = \frac{1}{2}(1-\sin\theta) = \sin^2\frac{\phi}{2} \; //$$

$$D_1^{AB} = 4 \times \frac{1}{8} \cos^2\theta = \frac{\cos^2\theta}{2} = \frac{\sin^2\phi}{2} \; //$$

(10)

Part e) $\quad 2 D_0^{AB} + 2 D_1^{AB} = 1$

$$2 \times \frac{\sin^2\theta}{2} + 2 \times \frac{\cos^2\theta}{2} = 1 \quad \checkmark$$

Part f) $\quad \sqrt{2 D_0^{AB}} + 2 D_1^{AE} = 1$

$$\sqrt{2 \times \frac{\sin^2\theta}{2}} + 2 \times \frac{1}{2}(1-\sin\theta)$$

$$= \sin\theta + 1 - \sin\theta = 1 \quad \checkmark$$

Part g)

	$\phi = \pi/2$ $\theta = 0$	$\phi = 0$ $\theta = \frac{\pi}{2}$
$D_0^{AE} = (\sin^2\frac{\theta}{2})$	0	$\frac{1}{2}$
$D_0^{AB} = \frac{1}{2}\sin^2\theta$	0	$\frac{1}{2}$
$D_1^{AE} = \sin^2\frac{\phi}{2}$	$\frac{1}{2}$	0
$D_1^{AB} = \frac{\sin^2\phi}{2}$	$\frac{1}{2}$	0