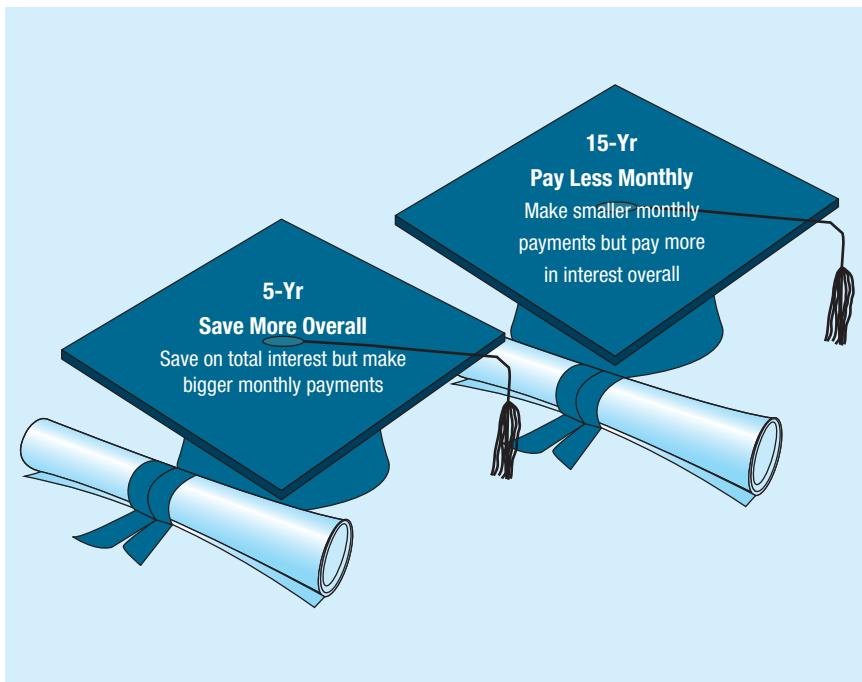


# Understanding Money Management

## **Refinance Your Student Loans—More Options and Better Fit**

Edward Johns is a senior who is about to graduate in the spring. He has already received a couple of job offers and has been thinking about either refinancing or paying off his student loans upon graduation. Edward has received the following flyer in the mail, and he feels he needs to do some homework in coming up with the right strategy, if he decides to refinance his loans.



*Source: College Ave Student Loans, LLC, ©2017 All Rights Reserved.  
(<https://www.collegeavestudentloans.com>).*



Dear Edward,

With College Ave Student Loans Refi, you could reduce the total cost of your existing student loans or your current monthly payment! We will help you find your perfect fit. With no origination fees and a choice of variable rates from 2.5% to 7.25% APR or fixed rates from 4.74% to 8.5% APR, you can create a loan that fits your budget and your life. Plus we make it fast and easy.

You choose whether to start making full payments right away or pay just the interest charges each month for the first two years. You also decide how many years you take to repay the loan.

You're pre-qualified to refinance up to \$150,000 in existing student loans. All it takes is 3 minutes to apply, and there are no application fees.

Sincerely,

Jennifer P. Astle  
Chief Marketing Officer

Edward needs to decide three things: (1) whether or not to lock the interest rate over the life of the loan (fixed vs variable interest rate), (2) how long he wants to stretch his payments over, say five years or 10 years, and (3) making full payments or interest payments for the first two years.

**C**an you explain the meaning of a 2.5% annual percentage rate (APR) quoted by the lending company? And how the lending company calculates the interest payment? In this chapter, we will consider several concepts crucial to managing money. In Chapter 2, we examined how time affects the value of money and developed various interest formulas for these calculations. Using these basic formulas, we will now extend the concept of equivalence to determine interest rates implicit in many financial contracts. To this end, we will introduce several examples in the area of loan transactions. For instance, many commercial loans require interest to compound more frequently than once a year—be it monthly or daily. Once you understand the contents presented in this chapter, you should be able to answer the three questions raised by Edward Jones in the cover story. To appreciate the effects of more frequent compounding, we must begin with an understanding of the concepts of nominal and effective interest.

## 3.1 Market Interest Rates

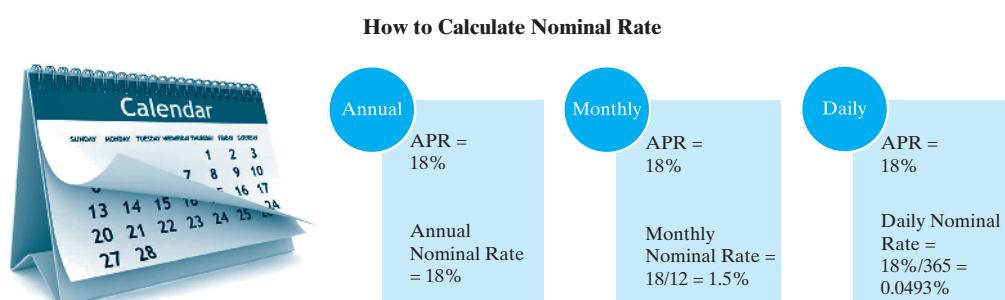
In Chapter 2, the market interest rate is defined as the interest rate established by the financial market, such as banks and financial institutions. This interest rate is supposed to reflect any anticipated changes in earning power as well as purchasing power (inflation) in the economy. In this section, we will review the nature of this interest rate in more detail.

### 3.1.1 Nominal Interest Rates

Take a closer look at the billing statement for any credit card or the loan contract for a newly financed car. You should be able to find the interest that the bank charges on your unpaid balance. Even if a financial institution uses a unit of time other than a year—for example, a month or a quarter—when calculating interest payments, the institution usually quotes the interest rate on an *annual basis*, commonly known as “**the nominal interest rate or annual percentage rate (APR)**.” Many banks, for example, state a typical interest arrangement for credit cards in the following manner:

“APR of 18%.”

This statement means simply that each month the bank will charge 1.5% interest ( $12 \text{ months per year} \times 1.5\% \text{ per month} = 18\% \text{ per year}$ ) on the unpaid balance. As shown in Figure 3.1, we can express the nominal rate over any time interval other than



**Figure 3.1** Relationship between APR and interest period.

annual, say monthly or daily and that the time interval indicates the compounding frequency implied in the financial transactions.

Although APR is commonly used by financial institutions and is familiar to many customers, the term does not explain precisely the amount of interest that will accumulate in a year. To explain the true effect of more frequent compounding on annual interest amounts, we will introduce the term *effective interest rate*, commonly known as *annual effective yield*, or annual percentage yield (APY).

### 3.1.2 Effective Annual Interest Rates

The **effective annual interest rate** (or **annual effective yield**) is the one rate that truly represents the interest earned in a year. On a yearly basis, you are looking for a cumulative rate—1.5% each month for 12 times in our previous example. This cumulative rate predicts the actual interest payment on your outstanding credit card balance.

We could calculate the total annual interest payment for a credit card debt of \$1,000 by using the future value formula given in Eq. (2.3). If  $P = \$1,000$ ,  $i = 1.5\%$ , and  $N = 12$ , we obtain

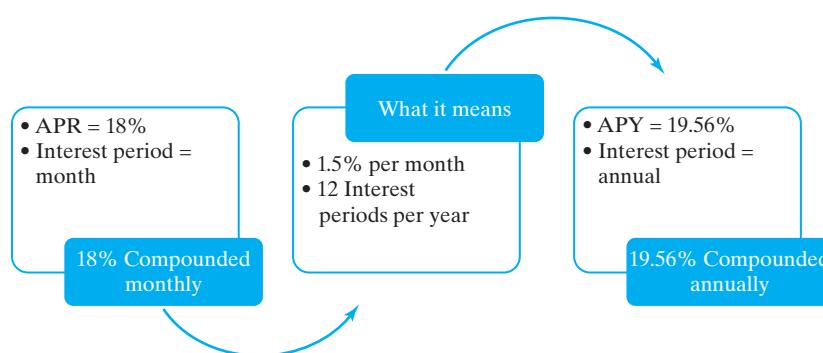
$$\begin{aligned} F &= P(1 + i)^N \\ &= \$1,000(1 + 0.015)^{12} \\ &= \$1,195.62. \end{aligned}$$

In effect, the bank is earning more than the stated 18% APR on your original credit card debt. In fact, since you are paying \$195.62, the implication is that, for each dollar owed, you are paying an equivalent annual interest of 19.56 cents. In terms of an effective annual interest rate ( $i_a$ ), the interest payment can be rewritten as a percentage of the principal amount:

$$i_a = \$195.62/\$1,000 = 0.19562 \text{ or } 19.562\%.$$

In other words, paying 1.5% interest per month for 12 months is equivalent to paying 19.56% interest just one time each year. This relationship is depicted in Figure 3.2.

Table 3.1 shows effective interest rates at various compounding intervals for 4%–12% APRs. Depending on the frequency of compounding, the effective interest earned (or paid by the borrower) can differ significantly from the APR. Therefore,



**Figure 3.2** Relationship between nominal and effective interest rates.

**TABLE 3.1** Annual Yields at Various Compounding Intervals

Nominal Rate	Compounding Frequency				
	Annually	Semiannually	Quarterly	Monthly	Daily
4%	4.00%	4.04%	4.06%	4.07%	4.08%
5%	5.00%	5.06%	5.09%	5.12%	5.13%
6%	6.00%	6.09%	6.14%	6.17%	6.18%
7%	7.00%	7.12%	7.19%	7.23%	7.25%
8%	8.00%	8.16%	8.24%	8.30%	8.33%
9%	9.00%	9.20%	9.31%	9.38%	9.42%
10%	10.00%	10.25%	10.38%	10.47%	10.52%
11%	11.00%	11.30%	11.46%	11.57%	11.63%
12%	12.00%	12.36%	12.55%	12.68%	12.75%

**truth-in-lending laws**<sup>1</sup> require that financial institutions quote both nominal and effective interest rates when you deposit or borrow money.

More frequent compounding increases the amount of interest paid over a year at the same nominal interest rate. Assuming that the nominal interest rate is  $r$  and that  $M$  compounding periods occur during the year, the effective annual yield  $i_a$  can be calculated as follows:

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1. \quad (3.1)$$

When  $M = 1$ , we have the special case of annual compounding. Substituting  $M = 1$  into Eq. (3.1) reduces it to  $i_a = r$ . That is, when compounding takes place once annually, effective interest is equal to nominal interest. Thus, in most of our examples in Chapter 2, where only annual interest was considered, we were, by definition, using effective annual yields.

### EXAMPLE 3.1 Comparing Two Different Financial Products

You want to choose between the following investment alternatives:

- Option A: an investment pays 9% interest compounded monthly.
- Option B: an investment pays 9.2% interest compounded semi-annually.
  - Determine the nominal interest rate for each investment.
  - Determine the effective annual interest rate for each investment.
  - Select the best investment option.

<sup>1</sup>The Truth in Lending Act (TILA), passed in 1968, is a federal law that regulates the credit market and sets minimum standards for the information that a creditor must provide in an installment credit contract.

## DISSECTING THE PROBLEM

### METHODOLOGY

- The nominal interest rate is the stated rate on the financial product, which is the same as APR.
- The effective annual interest rate is calculated by taking the nominal interest rate and adjusting it for the number of compounding periods the financial product will experience in the given period of time.
- Select the investment option with highest effective annual interest rate.

**Given:** APRs and compounding frequencies.

**Find:** (a) nominal interest rates, (b) effective interest rates, and (c) select the best investment plan.

### SOLUTION

- Nominal interest rates
  - Option A: 9.0%
  - Option B: 9.2%

- Effective annual interest rates
  - Option A:

$$i_a = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 9.38\%.$$

- Option B:

$$i_a = \left(1 + \frac{0.092}{2}\right)^2 - 1 = 9.25\%.$$

- Best investment option

Select Option A.

**COMMENTS:** As can be seen, even though investment B has a higher stated nominal interest rate, we see that the effective annual interest rate is lower than the effective rate for investment A due to the fact that it compounds less times over the year. As the number of compounding periods increases so does the effective annual interest rate.

## EXAMPLE 3.2 Determining a Compounding Period

Consider the following bank advertisement that appeared in a local newspaper:

“Open a Liberty Bank Certificate of Deposit (CD) and get a guaranteed rate of return on as little as \$500. It’s a smart way to manage your money for months.”

In this advertisement, no mention is made of specific interest compounding frequencies. Find the compounding period for each CD.

Type of Certificate	Interest Rate (APR)	Annual Percentage Yield (APY)	Minimum Required to Open
1-Year Certificate	2.23%	2.25%	\$500
2-Year Certificate	3.06%	3.10%	\$500
3-Year Certificate	3.35%	3.40%	\$500
4-Year Certificate	3.45%	3.50%	\$500
5- to 10-Year Certificates	4.41 %	4.50 %	\$500

<p><b>DISSECTING THE PROBLEM</b></p> <p><b>METHODOLOGY</b></p> <p><i>Method 1: By Trial and Error.</i></p> <p><i>Method 2: Using an Excel Function.</i></p>	<p><b>Given:</b> <math>r = 4.41\%</math> per year, <math>i_a = 4.50\%</math>.  <b>Find:</b> <math>M</math>.</p> <p><b>SOLUTION</b></p> <p>First, we will consider the 5-to-10-year CD. The nominal interest rate is 4.41% per year, and the effective annual interest rate (or APY) is 4.50%. Using Eq. (3.1), we obtain the expression</p> $0.0450 = (1 + 0.0441/M)^M - 1.$ <p>By trial and error, we find that <math>M = 12</math>. Thus, the 5-to-10-year CD earns 4.41% interest compounded monthly. Similarly, we can find that the interest periods for the other CDs are monthly as well.</p> <p>We can also find the compounding period in much quicker way using one of the following financial functions in Excel.</p> <p>Effective rate “=EFFECT(<math>r, M</math>)”  Nominal rate “=NOMINAL(<math>i_a, M</math>)”</p> <p>Taking the 5-to-10-year CD example, we create a worksheet where Cell B3 is a function of <math>M</math> (Cell B4). We vary the value in B4 until we see a close match to the published APY.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 10%;"></th><th style="width: 45%;">A</th><th style="width: 45%;">B</th></tr> </thead> <tbody> <tr> <td>1</td><td></td><td></td></tr> <tr> <td>2</td><td>Annual percentage rate (<math>r</math>)</td><td>0.0441</td></tr> <tr> <td>3</td><td>Effective annual rate (<math>i_a</math>)</td><td>0.045002</td></tr> <tr> <td>4</td><td>Number of compounding periods per year (<math>M</math>)</td><td>12</td></tr> <tr> <td>5</td><td></td><td></td></tr> <tr> <td></td><td colspan="2" style="border-top: none;">=EFFECT(B2,B4)</td></tr> </tbody> </table>		A	B	1			2	Annual percentage rate ( $r$ )	0.0441	3	Effective annual rate ( $i_a$ )	0.045002	4	Number of compounding periods per year ( $M$ )	12	5				=EFFECT(B2,B4)	
	A	B																				
1																						
2	Annual percentage rate ( $r$ )	0.0441																				
3	Effective annual rate ( $i_a$ )	0.045002																				
4	Number of compounding periods per year ( $M$ )	12																				
5																						
	=EFFECT(B2,B4)																					

## 3.2 Calculating Effective Interest Rates Based on Payment Periods

We can generalize the result of Eq. (3.1) to compute the effective interest rate for *any duration of time*. As you will see later, the effective interest rate is usually computed according to the payment (transaction) period. We will look into two types of compounding situations: (1) discrete compounding and (2) continuous compounding.

### 3.2.1 Discrete Compounding

If cash flow transactions occur *quarterly* but interest is compounded *monthly*, we may wish to calculate the effective interest rate on a *quarterly basis*. In this case, we may redefine Eq. (3.1) as

$$\begin{aligned} i &= (1 + r/M)^C - 1 \\ &= (1 + r/CK)^C - 1, \end{aligned} \quad (3.2)$$

where

- $M$  = the number of interest periods per year,
- $C$  = the number of interest periods per payment period, and
- $K$  = the number of payment periods per year.

Note that  $M = CK$  in Eq. (3.2).

### EXAMPLE 3.3 Effective Rate per Payment Period

Suppose that you make quarterly deposits into a savings account that earns 8% interest compounded monthly. Compute the effective interest rate per quarter.

#### DISSECTING THE PROBLEM

**Given:**  $r = 8\%$  per year,  $C = 3$  interest periods per quarter,  $K = 4$  quarterly payments per year, and  $M = 12$  interest periods per year.

**Find:**  $i$ .

#### METHODOLOGY

*Method 1: Calculate the Effective Interest Rate Based on the Payment Period.*

*Method 2: Calculate the Effective Interest Rate per Payment Period Using Excel.*

#### SOLUTION

Using Eq. (3.2), we compute the effective interest rate per quarter as

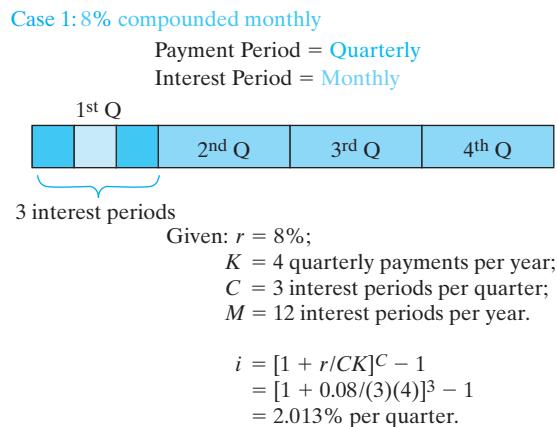
$$\begin{aligned} i &= (1 + 0.08/12)^3 - 1 \\ &= 2.0134\% \text{ per quarter.} \end{aligned}$$

Using Excel,

	A	B
1	<b>Inputs</b>	
2		
3	( $r$ ) APR (%)	8%
4	( $K$ ) Payment periods per year	4
5	( $C$ ) Interest periods per payment period	3
6	<b>Output</b>	
7	( $M$ ) Compounding periods per year	=B4*B5 12
8	( $i$ ) Effective interest rate per payment period	2.0134%
9		

=EFFECT(B3,B7)

**COMMENTS:** The effective annual interest rate  $i_a$  is  $(1 + 0.02013)^4 - 1 = 8.30\%$ . For the special case of annual payments with annual compounding, we obtain  $i = i_a$  with  $C = M$  and  $K = 1$ . Figure 3.3 illustrates the relationship between the nominal and effective interest rates per payment period.



**Figure 3.3** Computing the effective interest rate per quarter.

### 3.2.2 Continuous Compounding

To be competitive in the financial market, or to entice potential depositors, some financial institutions offer more frequent compounding. As the number of compounding periods ( $M$ ) becomes very large, the interest rate per compounding period ( $r/M$ ) becomes very small. As  $M$  approaches infinity and  $r/M$  approaches zero, we approximate the situation of **continuous compounding**.

By taking limits on the right side of Eq. (3.2), we obtain the effective interest rate per payment period as

$$\begin{aligned} i &= \lim_{CK \rightarrow \infty} \left[ (1 + r/CK)^C - 1 \right] \\ &= \lim_{CK \rightarrow \infty} (1 + r/CK)^C - 1 \\ &= (e^r)^{1/K} - 1. \end{aligned}$$

In sum, the effective interest rate per payment period is

$$i = (e^{r/K}) - 1. \quad (3.3)$$

To calculate the effective *annual* interest rate for continuous compounding, we set  $K$  equal to 1, resulting in

$$i_a = e^r - 1. \quad (3.4)$$

As an example, the effective annual interest rate for a nominal interest rate of 12% compounded continuously is  $i_a = e^{0.12} - 1 = 12.7497\%$ .

## EXAMPLE 3.4 Calculating an Effective Interest Rate

Find the effective interest rate per quarter at a nominal rate of 8% compounded  
 (a) weekly, (b) daily, and (c) continuously.

DISSECTING THE PROBLEM	
	<b>Given:</b> $r = 8\%$ per year, $K = 4$ payments per year. <b>Find:</b> $i$ per quarter.
METHODOLOGY	<b>SOLUTION</b> <p>(a) Weekly compounding:            With <math>r = 8\%</math>, <math>M = 52</math>, and <math>C = 13</math> periods per quarter, we have</p> $i = (1 + 0.08/52)^{13} - 1$ $= 2.0186\% \text{ per quarter.}$ <p>Figure 3.4 illustrates this result.</p> <p style="color: #00AEEF;">Case 2: 8% compounded weekly</p> <p>Payment Period = <b>Quarterly</b>    Interest Period = <b>Weekly</b></p> <p>The diagram shows four vertical bars labeled 1<sup>st</sup> Q, 2<sup>nd</sup> Q, 3<sup>rd</sup> Q, and 4<sup>th</sup> Q from left to right. The first bar, 1<sup>st</sup> Q, contains 13 smaller blue squares arranged horizontally, with a brace underneath it labeled '13 interest periods'. The other three quarters are represented by single vertical bars.</p> <p><b>Figure 3.4</b> Effective interest rate per payment period: quarterly payments with weekly compounding.</p> <p>(b) Daily compounding:            With <math>r = 8\%</math>, <math>M = 365</math>, and <math>C = 91.25</math> per quarter, we have</p> $i = (1 + 0.08/365)^{91.25} - 1$ $= 2.0199\% \text{ per quarter.}$ <p>(c) Continuous compounding:            With <math>r = 8\%</math>, <math>M \rightarrow \infty</math>, <math>C \rightarrow \infty</math>, and <math>K = 4</math> using Eq. (3.3), we obtain</p> $i = e^{0.08/4} - 1 = 2.0201\% \text{ per quarter.}$

**COMMENTS:** Note that the difference between daily compounding and continuous compounding is often negligible. Many banks offer continuous compounding to entice deposit customers, but the extra benefits are small. Table 3.2 summarizes the varying effective interest rates per payment period (quarterly, in this case) under various compounding frequencies.

**TABLE 3.2** Effective Interest Rates per Payment Period

	Base	Case 1	Case 2	Case 3	Case 4
Interest Rate	8% compounded quarterly	8% compounded monthly	8% compounded weekly	8% compounded daily	8% compounded continuously
Payment Period	Payments occur quarterly				
Effective Interest Rate per Payment Period	2.000% per quarter	2.013% per quarter	2.0186% per quarter	2.0199% per quarter	2.0201% per quarter

### 3.3 Equivalence Calculations with Effective Interest Rates

When calculating equivalent values, we need to identify both the interest period and the payment period. If the time interval for compounding is different from the time interval for cash transaction (or payment), we need to find *the effective interest rate that covers the payment period*. We illustrate this concept with specific examples.

#### 3.3.1 Compounding Period Equal to Payment Period

All examples in Chapter 2 assumed annual payments and annual compounding. Whenever a situation occurs where the compounding and payment periods are equal ( $M = K$ ), no matter whether the interest is compounded annually or at some other interval, the following solution method can be used:

1. Identify the number of compounding (or payment) periods ( $M = K$ ) per year.
2. Compute the effective interest rate per payment period; that is,

$$\begin{aligned} i &= (1 + r/M)^C - 1 \\ &= r/M \text{ (with } C = 1\text{).} \end{aligned}$$

3. Determine the number of payment periods:

$$N = M \times (\text{number of years}).$$

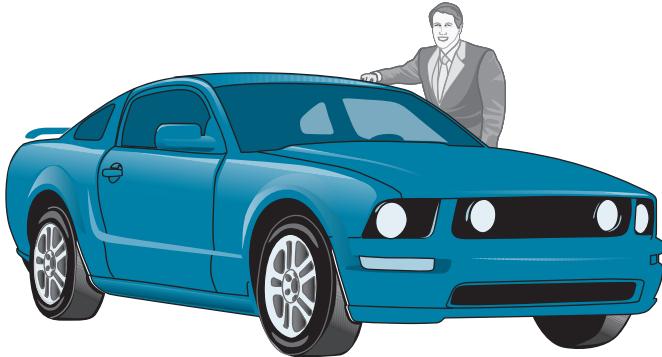
4. Use  $i$  and  $N$  in the appropriate formulas in Table 2.11.

#### EXAMPLE 3.5 Calculating Auto Loan Payments

Suppose you want to buy a car. You have surveyed the dealers' newspaper advertisements, and the one in Figure 3.5 has caught your attention. You can afford to make a down payment of \$2,678.95, so the net amount to be financed is \$20,000.

- What would the monthly payment be?
- After the 25th payment, you want to pay off the remaining loan in a lump-sum amount. What is this lump sum?

- 8.5% Annual Percentage Rate!  
48 month financing on all Mustangs in stock. 60 to choose from.
- ALL READY FOR DELIVERY!  
Prices starting as low as \$21,599.
- You just add sales tax and 1% for dealer's freight. We will pay the tag, title, and license.
- Add 4% sales tax = \$863.96
- Add 1% dealer's freight = \$215.99
- Total purchase price = **\$22,678.95**



**Figure 3.5** Financing an automobile.

### DISSECTING THE PROBLEM

The advertisement does not specify a compounding period, but in automobile financing, the interest and the payment periods are almost always monthly.

In the second part of the example, we need to calculate the remaining balance after the 25th payment.

### METHODOLOGY

**Step 1:** In this situation, we can easily compute the monthly payment by using Eq. (2.9). The 8.5% APR means 8.5% compounded monthly.

**Given:**  $P = \$20,000$ ,  $r = 8.5\%$  per year,  $K = 12$  payments per year,  $N = 48$  months, and  $C = 1$ , and  $M = 12$ .

**Find:**  $A$ .

### SOLUTION

$$i = \left(1 + \frac{0.085}{12}\right)^1 - 1 = 0.7083\% \text{ per month}$$

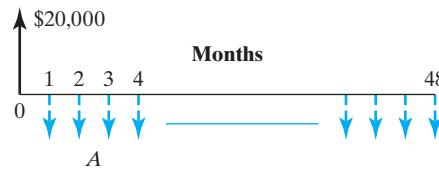
and

$$N = (12)(4) = 48 \text{ months},$$

we have

$$A = \$20,000 (A/P, 0.7083\%, 48) = \$492.97.$$

Figure 3.6 shows the cash flow diagram for this part of the example.



**Figure 3.6** Cash flow diagram for part (a).

**Step 2:** We can compute the amount you owe after you make the 25th payment by calculating the equivalent worth of the remaining 23 payments at the end of the 25th month, with the time scale shifted by 25 months.

**Given:**  $A = \$492.97$ ,  $i = 0.7083\%$ , and  $N = 23$  months.

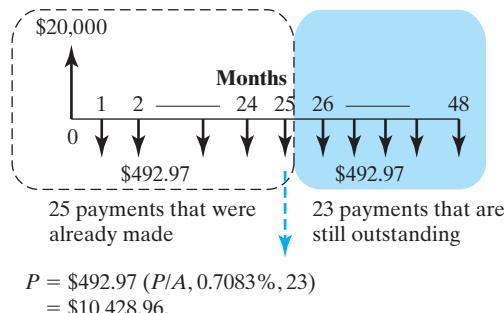
**Find:** Remaining balance after 25 months ( $B_{25}$ ).

The balance is calculated as follows:

$$B_{25} = \$492.97(P/A, 0.7083\%, 23) = \$10,428.96.$$

So, if you desire to pay off the remainder of the loan at the end of the 25th payment, you must come up with \$10,428.96, in addition to the payment for that month of \$492.97. (See Figure 3.7)

Suppose you want to pay off the remaining loan in a lump sum right after making the 25th payment. How much would this payoff amount be?



**Figure 3.7** Process of calculating the remaining balance of the auto loan.

### 3.3.2 Compounding Occurs at a Different Rate than That at Which Payments Are Made

The computational procedure for dealing with compounding periods and payment periods that cannot be compared is as follows:

1. Identify the number of compounding periods per year ( $M$ ), the number of payment periods per year ( $K$ ), and the number of interest periods per payment period ( $C$ ).
2. Compute the effective interest rate per payment period:
  - For discrete compounding, compute

$$i = (1 + r/M)^C - 1.$$

- For continuous compounding, compute

$$i = e^{r/K} - 1.$$

3. Find the total number of payment periods:

$$N = K \times (\text{number of years}).$$

4. Use  $i$  and  $N$  in the appropriate formulas in Table 2.11.

### EXAMPLE 3.6 Compounding Occurs More Frequently than Payments Are Made

Suppose you make equal quarterly deposits of \$1,000 into a fund that pays interest at a rate of 12% compounded monthly. Find the balance at the end of year 3.

#### DISSECTING THE PROBLEM

**Given:**  $A = \$1,000$  per quarter,  $r = 12\%$  per year,  $M = 12$  compounding periods per year,  $N = 12$  quarters, and the cash flow diagram in Figure 3.8.

**Find:**  $F$ .

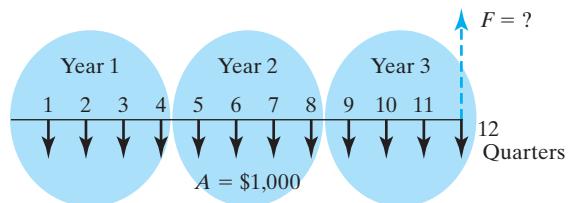


Figure 3.8 Cash flow diagram.

#### METHODOLOGY

*Method 1: Noncomparable Compounding and Payment Periods*

We follow the procedure for non-comparable compounding and payment periods, as described previously.

#### SOLUTION

1. Identify the parameter values for  $M$ ,  $K$ , and  $C$ , where

$M = 12$  compounding periods per year,

$K = 4$  payment periods per year, and

$C = 3$  interest periods per payment period.

2. Use Eq. (3.2) to compute effective interest:

$$\begin{aligned} i &= (1 + 0.12/12)^3 - 1 \\ &= 3.030\% \text{ per quarter.} \end{aligned}$$

3. Find the total number of payment periods,  $N$ , where

$$N = K(\text{number of years}) = 4(3) = 12 \text{ quarters.}$$

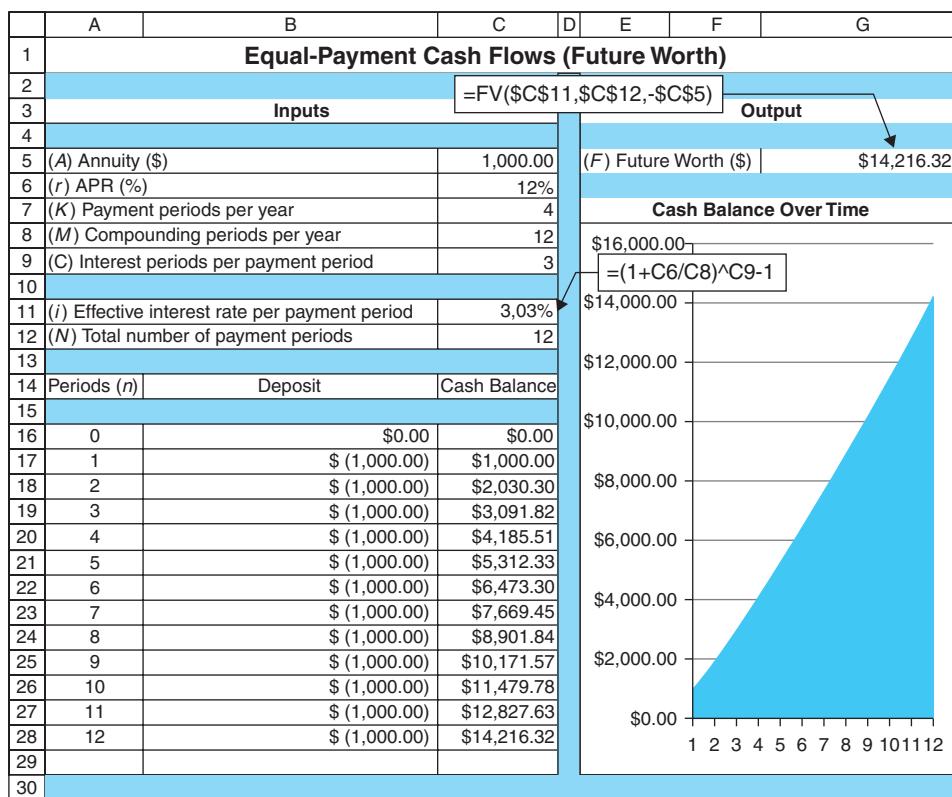
4. Use  $i$  and  $N$  in the appropriate equivalence formulas:

$$F = \$1,000 (F/A, 3.030\%, 12) = \$14,216.24.$$

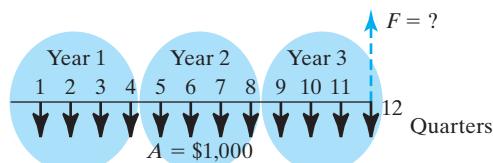
**Method 2: Excel Worksheet**

Table 3.3 illustrates the process of obtaining the future worth of the quarterly payment series in Excel format.

**TABLE 3.3** An Excel Worksheet to Illustrate the Process of Accumulating the Future Worth



**COMMENTS:** Appendix B does not provide interest factors for  $i = 3.03\%$ , but the interest factor can still be evaluated by  $F = \$1,000(A/F, 1\%, 3)(F/A, 1\%, 36)$ , where the first interest factor finds its equivalent monthly payment and the second interest factor converts the monthly payment series to an equivalent lump-sum future payment. If continuous compounding is assumed, the accumulated balance would be \$14,228.37, which is about \$12 more than the balance for the monthly compounding situation. (See Figure 3.9.)

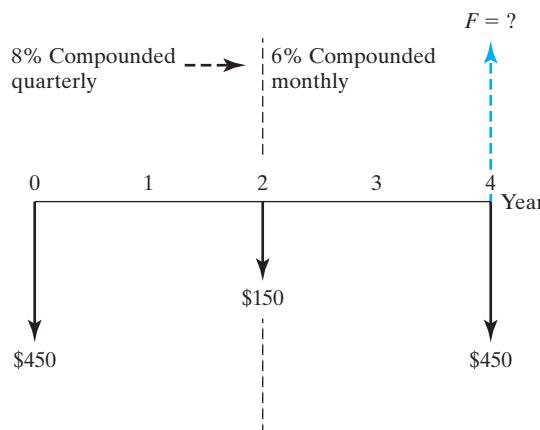


- Step 1:  $K = 4$  payment periods/year;  
 $C = \infty$  interest periods per quarter.  
Step 2:  $i = e^{0.12/4} - 1$   
 $= 3.045\%$  per quarter.  
Step 3:  $N = 4(3) = 12$ .  
Step 4:  $F = \$1,000 (F/A, 3.045\%, 12)$   
 $= \$14,228.37$ .

**Figure 3.9** Equivalence calculation for continuous compounding.

### EXAMPLE 3.7 Equivalence Calculations with Changing Interest Rates

Compute the future worth ( $F$ ) for the cash flows with the different interest rates specified in Figure 3.10. The cash flows occur at the end of each year over four years.



**Figure 3.10** Cash flow diagram for a deposit series with changing interest rates.

#### DISSECTING THE PROBLEM

**Given:**  $r_1 = 8\%$  per quarter,  $r_2 = 6\%$  per year,  $N = 4$  years, and payment period = annual.  
**Find:**  $F$ .

**METHODOLOGY**

Calculate the effective interest rates per payment period, altering the nominal interest rate. Then, calculate the balance at the end of each deposit period.

**SOLUTION**

- (a) Years 0–2:

With  $r_1 = 8\%$ ,  $M = 1$ , and  $C = 4$ , we have

$$i = i_a = (1 + 0.08/4)^4 - 1 = 8.243\% \text{ per year.}$$

The balance at the end of year 2 is

$$B_2 = \$450(F/P, 8.243\%, 2) + \$150 = \$677.24.$$

- (b) Years 2–4:

With  $r_2 = 6\%$ ,  $M = 12$ , and  $B_2$ , we have

$$\begin{aligned} i &= i_a = (1 + 0.06/12)^{12} - 1 \\ &= 6.168\% \text{ per year.} \end{aligned}$$

The balance at the end of year 4 will be

$$\begin{aligned} F &= B_4 = \$677.24(F/P, 6.168\%, 2) + \$450 \\ &= \$1,213.36. \end{aligned}$$

## 3.4 Debt Management

Credit card debt and commercial loans are easily among the most significant and familiar financial obligations that involve interest. Many types of loans are available, but here we will focus on those most frequently used by individuals and in business.

### 3.4.1 Borrowing with Credit Cards

When credit cards were introduced in 1959, they offered people the ability to handle their personal finances in a dramatically different way. From a consumer's perspective, your ability to use credit cards means that you do not have to wait for a paycheck to reach the bank before you can make a purchase. Most credit cards operate on *revolving credit*. With revolving credit, you have a line of borrowing that you can tap into at will and pay back as quickly or slowly as you want—as long as you pay the minimum required each month.

Your monthly bill is an excellent source of information about what your card really costs. Four things affect your card-based credit costs: the annual fees, the finance charges, the grace period, and the method of calculating interest. In fact, there are three different ways to compute interest charges, as summarized in Table 3.4. The average-daily-balance approach is the most common.

**TABLE 3.4** Methods of Calculating Interests on Your Credit Cards

Method	Description	Example of the Interest You Owe, Given a Beginning Balance of \$3,000 at 18%
Adjusted Balance	The bank subtracts the amount of your payment from the beginning balance and charges you interest on the remainder. This method costs you the least.	With a \$1,000 payment, your new balance will be \$2,000. You pay 1.5% interest for the month on this new balance, which comes out to $(1.5\%) (\$2,000) = \$30$ .
Average Daily Balance	The bank charges you interest on the average of the amount you owe each day during the period. So, the larger the payment you make, the lower the interest you pay.	With a \$1,000 payment on the 15th day, your balance reduced to \$2,000. Therefore, the interest on your average daily balance for the month will be $(1.5\%) (\$3,000 + \$2,000)/2 = \$37.50$ .
Previous Balance	The bank does not subtract any payments you make from your previous balance. You pay interest on the total amount you owe at the beginning of the period. This method costs you the most.	The annual interest rate is 18% compounded monthly. Regardless of your payment amount, the bank will charge 1.5% on your beginning balance of \$3,000. Therefore, your interest for the month will be $(1.5\%) (\$3,000) = \$45$ .

### EXAMPLE 3.8 Paying Off Cards Saves a Bundle

Suppose that you owe \$2,000 on a credit card that charges 18% APR and you pay either the minimum 10% or \$20, whichever is higher, every month. How long will it take you to pay off the debt? Assume that the bank uses the previous-balance method to calculate your interest, meaning that the bank does not subtract the amount of your payment from the beginning balance but charges you interest on the previous balance.

<b>DISSECTING THE PROBLEM</b>	<b>Given:</b> APR = 18% (or 1.5% per month), beginning balance = \$2,000, and minimum monthly payment = 10% of outstanding balance or \$20, whichever is higher. <b>Find:</b> The number of months to pay off the loan, assuming that no new purchases are made during this payment period.
<b>METHODOLOGY</b> Use Excel to calculate the number of payments.	<b>SOLUTION</b> With the initial balance of \$2,000 ( $n = 0$ ), the interest for the first month will be \$30 ( $= \$2,000(0.015)$ ), so you will be billed \$2,030. Then you make a \$203 payment (10% of the outstanding balance), so the remaining balance will be \$1,827. At the end of the second month, the billing statement will show that you owe the bank the amount of \$1,854.41 of which \$2741 is interest. With a \$185.44 payment, the balance is reduced to \$1,668.96. This process repeats until the 26th payment. For the 27th payment and all those thereafter, 10% of the outstanding balance is less than \$20, so you pay \$20. As shown in Table 3.5, it would take 37 months to pay off the \$2,000 debt with total interest payments of \$330.42.

**TABLE 3.5** Creating a Loan Repayment Schedule

	A	B	C	D	E	F
1						
2	Period	Beg.	Interest	Amount	Payment	End.
3	(n)	Balance	Charged	Billed	Required	Balance
4						
5	0					\$2,000.00
6	1	\$2,000.00	\$30.00	\$2,030.00	\$203.00	\$1,827.00
7	2	\$1,827.00	\$27.41	\$1,854.41	\$185.44	\$1,668.96
8	3	\$1,668.96	\$25.03	\$1,694.00	\$169.40	\$1,524.60
9	4	\$1,524.60	\$22.87	\$1,547.47	\$154.75	\$1,392.72
10	5	\$1,392.72	\$20.89	\$1,413.61	\$141.36	\$1,272.25
11	6	\$1,272.25	\$19.08	\$1,291.33	\$129.13	\$1,162.20
12	7	\$1,162.20	\$17.43	\$1,179.63	\$117.96	\$1,061.67
13	8	\$1,061.67	\$15.93	\$1,077.60	\$107.76	\$969.84
14	9	\$969.84	\$14.55	\$984.38	\$98.44	\$885.95
15	10	\$885.95	\$13.29	\$899.23	\$89.92	\$809.31
16	11	\$809.31	\$12.14	\$821.45	\$82.15	\$739.31
17	12	\$739.31	\$11.09	\$750.40	\$75.04	\$675.36
18	13	\$675.36	\$10.13	\$685.49	\$68.55	\$616.94
19	14	\$616.94	\$9.25	\$626.19	\$62.62	\$563.57
20	15	\$563.57	\$8.45	\$572.03	\$57.20	\$514.82
21	16	\$514.82	\$7.72	\$522.55	\$52.25	\$470.29
22	17	\$470.29	\$7.05	\$477.35	\$47.73	\$429.61
23	18	\$429.61	\$6.44	\$436.06	\$43.61	\$392.45
24	19	\$392.45	\$5.89	\$398.34	\$39.83	\$358.50
25	20	\$358.50	\$5.38	\$363.88	\$36.39	\$327.49
26	21	\$327.49	\$4.91	\$332.40	\$33.24	\$299.16
27	22	\$299.16	\$4.49	\$303.65	\$30.37	\$273.29
28	23	\$273.29	\$4.10	\$277.39	\$27.74	\$249.65
29	24	\$249.65	\$3.74	\$253.39	\$25.34	\$228.05
30	25	\$228.05	\$3.42	\$231.47	\$23.15	\$208.33
31	26	\$208.33	\$3.12	\$211.45	\$21.15	\$190.31
32	27	\$190.31	\$2.85	\$193.16	\$20.00	\$173.16
33	28	\$173.16	\$2.60	\$175.76	\$20.00	\$155.76
34	29	\$155.76	\$2.34	\$158.09	\$20.00	\$138.09
35	30	\$138.09	\$2.07	\$140.17	\$20.00	\$120.17
36	31	\$120.17	\$1.80	\$121.97	\$20.00	\$101.97
37	32	\$101.97	\$1.53	\$103.50	\$20.00	\$83.50
38	33	\$83.50	\$1.25	\$84.75	\$20.00	\$64.75
39	34	\$64.75	\$0.97	\$65.72	\$20.00	\$45.72
40	35	\$45.72	\$0.69	\$46.41	\$20.00	\$26.41
41	36	\$26.41	\$0.40	\$26.80	\$20.00	\$6.80
42	37	\$6.80	\$0.10	\$6.91	\$6.91	\$0.00
43						

=IF(D41\*0.1>20,D41\*0.1,IF(F40<20,D41,20))

In developing the Excel worksheet in Table 3.5, you may use a nested IF function, one of Excel's most useful functions. What it does is test to see whether a certain condition is true or false. In our example, the values in column E are obtained by using the nested IF function. For example, to calculate the value in Cell E6, the IF logic function looks like

$$= \text{IF}(\text{D6}*0.1 > 20, \text{D6}*0.1, \text{IF}(\text{F5} < 20, \text{D6}, 20)).$$

The reason why you need a nested IF function is to avoid the situation where you do not want to pay more than what you owe, like in Cell E42.

**COMMENTS:** If the bank uses the average-daily-balance method and charges you interest on the average of the amount you owe each day during the period, it would take a little longer to pay off the debt. For example, if you make a \$200 payment on the 15th day, your balance is reduced to \$1,800. Therefore, the interest on your average daily balance for the month will be  $(1.5\%) (\$2,000 + \$1,800)/2 = \$28.50$ , and the ending balance will be \$1,828.50 instead of \$1,827.00.

### 3.4.2 Commercial Loans—Calculating Principal and Interest Payments

One of the most important applications of compound interest involves loans that are paid off in **installments** over time. If a loan is to be repaid in equal periodic amounts (e.g., weekly, monthly, quarterly, or annually), it is said to be an **amortized loan**. Examples include automobile loans, loans for appliances, home mortgage loans, and most business debts other than very short-term loans. Most commercial loans have interest that is compounded monthly. With a car loan, for example, a local bank or a dealer advances you the money to pay for the car, and you repay the principal plus interest in monthly installments, usually over a period of three to five years. The car is your collateral. If you don't keep up with your payments, the lender can repossess, or take back, the car and keep all the payments you have made.

Two factors determine what borrowing will cost you: the finance charge and the length of the loan. The cheapest loan is not necessarily the loan with the lowest payments or even the loan with the lowest interest rate. Instead, you have to look at the total cost of borrowing, which depends on the interest rate, fees, and the term (i.e., the length of time it takes you to repay the loan). While you probably cannot influence the rate and fees, you may be able to arrange for a shorter term.

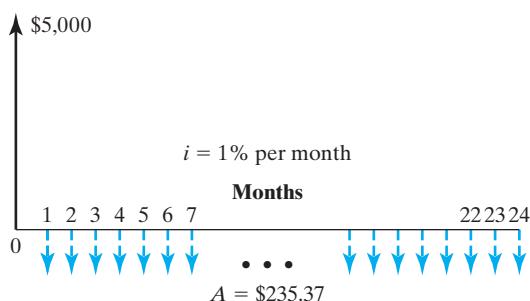
So far, we have considered many instances of amortized loans in which we calculated present or future values of the loans or the amounts of the installment payments. Also of great interest to us is calculating the amount of interest contained versus the portion of the principal that is paid off in each installment. As we shall explore more fully in Chapter 10, the interest paid on a business loan is an important element in calculating taxable income. We will further show how we may calculate the interest and principal paid at any point in the life of a loan using Excel's financial functions. As Example 3.8 illustrates, the amount of interest owed for a specified period is calculated based on the *remaining balance* of the loan at the beginning of the period.

### EXAMPLE 3.9 Using Excel to Determine a Loan's Balance, Principal, and Interest

Suppose you secure a home improvement loan in the amount of \$5,000 from a local bank. The loan officer gives you the following loan terms:

- Contract amount = \$5,000
- Contract period = 24 months
- Annual percentage rate = 12%
- Monthly installment = \$235.37

Figure 3.11 shows the cash flow diagram for this loan transactions. Construct the loan payment schedule by showing the remaining balance, interest payment, and principal payment at the end of each period over the life of the loan.



**Figure 3.11** Cash flow diagram.

#### DISSECTING THE PROBLEM

**Given:**  $P = \$5,000$ ,  $A = \$235.37$  per month,  $r = 12\%$  per year,  $M = 12$  compounding periods per year, and  $N = 24$  months.  
**Find:**  $B_n$  and  $I_n$  for  $n = 1$  to 24.

#### METHODOLOGY

We can easily see how the bank calculated the monthly payment of \$235.37.

#### SOLUTION

Since the effective interest rate per payment period on this loan transaction is 1% per month, we establish the following equivalence relationship:

$$\$5,000(A/P, 1\%, 24) = \$5,000(0.0471) = \$235.37.$$

The loan payment schedule can be constructed using Excel as in Table 3.6. The interest due at  $n = 1$  is \$50.00, 1% of the \$5,000 outstanding during the first month. The \$185.37 left over is applied to the principal, reducing the amount outstanding in the second month to \$4,814.63. The interest due in the second month is 1% of \$4,814.63, or \$48.15, leaving \$187.22 for repayment of the principal. At  $n = 24$ , the last \$235.37 payment is just sufficient to pay the interest on the unpaid loan principal and to repay the remaining principal.

**COMMENTS:** We can use the special financial functions provided in Excel to determine the total interest (or principal) paid between any two payment periods. Two financial functions are “=CUMIPMT(.)” and “=CUMPRINC(.)”. Using the functional arguments in Table 3.7, you may determine the total interest and principal paid in the second year as follows:

- =CUMIPMT(1%,24,5000,13,24,0) → (\$175.33)
- =CUMPRINC(1%,24,5000,13,24,0) → (\$2,649.08)

**TABLE 3.6** Loan Repayment Schedule Generated by Excel

A	B	C	D	E	F	G
1	Example 3.9 Loan Repayment Schedule					
2						
3						
4	Contract amount	\$ 5,000.00		Total payment	\$ 5,648.82	
5	Contract period	24		Total interest	\$648.82	
6	APR (%)	12		=C7*C5		
7	Monthly Payment	(\$235.37)		=CUMIPMT(1%,C5,C4,1,24,0)		
8						
9						
10		Payment No.	Payment Size	Principal Payment	Interest Payment	Loan Balance
11		1	(\$235.37)	(\$185.37)	(\$50.00)	\$4,814.63
12		2	(\$235.37)	(\$187.22)	(\$48.15)	\$4,627.41
13		3	(\$235.37)	(\$189.09)	(\$46.27)	\$4,438.32
14	=PMT(\$C\$6/1200,\$C\$5,\$C\$4,0)	4	(\$235.37)	(\$190.98)	(\$44.38)	\$4,247.33
15		5	(\$235.37)	(\$192.89)	(\$42.47)	\$4,054.44
16		6	(\$235.37)	(\$194.82)	(\$40.54)	\$3,859.62
17		7	(\$235.37)	(\$196.77)	(\$38.60)	\$3,662.85
18		8	(\$235.37)	(\$198.74)	(\$36.63)	\$3,464.11
19		9	(\$235.37)	(\$200.73)	(\$34.64)	\$3,263.38
20		10	(\$235.37)	(\$202.73)	(\$32.63)	\$3,060.65
21		11	(\$235.37)	(\$204.76)	(\$30.61)	\$2,855.89
22		12	(\$235.37)	(\$206.81)	(\$28.56)	\$2,649.08
23		13	(\$235.37)	(\$208.88)	(\$26.49)	\$2,440.20
24		14	(\$235.37)	(\$210.97)	(\$24.40)	\$2,229.24
25		15	(\$235.37)	(\$213.08)	(\$22.29)	\$2,016.16
26		16	(\$235.37)	(\$215.21)	(\$20.16)	\$1,800.96
27		17	(\$235.37)	(\$217.36)	(\$18.01)	\$1,583.60
28		18	(\$235.37)	(\$219.53)	(\$15.84)	\$1,364.07
29		19	(\$235.37)	(\$221.73)	(\$13.64)	\$1,142.34
30		20	(\$235.37)	(\$223.94)	(\$11.42)	\$918.40
31		21	(\$235.37)	(\$226.18)	(\$9.18)	\$692.21
32		22	(\$235.37)	(\$228.45)	(\$6.92)	\$463.77
33		23	(\$235.37)	(\$230.73)	(\$4.64)	\$233.04
34		24	(\$235.37)	(\$233.04)	(\$2.33)	\$0.00
35						
36						
37						
38	=PPMT(\$C\$6/1200,C33,\$C\$5,\$C\$4,0)					
39						
40						
41						

$$=IPMT(\$C\$6/1200,C33,\$C\$5,\$C\$4,0)$$

$$=G32+E33$$

**TABLE 3.7** Excel's Financial Functions to Determine a Loan's Principal and Interest Payments between Two Payment Periods

	A	B
1		
2	<b>EXCEL FUNCTIONS</b>	<b>DESCRIPTION</b>
3		
4	=IMPT(i%,n,N,P)	Interest payment
5		
6	=PPMT(i%,n,N,P)	Principal payment
7		
8	=CUMIPMT(i%,N,P,start_period,end_period,type)	Cumulative interest payment
9		
10	=CUMPRINC(i%,N,P,start_period,end_period,type)	Cumulative principal payment
11		
12	<b>Data</b>	<b>DESCRIPTION</b>
13	12%	Annual percentage rate
14	5	Period for which you want to find the interest
15	2	Years of loan
16	\$5,000.00	Present value of loan
17	<b>Result</b>	<b>DESCRIPTION</b>
18	=IPMT(A13/12,A14,A15*12,A16)	(\$42.47) Interest due in the fifth month for a loan with the terms above
19	=PPMT(A13/12,A14,A15*12,A16,1)	(\$192.93) Principal payment due in the fifth month for a loan with the terms above
20	=CUMIPMT(A13/12,A15*12,A16,13,24,0)	(\$175.33) Total Interest paid in the 2nd year for a loan with the terms above, where payments are made monthly
21	=CUMPRINC(A13/12,A15*12,A16,13,24,0)	Total principal paid in the 2nd year for a loan with the terms above, where payments are made monthly (\$2,649.08)
22		

### 3.4.3 Comparing Different Financing Options

When you purchase a car, you also choose how to pay for it. If you do not have the cash on hand to buy a new car outright—as most of us do not—you can consider taking out a loan or leasing the car in order to spread out the payments over time. Your decision to pay cash, take out a loan, or sign a lease depends on a number of personal as well as economic factors. Leasing is an option that lets you pay for the portion of a vehicle you expect to use over a specified term plus rent charge, taxes, and fees. For example, you

might want a \$20,000 vehicle. Assume that the vehicle might be worth about \$9,000 (its residual value) at the end of a three-year lease. Your options are as follows:

- If you have enough money to buy the car, you could purchase the car with cash. If you pay cash, however, you will lose the opportunity to earn interest on the amount you spend. That could be substantial if you have access to investments paying good returns.
- If you purchase the vehicle via debt financing, your monthly payments will be based on the entire \$20,000 value of the vehicle. You will own the vehicle at the end of your financing term, but the interest you will pay on the loan will drive up the real cost of the car ownership.
- If you lease the vehicle, your monthly payments will be based on the amount of the vehicle you expect to “use up” over the lease term. This value (\$11,000 in our example) is the difference between the original cost (\$20,000) and the estimated value at lease end (\$9,000). With leasing, the length of your agreement, the monthly payments, and the yearly mileage allowance can be tailored to your driving needs. The greatest financial appeal for leasing is its low initial outlay costs: usually, you pay only an administrative fee, one month’s lease payment, and a refundable security deposit. The terms of your lease will include a specific mileage allowance; if you use additional miles, you will have to pay an additional charge for each extra mile. Of course, you can make some cash down payment up-front to reduce the lease payment.

### Which Interest Rate Do We Use in Comparing Different Financing Options?

The dealer’s (bank’s) interest rate is supposed to reflect the time value of money of the dealer (or the bank) and is factored into the required payments. However, the correct interest rate for us to use when comparing financing options is the interest rate that reflects *your* earning opportunity. For most individuals, this interest rate is equivalent to the savings rate from their deposits. To illustrate, we provide two examples. Example 3.10 compares two different financing options for an automobile. Example 3.11 explores a lease-versus-buying decision on an automobile.

### EXAMPLE 3.10 Buying a Car: Paying in Cash versus Taking a Loan

Consider the following two options proposed by an auto dealer:

- **Option A:** Purchase the vehicle at the dealer’s suggested price of \$26,200 and pay for the vehicle over 36 months with equal monthly payments at 1.9% APR financing.
- **Option B:** Purchase the vehicle at a discounted price of \$24,048 to be paid immediately. The funds that would be used to purchase the vehicle are presently earning 5% annual interest compounded monthly.

Which option is more economically sound?

### DISSECTING THE PROBLEM

In calculating the net cost of financing the car, we need to decide which interest rate to use in discounting the loan repayment series. Note that the 1.9% APR represents the dealer's interest rate to calculate the loan payments. With the 1.9% interest, your monthly payments will be  $A = \$26,200 (A/P, 1.9\% / 12, 36) = \$749.29$ . On the other hand, the 5% APR represents your earning opportunity rate. Thus, if you do not buy the car, your money continues to earn 5% APR. Therefore, this 5% rate also represents your opportunity cost of purchasing the car. Which interest rate should we use in this analysis? Since we wish to calculate each option's present worth to you, given your money and financial situation, we must use your 5% interest rate to value these cash flows.

### METHODOLOGY

For each option, we will calculate the net equivalent cost (present worth) at  $n = 0$ . Since the loan payments occur monthly, we need to determine the effective interest rate per month, which is  $5\%/12$ .

**Given:** The loan payment series shown in Figure 3.12,  $r = 5\%$  per year, payment period = monthly, and compounding period = monthly.

**Find:** The most economical financing option.

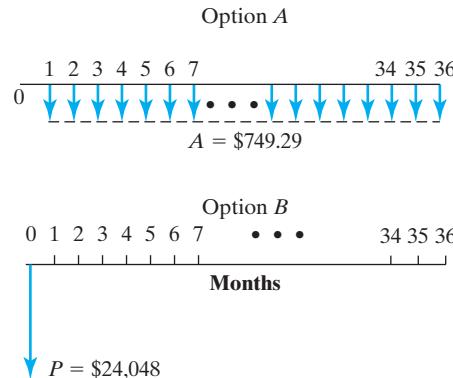


Figure 3.12 Cash flow diagram.

### SOLUTION

- Option A (conventional financing): The equivalent present cost of the total loan repayments is calculated as

$$\begin{aligned} P_A &= \$749.29(P/A, 5\% / 12, 36) \\ &= \$25,000. \end{aligned}$$

- Option B (cash payment): Since the cash payment is a lump sum to be paid presently, its equivalent present cost is equal to its value:

$$P_B = \$24,048.$$

Thus, there would be \$952 of savings in present value with the cash payment option.

### EXAMPLE 3.11 Buying versus Leasing a Car

Two types of financing options are offered for an automobile by a local dealer, as shown in the following table. The calculations are based on special financing programs available at participating dealers for a limited time. For each option, you pay the same amount for license, title, registration fees, taxes, and insurance, so they are irrelevant in our comparison. For the lease option, the lessee must come up with \$731.45 at signing. This cash due at signing includes the first month's lease payment

of \$236.45 and a \$495 administrative fee. No security deposit is required. However, a \$300 disposition fee is due at lease end. The lessee has the option to purchase the vehicle at lease end for \$8,673.10. The lessee is also responsible for excessive wear and use. If your earning interest rate is 6% compounded monthly, which financing option is a better choice?

### Buying versus Leasing

	Option 1 Debt Financing	Option 2 Lease Financing
Price	\$14,695	\$14,695
Down payment	\$2,000	\$0
APR (%)	3.6%	
Monthly payment	\$372.55	\$236.45
Length	36 months	36 months
Fees		\$495
Cash due at lease end		\$300
Purchase option at lease end		\$8,673.10
Cash due at signing	\$2,000	\$731.45

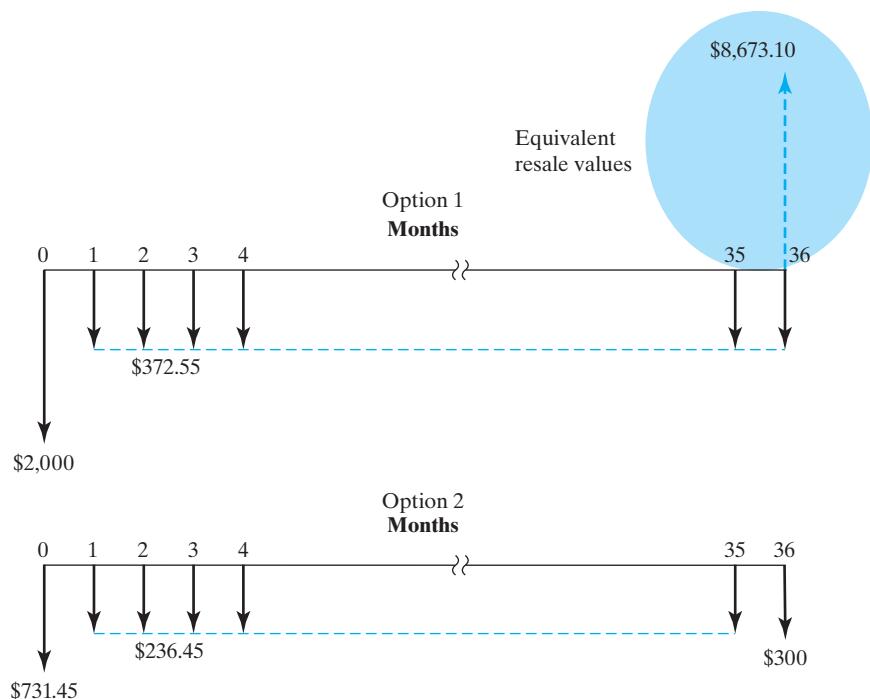
### DISSECTING THE PROBLEM

With a lease payment, you pay for the portion of the vehicle you expect to use. At the end of the lease, you simply return the vehicle to the dealer and pay the agreed-upon disposal fee. With traditional financing, your monthly payment is based on the entire \$14,695 value of the vehicle, and you will own the vehicle at the end of your financing terms. Since you are comparing the options over three years, you must explicitly consider the unused portion (resale value) of the vehicle at the end of the term. For comparison purposes, you must consider the resale value of the vehicle in order to figure out the net cost of owning the vehicle. You could use the \$8,673.10 quoted by the dealer in the lease option as the resale value. Then you have to ask yourself if you can get that kind of resale value after three years of ownership. (See Figure 3.13.)

**Given:** The lease payment series shown in Figure 3.13,  $r = 6\%$  per year, payment period = monthly, and compounding period = monthly.

**Find:** The most economical financing option, assuming that you will be able to sell the vehicle for \$8,673.10 at the end of three years.

For each option, we will calculate the net equivalent total cost at  $n = 0$ . Since the loan payments occur monthly, we need to determine the effective interest rate per month, which is 0.5%.



**Figure 3.13** Cash flow diagrams for buying and leasing the car.

## METHODOLOGY

*Method 1: Conventional Financing*

## SOLUTION

The equivalent present cost of the total loan payments is calculated as

$$\begin{aligned} P_1 &= \$2,000 + \$372.55 (P/A, 0.5\%, 36) \\ &= \$14,246.10. \end{aligned}$$

The equivalent present worth of the resale value is calculated as

$$P_2 = \$8,673.10(P/F, 0.5\%, 36) = \$7,247.63.$$

The equivalent present net financing cost is therefore

$$\begin{aligned} P &= P_1 - P_2 = \$14,246.10 - \$7,247.63 \\ &= \$6,998.47. \end{aligned}$$

*Method 2: Lease Financing*

The equivalent present cost of the total lease payments is calculated as

$$\begin{aligned} P_1 &= \$731.45 + \$236.45 (P/A, 0.5\%, 35) \\ &= \$731.45 + \$7,574.76 \\ &= \$8,306.21. \end{aligned}$$

The equivalent present cost of the disposition fee is calculated as

$$P_2 = \$300(P/F, 0.5\%, 36) = \$250.69.$$

The equivalent present net lease cost is therefore

$$\begin{aligned} P &= P_1 + P_2 = \$8,306.21 + \$250.69 \\ &= \$8,556.90. \end{aligned}$$

It appears that the traditional financing program to purchase the car is more economical at 6% interest compounded monthly.

**COMMENTS:** By varying the resale value  $S$ , we can find the break-even resale value that makes traditional financing equivalent to lease financing for this case:

$$\$8,556.90 = \$14,246.10 - S(P/F, 0.5\%, 36).$$

Thus, the break-even resale value is

$$\begin{aligned} S &= (\$14,246.10 - \$8,556.90)/0.8356 \\ &= \$6,808.14. \end{aligned}$$

So, at a resale value greater than \$6,808.14, the conventional financing plan would be the more economical choice. Table 3.8 illustrates how we may create an Excel worksheet to determine the break-even resale value by using the **Goal Seek** function. To perform this function, we calculate the equivalent debt financing cost at cell B18 and the equivalent leasing financing cost at cell D20. Next, we enter the differential amount in cell B22. Then we are looking for a resale value (by changing cell B17) to make this differential cost (set cell B22) to be zero (To value). When you click OK, Excel will find the break-even resale value at \$6,808.14.

## SUMMARY

■ Interest is most frequently quoted by financial institutions as an **APR**. However, compounding often occurs more frequently. Simply multiplying the APR by the amount of debt does not account for the effect of this more frequent compounding. This situation leads to the distinction between nominal and effective interest.

■ **Nominal interest** is a stated rate of interest for a given period (usually, a year).

■ **Effective interest** is the actual rate of interest, which accounts for the interest amount accumulated over a given period. The **effective rate** is related to the APR by

$$i = (1 + r/M)^M - 1,$$

where  $r$  = the APR,  $M$  = the number of compounding periods, and  $i$  = the effective interest rate.

**TABLE 3.8** An Excel Worksheet to Determine the Break-Even Resale Value

	A	B	C	D	E	F	G	H
1								
2		Option 1		Option 2			Option 1	Option 2
3		Debt Financing		Lease Financing			Debt Financing	Lease Financing
4								
5	Price	\$14,695.00		\$14,695.00	0	\$2,000.00	\$731.45	
6	Down payment	\$2,000.00			1	\$372.55	\$236.45	
7	APR(%)	3.60%			2	\$372.55	\$236.45	
8	Length (months)	36		36	3	\$372.55	\$236.45	
9	Monthly payment	\$372.55		\$236.45	4	\$372.55	\$236.45	
10	Fees			\$495.00	5	\$372.55	\$236.45	
11	Cash Due at lease end			\$300.00	6	\$372.55	\$236.45	
12	Purchase option at lease end			\$8,673.10	7	\$372.55	\$236.45	
13	Cash due at signing	\$2,000.00		\$731.45	8	\$372.55	\$236.45	
14					9	\$372.55	\$236.45	
15					10	\$372.55	\$236.45	
16	Interest rate per month	0.50%			11	\$372.55	\$236.45	
17	Estimated resale value	\$6,808.14			12	\$372.55	\$236.45	
18	Equivalent debt financing cost	\$8,556.91			13	\$372.55	\$236.45	
19					14	\$372.55	\$236.45	
20	Equivalent lease financing cost			\$8,556.91	15	\$372.55	\$236.45	
21					16	\$372.55	\$236.45	
22	Differential cost	\$0.00			17	\$372.55	\$236.45	
23					18	\$372.55	\$236.45	
24					19	\$372.55	\$236.45	
25	=NPV(0.5%,G6:G41)+G5				20	\$372.55	\$236.45	
26					21	\$372.55	\$236.45	
27					22	\$372.55	\$236.45	
28					23	\$372.55	\$236.45	
29					24	\$372.55	\$236.45	
30					25	\$372.55	\$236.45	
31					26	\$372.55	\$236.45	
32					27	\$372.55	\$236.45	
33					28	\$372.55	\$236.45	
34					29	\$372.55	\$236.45	
35					30	\$372.55	\$236.45	
36					31	\$372.55	\$236.45	
37					32	\$372.55	\$236.45	
38					33	\$372.55	\$236.45	
39					34	\$372.55	\$236.45	
40					35	\$372.55	\$236.45	
41					36	(\$6,435.59)	\$300.00	
42								

- In any equivalence problem, the interest rate to use is the effective interest rate per payment period, which is expressed as

$$i = [1 + r/CK]^C - 1,$$

where  $C$  = the number of interest periods per payment period,  $K$  = the number of payment periods per year, and  $r/K$  = the nominal interest rate per payment period.

- The equation for determining the effective interest of continuous compounding is as follows:

$$i = e^{r/K} - 1.$$

- The difference in accumulated interest between continuous compounding and daily compounding is relatively small.

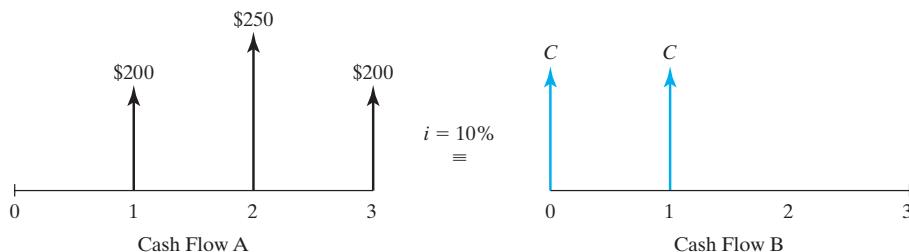
- Whenever payment and compounding periods differ from each other, it is recommended to compute the effective interest rate per payment period. The reason is that, to proceed with equivalency analysis, the compounding and payment periods must be the same.
- The cost of a loan will depend on many factors, such as loan amount, loan term, payment frequency, fees, and interest rate.
- In comparing different financing options, the interest rate we use is the one that reflects the decision maker's time value of money, not the interest rate quoted by the financial institution(s) lending the money.

## SELF-TEST QUESTIONS

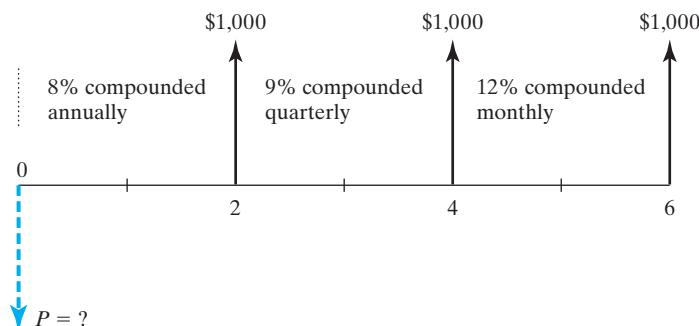
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- 3s.1 You make \$500 monthly deposits into a fund that pays interest at a rate of 9% compounded *monthly*. What would be the balance at the end of 20 years?
- \$163,879
  - \$258,169
  - \$327,200
  - \$333,943
- 3s.2 Two banks offer the following interest rates on your deposit:
- Bank A: 8% interest compounded *quarterly*
  - Bank B: 7.9% interest compounded *continuously*
- Which of the following statements is *not* true?
- The annual percentage yield (APY) for Bank A is 8.24%.
  - The effective annual interest rate for Bank B is 8.22%.
  - Bank B offers a better deal as your money earns interest continuously.
  - The annual percentage rate (APR) for Bank B is 7.9%.
- 3s.3 You are making semiannual deposits into a fund that pays interest at a rate of 6% compounded *continuously*. What is the effective *semiannual* interest rate?
- 3.045%
  - 3.681%
  - 4.081%
  - 4.128%
- 3s.4 Calculate the future worth of 18 annual \$5,000 deposits in a savings account that earns 8% compounded monthly. Assume that all deposits are made at the *beginning* of each year.
- \$196,010
  - \$199,884
  - \$208,811
  - \$282,693
- 3s.5 You borrow \$20,000 from a bank to be repaid in monthly installments for three years at 9% interest compounded monthly. What is the portion of interest payment for the 18th payment?
- \$150.00
  - \$88.28

- (c) \$80.04  
 (d) \$84.17
- 3s.6 You borrowed \$10,000 from a bank at an interest rate of 9%, compounded *monthly*. This loan will be repaid in 48 equal monthly installments over four years. Immediately after your 25th payment, if you want to pay off the remainder of the loan in a single payment, the amount is close to  
 (a) \$5,723  
 (b) \$5,447  
 (c) \$5,239  
 (d) \$5,029
- 3s.7 You borrowed \$100,000, agreeing to pay the balance in 10 equal annual installments at 8% annual interest. Determine the remaining loan balance right after the fifth payment.  
 (a) \$74,515  
 (b) \$68,894  
 (c) \$59,503  
 (d) \$49,360
- 3s.8 Consider the following two cash flow transactions. If they are economically equivalent at 10% interest, find the value of  $C$ .



- (a)  $C = \$325$   
 (b)  $C = \$282$   
 (c)  $C = \$310$   
 (d)  $C = \$277$
- 3s.9 Compute the present worth ( $P$ ) for the cash flows with the different periodic interest rates specified. The cash flows occur at the end of each year over six years.  
 (a)  $P = \$2,140$   
 (b)  $P = \$2,154$   
 (c)  $P = \$2,234$   
 (d)  $P = \$2,249$



- 3s.10 Suppose you purchased a corporate bond with a 10-year maturity, a \$1,000 par value, a 9% coupon rate (\$45 interest payment every six months), and semiannual interest payments. Five years after the bonds were purchased, the going rate of interest on new bonds fell to 6% (or 6% compounded semiannually). What is the current market value ( $P$ ) of the bond (five years after the purchase)?
- $P = \$890$
  - $P = \$1,223$
  - $P = \$1,090$
  - $P = \$1,128$
- 3s.11 Your company borrowed \$150,000, agreeing to pay the balance in 24 equal monthly installments at 9% compounded monthly. Determine the total interest payment during the first 12 months.
- \$2,346
  - \$10,592
  - \$13,500
  - \$6,343
- 3s.12 You are considering either buying or leasing a vehicle. The following data have been compiled:

	Buying	Leasing
Price of vehicle	\$22,000	\$22,000
Down payment required at year 0	\$2,000	0
Value of vehicle at the end of year 3 (unknown)	$S$	
36 Monthly payments	\$608 ( <i>end of each month</i> )	\$420 ( <i>beginning of each month</i> )
Documentation fee (one time, nonrecurring expense, not refundable)		\$400 ( <i>payable at the beginning of lease</i> )

If your interest rate is 6% compounded monthly, at what value of the vehicle at the end of three years ( $S$ ) would make the both options economically equivalent?

- \$7,711
- \$8,980
- \$9,228
- \$9,310

- 3s.13 To finance your car, you have decided to take a car loan in the amount of \$15,000 from your credit union. You will pay off this loan over 60 months. If the required monthly payment is \$322.41, what is the effective annual interest rate on this car loan?
- (a) 10.54%
  - (b) 11.02%
  - (c) 11.58%
  - (d) 11.64%
- 3s.14 You obtained a loan of \$20,000 to finance your home improvement project. Based on monthly compounding over 24 months, the end-of-the-month equal payment was figured to be \$922.90. What is the APR used for this loan?
- (a) 9.5%
  - (b) 9.8%
  - (c) 10%
  - (d) 10.4%
- 3s.15 What is the effective interest rate per quarter if the interest rate is 8% compounded continuously?
- (a) 1.035%
  - (b) 1.235%
  - (c) 1.511%
  - (d) 2.02%
- 3s.16 You are in financial trouble and are delinquent on your mortgage payment. Your bank has agreed to a repayment schedule of \$1,600 per month, and it will charge 0.5% per month interest on the outstanding balance. If the current outstanding balance is \$250,000, how long will it take for you to pay off the loan?
- (a) 22.45 years
  - (b) 23.33 years
  - (c) 24.85 years
  - (d) 25.39 years
- 3s.17 Today is your birthday and you decide to start saving for your retirement. You will retire on your 67th birthday and need \$50,000 per year at the end of each of following 20 years. You will make a first deposit one year from today in an account paying 8% interest *annually* and continue to make an equal amount of deposit each year up to the year on your 67th birthday. If an annual deposit of \$6,715 will allow you to reach your goal, what birthday are you celebrating today?
- (a) 37
  - (b) 40
  - (c) 42
  - (d) 48

## PROBLEMS

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### Market Interest Rates (Nominal versus Effective Interest Rates)

- 3.1 Suppose you have received a credit card offer from a bank that charges interest at 1.5% per month compounded monthly. What is the nominal interest (annual percentage) rate for this credit card? What is the effective annual interest rate?

- 3.2 If your credit card calculates interest based on 19.25% APR, compounded monthly:
- What are your monthly interest rate and annual effective interest rate?
  - If your current outstanding balance is \$3,400 and you skip payments for four months, what would be the total balance four months from now?
- 3.3 A discount interest loan is a loan arrangement where the interest and any other related charges are calculated at the time the loan is closed. Suppose a one-year loan is stated as \$12,000 and the interest rate is 12%. The borrower then pays \$1,440 interest up front, thereby receiving net funds of \$10,560 and repaying \$12,000 in a year. What is the effective interest rate on this one-year loan?
- 3.4 A local bank advertised the following information: Interest 6.89%—effective annual yield 7.128%. No mention was made of the interest period in the advertisement. Can you figure out the compounding scheme used by the bank?
- 3.5 What is the effective annual yield of 7.8% compounded continuously?
- 3.6 A financial institution is willing to lend you \$400. However, \$480 is to be repaid at the end of one week.
- What is the nominal interest rate?
  - What is the effective annual interest rate?
- 3.7 A loan of \$25,000 is to be financed to assist a person's college education. Based upon monthly compounding for 60 months, the end-of-the-month equal payment is quoted as \$600. What nominal interest rate is being charged?
- 3.8 You are purchasing a \$25,000 used automobile, which is to be paid for in 48 monthly installments of \$712.52. What nominal interest rate are you paying on this financing arrangement?
- 3.9 You have three choices in placing your money in a bank account.
- Bank A** pays 6.24% compounded annually.
  - Bank B** pays 6.00% compounded quarterly.
  - Bank C** pays 6.10% compounded continuously.

Which bank would you open an account with?

### Calculating an Effective Interest Rate Based on a Payment Period

- 3.10 Find the effective interest rate per payment period for an interest rate of 9% compounded monthly for each of the given payment schedules:
- Monthly
  - Quarterly
  - Semiannually
  - Annually
- 3.11 What is the effective interest rate per quarter if the interest rate is 9% compounded monthly?
- 3.12 What is the effective interest rate per month if the interest rate is 12% compounded continuously?
- 3.13 James Hogan is purchasing a \$32,000 automobile, which is to be paid for in 60 monthly installments of \$575. What is the effective interest rate per month for this financing arrangement?

3.14 Find the APY in each of the following cases:

- (a) 14% compounded annually
- (b) 7% compounded semiannually
- (c) 8.4% compounded quarterly
- (d) 9.5% compounded daily

### Equivalence Calculations Using Effective Interest Rates

3.15 What will be the amount accumulated by each of these present investments?

- (a) \$9,545 in 12 years at 8.2% compounded semiannually.
- (b) \$6,500 in 10 years at 6% compounded quarterly.
- (c) \$42,800 in 8 years at 9% compounded monthly.

3.16 What is the future worth of each of the given series of payments?

- (a) \$4,000 at the end of each six-month period for 20 years at 4% compounded semi-annually.
- (b) \$6,000 at the end of each quarter for seven years at 10% compounded quarterly.
- (c) \$2,200 at the end of each month for 12 years at 6% compounded monthly.

3.17 What equal series of payments must be paid into a sinking fund in order to accumulate each given amount?

- (a) \$16,000 in eight years at 7% compounded semiannually when payments are semiannual.
- (b) \$12,000 in five years at 8% compounded quarterly when payments are quarterly.
- (c) \$35,000 in 10 years at 7.2% compounded monthly when payments are monthly.

3.18 A series of equal quarterly payments of \$10,000 for 15 years is equivalent to what future worth amount at an interest rate of 9% compounded at the given intervals?

- (a) Quarterly
- (b) Monthly

3.19 What is the amount of the quarterly deposits  $A$  such that you will be able to withdraw the following amounts?

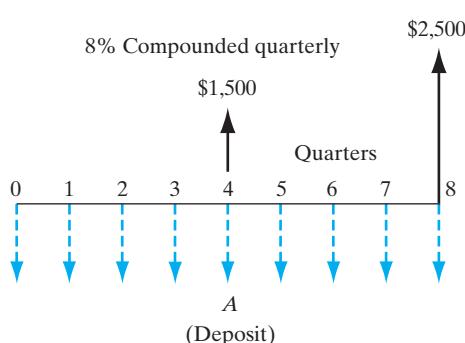
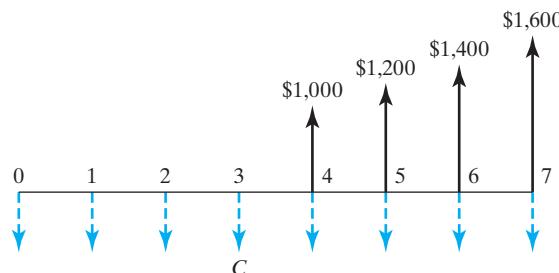


Figure P3.19

3.20 A series of equal end-of-quarter deposits of \$1,000 extends over a period of three years. It is desired to compute the future worth of this quarterly deposit series at 12% compounded monthly. Which of the following equations is correct?

- (a)  $F = 4(\$1,000)(F/A, 12\%, 3)$   
 (b)  $F = \$1,000(F/A, 3\%, 12)$   
 (c)  $F = \$1,000(F/A, 1\%, 12)$   
 (d)  $F = \$1,000(F/A, 3.03\%, 12)$
- 3.21 Suppose you deposit \$500 at the end of each quarter for five years at an interest rate of 8% compounded monthly. What equal end-of-year deposit over the five years would accumulate the same amount at the end of the five years under the same interest compounding? To answer the question, which of the following is correct?
- (a)  $A = [\$500(F/A, 2\%, 20)] \times (A/F, 8\%, 5)$   
 (b)  $A = \$500(F/A, 2.013\%, 4)$   
 (c)  $A = \$500\left(F/A, \frac{8\%}{12}, 20\right) \times (A/F, 8\%, 5)$   
 (d) None of the above.
- 3.22 Suppose a young newlywed couple is planning to buy a home three and half years from now. To save the down payment required at the time of purchasing a home worth \$420,000 (let's assume that the down payment is 20% of the sale price, which is \$84,000), the couple decides to set aside some money from each of their salaries at the end of every month. If each of them can earn 7.2% interest (compounded monthly) on his or her savings, determine the equal amount this couple must deposit each month until the point is reached where the couple can buy the home.
- 3.23 Joe Franklin deposits \$22,000 in a savings account that pays 6% interest compounded monthly. Three years later, he deposits \$16,000. Two years after the \$16,000 deposit, he makes another deposit in the amount of \$13,500. Four years after the \$13,500 deposit, half of the accumulated funds are transferred to a fund that pays 8% interest compounded quarterly. How much money will be in each account six years after the transfer?
- 3.24 A man is planning to retire in 25 years. He wishes to deposit a regular amount every three months until he retires, so that, beginning one year following his retirement, he will receive annual payments of \$80,000 for the next 15 years. How much must he deposit if the interest rate is 6% compounded quarterly?
- 3.25 Consider the following cash flow series. Determine the required annual deposits (end of year) that will generate the cash flows from years 4 to 7. Assume the interest rate is 6%, compounded monthly.



- 3.26 A building is priced at \$250,000. If a down payment of \$70,000 is made and a payment of \$5,000 every month thereafter is required, how many months will it take to pay for the building? Interest is charged at a rate of 6% compounded monthly.
- 3.27 A married couple is trying to finance their three-year-old son's college education. Money can be deposited at 6% compounded quarterly. What end-of-quarter deposit must be made from the son's 3rd birthday to his 18th birthday to provide \$60,000 on each birthday from the 18th to the 21st? (Note that the first deposit comes three months after his 3rd birthday and the last deposit is made on the date of the first withdrawal.)
- 3.28 Cory Manciagli is planning to retire in 20 years. Money can be deposited at 6% compounded quarterly. What quarterly deposit must be made at the end of each quarter until Cory retires so that he can make a withdrawal of \$40,000 semiannually over the first 10 years of his retirement? Assume that his first withdrawal occurs at the end of six months after his retirement.
- 3.29 You borrowed \$30,000 to buy a new car from a bank at an interest rate of 9% compounded monthly. This loan will be repaid in 84 equal monthly installments over seven years. Immediately after the 30th payment, you desire to pay the remainder of the loan in a single payment. Compute this lump-sum amount.
- 3.30 Patricia French received \$800,000 from an insurance company after her husband's death. Patricia wants to deposit this amount in a savings account that earns interest at a rate of 6% compounded monthly. Then she would like to make 144 equal monthly withdrawals over the 12-year period such that, when she makes the last withdrawal, the savings account will have a balance of zero. How much can she withdraw each month?
- 3.31 Jenny Walters, who owns a real estate agency, bought an old house to use as her business office. She found that the ceiling was poorly insulated and that the heat loss could be cut significantly if six inches of foam insulation were installed. She estimated that with the insulation, she could cut the heating bill by \$80 per month and the air-conditioning cost by \$70 per month. Assuming that the summer season is three months (June, July, and August) of the year and that the winter season is another three months (December, January, and February) of the year, how much can Jenny spend on insulation if she expects to keep the property for five years? Assume that neither heating nor air-conditioning would be required during the fall and spring seasons. If she decides to install the insulation, it will be done at the beginning of May. Jenny's interest rate is 6% compounded monthly.
- 3.32 You want to open a savings plan for your future retirement. You are considering the following two options:
- Option 1: You deposit \$1,000 at the end of each quarter for the first 10 years. At the end of 10 years, you make no further deposits, but you leave the amount accumulated at the end of 10 years for the next 15 years.
  - Option 2: You do nothing for the first 10 years. Then you deposit \$6,000 at the end of each year for the next 15 years.
- If your deposits or investments earn an interest rate of 6% compounded quarterly and you choose Option 2 over Option 1, then at the end of 25 years from now, you will have accumulated
- (a) \$7,067 more.
  - (b) \$8,523 more.

- (c) \$14,757 less.  
 (d) \$13,302 less.
- 3.33 Maria Anguiano's current salary is \$65,000 per year, and she is planning to retire 25 years from now. She anticipates that her annual salary will increase by \$3,000 each year. (That is, in the first year she will earn \$65,000, in the second year \$68,000, in the third year \$71,000, and so forth.) She plans to deposit 5% of her yearly salary into a retirement fund that earns 7% interest compounded daily. What will be the amount accumulated at the time of her retirement?

### Equivalence Calculations with Continuous Compounding

- 3.34 How many years will it take an investment to triple if the interest rate is 9% compounded at the given intervals?  
 (a) Quarterly  
 (b) Monthly  
 (c) Continuously
- 3.35 A series of equal quarterly payments of \$10,000 for 15 years is equivalent to what future worth amount at an interest rate of 6% compounded at the given intervals?  
 (a) Quarterly  
 (b) Monthly  
 (c) Continuously
- 3.36 How much money will you have in 10 years if you deposit \$8,000 in the bank at 7.5% interest compounded continuously?
- 3.37 Suppose that \$4,000 is placed in a bank account at the end of each quarter over the next 10 years. What is the future worth at the end of 10 years when the interest rate is 9% compounded at the given intervals?  
 (a) Quarterly  
 (b) Monthly  
 (c) Continuously
- 3.38 If the interest rate is 7.2% compounded continuously, what is the required quarterly payment to repay a loan of \$16,000 in four years?
- 3.39 What is the future worth of a series of equal end-of-month payments of \$1,500 if the series extends over a period of eight years at 9% interest compounded at the given intervals?  
 (a) Quarterly  
 (b) Monthly  
 (c) Continuously
- 3.40 What will be the required monthly payment to repay a loan of \$65,000 in eight years if the interest rate is 11.25% compounded continuously?
- 3.41 A series of equal quarterly payments of \$3,800 extends over a period of 10 years. What is the present worth of this quarterly payment series at 7.8% interest compounded continuously?
- 3.42 A series of equal quarterly payments of \$6,400 for 12 years is equivalent to what future lump-sum amount at the end of 16 years at an interest rate of 8.8% compounded continuously?

### Borrowing with Credit Cards

- 3.43 Your bank calculates the interest based on 18% APR on your credit card balance (monthly compounding). Suppose that your current outstanding balance is \$1,800 and you skip payments for three months. What would be the total balance two months from now?
- 3.44 You have just received credit card applications from two banks, *A* and *B*. The interest terms on your unpaid balance are stated as follows:
1. Bank *A*: 18% compounded quarterly.
  2. Bank *B*: 17.5% compounded daily.
- Which of the following statements is *incorrect*?
- (a) The effective annual interest rate for Bank *A* is 19.25%.
  - (b) The nominal annual interest rate for Bank *B* is 17.5%.
  - (c) Bank *B*'s term is a better deal because you will pay less interest on your unpaid balance.
  - (d) Bank *A*'s term is a better deal because you will pay less interest on your unpaid balance.
- 3.45 You received a credit card application from Sun Bank offering an introductory rate of 2.9% per year compounded monthly for the first six months, increasing thereafter to 17% compounded monthly. This offer is good as long as you transfer your current debt from your existing card. Assuming that you will transfer \$3,000 balance and that you will continue to make \$100 monthly payment (without making any subsequent purchases), what would be the credit card balance at the end of the first year?
- 3.46 Jennifer Lee, an engineering major in her junior year, has received in the mail two guaranteed line-of-credit applications from two different banks. Each bank offers a different annual fee and finance charge. Jennifer expects her average monthly balance after payment to be \$300 and plans to keep the card she chooses for only 24 months. (After graduation, she will apply for a new card.) Jennifer's interest rate (on her savings account) is 6% compounded daily.

Terms	Bank A	Bank B
Annual fee	\$20	Free
Finance charge	1.55% monthly interest rate	19.5% annual percentage rate

- (a) Compute the effective annual interest rate for each card.
- (b) Which bank's credit card should Jennifer choose?

### Commercial Loans

- 3.47 Suppose you take out a car loan of \$10,000 with an interest rate of 12% compounded monthly. You will pay off the loan over 48 months with equal monthly payments.
- (a) What is the monthly interest rate?
  - (b) What is the amount of the equal monthly payment?
  - (c) What is the interest payment for the 20th payment?
  - (d) What is the total interest paid over the life of the loan?

- 3.48 An automobile loan of \$20,000 at a nominal rate of 6% compounded monthly for 36 months requires equal end-of-month payments of \$608.44. Complete Table P3.48 for the first six payments, as you would expect a bank to calculate the values.

**TABLE P3.48**

End of Month ( <i>n</i> )	Interest Payment	Repayment of Principal	Remaining Loan Balance
1			\$19,491.56
2		\$510.98	
13	\$68.64		
24			\$7,069.38
36			\$0

- 3.49 You borrow \$120,000 with a 30-year term at a 9% (APR) variable rate and the interest rate can be changed every five years.
- What is the initial monthly payment?
  - If the lender's interest rate is 9.75% (APR) at the end of five years, what will the new monthly payments be?
- 3.50 Mr. Smith wants to buy a new car that will cost \$18,000. He will make a down payment in the amount of \$8,000. He would like to borrow the remainder from a bank at an interest rate of 9% compounded monthly. He agrees to pay off the loan monthly for a period of two years. Select the correct answer for the following questions:
- What is the amount of the monthly payment *A*?
    - $A = \$10,000(A/P, 0.75\%, 24)$
    - $A = \$10,000(A/P, 9\%, 2)/12$
    - $A = \$10,000(A/F, 0.75\%, 24)$
    - $A = \$12,500(A/F, 9\%, 2)/12$
  - Mr. Smith has made 12 payments and wants to figure out the balance remaining immediately after the 12th payment. What is that balance?
    - $B_{12} = 12A$
    - $B_{12} = A(P/A, 9\%, 1)/12$
    - $B_{12} = A(P/A, 0.75\%, 12)$
    - $B_{12} = 10,000 - 12A$
- 3.51 Frederic Polanski is considering purchasing a used automobile. The price including the title and taxes is \$16,540. Frederic is able to make a \$2,040 down payment. The balance of \$14,500 will be borrowed from her credit union at an interest rate of 8.25% compounded daily. The loan should be paid in 48 equal monthly payments. Compute the monthly payment. What is the total amount of interest Frederic has to pay over the life of the loan?
- 3.52 Clay Harden borrowed \$36,000 from a bank at an interest rate of 6% compounded monthly. The loan will be repaid in 30 equal monthly installments over two and half years. Immediately after his 24th payment, Clay desires to pay the remainder of the loan in a single payment. Compute the total amount he must pay.

- 3.53 You plan to buy a \$250,000 home with a 20% down payment. The bank you want to finance the loan through suggests two options: a 15-year mortgage at 4.25% APR and a 30-year mortgage at 5% APR. What is the difference in monthly payments between these two options?
- 3.54 On a \$400,000 home mortgage loan with a 15-year term at 9% APR compounded monthly, compute the total payments on principal and interest over the first five years of ownership.
- 3.55 A lender requires that monthly mortgage payments be no more than 25% of gross monthly income with a maximum term of 30 years. If you can make only a 15% down payment, what is the minimum monthly income needed to purchase a \$300,000 house when the interest rate is 6% compounded monthly?
- 3.56 To buy a \$150,000 house, you take out a 9% (APR) mortgage for \$120,000. Five years later, you sell the house for \$185,000 (after all other selling expenses). What equity (the amount that you can keep before tax) would you realize with a 30-year repayment term?
- 3.57 Consider the following three individuals. Just after their 19th payment:
- Robert Dixon had a balance of \$80,000 on a 9%, 15-year mortgage;
  - Wanda Harper had a balance of \$80,000 on a 9%, 20-year mortgage; and
  - Tony Zang had a balance of \$80,000 on a 9%, 30-year mortgage.
- How much interest did each individual pay on the 20th payment?
- 3.58 Home mortgage lenders usually charge points on a loan to avoid exceeding a legal limit on interest rates or to be competitive with other lenders. As an example, for a two-point loan, the lender would lend only \$98 for each \$100 borrowed. The borrower would receive only \$98, but would have to make payments just as if he or she had received \$100. Suppose that you receive a loan of \$130,000, payable at the end of each month for 30 years with an interest rate of 9% compounded monthly, but you have been charged three points. What is the effective borrowing rate on this home mortgage loan?
- 3.59 You are considering purchasing a lot adjacent to your laundry business to provide adequate parking space for your customers. You need to borrow \$50,000 to secure the lot. You have made a deal with a local bank to pay the loan back over a five-year period with the following payment terms: 15%, 20%, 25%, 30%, and 35% of the initial loan at the end of first, second, third, fourth, and fifth years, respectively.
- (a) What rate of interest is the bank earning from this loan?
  - (b) What would be the total interest paid over the five-year period?
- 3.60 Kathy Stonewall bought a new car for \$15,458. A dealer's financing was available through a local bank at an interest rate of 11.5% compounded monthly. Dealer financing required a 10% down payment and 60 equal monthly payments. Because the interest rate was rather high, Kathy checked her credit union for possible financing. The loan officer at the credit union quoted a 9.8% interest rate for a new-car loan and 10.5% for a used car. But to be eligible for the loan, Kathy has to be a member of the union for at least six months. Since she joined the union two months ago, she has to wait four more months to apply for the loan. Consequently, she decided to go ahead with the dealer's financing, and four months later she refinanced the balance through the credit union at an interest rate of 10.5%.

- (a) Compute the monthly payment to the dealer.  
 (b) Compute the monthly payment to the union.  
 (c) What is the total interest payment on each loan?
- 3.61 Antonio Menifee borrowed money from a bank to finance a small fishing boat. The bank's terms allowed him to defer payments (including interest) on the loan for six months and to make 36 equal end-of-month payments thereafter. The original bank note was for \$16,000 with an interest rate of 9% compounded monthly. After 16 monthly payments, Antonio found himself in a financial bind and went to a loan company for assistance in lowering his monthly payments. Fortunately, the loan company offered to pay his debts in one lump sum if he would pay the company \$308.29 per month for the next 36 months. What monthly rate of interest is the loan company charging on this transaction?
- 3.62 A loan of \$18,000 is to be financed over a period of 24 months. The agency quotes a nominal interest rate of 8% for the first 12 months and a nominal interest rate of 9% for any remaining unpaid balance after 12 months with both rates compounded monthly. At these rates, what equal end-of-the-month payment for 24 months would be required in order to repay the loan?
- 3.63 Emily Wang financed her office furniture from a furniture dealer. The dealer's terms allowed her to defer payments (including interest) for six months and to make 36 equal end-of-month payments thereafter. The original note was for \$15,000, with interest at 9% compounded monthly. After 26 monthly payments, Emily found herself in a financial bind and went to a loan company for assistance. The loan company offered to pay her debts in one lump sum if she would pay the company \$186 per month for the next 30 months.
- (a) Determine the original monthly payment made to the furniture store.  
 (b) Determine the lump-sum payoff amount the loan company will make.  
 (c) What monthly rate of interest is the loan company charging on this loan?

### Comparing Different Financing Options

- 3.64 Suppose you are in the market for a new car worth \$22,000. You are offered a deal to make a \$2,000 down payment now and to pay the balance in equal end-of-month payments of \$505.33 over a 48-month period. Consider the following situations.
- (a) Instead of going through the dealer's financing, you want to make a down payment of \$1,800 and take out an auto loan from a bank at 9.2% compounded monthly. What would be your monthly payment to pay off the loan in four years?  
 (b) If you were to accept the dealer's offer, what would be the effective rate of interest per month the dealer charges on your financing?
- 3.65 A local dealer is advertising a 24-month lease of a sport utility vehicle for \$520 payable at the beginning of each month. The lease requires a \$2,500 down payment plus a \$500 refundable security deposit. As an alternative, the company offers a 24-month lease with a single up-front payment of \$12,780 plus a \$500 refundable security deposit. The security deposit will be refunded at the end of the 24-month lease. Assuming you have access to a deposit account that pays an interest rate of 6% compounded monthly, which lease is more favorable?

3.66 A house can be purchased for \$155,000, and you have \$25,000 cash for a down payment. You are considering the following two financing options:

- **Option 1.** Getting a new standard mortgage with a 7.5% (APR) interest and a 30-year term.
- **Option 2.** Assuming the seller's old mortgage, which has an interest rate of 5.5% (APR), a remaining term of 25 years (the original term was 30 years), a remaining balance of \$97,218, and payments of \$597 per month. You can obtain a second mortgage for the remaining balance (\$32,782) from your credit union at 9% (APR) with a 10-year repayment period.
  - (a) What is the effective interest rate of the combined mortgage?
  - (b) Compute the monthly payments for each option over the life of the mortgage.
  - (c) Compute the total interest payment for each option.
  - (d) What homeowner's interest rate makes the two financing options equivalent?

### Short Case Studies with Excel

3.67 Kevin Moore received his monthly credit card statement from his bank. His current outstanding balance is \$3,168.97. The minimum monthly payment due is 1% of the outstanding balance or \$20, whichever is higher. The bank uses the Average Daily Balance method to calculate the periodic interest charges. Show how the bank calculated the total estimated payments for the two scenarios: (a) the minimum payment only option and (b) \$111 per month.

Payment Information		
New Balance .....		\$3,168.97
Minimum Payment Due (Current Month)		\$32.00
Minimum Payment Due (Past Due)		\$0.00
<b>Total New Minimum Payment Due</b>		<b>\$32.00</b>
Payment Due Date .....		Jun 22, 2017
<b>Late Payment Warning:</b> If we do not receive your minimum payment by the date listed above, you may have to pay up to a \$35.00 Late Fee.		
<b>Minimum Payment Warning:</b> If you make only the minimum payment each period, you will pay more in interest and it will take you longer to pay off your balance. For example:		
If you make no additional charges using this card and each month you pay...	You will pay off the balance shown on this statement in about...	And you will end up paying an estimated total of...
Only the minimum payment	9 years	\$5,408
\$111	3 years	\$4,010 ( <i>Savings = \$1,398</i> )

- 3.68 Suppose you purchased a corporate bond with a 10-year maturity, a \$1,000 par value, a 10% coupon rate, and semiannual interest payments. What all this means that you receive \$50 interest payment at the end of each six-month period for 10 years (20 times). Then, when the bond matures, you will receive the principal amount (the face value) in a lump sum. Three years after the bonds were purchased, the going rate of interest (coupon rate) on new bonds fell to 6% (or 6% compounded semiannually). What is the current market value ( $P$ ) of the bond (3 years after the purchase)?
- 3.69 You are considering buying a new car worth \$15,000. You can finance the car either by withdrawing cash from your savings account, which earns 8% interest compounded monthly, or by borrowing \$15,000 from your dealer for four years at 11% interest compounded monthly. You could earn \$5,635 in interest from your savings account in four years if you leave the money in the account. If you borrow \$15,000 from your dealer, you pay only \$3,609 in interest over four years, so it makes sense to borrow for your new car and keep your cash in your savings account. Do you agree or disagree with the foregoing statement? Justify your reasoning with a numerical calculation.
- 3.70 Suppose you are going to buy a home worth \$110,000, making a down payment in the amount of \$50,000. The balance will be borrowed from the Capital Savings and Loan Bank. The loan officer offers the following two financing plans for the property:
- Option 1: A conventional fixed loan at an interest rate of 13% compounded monthly over 30 years with 360 equal monthly payments.
  - Option 2: A graduated payment schedule (FHA 235 plan) at 11.5% interest compounded monthly with the following monthly payment schedule:

Year (n)	Monthly Payment	Monthly Mortgage Insurance
1	\$497.76	\$25.19
2	\$522.65	\$25.56
3	\$548.78	\$25.84
4	\$576.22	\$26.01
5	\$605.03	\$26.06
6–30	\$635.28	\$25.96

For the FHA 235 plan, mortgage insurance is a must.

- (a) Compute the monthly payment for Option 1.
- (b) What is the effective annual interest rate you would pay under Option 2?
- (c) Compute the outstanding balance for each option at the end of five years.
- (d) Compute the total interest payment for each option.
- (e) Assuming that your only investment alternative is a savings account that earns an interest rate of 6% compounded monthly, which option is a better deal?