CHAPTER ELEVEN

Handling Project Uncertainty

Eagle Mountain Pumped Storage Development Project[1]

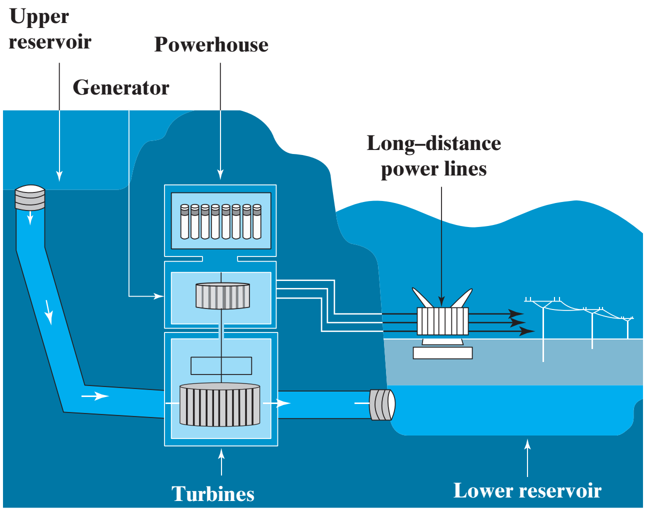
With the growing demand for renewable energy sources in California, Eagle Crest Energy Company obtained the license to construct a hydroelectric project from the Federal Energy Regulatory Commission (FERC) in 2014. Using two of the excavated open mine pits as reservoirs, the Eagle Mountain Pumped Storage Project, in Palm Spring, California, will store water in an upper reservoir for later release through an underground power plant to the lower reservoir. This would allow 1,300 MW of electricity to be generated during peak hours when it is needed most. Unlike other renewable energy production methods, pumped storage gives producers a way to bank energy for future use. In particular, it helps solve a big problem as power companies invest in renewable energy, which doesn't always produce electricity when it is most needed. For example, solar power production sets a record in California, but peak demand for electricity comes in the early evening, long after solar production has peaked.

Steve Lowe, president of Eagle Crest Energy, estimates that he needs six years and $2 billion to get it built. He hopes it will serve as the battery pack for nearby wind turbines whose production peaks at night. He is currently seeking financing and customers.[2]

Even though the pumped storage project is attracting renewed respect from utilities and environmentalists aiming to fight climate change, the company faces numerous challenges even after obtaining a license from FERC as well as multiple other state and Federal permits. First,

[1] Eagle Crest Energy, "Eagle Mountain Pumped Storage Project." (http://www.eaglecrestenergy.com/index.html).

[2] Rebecca Smith, "Pumped Up: Renewables Growth Revives Old Energy-Storage Method," *The Wall Street Journal*, July 22, 2016. (https://www.wsj.com/articles/pumped-up-renewables-growth-revives-old-energy-storage-method-1469179801).



typically, a three- to five-year construction period is common for most large projects of this nature. Furthermore, even projects that do not pose any serious environmental concerns could take 6–10 years or longer to construct and support renewable energy integration. Second, very few financial institutions are willing to finance these types of long-lead time projects, unless they see a clear financial payback plan. Third, the prediction of supply and demand of future energy needs is hard to predict so that it is not clear how long it would take to recover the investment cost. The overriding issue is clearly, how do we consider all these risks and uncertainties in evaluating a long-term project?

In previous chapters, cash flows from projects were assumed to be known with complete certainty; our analysis was concerned with measuring the economic worth of projects and selecting the best investment projects. Although these types of analyses can provide a reasonable decision basis in many investment situations, we should admit that most projects involve uncertainty. In this type of situation, management rarely has precise expectations about the future cash flows to be derived from a particular project. In fact, the best that a firm can reasonably expect is to estimate the range of possible future costs and benefits and the relative chances of achieving a reasonable return on the investment.

We use the term **risk** to describe an investment project where cash flows are not known in advance with certainty but for which an array of outcomes and their probabilities (odds) can be considered. We will also use the term **project risk** to refer to variability in a project's net present worth. Higher project risk reflects a higher anticipated variability in a project's NPW. In essence, we can see *risk as the potential for loss.* This chapter begins by exploring the origins of project risk.

11.1 Origins of Project Risk

When deciding whether to make a major capital investment, a firm must consider and estimate a number of issues. The factors to be estimated include the total market size for the product, the market share that the firm can attain, the growth of the market, the cost of production, the selling price of the product, the life span of the product, the cost and life span of the equipment needed, and the effective tax rates. Many of these factors are subject to substantial uncertainty. A common approach is to generate single-number "best estimates" for each of the uncertain factors and then to calculate measures of profitability, such as NPW or rate of return for the project. This approach has two drawbacks:

1. No guarantee can ensure that the best estimates will ever match the actual values.
2. No provision is made to measure the risk associated with the investment or the project risk. In particular, managers have no way of determining either the probability that a project will lose money or the probability that it will generate large profits.

Because cash flows can be so difficult to estimate accurately, project managers frequently consider a range of possible values for cash flow elements. If a range of values for individual cash flows is possible to make, then logically determining a range of values for the NPW of a given project is also possible. Clearly, the analyst will want to try to gauge the probability and reliability of individual cash flows occurring and, consequently, the level of certainty about overall project worth.

Quantitative statements about risk are given as numerical probabilities or as values for the likelihood (odds) of occurrence. Probabilities are given as decimal fractions in the interval 0.0 to 1.0. An event or outcome that is certain to occur has a probability of 1.0. As the probability of an event approaches 0, the event becomes increasingly less likely to occur. In general, **risk analysis** is the process of assigning probabilities to the various investment outcomes and estimating the overall probability of a project's financial success or failure.

11.2 Methods of Describing Project Risk

We may begin analyzing project risk by first determining the uncertainty inherent in a project's cash flows. We can do this analysis in a number of ways, which range from making informal judgments to performing complex economic and statistical analyses. In this section, we will introduce three methods of determining project risk: (1) sensitivity analysis, (2) break-even analysis, and (3) scenario analysis. Each method will be explained with reference to a single example. We also introduce the method for conducting sensitivity analysis for mutually exclusive alternatives.

11.2.1 Sensitivity Analysis

One way to glean a sense of all possible outcomes of an investment is to perform a **sensitivity analysis**, a technique whereby different values of certain key variables are tested to see how sensitive investment results are to possible change in assumptions. It is a method of evaluating the riskiness of an investment. In calculating cash flows, some items have more influence on the final result (NPW) than others. In some problems, the most significant item is easily identified. For example, the estimate of sales volume can have a major impact in a project's NPW, especially in new product introductions.

We may want to identify the items that have an important influence on the final results so that they can be subjected to special scrutiny. Sensitivity analysis is sometimes called "**what-if analysis**" because it answers questions such as, **if** incremental sales are only 1,000 units, rather than 2,000 units, **what** will the NPW be?

- Sensitivity analysis begins with a base-case situation, which we develop by using the most likely value for each input. We then change the specific variable of interest by several specified percentages above and below the most likely value while holding other variables constant.
- Next, we calculate a new NPW for each of these values. A convenient and useful way to present the results of a sensitivity analysis is to plot **sensitivity graphs**. The slopes of the lines show how sensitive the NPW will be to changes in each of the inputs: the steeper the slope, the more sensitive the NPW will be to a change in a particular variable. Sensitivity graphs identify the crucial variables that affect the final outcome the most.

We will use Example 11.1 to illustrate the concept of sensitivity analysis.

EXAMPLE 11.1 Sensitivity Analysis—Know Which Variable Is Most Critical to Your Bottom Line

Capstone Turbine Corporation is the world's leading provider of micro-turbine by integrating an innovative turbine engine, magnetic generator, advanced power electronics, and air bearing technology for clean, continuous, distributed-generation electricity. Capstone micro-turbines are the ideal solution to meet electricity on-site or close to the point where it is needed. Designed to operate on a variety of gaseous and liquid fuels, this form of distributed-generation technology first debuted in 1998 for commercial use.

Capstone is considering marketing a modified but downsized version of the system unit named C30X for residential use, primarily for vacation properties in remote places. The project requires an initial investment of \$55 million, but Capstone managers are uneasy about this project because too many uncertain elements have not been considered in the analysis. Two primary factors that are difficult to estimate are the initial market size and how the market size will grow over the life of the project. The company has prepared the following financial data related to the project:

Key Cash Flow Variables	Low	Most Likely	High
Initial market size (units), year 1	1,000	1,500	2,000
Market growth rate (annual)	3%	5%	8%
Unit price	\$72,000	\$80,000	\$86,000
Unit variable cost	\$56,000	\$60,000	\$65,000
Fixed cost (annual) excluding depreciation	\$6,500,000	\$8,000,000	\$9,000,000
Salvage value	\$4,000,000	\$7,000,000	\$8,000,000

The initial investment can be depreciated on a seven-year MACRS, and the project is expected to have an economic service life of five years. The product life is relatively short, as the technology changes in the energy sector are evolving rapidly. The firm's marginal tax rate is 25%, and its MARR is known to be 15%.

(a) Develop the cash flow series over the project life based on the assumption of most likely estimates.

(b) Conduct a sensitivity analysis for each variable and develop a sensitivity graph.

DISSECTING THE PROBLEM

Table 11.1 shows Capstone's expected but ultimately uncertain cash flows based on the most likely estimates. The annual revenues and expenses are estimated with the following relationship:

- Revenues = market size × unit price × $(1 + \text{growth rate})^{(n-1)}$.
- Costs = market size × variable unit cost × $(1 + \text{growth rate})^{(n-1)}$ + fixed cost (excluding depreciation).

If everything goes as well as expected, it appears that the project is worth undertaking with NPW = \$19,202 (or \$19,202,000).

Suppose that Capstone is not confident in its revenue forecasts in particular. The managers think that if the product is not well accepted in the U.S. marketplace, Capstone will need to sell the units outside the United States, primarily in the European market. However, the company is not guaranteed success in that market either. Before undertaking the project described, the company needs to identify the key variables that will determine whether the project will succeed or fail.

After defining the market size, growth rate, unit price, unit variable cost, fixed cost, and salvage value, we conduct a sensitivity analysis with respect to these key input variables. We do this by varying each of the estimates by a given percentage and determining what effect the variation in that item will have on the final results. If the effect is large, the result is sensitive to that item. Our objective is to locate the most sensitive item(s).

Given: Range of estimates of key input variables and the cash flow estimates based on most likely estimates given in Table 11.1.

Find: (a) Which input variable is the most critical? (b) Plot a sensitivity graph.

METHODOLOGY

Develop a cash flow series in Excel and plot the sensitivity graph.

SOLUTION

(a) **Sensitivity analysis:** We begin the sensitivity analysis with a consideration of the base-case situation, which reflects the most likely estimate (expected value) for each input variable. In developing Table 11.2, we changed a given variable by 20% in 5% increments above and below the base-case value and calculated new NPWs while other variables were held constant. Now we ask a series of what-if questions: What if sales are 20% below the expected level? What if operating costs rise? What if the unit price drops from \$80,000 to \$64,000 (20% drop)? Table 11.2 summarizes the results of our varying the values of the key input variables.

TABLE 11.1 **Cash Flows for Capstone's C30X Project, Based on Most Likely Estimates (monetary unit: $000)**

	A	B	C	D	E	F	G
3							
4	**Input Data (Base):**			**Sensitivity Analysis:**			
5							
6	Unit Price ($)	80.00		Category		% Change	
7	Demand	1500		Unit price		0%	
8	Growth rate	5%		Growth rate		0%	
9	Var. cost ($/unit)	60.00		Demand		0%	
10	Fixed cost ($)	8000		Var. cost (unit)		0%	
11	Salvage ($)	7000		Fixed cost		0%	
12	Tax rate (%)	25%		Salvage		20%	
13	MARR (%)	15%					
14				Output (PW)		$19,202	
15							
16							
17		0	1	2	3	4	5
18	**Income Statement**						
19							
20	Revenues:						
21	Unit Price		$ 80	$ 80	$ 80	$ 80	$ 80
22	Demand (units)		1500	1575	1654	1736	1823
23	Sales Revenue		$ 120,000	$ 126,000	$ 132,300	$ 138,915	$ 145,861
24	Expenses:						
25	Unit Variable Cost		$ 60	$ 60	$ 60	$ 60	$ 60
26	Variable Cost		90,000	94,500	99,225	104,186	109,396
27	Fixed Cost		8,000	8,000	8,000	8,000	8,000
28	Depreciation		7,860	13,470	9,620	6,870	2,453
30	Taxable Income		$ 14,141	$ 10,031	$ 15,456	$ 19,859	$ 26,012
31	Income Taxes (40%)		3,535	2,508	3,864	4,965	6,503
33	Net Income		$ 10,605	$ 7,523	$ 11,592	$ 14,894	$ 19,509
34							
35	**Cash Flow Statement**						
36							
37	Operating Activities:						
38	Net Income		$ 10,605	$ 7,523	$ 11,592	$ 14,894	$ 19,509
39	Depreciation		$ 7,860	$ 13,470	$ 9,620	$ 6,870	$ 2,453
40	Investment Activities:						
41	Investment	$ (55,000)					
42	Salvage						$ 8,400
43	Gains Tax						$ 1,582
45	Net Cash Flow	$ (55,000)	$ 18,465	$ 20,992	$ 21,211	$ 21,764	$ 31,944
46							
47	IRR=	28%					
48	PW(15%) =	$ 19,202					
49							
50							
51							

=C22*(1+$B$8*(1+$F$8))

=-$B$41*0.1429

=-0.25*(G42-(-B41-SUM(C28:G28)))

=IRR(B45:G45,10%)

=NPV(15%,C45:G45)+B45

(b) **Sensitivity graph:** Figure 11.1 shows the Capstone project's sensitivity graphs for six of the key input variables. The base-case NPW is plotted on the ordinate of the graph at the value 1.0 on the abscissa (or 0% deviation). Next, the value of product demand is reduced to 95% of its base-case value, and the

TABLE 11.2 **Sensitivity Analysis for Six Key Input Variables (Example 11.1)**

Key Input Variables	NPW as a Function of Percent Deviation from the Most Likely Estimates								
	−20%	−15%	−10%	−5%	Base 0%	5%	10%	15%	20%
Unit price	($47,103)	($30,656)	($14,212)	($2,234)	$18,680	$35,126	$51,572	$68,017	$84,463
Demand	$2,234	$6,346	$10,457	$14,568	$18,680	$22,791	$26,903	$31,014	$35,126
Growth rate	$17,258	$17,619	$17,971	$18,325	$18,680	$19,037	$19,395	$19,755	$20,117
Variable cost	$68,017	$55,683	$43,349	$31,014	$18,680	$6,346	($5,989)	($18,323)	($30,658)
Fixed cost	$22,702	$21,697	$20,691	$19,686	$18,680	$17,674	$16,669	$15,663	$14,657
Salvage value	$18,158	$18,288	$18,419	$18,549	$18,680	$18,818	$18,941	$19,071	$19,202

NPW is recomputed with all other variables held at their base-case value. We repeat the process by either decreasing or increasing the relative deviation from the base case. The lines for the variable unit price, variable unit cost, fixed cost, and salvage value are obtained in the same manner.

COMMENTS: In Figure 11.1, we see that the project's NPW is (1) very sensitive to changes in the unit price and variable cost, (2) fairly sensitive to changes in demand, and (3) relatively insensitive to changes in growth rate, fixed cost, and salvage value. Graphic displays such as the one in Figure 11.1 provide a useful

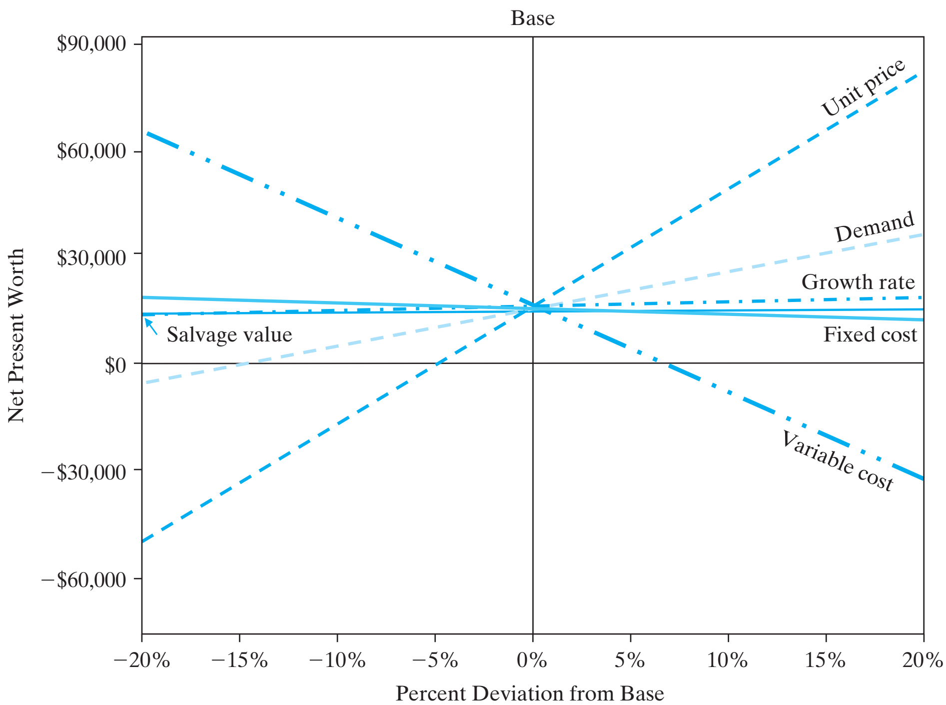


Figure 11.1 Sensitivity graph for Capstone's C30X project.

means to communicate the relative sensitivities of the different variables on the corresponding NPW value. However, sensitivity graphs do not explain any interactions among the variables or the likelihood of realizing any specific deviation from the base case. Certainly, it is conceivable that a project might not be very sensitive to changes in either of two items but might be very sensitive to combined changes in them.

11.2.2 Sensitivity Analysis for Mutually Exclusive Alternatives

In Figure 11.1, each variable is uniformly adjusted by ±20% and all variables are plotted on the same chart. This uniform adjustment can be too simplistic an assumption; in many situations, each variable can have a different range of uncertainty. Also, plotting all variables on the same chart could be confusing if there are too many variables to consider. When we perform sensitivity analysis for mutually exclusive alternatives, it may be more effective to plot the NPWs (or any other measures, such as AEs) of all alternatives over the range of each input; in other words, create one plot for each input, with units of the input on the horizontal axis. Example 11.2 illustrates this approach.

EXAMPLE 11.2 Sensitivity Analysis for Mutually Exclusive Alternatives

A local U.S. Postal Service office is considering purchasing a 4,000 lb forklift truck, which will be used primarily for processing incoming as well as outgoing postal packages. Forklift trucks traditionally have been fueled by either gasoline, liquid propane gas (LPG), or diesel fuel. Battery-powered electric forklifts, however, are increasingly popular in many industrial sectors because of their economic and environmental benefits. Therefore, the Postal Service is interested in comparing the four different types of forklifts. Purchase costs as well as annual operating and maintenance costs for each type of forklift are provided by a local utility company. Annual fuel and maintenance costs are measured in terms of number of shifts per year, where one shift is equivalent to eight hours of operation. A comparison of the variables of the four forklift types is given in the following table:

	Electrical Power	LPG	Gasoline	Diesel Fuel
Life expectancy	7 years	7 years	7 years	7 years
Initial cost	\$30,000	\$21,000	\$20,000	\$25,000
Salvage value	\$3,000	\$2,000	\$2,000	\$2,200
Maximum shifts per year	260	260	260	260
Fuel consumption/shift	32 kWh	12 gal	11 gal	7 gal
Fuel cost/unit	\$0.20/kWh	\$3.15/gal	\$3.60/gal	\$3.95/gal
Fuel cost/shift	\$6.40	\$37.80	\$39.60	\$27.65
Annual maintenance cost:				
Fixed cost	\$500	\$1,000	\$1,200	\$1,500
Variable cost/shift	\$5	\$6	\$7	\$9

The Postal Service is unsure of the number of shifts per year, but it expects them to be somewhere between 200 and 260 shifts. Since the U.S. Postal Service does not pay income taxes, no depreciation or tax information is required. The U.S. government uses 10% as the discount rate for any project evaluation of this nature. Develop a sensitivity graph that shows how the best choice from the alternatives changes as a function of the number of shifts per year.

DISSECTING THE PROBLEM

Two annual cost components are pertinent to this problem: (1) ownership cost (capital cost) and (2) operating cost (fuel and maintenance cost). Since the operating cost is already given on an annual basis, we need only determine the equivalent annual ownership cost for each alternative.

Given: Financial data as given in the table, interest rate $=10\%$, and range of operating shifts.

Find: Which alternative is the best as a function of the number of shifts?

METHODOLOGY

Calculate ownership (capital) cost, annual operating cost, and equivalent annual cost.

SOLUTION

(a) **Ownership cost (capital cost):** Using the capital-recovery-with-return formula developed in Eq. (6.2), we compute the following:

Electrical power:

$$\begin{aligned}\text{CR }(10\%) &= (\$30{,}000 - \$3{,}000)(A/P, 10\%, 7) \\ &\quad + (0.10)\$3{,}000 \\ &= \$5{,}845;\end{aligned}$$

$$\begin{aligned}\text{LPG: CR}(10\%) &= (\$21{,}000 - \$2{,}000)(A/P, 10\%, 7) \\ &\quad + (0.10)\$2{,}000 \\ &= \$4{,}103;\end{aligned}$$

$$\begin{aligned}\text{Gasoline: CR}(10\%) &= (\$20{,}000 - \$2{,}000)(A/P, 10\%, 7) \\ &\quad + (0.10)\$2{,}000 \\ &= \$3{,}897;\end{aligned}$$

$$\begin{aligned}\text{Diesel fuel: CR}(10\%) &= (\$25{,}000 - \$2{,}200)(A/P, 10\%, 7) \\ &\quad + (0.10)\$2{,}200 \\ &= \$4{,}903.\end{aligned}$$

(b) **Annual operating cost:** We can express the annual operating cost as a function of the number of shifts per year (M) by combining the variable- and fixed-cost portions of the fuel and maintenance expenditures:

$$\begin{aligned}\text{Electrical power: } \$500 + (6.40 + 5)M &= \$500 + 11.40M; \\ \text{LPG: } \$1{,}000 + (37.80 + 6)M &= \$1{,}000 + 43.80M; \\ \text{Gasoline: } \$1{,}200 + (39.60 + 7)M &= \$1{,}200 + 46.60M; \\ \text{Diesel fuel: } \$1{,}500 + (27.65 + 9)M &= \$1{,}500 + 36.65M.\end{aligned}$$

(c) **Total equivalent annual cost:** This value is the sum of the ownership cost and operating cost:

$$\begin{aligned} \text{Electrical power: AEC}(10\%) &= \$6{,}345 + 11.40M; \\ \text{LPG: AEC}(10\%) &= \$5{,}103 + 43.80M; \\ \text{Gasoline: AEC}(10\%) &= \$5{,}097 + 46.60M; \\ \text{Diesel fuel: AEC}(10\%) &= \$6{,}403 + 36.65M. \end{aligned}$$

In Figure 11.2, these four annual-equivalence costs are plotted as a function of the number of shifts, M. It appears that the economics of the electric forklift truck can be justified as long as the number of annual shifts exceeds approximately 39. If the number of shifts is fewer than 39, the LPG truck becomes the most economically viable option. In terms of a pairwise comparison with the electric lift truck, the diesel option is not a viable alternative for any range of M.

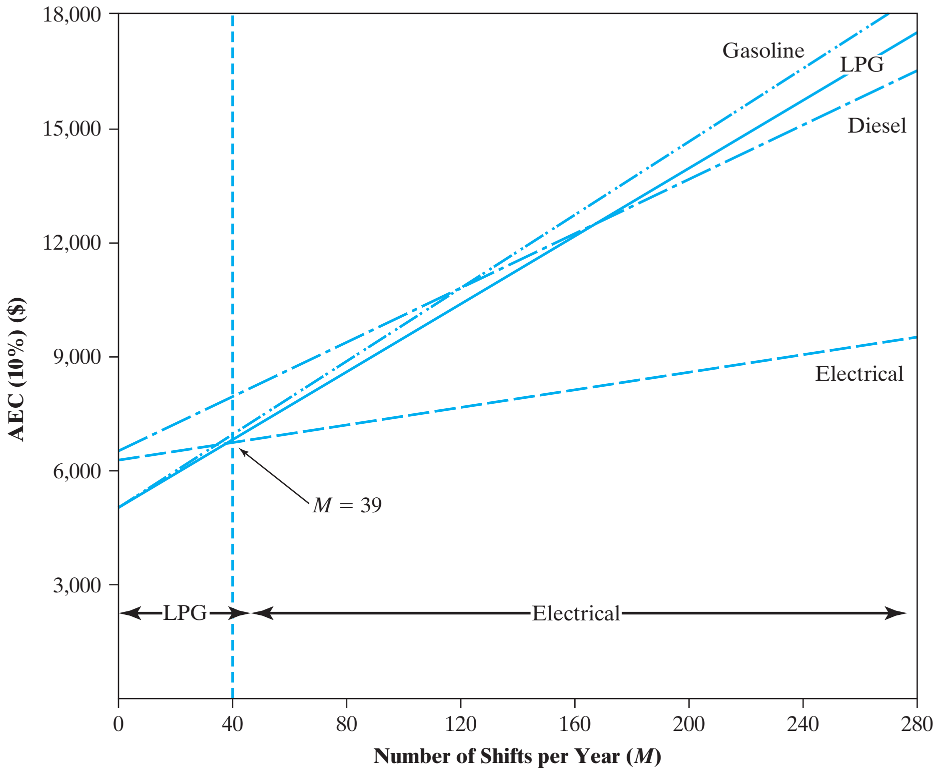


Figure 11.2 Sensitivity analyses for mutually exclusive alternatives. With $2 < M < 39$, select LPG lift truck; otherwise, select electrical power lift truck.

11.2.3 Break-Even Analysis

When we perform a sensitivity analysis for a project, we are asking how serious the effect of lower revenues or higher costs will be on the project's profitability. Managers sometimes prefer to ask how much sales can decrease below forecasts before the project begins to lose money. This type of analysis is known as **break-even analysis**. To illustrate the procedure of break-even analysis based on NPW, we use the Goal Seek function from Excel. Note that this break-even value calculation is similar to the calculation we use for the internal rate of return when we want to find the interest rate that makes the NPW equal zero as well as when we want to find many other "cutoff values" when a choice changes.

EXAMPLE 11.3 Break-Even Analysis with Excel

From the sensitivity analysis discussed in Example 11.1, Capstone's managers are convinced that the NPW is most sensitive to changes in unit price. Determine the unit price to break even.

DISSECTING THE PROBLEM

The after-tax cash flows shown in Table 11.3 are basically the same as those in Table 11.1. The table is simply an Excel spreadsheet in which the cash flow entries are a function of the input variables. Here, what we are looking for is the minimum amount of unit price that makes the NPW zero.

Given: Cash flow statement given in Table 11.1.

Find: Break-even unit price.

METHODOLOGY

Use Excel's Goal Seek function to find the break-even unit price.

SOLUTION

Using the Goal Seek function, we want to set the NPW (cell \$F\$7) to zero by changing the unit price value (cell \$B\$6). Pressing the OK button will produce the results shown in Table 11.3, indicating that the project will break even when the unit price reaches exactly \$75.46 (\$75,620), or a mere 5.68% lower than the most likely estimate of \$80.

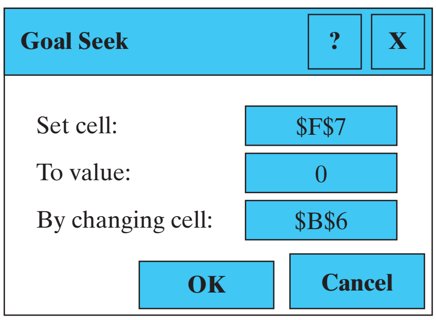


TABLE 11.3 Break-Even Analysis Using Excel's Goal Seek Function (unit: $000)

	A	B	C	D	E	F	G
3							
4	**Input Data (Base):**			**Output Analysis**			
5							
6	Unit Price ($)	75.46			IRR=	15%	
7	Demand	1500			PW(15%) =	$ -	
8	Growth rate	5%					
9	Var. cost ($/unit)	60.00					
10	Fixed cost ($)	8000		By changing cell			Set cell
11	Salvage ($)	7000					
12	Tax rate (%)	25%					
13	MARR (%)	15%			=NPV(15%,C45:G45)+B45		
14							
15							
16							
17		0	1	2	3	4	5
18	**Income Statement**						
19							
20	Revenues:						
21	Unit Price		$ 75.46	$ 75.46	$ 75.46	$ 75.46	$ 75.46
22	Demand (units)		1500	1575	1654	1736	1823
23	Sales Revenue		$ 113,185	$ 118,844	$ 124,786	$ 131,026	$ 137,577
24	Expenses:						
25	Unit Variable Cost		$ 60	$ 60	$ 60	$ 60	$ 60
26	Variable Cost		90,000	94,500	99,225	104,186	109,396
27	Fixed Cost		8,000	8,000	8,000	8,000	8,000
28	Depreciation		7,860	13,470	9,620	6,870	2,453
30	Taxable Income		$ 7,325	$ 2,875	$ 7,942	$ 11,970	$ 17,728
31	Income Taxes (25%)		1,831	719	1,985	2,992	4,432
33	Net Income		$ 5,494	$ 2,156	$ 5,956	$ 8,977	$ 13,296
34							
35	**Cash Flow Statement**						
36							
37	Operating Activities:						
38	Net Income		$ 5,494	$ 2,156	$ 5,956	$ 8,977	$ 13,296
39	Depreciation		$ 7,860	$ 13,470	$ 9,620	$ 6,870	$ 2,453
40	Investment Activities:						
41	Investment	$ (55,000)					
42	Salvage						$ 7,000
43	Gains Tax						$ 1,932
45	Net Cash Flow	$ (55,000)	$ 13,354	$ 15,626	$ 15,576	$ 15,847	$ 24,682
46							
47							
48							

11.2.4 Scenario Analysis

Although both sensitivity and break-even analyses are useful, they have limitations. Often, it is difficult to specify precisely the relationship between a particular variable and the NPW. The relationship is further complicated by interdependencies among the variables. Holding operating costs constant while varying unit sales may ease the analysis, but in reality, operating costs do not behave in this manner. Yet, it may complicate the analysis too much to permit movement in more than one variable at a time.

Scenario analysis is a technique that considers the sensitivity of the NPW to changes in both key variables and the range of likely variable values. For example, the decision maker may consider two extreme cases: a worst-case scenario (low unit sales, low unit price, high variable cost per unit, high fixed cost, and so on) and a best-case scenario. The NPWs under the worst and the best conditions are then calculated and compared with the expected, or base-case, NPW. Example 11.4 illustrates a plausible scenario analysis for Capstone's C30X project.

EXAMPLE 11.4 Scenario Analysis

Consider again Capstone's C30X project from Example 11.1. Given the three-point estimates for six key input variables (market size, market growth rate, unit price, unit variable cost, fixed cost, and salvage value), the marketing and engineering staffs come up with the scenarios shown in the following table.

Assume that the company's managers are fairly confident of other estimates such as project life, tax rate, and MARR. Furthermore, assume that they regard a decline in unit sales to below 1,000 or an increase above 2,000 as extremely unlikely. Thus, decremental annual sales of 500 units define the lower bound, or the worst-case scenario, whereas incremental annual sales of 500 units define the upper bound, or the best-case scenario. The same logic applies to other input estimates. Discuss the worst- and best-case scenarios.

Variable Considered	Worst-Case Scenario	Most Likely Scenario	Best-Case Scenario
Market size	1,000	1,500	2,000
Market growth rate	3%	5%	8%
Unit price	\$72,000	\$80,000	\$86,000
Unit variable cost	\$65,000	\$60,000	\$56,000
Fixed cost	\$9,000,000	\$8,000,000	\$6,500,000
Salvage value	\$4,000,000	\$7,000,000	\$8,000,000

DISSECTING THE PROBLEM

Given: Three-point estimates for the six key input variables in Example 11.1.

Find: NPW of the project under each scenario.

METHODOLOGY

To carry out the scenario analysis, we use the worst-case variable values to obtain the worst-case NPW and the best-case variable values to obtain the best-case NPW.

SOLUTION

The results of our analysis are summarized as follows:

(a) Worst-case scenario: With the input parameters under the worst-case scenario, the project cash flow statement would look like the results in Table 11.4. If this were to happen, Capstone would lose the entire investment made in the project to the extent of suffering a negative rate of return on its investment (−32%).

TABLE 11.4 **Capstone's C30X Project Cash Flows under the Worst-Case Scenario (monetary unit: $000)**

	A	B	C	D	E	F	G
15							
16							
17		0	1	2	3	4	5
18	**Income Statement**						
19							
20	Revenues:						
21	Unit Price		$ 72.00	$ 72.00	$ 72.00	$ 72.00	$ 72.00
22	Demand (units)		1000	1030	1061	1093	1126
23	Sales Revenue		$ 72,000	$ 74,160	$ 76,385	$ 78,676	$ 81,037
24	Expenses:						
25	Unit Variable Cost		$ 65	$ 65	$ 65	$ 65	$ 65
26	Variable Cost		65,000	66,950	68,959	71,027	73,158
27	Fixed Cost		9,000	9,000	9,000	9,000	9,000
28	Depreciation		7,860	13,470	9,620	6,870	2,453
30	Taxable Income		$ (9,860)	$ (15,260)	$ (11,193)	$ (8,220)	$ (3,574)
31	Income Taxes (25%)		(2,465)	(3,815)	(2,798)	(2,055)	(894)
33	Net Income		$ (7,395)	$ (11,445)	$ (8,395)	$ (6,165)	$ (2,681)
34							
35	**Cash Flow Statement**						
36							
37	Operating Activities:						
38	Net Income		$ (7,395)	$ (11,445)	$ (8,395)	$ (6,165)	$ (2,681)
39	Depreciation		$ 7,860	$ 13,470	$ 9,620	$ 6,870	$ 2,453
40	Investment Activities:						
41	Investment	$ (55,000)					
42	Salvage						$ 4,000
43	Gains Tax						$ 2,682
45	Net Cash Flow	$ (55,000)	$ 465	$ 2,025	$ 1,225	$ 704	$ 6,454
46							
47							
48							
49		IRR =	-32%				
50		PW(15%) =	$ (48,648)				
51							
52							

=-0.25*(G42-(-B41-SUM(C28:G28)))

=NPV(15%,C45:G45)+B45

(b) Best-case scenario: With the best-case scenario, Capstone would make a significant profit from the project, earning more than $113 million surplus after recovering all the investment made in the project. (See Table 11.5.) This is equivalent to realizing a 80% rate of return on the investment.

COMMENTS: We see that the base case produces a positive NPW ($19,202; see Table 11.1), the worst case produces a negative NPW (−$48,648), and the best case produces a large positive NPW ($113,835). Still, just by looking at the results in the table, it is not easy to interpret the scenario analysis or to make a decision. For example, we could say that there is a chance of losing money on the project, but we do not yet have a specific probability for that possibility. Clearly, we need estimates of the probabilities of occurrence of the worst case, the best case, the base (most likely) case, and all other possibilities.

TABLE 11.5 **Capstone's C30X Project Cash Flows under the Best-Case Scenario (monetary unit: $000)**

	A	B	C	D	E	F	G
15							
16							
17		0	1	2	3	4	5
18	**Income Statement**						
19							
20	Revenues:						
21	Unit Price		$ 86.00	$ 86.00	$ 86.00	$ 86.00	$ 86.00
22	Demand (units)		2000	2160	2333	2519	2721
23	Sales Revenue		$ 172,000	$ 185,760	$ 200,621	$ 216,670	$ 234,004
24	Expenses:						
25	Unit Variable Cost		$ 56	$ 56	$ 56	$ 56	$ 56
26	Variable Cost		112,000	120,960	130,637	141,088	152,375
27	Fixed Cost		6,500	6,500	6,500	6,500	6,500
28	Depreciation		7,860	13,470	9,620	6,870	2,453
30	Taxable Income		$ 45,641	$ 44,831	$ 53,865	$ 62,213	$ 72,676
31	Income Taxes (25%)		11,410	11,208	13,466	15,553	18,169
33	Net Income		$ 34,230	$ 33,623	$ 40,398	$ 46,660	$ 54,507
34							
35	**Cash Flow Statement**						
36							
37	Operating Activities:						
38	Net Income		$ 34,230	$ 33,623	$ 40,398	$ 46,660	$ 54,507
39	Depreciation		$ 7,860	$ 13,470	$ 9,620	$ 6,870	$ 2,453
40	Investment Activities:						
41	Investment	$ (55,000)					
42	Salvage			=-0.25*(G42-(-B41-SUM(C28:G28)))			$ 8,000
43	Gains Tax						$ 1,682
45	Net Cash Flow	$ (55,000)	$ 42,090	$ 47,092	$ 50,018	$ 53,529	$ 66,643
46							
47							
48							
49		IRR =	80%				
50		PW(15%) =	$ 113,835		=NPV(15%,C45:G45)+B45		
51							

The need to estimate probabilities leads us directly to our next step: developing a probability distribution (or, put another way, the probability that the variable in question takes on a certain value). We will consider this issue in the next section.

11.3 Probabilistic Cash Flow Analysis

Once you have an idea of the degree of risk inherent in an investment, the next step is to incorporate this information into your evaluation of the proposed project. There are two fundamental approaches: (1) consider the risk elements directly through probabilistic assessments and (2) adjust the discount rate to reflect any perceived risk in the project's cash flows. We will consider both approaches briefly. However, the risk-adjusted discount-rate approach is more commonly practiced in the real world, as the method is mathematically much simpler than the probabilistic approach.

11.3.1 Including Risk in Investment Evaluation

In principle, investment risk is concerned with the range of possible outcomes from an investment: the wider this range, the higher is the risk. Figure 11.3 illustrates this intuitive notion. It shows, in the form of bell-shaped curves, the possible rates of return that might be earned on two investments. According to the figure, the expected return on investment *A* is about 10%, and the corresponding figure for investment *B* is about 20%.

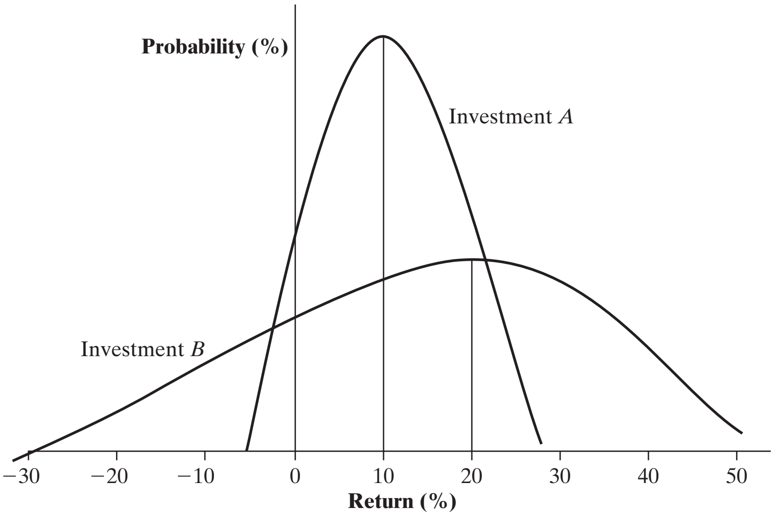


Figure 11.3 Illustration of investment risk: Investment *A* has a lower expected return but a lower risk than investment *B*.

Because we would define **expected return** as the probability-weighted average of possible returns, this expected figure represents the central tendency of the random outcome (in our case, the return). To take a simple example, if three returns are possible—6%, 9%, and 18%—and if the chance of each occurring is 0.40, 0.30, and 0.30, respectively, the investment's expected return is calculated as follows:

$$\begin{aligned}\text{Expected return } (\mu) &= (0.40 \times 6\%) + (0.30 \times 9\%) + (0.30 \times 18\%) \\ &= 10.5\%.\end{aligned}$$

Basically, **risk** refers to the bunching of possible returns around an investment's expected return. If there is considerable bunching, as with investment *A,* the investment risk is low. With investment *B,* there is considerably less clustering of returns around the expected return, so it has a higher risk. The way to measure this clustering tendency is to calculate a probability-weighted average of the deviations of possible returns from the expected return. One such average is the **standard deviation** (σ) of returns.

To illustrate the calculation of the standard deviation of returns, we calculate the differences between the possible returns and the expected return, in the foregoing example, as $(6\% - 10.5\%)$, $(9\% - 10.5\%)$, and $(18\% - 10.5\%)$. Because some of these deviations are positive and others are negative, they would tend to cancel one another out if we added them directly. So, we square them to ensure the same sign, calculate the probability-weighted average of the squared deviations [a value known as the variance (σ^2)], and then find the square root:

$$\text{Standard deviation } (\sigma) = (25.65)^{1/2} = 5.065\%.$$

Event	Deviations	Weighted Deviations
1	$(6\% - 10.5\%)^2$	$0.40 \times (6\% - 10.5\%)^2$
2	$(9\% - 10.5\%)^2$	$0.30 \times (9\% - 10.5\%)^2$
3	$(18\% - 10.5\%)^2$	$0.30 \times (18\% - 10.5\%)^2$
		$(\sigma^2) = 25.65$

What we can tell here is that risk corresponds to the *dispersion,* or *uncertainty,* in possible outcomes. We also know that statistical techniques exist to measure this dispersion. In our example, the smaller standard deviation means a considerable bunching, or less risk. When comparing investments with the same expected returns, conservative, or risk-averse, investors would prefer the investment with the smaller standard deviation of return.

11.3.2 Aggregating Risk over Time

Having defined risk and risk aversion in at least a general sense, we might be interested in estimating the amount of risk present in a particular investment opportunity. For an investment project, if we can determine the expected cash flow, as well as the variability of the cash flow, in each period over the project life, we may be able to aggregate the risk over the project life in terms of net present value PW(r), as

$$E[\text{PW}(r)] = \sum_{n=0}^{N} \frac{E(A_n)}{(1 + r)^n} \tag{11.1}$$

and

$$V[\text{PW}(r)] = \sum_{n=0}^{N} \frac{V(A_n)}{(1 + r)^{2n}}, \tag{11.2}$$

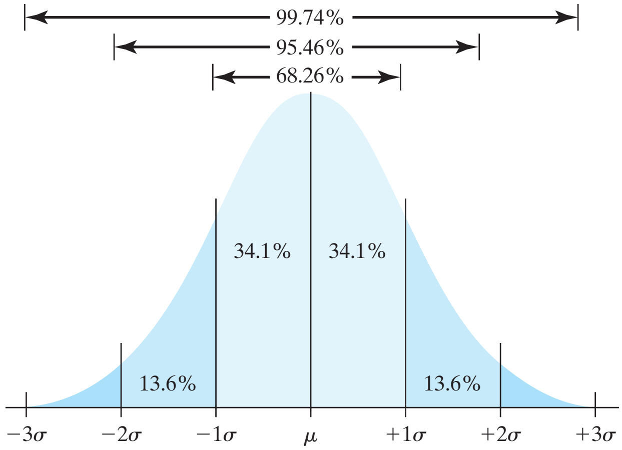
where

$$
\begin{aligned}
r &= \text{risk-free discount rate,} \\
A_n &= \text{cash flow in period } n, \\
E(A_n) &= \text{expected cash flow in period } n, \\
V(A_n) &= \text{variance of the cash flow in period } n, \\
E[\text{PW}(r)] &= \text{expected net present value of the project, and} \\
V[\text{PW}(r)] &= \text{variance of the net present value of the project.}
\end{aligned}
$$ [3]

It is important to observe that we use a risk-free interest rate to discount the project cash flows, as the riskiness of project cash flows is reflected in the probability distributions.

In defining Eq. (11.2), we are also assuming the independence of cash flows, meaning that knowledge of what will happen to one particular period's cash flow will not allow us to predict what will happen to cash flows in other periods. Borrowing again from statistics, we are assuming mutually independent project cash flows. In the event that we cannot assume such a statistical independence among cash flows, we need to consider the degree of dependence among the cash flows.

[3] For a random variable Y, which can be expressed as a linear function of another random variable X (say $Y = aX$, where a is constant), the variance Y can be calculated as a function of variance of X, $\text{Var}[Y] = a^2 \text{Var}[X]$. When deriving Eq. (11.2), we used this algebra of random variable.



- The area under the normal curve equals 1.0, or 100%.
- Of the area under the curve, 99.74% is within $\pm\, 3\sigma$ of the mean (μ), indicating that the probability is 99.74% that actual outcome will be within the range $\mu - 3\sigma$ to $\mu + 3\sigma$.
- The larger the value of σ, the higher the probability will be the actual outcome will vary widely from the expected or most-likely estimate.
- Tables exist for finding the probability of other ranges, as shown in Appendix C.

Figure 11.4 Probability ranges for a normal distribution.

The normal distribution has the very important property that, under certain conditions, the distribution of a sum of a large number of *independent variables* is approximately normal. This is the central limit theorem. Since PW(r) is the sum of discounted random cash flows (A_n's), it follows that, under certain conditions, PW(r) is also closely approximated by a normal distribution with the mean and variance as calculated in Eqs. (11.1) and (11.2). A normal distribution is a bell-shaped probability distribution with many practical applications found in the real world. If a probability distribution is normal, the actual cash flow will be within ± 1 standard deviation (σ) of the expected cash flow 68.26% of the time. Figure 11.4 illustrates this point, and it also shows the situation for $\pm 2\sigma$ and $\pm 3\sigma$.

EXAMPLE 11.5 Computing the Mean and Variance of an Investment Opportunity

Assume that a project is expected to produce the following cash flows in each year, each periodic cash flow is independent of every other, and the risk-free rate is 6%:

Period	Expected Cash Flow	Estimated Standard Deviation
0	−$2,000	$100
1	$1,000	$200
2	$2,000	$500

Find the expected NPW as well as the variance of the NPW.

DISSECTING THE PROBLEM	
	Given: Periodic estimated project cash flows (means and variances); risk-free interest rate. **Find:** $E[\text{PW}(r)]$ and $V[\text{PW}(r)]$.
METHODOLOGY	**SOLUTION**
Calculate the expected NPW and the variance of the NPW.	Using Eqs. (11.1) and (11.2), we find that the expected NPW and the variance of the NPW are $E[\text{PW}(6\%)] = -\$2{,}000 + \frac{\$1{,}000}{1.06} + \frac{\$2{,}000}{1.06^2} = \$723$ and $Var[\text{PW}(6\%)] = 100^2 + \frac{200^2}{1.06^2} + \frac{500^2}{1.06^4} = 243{,}623,$ respectively. Thus, the standard deviation is \$494.

COMMENTS: How is this information used in decision making? It is assumed that most probability distributions are characterized by six standard deviations—three standard deviations above and three standard deviations below the mean. Therefore, the actual NPW of this project would almost certainly fall between −\$759 and \$2,205, as shown in Figure 11.5. If the NPW below 3σ from the mean is still positive, we may say that the project is quite safe. If that figure is negative, then it is up to the decision maker to determine whether the project is worth investing in given the expected mean and standard deviation of the project. If we assume a normal probability distribution, we could find the probability that the project's NPW will be negative, which is only 7.17%.

$$\begin{aligned}\Pr(X \leq x) &= \Pr(\text{PW}(6\%) \leq 0)\\ &= \Phi\left(\frac{X-\mu}{\sigma}\right) = \Phi\left(\frac{0-723}{494}\right) = \Phi(-1.4636)\end{aligned}$$

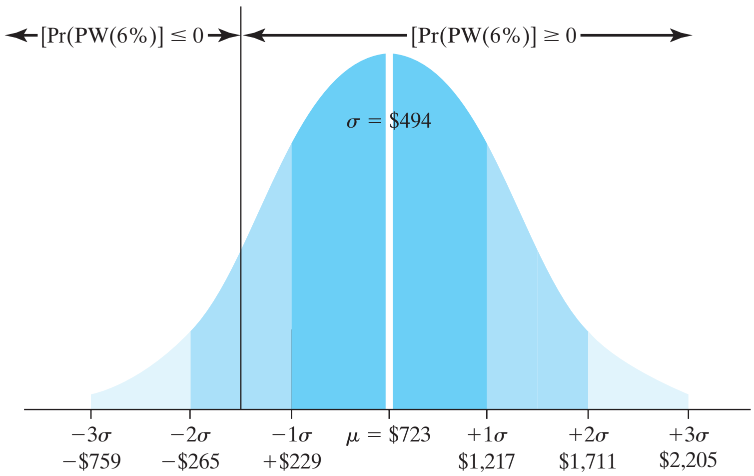


Figure 11.5 NPW distribution. With a normal probability assumption, the probability that the NPW will be negative is found to be 7.17%.

$$= 1 - \Phi(1.4636)$$
$$= 1 - 0.9283$$
$$= 7.17\%.$$

See Appendix C for procedures on how to use the normal probability distribution.

11.3.3 Estimating Risky Cash Flows

In the previous section, we derived the expression of mean and variance of the net present worth distribution. These calculations, however, involve the mean and variance of the various individual cash flows and the statistical relationships among them. Knowledge of this information is a prerequisite to our analysis. The estimating procedure that is common in practice is to make an "optimistic" estimate, a "pessimistic" estimate, and a "most likely" estimate for each cash flow. Here, the meaning of these three estimates is as follows:

- Optimistic estimate: Everything will go as well as is reasonably possible.
- Pessimistic estimate: Everything will go as poorly as is reasonably possible.
- Most-likely estimate: Everything will go as most likely as is initially projected.

Then, these three estimates are used as the upper bound, the lower bound, and the mode of the corresponding cash flow probability distribution. The probability distribution itself is assumed to be a beta distribution[4] with a standard deviation of one-sixth of the spread between the upper and lower bounds (the optimistic and pessimistic estimates). With these assumptions, the mean and variance become explicit functions of the bounds and mode. Given the estimates

$$\text{Est}_o(A_n) = H(\text{optimistic estimate}),$$

$$\text{Est}_o(A_n) = L(\text{pessimistic estimate}), \text{ and}$$

$$\text{Est}_o(A_n) = M_o(\text{most likely estimate}),$$

of the bounds and mode of cash flow distribution of A_n at the end of period n, we find that

$$E[A_n] = \frac{H + 4M_o + L}{6} \tag{11.3}$$

and

$$Var[A_n] = \left(\frac{H - L}{6}\right)^2. \tag{11.4}$$

The model of an underlying beta distribution has been assumed primarily for convenience here, but these estimating formulas are based on a system developed for Program Evaluation and Review Technique (PERT) for network planning and scheduling. It is

[4] The beta distribution models events that are constrained to take place within an interval defined by a minimum and maximum value, and it is often used in the absence of data. The beta distribution has two shape parameters, α and β. When the two parameters are equal, the distribution is symmetrical. For example, when both α and β are equal to 1, the distribution becomes uniform. If α is less than β, the distribution is skewed to the left. And if α is more than β, the distribution is skewed to the right. Because of this modeling flexibility, the beta distribution is used extensively in estimating project cash flows.

important to note that the difference between the approximate expected values as just calculated and those resulting from the exact formula is relatively small for a wide range of beta-distribution conditions.

EXAMPLE 11.6 Developing a Present Worth Distribution for Capstone's Investment Project

Let's recall Example 11.4, where Capstone engineers have created three different scenarios based on the key input variables expressed in terms of three-point estimates in Example 11.1. If we examine the annual cash flows under each scenario, we find the following values:

Annual Cash Flow Estimates (unit: $000)

n	Worst-Case Scenario (Table 11.4) L	Most Likely Scenario (Table 11.1) M_o	Best-Case Scenario (Table 11.5) H
0	($55,000)	($55,000)	($55,000)
1	$465	$18,465	$42,090
2	$2,025	$20,992	$47,092
3	$1,225	$21,211	$50,018
4	$704	$21,764	$53,529
5	$6,454	$31,944	$66,643

Using these annual cash flow estimates as three-point estimates for each period, compute the mean and variance of the NPW distribution.

DISSECTING THE PROBLEM

Given: Three-point cash flow estimates for each period, interest rate $= 15\%$ per year.

Find: Mean and variance of the project, probability that the NPW will be negative.

METHODOLOGY

Calculate the mean and variance.

SOLUTION

$n = 0$: Since there is no variability in the initial investment amount, the variance is zero.

$n = 1$: Using Eqs. (11.3) and (11.4), we find that

$$E[A_1] = \frac{\$465 + 4(\$18{,}465) + \$42{,}090}{6} = \$19{,}403,$$

$$Var[A_1] = \left(\frac{\$42{,}090 - 465}{6}\right)^2 = 48{,}128{,}906.$$

Similarly, we can calculate the means and variances for other periods as follows:

n	L	M_o	H	$E[A_n]$	$Var[A_n]$
0	($55,000)	($55,000)	($55,000)	($55,000)	–
1	$465	$18,465	$42,090	$19,403	48,128,906
2	$2,025	$20,992	$47,092	$22,181	56,417,625
3	$1,225	$21,211	$50,018	$22,681	66,132,135
4	$704	$21,764	$53,529	$23,548	77,513,351
5	$6,454	$31,944	$66,643	$33,479	100,630,992

Now we are ready to find the mean and variance of the NPW by using Eqs. (11.1) and (11.2). We have

$$\begin{aligned} E[\text{PW}(15\%)] &= -\$55{,}000 + \frac{\$19{,}403}{(1+0.15)} + \frac{\$22{,}181}{(1+0.15)^2} + \cdots \\ &\quad + \frac{\$33{,}479}{(1+0.15)^5} \\ &= \$23{,}666 \end{aligned}$$

$$\begin{aligned} Var[\text{PW}(15\%)] &= \frac{48{,}128{,}906}{(1+0.15)^2} + \frac{56{,}417{,}625}{(1+0.15)^4} + \frac{66{,}132{,}135}{(1+0.15)^6} \\ &\quad + \frac{77{,}513{,}351}{(1+0.15)^8} + \frac{100{,}630{,}992}{(1+0.15)^{10}} \\ &= 147{,}453{,}769 \end{aligned}$$

$$\begin{aligned} \sigma[\text{PW}(15\%)] &= \sqrt{147{,}453{,}769} \\ &= \$12{,}143. \end{aligned}$$

Once again, if we assume that the PW(15%) is normally distributed with the mean and variance as just calculated, we can determine the probability that the NPW will be negative:

$$\begin{aligned} &\Pr(X \le x) = \Pr(\text{PW}(15\%) \le 0) \\ &= \Phi\left(\frac{X-\mu}{\sigma}\right) = \Phi\left(\frac{0-23{,}666}{12{,}143}\right) = \Phi(-1.9489) \\ &= 1 - \Phi(1.9489) \\ &= 1 - 0.9743 \\ &= 2.57\%. \end{aligned}$$

With the scenario approach in Example 11.4, the worst-case scenario resulted in PW(15%) = −$48,648. However, this possibility is truly remote with independent cash flow assumptions because this value is equivalent to 4σ below the mean, which is practically non-existent. We can say the same thing for the best-case scenario with

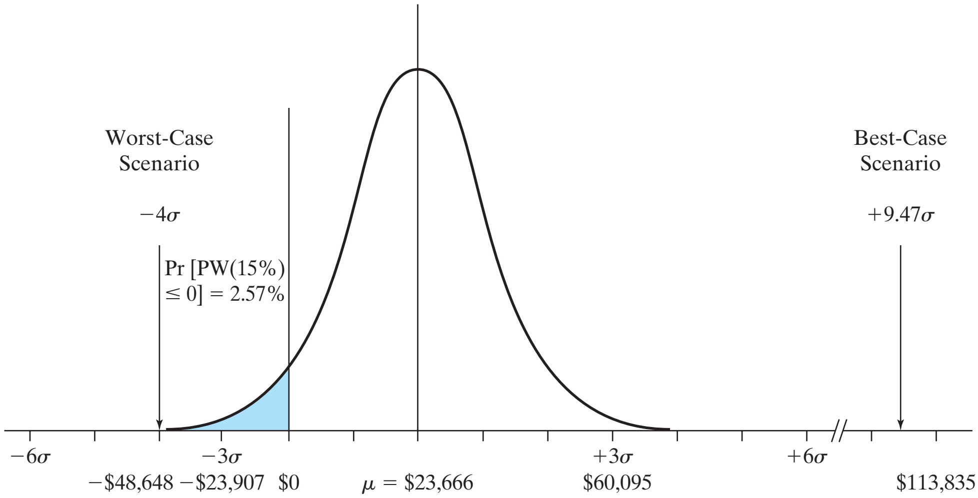


Figure 11.6 The NPW probability distribution for Capstone's C30X project.

$PW(15\%) = \$113{,}835$, which is equivalent to 9.47σ above the mean. (See Figure 11.6.) Clearly, the probabilistic approach outlined in this section provides much more meaningful information regarding the assessment of financial risk in investment evaluation.

COMMENTS: Recall that in deriving the mean and variance of the NPW, we assumed that the cash flows in each period are mutually independent.[5] In fact, these cash flows are not independent. They are perfectly positively correlated as these cash flow estimates are taken from the scenario analysis. In other words, if you assume a worst-case scenario, the cash flow in each period presents a worst case as well, indicating a perfect correlation. Similarly, if we assume a best-case scenario, the cash flow in each period reflects a best case as well. Then, we may easily determine the mean and variance of the NPW on the basis of the three NPW estimates as follows:

$$E[PW(15\%)] = \frac{-\$48{,}648 + 4(\$19{,}202) + (\$113{,}835)}{6}$$
$$= \$23{,}666$$
$$Var[PW(15\%)] = \left(\frac{113{,}835 - (-48{,}648)}{6}\right)^2$$
$$= (27{,}080)^2.$$

With this mean and this variance, the range of possible NPWs extends from $-\$57{,}574(-3\sigma)$ to $\$104{,}906(+3\sigma)$. What it implies is that the worst-case scenario ($-\$42{,}755$, or -1.80σ below the mean) cannot be ruled out completely, but the best-case scenario ($\$113{,}835$, or $+4.20\sigma$ above the mean) may be.

[5] Again, the analytical treatment of this type of dependent relationship is beyond the scope of this introductory text but can be found in Chan S. Park, *Contemporary Engineering Economics,* Prentice Hall, 2011 (Chapter 12).

11.4 Considering the Project Risk by Discount Rate

Without resorting to sophisticated financial tools, financial managers understand intuitively that, other things equal, risky projects are less desirable than safe ones and must provide a higher return. Therefore, an alternative approach to considering risk in project evaluation is to adjust the discount rate to reflect the degree of perceived investment risk. How do we do this?

Many companies estimate the rate of return required by investors in their securities and use this company cost of capital to discount the cash flows on all new projects. Knowing that investors expect a higher rate of return from a risky company, consequently, risky firms will have a higher company cost of capital and will set a higher discount rate for their new investment opportunities. We will first review how managers can estimate the opportunity cost of capital for a company and then how they use this company cost of capital as the basis for determining the discount rate to use in project evaluation.

11.4.1 Determining the Company Cost of Capital

When you invest money in a company, you incur an opportunity equal to the return you could have earned on an alternative, similar risky investment. The opportunity cost is the firm's **cost of capital**—it is the required rate of return the company must earn from existing assets and still meet the expectations of its capital providers. In most of the investment examples in the earlier chapters, we assumed that the projects under consideration were financed entirely with equity funds. In those cases, the cost of capital may have represented only the firm's required return on equity. However, most firms finance a substantial portion of their capital budget with long-term debt (bonds), and many also use common stock as a source of capital. In these cases, a firm's cost of capital must reflect the average cost of the various sources of long-term funds that the firm uses, not only the cost of equity. Therefore, we will examine cost of capital from two sources.

Cost of Equity Capital

Although debt and preferred stocks are contractual obligations that have easily determined costs, it is not easy to measure the cost of equity. In principle, the cost of equity capital involves an **opportunity cost**. In fact, the firm's after-tax cash flows belong to the stockholders. Management may either pay out these earnings in the form of dividends or retain the earnings and reinvest them in the business. If management decides to retain earnings, an opportunity cost is involved; stockholders could have received the earnings as dividends and invested this money in other financial assets. Therefore, the firm should earn on its retained earnings at least as much as the stockholders themselves could earn in alternative, but comparable, investments.

What rate of return can stockholders expect to receive on retained earnings? This question is difficult to answer, but the value sought is often regarded as the rate of return stockholders require on a firm's common stock. If a firm cannot invest retained earnings so as to earn at least the rate of return on equity, it should pay these funds to these stockholders and let them invest directly in other assets that do provide this return. In general, the expected return on any risky asset is composed of three factors:[6]

$$\begin{pmatrix}\text{Expected return}\\ \text{on risky asset}\end{pmatrix} = \begin{pmatrix}\text{Risk-free}\\ \text{interest rate}\end{pmatrix} + \begin{pmatrix}\text{Inflation}\\ \text{premium}\end{pmatrix} + \begin{pmatrix}\text{Risk}\\ \text{premium}\end{pmatrix}.$$

[6] Robert C. Higgins, *Analysis for Financial Management,* 5th ed., New York: Irwin/McGraw-Hill, 1998.

This equation says that the owner of a risky asset should expect to earn a return from three sources:

1. Compensation from the opportunity cost incurred in holding the asset. This is the risk-free interest rate.
2. Compensation for the declining purchasing power of the investment over time, known as the inflation premium.
3. Compensation for bearing risk, known as the risk premium.

Fortunately, we do not need to treat the first two terms as separate factors because together they equal the expected return on a default-free bond such as a government bond. In other words, owners of government bonds expect a return from the first two sources but not the third. So, we can simplify the previous equation as

$$\begin{pmatrix}\text{Expected return}\\ \text{on risky asset}\end{pmatrix} = \begin{pmatrix}\text{Interest rate on a}\\ \text{government bond}\end{pmatrix} + \begin{pmatrix}\text{Risk}\\ \text{premium}\end{pmatrix}.$$

When investors are contemplating buying a firm's stock, they have two primary things in mind: (1) cash dividends and (2) gains (share appreciation) at the time of sale. From a conceptual standpoint, investors determine market values of stocks by discounting expected future dividends at a rate that takes into account any future growth. Since investors seek growing and profitable companies, a desired growth factor for future dividends is usually included in the calculation.

The cost of equity is the risk-free interest rate (for example, a 20-year U.S. Treasury bond that returns around 3%) plus a premium for taking a risk as to whether a return will be received. The risk premium is the average return on the market—typically, the return for Standard & Poor's 500 largest U.S. stocks, or the S&P 500 (say, 12.5%), less the risk-free interest rate. This premium is multiplied by *beta,* an approximate measure of stock price volatility. **Beta** (β) quantifies risk by measuring one firm's stock price relative to all of the market's stock prices as a whole.

- If $\beta > 1$, it means that, on average, the stock is more volatile than the market.
- If $\beta < 1$, it means that, on average, the stock is less volatile than the market.

The values for beta are commonly found for most publicly traded stocks in various sources such as Value Line.[7] The cost of equity (i_e) is quantified by

$$i_e = r_f + \beta[r_M - r_f], \tag{11.5}$$

where

r_f = risk-free interest rate (commonly referenced to U.S. Treasury bond yield, in flation adjusted) and

r_M = market rate of return (commonly referenced to average return on S&P 500 stock index funds, inflation adjusted).

Note that this amount is almost always higher than the cost of debt. This is so because the U.S. Tax Code allows the deduction of interest expense but does not allow the deduction of the cost of equity, which could be considered more subjective and complex. Example 11.7 illustrates how we may determine the cost of equity.

[7] Value Line reports are presently available for more than 5,000 public companies, and that number is growing. The Value Line reports contain the following information: (1) total assets, (2) total liabilities, (3) total equity, (4) long-term debt as a percent of capital, (5) equity as a percent of capital, (6) financial strength (which is used to determine interest rates), (7) beta, and (8) return on invested capital.

EXAMPLE 11.7 Determining the Cost of Equity

Capstone Corporation needs to raise \$55 million for the C30X project described in Example 11.1. Capstone's target capital structure calls for a debt ratio of 0.4, indicating that \$33 million has to be financed from equity. The pertinent information is as follows:

- Capstone is planning to raise \$33 million from the financial markets.
- Capstone's beta is known to be 2.0, which is higher than 1, indicating that the firm is perceived to be riskier than the market average.
- The risk-free interest rate is 2.57%, and the average market return is 10.99%. (These interest rates are adjusted to reflect inflation in the economy.)
- The C30X project is a normal risky project comparable to the firm's market risk.

Determine the cost of equity to finance the plant modernization.

DISSECTING THE PROBLEM	**Given:** $r_M = 10.99\%$, $r_f = 2.57\%$, and $\beta = 2.0$. **Find:** i_e.
METHODOLOGY Compute i_e.	**SOLUTION** We calculate i_e as follows: $i_e = 0.0257 + 2.0(0.1099 - 0.0257) = 19.41\%.$

COMMENTS: What does this 19.41% represent? If Capstone finances the project entirely from its equity funds, the project must earn at least a 19.41% return on investment to be worthwhile, assuming that the project's risk is about the same as the average risk of the company's assets and operations.

Cost of Debt Capital

Now let us consider the calculation of the specific cost that is to be assigned to the debt financing. The calculation is relatively straightforward and simple. Two types of debt financing are term loans and bonds. Because the interest payments on both are tax deductible, they reduce the effective cost of debt. To determine the after-tax cost of debt (i_d), we can evaluate the expression:

$$\text{After-tax cost of debt} = \text{pretax cost} \times (1 - \text{tax rate})$$

or,

$$i_d = \left(\frac{c_s}{c_d}\right)k_s(1 - t_m) + \left(\frac{c_b}{c_d}\right)k_b(1 - t_m), \qquad (11.6)$$

where

$c_s =$ the amount of the term loan,
$k_s =$ the before-tax interest rate on the term loan,
$t_m =$ the firm's marginal tax rate,
$k_b =$ the before-tax interest rate on the bond, and
$c_b =$ the amount of bond financing and $c_s + c_b = c_d$.

Example 11.8 illustrates the process of computing the cost of debt for the Capstone Corporation scenario introduced in Example 11.7.

EXAMPLE 11.8 Determining the Cost of Debt

For the case in Example 11.7, suppose that Capstone decided to finance the remaining $22 million by securing a term loan and issuing 20-year $1,000 par bonds under the following conditions:

		Interest	
Source	**Amount**	**Fraction**	**Rate**
Term loan	$6.6 million	0.30	12.16% per year
Bonds	$15.4 million	0.70	10.74% per year

Capstone's marginal tax rate is 25%, which is expected to remain constant in the future. Determine the after-tax cost of debt.

DISSECTING THE PROBLEM

Given: $k_s = 12.16\%$, $k_b = 10.74\%$, $c_s/c_d = 0.30$, $c_b/c_d = 0.70$, $t_m = 25\%$.
Find: i_d.

METHODOLOGY

Solve for i_d.

SOLUTION

The after-tax cost of debt is the interest rate on debt multiplied by $(1 - t_m)$. In effect, the government pays part of the cost of debt because interest is tax deductible. Now we are ready to compute the after-tax cost of debt as follows:

$$\begin{aligned} i_d &= (0.30)(0.1216)(1 - 0.25) + (0.70)(0.1074)(1 - 0.25) \\ &= 8.38\%. \end{aligned}$$

COMMENTS: What does this 8.38% interest really mean? Even though you are borrowing the money at 12% and 10.74% from two different sources, your net cost of composite borrowing will be just 8.38%, as debt interest payments are tax deductible expenses.

Calculating the Company Cost of Capital

With the specific cost of each financing component determined, we are now ready to calculate the weighted-average cost of capital on the basis of total capital. Assuming that a firm raises capital on the basis of the target capital structure and that the target capital structure remains unchanged in the future, we can determine a **company cost of capital**, which is a **weighted-average cost of capital** (or, simply stated, the **cost of capital, k**). As illustrated in Figure 11.7, this cost of capital represents a composite

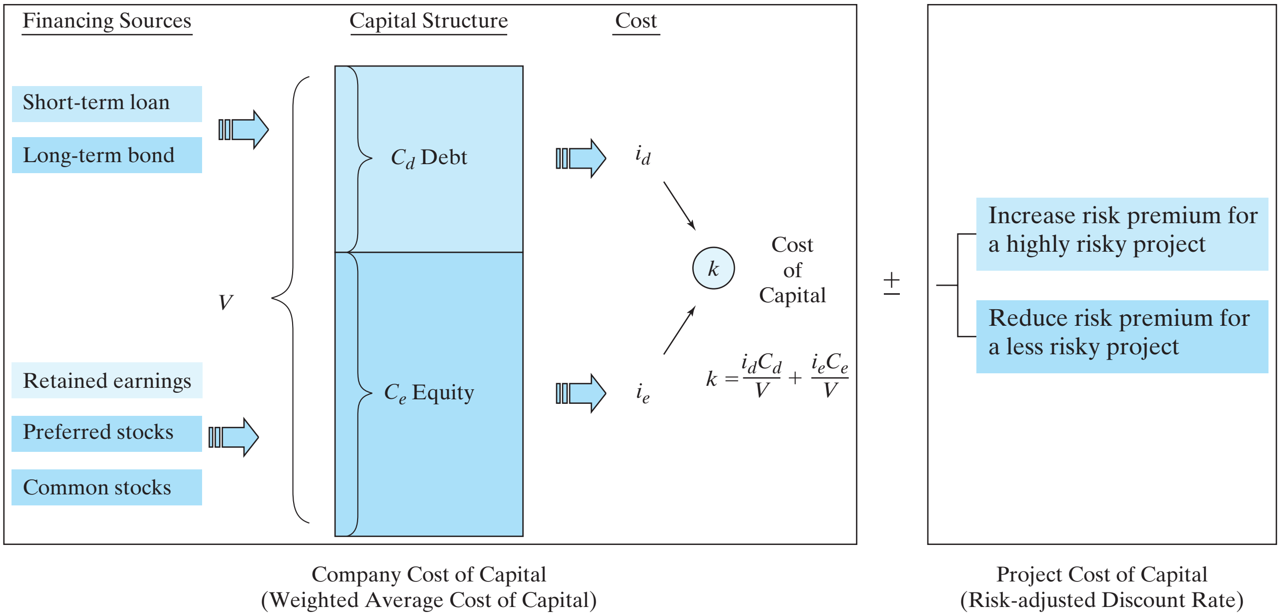


Figure 11.7 Company cost of capital versus project cost of capital. In project evaluation, we use the project cost of capital.

index reflecting the cost of raising funds from different sources. The cost of capital is defined as

$$k = i_d\left(\frac{c_d}{V}\right) + i_e\left(\frac{c_e}{V}\right), \tag{11.7}$$

where

c_d = total debt capital (such as bonds) in dollars,
c_e = total equity capital in dollars,
$V = c_d + c_e$,
i_e = average equity interest rate per period, considering all equity sources,
i_d = after-tax average borrowing interest rate per period, considering all debt sources, and
k = weighted-average cost of capital.

Note that the cost of equity is already expressed in terms of after-tax cost because any return to holders of either common stock or preferred stock is made after the payment of income taxes.

In evaluating an investment project, the costs of debt and equity in Eq. (11.7) are the interest rates on *new debt* and *equity*,[8] not existing debt or equity, respectively. Our primary concern with the cost of capital is to use it in evaluating a new investment project. The rate at which the firm has borrowed in the past is less important for this purpose. Example 11.9 works through the computations for finding the cost of capital (k).

[8] This is known as the **marginal cost of capital**, the cost of obtaining another dollar of new capital, and this value rises as more and more capital is raised during a given period.

EXAMPLE 11.9 Calculating the Company Cost of Capital

Reconsider Examples 11.7 and 11.8. The marginal income-tax rate (t_m) for Capstone is expected to remain at 25% in the future. Assuming that Capstone's capital structure (debt ratio) also remains unchanged in the future, determine the cost of capital (k) of raising $55 million in addition to Capstone's existing capital.

DISSECTING THE PROBLEM

Given: With $c_d = \$22$ million, $c_e = \$33$ million, $V = \$55$ million, $i_d = 8.38\%,\ i_e = 19.41\%$, and Eq. (11.7).

Find: Marginal cost of capital (k).

METHODOLOGY

Solve for cost of capital (k).

SOLUTION

We calculate the marginal cost of capital as follows:

$$k = \frac{(0.0838)(4)}{10} + \frac{(0.1941)(6)}{10} = 15\%.$$

This 15% would be the marginal cost of capital that a company with this capital structure would expect to pay to raise $55 million, and this is the discount rate to be used in evaluating the Capstone's new investment project. In other words, the project must return more than 15% in order to be justified.

11.4.2 Project Cost of Capital: Risk-Adjusted Discount Rate Approach

It is important to distinguish between the *risk of the project* and the *risk of the company*. The risk of the company is reflected in the β value. So even if a risky company invests in a low-risk project, it should discount the cash flows at a correspondingly low cost of capital. If it invests in a high-risk project, those cash flows should be discounted at a high cost of capital. The most common way to do this is to add an increment to the discount rate—that is, discount the expected value of the risky cash flows at a rate that includes a premium for risk. The size of the risk premium naturally increases with the perceived risk of the investment. This discussion implies a simple rule—the discount rate of a project should be the expected return on a financial asset of comparable risk known as the **project cost of capital**.

What makes the risk-adjusted discount rate approach appealing in practical application is its simplicity. Most chief financial officers (CFOs) have at least a rough idea of how an investment's required rate of return should vary with perceived risk. For example, they know from the historical data that, over many years, common stocks have yielded an average annual return about 7% higher than the return on government bonds (risk-free return). If the present return on government bonds is 8%, it is plausible to expect an investment that is about as risky as common stocks to yield a return of about 15%. Similarly, CFOs know that an investment promising a return of 40% is attractive unless its risk is extraordinarily high. Granted, such reasoning is imprecise; nonetheless, it does lend some objectivity to risk assessment. To illustrate the use of such risk-adjusted discount rates, consider Example 11.10.

EXAMPLE 11.10 Risk-Adjusted Discount Rate Approach

Reconsider Example 11.6, where the expected cash flows for the Capstone project are

Period	0	1	2	3	4	5
Cash Flow	−$55,000	$19,403	$22,181	$22,681	$23,548	$33,479

Suppose that Capstone consider the C30X project to be just one of their normal risky projects. Then the appropriate discount rate to use is 15%, as calculated in Example 11.9. However, Capstone considers the C30X project to be much riskier than normal projects, so it believes an additional risk premium of 6.93% should be added. If management has decided to use a risk-adjusted discount rate of 21.93% to compensate for the uncertainty of the cash flows, is this project acceptable?

DISSECTING THE PROBLEM

You need to determine whether the project under consideration is of normal risk. If not, you need to come up with a desired risk premium to add.

Given: Annual cash flows given in most likely values, $N = 5$ years, $k = 15\%$, and additional risk premium $= 6.93\%$.

Find: PW.

METHODOLOGY

Use the risk-adjusted discount rate to calculate the net present value.

SOLUTION

First find the project cost of capital (risk-adjusted discount rate):

$$15\% + 6.93\% = 21.93\%.$$

Then calculate the net present value, using this project cost of capital:

$$\begin{aligned} PW(21.93\%) &= -\$55{,}000 + \frac{\$19{,}403}{1.2193} + \frac{\$22{,}181}{1.2193^2} + \cdots + \frac{\$33{,}479}{1.2193^5} \\ &= \$11{,}421 > 0. \end{aligned}$$

Because the PW is still positive, the investment is attractive even after we adjust for further risk.

COMMENTS: Note that the risk-adjusted discount rate reduces the investment's appeal. If the investment were normal risk, its NPW at a 15% discount rate would be $23,666; but because a higher risk-adjusted rate is deemed appropriate, the NPW falls by $12,245. Certainly, as the perceived project risk increases beyond the 6.93% risk premium, management would require an inducement higher than this amount before it would be willing to make the investment.

11.5 Value at Risk

Thus far, we have considered the risk element for a single project. For a firm with many ongoing projects or a financial portfolio, they may be interested in knowing the company-wide risk exposure. As introduced in Section 11.3.1, the most common and traditional measure of risk is volatility as expressed in terms of the dispersion or standard deviation in possible outcomes. The main problem with volatility, however, is that it views any deviation from the mean undesirable; in other words, the direction of an investment's movement is not reflected. For any investor, unexpected gains above the mean value are no cause of concern. However, for most investors, risk is about the odds of losing money, and they would be interested in knowing the answer to "What is the most I can lose on my investment?" The **value at risk** (VaR) tries to provide an answer, at least within a reasonable bound. In its most general form, the VaR measures the potential loss in value of a risky asset or portfolio over a target horizon within a given confidence interval. A VaR statistic has three components: a time period, a confidence level, and a loss amount (or loss percentage). For example, we may rephrase our question in terms of dollar amount or percentage:

- What is the most I expect to lose in dollars over the next month with 95% confidence?
- What is the maximum percentage return I can expect to lose over the next year with 95% confidence?

While VaR can be used by any entity to measure its risk exposure, it is used most often by financial institutions to capture the potential loss in value of their traded portfolios from market movements over a specified period. Once the potential loss amount is determined, then this can be compared to their available capital and cash reserves to see if the losses can be covered without putting the firms at risk. However, we need to understand the limitation of the VaR measure. The problem with relying solely on the VaR model is that the scope of risk assessed is limited, since the tail end of the distribution of loss is not typically assessed. Seeking refinement to the VaR, we may calculate the mean excess loss, commonly known as **conditional value at risk (CVaR)**. If we use CVaR as a risk measure, the smaller the value of the CVaR, the better. In this introductory text, we will just focus on the VaR measure.

11.5.1 Calculation of VaR

Suppose you hold \$100 million in traded securities. How much could you lose in a month? To answer this question, you need to obtain monthly returns on those traded securities. Suppose that returns ranged from a low of −10.5% to a high of +9.0%. Now construct evenly spaced "intervals" going from the lowest to the highest number and count how many observations fall into each interval. Basically you are constructing a histogram of the returns. If you have enough observations, you may attempt to fit a well-known probability distribution based on the shape of the histogram. Once a probability distribution of return is defined, we can easily compute the probability that the return (or value) of an asset or portfolio will drop below a critical value in a particular time period.

A normal distribution as shown in Figure 11.8 is a bell-shaped probability distribution with many practical applications found in investment analysis. The advantage of the normal curve is that we automatically know where the worst 5% and 1% lie on the curve. They are a function of the confidence level and the standard deviation of the return distribution. With known mean (μ) and variance (σ^2) of the return distribution, we can easily determine that critical value ($x = x_c$), which is known as VaR. If a return

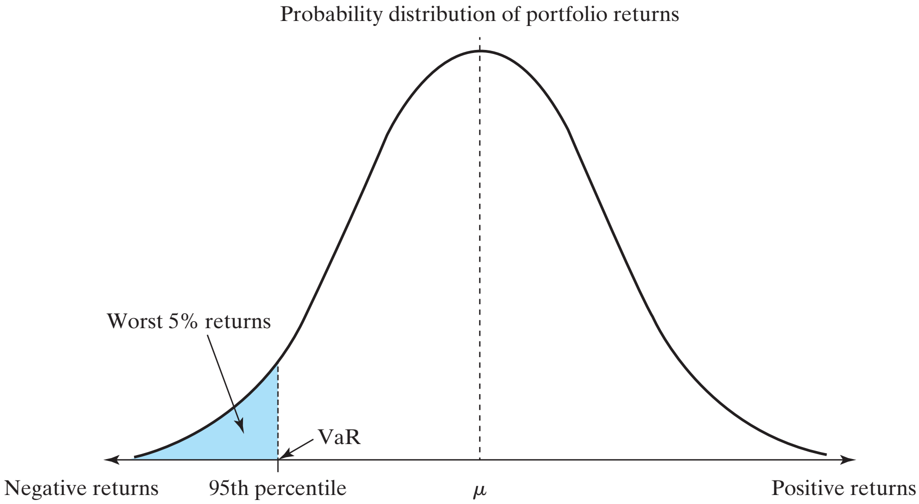


Figure 11.8 Value at risk at a 95% confidence level.

variable X is not "standard," its value must be standardized: $z = \dfrac{x - \mu}{\sigma}$. For example, with a 95% confidence level, the cumulative probabilistic statement is as follows:

$$P(X \geq x) = 1 - P(X \leq x) = 1 - \Phi(z) = 0.95.$$

The unshaded area in Figure 11.8 represents the cumulative probability that the return would exceed the critical value. From the normal distribution table, we find

Confidence	*Z*-value
95%	−1.647
99%	−2.325

Then, the VaR is calculated by plugging μ and σ into the $z = \dfrac{x - \mu}{\sigma}$ formula and solve for x.

Confidence	Critical Value (VaR)
95%	$-1.647 \times \sigma + \mu$
99%	$-2.325 \times \sigma + \mu$

11.5.2 Conversion of VaR Parameters

With a normal distribution for the asset or portfolio returns, we can easily convert one horizon or confidence level to another. As returns across different periods are close to uncorrelated, the variance of an N-day return should N times the variance of a 1-day return.

$$\text{VaR}\ (N\ \text{days}) = \text{VaR}\ (1\text{-day}) \times \text{SQRT}(N).$$

Once again, with a normal distribution assumption, conversion across confidence levels is

$$\text{VaR}(99\%) = \text{VaR}(95\%) \times 2.325/1.645.$$

In summary, the VaR summarizes the portfolio's exposure to market risk as well as the probability of an adverse move. Knowing this information, the investor (or the firm) would be able to set aside an appropriate amount of contingency funds to cover the worst case.

EXAMPLE 11.11 Computing the Value at Risk for AAPL

Consider the Apple stock (AAPL). If we calculate each daily return between May 23, 2016, and May 17, 2017, we produce a data set of 250 points. If we put them in a histogram that compares the frequency of return "buckets," it looks like Figure 11.9. The mean and standard deviation of the daily return (x_i) based on 250 trading days are as follows:

$$\text{Mean } (\mu) = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{250}\sum_{i=1}^{250} x_i = 0.1912\%;$$

$$\text{Standard deviation } (\sigma) = \sqrt{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{n-1}} = \sqrt{\sum_{i=1}^{250}\frac{(x_i-\mu)^2}{250-1}} = 1.1268\%.$$

If we can assume the daily return histogram is approximately normally distributed with the mean and standard deviation as shown above, determine the VaR per day, with 95% confidence level, for an investor who holds the AAPL stock in the amount of $1,000,000.

DISSECTING THE PROBLEM

We need to determine the amount of deviation from the mean that is corresponding to the 95% confidence interval with a normal distribution.

Given: $\mu = 0.001912$, $\sigma = 0.011268$, $N = 20$ trading days (1 month), $\alpha = 95\%$, investment amount at risk = $1,000,000

Find: VaR at 95%

	A	B
1	Daily Return	Frequency
2	−3.36%	1
3	−2.70%	2
4	−2.04%	5
5	−1.39%	8
6	−0.73%	17
7	−0.07%	66
8	0.58%	79
9	1.24%	40
10	1.90%	20
11	2.56%	6
12	3.21%	2
13	3.87%	2
14	4.53%	0
15	5.18%	0
16	5.84%	0
17	6.50%	2

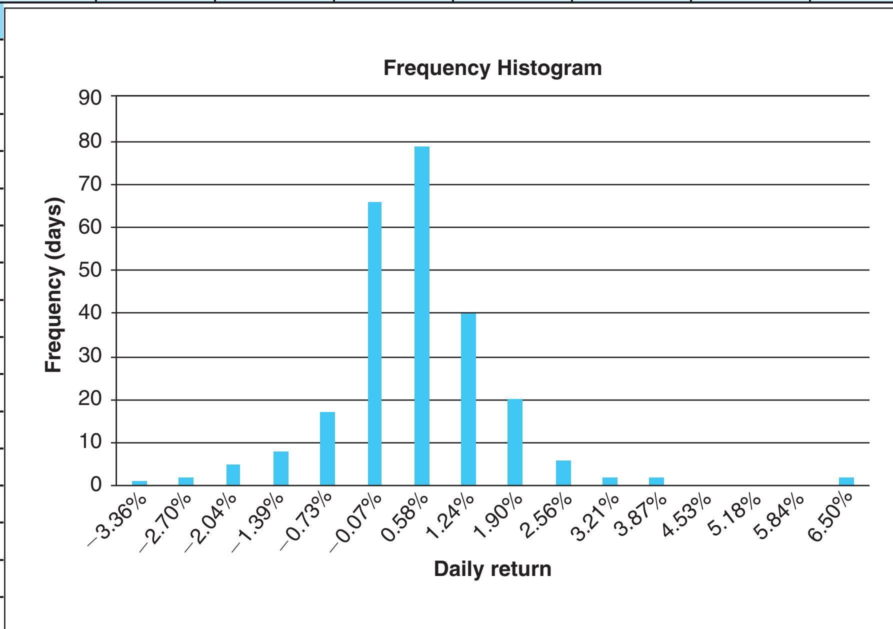


Figure 11.9 Frequency histogram for daily return (AAPL) based on 250 trading days.

METHODOLODY

Since the return variable X is not "standard," its value must be standardized: $z = \frac{x - \mu}{\sigma}$.

At a 95% confidence level, Z-value is -1.647.

SOLUTION

$$P(X \le x) = \Phi\left(z = \frac{x - \mu}{\sigma}\right)$$
$$= \Phi\left(z = \frac{x - 0.001912}{0.011268}\right).$$
$$P(X \ge x) = 1 - P(X \le x) = 1 - \Phi\left(z = \frac{x - 0.001912}{0.011268}\right) = 0.95.$$

From the normal table, we read that $z = -1.647$.
Then, we find

$$x = -1.647 \times 0.011268 + 0.001912 = -0.016646;$$
$$\text{VaR} = \$1{,}000{,}000 \times -0.016646 = -\$16{,}646 \text{ per day}.$$

If we invest \$1,000,000, we are 95% confident that our worst daily loss will not exceed \$16,646. This is shown in Figure 11.10.

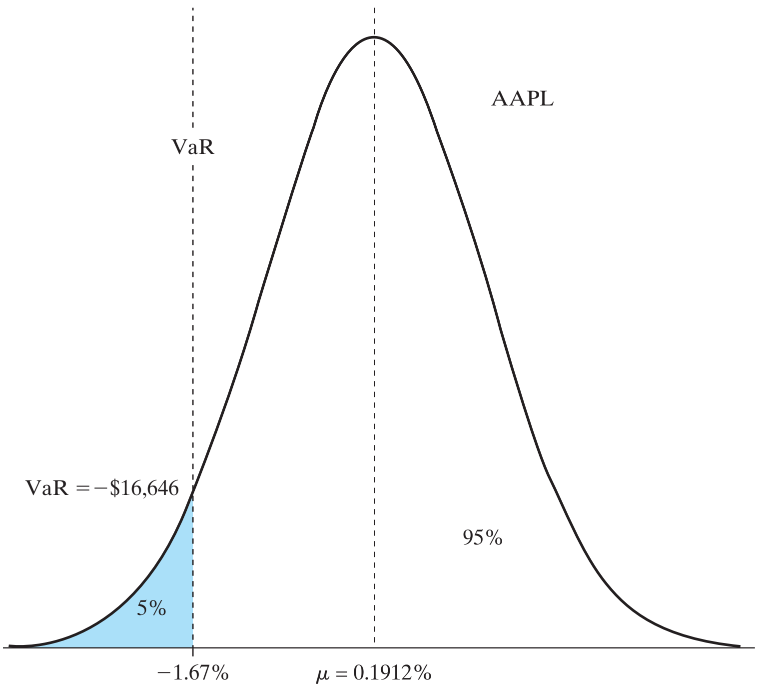


Figure 11.10 Value at risk for AAPL

COMMENTS: For the same investor, the VaR over 1-month holding period (or with 20 trading days per months) is determined as follows:

$$\text{VaR(month)} = \$16{,}646 \times \sqrt{20} = \$74{,}443.$$

EXAMPLE 11.12 Computing the Value at Risk for an Investment Project

Reconsider the net present worth distribution for Capstone's investment project developed in Example 11.6. Assuming that the NPW distribution be normally distributed with

$$E[PW(15\%)] = \$14{,}817$$
$$Var[PW(15\%0)] = (9{,}714)^2$$

Compute the net present worth at risk at a 95% confidence level for the duration of the project.

DISSECTING THE PROBLEM

Given: $\mu = \$14{,}817$, $\sigma^2 = (9{,}714)^2$, $\alpha = 95\%$

Find: VaR

METHODOLOGY

The NPW variable is not standard, its value must be standardized:

$$z = \frac{x - \mu}{\sigma}.$$

At a 95% confidence level, z value is -1.647.

SOLUTION

We calculate the VaR for the project as follows:

$$P(X \leq x) = \Phi\left(z = \frac{x - \mu}{\sigma}\right)$$
$$= \Phi\left(z = \frac{x - 14{,}817}{9{,}714}\right).$$

$$P(X \geq x) = 1 - P(X \leq x) = 1 - \Phi\left(z = \frac{x - 14{,}817}{9{,}714}\right) = 0.95.$$

From the normal table, we read that $z = -1.647$.

Then, we find

$$x = -1.647 \times \$9{,}714 + \$14{,}817 = -\$1{,}181.96$$

$$\text{Value at Risk} = (\$1{,}181.96) \text{ per investment as a whole}$$

COMMENTS: Unlike the financial asset in Example 11.11, the risk exposure is defined over the project life, assuming that you will held the project over the entire project life.

SUMMARY

- Often, cash flow amounts and other aspects of investment project analysis are uncertain. Whenever such uncertainty exists, we are faced with the difficulty of **project risk**—the possibility that an investment project will not meet our minimum requirements for acceptability and success.
- Three of the most basic tools for assessing project risk are as follows:
  1. **Sensitivity analysis**—identifying the project variables that, when varied, have the greatest effect on project acceptability.
  2. **Break-even analysis**—identifying the value of a particular project variable that causes the project to exactly break even.
  3. **Scenario analysis**—comparing a base-case, or expected, project measurement (such as PW) with the measurement(s) for one or more additional scenarios, such as best and worst case, to identify the extreme and most likely project outcomes.
- In considering the risk elements in project evaluation, there are two common approaches.
  1. The first approach is to describe the riskiness of the project cash flows in terms of probability distributions and then to use the risk-free interest rate to determine the net-present-value distribution. Once you obtain the PW distribution (mean and variance), you need to determine whether the expected value of the PW distribution is large enough to undertake the risk perceived in the project, which is revealed by the variance of the PW distribution.
  2. The second approach is to adjust the discount rate to reflect your perceived risk in terms of the risk premium and then use this adjusted rate to discount the expected cash flows. In practice, the risk-adjusted discount-rate approach is much more popular, as the risk-assessment process is much simpler than the probabilistic approach.
- The company cost of capital (or **weighted-average cost of capital,** or simply **cost of capital, *k***) is the rate of return that the firm must expect to earn on its average-risk investments in order to provide a fair expected return to all its stockholders. Therefore, if the firm uses the weighted-average cost of capital as a discount rate for project evaluation, it assumes that the project risk is about the same as the risk of the firm's existing business.
- The **project cost of capital** can be different from the company cost of capital if the firm uses the capital raised from all sources put in a project whose risk is different from the risk of existing business operation. The risk-adjusted discount rate refers to this project cost of capital.
- The VaR estimates how much a set of investments might lose, given normal market conditions, in a set time period such as a day. VaR is typically used by firms and financial institutions to gauge the amount of assets needed to cover possible losses.

SELF-TEST QUESTIONS

11s.1 For a certain investment project, the net present worth can be expressed as functions of sales price (X) and variable production cost Y of PW $= 12{,}350\ (2X - Y) - 8{,}480$. The base values for X and Y are \$30 and \$20, respectively. If the sales price is increased 10% above the base price, how much change in NPW can be expected?

(a) 20.82% (b) 15.26% (c) 13.68% (d) 12.32%

11s.2 An investor bought 100 shares of stock at a cost of \$10 per share. He held the stock for 15 years and wants to sell it now. For the first three years, he received no dividends. For each of the next seven years, he received total dividends of \$100 per year. For each of the remaining five years, no dividends were paid. In the last 15 years, the investor's marginal tax rate and capital gain tax rate were averaging about 30% and 20%, respectively. What would be the break-even selling price to earn a 15% return on investment after tax?

(a) \$6,579 (b) \$7,977 (c) \$8,224 (d) \$9,398

11s.3 A company is currently paying a sales representative \$0.54 per mile to drive her car for company business. The company is considering supplying the representative with a car, which would involve the following:

- Option 1: Continue to pay at the rate of 54 cents per mile.
- Option 2: Provide a company vehicle to the sales representative. A car costs \$24,000 and has a service life of five years and a market value of \$7,000 at the end of that time. The cost of keeping the car in the garage during the off-hours amounts to \$2,500 a year, and the cost of fuel, tires, and maintenance is 30 cents per mile. The car will be depreciated by MACRS using a recovery period of five years (20%, 32%, 19.20%, 11.52%, 11.52%).

The firm's marginal tax rate is 25%. What annual mileage must the sales representative travel by car for the cost of the two options of providing transportation to be equal if the interest rate is 15%?

(a) 36,345 miles (b) 38,966 miles (c) 45,233 miles (d) 47,518 miles

11s.4 A manufacturing company is considering a capacity expansion investment at the cost of \$220,000. The expansion would enable the company to produce up to 85,000 more parts and the useful life of the additional capacity is six years. Each part would generate \$3 net profit and annual operating and maintenance costs are estimated at \$27,000 per year. If the MARR of the firm is 12%, what is the minimum yearly production rate to make this investment justifiable? Assume a salvage value of 0.

(a) Less than 27,000 (b) Between 27,000 and 30,000
(c) Between 30,000 and 34,000 (d) More than 34,000

11s.5 Project A has the following probability distribution of net future returns:

Probability	Net Future Worth
0.1	−\$16,000
0.2	\$12,000
0.4	\$15,000
0.3	\$18,000

What is the expected net future worth for Project A?
(a) \$9,450 (b) \$10,800 (c) \$11,400 (d) \$12,200

11s.6 In Problem 11s.5, what is the standard deviation of net future worth for Project A?
(a) \$9,843.54 (b) \$2,987.60 (c) \$10,800 (d) \$8,945.21

11s.7 The Arizona Mining Company contemplates investing \$4 million in new sets of ripping equipment to expand its copper mining operation. The management of the company forecasts that the new investment will generate incremental net cash inflows of $A_n(n = 1, 2, \ldots, 5)$, where each of the A_i is a random variable with a mean of \$2 million and a standard deviation of \$400,000. The salvage value of the mining equipment at the end of year 5 will be also a random variable with a mean of \$1 million and a standard deviation of \$300,000. Compute the *mean* of the present value of this investment. Assume that the cash inflows of A_n are mutually independent random variables and the risk-free interest rate is 10%.
(a) \$2.9607 (b) \$3.235 (c) \$4.239 (d) −\$3.276

11s.8 In Problem 11s.7, compute the variance of the present value of the investment.
(a) −0.2220 (b) 0.2220 (c) 0.4711 (d) 0.5029

11s.9 In Problem 11s.7, the degree of project cash flow uncertainty is instead captured by adjusting the discount rate from 10% to 15%. Determine the certainty equivalent present value of the project at this risk-adjusted discount rate.
(a) \$2.5684 (b) \$2.980 (c) \$3.2015 (d) \$3.674

11s.10 Harry Wilson, a mechanical engineer at Lehigh Manufacturing, has found that the anticipated profitability of a newly developed motion detector for its popular home security device product line can be estimated as follows:

$$\text{PW} = 40.28W(2X - 11) - 77{,}860.$$

where W is the number of units produced and sold and X is the sales price per unit. Harry also found that W parameter value could occur anywhere over a range of 1,000 to 6,000 units and the X parameter value anywhere between \$20 and \$40 per unit.

Suppose both W and X are statistically independent continuous random variables with the following means and variances:

$$E[W] = 3{,}500,\ V[W] = 2{,}083{,}333$$
$$E[X] = 30,\ V[X] = 33.$$

What is the expected PW?
(a) \$5,932 (b) \$6,321 (c) \$6,830 (d) \$7,348

11s.11 In 11s.10, W and X are mutually independent discrete random variables with the following probabilities:

W		*X*	
Event	**Probability**	**Event**	**Probability**
1,000	0.4	\$20	0.7
6,000	0.6	\$40	0.3

What is the probability that the PW would exceed \$6,000,000?
(a) 0.28 (b) 0.40 (c) 0.60 (d) 0.82

PROBLEMS

Sensitivity Analysis

11.1 A machine costing $23,000 to buy and $5,000 per year to operate will save mainly labor expenses in packaging over five years. The anticipated salvage value of the machine at the end of six years is $4,000.

(a) If a 14% return on investment (rate of return) is desired, what is the minimum required annual savings in labor from this machine?

(b) If the service life is four years instead of five years, what is the minimum required annual savings in labor for the firm to realize a 14% return on investment?

(c) If the annual operating cost increases 10%, say, from $5,000 to $5,500, what will happen to the answer to (a)?

11.2 The Wellington Construction Company is considering acquiring a new earthmover. The mover's basic price is $100,000, and it will cost another $24,000 to modify it for special use by the company. This earthmover falls into the MACRS five-year class. It will be sold after four years for $30,000. The purchase of the earthmover will have no effect on revenues, but it is expected to save the firm $35,000 per year in before-tax operating costs, mainly labor. The firm's marginal tax rate (federal plus state) is 25%, and its MARR is 8%.

(a) Is this project acceptable, based on the most likely estimates given?

(b) Suppose that the project will require an increase in net working capital (spare parts inventory) of $6,000, which will be recovered at the end of year 4. Taking this new requirement into account, would the project still be acceptable?

11.3 The Minnesota Metal Forming Company has just invested $500,000 of fixed capital in a manufacturing process that is estimated to generate an after-tax annual cash flow of $200,000 in each of the next five years. At the end of year 5, no further market for the product and no salvage value for the manufacturing process is expected. If a manufacturing problem delays the start-up of the plant for one year (leaving only four years of process life), what additional after-tax cash flow will be needed to maintain the same internal rate of return as would be experienced if no delay occurred?

11.4 You are considering an investment project with the following financial information:

(a) Required investment = $500,000

(b) Project life = 5 years

(c) Salvage value = $50,000

(d) Depreciation method = straight-line depreciation (no half-year convention)

(e) Unit price = $40

(f) Unit variable cost = $18

(g) Fixed annual cost = $230,000

(h) Annual sales volume = 100,000 units

(i) Tax rate = 35%

(j) MARR = 15%

Suppose the company is most concerned about the impact of its price estimate on the project's rate of return. How would you address this concern?

11.5 A real-estate developer seeks to determine the most economical height for a new office building, which will be sold after five years. The relevant net annual revenues and salvage values on after-tax basis are as given in Table P11.5.

TABLE P11.5

	Height			
	2 Floors	**3 Floors**	**4 Floors**	**5 Floors**
First cost	\$500,000	\$750,000	\$1,250,000	\$2,000,000
Lease revenue	\$199,100	\$169,200	\$149,200	\$378,150
Net resale value	\$600,000	\$900,000	\$2,000,000	\$3,000,000

(a) The developer is uncertain about the interest rate (i) to use, but is certain that it is in the range from 5% to 30%. For each building height, find the range of values of i for which that building height is the most economical.

(b) Suppose that the developer's interest rate is known to be 15%. What would be the cost (in terms of net present value) of a 10% overestimation of the resale value? (In other words, the true value was 10% lower than that of the original estimate.)

11.6 Mike Lazenby, an industrial engineer at Energy Conservation Service, has found that the anticipated profitability of a newly developed water-heater temperature-control device can be measured by present worth with the formula

$$\text{NPW} = 4.028V(2X - \$11) - 77{,}860.$$

where V is the number of units produced and sold and X is the sales price per unit. Mike also has found that the value of the parameter V could occur anywhere over the range from 1,000 to 6,000 units and that of the parameter X anywhere between \$20 and \$45 per unit. Develop a sensitivity graph as a function of the number of units produced and the sales price per unit.

11.7 Burlington Motor Carriers, a trucking company, is considering the installation of a two-way mobile satellite messaging service on its 2,000 trucks. From tests done last year on 120 trucks, the company found that satellite messaging could cut 60% of its \$5 million bill for long-distance communications with truck drivers. More importantly, the drivers who used this system reduced the number of "deadhead" miles—those driven without paying loads—by 0.5%. Applying that improvement to all 230 million miles covered by the Burlington fleet each year would produce an extra \$1.25 million in savings.

Equipping all 2,000 trucks with the satellite hookup will require an investment of \$8 million and the construction of a message-relaying system costing \$2 million. The equipment and onboard devices will have a service life of eight years and negligible salvage value; they will be depreciated under the five-year MACRS class. Burlington's marginal tax rate is about 25%, and its required minimum attractive rate of return is 18%.

(a) Determine the annual net cash flows from the project.

(b) Perform a sensitivity analysis on the project's data, varying savings on the telephone bill and savings in deadhead miles. Assume that each of these variables can deviate from its base-case expected value by ±10%, ±20%, and ±30%.

(c) Prepare sensitivity diagrams and interpret the results.

Break-Even Analysis

11.8 Susan Campbell is thinking about going into the motel business near Disney World in Orlando, Florida. The cost to build a motel is $2,200,000. The lot costs $600,000. Furniture and furnishings cost $400,000 and should be recovered in eight years (seven-year MACRS property), while the motel building should be recovered in 39 years (39-year MACRS real property placed in service on January 1st). The land will appreciate at an annual rate of 5% over the project period, but the building will have a zero salvage value after 25 years. When the motel is full (100% capacity), it takes in (receipts) $4,000 per day, 365 days per year. Exclusive of depreciation, the motel has fixed operating expenses of $230,000 per year. The variable operating expenses are $170,000 at 100% capacity, and these vary directly with percent capacity down to zero at 0% capacity. If the interest is 10% compounded annually, at what percent capacity over 25 years must the motel operate in order for Susan to break even? (Assume that Susan's tax rate is 31%.)

11.9 D&D Machinery has been making a part for its industrial rotary gear shaving machine. D&D engineers are asked to investigate alternative ways of obtaining the part, as the unit cost for the part currently is not competitive in the marketplace. D&D needs 25,000 parts per year for the next three years. At that point, any capital equipment could be sold. D&D's tax rate is 25%, and its MARR is 12%.

- Option A: Continue to produce the part with the old machine. The machine has been fully depreciated. The current machine could be sold for $6,000 in three years. Making the part with the old machine involves the following: Variable costs for the part are $4 for direct materials, $3 for direct labor, and $2 for variable manufacturing overhead.
- Option B: Purchase the part from outside for $13 per part, including shipping.
- Option C: Replace the old machine with the new model. The newer model would cost $55,000 and would depreciate according to a seven-year MACRS method. The new machine, if purchased, could be sold for $15,000 in three years. It would cut direct labor costs to $1.50 per unit and variable costs to $0.75 per part. If the new model is acquired, the old machine would be sold for $25,000.

(a) When Option B is compared with Option A, determine the break-even outsourcing unit cost.

(b) When Option B is compared with Option C, determine the break-even outsourcing cost per unit.

(c) Which alternative is the most economical?

11.10 A plant engineer wishes to know which of two types of light bulbs should be used to light a warehouse. The bulbs that are currently used cost $46.20 per bulb and last 26,300 hours before burning out. The new bulb (at $62 per bulb) provides the same amount of light and consumes the same amount of energy, but it lasts six years. The labor cost to change a bulb is $20. The lights are on 18 hours a day, 365 days a year. If the firm's MARR is 10%, what is the maximum price (per bulb) the engineer should be willing to pay to switch to the new bulb? (Assume that the firm's marginal tax rate is 25%.)

11.11 Robert Cooper is considering purchasing a piece of business rental property containing stores and offices at a cost of $250,000. Cooper estimates that annual disbursements (other than income taxes) will be about $16,000. The property is expected to appreciate at the annual rate of 6%. Cooper expects to retain the property for 16 years once it is acquired. Then it will be depreciated as a 39-year real-property class (MACRS), assuming that the property will be placed in service on January 1. Cooper's marginal tax rate is 30% and his MARR is 14%. What would be the minimum annual total of rental receipts that would make the investment break even?

11.12 Two different methods of solving a production problem are under consideration. Both methods are expected to be obsolete in six years. Method A would cost $80,000 initially and have annual operating costs of $22,000 per year. Method B would cost $52,000 and costs $17,000 per year to operate. The salvage value realized would be $20,000 with Method A and $15,000 with Method B. Investments in both methods are subject to a five-year MACRS property class. The firm's marginal income tax rate is 25%. The firm's MARR is 20%. What would be the required additional annual revenue for Method A such that an engineer would be indifferent to choosing one method over the other?

11.13 The Rocky Mountain Publishing Company is considering introducing a new morning newspaper in Denver. Its direct competitor charges $0.25 at retail with $0.05 going to the retailer. For the level of news coverage the company desires, it determines the fixed cost of editors, reporters, rent, pressroom expenses, and wire-service charges to be $300,000 per month. The variable cost of ink and paper is $0.10 per copy, but advertising revenues of $0.05 per paper will be generated. To print the morning paper, the publisher has to purchase a new printing press, which will cost $600,000. The press machine will be depreciated according to a seven-year MACRS class. The press machine will be used for 10 years, at which time its salvage value would be about $100,000. Assume 300 issues per year, a 25% tax rate, and a 13% MARR. How many copies per day must be sold to break even at a retail selling price of $0.25 per paper?

11.14 A small manufacturing firm is considering purchasing a new boring machine to modernize one of its production lines. Two types of boring machines are available on the market. The lives of Machine A and Machine B are 8 years and 10 years, respectively. The machines have the following receipts and disbursements:

Use a MARR (after tax) of 10% and a marginal tax rate of 30%, and answer the following questions.

(a) Which machine would be most economical to purchase under an infinite planning horizon? Explain any assumption that you need to make about future alternatives.

(b) Determine the break-even annual O&M costs for Machine A so that the annual equivalent cost of Machine A is the same as that of Machine B.

Item	Machine A	Machine B
Depreciation (MACRS)	7 years	7 years

(c) Suppose that the required service life of the machine is only five years. The salvage values at the end of the required service period are estimated to be \$3,000 for Machine A and \$3,500 for Machine B. Which machine is more economical?

Item	Machine A	Machine B
First cost	\$6,000	\$8,500
Service life	8 years	10 years
Salvage value	\$500	\$1,000
Annual O&M costs	\$700	\$520

Scenario Analysis

11.15 Peabody Corporation has the following base-case estimates for its new small engine assembly project:

- Price per unit = \$500
- Variable costs = \$120 per unit
- Fixed costs = \$2.5 million
- Demand = 20,000 units per year
- Capital investment = \$8 million at year 0
- Product life = 8 years
- Salvage value = \$500,000
- Depreciation method = seven-year MACRS
- Tax rate = 35%
- MARR = 12%

Suppose the company believes that all of its estimates (except the product life, depreciation method, tax rate, and MARR) are accurate only to within ±20%.

(a) What is the NPW of the project based on its base-case scenario?
(b) What is the NPW of the project based on its best-case scenario?
(c) What is the worst-case scenario?
(d) What conclusion would you make about the project after seeing the scenario analyses?

11.16 Suppose you are considering an investment project that requires \$800,000, has a six-year life, and has a salvage value of \$100,000. Sales volume is projected to be 65,000 units per year. Price per unit is \$63, variable cost per unit is \$42, and fixed costs are \$532,000 per year. The depreciation method is a five-year MACRS. The tax rate is 35% and you expect a 20% return on this investment.

(a) Determine the break-even sales volume.
(b) Calculate the cash flows of the base case over six years and its NPW.
(c) If the sales price per unit increases to \$400, what is the required break-even volume?
(d) Suppose the projections given for price, sales volume, variable costs, and fixed costs are all accurate to within ±15%. What would be the NPW figures of the best-case and worst-case scenarios?

Probabilistic Analysis

11.17 The following information is available about Henson Manufacturing Co.:

Probability	Return on Investment
0.10	8%
0.25	16%
0.30	25%
0.20	32%
0.15	40%

Compute Henson's expected return and standard deviation.

11.18 Suppose that you are considering an investment project with a two-year life. If the annual cash flows are given in terms of three-point estimates and these cash flows are statistically independent of each other, compute the mean and variance of the NPW distribution. Use a risk-free discount rate of 10%.

Period (n)	Pessimistic	Most Likely	Optimistic
0	−$10,000	−$8,000	−$7,000
1	$5,000	$12,000	$15,000
2	$4,000	$10,000	$13,000

11.19 A corporation is trying to decide whether to buy the patent for a product designed by another company. The decision to buy will mean an investment of $7 million, and the demand for the product is not known. If demand is light, the company expects a return of $1.1 million each year for three years. If demand is moderate, the return will be $2.4 million each year for four years, and high demand means a return of $4.8 million each year for four years. It is estimated the probability of a high demand is 0.45, and the probability of a light demand is 0.25. The firm's (risk-free) interest rate is 14%. Calculate the expected present worth of the patent. On this basis, should the company make the investment? (All figures represent after-tax values.)

11.20 Consider the following investment cash flows over a two-year life:

Period	Cash Flow	Mean (μ)	Variance (σ^2)
0	A_0	−$500	0
1	A_1	$200	50^2
2	A_2	$500	50^2

(a) Compute the mean and variance of PW of this project at $i = 10\%$ if A_1 and A_2 are mutually independent.

(b) In part (a)(1), if random variables (A_1 and A_2) are normally distributed with the mean and variance as specified in the table, compute the probability that the PW will be negative. (See Appendix C.)

11.21 Assume that we can estimate a project's cash flows as follows:

n	Expected Flow $E(A_n)$	Estimate of Standard Deviation σ_n
0	−\$300	\$20
1	\$120	\$10
2	\$150	\$15
3	\$150	\$20
4	\$110	\$25
5	\$100	\$30

In this case, each annual flow can be represented by a random variable with known mean and variance. Further assume complete independence among the cash flows.

(a) Find the expected NPW and the variance of this project at $i = 10\%$.

(b) If all cash flows are normally distributed with the given means and variances, what is the probability that the NPW will exceed \$200?

11.22 A consumer electronics firm is trying to decide whether to invest \$18M in a new plant for expanding its output of cell phones. Management forecasts that the new expansion will generate incremental net cash flows over the next five years as follows:

n	Expected Flow $E(A_n)$	Estimate of Standard Deviation σ_n
0	−\$18M	\$0
1	\$5M	\$8M
2	\$8M	\$9M
3	\$12M	\$10M
4	\$10M	\$5M
5	\$5M	\$3M

(a) If the firm's discount rate is 12%, determine the mean and variance of the NPW, assuming statistical independence among cash flows..

(b) If all annual cash flows are normally distributed with the means and variances as previously specified, compute the probability that the project will lose money.

11.23 Consider the following investment cash flows over a two-year life:

Period	Cash Flow	Mean (μ)	Variance (σ^2)
0	A_0	−\$500	100^2
1	A_1	\$200	50^2
2	A_2	\$500	50^2

(a) Compute the mean and variance of PW of this project at $i = 10\%$ if A_0, A_1 and A_2 are mutually independent.

(b) In part (a)(1), if random variables (A_0, A_1 and A_2) are normally distributed with the mean and variance as specified in the table, compute the probability that the PW will be greater than \$100.

Comparing Risky Projects

11.24 You are considering investing in one of two projects, which have the following returns and probabilities of occurrence. Assume statistical independence among returns of two projects:

	Return on Investment	
Probability	**Project *A***	**Project *B***
0.10	−20%	−35%
0.20	0	−10%
0.25	10%	15%
0.30	15%	25%
0.10	20%	40%
0.05	40%	50%

(a) Compute the mean return for each project.
(b) Compute the variance of return for each project.
(c) Which project would you prefer, and why?

11.25 You are considering two mutually exclusive projects with the following cash flows:

		Projected Cash Flows	
	Probabilities	**Project *A***	**Project *B***
Investment required	1.0	\$150,000	\$180,000
Annual cash flows and probabilities of occurrence for each of five years	0.3	\$35,000	\$45,000
	0.5	\$40,000	\$55,000
	0.2	\$50,000	\$67,000

(a) Compute the mean and variance of NPW distribution for each project using $i = 12\%$. Assume that project cash flows are mutually independent.
(b) Which project has a higher probability of losing money?

11.26 Juan Carols is considering two investment projects whose PWs are described as follows:

- Project 1: $PW(10\%) = 2X(X - Y)$, where X and Y are statistically independent discrete random variables with the following distributions:

X		*Y*	
Event	**Probability**	**Event**	**Probability**
\$20	0.30	\$10	0.60
\$40	0.70	\$20	0.40

- Project 2:

PW(10%)	Probability
0	0.24
$400	0.20
$1,600	0.36
$2,400	0.20

The cash flows between the two projects are assumed to be statistically independent.

(a) Develop the NPW distribution for Project 1.

(b) Compute the mean and variance of the NPW for Project 1.

(c) Compute the mean and variance of the NPW for Project 2.

(d) Suppose that Projects 1 and 2 are mutually exclusive. Which project would you select?

11.27 A financial investor has an investment portfolio worth $350,000. A bond in the portfolio will mature next month and provide him with $25,000 to reinvest. The choices have been narrowed down to the following two options.

- **Option 1:** Reinvest in a foreign bond that will mature in one year. This will entail a brokerage fee of $150. For simplicity, assume that the bond will provide interest of $2,450, $2,000, or $1,675 over the one-year period and that the probabilities of these occurrences are assessed to be 0.25, 0.45, and 0.30, respectively.
- **Option 2:** Reinvest in a $25,000 certificate with a savings-and-loan association. Assume that this certificate has an effective annual rate of 7.5%.

(a) Which form of reinvestment should the investor choose in order to maximize his expected financial gain?

(b) If the investor can obtain professional investment advice from Salomon Brothers, Inc., what would be the maximum amount the investor should pay for this service?

11.28 A manufacturing firm is considering two mutually exclusive projects, both of which have an economic service life of one year with no salvage value. The initial cost and the net year-end revenue for each project are given in Table P11.28.

TABLE P11.28 **Comparison of Mutually Exclusive Projects**

First Cost	Project 1 ($1,000)		Project 2 ($800)	
	Probability	Revenue	Probability	Revenue
	0.2	$2,000	0.3	$1,000
Net revenue, given in PW	0.6	$3,000	0.4	$2,500
	0.2	$3,500	0.3	$4,500

Assume that both projects are statistically independent of each other.

(a) If you are an expected-value maximizer, which project would you select?

(b) If you also consider the variance of the project, which project would you select?

11.29 A business executive is trying to decide whether to undertake one of two contracts or neither one. He has simplified the situation somewhat and feels that it is sufficient to imagine that the contracts provide alternatives as given in Table P11.29.

TABLE P11.29

Contract A		Contract B	
NPW	**Probability**	**NPW**	**Probability**
\$100,000	0.2	\$40,000	0.3
\$50,000	0.4	\$10,000	0.4
\$0	0.4	−\$10,000	0.3

(a) Should the executive undertake either one of the contracts? If so, which one? What would he do if he made decisions with an eye toward maximizing his expected NPW?

(b) What would be the probability that Contract A would result in a larger profit than that of Contract B?

11.30 Two alternative machines are being considered for a cost-reduction project.

- Machine A has a first cost of \$60,000 and a salvage value (after tax) of \$22,000 at the end of six years of service life. The probabilities of annual after-tax operating costs of this machine are estimated as follows:

Annual O&M Costs	Probability
\$5,000	0.20
\$8,000	0.30
\$10,000	0.30
\$12,000	0.20

- Machine B has an initial cost of \$35,000, and its estimated salvage value (after tax) at the end of four years of service is negligible. The annual after-tax operating costs are estimated to be as follows:

Annual O&M Costs	Probability
\$8,000	0.10
\$10,000	0.30
\$12,000	0.40
\$14,000	0.20

The MARR on this project is 10%. The required service period of these machines is estimated to be 12 years, and no technological advance in either machine is expected.

(a) Assuming independence, calculate the mean and variance for the equivalent annual cost of operating each machine.

(b) From the results of part (a), calculate the probability that the annual cost of operating Machine A will exceed the cost of operating Machine B.

11.31 Two mutually exclusive investment projects are under consideration. It is assumed that the cash flows are statistically independent random variables with means and variances estimated as given in Table P11.31.

TABLE P11.31

	Project *A*		Project *B*	
End of Year	**Mean**	**Variance**	**Mean**	**Variance**
0	−\$5,000	$1{,}000^2$	−\$10,000	$2{,}000^2$
1	\$4,000	$1{,}000^2$	\$6,000	$1{,}500^2$
2	\$4,000	$1{,}500^2$	\$8,000	$2{,}000^2$

(a) For each project, determine the mean and standard deviation of the NPW, using an interest rate of 15%.

(b) On the basis of the results of part (a), which project would you recommend?

Cost of Capital

11.32 Calculate the after-tax cost of debt under each of the following conditions:

(a) Interest rate, 12%; tax rate, 30%

(b) Interest rate, 15%; tax rate, 34%

(c) Interest rate, 16%; tax rate, 25%

11.33 The estimated beta (β) of a firm is 1.6. The market return (r_m) is 15%, and the risk-free rate (r_f) is 7%. Estimate the cost of equity (i_e).

11.34 The Callaway Company's cost of equity is 20%. Its before-tax cost of debt is 11%, and its marginal tax rate is 30%. The firm's capital structure calls for a debt-to-equity ratio of 40%. Calculate Callaway's cost of capital.

11.35 Consider the estimated beta values as of August 12, 2018, for the following companies:

Alpha Ltd.	0.72
Gamma Co.	1.00
Delta Inc.	1.56
Zee Co.	1.25

If the risk-free rate is 2.3% and the average market return is 10.8%, estimate each company's cost of equity capital.

11.36 An automobile company is contemplating issuing stock to finance an investment in producing a new sport-utility vehicle. The annual return to the market portfolio is expected to be 17% and the current risk-free interest rate is 3%. The company's analysts further believe that the expected return to the project will be 20% annually. What is the maximum beta value that would induce the auto maker to issue the stock?

Risk-Adjusted Discount Rate Approach

11.37 Consider Problem 11.20. If you use a 20% risk-adjusted discount rate approach, would this project be justified?

11.38 Consider Problem 11.21. If your risk-adjusted discount rate is 16%, is this project justifiable?

Value at Risk

11.39 Consider hypothetical Google stock traded between January 3, 2018, and June 23, 2018. With 120 trading days, the average daily return and standard deviation was 0.1679% and 0.6961%, respectively.

(a) Determine the degree of daily loss at 95% confidence level.

(b) If you invested $1,500,000 in the stock, what are the maximum daily losses at 95% confidence level?

(c) If you intend to hold the stock for 3 months, what are the maximum losses at 95% confidence level?

11.40 Consider a risky investment project. The expected NPW of the project is $120 million and its standard deviation is estimated to be $50 million. Assuming that the NPW is distributed with the mean and standard deviation above,

(a) Calculate the probability that the NPW would be negative.

(b) Determine the VaR at 95% confidence level. How would you interpret the VaR of this project, if the project is a part of investment portfolio of the firm?

11.41 Consider any choice of your favorite stock, for example, Netflix (NFLX).

(a) Collect the closing prices of the stock for a year.

(b) Calculate the daily return over the period.

(c) Determine the average daily return.

(d) Determine the variance of the daily return.

(e) Determine the VaR (per day) at 95% confidence, with $100,000 investment in the stock.

Short Case Studies with Excel

11.42 Boston Metal Company (BMC), a small manufacturer of fabricated metal parts, must decide whether to compete to become the supplier of transmission housings for Gulf Electric. Gulf Electric produces transmission housings in its own in-house manufacturing facility, but it has almost reached its maximum production capacity. Therefore, Gulf is looking for an outside supplier. To compete, BMC must design a new fixture for the production process and purchase a new forge. The available details for this purchase are as follows:

- The new forge would cost $125,000. This total includes retooling costs for the transmission housings.
- If BMC gets the order, it may be able to sell as many as 2,000 units per year to Gulf Electric for $50 each, and variable production costs (such as direct-labor and direct-material costs) will be $15 per unit. The increase in fixed costs, other than depreciation, will amount to $10,000 per year.
- The firm expects that the proposed transmission-housings project will have about a five-year project life. The firm also estimates that the amount ordered by Gulf Electric for the first year will also be ordered in each of the

subsequent four years. (Due to the nature of contracted production, the annual demand and unit price would remain the same over the project after the contract is signed.)

- The initial investment can be depreciated on a MACRS basis over a seven-year period, and the marginal income-tax rate is expected to remain at 25%. At the end of five years, the forge is expected to retain a market value of about 32% of the original investment. BMC's MARR is known to be 15%.

What Makes BMC Managers Worry: BMC's managers are uneasy about this project because too many uncertain elements have not been considered in the analysis:

- If it decides to compete for the project, BMC must invest in the forging machine in order to provide Gulf Electric with samples as a part of the bidding process. If Gulf Electric does not like BMC's samples, BMC stands to lose its entire investment in the forging machine.
- If Gulf likes BMC's samples, but feels that they are overpriced, BMC would be under pressure to bring the price in line with those of competing firms. Even the possibility that BMC would get a smaller order must be considered, as Gulf may use its overtime capacity to produce some units in-house instead of purchasing the entire number of units it needs. BMC is also not certain about the variable- and fixed-cost projections.

Recognizing these uncertainties, the managers want to assess a variety of possible scenarios before making a final decision. Put yourself in BMC's management's position mentally, and describe how you might address the uncertainty associated with the project. In doing so, perform a sensitivity analysis for each variable and develop a sensitivity graph.

Annual Revenue *(X)*	Probability	General Inflation Rate *(Y)*	Probability
$15,000	0.20	3%	0.25
$25,000	0.50	5%	0.50
$35,000	0.30	7%	0.25

11.43 MG Cutting Systems is considering an investment project with the following parameters, where all cost and revenue figures are estimated in constant dollars:

- The project requires the purchase of a $23,000 asset, which will be used for only two years (project life). The project also requires an investment of $3,000 in working capital, and this amount will be fully recovered at the end of year 2.
- The salvage value of this asset at the end of two years is expected to be $6,000.
- The annual revenue and the general inflation rate are discrete random variables but can be described by the following probability distributions:

Both random variables are statistically independent.

- The investment will be classified as a three-year MACRS property (tax life).
- It is assumed that the revenues, salvage value, and working capital are responsive to the general inflation rate.
- The revenue and inflation rate dictated during the first year will prevail over the remaining project period.

The marginal income-tax rate for the firm is 25%. The firm's inflation-free interest rate (i') is 10%.

(a) Determine the PW as a function of X.

(b) Compute the expected NPW of this investment.

(c) Compute the variance of the PW of this investment.

11.44 Mount Manufacturing Company produces industrial and public-safety shirts. As is done in most apparel manufacturing, the cloth must be cut into shirt parts in accordance with patterns marked on sheets of paper, which indicate the way that the particular cloth is to be cut. At present, these patterns are marked manually, and the annual labor cost of the process is running around $103,718. Mount has the option of purchasing one of two automated marking systems. The two systems are the Lectra System 305 and the Tex Corporation Marking System. The comparative characteristics of the two systems are as follows:

Most Likely Estimates

	Lectra System	Tex System
Annual labor cost	$51,609	$51,609
Annual material savings	$230,000	$274,000
Investment cost	$136,150	$195,500
Estimated life	6 years	6 years
Salvage value	$20,000	$15,000
Depreciation method (MACRS)	5 year	5 year

The firm's marginal tax rate is 40%, and the interest rate used for project evaluation is 12% after taxes.

(a) Based on the most likely estimates, which alternative is better?

(b) Suppose that the company estimates the material savings during the first year for each system on the basis of the following probability distributions:

Lectra System

Material Savings	Probability
$150,000	0.25
$230,000	0.40
$270,000	0.35

Tex System

Material Savings	Probability
$200,000	0.30
$274,000	0.50
$312,000	0.20

Further assume that the annual material savings for both Lectra and Tex are statistically independent. Compute the mean and variance for the equivalent annual value of operating each system.

11.45 The following is a comparison of the cost structure of a conventional manufacturing technology (CMT) system with that of a flexible manufacturing system (FMS) at one U.S. firm:

	Most Likely Estimates	
	CMT	**FMS**
Number of part types	3,000	3,000
Number of pieces produced/year	544,000	544,000
Variable labor cost/part	$2.15	$1.30
Variable material cost/part	$1.53	$1.10
Total variable cost/part	$3.68	$2.40
Annual overhead costs	$3.15M	$1.95M
Annual tooling costs	$470,000	$300,000
Annual inventory costs	$141,000	$31,500
Total annual fixed operating costs	$3.76M	$2.28M
Investment	$3.5M	$10M
Salvage value	$0.5M	$1M
Service life	10 years	10 years
Depreciation method (MACRS)	7 year	7 year

(a) The firm's marginal tax rate and MARR are 40% and 15%, respectively. Determine the incremental cash flow (FMS − CMT) based on the most likely estimates.

(b) Management feels confident about all input estimates for the CMT. However, the firm does not have any previous experience in operating an FMS. Therefore, many of the input estimates for that system, except the investment and salvage value, are subject to variation. Perform a sensitivity analysis on the project's data, varying the elements of the operating costs. Assume that each of these variables can deviate from its base-case expected value by $\pm 10\%$, $\pm 20\%$, and $\pm 30\%$.

(c) Prepare sensitivity diagrams and interpret the results.

(d) Suppose that probabilities of the variable material cost and the annual inventory cost for the FMS are estimated as follows:

Material Cost

Cost per Part	Probability
$1.00	0.25
$1.10	0.30
$1.20	0.20
$1.30	0.20
$1.40	0.05

Inventory Cost

Annual Inventory Cost	Probability
$25,000	0.10
$31,000	0.30
$50,000	0.20
$80,000	0.20
$100,000	0.20

What are the best and the worst cases of incremental PW?

(e) In part (d), assuming that the random variables of the cost per part and the annual inventory cost are statistically independent, find the mean and variance of the PW for the incremental cash flows.

(f) In parts (d) and (e), what is the probability that the FMS would be a more expensive investment option?