CHAPTER SIX

Annual-Equivalence Analysis

Owning and Operating a Dump Truck. The City of Flagstaff's capital budget in 2017 included a line item of $400,000 on heavy equipment and vehicles. Of the proposed purchases, the most expensive would be a dump truck, on which the lowest bid received was $100,206. A 2000 GMC dump truck that the city has relied on for 17 years is no longer reliable and would cost more to repair than is worthwhile. Certainly initial price is one thing but the city also has to consider how much that equipment will cost over time, such that the future operating budgets include these recurring expenses as well. Since the city mandates a life cycle cost analysis for any equipment purchase of this size of expenditure, the engineer contacted the dump truck dealer to find out any additional information regarding the resale value as well as other typical operating characteristics of the dump truck. The dealer provided the engineer with the following financial data complied based on an annual usage of 56,000 miles over a seven-year life cycle:

- Purchase price: $100,206
- Life cycle: 7 years
- Resale value: $60,000
- Operating costs per seven-year life cycle:
  - Diesel fuel at $3.00 per gallon: $169,355
  - Diesel exhaust fluid: $3,387
  - Tires: 10 $\times$ 4.375 sets tires at $400 per tire: $17,500
  - Driver wages and benefits: $35/h: $450,000
  - Scheduled maintenance, lubes, filters, and grease: $12,600
  - Repair costs: $26,600

Now the engineer needed to figure out how much it costs to own and operate the dump truck on an annual basis and also the cost per mile so that it can be compared with other bids. This information was needed before presenting the findings at the upcoming City Council meeting for final approval of the purchase.

The engineer will also consider suggesting that the city leases the dump truck instead of purchasing it. Certainly if the city's capital budget is tight, lease financing is an option. How does this lease financing affect the operating cost? The main issue is: *How much will it really cost per hour to own and operate* the dump truck? How does the hourly cost change with increased operating hours per year? For any major equipment acquisition, we need to consider both the ownership cost (or commonly known as capital costs) and the operating costs.

Suppose you are considering buying a new car. If you expect to drive 12,000 miles per year, can you figure out the cost of owning and operating the car per mile? You would have good reason to want to know this cost if you were being reimbursed by your employer on a per-mile basis for the business use of your car. Or consider a real-estate developer who is planning to build a shopping center of 500,000 square feet. What would be the minimum annual rental fee per square foot required in order to recover the initial investment?

Annual cash flow analysis is the method by which these and other unit costs can be calculated. Annual-equivalence analysis (along with present-worth analysis) is the second major equivalence technique for translating alternatives into a common basis of comparison. In this chapter, we develop the annual-equivalence criterion and demonstrate a number of situations in which annual-equivalence analysis is preferable to other methods of comparison.

6.1 Annual-Equivalent Worth Criterion

The **annual-equivalent worth (AE) criterion** provides a basis for measuring investment worth by determining equal payments on an annual basis. Knowing that any lump-sum amount can be converted into a series of equal annual payments, we may first find the net present worth of the original series and then multiply this amount by the capital-recovery factor:

$$\text{AE}(i) = \text{PW}(i)\,(A/P, i, N). \tag{6.1}$$

We use this formula to evaluate the investment worth of projects as follows:

- **Evaluating a Single Project:** The accept–reject decision rule for a single *revenue* project is as follows:

  If $\text{AE}(i) > 0$, accept the investment.
  If $\text{AE}(i) = 0$, remain indifferent to the investment.
  If $\text{AE}(i) < 0$, reject the investment.

  Notice that the factor $(A/P, i, N)$ in Eq. (6.1) is positive for $-1 < i < \infty$, which indicates that the AE(i) value will be positive if and only if PW(i) is positive. In other words, accepting a project that has a positive AE(i) value is equivalent to accepting a project that has a positive PW(i) value. Therefore, the AE criterion provides a basis for evaluating a project that is consistent with the PW criterion.
- **Comparing Multiple Alternatives.** When you compare mutually exclusive revenue projects, you select the project with the largest AE value. If you are comparing mutually exclusive *service* projects that have equivalent revenues, you may compare them on a *cost-only* basis. In this situation, the alternative with the least annual equivalent cost (AEC) or least negative annual equivalent worth is selected.

EXAMPLE 6.1 Finding Annual Equivalent Worth by Conversion from Net Present Worth (NPW)

A hospital uses four coal-fired boilers to supply steam for space heating, domestic hot water, and the hospital laundry. One boiler is operated at times of low load and on weekends, two are operated during the week, and the fourth boiler is normally off-line. The design efficiency on a steady load is generally about 78%. The boilers at the hospital were being run at between 70% and 73% efficiency, due to inadequate instrumentation and controls. Engineers have proposed that the boiler controls be upgraded. The upgrade would consist of installing variable-speed drives for the boiler fans and using the fans in conjunction with oxygen trim equipment for combustion control.

- The cost of implementing the project is $159,000. The boilers have a remaining service life of 12 years. Any upgrade will have no salvage value at the end of 12 years.
- The annual electricity use in the boiler house is expected to be reduced from 410,000 kWh to 180,000 kWh as a result of variable speed control of the boiler fan (because with the variable-speed drives, the fan motors draw only the power actually required to supply air to the boilers). This is equivalent to $14,000 per year. This savings is expected to increase at an annual rate of 4% as the cost of electricity increases over time.
- Coal use will be 2% lower due to the projected improvement in boiler efficiency. This corresponds to a cost reduction of $40,950 per year. This savings is projected to increase as the coal price increases at an annual rate of 5%.

If the hospital uses a 10% interest rate for any project justification, what would be the annual equivalent energy savings due to the improvement?

DISSECTING THE PROBLEM

When a cash flow has no special pattern, it is easiest to find the AE in two steps: (1) Find the PW of the flow and (2) find the AE of the PW.

Given: The cash flow diagram in Figure 6.1; $i = 10\%$ per year.

Find: The AE.

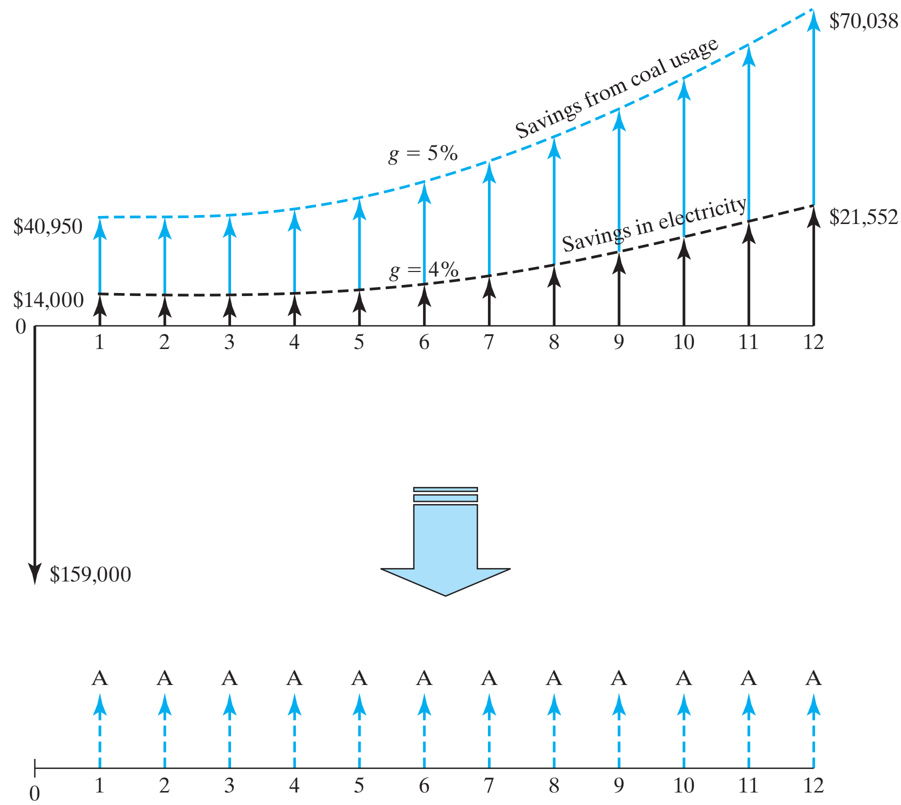


Figure 6.1 Computing equivalent annual worth—First convert each of the two different geometric gradient series into its equivalent present worth, determine the NPW of the boiler improvement project, and then find the equivalent annual worth of the energy savings.

METHODOLOGY

Find the present worth of the cash flows from fuel savings and then find the AE of the present worth.

SOLUTION

Since energy savings are in two different geometric gradient series, we calculate the equivalent present worth in the following steps:

- Savings in electricity:

$$
\begin{aligned}
P_{\text{Savings in electricity}} &= \$14{,}000(P/A_1, 4\%, 10\%, 12) \\
&= \$14{,}000\left[\frac{1 - (1 + 0.04)^{12}(1 + 0.10)^{-12}}{0.10 - 0.04}\right] \\
&= \$114{,}301.
\end{aligned}
$$

- Savings in coal usage:

$$P_{\text{Savings in coal usage}} = \$40{,}950(P/A_1, 5\%, 10\%, 12)$$
$$= \$40{,}950\left[\frac{1-(1+0.05)^{12}(1+0.10)^{-12}}{0.10-0.05}\right]$$
$$= \$350{,}356.$$

- Net present worth calculation:

$$\text{PW}(10\%) = \$114{,}301 + \$350{,}356 - \$159{,}000$$
$$= \$305{,}657.$$

Since $\text{PW}(10\%) > 0$, the project would be acceptable under the PW analysis.

Now, spreading the NPW over the project life gives

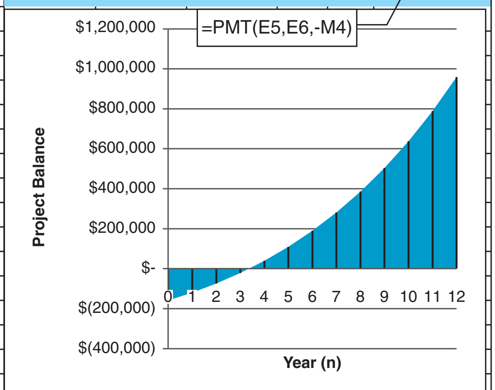
$$\text{AE}(10\%) = \$305{,}657(A/P, 10\%, 12)$$
$$= \$44{,}859.$$

Since $\text{AE}(10\%) > 0$, the project is also worth undertaking. The positive AE value indicates that the project is expected to bring in a net annual benefit of $44,859 over the life of the project.

TABLE 6.1 **An Excel Worksheet to Calculate the Annual Equivalent Worth (Example 6.1)**

	A	B	C	D	E	G–K	M
1							
2	Input					Output	
3							
4	(*I*) Investment				$ 159,000	Net Present Worth	$ 305,657
5	(*i*) MARR (%)				10%	Net Future Worth	$ 959,282
6	(*N*) Project life				12	Annual Equivalent Worth	$ 44,859
7							
8		Fuel Savings		Net	Project		
9	Period	Electricity	Coal Usage	Cash Flow	Balance		
10							
11	0			$ (159,000)	$ (159,000)		
12	1	$ 14,000	$ 40,950	$ 54,950	$ (119,950)		
13	2	$ 14,560	$ 42,998	$ 57,558	$ (74,388)		
14	3	$ 15,142	$ 45,147	$ 60,290	$ (21,536)		
15	4	$ 15,748	$ 47,405	$ 63,153	$ 39,463		
16	5	$ 16,378	$ 49,775	$ 66,153	$ 109,562		
17	6	$ 17,033	$ 52,264	$ 69,297	$ 189,815		
18	7	$ 17,714	$ 54,877	$ 72,591	$ 281,388		
19	8	$ 18,423	$ 57,621	$ 76,044	$ 385,571		
20	9	$ 19,160	$ 60,502	$ 79,662	$ 503,789		
21	10	$ 19,926	$ 63,527	$ 83,453	$ 637,622		
22	11	$ 20,723	$ 66,703	$ 87,427	$ 788,810		
23	12	$ 21,552	$ 70,038	$ 91,591	$ 959,282		
24							

=PMT(E5,E6,-M4)



=E22*(1+E5)+D23

COMMENTS: Table 6.1 illustrates how we might calculate the annual equivalent value by using Excel. To speed up the cash flow input process, we may enter the two different geometric series by using Columns B and C. In Column D, these two cash flows are combined and the project balances are calculated in Column E. Then, we calculate the net present worth first at 10%, followed by the annual equivalent value. These are shown in the outputs section.

6.1.1 Benefits of AE Analysis

Example 6.1 should look familiar to you. It is exactly the situation we encountered in Chapter 2 when we converted an uneven cash flow series into a single present value and then into a series of equivalent cash flows. In the case of Example 6.1, you may wonder why we bother to convert NPW to AE at all, since we already know from the NPW analysis that the project is acceptable. In fact, the example was mainly an exercise to familiarize you with the AE calculation.

In the real world, a number of situations can occur in which AE analysis is preferred, or even demanded, over NPW analysis. Some of typical situations include the following:

1. **When life-cycle-cost analysis is desired on an annual basis.** A life-cycle-cost analysis is useful when project alternatives fulfill the same performance requirements but differ with respect to initial costs and operating costs. **Life-cycle-cost analysis** enables the analysts to make sure that the selection of a design alternative is not based solely on the lowest initial costs but also to take into account all the future costs over the project's useful life.
2. **When there is a need to determine unit costs or profits.** In many situations, projects must be broken into unit costs (or profits) for ease of comparison with alternatives. Outsourcing decisions such as "make-or-buy" and pricing the usage of an asset (rental charge per hour) reimbursement analyses are key examples of such situations and will be discussed in this chapter.
3. **When project lives are unequal.** As we saw in Chapter 5, comparison of projects with unequal service lives is complicated by the need to determine the common lifespan. For the special situation of an indefinite service period and replacement with identical projects, we can avoid this complication by using AE analysis. This situation will also be discussed in more detail in this chapter.

6.1.2 Capital (Ownership) Costs versus Operating Costs

When only costs are involved, the AE method is sometimes called the **annual-equivalent cost method**. In this case, revenues must cover two kinds of costs: **operating costs** and **capital costs**. Normally, capital costs are nonrecurring (i.e., one-time costs) whereas operating costs recur for as long as an asset is being utilized.

Capital (Ownership) Costs

Capital costs (or *ownership costs*) are incurred by the purchase of assets to be used in production and service. Because capital costs tend to be one-time costs (buying and selling), when conducting an annual equivalent cost analysis, we must convert these

- Definition: The cost of owning a piece of equipment is associated with two amounts: (1) the equipment's initial cost (I) and (2) its salvage value (S).
- Capital costs: Taking these amounts into account, we calculate the capital costs as follows:

$$\begin{aligned}\mathrm{CR}(i) &= I(A/P, i, N) - S(A/F, i, N)\\ &= (I - S)(A/P, i, N) + iS\end{aligned}$$

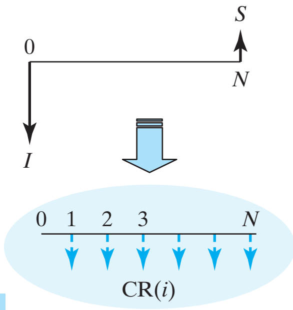


Figure 6.2 Calculation of capital-recovery cost (with return).

one-time costs into their annual equivalent over the life of the project. Suppose you purchased an equipment costing \$10,000. If you keep the asset for 5 years, your cost of owning the asset should include the opportunity cost of \$10,000. So, the **capital cost** is defined as the net cost of purchasing (after any salvage value adjustment) plus the interest cost over the life of the ownership.

The annual equivalent of a capital cost is given a special name: **capital-recovery cost**, designated CR(i). As shown in Figure 6.2, two general monetary transactions are associated with the purchase and eventual retirement of a capital asset: the asset's initial cost (I) and its salvage value (S). Taking these amounts into account, we calculate the capital-recovery cost as follows:

$$\mathrm{CR}(i) = I(A/P, i, N) - S(A/F, i, N).$$

If we recall the algebraic relationships between factors shown in Table 2.11 and notice that the ($A/F, i, N$) factor can be expressed as

$$(A/F, i, N) = (A/P, i, N) - i,$$

then we may rewrite the expression for CR(i) as

$$\begin{aligned}\mathrm{CR}(i) &= I(A/P, i, N) - S[(A/P, i, N) - i]\\ &= (I - S)(A/P, i, N) + iS. \qquad (6.2)\end{aligned}$$

Basically, to obtain the machine, one borrows a total of I dollars, S dollars of which are returned at the end of the Nth year. The first term, $(I - S)(A/P, i, N)$, implies that the balance $(I - S)$ will be paid back in equal installments over the N-year period at a rate of i, and the second term implies that *simple interest* in the amount of iS is paid on S until S is repaid. Many auto leases are based on this arrangement, in that most require a guarantee of S dollars in salvage (or market value at the time of restocking the returned vehicle).

Equivalent Annual Operating Costs

Once you place an asset in service, *operating costs* will incur by the operation of physical plants or equipment needed to provide service; examples include the costs of items such as labor and raw materials. Because operating costs recur over the life of a project, they tend to be estimated on an annual basis; so, for the purposes of annual-equivalent cost analysis, no special calculation is required unless the annual amount keeps changing. In that case, we need to find the *equivalent* present worth of the operating costs and spread the present worth over the asset life on an annual basis.

$$\mathrm{OC}(i) = \overbrace{\underbrace{\left(\sum_{n=1}^{N} \mathrm{OC}_n (1+i)^{-n}\right)}_{\text{Total present worth of operating cost}} \quad (A/P, i, N)}^{\text{Equivalent annual operating cost}}. \tag{6.3}$$

Examples 6.2 and 6.3 will illustrate the process of computing the capital costs and equivalent annual operating costs.

EXAMPLE 6.2 Will Your Car Hold Its Value? Costs of Owning a Vehicle

Consider two vehicles whose values are expected to hold during three or five years of ownership:

		Percent of Total Value Retained	
Type of Vehicles	**MSRP**	**After three years**	**After five years**
2016 Jeep Wrangler	\$26,453	66.8%	55.9%
2016 Toyota Tacoma	\$28,614	72.9%	61.7%

MSRP: Manufacturer's suggested retail price.

Determine the annual ownership cost of each vehicle after three or five years, assuming an interest rate of 6% compounded annually.

DISSECTING THE PROBLEM

Given: I = \$26,453, S = \$14,787, N = 5 years, and i = 6% per year.
Find: CR(6%) for each vehicle after five years of ownership.

METHODOLOGY

SOLUTION

For a Jeep Wrangler, buying it at \$26,453 and selling it at \$14,787 (55.9% of MSRP) after five years, your annual ownership cost (capital cost) would be:

$$\mathrm{CR}(6\%) = \underbrace{(\$26{,}453 - \$14{,}787)(A/P, 6\%, 5)}_{\$2{,}769} + (0.06)\$14{,}787$$

$$= \$3{,}657.$$

Costs of owning the vehicles as a function of ownership period are then as follows:

Type of Vehicles	MSRP	CR(6%) After three years	CR(6%) After five years
2016 Jeep Wrangler	\$26,453	\$4,346	\$3,657
2016 Toyota Tacoma	\$28,614	\$4,213	\$3,661

Clearly, the longer you keep the vehicle, the smaller the ownership cost.

From an industry viewpoint, CR(i) is the annual cost to the firm of owning the asset. With this information, the amount of annual savings or revenues required in order to recover the capital and operating costs associated with a project can be determined. As an illustration, consider Example 6.3.

EXAMPLE 6.3 Required Annual Savings to Justify the Purchase of Equipment

Ferguson Company is considering an investment in computer-aided design equipment. The equipment will cost \$110,000 and will have a five-year useful economic life. It has a salvage value of \$10,000. The expected annual operating costs for the equipment would be \$20,000 for the first two years and \$25,000 for the remaining three years. Assuming that Ferguson's desired return on its investment (MARR) is 15%, what is the required annual savings to make the investment worthwhile?

DISSECTING THE PROBLEM

Given: $I = \$110{,}000$, $S = \$10{,}000$, yearly operating costs, $N = 5$ years, and $i = 15\%$ per year.

Find: AEC, and determine the required annual savings.

METHODOLOGY

Separate cash flows into two parts: (1) cash flows related to the purchase of equipment and (2) cash flows related to operating activities.

SOLUTION

As shown in Figure 6.3, we separate cash flows associated with the asset acquisition and disposal from the normal operating cash flows. Since the operating cash flows are not uniform, we need to convert the series into equivalent annual flows.

- Capital costs:

$$\begin{aligned}\text{CR}(15\%) &= (\$110{,}000 - \$10{,}000)(A/P, 15\%, 5) \\ &\quad + (0.15)\$10{,}000 \\ &= \$31{,}332.\end{aligned}$$

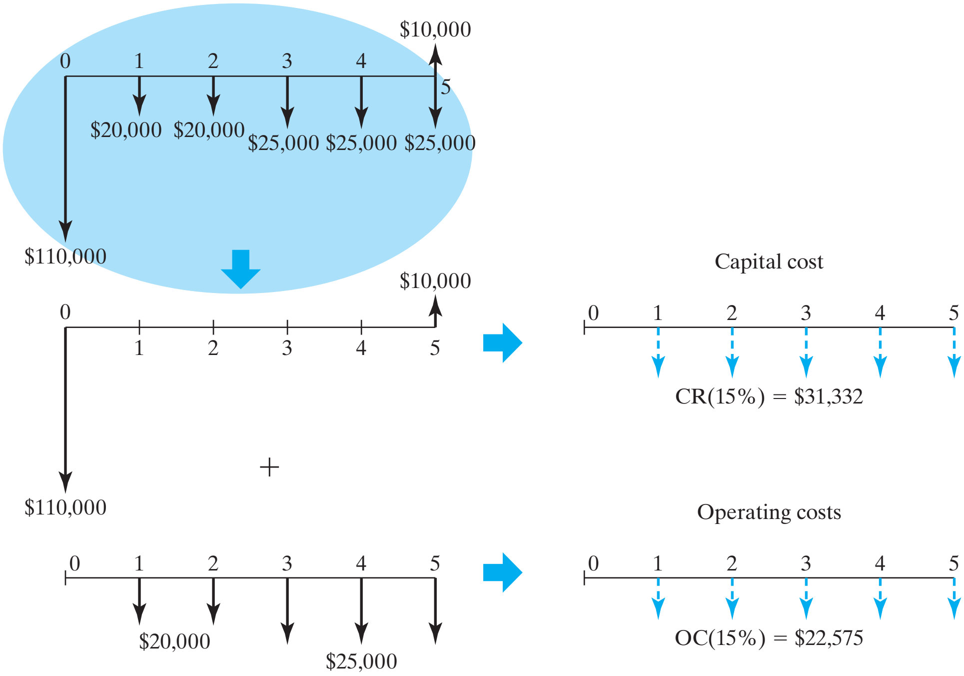


Figure 6.3 Computing the required annual savings to cover the capital and operating costs.

- Operating costs:

$$OC(15\%) = \Bigg[\overbrace{\underbrace{\begin{pmatrix} \$20{,}000(P/A, 15\%, 2) \\ +\$25{,}000\ (P/A, 15\%, 3)(P/F, 15\%, 2) \end{pmatrix}}_{\$75{,}675}}^{\text{Total present worth of operating cash flows}} \Bigg]$$

$$\times (A/P, 15\%, 5)$$

$$= \$22{,}575.$$

- Annual equivalent cost:

$$\begin{aligned} AEC(15\%) &= CR(15\%) + OC(15\%) \\ &= \$31{,}332 + \$22{,}575 \\ &= \$53{,}907. \end{aligned}$$

The required annual savings must be at least $53,907 to recover the investment made in the asset with the desired return of 15% and cover the annual operating expenses.

6.2 Applying Annual-Worth Analysis

In general, most engineering economic analysis problems can be solved by the present-worth methods that were introduced in Chapter 5. However, some economic analysis problems can be solved more efficiently by annual-worth analysis. In this section, we introduce several applications that call for annual-worth analysis techniques.

6.2.1 Unit-Profit or Unit-Cost Calculation

In many situations, we need to know the *unit profit* (or *unit cost*) of operating an asset. To obtain a unit profit (or cost), we may proceed as follows:

Step 1: Determine the number of units to be produced (or serviced) each year over the life of the asset.

Step 2: Identify the cash flow series associated with production or service over the life of the asset.

Step 3: Calculate the present worth of the project's cash flow series at a given interest rate, and then determine the equivalent annual worth or cost.

Step 4: Divide the equivalent annual worth or cost by the number of units to be produced or serviced during each year. When the number of units varies each year, you may need to convert the varying annual cash flows into equivalent annual worth or cost.

To illustrate the procedure, we will consider Example 6.4, in which the annual-equivalence concept is useful in estimating the savings per machine-hour for a proposed machine acquisition.

EXAMPLE 6.4 Unit Profit per Machine-Hour with Constant or Varying Annual Operating Hours

Harrison Company experiences frequent industrial accidents involving workers who perform spot-welding. The firm is looking into the possibility of investing in a specific robot for welding tasks. The required investment will cost Harrison \$1 million upfront, and this robot has a five-year useful life and a salvage value of \$100,000. The robot will reduce labor costs, worker insurance costs, and materials usage cost and will eliminate accidents involving workers at the spot-welding operations. The savings figure translates into a total of \$800,000 a year. The additional operating and maintenance costs associated with the robot amount to \$300,000 annually. Compute the equivalent savings per machine-hour at $i = 15\%$ compounded annually for the following two situations:

(a) Suppose that this robot will be operated for 2,000 hours per year.

(b) Suppose that the robot will be operated according to varying hours: 1,500 hours in the first year, 2,500 hours in the second year, 2,500 hours in the third year, 2,000 hours in the fourth year, and 1,500 hours in the fifth year. The total number of operating hours is still 10,000 over five years.

DISSECTING THE PROBLEM

Given: $I = \$1{,}000{,}000$, $S = \$100{,}000$, $N = 5$ years, net savings per year $= \$500{,}000$ $(= \$800{,}000 - \$300{,}000)$, and there are 10,000 machine-hours over five years.

Find: Equivalent savings per machine-hour.

METHODOLOGY

(a) *Annual operating hours remain constant:* First compute the annual equivalent savings, and then calculate the equivalent savings per machine-hour.

SOLUTION

We first compute the annual equivalent savings from the use of the robot.

$$\begin{aligned} \text{PW}(15\%) &= -\$1{,}000{,}000 + \$500{,}000(P/A, 15\%, 5) \\ &\quad + \$100{,}000(P/F, 15\%, 5) \\ &= \$725{,}795. \\ \text{AE}(15\%) &= \$725{,}795(A/P, 15\%, 5) \\ &= \$216{,}516. \end{aligned}$$

With an annual usage of 2,000 hours, the equivalent savings per machine-hour would be calculated as follows:

$$\text{Savings per machine-hour} = \$216{,}516/2{,}000 \text{ hours} = \$108.26/\text{hour}.$$

See Figure 6.4.

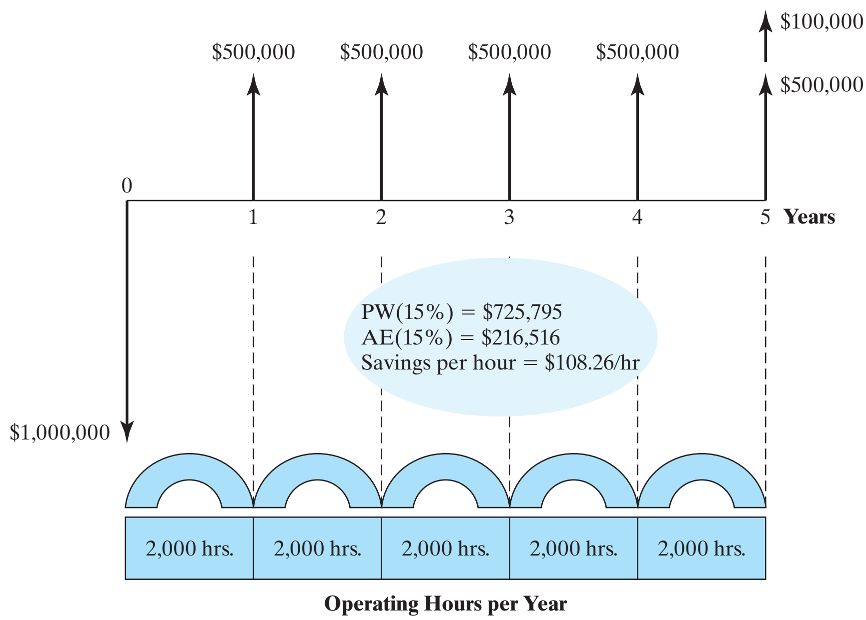


Figure 6.4 Computing equivalent savings per machine-hour (Example 6.4).

(b) *Annual operating hours fluctuate:* Calculate the equivalent annual savings as a function of C.

SOLUTION

Let C denote the equivalent annual savings per machine-hour that need to be determined. Now, with varying annual usages of the machine, we can set up the equivalent annual savings as a function of C:

$$\begin{aligned}\text{Equivalent annual savings} &= C[1{,}500(P/F, 15\%, 1) \\ &\quad + 2{,}500(P/F, 15\%, 2) \\ &\quad + 2{,}500(P/F, 15\%, 3) + 2{,}000(P/F, 15\%, 4) \\ &\quad + 1{,}500(P/F, 15\%, 5)](A/P, 15\%, 5) \\ &= 2{,}006.99C.\end{aligned}$$

We can equate this amount to \$216,516 and solve for C. This operation gives us

$$C = \$216{,}516/2{,}006.99 = \$107.88/\text{hour},$$

which is about \$0.38 less than in the situation in Example 6.4(a).

COMMENTS: Note that we cannot simply divide the NPW amount (\$725,795) by the total number of machine-hours over the five-year period (10,000 hours), which would result in \$72.58/hour. This \$72.58/hour figure represents the instant savings in present worth for each hour of use of the robot but *does not* consider the time over which the savings occur. Once we have the annual equivalent worth, we can divide by the desired time unit if the compounding period is one year. If the compounding period is shorter, then the equivalent worth should be calculated over the compounding period.

EXAMPLE 6.5 Cost to Fly per Hour

Lambert Manufacturing Corporation (LMC) is considering purchasing a business jet for domestic travels for their executives. Purchasing a business airplane for domestic travels. Key financial data are as follows:

- Cost of aircraft: \$1,282,035
- Market value of the aircraft after five years: \$832,000
- Total variable cost per flying hour: \$272 first year and increases at 6% per year thereafter
- Total fixed operating cost per flying hour: \$302.95 first year and increases at 4% per year
- Annual operating (flying) hours: 200 hours

Compute the equivalent annual cost of owning and operating the aircraft per flying hour over five years at an annual interest rate of 12%.

DISSECTING THE PROBLEM

Given: Financial data as summarized in Figure 6.5, and $i = 12\%$ per year.

Find: Hourly cost of owning and operating the business airplane.

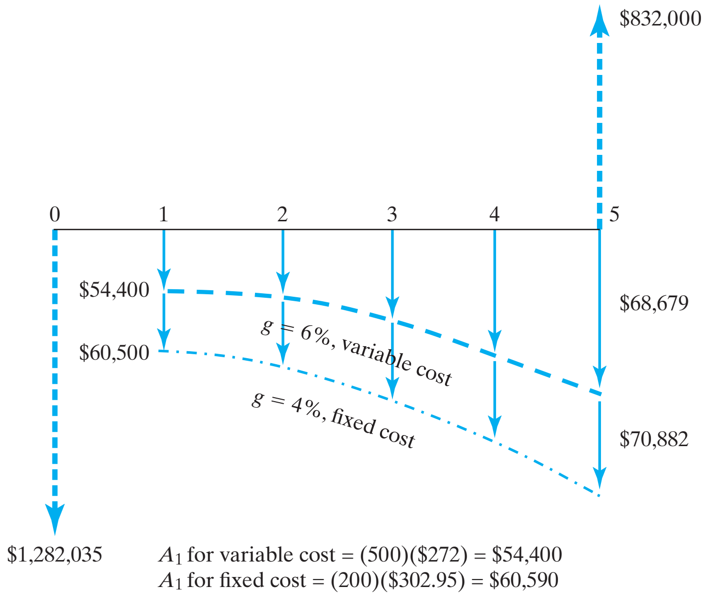


Figure 6.5 Cash flows associated with owning and operating a business airplane (Example 6.5).

METHODOLOGY

First compute the annual equivalent cost, then calculate the equivalent cost per flying hour.

SOLUTION

The two cost components are capital cost and operating cost. We calculate the annual equivalent cost for each as follows:

- **Capital costs:**

$$\begin{aligned}\text{CR}(12\%) &= (\$1{,}282{,}035 - \$832{,}000)(A/P, 12\%, 5)\\ &\quad + 0.12(\$832{,}000)\\ &= \$224{,}684.\end{aligned}$$

- **Operating costs:**
  - Variable cost:

$$\begin{aligned}\text{OC}(12\%)_{\text{Variable}} &= (200)(\$272)(P/A_1, 6\%, 12\%, 5)\\ &\quad \times (A/P, 12\%, 5)\\ &= \$60{,}529.\end{aligned}$$

  - Fixed cost:

$$\begin{aligned}\text{OC}(12\%)_{\text{Fixed}} &= (200)(\$302.95)(P/A_1, 4\%, 12\%, 5)\\ &\quad \times (A/P, 12\%, 5)\\ &= \$65{,}056.\end{aligned}$$

- Total operating costs:

$$OC(12\%) = OC(12\%)_{\text{Variable}} + OC(12\%)_{\text{Fixed}}$$
$$= \$125{,}585.$$

Then, the cost of flying is

$$\text{Flying cost per hour} = AEC(12\%)/200 = \frac{\$224{,}684 + \$125{,}585}{200}$$
$$= \$1{,}751.35.$$

COMMENTS: Note that the flying cost per hour when we considered the ownership cost (capital costs) along with the operating costs is $1,751.35. Now LMC needs to look at how much it has budgeted for its executives' travels next five years and can decide whether it is worth owning a corporate airplane.

6.2.2 Make-or-Buy Decision

Outsourcing decisions such as make-or-buy problems are among the most common business decisions. At any given time, a firm may have the option of either buying an item or producing it. *If either the "make" or the "buy" alternative requires the acquisition of machinery or equipment besides the item itself, then the problem becomes an investment decision*. Since the cost of an outside service (the "buy" alternative) is usually quoted in terms of dollars per unit, it is easier to compare the two alternatives if the differential costs of the "make" alternative are also given in dollars per unit. This unit-cost comparison requires the use of annual-worth analysis. The specific procedure is as follows:

- **Step 1:** Determine the time span (planning horizon) for which the part (or product) will be needed.
- **Step 2:** Determine the required annual quantity of the part (or product).
- **Step 3:** Obtain the unit cost of purchasing the part (or product) from the outside firm.
- **Step 4:** Determine the cost of the equipment, manpower, and all other resources required to make the part (or product).
- **Step 5:** Estimate the net cash flows associated with the "make" option over the planning horizon.
- **Step 6:** Compute the annual equivalent cost of producing the part (or product).
- **Step 7:** Compute the unit cost of making the part (or product) by dividing the annual equivalent cost by the required annual quantity.
- **Step 8:** Choose the option with the smallest unit cost.

EXAMPLE 6.6 Unit Cost: Make or Buy

B&S Company manufactures several lines of pressure washers. One unique part, an axial cam, requires specialized tools and equipment that need to be replaced. Management has decided that the only alternative to replacing these tools is to acquire the axial cam from an outside source. B&S's average usage of the axial cam is 120,000 units each year over the next five years.

- **Buy Option:** A supplier is willing to provide the axial cam at a unit sales price of \$35 if at least 100,000 units are ordered annually.
- **Make Option:** If the specialized tools and equipment are purchased, they will cost \$2,200,000 and will have a salvage value of \$120,000 after their expected economic life of five years. With these new tools, the direct labor and variable factory overhead will be reduced, resulting in the following estimated unit production cost:

Direct material	\$8.50
Direct labor	\$5.50
Variable factory overhead	\$4.80
Fixed factory overhead	\$7.50
Total unit cost	\$26.30

Assuming that the firm's interest rate is 12% per year, calculate the unit cost under each option and determine whether the company should replace the old tools or purchase the axial cam from an outside source.

DISSECTING THE PROBLEM

Given: Cash flows for both options as shown in Figure 6.6; $i = 12\%$.

Find: Unit cost for each option and which option is preferred.

METHODOLOGY

First determine the AEC for each option and then calculate the unit cost for each option.

SOLUTION

The required annual production volume is 120,000 units. We now need to calculate the annual equivalent cost under each option.

- **Buy Option:** Since we already know how much it would cost to buy the axial cam, we can easily find the annual equivalent cost:

$$\begin{aligned}\text{AEC}(12\%)_{\text{Buy}} &= (\$35/\text{unit}) \times (120{,}000 \text{ units/year}) \\ &= \$4{,}200{,}000/\text{year}.\end{aligned}$$

- **Make Option:** The two cost components are capital cost and operating cost.

Capital cost:

$$\begin{aligned}\text{CR}(12\%) &= (\$2{,}200{,}000 - \$120{,}000)(A/P, 12\%, 5) \\ &\quad + (0.12)(\$120{,}000) = \$591{,}412.\end{aligned}$$

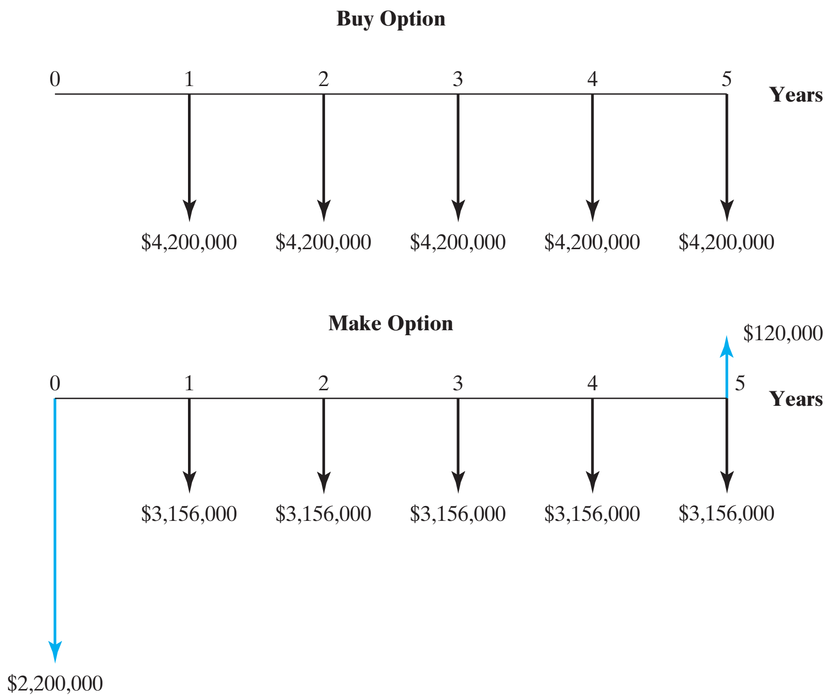


Figure 6.6 Cash flows associated with make-or-buy option in manufacturing axial cams.

Production cost:

$$\text{OC}(12\%)_{\text{Make}} = (\$26.30/\text{unit}) \times (120{,}000 \text{ units/year})$$
$$= \$\,3{,}156{,}000/\text{year}.$$

Total annual equivalent cost:

$$\text{AEC}(12\%)_{\text{Make}} = \$591{,}412 + \$3{,}156{,}000 = \$3{,}747{,}412.$$

Obviously, this annual-equivalence calculation indicates that B&S would be better off making the axial cam in-house rather than buying it from the outside vendor. However, B&S wants to know the unit costs in order to set a price for the product. For this situation, we need to calculate the unit cost of producing the axial cam under each option. We do this calculation by dividing the magnitude of the annual-equivalent cost for each option by the annual quantity required:

- **Buy Option:**

$$\text{Unit cost} = \$35.00/\text{unit}.$$

- **Make Option:**

$$\text{Unit cost} = \$3{,}747{,}412/120{,}000 = \$31.23/\text{unit}.$$

Making the axial cam in-house by replacing the tools and equipment will save B&S $3.77 per unit before any tax consideration.

COMMENTS: Two important noneconomic factors should also be considered. The first factor is the question of whether the quality of the supplier's component is better than, equal to, or worse than the quality of the component the firm is presently manufacturing. The second factor is the reliability of the supplier in terms of providing the needed quantities of the axial cams on a timely basis. A reduction in quality or reliability should virtually always rule out a switch from making to buying.

6.3 Comparing Mutually Exclusive Projects

In this section, we will consider a situation where two or more mutually exclusive alternatives need to be compared on the basis of annual equivalent worth. In Section 5.4, we discussed the general principle that should be applied when mutually exclusive alternatives with unequal service lives are compared. The same general principle should be applied when comparing mutually exclusive alternatives on annual—the basis of equivalent worth—that is, mutually exclusive alternatives in equal time spans must be compared. Therefore, we must give careful consideration to the time expended in the analysis process, called the **analysis period**. We will consider two situations: (1) the analysis period equals the project lives, and (2) the analysis period differs from the project lives.

6.3.1 Analysis Period Equals Project Lives

Let us begin our comparison with a simple situation where the length of the projects' lives equals the length of the analysis period. In this situation, we compute the AE value for each project and select the project that has the least AEC (for service projects) or the largest AE value (for revenue projects).

In many situations, we need to compare different design alternatives, each of which would produce the same number of units (constant revenues) but would require different amounts of investment and operating costs (because of different degrees of mechanization). This is commonly known as **life-cycle-cost analysis**. Example 6.7 illustrates the use of the annual equivalent cost concept to compare the cost of operating a conventional electric motor with that of operating a premium-efficiency motor in a strip-processing mill.

EXAMPLE 6.7 Life-Cycle Cost Analysis—How Premium Efficiency Motors Can Cut Your Electric Costs

Birmingham Steel Corporation is considering replacing 20 conventional 25-HP, 230-V, 60-Hz, 1,800-rpm induction motors in its plant with either brand new standard efficiency (SE) motors or modern premium-efficiency (PE) motors operating at 75% of full-rated load. Both types of motors have power outputs of 18.650 kW per motor $(25\text{ HP} \times 0.746\text{ kW/HP})$. Standard motors have a published efficiency of 89.5%, while the PE motors are 93.8% efficient. The initial cost of the SE motors is \$3,220 while the initial cost of the proposed PE motors is \$4,570. The motors are operated 12 hours per day, 5 days per week, 52 weeks per year with a local utility cost of \$0.09 per kilowatt-hour (kWh). The demand charges are \$8 per kW-month. The life cycle of both the SE motor and the PE motor is 20 years with no appreciable salvage value.

(a) At an interest rate of 10% compounded annually, what is the amount of savings per kWh gained by adopting the PE motors over the SE motors?

(b) At what operating hours are the two types of motors equally economical?

DISSECTING THE PROBLEM

Whenever we compare machines with different efficiency ratings, we need to determine the input powers required to operate the machines. Since percent efficiency is equal to the ratio of output power to input power, we can determine the required input power by dividing the output power by the motor's percent efficiency:

$$\text{Input power} = \frac{\text{ourput power}}{\text{percent efficiency}}.$$

For example, a 30-HP motor with 90% efficiency will require an input power calculated as follows:

$$\text{Input power} = \frac{(30 \text{ HP} \times 0.746 \text{ kW/HP})}{0.90}$$
$$= 24.87 \text{ kW}.$$

Given: Types of motors $= (\text{SE, PE})$, $I = (\$3{,}220,\ \$4{,}570)$, $S = (0, 0)$, $N = (20 \text{ years}, 20 \text{ years})$, rated power output $= (18.65 \text{ kW},\ 18.65 \text{ kW})$, efficiency rating $= (89.5\%,\ 93.8\%)$, $i = 10\%$, utility rate $= \$0.09/\text{kWh}$, demand charges $= \$8/\text{kW-month}$, operating hours $= 3{,}120$ hours per year, and number of motors required $= 20$.

Find: (a) The amount saved per kWh by operating the PE motor and (b) the range of operating hours for the PE motor to be more economical.

Mutually Exclusive Alternatives With Equal Project Lives

	Standard Motor	Premium-Efficiency Motor
Size	18.65 kW	18.65 kW
Cost	$3,220	$4,570
Life	20 years	20 years
Salvage Value	$0	$0
Efficiency	89.5%	93.8%
Energy Cost	$0.09/kWh	$0.09/kWh
Demand charge	$8/kW-month	$8/kW-month
Operating Hours	3,120 hrs/yr	3,120 hrs/yr

METHODOLOGY

First, calculate the operating cost per kWh per unit, and then determine the break-even number of operating hours for the PE motors.

Calculating annual energy consumption:

$$\text{kW}_{\text{Required}} = \text{hp} \times L \times 0.746 \times \frac{100}{\text{Eff}}$$

$$\text{kWh} = \text{kW}_{\text{Required}} \times \begin{array}{l}\text{annual}\\ \text{operating hours}\end{array}$$

Where:

hp = motor nameplate rating

L = Motor load in decimal format

Eff = Motor efficiency under actual operating conditions, %

SOLUTION

(a) Execute the following steps to determine the operating cost per kWh per unit:

- Determine total input power for both motor types.

SE motor:

$$\text{Input power} = \frac{25 \times 0.75 \times 0.746}{0.895} = 15.6285 \text{ kW}.$$

PE motor:

$$\text{Input power} = \frac{25 \times 0.75 \times 0.746}{0.938} = 14.9120 \text{ kW}.$$

Note that each PE motor requires 0.7165 kW less input power (or 14.33 kW for 20 motors), which results in energy savings.

- Determine the total kWh per year for each type of motor, assuming a total of 3,120 hours per year in motor operation.

SE motor:

$$3{,}120 \text{ hrs/year} \times 15.6285 \text{ kW} = 48{,}761 \text{ kWh/year}.$$

PE motor:

$$3{,}120 \text{ hrs/year} \times 14.9120 \text{ kW} = 46{,}525 \text{ kWh/year}.$$

- Determine the annual energy costs for both motor types. Since the utility rate is $0.09/kWh, the annual energy cost for each type of motor is calculated as follows:

SE motor:

$$\$0.09/\text{kWh} \times 48{,}761 \text{ kWh/year} = \$4{,}388/\text{year}.$$

PE motor:

$$\$0.09/\text{kWh} \times 46{,}525 \text{ kWh/year} = \$4{,}187/\text{year}.$$

$$\text{Total annual energy cost} = \text{kW}_{\text{Required}} \times \text{annual operating hours} \times \text{energy charge} + \text{annual demand charge}$$

where the demand charges are the surcharges for larger users of electricity levied by the power company.

- Determine the annual demand charges for each motor. The demand charge rate is normally given monthly, so we may calculate the annual demand charge as follows:

SE motor:

$$15.6285 \text{ kW} \times 12 \text{ months/yr} \times \$8/\text{kW-month} = \$1{,}500/\text{yr}.$$

PE motor:

$$14.9120 \text{ kW} \times 12 \text{ months/yr} \times \$8/\text{kW-month} = \$1{,}432/\text{yr}.$$

Calculating annual demand charge:

$$\text{Annual demand charge} = \text{kW}_{\text{Required}} \times 12 \times \text{monthly demand charge}$$

- Determine the capital costs for both types of motors. Recall that we assumed that the useful life for both motor types is 20 years. To determine the annualized capital cost at 10% interest, we use the capital-recovery factor.

SE motor:

$$(\$3{,}220)(A/P, 10\%, 20) = \$378.$$

PE motor:

$$(\$4{,}570)(A/P, 10\%, 20) = \$537.$$

- Determine the total equivalent annual cost, which is equal to the capital cost plus the annual energy cost. Then calculate the unit cost per kWh on the basis of output power. Note that the total output power is 43,641 kWh per year (25 HP $\times$ 0.75 $\times$ 0.746 kW/HP $\times$ 3,120 hours/year). We execute these steps as follows:

$$\text{AEC}(10\%) = \text{capital cost} + \text{annual total energy cost}$$

SE motor:

We calculate that

$$\text{AEC}(10\%) = \$4{,}388 + \$1{,}500 + \$378 = \$6{,}267.$$

So,

$$\begin{aligned}\text{Cost per kWh} &= \$6{,}267/43{,}641 \text{ kWh}\\ &= 14.36 \text{ cents/kWh.}\end{aligned}$$

PE motor:

We calculate that

$$\text{AEC}(10\%) = \$4{,}187 + \$1{,}432 + \$537 = \$6{,}156.$$

So,

$$\begin{aligned}\text{Cost per kWh} &= \$6{,}156/43{,}641 \text{ kWh}\\ &= 14.11 \text{ cents/kWh.}\end{aligned}$$

Clearly, PE motors are cheaper to operate if the motors are expected to run only 3,120 hours per year.

- Determine the savings (or loss) per operating hour obtained by switching from SE to PE motors.

Additional capital cost required from switching from SE to PE motors:

$$\text{Incremental capital cost} = \$537 - \$378 = \$159.$$

Additional energy-cost savings from switching from SE to PE motors:

$$\text{Incremental energy savings} = \$5{,}889 - \$5{,}619 = \$270.$$

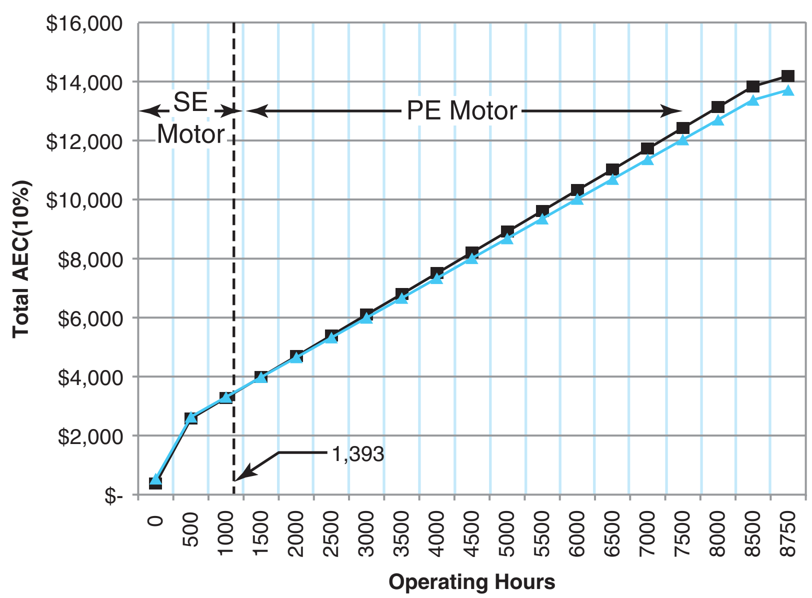
At 3,120 annual operating hours, it will cost the company an additional \$159 to switch to PE motors, but the energy savings are \$270, which results in a \$111 gain from each motor.

(b) Determine the break-even number of operating hours for the PE motors:

- Would the result found in part (a) change if the same motor were to operate just 1,200 hours per year? If a motor is to run around the clock, the savings in kWh would result in a substantial annual savings in electricity bills, which are an operating cost. As we calculate the annual equivalent cost by varying the number of operating hours, we obtain the situation shown in Table 6.2. Observe that if Birmingham Steel Corporation used the PE motors for more than 1,393 hours annually, adopting the PE motors would be justified.

TABLE 6.2 **Break-Even Number of Operating Hours as Calculated by Using Excel**

	A	B	C	D	E	F	G
2						AEC(10%)	
3–4		Standard Motor	Premium Efficiency Motor		Operating Hours	Standard Motor	PE Motor
5							
6	Output power (hp)	25	25		0	$ 378	$ 537
7	Operating hours per year	1,393	1,393		500	$ 2,582	$ 2,639
8	Efficiency (%)	89.5	93.8		1000	$ 3,285	$ 3,310
9	Load factor (%)	75	75		1500	$ 3,988	$ 3,981
10	Initial cost ($)	$ 3,220	$ 4,570		2000	$ 4,692	$ 4,653
11	Salvage value ($)	$ 0	$ 0		2500	$ 5,395	$ 5,324
12	Service life (year)	20	20		3000	$ 6,098	$ 5,995
13	Utility rate ($/kWh)	$ 0.09	$ 0.09		3500	$ 6,802	$ 6,666
14	Interest rate (%)	10	10		4000	$ 7,505	$ 7,337
15	Demand charge				4500	$ 8,208	$ 8,008
16	($/kW-month)	8	8		5000	$ 8,911	$ 8,679
17	Capital cost ($/year)	$ 378.22	$ 536.79		5500	$ 9,615	$ 9,350
18	Energy cost ($/year)	$ 1,958.71	$ 1,868.92		6000	$ 10,318	$ 10,021
19	Demand charges ($/year)	$ 1,500.34	$ 1,431.56		6500	$ 11,021	$ 10,692
20	Total equ. annual cost	$ 3,837.27	$ 3,837.27		7000	$ 11,725	$ 11,363
21	Cost per kWh	$ 0.1970	$ 0.1970		7500	$ 12,428	$ 12,034
22					8000	$ 13,131	$ 12,705
23	Break-even operating hrs	1,393	1,393		8500	$ 13,834	$ 13,376
24	AEC(10%)	$ 3,837.27	$ 3,837.27		8750	$ 14,186	$ 13,712
25	Differential cost	$	-				
26–45							



6.3.2 Analysis Period Differs from Project Lives

In Section 5.4, we learned that, in present-worth analysis we must have a common analysis period when mutually exclusive alternatives are compared. One of the approaches is the **replacement chain method** (or lowest common multiple), which assumes that each project can be repeated as many times as necessary to reach a common life span; the NPWs over this life span are then compared, and the project with the higher NPW over the common life is chosen. Annual-worth analysis also requires establishing common analysis periods, but AE analysis offers some computational advantages over present-worth analysis, provided the following criteria are met:

1. The service of the selected alternative is required on a continuous basis.
2. Each alternative will be replaced by an *identical* asset that has the same costs and performance.

When these two criteria are met, we may solve for the AE (or AEC for service projects) of each project on the basis of its initial life span rather than on the basis of the infinite streams of project cash flows.

EXAMPLE 6.8 Annual Equivalent Cost Comparison—Unequal Project Lives

You are running a small machine shop where you need to replace a worn-out sanding machine. Two different models have been proposed:

- Model A is semiautomated, requires an initial investment of $150,000, and has an annual operating cost of $55,000 for each of three years, at which time it will have to be replaced. The expected salvage value of the machine is just $15,000.
- Model B is an automated machine with a five-year life and requires an initial investment of $230,000 with an estimated salvage value of $35,000. Expected annual operating and maintenance cost of the B machine is $30,000.

Suppose that the current mode of operation is expected to continue for an indefinite period. You also think that these two models will be available in the future without significant changes in price and operating costs. At MARR = 15%, which model should you select? Apply the annual-equivalence approach to select the most economical machine.

DISSECTING THE PROBLEM

A required service period of infinity may be assumed if we anticipate that an investment project will be ongoing at roughly the same level of production for some indefinite period. This assumption certainly is possible mathematically although the analysis is likely to be complicated and tedious.

Given: Cash flows for Model A and Model B, and i = 15% compounded annually, required service period = indefinite.

Find: AE cost and which model is the preferred alternative.

Therefore, in the case of an indefinitely ongoing investment project, we typically select a finite analysis period by using the **lowest common multiple** of project lives (15 years). We would consider alternative A through five life cycles and alternative B through three life cycles; in each case, we would use the alternatives completely to serve out the 15 years of operation. We then accept the finite model's results as a good prediction of the economically wisest course of action for the foreseeable future. This example is a case in which we conveniently use the lowest common multiple of project lives as our analysis period. (See Figure 6.7.)

n	Model A	Model B
0	−\$150,000	−\$230,000
1	−\$55,000	−\$30,000
2	−\$55,000	−\$30,000
3	+\$15,000 − \$55,000	−\$30,000
4		−\$30,000
5		+\$35,000 − \$30,000

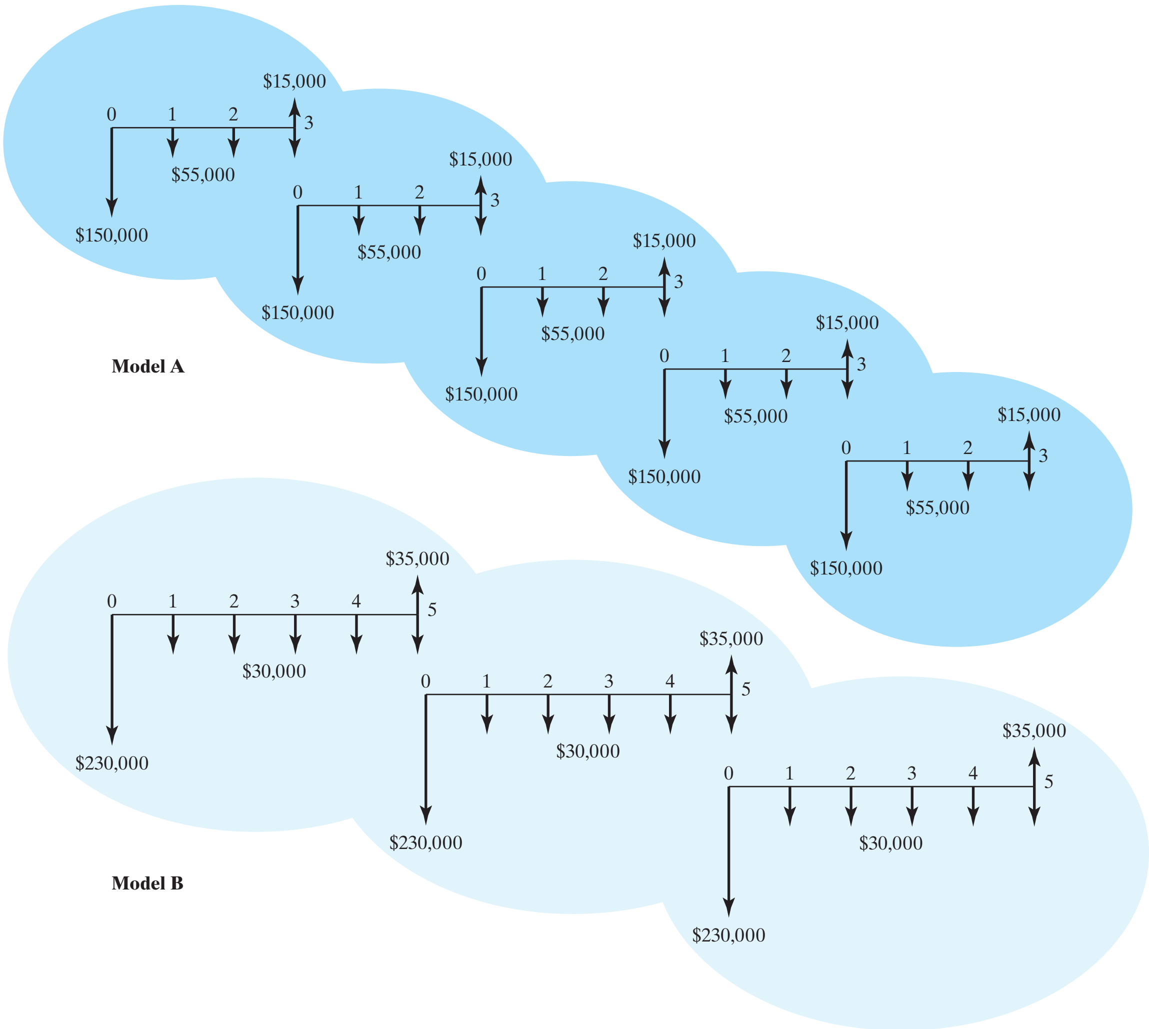


Figure 6.7 Comparing unequal-lived projects on the basis of the replacement chain approach—least common multiple service period of 15 years.

METHODOLOGY

Our objective is to determine the AE cost of each model over the lowest common multiple period of 15 years. In doing so, we will compute the PW cost of the first cycle, and then we convert it into its equivalent AE cost. We do the same for the entire cycle.

SOLUTION

Model A:

- For a three-year period (first cycle):

$$\begin{aligned} PW(15\%)_{\text{first cycle}} &= -\$150{,}000 - \$55{,}000(P/A, 15\%, 3) \\ &\quad + \$15{,}000\ (P/F, 15\%, 3) \\ &= -\$265{,}715. \end{aligned}$$

$$\begin{aligned} AEC(15\%)_{\text{first cycle}} &= \$265{,}715(A/P, 15\%, 3) \\ &= \$116{,}377. \end{aligned}$$

- For a 15-year period (five replacement cycles):

$$\begin{aligned} PW(15\%)_{\text{15-year period}} &= -\$265{,}715[1 + (P/F, 15\%, 3) \\ &\quad + (P/F, 15\%, 6) + (P/F, 15\%, 9) + (P/F, 15\%, 12)] \\ &= -\$680{,}499. \end{aligned}$$

$$\begin{aligned} AEC(15\%)_{\text{15-year period}} &= \$680{,}499(A/P, 15\%, 15) \\ &= \$116{,}377. \end{aligned}$$

Model B:

- For a five-year life (first cycle):

$$\begin{aligned} PW(15\%)_{\text{first cycle}} &= -\$230{,}000 - \$30{,}000(P/A, 15\%, 5) \\ &\quad + \$35{,}000(P/F, 15\%, 5) \\ &= -\$313{,}163. \end{aligned}$$

$$\begin{aligned} AEC(15\%)_{\text{first cycle}} &= \$313{,}163(A/P, 15\%, 5) \\ &= \$93{,}422. \end{aligned}$$

- For a 15-year period (three replacement cycles):

$$\begin{aligned} PW(15\%)_{\text{15-year period}} &= -\$313{,}163[1 + (P/F, 15\%, 5) \\ &\quad + (P/F, 15\%, 10)] \\ &= -\$546{,}270. \end{aligned}$$

$$\begin{aligned} AEC(15\%)_{\text{15-year period}} &= \$546, 270(A/P, 15\%, 15) \\ &= \$93, 422. \end{aligned}$$

We can see that the AE cost of Model A is much higher ($\$116{,}377 > \$93{,}422$); thus, we select Model B, despite its higher initial cost.

COMMENTS: Notice that the AE costs calculated on the basis of the lowest common multiple period are the same as those that were obtained over the initial life spans. Thus, for alternatives with unequal lives, comparing the AE cost of each project over its first cycle is sufficient in determining the best alternative.

SUMMARY

- Annual-equivalent worth analysis, or AE, and present-worth analysis are the two main analysis techniques determined on the concept of equivalence. The equation for AE is

$$\text{AE}(i) = \text{PW}(i)(A/P, i, N).$$

AE analysis yields the same decision result as PW analysis.

- The capital-recovery cost factor, or CR(i), is one of the most important applications of AE analysis in that it allows managers to calculate an annual equivalent cost of capital for ease of itemization with annual operating costs. The equation for CR(i) is

$$\text{CR}(i) = (I - S)(A/P, i, N) + iS,$$

where I = initial cost and S = salvage value.

- AE analysis is recommended over PW analysis in many key real-world situations for the following reasons:
  1. In many financial reports, an annual-equivalence value is preferred over a present-worth value for ease of use and its relevance to annual results.
  2. Calculation of unit costs is often required in order to determine reasonable pricing for sale items.
  3. Calculation of cost per unit of use is required in order to reimburse employees for business use of personal cars.
  4. Make-or-buy decisions usually require the development of unit costs so that "make" costs can be compared with prices for "buying."
  5. Comparisons of options with unequal service lives is facilitated by the AE method, assuming that the future replacements of the project have the same initial and operating costs. In general, this method is not practical, because future replacement projects typically have quite different cost streams. It is recommended that you consider various future replacement options by estimating the cash flows associated with each of them.

SELF-TEST QUESTIONS

6s.1 You are considering making an $80,000 investment in a process improvement project. Revenues are expected to grow from $50,000 in year 1 by $30,000 each year for next four years ($50,000 first year, $80,000 second year, $110,000 third year, and so forth) while costs are expected to increase from $20,000 in year 1 by $10,000 each year. If there is no salvage value at the end of five years, what is the annual equivalent worth of the project assuming an MARR of 12%?

(a) $65,492
(b) $53,300
(c) $47,785
(d) $43,300

6s.2 You are considering buying a 30-HP electric motor which has an efficiency rating of 89%. The motor costs $10,000 and will be used for 10 years. The expected salvage value at that time is $1,000. The cost to run the electric motor is $0.09 per kWh for 2,000 hours a year. (1 HP = 0.7457 kW.) What is the total annual equivalent cost of owning and operating the motor for 10 years at an interest rate of 12% per year?

(a) $6,237
(b) $4,581
(c) $5,739
(d) $3,969

6s.3 You purchased a drill press machine for $160,000. It is expected to have a useful life of 12 years. The accounting department tells you that the annual capital cost is $28,865 at $i = 12\%$. What salvage value is used in obtaining the annual capital cost of this machine?

(a) $50,100
(b) $55,300
(c) $29,970
(d) $73,454

6s.4 You are considering a project with the following financial data:

- Required initial investment at $n = 0$: $50 M
- Project life: 10 years
- Estimated annual revenue: $$X$ (unknown)
- Estimated annual operating cost: $15 M
- Required minimum return: 20% per year
- Salvage value of the project: 15% of the initial investment

What *minimum* annual revenue (in $M) must be generated to make the project worthwhile?

(a) $X = \$26.64\ M$
(b) $X = \$28.38\ M$
(c) $X = \$32.47\ M$
(d) $X = \$35.22\ M$

6s.5 Consider manufacturing equipment that has an installed cost of $120,000. The equipment is expected to generate $45,000 of the annual energy savings during its first year of installation. The value of these annual savings is expected to increase by 5% per year (over previous year) because of increased fuel costs. Assume that the equipment has a service life of 10 years (or 5,000 operating hours per year) with $20,000 worth of salvage value. Determine the equivalent dollar savings per each operating hour at $i = 10\%$ per year.

(a) $6.99 per hour
(b) $7.24 per hour
(c) $4.45 per hour
(d) $4.29 per hour

6s.6 The city of Atlanta is considering adding new buses for its current mass-transit system that links Hartsfield International Airport to major city destinations on nonstop basis. The total investment package is worth $8 million and is expected to last 10 years with a $750,000 salvage value. The annual

operating and maintenance costs for buses would be \$2 million. If the system is used for 600,000 trips per year, what would be the fair price to charge per trip? Assume that the city of Atlanta uses 5% interest rate for any city-sponsored projects.
(a) \$3.50 per trip
(b) \$4.00 per trip
(c) \$4.50 per trip
(d) \$5.00 per trip

6s.7 You are considering a luxury apartment building project that requires a capital investment of \$12,500,000. The building has 50 units. You expect the maintenance cost for the apartment building to be \$250,000 in the first year, \$300,000 in the second year, and increasing by \$50,000 in subsequent years. The cost to hire a manager for the building is estimated to be \$80,000 per year. After five years of operation, the apartment building can be sold for \$14,000,000. What is the annual rent per apartment unit that will provide a return on investment of 15% per year? Assume the building will remain fully occupied during the five years.
(a) \$36,445
(b) \$38,567
(c) \$41,373
(d) \$44,980

6s.8 Two options are available for painting your house: (1) Oil-based painting, which costs \$5,000, and (2) water-based painting, which costs \$3,000. The estimated lives are 10 years and 5 years, respectively. For either option, no salvage value will remain at the end of respective service lives. Assume that you will keep and maintain the house for 10 years. If your personal interest rate is 10% per year, which of the following statements is correct?
(a) On an annual basis, Option 1 will cost about \$850.
(b) On an annual basis, Option 2 is about \$22 less than Option 1.
(c) On an annual basis, both options cost about the same.
(d) On an annual basis, Option 2 will cost about \$820.

6s.9 A consumer product company is considering introducing a new shaving system called DELTA-4 in the market. The company plans to manufacture 75 million units of DELTA-4 a year. The investment at time 0 that is required for building the manufacturing plant is estimated as \$500 million, and the economic life of the project is assumed to be 10 years. The annual total operating expenses, including manufacturing costs and overhead, are estimated at \$175 million. The salvage value that can be realized from the project is estimated at \$120 million. If the company's MARR is 25%, determine the price that the company should charge for a DELTA-4 shaving system to break even.
(a) \$3.15
(b) \$4.15
(c) \$5.15
(d) \$2.80

6s.10 Consider the following cash flows and compute the equivalent annual worth at $i = 12\%$:

n	A_n Investment	Revenue
0	−\$34,000	
1	−\$10,000	\$15,000
2		\$14,000
3		\$13,000
4		\$13,000
5		\$8,000
6		\$5,500

(a) \$1,489
(b) \$1,572
(c) \$1,603
(d) \$1,647

6s.11 A newly constructed water treatment facility costs \$3.2 million. It is estimated that the facility will need revamping to maintain the original design specification every 25 years at a cost of \$1.5 million. Annual repairs and maintenance costs are estimated to be \$160,000. At an interest rate of 10%, determine the capitalized cost of the facility, assuming that it will be used for an indefinite period.
(a) \$268,800
(b) \$325,652
(c) \$412,668
(d) \$495,300

6s.12 You purchased a CNC machine for \$50,000. It is expected to have a useful life of 15 years and a salvage value of \$6,000. At $i = 14\%$, what is the annual capital cost of this machine?
(a) \$7,893
(b) \$8,003
(c) \$8,622
(d) \$8,850

6s.13 Radcliffe Electronics, Inc., just purchased a soldering machine to be used in its assembly cell for flexible disk drives. This machine costs \$284,000. Because of the specialized function it performs, its useful life is estimated to be seven years. At the end of that time, its salvage value is estimated to be \$48,000. What is the capital cost for this investment if the firm's interest rate is 16%?
(a) \$82,235
(b) \$73,299
(c) \$67,865
(d) \$66,114

6s.14 Consider a piece of industrial equipment that has an installed cost of \$100,000. The equipment is expected to generate \$30,000 worth of annual energy savings during its first year of installation. The value of these annual savings is expected to increase at the rate of 5% per year because of increased fuel costs. Assume that the equipment has a service life of five years (or 3,000 operating hours per year) with no appreciable salvage value. Determine the equivalent dollar savings per each operating hour at $i = 14\%$.

(a) \$1.20
(b) \$1.25
(c) \$1.34
(d) \$1,39

6s.15 You invest in a piece of equipment costing \$30,000. The equipment will be used for two years, at the end of which time the salvage value of the machine is expected to be \$10,000. The machine will be used for 5,000 hours during the first year and 8,000 hours during the second year. The expected annual net savings in operating costs will be \$25,000 during the first year and \$40,000 during the second year. If your interest rate is 10%, what would be the equivalent net savings per machine-hour?

(a) \$2.95
(b) \$3.05
(c) \$3.18
(d) \$3.30

6s.16 You are considering two types of electric motors for your paint shop. Financial information and operating characteristics are summarized as follows:

Summary Info and Characteristics	Brand X	Brand Y
Price	\$4,500	\$3,600
O&M cost per year	\$300	\$500
Salvage value	\$250	\$100
Capacity	150 HP	150 HP
Efficiency	83%	80%

If you plan to operate the motor for 2,000 hours annually, what would be the additional cost savings per operating hour associated with the more efficient brand (Brand X) at an interest rate of 12%? The motor will be needed for 10 years. Assume that power costs are 5 cents per kilowatt-hour (1 HP = 0.746 kW).

(a) Save \$0.28 per hour
(b) Save \$1.25 per hour
(c) Save \$1.53 per hour
(d) Cost \$0.30 more per hour

PROBLEMS

Note 1: Unless otherwise stated, all cash flows given in the problems represent after-tax cash flows in *actual dollars*. The MARR also represents a market interest rate, which considers any inflationary effects in the cash flows.

Note 2: Unless otherwise stated, all interest rates presented in this set of problems are based on annual compounding.

6.1 An engineering design firm needs to borrow $450,000 from a local bank at an interest rate of 10% over eight years. What is the required annual equal payment to retire the loan in eight years?

6.2 You are considering an investment costing $300,000. If you want to recover the initial investment, and also earn 11% interest while your money is being tied up within four years, what is the required equal annual net revenue that must be generated from the investment?

6.3 Consider the accompanying cash flow diagram. Compute the equivalent annual worth at $i = 10\%$.

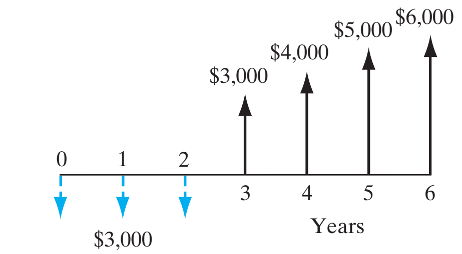
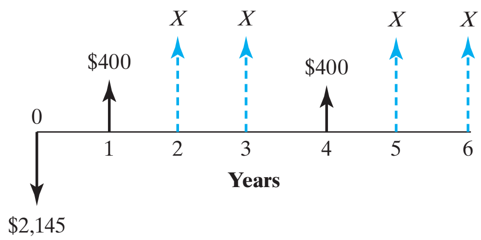


Figure P6.3

6.4 The investment shown in the following figure has an annual equivalent worth of $400 at $i = 10\%$. Determine the cash flows in periods 2, 3, 5, and 6.



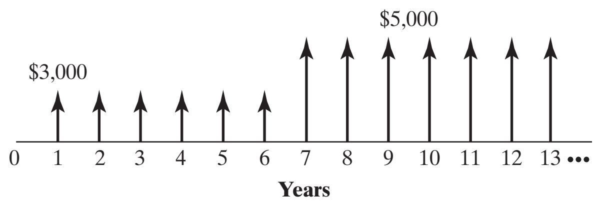
6.5 Consider the cash flows in Table P6.5 for the following investment projects (MARR = 15%).

TABLE P6.5

	Project's Cash Flow		
n	A	B	C
0	−$3,500	−$4,000	−$6,500
1	$1,500	$1,600	$2,000
2	$1,800	$1,500	$2,000
3	$1,000	$1,500	$2,500
4	$600	$1,500	$2,500

Determine the annual equivalent worth for each project at $i = 15\%$ and determine the acceptability of each project.

6.6 At $i = 16\%$, what is the annual-equivalence amount for the infinite series shown next?



6.7 Consider the cash flows in Figure P6.7 and compute the equivalent annual worth at $i = 13\%$.

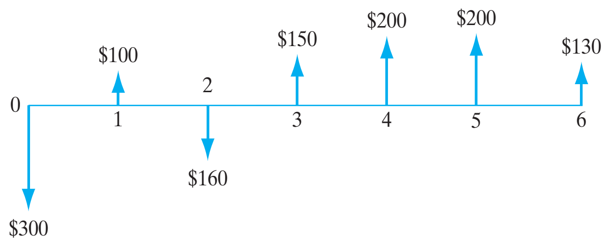


Figure P6.7

6.8 Consider the sets of investment projects from Table P6.8. Compute the equivalent annual worth of each project at $i = 9\%$, and determine the acceptability of each project.

TABLE P6.8

	Project Cash Flows			
n	A	B	C	D
0	−$5,800	−$4,500	−$3,200	−$5,500
1	$0	$1,150	$4,400	$4,100
2	$0	$1,250	$3,300	$4,100
3	$15,200	$1,350	$2,200	$4,100

6.9 The repeating cash flows for a certain project are as given in Table P6.9. Find the equivalent annual worth for this project at $i = 12\%$, and determine the acceptability of the project.

TABLE P6.9

n	Net Cash Flow: Investment	Net Cash Flow: Operating Income	
0	−$3,600	$0	1st cycle
1	$0	$2,100	
2	$0	$1,100	
3	−$3,600	$600	2nd cycle
4	$0	$2,100	
5	$0	$1,100	
6	−$3,600	$600	3rd cycle
7	$0	$2,100	
8	$0	$1,100	
9	$0	$600	

6.10 An airline is planning to make iPads® available on all of its Boeing 747 aircraft with in-flight e-mail and Internet service on transoceanic flights. Passengers on these flights will be able to rent iPads from the airline and use them to browse the Internet or to send and receive e-mail no matter where they are in the skies. A nominal charge of approximately $30 will be instituted for each rental. The airline has estimated the projected cash flows (in millions of dollars) for the systems in the first 10 aircraft as follows:

Year	A_n (Unit: Million Dollars)
2017	–$6.0
2018	$2.4
2019	$6.0
2020	$9.0
2021	$10.5
2022	$12.0

Determine whether this project can be justified at MARR = 7%, and calculate the annual benefit (or loss) that would be generated after installation of the systems.

Capital Recovery (Ownership) Cost

6.11 M.T. Labs bought a Gene gun for $33,000. The accounting department has estimated that the machine would have an annualized capital cost of $5,250 over its eight-year service life. What salvage value was assumed in calculating the capital cost? The firm's interest rate is known to be 11%.

6.12 Keiko is considering buying a 2017 Smart ForTwo costing $23,650 and finds that the retaining values of the vehicle over next five years are as follows:

- Percent of the total value retained after 36 months: 25%
- Percent of the total value retained after 60 months: 16%

If her interest rate is 5% compounded annually, what is the ownership cost of the vehicle over three years? five years?

6.13 You invest in a piece of equipment costing $40,000. The equipment will be used for two years, and it will be worth $15,000 at the end of two years. The machine will be used for 4,000 hours during the first year and 6,000 hours during the second year. The expected savings associated with the use of the piece of equipment will be $28,000 during the first year and $40,000 during the second year. Your interest rate is 10%.

(a) What is the capital recovery cost?

(b) What is the annual equivalent worth?

(c) What is net savings generated per machine-hour?

6.14 You are considering purchasing a dump truck. The truck will cost $75,000 and have operating and maintenance costs that start at $18,000 the first year and increases by $2,000 per year. Assume that the salvage value at the end of five years is $22,000 and interest rate is 12%. What is the equivalent annual cost of owning and operating the truck?

6.15 A construction firm is considering establishing an engineering computing center. The center will be equipped with three engineering workstations that cost $52,000 each, each having a service life of six years. The expected salvage value of each workstation is $2,000. The annual operating and maintenance cost would be $28,000 for each workstation. At a MARR of 16%, determine the equivalent annual cost for operating the engineering center.

6.16 Beginning next year, a foundation will support an annual seminar on campus with the earnings of a $200,000 gift it received this year. It is felt that 6% interest will be realized for the first 10 years, but that plans should be made to anticipate an interest rate of 4% after that time. What amount should be added to the foundation now to fund the seminar at the $20,000 level into infinity?

6.17 The Geo-Star Manufacturing Company is considering a new investment in a punch-press machine that will cost $138,000 and has an annual maintenance cost of $10,000. There is also an additional overhauling cost of $24,000 for the equipment once every five years. Assuming that this equipment will last infinitely under these conditions, what is the capitalized equivalent cost of this investment at an interest rate of 8%?

6.18 You are considering investing $82,000 in new equipment. You estimate that the net cash flows will be $21,000 during the first year, but will increase by $2,800 per year the next year and each year thereafter. The equipment is estimated to have a 12-year service life and a net salvage value of $7,000 at that time. Assume an interest rate of 10%.

(a) Determine the annual capital cost (ownership cost) for the equipment.

(b) Determine the equivalent annual savings (revenues).

(c) Determine whether this is a wise investment.

Annual Equivalent Worth Criterion

6.19 Company X has been contracting its overhauling work to Company Y for \$38,000 per machine per year. Company X estimates that by building a \$475,000 maintenance facility with a life of 10 years and a salvage value of \$95,000 at the end of its life, it could handle its own overhauling at a cost of only \$30,000 per machine per year. What is the minimum annual number of machines that Company X must operate to make it economically feasible to build its own facility? (Assume an interest rate of 15%.)

6.20 The cash flows for two investment projects are as given in Table P6.20.

(a) For project A, find the value of X that makes the equivalent annual receipts equal the equivalent annual disbursement at $i = 15\%$.

(b) For A to be preferred over project B, determine the minimum acceptable value of X in year 2 at $i = 12\%$ based on an AE criterion.

TABLE P6.20

	Project's Cash Flow	
n	A	B
0	−\$4,500	\$6,500
1	\$1,000	−\$1,400
2	X	−\$1,400
3	\$1,000	−\$1,400
4	\$1,000	−\$1,400

6.21 An industrial firm is considering purchasing several programmable controllers and automating the company's manufacturing operations. It is estimated that the equipment will initially cost \$120,000, and the labor to install it will cost \$25,000. A service contract to maintain the equipment will cost \$5,000 per year. Trained service personnel will have to be hired at an annual salary of \$50,000. Also estimated is an approximate \$10,000 annual income-tax savings (cash inflow). How much will this investment in equipment and services have to increase the annual revenues after taxes in order to break even? The equipment is estimated to have an operating life of 10 years with no salvage value (because of obsolescence). The firm's MARR is 12%.

6.22 A certain factory building has an old lighting system. Lighting the building currently costs, on average, \$25,000 a year. A lighting consultant tells the factory supervisor that the lighting bill can be reduced to \$8,200 a year if \$67,000 is invested in new lighting in the building. If the new lighting system is installed, an incremental maintenance cost of \$3,800 per year must be taken into account. The new lighting system has zero salvage value at the end of its life. If the old lighting system also has zero salvage value, and the new lighting system is estimated to have a life of 14 years, what is the net annual benefit for this investment in new lighting? Take the MARR to be 13%. Assume the old lighting system will last 14 years.

Unit-Profit or Unit-Cost Calculation

6.23 You have purchased a machine costing \$30,000. The machine will be used for two years, and at the end of this time, its salvage value is expected to be \$18,000.

The machine will be used 6,000 hours during the first year and 8,000 hours during the second year. The expected annual net savings will be $35,000 during the first year and $42,000 during the second year. If your interest rate is 12%, what would be the equivalent net savings per machine hour?

6.24 You just purchased a pin-inserting machine to relieve some bottleneck problems that have been created in manufacturing a PC board. The machine cost $65,000 and has an estimated service life of six years. At that time, the estimated salvage value would be $6,500. The machine is expected to operate 3,000 hours per year. The expected annual operating and maintenance cost would be $6,400. If your firm's interest rate is 14%, what would be the machine cost (owning and operating) per hour?

6.25 The General Mills Company (GMC) purchased a milling machine for $100,000, which it intends to use for the next five years. This machine is expected to save GMC $35,000 during the first operating year. Then the annual savings are expected to decrease by *3% each subsequent year over the previous year* due to increased maintenance costs.
Assuming that GMC would operate the machine for an average of 3,000 hours per year and that it would have no appreciable salvage value at the end of the five-year period, determine the equivalent dollar savings per operating hour at 14% interest compounded annually.

6.26 A company is currently paying its employees $0.56 per mile to drive their own cars on company business. The company is considering supplying employees with cars, which would involve purchasing at $25,000 with an estimated three-year life, a net salvage value of $8,000, taxes and insurance at a cost of $1,200 per year, and operating and maintenance expenses of $0.30 per mile. If the interest rate is 10% and the company anticipates an employee's annual travel to be 30,000 miles, what is the equivalent cost per mile (neglecting income taxes)?

6.27 The Novelty Company, a manufacturer of farm equipment, currently produces 20,000 units of gas filters per year for use in its lawn-mower production. The costs, based on the previous year's production, are reported in Table P6.27.

TABLE P6.27 Production Costs

Item	Expense ($)
Direct materials	$60,000
Direct labor	$180,000
Variable overhead (power and water)	$135,000
Fixed overhead (light and heat)	$70,000
Total cost	$445,000

It is anticipated that gas-filter production will last five years. If the company continues to produce the product in-house, annual direct material costs will increase at the rate of 5%. (For example, annual material costs during the first production year will be $63,000.) Direct labor will increase at the rate of 6% per year, and variable overhead costs will increase at the rate of 3%. However, the fixed overhead will remain at its current level over the next five years. The John Holland Company has offered to sell Novelty 20,000 units of gas filters for \$25 per unit. If Novelty accepts the offer, some of the manufacturing facilities currently used to manufacture the filter could be rented to a third party for \$35,000 per year. In addition, \$3.5 per unit of the fixed overheard applied to the production of gas filters would be eliminated. The firm's interest rate is known to be 15%. What is the unit cost of buying the gas filters from the outside source? Should Novelty accept John Holland's offer, and if so, why?

6.28 An automobile that runs on electricity can be purchased for \$25,000. The automobile is estimated to have a life of 12 years with annual travel of 20,000 miles. Every three years, a new set of batteries will have to be purchased at a cost of \$3,000. Annual maintenance of the vehicle is estimated to cost \$700. The cost of recharging the batteries is estimated at \$0.015 per mile. The salvage value of the batteries and the vehicle at the end of 12 years is estimated to be \$2,000. Suppose the MARR is 7%. What is the cost per mile to own and operate this vehicle based on the preceding estimates? The \$3,000 cost of the batteries is a net value with the old batteries traded in for the new ones.

6.29 A California utility firm is considering building a 50-megawatt geothermal plant that generates electricity from naturally occurring underground heat. The binary geothermal system will cost \$85 million to build and \$6 million (including any income-tax effect) to operate per year. (Virtually no fuel costs will accrue compared with fuel costs related to a conventional fossil-fuel plant.) The geothermal plant is to last for 25 years. At that time, its expected salvage value will be about the same as the cost to remove the plant. The plant will be in operation for 70% (plant utilization factor) of the year (or 70% of 8,760 hours per year). If the firm's MARR is 14% per year, determine the cost per kilowatt-hour of generating electricity.

Break-Even Analysis

6.30 A city has decided to build a softball complex, and the city council has already voted to fund the project at the level of \$800,000 (initial capital investment). The city engineer has collected the following financial information for the complex project:

- Annual upkeep costs: \$120,000
- Annual utility costs: \$13,000
- Renovation costs: \$50,000 for every five years
- Annual team user fees (revenues): \$32,000
- Useful life: Infinite
- Interest rate: 8%

If the city can expect 40,000 visitors to the complex each year, what should be the minimum ticket price per person so that the city can break even?

6.31 B&B Company is a real-estate developer considering a 40-unit apartment complex in a growing college town. As the area is also booming with foreign automakers locating their U.S. assembly plants, the firm expects that the apartment complex, once built, will enjoy a 90% occupancy for an extended period. The firm already compiled some of the critical financial information related to the development project as follows:

- Land price (1 acre) = \$1,200,000
- Building (40 units of single bedroom) = \$4,800,000

- Project life = 25 years
- Building maintenance per unit per month = \$100
- Annual property taxes and insurance = \$400,000

Assuming that the land will appreciate at an annual rate of 5%, but the building will have no value at the end of 25 years (it will be torn down and a new structure would be built), determine the minimum monthly rent that should be charged if a 12% return (or 0.9489% per month) before tax is desired.

6.32 The estimated cost of a completely installed and ready-to-operate 40-kilowatt generator is \$32,000. Its annual maintenance costs are estimated at \$800. The energy that can be generated annually at full load is estimated to be 100,000 kilowatt-hours. If the value of the energy generated is \$0.09 per kilowatt-hour, how long will it take before this machine becomes profitable? Take the MARR to be 10% and the salvage value of the machine to be \$2,000 at the end of its estimated life of 15 years.

6.33 A large land-grant university that is currently facing severe parking problems on its campus is considering constructing parking decks off campus. A shuttle service could pick up students at the off campus parking deck and transport them to various locations on campus. The university would charge a small fee for each shuttle ride, and the students could be quickly and economically transported to their classes. The funds raised by the shuttle would be used to pay for trolleys, which cost about \$170,000 each. Each trolley has a 12-year service life, with an estimated salvage value of \$12,000. To operate each trolley, additional expenses will be incurred, as given in Table P6.33.

TABLE P6.33

Item	Annual Expenses (\$)
Driver	\$70,000
Maintenance	\$15,000
Insurance	\$5,000

If students pay 10 cents for each ride, determine the annual ridership per trolley (number of shuttle rides per year) required to justify the shuttle project, assuming an interest rate of 6%.

6.34 Two 180-horsepower water pumps are being considered for installation in a farm. Financial data for these pumps are as follows:

Item	Pump I	Pump II
Initial cost	\$6,000	\$4,000
Efficiency	86%	80%
Useful life	12 years	12 years
Annual operating cost	\$500	\$440
Salvage value	\$0	\$0

If power cost is a flat 6 cents per kWh over the study period, determine the minimum number of hours of full-load operation per year that would justify the purchase of the more expensive pump at an interest rate of 8% (1 HP = 746 watts = 0.746 kilowatts).

6.35 Eradicator Food Prep, Inc. has invested $10 million to construct a food irradiation plant. This technology destroys organisms that cause spoilage and disease, thus extending the shelf life of fresh foods and the distances over which they can be shipped. The plant can handle about 300,000 pounds of produce in an hour, and it will be operated for 4,000 hours a year. The net expected operating and maintenance costs (taking into account income-tax effects) would be $4 million per year. The plant is expected to have a useful life of 15 years with a net salvage value of $800,000. The firm's interest rate is 15%.

(a) If investors in the company want to recover the plant investment within six years of operation (rather than 15 years), what would be the equivalent annual revenues that must be generated?

(b) To generate annual revenues determined in part (a), what minimum processing fee per pound should the company charge to its producers?

6.36 You are considering developing an 18-hole championship golf course that requires an investment of $20,000,000. This investment cost includes the course development, club house, and golf carts. Once constructed, you expect the maintenance cost for the golf course to be $650,000 in the first year, $700,000 in the second year and continue to increase by $50,000 in subsequent years. The net revenue generated from selling food and beverage will be about 15% of greens fees paid by the players. The cart fee per player is $15, and 40,000 rounds of golf are expected per year. You will own and operate the course complex for 10 years and expect to sell it for $25,000,000. What is the greens fee per round that will provide a return on investment of 15%? Assume that the greens fee will be increased at an annual rate of 5%.

6.37 A corporate executive jet with a seating capacity of 20 has the cost factors given in Table P6.37.

The company flies three round trips from Boston to London per week, which is a distance of 3,280 miles one way. How many passengers must be carried on an average trip in order to justify the use of the jet if the first-class round-trip fare is $3,400? The firm's MARR is 15%. (Ignore income-tax consequences.)

TABLE P6.37

Item	Cost
Initial cost	$12,000,000
Service life	15 years
Salvage value	$2,000,000
Crew costs per year	$225,000
Fuel cost per mile	$1.10
Landing fee	$250
Maintenance per year	$237,500
Insurance cost per year	$166,000
Catering per passenger trip	$75

Using the AE Method to Compare Mutually Exclusive Alternatives

6.38 An industrial firm can purchase a special machine for $70,000. A down payment of $5,000 is required, and the unpaid balance can be paid off in five equal year-end installments at 9% interest. As an alternative, the machine can be purchased for $66,000 in cash. If the firm's MARR is 10%, use the annual equivalent method to determine which alternative should be accepted.

6.39 Washington Air Company is considering the purchase of a helicopter for connecting services between the company's base airport and the new intercounty airport being built about 30 miles away. It is believed that the chopper will be needed only for six years until the Rapid Transit Connection is phased in. The estimates on two types of helicopters under consideration, the Whirl 2B and the ROT 8, are given in Table P6.39.

TABLE P6.39

	The Whirl 2B	The ROT 8
First cost	$95,000	$120,000
Annual maintenance	$3,000	$9,000
Salvage value	$12,000	$25,000
Useful life in years	3	6

Assuming that the Whirl 2B will be available in the future with identical costs, what is the annual cost advantage of selecting the ROT8? (Use an interest rate of 10%.)

6.40 You are asked to decide between two projects based on annual equivalent worth.

A	B
−$10,000	−$12,000
$6,000	$7,000
$5,000	$8,000
$4,000	

(a) What assumptions do you need to make in comparing these mutually exclusive revenue projects?

(b) Based on the assumptions in (a), which project should you choose at $i = 10\%$?

6.41 The cash flows in Table P6.41 represent the potential annual savings associated with two different types of production processes, each of which requires an investment of $40,000.

Assume an interest rate of 12%.

TABLE P6.41

n	Process A	Process B
0	−$40,000	−$40,000
1	$19,120	$17,350
2	$17,840	$17,350
3	$16,560	$17,350
4	$15,280	$17,350

(a) Determine the equivalent annual savings for each process.
(b) Determine the hourly savings for each process if it will be in operation of 3,000 hours per year.
(c) Which process should be selected?

6.42 Two 180-horsepower (HP) motors are being considered for installation at a municipal sewage-treatment plant. The first costs $5,800 and has an operating efficiency of 83%. The second costs $4,600 and has an efficiency of 80%. Both motors are projected to have zero salvage value after a life of 10 years. If all the annual charges, such as insurance, maintenance, etc., amount to a total of 15% of the original cost of each motor, and if power costs are a flat five cents per kilowatt-hour, how many minimum hours of full-load operation per year are necessary to justify purchasing the more expensive motor at $i = 6\%$? (A conversion factor you might find useful is 1 HP = 746 watts = 0.746 kilowatts.)

6.43 A chemical company is considering two types of incinerators to burn solid waste generated by a chemical operation. Both incinerators have a burning capacity of 20 tons per day. The data in Table P6.43 have been compiled for comparison.

TABLE P6.43

	Incinerator A	Incinerator B
Installed cost	$1,300,000	$850,000
Annual O&M costs	$70,000	$100,000
Service life	20 years	10 years
Salvage value	$60,000	$30,000
Income taxes	$40,000	$30,000

If the firm's MARR is known to be 13%, determine the processing cost per ton of solid waste incurred by each incinerator. Assume that incinerator B will be available in the future at the same cost.

6.44 An airline is considering two types of engine systems for use in its planes. Each has the same life and the same maintenance and repair record.

- **System A** costs $100,000 and uses 40,000 gallons per 1,000 hours of operation at the average load encountered in passenger service.

- **System B** costs \$200,000 and uses 32,000 gallons per 1,000 hours of operation at the same level.

Both engine systems have three-year lives before any major overhaul is required. On the basis of the initial investment, the systems have 10% salvage values. If jet fuel currently costs \$2.10 a gallon and fuel consumption is expected to increase at the rate of 6% per year because of degrading engine efficiency, which engine system should the firm install? Assume 2,000 hours of operation per year and an MARR of 10%. Use the AE criterion. What is the equivalent operating cost per hour for each engine?

6.45 Mustang Auto Parts, Inc. is considering one of two forklift trucks for its assembly plant.

- **Truck A** costs \$15,000 and requires \$3,000 annually in operating expenses. It will have a \$5,000 salvage value at the end of its three-year service life.
- **Truck B** costs \$20,000, but requires only \$2,000 annually in operating expenses; its service life is four years, at which time its expected salvage value will be \$8,000.

The firm's MARR is 12%. Assuming that the trucks are needed for 12 years and that no significant changes are expected in the future price and functional capacity of each truck, select the most economical truck on the basis of AE analysis.

6.46 A small manufacturing firm is considering purchasing a new machine to modernize one of its current production lines. Two types of machines are available on the market. The lives of machine A and machine B are four years and six years, respectively, but the firm does not expect to need the service of either machine for more than five years. The machines have the expected receipts and disbursements given in Table P6.46.

TABLE P6.46

Item	Machine A	Machine B
First cost	\$6,500	\$8,500
Service life	4 years	6 years
Estimated salvage value	\$600	\$1,000
Annual O&M costs	\$800	\$520
Change oil filter every other year	\$100	None
Engine overhaul	\$200 (every 3 years)	\$280 (every 4 years)

The firm also has another option: leasing a machine at \$3,000 per year, which is fully maintained by the leasing company. After four years of use, the salvage value for machine B will remain at \$1,000.

(a) How many decision alternatives are there?

(b) Which decision appears to be the best at $i = 10\%$?

6.47 A plastic-manufacturing company owns and operates a polypropylene production facility that converts the propylene from one of its cracking facilities to polypropylene plastics for outside sale. The polypropylene production facility is currently forced to operate at less than capacity due to an insufficiency of propylene production capacity in its hydrocarbon cracking facility. The chemical engineers are considering alternatives for supplying additional propylene to the polypropylene production facility. Two feasible alternatives are to build a pipeline to the nearest outside supply source and to provide additional propylene by truck from an outside source. The engineers also gathered the following projected cost estimates.

- Future costs for purchased propylene excluding delivery: $0.215 per lb
- Cost of pipeline construction: $200,000 per pipeline mile
- Estimated length of pipeline: 180 miles
- Transportation costs by tank truck: $0.05 per lb, utilizing a common carrier
- Pipeline operating costs: $0.005 per lb, excluding capital costs
- Projected additional propylene needs: 180 million lb per year
- Projected project life: 20 years
- Estimated salvage value of the pipeline: 8% of the installed costs

Determine the propylene cost per pound under each option if the firm's MARR is 18%. Which option is more economical?

Short Case Studies with Excel

6.48 The city of Peachtree is comparing the following two plans for supplying water to a newly developed subdivision:

- Plan A will manage requirements for the next 15 years; at the end of that period, the initial cost of $1,500,000 will have to be doubled to meet the requirements of subsequent years. The facilities installed in years 0 and 15 may be considered permanent; however, certain supporting equipment will have to be replaced every 30 years from the installation dates at a cost of $200,000. Operating costs are $91,000 a year for the first 15 years and $182,000 thereafter. Beginning in the 21st year, they will increase by $3,000 a year.
- Plan B will supply all requirements for water indefinitely into the future, although it will operate at only half capacity for the first 15 years. Annual costs over this period will be $105,000 and will increase to $155,000 beginning in the 16th year. The initial cost of Plan B is $1,950,000; the facilities can be considered permanent, although it will be necessary to replace $350,000 of equipment every 30 years after the initial installation.

The city will charge the subdivision the use of water calculated on the equivalent annual cost. At an interest rate of 10%, determine the equivalent annual cost for each plan and make a recommendation to the city as to the amount that should be charged to the subdivision.

6.49 Capstone Turbine Corporation is the world's leading provider of microturbine-based MicroCHP (combined heat and power) systems for clean, continuous, distributed-generation electricity. The MicroCHP unit is a compact turbine generator that delivers electricity on-site or close to the point where it is needed. This form of distributed-generation technology, designed to operate on a variety of gaseous and liquid fuels, first debuted in 1998. The microturbine is expected to operate on demand or continuously for up to a year between recommended maintenance (filter cleaning/replacement). The generator is cooled by airflow into the gas turbine, thus eliminating the need for liquid cooling. It can make electricity from a variety of fuels—natural gas, kerosene, diesel oil, and even waste gases from landfills, sewage plants, and oil fields.

Capstone's focus applications include combined heat and power, resource recovery of waste fuel from wellhead and biogas sites, and hybrid electric vehicles. And, unlike traditional backup power, this solution can support everyday energy needs and generate favorable payback. With the current design, which has a 60-kW rating, one of Capstone's generators would cost about $84,000. The expected annual expenses, including capital costs as well as operating costs, would run close to $19,000. These expenses yield an annual savings of close to $25,000 compared with the corresponding expenses for a conventional generator of the same size. The investment would pay for itself within three to four years.

One of the major questions among the Capstone executives is: How low does the microturbine's production cost need to be for it to be a sensible option in some utility operations? To answer this question, Capstone must first determine the cost per kilowatt of its generators.

How does Capstone come up with the capital cost of $1,400 per kilowatt? Suppose you plan to purchase the 60-kW microturbine and expect to operate it continuously for 10 years. How would you calculate the operating cost per kilowatt-hour?

6.50 Empex Corporation currently produces both videocassette cases (bodies) and metal-particle magnetic tape for commercial use. An increased demand for metal-particle videotapes is projected, and Empex is deciding between increasing the internal production of both empty cassette cases and magnetic tape or purchasing empty cassette cases from an outside vendor. If Empex purchases the cases from a vendor, the company must also buy specialized equipment to load the magnetic tape into the empty cases, since its current loading machine is not compatible with the cassette cases produced by the vendor under consideration. The projected production rate of cassettes is 79,815 units per week for 48 weeks of operation per year. The planning horizon is seven years. After considering the effects of income taxes, the accounting department has itemized the costs associated with each option as follows:

- "Make" Option:

Annual Costs	
Labor	$1,445,633
Materials	$2,048,511
Incremental overhead	$1,088,110
Total annual cost	$4,582,254

- "Buy" Option:

Capital Expenditure	
Acquisition of a new loading machine	$405,000
Salvage value at end of seven years	$45,000

Annual Operating Costs:	
Labor	$251,956
Purchase of empty cassette cases ($0.85/unit)	$3,256,452
Incremental overhead	$822,719
Total annual operating costs	$4,331,127

(Note the conventional assumption that cash flows occur in discrete lumps at the ends of years, as shown in Figure 6.5.) Assuming that Empex's MARR is 14%, calculate the unit cost under each option.

6.51 A Veterans Administration (VA) hospital needs to decide which type of boiler fuel system will most efficiently provide the required steam energy output for heating, laundry, and sterilization purposes. The present boilers were installed in the early 1970s and are now obsolete. Much of the auxiliary equipment is also old and requires repair. Because of these general conditions, an engineering recommendation was made to replace the entire plant with a new boiler-plant building that would house modern equipment. The cost of demolishing the old boiler plant would be almost a complete loss, as the salvage value of the scrap steel and used brick is estimated to be only about $1,000. The VA hospital's engineer finally selected two alternative proposals as being worthy of more intensive analysis. The hospital's annual energy requirement, measured in terms of steam output, is approximately 145,000,000 pounds of steam. As a general rule for analysis, 1 pound of steam is approximately 1,000 BTUs, and 1 cubic foot of natural gas is approximately 1,000 BTUs. The two alternatives are as follows:

- Proposal 1: Build a new coal-fired boiler plant, which would cost $2,570,300. To meet the requirements for particulate emission as set by the Environmental Protection Agency (EPA), the new coal-fired boiler, even if it burned low-sulfur coal, would need an electrostatic precipitator, which would cost approximately $145,000. The new plant would last for 20 years. One pound of dry coal yields about 14,300 BTUs. To convert the 145,000,000 pounds of steam energy into the common unit of BTUs, it is necessary to multiply by 1,000. To find BTU input requirements, it is necessary to divide by the relative boiler efficiency for type of fuel. The boiler efficiency for coal is 0.75. The coal price is estimated to be $125 per metric ton.
- Proposal 2: Build a gas-fired boiler plant with No. 2 fuel oil as a standby. This system would cost $1,289,340 with an expected service life of 20 years. Since small household or commercial users entirely dependent on gas for energy have priority, large plants must have oil switch-over capabilities. It has been estimated that 6% of 145,000,000 pounds (or 8,700,000 pounds) of steam energy would result from the switch between oil and gas. The boiler efficiency would be 0.78 for gas and 0.81 for oil. The heat value of natural gas is approximately 1,000,000 BTU/MCF (million cubic feet), and for No. 2 fuel oil it is 139,400 BTU/gal. The estimated gas price is $14.50/MCF, and the No. 2 fuel-oil price is $2.93 per gallon.

(a) Calculate the annual fuel costs for each proposal.

(b) Determine the unit cost per steam pound for each proposal. Assume $i = 10\%$.

(c) Which proposal is more economical?

CHAPTER SEVEN

Rate-of-Return Analysis

Is College Worth It?[1] For years, higher education was touted as a safe path to professional and financial success. Easy money, in the form of student loans, flowed to help parents and students finance degrees with the implication that in the long run, a bachelor's degree was a good bet. With this ever-itching curiosity, *The Economist* compiled an interesting statistic—the typical cumulative pay for graduates 20 years after they earn their bachelor's degree. The magazine looked at net pay, beyond what it cost the student to get their degree. Some private engineering graduates do display an impressive number; for example the median salary was $111,021 for 2016 graduates from MIT[2], but the typical price tag at this institution is about $240,000.

What happens when you look at earnings per dollar spent to get an education? According to *The Economist* based on 2013 dollars, leading

[1] "Is College Worth It?," *The Economist*, April 5, 2014. (http://www.economist.com/news/united-states/21600131-too-many-degrees-are-waste-money-return-higher-education-would-be-much-better).
[2] "Graduating Student Survey, June 2016 Survey Results, MIT Global Education & Career Development. (https://gecd.mit.edu/sites/default/files/about/files/2016-gss-survey.pdf).

is the University of Virginia's 17.60% return on investment, next is Georgia Institute of Technology's 17.10%. The top returns on investment (ROI) get a boost from the relatively modest costs for in-state students who spent $25,000 after financial aid to earn a degree at University of Virginia. In terms of majors,[3] engineers and computer scientists do best, earning an impressive 20-year annualized return of 12% on their college fees (the S&P 500 yielded just 7.8%). Business and economics degrees also pay well, delivering a solid 8.7% average return.

Total Cost of Degree, 2013

Institution	Cost to complete degree after financial aid	Annual return over 20 years (%)
University of Virgina	$25,000	17.60
Geogia Institute of Technology	$37,000	17.10
Harvard	$52,000	15.10
William and Mary	$38,000	14.80
University of Washington	$38,000	14.80
Stanford	$75,000	14.20
MIT	$80,000	13.90
U.C Berkeley	$62,000	13.50
Caltech	$85,000	13.30
Dartmouth	$76,000	13.30
Yale	$82,000	13.30
Princeton	$76,000	13.20
Purdue	$55,000	13.20
UCLA	$53,000	12.80
South Dakoda School of Mines	$73,000	12.70
Harvey Mudd College	$115,000	12.60
20-year U.S. Treasury Bill		3.40

Data from www.payscale.com
Note: Annual return based on earnings minus cost of college and earnings of a typical high-school graduate.

[3] "Is Your Degree Worth It? It Depends What You Study, Not Where," *The Economist*, March 12, 2015. (http://www.economist.com/news/united-states/21646220-it-depends-what-you-study-not-where).

What does the 17.60% rate-of-return figure represent for the University of Virginia graduates? If you look at your college degree from purely an investment point of view, what we are saying here is "investing $25,000" in four years will bring in additional earnings (when compared with the high school graduates), equivalent to generating 17.6% interest earned by your savings account. Is this a good investment?

How do we compute the figure? And once we have computed this figure, how do we use it when evaluating an investment alternative? Our consideration of the concept of rate of return in this chapter will answer these and other questions.

Along with the PW and AE concepts, the third primary measure of investment worth is **rate of return (ROR)**. As shown in Chapter 5, the PW measure is easy to calculate and apply. Nevertheless, many engineers and financial managers prefer rate-of-return analysis to the PW method because they find it intuitively more appealing to analyze investments in terms of percentage rates of return rather than in dollars of PW. Consider the following statements regarding an investment's profitability:

- This project will bring in a 15% rate of return on the investment.
- This project will result in a net surplus of $10,000 in terms of PW.

Neither statement describes the nature of an investment project in any complete sense. However, the rate-of-return figure is somewhat easier to understand because many of us are familiar with savings and loan interest rates, which are in fact rates of return.

In this chapter, we will examine four aspects of rate-of-return analysis: (1) the concept of return on investment; (2) calculation of a rate of return; (3) development of an internal rate-of-return criterion; and (4) comparison of mutually exclusive alternatives, based on rate of return.

7.1 Rate of Return

Many different terms are used to refer to **rate of return**, including **yield** (e.g., yield to maturity, commonly used in bond valuation), **internal rate of return**, and **marginal efficiency of capital**. We will first review three common definitions of rate of return. Then we will use the definition of internal rate of return as a measure of profitability for a single investment project, throughout the text.

7.1.1 Return on Investment

There are several ways of defining the concept of rate of return on investment. We will show two of them: the first is based on a typical loan transaction, and the second is based on the mathematical expression of the present-worth function.

Definition 1. *Rate of return is the rate of interest earned on the outstanding balance of an amortized loan.*

Suppose that a bank lends \$10,000, which is repaid in installments of \$4,021 at the end of each year for three years. How would you determine the interest rate that the bank charges on this transaction? As we learned in Chapter 3, you would set up the following equivalence equation and solve for i:

$$\$10{,}000 = \$4{,}021(P/A, i, 3).$$

It turns out that $i = 10\%$. In this situation, the bank will earn a return of 10% on its loan transactions. The bank calculates the loan balances over the life of the loan as follows:

Year	Unpaid Balance at Beginning of Year	Return on Unpaid Balance (10%)	Payment Received	Unpaid Balance at End of Year
0				−\$10,000
1	−\$10,000	−\$1,000	\$4,021	−\$6,979
2	−\$6,979	−\$698	\$4,021	−\$3,656
3	−\$3,656	−\$366	\$4,021	\$0

A negative balance indicates an unpaid balance.

Observe that, for the repayment schedule shown, the 10% interest is calculated only for each year's outstanding balance. In this situation, \$1,000 of the \$4,021 annual payment represents interest; the remainder (\$3,021) goes toward repaying the principal. In other words, the three annual payments repay the loan itself and provide a return of 10% on the *amount still outstanding each year.*

Note also that, when the last payment is made, the outstanding principal is eventually reduced to zero.[4] If we calculate the NPW of the loan transaction at its rate of return (10%), we see that

$$\mathrm{PW}(10\%) = -\$10{,}000 + \$4{,}021(P/A, 10\%, 3) = 0,$$

which indicates that the bank can break even at a 10% rate of interest. In other words, the rate of return becomes the rate of interest that equates the present value of future cash repayments to the amount of the loan. This observation prompts the second definition of rate of return:

Definition 2. *Rate of return is the break-even interest rate i* at which the net present worth of a project is zero, or*

$$\mathrm{PW}(i^*) = \mathrm{PW}_{\text{Cash inflows}} - \mathrm{PW}_{\text{Cash outflows}} = 0.$$

Note that the foregoing expression is equivalent to

$$\mathrm{PW}(i^*) = \frac{A_0}{(1+i^*)^0} + \frac{A_1}{(1+i^*)^1} + \cdots + \frac{A_N}{(1+i^*)^N} = 0. \tag{7.1}$$

Here, we know the value of A_N for each period, but not the value of i^*. Since it is the only unknown, we can solve for i^*. As we will discuss momentarily, this solution is not

[4] As we learned in Section 5.3.4, the terminal balance is equivalent to the net future worth of the investment. If the net future worth of the investment is zero, its PW should also be zero.

always straightforward due to the nature of the PW function in Eq. (7.1); it is certainly possible to have more than one rate of return for certain types of cash flows.[5]

Note that the i^* formula in Eq. (7.1) is simply the PW formula in Eq. (5.1) solved for the particular interest rate (i^*) at which PW(i) is equal to zero. By multiplying both sides of Eq. (7.1) by $(1 + i^*)^N$, we obtain

$$\text{PW}(i^*)(1 + i^*)^N = \text{FW}(i^*) = 0.$$

If we multiply both sides of Eq. (7.1) by the capital-recovery factor, $(A/P, i^*, N)$, we obtain the relationship $\text{AE}(i^*) = 0$. *Therefore, the i^* of a project may also be defined as the rate of interest that equates the net present worth, future worth, and annual equivalent worth of the entire series of cash flows to zero*. Or,

$$\text{PW}(i^*) = \text{FW}(i^*) = \text{AE}(i^*) = 0.$$

7.1.2 Return on Invested Capital

Investment projects can be viewed as analogous to bank loans. We will now introduce the concept of rate of return based on the return on invested capital (RIC) in terms of a project investment. A project's return is referred to as the internal rate of return (IRR), or the **yield** promised by an **investment project** over its **useful life**.

Definition 3. *The internal rate of return is the interest rate charged (or earned) on the unrecovered project balance of the investment such that, when the project terminates, the unrecovered project balance is zero.*

Suppose a company invests \$10,000 in a machine with a three-year useful life and equivalent annual labor savings of \$4,021. Here, we may view the investing firm as the lender and the project as the borrower. The cash flow transaction between them would be identical to the amortized loan transaction described under Definition 1:

Year	Beginning Project Balance	Return on Invested Capital (10%)	Cash Generated from Project	Project Balance at End of Year
0				−\$10,000
1	−\$10,000	−\$1,000	\$4,021	−\$6,979
2	−\$6,979	−\$698	\$4,021	−\$3,656
3	−\$3,656	−\$366	\$4,021	\$0

In our project-balance calculation, we see that 10% is earned (or charged) on \$10,000 during year 1, 10% is earned on \$6,979 during year 2, and 10% is earned on \$3,656 during year 3. This information indicates that the firm earns a 10% rate of return on funds that remain *internally* invested in the project. Since it is a return *internal* to the project, we refer to it as the **internal rate of return**, or IRR. This means that the machine project under consideration brings in enough cash to pay for itself in three years and to provide the firm with a return of 10% on its invested capital. To put it differently, if the machine is financed with funds costing 10% annually, the cash generated by the investment will be exactly sufficient to repay the principal and the annual interest charge on the fund in three years.

[5] You will always have N rates of return. The issue is whether these rates are real or imaginary numbers. If they are real, the question of whether they are in the $(-100\%, \infty)$ interval should be asked. A **negative rate of return** implies that you never recover your initial investment fully.

Notice also that only one cash outflow occurs in year 0, and the present worth of this outflow is simply \$10,000. There are three equal receipts, and the present worth of these inflows is $\$4{,}021(P/A, 10\%, 3) = \$10{,}000$. Since $PW = PW_{Inflow} - PW_{Outflow} = \$10{,}000 - \$10{,}000 = 0$, 10% also satisfies Definition 2 for rate of return. Even though this simple example implies that i^* coincides with IRR, only Definitions 1 and 3 correctly describe the true meaning of internal rate of return. As we will see later, if the cash expenditures of an investment are not restricted to the initial period, several break-even interest rates may exist that satisfy Eq. (7.1). However, there may not be a rate of return *internal* to the project.

7.2 Methods for Finding Rate of Return

We may find i^* by using several procedures, each of which has its advantages and disadvantages. To facilitate the process of finding the rate of return for an investment project, we will first classify the types of investment cash flow.

7.2.1 Simple versus Nonsimple Investments

We can classify an investment project by counting the number of sign changes in its net cash flow sequence. A change from either "−" to "+" or "+" to "−" is counted as one sign change. (We ignore a zero cash flow.) We can then establish the following categories:

- A **simple (or conventional) investment** is simply when one sign change occurs in the net cash flow series. If the initial cash flows are negative, we call them **simple-investment** cash flows. If the initial flows are positive, we call them **simple-borrowing** cash flows.
- A **nonsimple (or nonconventional) investment** is an investment in which more than one sign change occurs in the cash flow series.

Multiple i^*s, as we will see later, occur only in nonsimple investments. If there is no sign change in the entire cash flow series, no rate of return exists. The different types of investment possibilities may be illustrated as follows:

Investment Type	Cash Flow Sign at Period						The Number of Sign Changes
	0	1	2	3	4	5	
Simple	−	+	+	+	+	+	1
Simple	−	−	+	+	0	+	1
Nonsimple	−	+	−	+	+	−	4
Nonsimple	−	+	+	−	0	+	3

EXAMPLE 7.1 Investment Classification

Classify the following three cash flow series as either simple or nonsimple investments (borrowings):

Net Cash Flow

Period n	Project A	Project B	Project C
0	−\$1,000	−\$1,000	\$1,000
1	−\$500	\$3,900	−\$450
2	\$800	−\$5,030	−\$450
3	\$1,500	\$2,145	−\$450
4	\$2,000		

DISSECTING THE PROBLEM

Given: Cash flow sequences provided in the foregoing table.

Find: Classify the sequences as either simple or nonsimple investments.

METHODOLOGY

Apply the sign rule to classify the investments and draw a present worth profile as a function of interest rate.

SOLUTION

- Project A represents many common simple investments. This type of investment reveals the PW profile shown in Figure 7.1(a). The curve crosses the i-axis only once.
- Project B represents a nonsimple investment. The PW profile for this investment has the shape shown in Figure 7.1(b). The i-axis is crossed at 10%, 30%, and 50%.

Period (n)	Project A	Project B	Project C
0	−\$1,000	−\$1,000	\$1,000
1	−\$500	\$3,900	−\$450
2	\$800	−\$5,030	−\$450
3	\$1,500	\$2,145	−\$450
4	\$2,000		

Project A is a simple investment.
Project B is a nonsimple investment.
Project C is a simple-borrowing cash flow.

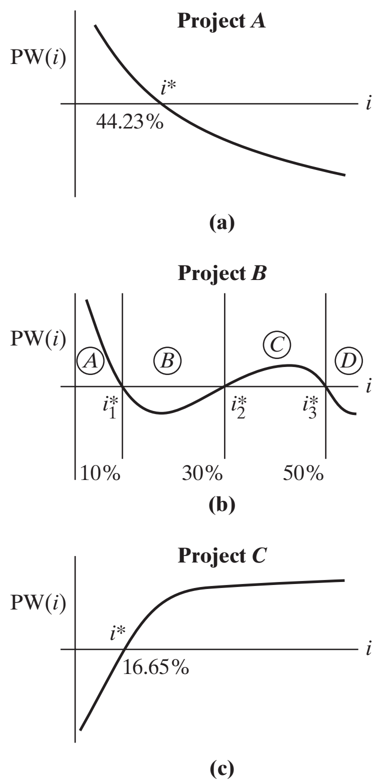


Figure 7.1 Classification of investments.

- Project *C* represents neither a simple nor a nonsimple investment, even though only one sign change occurs in the cash flow sequence. Since the first cash flow is positive, this flow is a **simple-borrowing** cash flow, not an investment flow. The PW profile for this type of investment looks like the one in Figure 7.1(c).

7.2.2 Computational Methods

Once we identify the type of an investment cash flow, there are several ways to determine its rate of return. We will discuss some of the most practical methods here. They are as follows:

- Direct-solution method
- Trial-and-error method
- Excel method

Direct-Solution Method

For the very special case of a project with only a two-non-zero-flow transaction (an investment followed by a single future payment) or a project with a service life of two years of return, we can seek a direct analytical solution for determining the rate of return. These two cases are examined in Example 7.2.

EXAMPLE 7.2 Finding i^* by Direct Solution: Two Non-zero Flows or Two Periods

Consider two investment projects with the following cash flow transactions:

n	Project 1	Project 2
0	−\$3,000	−\$2,000
1	\$0	\$1,300
2	\$0	\$1,500
3	\$0	
4	\$4,500	

Compute the rate of return for each project.

DISSECTING THE PROBLEM

Given: Cash flows for two projects.

Find: i^* for each project.

METHODOLOGY

Write out the net present (or future) worth expression and solve for i.

SOLUTION

Project 1: Solving for i^* in $\mathrm{PW}(i^*) = 0$ is identical to solving for i^* in $\mathrm{FW}(i^*) = 0$, because FW equals PW times a constant. We could use either method here, but we choose $\mathrm{FW}(i^*) = 0$. Using the single-payment future-worth relationship, we obtain

$$\mathrm{FW}(i) = -\$3{,}000(F/P, i, 4) + \$4{,}500 = 0.$$

Setting $\mathrm{FW}(i) = 0$, we obtain

$$\$4{,}500 = \$3{,}000(F/P, i, 4) = \$3{,}000(1 + i)^4,$$

or

$$1.5 = (1 + i)^4.$$

Solving for i yields

$$i^* = \sqrt[4]{1.5} - 1 = 0.1067, \text{ or } 10.67\%.$$

Project 2: We may write the PW expression for this project as follows:

$$\mathrm{PW}(i) = -\$2{,}000 + \frac{\$1{,}300}{(1 + i)} + \frac{\$1{,}500}{(1 + i)^2} = 0.$$

Let

$$X = \frac{1}{(1 + i)}.$$

We may then rewrite the PW(i) expression as a function of X and set it equal to zero, as follows:

$$\mathrm{PW}(i) = -\$2{,}000 + \$1{,}300X + \$1{,}500X^2 = 0.$$

This expression is a quadratic equation that has the following solution:[6]

$$\begin{aligned} X &= \frac{-1{,}300 \pm \sqrt{1{,}300^2 - 4(1{,}500)(-2{,}000)}}{2(1{,}500)} \\ &= \frac{-1{,}300 \pm 3{,}700}{3{,}000} \\ &= 0.8 \text{ or } -1.667. \end{aligned}$$

Replacing the X values and solving for i gives us

$$0.8 = \frac{1}{(1 + i)} \rightarrow i = 25\%$$

and

$$-1.667 = \frac{1}{(1 + i)} \rightarrow i = -160\%.$$

Since an interest rate less than -100% has no economic significance, we find that the project's unique i^* is 25%.

[6] Given $aX^2 + bX + c = 0$, the solution of the quadratic equation is $X = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

COMMENTS: In both projects, one sign change occurred in the net cash flow series, so we expected a unique i^*. Also, these projects had either two non-zero cash flows or two periods of cash flows. Generally, when cash flows are more complex, we must use a trial-and-error method or a computer program to find i^*.

Trial-and-Error Method

The first step in the trial-and-error method is to make an educated guess at the value of i^*.[7] For a simple investment, we use the guessed interest rate to compute the present worth of net cash flows and observe whether the result is positive, negative, or zero:

- **Case 1:** $PW(i) < 0$. Since we are aiming for a value of i that makes $PW(i) = 0$, we must raise the present worth of the cash flow. To do this, we lower the interest rate and repeat the process.
- **Case 2:** $PW(i) > 0$. We raise the interest rate in order to lower $PW(i)$. The process is continued until $PW(i)$ is approximately equal to zero.

Whenever we reach the point where $PW(i)$ is bounded by one negative value and one positive value, we use **linear interpolation** to approximate i^*. This technique was discussed in Chapter 2. This process is somewhat tedious and inefficient. (The trial-and-error method does not work for nonsimple investments in which the PW function is not, in general, a monotonically decreasing function of interest rate.)

EXAMPLE 7.3 Finding i^* by Trial and Error

ACME Corporation distributes agricultural equipment. The board of directors is considering a proposal to establish a facility to manufacture an electronically controlled "intelligent" crop sprayer invented by a professor at a local university. This crop-sprayer project would require an investment of $10 million in assets and would produce an annual after-tax net benefit of $1.8 million over a service life of eight years. All costs and benefits are included in these figures. When the project terminates, the net proceeds from the sale of the assets would be $1 million (Figure 7.2). Compute the rate of return of this project.

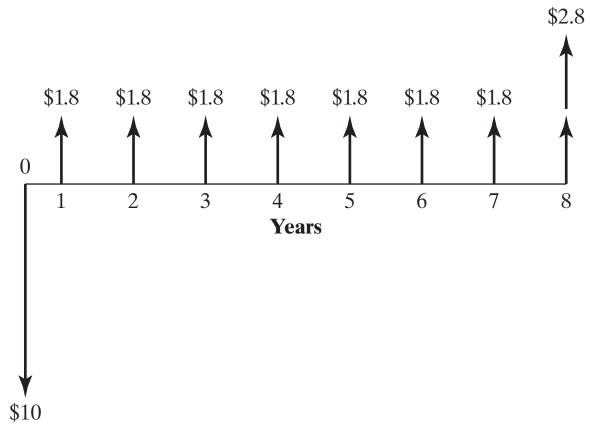


Figure 7.2 Cash flow diagram for a simple investment; all dollar amounts are in millions of dollars.

[7] As we will see later in this chapter, the ultimate objective of finding i^* is to compare it with the MARR. Therefore, it is a good idea to use the MARR as the initial guess value.

DISSECTING THE PROBLEM	**Given:** $I = \$10$ million, $A = \$1.8$ million, $S = \$1$ million, and $N = 8$ years. **Find:** i^*.
METHODOLOGY Find i^* by trial and error as this is a simple investment.	**SOLUTION** We start with a guessed interest rate of 8%. The net present worth of the cash flows in millions of dollars is $\text{PW}(8\%) = -\$10 + \$1.8(P/A, 8\%, 8) + \$1(P/F, 8\%, 8) = \$0.88.$ Since this net present worth is positive, we must raise the interest rate in order to bring this value toward zero. When we use an interest rate of 12%, we find that $\text{PW}(12\%) = -\$10 + \$1.8(P/A, 12\%, 8) + \$1(P/F, 12\%, 8) = -\$0.65.$

We have bracketed the solution. PW(i) will be zero at i somewhere between 8% and 12%. Using straight-line interpolation, we approximate that

$$\begin{aligned} i^* &\approx 8\% + (12\% - 8\%)\left[\frac{0.88 - 0}{0.88 - (-0.65)}\right] \\ &= 8\% + 4\%(0.5752) \\ &= 10.30\%. \end{aligned}$$

Now we will check to see how close this value is to the precise value of i^*. If we compute the net present worth at this interpolated value, we obtain

$$\begin{aligned} \text{PW}(10.30\%) &= -\$10 + \$1.8(P/A, 10.30\%, 8) + \$1(P/F, 10.30\%, 8) \\ &= -\$0.045. \end{aligned}$$

As this result is not zero, we may recompute i^* at a lower interest rate, say, 10%:

$$\begin{aligned} \text{PW}(10\%) &= -\$10 + \$1.8(P/A, 10\%, 8) + \$1(P/F, 10\%, 8) \\ &= \$0.069. \end{aligned}$$

With another round of linear interpolation, we approximate that

$$\begin{aligned} i^* &\approx 10\% + (10.30\% - 10\%)\left[\frac{0.069 - 0}{0.069 - (-0.045)}\right] \\ &= 10\% + 0.30\%(0.6053) \\ &= 10.18\%. \end{aligned}$$

At this interest rate,

$$\begin{aligned} \text{PW}(10.18\%) &= -\$10 + \$1.8(P/A, 10.18\%, 8) + \$1(P/F, 10.18\%, 8) \\ &= \$0.0007, \end{aligned}$$

which is practically zero, so we may stop here. In fact, there is no need to be more precise about these interpolations because the final result can be no more accurate than the basic data, which ordinarily are only rough estimates. Incidentally, computing the i^* for this problem on a computer gives us 10.1819%.

Rate-of-Return Calculation with Excel

Fortunately, we do not need to do laborious manual calculations to find i^*. Many financial calculators have built-in functions for calculating i^*. Microsoft Excel has an IRR financial function that analyzes investment cash flows, namely, $=\text{IRR(range, guess)}$. We will demonstrate the IRR function with an example involving an investment in a corporate bond.

You can trade bonds on the market just like stocks. Once you have purchased a bond, you can keep the bond until it reaches maturity or sell at any interest period before maturity. You can purchase or sell bonds at prices other than face value, depending on the economic environment, as bond prices change over time because of the risk of nonpayment of interest or face value, supply and demand, and the outlook for economic conditions. These factors affect the **yield to maturity** (or **return on investment**). The **yield to maturity** represents the actual interest earned from a bond over the holding period. In other words, the yield to maturity on a bond is the interest rate that establishes the equivalence between all future interest and face-value receipts and the market price of the bond.

EXAMPLE 7.4 Yield to Maturity

Consider buying a $1,000-denomination corporate bond at the market price of $996.25. The interest will be paid semiannually at an interest rate of 4.8125%. That is $48.13 every six-month period. Twenty interest payments over 10 years are required. We show the resulting cash flow to the investor in Figure 7.3. Find the return on this bond investment (or, commonly known as "yield to maturity").

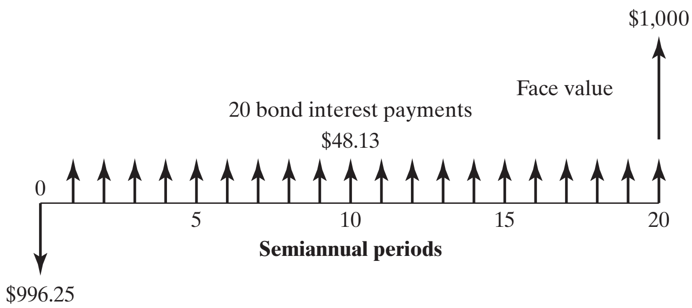


Figure 7.3 A typical cash flow transaction associated with an investment in the corporate bond.

DISSECTING THE PROBLEM

Some specific bond terms to understand are summarized as follows:

- **Face (par) value:** The corporate bond has a face value of \$1,000.
- **Maturity date:** The bonds that were issued on January 30, 2011, will mature on January 31, 2021; thus, they have a 10-year maturity at time of issue.
- **Coupon rate:** The bond's coupon rate is 9.625%, and interest is payable semiannually. For example, the bonds have a \$1,000 face value, and they pay \$96.25 in simple interest $(9\frac{5}{8}\%)$ each year (or \$48.13 every six months).
- **Discount bond:** The bonds are offered at less than the face value at 99.625%, or 0.375% discount. For example, a bond with a face value of \$1,000 can be purchased for just \$996.25, which is known as the **market price** (or value) of the bond.

Given: Initial purchase price = \$996.25, coupon rate = 9.625% per year paid semiannually, and 10-year maturity with a face value of \$1,000.

Find: Yield to maturity.

METHODOLOGY

Determine the interest rate that makes the PW of receipts equal to the bond's market price.

SOLUTION

We find the yield to maturity by determining the interest rate that makes the present worth of the receipts equal to the market price of the bond:

$$\$996.25 = \$48.13(P/A, i, 20) + \$1000(P/F, i, 20).$$

The value of i that makes the present worth of the receipts equal to \$996.25 lies between 4.5% and 5%. Solving for i by interpolation yields $i = 4.84\%$.

Using the IRR function in Excel, we may easily calculate the yield to maturity, as shown in Table 7.1. The initial guess value used in this calculation is 4% with the cell range of B10:B30.

$$= \text{IRR}(\text{cash flow series, guessed value})$$

or

$$= \text{IRR}(\text{B10:B30,4\%}).$$

COMMENTS: Note that this result is a 4.84% yield to maturity per semiannual period. The nominal (annual) yield is $2(4.84\%) = 9.68\%$ compounded semiannually. When compared with the coupon rate of $9\frac{5}{8}\%$ (or 9.625%), purchasing the bond with the price discounted at 0.375% brings about an additional 0.055% yield. The effective annual coupon rate is then

$$i_a = (1 + 0.0484)^2 - 1 = 9.91\%.$$

This 9.91% represents the **effective annual yield** to maturity on the bond. Notice that when you purchase a bond at face value and sell at face value, the yield to maturity will be the *same* as the coupon rate of the bond.

TABLE 7.1 **Yield-to-Maturity Calculation for the Corporate Bond**

	A	B	C	D
1	**Example 7.4 Yield to Maturity Calculation**			
2				
3	Market price			$ 996.25
4	Maturity (semiannual periods)			20
5	Face value			$ 1,000.00
6	Coupon rate (per semiannual period)			4.813%
7	Yield to maturity			4.842%
8				
9	Period	Cash Flow		
10	0	$ (996.25)		
11	1	$ 48.13		
12	2	$ 48.13	=IRR(B10:B30,4%)	
13	3	$ 48.13		
14	4	$ 48.13		
15	5	$ 48.13		
16	6	$ 48.13		
17	7	$ 48.13		
18	8	$ 48.13		
19	9	$ 48.13		
20	10	$ 48.13		
21	11	$ 48.13		
22	12	$ 48.13		
23	13	$ 48.13		
24	14	$ 48.13		
25	15	$ 48.13		
26	16	$ 48.13		
27	17	$ 48.13		
28	18	$ 48.13		
29	19	$ 48.13		
30	20	$ 1,048.13		
31				

7.3 Internal-Rate-of-Return Criterion

Now that we have classified investment projects and learned methods to determine the i^* value for a given project's cash flows, our objective is to develop an accept-or-reject decision rule that gives results consistent with those obtained from PW analysis.

7.3.1 Relationship to the PW Analysis

As we already observed in Chapter 5, PW analysis is dependent on the rate of interest used for the PW computation. A different rate may change a project from being considered acceptable to being considered unacceptable, or it may change the priority of several projects.

Consider again the PW profile as drawn for the simple project in Figure 7.1(a). For interest rates below i^*, this project should be accepted, as PW > 0; for interest rates above i^*, it should be rejected.

On the other hand, for certain nonsimple projects, the PW may look like the one shown in Figure 7.1(b). Use of PW analysis would lead you to accept the projects in regions A and C but reject those in regions B and D. Of course, this result goes against intuition: A higher interest rate would change an unacceptable project into an acceptable one. The situation graphed in Figure 7.1(b) is one of the cases of multiple i^*s mentioned earlier.

For the simple investment situation in Figure 7.1(a), the i^* can serve as an appropriate index for either accepting or rejecting the investment. However, for the nonsimple investment in Figure 7.1(b), it is not clear which i^* to use in order to make an accept-or-reject decision. Therefore, the i^* value fails to provide an appropriate measure of profitability for an investment project with multiple rates of return. In this section, we will provide a decision rule to come up with a correct decision for a project with multiple rates of return.

7.3.2 Decision Rule for Simple Investments

Suppose we have a simple investment. Why should we be interested in finding the interest rate that equates a project's investment cost with the present worth of its receipts? Again, we may easily answer this question by examining Figure 7.1(a). In this figure, we notice two important characteristics of the PW profile. First, as we compute the project's PW(i) at a varying interest rate i, we see that the PW is positive for $i < i^*$, indicating that the project would be acceptable under the PW analysis for those values of i. Second, the PW is negative for $i > i^*$, indicating that the project is unacceptable for those values of i. Therefore, the i^* serves as a **benchmark** interest rate. By knowing this benchmark rate, we will be able to make an accept-or-reject decision consistent with the PW analysis:

- **Evaluating a Single Project:** For a simple investment, i^* is indeed the IRR of the investment. (See Section 7.1.2.) Merely knowing i^* is not enough to apply this method, however. Because firms typically wish to do better than break even (recall that at PW $= 0$, we were indifferent to the project), a minimum attractive rate of return (MARR) is indicated by company policy, management, or the project decision maker. If the IRR exceeds this MARR, we are assured that the company will more than break even. Thus, the IRR becomes a useful gauge against which to judge project acceptability (see Figure 7.4), and the decision rule for a simple project is as follows:

$$\text{If IRR} > \text{MARR, accept the project.}$$
$$\text{If IRR} = \text{MARR, remain indifferent.}$$
$$\text{If IRR} < \text{MARR, reject the project.}$$

- **Evaluating Mutually Exclusive Projects:** Note that the foregoing decision rule is designed for a single-project evaluation. When we have to compare mutually exclusive investment projects, we need to apply the **incremental analysis approach**, as we shall see in Section 7.4.2. At this point, we just want to mention that you can't

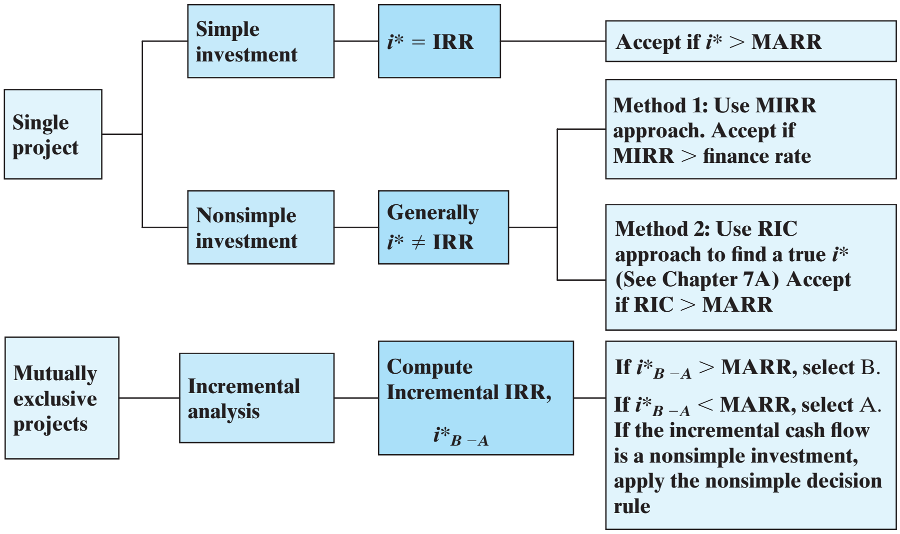


Figure 7.4 Project selection rules under the IRR criterion.

select the best project by merely ranking the IRRs. For now, we will consider the single-project evaluations.

EXAMPLE 7.5 Investment Decision for a Simple Investment—Economics of Wind

An energy firm is betting on wind power's long-term viability in Texas and plans to erect what would be one of the biggest wind farms in the world with 200 wind turbines costing some \$1.69 million each. Energy companies investing in wind power are also expecting governments to toughen rules relating to traditional energy sources, as part of long-term efforts to reduce global-warming emissions. But generating power from wind is not profitable for companies without government tax breaks. The following financial and technical data have been compiled for further consideration:

- Number of wind turbines to be built: 200 units
- Power capacity: 310,000 kW
- Capital investment required: \$338,000,000
- Project life: 20 years
- Salvage value of the wind turbines after 20 years: \$0
- Annual net cash flows (after all deductions): \$41,391,160

According to the data provided, answer the following questions:

(a) What is the projected IRR on this investment?

(b) If the company's MARR is known to be 10%, is the investment justified?

DISSECTING THE PROBLEM

We assumed the following in obtaining the annual net cash flows ($41,391,160):

- Average load factor: 35% (site specific, but use this value for analysis purposes)
- Power generated per year:

$$\text{Power generated per year} = 310{,}000 \text{ kW} \times 0.35 \times 24 \text{ hrs/day} \times 365 \text{ days/yr} = 950{,}460{,}000 \text{ kWh/yr}$$

- Selling price of power generated from the wind turbines: $0.034/kWh:

$$\text{Operating revenues} = \$0.034/\text{kWh} \times 950{,}460{,}000 \text{ kWh/yr} = \$32{,}315{,}640/\text{yr}$$

- Federal tax credit: $0.018/kWh sold to utility customers:

$$\text{Tax credit} = \$0.018/\text{kWh} \times 950{,}460{,}000 \text{ kWh/yr} = \$17{,}108{,}280/\text{yr}$$

- Annual easement cost: $4,000 per turbine per year (or $800,000)
- Annual operating and maintenance cost: $16,300 per turbine per year or ($3,260,000)
- Annual taxes paid: $3,972,760 (We will discuss how we calculate this amount in Chapter 9.)

Given: Financial as well as the technical data as given above; MARR = 10%.

Find: (a) IRR and (b) whether to accept or reject the investment.

Cash Flow Summary:
Total investment: $338,000,000
Total annual cash inflow:

$$\$32{,}315{,}640 + \$17{,}108{,}280 = \$49{,}423{,}920$$

Total annual cash outflow:

$$\$800{,}000 + \$3{,}260{,}000 + \$3{,}972{,}760 = \$7{,}972{,}760$$

METHODOLOGY

Use Excel to calculate IRR.

SOLUTION

(a) Since only one sign change occurs in the net cash flow series, the project is a simple investment. This factor indicates that there will be a unique rate of return that is internal to the project:

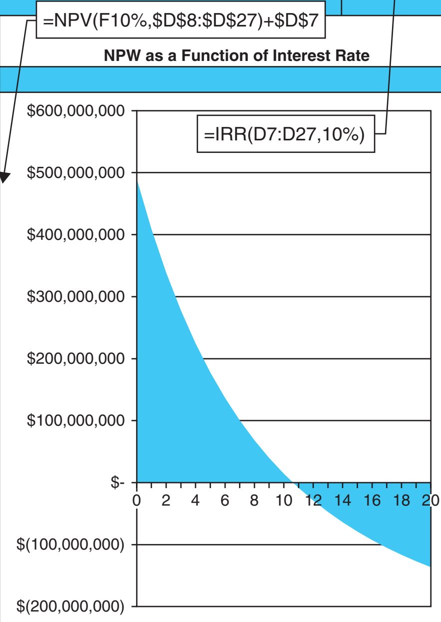
$$\text{PW}(i) = -\$338{,}000{,}000 + \$41{,}391{,}160(P/A, i, 20) = 0.$$

We could use the trial-and-error approach outlined in Section 7.2.2 to find the IRR, but the IRR function in Excel would be a more convenient way to calculate the internal rate of return. Table 7.2 shows that the rate of return for the project is 10.62%, which barely exceeds the MARR of 10%. (This project is not economical without the federal tax credit.)

(b) The IRR figure exceeds the required MARR, indicating that the project is an economically attractive one and could proceed. However, there is no doubt that the installation of the wind turbines system would pose a great deal of financial risk to the firm if any climate changes in the region lowered the capacity (or load) factor.

TABLE 7.2 **An Excel Worksheet to Illustrate How to Determine the IRR (Example 7.5)**

	A	B	C	D	E	F	G	H	I	J	K	L
1	*(I)* Investment			$ 338,000,000		Net Present Worth						$ 14,386,278
2	*(i)* MARR (%)			10%		Internal Rate of Return						10.62%
3												
4		Cash	Cash	Net		MARR						
5	*n*	Inflows	Outflows	Cash Flows		(%)	NPW	NPW as a Function of Interest Rate				
6												
7	0			$ (338,000,000)		0	$ 489,823,200					
8	1	$ 49,423,920	$ 8,032,760	$ 41,391,160		1	$ 408,926,370					
9	2	$ 49,423,920	$ 8,032,760	$ 41,391,160		2	$ 338,804,794					
10	3	$ 49,423,920	$ 8,032,760	$ 41,391,160		3	$ 277,795,942					
11	4	$ 49,423,920	$ 8,032,760	$ 41,391,160		4	$ 224,519,372					
12	5	$ 49,423,920	$ 8,032,760	$ 41,391,160		5	$ 177,825,342					
13	6	$ 49,423,920	$ 8,032,760	$ 41,391,160		6	$ 136,753,344					
14	7	$ 49,423,920	$ 8,032,760	$ 41,391,160		7	$ 100,498,539					
15	8	$ 49,423,920	$ 8,032,760	$ 41,391,160		8	$ 68,384,510					
16	9	$ 49,423,920	$ 8,032,760	$ 41,391,160		9	$ 39,841,094					
17	10	$ 49,423,920	$ 8,032,760	$ 41,391,160		10	$ 14,386,278					
18	11	$ 49,423,920	$ 8,032,760	$ 41,391,160		11	$ (8,388,612)					
19	12	$ 49,423,920	$ 8,032,760	$ 41,391,160		12	$ (28,831,064)					
20	13	$ 49,423,920	$ 8,032,760	$ 41,391,160		13	$ (47,237,383)					
21	14	$ 49,423,920	$ 8,032,760	$ 41,391,160		14	$ (63,860,944)					
22	15	$ 49,423,920	$ 8,032,760	$ 41,391,160		15	$ (78,919,009)					
23	16	$ 49,423,920	$ 8,032,760	$ 41,391,160		16	$ (92,598,398)					
24	17	$ 49,423,920	$ 8,032,760	$ 41,391,160		17	$ (105,060,182)					
25	18	$ 49,423,920	$ 8,032,760	$ 41,391,160		18	$ (116,443,613)					
26	19	$ 49,423,920	$ 8,032,760	$ 41,391,160		19	$ (126,869,399)					
27	20	$ 49,423,920	$ 8,032,760	$ 41,391,160		20	$ (136,442,446)					
28												



7.3.3 Decision Rule for Nonsimple Investments

When applied to simple projects, the i^* provides an unambiguous criterion for measuring profitability. However, when multiple rates of return occur, none of them is an accurate portrayal of project acceptability or profitability. Clearly, then, we should place a high priority on discovering this situation early in our analysis of a project's cash flows. The quickest way to predict the existence of multiple i^*s is to generate a PW profile on the computer and check if it crosses the horizontal axis more than once.

When we are dealing with a nonsimple investment, we may adopt one of the following two approaches to make an accept–reject decision that is consistent with the NPW criterion:

1. The modified internal rate of return (MIRR) method or
2. The Return on invested capital (RIC) method.

The MIRR approach is much simpler than the second approach, so it is more widely practiced. However, the RIC method is more conceptually appealing, so it is well received in the academic community. In our introductory text, we will give more detail treatment on the first approach, but for anyone who is interested in more rigorous analytical treatment of the nonsimple investment, refer to Chapter 7A.

Modified Internal Rate of Return

The **MIRR** is a variation of the traditional Internal Rate of Return (IRR) calculation in that it computes IRR with explicit reinvestment rate and finance rate assumptions. The reason why these two rates are used is because it allows for any positive cash flows from an investment over the holding period to be returned to the company's investment pool and earn at the rate of "MARR." It also allows any future investment cash flows to be financed at the borrowing rate and discounted back to the present time at the "borrowing rate" to determine how much needs to be set aside today in order to fund the future cash outflows.

By using this approach, we convert a nonsimple investment to a simple investment of having two numbers as shown in Figure 7.5: 1) a single equivalent initial investment amount at the present time and 2) a total accumulated cash amount at the end of the holding period. Since this modified cash flow becomes a simple investment, we should be able to find a unique rate of return, which results in what's known as the MIRR:

$$\frac{\begin{pmatrix}\text{Future value of all cash inflows}\\ \text{at the end of project life}\end{pmatrix}}{\begin{pmatrix}\text{Present value of all cash outflows}\\ \text{at the beginning of projects life}\end{pmatrix}} = (1 + \text{MIRR})^N. \tag{7.2}$$

Then, the decision rules are

$$\text{If MIRR} > \text{borrowing rate, accept the project.}$$
$$\text{If MIRR} = \text{borrowing rate, indifferent.}$$
$$\text{If MIRR} < \text{borrowing rate, reject the project.}$$

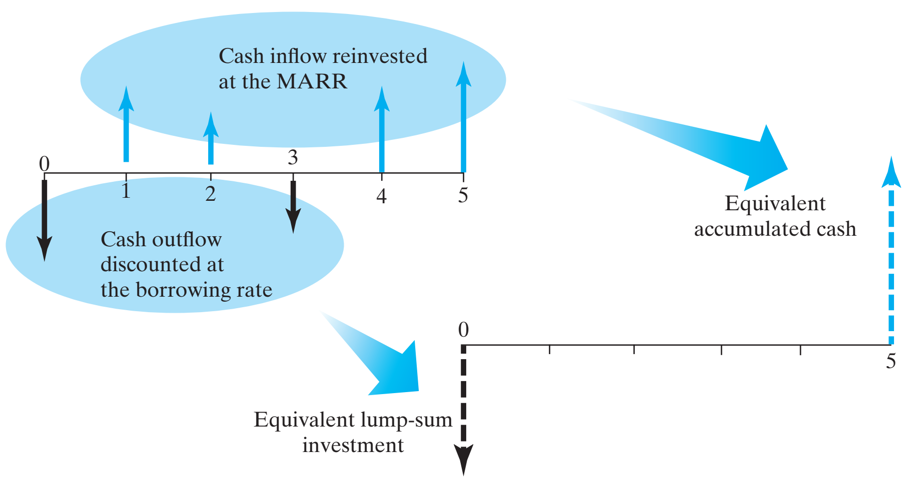


Figure 7.5 Illustration of MIRR Concept.

In pure economic terms, we are asking ourself that, if an investment requires a lump-sum amount of P at time 0 but it will generate F dollars at the end of investment period (holding period) N, what is the return on that investment? One critical point is that, if we are looking for a consistent result with the NPW criterion, we must assume that the reinvestment rate (typically MARR in NPW analysis) should be higher or equal to the borrowing rate.

Let's take two examples of the MIRR to see how this works.

EXAMPLE 7.6 Finding MIRR for a Simple Investment

Let's reconsider the project cash flows in Example 7.3. Calculate the MIRR at a reinvestment rate of 10% and a finance rate of 7%.

DISSECTING THE PROBLEM

We invest \$10M today and in return we receive \$1.8M per year for 8 years, plus at the end of year 8 we sell the asset and get back \$1M. If we use the traditional Internal Rate of Return (IRR) calculation, we get an IRR of 10.18%.

Given: $I = \$10$ million, $A = \$1.8$ million, $S = \$1$ million, and $N = 8$ years. Reinvestment rate $= 10\%$, Finance rate $= 7\%$

Find: MIRR

METHODOLOGY

Convert the original cash flows into a two-flow simple investment. We simply take each of our interim cash flows of \$1.8M and then compound them forward at a rate of 10% to the end of year 8. When we add up all of our positive cash flows at the end of year 8, we get a total of \$21.58M. By doing this, we have transformed our initial set of cash flows into a different **time value of money** problem, which takes into account the yield we earn on interim cash flows that are reinvested elsewhere. Now we can simply take our new set of cash flows and solve for the IRR, which in this case is actually the MIRR since it's based on our modified set of cash flows.

SOLUTION

We calculate the accumulated cash amount from reinvesting the cash receipts from $n = 1$ to $n = 8$ as follows. The future worth of the cash flows in millions of dollars is

$$F = \$1.8(F/A, 10\%, 8) + \$1 = \$21.58.$$

The only outflow occurs at $n = 0$, so we don't really need the finance rate to discount any future cash outflows.

$$P = \$10.$$

The MIRR is

$$\frac{\$21.58}{\$10} = (1 + \text{MIRR})^8$$

$$\text{MIRR} = 10.09\% > 7\%.$$

So we may accept the investment. Note that the traditional IRR is 10.18%, which is higher than the MIRR.

Let's consider the situation where the cash flow series contains one or more outflows after the initial investment period. Example 7.7 illustrates how we may determine the MIRR for such a nonsimple investment.

EXAMPLE 7.7 Investment Decision for a Nonsimple Project

By outbidding its competitors, Turbo Image Processing (TIP), a defense contractor, has received a contract worth $7,300,000 to build navy flight simulators for U.S. Navy pilot training over two years. For some defense contracts, the U.S. government makes an advance payment when the contract is signed, but in this case, the government will make two progressive payments: $4,300,000 at the end of the first year and the $3,000,000 balance at the end of the second year. The expected cash outflows required in order to produce these simulators are estimated to be $1,000,000 now, $2,000,000 during the first year, and $4,320,000 during the second year. The expected net cash flows from this project are summarized as follows:

Year	Cash Inflow	Cash Outflow	Net Cash Flow
0		$1,000,000	−$1,000,000
1	$4,300,000	$2,000,000	$2,300,000
2	$3,000,000	$4,320,000	−$1,320,000

In normal situations, TIP would not even consider a marginal project such as this one in the first place. However, hoping that TIP can establish itself as a technology leader in the field, management felt that it was worth outbidding its competitors by providing the lowest bid. Financially, what is the economic worth of outbidding the competitors for this project?

(a) Compute the values of the i^*s for this project.
(b) Assuming a financing rate of 12% and reinvestment rate of 15%, compute the MIRR.
(c) Make an accept-or-reject decision on the basis of results of part (b).

DISSECTING THE PROBLEM

Given: Cash flow shown in the foregoing table; financing rate = 12%, reinvestment rate = 15%.

Find: (a) i^* (b) MIRR, and (c) determine whether to accept the project.

METHODOLOGY

(a) Quadratic method.

SOLUTION

(a) Since this project has a two-year life, we may solve the NPW equation directly via the quadratic-formula method:

$$-\$1,000,000 + \frac{\$2,300,000}{(1 + i^*)} - \frac{\$1,320,000}{(1 + i^*)^2} = 0.$$

(b) MIRR method

If we let $X = 1/(1 + i^*)$, we can rewrite the expression as

$$-1{,}000{,}000 + 2{,}300{,}000X - 1{,}320{,}000X^2 = 0.$$

Solving for X gives $X = \frac{10}{11}$ and $\frac{10}{12}$ or $i^* = 10\%$ and 20%, respectively. As shown in Figure 7.6, the PW profile intersects the horizontal axis twice, once at 10% and again at 20%. The investment is obviously not a simple one, and thus neither 10% nor 20% represents the true internal rate of return of this government project.

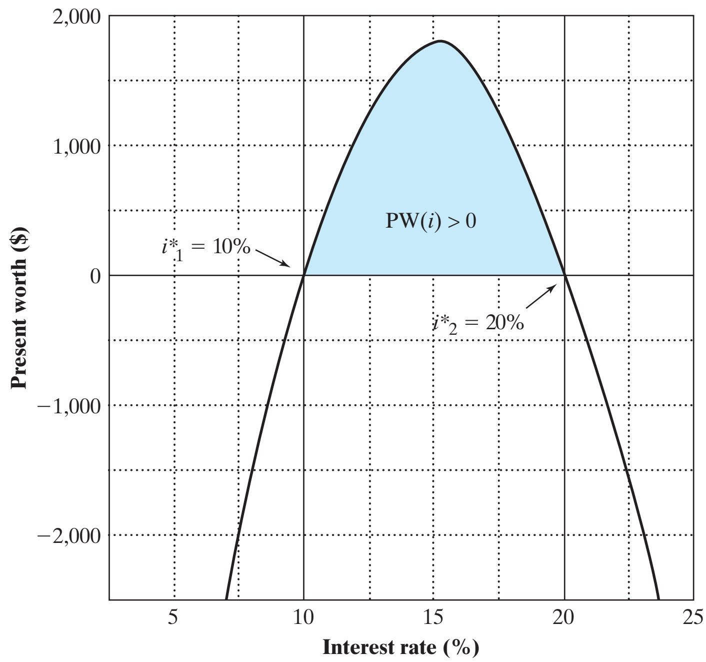


Figure 7.6 NPW plot for a nonsimple investment with multiple rates of return.

With a reinvestment rate of 15%, the future value of all cash inflows at the end of year 2 is

$$\$2{,}300{,}000(1.15) = \$2{,}645{,}000.$$

With a borrowing rate of 12%, the present value of all cash outflows at year 0 is

$$\$1{,}000{,}000 + \$1{,}320{,}000(P/F, 12\%, 2) = \$2{,}052{,}296.$$

The MIRR is

$$\frac{\$2{,}645{,}000}{\$2{,}052{,}296} = (1 + \text{MIRR})^2$$

$$\text{MIRR} = 13.53\% > 12\%.$$

At a 12% borrowing rate, the project is worth undertaking. Now if we assume a borrowing rate equal to the reinvestment rate of 15%, the MIRR would be 15.05% > 15%, still a marginally acceptable project.

COMMENTS: Since the project is a nonsimple project, we may abandon the IRR criterion for practical purposes and use the PW criterion. If we use the present-worth method at MARR = 15%, we obtain

$$\begin{aligned} PW(15\%) &= -\$1{,}000{,}000 + \$2{,}300{,}000(P/F, 15\%, 1) \\ &\quad -\$1{,}320{,}000(P/F, 15\%, 2) \\ &= \$1{,}890, \end{aligned}$$

which verifies that the project is marginally acceptable, and it is thus not as bad as we initially believed.

Return on Invested Capital

The MIRR approach is rather simple and easy to implement but its method is based on the assumptions that the firm would be able to reinvest all future proceeds from the project at the MARR and would be able to finance all future cash needs at a fixed financing rate. If these assumptions are not met, an appropriate adjustment has to be made in calculating the MIRR. As an alternative, we may attempt to calculate the true rate of return for a typical non-simple investment. The **return on invested capital** (RIC) technique is to answer this question by introducing an external rate of return concept, as discussed in Chapter 7A, Once again for our introductory text, we will limit our discussion to the MIRR approach.

7.4 Incremental Analysis for Comparing Mutually Exclusive Alternatives

In this section, we present the decision procedures that should be used when comparing two or more mutually exclusive projects on the basis of the rate-of-return measure. We will consider two situations: (1) alternatives that have the same economic service life and (2) alternatives that have unequal service lives.

7.4.1 Flaws in Project Ranking by IRR

Under PW or AE analysis, the mutually exclusive project with the highest worth figure was preferred. (This approach is known as the "total investment approach.") Unfortunately, the analogy does not carry over to IRR analysis. The project with the highest IRR may ***not*** be the preferred alternative. To illustrate the flaws of comparing IRRs when selecting between mutually exclusive projects, suppose you have two mutually exclusive alternatives, each with a one-year service life. One requires an investment of \$1,000 with a return of \$2,000, and the other requires \$5,000 with a return of \$7,000. You have already obtained the IRRs and PWs at MARR = 10% as follows:

Comparing Mutually Exclusive Alternatives, Based on IRR

• Issue: Can we rank the mutually exclusive projects by the magnitude of IRR?

n	Project *A*1		Project *A*2
0	−\$1,000		−\$5,000
1	\$2,000		\$7,000
IRR	100%	>	40%
PW(10%)	\$818	<	\$1,364

Assuming that you have exactly \$5,000 in your investment pool to select either project, would you prefer the first project simply because you expect a higher rate of return?

We can see that Project $A2$ is preferred over Project $A1$ by the PW measure. On the other hand, the IRR measure gives a numerically higher rating for Project $A1$. This inconsistency in ranking occurs because the PW, FW, and AE are **absolute (dollar)** measures of investment worth, whereas the IRR is a **relative (percentage)** measure and cannot be applied in the same way. Consequently, the IRR measure ignores the **scale** of the investment. Clearly, the answer is no; instead, you would prefer the second project with the lower rate of return but higher PW. Either the PW or the AE measure would lead to that choice, but comparison of IRRs would rank the smaller project higher. Another approach, referred to as **incremental analysis**, is needed.

7.4.2 Incremental-Investment Analysis

In our previous example, the costlier option requires \$4,000 more than the other option while it provides an additional return of \$5,000. To compare these directly, we must look at the total impact on our investment pool using common terms. Here we look at each project's impact on an investment pool of \$5,000. You must consider the following factors:

- If you decide to invest in Project $A1$, you will need to withdraw only \$1,000 from your investment pool. The remaining \$4,000 that is not committed will continue to earn 10% interest. One year later, you will have \$2,000 from the outside investment and \$4,400 from the investment pool. Thus, with an investment of \$5,000, in one year you will have \$6,400 (or a 28% return on the \$5,000). The equivalent present worth of this wealth change is

$$\text{PW}(10\%) = -\$5{,}000 + \$6{,}400(P/F, 10\%, 1) = \$818.$$

- If you decide to invest in Project $A2$, you will need to withdraw \$5,000 from your investment pool, leaving no money in the pool, but you will have \$7,000 from your outside investment. Your total wealth changes from \$5,000 to \$7,000 in a year. The equivalent present worth of this wealth change is

$$\text{PW}(10\%) = -\$5{,}000 + \$7{,}000(P/F, 10\%, 1) = \$1{,}364.$$

In other words, if you decide to take the more costly option, certainly you would be interested in knowing that this additional investment can be justified at the MARR. The MARR value of 10% implies that you can always earn that rate from other investment sources (e.g., \$4,400 at the end of one year for a \$4,000 investment). However, in the second option, by investing the additional \$4,000, you would make an additional \$5,000, which is equivalent to earning at the rate of 25%. Therefore, the higher-cost investment can be justified. Figure 7.7 summarizes the process of performing incremental-investment analysis.

Now we can generalize the decision rule for comparing mutually exclusive projects. For a pair of mutually exclusive projects (A and B, with B defined as a more costly option), we may rewrite B as

$$B = A + (B - A).$$

In other words, B has two cash flow components: (1) the same cash flow as A and (2) the incremental component $(B - A)$. Therefore, the only situation in which B is preferred to A is when the rate of return on the incremental component $(B - A)$ exceeds the MARR. Therefore, for two mutually exclusive projects, we use rate-of-return

n	Project $A1$	Project $A2$	Incremental Investment ($A2 - A1$)
0	−\$1,000	−\$5,000	−\$4,000
1	\$2,000	\$7,000	\$5,000
IRR	100%	40%	25%
PW(10%)	\$818	\$1,364	\$546

- Assuming a MARR of 10%, you can always earn that rate from another investment source—e.g., \$4,400 at the end of one year for a \$4,000 investment.
- By investing the additional \$4,000 in $A2$, you would make an additional \$5,000, which is equivalent to earning at the rate of 25%. Therefore, the incremental investment in $A2$ is justified.

Figure 7.7 Illustration of incremental analysis.

analysis by computing the *internal rate of return on incremental investment* (IRR_{B-A}) between the projects. Since we want to consider increments of investment, we compute the cash flow for the difference between the projects by subtracting the cash flow for the lower investment-cost project (A) from that of the higher investment-cost project (B). Then the decision rule is as follows, where ($B - A$) is an investment increment. (The sign of the first cash flow should be always negative.)

If $\text{IRR}_{B-A} > \text{MARR}$, select B (higher first-cost alternative).
If $\text{IRR}_{B-A} = \text{MARR}$, select either one.
If $\text{IRR}_{B-A} < \text{MARR}$, select A (lower first-cost alternative).

If a "do-nothing" alternative is allowed, the smaller cost option must be profitable (its IRR must be higher than the MARR) at first. This means that we compute the rate of return for each alternative in the mutually exclusive group and then eliminate the alternatives that have IRRs less than the MARR before applying the incremental-investment analysis. It may seem odd to you how this simple rule allows us to select the right project. Example 7.8 will illustrate the incremental-investment decision rule for you.

EXAMPLE 7.8 IRR on Incremental Investment: Two Alternatives

John Covington, a college student, wants to start a small-scale painting business during his off-school hours. To economize the start-up business, he decides to purchase some used painting equipment. He has two mutually exclusive options: ($B1$) Do most of the painting by himself by limiting his business to only residential painting jobs or ($B2$) purchase more painting equipment and hire some helpers to do both residential and commercial painting jobs. He expects option $B2$ will have a higher equipment cost but provide higher revenues as well. In either case, he expects to fold the business in three years when he graduates from college.

The cash flows for the two mutually exclusive alternatives are given as follows:

n	$B1$	$B2$	$B2 - B1$
0	−\$3,000	−\$12,000	−\$9,000
1	\$1,350	\$4,200	\$2,850
2	\$1,800	\$6,225	\$4,425
3	\$1,500	\$6,330	\$4,830
IRR	25%	17.43%	

With the knowledge that both alternatives are revenue projects, which project would John select at MARR = 10%? (Note that both projects are profitable at 10%.)

DISSECTING THE PROBLEM

Given: Incremental cash flow between two alternatives; MARR = 10%.

Find: (a) The IRR on the increment and (b) which alternative is preferable.

METHODOLOGY

Compute incremental cash flow, then IRR.

SOLUTION

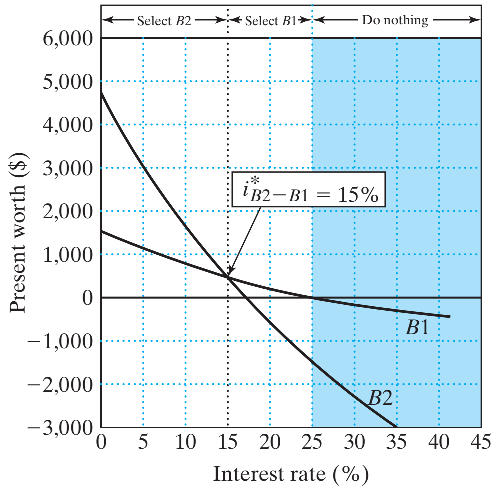
(a) To choose the best project, we compute the incremental cash flow for $B2 - B1$. Then we compute the IRR on this increment of investment by solving

$$-\$9{,}000 + \$2{,}850(P/F, i, 1) + \$4{,}425(P/F, i, 2) + \$4{,}830(P/F, i, 3) = 0.$$

(b) We obtain $i^*_{B2-B1} = 15\%$ as plotted in Figure 7.8. By inspection of the incremental cash flow, we know it is a simple investment; so, $\mathrm{IRR}_{B2-B1} = i^*_{B2-B1}$. Since $\mathrm{IRR}_{B2-B1} > \mathrm{MARR}$, we select B2, which is consistent with the PW analysis. Note that, at MARR > 25% neither project would be acceptable.

COMMENTS: Why did we choose to look at the increment $B2 - B1$ instead of $B1 - B2$? We want the first flow of the incremental cash flow series to be negative (investment flow) so that we can calculate an IRR. By subtracting the lower initial-investment project from the higher, we guarantee that the first increment will be an investment flow. If we ignore the investment ranking, we might end up with an increment that involves borrowing cash flow and has no rate of return internal to the investment.

n	$B1$	$B2$	$B2 - B1$
0	−\$3,000	−\$12,000	−\$9,000
1	\$1,350	\$4,200	\$2,850
2	\$1,800	\$6,225	\$4,425
3	\$1,500	\$6,330	\$4,830
IRR	25%	17.43%	15%



Given MARR = 10%, which project is a better choice?
Since $\text{IRR}_{B2-B1} = 15\% > 10\%$ and $\text{IRR}_{B2} > 10\%$, select $B2$.

Figure 7.8 NPW profiles for $B1$ and $B2$.

The next example indicates that the ranking inconsistency between PW and IRR can also occur when differences in the timing of a project's future cash flows exist, even if their initial investments are the same.

EXAMPLE 7.9 IRR on Incremental Investment When Initial Flows Are Equal

Consider the following two mutually exclusive investment projects that require the same amount of investment:

n	$C1$	$C2$
0	−\$9,000	−\$9,000
1	\$480	\$5,800
2	\$3,700	\$3,250
3	\$6,550	\$2,000
4	\$3,780	\$1,561
IRR	18%	20%

Which project would you select on the basis of rate of return on incremental investment, assuming that MARR = 12%? (Once again, both projects are profitable at 12%.)

DISSECTING THE PROBLEM

Given: Cash flows for two mutually exclusive alternatives as shown; MARR = 12%.

Find: (a) The IRR on incremental investment and (b) which alternative is preferable.

METHODOLOGY

(a) When initial investments are equal, we progress through the cash flows until we find the first difference and then set up the increment so that this first non-zero flow is negative (i.e., an investment).

SOLUTION

(a) We set up the incremental investment by taking $(C1 - C2)$:

n	$C1 - C2$
0	$0
1	−$5,320
2	$450
3	$4,550
4	$2,219

We next set the PW equation equal to zero, as follows:

$$-\$5{,}320 + \$450(P/F, i, 1) + \$4{,}550(P/F, i, 2) + \$2{,}219(P/F, i, 3) = 0.$$

(b) Solving for i yields $i^* = 14.71\%$, which is also an IRR, since the increment is a simple investment. Since $\text{IRR}_{C1-C2} = 14.71\% > \text{MARR}$, we would select $C1$. If we used PW analysis, we would obtain $\text{PW}(12\%)_{C1} = \$1{,}443$ and $\text{PW}(12\%)_{C2} = \$1{,}185$, confirming the preference of $C1$ over $C2$.

COMMENTS: When you have more than two mutually exclusive alternatives, they can be compared in pairs by successive examination, as shown in Figure 7.9.

n	$D1$	$D2$	$D3$
0	−$2,000	−$1,000	−$3,000
1	$1,500	$800	$1,500
2	$1,000	$500	$2,000
3	$800	$500	$1,000
IRR	34.37%	40.76%	24.81%

Step 1: Examine the IRR for each project in order to eliminate any project that fails to meet the MARR(= 15%).

Step 2: Compare $D1$ and $D2$ in pairs:
$\text{IRR}_{D1-D2} = 27.61\% > 15\%$,
so select $D1$.

Step 3: Compare $D1$ and $D3$:
$\text{IRR}_{D3-D1} = 8.8\% < 15\%$,
so select $D1$.

Here, we conclude that $D1$ is the best alternative.

Figure 7.9 IRR on incremental investment: Three alternatives.

EXAMPLE 7.10 Incremental Analysis for Cost-Only Projects

Falk Corporation is considering two types of manufacturing systems to produce its shaft couplings over six years: (1) a cellular manufacturing system (CMS) and (2) a flexible manufacturing system (FMS). The average number of pieces to be produced on either system would be 544,000 per year. Operating costs, initial investment, and salvage value for each alternative are estimated as follows:

Items	CMS Option	FMS Option
Investment	\$4,500,000	\$12,500,000
Total annual operating costs	\$7,412,920	\$5,504,100
Net salvage value	\$500,000	\$1,000,000

The firm's MARR is 15%. Which alternative would be a better choice based on the IRR criterion?

DISSECTING THE PROBLEM

Since we can assume that both manufacturing systems would provide the same level of revenues over the analysis period, we can compare these alternatives on the basis of cost only. (These systems are service projects.) Although we cannot compute the IRR for each option without knowing the revenue figures, we can still calculate the IRR on incremental cash flows. Since the FMS option requires a higher initial investment than that of the CMS, the incremental cash flow is the difference (FMS − CMS).

Given: Cash flows shown in Figure 7.10 and $i = 15\%$ per year.

Find: Incremental cash flows, and select the better alternative based on the IRR criterion.

n	CMS Option	FMS Option	Incremental (FMS − CMS)
0	−\$4,500,000	−\$12,500,000	−\$8,000,000
1	−\$7,412,920	−\$5,504,100	\$1,908,820
2	−\$7,412,920	−\$5,504,100	\$1,908,820
3	−\$7,412,920	−\$5,504,100	\$1,908,820
4	−\$7,412,920	−\$5,504,100	\$1,908,820
5	−\$7,412,920	−\$5,504,100	\$1,908,820
6	−\$7,412,920 +\$500,000	−\$5,504,100 +\$1,000,000	\$2,408,820

$$\mathrm{PW}(i)_{\mathrm{FMS-CMS}} = -\$8{,}000{,}000 + \$1{,}908{,}820(P/A, i, 5) + \$2{,}408{,}820(P/F, i, 6) = 0.$$

METHODOLOGY

Solve for i using Excel.

SOLUTION

Solving for i by Excel yields 12.43%. Since $\mathrm{IRR}_{\mathrm{FMS-CMS}} = 12.43\% < 15\%$ we would select CMS. Although the FMS would provide an incremental annual savings of \$1,908,820 in operating costs, the savings do not justify the incremental investment of \$8,000,000.

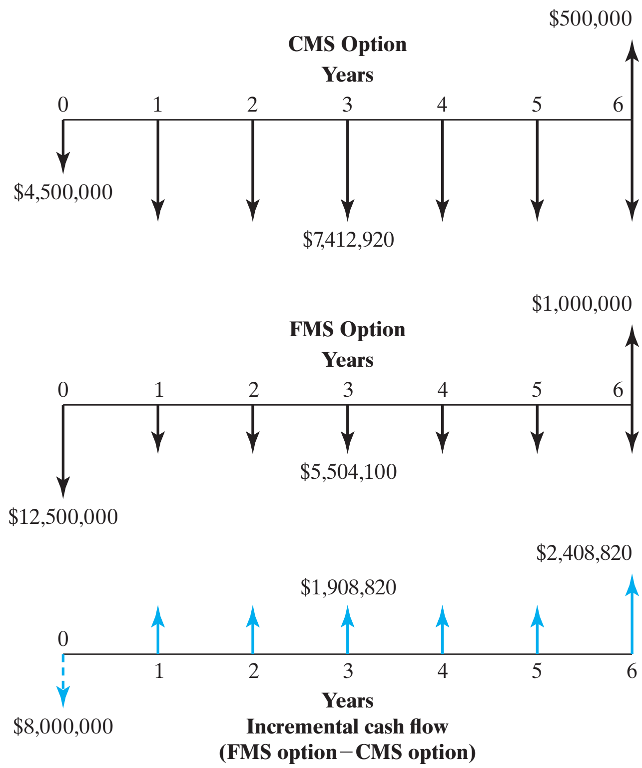


Figure 7.10 Cash flow diagrams for comparing cost-only projects.

COMMENTS: Note that the CMS option was marginally preferred to the FMS option. However, there are dangers in relying solely on the easily quantified savings in input factors—such as labor, energy, and materials—from FMS and in not considering gains from improved manufacturing performance that are more difficult and subjective to quantify. Factors such as improved product quality, increased manufacturing flexibility (rapid response to customer demand), reduced inventory levels, and increased capacity for product innovation are frequently ignored in financial analysis because we have inadequate means for quantifying their benefits. If these intangible benefits were considered, as they ought to be, however, the FMS option could come out better than the CMS option.

7.4.3 Handling Unequal Service Lives

In Chapters 5 and 6, we discussed the use of the PW and AE criteria as bases for comparing projects with unequal lives. The IRR measure can also be used to compare projects with unequal lives as long as we can establish a common analysis period. The decision procedure is then exactly the same as for projects with equal lives.

It is likely, however, that we will have a multiple-root problem, which creates a substantial computational burden. For example, suppose we apply the IRR measure to a case in which one project has a five-year life and the other project has an eight-year life, resulting in a least common multiple of 40 years. Moreover, when we determine the incremental cash flows over the analysis period, we are bound to observe many sign changes.[8]

Example 7.11 compares mutually exclusive projects where the incremental cash flow series results in several sign changes.

EXAMPLE 7.11 Comparing Unequal-Service-Life Problems

Reconsider the unequal-service-life problem given in Example 5.8. Select the desired alternative on the basis of the rate-of-return criterion.

DISSECTING THE PROBLEM

Given: Cash flows for unequal-service-life projects as shown in Figure 5.13, and MARR = 12%.

Find: Which project should be selected?

METHODOLOGY

Use an Excel worksheet to prepare incremental cash flows.

SOLUTION

Since each model has a shorter life than the required service period (10 years), we need to make an explicit assumption of how the service requirement is to be met. With the leasing assumption given in Figure 7.11, the incremental cash flows would look like Table 7.3.

Note that there are three sign changes in the incremental cash flows, indicating a possibility of multiple rates of return. indicating that we may need to calculate the MIRR on the incremental cash flows.[9]

[8] This factor leads to the possibility of having many i^*s. Once again, if you desire to find the true rate of return on this incremental cash flow series, you need to refer to Chapter 7A.

[9] In Chapter 7A, a project with this type of cash flow series is known as a **pure investment**. However, there is a unique rate of return on this incremental cash flow series, namely, 3.90%. Since this rate of return on incremental cash flow is less than the MARR of 12%, Option 2 is a better choice. This was the same conclusion that we reached in Example 5.8.

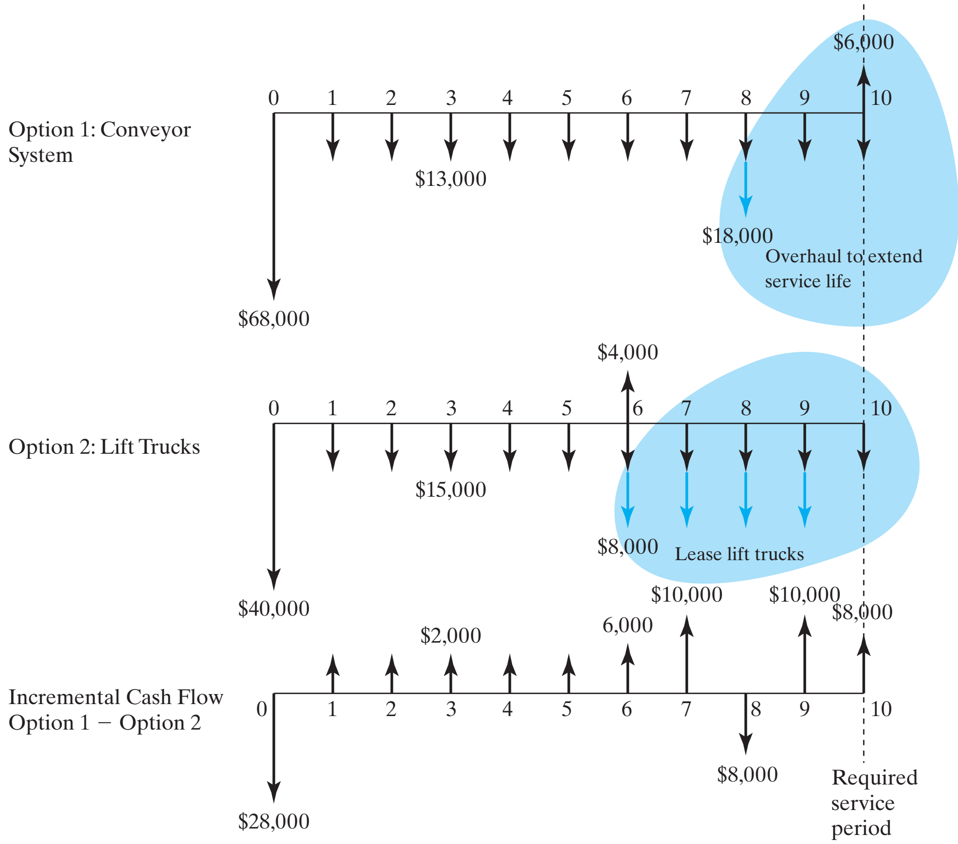


Figure 7.11 Incremental analysis (Option 1 – Option 2)—here we subtract Option 2 from Option 1 because Option 1 is a more costly alternative.

TABLE 7.3 An Excel Worksheet to Illustrate How to Prepare the Incremental Cash Flows

	A	B	C	D
1	Example 7.10 - Comparing Unequal-Service-Life Problems			
2				
3		Option 1	Option 2	Incremental
4		Conveyor Systems	Lift Trucks	Option 1 – Option 2
5				
6	0	$ (68,000)	$ (40,000)	$ (28,000)
7	1	$ (13,000)	$ (15,000)	$ 2,000
8	2	$ (13,000)	$ (15,000)	$ 2,000
9	3	$ (13,000)	$ (15,000)	$ 2,000
10	4	$ (13,000)	$ (15,000)	$ 2,000
11	5	$ (13,000)	$ (15,000)	$ 2,000
12	6	$ (13,000)	$ (19,000)	$ 6,000
13	7	$ (13,000)	$ (23,000)	$ 10,000
14	8	$ (31,000)	$ (23,000)	$ (8,000)
15	9	$ (13,000)	$ (23,000)	$ 10,000
16	10	$ (7,000)	$ (15,000)	$ 8,000
17				

Since the cash flow on incremental investment indicates a non-simple investment, we may use the MIRR approach to make the final selection. Assuming that financing rate and reinvestment rate are the same at MARR = 12%, we can calculate the MIRR as follows:

$$\text{Equivalent cash outlay at } n = 0\text{: } \$28{,}000 + \$8{,}000(P/F, 12\%, 8) = \$31{,}231.$$

$$\text{Equivalent cash inflow at } n = 10\text{: } \$2{,}000(F/P, 12\%, 9) + \cdots + \$8{,}000 = \$65{,}082.$$

$$\$65{,}082 = \$31{,}231(1 + \text{MIRR})^{10}.$$

$$\text{MIRR} = 7.62\% < 12\% \leftarrow \text{Select Option 2}$$

SUMMARY

- **Rate of return (ROR)** is the interest rate earned on unrecovered project balances such that an investment's cash receipts make the terminal project balance equal to zero. Rate of return is an intuitively familiar and understandable measure of project profitability that many managers prefer over PW or other equivalence measures.

- Mathematically, we can determine the rate of return for a given project's cash flow series by identifying an interest rate that equates the present worth (annual equivalent or future worth) of its cash flows to zero. This break-even interest rate is denoted by the symbol i^*.

- **Internal rate of return (IRR)** is another term for ROR that stresses the fact that we are concerned with the interest earned on the portion of the project that is *internally* invested, not with those portions that are released by (borrowed from) the project.

- To apply rate-of-return analysis correctly, we need to classify an investment as either a simple investment or a nonsimple investment. A **simple investment** is defined as an investment in which the initial cash flows are negative and only one sign change in the net cash flow occurs whereas a **nonsimple investment** is an investment for which more than one sign change in the cash flow series occurs. Multiple i^*s occur only in nonsimple investments. However, not all nonsimple investments will have multiple i^*s as shown in Example 7.11.

- For a simple investment, the calculated rate of return (i^*) is the rate of return internal to the project, so the decision rule is as follows:

  If IRR > MARR, accept the project.
  If IRR = MARR, remain indifferent.
  If IRR < MARR, reject the project.

  IRR analysis yields results consistent with PW and other equivalence methods.

- For a nonsimple investment, because of the possibility of having multiple rates of return, it is recommended that the IRR analysis be abandoned and either the PW or AE analysis be used to make an accept-or-reject decision. As an alternative, you may use the MIRR or RIC method. Procedures are outlined in Chapter 7A for determining the rate of return internal to nonsimple investments. Once you find the IRR (or RIC), you can use the same decision rule for simple investments.

- When properly selecting among alternative projects by IRR analysis, you must use **incremental investment**.

- Our purpose is not to encourage you to use the IRR approach to compare projects with unequal lives; rather, it is to show the correct way to compare them if the IRR approach must be used.

SELF-TEST QUESTIONS

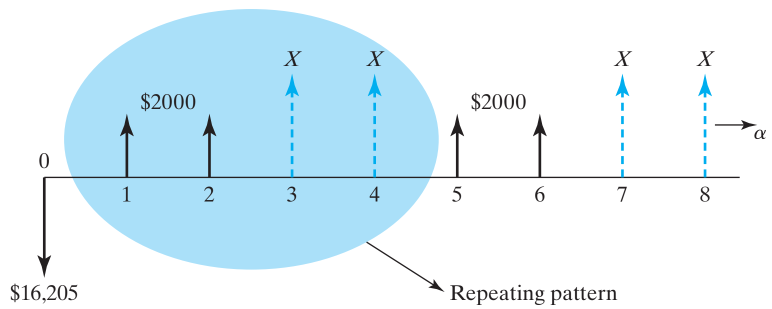
7s.1 An investor purchased Alphabet stock (GOOG) at $236.70 per share on October 31, 2012. The closing price of the stock on October 31, 2018 was $965.34. What would the compound annual return on investment be for the investors who purchased the stock and held it for 6 years?
(a) 47.8%
(b) 26.4%
(c) 24.6%
(d) 18.9%

7s.2 You are considering an open-pit mining operation. The cash flow pattern is somewhat unusual since you must invest in some mining equipment, conduct operations for two years, and then close and restore the sites to their original condition. You estimate the net cash flows to be as follows:

n	Cash Flows
0	−$1,850,000
1	$1,700,000
2	$1,620,000
3	−$850,000

What is the approximate rate of return of this investment?
(a) 32%
(b) 38%
(c) 42%
(d) 62%

7s.3 The following infinite cash flow has a rate of return of 16%. Compute the unknown value of X.



(a) $3,390
(b) $5,510
(c) $5,028
(d) $5,236

7s.4 Consider the investment project with the net cash flows as shown in the following table. What would be the value of X if the project's IRR is 20%?

End of Year (n)	Net Cash Flow
0	−\$25,000
1	\$5,000
2	\$12,000
3	\$$X$
4	\$$X$

(a) \$11,782
(b) \$9,755
(c) \$6,890
(d) \$6,500

7s.5 You purchased a stamping machine for \$100,000 to produce a new line of products. The stamping machine will be used for five years, and the expected salvage value of the machine is 20% of the initial cost. The annual operating and maintenance costs amount to \$30,000. If each part stamped generates \$12 revenue, how many parts should be stamped each year just to break even? Assume that you require a 15% return on your investment.
(a) 5,000
(b) 4,739
(c) 4,488
(d) 2,238

7s.6 You are considering purchasing a new punch press machine. This machine will have an estimated service life of 10 years. The expected after-tax salvage value at the end of service life will be 10% of the purchase cost. Its annual after-tax operating cash flows are estimated to be \$60,000. If you can purchase the machine at \$308,758, what is the expected rate of return on this investment?
(a) 12%
(b) 13.6%
(c) 15%
(d) 17.2%

7s.7 Consider the following projects' cash flows:

End of Year (n)	Project A	Project B	Project C	Project D
0	−\$1,000	−\$3,000	\$2,000	−\$4,000
1	\$200	\$2,000	−\$600	−\$1,000
2	\$500	\$2,000	−\$600	\$6,000
3	\$600	−\$1,500	−\$600	\$0
4	\$400	\$2,000	−\$600	\$2,000

Which of the following statements is *correct*?

(a) Project A is the only simple investment.

(b) Project A and D are the only projects that will have a unique positive rate of return.

(c) Project B is the only nonsimple investment.

(d) None of the above.

7s.8 You are evaluating five investment projects. You have already calculated the rate of return for each alternative investment and incremental rate of return for the two alternatives. In calculating the incremental rate of return, a lower cost investment project is subtracted from the higher cost investment project. All rate of return figures are rounded to the nearest integers.

Investment Alternative	Initial Investment ($)	Rate of Return (%)	Rate of Return on Incremental Investment (%) When Compared with Alternative				
			A	*B*	*C*	*D*	*E*
A	35,000	12		28	20	36	27
B	45,000	15			12	40	22
C	50,000	13				42	25
D	65,000	20					−5
E	80,000	18					

If all investment alternatives are mutually exclusive and the MARR is 12%, which alternative should be chosen?

(a) D

(b) E

(c) B

(d) Do nothing

7s.9 Consider the following *project balance profiles* for proposed investment projects.

Project Balances

n	Project *A*	Project *B*	Project *C*
0	−$600	−$500	−$200
1	$200	$300	$0
2	$300	$650	$150
NPW	–	$416.00	–
Rate used	15%	?	–

- Statement 1—For Project A, the cash flow at the end of year 2 is $100.
- Statement 2—For Project C, its net future worth at the end of year 2 is $150.
- Statement 3—For Project B, the interest rate used is 25%.
- Statement 4—For Project A, the rate of return should be greater than 15%.

Which of the statement(s) above is (are) *correct*?
(a) Just Statements 1 and 2
(b) Just Statements 2 and 3
(c) Just Statements 1 and 3
(d) Just Statements 2, 3, and 4

7s.10 Consider the following investment projects.

Year (n)	Net Cash Flow	
	Project 1	Project 2
0	−$1,200	−$2,000
1	$600	$1,500
2	$1,000	$1,000
IRR	19.65%	17.54%

Determine the range of MARR for which Project 2 would be preferred over Project 1.
(a) MARR ≤ 12.5%
(b) 13% ≤ MARR ≤ 15%
(c) 16% ≤ MARR
(d) Not enough information to determine

7s.11 The following information on two mutually exclusive projects is given below:

n	Project A	Project B
0	−$3,000	−$5,000
1	$1,350	$1,350
2	$1,800	$1,800
3	$1,500	$5,406
IRR	25%	25%

Which of the following statements is *correct?*
(a) Since the two projects have the same rate of return, they are indifferent.
(b) Project A would be a better choice because the required investment is smaller with the same rate of return.
(c) Project B would be a better choice as long as the investor's MARR is less than 25%.
(d) Project B is a better choice regardless of the investor's MARR.

7s.12 You purchased a piece of property at $360,000 five years ago. You can sell the property at $450,000. What is the rate of return on this real estate investment?
(a) 3.55%
(b) 4.08%
(c) 4.56%
(d) 5.22%

7s.13 The city of Houston made a \$10 million public investment in the arts five years ago and the citizens of Houston are getting a 54% rate of return on their tax investment. What kind of proceeds from the investment in the arts is being made?
(a) \$27.00M
(b) \$53.35M
(c) \$86.62M
(d) \$92.23M

7s.14 Best Foods forecasts the following cash flows on a special meat packing operation under consideration.

A_0	A_1	A_2	A_3
(\$15,000)	\$0	\$8,200	\$9,500

What is the rate of return on this investment?
(a) 11.34%
(b) 6.76%
(c) 13.02%
(d) 14.17%

7s.15 You are considering a CNC machine. This machine will have an estimated service life of 10 years with a salvage value of 10% of the investment cost. Its expected savings from annual operating and maintenance costs are estimated to be \$60,000. To expect a 15% rate of return on investment, what would be the maximum amount that you are willing to pay for the machine?
(a) \$287,348
(b) \$299,225
(c) \$321,888
(d) \$308,761

7s.16 Consider two investments with the following sequences of cash flows:

	Net Cash Flow	
n	Project *A*	Project *B*
0	−\$3,000	−\$3,000
1	\$1,500	\$300
2	\$1,200	\$600
3	\$600	\$600
4	\$600	\$1,200
5	\$300	\$2,100

At MARR of 12%, which project would you select?
(a) Select A as it has a higher rate of return.
(b) Select A as the return on incremental investment of Project B is less than 12%.

(c) Select B as it generates more cash for the same amount of investment.
(d) Neither

7s.17 Consider the following two mutually exclusive investment projects:

	Net Cash Flow	
Year (n)	Project 1	Project 2
0	−$2,200	−$2,000
1	$1,200	$1,200
2	$1,650	$1,400
IRR	18.07%	18.88%

Determine the range of MARR where Project 2 would be preferred over Project 1 with "do-nothing" alternative.
(a) MARR $\leq$ 11.80%
(b) MARR $\geq$ 11.80%
(c) 11.80% $\leq$ MARR $\leq$ 18.88%
(d) MARR $\leq$ 18.88%

PROBLEMS

Note: The symbol i^* represents the interest rate that makes the present worth of the project equal to zero. The acronym IRR represents the *internal rate of return* of the investment. For a simple investment, IRR $= i^*$. For a nonsimple investment, i^* generally is not equal to IRR.

Concept of Rate of Return

7.1 If you invest $1,000 in a stock and its value is doubled in six years, what is the return on your stock investment?

7.2 Suppose you deposited $2,000 seven years ago and another $2,000 four years ago in your savings account. If your current account balance shows $5,833.2, what is the rate of return on your deposit?

7.3 You are going to buy a new car worth $25,400. The dealer computes your monthly payment to be $515.45 for 60 months of financing. What is the dealer's effective rate of return on this loan transaction?

7.4 You wish to sell a bond that has a face value of $1,000. The bond bears an interest rate of 8%, which is payable semi-annually. Five years ago, the bond was purchased at $980. At least a 6% annual return on the investment is desired. What must be the minimum selling price of the bond now in order to make the desired return on the investment?

7.5 A painting of the Baroque era was purchased by an art collector for $212 million in 2017. The painting was bought at auction by a private buyer in 1984. That time the buyer paid just US$12,000. If the original buyer of 1984 had invested his $12,000 in another investment vehicle (such as stock), how much annual return would he need to have earned in order to accumulate the same wealth as he did from the painting investment? Assume for simplicity that the investment period is 32 years.

Methods for Finding Rate of Return

7.6 Suppose that you invest $40,000 in a restaurant business. One year later, you sell half of this business to a partner for $110,000. Then, a year later, the business is in the red, and you have to pay $50,000 to close the business. What is the rate of return on your investment from this restaurant business?

7.7 Consider a project that will bring in upfront cash inflows for the first two years but require paying some money to close the project in the third year.

A_0	A_1	A_2
$6,800	$5,200	($15,500)

This is a simple borrowing project. Determine the borrowing rate of return.

7.8 Consider four investments with the sequences of cash flows given in Table P7.8.

TABLE P7.8

	Net Cash Flow			
n	Project A	Project B	Project C	Project D
0	−$32,000	−$38,000	$45,000	−$46,300
1	$30,000	$32,000	−$18,000	$2,500
2	$20,000	$32,000	−$18,000	$6,459
3	$10,000	−$22,000	−$18,000	$78,345

(a) Identify all the simple investments.
(b) Identify all the nonsimple investments.
(c) Compute i^* for each investment.
(d) Which project has no rate of return?

7.9 You are considering an investment that costs $4,800. It is expected to have a useful life of three years. You are very confident about the revenues during the first two years but you are unsure about the revenue in year 3. If you hope to make at least a 12% rate of return on your investment ($4,800), what should be the minimum revenue in year 3?

Year	Cash Flow
0	−$4,800
1	$2,300
2	$2,800
3	$*X*

7.10 Consider the investment project with the following net cash flows:

End of Year (n)	Net Cash Flow
0	−$12,500
1	$3,000
2	$6,500
3	$X
4	$X

What would be the value of X if the project's IRR is 20%?

7.11 An investor bought 100 shares of stock at a cost of $10 per share. He held the stock for 15 years and then sold it for a total of $4,000. For the first three years, he received no dividends. For each of the next seven years, he received total dividends of $50 per year. For the remaining period, he received total dividends of $100 per year. What rate of return did he make on the investment?

7.12 Consider the investment projects given in Table P7.12.

TABLE P7.12

	Project Cash Flow				
n	A	B	C	D	E
0	−$250	−$300	−$400	−$100	−$60
1	$450	$100	$200	$120	−$120
2	−$120	$110	$15	$40	−$50
3	—	$180	$50	$40	$0
4	—	$75	−$185	−$200	$160
5	—	—	$100	$40	$150
6	—	—	$45	$30	$140
7	—	—	$500	—	$130

(a) Classify each project as either simple or nonsimple.

(b) Use the quadratic equation to compute i^* for Project A.

(c) Obtain the rate(s) of return for each project by plotting the NPW as a function of the interest rate.

7.13 Consider the projects given in Table P7.13.

TABLE P7.13

	Net Cash Flow			
n	A	B	C	D
0	−\$2,500	−\$2,000	−\$10,000	−\$2500
1	\$100	\$800	\$5,600	−\$1,360
2	\$100	\$600	\$4,900	\$4,675
3	\$100	\$500	−\$3,500	\$2,288
4	\$2,100	\$700	\$7,000	—
5	—	—	−\$1,400	—
6	—	—	\$2,100	—
7	—	—	\$900	—

(a) Classify each project as either simple or nonsimple.
(b) Identify all positive i^*s for each project.
(c) For each project, plot the present worth as a function of the interest rate (i).

7.14 Consider the financial data for a project given in Table P7.14.

TABLE P7.14

Initial investment	\$90,000
Project life	6 years
Salvage value	\$10,000
Annual revenue	\$27,000
Annual expenses	\$8,000

(a) What is i^* for this project?
(b) If the annual expense increases at a 7% rate over the previous year's expenses, but the annual income is unchanged, what is the new i^*?
(c) In part (b), at what annual rate will the annual income have to increase to maintain the same i^* obtained in part (a)?

Internal-Rate-of-Return Criterion

7.15 A corporation is considering purchasing a machine that will save \$210,000 per year before taxes. The cost of operating the machine, including maintenance, is \$60,000 per year. The machine, costing \$120,000, will be needed for four years after which it will have a salvage value of \$16,000. If the firm wants a 12% rate of return before taxes, what is the net present value of the cash flows generated from this machine?

7.16 Consider an investment project with the following net cash flows:

TABLE P7.16

n	Cash Flow
0	−\$38,500
1	\$13,800
2	\$14,250
3	\$15,000

Compute the IRR for this investment. Is the project acceptable at MARR = 8%?

7.17 Consider the cash flow of a certain project given in Table P7.17.

TABLE P7.17

n	Net Cash Flow
0	−\$16,800
1	\$10,200
2	\$6,000
3	\$$X$

If the project's IRR is 8%:

(a) Find the value of X.

(b) Is this project acceptable at MARR = 9%?

7.18 You are considering a luxury apartment building project that requires an investment of \$14,500,000. The building has 50 units. You expect the maintenance cost for the apartment building to be \$350,000 the first year and \$400,000 the second year. The maintenance cost will continue to increase by \$50,000 in subsequent years. The cost to hire a manager for the building is estimated to be \$85,000 per year. After five years of operation, the apartment building can be sold for \$16,000,000. What is the annual rent per apartment unit that will provide a return on investment of 15%? Assume that the building will remain fully occupied during its five years of operation.

7.19 A machine costing \$32,000 to buy and \$5,000 per year to operate will mainly save labor expenses in packaging over 10 years. The anticipated salvage value of the machine at the end of the six years is \$4,000. To receive a 10% return on investment (rate of return), what is the minimum required annual savings in labor from this machine?

7.20 You are considering purchasing a CNC machine that costs \$180,000. This machine will have an estimated service life of eight years with a net after-tax salvage value of \$20,000. Its annual after-tax operating and maintenance costs are estimated to be \$48,000. To expect an 11% rate of return on investment, what would be the required minimum annual after-tax revenues?

7.21 Champion Chemical Corporation is planning to expand one of its propylene manufacturing facilities. At $n = 0$, a piece of property costing \$1.5 million must be purchased to build a plant. The building, which needs to be expanded during the first year, costs \$3 million. At the end of the first year, the company needs to spend about \$4 million on equipment and other start-up costs. Once the building becomes operational, it will generate revenue in the amount of \$3.5 million during the first operating year. This will increase at the annual rate of 5% over the previous year's revenue for the next nine years. After 10 years, the sales revenue will stay constant for another three years before the operation is phased out. (It will have a project life of 13 years after construction.) The expected salvage value of the land at the end of the project's life would be about \$2 million, the building about \$1.4 million, and the equipment about \$500,000. The annual operating and maintenance costs are estimated to be approximately 40% of the sales revenue each year. What is the IRR for this investment? If the company's MARR is 15%, determine whether the investment is a good one. (Assume that all figures represent the effect of the income tax.)

7.22 Recent technology has made possible a computerized vending machine that can grind coffee beans and brew fresh coffee on demand. The computer also makes possible such complicated functions as changing \$5 and \$10 bills, tracking the age of an item, and moving the oldest stock to the front of the line, thus cutting down on spoilage. With a price tag of \$4,500 for each unit, Easy Snack has estimated the cash flows in millions of dollars over the product's six-year useful life, including the initial investment, as given in Table P7.22.

TABLE P7.22

n	Net Cash Flow
0	−\$30
1	\$9
2	\$18
3	\$20
4	\$18
5	\$10
6	\$5

(a) On the basis of the IRR criterion, if the firm's MARR is 18%, is this product worth marketing?

(b) If the required investment remains unchanged, but the future cash flows are expected to be 10% higher than the original estimates, how much of an increase in IRR do you expect?

(c) If the required investment has increased from \$30 million to \$35 million, but the expected future cash flows are projected to be 10% smaller than the original estimates, how much of a decrease in IRR do you expect?

7.23 Consider an investment project with the cash flows given in Table P7.23.

TABLE P7.23

n	Net Cash Flow
0	−\$220,000
1	\$94,000
2	\$144,000
3	−\$72,000

(a) Is this a simple investment?
(b) At the reinvestment rate of 15% and 12% of financing rate, determine MIRR.
(c) On the basis of the MIRR criterion, should the project be accepted?

Incremental-Investment Analysis for Comparing Mutually Exclusive Alternatives

7.24 Consider two investments, A and B, with the sequences of cash flows given in Table P7.24.

TABLE P7.24

	Net Cash Flow	
n	Project A	Project B
0	−\$35,000	−\$35,000
1	\$2,000	\$11,000
2	\$6,000	\$10,000
3	\$10,000	\$10,000
4	\$24,800	\$8,000
5	\$27,050	\$5,000

(a) Compute i^* for each investment.
(b) Plot the present-worth curve for each project on the same chart, and find the interest rate that makes the two projects equivalent.

7.25 Consider two investments A and B with the sequences of cash flows given in Table P7.25.

TABLE P7.25

	Net Cash Flow	
n	Project A	Project B
0	−\$125,000	−\$110,000
1	\$30,000	\$20,000
2	\$30,000	\$20,000
3	\$120,000	\$130,000

(a) Compute the IRR for each investment.
(b) At MARR = 15% and financing rate = 12%, determine the MIRR of each project.
(c) If A and B are mutually exclusive projects, which project would you select on the basis of the MIRR on incremental investment?

7.26 Consider two investments with the following sequences of cash flows:

	Net Cash Flow	
n	**Project A**	**Project B**
0	−$125,000	−$125,000
1	$34,000	$85,000
2	$40,500	$62,750
3	$96,800	$36,500

(a) Compute the IRR for each investment.
(b) At MARR = 15%, determine the acceptability of each project.

7.27 With $10,000 available, you have two investment options. The first is to buy a certificate of deposit from a bank at an interest rate of 9% annually for five years. The second choice is to purchase a bond for $10,000 and invest the bond's interest in the bank at an interest rate of 5%. The bond pays 6.5% interest annually and will mature to its face value of $10,000 in five years. Which option is better? Assume that your MARR is 5% per year.

7.28 A manufacturing firm is considering the mutually exclusive alternatives given in Table P7.28. Determine which project is a better choice at a MARR = 15% based on the IRR criterion.

TABLE P7.28

	Net Cash Flow	
n	**Project A1**	**Project A2**
0	−$4,000	−$5,000
1	$2,600	$3,600
2	$2,800	$3,200

7.29 Consider the two mutually exclusive alternatives given in Table P7.29.

TABLE P7.29

	Net Cash Flow	
n	**Project A1**	**Project A2**
0	−$12,000	−$14,000
1	$5,000	$6,200
2	$5,000	$6,200
3	$5,000	$6,200

(a) Determine the IRR on the incremental investment in the amount of \$2,000.
(b) If the firm's MARR is 10%, which alternative is the better choice?

7.30 Consider the following two mutually exclusive investment alternatives:

	Net Cash Flow	
n	Project $A1$	Project $A2$
0	−\$16,000	−\$20,000
1	\$7,500	\$5,000
2	\$7,500	\$15,000
3	\$7,500	\$8,000
IRR	19.19%	17.65%

(a) Determine the IRR on the incremental investment in the amount of \$4,000. (Assume that MARR = 10%.)
(b) If the firm's MARR is 10%, which alternative is the better choice?

7.31 Consider the two investment alternatives given in Table P7.31.

TABLE P7.31

	Net Cash Flow	
n	Project A	Project B
0	−\$10,000	−\$20,000
1	\$5,500	\$0
2	\$5,500	\$0
3	\$5,500	\$40,000
IRR	30%	$\boxed{?}$
PW(15%)	$\boxed{?}$	\$6301

The firm's MARR is known to be 15%.
(a) Compute the IRR of project B.
(b) Compute the NPW of project A.
(c) Suppose that projects A and B are mutually exclusive. Using the IRR, which project would you select?

7.32 You are considering two types of automobiles. Model A costs \$22,556 and model B costs \$20,000. Although the two models are essentially the same, after five years of use, model A can be sold for \$7,500, while model B can be sold for \$4,200. Model A commands a better resale value because its styling is popular among young college students. Determine the rate of return on the incremental investment of \$2,556. For what range of values of your MARR is model A preferable?

7.33 A plant engineer is considering the two types of solar water heating system given in Table P7.33.

TABLE P7.33

	Model A	Model B
Initial cost	$8,000	$12,000
Annual savings	$900	$1,100
Annual maintenance	$150	$100
Expected life	20 years	20 years
Salvage value	$500	$600

The firm's MARR is 12%. On the basis of the IRR criterion, which system is the better choice? "Do nothing" is not an option.

7.34 Fulton National Hospital is reviewing ways of cutting the cost of stocking medical supplies. Two new systems are being considered to lower the hospital's holding and handling costs. The hospital's industrial engineer has compiled the relevant financial data for each system as given in Table P7.34 (dollar values are in millions).

TABLE P7.34

	Current Practice	Just-in-Time System	Stockless Supply System
Start-up cost	$0	$3	$6
Annual stock holding cost	$6	$1.8	$0.4
Annual operating cost	$3	$2	$1.5
System life	8 years	8 years	8 years

The system life of eight years represents the period that the contract with the medical suppliers is in force. If the hospital's MARR is 10%, which system is the most economical?

7.35 Consider the investment projects given in Table P7.35.

TABLE P7.35

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−$1,500	−$5,000	−$2,200
1	$700	$7,500	$1,600
2	$2,500	$600	$2,000

Assume that MARR = 15% and a financing rate of 12%.

(a) Compute the IRR for each project.

(b) On the basis of the IRR criterion, if the three projects are mutually exclusive investments, which project should be selected?

7.36 Consider the following two investment situations:

- In 1970, when W.M. Company went public, 100 shares cost $1,650. That investment was worth $15,337,472 after 47 years (2017) with a rate of return of around 32%.
- In 1990, if you bought 100 shares of First Mutual Funds it would have cost $5,245. That investment would have been worth $289,556 after 27 years.

Which of the following statements is correct?

(a) If you had bought only 50 shares of the W.M. Company stock in 1970 and kept them for 47 years, your rate of return would be 0.5 times 21.46%.

(b) If you had bought 100 shares of First Mutual Funds in 1990, you would have made a profit at an annual rate of 20.45% on the funds remaining invested.

(c) If you had bought 100 shares of W.M. Company in 1970 but sold them after 20 years at $238,080 and immediately put all the proceeds into First Mutual Funds, then after 27 years the total worth of your investment would be around $13,155,585.

(d) None of the above.

7.37 The GeoStar Company, a leading manufacturer of wireless communication devices, is considering three cost-reduction proposals in its batch job-shop manufacturing operations. The company has already calculated rates of return for the three projects, along with some incremental rates of return, as given in Table P7.37.

TABLE P7.37

Incremental Investment	Incremental Rate of Return (%)
$A_1 - A_0$	18
$A_2 - A_0$	20
$A_3 - A_0$	25
$A_2 - A_1$	10
$A_3 - A_1$	18
$A_3 - A_2$	23

A_0 denotes the do-nothing alternative. The required investments are $420,000 for A_1 $550,000 for A_2 and $720,000 for A_3. If the MARR is 15%, what system should be selected?

7.38 A manufacturer of electronic circuit boards is considering six mutually exclusive cost-reduction projects for its PC-board manufacturing plant. All have lives of 10 years and zero salvage values. The required investment and the estimated after-tax reduction in annual disbursements for each alternative are given in Table P7.38a, along with computed rates of return on incremental investments in Table P7.38b.

TABLE P7.38A

Proposal A_j	Required After-Tax Investment	Savings	Rate of Return (%)
A_1	$60,000	$22,000	34.8
A_2	$100,000	$28,200	25.2
A_3	$110,000	$32,600	26.9
A_4	$120,000	$33,600	25.0
A_5	$140,000	$38,400	24.3
A_6	$150,000	$42,200	25.1

TABLE P7.38B

Incremental Investment	Incremental Rate of Return (%)
$A_2 - A_1$	8.9
$A_3 - A_2$	42.7
$A_4 - A_3$	0.0
$A_5 - A_4$	20.2
$A_6 - A_5$	36.3

If the MARR is 15%, which project would you select based on the rate of return on incremental investment?

7.39 The following information on four mutually exclusive projects is given here. All four projects have the same service life and require investment in year 0 only one. Suppose that you are provided with the following additional information about incremental rates of return between projects.

IRR(B − A) = 85%
IRR(D − C) = 25%
IRR(B − C) = 30%
IRR(A − D) = 50%

Which project would you choose based on the rate of-return criterion at a MARR of 29%?

TABLE P7.39

Project	Required Investment at Year 0	IRR
A	$1,000	56%
B	$1,200	67%
C	$1,500	43%
D	$2,500	49%

7.40 Fulton National Hospital is reviewing ways of cutting the costs for stocking medical supplies. Two new stockless systems are being considered to lower the hospital's holding and handling costs. The hospital's industrial engineer has compiled the relevant financial data for each system, as follows, where dollar values are in millions:

TABLE P7.40

Item	Current Practice	Just-in-Time System	Stockless Supply
Start-up cost	\$0	\$2.5	\$5
Annual stock-holding cost	\$3	\$1.4	\$0.2
Annual operating cost	\$2	\$1.5	\$1.2
System life	8 years	8 years	8 years

The system life of eight years represents the contract period with the medical suppliers. If the hospital's MARR is 10%, which system is more economical?

7.41 An oil company is considering changing the size of a small pump that currently is in operation in an oil field. If the current pump is kept, it will extract 50% of the known crude oil reserve in the first year of operation and the remaining 50% in the second year. A pump larger than the current pump will cost \$1.6 million, but it will extract 100% of the known reserve in the first year. The total oil revenues over the two years are the same for both pumps: \$20 million. The advantage of the large pump is that it allows 50% of the revenue to be realized a year earlier than the small pump. The two options are summarized as follows:

Item	Current Pump	Larger Pump
Investment, year 0	\$0	\$1.6 million
Revenue, year 1	\$10 million	\$20 million
Revenue, year 2	\$10 million	\$0

If the firm's MARR is known to be 20% and financing rate of 15%, what do you recommend, according to the IRR criterion?

Unequal Service Lives

7.42 Consider the two mutually exclusive investment projects given in Table P7.42 for which MARR = 15%. On the basis of the IRR criterion, which project would be selected under an infinite planning horizon with project repeatability likely? Assume a financing rate of 10%.

TABLE P7.42

	Net Cash Flow	
n	Project A	Project B
0	−\$5,000	−\$10,000
1	\$3,000	\$8,000
2	\$4,000	\$8,000
3	\$4,000	—
IRR	49.49%	39.97%

7.43 A company has to decide between two machines that do the same job but have different lives.

Net Cash Flow	
Machine A	Machine B
−\$40,000	−\$55,000
−\$15,000	−\$10,000
−\$15,000	−\$10,000
−\$15,000	−\$10,000
	−\$10,000

Which machine should the company buy, at an interest rate of 10%, based on the principle of internal rate of return? Assume a financing rate of 8%.

7.44 Consider the two mutually exclusive investment projects given in Table P7.44.

TABLE P7.44

	Net Cash Flow	
n	Project A1	Project A2
0	−\$10,000	−\$15,000
1	\$5,000	\$20,000
2	\$5,000	—
3	\$5,000	—

(a) To use the IRR criterion, what assumption must be made in comparing a set of mutually exclusive investments with unequal service lives?

(b) With the assumption made in part (a), determine the range of MARRs which will indicate that project A1 should be selected.

Short Case Studies with Excel

7.45 Merco, Inc., a machinery builder in Louisville, Kentucky, is considering making an investment of $1,250,000 in a complete structural-beam-fabrication system. The increased productivity resulting from the installation of the drilling system is central to the project's justification. Merco estimates the following figures as a basis for calculating productivity:

- Increased fabricated-steel production: 2,000 tons/year
- Average sales price per ton of fabricated steel: $2,566.50
- Labor rate: $10.50/hour
- Tons of steel produced in a year: 15,000
- Cost of steel per ton (2,205 lb): $1,950
- Number of workers on layout, hole making, sawing, and material handling: 17
- Additional maintenance cost: $128,500/year

With the cost of steel at $1,950 per ton and the direct-labor cost of fabricating 1 lb at 10 cents, the cost of producing a ton of fabricated steel is about $2,170.50. With a selling price of $2,566.50 per ton, the resulting contribution to overhead and profit becomes $396 per ton. Assuming that Merco will be able to sustain an increased production of 2,000 tons per year by purchasing the system, the projected additional contribution has been estimated to be 2,000 tons $\times$ \$396 $=$ \$792,000.

Since the drilling system has the capacity to fabricate the full range of structural steel, two workers can run the system, one operating the saw and the other operating the drill. A third worker is required to operate a crane for loading and unloading materials. Merco estimates that to perform equivalent work with a conventional manufacturing system would require, on average, an additional 14 people for center punching, hole making with a radial or magnetic drill, and material handling. This factor translates into a labor savings in the amount of $294,000 per year $(14 \times \$10.50 \times 40\text{ hours/week} \times 50\text{ weeks/year})$. The system can last for 15 years with an estimated after-tax salvage value of $80,000. However, after an annual deduction of $226,000 in corporate income taxes, the net investment costs, as well as savings, are as follows:

- Project investment cost: $1,250,000
- Projected annual net savings:

$$(\$792{,}000 + \$294{,}000) - \$128{,}500 - \$226{,}000 = \$731{,}500$$

- Projected after-tax salvage value at the end of year 15: $80,000

(a) What is the projected IRR on this investment?

(b) If Merco's MARR is known to be 18%, is this investment justifiable?

7.46 Critics have charged that the commercial nuclear power industry does not consider the cost of "decommissioning," or "mothballing," a nuclear power plant when doing an economic analysis and that the analysis is therefore unduly optimistic. As an example, consider the Tennessee Valley Authority's Bellefont twin nuclear generating facility under construction at Scottsboro in northern

Alabama. The initial cost is \$1.5 billion, and the estimated life is 40 years. The annual operating and maintenance costs the first year are assumed to be 4.6% of the first cost in the first year and are expected to increase at the fixed rate of 0.05% of the first cost each year. Annual revenues have been estimated to be three times the annual operating and maintenance costs throughout the life of the plant. From this information, comment on the following statements:

(a) The criticism of excessive optimism in the economic analysis caused by the omission of mothballing costs is not justified since the addition of a cost to mothball the plant equal to 50% of the initial cost decreases the 10% rate of return only to approximately 9.9%.

(b) If the estimated life of the plant is more realistically taken as 25 years instead of 40 years, then the criticism is justified. When the life is reduced to 25 years, the rate of return of approximately 9% without a mothballing cost drops to approximately 7.7% with a cost to mothball the plant equal to 50% of the initial cost added to the analysis.

7.47 Reconsider the chapter opening story about the return on investment on college education. According to a recent study, attending a university full-time costs the average student more than \$17,000 a year. But here's the magic number that makes it all worthwhile: Over the years, university graduates earn an average of almost \$10,000 a year more than high-school graduates do—and that's an after-tax figure. One recent attempt was made to estimate the median future earnings by two groups—high-school graduates and college graduates. Suppose that net incomes after n years from high-school graduation are as follows:

- High-school graduates: Net income $= \$13{,}529 \ln(n + 17) - \$14{,}788$
- College graduates: Net income $= \$36{,}529 \ln(n + 17) - \$64{,}021, (n > 4)$

Here n represents the year after high school graduation. Now estimate your college expenses for four years and determine the rate of return on your college education.

But you would expect that some degrees to lead to more lucrative occupations than others. It is the right time of year to take a fresh look at an old question: Why bother going to college? With all the accompanying fees and debts, is doing so still worth it?