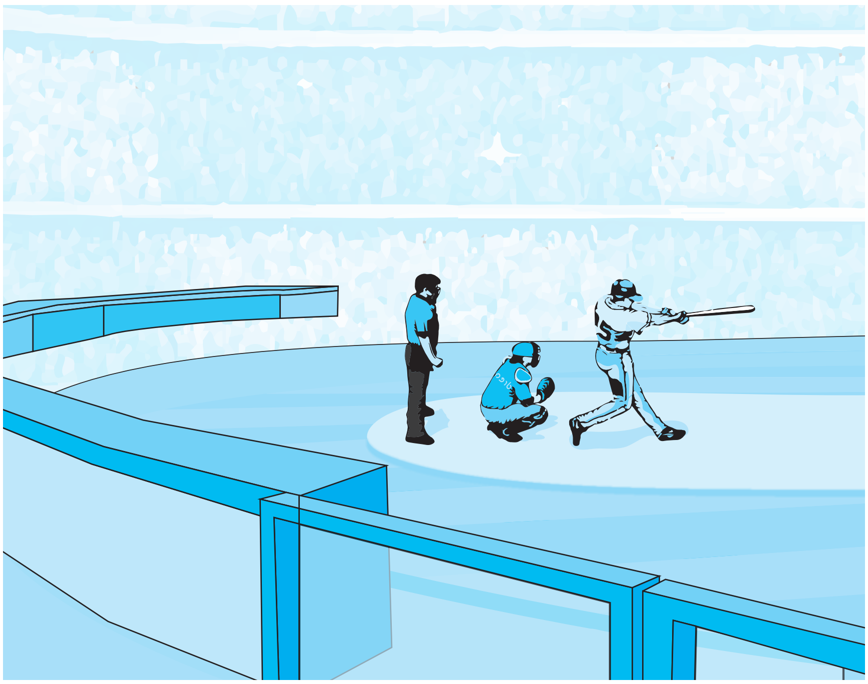
CHAPTER FOUR

Equivalence Calculations under Inflation

How Much Will It Cost to Buy Baseball Tickets in 2020?

When was the last time you bought a ticket for a professional baseball game? With almost everything going up in price due to inflation, baseball tickets are no exception. In 2007, it cost about $47.71 to purchase a ticket for a Boston Red Sox game. In 2016, it cost $54.79, representing an almost 14.84% increase in price. TMR's (Team Marketing Report, Inc.) exclusive Fan Cost Index™[1] (FCI) survey tracks the cost of attendance for a family of four. The FCI comprises



[1] Team Marketing Report, Inc. Chicago, Illinois.

the prices of four adult average-price tickets, two small draft beers, four small soft drinks, four regular size hot dogs, parking for one car, and two least expensive, adult-size caps.

In developing the average ticket price, TMR uses a weighted average of season ticket prices for general seating categories determined by factoring the tickets in each price range as a percentage of the total number of seats in each ballpark. The 2016 FCIs for three teams and the MLB League average were as follows:

Team	Avg. Ticket	Annual Percent Change	Avg. Premium Ticket	Beer	Soft Drink	Hot Dog	Parking	Cap	FCI	Annual Percent Change
Boston Red Sox	$54.79	4.70%	$180.37	$7.75	$5.00	$5.25	$35.00	$25.00	$360.66	2.80%
Los Angeles Angels	$32.70	18.70%	$82.79	$4.50	$2.75	$4.50	$10.00	$16.00	$210.80	10.90%
Minnesota Twins	$33.28	2.10%	$75.43	$7.50	$4.50	$4.00	$6.00	$12.00	$212.12	−7.50%
MLB League Average	$31.00	7.10%	$95.42	$5.90	$4.19	$4.52	$16.32	$16.48	$219.53	3.70%

The MLB average price increase over 2015 is just 3.70%. Essentially, fans could buy 3.70% less with the same amount of dollars in 2015. If this trend continues, the purchasing power of future dollars will also continue to decline. In addition, we may be able to predict what the future MLB ticket price would be for a typical team. Here the purchasing power reflects the value of a currency expressed in terms of the amount of goods or services that one unit of money can buy. Purchasing power is important because, all else being equal, inflation decreases the amount of goods or services that the same amount of money can normally purchase. Our interest in this case is how we may incorporate this loss of purchasing power into our dollar comparison from Chapters 2 and 3.

Up to this point, we have demonstrated how to compute equivalence values under constant conditions in the general economy. We have assumed that prices remain relatively unchanged over long periods. As you know from personal experience, this is not a realistic assumption. In this chapter, we define and quantify the loss of purchasing power, or **inflation**, and then go on to apply it in several equivalence analyses.

4.1 Measure of Inflation

Historically, the general economy has usually fluctuated in such a way as to experience **inflation**, a loss in the purchasing power of money over time. Inflation means that the cost of an item tends to increase over time; or, to put it another way, the same dollar amount buys less of an item over time. **Deflation** is the opposite of inflation, in that prices decrease over time, and hence a specified dollar amount gains purchasing power. Inflation is far more common than deflation in the real world, so our consideration in this chapter will be restricted to accounting for inflation in economic analyses.

4.1.1 Consumer Price Index

Before we can introduce inflation into an equivalence calculation, we need a means of isolating and measuring its effect. Consumers usually have a relative and imprecise sense of how their purchasing power is declining based on their experience of shopping for food, clothing, transportation, and housing over the years. Economists have developed a measure called the **consumer price index** (CPI), which is based on a typical **market basket** of goods and services required by the average consumer. This market basket normally consists of items from eight major groups: (1) food and alcoholic beverages, (2) housing, (3) apparel, (4) transportation, (5) medical care, (6) entertainment, (7) personal care, and (8) other goods and services.

The CPI compares the cost of the typical market basket of goods and services in a current month with its cost at a previous time, such as one month ago, one year ago, or 10 years ago. The point in the past with which current prices are compared is called the **base period**. The index value for this base period is set at \$100. There are two different types of CPIs kept by the Bureau of Labor and Statistics (BLS) of the U.S. Department of Labor:

- **Original Measure (*Base Period* = 1967):** The original base period used by the BLS for the CPI index is 1967. For example, let us say that, in 1967, the prescribed market basket could have been purchased for \$100. Suppose the same combination of goods and services costs \$722.04 in 2016. We can then compute the CPI for 2016 by multiplying the ratio of the current price to the base-period price by 100. In our example, the price index is $(\$722.04/\$100)100 = 722.04$, which means that the 2016 price of the contents of the market basket is 722.04% of its base-period price. (See Figure 4.1.)
- **Revised Measure (*Base Period* = 1982–1984):** The revised CPI introduced by the BLS in 1987 includes indices for two populations: (1) urban wage earners and clerical workers (CW) and (2) all urban consumers (CU). This change reflected the fact that differing populations had differing needs and thus differing market baskets. Both the CW and the CU indices use updated expenditure weights based upon data tabulated from the three years of the Consumer Expenditure Survey (1982, 1983, and 1984) and incorporate a number of technical improvements. As shown in Figure 4.1, the new CPI measure for 2016 is 241.04.

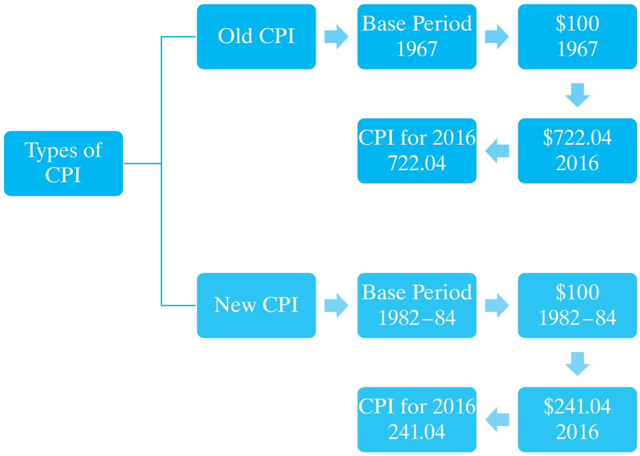


Figure 4.1 Comparison between old CPI and new CPI measures.

Basically, these two indices measure the same degree of changes in purchasing power as long as we use them consistently in any economic study. For example, the CPIs for 2001 and 2016 are 176.2 and 241.04, respectively. For the same period, the original CPIs are 528.0 and 722.04, respectively. So, the price change over a 15-year period is 36.80% and 36.75%, which is basically the same value.

However, the BLS method of assessing inflation does not imply that consumers actually purchase the same goods and services year after year. Consumers tend to adjust their shopping practices to changes in relative prices and to substitute other items for those whose prices have greatly increased in relative terms. We must understand that the CPI does not take into account this sort of consumer behavior because it is predicated on the purchase of a fixed market basket of the same goods and services in the same proportions, month after month. For this reason, the CPI is called a **price index** rather than a **cost-of-living index**, although the general public often refers to it as a cost-of-living index.

4.1.2 Producer Price Index

The consumer price index is an appropriate measure of the general price increase of consumer products, but it is not a good measure of industrial price increases. When performing engineering economic analysis, the appropriate price indices must be selected to accurately estimate the price increases of raw materials, finished products, and operating costs. For example, the cost to produce and deliver gasoline to consumers includes the cost of crude oil to refiners, refinery processing costs, marketing and distribution costs, and finally, the retail station costs and taxes. As shown in Figure 4.2, the prices paid by consumers at the pump reflect these costs as well as the profits (and sometimes losses) of refiners, marketers, distributors, and retail station owners.

In 2011, the price of crude oil averaged $109.92 per barrel, and crude oil accounted for about 69% of the cost of a gallon of regular-grade gasoline. In comparison, the average price for crude oil in 2016 was $44.13 per barrel, and it composed 46% of the cost of a gallon of regular gasoline.

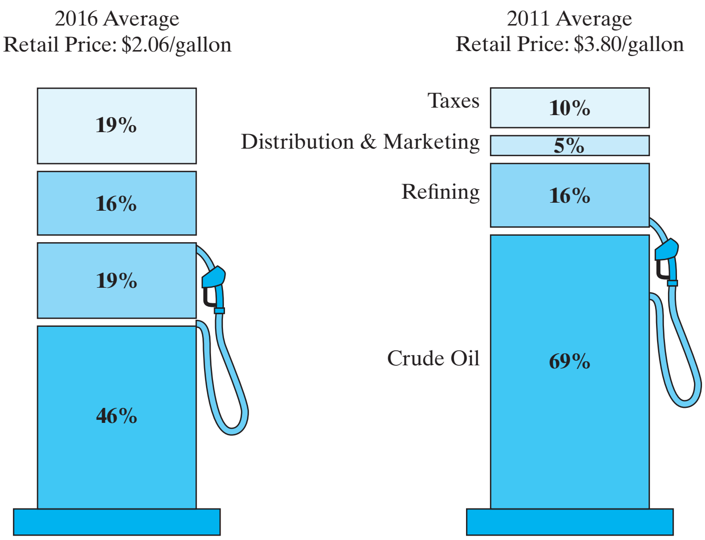


Figure 4.2 Various cost components that affect the retail gasoline price in the United States (*Source:* Energy Information Administration, Washington, DC.).

The producer price index is calculated to capture this type of price changes over time for a specific commodity or industry. The *Survey of Current Business,* a monthly publication prepared by the BLS, provides the industrial-product price index for various industrial goods. Table 4.1 lists the CPI together with several price indexes over a number of years.[2]

From Table 4.1, we can easily calculate the price change (or inflation rate) of gasoline from 2015 to 2016 as follows:

$$\frac{181.2 - 208.5}{208.5} = -0.1309 = -13.09\%.$$

Since the price index calculated is negative, the price of gasoline actually decreased at an annual rate of 13.09% over the year 2015, which was one of the best years for consumers who drive. On the other hand, in 2016, the price of lumber actually increased at an annual rate of 1.3% over the price in 2015.

4.1.3 Average Inflation Rate

To account for the effect of varying yearly inflation rates over a period of several years, we can compute a single rate that represents an **average inflation rate**. Since each year's inflation rate is based on the previous year's rate, these rates have a compounding effect. Suppose we want to calculate the average inflation rate for a two-year period. The first year's inflation rate is 4%, and the second year's rate is 8%, with a base price of $100.

[2] Most up-to-date CPI data are available at http://stats.bls.gov.

TABLE 4.1 **Selected Price Indices between 2006 and 2016**

	Consumer Price Index			Producer Price Index		
Base Year	New (1982–84)	Old 1967	Gasoline 1982	All Commodities 1982	Metals 1982	Lumber 1982
2006	\$201.5	\$600.9	\$218.0	\$166.8	\$193.9	\$187.3
2007	\$206.7	\$619.1	\$247.1	\$175.1	\$204.3	\$179.2
2008	\$214.8	\$643.5	\$291.7	\$205.5	\$293.1	\$166.9
2009	\$213.2	\$638.8	\$222.4	\$172.5	\$178.2	\$150.9
2010	\$218.0	\$653.1	\$240.0	\$184.1	\$224.8	\$167.4
2011	\$225.9	\$676.8	\$300.4	\$204.6	\$256.9	\$165.9
2012	\$229.1	\$686.3	\$308.2	\$200.1	\$232.1	\$170.7
2013	\$233.6	\$699.8	\$296.6	\$204.4	\$225.3	\$190.5
2014	\$238.3	\$713.7	\$276.4	\$208.0	\$232.7	\$215.6
2015	\$238.7	\$714.9	\$208.5	\$193.9	\$198.2	\$200.1
2016	\$240.6	\$720.9	\$181.2	\$187.3	\$194.1	\$202.7

Source: U.S. Bureau of Labor Statistics.

To calculate the average inflation rate for the two years, we employ the following procedure:

- **Step 1:** To find the price at the end of the second year, we use the process of compounding:

$$\underbrace{\overbrace{\$100(1 + 0.04)}^{\text{First year}}(1 + 0.08)}_{\text{Second year}} = \$112.32.$$

- **Step 2:** To find the average inflation rate f, we establish the following equivalence equation:

$$\$100(1 + f)^2 = \$112.32, \text{ or } \$100(F/P, f, 2) = \$112.32.$$

Solving for f yields

$$f = 5.98\%.$$

Thus, we can say that the price increases in the last two years are equivalent to an average rate of 5.98% per year. Note that the average is a geometric average, not an arithmetic average, over a two-year period. *Why do we need to calculate this average inflation rate?* If we want to estimate the future prices on the basis of the historical data, it simplifies our economic analysis to have a single average rate such as this. Otherwise, it can be a challenging task to estimate such a specific yearly inflation rate over the study period.

EXAMPLE 4.1 Calculating an Average Inflation Rate

Consider the price increases for the 11 items in the following table over the last 16 years:

Category	2016 Price	2000 Price	Average Inflation Rate
Postage	\$0.49	\$0.33	2.50%
Homeowners insurance (per year)	\$1,198.00	\$500.00	5.61%
Auto insurance (per year)	\$1,325.00	\$687.00	4.19%
Private college tuition and fees	\$32,405.00	\$15,518.00	4.71%
Gasoline (per gallon)	\$2.67	\$ 1.56	3.42%
Haircut	\$32.00	\$10.50	7.21%
Car (Toyota Camry)	\$25,560.00	\$21,000.00	1.24%
Natural gas (per million BTUs)	\$2.59	\$ 3.17	−1.26%
Baseball tickets (family of four)	\$219.53	\$131.88	3.24%
Movies (average ticket)	\$8.66	\$ 5.39	3.01%
Healthcare (per year)	\$4,316.00	\$1,656.00	6.17%
Consumer price index (CPI) Base period (1982–84): index = 100	**241.04**	**171.20**	**2.16%**

Explain how the average inflation rates are calculated in the table.

DISSECTING THE PROBLEM

Let's take the fourth item, the cost of private college tuition, for a sample calculation. Since we know the prices during both 2000 and 2016, we can use the appropriate equivalence formula (single-payment compound amount factor or growth formula).

Given: $P = \$15{,}518$, $F = \$32{,}405$, and $N = 2016 - 2000 = 16$.
Find: f.

METHODOLOGY

Compute for average inflation rate.

SOLUTION

We use the equation $F = P(1 + f)^N$:

$$\$32{,}405 = \$15{,}518(1 + f)^{16}.$$

Solving for f yields

$$f = \sqrt[16]{2.0882} - 1$$
$$= 0.00471 = 4.71\%$$

This 4.71% means that the private college tuition has outpaced the overall inflation (2.16%) by more than 100% over the last

16 years. If the past trend continues into the future, the private college tuition in 2025 may be estimated as follows:

$$\begin{aligned}\text{Private tuition in year 2025} &= \$32{,}405(1 + 0.0471)^9 \\ &= \$49{,}035.\end{aligned}$$

In a similar fashion, we can obtain the average inflation rates for the remaining items as shown in the table. Clearly, the cost of natural gas increased the least (actually decreased) among the items listed in the table.

4.1.4 General Inflation Rate (f) versus Specific Inflation Rate (f_j)

When we use the CPI as a base to determine the average inflation rate, we obtain the **general inflation rate**. We need to distinguish carefully between the general inflation rate and the average inflation rate for specific goods:

- **General inflation rate ($\bar{f}$):** This average inflation rate is calculated on the basis of the CPI for all items in the market basket. The market interest rate is expected to respond to this general inflation rate.

  In terms of CPI, we define the general inflation rate as

$$\text{CPI}_n = \text{CPI}_0(1 + \bar{f})^n, \tag{4.1}$$

or

$$\bar{f} = \left[\frac{\text{CPI}_n}{\text{CPI}_0}\right]^{1/n} - 1, \tag{4.2}$$

where $\bar{f}$ = the general inflation rate,
CPI_n = the consumer price index at the end period n, and
CPI_0 = the consumer price index for the base period.

If we know the CPI values for two consecutive years, we can calculate the annual general inflation rate as

$$\bar{f}_n = \frac{\text{CPI}_n - \text{CPI}_{n-1}}{\text{CPI}_{n-1}}, \tag{4.3}$$

where $\bar{f}_n$ = the general inflation rate for period n.

As an example, let us calculate the general inflation rate for the year 2016, where $\text{CPI}_{2015} = 238.7$ and $\text{CPI}_{2016} = 240.6$:

$$\frac{240.6 - 238.7}{238.7} = 0.008 = 0.8\%.$$

This calculation demonstrates that 2016 was an unusually good year for the U.S. consumers, as its 0.8% general inflation rate is significantly lower than the average general inflation rate of 2.72% over the last 34 years.[3]

[3] To calculate the average general inflation rate from the base period (1982) to 2016, we need to know the CPI for year 1982, which is 96.5.

$$f = \left[\frac{240.6}{96.5}\right]^{1/34} - 1 = 2.72\%.$$

- **Specific inflation rate** (f_j)**:** This rate is based on a price index (other than the CPI) specific to segment j of the economy. For example, we often must estimate the future cost for an item such as labor, material, housing, or gasoline. (When we refer to the average inflation rate for just one item, we will drop the subscript j for simplicity.) All average inflation rates (except the CPI) calculated in Example 4.1 are specific inflation rates for each individual price item.

EXAMPLE 4.2 Developing Specific Inflation Rates for Baseball Tickets

The accompanying table shows the average cost since 2010 for a family of four to attend a Boston Red Sox game. Determine the specific inflation rate for each period, and calculate the average inflation rate over the six years.

Year	Cost	Yearly Inflation Rate
2016	\$360.66	?
2015	\$350.86	?
2014	\$350.78	?
2013	\$325.17	?
2012	\$336.99	?
2011	\$339.01	?
2010	\$334.78	

DISSECTING THE PROBLEM

Given: History of baseball ticket prices.
Find: The yearly inflation rate (f_j) and the average inflation rate over the six-year time period (f).

METHODOLOGY

Calculate the inflation rate for each year and the average inflation rate over the six-year period.

SOLUTION

The inflation rate between year 2010 and year 2011 (f_1) is

$$(\$339.01 - \$334.78)/\$334.78 = 1.26\%.$$

The inflation rate during year 2011 and year 2012 (f_2) is

$$(\$336.99 - \$339.01)/\$339.01 = -0.60\%.$$

The inflation rate during year 2012 and year 2013 (f_3) is

$$(\$325.17 - \$336.99)/\$336.99 = -3.51\%.$$

Continue these calculations through 2014–2016. The inflation rates will be 7.88%, 0.02% and 2.79%, respectively. The average inflation rate over the six years is

$$f = \left(\frac{\$360.66}{\$334.78}\right)^{1/6} - 1 = 1.25\%.$$

Note that, although the average inflation rate is 1.25% for the period taken as a whole, none of the years within the period had this rate.[4]

COMMENTS: Let's see how FCI works out for the National Football League. The average FCI was $420.54 for 2010 and $502.84 for 2016, respectively. The average inflation rate is 3.02% over this six-year period. Clearly, football fans have experienced much lager price increase in attending the game during the same period.

4.2 Actual Versus Constant Dollars

Due to inflation, the purchasing power of the dollar changes over time. To compare dollar values of different purchasing power from one period to another, they need to be converted to dollar values of *common* purchasing power—converting from actual to constant dollars or from constant to actual dollars. To introduce the effect of inflation into our economic analysis, we need to define the following two inflation-related terms:[5]

- **Actual (current) dollars (A_n):** Actual dollars are estimates of future cash flows for year n that take into account any anticipated changes in amount caused by inflationary or deflationary effects. Actual dollars are the number of dollars that will be paid or received, regardless of how much these dollars are worth. Usually, these numbers are determined by applying an inflation rate to base-year dollar estimates.
- **Constant (real) dollars (A'_n):** Constant dollars reflect constant purchasing power independent of the passage of time. Constant dollars are a measure of worth, not an indicator of the number of dollars paid or received in physical transactions. In situations where inflationary effects were assumed when cash flows were estimated, we can convert these estimates to constant dollars (base-year dollars) by adjustment, using some readily accepted **general inflation rate**. *Unless specified otherwise, we will always assume that the base year is at time zero.*

4.2.1 Conversion from Constant to Actual Dollars

Since constant dollars represent dollar amounts expressed in terms of the purchasing power of the base year, we may find the equivalent dollars in year n by using the general inflation rate $\bar{f}$ in the equation

$$A_n = A'_n(1 + \bar{f})^n = A'_n(F/P, \bar{f}, n), \tag{4.4}$$

where $A'_n =$ the constant-dollar expression for the cash flow occurring at the end of year n and $A_n =$ the actual-dollar expression for the cash flow occurring at the end of year n. For example, as shown in Figure 4.3, $1,260 received in year 3 is only worth $1,000 in terms of value in year 0. Or $1,000 expressed in terms of purchasing power in year 0 is equivalent to $1,260 in terms of purchasing power in year 3.

[4] Since we obtained this average rate on the basis of costs that are specific to the baseball industry, this rate is not the general inflation rate. It is a specific inflation rate for the Boston Red Sox.

[5] Based on the ANSI Z94 Standard Committee on Industrial Engineering Terminology, *The Engineering Economist,* 33(2), 1988, pp. 145–171.

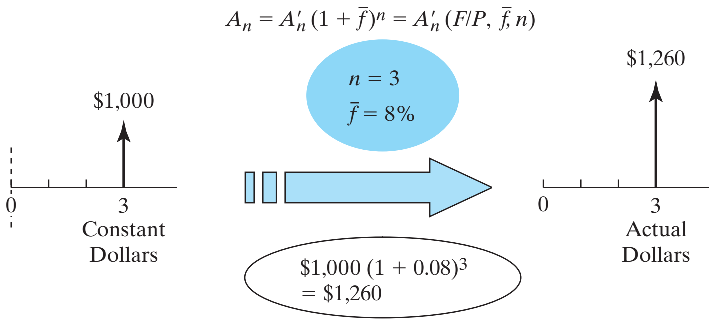


Figure 4.3 Conversion from constant to actual dollars.

If the future price of a specific cost element (j) is not expected to follow the general inflation rate, we will need to use the appropriate average inflation rate applicable to this cost element, f_j instead of $\bar{f}$.

EXAMPLE 4.3 Conversion from Constant to Actual Dollars

The average starting salaries for mechanical engineers were $51,340 in year 2000 and $62,527 in year 2016, respectively. Can we say that the engineers' salaries kept pace with the general inflation over the last 16 years? The CPIs for the two specific years are 171.20 and 241.04, respectively.

DISSECTING THE PROBLEM

Given: $51,340 in 2000, $\text{CPI}_{2000} = 171.20$, and $\text{CPI}_{2016} = 241.04$.
Find: The actual dollar amount in 2016.

METHODOLOGY

Compute the equivalent actual dollars and then compare them with the actual salaries in year 2016.

SOLUTION

By Eq. (4.2), the general inflation rate is calculated as

$$\bar{f} = \left[\frac{\text{CPI}_{2016}}{\text{CPI}_{2000}}\right]^{1/16} - 1 = \left[\frac{241.04}{171.2}\right]^{1/16} - 1 = 2.1613\%,$$

and the equivalent actual dollars in 2016 is

$$\$51{,}340(1 + 0.021613)^{16} = \$72{,}283.$$

Since $62,527 < $72,283, we can say that the inflation outpaced the engineers' salaries over the last 16 years.

4.2.2 Conversion from Actual to Constant Dollars

As shown in Figure 4.4, this process is the reverse of converting from constant to actual dollars. Instead of using the compounding formula, we use a discounting formula (single-payment present-worth factor):

$$A'_n = \frac{A_n}{(1 + \bar{f})^n} = A_n(P/F, \bar{f}, n). \tag{4.5}$$

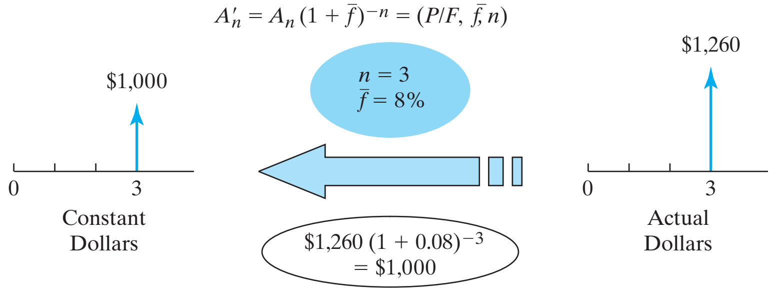


Figure 4.4 Conversion from actual to constant dollars: $1,260 three years from now will have a purchasing power of $1,000 in terms of base dollars (year 0).

Once again, we may substitute f_j for $\bar{f}$ if future prices are not expected to follow the general inflation rate.

The constant dollar is often used by companies to compare the performance of recent years with past performance. Governments also use the constant dollar to track changes in economic indicators, such as wages, over time. Any kind of financial data that is represented in dollar terms can be converted into constant dollars based on the CPIs of various years. As an example, we may track U.S. gasoline prices between 1960 and 2017. Since the actual prices and the consumer price indices are available over this period, we can develop a chart in which the gasoline prices are expressed in both actual dollars and constant dollars (expressed in terms of 2017 dollars) from 1960 to 2017. Even though consumers experienced a sharp price increase in 2010, the price, in fact, closely matches the 1981 price, which rose sharply due to an oil embargo imposed by some Middle Eastern countries.

In Examples 4.4 and 4.5, we will illustrate how we may convert actual dollars into constant dollars of the base year or vice versa.

EXAMPLE 4.4 Comparing Prize Monies Earned at Different Points in Time

The next table lists the winners and their prize monies in actual dollars from the Masters Golf Championship from 2012–2017 along with the CPIs. Convert the prize monies into equivalent dollars of 2017.

Prize Money of the Masters Golf Championship

Year	Winner	Prize Money (in Actual Dollars)	CPI
2012	Bubba Watson	$1,440,000	229.1
2013	Adam Scott	$1,440,000	233.6
2014	Bubba Watson	$1,620,000	238.3
2015	Jordan Spieth	$1,800,000	238.7
2016	Danny Willett	$1,800,000	240.6
2017	Sergio Garcia	$1,980,000	244.9

In doing so,

(a) Determine the growth rate of the prize money in actual dollars over the five-year period.
(b) Compute the general inflation rate each year based on CPI.
(c) Find the equivalent prize money for each, stated in terms of year 2017 dollars.
(d) Determine the growth rate of the prize money in constant (2017) dollars.
(e) If the current trend continues, what would the expected prize money be in actual dollars for the winner in 2020?

DISSECTING THE PROBLEM

You need to find out the specific general inflation rate for each past year to determine the loss of purchasing power. This requires the CPI information from 2012 to 2017.

Given: Prize history.

Find: Growth rate of the prize monies in actual as well as constant dollars and projection of the prize money for year 2020.

METHODOLOGY

Calculate the growth rate of prize monies in actual and constant dollars and project the 2020 prize money value.

SOLUTION

(a) Growth of the prize money in actual dollars:

$$\$1{,}980{,}000 = \$1{,}440{,}000(1 + f)^5$$
$$f = 6.58\%.$$

(b) General inflation rate:

$$f_{2017} = (244.9 - 240.6)/240.6 = 0.0179$$
$$f_{2016} = (240.6 - 238.7)/238.8 = 0.0080$$

(c) Prize money, stated in terms of 2017 dollars. In doing so, you first need to calculate the general inflation rate for each year which is shown in the table:

- Danny Willet (2016):

$$\$1{,}800{,}000(1 + 0.0179) = \$1{,}832{,}170.$$

- Jordan Spieth (2015):

$$\$1{,}800{,}000(1 + 0.0080)(1 + 0.0179) = \$1{,}846{,}753.$$

Other Masters Champions' prize monies can be converted into the base year dollars of 2017 as shown in the table below.

(d) Real growth of the prize money:

$$\$1{,}980{,}000 = \$1{,}539{,}310(1 + g)^5$$
$$g = 5.16\%.$$

(e) Anticipated 2020 prize money in actual dollars:

$$F_{2020} = \$1{,}980{,}000(1 + 0.0658)^3$$
$$= \$2{,}397{,}134.$$

Year	Winner	Prize Money (in Actual Dollars)	CPI	General Inflation Rate	Equivalent Prize Money (in 2017 Dollars)
2012	Bubba Watson	\$1,440,000	229.1		\$1,539,310
2013	Adam Scott	\$1,440,000	233.6	1.96%	\$1,509,658
2014	Bubba Watson	\$1,620,000	238.3	2.01%	\$1,664,868
2015	Jordan Spieth	\$1,800,000	238.7	0.17%	\$1,846,753
2016	Danny Willett	\$1,800,000	240.6	0.80%	\$1,832,170
2017	Sergio Garcia	\$1,980,000	244.9	1.79%	\$1,980,000

COMMENTS: The annual growth rate of the prize monies in actual dollars between 2012 and 2017 is just 6.58%. However, if you look at the entire history of the prize monies over 83 years and know that the prize money for the first winner in 1934 was \$1,500, the growth rate is 9.04%.

EXAMPLE 4.5 How Much Does It Cost to Go to College?

Tuition and fees for colleges and universities across the U.S. have consistently risen higher per year than inflation over the same period of time. The following table summarizes tuition and fees for four-year public institutions for the last 15 years. Cost is the value in actual dollars. With 2015 as the base year, adjust the college cost in actual dollars to the college cost in constant dollars of 2015 in each year.

DISSECTING THE PROBLEM

The first task is to determine the general inflation $(\bar{f})$ between each period. Then we convert the actual dollars in each period into the respective constant 2015\$ using this periodic general inflation rate.

Given: College costs in actual dollars.
Find: Equivalent constant dollars in year 2015.

	in Actual Dollars		Consumer Price Index	
Academic Year	Four-Year Public	One-Year % Change	Base Year (1982–84) = 100	One-Year % Change ($\bar{f}_j$)
00–01	\$3,508		172.20	
01–02	\$3,766	7.35%	177.10	2.85%
02–03	\$4,098	8.82%	179.90	1.58%
03–04	\$4,645	13.35%	184.00	2.28%
04–05	\$5,126	10.36%	188.90	2.66%
05–06	\$5,492	7.14%	195.30	3.39%
06–07	\$5,804	5.68%	201.50	3.17%
07–08	\$6,191	6.67%	206.70	2.58%
08–09	\$6,599	6.59%	214.80	3.92%
09–10	\$7,073	7.18%	213.20	−0.74%
10–11	\$7,629	7.86%	218.00	2.25%
11–12	\$8,276	8.48%	225.90	3.62%
12–13	\$8,646	4.47%	229.10	1.42%
13–14	\$8,885	2.76%	233.60	1.96%
14–15	\$9,145	2.93%	238.30	2.01%
15–16	\$9,410	2.90%	238.70	0.17%

METHODOLOGY

Calculate the equivalent college cost in constant 2015\$ for each year.

Since the base period is set at year 2015, we are calculating the constant dollars of 2015. To do so, we are calculating the future worth of the tuition and fees in each year at the end of year 2015. We know the general inflation rate each year, it is equivalent to using multiple interest rates to find the future worth.

SOLUTION

Using Eq. (4.5), we determine the equivalent college tuition and fees in constant 2015 dollars as follows:

Average Tuition and Fees for Four-Year Public Institutions				
		CPI		
Academic Year	Actual Dollars	General Inflation Rate	Equivalent Compounding Factor	Constant 2015 Dollars
00–01	\$3,508		1.3839	\$4,855
01–02	\$3,766	2.85%	1.3839	\$5,212
02–03	\$4,098	1.58%	1.3456	\$5,514
03–04	\$4,645	2.28%	1.3246	\$6,153
04–05	\$5,126	2.66%	1.2951	\$6,639
05–06	\$5,492	3.39%	1.2615	\$6,928
06–07	\$5,804	3.17%	1.2202	\$7,082
07–08	\$6,191	2.58%	1.1826	\$7,322

08–09	\$6,599	3.92%	1.1529	\$7,608
09–10	\$7,073	−0.74%	1.1094	\$7,847
10–11	\$7,629	2.25%	1.1177	\$8,527
11–12	\$8,276	3.62%	1.0931	\$9,047
12–13	\$8,646	1.42%	1.0549	\$9,121
13–14	\$8,885	1.96%	1.0402	\$9,242
14–15	\$9,145	2.01%	1.0201	\$9,329
15–16	\$9,410	0.17%	1.0000	\$9,410

For example, public four-year colleges charged, on average, \$3,508 in tuition and fees in actual dollars for the 2000–2001 year. To determine its equivalent constant dollars in the year 2015, we need to compound the actual dollars using the multiple inflation rates calculated in each period, as follows:

$$\begin{aligned} A'_{2015} &= \$3{,}505(1+0.0285)(1+0.0158)(1+0.0228)\cdots(1+0.0017) \\ &= \$3{,}505(1.3839) \\ &= \$4{,}855 \end{aligned}$$

Here we call the aggregated value of "1.3839" the equivalent compounding factor.
The average general infaltion rate over a 15-year period is

$$\begin{aligned} \bar{f} &= \left[\frac{\text{CPI}_{2015}}{\text{CPI}_{2000}}\right]^{1/15} - 1 = \left[\frac{238.70}{172.20}\right]^{1/15} - 1 \\ &= 2.20\% \end{aligned}$$

The average price index (inflation) for public 4-year colleges over the same period is

$$f = \left[\frac{9{,}410}{3{,}508}\right]^{1/15} - 1 = 6.80\%.$$

This indicates that the average price of public college tuition and fees has been increasing at a much faster rate than the general inflation in the economy.

COMMENTS: In determining the equivalent college cost in constant dollars, we used the periodic annual general inflation rates, instead of the average general inflation rate of 2.20%. This is possible only because we know precisely the actual CPIs in each year. However, if our task is to estimate the future college costs in, say, 2025, then we need to use the average inflation rate specific to the college cost, which is 6.8%. For example, the projected public four-year college tuition and fees in year 2025 is $\$9{,}410(1 + 0.068)^{10} = \$18{,}168$ in actual dollars. If we want to use the price of 2000 as a base (\$3,508), the projected price in year 2025 is $\$3{,}508(1 + 0.068)^{25} = \$18{,}154$, (rounding error) which is the same value we obtained previously.

4.3 Equivalence Calculations under Inflation

In previous chapters, our equivalence analyses took changes in the **earning power** of money into consideration. To factor in changes in **purchasing power**—that is, inflation—we may use either (1) constant-dollar analysis or (2) actual-dollar analysis. Either method produces the same solution; however, each method requires the use of a different interest rate and procedure. Before presenting the two procedures for integrating earning and purchasing power, we will give a precise definition of the two interest rates used in them.

4.3.1 Market and Inflation-Free Interest Rates

Two types of interest rates are used in equivalence calculations: (1) the market interest rate and (2) the inflation-free interest rate. The difference between the two is analogous to the relationship between actual and constant dollars.

- **Market interest rate (i):** This rate, commonly known as the **nominal interest rate**, takes into account the combined effects of the earning value of capital (earning power) and any anticipated inflation or deflation (purchasing power). Virtually all interest rates stated by financial institutions for loans and savings accounts are market interest rates. Most firms use a market interest rate (also known as **inflation-adjusted required rate of return**) in evaluating their investment projects, as well. In fact, all interest rates mentioned in Chapters 2 and 3 are market interest rates.
- **Inflation-free interest rate (i'):** This rate is an estimate of the true earning power of money when the effects of inflation have been removed. Commonly known as the **real interest rate**, it can be computed if the market interest rate and the inflation rate are known. As you will see later in this chapter, in the absence of inflation, the market interest rate is the same as the inflation-free interest rate.

In calculating any economic equivalence, we need to identify the nature of the cash flows. The three common cases are as follows:

Case 1: All cash flow elements are estimated in constant dollars.

Case 2: All cash flow elements are estimated in actual dollars.

Case 3: Some of the cash flow elements are estimated in constant dollars, and others are estimated in actual dollars.

For Case 3, we simply convert all cash flow elements into one type—either constant or actual dollars. Then we proceed with either constant-dollar analysis, as for Case 1, or actual-dollar analysis, as for Case 2. Constant-dollar analysis is common in the evaluation of many long-term public projects because governments do not pay income taxes. Typically, income taxes are levied on the basis of taxable incomes in actual dollars, so actual-dollar analysis is more common in the private sector.

4.3.2 Constant-Dollar Analysis

Suppose that all cash flow elements are already given in constant dollars and that we want to compute the equivalent present worth of the constant dollars in year n (A'_n). In the absence of any inflationary effect, we should use i' to account for only the earning power of the money. To find the present-worth equivalent of this constant-dollar amount at i', we use

$$P_n = \frac{A'_n}{(1 + i')^n}. \tag{4.6}$$

4.3.3 Actual-Dollar Analysis

Now let us assume that all cash flow elements are estimated in actual dollars. To find the equivalent present worth of this actual-dollar amount in year n (A_n), we may use two steps to convert actual dollars into equivalent present-worth dollars. First, we convert actual dollars into equivalent constant dollars by discounting with the general inflation rate, a step that removes the inflationary effect. Now we can use i' to find the equivalent present worth. However, the two-step process can be greatly streamlined by the efficiency of the **market interest rate**, which performs deflation and discounting in one step. Mathematically, the two steps can be expressed as

$$
\begin{aligned}
P_n &= \underbrace{\overbrace{A_n(1+\bar{f})^{-n}}^{\text{Convert to constant \$}}(1+i')^{-n}}_{\text{Discount the constant \$}} \\
&= A_n[(1+\bar{f})(1+i')]^{-n} \qquad (4.7)\\
&= A_n\Big(1+\underbrace{i'+\bar{f}+i'\bar{f}}_{\text{market interest rate}}\Big)^{-n} \\
&= A_n(1+i)^{-n}.
\end{aligned}
$$

This equation leads to the following relationship among $\bar{f}$, i', and i:

$$i = i' + \bar{f} + i'\bar{f}. \qquad (4.8)$$

This equation also implies that the market interest rate is a function of two terms, i' and $\bar{f}$.

Note that, without an inflationary effect, the two interest rates are the same. (If $\bar{f} = 0$, then $i = i'$.) As either i' or $\bar{f}$ increases, i also increases. When prices increase due to inflation, bond rates climb because promises of future payments from debtors are worth relatively less to lenders (i.e., banks, bondholders, money market investors, CD holders, etc.). Thus, lenders demand and set higher interest rates. Similarly, if inflation were to remain at 3%, we might be satisfied with an interest rate of 7% on a bond because our return would more than beat inflation. If inflation were running at 10%, however, we would not buy a 7% bond; we might insist instead on a return of at least 14%. On the other hand, when prices are coming down or at least are stable, lenders do not fear the loss of purchasing power with the loans they make, so they are satisfied to lend at lower interest rates.

In practice, we often approximate the market interest rate i by simply adding the inflation rate $\bar{f}$ to the real interest rate i' and ignoring the product term $(i'\bar{f})$. *This practice is okay as long as either i' or $\bar{f}$ is relatively small.* With continuous compounding, however, the relationship among i, i', and $\bar{f}$ becomes precisely

$$i' = i - \bar{f}. \qquad (4.9)$$

So, if we assume an annual percentage rate (APR) (market interest rate) of 6% per year compounded continuously and an inflation rate of 4% per year compounded continuously, the inflation-free interest rate is exactly 2% per year compounded continuously.

EXAMPLE 4.6 Equivalence Calculations under Inflation—Constant- and Actual-Dollar Analysis

Chena Hot Springs Resort built the very first ice museum in the U.S. Chena is located 60 miles (100 km) northeast of Fairbanks, Alaska, which is the traditional world capital of ice art. To save the cooling cost for the ice museum in 2017, Chena installed an absorption chiller, which requires 258 HP (horsepower) to operate. This saves \$244,560 in fuel costs per year at the 2017 diesel fuel price of \$2.35 per gallon. The expected life of the plant is 15 years, and the diesel price is expected to increase at an annual rate of 5% during the life of operation of the absorption chiller. If Chena's market interest rate is 12%, which would account for the expected general inflation rate of 3% during this project period, what is the value of installing the absorption chiller in 2017 dollars?

DISSECTING THE PROBLEM

In solving this problem, we demonstrate that we will obtain the same result whether we use the constant-dollar analysis or the actual dollar analysis.

Given: Cash flows stated in constant dollars, $i = 12\%$ per year.

Find: Equivalent present worth of the cash flow series.

METHODOLOGY

Method 1: Constant-Dollar Analysis

SOLUTION

Since the diesel fuel price increases at a different rate (5%) from the general inflation rate (3%), we first need to find the fuel savings in actual dollars at each period. For example, equivalent fuel savings in actual dollars are determined by

$$A_1 = \$244{,}560(1 + 0.05) = \$\$256{,}788$$
$$A_2 = \$244{,}560(1 + 0.05)^2 = \$269{,}627$$
$$\vdots$$
$$A_{15} = \$244{,}560(1 + 0.05)^{15} = \$508{,}423.$$

Then we convert these fuel savings in actual dollars into equivalent constant dollars.

$$A'_1 = \$256{,}788(1 + 0.03)^{-1} = \$249{,}309$$
$$A'_2 = \$269{,}627(1 + 0.03)^{-2} = \$254{,}150$$
$$\vdots$$
$$A'_{15} = \$508{,}423(1 + 0.03)^{-15} = \$326{,}337.$$

Now we have the fuel savings expressed in 2017 constant dollars, we can adopt the constant dollar analysis. To do so, we first need to find the inflation-free interest rate. Using Eq. (4.8), we find i' as follows:

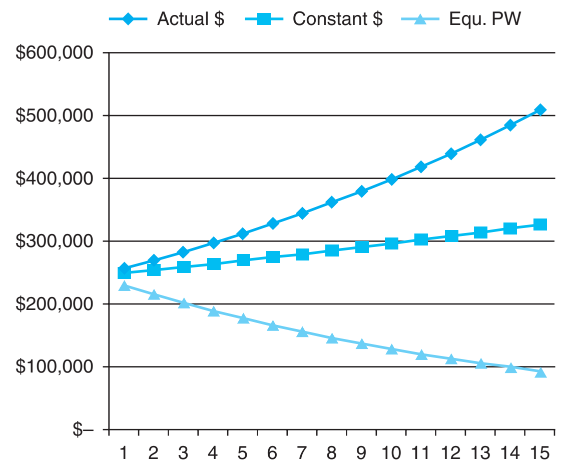
$$0.12 = i' + 0.03 + 0.03i'$$
$$i' = 8.74\%.$$

METHODOLOGY	SOLUTION
The fuel savings increases at the rate of 5% per year but the general inflation rate is 3%, the net annual growth factor for fuel savings in 2017 dollars is determined by $$g = \frac{1 + 0.05}{1 + 0.03} - 1$$	Then, the total present equivalent fuel savings in 2017 can be calculated using the geometric series present-worth factor with g calculated as $$g = 1.94\%,$$ $$\begin{aligned} P &= \$249{,}309\,(P/A_1, 1.94\%, 8.74\%, 15) \\ &= \$2{,}275{,}096. \end{aligned}$$ Table 4.2 illustrates the process of computing the equivalent present worth when all cash flows are given in actual dollars. Once the fuel savings figures in actual dollars are entered in cells B12 through B27, we need to convert them into equivalent constant dollars as shown in cells C12 through C27. Once the cash flow series is expressed in constant dollars, we use the inflation-free (real) interest rate to find the equivalent present worth. This is shown in column D, or cells D12 through D27. We can also easily plot the expected fuel savings in terms of both constant and actual dollars.
Method 2: Actual-Dollar Analysis Since the market interest rate reflects the effect of inflation, there is no adjustment in the growth of fuel savings.	Since fuel savings are given in actual dollars of 2017, we need to convert them into equivalent present worth using the market interest rate. Since the fuel savings are growing at the annual rate of 5%, the fuel savings series in actual dollars forms a geometric gradient series. To find the equivalent total savings in 2017 dollars, we use the market interest rate of 12% to discount this geometric series: $$\begin{aligned} P &= \$256{,}788(P/A_1, 5\%, 12\%, 15) \\ &= \$2{,}275{,}096 \text{ (in year 2017 dollars)}. \end{aligned}$$ Note that the result is essentially the same as before. (We might observe some slight discrepancy due to rounding errors introduced when we apply the different interest factors.) Table 4.3 illustrates the tabular approach to obtaining the equivalent present worth in Excel. We first enter the fuel savings in actual dollars in cells B12 through B27. We could calculate the present worth of the cash flow series in actual dollars, using the market interest rate. This is shown in cells C12 through C27. We may also display the cumulative present worth (PW) of fuel savings as a function of operating year as shown in cells D12 through D27. So, the figure in cell D27 represents the total fuel savings in present worth over 15 years.

TABLE 4.2 An Excel Worksheet to Perform a Constant-Dollar Analysis

	A	B	C	D
1				
2	Input			
3				
4	(*i*) Market interest rate (%)			12%
5	(*f*) General inflation rate (%)			3%
6	(*fj*) Inflation rate for diesel fuel (%)			5%
7	(*A*1) Fuel savings in year 1 (constant $)			$ 244,560
8				
9		Fuel savings	Fuel savings	Present
10	Period	in Actual $	in Constant $	worth
11				
12	0			
13	1	$ 256,788	$ 249,309	$ 229,275
14	2	$ 269,627	$ 254,150	$ 214,945
15	3	$ 283,109	$ 259,085	$ 201,511
16	4	$ 297,264	$ 264,115	$ 188,917
17	5	$ 312,127	$ 269,244	$ 177,109
18	6	$ 327,734	$ 274,472	$ 166,040
19	7	$ 344,120	$ 279,801	$ 155,663
20	8	$ 361,327	$ 285,234	$ 145,934
21	9	$ 379,393	$ 290,773	$ 136,813
22	10	$ 398,362	$ 296,419	$ 128,262
23	11	$ 418,281	$ 302,175	$ 120,246
24	12	$ 439,195	$ 308,042	$ 112,730
25	13	$ 461,154	$ 314,024	$ 105,685
26	14	$ 484,212	$ 320,121	$ 99,079
27	15	$ 508,423	$ 326,337	$ 92,887
28				

	F–K	L
2	Output	
4	(*I*′) Inflation-free interest rate (%)	8.738%
5	(*P*) Present worth ($)	$ 2,275,096



=$D$7*(1+$D$6)^A27

=B27*(1+$D$5)^(-A27)

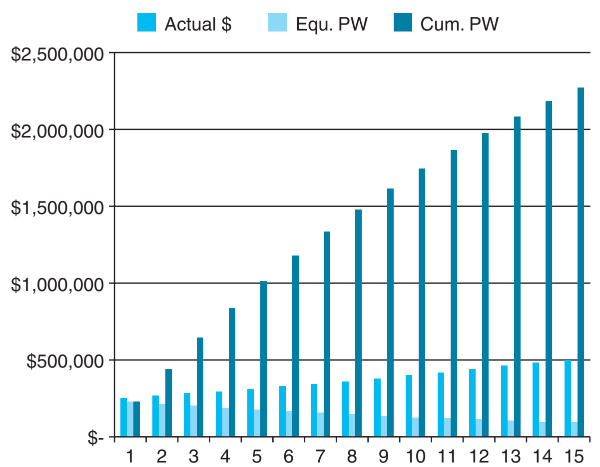
=C27*(1+$L$4)^(-A27)

TABLE 4.3 An Excel Worksheet to Perform an Actual-Dollar Analysis

	A	B	C	D
1				
2	Input			
3				
4	(*i*) Market interest rate (%)			12%
5	(*f*) General inflation rate (%)			3%
6	(*fj*) Inflation rate for diesel fuel (%)			5%
7	(*A*1) Fuel savings in year 1 (constant $)			$ 244,560
8				
9		Fuel savings	Equivalent	Cum.
10	Period	in Actual $	Present Worth	PW
11				
12	0			
13	1	$ 256,788	$ 229,275	$ 229,275
14	2	$ 269,627	$ 214,945	$ 444,220
15	3	$ 283,109	$ 201,511	$ 645,732
16	4	$ 297,264	$ 188,917	$ 834,648
17	5	$ 312,127	$ 177,109	$ 1,011,758
18	6	$ 327,734	$ 166,040	$ 1,177,798
19	7	$ 344,120	$ 155,663	$ 1,333,461
20	8	$ 361,327	$ 145,934	$ 1,479,394
21	9	$ 379,393	$ 136,813	$ 1,616,207
22	10	$ 398,362	$ 128,262	$ 1,744,469
23	11	$ 418,281	$ 120,246	$ 1,846,715
24	12	$ 439,195	$ 112,730	$ 1,977,445
25	13	$ 461,154	$ 105,685	$ 2,083,130
26	14	$ 484,212	$ 99,079	$ 2,182,209
27	15	$ 508,423	$ 92,887	$ 2,275,096
28				

	F–K	L
2	Output	
4	(*I*′) Inflation-free interest rate (%)	8.738%
5	(*P*) Present worth ($)	$ 2,275,096

=(D4-D5)/(1+D5)



=$D$7*(1+$D$6)^A27

=B27*(1+$D$4)^(-A27)

=D26+C27

4.3.4 Mixed-Dollar Analysis

Let's examine a situation in which some cash flow elements are expressed in constant (or today's) dollars and other elements in actual dollars. In this situation, we convert all cash flow elements into the same dollar units (either constant or actual). If the cash flow elements are all converted into actual dollars, we can use the market interest rate i in calculating the equivalence value. If the cash flow elements are all converted into constant dollars, we use the inflation-free interest rate i'. Example 4.7 illustrates this situation.

EXAMPLE 4.7 Equivalence Calculations with Composite Cash Flow Elements

A couple wishes to establish a college fund at a bank for their five-year-old child. The college fund will earn an 8% interest compounded quarterly. Assuming that the child enters college at age 18, the couple estimates that an amount of $30,000 per year, in terms of today's dollars (dollars at child's age of five), will be required to support the child's college expenses for four years. College expenses are estimated to increase at an annual rate of 6%. Determine the equal quarterly deposits the couple must make until they send their child to college. Assume that the first deposit will be made at the end of the first quarter and that deposits will continue until the child reaches age 17. The child will enter college at age 18, and the annual college expense will be paid at the beginning of each college year. In other words, the first withdrawal will be made when the child is 18.

DISSECTING THE PROBLEM

In this problem, future college expenses are expressed in terms of today's dollars whereas the quarterly deposits are in actual dollars. Since the interest rate quoted for the college fund is a market interest rate, we may convert the future college expenses into actual dollars.

Given: A college savings plan, $i = 2\%$ per quarter, $f = 6\%$, and $N = 12$ years.

Find: Amount of quarterly deposit in actual dollars.

Equivalence Calculation with Composite Cash Flow Elements

Age	College Expenses (in Today's Dollars)	College Expenses (in Actual Dollars)
18 (freshman)	$30,000	$\$30{,}000(F/P, 6\%, 13) = \$63{,}988$
19 (sophomore)	$30,000	$\$30{,}000(F/P, 6\%, 14) = \$67{,}827$
20 (junior)	$30,000	$\$30{,}000(F/P, 6\%, 15) = \$71{,}897$
21 (senior)	$30,000	$\$30{,}000(F/P, 6\%, 16) = \$76{,}211$

METHODOLOGY

Convert any *cash flow elements* in *constant dollars* into *actual dollars.* Then use the market interest rate to find the equivalent present value.

SOLUTION

The college expenses as well as the quarterly deposit series in actual dollars are shown in Figure 4.5. We first select $n = 12$, or age 17, as the base period for our equivalence calculation. (Note: Inflation is compounded annually; thus, the n we use here differs from the quarterly n we use next.) Then we calculate the accumulated total amount at the base period at 2% interest per quarter (8% APR/4 = 2% per quarter).

Since the deposit period is 12 years and the first deposit is made at the end of the first quarter, we have a 48-quarter deposit period. Therefore, the total balance of the deposits when the child is 17 would be

$$\begin{aligned} V_1 &= C(F/A, 2\%, 48) \\ &= 79.3535C. \end{aligned}$$

The equivalent lump-sum worth of the total college expenditure at the base period (at age of 17) would be

$$\begin{aligned} V_2 = {} & \$63{,}988(P/F, 2\%, 4) + \$67{,}827(P/F, 2\%, 8) \\ & + \$71{,}897(P/F, 2\%, 12) + \$76{,}211(P/F, 2\%, 16) \\ = {} & \$229{,}211. \end{aligned}$$

By setting $V_1 = V_2$ and solving for C, we obtain $C = \$2{,}888.48$.

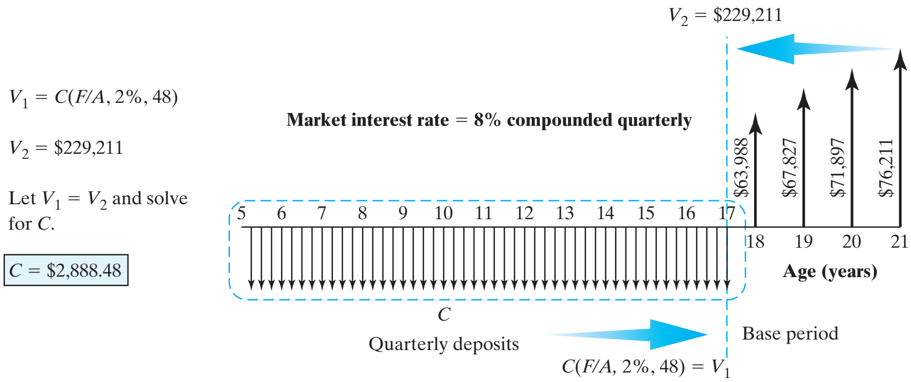


Figure 4.5 Establishing a college fund under an inflationary economy for a five-year-old child by making 48 quarterly deposits.

COMMENTS: This is a good example for which Excel would help us understand the effects of variations in the circumstances of the situation. For example, how does changing the inflation rate affect the required quarterly savings plan? To begin, we must set up an Excel spreadsheet and utilize the Goal Seek function. Table 4.4 is a sample spreadsheet that shows the deposit and withdrawal schedule for this scenario. After specifying the interest rate in E6 and using Excel functions to calculate equivalent total deposits and withdrawals in present-worth terms, we designate cell E58 as the difference between E56 and E57 (i.e., E56−E57). To ensure that the accumulated balance of deposits is exactly sufficient to meet projected withdrawals, we specify that this target cell be zero (i.e., E56−E57 = 0) and command Goal Seek to adjust the quarterly deposit (E55) accordingly. Note that the quarterly deposit is linked to the schedule of deposits in column B. The Goal Seek function finds the required quarterly deposit amount to be \$2,888.47, in Cell E55.

Sensitivity Analysis: Using the spreadsheet displayed in Table 4.4, we can then adjust the interest rate and see the change in quarterly deposits required. If we adjust annual inflation from 6% to 4%, we will find that the required quarterly deposit amount is \$2,192.96, which is \$695.51 less than in the 6% case. The result is shown in Table 4.5.

TABLE 4.4 Excel Solution for Finding the Required Quarterly Deposits (Example 4.7)

	A	B	C	D	E
1					
2	Quarter	Deposits	Withdrawals	Intput	
3					
4	0			College Expense in Constant $	$30,000.00
5	1	($2,888.47)		Inflation Rate	6.00%
6	2	($2,888.47)		Interest Rate per Quarter	2.00%
7	3	($2,888.47)			
8	4	($2,888.47)		College Expense in Actual $	
9	5	($2,888.47)		18 (Freshman)	$63,988
41	37	($2,888.47)		19 (Sophomore)	$67,827
42	38	($2,888.47)		20 (Junior)	$71,897
43	39	($2,888.47)		21 (Senior)	$76,211
44	40	($2,888.47)			
45	41	($2,888.47)			
46	42	($2,888.47)		=FV(E5,16,,-E4)	
47	43	($2,888.47)			
48	44	($2,888.47)			
49	45	($2,888.47)			
50	46	($2,888.47)		Base Period	
51	47	($2,888.47)			
52	48	($2,888.47)			
53	49			Output	
54	50				
55	51			Required Quarterly Deposit	$2,888.47
56	52		$63,988	Equ. Total Deposit	$229,210.02
57	53			Equ. Total Withdrawals	$229,210.02
58	54			Target cell	$0.00
59	55				
60	56		$67,827		
61	57			=FV($E$6,48,-1*E55)	
62	58				
63	59				
64	60		$71,897	=PV($E$6,4,,-C56)	
65	61			+PV($E$6,8,,-C60)	
66	62			+PV($E$6,12,,-C64)	
67	63			+PV($E$6,16,,-C68)	
68	64		$76,211		
69					
70	Note: Row 10 through Row 40 are hidden.				
71					

=E56-E57

TABLE 4.5 Required Quarterly Savings at Varying Interest Rates and Inflation Rates

Varying the annual inflation rate when the savings rate is fixed at 2% per quarter		Varying the quarterly savings rate when the inflation rate is fixed at 6% per year	
Annual Inflation Rate	Required Deposit Amount	Quarterly Savings Rate	Required Deposit Amount
2%	$1,657	1%	$4,131
4%	$2,193	1.5%	$3,459
6%	$2,888	2.0%	$2,888
8%	$3,787	2.5%	$2,404
10%	$4,942	3.0%	$1,995

SUMMARY

- The **Consumer Price Index (CPI)** is a statistical measure of change, over time, of the prices of goods and services in major expenditure groups—such as food, housing, apparel, transportation, and medical care—typically purchased by urban consumers. The CPI compares the cost of a sample "market basket" of goods and services in a specific period with the cost of the same market basket in an earlier reference period. This reference period is designated as the **base period**.
- **Inflation** is the term used to describe a **decline in purchasing power** evidenced in an economic environment of rising prices.
- **Deflation** is the opposite of inflation: It is an increase in purchasing power evidenced by falling prices.
- The **general inflation rate** $\bar{f}$ is an average inflation rate based on the CPI. An annual general inflation rate $\bar{f}_n$ can be calculated by the following equation:

$$\bar{f}_n = \frac{\text{CPI}_n - \text{CPI}_{n-1}}{\text{CPI}_{n-1}}.$$

- The price changes of specific, individual commodities do not always reflect the general inflation rate. We can calculate an **average inflation rate** $\bar{f}$ for a specific commodity (j) if we have an index (i.e., a record of historical costs) for that commodity.
- Project cash flows may be stated in one of two forms:
  1. **Actual dollars (A_n):** Dollar amounts that reflect the inflation or deflation rate.
  2. **Constant dollars (A'_n):** Dollar amounts that reflect the purchasing power of year zero dollars.
- Interest rates for the evaluation of cash flows may be stated in one of two forms:
  1. **Market interest rate (i):** A rate that combines the effects of interest and inflation; this rate is used with actual-dollar analysis. Unless otherwise mentioned, the interest rates used in the remainder of this text are the market interest rates.
  2. **Inflation-free interest rate (i'):** A rate from which the effects of inflation have been removed; this rate is used with constant-dollar analysis.
- To calculate the present worth of actual dollars, we can use either a two-step or a one-step process:
  - **Deflation method—two steps:**
    1. Convert actual dollars to constant dollars by deflating with the general inflation rate of $\bar{f}$.
    2. Calculate the present worth of constant dollars by discounting at i'.
  - **Adjusted-discount method—one step (use the market interest rate):**

$$P_n = \frac{A_n}{[(1+\bar{f})(1+i')]^n} = \frac{A_n}{(1+i)^n},$$

where

$$i = i' + \bar{f} + i'\bar{f}.$$

Alternatively, just use the market interest rate to find the net present worth.

TABLE 4.6 **Summary of Inflation Terminologies**

Category	Terms Used in This Text	Other Terms Used
Dollars	Actual dollars	Current dollars
	Constant dollars	Real dollars
Cash flow	Cash flow in actual dollars	Nominal cash flow
	Cash flow in constant dollars	Real cash flow
Interest rate	Market interest rate	Nominal interest rate
	Inflation-free interest rate	Real interest rate
Inflation rate	General inflation rate	Average rate of inflation, annual rate of inflation
	Average inflation rate	Escalation rate, growth rate

- In this chapter we have introduced many inflation-related terminologies. Since some of them are different from those commonly found in other publications, Table 4.6 summarizes other terms used in other publications or business.

SELF-TEST QUESTIONS

4s.1 How many years will it take for the dollar's purchasing power to be one-half of what it is now if the general inflation rate is expected to continue at the rate of 9% for an indefinite period?

(a) About 7 years
(b) About 8 years
(c) About 11 years
(d) About 12 years

4s.2 An engineer's salary was $50,000 in 2012. The same engineer's salary in 2018 is $75,000. If the company's salary policy dictates that a yearly raise in salaries should reflect the cost-of-living increase due to inflation, what is the average inflation rate for the period 2012–2018?

(a) 4.5%
(b) 5%
(c) 6.5%
(d) 7%

4s.3 Suppose that you borrow $20,000 at 9%, compounded monthly, over five years. Knowing that the 9% represents the market interest rate, the monthly payment in actual dollars will be $415.17. If the average monthly general inflation rate is expected to be 0.5%, what is the equivalent equal monthly payment series in constant dollars?

(a) $359
(b) $375
(c) $405
(d) $415

4s.4 A couple wants to save for their daughter's college expenses. The daughter will enter college eight years from now and will need \$40,000, \$41,000, \$42,000 and \$43,000 in *actual dollars* over four college years. Assume that these college payments will be made at the beginning of the school year. The future general inflation rate is estimated to be 6% per year and the annual inflation-free interest rate is 5%. What is the equal amount, in *actual dollars,* the couple must save each year until their daughter goes to college?
(a) \$11,838
(b) \$11,945
(c) \$12,142
(d) \$12,538

4s.5 You just signed a business consulting contract with one of your clients, who will pay you \$40,000 a year for seven years for the service you will provide over this period. You anticipate the general inflation rate over this period to be 6% annually. If your desired inflation-free interest rate is 4%, what is the worth of the seventh payment in today's dollars? The client will pay the consulting fee at the end of each year.
(a) \$25,856
(b) \$24,506
(c) \$20,216
(d) \$19,320

4s.6 A company is considering an investment with the following expected cash flows in *constant* dollars over three years. If the company's market interest is known to be 15% and the expected general inflation rate ($\bar{f}$) is 6% during this project period, determine the equivalent present worth of the project at period 0.

Year	0	1	2	3
Cash flow (\$)	−30,000	15,000	15,000	15,000

(a) \$ 4,248
(b) \$ 3,567
(c) \$10,095
(d) \$ 8,317

4s.7 A series of five constant-dollar (or real-dollar) payments, beginning with \$6,000 at the end of the first year is increasing at the rate of 5% per year. Assume that the average general inflation rate is 4% and the market interest rate is 11% during this inflationary period. What is the equivalent present worth of the series?
(a) \$24,259
(b) \$25,892
(c) \$27,211
(d) \$29,406

4s.8 "At a market interest rate of 7% per year and an inflation rate of 5% per year, a series of three equal annual receipts of \$100 in constant dollars is equivalent to a series of three annual receipts of \$108 in actual dollars." Which of the following statements is correct?

(a) The amount of actual dollars is overstated.
(b) The amount of actual dollars is understated.
(c) The amount of actual dollars is about right.
(d) Sufficient information is not available to make a comparison.

4s.9 A father wants to save in advance for his eight-year-old daughter's college expenses. The daughter will enter the college 10 years from now. An annual amount of $20,000 in today's dollars (constant dollars) will be required to support her college expenses for four years. Assume that these college payments will be made at the *beginning* of each school year. (The first payment occurs at the end of 10 years.) The future general inflation rate is estimated to be 5% per year, and the interest rate on the savings account will be 8% compounded quarterly (market interest rate) during this period. If the father has decided to save only $500 (actual dollars) each quarter, how much will the daughter have to borrow to cover her *freshman* expenses?
(a) $1,920
(b) $2,114
(c) $2,210
(d) $2,377

4s.10 The average unleaded gasoline price for California residents on June 20, 2017, was $3.35 per gallon. Assuming that the base period (price index = 100) is 1996 and that the unleaded gasoline price for that year was $1.50 per gallon, compute the average price index for the unleaded gasoline price for the year 2017.
(a) 223.33
(b) 304.77
(c) 315.33
(d) 324.62

4s.11 An engineer's salary was $55,000 in 2002. The same engineer's salary in 2017 is $99,500. The CPIs in 1996 and 2017 were 158.6 and 246.6 respectively. Which of the following statements is correct?
(a) The engineer's salary kept pace with the inflation.
(b) The engineer's salary did not keep pace with inflation.
(c) The general inflation rate for the period 1996–2017 is higher than 2% per year.
(d) The engineer could expect his salary to be higher than $108,000 by 2020.

PROBLEMS

Note: In these problems, the term "market interest rate" represents the inflation-adjusted interest rate for equivalence calculations or the APR quoted by a financial institution for commercial loans. Unless otherwise mentioned, all stated interest rates will be compounded annually.

Measure of Inflation

4.1 The current gasoline price is $4.50 per gallon, and it is projected to increase by 5% the next year, 7% the following year, and 8% the third year. What is the average inflation rate for the projected gasoline price for the next three years?

4.2 The following data indicate the price indexes of credit rating services (base period 1982 = 100) between 2013 and 2017:

Period	Price Index
2013	456.75
2014	467.60
2015	482.50
2016	498.64
2017	510.12
2022	?

(a) Assuming that the base period (price index = 100) is reset to the year 2013, compute the average (geometric) price index for credit rating services between 2013 and 2017.

(b) If the past trend is expected to continue, how would you estimate the credit rating services in 2022?

4.3 For prices that are increasing at an annual rate of 4% the first year and 7.5% the second year, determine the average inflation rate $(\bar{f})$ over the two years.

4.4 The following table shows a utility company's cost to supply a fixed amount of power to a new housing development; the indices are specific to the utility industry. Assume that year 0 is the base period. Determine the specific inflation for each period and calculate the average inflation rate over the three-year period.

Year	Cost
0	\$468,000
1	\$487,000
2	\$513,000
3	\$532,400

4.5 Because of general price inflation in our economy, the purchasing power of the dollar shrinks with the passage of time. If the average general inflation rate is expected to be 8% per year for the foreseeable future, how many years will it take for the dollar's purchasing power to be one-half of what it is now?

Actual versus Constant Dollars

4.6 The average starting salary for engineers was \$5,000 a year in 1955. Sohrab, a mechanical engineer, got an offer for \$78,500 a year in 2018. Knowing that the CPIs for 1955 and 2018 are 26.87 and 256.46, respectively, what is Sohrab's real salary in terms of constant 1955 dollars?

4.7 A company is considering buying a CNC machine. In today's dollars, it is estimated that the maintenance costs for the machine (paid at the end of each year) will be \$25,000, \$26,000, \$28,000, \$30,000, and \$32,000 for years 1 to 5, respectively. The general inflation rate $(\bar{f})$ is estimated to be 5% per year, and the company will receive 13% return (interest) per year on its invested funds during the inflationary period. The company wants to pay for maintenance expenses in equivalent equal payments (in actual dollars) at the end of each of the five years. Find the amount of the company's payments.

4.8 The following cash flows are in actual dollars:

n	Cash Flow (in Actual $)
0	$25,000
4	$35,000
5	$45,000
7	$55,000

Convert to an equivalent cash flow in constant dollars if the base year is time 0. Keep cash flows at the same point in time—that is, years 0, 4, 5, and 7. Assume that the market interest rate is 14% and that the general inflation rate $(\bar{f})$ is 5% per year.

4.9 The purchase of a car requires a $32,000 loan to be repaid in monthly installments for five years at 9% interest compounded monthly. If the general inflation rate is 0.5% compounded monthly, find the actual- and constant-dollar value of the 20th payment.

4.10 Consider the accompanying cash flow diagrams, where the equal-payment cash flow in constant dollars is converted (a) from the equal-payment cash flow in actual dollars, and (b) at an annual general inflation rate of $\bar{f} = 3.8\%$ and $i = 9\%$. What is the amount A in actual dollars equivalent to $A' = \$1{,}000$ in constant dollars? Assume base year is $n = 0$.

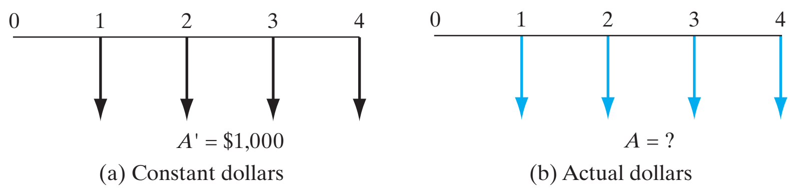


Figure P4.11

4.11 A 10-year $1,000 bond pays a nominal rate of 9% compounded semi-annually. If the market interest rate is 12% compounded annually and the general inflation rate is 6% per year, find the actual- and constant-dollar amounts (in time-0 dollars) of the 15th interest payment on the bond.

Equivalence Calculation under Inflation

4.12 If you deposit $8,000 in a bank account that pays a 3% interest compounded monthly for six years, what would be your economic loss if the general inflation rate is 4% during that period?

4.13 If you are looking for a 5% real return (inflation-free interest) on your investment, would you be interested in an investment opportunity that produces a 12% return on investment (market interest rate) if the inflation rate is 7%?

4.14 An annuity provides for 8 consecutive end-of-year payments of \$50,000. The average general inflation rate is estimated to be 4% annually, and the market interest rate is 10% annually. What is the annuity worth in terms of a single equivalent amount in today's dollars?

4.15 A series of four annual constant-dollar payments beginning with \$10,000 at the end of the first year is growing at the rate of 8% per year. Assume that the base year is the current year $(n = 0)$. If the market interest rate is 15% per year and the general inflation rate $(\bar{f})$ is 7% per year, find the present worth of this series of payments, based on

(a) Constant-dollar analysis

(b) Actual-dollar analysis

4.16 You will receive \$50 interest every six months from your investment in a corporate bond. The bond will mature in five years from now and has a face value of \$1,000. This means that if you hold the bond until its maturity, you will continue to receive \$50 interest semiannually and \$1,000 face value at the end of five years.

(a) What is the present value of the bond in the absence of inflation if the market interest rate is 8%?

(b) What would happen to the value of the bond if the inflation rate over the next five years is expected to be 3%?

4.17 You just signed a business consulting contract with one of your clients. The client will pay you \$50,000 a year for five years for the service you will provide over this period. You anticipate the general inflation rate over this period to be 6%. If your desired inflation-free interest rate (real interest rate) is to be 4%, what is the worth of the fifth payment in present dollars? The client will pay the consulting fee at the end of each year.

4.18 Suppose you borrow \$28,000 at 12% compounded monthly over four years. Knowing that the 12% represents the market interest rate, you realize that the monthly payment in actual dollars will be \$556.11. If the average monthly general inflation rate is expected to be 0.6%, determine the equivalent equal monthly payment series in constant dollars.

4.19 The annual fuel costs required to operate a small solid-waste treatment plant are projected to be \$2.2 million without considering any future inflation. The best estimates indicate that the annual inflation-free interest rate (i') will be 6% and the general inflation rate $(\bar{f})$ will be 5%. If the plant has a remaining useful life of five years, what is the present equivalent of its fuel costs? Use actual-dollar analysis.

4.20 Suppose that you just purchased a used car worth \$8,000 in today's dollars. Suppose also that you borrowed \$8,000 from a local bank at 9% compounded monthly over two years. The bank calculated your monthly payment at \$365.48. Assuming that average general inflation will run at 0.5% per month over the next two years,

(a) Determine the monthly inflation-free interest rate (i') for the bank.

(b) What equal monthly payments (in terms of constant dollars over the next two years) are equivalent to the series of actual payments to be made over the life of the loan?

4.21 A man is planning to retire in 20 years. Money can be deposited at 6% interest compounded monthly, and it is also estimated that the future general inflation ($\bar{f}$) rate will be 4% compounded annually. What amount of end-of month deposit must be made each month until the man retires so that he can make annual withdrawals of \$60,000 in terms of today's dollars over the 15 years following his retirement? (Assume that his first withdrawal occurs at the end of the first six months after his retirement.)

4.22 On her 25th birthday, a young woman engineer decides to start saving toward building up a retirement fund that pays 6% interest compounded monthly (the market interest rate). She feels that \$1,000,000 worth of purchasing power in today's dollars will be adequate to see her through her sunset years after her 65th birthday. Assume a general inflation rate of 4% per year.

(a) If she plans to save by making 480 equal monthly deposits, what should be the amount of her monthly deposit in actual dollars? Assume the first deposit is made at the end of first month.

(b) If she plans to save by making end-of-the-year deposits, increasing by \$1,000 over each subsequent year, how much would her first deposit be in actual dollars?

4.23 A couple wants to save for their daughter's college expense. The daughter will enter college eight years from now, and she will need \$40,000, \$41,000, \$42,000, and \$43,000 in *actual dollars* for four school years. Assume that these college payments will be made at the beginning of each school year. The future general inflation rate is estimated to be 6% per year, and the annual inflation-free interest rate is 5%.

(a) What is the market interest rate to use in the analysis?

(b) What is the equal amount, in *actual dollars,* the couple must save each year until their daughter goes to college?

4.24 A father wants to save for his 8-year-old son's college expenses. The son will enter college 10 years from now. An annual amount of \$40,000 in today's constant dollars will be required to support the son's college expenses for four years. Assume that these college payments will be made at the beginning of the school year. The future general inflation rate is estimated to be 6% per year, and the market interest rate on the savings account will average 8% compounded annually. Given this information,

(a) What is the amount of the son's freshman-year expense in terms of actual dollars?

(b) What is the equivalent single-sum amount at the present time for these college expenses?

(c) What is the equal amount, in actual dollars, the father must save each year until his son goes to college?

4.25 Consider the following project's after-tax cash flow and the expected annual general inflation rate during the project period.

End of Year	Expected Cash Flow (in Actual \$)	General Inflation Rate
0	−\$45,000	
1	\$32,000	3.5%
2	\$32,000	4.2
3	\$32,000	5.5

(a) Determine the average annual general inflation rate over the project period.

(b) Convert the cash flows in actual dollars into equivalent constant dollars with the base year 0.

(c) If the annual inflation-free interest rate is 5%, what is the present worth of the cash flow? Is this project acceptable?

Short Case Studies with Excel

4.26 Suppose you have three goals in your financial planning for saving money. First, you would like to be able to retire 25 years from now with a retirement income of $10,000 (today's dollar) per month for 20 years. Second, you would like to purchase a vacation home in Sedona in 10 years at an estimated cost of $500,000 (today's dollars). Third, assuming that you will live until your life expectancy, say 20 years after your retirement, you would like to leave a cash contribution to your college in the amount of $1,000,000 (actual dollars). You can afford to save $2,000 (actual dollars) per month for the next 10 years. Assume that the general inflation rate is 4% and the property value in Sedona increases at an annual rate of 5%. Your first retirement withdrawal will be made 25 years and 1 month from now. Before retirement, you would be able to invest your money at an annual rate of 10%. But after retirement, you will invest your assets in more conservative financial assets at an annual rate of 6%. What is the required savings in each month in years 11 through 25?

4.27 If we were to invest $10,000 in an S&P 500 index fund, pay 0.2% annual fees, and add $100 a month to it for 40 years, receiving 10.4% annual returns, what, theoretically, would happen to our investment with 3.43% annual inflation? What would it be worth in terms of today's dollars? That's 10.4% − 0.2% − 3.43% = 6.77% over 40 years.

4.28 You have $10,000 cash that you want to invest. Normally, you would deposit the money in a savings account that pays an annual interest rate of 6%. However, you are now considering the possibility of investing in a bond. Your alternatives are either a nontaxable municipal bond paying 9% or a taxable corporate bond paying 12%. Your marginal tax rate is 30% for both ordinary income and capital gains. (The marginal tax rate of 30% means that you will keep only 70% of your bond interest income.) You expect the general inflation rate to be 3% during the investment period. You can buy a high-grade municipal bond costing $10,000 that pays interest of 9% ($900) per year. This interest is not taxable. A comparable high-grade corporate bond for the same price is also available. This bond is just as safe as the municipal bond but pays an interest rate of 12% ($1,200) per year. The interest for this bond is taxable as ordinary income. Both bonds mature at the end of year 5.

(a) Determine the real (inflation-free) rate of return for each bond.

(b) Without knowing your earning-interest rate, what choice would you make between these two bonds?