

CHAPTER 7A

Resolution of Multiple Rates of Return

To comprehend the nature of multiple i^* 's, we need to understand the investment situation represented by any cash flow. The net-investment test will indicate whether the i^* computed represents the true rate of return earned on the money invested in a project while the money is actually in the project. As we shall see, the phenomenon of multiple i^* 's occurs only when the net-investment test fails. When multiple positive rates of return for a cash flow are found, in general, none is suitable as a measure of project profitability, and we must proceed to the next analysis step: introducing an external rate of return.

7A-1 Net-Investment Test

A project is said to be a **net investment** when the project balances computed at the project's i^* values, $PB(i^*)_n$, are all less than or equal to zero throughout the life of the investment with $A_0 < 0$. The investment is *net* in the sense that the firm does not overdraw on its return at any point and hence is *not indebted* to the project. This type of project is called a **pure investment**. [On the other hand, **pure borrowing** is defined as the situation where $PB(i^*)_n$ values are all positive or zero throughout the life of the loan, with $A_0 > 0$.] *Simple investments will always be pure investments.* Therefore, if a nonsimple project passes the net-investment test (i.e., it is a pure investment), then the accept-or-reject decision rule will be the same as in the simple-investment case given in Section 7.3.2.

If any of the project balances calculated at the project's i^* is positive, the project is not a pure investment. A positive project balance indicates that, at some time during the project life, the firm acts as a borrower [$PB(i^*)_n > 0$] rather than an investor [$PB(i^*)_n < 0$] in the project. This type of investment is called a **mixed investment**.

EXAMPLE 7A.1 Pure versus Mixed Investments

Consider the following four investment projects with known i^* values:

| <i>n</i> | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0 | -\$1,000 | -\$1,000 | -\$1,000 | -\$1,000 |
| 1 | -\$1,000 | \$1,600 | \$500 | \$3,900 |
| 2 | \$2,000 | -\$300 | -\$500 | -\$5,030 |
| 3 | \$1,500 | -\$200 | \$2,000 | \$2,145 |
| i^* | 33.64% | 21.95% | 29.95% | (10%, 30%, 50%) |

Determine which projects are pure investments.

SOLUTION

Given: Four projects with cash flows and i^* s as shown in the preceding table.

Find: Which projects are pure investments.

We will first compute the project balances at the projects' respective i^* s. If multiple rates of return exist, we may use the largest value of i^* greater than zero. (In fact, it does not matter which rate we use in applying the net-investment test. If one value passes the net-investment test, they will all pass. If one value fails, they will all fail.)

Project A:

$$PB(33.64\%)_0 = -\$1,000;$$

$$PB(33.64\%)_1 = -\$1,000(1 + 0.3364) + (-\$1,000) = -\$2,336.40;$$

$$PB(33.64\%)_2 = -\$2,336.40(1 + 0.3364) + \$2,000 = -\$1,122.36;$$

$$PB(33.64\%)_3 = -\$1,122.36(1 + 0.3364) + \$1,500 = 0.$$

(-, -, -, 0): Passes the net-investment test (pure investment).

Project B:

$$PB(21.95\%)_0 = -\$1,000;$$

$$PB(21.95\%)_1 = -\$1,000(1 + 0.2195) + \$1,600 = \$380.50;$$

$$PB(21.95\%)_2 = +\$380.50(1 + 0.2195) - \$300 = \$164.02;$$

$$PB(21.95\%)_3 = +\$164.02(1 + 0.2195) - \$200 = 0.$$

(-, +, +, 0): Fails the net-investment test (mixed investment).

Project C:

$$PB(29.95\%)_0 = -\$1,000;$$

$$PB(29.95\%)_1 = -\$1,000(1 + 0.2995) + \$500 = -\$799.50;$$

$$PB(29.95\%)_2 = -\$799.50(1 + 0.2995) - \$500 = -\$1,538.95;$$

$$PB(29.95\%)_3 = -\$1,538.95(1 + 0.2995) + \$2,000 = 0.$$

(-, -, -, 0): Passes the net-investment test (pure investment).

Project D:

There are three rates of return. We can use any of them for the net-investment test.

We use the third rate given, 50%, as follows:

$$PB(50\%)_0 = -\$1,000;$$

$$PB(50\%)_1 = -\$1,000(1 + 0.50) + \$3,900 = \$2400;$$

$$PB(50\%)_2 = +\$2,400(1 + 0.50) - \$5,030 = -\$1,430;$$

$$PB(50\%)_3 = -\$1,430(1 + 0.50) + \$2,145 = 0.$$

(-, +, -, 0): Fails the net-investment test (mixed investment).

COMMENTS: As shown in Figure 7A.1, Projects A and C are the only pure investments. Project B demonstrates that the existence of a unique i^* is a necessary, but not sufficient, condition for a pure investment.

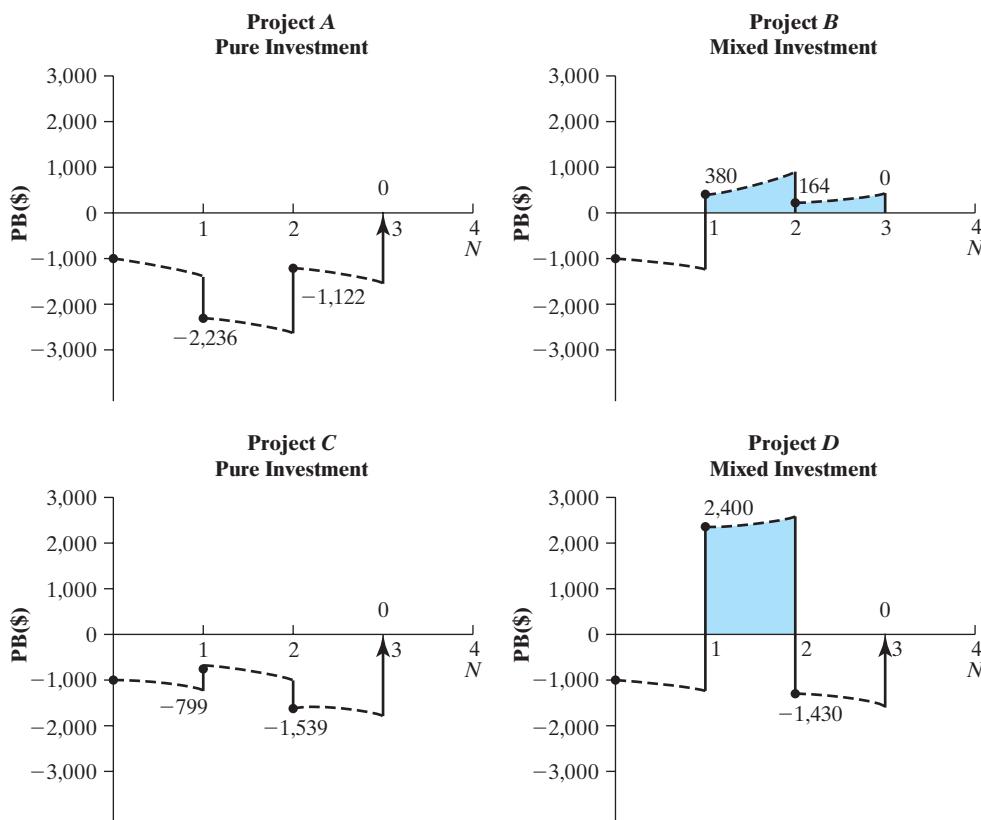


Figure 7A.1 Net-investment test.

7A-2 The Need for an External Interest Rate

Even for a nonsimple investment, in which there is only one positive rate of return, the project may fail the net-investment test, as demonstrated by Project B in Example 7A.1. In this case, the unique i^* still may not be a true indicator of the project's profitability. That is, when we calculate the project balance at an i^* for mixed investments, we notice an important point. Cash borrowed (released) from the project is assumed to earn the same interest rate through external investment as money that remains internally invested. In other words, in solving for a cash flow for an unknown interest rate, it is assumed that money released from a project can be reinvested to yield a rate of return equal to that received from the project. In fact, we have been making this assumption whether a cash flow produces a unique positive i^* or not. Note that money is borrowed from the project only when $PB(i^*) > 0$, and the magnitude of the borrowed amount is the project balance. When $PB(i^*) < 0$, no money is borrowed even though the cash flow may be positive at that time.

In reality, it is not always possible for cash borrowed (released) from a project to be reinvested to yield a rate of return equal to that received from the project. Instead, it is likely that the rate of return available on a capital investment in the business is much different—usually higher—from the rate of return available on other external investments. Thus, it may be necessary to compute project balances for a project's cash flow

at two different interest rates—one on the internal investment and one on the external investments. As we will see later, by separating the interest rates, we can measure the **true rate of return** of any internal portion of an investment project.

Because the net-investment test is the only way to accurately predict project borrowing (i.e., external investment), its significance now becomes clear. In order to calculate accurately a project's true IRR, we should always test a solution by the net-investment test and, if the test fails, take the further analytical step of introducing an external interest rate. Even the presence of a unique positive i^* is a necessary, but not sufficient, condition to predict net investment, so if we find a unique value, we should still subject the project to the net-investment test.

7A-3 Calculation of Return on Invested Capital for Mixed Investments

A failed net-investment test indicates a combination of internal and external investment. When this combination exists, we must calculate a rate of return on the portion of capital that remains invested internally. This rate is defined as the **true IRR** for the mixed investment and is commonly known as the **RIC**.

How do we determine the true IRR of a mixed investment? Insofar as a project is not a net investment, the money from one or more periods when the project has a net outflow of money (positive project balance) must later be returned to the project. This money can be put into the firm's investment pool until such time when it is needed in the project. The interest rate of this investment pool is the interest rate at which the money can, in fact, be invested outside the project.

Recall that the PW method assumed that the interest rate charged to any funds withdrawn from a firm's investment pool would be equal to the MARR. In this book, we use the MARR as an established external interest rate (i.e., the rate earned by money invested outside of the project). We can then compute the true IRR, or RIC, as a function of the MARR by finding the value of IRR that will make the terminal project balance equal to zero. (This definition implies that the firm wants to fully recover any investment made in the project and pays off any borrowed funds at the end of the project life.) This way of computing rate of return is an accurate measure of the profitability of the project represented by the cash flow. The following procedure outlines the steps for determining the IRR for a mixed investment:

Step 1: Identify the MARR (or external interest rate).

Step 2: Calculate $PB(i, \text{MARR})_n$ or simply PB_n , according to the following rule:

$$\begin{aligned} PB(i, \text{MARR})_0 &= A_0. \\ PB(i, \text{MARR})_1 &= \begin{cases} PB_0(1 + i) + A_1, & \text{if } PB_0 < 0. \\ PB_0(1 + \text{MARR}) + A_1, & \text{if } PB_0 > 0. \end{cases} \\ &\vdots \\ PB(i, \text{MARR})_n &= \begin{cases} PB_{n-1}(1 + i) + A_n, & \text{if } PB_{n-1} < 0. \\ PB_{n-1}(1 + \text{MARR}) + A_n, & \text{if } PB_{n-1} > 0. \end{cases} \end{aligned}$$

(As defined in the text, A_n stands for the net cash flow at the end of period n .

Note also that the terminal project balance must be zero.)

Step 3: Determine the value of i by solving the terminal-project-balance equation:

$$PB(i, \text{MARR})_n = 0.$$

That interest rate is the true IRR for the mixed investment.

Using the MARR as an external interest rate, we may accept a project if the IRR exceeds MARR and should reject the project otherwise. Figure 7A.2 summarizes the IRR computation for a mixed investment.

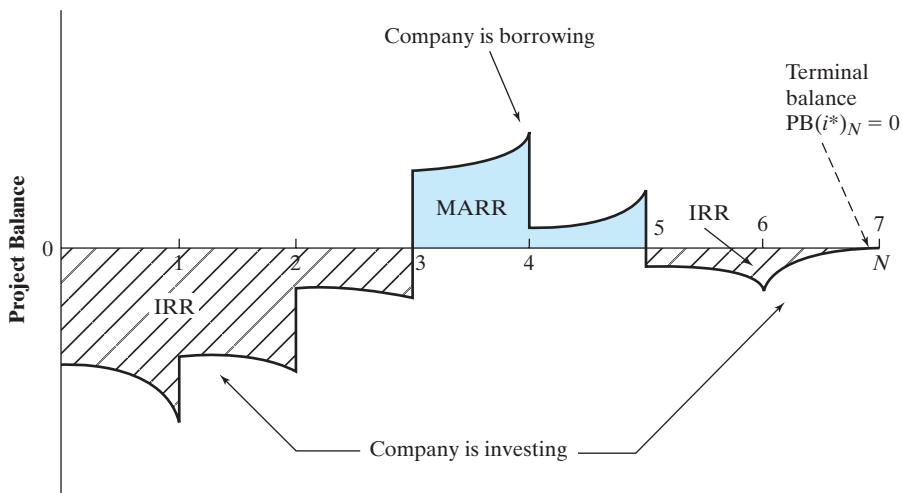


Figure 7A.2 Computational logic to find the true IRR for a mixed investment.

EXAMPLE 7A.2 IRR for a Nonsimple Project: Mixed Investment

Reconsider the defense contractor's flight-simulator project in Example 7.7. The project was a nonsimple and mixed investment. To simplify the decision-making process, we abandoned the IRR criterion and used the PW to make an accept-or-reject decision. Apply the procedures outlined in this chapter to find the true IRR of this mixed investment:

- (a) Compute the IRR (RIC) for this project, assuming MARR = 15%.
- (b) Make an accept-or-reject decision based on the results in part (a).

SOLUTION

Given: Cash flow shown in Example 7.7; MARR = 15%.

Find: (a) IRR and (b) determine whether to accept the project.

- (a) As calculated in Example 7.7, the project has multiple rates of return (10% and 20%). This project is obviously not a pure investment as shown in the following table:

Net-Investment Test Using $i^* = 20\%$

| <i>n</i> | 0 | 1 | 2 |
|----------------------|----------|----------|----------|
| Beginning balance | \$0 | -\$1,000 | \$1,100 |
| Return on investment | \$0 | -\$200 | -\$220 |
| Payment | -\$1,000 | \$2,300 | -\$1,320 |
| Ending balance | -\$1,000 | \$1,100 | \$0 |
| (Unit: \$1,000) | | | |

Because the net-investment test indicates external as well as internal investment, neither 10% nor 20% represents the true internal rate of return of this project. Since the project is a mixed investment, we need to find the true IRR by applying the steps shown previously.

At $n = 0$, there is a net investment for the firm so that the project-balance expression becomes

$$PB(i, 15\%)_0 = -\$1,000,000.$$

The net investment of \$1,000,000 that remains invested internally grows at i for the next period. With the receipt of \$2,300,000 in year 1, the project balance becomes

$$\begin{aligned} PB(i, 15\%)_1 &= -\$1,000,000(1 + i) + \$2,300,000 \\ &= \$1,300,000 - \$1,000,000i \\ &= \$1,000,000(1.3 - i). \end{aligned}$$

At this point, we do not know whether $PB(i, 15\%)_1$ is positive or negative. We need to know this information in order to test for net investment and the presence of a unique i^* . Net investment depends on the value of i , which we want to determine. Therefore, we need to consider two situations, (1) $i < 1.3$ and $i > 1.3$:

- **Case 1:** $i < 1.3 \rightarrow PB(i, 15\%)_1 > 0$.

Since this condition indicates a positive balance, the cash released from the project would be returned to the firm's investment pool to grow at the MARR until it is required to be put back into the project. By the end of year 2, the cash placed in the investment pool would have grown at the rate of 15% [to $\$1,000,000(1.3 - i)(1 + 0.15)$] and must equal the investment into the project of \$1,320,000 required at that time. Then the terminal balance must be

$$\begin{aligned} PB(i, 15\%)_2 &= \$1,000,000(1.3 - i)(1 + 0.15) - \$1,320,000 \\ &= \$175,000 - \$1,150,000i = 0. \end{aligned}$$

Solving for i yields

$$IRR = 0.1522, \text{ or } 15.22\%.$$

- **Case 2:** $i > 1.3 \rightarrow PB(i, 15\%)_1 < 0$.

The firm is still in an investment mode. Therefore, the balance at the end of year 1 that remains invested will grow at a rate of i for the next period. Because of

the investment of \$1,320,000 required in year 2 and the fact that the net investment must be zero at the end of the project life, the balance at the end of year 2 should be

$$\begin{aligned} \text{PB}(i, 15\%)_2 &= \$1,000,000(1.3 - i)(1 + i) - \$1,320,000 \\ &= -\$20,000 + \$300,000i - \$1,000,000i^2 = 0. \end{aligned}$$

Solving for i gives

$$\text{IRR} = 0.1 \text{ or } 0.2 < 1.3,$$

which violates the initial assumption ($i > 1.3$). Therefore, Case 1 is the correct situation.

- (b) Case 1 indicates that $\text{IRR} > \text{MARR}$, so the project would be acceptable, resulting in the same decision obtained in Example 7.6 by applying the PW criterion.

COMMENTS: In this example, we could have seen by inspection that Case 1 was correct. Since the project required an investment as the final cash flow, the project balance at the end of the previous period (year 1) had to be positive in order for the final balance to equal zero. However, inspection does not typically work for more complex cash flows. In general, it is much easier to find the true IRR by using the Goal Seek function in Excel. Table 7A.1 illustrates how you may obtain the true IRR by using the Goal Seek function. In doing so, you may need the following steps:

- **Step 1:** Specify the MARR in cell B4 and the guessed RIC in cell B5. Enter cash flow information into the cells B9 through B11.
- **Step 2:** Designate cells C9 through C11 as project balance for each period. To calculate the project balance figure as a function of both MARR and RIC, we need an “IF” statement in Excel. In a formula, the arguments are separated by commas, so for this example, let us put our formula in cell C10:

$$= \text{IF}(C9 < 0, C9*(1 + \$B\$5) + B10, C9*(1 + \$B\$4) + B10).$$

This says, IF the value in C9 is less than 0, calculate $C9*(1 + \$B\$5) + B10$ and enter this value in C10 and if it is greater than zero, calculate $C9*(1 + \$B\$4) + B10$ and enter this value in C10. Repeat the process for cell C11.

- **Step 3:** Go to the “Tools” menu, and click on the “Goal Seek” command. Then you will see the “Goal Seek” dialog box as shown in Table 7A.1. Make \$C\$11 as the “Set Cell,” enter “0” to “To value,” and \$B\$5 as “By changing cell.” Then click “OK.”
- **Step 4:** Check the RIC displayed in cell B5. In our example, this value is 15.22%. If this figure is the same as the rate of return, you will have a pure investment. For a mixed investment, the RIC will be different from any of the rate-of-return figures.

TABLE 7A.1 Calculating the Return on Invested Capital (or True IRR) by Using Excel

| | A | B | C | D | E | F | G | H |
|----|--|-------------|-----------------|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | Example 7A.2 Calculating the Return on Invested Capital (RIC) | | | | | | | |
| 3 | | | | | | | | |
| 4 | MARR = | 15% | | | | | | |
| 5 | Guess RIC | 15.217% | | | | | | |
| 6 | | | | | | | | |
| 7 | Period | Cash Flow | PB(i , MARR) | | | | | |
| 8 | | | | | | | | |
| 9 | 0 | -\$1,000.00 | -\$1,000.00 | | | | | |
| 10 | 1 | \$2,300.00 | \$1,147.83 | | | | | |
| 11 | 2 | -\$1,320.00 | \$0.00 | | | | | |
| 12 | | | | | | | | |
| 13 | | | | | | | | |
| 14 | Goal Seek | | | | | | | |
| 15 | Set cell: | \$C\$11 | | | | | | |
| 16 | To value: | 0 | | | | | | |
| 17 | By changing cell: | \$B\$5 | | | | | | |
| 18 | | | | | | | | |
| 19 | | | | | | | | |
| 20 | | | | | | | | |
| 21 | OK | Cancel | | | | | | |
| 22 | | | | | | | | |
| 23 | | | | | | | | |
| 24 | | | | | | | | |
| 25 | | | | | | | | |

=IF(C9<0,C9*(1+\$B\$5)+B10,C9*(1+\$B\$4)+B10)
=IF(C10<0,C10*(1+\$B\$5)+B11,C10*(1+\$B\$4)+B11)

Mixed Investment

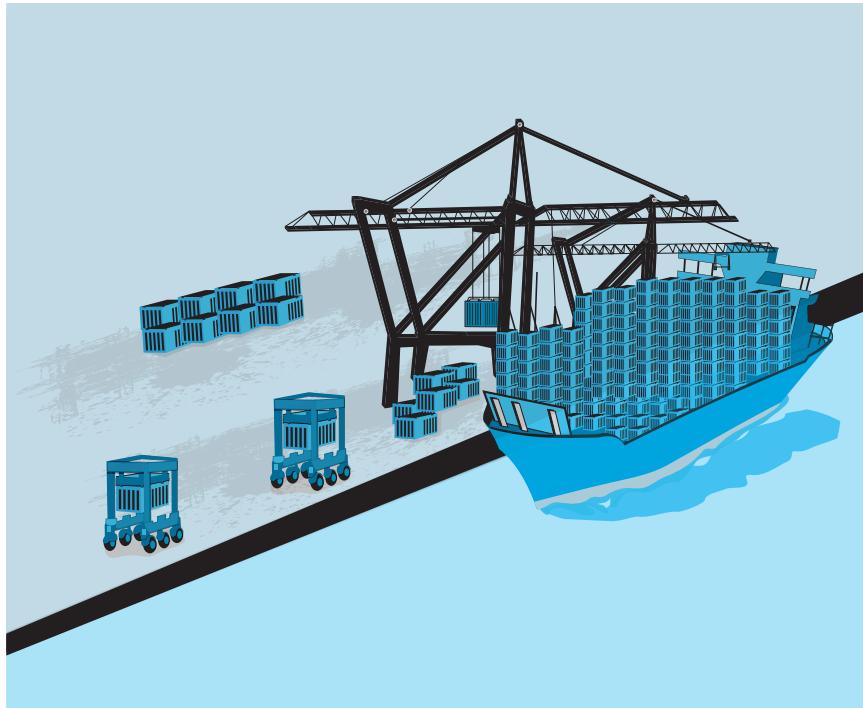
| Point | Project Balance |
|-------|-----------------|
| 0 | 0 |
| 1 | ~\$1,147.83 |
| 2 | 0 |

CHAPTER

EIGHT

Benefit–Cost Analysis

Robot Cargo Handling at Port¹—The Port of Los Angeles' TraPac terminal is one of the busiest shipping terminals in the United States. You can easily spot more than two dozen giant red robots wheeled cargo containers along the docks using highly automated systems and machinery to handle the flood of goods from around the world. Such automation allows the terminal to handle greater volumes of goods in a tighter, more efficient space. The \$1.3 billion phased-in opening of an automated terminal at the Port of Long Beach will be another showcase for the supporters of automated cargo handling. It has the capacity to handle 3.3 million 20-foot container units—nearly half of the entire port's volume in 2016. Operated by the Long Beach



¹ Erica E. Phillips, "Massive Robots Keep Docks Shipshape," *The Wall Street Journal*, March 27, 2016. (<https://www.wsj.com/articles/massive-robots-keep-docks-shipshape-1459104327>).



Container Terminal, the container yard is the port's largest development project. The appeal of robotics to operators is clear.

- It drives down labor costs, enhances safety, and provides faster service to truckers streaming in and out of ports. Automation cut in half the time big-rig drivers idled while waiting to pick up loads. Some studies have shown robotic cargo handling can reduce the need for longshore labor by as much as 50%.
- The electric-and hybrid-powered equipment also produces lower emissions.
- Wait times are down at TraPac, too. That cuts congestion at the port where volumes are expected to rise steeply in the coming decades.
- Analysts estimate the technology can reduce the amount of time ships spend in port and improve productivity by as much as 30%.

Overall, Long Beach automation will cost more than \$1.3 billion in public and private funds once the entire terminal is automated, but executives say they aren't sure when the investment will pay off. It takes a long time to realize the return.

Many engineers are employed in public-works areas such as sea port construction, highway construction, and water projects. One of the most important aspects of any public project is to quantify the benefits in dollar terms. For example, in any airport runway expansion, what is the economic benefit of reducing airport delays? From the airline's point of view, taxiing and arrival delays mean added fuel costs. From the airport's point of view, delays mean lost revenues in landing and departure fees. From the public's point of view, delays mean lost earnings, as people spend more time traveling and less time earning a living. Comparison of the investment costs of a project with the project's potential benefits, a process known as **benefit-cost analysis**, is an important feature of the economic analysis method.

Up to this point, we have focused our attention on investment decisions in the private sector; the primary objective of these investments was to increase the wealth of corporations. In the public sector, federal, state, and local governments spend hundreds of billions of dollars annually on a wide variety of public activities, such as expanding airport runways. In addition, governments at all levels regulate the behavior of individuals and businesses by influencing the use of enormous quantities of productive

resources. How can public decision makers determine whether their decisions, which affect the use of these productive resources, are, in fact, in the best public interest?

Benefit–cost analysis is a decision-making tool for systematically developing useful information about the desirable and undesirable effects of public projects. In a sense, we may view benefit–cost analysis in the public sector as profitability analysis in the private sector. In other words, benefit–cost analyses attempt to determine whether the social benefits of a proposed public activity outweigh the social costs. Usually, public investment decisions involve a great deal of expenditure, and their benefits are expected to occur over an extended time. Examples of benefit–cost analyses include studies of public transportation systems, environmental regulations, public safety programs, education and training programs, public health programs, flood control systems, water resource development projects, and national defense programs.

Benefit–cost analysis problems have three goals: (1) maximize the benefits for any given set of costs (or budgets), (2) maximize the net benefits when both benefits and costs vary, and (3) minimize costs in order to achieve any given level of benefits (often called cost-effectiveness analysis). However, the goal of cost-effectiveness analysis is NOT to determine which alternative is the least costly, but whether the added benefit of a more costly option is worth the added cost. These types of decision problems will be considered in this chapter.

8.1 Evaluation of Public Projects

To evaluate public projects designed to accomplish widely differing tasks, we need to measure the benefits or costs in the same units in all projects so that we have a common perspective by which to judge them. In practice, this requirement means expressing benefits and costs in monetary units, a process that often must be performed without accurate data. In performing benefit–cost analysis, we define **users** as the public and **sponsors** as the government.

The general framework for benefit–cost analysis can be summarized as follows:

1. Identify all users' **benefits** (favorable outcomes) and **disbenefits** (unfavorable outcomes) expected to arise from the project.
2. Quantify, as well as possible, these benefits and disbenefits in dollar terms so that different benefits and the respective costs of attaining them may be compared.
3. Identify the sponsor's costs and any fees (revenues) collected by providing the services, and quantify them.
4. Determine the equivalent net benefits (= benefits – disbenefits) and net costs at the base period; use a discount rate appropriate for the project.
5. Accept the project if the equivalent users' net benefits exceed the equivalent sponsor's net costs.

We can use benefit–cost analysis to choose among alternatives in allocating funds for such projects as the construction of a mass-transit system, building an irrigation dam, highway maintenance, or implementing an air-traffic control system. If the projects are on the same scale with respect to cost, it is merely a question of choosing the project where the benefits exceed the costs by the greatest amount. The steps previously outlined are for a single (or independent) project evaluation. As in the case for the internal rate of return criterion, when comparing mutually exclusive alternatives, an **incremental benefit–cost ratio** must be used. Section 8.2.2 illustrates this important issue in detail.

8.1.1 Valuation of Benefits and Costs

In the abstract, the framework we just developed for benefit–cost analysis is no different from the one we have used throughout this text to evaluate private investment projects. The complications, as we shall discover in practice, arise in trying to identify and assign values to all the benefits and costs of a public project.

8.1.2 Users' Benefits

To begin a benefit–cost analysis, we identify all project benefits and disbenefits to the users, bearing in mind the indirect consequences resulting from the project—the so-called **secondary effects**. For example, the construction of a new highway will create new businesses such as gas stations, restaurants, and motels (benefits), but it will also divert some traffic from the old roads, and, as a consequence, some businesses may be lost (disbenefits). Once the benefits and disbenefits are quantified, we define the overall user's net benefit B as follows:

$$B = \text{benefits} - \text{disbenefits}.$$

In identifying the user's benefits, we classify each as a **primary benefit**—a benefit directly attributable to the project—or a **secondary benefit**—a benefit indirectly attributable to the project. As an example, the U.S. government at one time was considering building the Superconducting Super Collider (SSC)² in Texas. Such a move would bring many scientists and engineers, along with other supporting people, to the region. Primary national benefits may include the long-term benefits that accrue as a result of various applications of the research to U.S. businesses. Primary regional benefits may include economic benefits created by the research laboratory activities, which would generate many new supporting businesses. The secondary benefits might include the creation of new economic wealth as a consequence of any increase in international trade and the income of various regional producers attributable to a growing population.

The reason for making this distinction between primary and secondary benefits is that it may make our analysis more efficient. If primary benefits alone are sufficient to justify project costs, the secondary benefits need not be quantified.

8.1.3 Sponsor's Costs

We determine the cost to the sponsor by identifying and classifying the expenditures required and any savings (or revenues) to be realized. The sponsor's costs should include both capital investments and annual operating costs. Any sales of products or services that take place on completion of the project will generate some revenues—for example, toll revenues on highways. These revenues reduce the sponsor's costs. Therefore, we calculate the sponsor's costs by combining these cost and revenue elements:

$$\text{Sponsor's costs} = \text{capital costs} + \text{operating and maintenance costs} - \text{revenues}.$$

8.1.4 Social Discount Rate

As we briefly mentioned in Chapter 5, the selection of an appropriate MARR for evaluating an investment project is a critical issue in the private sector. In public-project analyses, we also need to select an interest rate, called the **social discount rate**, in order to determine

² The project was canceled in 1993 due to budget problems.

equivalent benefits as well as the equivalent costs. Selection of the social discount rate in public-project evaluation is as critical as the selection of an MARR in the private sector.

Historically, present-worth calculations were initiated to evaluate public water resources and related land-use projects in the 1930s. Since then, the growing tendency has been for analysts to use relatively low rates of discount compared with the rates existing in markets for private assets. During the 1950s and into the 1960s, the rate for water resource projects was 2.63%, which, for much of this period, was even lower than the yield on long-term government securities. The persistent use of a lower interest rate for water resource projects has been a political issue.

In recent years, with the growing interest in performance budgeting and systems analysis that started in the 1960s, government agencies have begun to examine the appropriateness of the discount rate in the public sector in relation to the efficient allocation of resources in the economic system as a whole. Two views of the basis for determining the social discount rate prevail:

- 1. Projects without private counterparts:** *The social discount rate should reflect only the prevailing government borrowing rate.* Projects such as dams designed purely for flood control, access roads for noncommercial uses, and reservoirs for community water supply may not have corresponding private counterparts. In such areas of government activity, the rate of discount traditionally used is benefit–cost analysis, which has been the cost of government borrowing. In fact, water resource project evaluations follow this view exclusively.
- 2. Projects with private counterparts:** *The social discount rate should represent the rate that could have been earned had the funds not been removed from the private sector.* If all public projects were financed by borrowing at the expense of private investment, we may have focused on the opportunity cost of capital in alternative investments in the private sector in order to determine the social discount rate. So, in the case of public capital projects similar to projects in the private sector that produce a commodity or a service to be sold on the market (such as electric power), the discount rate employed is the average cost of capital as will be discussed in Chapter 11. The reasons for using the private rate of return as the opportunity cost of capital for projects similar to those in the private sector are (1) to prevent the public sector from transferring capital from higher yielding to lower yielding investments and (2) to force public-project evaluators to employ market standards when justifying projects.

The Office of Management and Budget (OMB) holds the second view. Since 1972, the OMB has annually updated the discount rates to be used in various federal projects. For calendar year 2017, discount rates for public investment and regulatory analyses³ are as follows:

- **Base-Case Analysis.** Constant-dollar benefit–cost analyses of proposed investments and regulations for a 30-year long-term project should report net present value and other outcomes determined using *a real discount rate of 0.7% (or a nominal rate of 2.8%)*. This rate approximates the marginal pretax rate of return on an average investment in the private sector in recent years. Significant changes in this rate will be reflected in future updates of this circular.
- **Other Discount Rates.** Analyses should show the sensitivity of the discounted net present value and other outcomes to variations in the discount rate. The importance of these alternative calculations will depend on the specific economic

³ The Office of Management and Budget, The White House, www.whitehouse.gov/omb/circulars_a094#8 (2017 Discount Rates for OMB Circular No. A-94).

characteristics of the program under analysis. For example, in analyzing a regulatory proposal whose main cost is to reduce business investment, net present value should also be calculated using a discount rate higher than the real discount rate.

Many state and city governments use nominal discount rates different from the federal government, typically ranging from 3% to 8%, depending on the nature of the project. So, it is important to check the appropriate interest rate to use before conducting any public project analyses. Figure 8.1 summarizes the key steps involved in any benefit–cost analysis for a typical public project.

Step 1

Identify user's benefits

- Calculate all users' benefits in dollar terms
- Calculate all users' disbenefits in dollar terms
- Compute the net users' benefits (= benefit – disbenefits)

Step 2

Identify the sponsor's costs

- Determine the non-recurring costs such as capital expenditure and residual value at the end of project period
- Determine the recurring costs such as operating and maintenance costs, renovation costs
- Determine any annual revenues (fees) generated from the project such as toll revenues in providing service
- Calculate the net annual sponsor's cost (= non-recurring costs + recurring costs – revenues or fees charged – residual value)

Step 3

Perform the benefit–cost analysis

- Determine the interest rate to use in benefit–cost analysis
- Determine the equivalent net benefits
- Determine the equivalent sponsor's cost
- Calculate the benefit–cost ratio

Figure 8.1 Key steps in performing benefit–cost analysis for a typical public project.

8.2 Benefit–Cost Analysis

An alternative way of expressing the worthiness of a public project is to compare the user's benefits (B) with the sponsor's costs (C) by taking the ratio B/C . In this section, we shall define the benefit–cost (B/C) ratio and explain the relationship between the conventional PW criterion and the B/C ratio.

8.2.1 Definition of Benefit–Cost Ratio

For a given benefit–cost profile, let B and C be the present worth of benefits and costs, defined by

$$B = \sum_{n=0}^N b_n(1+i)^{-n}, \quad (8.1)$$

$$C = \sum_{n=0}^N c_n(1+i)^{-n} - S(1+i)^{-N}, \quad (8.2)$$

where

- b_n = benefits at the end of period n ,
- c_n = expenses at the end of period n ,
- N = project life,
- S = salvage value at the end of N , and
- i = sponsor's interest rate (social discount rate).

The sponsor's costs (C) consist of the capital expenditure (I), salvage value (S), and the annual operating costs (C') accrued in each successive period. (Note the sign convention we use in calculating a benefit–cost ratio: Since we are using a ratio, all benefit and cost flows are expressed in positive units. Recall that in previous equivalent worth calculations, our sign convention was to explicitly assign “+” for cash inflows and “−” for cash outflows.) Let us assume that a series of initial investments is required during the first K periods while annual operating and maintenance costs accrue in each following period. Then the equivalent present worth for each component is

$$I = \sum_{n=0}^K c_n(1+i)^{-n} - S(1+i)^{-N}, \quad (8.3)$$

$$C' = \sum_{n=K+1}^N c_n(1+i)^{-n}, \quad (8.4)$$

and $C = I + C'$.

The B/C ratio is defined as

$$BC(i) = \frac{B}{C} = \frac{B}{I + C'} \text{ where } I + C' > 0. \quad (8.5)$$

If we are to accept a project, $BC(i)$ must be higher than 1. Note that the acceptance rule by the B/C -ratio criterion is equivalent to that for the PW criterion as illustrated in Figure 8.2. Note also that we must express the values of B , C' , and I in present-worth equivalents. Alternatively, we can compute these values in terms of annual equivalents and use them in calculating the B/C ratio. The resulting B/C ratio is not affected.

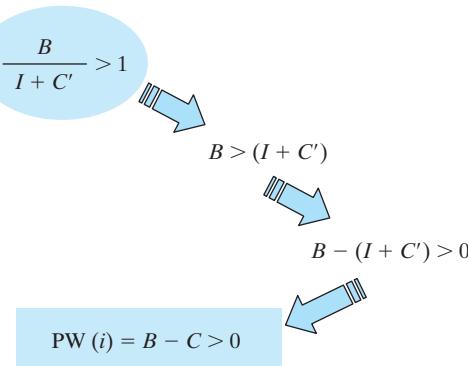


Figure 8.2 Relationship between B/C ratio and PW criterion.

EXAMPLE 8.1 Benefit–Cost Ratio

A public project being considered by a local government has the following estimated benefit–cost profile (see Figure 8.3):

Assume that $i = 10\%$, $N = 5$, and $K = 1$. Compute B , C , I , C' , and $BC(10\%)$.

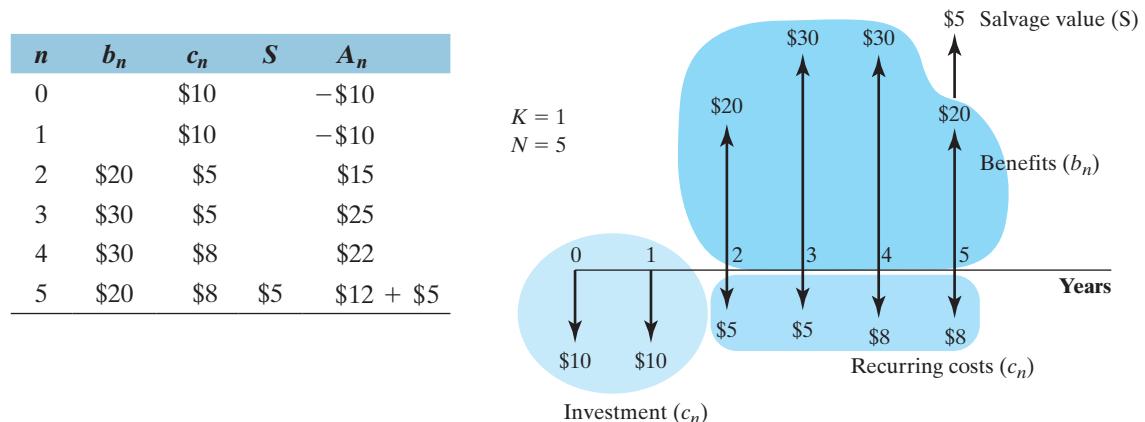


Figure 8.3 Classification of a project’s cash flow elements into benefits and costs.

DISSECTING THE PROBLEM

Given: b_n , c_n , S , $K = 1$, $N = 5$, and $i = 10\%$ per year
Find: $BC(i)$.

METHODOLOGY

SOLUTION

We calculate B as follows:

$$\begin{aligned} B &= \$20(P/F, 10\%, 2) + \$30(P/F, 10\%, 3) \\ &\quad + \$30(P/F, 10\%, 4) + \$20(P/F, 10\%, 5) \\ &= \$71.98. \end{aligned}$$

We calculate C as follows:

$$\begin{aligned} C &= \$10 + \$10(P/F, 10\%, 1) + \$5(P/F, 10\%, 2) \\ &\quad + \$5(P/F, 10\%, 3) + \$8(P/F, 10\%, 4) \\ &\quad + \$8(P/F, 10\%, 5) - \$5(P/F, 10\%, 5) \\ &= \$34.31. \end{aligned}$$

We calculate I as follows:

$$\begin{aligned} I &= \$10 + \$10(P/F, 10\%, 1) - \$5(P/F, 10\%, 5) \\ &= \$15.99. \end{aligned}$$

We calculate C' as follows:

$$\begin{aligned} C' &= C - I \\ &= \$18.32. \end{aligned}$$

Using Eq. (8.5), we can compute the B/C ratio as

$$\begin{aligned} BC(10\%) &= \frac{\$71.98}{\$15.99 + \$18.32} = \frac{\$71.98}{\$34.31} \\ &= 2.10 > 1. \end{aligned}$$

The B/C ratio exceeds 1, so the user's benefits exceed the sponsor's costs.

COMMENTS: Since governments do not pay taxes, depreciation and income taxes are not issues in BC analysis.

8.2.2 Incremental B/C-Ratio Analysis

Let us now consider how we choose among mutually exclusive public projects. As we explained in Chapter 7, we must use the incremental-investment approach in comparing alternatives on the basis of any relative measure, such as IRR or B/C .

To apply incremental analysis, we compute the incremental differences for each term (B , I , and C') and take the B/C ratio based on these differences. To use $BC(i)$ on an incremental investment, we may proceed as follows:

1. Eliminate any alternatives with a B/C ratio less than 1.
2. Arrange the remaining alternatives in increasing order of the denominator ($I + C'$). Thus, the alternative with the smallest denominator should be the first (j), the alternative with the second smallest (k) should be the second, and so forth.
3. Compute the incremental differences for each term (B , I , and C') for the paired alternatives (j , k) in the list:

$$\Delta B = B_k - B_j;$$

$$\Delta I = I_k - I_j;$$

$$\Delta C' = C'_k - C'_j.$$

4. Compute $BC(i)$ on incremental investment by evaluating

$$BC(i)_{k-j} = \frac{\Delta B}{\Delta I + \Delta C'}.$$

If $BC(i)_{k-j} > 1$, select the k alternative. Otherwise, select the j alternative.

5. Compare the selected alternative with the next one on the list by computing the incremental benefit–cost ratio. Continue the process until you reach

the bottom of the list. The alternative selected during the last pairing is the best one.

We may modify the decision procedures when we encounter the following situations:

- If $\Delta I + \Delta C' = 0$, we cannot use the benefit–cost ratio because this relationship implies that both alternatives require the same initial investment and operating expenditures. In such cases, we simply select the alternative with the largest B value.
- In situations where public projects with *unequal service lives* are to be compared but the alternative can be repeated, we may compute all component values (B , C' , and I) on an *annual basis* and use them in incremental analysis.

EXAMPLE 8.2 Comparing Mutually Exclusive Alternatives—Two Projects

Federal Aviation Administration (FAA) is considering two types of public projects where one project is capital intensive (A1) and the other project is operating intensive (A2). Each project has the same service life, and the present worth of each component value (B , I , and C') is computed at 7% as follows:

| Project Element | Type of Project | |
|-----------------------------------|------------------------|--------------------------|
| | Capital Intensive (A1) | Operating Intensive (A2) |
| Initial investment (I) | \$1,000,000 | \$1,000,000 |
| Annual costs | \$50,000 | \$500,000 |
| Annual benefits | \$250,000 | \$700,000 |
| Annual net benefits | \$200,000 | \$200,000 |
| Useful life | 10 years | 10 years |
| Salvage value | \$0 | \$0 |
| Total benefits (B) at 7% | \$1,755,895 | \$4,916,507 |
| Total annual costs (C') at 7% | \$351,179 | \$3,511,791 |
| Total costs ($I + C'$) at 7% | \$1,351,179 | \$4,511,791 |
| Benefit–cost ratio | 1.30 | 1.09 |
| Internal rate of return | 15.10% | 15.10% |
| Net present worth | \$404,716 | \$404,716 |

As shown in Figure 8.4, clearly both projects are exactly the same in terms of net cash flows, so their IRR and NPW should be the same, as expected. However, when we calculate the B/C ratio for each project, we obtain 1.30 for A1 and 1.09 for A2, respectively. Use the B/C ratio on incremental investment to demonstrate that both projects are indeed identical in terms of economic merits.

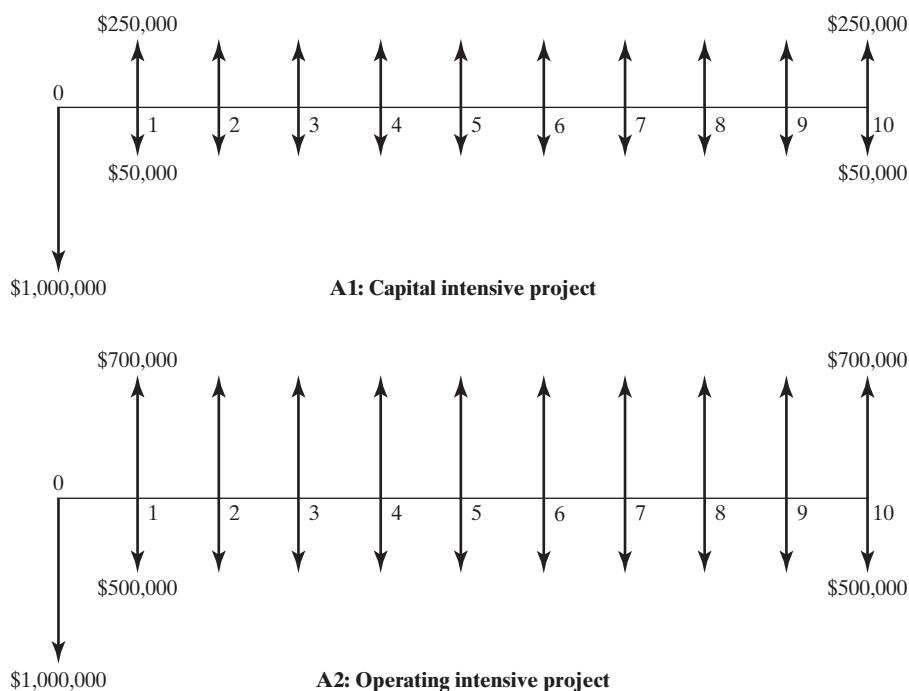


Figure 8.4 Public project cash flow series: A1—Capital intensive projects, A2—Operating intensive project. The net cash flows are the same for both projects.

| | |
|--|--|
| DISSECTING THE PROBLEM | Given: I , B , and C' for each project, $i = 7\%$ per year. Find: Select the best project. |
| METHODOLOGY Calculate the $BC(i)$ for each project and compare the projects incrementally. | SOLUTION <p>(a) Since $PW(i)_1$ and $PW(i)_2$ are positive, all of the projects would be acceptable if they were independent. Also, the $BC(i)$ value for each project is higher than 1, so the use of the benefit–cost ratio criterion leads to the same accept–reject conclusion as under the PW criterion.</p> <p>(b) If these projects are mutually exclusive, we must use the principle of incremental analysis. If we attempt to rank the projects according to the size of the B/C ratio, obviously, we will observe a different project preference. For example, if we use $BC(i)$ on the total investment, we see that $A1$ appears to be more desirable than $A2$; however, by computing the incremental B/C ratios, we will see the result that is consistent with the NPW criterion. We will first arrange the projects by increasing order of their denominator ($I + C'$) for the $BC(i)$ criterion:</p> |

| Ranking Base | A1 | A2 |
|--------------|-------------|-------------|
| $I + C'$ | \$1,351,179 | \$4,511,791 |

With the do-nothing alternative, we first drop from consideration any project that has a B/C ratio smaller than 1. In our example, the B/C ratios of all three projects exceed 1, so the first incremental comparison is between A_1 and A_2 :

$$\begin{aligned}\Delta BC_{A_2-A_1} &= \frac{\$4,916,507 - \$1,755,895}{\$4,511,791 - \$1,351,179} \\ &= \frac{\$3,160,612}{\$3,160,612} \\ &= 1,\end{aligned}$$

which indicates that A_1 and A_2 are equally likely. This inconsistency in ranking is another reason why we need to use the incremental analysis to select the correct proper project using the benefit–cost criterion.

EXAMPLE 8.3 Incremental Benefit–Cost Ratio Analysis for Multiple Alternatives

Consider three investment projects: A_1 , A_2 , and A_3 . Each project has the same service life, and the present worth of each component value (B , I , and C') is computed at 10% as follows:

| | Projects | | |
|---------|-------------------------|-------------------------|-------------------------|
| | A_1 | A_2 | A_3 |
| B | \$12,000 | \$35,000 | \$21,000 |
| I | \$5,000 | \$20,000 | \$14,000 |
| C' | \$4,000 | \$8,000 | \$1,000 |
| $PW(i)$ | \$3,000 | \$7,000 | \$6,000 |

- (a) If all three projects are independent, which projects would be selected, based on $BC(i)$?
- (b) If the three projects are mutually exclusive, which project would be the best alternative? Show the sequence of calculations that would be required in order to produce the correct results. Use the B/C ratio on incremental investment.

DISSECTING THE PROBLEM

Given: I , B , and C' for each project, $i = 10\%$ per year.
Find: Select the best project.

METHODOLOGY

Calculate the $BC(i)$ for each project and compare the projects incrementally.

SOLUTION

- (a) Since $PW(i)_1$, $PW(i)_2$, and $PW(i)_3$ are positive, all of the projects would be acceptable if they were independent. Also, the $BC(i)$ value for each project is higher than 1, so the use of the benefit–cost ratio criterion leads to the same accept–reject conclusion as under the PW criterion:

| | A1 | A2 | A3 |
|----------------|-----------|-----------|-----------|
| BC(<i>i</i>) | 1.33 | 1.25 | 1.40 |

Benefit–cost analysis may be graphically illustrated as shown in Figure 8.5. $B/C = 1$ represents the 45° line where any project located above this line is selected. The slope made by connecting the origin $(0, 0)$ with each point of the project location represents the B/C ratio.

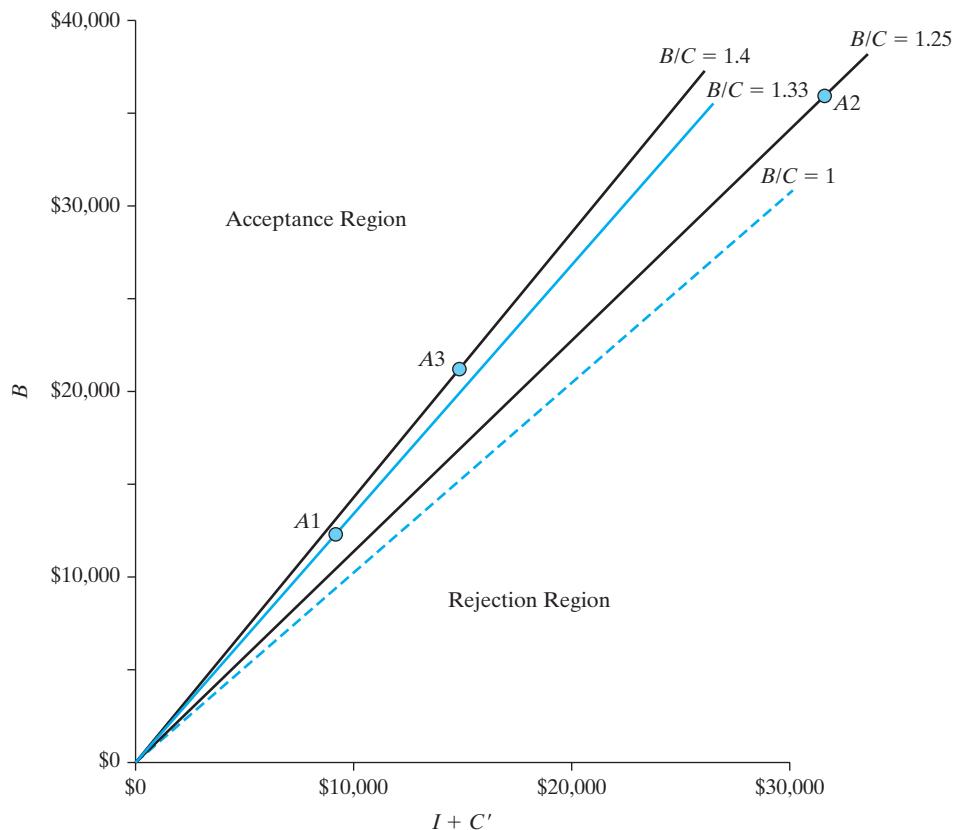


Figure 8.5 Benefit–cost ratio graph of Example 8.3.

(b) If these projects are mutually exclusive, we must use the principle of incremental analysis as shown in Figure 8.6. If we attempt to rank the projects according to the size of the B/C ratio, obviously, we will observe a different project preference. For example, if we use $BC(i)$ on the total investment, we see that A_3 appears to be the most desirable and A_2 the least desirable project; however, selecting mutually exclusive projects on the basis of B/C ratios is incorrect. Certainly, with $PW(i)_2 > PW(i)_3 > PW(i)_1$, project A_2 would be selected under the PW criterion. By computing the incremental B/C ratios, we will select a project that is consistent with the PW criterion. We will first arrange the projects by increasing order of their denominator ($I + C'$) for the $BC(i)$ criterion:

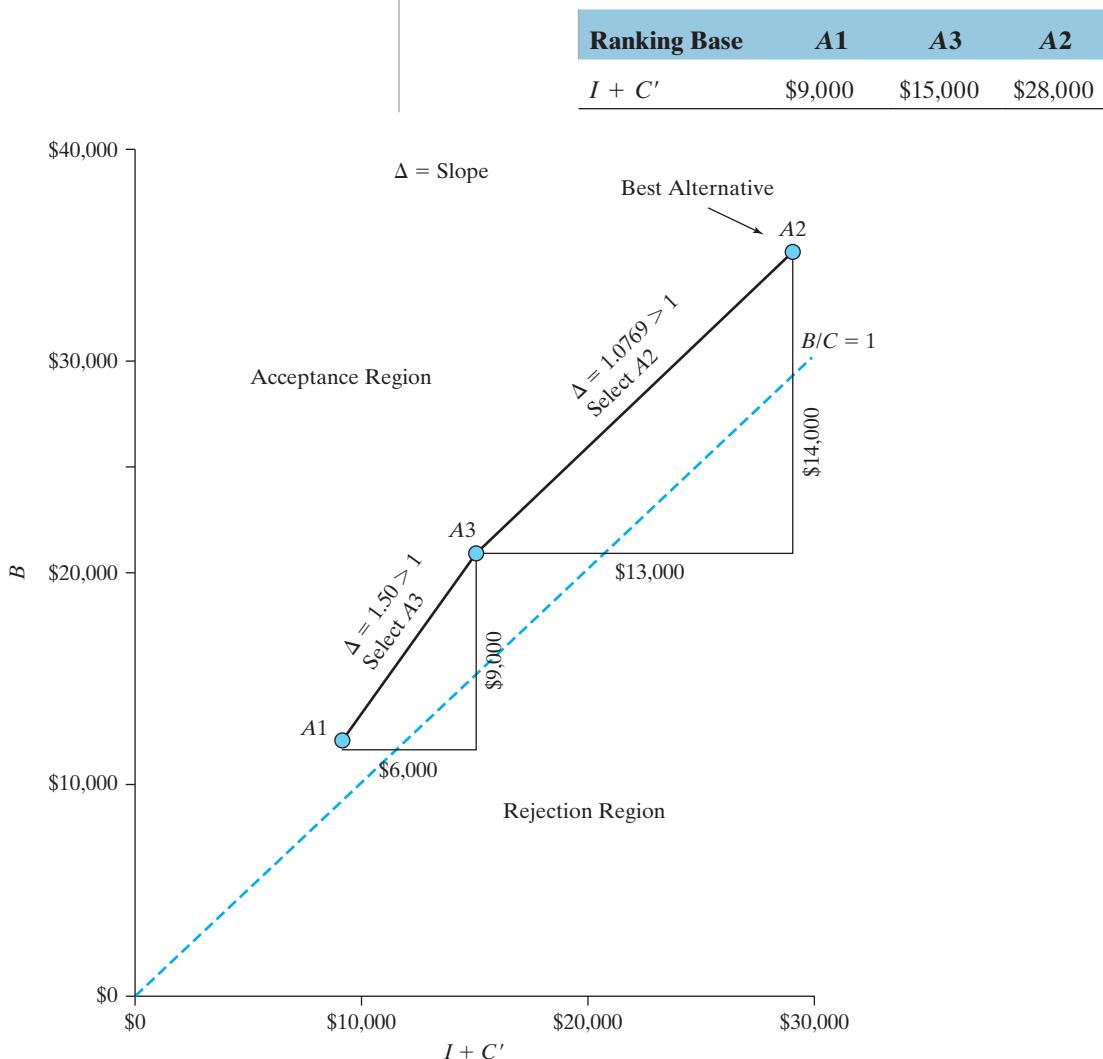


Figure 8.6 Incremental benefit–cost analyses. The slope of line $A_3 - A_1$ indicates a $\Delta B/\Delta C' > 1$. This is also true for line $A_2 - A_3$, indicating A_2 will be the ultimate choice.

We now compare the projects incrementally as follows:

- A_1 versus A_3 : With the do-nothing alternative, we first drop from consideration any project that has a B/C ratio smaller than 1. In our example, the B/C ratios of all three projects exceed 1, so the first incremental comparison is between A_1 and A_3 :

$$\begin{aligned} BC(i)_{3-1} &= \frac{\$21,000 - \$12,000}{(\$14,000 - \$5,000) + (\$1,000 - \$4,000)} \\ &= 1.50. \end{aligned}$$

Since the ratio is higher than 1, we prefer A_3 over A_1 . Therefore, A_3 becomes the “current best” alternative.

- A_3 versus A_2 : Next we must determine whether the incremental benefits to be realized from A_2 would justify the additional expenditure. Therefore, we need to compare A_2 and A_3 as follows:

$$\begin{aligned} BC(i)_{2-3} &= \frac{\$35,000 - \$21,000}{(\$20,000 - \$14,000) + (\$8,000 - \$1,000)} \\ &= 1.0769. \end{aligned}$$

The incremental B/C ratio again exceeds 1, and therefore, we prefer A_2 over A_3 . With no further projects to consider, A_2 becomes the ultimate choice.

COMMENTS: Figure 8.6 illustrates also elimination of inferior alternatives. When we compare A_3 with A_1 , where A_3 has a higher sum of $I + C'$, we are basically attempting to determine the slope (Δ) created by connecting A_1 and A_3 . If this slope is higher than 1, the higher cost alternative ($I + C'$) is preferred. In our case, this slope happens to be $\Delta = 1.5$, so A_3 is selected. Then we compare A_3 with A_2 . The slope created by connecting A_3 with A_2 gives $\Delta = 1.0769$, which is higher than 1; so A_2 is the better choice.

8.3 Profitability Index

Another method similar to the benefit–cost ratio is called the profitability index. This index attempts to identify the relationship between the costs and benefits of a proposed project through the use of a ratio.

8.3.1 Definition of Profitability Index

Unlike the benefit–cost ratio, the **profitability index**, $PI(i)$, considers only the initial capital expenditure (less the salvage value, if any) as cash outlay, and annual net benefits are used—the ratio of the present value of the future expected net benefit cash flows divided by the amount of the equivalent initial investment.

$$PI(i) = \frac{B - C'}{I}, I > 0. \quad (8.6)$$

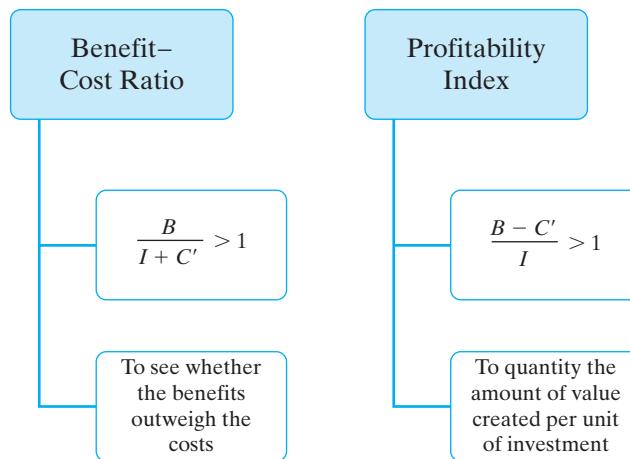


Figure 8.7 Conceptual difference between the benefit-cost ratio and the profitability index.

The profitability index measures the bang for the buck invested. In a way, it is a measure of capital efficiency. A ratio of 1.0 is logically the lowest acceptable measure on the index. With a $PI(i) < 1$, the project's present value of the future cash flow streams is less than the initial investment. As the values on the profitability index increase, so does the financial attractiveness of the proposed project. In fact, this profitability index is not limited to the public project evaluation. It is also commonly used in ranking investments in the private sector. As is the case with the benefit-cost ratio, we select all independent projects whose PI ratio is higher than 1 as long as the budget permits. Figure 8.7 illustrates the conceptual differences between the benefit-cost ratio and the profitability index.

EXAMPLE 8.4 Profitability Index Ratio

Assume a five-year program that will commit the government to the stream of real (or constant-dollar) expenditures to generate a series of real benefits for a public project. Assume that $i = 7\%$, $N = 5$, and $K = 1$. Compute $PI(7\%)$ and determine whether it is worth funding the program.

| n | b_n | c_n | S | A_n |
|-----|-------|-------|-----|-----------|
| 0 | | \$10 | | -\$10 |
| 1 | | \$20 | | -\$20 |
| 2 | \$35 | \$5 | | \$30 |
| 3 | \$30 | \$5 | | \$25 |
| 4 | \$20 | \$7 | | \$13 |
| 5 | \$10 | \$7 | \$5 | \$3 + \$5 |

| | |
|--|---|
| <p>DISSECTING THE PROBLEM</p> <p>METHODOLOGY</p> <p>Use Eq. (8.6).</p> | <p>GIVEN: $b_n, c_n, S, N = 5$, and $i = 7\%$ per year.</p> <p>Find: $PI(i)$.</p> <p>SOLUTION</p> <p>We calculate $B - C'$ as follows:</p> $\begin{aligned} B - C' &= \$30(P/F, 7\%, 2) + \$25(P/F, 7\%, 3) \\ &\quad + \$13(P/F, 7\%, 4) + \$3(P/F, 7\%, 5) \\ &= \$58.66. \end{aligned}$ <p>We calculate I as follows:</p> $\begin{aligned} I &= \$10 + \$20(P/F, 7\%, 1) - \$5(P/F, 7\%, 5) \\ &= \$25.13. \end{aligned}$ <p>Using Eq. (8.6), we can compute the PI as</p> $PI(7\%) = \frac{\$58.66}{\$25.13} = 2.33 > 1.$ <p>The PI exceeds 1, so the project is worth undertaking.</p> |
|--|---|

COMMENTS: The PI value of 2.33 indicates that for every dollar spent by the government, the project will generate \$2.33 benefits. What does our conclusion have to say about the PI rule? In the case of limited funds, the PI measures the “bang for the buck,” so it could be used as a tool to rank the projects according to its capital efficiency.

8.3.2 Incremental PI Ratio Analysis for Mutually Exclusive Alternatives

Because PI is a ratio, like $BC(i)$ and IRR, it ignores differences of scale for mutually exclusive projects. This, however, can be corrected by using incremental analysis. We create an incremental PI by subtracting the lower cost investment alternative from the higher cost alternative. If the incremental PI is higher than 1, we choose the alternative with the higher cost of investment. This way of selecting the alternative will ensure that we will pick the same project that would be selected under the NPW analysis.

In situations where we need to compare projects with *unequal service lives*, we may compute all component values (B , C' , and I) on an *annual basis* and use them in incremental analysis with the assumption that the alternative can be repeated.

EXAMPLE 8.5 Incremental Profitability Index

Reconsider the three investment projects: A_1 , A_2 , and A_3 in Example 8.3. Each project has the same service life, and the present worth of each component value (B , I , and C') is computed at 10% as follows:

| Projects | | | |
|-----------|----------|----------|----------|
| | A1 | A2 | A3 |
| B | \$12,000 | \$35,000 | \$21,000 |
| I | \$5,000 | \$20,000 | \$14,000 |
| C' | \$4,000 | \$8,000 | \$1,000 |
| PW(i) | \$3,000 | \$7,000 | \$6,000 |

- (a) If all three projects are independent, which projects would be selected based on PI(i)?
- (b) If the three projects are mutually exclusive, which project would be the best alternative? Show the sequence of calculations that would be required in order to produce the correct results. Use the PI on incremental investment.

DISSECTING THE PROBLEM

Given: I , B , and C' for each project, $i = 10\%$ per year.

Find: Select the best project based on PI.

METHODOLOGY

Calculate the PI(i) for each project and compare the projects incrementally.

SOLUTION

- (a) As before, all of the projects would be acceptable if they were independent. Also, the PI(i) value for each project is higher than 1, so the use of the profitability index criterion leads to the same accept-reject conclusion as under the PW criterion:

| | A1 | A2 | A3 |
|-----------|-----|------|------|
| PI(i) | 1.6 | 1.35 | 1.43 |

- (b) If these projects are mutually exclusive, we must use the principle of incremental analysis. If we attempt to rank the projects according to the size of the PI ratio, obviously, we will observe a different project preference. For example, if we use PI(i) on the total investment, we see that A_1 appears to be the most desirable and A_2 the least desirable project; however, selecting mutually exclusive projects on the basis of PI ratios is incorrect. Certainly, with $PW(i)_2 > PW(i)_3 > PW(i)_1$, project A_2 would be selected under the PW criterion. By computing the incremental PI ratios, we will select a project that is consistent with the PW criterion. We will first arrange the projects by increasing order of their denominator I for the PI(i) criterion:

| Ranking Base | A1 | A3 | A2 |
|--------------|---------|----------|----------|
| I | \$5,000 | \$14,000 | \$20,000 |

With the do-nothing alternative, we first drop from consideration any project that has a PI ratio less than 1. In our example, the PI ratios of all three projects exceed 1, so the first incremental comparison is between A_1 and A_3 . We now compare the projects incrementally as follows:

- A_1 versus A_3 :

$$\begin{aligned} \text{PI}(i)_{3-1} &= \frac{(\$21,000 - \$1,000) - (\$12,000 - \$4,000)}{(\$14,000 - \$5,000)} \\ &= 1.33 > 1. \end{aligned}$$

Since the ratio is higher than 1, we prefer A_3 over A_1 . Therefore, A_3 becomes the “current best” alternative.

- A_3 versus A_2 : Next we must determine whether the incremental benefits to be realized from A_2 would justify the additional expenditure. Therefore, we need to compare A_2 and A_3 as follows:

$$\begin{aligned} \text{PI}(i)_{2-3} &= \frac{(\$35,000 - \$8,000) - (\$21,000 - \$1,000)}{(\$20,000 - \$14,000)} \\ &= 1.17 > 1. \end{aligned}$$

The incremental PI ratio again exceeds 1, and therefore, we prefer A_2 over A_3 . With no further projects to consider, A_2 becomes the ultimate choice.

Note that this is exactly the same conclusion that we obtained under the benefit–cost analysis.

8.4 Highway Benefit–Cost Analysis⁴

In this section, we will examine how a benefit–cost analysis may be used in making investments in highway improvement projects. The improvement may reduce the number or severity of crashes, eliminate long delays during peak hours, or reduce travel time (by providing a shorter route). In highway benefit–cost analysis, the usual procedure is that benefits are first estimated in physical terms and then valued in economic terms. This means that the analyst has to first estimate the number of crashes eliminated, the amount of travel time saved, and/or the number of vehicle-miles reduced before assigning or calculating monetary values.

8.4.1 Define the Base Case and the Proposed Alternatives

Every analysis requires a definition of the base case and proposed alternative(s).

- The **base case** is not necessarily a “do-nothing” alternative, but it is generally the “lowest” capital cost alternative that maintains the serviceability of the existing facility. In other words, the base case should include an estimate of any physical

⁴ This section is based on materials from “Benefit–Cost Analysis for Transportation Projects,” *Benefit Cost Analysis Guidance*, Minnesota Department of Transportation, June 2005.

and operational deterioration in the condition of the facility and the costs associated with the periodic need to rehabilitate the major elements of the facility through the analysis period.

- The **proposed alternatives** are specific and discrete sets of highway improvements that can be undertaken. These improvements generally change travel times, vehicle operating costs, and/or safety characteristics from the base case. Proposed alternatives must also be reasonably distinct from one another, and each alternative should be specified with as much detail as possible to estimate costs (capital and maintenance) and effects on travel time, operating costs, and safety.
- We also need to define the **analysis period** that will be used in the benefit–cost analysis. For many highway improvement projects, it is quite common to use an analysis period of 20 years or longer.

8.4.2 Highway User Benefits

The benefits of a transportation investment are typically estimated by comparing the amount of travel time, vehicle miles traveled, and expected number of crashes for the alternative with the base case. Specifically, consider the following:

- 1. Travel-Time Savings:** These typically generate the greatest benefit. Savings are calculated on the basis of the difference in travel time between the base case and an alternative. Travel time is often expressed as vehicle-hours traveled (VHT) and can be estimated with various computer models. The estimation of travel-time savings should include both the driver and passengers (vehicle occupancy rates). In many cases, vehicle occupancy rates vary between peak and off-peak hours as well as between alternatives. The valuation of travel-time savings is calculated by standardized cost-per-hour-person figures for different types of vehicles (auto or truck).
- 2. Vehicle Operating Cost Savings:** When transportation improvements are made, the cost of operating vehicles along a particular facility or set of facilities can change. Operating costs can change because the number of miles driven varies (as in the case of a shorter bypass or a reduction in diversion of trips) or because of variation in the number of stops or speed-cycle changes. The number of vehicle-miles traveled (VMT) is the most common variable that affects vehicle operating costs. Once the change in vehicle miles is estimated, the valuation of vehicle operating costs is calculated for standardized cost-per-mile figures for different types of vehicles (auto or truck).
- 3. Safety Benefits:** Safety benefits are one of the principal benefits that can result from transportation improvements. Benefits occur when the number of crashes is reduced and/or the severity of the crashes is diminished on a facility or set of facilities because of the transportation improvement. Standard engineering methods can be used to evaluate both the potential crash reductions and/or changes in severity.

8.4.3 Sponsors' Costs

In economic terms, the transportation investment cost is determined by the amount of resources used/consumed over the course of a project. The total value of construction and any additional maintenance costs must be estimated. It is important to note that the analysis does not emphasize who incurs the cost but rather aims to include any and all costs that are involved in developing the project.

- 1. Capital Costs:** These costs make up the total investment required to prepare a highway improvement for service from engineering through landscaping. When possible, capital costs should be grouped into similar life-cycle categories, including engineering, right of way, major structures, grading, subbase and base, surfacing, and miscellaneous items.
- 2. Major Rehabilitation Costs:** Within a benefit–cost analysis period, future investments may be needed to maintain the serviceability of a major transportation facility. For example, with a new or reconstructed highway, pavement overlays may be required 8, 12, or 15 years after the initial construction year. The cost of overlays or other major preservation activities should be included in the analysis and allocated to the year in which they are anticipated to occur.
- 3. Routine Annual Maintenance Costs:** When evaluating transportation investments, it is important to account for the future operating and maintenance costs of the facility. Bridges require preventive maintenance, and roadway lanes have to be plowed and patched each year. In the case of upgraded roadways, it is necessary to estimate the marginal or additional maintenance costs that would be required for the alternative, compared with the base case. For a new facility, the entire additional maintenance costs should be included as the incremental increase in costs.
- 4. Remaining Capital Value (RCV):** Many components of a project retain some residual useful life beyond the benefit–cost analysis period (typically, 20 years). At the end of the analysis period, the infrastructure that has been put in place generally has not been completely worn out and will continue to provide benefits to drivers and travelers into the future. It is important to reflect this value in the analysis. Typically, we calculate the remaining capital value by determining the percentage of useful life remaining beyond the analysis period and multiplying that percentage by the construction cost for that component. Figure 8.8 summarizes the framework to be used in estimating the user’s benefits and sponsor’s cost in a highway improvement project.

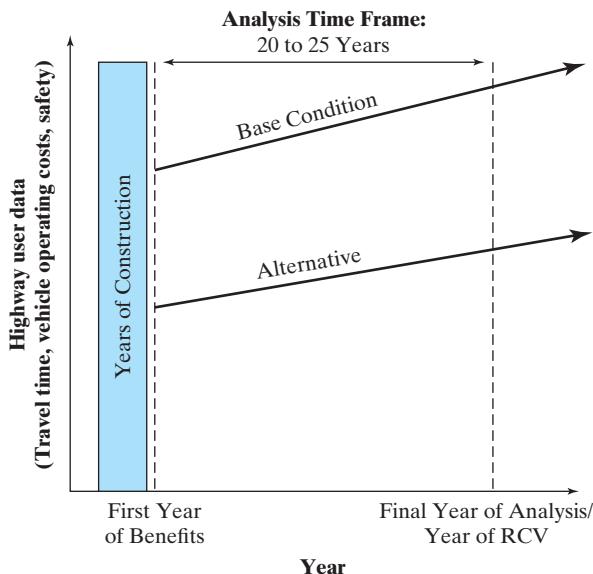


Figure 8.8 Time-dependent elements in a benefit–cost analysis.

8.4.4 Illustrating Case Example

This case example illustrates how the Minnesota Department of Transportation (Mn/DOT) made a decision to implement a highway improvement project. The base case was just maintaining the existing highway, and the proposed alternative case was to widen the existing highways from four lanes to six lanes. In doing economic analysis, Mn/DOT assumed that the construction would begin in 2007 and completed by the end of 2010. Mn/DOT used a study period of 20 years in justifying the improvement project.

- **Users' benefits:** In estimating the technical as well as financial data related to the project, we may proceed as follows:
 1. First, it is necessary to assemble highway user data from the database kept by the Mn/DOT (VMT, VHT, other operating costs and safety information) for the first year of benefits and the final year of analysis at a minimum. (If detailed annual estimates are not available, we could interpolate between these two data points to compute information for each year in the analysis time frame.)
 2. Compute the difference in travel time, vehicle operating costs, and safety between the base case and the alternative for each year in the analysis. Table 8.1 summarizes the VMT benefits, VHT benefits, and accident reduction savings due to the proposed highway improvement project. Note that these benefit figures are incremental benefits over the base case.
- **Total Sponsor's Costs:** In estimating the sponsor's cost (in this case, the state of Minnesota), the following procedures are assumed:
 1. Construction costs for the proposed alternative are estimated and allocated to the anticipated year of expenditure. In this case example, it is assumed that the base case has construction costs in the 10th and 11th years to maintain the existing traffic flows.
 2. Maintenance costs for each year are estimated over the analysis period, as the proposed alternative has a significant effect on maintenance costs (much different from the base case).
 3. Existing infrastructure typically retains some remaining capital value (RCV) at the end of the benefit–cost analysis period. When calculating the remaining capital value, we must estimate the useful life of the investment elements. The capital costs are broken into elements such as preliminary engineering, right of way, major structures, roadway grading and drainage, roadway sub-base and base, and roadway surface. The remaining capital values are estimated at the end of their useful lives. Since all investment elements last more than the analysis period of 20 years, we estimated these values at the end of 20 years, using the guideline suggested by the Mn/DOT.
 4. Table 8.2 shows the total present cost as the sum of the discounted annual costs found for each year in the analysis period. We calculate annual costs by adding the construction and recurring maintenance costs and subtracting the discounted remaining capital value for each year in the analysis.

TABLE 8.1 Example of a Total Highway User Benefit Tallying Spreadsheet⁵

| Year | Present Value of User Benefits | | | Present Value of Total User Benefits |
|-----------|--------------------------------|---------------|----------------------------|--------------------------------------|
| | VMT Benefits | VHT Benefits | Accident Reduction Savings | |
| 2011 | \$5,373,985 | \$22,348,044 | \$4,079,014 | \$31,801,043 |
| 2012 | \$5,222,634 | \$24,236,203 | \$3,774,352 | \$33,233,189 |
| 2013 | \$5,075,546 | \$26,049,598 | \$3,485,631 | \$34,610,775 |
| 2014 | \$4,932,600 | \$27,790,302 | \$3,212,120 | \$35,935,022 |
| 2015 | \$4,793,680 | \$29,460,334 | \$2,953,122 | \$37,207,136 |
| 2016 | \$4,658,672 | \$31,061,668 | \$2,707,967 | \$38,428,308 |
| 2017 | \$4,527,467 | \$32,596,229 | \$2,476,016 | \$39,599,712 |
| 2018 | \$4,399,957 | \$34,065,893 | \$2,256,656 | \$40,722,506 |
| 2019 | \$4,276,038 | \$35,472,493 | \$2,049,300 | \$41,797,832 |
| 2020 | \$4,155,609 | \$36,817,816 | \$1,853,388 | \$42,826,813 |
| 2021 | \$4,038,572 | \$38,103,604 | \$1,668,382 | \$43,810,559 |
| 2022 | \$3,924,831 | \$39,331,558 | \$1,493,770 | \$44,750,159 |
| 2023 | \$3,814,294 | \$40,503,338 | \$1,329,059 | \$45,646,690 |
| 2024 | \$3,706,869 | \$41,620,560 | \$1,173,779 | \$46,501,208 |
| 2025 | \$3,602,470 | \$42,684,802 | \$1,027,482 | \$47,314,754 |
| 2026 | \$3,501,011 | \$43,697,603 | \$889,737 | \$48,088,352 |
| 2027 | \$3,402,410 | \$44,660,465 | \$760,133 | \$48,823,008 |
| 2028 | \$3,306,585 | \$45,574,849 | \$638,277 | \$49,519,712 |
| 2029 | \$3,213,460 | \$46,442,185 | \$523,794 | \$50,179,438 |
| 2030 | \$3,122,957 | \$47,263,862 | \$416,323 | \$50,803,142 |
| 2011–2030 | \$83,049,648 | \$729,781,406 | \$38,768,303 | \$851,599,358 |

- **Benefit–Cost Ratio Calculation:** With the benefits and sponsor’s costs determined, we are ready to calculate the benefit–cost ratio by using Eq. (8.5). This is summarized in Table 8.3. The *B/C* ratio is 6.10, indicating that for each dollar of expenditure, it is generating \$6.10 of benefits in present worth. This is an exceptionally good highway improvement project in terms of spending taxpayers’ money. Of course, there are many uncertain parameters assumed in this analysis, so it is important to conduct a series of sensitivity as well as risk analyses before undertaking a large-scale investment such as this highway project.

⁵ The detailed procedure to obtain these savings figures is not presented here but can be found in the Benefit–Cost Analysis Guidance by the State of Minnesota, Department of Transportation.

TABLE 8.2 Calculate Total Present Cost for the Base Case and Alternative(s)

| Year | Capital Cost | | Maintenance Cost | | Remaining Capital Value | | Present Value of Net Annual Costs |
|------------------|--------------|--------------|------------------|-------------|-------------------------|-------------|-----------------------------------|
| | Base Case | Alternative | Base Case | Alternative | Base Case | Alternative | |
| 2008 | - | \$0 | \$94,115,754 | \$0 | \$0 | \$0 | \$0 |
| 2009 | - | \$0 | \$90,933,096 | \$0 | \$0 | \$0 | \$90,933,096 |
| 2010 | - | \$0 | \$87,858,064 | \$0 | \$0 | \$0 | \$87,858,064 |
| 2011 | 1 | \$0 | \$0 | \$391,887 | \$218,226 | \$0 | -\$173,662 |
| 2012 | 2 | \$0 | \$0 | \$378,635 | \$210,846 | \$0 | -\$167,789 |
| 2013 | 3 | \$0 | \$0 | \$384,120 | \$203,716 | \$0 | -\$180,404 |
| 2014 | 4 | \$0 | \$0 | \$353,460 | \$196,827 | \$0 | -\$156,633 |
| 2015 | 5 | \$0 | \$0 | \$341,507 | \$190,171 | \$0 | -\$151,336 |
| 2016 | 6 | \$0 | \$0 | \$329,959 | \$183,740 | \$0 | -\$146,218 |
| 2017 | 7 | \$0 | \$0 | \$470,084 | \$177,527 | \$0 | -\$292,557 |
| 2018 | 8 | \$0 | \$0 | \$448,041 | \$171,523 | \$0 | -\$276,518 |
| 2019 | 9 | \$0 | \$0 | \$297,604 | \$165,723 | \$0 | -\$131,881 |
| 2020 | 10 | \$7,497,177 | \$0 | \$37,080 | \$160,119 | \$0 | -\$7,374,138 |
| 2021 | 11 | \$7,243,649 | \$0 | \$27,075 | \$154,704 | \$0 | -\$7,116,019 |
| 2022 | 12 | \$0 | \$0 | \$268,421 | \$149,473 | \$0 | -\$118,949 |
| 2023 | 13 | \$0 | \$0 | \$259,344 | \$144,418 | \$0 | -\$114,926 |
| 2024 | 14 | \$0 | \$0 | \$250,574 | \$139,534 | \$0 | -\$111,040 |
| 2025 | 15 | \$0 | \$0 | \$242,101 | \$134,816 | \$0 | -\$107,285 |
| 2026 | 16 | \$0 | \$0 | \$503,821 | \$130,257 | \$0 | -\$373,564 |
| 2027 | 17 | \$0 | \$0 | \$226,004 | \$125,852 | \$0 | -\$100,152 |
| 2028 | 18 | \$0 | \$0 | \$218,361 | \$121,596 | \$0 | -\$96,765 |
| 2029 | 19 | \$0 | \$0 | \$364,368 | \$117,484 | \$0 | -\$246,883 |
| 2030 | 20 | \$0 | \$0 | \$203,842 | \$113,511 | \$9,673,099 | \$91,944,562 |
| 2011–2030 | | \$14,740,826 | \$272,906,915 | \$5,996,288 | \$3,210,064 | \$9,673,099 | \$91,944,562 |
| | | | | | | | \$173,108,403 |

TABLE 8.3 Summary of Benefit–Cost Analysis for the Proposed Highway Improvement Project

| BENEFIT–COST ANALYSIS | |
|--|------------|
| SUMMARY RESULTS | |
| Net Cost of Project (mil. \$) | \$173.11 |
| Present Value of Benefits (mil. \$) | \$1,055.86 |
| Net Present Value (mil. \$) | \$882.75 |
| Benefit/Cost Ratio | 6.10 |
| PRESENT VALUE OF ITEMIZED BENEFITS (mil. \$) | |
| VMT Savings | \$96.97 |
| VHT Savings | \$915.47 |
| Accident Reduction Benefits | \$43.42 |
| PRESENT VALUE OF TOTAL BENEFITS (mil. \$) | \$1,055.86 |
| PRESENT VALUE OF ITEMIZED COSTS (mil. \$) | |
| Capital Cost Differential | \$258.17 |
| Maintenance Cost Differential | −\$2.79 |
| Remaining Capital Value Differential | \$82.27 |
| PRESENT VALUE OF TOTAL COSTS (mil. \$) | \$173.11 |

SUMMARY

■ **Benefit–cost analysis** is commonly used to evaluate public projects. Several facets unique to public-project analysis are addressed by benefit–cost analysis:

1. Benefits of a nonmonetary nature can be quantified and factored into the analysis.
 2. A broad range of project users distinct from the sponsor are considered; benefits and disbenefits to *all* these users can (and should) be taken into account.
- Difficulties involved in public-project analysis include
1. Identifying all the users of the project.
 2. Identifying all the benefits and disbenefits of the project.
 3. Quantifying all the benefits and disbenefits in dollars or some other unit of measure.
 4. Selecting an appropriate interest rate at which to discount benefits and costs to a present value.

■ The B/C ratio is defined as

$$BC(i) = \frac{B}{C} = \frac{B}{I + C'} \text{ where } I + C' > 0.$$

The decision rule is that, if $BC(i) > 1$, the project is acceptable.

■ The profitability index is defined as

$$PI(i) = \frac{B - C'}{I}, I > 0.$$

The profitability index measures the bang for the buck invested. In a way, it is a measure of capital efficiency.

- When comparing mutually exclusive projects based on either B/C or PI , we need to use the incremental analysis.

SELF-TEST QUESTIONS

- 8s.1 A city is trying to decide whether to build a parking garage. An engineering plan calculates that it takes a year to build at a cost of \$2 million and \$200,000 per year to operate. An in-depth analysis of operating revenue determines that the garage will start to earn revenues of \$500,000 per year in the second year. The city is interested in knowing whether this project will be profitable ($BC > 1$) over the next eight years (counting the year of construction) at 10%. The project's B/C ratio over the eight-year period is closest to
- 0.77
 - 0.87
 - 1.33
 - 2.50
- 8s.2 A city government is considering increasing the capacity of the current wastewater treatment plant. The estimated financial data for the project are as follows:

| Description | Data |
|-----------------------------|-------------|
| Capital investment | \$1,200,000 |
| Project life | 25 years |
| Incremental annual benefits | \$250,000 |
| Incremental annual costs | \$100,000 |
| Salvage value | \$50,000 |
| Discount rate | 6% |

What would be the benefit–cost ratio for this expansion project?

- 3.26
 - 3.12
 - 1.30
 - 2.23
- 8s.3 Auburn Recreation and Parks Department is considering two mutually exclusive proposals for a new softball complex on a city-owned lot.

| Alternative Design | Seating Capacity | Annual Benefits | Annual Costs | Required Investment |
|--------------------|------------------|-----------------|--------------|---------------------|
| A1 | 3,000 | \$194,000 | \$87,500 | \$800,000 |
| A2 | 4,000 | \$224,000 | \$105,000 | \$1,000,000 |

The complex will be useful for 30 years and has no appreciable salvage value (regardless of seating capacity). Assuming an 8% discount rate, which of the following statements is *incorrect*?

- Select A1 because it has the largest B/C ratio.
- Select A1 because it has the most benefits per seating capacity.
- Select A1 because it has the largest PW.
- Select A1 because the incremental benefits generated from A2 are not large enough to offset the additional investment (\$200,000 more than A1).

8s.4 The city of Jefferson is reviewing the benefits and costs of a potential universal water-metering program. Two options are considered:

- **Option 1: Treated Surface Water Only** Benefits of Deferring/Downsizing Water and Sewer Projects—\$28.1 million
Costs of Metering—\$9.3 million Benefit–Cost Ratio—3.02
- **Option 2: Treated Surface Water Supplemented By 40 ML/Day Groundwater**
Benefits of Deferring/Downsizing Water and Sewer Projects—\$38.9 million
Costs of Metering (and groundwater development)—\$14.7 million
Benefit–Cost Ratio—2.65

Which option should the city implement, assuming that the city has enough money to fund either project?

- Option 1
- Option 2
- Neither
- Both options

8s.5 The Texas Department of Transportation is considering in improving accident prevention countermeasures on the state's accident-prone public highways and bridges. The following set of projects has been recommended for evaluation at three different locations and assumes the budget is \$25 million. All alternatives are mutually *independent* projects.

| Location | Alternative | Benefit ($B - C'$) | Cost (I) | PI Ratio |
|----------|-------------|----------------------|--------------|----------|
| I | I-A | \$45 | 12 | 3.75 |
| | I-B | 30 | 9 | 3.33 |
| II | II-A | 35 | 6 | 5.83 |
| | II-B | 20 | 12 | 1.67 |
| III | III-A | 25 | 2 | 12.5 |
| | III-B | 30 | 7 | 4.29 |

Determine the best combination of projects within the budget constraint.

- II-B and III-B only
- I-A, II-A, and III-B
- I-B, III-A, and III-B
- II-B and III-B

8s.6 The Ulrich Corporation is trying to choose between the following two mutually exclusive design projects. If the required rate of return is 10%, which of the following statements is correct?

| Period | Project A1 | Project A2 |
|----------|------------|------------|
| 0 | -\$55,000 | -\$18,500 |
| 1 | \$38,000 | \$15,000 |
| 2 | \$38,000 | \$15,000 |
| PI index | 1.19 | 1.40 |
| ROR | 24.56% | 39.29% |

- (a) Select A2 because it has a higher PI.
 (b) Select A2 because it has a higher IRR.
 (c) Select A1 because its incremental PI exceeds 1.
 (d) Not enough information to decide.
- 8s.7 A regional airport is considering installing a new baggage handling system. Two different vendors submitted bids on the system. The airport is to replace its current system. The cash flows that describe each baggage handling system with respect to the current system are given below.

| Years (n) | Vendor A | Vendor B | Vendor B--Vendor A |
|------------------|-----------------|-----------------|---------------------------|
| 0 | -\$500,000 | -\$600,000 | -\$100,000 |
| 1–15 | \$48,170 | \$65,880 | \$17,710 |
| IRR | 5% | 7% | 15.73% |
| PI (4%) | 1.07 | 1.22 | 1.97 |

- Which of the following statements is correct?
- (a) Vendor B is preferred as its PI is higher than that of Vendor A.
 (b) Vendor B is preferred as long as the airport's interest rate is more than 5%.
 (c) Vendor B is preferred as long as the airport's interest rate is less than 7%.
 (d) Vendor A is preferred if the airport's interest rate is more than 15.73%.
- 8s.8 The U.S. Department of Interior is planning to build a dam and construct a hydroelectric power plant. In addition to producing electric power, this project will provide flood control, irrigation, and recreational benefits. The estimated benefits and costs expected to be derived from the three alternatives under consideration are listed as follows:

| | Design Alternatives | | |
|---------------------------|----------------------------|----------|----------|
| | A | B | C |
| Initial investment | \$100M | \$160M | \$220M |
| Annual recurring benefits | \$20M | \$40M | \$55M |
| Annual recurring costs | \$8M | \$18M | \$25M |
| Salvage value | 0 | 0 | 0 |

The interest rate is 5%, and the life of each project is estimated to be 40 years. Which one of the followings is most correct?

- (a) Select B as its BC ratio is the largest among the three alternatives.
 (b) Select C as its incremental BC ratio over either A or B exceeds 1.
 (c) Select A as it requires the least investment and recurring annual cost.
 (d) Select B as its incremental BC ratio over A exceeds 1.

PROBLEMS

Benefits–Cost Analyses

- 8.1 A local government is considering promoting tourism in the city. It will cost \$5,000 to develop a plan. The anticipated annual benefits and costs are as follows:

| | |
|---|-----------|
| Annual benefits: Increased local income and tax collections | \$117,400 |
| Annual support service: Parking lot expansion, rest room, patrol car, and street repair | \$48,830 |

If the city government uses a discount rate of 6% and a study period of five years, is this tourism project justifiable according to the benefit–cost analysis?

- 8.2 A city government is considering increasing the capacity of the current waste-water treatment plant. The estimated financial data for the project are as follows:

| Description | Data |
|-----------------------------|-------------|
| Capital investment | \$1,200,000 |
| Project life | 25 years |
| Incremental annual benefits | \$250,000 |
| Incremental annual costs | \$100,000 |
| Salvage value | \$50,000 |
| Discount rate | 6% |

Calculate the benefit–cost ratio for this capacity expansion project.

Incremental Benefit–Cost Analysis

- 8.3 A city government is considering two types of town-dump sanitary systems. Design A requires an initial outlay of \$400,000 with annual operating and maintenance costs of \$50,000 for the next 15 years; design B calls for an investment of \$300,000 with annual operating and maintenance costs of \$80,000 per year for the next 15 years. Fee collections from the residents would again be \$85,000 per year. The interest rate is 8%, and no salvage value is associated with either system.
- (a) Using the benefit–cost ratio $BC(i)$, which system should be selected?
 - (b) If a new design (design C), which requires an initial outlay of \$350,000 and annual operating and maintenance costs of \$65,000, is proposed, would your answer in part (a) change?
- 8.4 The U.S. government is considering building apartments for government employees working in a foreign country and living in locally owned housing. A comparison of two possible buildings is given in Table P8.4.

TABLE P8.4

| | Building X | Building Y |
|--|-------------------|-------------------|
| Original investment by government agencies | \$8,000,000 | \$12,000,000 |
| Estimated annual maintenance costs | \$240,000 | \$180,000 |
| Savings in annual rent now being paid to house employees | \$1,960,000 | \$1,320,000 |

Assume the salvage or sale value of the apartments to be 60% of the first investment. Use 10% and a 20-year study period to compute the B/C ratio on incremental investment, and make a recommendation. (Assume no do-nothing alternative.)

- 8.5 Two different routes are under consideration for a new interstate highway:

TABLE P8.5

| | Length of Highway | First Cost | Annual Upkeep |
|------------------------|--------------------------|-------------------|----------------------|
| The “long” route | 22 miles | \$21 million | \$140,000 |
| Transmountain shortcut | 10 miles | \$45 million | \$165,000 |

For either route, the volume of traffic will be 400,000 cars per year. These cars are assumed to operate at \$0.25 per mile. Assume a 40-year life for each road and an interest rate of 10%. Determine which route should be selected.

- 8.6 Three public-investment alternatives with the same service life are available: A1, A2, and A3. Their respective total benefits, costs, and first costs are given in present worth as follows:

TABLE P8.6

| Present worth | Proposals | | |
|----------------------|------------------|-----------|-----------|
| | A1 | A2 | A3 |
| <i>I</i> | 100 | 300 | 200 |
| <i>B</i> | 400 | 700 | 500 |
| <i>C'</i> | 100 | 200 | 150 |

Assuming no do-nothing alternative, which project would you select on the basis of the benefit–cost ratio $BC(i)$ on incremental investment?

- 8.7 A local city that operates automobile parking facilities is evaluating a proposal that it will erect and operate a structure for parking in the city’s downtown area. Three designs for a facility to be built on available sites have been identified in Table P8.7 (all dollar figures are in thousands).

TABLE P8.7

| | Design A | Design B | Design C |
|-------------------------|-----------------|-----------------|-----------------|
| Cost of site | \$240 | \$180 | \$200 |
| Cost of building | \$2,200 | \$700 | \$1,400 |
| Annual fee collection | \$830 | \$750 | \$600 |
| Annual maintenance cost | \$410 | \$360 | \$310 |
| Service life (years) | 30 | 30 | 30 |

At the end of the estimated service life, whichever facility had been constructed would be torn down, and the land would be sold. It is estimated that the proceeds from the resale of the land will be equal to the cost of clearing the site. If the city's interest rate is known to be 10%, which design alternative would be selected on the basis of the benefit–cost criterion?

- 8.8 The federal government is planning a hydroelectric project for a river basin. In addition to producing electric power, this project will provide flood control, irrigation, and recreational benefits. The estimated benefits and costs expected to be derived from the three alternatives under consideration are listed in Table P8.8.

TABLE P8.8

| | Decision Alternatives | | |
|---------------------------|------------------------------|--------------|--------------|
| | A | B | C |
| Initial cost | \$8,000,000 | \$10,000,000 | \$15,000,000 |
| Annual benefits or costs: | | | |
| Power sales | \$1,000,000 | \$1,200,000 | \$1,800,000 |
| Flood-control savings | \$250,000 | \$350,000 | \$500,000 |
| Irrigation benefits | \$350,000 | \$450,000 | \$600,000 |
| Recreation benefits | \$100,000 | \$200,000 | \$350,000 |
| O&M costs | \$200,000 | \$250,000 | \$350,000 |

The interest rate is 10%, and the life of each of the projects is estimated to be 50 years.

- (a) Find the benefit–cost ratio for each alternative.
 (b) Select the best alternative on the basis of $BC(i)$.
- 8.9 The U.S. Department of Interior is planning to build a dam and construct a hydroelectric power plant. In addition to producing electric power, this project will provide flood control, irrigation, and recreational benefits. The estimated benefits and costs expected to be derived from the three alternatives under consideration are listed as follows:

TABLE P8.9

| | Design Alternatives | | |
|---------------------------|---------------------|--------|--------|
| | A | B | C |
| Initial investment | \$100M | \$160M | \$220M |
| Annual recurring benefits | \$20M | \$40M | \$60M |
| Annual recurring costs | \$8M | \$18M | \$25M |
| Salvage value | 0 | 0 | 0 |

The interest rate is 5%, and the life of each project is estimated to be 40 years.

- (a) Find the benefit–cost ratio for each alternative.
- (b) Select the best alternative, according to $BC(i)$.

- 8.10 The government is considering undertaking four projects. These projects are mutually exclusive, and the estimated present worth of their costs and the present worth of their benefits are shown in millions of dollars in Table P8.10. All of the projects have the same duration.

TABLE P8.10

| Projects | PW of Benefits | PW of Costs |
|----------|----------------|-------------|
| A1 | \$40 | \$85 |
| A2 | \$150 | \$110 |
| A3 | \$70 | \$25 |
| A4 | \$120 | \$73 |

Assuming no do-nothing alternative, which alternative would you select? Justify your choice by using a benefit–cost ($BC(i)$) analysis on incremental investment.

Profitability Index

- 8.11 Consider the following two projects:

| | Project A1 | Project A2 |
|---------------------|------------|-------------|
| Investment | | |
| Required at $n = 0$ | \$900,000 | \$1,200,000 |
| | | |
| 1 | \$400,000 | \$200,000 |
| 2 | \$340,000 | \$300,000 |
| 3 | \$300,000 | \$350,000 |
| 4 | \$240,000 | \$440,000 |
| 5 | \$200,000 | \$400,000 |
| 6 | \$150,000 | \$350,000 |

- (a) Calculate the profitability index for A1 and A2 at an interest rate of 6%.
 (b) Determine which project(s) you should accept (1) if you have enough money to undertake both and (2) if you could take only one due to a budget limit.

8.12 Consider the following two mutually exclusive projects:

| | Project A1 | Project A2 |
|--------------------------------------|------------|-------------|
| Investment Required at $n = 0$ | \$750,000 | \$1,000,000 |
| 1 | \$100,000 | \$200,000 |
| 2 | \$100,000 | \$200,000 |
| 3 | \$100,000 | \$200,000 |
| 4 | \$240,000 | \$300,000 |
| 5 | \$200,000 | \$300,000 |
| 6 | \$180,000 | \$250,000 |
| 7 | \$180,000 | \$250,000 |
| 8 | \$180,000 | \$150,000 |
| 9 | \$180,000 | \$100,000 |
| 10 | \$180,000 | \$50,000 |

- (a) Which project has the higher profitability index at $i = 7\%$?
 (b) Which project would you choose if you can raise an unlimited amount of funds in financing either project at 7%? Use the profitability index rule.

Short Case Studies

8.13 The city of Portland Sanitation Department is responsible for the collection and disposal of all solid waste within the city limits. The city must collect and dispose of an average of 300 tons of garbage each day. The city is considering ways to improve the current solid-waste collection and disposal system:

- The present collection and disposal system uses Dempster Dumpmaster front-end loaders for collection and disposal. Each collecting vehicle has a load capacity of 10 tons, or 24 cubic yards, and dumping is automatic. The incinerator in use was manufactured in 1962. It was designed to incinerate 150 tons every 24 hours. A natural gas afterburner has been added in an effort to reduce air pollution; however, the incinerator still does not meet state air-pollution requirements. Prison-farm labor is used for the operation of the incinerator. Because the capacity of the incinerator is relatively low, some trash is not incinerated but is taken to the city landfill instead. The trash landfill and the incinerator are located approximately 11 miles and

5 miles, respectively, from the center of the city. The mileage and costs in person-hours for delivery to the disposal sites are excessive; a high percentage of empty vehicle miles and person-hours are required because separate methods of disposal are used and the destination sites are remote from the collection areas. The operating cost for the present system is \$905,400. This figure includes \$624,635 to operate the prison-farm incinerator, \$222,928 to operate the existing landfill, and \$57,837 to maintain the current incinerator.

- The proposed system calls for a number of portable incinerators, each with 100-ton-per-day capacity for the collection and disposal of refuse waste from three designated areas within the city. Collection vehicles will also be staged at these incineration disposal sites with the necessary plant and support facilities for incinerator operation and collection-vehicle fueling and washing, and with support building for stores as well as shower and locker rooms for collection and site personnel. The pickup and collection procedure remains essentially the same as in the existing system. The disposal-staging sites, however, are located strategically in the city according to the volume and location of wastes collected; thus, long hauls are eliminated and the number of miles the collection vehicles must travel from pickup to disposal site and back is reduced.

Four variations of the proposed system are being considered, containing one, two, three, and four incinerator-staging areas, respectively. The type of incinerator used in each variation is a modular prepackaged unit, which can be installed at several sites in the city. All of the four units meet state and federal standards on exhaust emissions. The city of Portland needs 24 units, each with a rated capacity of 8.5 tons of garbage per 24 hours. The price per unit is \$137,600, which means a capital investment of about \$3,302,000. The estimated plant facilities, such as housing and foundation, are estimated to cost \$200,000 per facility. This figure is based on a plan incorporating four incinerator plants strategically located around the city. Each plant would house eight units and would be capable of handling 100 tons of garbage per day. Additional plant features, such as landscaping, are estimated to cost \$60,000 per plant.

The annual operating cost of the proposed system would vary according to the type of system configuration. It takes about 1.5 to 1.7 MCF of fuel to incinerate 1 ton of garbage. The conservative 1.7 MCF figure was used for total cost, resulting in a fuel cost of \$4.25 per ton of garbage at a cost of \$2.50 per MCF. Electric requirements at each plant will be 230 kW per day. For a plant operating at full capacity, that requirement means a \$0.48-per-ton cost for electricity. Two workers can easily operate one plant, but safety factors dictate that there will be three operators at a cost of \$7.14 per hour per operator. This translates to a cost of \$1.72 per ton. The maintenance cost of each plant is estimated to be \$1.19 per ton. Since the proposal for three plants will require fewer transportation miles, it is necessary to consider the savings accruing from this operating advantage. For example, the configuration with three plant locations will save 6.14 miles per truck per day on average. At an estimated \$0.30-per-mile cost, this would mean that an annual savings of \$15,300 would be realized from minimum trips to the landfill. Labor savings are also realized because the shorter routes would permit more pickups per day. This plan thus offers an annual labor savings of \$103,500 for three incinerators. The following table summarizes all costs, in thousands of dollars, associated with the present and proposed systems.

| Item | Present System | Number of Incinerator Sites | | | |
|-----------------------------|----------------|-----------------------------|---------|---------|----------|
| | | 1 | 2 | 3 | 4 |
| Capital costs | | | | | |
| Incinerators | | \$3,302 | \$3,302 | \$3,302 | \$3,302 |
| Plant facilities | | \$600 | \$900 | \$1,260 | \$1,920 |
| Annex buildings | | \$91 | \$102 | \$112 | \$132 |
| Additional features | | \$60 | \$80 | \$90 | \$100 |
| Total | | \$4,053 | \$4,384 | \$4,764 | \$5,454 |
| Annual O&M costs | \$905.4 | \$342 | \$480 | \$414 | \$408 |
| Annual savings | | | | | |
| Pickup transportation | | \$13.2 | \$14.7 | \$15.3 | \$17.1 |
| Labor | | \$87.6 | \$99.3 | \$103.5 | \$119.40 |

A bond will be issued to provide the necessary capital investment at an interest rate of 8% with a maturity date 20 years.

The proposed systems are expected to last 20 years with negligible salvage values. If the current system is to be retained, the annual O&M costs would be expected to increase at an annual rate of 10%. The city will use the bond interest rate as the interest rate for any public-project evaluation.

- (a) Determine the operating cost of the current system in terms of dollars per ton of solid waste.
- (b) Determine the economics of each solid-waste disposal alternative in terms of dollars per ton of solid waste.
- (c) Select the best alternative based on the benefit–cost ratio.