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2.1.2

a) $a(ba)^*$

b) a^*b

c) aynı sayıda a's ve b'si de sahip olan ve hiçbir öneki a's den 2 reya daha fazla b's içermeyen tüm dizeeler

d) aynı sayıda a's ve b'si de sahip olan ve hiçbir öneki bir'den fazla a'ya sahip olmamıştır tüm dizeeler veya tersi

e) aab veya bba alt dizesini içeren tüm dizeeler

2.1.3

a) $\{w : \text{each } a \text{ in } w \text{ immediately preceded by } ab\}$:

$$\mathcal{L} = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$\delta = q_0$$

$$F = \{q_1\}$$

	a	σ	$\delta(q, \sigma)$
q_0	a	q_1	
q_0	b	q_1	
q_1	a	q_0	
q_1	b	q_1	
q_1	a	q_1	
q_2	b	q_2	

b) $\{w : \text{neither abab nor babb is a substring of } w\}$:

$$\mathcal{L} = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b\}$$

$$\delta = q_0$$

$$F = \{q_4\}$$

	a	σ	$\delta(q, \sigma)$
q_0	a	q_1	
q_0	b	q_0	
q_1	a	q_1	
q_1	b	q_2	
q_2	a	q_2	
q_2	b	q_0	
q_3	a	q_3	
q_3	b	q_1	
q_4	a	q_4	
q_4	b	q_1	

c) $\{w : \text{neither aa nor bb is a substring of } w\}$:

$$\mathcal{L} = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$$\delta = q_0$$

$$F = \{q_0, q_1, q_2\}$$

	a	σ	$\delta(q, \sigma)$
q_0	a	q_1	
q_0	b	q_2	
q_1	a	q_3	
q_1	b	q_1	
q_2	a	q_1	
q_2	b	q_3	
q_3	a	q_1	
q_3	b	q_3	

d-) $\{w : w \text{ has an odd number of } a's \text{ and even number of } b's\}$:

$$L = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$$q = q_0$$

$$F = \{q_3\}$$

q	σ	$\delta(q, \sigma)$
q_0	a	q_1
q_0	b	q_2
q_1	a	q_0
q_1	b	q_3
q_2	a	q_3
q_2	b	q_0
q_3	a	q_2
q_3	b	q_1

e-) $\{w : w \text{ has both } ab \text{ and } ba \text{ as substrings}\}$

$$L = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

$$\Sigma = \{a, b\}$$

$$q = q_0$$

$$F = \{q_5\}$$

q	σ	$\delta(q, \sigma)$
q_0	a	q_1
q_0	b	q_1
q_1	a	q_2
q_1	b	q_1
q_2	a	q_3
q_2	b	q_4
q_3	a	q_2
q_3	b	q_5
q_4	a	q_5
q_4	b	q_5
q_5	a	q_5
q_5	b	q_5

2.1.7

Let M' be the machine formed from M by deleting the unreachable state p . We show $L(M) = L(M')$. As a lemma, we will show that (q_1, π_2) $\xrightarrow{*M} (q_2, y)$ implies $(q, x y) \xrightarrow{*M} (q_2, y)$, where q_1 is any reachable state of M (q_2 is trivially reachable, as well, given the hypothesis). We prove this lemma by induction on $|w|$.

Base step: $|x|=0$ then $x=\epsilon$, so that $(q, y) \xrightarrow{*M} (q_2, y)$ which forces $q=q_2$.
Thus by reflexivity $(q_1, y) \xrightarrow{*M'} (q_2, y)$.

Induction step: suppose $(q, x y) \xrightarrow{*M} (q_2, y)$ where $|x|=n+1$. Then $x=w\sigma$, where $\sigma \in \Sigma$ and $|w|=n$. Let r be the state such that $(q_1, x\sigma) \xrightarrow{*M} r$. Because q_1 is reachable there is some string $(r, \sigma y) \xrightarrow{*M} (q_2, y)$. Because q_1 is reachable there is some string $(s, \sigma y) \xrightarrow{*M} (q_1, \epsilon)$. Therefore $(s, \sigma y) \xrightarrow{*M} r$ and r is a state of M' and $s \in F$. Whenever it exists is the same as δ_M , $\delta_{M'}(r, \sigma) = \delta_M(r, \sigma) = q_2$, so that $(r, \sigma y) \xrightarrow{*M'} (q_2, y)$ and thus $(q, x y) \xrightarrow{*M'} (q_2, y)$.

Suppose now that $w \in L(M)$. By definition of $L(M)$, $(s, w) \xrightarrow{*M} (q, \epsilon)$ for some $s \in F$ and $q \in Q$. By the lemma $(s, w) \xrightarrow{*M'} (q, \epsilon)$ so that $w \in L(M')$.