Report of Computing Assignment 8 of MACM 316

The first method is the standard Euler's method, and the approximate position of the moving planet at time $t_n \in [0,200]$ is shown in Figure 1. It is clear that the orbit is not closed but gradually enlarging (regarding the orbit as ellipses, the major and minor axes are increasing). As for the angular momentum and Hamiltonian, as shown in Figure 2 and 3, the numerical solution does not conserve these two quantities at all, and it is obvious that both of A(t) and H(t) are gradually increasing (stepped increasing) when t gets larger.

The second method is the symplectic Euler's method. The orbit is shown in Figure 4, and the plots of the two quantities are shown in Figure 5 and 6. It is clear that, in this case, the orbit is closed and much like an ellipse. As for the two quantities, when t is between 0 and 200 (200 is a large enough upper bound to see the behaviours), angular momentum is oscillating around 0.8, with the amplitude of less than 4×10^{-14}, and Hamiltonian is oscillating around -0.5, with the amplitude of less than 8×10^{-4}.

Generally, the symplectic Euler's method is better than the standard one. Because when using the first method, the planetary orbit is not closed but enlarging, which is impossible and makes no sense in the two-body problem. Even though the two quantities A(t) and H(t) are oscillating when adapting the symplectic method, the amplitudes are quite small (similar to dynamic balance). Such phenomenon shows that when using the symplectic method, these two quantities are much more like conserved than using the standard method. To conclude, the symplectic Euler's method is better than the standard one.

Figure 1: Standard Euler's Method

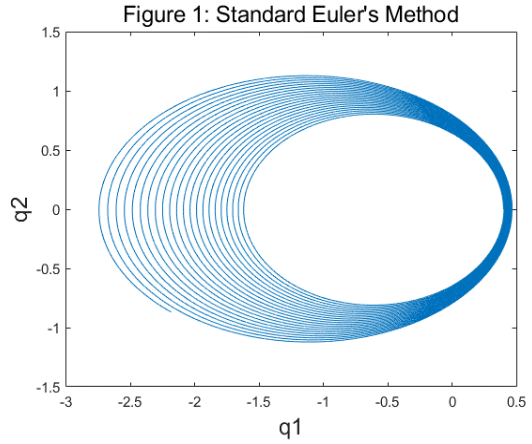


Figure 2: Augular Momentum

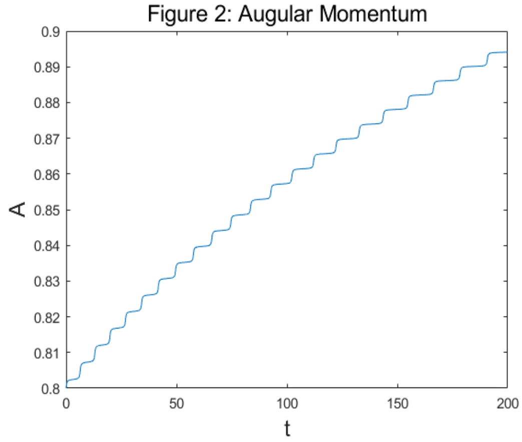


Figure 3: Hamitonian

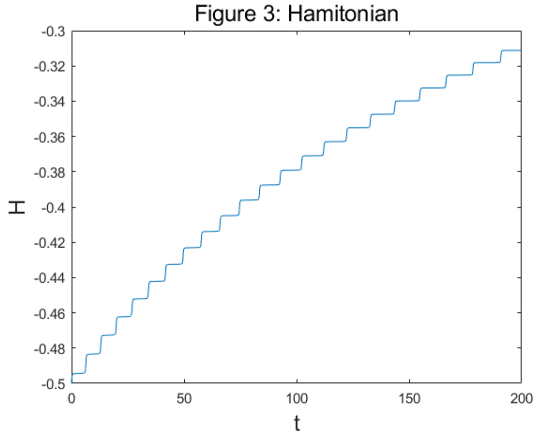


Figure 4: Symplectic Euler's Method

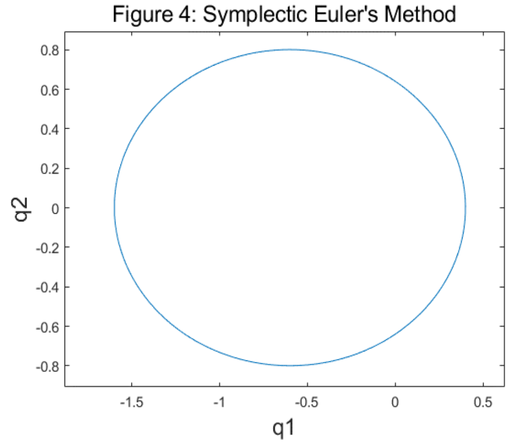


Figure 5: New Augular Momentum

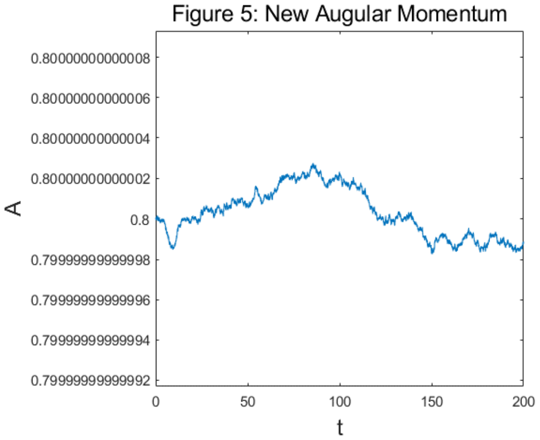
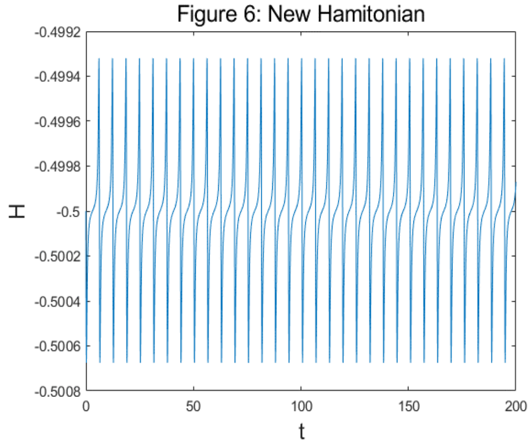


Figure 6: New Hamitonian



Codes for Standard Euler's Method:

```
a = 0; % Start time
b = 200; % End time
e = 0.6;

%Initial conditions
q10 = 1-e;
q20 = 0;
p10 = 0;
p20 = sqrt((1+e)/(1-e));


%Stepsize and mesh
h = 0.0005;

t = a:h:b; % Mesh
N = length(t);
q1 = zeros(N,1); % Values for q1
q2 = zeros(N,1); % Values for q2
p1 = zeros(N,1); % Values for p1
p2 = zeros(N,1); % Values for p2

q1(1) = q10; % Initial values
q2(1) = q20; % Initial values
p1(1) = p10; % Initial values
p2(1) = p20; % Initial values

%Euler steps

for i = 1: N-1
    q1(i+1) = q1(i) + h * p1(i);
    q2(i+1) = q2(i) + h * p2(i);
    q = [q1(i),q2(i)];
    p1(i+1) = p1(i) - h * q1(i)/norm(q)^3;
    p2(i+1) = p2(i) - h * q2(i)/norm(q)^3;
%     p1(i+1) = p1(i) + h * (-(q1(i))/(((q1(i))^2+(q2(i))^2)^(3/2)));
%     p2(i+1) = p2(i) + h * (-(q2(i))/(((q1(i))^2+(q2(i))^2)^(3/2)));
end

A = q1.*p2 - q2.*p1;
H = 0.5*(p1.^2+p2.^2)-1./(q1.^2+q2.^2).^(1/2);

figure(1);
plot(q1,q2)
title('Figure 1: Standard Euler''s Method','fontsize',16)
xlabel('q1','fontsize',16)
ylabel('q2','fontsize',16)

figure(2);
plot(t,A);
```

```
title('Figure 2: Augular Momentum','fontsize',16)
xlabel('t','fontsize',16)
ylabel('A','fontsize',16)

figure(3);
plot(t,H);
title('Figure 3: Hamitonian','fontsize',16)
xlabel('t','fontsize',16)
ylabel('H','fontsize',16)
```

Codes for Symplectic Euler's Method:

```
a = 0; % Start time
b = 200; % End time
e = 0.6;

%Initial conditions
q10 = 1-e;
q20 = 0;
p10 = 0;
p20 = sqrt((1+e)/(1-e));


%Stepsize and mesh
h = 0.0005;

t = a:h:b; % Mesh
N = length(t);
q1 = zeros(N,1); % Values for q1
q2 = zeros(N,1); % Values for q2
p1 = zeros(N,1); % Values for p1
p2 = zeros(N,1); % Values for p2

q1(1) = q10; % Initial values
q2(1) = q20; % Initial values
p1(1) = p10; % Initial values
p2(1) = p20; % Initial values

%Euler steps

for i = 1: N-1
    q1(i+1) = q1(i) + h * p1(i);
    q2(i+1) = q2(i) + h * p2(i);
    q = [q1(i+1),q2(i+1)];
    p1(i+1) = p1(i) - h * q1(i+1)/norm(q)^3;
    p2(i+1) = p2(i) - h * q2(i+1)/norm(q)^3;
    % p1(i+1) = p1(i) - h * ((q1(i+1))/(((q1(i+1))^2+(q2(i+1))^2)^(3/2)));
    % p2(i+1) = p2(i) - h * ((q2(i+1))/(((q1(i+1))^2+(q2(i+1))^2)^(3/2)));
end

A = q1.*p2 - q2.*p1;
```

```
H = 0.5*(p1.^2+p2.^2)-1./(q1.^2+q2.^2).^(1/2);

figure(1);
plot(q1,q2)
title('Figure 4: Symplectic Euler''s Method','fontsize',16)
xlabel('q1','fontsize',16)
ylabel('q2','fontsize',16)

figure(2);
plot(t,A);
title('Figure 5: New Augular Momentum','fontsize',16)
xlabel('t','fontsize',16)
ylabel('A','fontsize',16)

figure(3);
plot(t,H);
title('Figure 6: New Hamitonian','fontsize',16)
xlabel('t','fontsize',16)
ylabel('H','fontsize',16)
```