

Static Program Analysis For Security

Cambridge IB Tech Talks

Zayne Zhang

zz513@cam.ac.uk

March 12, 2025

Overview

1. Introduction to Static Program Analysis
2. Lattices and Fixed Points
3. Data Flow Analysis
4. CodeQL

How to find bugs in code?

Dynamic Analysis (e.g. Unit Testing, Fuzzing)

- Done on a limited number of different inputs
- Often reveals the presence of errors but **cannot guarantee their absence**
- 100% test coverage \neq bug-free code
- Many tests are regressive and only added after a bug is found

Static Analysis: analyze code without executing it

- Can check all possible executions and provide guarantees about its behavior
- With the right tools, can catch bugs early in the development process
- Particularly useful for testing the absence of security vulnerabilities

How to find bugs in code?



Brenan Keller
@brenankeller



A QA engineer walks into a bar.
Orders a beer. Orders 0 beers.
Orders 999999999999 beers.
Orders a lizard. Orders -1 beers.
Orders a ueicbksjdhd.

First real customer walks in
and asks where the bathroom
is. The bar bursts into flames,
killing everyone.

1:21 PM · 30 Nov 18

Data Flow Analysis

A particular form of static analysis that **examines how data moves through a program** to answer questions such as:

- What values can reach this point in the code?
- Is this variable always initialized before it is used?
- **Does untrusted data ever reach an unsafe function?**

Partial Orders

Definition

A **partial order** (S, \sqsubseteq) is a set S equipped with a binary relation \sqsubseteq that is:

- **Reflexive:** $\forall x \in S, x \sqsubseteq x$
 - **Transitive:** $\forall x, y, z \in S, x \sqsubseteq y \wedge y \sqsubseteq z \Rightarrow x \sqsubseteq z$
 - **Antisymmetric:** $\forall x, y \in S, x \sqsubseteq y \wedge y \sqsubseteq x \Rightarrow x = y$
-
- $y \in S$ is an upper bound for X ($X \sqsubseteq y$) if $\forall x \in X, x \sqsubseteq y$
 - $y \in S$ is the least upper bound for X ($X \sqcup y$) if y is an upper bound for X and $\forall z \in S, X \sqsubseteq z \Rightarrow y \sqsubseteq z$
 - $y \in S$ is a lower bound for X ($y \sqsubseteq X$) if $\forall x \in X, y \sqsubseteq x$
 - $y \in S$ is the greatest lower bound for X ($y \sqcap X$) if y is a lower bound for X and $\forall z \in S, z \sqsubseteq X \Rightarrow z \sqsubseteq y$

Lattices

Definition

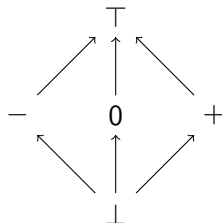
A **lattice** (L, \sqsubseteq) is a partial order (L, \sqsubseteq) in which every pair of elements $x, y \in L$ has a least upper bound $x \sqcup y$ (join) and a greatest lower bound $x \sqcap y$ (meet).

A **complete lattice** is a lattice in which every subset has a least upper bound and a greatest lower bound.

Sign Analysis

As an example, we want to find the possible signs of integer variables and expressions. Consider the following abstract values for the sign of an integer:

- \top : unknown sign
- $+$: positive
- $-$: negative
- 0 : zero
- \perp : not an integer, or unreachable code



This partial order, with edges for \sqsubseteq , forms a complete lattice. e.g. $+\sqsubseteq\top$ means $+$ is *at least as precise as* \top

Sign Analysis

Let's create a **map lattice** $State = Var \rightarrow Sign$ that describes the sign of each variable. Derive a system of equations, one per line, using values from the lattice.

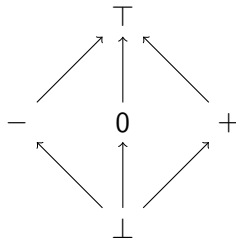
```
var a, b; // 1
a = 42; // 2
b = a + input(); // 3
a = a - b; // 4
```

$$x_1 = [a \mapsto \top, b \mapsto \top]$$

$$x_2 = x_1[a \mapsto +]$$

$$x_3 = x_2[b \mapsto x_2(a) + \top]$$

$$x_4 = x_3[a \mapsto x_3(a) - x_3(b)]$$



Sign

Sign Analysis

$$x_1 = [a \mapsto \top, b \mapsto \top]$$

$$x_2 = x_1[a \mapsto +]$$

$$x_3 = x_2[b \mapsto x_2(a) + \top]$$

$$x_4 = x_3[a \mapsto x_3(a) - x_3(b)]$$

$$f_1(x_1, \dots, x_n) = [a \mapsto \top, b \mapsto \top]$$

$$f_2(x_1, \dots, x_n) = x_1[a \mapsto +]$$

$$f_3(x_1, \dots, x_n) = x_2[b \mapsto x_2(a) + \top]$$

$$f_4(x_1, \dots, x_n) = x_3[a \mapsto x_3(a) - x_3(b)]$$

Generalised equation system over a lattice L , with functions $f_i : L^n \rightarrow L$:

$$x_1 = f_1(x_1, \dots, x_n)$$

$$x_2 = f_2(x_1, \dots, x_n)$$

$$\vdots$$

$$x_n = f_n(x_1, \dots, x_n)$$

Monotonicity and Fixed Points

Generalised equation system over a lattice L , with functions $f_i : L^n \rightarrow L$:

$$x_1 = f_1(x_1, \dots, x_n)$$

$$x_2 = f_2(x_1, \dots, x_n)$$

$$\vdots$$

$$x_n = f_n(x_1, \dots, x_n)$$

Combine the n functions into $F : L^n \rightarrow L$:

$$\begin{aligned} F(x_1, \dots, x_n) &= (f_1(x_1, \dots, x_n), \dots, f_n(x_1, \dots, x_n)) \\ &= (x_1, \dots, x_n) \end{aligned}$$

Then we are looking for $x = F(x)$, i.e. a fixed point of F .

Monotonicity and Fixed Points

Definition

A function $f : L_1 \rightarrow L_2$ is **monotone** if $\forall x, y \in L_1, x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$

More precise input leads to more precise output

Theorem

Kleene's Fixed Point Theorem: *In a complete lattice L with finite height, every monotone function $f : L \rightarrow L$ has a unique least fixed point $\bigsqcup_{i=0}^{\infty} f^i(\perp)$*

These results generalise to functions that take multiple arguments $f : L^n \rightarrow L$ that are monotone in each argument – such as the ones we derived for sign analysis.

Corollary

*For an equation system over complete lattices of finite height with monotone constraint functions, a **unique, most precise solution** always exists*

Computing the Least Fixed Point

Algorithm 1 Naive Fixed Point Algorithm

```
1: procedure NAIVEFIXEDPOINT( $F$ )  
2:    $x := \perp$   
3:   while  $x \neq F(x)$  do  
4:      $x := F(x)$   
5:   end while  
6:   return  $x$   
7: end procedure
```

In each iteration, all of f_1, \dots, f_4 are applied. But f_2 depends only on x_1 , and the value of x_1 is unchanged in most iterations. We'll see a more efficient way later.

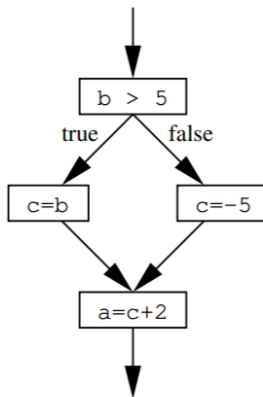
Data Flow Analysis

Main idea: we want to **find the possible values of variables at each point in the program.**

- In compilers: used for optimisations (e.g. constant propagation)
- In security: used to find vulnerabilities (e.g. untrusted data reaching a sink)

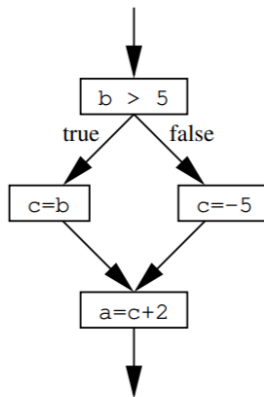
Control Flow Graph

- In our previous example, we had a sequence of statements with no branches
- In general, we have a **control flow graph (CFG)** with basic blocks and edges



Abstract States

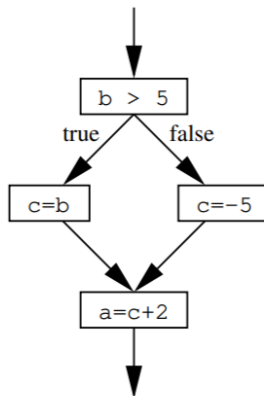
- Recall: each element of the lattice $State = Var \rightarrow Sign$ is an abstract state that maps variables to signs
- For each CFG node v , let the **constraint variable** $[[v]]$ be the abstract state at the program point *immediately after* v
- We have a lattice $State^n$ of abstract states, where n is the number of CFG nodes



Constraint Rules

We need to combine the abstract states of the predecessors of a node to get the abstract state of the node itself.

$$\text{JOIN}(v) = \bigsqcup_{u \in \text{pred}(v)} [[u]]$$



$$\text{JOIN}([[a = c + 2]]) = [[c = b]] \sqcup [[c = -5]]$$

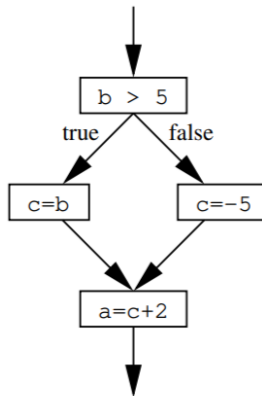
Constraint Rules

$$[[c = b]] = [b \mapsto +, c \mapsto +]$$

$$[[c = -5]] = [b \mapsto \top, c \mapsto -]$$

$$\begin{aligned} \text{JOIN}([a = c + 2]) &= [[c = b]] \sqcup [[c = -5]] \\ &= [b \mapsto \top, c \mapsto \top] \end{aligned}$$

$$\begin{aligned} [[a = c + 2]] &= \text{JOIN}([a = c + 2])[a \mapsto \text{eval}(\text{JOIN}([a = c + 2]), c + 2)] \\ &= [a \mapsto \top, b \mapsto \top, c \mapsto \top] \end{aligned}$$



Solving Data Flow Equations

Generalised equation system over a lattice L , with functions $f_i : L^n \rightarrow L$:

$$\begin{aligned} [[v_1]] &= f_{v_1}([v_1], \dots, [v_n]) \\ [[v_2]] &= f_{v_2}([v_1], \dots, [v_n]) \\ &\vdots \\ [[v_n]] &= f_{v_n}([v_1], \dots, [v_n]) \end{aligned}$$

Combine the n functions into $F : L^n \rightarrow L$:

$$\begin{aligned} F([v_1], \dots, [v_n]) &= (f_{v_1}([v_1], \dots, [v_n]), \dots, f_{v_n}([v_1], \dots, [v_n])) \\ &= ([v_1], \dots, [v_n]) \end{aligned}$$

Then we are looking for $x = F(x)$, i.e. a fixed point of F .

A More Efficient Algorithm

Algorithm 2 Simple Worklist Algorithm

```
1: procedure SIMPLEWORKLIST( $F$ )
2:    $(x_1, \dots, x_n) := (\perp, \dots, \perp)$ 
3:    $W := \{v_1, \dots, v_n\}$ 
4:   while  $W \neq \emptyset$  do
5:      $v_i := W.\text{pop}()$ 
6:      $y := f_{v_i}(x_1, \dots, x_n)$ 
7:     if  $y \neq x_i$  then
8:        $x_i := y$ 
9:       for  $v_j \in \text{dep}(v_i)$  do
10:         $W.\text{add}(v_j)$ 
11:       end for
12:     end if
13:   end while
14:   return  $(x_1, \dots, x_n)$ 
15: end procedure
```

Insight: most f_{v_i} will only read the values from a few other variables, instead of all $[[v_1]], \dots, [[v_n]]$.

$\text{dep}(v_i)$ is the set of nodes that depend on v_i (i.e. the successors of v_i)

Taint Tracking

- **Can we tell which computations may involve “tainted” data?**
- i.e. data that comes from an untrusted source
- e.g. user input, HTTP responses, environment variables

Taint Tracking

The approach is similar! We just define different abstract values and equations.

- **Abstract taint values:**
 - \top : Unknown taint status.
 - T : Tainted.
 - U : Untainted.
 - \perp : Unreachable.
- **Ordering:** $\perp \sqsubseteq U \sqsubseteq T \sqsubseteq \top$
- **Abstract state:** A mapping $\sigma : \text{Var} \rightarrow \{\perp, U, T, \top\}$.
- **Transfer functions:** e.g. for an assignment $x := y \text{ op } z$, define

$$f(\sigma) = \sigma[x \mapsto \sigma(y) \sqcup \sigma(z)]$$

i.e. if either operand is tainted, the result is tainted

In Practice: CodeQL

A tool developed by Semmle (a spin-out company from Oxf*rd), now acquired by GitHub. Used with CLI or GitHub integration (free for all public repos!)

- The source code is compiled into a relational database, which includes information about the control flow graph, data flow, and other properties of the code.
- The user writes queries in a high-level language called QL, which is executed by the CodeQL engine.
- The engine uses fixed-point algorithms to perform data flow analysis.
- Results are exported into the SARIF format which can be consumed by CI tools or custom integrations.

In Practice: CodeQL

We want to find all instances where untrusted user input (source) reaches a sensitive function (sink) without being sanitized.

○ ○ ○

```
1 class UnsafeDOMManipulationConfiguration extends TaintTracking::Configuration {
2   UnsafeDOMManipulationConfiguration() { this = "UnsafeDOMManipulationConfiguration" }
3
4   override predicate isSource(DataFlow::Node source) {}
5
6   override predicate isSink(DataFlow::Node sink){}
7 }
8
9 from DataFlow::PathNode source, DataFlow::PathNode sink, UnsafeDOMManipulationConfiguration config
10 where config.hasFlowPath(source, sink)
11 select sink.getNode(), "Potentially unsafe DOM manipulation with $@.", source.getNode(), "untrusted data"
```


Defining Sources

○ ○ ○

```
1 override predicate isSource(DataFlow::Node source) {  
2   source instanceof RemoteFlowSource or    // user input, e.g. https://example.com?x=1  
3   source instanceof ClientRequest::Range  // results of HTTP requests  
4 }
```

You can also extend this with custom logic, to incorporate codebase-specific patterns – e.g. RPC calls, deserialization, etc.

Defining Sinks

○ ○ ○

```
1 override predicate isSink(DataFlow::Node sink) {
2   // Direct assignment to innerHTML or outerHTML
3   exists(DataFlow::PropWrite pw |
4     pw.getPropertyName() in ["innerHTML", "outerHTML"] and
5     sink = pw.getRhs()
6   )
7   or
8   // Element.insertAdjacentHTML()
9   exists(DataFlow::MethodCallNode call |
10    call.getMethodName() = "insertAdjacentHTML" and
11    sink = call.getArgument(1)
12  )
13  or
14  [...]
15  or
16  // DOMParser.parseFromString()
17  exists(DataFlow::MethodCallNode call |
18    call.getMethodName() = "parseFromString" and
19    call.getReceiver().getALocalSource() instanceof DataFlow::NewNode and
20    call.getReceiver().getALocalSource().(DataFlow::NewNode).getCalleeName() = "DOMParser" and
21    sink = call.getArgument(0)
22  )
23 }
```

Changing the Transfer Function

○ ○ ○

```
1 override predicate isAdditionalTaintStep(DataFlow::Node pred, DataFlow::Node succ) {  
2   exists(DataFlow::ArrayCreationNode array |  
3     pred = array.getAnElement() and  
4     succ = array  
5   )  
6   or  
7   exists(DataFlow::MethodCallNode find |  
8     find.getMethodName().regexMatch("find|filter|some|every|map") and  
9     pred = find.getReceiver() and  
10    succ = find.getCallback(0).getParameter(0)  
11  )  
12 }
```

Changing the Transfer Function

○ ○ ○

```
1 override predicate isSanitizer(DataFlow::Node node) {  
2   node = DataFlow::moduleImport("dompurify").getAMemberCall("sanitize")  
3 }
```

Any results from `DOMPurify.sanitize` are treated as untainted. Know your assumptions!

Limitations

- We need to create custom taint specifications for third-party library APIs.
- False positives: even if tainted data reaches a sink, it may not always be exploitable – some other conditions may need to be met
- Requires a good understanding of the codebase and the problem domain, and lots of fine-tuning to get good results – only as good as the queries you write

Alternative Approaches

- **Symbolic execution:** represent the program inputs symbolically and explore all possible paths through the program, generating constraints on the inputs such that a certain path is taken
- Ziyang Li, Saikat Dutta, and Mayur Naik. LLM-assisted static analysis for detecting security vulnerabilities, 2024

IRIS leverages LLMs to infer taint specifications and perform contextual analysis, alleviating needs for human specifications and inspection . . .

A state-of-the-art static analysis tool CodeQL detects only 27 of these vulnerabilities whereas IRIS with GPT-4 detects 55 (+28) and improves upon CodeQL's average false discovery rate by 5% points.

References

- [1] Anders Møller and Michael I. Schwartzbach. *Static Program Analysis*. November 2020.
- [2] Oege de Moor, Mathieu Verbaere, Elnar Hajiyeu, Pavel Avgustinov, Torbjorn Ekman, Neil Ongkingco, Damien Sereni, and Julian Tibble. Keynote address: .ql for source code analysis. In *Seventh IEEE International Working Conference on Source Code Analysis and Manipulation (SCAM 2007)*, pages 3–16, 2007.
- [3] Ziyang Li, Saikat Dutta, and Mayur Naik. LLM-assisted static analysis for detecting security vulnerabilities, 2024.