Notes on "Estimates of the Trade and Welfare Effects of NAFTA"

- Paper: Caliendo, L., & Parro, F. (2015). Estimates of the Trade and Welfare Effects of NAFTA. *The Review of Economic Studies*, *82*(1), 1–44.
- This paper extends the EK model by considering the heterogeneity of productivity across sectors. In CP2015, heterogeneity in the dispersion of productivity together with the share of intermediate goods in production and sectoral interrelations are portrayed to capture how the reduction of tariff has different influences across sectors.

1. Model

- Perfect competition.
- N countries denoted by i (origin) and n (destination). Please note that my notation for trade flows differs slightly from the original paper. For instance, while the authors use ni to denote trades from country i to n, I will use in, which I find more intuitive. For convenience, readers can simply remember that i generally refers to the origin country and j to the destination country. But this distinction warrants particular attention when estimating θ^j, as both trade shares from i to n and from n to i are utilized.
- J sectors denoted by j and k (including tradable sector and non-tradable sector).
- One factor input: labor (mobile across sectors but not across countries).
- Intermediate goods (corresponding to the goods in EK) are partly tradable.
- Composite goods (called "composite intermediate goods" in original text) are non-tradable, which are either consumed by households or used as inputs (materials) for the production of intermediate goods.

1.1. Households

- Endowment: L_n representative households.
- C-D type utility function for households in country n:

$$u(c_n) = \prod_{j=1}^{J} (c_n^j)^{\alpha_n^j}, \tag{1}$$

  where $\sum_{j=1}^{J} \alpha_n^j = 1$ and c_n^j represents the consumption of the composite good from sector j.
- Household income: $I_n = w_n L_n +$ received transfers (described below).

1.2. Intermediate goods

- Intermediate goods are portrayed as a continuum and are sector-specific produced: $u^j \in [0, 1]$.
- Since some sectors of production are defined as tradable sector, intermediate goods that belongs to these sectors are tradable.
- The production of intermediate goods requires two types of inputs: labor and composite goods from all sectors.
- C-D type production function for intermediate goods:

$$q_n^j(u^j) = Z_n^j(u^j)\left[l_n^j(u^j)\right]^{\gamma_n^j} \prod_{k=1}^{J} \left[m_n^{k,j}(u^j)\right]^{\gamma_n^{k,j}},$$

where $\sum_{k=1}^{J}\gamma_n^{k,j}=1-\gamma_n^j$, and l_n^j and $m_n^{k,j}$ denote the input of labor and input of the composite good produced in sector k, respectively.

- Since the production function is constant returns to scale, the cost of an input bundle is

$$c_n^j=\Upsilon_n^j w_n^{\gamma_n^j}\prod_{k=1}^{J}\left(P_n^k\right)^{\gamma_n^{k,j}}, \tag{2}$$

where Υ_n^j is a constant, and w_n and P_n^k is the wage and the price of composite good k, respectively.
Proof. According to the cost minimization, we can derive the input of composite goods as a function of labor input:

$$\forall k,\ m_n^{k,j}(u^j)=\frac{\gamma_n^{k,j}}{\gamma_n^j}\frac{w_n}{P_n^k}l_n^j(u^j),$$

substitute the above equation to the production function to yield the labor input for producing $Z_n^j(u^j)$ units of intermediate good u^j:

$$l_n^j(u^j)=\left(\frac{w_n}{\gamma_n^j}\right)^{\gamma_n^j-1}\prod_{k=1}^{J}\left(\frac{P_n^k}{\gamma_n^{k,j}}\right)^{\gamma_n^{k,j}},$$

and thus the composite good input is

$$m_n^{k,j}(u^j)=\left(\frac{w_n}{r_n^j}\right)^{\gamma_n^j}\left(\frac{P_n^k}{\gamma_n^{k,j}}\right)^{-1}\prod_{k=1}^{J}\left(\frac{P_n^k}{\gamma_n^{k,j}}\right)^{\gamma_n^{k,j}}.$$

Accordingly, the minimal cost to produce $Z_n^j(u^j)$ units of intermediate good u_j is (remembering $\gamma_n^j+\sum_{k=1}^{J}\gamma_n^{k,j}=1$):

$$\begin{aligned}c_n^j&=w_n l_n^j(u^j)+\sum_{k=1}^{J}P_n^k m_n^{k,j}(u^j)\\&=\Upsilon_n^j w_n^{\gamma_n^j}\prod_{k=1}^{J}\left(P_n^k\right)^{\gamma_n^j},\end{aligned}$$

where $\Upsilon_n^j\equiv\left(\gamma_n^j\right)^{-\gamma_n^j}\prod_{k=1}^{J}\left(\gamma_n^{k,j}\right)^{-\gamma_n^{k,j}}$. ■

1.3. Composite goods

- CES type production function for composite goods:

$$Q_n^j=\left(\int_0^1 r_n^j(u^j)^{\frac{\sigma^j-1}{\sigma^j}}du^j\right)^{\frac{\sigma^j}{\sigma^j-1}},$$

where $r_n^j(u^j)$ is the demand of intermediates u^j from the lowest cost suppliers.

- According to the cost minimization, we have

$$r_n^j(u^j)=\left(\frac{P_n^j(u^j)}{P_n^j}\right)^{-\sigma^j}Q_n^j,$$

where $P_n\triangleq\left(\int_0^1 P_n^j(w^j)^{1-\sigma^j}du^j\right)^{\frac{1}{1-\sigma^j}}$ is the price of the composite good produced in sector j.

1.4. International trade costs and price

- Assume there are two types of trade costs: ice-berg trade cost d_{in} and ad-valorem tariff τ_{in}. The key difference between these two kinds of trade costs is that the latter will become parts of the importers' national income. The model allows trade costs have heterogeneity across sectors.
- Define κ_{in}^j as the trade costs for goods traded from country i to country n:

$$\kappa_{in}^j \equiv (1+\tau_{in}^j)d_{in}^j \equiv \tilde{\tau}_{in}^j d_{in}^j, \tag{3}$$

  where τ_{in}^j is an ad-valorem flat-rate tariffs and d_{in}^j denotes iceberg trade costs. Assume that the triangle inequality still holds, i.e., $\kappa_{in}^j \leq \kappa_{ih}^j \kappa_{hn}^j$.
- Under the perfect competition and international trade of intermediates, country n's price of intermediates produced in sector j is: $P_n^j(u^j) = \min_{i\in N}\left\{\frac{c_i\kappa_{in}^j}{Z_i^j(u^j)}\right\}$.
- Like EK, assume $Z_n^j(u^j)$, the productivity of intermediates in sector j of country n, independent and identically obeys the Fréchet distribution with a scale parameter $T_n^j > 0$ and shape parameter $\theta^j > 1$ (the important restriction $\theta^j + 1 > \sigma^j$ still needs to be held). According to EK, we can derive the price index of sector j (as well the price of the composite good in sector j):

$$P_n^j = A^j \left[\sum_{i=1}^N T_i^j \left(c_i^j \kappa_{in}^j\right)^{-\theta^j}\right]^{-\frac{1}{\theta^j}}, \tag{4}$$

  where A^j is a constant.
- For non-tradable sectors, assume that $\kappa_{in}^j \to \infty$.
  - The price of intermediate goods: $P_n^j(u^j) = c_n^j / Z_n^j(u^j)$.
  - The demand of intermediate goods: $r_n^j(u^j) = q_n^j(u^j)$.
  - The price of the composite good: $P_n^j = A^j (T_n^j)^{-\frac{1}{\theta^j}} c_n^j$.
- With C-D preference, the price index for consumers are:

$$P_n = \prod_{j=1}^J \left(\frac{P_n^j}{\alpha_n^j}\right)^{\alpha_n^j} \tag{5}$$

  Proof. Using the property of C-D preference, it is easy to write the demand function of each composite good:

$$\forall j,\ c_n^j(P_n^j) = \frac{\alpha_n^j I_n}{P_n^j}.$$

  Then substitute the demand functions to the utility function to yield the indirect utility function:

$$V_n(I_n, P_n) = \prod_{j=1}^J \left(\frac{\alpha_n^j I_n}{P_n^j}\right)^{\alpha_n^j} = \frac{I_n}{P_n},$$

  where we define $P_n \equiv \prod_{j=1}^J \left(\frac{P_n^j}{\alpha_n^j}\right)^{\alpha_n^j}$ as the price index to let the real income reflect the households' welfare.

1.5. Expenditure share

- Total expenditure on sector j in country n is $X_n^j = P_n^j Q_n^j$. This is also the total expenditure for intermediate goods produced in sector j.
- Define the expenditure share $\pi_{in}^j = X_{in}/X_n$ as the share of intermediate goods traded from country i, according to EK, we have:

$$\pi_{in}^j = \frac{T_i^j \left(c_i^j \kappa_{in}^j\right)^{-\theta^j}}{\sum_{h=1}^N T_h^j \left(c_h^j \kappa_{hn}^j\right)^{-\theta^j}}. \tag{6}$$

1.6. Total expenditure and trade balance

- Goods market clearing:

$$X_n^j = \sum_{k=1}^J \gamma_n^{k,j} \sum_{i=1}^N X_i^k \frac{\pi_{ni}^k}{1+\tau_{ni}^k} + \alpha_n^j I_n, \tag{7}$$

  where the first term represents the composite goods produced in sector j that are used as inputs for the production of intermediates, and the second term represents those that are used as consumption goods.
- Define the composition of the national income:

$$I_n = w_n L_n + R_n + D_n, \tag{8}$$

  where $w_n L_n$ is the wage income, R_n is the tariff income, and D_n denotes the deficit.
- The tariff income can be further expressed as:

$$R_n = \sum_{j=1}^J \sum_{i=1}^N \tau_{in}^j M_{in}^j,$$

  where $M_{in}^j \triangleq X_n^j \pi_{in}^j / (1+\tau_{in}^j)$.
- Let the sum of deficit across all countries equals to be zero:

$$\sum_{n=1}^N D_n = 0.$$

  A country's total deficit equals to the sum of deficit of each sector: $D_n = \sum_{j=1}^J D_n^j$, and deficit of each sector is:

$$\begin{aligned} D_n &= \sum_{j=1}^J D_n^j \\ &= \sum_{j=1}^J \left(\sum_{i=1}^N M_{ni}^j - \sum_{i=1}^N X_{in}^j \right) \\ &= \sum_{j=1}^J \sum_{i=1}^N X_n^j \frac{\pi_{in}^j}{1+\tau_{in}^j} - \sum_{j=1}^J \sum_{i=1}^N X_i^j \frac{\pi_{ni}^j}{1+\tau_{ni}^j}. \end{aligned}$$

  Thus we can yield the trade balance condition:

$$\sum_{j=1}^J \sum_{i=1}^N X_n^j \frac{\pi_{in}^j}{1+\tau_{in}^j} - D_n = \sum_{j=1}^J \sum_{i=1}^N X_i^j \frac{\pi_{ni}^j}{1+\tau_{ni}^j}. \tag{9}$$

- **Definition 1.** Given L_n, D_n, T_n^j, and d_{in}^j, an equilibrium under tariff structure τ is a wage vector $w \in R_{++}^N$ and price $\{P_n^j\}_{j=1,n=1}^{J,N}$ that satisfy equilibrium conditions (2), (4), (6), (7), and (9) for all j and n.

1.7. Equilibrium in relative changes

- **Definition 2.** Let (w, P) be an equilibrium under tariff structure τ and let (w', P') be an equilibrium under tariff structure τ'. Define $(\hat{w}, \hat{P})$ as an equilibrium under τ relative to τ', where a variable with a hat "$\hat{x}$" represents the relative change of the variable, namely $\hat{x} = x'/x$. Using equation (2), (4), (6), (7), and (9), the equilibrium conditions in relative changes satisfy:
Cost of input bundle:
$$\hat{c}_n^j = \hat{w}_n^{\gamma_n^j} \prod_{k=1}^J \left(\hat{P}_n^k\right)^{\gamma_n^{k,j}}, \tag{10}$$
Price index:
$$\hat{P}_n^j = \left[\sum_{i=1}^N \pi_{in}^j (\hat{c}_i^j \hat{\kappa}_{in}^j)^{-\theta^j}\right]^{-\frac{1}{\theta^j}}, \tag{11}$$
Bilateral trade share:
$$\pi_{in}^j = \left(\frac{\hat{c}_i^j \hat{\kappa}_{ni}^j}{\hat{P}_n^j}\right)^{-\theta^j}, \tag{12}$$
Total expenditure in each country n and sector j:
$$X_n^{j\prime} = \sum_{k=1}^J \gamma_n^{k,j} \sum_{i=1}^N X_i^{k\prime} \frac{\pi_{ni}^{k\ \prime}}{1+\tau_{ni}^{k\ \prime}} + \alpha_n^k I_n{}', \tag{13}$$
Trade balance:
$$\sum_{j=1}^J \sum_{i=1}^N X_n^{j\prime} \frac{\pi_{in}^{j\ \prime}}{1+\tau_{in}^{j\ \prime}} - D_n = \sum_{j=1}^J \sum_{i=1}^N X_i^{j\prime} \frac{\pi_{ni}^{j\ \prime}}{1+\tau_{ni}^{j\ \prime}}, \tag{14}$$
where $\hat{\kappa}_{in}^j = (1+\tau_{in}^{j\ \prime})/(1+\tau_{in}^j)$ and $I_n{}' = \hat{w}_n w_n L_n + \sum_{j=1}^J \sum_{i=1}^N \tau_{in}^{j\ \prime} X_n^{j\prime} \frac{\pi_{in}^{j\ \prime}}{1+\tau_{in}^{j\ \prime}} + D_n$.
- From inspecting equilibrium conditions (10) through (13), we can observe that the focus on relative changes allows us to perform policy experiments without relying on estimates of total factor productivity or transport costs. We only need:
    - two sets of tariff structure: τ and τ' (from data);
    - bilateral trade share: π_{in}^j (from data);
    - share of value added in production: γ_n^j (from data);
    - value added: $w_n L_n$ (from data);
    - share of intermediate consumption: $\gamma_n^{k,j}$ (from data);
    - the share of each sector in final demand: α_n^j (from data);
    - the sectoral dispersion of productivity (trade elasticity): θ^j (to be estimated later).

1.8. Relative change in real wages

- **Proposition 1.** The relative change in real wages can be decomposed into three components:

$$\ln \frac{\hat{w}_n}{\hat{P}_n} = \underbrace{-\sum_{j=1}^{J} \frac{\alpha_n^j}{\theta^j} \ln \hat{\pi}_{nn}^j}_{\text{Final goods}} \underbrace{-\sum_{j=1}^{J} \frac{1-\gamma_n^j}{\gamma_n^j} \frac{\alpha_n^j}{\theta^j} \ln \hat{\pi}_{nn}^j}_{\text{Intermediate goods}} \underbrace{-\sum_{j=1}^{J} \frac{\alpha_n^j}{\gamma_n^j} \ln \prod_{k=1}^{J} \left(\frac{\hat{P}_n^k}{\hat{P}_n^j}\right)^{\gamma_n^{k,j}}}_{\text{Sectoral linkages}}. \tag{15}$$

Proof. let $i = n$ in equation (12) to yield $\hat{\pi}_{nn}^j = (\hat{c}_n^j / \hat{P}_n^j)^{-\theta^j}$ and substitute (10) in it to yield:

$$\hat{\pi}_{nn}^j = \left[\frac{\hat{w}_n^{\gamma_n^j} \prod_{k=1}^{J} \left(\hat{P}_n^k\right)^{\gamma_n^{k,j}}}{\hat{P}_n^j}\right]^{-\theta^j}.$$

Equivalently, we can derive the expression of the change in real wages:

$$\begin{aligned} \frac{\hat{w}_n}{\hat{P}_n^j} &= \left(\hat{\pi}_{nn}^j\right)^{-\frac{1}{\theta^j \gamma_n^j}} \left(\hat{P}_n^j\right)^{\frac{1-\gamma_n^j}{\gamma_n^j}} \prod_{k=1}^{J} \left(P_n^k\right)^{-\frac{\gamma_n^{k,j}}{\gamma_n^j}} \\ &= \left(\hat{\pi}_{nn}^j\right)^{-\frac{1}{\theta^j \gamma_n^j}} \left(\prod_{k=1}^{J} \left(\frac{P_n^k}{P_n^j}\right)^{\gamma_n^{k,j}}\right)^{-\frac{1}{\gamma_n^j}}. \end{aligned}$$

Since $P_n = \prod_{j=1}^{J} (P_n^j / \alpha_n^j)^{\alpha_n^j}$, we have $\hat{P}_n = \prod_{k=1}^{J} (\hat{P}_n^j)^{\alpha_n^j}$, then take the two sides of above equation to the α_n^j power and find the product:

$$\prod_{j=1}^{J} \left(\frac{\hat{w}_n}{\hat{P}_n^j}\right)^{\alpha_n^j} = \prod_{j=1}^{J} \left(\hat{\pi}_{nn}^j\right)^{-\frac{\alpha_n^j}{\theta^j \gamma_n^j}} \left(\prod_{k=1}^{J} \left(\frac{P_n^k}{P_n^j}\right)^{\gamma_n^{k,j}}\right)^{-\frac{\alpha_n^j}{\gamma_n^j}}$$

$$\Rightarrow \quad \frac{\hat{w}_n}{\hat{P}_n} = \prod_{j=1}^{J} \left(\hat{\pi}_{nn}^j\right)^{-\frac{\alpha_n^j}{\theta^j}} \left(\hat{\pi}_{nn}^j\right)^{-\frac{\alpha_n^j}{\theta^j} \frac{1-\gamma_n^j}{\gamma_n^j}} \left(\prod_{k=1}^{J} \left(\frac{P_n^k}{P_n^j}\right)^{\gamma_n^{k,j}}\right)^{-\frac{\alpha_n^j}{\gamma_n^j}}.$$

Take the logarithm to yield:

$$\ln \frac{\hat{w}_n}{\hat{P}_n} = -\sum_{j=1}^{J} \frac{\alpha_n^j}{\theta^j} \ln \hat{\pi}_{nn}^j - \sum_{j=1}^{J} \frac{1-\gamma_n^j}{\gamma_n^j} \frac{\alpha_n^j}{\theta^j} \ln \hat{\pi}_{nn}^j - \sum_{j=1}^{J} \frac{\alpha_n^j}{\gamma_n^j} \ln \prod_{k=1}^{J} \left(\frac{\hat{P}_n^k}{\hat{P}_n^j}\right)^{\gamma_n^{k,j}}.$$

■

- In equation (15), the first term represents the aggregate effects of trade in final goods, depending on α_n^j and trade elasticity θ^j in each sector j. The second term represents the additional effect on real wages compared to a model with no intermediate goods (i.e., $\gamma_n^j = 1$). The third term reflects the importance of the I-O structure of the economy compared to a model with no sectoral linkages (i.e., $\gamma_n^{j,j} = 1 - \gamma_n^j$).

2. A new approach to estimate trade elasticities

- In Eaton and Kortum (2002), the authors construct a measure for bilateral trade resistance using price data, but this measure is imprecise. Therefore, it raises concerns about underestimation, which could eventually lead to the overestimation of trade elasticity.

- In Caliendo and Parro (2015), the authors propose a new approach to estimate θ^j with assuming tariff .
- Consider three countries indexed by n, i, and h. Using Equation (6), we have

$$\frac{\pi_{in}^j \pi_{nh}^j \pi_{hi}^j}{\pi_{ih}^j \pi_{hn}^j \pi_{ni}^j} = \left(\frac{\kappa_{in}^j \kappa_{nh}^j \kappa_{hi}^j}{\kappa_{ih}^j \kappa_{hn}^j \kappa_{ni}^j} \right)^{-\theta^j} . \tag{16}$$

- Recap that trade costs are composed of asymmetric tariffs and asymmetric ice-berg costs, namely $\ln \kappa_{in}^j = \ln \tilde{\tau}_{in}^j + \ln d_{in}^j$. The authors model the ice-berg costs with a linear specification, given by

$$\ln \kappa_{in}^j = \ln \tilde{\tau}_{in}^j + \ln d_{in}^j = \ln \tilde{\tau}_{in}^j + \nu_{in}^j + \delta_i^j + \mu_n^j + \varepsilon_{in}^j, \tag{17}$$

  where $\nu_{in} = \nu_{ni}$ is the trade-pair fixed effects capturing symmetric bilateral trade costs like distance, language, and common border. δ_i^j and μ_n^j are exporter country fixed effects and importer country fixed effects.
- Taking (17) into (16) yields

$$\frac{\pi_{in}^j \pi_{nh}^j \pi_{hi}^j}{\pi_{ih}^j \pi_{hn}^j \pi_{ni}^j} = \left(\frac{\kappa_{in}^j \kappa_{nh}^j \kappa_{hi}^j}{\kappa_{ih}^j \kappa_{hn}^j \kappa_{ni}^j} \right)^{-\theta^j} \mathrm{e}^{-\theta^j \left(\varepsilon_{in}^j + \varepsilon_{nh}^j + \varepsilon_{hi}^j - \varepsilon_{ih}^j - \varepsilon_{hn}^j - \varepsilon_{ni}^j \right)}.$$

  Take the logarithm:

$$\ln \left(\frac{\pi_{in}^j \pi_{nh}^j \pi_{hi}^j}{\pi_{ih}^j \pi_{hn}^j \pi_{ni}^j} \right) = -\theta^j \ln \left(\frac{\kappa_{in}^j \kappa_{nh}^j \kappa_{hi}^j}{\kappa_{ih}^j \kappa_{hn}^j \kappa_{ni}^j} \right) + \tilde{\varepsilon}_{inh}^j, \tag{18}$$

  where $\tilde{\varepsilon}_{inj}^j \equiv -\theta^j \left(\varepsilon_{in}^j - \varepsilon_{ni}^j + \varepsilon_{nh}^j - \varepsilon_{hn}^j + \varepsilon_{hi}^j - \varepsilon_{hi}^j \right)$. The only identification assumption is that those error terms are orthogonal to tariffs.
- By estimating Equation (18), the authors yield estimates of trade elasticity θ^j for each sector, which exhibit significant divergences across sectors. The authors also provide an aggregate elasticity estimates of 4.49. Readers might find this aggregate estimate directly used in some subsequent studies.