Notes on "The Economics of Density: Evidence From the Berlin Wall"

- Paper: Ahlfeldt, Gabriel M., Stephen J. Redding, Daniel M. Sturm, and Nikolaus Wolf, "The Economics of Density: Evidence From the Berlin Wall," *Econometrica*, 83 (2015), 2127–2189.
- This paper extends trade models by incorporating the disparities of places where people residing and working within a city, wherein the role of commuting in shaping the structure of a city is emphasized.

1. Settings

- A city embedded with a wider economy.
- The city consists of a set of discrete locations or blocks, indexed by $i, j \in \{1, \cdots, S\}$.
- Each block has an effective supply of floor space L_i that can be used commercially and residentially with an endogenous share θ_i and $1-\theta_i$, respectively.
- The city is populated by an endogenous measure of H workers, who are perfectly mobile within the city and the larger economy that provides a reservation utility $\bar{U}$.
- Firms produce single final good, which is traded costless within the city and the larger economy, and is chosen as the numeraire (i.e., $p=1$).

2. Workers

- The utility for worker o residing in block i and work in block j:

$$U_{ijo} = \frac{B_i z_{ijo}}{d_{ij}} \left(\frac{c_{ijo}}{\beta}\right)^{\beta} \left(\frac{l_{ijo}}{1-\beta}\right)^{1-\beta}, \tag{1}$$

  where c_{ijo} is the consumption of the single final good, l_{ijo} is the consumption of residential floor space (l_{ijo}), B_i represents residential amenities in block i that capture common characteristics that make a block a more or less attractive place to live (e.g., leafy street and scenic views), d_{ij} is the disutility from commuting from residence block i to workplace block j ($d_{ij} > 1$), and z_{ijo} is an idiosyncratic shock that is specific to individual workers and varies with the worker's blocks of employment and residence.
- The commuting cost d_{ij} takes a ice-berg form and increases with the travel time τ_{ij} between blocks i and j: $d_{ij} = e^{\kappa \tau_{ij}} \in [1, \infty)$. The parameter κ controls the size of commuting costs.
- The idiosyncratic shock is modelled to be drawn from an independent Fréchet distribution:

$$F(z_{ijo}) = e^{-T_i E_j z_{ijo}^{-\varepsilon}}, \text{ with } T_i, E_j > 0 \text{ and } \varepsilon > 1, \tag{2}$$

  where the scale parameter T_i determines the average utility derived from living in block i, the scale parameter E_j determines the average utility derived from working in block j, and the shape parameter ε controls the dispersion of idiosyncratic utility.
- Given residing in block i and working in block j, the optimization of utility yields the indirect utility function:

$$u_{ijo} = \frac{z_{ijo} B_i w_j}{Q_i^{1-\beta} d_{ij}}. \tag{3}$$

- Each worker chooses where to live and where to work to maximize his/her utility. Using the property of Fréchet distribution (analogous to what we have done in the previous trade models), one can derive the probability of a worker chooses to live in block i and work in block j:

$$\pi_{ij} = \frac{T_i E_j (d_{ij} Q_i^{1-\beta})^{-\varepsilon} (B_i w_j)^{\varepsilon}}{\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s (d_{rs} Q_r^{1-\beta})^{-\varepsilon} (B_r w_s)^{\varepsilon}} \equiv \frac{\Phi_{ij}}{\Phi}. \tag{4}$$

- Accordingly, the overall probability π_{Ri} that a worker resides in block i and the probability π_{Mj} that a worker works in block j are respectively:

$$\begin{aligned} \pi_{Ri} &= \sum_{j=1}^{S} \pi_{ij} = \frac{\sum_{j=1}^{S} \Phi_{ij}}{\Phi}, \\ \pi_{Mi} &= \sum_{i=1}^{S} \pi_{ij} = \frac{\sum_{i=1}^{S} \Phi_{ij}}{\Phi}. \end{aligned} \tag{5}$$

- Conditional on living in block i, the probability that a worker commutes to block j is

$$\pi_{ij|i} = \frac{\pi_{ij}}{\pi_{Ri}} = \frac{E_j (w_j / d_{ij})^{\varepsilon}}{\sum_{s=1}^{S} E_s (w_s / d_{ij})^{\varepsilon}}. \tag{6}$$

- Given above, the commuting market clearing condition should holds, that is

$$H_{Mj} = \sum_{i=1}^{S} \frac{E_j (w_j / d_{ij})^{\varepsilon}}{\sum_{s=1}^{S} E_s (w_s / d_{ij})^{\varepsilon}} H_{Ri}. \tag{7}$$

- Expected worker income conditional on living in block i is

$$\mathbb{E}[w_j | i] = \sum_{j=1}^{S} \frac{E_j (w_j / d_{ij})^{\varepsilon}}{\sum_{s=1}^{S} E_s (w_s / d_{ij})^{\varepsilon}} w_j. \tag{8}$$

- The expected utility of people living in the city is equal to the reservation utility $\bar{U}$ in the wider economy:

$$\mathbb{E}[u] = \gamma \left[\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s (d_{rs} Q_r^{1-\beta})^{-\varepsilon} (B_r w_s)^{\varepsilon} \right]^{1/\varepsilon} = \bar{U}, \tag{9}$$

where the expected utility derived using the property of Fréchet distribution (a basic prove in spatial model).

3. Production

- Monopolistic competition.
- A Cobb–Douglas form production function is assumed. The output of the final good in block j (y_j) is

$$y_j = A_j H_{Mj}^{\alpha} L_{Mj}^{1-\alpha}, \tag{10}$$

where A_j is the final goods productivity and L_{Mj} is the measure of floor space used commercially.
- Given commercially floor space fixed in block j, the optimization problem yields the equilibrium employment in block j:

$$H_{Mj} = \left(\frac{\alpha A_j}{w_j} \right)^{1/(1-\alpha)} L_{Mj}. \tag{11}$$

- The price of factors of production also serves as a force to control the zero profits, which satisfy:

$$q_j = (1-\alpha)\left(\frac{\alpha}{w_j}\right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}}. \tag{12}$$

4. Land marker clearing

- Land market clearing requires no-arbitrage between the commercial and residential use of floor space after the tax equivalent of land use regulations. The share of floor space used commercially θ_i is

$$\begin{aligned} &\theta_i = 1 \;\text{ if }\; q_i > \xi_i Q_i, \\ &\theta_i \in [0,1] \;\text{ if }\; q_i = \xi_q Q_i, \\ &\theta_i = 0 \;\text{ if }\; q_i < \xi_i Q_i, \end{aligned} \tag{13}$$

where $\xi_i \geq 1$ captures one plus the tax equivalent of land use regulations that restrict commercial land use relative to residential land use. The observed price of floor space in the data is assumed to be the maximum of the commercial and residential price: $\mathbb{Q}_i = \max\{q_i, Q_i\}$. Hence, we have:

$$\begin{aligned} &\mathbb{Q}_i = q_i, \;\; q_i > \xi_i Q_i, \;\; \theta_i = 1, \\ &\mathbb{Q}_i = q_i, \;\; q_i = \xi_i Q_i, \;\; \theta_i \in [0,1], \\ &\mathbb{Q}_i = Q_i, \;\; q_i < \xi_i Q_i, \;\; \theta_i = 0. \end{aligned} \tag{14}$$

- Floor space is depicted to be supplied by a competitive construction sector that uses land K and capital M as inputs: $L_i = M_i^{\mu} K_i^{1-\mu}$. The corresponding cost function is $\mathbb{Q}_i = \mu^{-\mu}(1-\mu)^{-(1-\mu)}\mathbb{P}^{\mu}\mathbb{R}_i^{1-\mu}$, where $\mathbb{P}$ is the common price for capital across all blocks, and $\mathbb{R}_i$ is the land price in block i. Since the price for capital is the same across all blocks, the following relationship holds:

$$L_i = \varphi_i K_i^{1-\mu}, \tag{15}$$

$$\mathbb{Q}_i = \chi \mathbb{R}_i^{1-\mu}, \tag{16}$$

where φ_i is the density of development which determines the relationship between floor space and land area, and χ is a constant.
- Residential land market clearing implies the demand of residential floor space equals the supply of floor space allocated to residential use in each location: $(1-\theta_i)L_i$, that is

$$(1-\beta)\frac{\mathbb{E}[w_j|i]H_{Ri}}{Q_i} = (1-\theta_i)L_i. \tag{17}$$

- Commercial land market clearing requires that the demand for commercial floor space equals the supply of floor space allocated to commercial use in each location: $\theta_j L_j$, that is

$$\left(\frac{(1-\alpha)A_j}{q_j}\right)^{1/\alpha} H_{Mj} = \theta_j L_j. \tag{18}$$

5. General equilibrium with exogenous location characteristics

- We begin by characterizing the properties of a benchmark version of the model in which location characteristics are exogenous, before relaxing this assumption to introduce endogenous agglomeration forces. Given the model's parameters $\{\alpha, \beta, \mu, \varepsilon, \kappa\}$, the reservation level of utility in the wider economy $\bar{U}$, and vectors of exogenous location characteristic vector $\{\mathbf{T}, \mathbf{E}, \mathbf{A}, \mathbf{B}, \boldsymbol{\varphi}, \mathbf{K}, \boldsymbol{\xi}, \boldsymbol{\tau}\}$, the general equilibrium of the model is referenced by the six vectors $\{\boldsymbol{\pi_M}, \boldsymbol{\pi_R}, \mathbf{Q}, \mathbf{q}, \mathbf{w}, \boldsymbol{\theta}\}$ and total city population H.

- **Proposition 1.** Assuming exogenous, finite, and strictly positive location characteristics ($T_i \in (0,\infty)$, $E_i \in (0,\infty)$, $\varphi_i \in (0,\infty)$, $K_i \in (0,\infty)$, $\xi_i \in (0,\infty)$, $\tau_{ij} \in (0,\infty) \times (0,\infty)$), and exogenous, finite, and nonnegative final goods productivity $A_i \in [0,\infty)$ and residential amenities $B_i \in [0,\infty)$, there exists a unique general equilibrium vector $\{\boldsymbol{\pi_M}, \boldsymbol{\pi_R}, H, \mathbf{Q}, \mathbf{q}, \mathbf{w}, \boldsymbol{\theta}\}$.
  Proof.
  - See paper's Appendix.
- In this case of exogenous location characteristics, there are no agglomeration forces, and hence the model's congestion forces of commuting costs and an inelastic supply of land ensure the existence of a unique equilibrium.

6. Introducing agglomeration forces

- Final goods productivity depends on production fundamentals (a_j) and production externalities Y_j. These externalities depend on the travel-time weighted sum of workplace employment density in surrounding blocks:

$$\begin{aligned} A_j &= a_j Y_j^{\gamma}, \\ Y_j &\equiv \sum_{s=1}^{S} e^{-\delta\tau_{js}} \left(\frac{H_{Ms}}{K_s}\right), \end{aligned} \tag{19}$$

  where H_{Ms}/K_s is workplace employment density per unit of land area, δ determines the rate of spatial decay ($e^{-\delta\tau_{js}} > 0$, which implies the externalities are always beneficial), λ controls the effects of agglomeration on production. The canonical interpretation of these production externalities in the urban economics literature is knowledge spillovers.
- Externalities in workers' residential choices depend on residential fundamentals (b_i) and residential externalities (Ω_i):

$$\begin{aligned} B_i &= b_i \Omega_i^{\eta}, \\ \Omega_i &\equiv \sum_{r=1}^{S} e^{-\rho\tau_{ir}} \left(\frac{H_{Rr}}{K_r}\right), \end{aligned} \tag{20}$$

  where η captures the net effect of residence employment density on amenities, including negative spillovers and positive externalities.

7. Recovering location characteristics

- For simplicity, the following composites denoted by a tilde was defined:

$$\begin{aligned} &\tilde{A}_i = A_i E_i^{\alpha/\varepsilon},\ \tilde{a}_i = a_i E_i^{\alpha/\varepsilon},\ \tilde{B}_i = B_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta},\ \tilde{b}_i = b_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta}, \\ &\tilde{w}_i = w_i E_i^{1/\varepsilon},\ \tilde{\varphi}_i = \tilde{\varphi}_i(\varphi_i, E_i^{1/\varepsilon}, \xi_i), \end{aligned}$$

  where ζ_{Ri} is specific to residential block and the function $\tilde{\varphi}_i$ is defined in the Appendix.
- **Proposition 2.**
  (i) Given known values for the parameters $\{\alpha, \beta, \mu, \varepsilon, \kappa\}$ and the observed data $\{\mathbb{Q}, \mathbf{H_M}, \mathbf{H_R}, \mathbf{K}, \boldsymbol{\tau}\}$, there exist unique vectors of the unobserved location characteristics $\{\tilde{\mathbf{A}}^*, \tilde{\mathbf{B}}^*, \tilde{\boldsymbol{\varphi}}^*\}$ that are consistent with the data being an equilibrium of the model.
  (ii) Given known values for the parameters $\{\alpha, \beta, \mu, \varepsilon, \kappa, \lambda, \delta, \eta, \rho\}$ and the observed data $\{\mathbb{Q}, \mathbf{H_M}, \mathbf{H_R}, \mathbf{K}, \boldsymbol{\tau}\}$, there exist unique vectors of the unobserved location characteristics $\{\tilde{\mathbf{a}}^*, \tilde{\mathbf{b}}^*, \tilde{\boldsymbol{\varphi}}^*\}$

that are consistent with the data being an equilibrium of the model.
Proof. See paper's Appendix.

8. Berlin's division and reunification

- The model captures the division of Berlin by assuming infinite costs of trading of final good, infinite commuting costs ($\kappa \to \infty$), infinite rates of decay of production externalities ($\delta \to \infty$), and infinite rate of decay of residential externalities ($\rho \to \infty$) across the Berlin Wall.
- The model points to four key channels through which division affects the distribution of economic activity within West Berlin: a loss of employment opportunities in East Berlin, a loss of commuters from East Berlin, a loss of production externalities from East Berlin, and a loss of residential externalities from East Berlin. Each of these four effects reduces the expected utility from living in West Berlin, and hence reduces its overall population, as workers out migrate to West Germany.
- As both commuting and externalities decay with travel time, each of these effects is stronger for parts of West Berlin close to employment and residential concentrations in East Berlin, reducing floor prices, workplace employment, and residence employment in these parts of West Berlin relative to those elsewhere in West Berlin. The mechanisms that restore equilibrium in the model are changes in wages and floor prices. Workplace and residence employment reallocate across locations within West Berlin and to West Germany.
- Since reunification involves a reintegration of West Berlin with employment and residential concentrations in East Berlin, we would expect to observe the reverse pattern of results in response to reunification. But reunification need not necessarily have exactly the opposite effects from division. As discussed above, if agglomeration forces are sufficiently strong relative to the differences in fundamentals across locations, there can be multiple equilibria in the model. In this case, division could shift the distribution of economic activity in West Berlin between multiple equilibria, and reunification need not necessarily reverse the impact of division.
- More generally, the level and distribution of economic activity within East Berlin could have changed between the pre-war and division periods, so that reunification is a different shock from division. Notwithstanding these points, reintegration with employment and residential concentrations in East Berlin is predicted to raise relative floor prices, workplace employment, and residence employment in the areas of West Berlin close to those concentrations.