Model with heterogeneous firms: Perfect competition (from Ricardo to DFS to EK)

- These notes draw on Chapter 3 & 4 of the notes by Allen and Arkolakis and slides for lecture 4 in 14.581 International Trade by Dave Donaldson for reference.
- Models with homogeneous firms do not explicitly explain the pattern of division of labor. In the Armington model, what good produced in each country is given. The Krugman model considers varieties within a country by introducing firms' entry decisions. However, the model portrays consumer to treat the same variety of goods produced in different countries as two totally different goods, therefore we could equivalently regard each country produces a given country-specific continuum of goods (that goes back to the Armington model).
- In this note, we take the journey from the naive 2 × 2 × 1 Ricardian model to the DFS model that allows infinite goods considered, and finally, to the EK model to further allow more than two countries.
- **What causes division of labor and trade?** In heterogeneous model, firms in different regions exhibit different levels of productivity, resulting in disparate production cost for identical goods among regions. Consumers in each region seek exporters offering the most competitive prices. This dynamics fosters a division of labor and promotes trade.

1. Ricardian model: pattern of division of labor

- The Ricardian model simply predict that countries may specialize in the production of certain ranges of goods based on the productivity/technology.

1.1. Settings

- Perfect competition.
- 2 countries, 2 goods, and 1 factor input (labor).
- Production function for good u in country i: $q^i(u) = Z^i(u)l^i(u),\ i, u \in \{1, 2\}$.
- Technology/productivity:
  - Assume country 1 has both absolute advantages in producing goods 1 and 2:

$$\begin{aligned} Z^1(1) &> Z^2(1), \\ Z^1(2) &> Z^2(2). \end{aligned}$$

  - And assume country 1 has the comparative advantage in producing good 1:

$$\frac{Z^1(1)}{Z^1(2)} > \frac{Z^2(1)}{Z^2(2)}. \tag{1}$$

- Consumer decision with a C-D preference:

$$\max a(1) \ln c^i(1) + a(2) \ln c^i(2), \quad \text{s.t.} \quad p(1)c^i(1) + p(2)c^i(2) \le w^i L^i.$$

  The optimization implies:

$$
\begin{aligned}
c^i(1) &= \frac{w_i L^i}{a(1)p(1)}, \\
c^i(2) &= \frac{w_i L^i}{a(2)p(2)}.
\end{aligned}
\tag{2}
$$

1.2. Autarky

- Under autarky, domestic market satisfies:

$$p^i(1)Z^i(1) = w^i = p^i(2)Z^i(2). \tag{3}$$

- Using the goods market clearing condition

$$c^i(u) = Z^i(u)l^i(u),\ u \in \{1,2\}, \tag{4}$$

  and the labor market clearing condition

$$w^i l^i(u) = a(u)w^i L^i,\ u \in \{1,2\}, \tag{5}$$

  we can solve the equilibrium domestic goods' prices, wage, and labor input in different sectors.

1.3. Free trade

- Under free trade, each good's price equals in each country.
- **Proposition 1.** At least one country specializes in the free trade equilibrium.
  Proof. According to (4), if two countries both produce two goods, then we have

$$
\begin{aligned}
\frac{p^1(1)}{p^1(2)} &= \frac{Z^1(1)}{Z^1(2)}, \\
\frac{p^2(1)}{p^2(2)} &= \frac{Z^2(1)}{Z^2(2)}.
\end{aligned}
$$

  Since $p^1(1) = p^2(1)$ and $p^1(2) = p^2(2)$, equations above imply $\frac{Z^1(1)}{Z^1(2)} = \frac{Z^2(1)}{Z^2(2)}$, which is contrary to the comparative advantage assumption. ■
- Assume factor input does not flow across countries, using the two market clearing conditions, the equilibrium can be solved.

2. DFS model

- Paper: Dornbusch, R., Fischer, S., & Samuelson, P. A. (1977). Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods. *American Economic Review*, *67*(5), 823–839.
- The naive Ricardian model only contains two goods. DFS use a continuum of sectors to make the equilibrium in a large number of industries being easy.

2.1. Settings

- Perfect competition.
- 2 countries denoted as H and F.
- Continuum of goods: $u \in [0,1]$.
- Constant returns to scale technology (labor is the only input).
- C-D preference with equal share of each good.

- Ice-berg trade cost: d_{HF} and d_{FH}.

2.2. Equilibrium

- Order all goods in a decreasing order of relative productivity Z_H/Z_F.
- Define u_H as the good with the lowest relative productivity produced by country H, and define u_F as the good with the lowest relative productivity produced by country F (because of the trade cost, $u_H \neq u_F$). Accordingly, country H produces goods in $[0, u_H]$ and country F produces goods in $[u_F, 1]$.
- It is easy to give the expression of u_H and u_F, which is where the good's domestic price (if produced) equals to its import price:

$$
\begin{aligned}
\frac{w_H}{Z_H(u_H)} = \frac{w_F}{Z_F(u_H)} \cdot \frac{1}{d_{FH}}, \\
\frac{w_H}{Z_H(u_F)} \cdot d_{HF} = \frac{w_F}{Z_F(u_F)}.
\end{aligned} \tag{6}
$$

  Thus, u_H and u_F are both a function of the endogenous relative wage.
- Using labor market clearing condition for country H to get the equilibrium wage w_H:

$$
w_H L_H = u_F w_F L_F + u_H w_H L_H. \tag{7}
$$

  where the first term at the right hand side represents the expenditure of country F on goods only produced in country H (remembering that we have assumed C-D preference with equal share of each good), and the second term represents the domestic expenditure of country H.

2.3. Where DFS stop and EK start?

- Although DFS has extended the model to a case with infinite goods types, it is still restricted between two countries, which makes it with little empirical content.
- EK develop a probabilistic model based on the property of Fréchet distribution to allow more than two countries and provides a richer model for counterfactual analysis.

3. EK model

- Paper: Eaton, J., & Kortum, S. (2002). Technology, Geography, and Trade. *Econometrica*, *70*(5), 1741–1779.
- Dave Donaldson: "The Ricardian model has long been perceived as a useful pedagogic tool with little empirical content until the appearance of the EK model, which has led to a 'Ricardian revival.'"

3.1. Settings

- Perfect competition.
- N countries: $i = 1, 2, \ldots, N$.
- Continuum of goods: $u \in [0, 1]$.
- CES preference with elasticity of substitution σ:

$$
U_i = \left(\int_0^1 q_i(u)^{\frac{\sigma-1}{\sigma}} \mathrm{d}u \right)^{\frac{\sigma}{\sigma-1}}. \tag{8}
$$

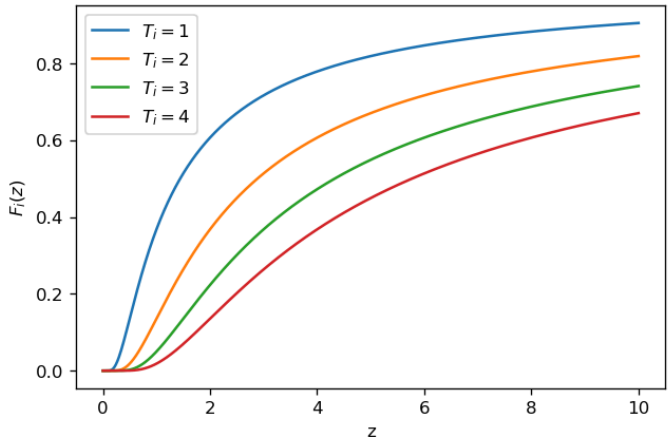
- One factor input: labor. The model takes into account of intermediate goods later.
- Input cost c_i:
  - Without intermediate goods: $c_i = w_i$.

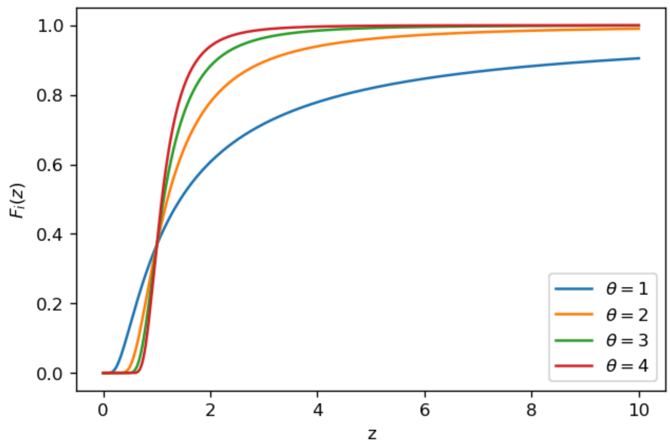
- With intermediate goods: $c_i = w_i^{\beta} p_i^{1-\beta}$ (assuming labor has a constant share β).

- Constant returns to scale:
  - Assume country i's productivity in producing good u is the realization of a random variable Z_i (drawn independently for each u) from country-specific probability distribution $F_i(z) = \Pr\{Z_i \le z\}$.
  - By the law of large number, $F_i(z)$ is also the fraction of goods for which country i's productivity below z.
  - Assume country i's productivity distribution is Fréchet:

$$F_i(z) = \mathrm{e}^{-T_i z^{-\theta}}, \tag{2}$$

  where $T_i > 0$, $\theta > 1$, and $\theta > \sigma - 1$ (important restriction, see below).
  - Roughly, T_i is a scale parameter (that reflects the mean of country i's productivity and thus the absolute advantage) and θ is a shape parameter (that reflects the fluctuation of productivity across goods and thus the comparative advantage). We also call θ the trade elasticity.





- Trade is subject to ice-berg costs: d_{ij} units need to be shipped from i so that 1 unit makes it to j $(d_{ij} \ge 1)$.

3.2. Price and three key propositions

- Under the competitive market assumption, the price of the good traded from i to j is $p_{ij} = c_i d_{ij}/Z_i$, which is also a random variable. Let $G_{ij}(p) \equiv \Pr\{p_{ij} \le p\}$, then we have

$$G_{ij}(p) = \Pr\{Z_i \ge c_i d_{ij}/p\} = 1 - \mathrm{e}^{-T_i (w_i d_{ij})^{-\theta} p^{\theta}}. \tag{3}$$

- Let $P_j \equiv \min\{p_{1j}, p_{2j}, \ldots, p_{Nj}\}$ and let $G_j(p) \equiv \Pr\{P_j \le p\}$, then we have

$$G_j(p) = 1 - \prod_{i=1}^{N} (1 - G_{ij}(p)) = 1 - \mathrm{e}^{-\Phi_j p^\theta}, \tag{4}$$

where $\Phi_j = \sum_{i=1}^{N} T_i (c_i d_{ij})^{-\theta}$.

- **Proposition 2.** The probability that country i provides a good at the lowest price in country j is:

$$\pi_{ij} = T_i (c_i d_{ij})^{-\theta} / \Phi_j \tag{5}$$

Proof. Based on the definition of π_{ij}, we have:

$$\begin{aligned}
\pi_{ij} =& \Pr\{p_{ij} \le \min_{k \ne i}\{p_{kj}\}\} \\
=& \int_0^\infty \Pr\{\min_{k \ne i}\{p_{kj}\} \ge p\} \mathrm{d}\left(G'_{ij}(p)\right) \\
=& \int_0^\infty \left(\prod_{k \ne i} \Pr\{p_{kj} \ge p\}\right) T_i (w_i d_{ij})^{-\theta} \mathrm{e}^{-T_i (w_i d_{ij})^{-\theta} p^\theta} \mathrm{d}(p^\theta) \\
=& \frac{T_i (w_i d_{ij})^{-\theta}}{\Phi_j} \int_0^\infty \mathrm{e}^{-\Phi_j p^\theta} \mathrm{d}(\Phi_j p^\theta) \\
=& \frac{T_i (w_i d_{ij})^{-\theta}}{\Phi_j}.
\end{aligned}$$

■

- **Proposition 3.** The price of a good that country j actually buys from any country i also has the distribution $G_j(p)$.
Proof. This proposition requires us to prove that

$$\Pr\{p_{ij} \le p \mid p_{ij} \le \min_{k \ne i}\{p_{kj}\}\} = G_j(p).$$

We can prove it by showing that

$$\begin{aligned}
\Pr\{p_{ij} \le p \mid p_{ij} \le \min_{k \ne i}\{p_{kj}\}\} &= \frac{\Pr\{p_{ij} \le p,\ p_{ij} \le \min_{k \ne i}\{p_{kj}\}\}}{\Pr\{p_{ij} \le \min_{k \ne i}\{p_{kj}\}\}} \\
&= \frac{\int_o^p \Pr\{x \le \min_{k \ne i}\{p_{kj}\}\} \mathrm{d}G_{ij}(x)}{\pi_{ij}} \\
&= 1 - \mathrm{e}^{-\Phi_j p^\theta}.
\end{aligned}$$

■

- **Proposition 4.** The exact price index for the CES objective function, assuming $\sigma < 1 + \theta$ is:

$$P_j = \gamma \Phi_j^{-\frac{1}{\theta}}, \tag{6}$$

where $\gamma = \left[\Gamma(\frac{\theta+1-\sigma}{\theta})\right]^{\frac{1}{1-\sigma}}$ and $\Gamma(\cdot)$ denotes for Gamma Function.
Proof. According to the Dixit-Stigliz price index derived in previous notes, we can similarly obtain the price index for the goods continuum case:

$$P_j \equiv \left(\int_0^1 P_j(u)^{1-\sigma} \mathrm{d}u\right)^{\frac{1}{1-\sigma}}.$$

Since we assume the same distribution of firms' productivity for any good within a country, in other words, $P_j(u) \overset{\text{i.i.d}}{\sim} G_j(p)$. Therefore, according to the Law of Large Number, we have:

$$P_j \to_p \mathbb{E}\left[P_j(u)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Then we show that

$$\mathbb{E}\left[P_j(u)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\int_o^\infty x^{1-\sigma}\mathrm{d}G_j(x)\right]^{\frac{1}{1-\sigma}} = \gamma\Phi_j^{-\frac{1}{\theta}},$$

where $\gamma = \left[\Gamma(\frac{\theta+1-\sigma}{\theta})\right]^{\frac{1}{1-\sigma}}$. ■

3.3. Trade flows and gravity

- By the Law of Large Number, π_{ij} is also the fraction of goods traded from country i to j. According to Proposition 3, country j's average expenditure per good does not vary by source, hence π_{ij} is also the fraction of j's expenditure on goods from country i, that is

$$\frac{X_{ij}}{X_j} = \frac{T_i(c_i d_{ij})^{-\theta}}{\Phi_j} = \frac{T_i(c_i d_{ij})^{-\theta}}{\sum_k T_k(c_k d_{kj})^{-\theta}}. \tag{7}$$

- Consider a world with the only input of labor. Define Y_i as the income of country i, then:

$$Y_i = \sum_j X_{ij} = \sum_j \frac{T_i(c_i d_{ij})^{-\theta}}{\Phi_j}X_j = T_i c_i^{-\theta}\sum_j \frac{d_{ij}^{-\theta}}{\Phi_j}X_j. \tag{8}$$

Denote $\Pi_i \equiv \sum_j \frac{d_{ij}^{-\theta}}{\Phi_j}X_j$ to transform (8) into

$$T_i c_i^{-\theta} = \frac{Y_i}{\Pi_i}. \tag{9}$$

- Substituting (9) in (7) yields

$$X_{ij} = d_{ij}^{-\theta}\frac{Y_i}{\Pi_i}\frac{X_j}{\Phi_j}. \tag{10}$$

We again derive a structural gravity equation. It shares the same structure with those derived from models with homogeneous firms, consisting of terms standing for bilateral resistance, income of the exporter and the importer, and multilateral resistance, but has different structural interpretations for these terms. For example, the trade elasticity in the EK model depends on the technology parameter rather than preference parameter in the Armington model.

3.4. Wage in equilibrium

- In the case with the only input of labor, $c_i = w_i$. Labor market clearing condition in country i:

$$\begin{aligned} w_i L_i &= \sum_j X_{ij} \\ \Rightarrow\ w_i L_i &= \sum_j \frac{T_i(w_i d_{ij})^{-\theta}}{\sum_k T_k(w_k d_{kj})^{-\theta}} w_j L_j. \end{aligned} \tag{11}$$

This provides system of $N-1$ independent equations that can be solved for wage $(w_1, w_2, \dots, w_N)$ after choosing a numeraire and estimating some key parameters (such as θ).

3.5. Welfare and gains from trade

- Case 1: the only input of labor.
  - Combine (5) and (6), we have

$$\pi_{ij} = \frac{T_i (c_i d_{ij})^{-\theta} \gamma^{\theta}}{P_j^{-\theta}}. \tag{12}$$

    Use $c_i = w_i$ and choose $i = j$ to get:

$$\frac{w_j}{P_j} = \frac{\pi_{jj}^{-\frac{1}{\theta}} T_j^{\frac{1}{\theta}}}{\gamma}. \tag{13}$$

    Recap that the indirect utility function of CES preference under perfect competition is the real wage, hence (12) gives the determinants of welfare. In general, welfare increases with the increase of trade share (i.e., the decrease of π_{jj}) and the increase of average productivity (i.e., the increase of T_j).
  - Define $W_j \equiv w_j / P_j$, thus the gains from trade are given by $GT_j = W_n / W_n^A$, where the superscript A denotes for autarky. According to (13), we have:

$$GT_j = \pi_{jj}^{-\frac{1}{\theta}}, \tag{14}$$

    which means trade elasticity θ and share of expenditure on domestic goods π_{nn} are sufficient statistics to compute GT_j.
- Case 2: considering intermediate goods as another input.
  - Now we add the intermediate goods as another input for final goods. The only difference is that c_i is no longer equals to w_i but $w_i^{\beta} P_i^{1-\beta}$. We assume intermediate goods comprise the full set of goods combined according to the CES aggregator, so the price of a bundle of intermediate goods is also P_i given by equation (6). According to (12) and also choose $i = j$, the welfare in the intermediate case is given by

$$\frac{w_j}{P_j} = \left(\frac{\pi_{jj}^{-\frac{1}{\theta}} T_j^{\frac{1}{\theta}}}{\gamma} \right)^{\frac{1}{\beta}}. \tag{15}$$

  - The gains from trade in this case is

$$GT_j = \pi_{jj}^{-\frac{1}{\theta\beta}}. \tag{16}$$

    Compared to (14), we only need to estimate another parameter that reflects labor share. Since $\beta < 1$, considering intermediate goods bring a higher GT_j, which is because the trade of intermediate goods across countries further deepens the international trades.
- Case 3: further considering non-tradable goods.
  - Assume now composite goods (i.e., $u \in [0, 1]$ above) cannot be consumed directly, instead, it can either be used to produce intermediates or to produce a consumption good (together with labor).
  - Assume the intermediate/composite goods are tradable while consumption goods are non-tradable.
  - The production function for the consumption goods is C-D with labor share α.
  - Thus, the price index for composite goods as well intermediates bundle denoted as P_j is calculated like above. In this case, composite goods need another production process to be

consumable. Since the final/consumption goods are non-tradable, the domestic price equals to the domestic production cost rather than the minimal production costs across countries. As the labor share in this production process is α, the price index for consumption goods is

$$P_j' = w_j^{\alpha} P_j^{1-\alpha}. \tag{17}$$

- Now the welfare is

$$W_j = \frac{w_j}{P_j'} = \left(\frac{w_j}{P_j}\right)^{1-\alpha} = \left(\frac{\pi_{jj}^{-\frac{1}{\theta}} T_j^{\frac{1}{\theta}}}{\gamma}\right)^{\frac{1-\alpha}{\beta}}. \tag{18}$$

- The gains from trade in this case is

$$GT_j = \pi_{jj}^{-\frac{1-\alpha}{\theta\beta}}.$$

  We find gains here are less than those in the second case.

- As we can see, trade elasticity θ is the key structural parameter for welfare and counterfactual analysis in EK model, therefore next we turn to the discussion of how to estimate θ.

3.6. How to estimate the trade elasticity?

- From (6) and (7) we get the country i's share in country j's expenditures normalized by its own share is

$$S_{ij} \equiv \frac{X_{ij}/X_j}{X_{ii}/X_i} = d_{ij}^{-\theta} \cdot \frac{\Phi_i}{\Phi_j} = \left(d_{ij} \cdot \frac{P_i}{P_j}\right)^{-\theta}. \tag{19}$$

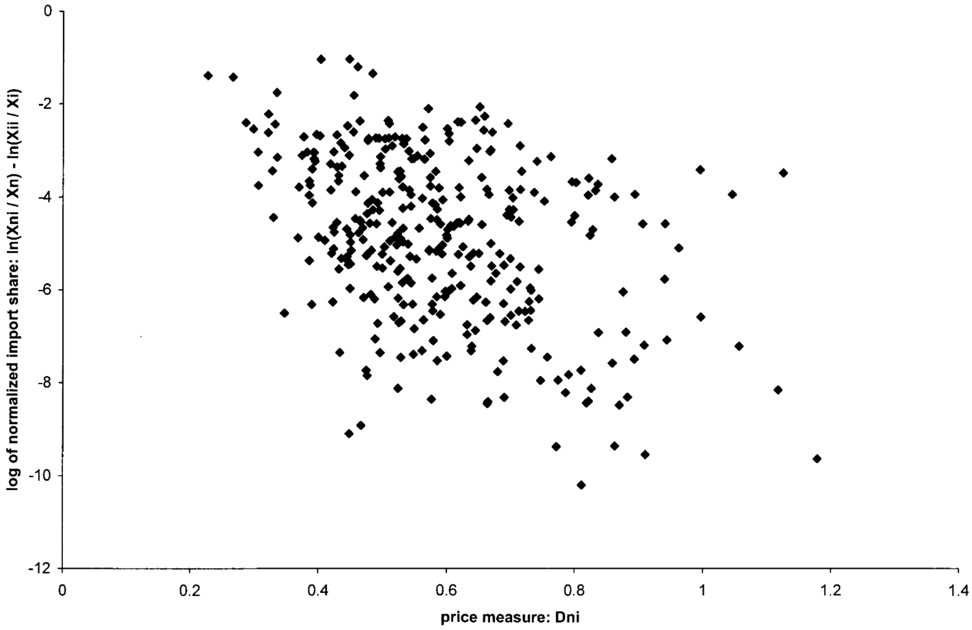
  We can obtain the data of $\{X_{\cdot\cdot}, X_{\cdot}, P_{\cdot}\}$ but not d_{ij}. If we have data of d_{ij}, we can run a regression of $\ln S_{ij}$ on $\ln(P_i d_{ij}/P_j)$ with importer and exporter dummies to estimate θ.

- EK use the price data to measure d_{ij}:
  - If good u is traded from i to j, that implies $P_i(u)d_{ij} = P_j(u)$; If good u is not traded from i to j, that implies $P_i(u)d_{ij} > P_j(u)$. Cases above can be summarized by $d_{ij} \geq P_j(u)/P_i(u), \forall u$.
  - Accordingly, the highest value of $P_j(u)/P_i(u)$ provides a measure of d_{ij}.
  - EK use a data set of retail prices in 19 countries of 50 manufactured products, where every country does in fact import from every other. EK take the second highest value of $P_j(u)/P_i(u)$ to deal with potential measure error bias.
  - Define $D_{ij} \equiv \ln(P_i d_{ij}/P_j) = \ln d_{ij} - \ln(P_j/P_i)$ and EK's measure for D_{ij} is

$$\max 2_u \left\{\ln \frac{P_j(u)}{P_i(u)}\right\} - \frac{1}{50}\sum_{u=1}^{50} \ln \frac{P_j(u)}{P_i(u)}.$$

    where $\max 2_u\{\cdot\}$ means the second highest. (EK's original text has some typos here and I have corrected them.)
  - The following figure shows the relationship between S_{ij} and EK's measure of D_{ij}, where there exists a significant negative slope (consistent with what theories predict). According to the simple

method-of-moments estimator, EK yield a estimate for θ of 8.28.



- Simonovska and Waugh (2014, *JIE*) argue that EK's procedure suffers from upward bias.
    - Since EK are only considering 50 goods, maximum price gap may still be strictly lower than trade cost.
    - If we underestimate trade costs, we overestimate trade elasticity.
    - Simulation based method of moments leads to a θ closer to 4.
- We will see an alternative approach developed by Caliendo and Parro (2015, *RES*) in later notes.

3.7. Estimate the gains from trade

- Head and Mayer (2015) offer a review of trade elasticity estimates. They find the typical value of θ is around 5.
- A typical value for π_{jj} (manufacturing) is 0.7. With $\theta = 5$, this implies GT_j in case 1 is $GT_j = 0.7^{-1/5} = 1.074$ (i.e., 7.4% gains).
- Standard value for labor share β is 0.5, thus with other parameters the same, GT_j in case 2 is $GT_j = 0.7^{-2/5} = 1.15$ (i.e., 15% gains).
- Additionally, we use the share of labor in service industries (always considered as non-tradable industries) as α, that is 0.75, to yield GT_j in case 3: $GT_j = 0.7^{1/10} = 1.036$ (i.e., 3.6% gains).
- Keep $\theta = 5$, $\beta = 0.5$, and $\alpha = 0.75$, the following figure shows the relationship between GT_j and π_{jj} in three cases.