The paper I choose is Brumm and Scheidegger (2017), which focused on introducing a method to solve high-dimensional dynamic problems. The research question is why the adaptive sparse grids (ASGs) method is suitable and useful to solve high-dimensional dynamic problems.

The research question is well defined. In the first section, Brumm and Scheidegger (2017) showed the importance of finding a method to solve high-dimensional dynamic problems, i.e., the complicated cross influence between subjects in real worlds make it necessary to solve dynamic models with many states to match the real world better (p.1576). Since the most accurate grid-based method is faced with "curse of dimensionality", a good method should make a balance between resolution and computational costs (Brumm & Scheidegger, 2017, p.1576). Brumm and Scheidegger (2017) proposed to use ASGs instead of standard sparse grids (SGs) method for the improved efficiency and accuracy, especially for solving nonsmooth models. In other words, the ASGs is a good method because it can reach the same accuracy with the least computational cost and have stable behaviors in different types of models.

The structure of the paper is well knit. After the introduction of topic and review of literature, section 3 introduced ASGs in detail and compared it with grid-based algorithm, the most accurate but most costly method, and SGs, the widely used method at present (Brumm & Scheidegger, 2017, p.1579). With graphs, formulas and explanations, it's easy for readers to understand the mechanism of those 3 methods even if they have no idea about grid-based computation before. Section 4 and section 5 are applications on 2 different economic dynamic models (Brumm & Scheidegger, 2017, p.1577). The major method the authors used to prove the advantages of ASGs is comparison. For both models, they compared ASGs and SGs in the trend of accuracy with the growth of dimensions, the required number of grids for the given resolution and the behavior in

nonsmooth version of the model. The authors used numerous graphs and tables to show advantages of ASGs in applications to high-dimensional dynamic models visually, which make their argument more persuasive. Besides, showing 2 different applications rather than a single one shows the broad applicability of ASGs, which is also a criterion of a good method. Section 6 make a conclusion of the whole paper and emphasize the advantages of ASGs, i.e., efficiency, accuracy, flexibility and stability in nonsmooth models. The paper gave a convincible answer with abundant and persuasive evidence to its research question.

However, there is still some space for improvement. As is mentioned, the major method Brumm and Scheidegger used to support their argument is comparison. They focused on SGs while ignoring other methods attempting to solve "curse of dimensionality", which can be improved by mentioned some other papers related to solving high-dimensional problems. A good example is a paper related to solving international real business cycle (IRBC) model with many states, Pichler (2011), which introduced how to reduce the computational cost of using Galerkin method with "non-product monomial cubature rules" (Pichler, 2011, p.241). Another example is White (2015), which provided an interpolation technique to apply endogenous gridpoints (ENDG) to high-dimensional models to speed up solutions to dynamic problems (White, 2015, p.26). The mechanism of reducing computational costs of that method is different to that of ASGs, for it avoids repeated computations of expected outcomes rather than reducing the number of grids to be computed (White, 2015, p.26). The former example is a comparison between techniques with similar mechanisms and the latter one is that between techniques with different mechanisms. The paper will be more persuasive with such comprehensive comparisons.

The paper has almost no spelling, grammatical or style errors, but there

are still some expressions that can be improved. For example, there is a sentence saying "In addition, we show that for smooth models too, in particular, if they are very high dimensional, ASGs can substantially improve upon standard SGs" (Brumm & Scheidegger, 2017, p.1576). It's better to show what the improvement is. It can be rewritten like "In addition, we show that for smooth models, especially for high-dimensional ones, the accuracy and efficiency of computation can be substantially improved by using ASGs rather than standard SGs." It can show the advantages of ASGs visually.

The paper is a heuristic one. It put forward a hot issue in economics, i.e., how to effectively solve high-dimensional dynamic problems. The authors proposed to use ASGs method and supported their answer with 2 applications, IRBC model and menu-cost model. However, they focused on the comparison between ASGs and standard SGs while ignoring many other methods to reduce computational costs. There are many questions waiting to be answered, like "what's the relative performance of ASGs to solve high-dimensional dynamic problems compared to non-grid-based methods?", "what type of dynamic models is not suitable to use ASGs" or "what technique can be combined with ASGs that can further reduce the computational cost of solving high-dimensional dynamic problems".

I'm interested in the possible technique that can be combined with ASGs to further reduce the computational cost. Brumm and Scheidegger (2017) proposed to distribute grid points "among multiple processes by message passing interface" (p.1592), which is a good example. That is a technique related to computer science, while I'm more interested to find a technique related to algorithm. As mentioned before, White (2015) proposed a method with another mechanism of reducing computational costs, i.e., avoid repeated computations of expected outcomes (p.26). If we can mix different mechanisms reducing computational costs in a way that they

don't affect the effects of other mechanisms, we may find a more efficient algorithm to solve dynamic problems with higher dimensions, which is supposed to contribute a lot to the simulation of the real world and the policy-making process.

Reference

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