hw4

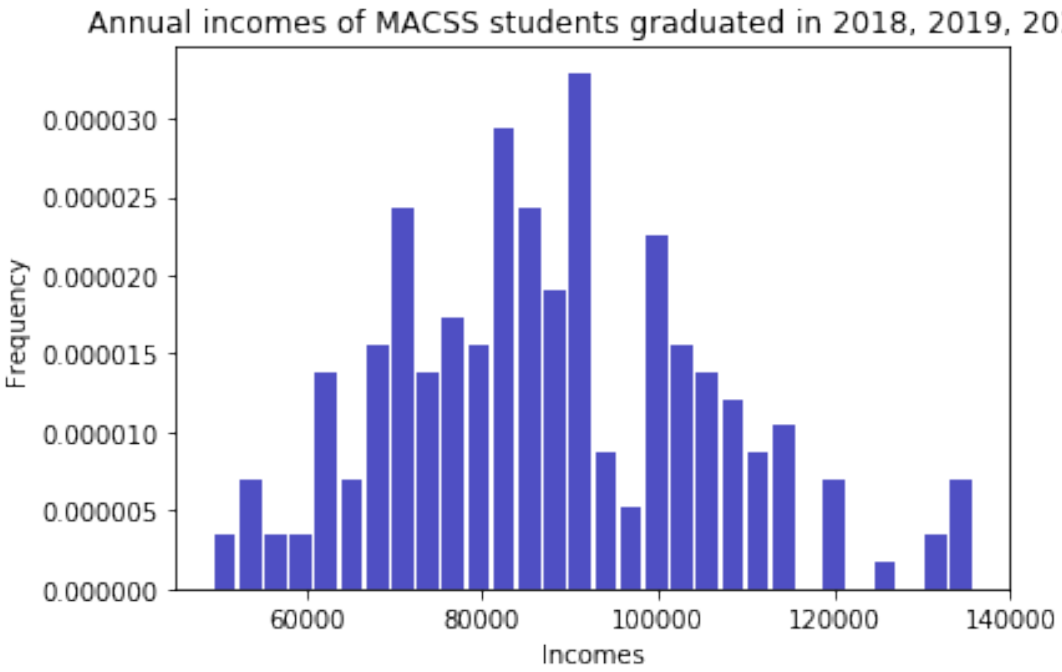
February 10, 2019

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.stats as sts
from scipy.integrate import quad
```

0.1 1

0.1.1 a

In [2]:
```
Incomes = np.loadtxt('data/incomes.txt')
fig,ax = plt.subplots()
ax.set_xlabel('Incomes')
ax.set_ylabel('Frequency')
ax.set_title('Annual incomes of MACSS students graduated in 2018, 2019, 2020')
ax.hist(x=Incomes, bins=30, color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
plt.show()
```



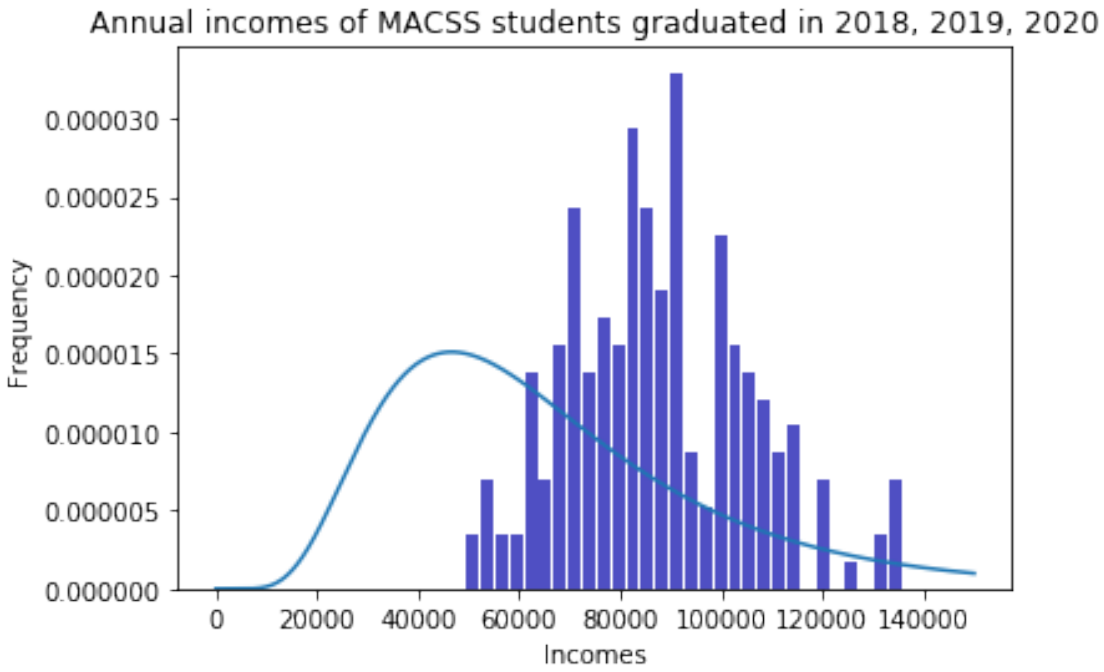
0.1.2 b

```
In [3]: def lognorm(x, mu=11, sigma=0.5):
            return 1/(x*sigma * np.sqrt(2 * np.pi))*np.e**(-(np.log(x) - mu)**2 / (2 * sigma**
        def log_lik_truncnorm(x, mu=11, sigma=0.5):
            pdf_vals = lognorm(x, mu, sigma)
            polishedpdf = pdf_vals
            ln_pdf_vals = np.log(polishedpdf)
            log_lik_val = ln_pdf_vals.sum()
            return log_lik_val
        total = 1
        X = np.linspace(0,150000, 150001)
        fig,ax = plt.subplots()
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('Annual incomes of MACSS students graduated in 2018, 2019, 2020')
        ax.plot(X, lognorm(X)/total)
        ax.hist(x=Incomes, bins=30, color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
        plt.show()
        LLV = log_lik_truncnorm(Incomes)
        print('The value of the log likelihood value is', LLV)
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: invalid val
```




```
The value of the log likelihood value is -2385.856997808558
```

0.1.3 c

```
In [4]: def crit(params):
            mu, sigma = params
            log_lik_val = log_lik_truncnorm(Incomes, mu, sigma)
            neg_log_lik_val = -log_lik_val
            return neg_log_lik_val
```

```
In [5]: import scipy.optimize as opt
        mu_init = 11
        sig_init = 0.5
        params_init = np.array([mu_init, sig_init])

        bnds = ((None, None), (1e-10, None))
        results = opt.minimize(crit, params_init, tol=1e-14, method='L-BFGS-B', bounds = bnds)
        mu_MLE, sig_MLE = results.x
        LLV_MLE = log_lik_truncnorm(Incomes, mu_MLE, sig_MLE)
        VCV_MLE = results.hess_inv

        print('mu_MLE=', mu_MLE, ' sig_MLE=', sig_MLE)
        print('The value of the log likelihood value is', LLV_MLE)
        print('The variance-covariance matrix is', VCV_MLE)
```

```
mu_MLE= 11.495016102470231  sig_MLE= 0.3108792150154579
The value of the log likelihood value is -2285.8999545081624
The variance-covariance matrix is <2x2 LbfgsInvHessProduct with dtype=float64>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C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:643: RuntimeWarning: inva
  grad[k] = (f(*((xk + d,) + args)) - f0) / d[k]
```
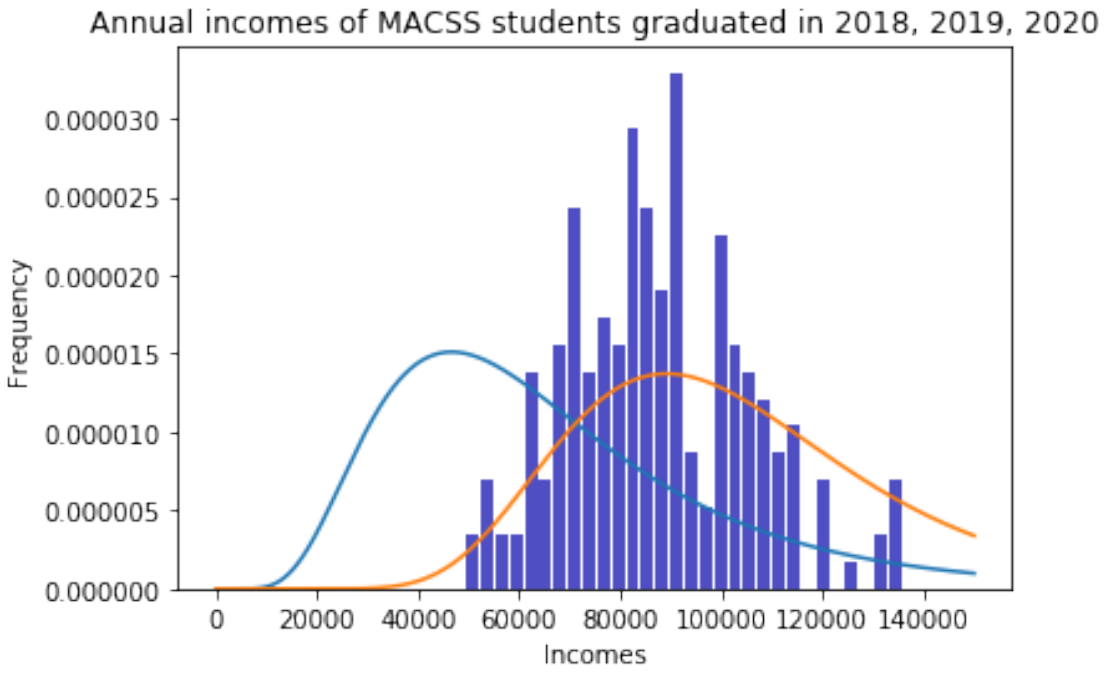

```
In [6]: X = np.linspace(0,150000, 150001)
        fig,ax = plt.subplots()
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('Annual incomes of MACSS students graduated in 2018, 2019, 2020')
        ax.plot(X, lognorm(X), label = 'initial guess')
        ax.plot(X, lognorm(X, mu=mu_MLE,sigma=sig_MLE), label = 'MLE result')
        ax.hist(x=Incomes, bins=30, color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
        plt.show()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: invalid val
```



0.1.4 d

```
In [7]: mu_new, sig_new = np.array([11, 0.5])
        log_lik_h0 = log_lik_truncnorm(Incomes, mu_new, sig_new)
        print('hypothesis value log likelihood', log_lik_h0)
        log_lik_mle = log_lik_truncnorm(Incomes, mu_MLE, sig_MLE)
        print('MLE log likelihood', log_lik_mle)
        LR_val = 2 * (log_lik_mle - log_lik_h0)
        print('likelihood ratio value', LR_val)
        pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
        print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2385.856997808558
MLE log likelihood -2285.8999545081624
likelihood ratio value 199.91408660079105
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

0.1.5 e

```
In [8]: print("The probability that you will earn more than $100,000 is",
              quad(lambda x: lognorm(x, mu=mu_MLE,sigma=sig_MLE), 100000, np.inf)[0])
        print("The probability that you will earn less than $75,000 is",
              quad(lambda x: lognorm(x, mu=mu_MLE,sigma=sig_MLE), 0, 75000)[0])
```

```
The probability that you will earn more than $100,000 is 0.47703014049157744
The probability that you will earn less than $75,000 is 0.19275920035472388
```

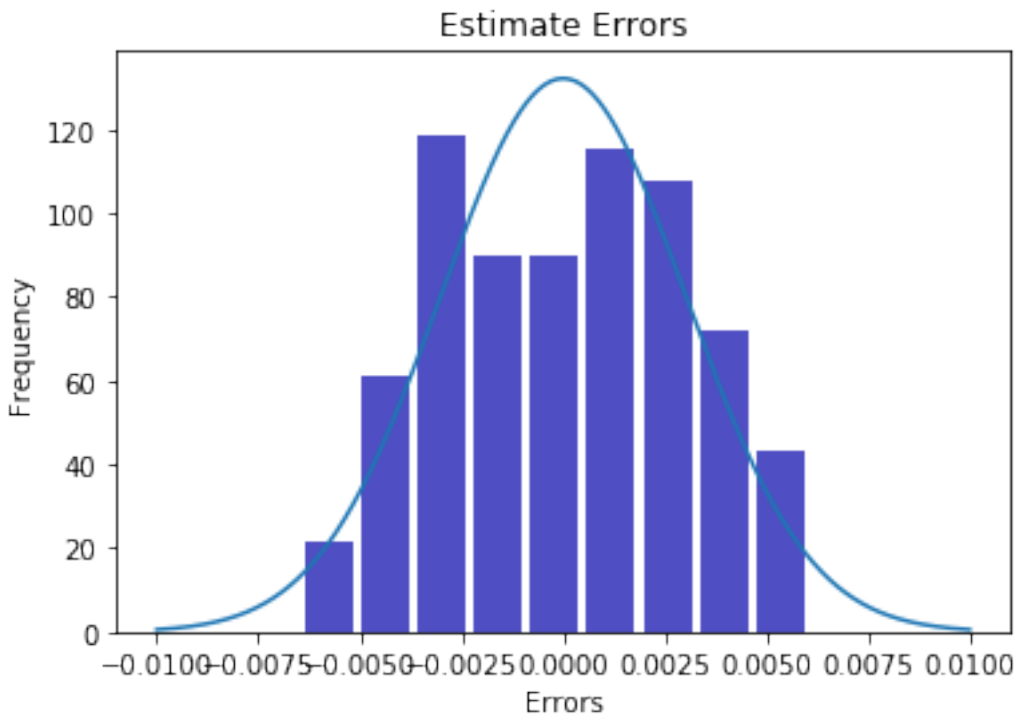
0.2 2

0.2.1 a

```
In [9]: def norm_pdf(x, sigma):
            sigma = np.abs(sigma)
            pdf_vals = (1/(sigma * np.sqrt(2 * np.pi)) *
                           np.exp( - x**2 / (2 * sigma**2)))
            return pdf_vals
        def log_lik_norm(x, beta0, beta1, beta2, beta3, sigma):
            error = x[:,0] - beta0 - beta1*x[:,1] - beta2*x[:,2] - beta3*x[:,3]
            pdf_vals = norm_pdf(error, sigma)
            ln_pdf_vals = np.log(pdf_vals)
            log_lik_val = ln_pdf_vals.sum()
            return log_lik_val
        def new_crit(params, *args):
            beta0, beta1, beta2, beta3, sigma = params
```

```
        x = args[0]
        log_lik_val = log_lik_norm(x,beta0, beta1, beta2, beta3, sigma)
        neg_log_lik_val = -log_lik_val
        return neg_log_lik_val
    Data = []
    with open('data/sick.txt') as f:
        for line in f:
            line = line.strip()
            data = line.split(',')
            Data.append(data)
    Data.pop(0)
    for i in range(len(Data)):
        Data[i] = [float(x) for x in Data[i]]
    Data = np.array(Data)
    new_Data = np.ones_like(Data)
    new_Data[:,1:] = Data[:,1:]
    beta0_init, beta1_init, beta2_init, beta3_init = np.linalg.lstsq(new_Data,Data[:,0])[0]
    sigma_init = 0.01
    parameters_init = np.array([beta0_init, beta1_init, beta2_init, beta3_init, sigma_init])
    results2 = opt.minimize(new_crit, parameters_init, args=(Data),tol=1e-14, method='L-BFG
    beta0_MLE, beta1_MLE, beta2_MLE, beta3_MLE, sigma_MLE = results2.x
    new_LLV_MLE = log_lik_norm(Data, beta0_MLE, beta1_MLE, beta2_MLE, beta3_MLE, sigma_MLE)
    new_VCV_MLE = results2.hess_inv
    print('beta0_MLE =',beta0_MLE,
          'beta1_MLE =',beta1_MLE,
          'beta2_MLE =',beta2_MLE,
          'beta3_MLE =',beta3_MLE,
          'sigma_MLE =',sigma_MLE)
    print('The value of the log likelihood value is', new_LLV_MLE)
    print('The variance-covariance matrix is', new_VCV_MLE)
    Error = Data[:,0] - beta0_MLE - beta1_MLE*Data[:,1] - beta2_MLE*Data[:,2] - beta3_MLE*D
    X2 = np.linspace(-0.01,0.01,10001)
    fig,ax = plt.subplots()
    ax.set_xlabel('Errors')
    ax.set_ylabel('Frequency')
    ax.set_title('Estimate Errors')
    ax.plot(X2, norm_pdf(X2,sigma_MLE))
    ax.hist(x=Error, bins='auto', color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
    plt.show()
```

```
beta0_MLE = 0.25164168313073476 beta1_MLE = 0.01293361306625627 beta2_MLE = 0.4004998342959863
The value of the log likelihood value is 876.8650774281919
The variance-covariance matrix is <5x5 LbfgsInvHessProduct with dtype=float64>


C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:30: FutureWarning: `rcond` par
To use the future default and silence this warning we advise to pass `rcond=None`, to keep usin
```



0.2.2 b

```
In [10]: parameters_new = np.array([1,0,0,0,0.1])
         log_lik_h0 = log_lik_norm(Data, *parameters_new)
         print('hypothesis value log likelihood', log_lik_h0)
         log_lik_mle = log_lik_norm(Data, *results2.x)
         print('MLE log likelihood', log_lik_mle)
         LR_val = 2 * (log_lik_mle - log_lik_h0)
         print('likelihood ratio value', LR_val)
         pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 5)
         print('chi squared of H0 with 5 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2253.700688042125
MLE log likelihood 876.8650774281919
likelihood ratio value 6261.131530940634
chi squared of H0 with 5 degrees of freedom p-value =  0.0
```

The results show that it's not likely that age, number of children, and average winter temperature have no effect on the number of sick days.