

# hw3

January 29, 2019

Import Packages

```
In [1]: import numpy as np
import scipy.optimize as opt
import sympy as sp
import matplotlib.pyplot as plt
from scipy.stats import norm
from mpl_toolkits.mplot3d import Axes3D
```

## 0.0.1 5.1

The problem is:

$$\max_{W_2 \in [0, W_1]} u(W_1 - W_2)$$

## 0.0.2 5.2

The condition that characterizes the optimal amount of cake to leave for the next period  $W_3$  in period 2 is:

$$\max_{W_3 \in [0, W_2]} u(W_2 - W_3)$$

The condition that characterizes the optimal amount of cake to leave for the next period  $W_2$  in period 1 is:

$$\max_{W_2 \in [0, W_1]} [u(W_1 - W_2) + \max_{W_3 \in [0, W_2]} \beta u(W_2 - W_3)]$$

## 0.0.3 5.3

The condition that characterizes the optimal amount of cake to leave for the next period  $W_2, W_3, W_4$  in period 1, 2, 3 are respectively:

$$\begin{aligned} \max_{W_2 \in [0, W_1]} \{ & u(W_1 - W_2) + \max_{W_3 \in [0, W_2]} \beta [u(W_2 - W_3) + \max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4)] \} \\ & \max_{W_3 \in [0, W_2]} \beta [u(W_2 - W_3) + \max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4)] \\ & \max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4) \end{aligned}$$

From the 3rd condition, we know that  $W_4 = 0$ , from the 1st and 2nd condition, we know that if we differentiate the left hand side of 1st condition with respect to  $W_2$  and  $W_3$ , every derivatives should equal to 0 to maximise it, i.e.,

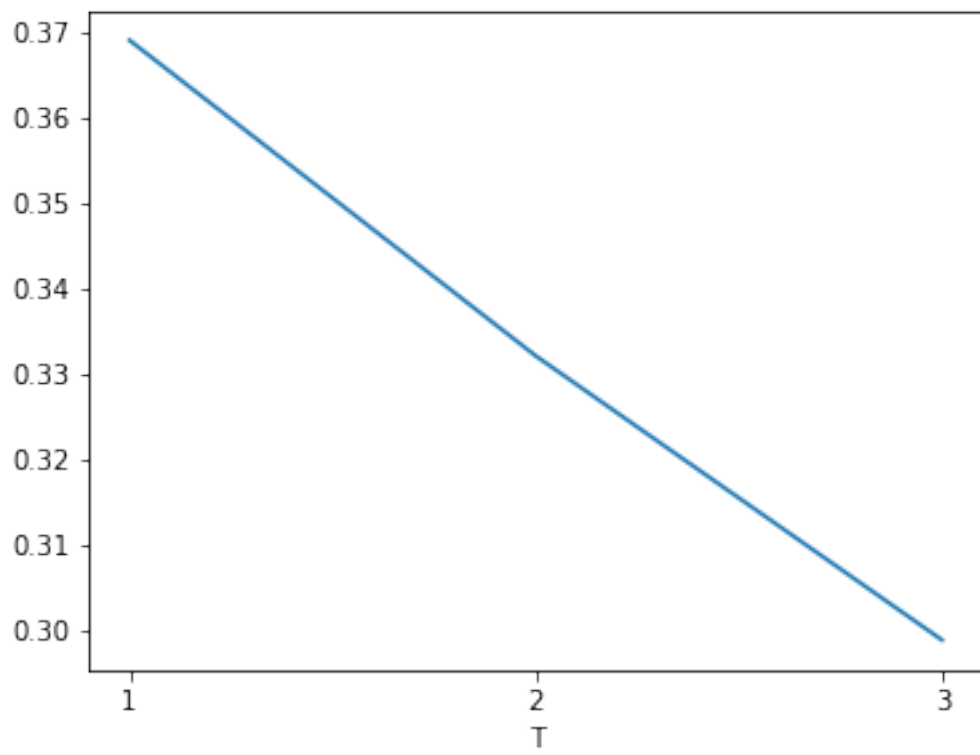
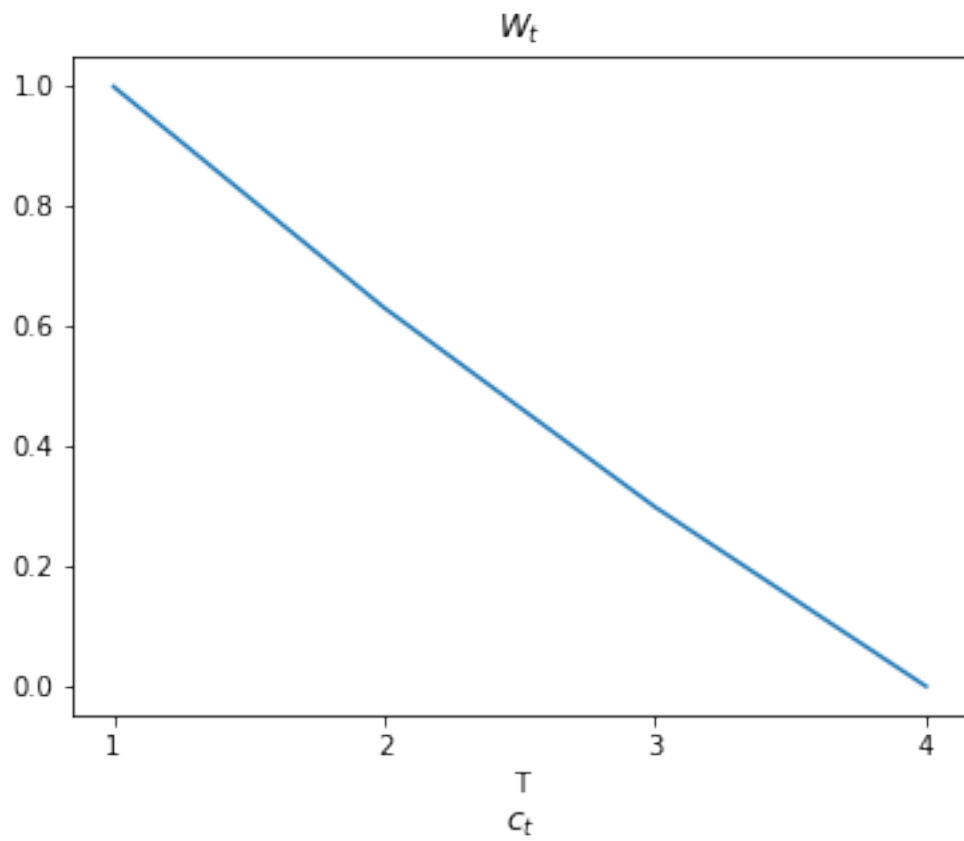
$$-u'(W_1 - W_2) + \beta u'(W_2 - W_3) = 0$$

$$-\beta u'(W_2 - W_3) + \beta^2 u'(W_3 - W_4) = 0$$

We also know that  $u(x) = \ln(x)$ ,  $W_1 = 1$  and  $W_4 = 0$ , so we can solve the equations to get  $W_2 = 0.631$ ,  $W_3 = 0.299$ , we can then find out  $c_1 = W_1 - W_2 = 0.369$ ,  $c_2 = W_2 - W_3 = 0.332$ ,  $c_3 = W_3 - W_4 = 0.299$  The evolve of  $\{c_t\}_{t=1}^3$  and  $\{W_t\}_{t=1}^4$  is as follows:

```
In [2]: W = [1, 1-1/(1+0.9+0.81), 1-1.9/(1+0.9+0.81), 0]
c = [1/(1+0.9+0.81), 0.9/(1+0.9+0.81), 0.81/(1+0.9+0.81)]
T = [1,2,3,4]
fig, ax=plt.subplots(2,1,figsize=(6,10))
ax[0].plot(T, W)
ax[1].plot(T[:-1], c)
ax[0].set_title(r"$W_t$")
ax[0].set_xlabel('T')
ax[0].set_xticks([1,2,3,4])
ax[1].set_title(r"$c_t$")
ax[1].set_xlabel('T')
ax[1].set_xticks([1,2,3])
```

```
Out[2]: [<matplotlib.axis.XTick at 0x16cac5f1eb8>,
<matplotlib.axis.XTick at 0x16cac5f1d68>,
<matplotlib.axis.XTick at 0x16cac5da630>]
```



#### 0.0.4 5.4

The condition that characterizes the optimal choice in period T-1 is:

$$-u'(W_{T-1} - \psi_{T-1}(W_{T-1})) + \beta u'(\psi_{T-1}(W_{T-1})) = 0$$

Then we can express  $V_{T-1}$  as follows:

$$V_{T-1}(W_{T-1}) = u(W_{T-1} - \psi_{T-1}(W_{T-1})) + \beta u(\psi_{T-1}(W_{T-1}))$$

#### 0.0.5 5.5

According to the former question, we know  $V_T(\bar{W}) = u(\bar{W})$ , suppose  $V_{T-1}(\bar{W}) = V_T(\bar{W})$ , that is:

$$u(\bar{W}) = u(\bar{W} - \psi_{T-1}(\bar{W})) + \beta u(\psi_{T-1}(\bar{W}))$$

$$-u'(\bar{W} - \psi_{T-1}(\bar{W})) + \beta u'(\psi_{T-1}(\bar{W})) = 0$$

Since  $u(x) = \ln(x)$  and we can solve former equations:

$$\psi_{T-1}(\bar{W}) = \frac{\beta}{1+\beta} \bar{W} > 0$$

$$\psi_T(\bar{W}) = 0$$

$$V_{T-1}(\bar{W}) = \ln\left(\frac{\bar{W}}{1+\beta}\right) + \beta \ln\left(\frac{\beta \bar{W}}{1+\beta}\right)$$

$$V_T(\bar{W}) = \ln(\bar{W})$$

#### 0.0.6 5.6

The finite horizon Bellman equation for the value function at time T-2 is:

$$V_{T-2}(W_{T-2}) = \max_{W_{T-1}} \ln(W_{T-2} - W_{T-1}) + \beta \ln\left(\frac{W_{T-1}}{1+\beta}\right) + \beta^2 \ln\left(\frac{\beta W_{T-1}}{1+\beta}\right)$$

The condition that characterizes the optimal choice in period T-2 is:

$$-\frac{1}{(W_{T-2} - \psi_{T-2}(W_{T-2}))} + (\beta + \beta^2) \frac{1}{\psi_{T-2}(W_{T-2})} = 0$$

The analytical solution for  $\psi_{T-2}(W_{T-2})$  and  $V_{T-2}(W_{T-2})$  is:

$$\psi_{T-2}(W_{T-2}) = \frac{\beta + \beta^2}{1 + \beta + \beta^2} W_{T-2}$$

$$V_{T-2}(W_{T-2}) = \ln\left(\frac{W_{T-2}}{1 + \beta + \beta^2}\right) + \beta \ln\left(\frac{\beta W_{T-2}}{1 + \beta + \beta^2}\right) + \beta^2 \ln\left(\frac{\beta^2 W_{T-2}}{1 + \beta + \beta^2}\right)$$

### 0.0.7 5.7

By induction, the analytical solution for  $\psi_{T-s}(W_{T-s})$  and  $V_{T-s}(W_{T-s})$  is:

$$\psi_{T-s}(W_{T-s}) = \frac{\sum_{i=1}^s \beta^i}{1 + \sum_{i=1}^s \beta^i} W_{T-s}$$
$$V_{T-s}(W_{T-s}) = \left[ \sum_{i=0}^{s-1} \beta^i \ln \left( \frac{\beta^i W_{T-s}}{1 + \sum_{i=1}^s \beta^i} \right) \right] + \beta^s \ln \left( \frac{\beta^s W_{T-s}}{1 + \sum_{i=1}^s \beta^i} \right)$$

Take limits of s tend to infinite, we have:

$$\psi(W_{T-s}) = \beta W_{T-s}$$
$$V(W_{T-s}) = \left( \frac{1}{1-\beta} \right) \ln((1-\beta)W_{T-s}) + \frac{\beta}{(1-\beta)^2} \ln(\beta)$$

### 0.0.8 5.8

$$V(W) = \max_{w \in [0, W]} u(W-w) + \beta V(w)$$

### 0.0.9 5.9

The code is as follows:

```
In [3]: W = np.linspace(0.01, 1, 100)
```

### 0.0.10 5.10

```
In [4]: c_mat = W.reshape(-1,1)-W
        c_mat[c_mat<=0] = 1e-7
        u_mat = np.log(c_mat)
        N = 100
        beta = 0.9

In [5]: psi_T = np.empty(100)
        Value_T = np.empty(100)
        for i in range(100):
            w = W[i]
            value_func = lambda x: -np.log(w-x)
            psi_T[i] = max(float(opt.fmin(value_func, 0, disp = 0)),0)
            Value_T[i] = np.log(w-psi_T[i])
        print("The policy function W' is\n", psi_T)
        print("The value function V_T is\n", Value_T)
```

The policy function  $W'$  is

[illegible]

The value function  $V_T$  is

```
[-4.60517019 -3.91202301 -3.5065579 -3.21887582 -2.99573227 -2.81341072
-2.65926004 -2.52572864 -2.40794561 -2.30258509 -2.20727491 -2.12026354
-2.04022083 -1.96611286 -1.89711998 -1.83258146 -1.77195684 -1.71479843
-1.66073121 -1.60943791 -1.56064775 -1.51412773 -1.46967597 -1.42711636
-1.38629436 -1.34707365 -1.30933332 -1.27296568 -1.23787436 -1.2039728
-1.17118298 -1.13943428 -1.10866262 -1.07880966 -1.04982212 -1.02165125
-0.99425227 -0.96758403 -0.94160854 -0.91629073 -0.89159812 -0.86750057
-0.84397007 -0.82098055 -0.7985077 -0.77652879 -0.75502258 -0.73396918
-0.71334989 -0.69314718 -0.67334455 -0.65392647 -0.63487827 -0.61618614
-0.597837 -0.5798185 -0.56211892 -0.54472718 -0.52763274 -0.51082562
-0.49429632 -0.4780358 -0.46203546 -0.4462871 -0.43078292 -0.41551544
-0.40047757 -0.38566248 -0.37106368 -0.35667494 -0.34249031 -0.32850407
-0.31471074 -0.301110509 -0.28768207 -0.27443685 -0.26136476 -0.24846136
-0.23572233 -0.22314355 -0.21072103 -0.19845094 -0.18632958 -0.17435339
-0.16251893 -0.15082289 -0.13926207 -0.12783337 -0.11653382 -0.10536052
-0.09431068 -0.08338161 -0.07257069 -0.0618754 -0.05129329 -0.04082199
-0.03045921 -0.02020271 -0.01005034 0.]
```

**0.0.11 5.11**

```
In [6]: Value_T_plus_1 = np.zeros(100)
        delta_T = np.sum((Value_T-Value_T_plus_1)**2)
        print("The distance metric is", delta_T)
```

The distance metric is 178.92611065972804

**0.0.12    5.12**

```
In [7]: Value_T_minus_1 = np.array(Value_T)
psi_T_minus_1 = np.array(psi_T)
VT1_matrix = np.tile(Value_T.reshape(1,100),(100,1))
VT1_matrix[c_mat<0] = -9e+4
for i in range(N):
    Value_T_minus_1[i] = -9e+4
    for j in range(100):
        if u_mat[i,j]+beta*VT1_matrix[i,j] > Value_T_minus_1[i]:
            psi_T_minus_1[i] = W[j]
            Value_T_minus_1[i] = u_mat[i,j]+beta*VT1_matrix[i,j]
    for i in range(100):
```

```

        if psi_T_minus_1[i] >= W[i]:
            psi_T_minus_1[i] = W[i]-0.01
    delta_T_minus_1 = np.sum((Value_T-Value_T_minus_1)**2)
    print("The policy function is\n", psi_T_minus_1)
    print("The value function is\n", Value_T_minus_1)
    print("The distance metric is", delta_T_minus_1)

```

The policy function is

```

[0.  0.01 0.01 0.02 0.02 0.03 0.03 0.04 0.04 0.05 0.05 0.06 0.06 0.07
 0.07 0.08 0.08 0.09 0.09 0.09 0.1  0.1  0.11 0.11 0.12 0.12 0.13 0.13
 0.14 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.18 0.18 0.19 0.19 0.2
 0.2  0.21 0.21 0.22 0.22 0.23 0.23 0.24 0.24 0.25 0.25 0.26 0.26 0.27
 0.27 0.27 0.28 0.28 0.29 0.29 0.3  0.3  0.31 0.31 0.32 0.32 0.33 0.33
 0.34 0.34 0.35 0.35 0.36 0.36 0.36 0.37 0.37 0.38 0.38 0.39 0.39 0.4
 0.4  0.41 0.41 0.42 0.42 0.43 0.43 0.44 0.44 0.45 0.45 0.45 0.46 0.46
 0.47 0.47]

```

The value function is

```

[-16.11809565 -8.74982335 -8.05667617 -7.43284371 -7.0273786
 -6.66246      -6.37477793 -6.11586407 -5.89272052 -5.69189132
 -5.50956976 -5.34548036 -5.19132968 -5.05259407 -4.91906268
 -4.79888442 -4.68110139 -4.57509666 -4.46973614 -4.37442596
 -4.2796015   -4.19259012 -4.10681096 -4.02676825 -3.94845801
 -3.87435004 -3.8023116  -3.73331873 -3.66662156 -3.60208303
 -3.53998945 -3.47936483 -3.42128016 -3.36412175 -3.30955959
 -3.25549236 -3.20404979 -3.1527565  -3.10396633 -3.05530583
 -3.00878582 -2.96262185 -2.91817009 -2.87425894 -2.83169933
 -2.78983132 -2.74900932 -2.70900273 -2.66978202 -2.63147837
 -2.59373804 -2.55699824 -2.5206306  -2.48533196 -2.45024064
 -2.41627434 -2.38237279 -2.34958297 -2.31685209 -2.28510339
 -2.2535212  -2.22274954 -2.19223815 -2.16238519 -2.13287434
 -2.10388681 -2.07531298 -2.0471421  -2.01944761 -1.99204864
 -1.96518097 -1.93851272 -1.91242394 -1.88644845 -1.86109466
 -1.83577685 -1.81108424 -1.78642517 -1.76232761 -1.73832619
 -1.71479569 -1.69141776 -1.66842824 -1.64564221 -1.62316935
 -1.600946    -1.5789671  -1.5572793  -1.5357731  -1.51459565
 -1.49354224 -1.47285167 -1.45223238 -1.43200681 -1.41180411
 -1.39200148 -1.37222046 -1.35280238 -1.33344679 -1.3143986 ]

```

The distance metric is 650.6032364682843

## 0.0.13 5.13

```

In [8]: Value_T_minus_2=np.array(Value_T_minus_1)
        psi_T_minus_2 = np.array(psi_T_minus_1)
        VT2_matrix = np.tile(Value_T_minus_1.reshape(1,100),(100,1))
        VT2_matrix[c_mat<0] = -9e+4
        for i in range(N):
            Value_T_minus_2[i] = -9e+4

```

```

for j in range(100):
    if u_mat[i,j]+beta*VT2_matrix[i,j] > Value_T_minus_2[i]:
        psi_T_minus_2[i] = W[j]
        Value_T_minus_2[i] = u_mat[i,j]+beta*VT2_matrix[i,j]
for i in range(100):
    if psi_T_minus_2[i] >= W[i]:
        psi_T_minus_2[i] = W[i]-0.01
delta_T_minus_2 = np.sum((Value_T_minus_1-Value_T_minus_2)**2)
print("The policy function is\n", psi_T_minus_2)
print("The value function is\n", Value_T_minus_2)
print("The distance metric is", delta_T_minus_2)

```

The policy function is

```

[0.  0.01 0.02 0.02 0.03 0.04 0.04 0.05 0.06 0.06 0.07 0.08 0.08 0.09
 0.09 0.1  0.11 0.11 0.12 0.13 0.13 0.14 0.15 0.15 0.16 0.17 0.17 0.18
 0.18 0.19 0.19 0.2  0.21 0.21 0.22 0.23 0.23 0.24 0.25 0.25 0.26 0.27
 0.27 0.28 0.28 0.29 0.3  0.3  0.31 0.32 0.32 0.33 0.34 0.34 0.35 0.35
 0.36 0.36 0.37 0.38 0.38 0.39 0.4  0.4  0.41 0.42 0.42 0.43 0.44 0.44
 0.45 0.45 0.46 0.47 0.47 0.48 0.49 0.49 0.5  0.51 0.51 0.52 0.52 0.53
 0.54 0.54 0.55 0.55 0.56 0.57 0.57 0.58 0.59 0.59 0.6  0.61 0.61 0.62
 0.63 0.63]

```

The value function is

```

[-17.30105439 -17.30105439 -12.4800112  -11.78686402 -11.16303156
 -10.60158234 -10.19611724  -9.83119864  -9.5027719  -9.21508983
  -8.95617596  -8.72315349  -8.50000993  -8.29918074  -8.11685918
  -7.9361129   -7.7720235   -7.61787282  -7.47019236  -7.33145675
  -7.19792536  -7.07306331  -6.95288505  -6.83510202  -6.72694159
  -6.62093686  -6.51557634  -6.42017208  -6.3248619   -6.23003744
  -6.14302606  -6.0572469   -5.97190488  -5.89186218  -5.81355194
  -5.73635069  -5.66224272  -5.59020428  -5.51972507  -5.45073219
  -5.38403502  -5.31920043  -5.25466191  -5.19256832  -5.1319437
  -5.07191624  -5.01383157  -4.95667316  -4.90078893  -4.84622677
  -4.79215955  -4.73988335  -4.68844078  -4.63714748  -4.58804154
  -4.53925138  -4.49059088  -4.44407086  -4.39777255  -4.35160858
  -4.30715682  -4.26324567  -4.21945122  -4.17689161  -4.13502359
  -4.09347602  -4.05265403  -4.01264744  -3.97312741  -3.9339067
  -3.89560304  -3.85786272  -3.8201815   -3.78344171  -3.74707406
  -3.71106814  -3.67576949  -3.64067817  -3.60620489  -3.57223859
  -3.53833704  -3.50527122  -3.4724814   -3.43975052  -3.40798174
  -3.37623305  -3.34465086  -3.3138792   -3.28330953  -3.25279814
  -3.22294517  -3.19343433  -3.16397654  -3.13498901  -3.10641518
  -3.07799121  -3.04982033  -3.02212584  -2.99466558  -2.96726661]

```

The distance metric is 673.4368745458985



## 0.0.14 5.14

```
In [9]: def optimize(init=W, u = lambda x: np.log(x),
        beta = 0.9, error = 1e-9, maxiter = 1000):
    W = init
    V = np.log(W)
    N = 100
    c_mat = W.reshape(-1,1)-W
    c_mat[c_mat<=0] = 1e-7
    u_mat = u(c_mat)
    Error = 1
    count = 0
    while Error>error and count <= maxiter:
        count += 1
        new_W = np.array(W)
        new_V = np.array(V)
        V_matrix = np.tile(V.reshape(1,100),(100,1))
        V_matrix[c_mat<=0] = -9e+4
        for i in range(N):
            new_V[i] = -9e+4
            for j in range(N):
                if u_mat[i,j]+beta*V_matrix[i,j] > new_V[i]:
                    new_W[i] = W[j]
                    new_V[i] = u_mat[i,j]+beta*V_matrix[i,j]
            for i in range(N):
                if new_W[i] > W[i]:
                    new_W[i] = W[i]-0.01
        Error = ((V-new_V)**2).sum()
        V = new_V
    return new_V,new_W
V, new_W = optimize()
print("The value function is\n", V)
print("The policy function is\n", new_W)
```

The value function is

```
[-42.67084199 -42.67084199 -42.31577781 -41.91031271 -41.62263063
-41.39948708 -41.21716553 -40.93815432 -40.67924046 -40.45609691
-40.25526771 -40.06321183 -39.83018935 -39.6070458 -39.40621661
-39.22389505 -39.04314877 -38.84289983 -38.64207063 -38.45974907
-38.2790028 -38.1149134 -37.95224175 -37.77201774 -37.59127146
-37.42718206 -37.26451041 -37.11035973 -36.96267927 -36.80822395
-36.6455523 -36.49140162 -36.34372116 -36.19731667 -36.05858106
-35.92504967 -35.78665896 -35.64025447 -35.50151886 -35.36798747
-35.23507505 -35.10331102 -34.97844897 -34.85827071 -34.73371918
-34.60195515 -34.4770931 -34.35691484 -34.23729367 -34.11870604
-34.000923 -33.88854716 -33.78038673 -33.66748577 -33.54970274
-33.43732689 -33.32916646 -33.22150741 -33.11477854 -33.00877381
-32.90341329 -32.80227503 -32.70493065 -32.60267565 -32.49731514
-32.39617688 -32.29883249 -32.20193934 -32.10588336 -32.0104791
```

```

-31.91516892 -31.82034446 -31.72932002 -31.64171007 -31.54967981
-31.45485534 -31.36383091 -31.27622096 -31.18901713 -31.10200575
-31.01555536 -30.92969153 -30.84391237 -30.75857035 -30.67664836
-30.59660565 -30.51497226 -30.42963024 -30.34770825 -30.26766554
-30.18881659 -30.11033313 -30.0320229 -29.95421755 -29.8769401
-29.79973885 -29.72293104 -29.64882307 -29.57509327 -29.50305484]

```

The policy function is

```

[0.  0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.04 0.05 0.05 0.06 0.08 0.09 0.09
0.1  0.1  0.11 0.13 0.14 0.14 0.15 0.16 0.17 0.18 0.19 0.2  0.21 0.21
0.22 0.24 0.25 0.25 0.26 0.27 0.28 0.28 0.3  0.31 0.32 0.32 0.33 0.34
0.35 0.36 0.37 0.38 0.39 0.4  0.41 0.42 0.42 0.43 0.44 0.46 0.46 0.47
0.48 0.49 0.5  0.51 0.51 0.52 0.53 0.55 0.55 0.56 0.57 0.58 0.59 0.6
0.6  0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.68 0.69 0.7  0.71 0.72
0.73 0.73 0.75 0.76 0.77 0.77 0.78 0.79 0.8  0.81 0.82 0.83 0.84 0.84
0.85 0.86]

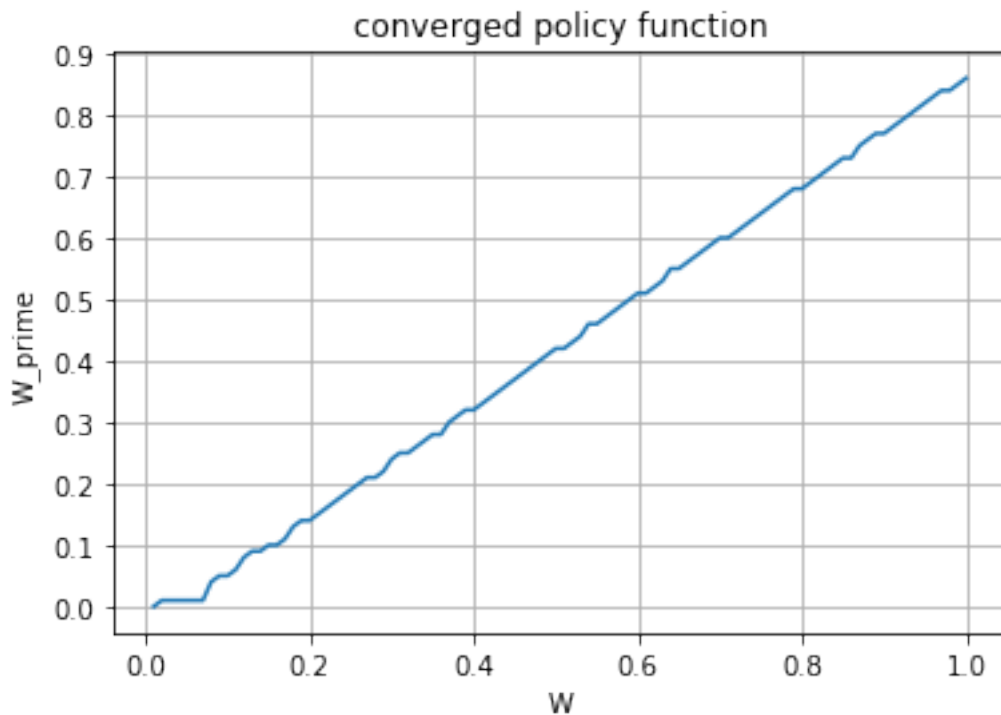
```

### 0.0.15 5.15

```

In [10]: fig,ax = plt.subplots()
         ax.plot(W,new_W)
         ax.set_xlabel("W")
         ax.set_ylabel("W_prime")
         ax.set_yticks([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9])
         ax.set_title("converged policy function")
         ax.grid()

```



0.0.16 5.16

```
In [11]: sigma = 0.5
varepsilon = np.array([i*sigma for i in range(1,8)])
Gamma_func = lambda x: norm(loc = 4*sigma, scale = sigma).pdf(x)
Gamma = Gamma_func(varepsilon)
```

**0.0.17    5.17**

```
In [12]: new_psi_T = np.zeros((100,7))
new_Value_T = np.empty((100,7))
for j in range(7):
    e = varepsilon[j]
    for i in range(100):
        w = W[i]
        new_value_func = lambda x: -e*np.log(w-x)
        new_psi_T[i,j] = max(float(opt.fmin(new_value_func, 0, disp = 0)),0)
        new_Value_T[i,j] = e*np.log(w-psi_T[i])
print("The policy function W' is\n", new_psi_T)
print("The value function V_T is\n", new_Value_T)
```

The policy function  $W'$  is

[illegible]

[illegible]

The value function  $V_T$  is

13

[-1.15129255e+00 -2.30258509e+00 -3.45387764e+00 -4.60517019e+00  
 -5.75646273e+00 -6.90775528e+00 -8.05904783e+00]  
 [-1.10363746e+00 -2.20727491e+00 -3.31091237e+00 -4.41454983e+00  
 -5.51818728e+00 -6.62182474e+00 -7.72546220e+00]  
 [-1.06013177e+00 -2.12026354e+00 -3.18039530e+00 -4.24052707e+00  
 -5.30065884e+00 -6.36079061e+00 -7.42092238e+00]  
 [-1.02011041e+00 -2.04022083e+00 -3.06033124e+00 -4.08044166e+00  
 -5.10055207e+00 -6.12066249e+00 -7.14077290e+00]  
 [-9.83056428e-01 -1.96611286e+00 -2.94916928e+00 -3.93222571e+00  
 -4.91528214e+00 -5.89833857e+00 -6.88139500e+00]  
 [-9.48559992e-01 -1.89711998e+00 -2.84567998e+00 -3.79423997e+00  
 -4.74279996e+00 -5.69135995e+00 -6.63991995e+00]  
 [-9.16290732e-01 -1.83258146e+00 -2.74887220e+00 -3.66516293e+00  
 -4.58145366e+00 -5.49774439e+00 -6.41403512e+00]  
 [-8.85978421e-01 -1.77195684e+00 -2.65793526e+00 -3.54391368e+00  
 -4.42989210e+00 -5.31587053e+00 -6.20184895e+00]  
 [-8.57399214e-01 -1.71479843e+00 -2.57219764e+00 -3.42959686e+00  
 -4.28699607e+00 -5.14439528e+00 -6.00179450e+00]  
 [-8.30365603e-01 -1.66073121e+00 -2.49109681e+00 -3.32146241e+00  
 -4.15182802e+00 -4.98219362e+00 -5.81255922e+00]  
 [-8.04718956e-01 -1.60943791e+00 -2.41415687e+00 -3.21887582e+00  
 -4.02359478e+00 -4.82831374e+00 -5.63303269e+00]  
 [-7.80323874e-01 -1.56064775e+00 -2.34097162e+00 -3.12129550e+00  
 -3.90161937e+00 -4.68194324e+00 -5.46226712e+00]  
 [-7.57063866e-01 -1.51412773e+00 -2.27119160e+00 -3.02825547e+00  
 -3.78531933e+00 -4.54238320e+00 -5.29944706e+00]  
 [-7.34837985e-01 -1.46967597e+00 -2.20451396e+00 -2.93935194e+00  
 -3.67418993e+00 -4.40902791e+00 -5.14386590e+00]  
 [-7.13558178e-01 -1.42711636e+00 -2.14067453e+00 -2.85423271e+00  
 -3.56779089e+00 -4.28134907e+00 -4.99490724e+00]  
 [-6.93147181e-01 -1.38629436e+00 -2.07944154e+00 -2.77258872e+00  
 -3.46573590e+00 -4.15888308e+00 -4.85203026e+00]  
 [-6.73536824e-01 -1.34707365e+00 -2.02061047e+00 -2.69414730e+00  
 -3.36768412e+00 -4.04122094e+00 -4.71475777e+00]  
 [-6.54666660e-01 -1.30933332e+00 -1.96399998e+00 -2.61866664e+00  
 -3.27333330e+00 -3.92799996e+00 -4.58266662e+00]  
 [-6.36482838e-01 -1.27296568e+00 -1.90944851e+00 -2.54593135e+00  
 -3.18241419e+00 -3.81889703e+00 -4.45537987e+00]  
 [-6.18937178e-01 -1.23787436e+00 -1.85681153e+00 -2.47574871e+00  
 -3.09468589e+00 -3.71362307e+00 -4.33256025e+00]  
 [-6.01986402e-01 -1.20397280e+00 -1.80595921e+00 -2.40794561e+00  
 -3.00993201e+00 -3.61191841e+00 -4.21390482e+00]  
 [-5.85591491e-01 -1.17118298e+00 -1.75677447e+00 -2.34236596e+00  
 -2.92795745e+00 -3.51354894e+00 -4.09914044e+00]  
 [-5.69717142e-01 -1.13943428e+00 -1.70915142e+00 -2.27886857e+00  
 -2.84858571e+00 -3.41830285e+00 -3.98801999e+00]  
 [-5.54331312e-01 -1.10866262e+00 -1.66299394e+00 -2.21732525e+00  
 -2.77165656e+00 -3.32598787e+00 -3.88031919e+00]

[-5.39404831e-01 -1.07880966e+00 -1.61821449e+00 -2.15761932e+00  
 -2.69702415e+00 -3.23642898e+00 -3.77583381e+00]  
 [-5.24911062e-01 -1.04982212e+00 -1.57473319e+00 -2.09964425e+00  
 -2.62455531e+00 -3.14946637e+00 -3.67437744e+00]  
 [-5.10825624e-01 -1.02165125e+00 -1.53247687e+00 -2.04330250e+00  
 -2.55412812e+00 -3.06495374e+00 -3.57577937e+00]  
 [-4.97126137e-01 -9.94252273e-01 -1.49137841e+00 -1.98850455e+00  
 -2.48563068e+00 -2.98275682e+00 -3.47988296e+00]  
 [-4.83792013e-01 -9.67584026e-01 -1.45137604e+00 -1.93516805e+00  
 -2.41896007e+00 -2.90275208e+00 -3.38654409e+00]  
 [-4.70804270e-01 -9.41608540e-01 -1.41241281e+00 -1.88321708e+00  
 -2.35402135e+00 -2.82482562e+00 -3.29562989e+00]  
 [-4.58145366e-01 -9.16290732e-01 -1.37443610e+00 -1.83258146e+00  
 -2.29072683e+00 -2.74887220e+00 -3.20701756e+00]  
 [-4.45799060e-01 -8.91598119e-01 -1.33739718e+00 -1.78319624e+00  
 -2.22899530e+00 -2.67479436e+00 -3.12059342e+00]  
 [-4.33750284e-01 -8.67500568e-01 -1.30125085e+00 -1.73500114e+00  
 -2.16875142e+00 -2.60250170e+00 -3.03625199e+00]  
 [-4.21985035e-01 -8.43970070e-01 -1.26595511e+00 -1.68794014e+00  
 -2.10992518e+00 -2.53191021e+00 -2.95389525e+00]  
 [-4.10490276e-01 -8.20980552e-01 -1.23147083e+00 -1.64196110e+00  
 -2.05245138e+00 -2.46294166e+00 -2.87343193e+00]  
 [-3.99253848e-01 -7.98507696e-01 -1.19776154e+00 -1.59701539e+00  
 -1.99626924e+00 -2.39552309e+00 -2.79477694e+00]  
 [-3.88264395e-01 -7.76528789e-01 -1.16479318e+00 -1.55305758e+00  
 -1.94132197e+00 -2.32958637e+00 -2.71785076e+00]  
 [-3.77511292e-01 -7.55022584e-01 -1.13253388e+00 -1.51004517e+00  
 -1.88755646e+00 -2.26506775e+00 -2.64257904e+00]  
 [-3.66984588e-01 -7.33969175e-01 -1.10095376e+00 -1.46793835e+00  
 -1.83492294e+00 -2.20190753e+00 -2.56889211e+00]  
 [-3.56674944e-01 -7.13349888e-01 -1.07002483e+00 -1.42669978e+00  
 -1.78337472e+00 -2.14004966e+00 -2.49672461e+00]  
 [-3.46573590e-01 -6.93147181e-01 -1.03972077e+00 -1.38629436e+00  
 -1.73286795e+00 -2.07944154e+00 -2.42601513e+00]  
 [-3.36672277e-01 -6.73344553e-01 -1.01001683e+00 -1.34668911e+00  
 -1.68336138e+00 -2.02003366e+00 -2.35670594e+00]  
 [-3.26963234e-01 -6.53926467e-01 -9.80889701e-01 -1.30785293e+00  
 -1.63481617e+00 -1.96177940e+00 -2.28874264e+00]  
 [-3.17439136e-01 -6.34878272e-01 -9.52317409e-01 -1.26975654e+00  
 -1.58719568e+00 -1.90463482e+00 -2.22207395e+00]  
 [-3.08093070e-01 -6.16186139e-01 -9.24279209e-01 -1.23237228e+00  
 -1.54046535e+00 -1.84855842e+00 -2.15665149e+00]  
 [-2.98918500e-01 -5.97837001e-01 -8.96755501e-01 -1.19567400e+00  
 -1.49459250e+00 -1.79351100e+00 -2.09242950e+00]  
 [-2.89909248e-01 -5.79818495e-01 -8.69727743e-01 -1.15963699e+00  
 -1.44954624e+00 -1.73945549e+00 -2.02936473e+00]  
 [-2.81059459e-01 -5.62118918e-01 -8.43178377e-01 -1.12423784e+00  
 -1.40529730e+00 -1.68635675e+00 -1.96741621e+00]

[-2.72363588e-01 -5.44727175e-01 -8.17090763e-01 -1.08945435e+00  
 -1.36181794e+00 -1.63418153e+00 -1.90654511e+00]  
 [-2.63816371e-01 -5.27632742e-01 -7.91449113e-01 -1.05526548e+00  
 -1.31908186e+00 -1.58289823e+00 -1.84671460e+00]  
 [-2.55412812e-01 -5.10825624e-01 -7.66238436e-01 -1.02165125e+00  
 -1.27706406e+00 -1.53247687e+00 -1.78788968e+00]  
 [-2.47148161e-01 -4.94296322e-01 -7.41444483e-01 -9.88592644e-01  
 -1.23574080e+00 -1.48288897e+00 -1.73003713e+00]  
 [-2.39017900e-01 -4.78035801e-01 -7.17053701e-01 -9.56071602e-01  
 -1.19508950e+00 -1.43410740e+00 -1.67312530e+00]  
 [-2.31017730e-01 -4.62035460e-01 -6.93053189e-01 -9.24070919e-01  
 -1.15508865e+00 -1.38610638e+00 -1.61712411e+00]  
 [-2.23143551e-01 -4.46287103e-01 -6.69430654e-01 -8.92574205e-01  
 -1.11571776e+00 -1.33886131e+00 -1.56200486e+00]  
 [-2.15391458e-01 -4.30782916e-01 -6.46174374e-01 -8.61565832e-01  
 -1.07695729e+00 -1.29234875e+00 -1.50774021e+00]  
 [-2.07757722e-01 -4.15515444e-01 -6.23273166e-01 -8.31030888e-01  
 -1.03878861e+00 -1.24654633e+00 -1.45430405e+00]  
 [-2.00238783e-01 -4.00477567e-01 -6.00716350e-01 -8.00955133e-01  
 -1.00119392e+00 -1.20143270e+00 -1.40167148e+00]  
 [-1.92831240e-01 -3.85662481e-01 -5.78493721e-01 -7.71324962e-01  
 -9.64156202e-01 -1.15698744e+00 -1.34981868e+00]  
 [-1.85531841e-01 -3.71063681e-01 -5.56595522e-01 -7.42127363e-01  
 -9.27659203e-01 -1.11319104e+00 -1.29872288e+00]  
 [-1.78337472e-01 -3.56674944e-01 -5.35012416e-01 -7.13349888e-01  
 -8.91687360e-01 -1.07002483e+00 -1.24836230e+00]  
 [-1.71245154e-01 -3.42490309e-01 -5.13735463e-01 -6.84980618e-01  
 -8.56225772e-01 -1.02747093e+00 -1.19871608e+00]  
 [-1.64252033e-01 -3.28504067e-01 -4.92756100e-01 -6.57008134e-01  
 -8.21260167e-01 -9.85512201e-01 -1.14976423e+00]  
 [-1.57355372e-01 -3.14710745e-01 -4.72066117e-01 -6.29421490e-01  
 -7.86776862e-01 -9.44132235e-01 -1.10148761e+00]  
 [-1.50552546e-01 -3.01105093e-01 -4.51657639e-01 -6.02210186e-01  
 -7.52762732e-01 -9.03315278e-01 -1.05386782e+00]  
 [-1.43841036e-01 -2.87682072e-01 -4.31523109e-01 -5.75364145e-01  
 -7.19205181e-01 -8.63046217e-01 -1.00688725e+00]  
 [-1.37218423e-01 -2.74436846e-01 -4.11655269e-01 -5.48873691e-01  
 -6.86092114e-01 -8.23310537e-01 -9.60528960e-01]  
 [-1.30682382e-01 -2.61364764e-01 -3.92047146e-01 -5.22729528e-01  
 -6.53411910e-01 -7.84094292e-01 -9.14776674e-01]  
 [-1.24230680e-01 -2.48461359e-01 -3.72692039e-01 -4.96922719e-01  
 -6.21153398e-01 -7.45384078e-01 -8.69614758e-01]  
 [-1.17861167e-01 -2.35722334e-01 -3.53583500e-01 -4.71444667e-01  
 -5.89305834e-01 -7.07167001e-01 -8.25028167e-01]  
 [-1.11571776e-01 -2.23143551e-01 -3.34715327e-01 -4.46287103e-01  
 -5.57858878e-01 -6.69430654e-01 -7.81002430e-01]  
 [-1.05360516e-01 -2.10721031e-01 -3.16081547e-01 -4.21442063e-01  
 -5.26802578e-01 -6.32163094e-01 -7.37523610e-01]



```

[-9.92254694e-02 -1.98450939e-01 -2.97676408e-01 -3.96901877e-01
 -4.96127347e-01 -5.95352816e-01 -6.94578286e-01]
[-9.31647891e-02 -1.86329578e-01 -2.79494367e-01 -3.72659156e-01
 -4.65823945e-01 -5.58988735e-01 -6.52153524e-01]
[-8.71766936e-02 -1.74353387e-01 -2.61530081e-01 -3.48706774e-01
 -4.35883468e-01 -5.23060161e-01 -6.10236855e-01]
[-8.12594647e-02 -1.62518929e-01 -2.43778394e-01 -3.25037859e-01
 -4.06297324e-01 -4.87556788e-01 -5.68816253e-01]
[-7.54114449e-02 -1.50822890e-01 -2.26234335e-01 -3.01645779e-01
 -3.77057224e-01 -4.52468669e-01 -5.27880114e-01]
[-6.96310337e-02 -1.39262067e-01 -2.08893101e-01 -2.78524135e-01
 -3.48155168e-01 -4.17786202e-01 -4.87417236e-01]
[-6.39166858e-02 -1.27833372e-01 -1.91750057e-01 -2.55666743e-01
 -3.19583429e-01 -3.83500115e-01 -4.47416800e-01]
[-5.82669081e-02 -1.16533816e-01 -1.74800724e-01 -2.33067633e-01
 -2.91334541e-01 -3.49601449e-01 -4.07868357e-01]
[-5.26802578e-02 -1.05360516e-01 -1.58040773e-01 -2.10721031e-01
 -2.63401289e-01 -3.16081547e-01 -3.68761805e-01]
[-4.71553397e-02 -9.43106795e-02 -1.41466019e-01 -1.88621359e-01
 -2.35776699e-01 -2.82932038e-01 -3.30087378e-01]
[-4.16908045e-02 -8.33816089e-02 -1.25072413e-01 -1.66763218e-01
 -2.08454022e-01 -2.50144827e-01 -2.91835631e-01]
[-3.62853464e-02 -7.25706928e-02 -1.08856039e-01 -1.45141386e-01
 -1.81426732e-01 -2.17712079e-01 -2.53997425e-01]
[-3.09377019e-02 -6.18754037e-02 -9.28131056e-02 -1.23750807e-01
 -1.54688509e-01 -1.85626211e-01 -2.16563913e-01]
[-2.56466472e-02 -5.12932944e-02 -7.69399416e-02 -1.02586589e-01
 -1.28233236e-01 -1.53879883e-01 -1.79526530e-01]
[-2.04109973e-02 -4.08219945e-02 -6.12329918e-02 -8.16439890e-02
 -1.02054986e-01 -1.22465984e-01 -1.42876981e-01]
[-1.52296037e-02 -3.04592075e-02 -4.56888112e-02 -6.09184150e-02
 -7.61480187e-02 -9.13776225e-02 -1.06607226e-01]
[-1.01013537e-02 -2.02027073e-02 -3.03040610e-02 -4.04054146e-02
 -5.05067683e-02 -6.06081220e-02 -7.07094756e-02]
[-5.02516793e-03 -1.00503359e-02 -1.50755038e-02 -2.01006717e-02
 -2.51258396e-02 -3.01510076e-02 -3.51761755e-02]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00]]

```

## 0.0.18 5.18

```

In [13]: new_Value_T_plus_1 = np.zeros((100,7))
         new_delta_T = np.sum((new_Value_T-new_Value_T_plus_1)**2)
         print("The distance metric is", new_delta_T)

```

The distance metric is 6262.4138730904815

## 0.0.19 5.19

```
In [14]: new_Value_T_minus_1 = np.array(new_Value_T)
        new_psi_T_minus_1 = np.array(new_psi_T)
        for j in range(7):
            e = varepsilon[j]
            for i in range(100):
                w = W[i]
                new_Value_T_minus_1[i,j] = -9e+4
                for k in range(i):
                    v_value = e*u_mat[i,k] + beta*\
                        sum(Gamma[p]*new_Value_T[k,p] for p in range(7))
                    if v_value > new_Value_T_minus_1[i,j]:
                        new_psi_T_minus_1[i,j] = W[k]
                        new_Value_T_minus_1[i,j] = v_value
        new_delta_T_minus_1 = np.sum((new_Value_T-new_Value_T_minus_1)**2)
        print("The policy function is\n", new_psi_T_minus_1)
        print("The value function is\n", new_Value_T_minus_1)
        print("The distance metric is", new_delta_T_minus_1)
```

The policy function is

```
[[0.  0.  0.  0.  0.  0.  0. ]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.02 0.02 0.02]
 [0.04 0.04 0.03 0.03 0.03 0.03 0.03]
 [0.05 0.05 0.04 0.04 0.04 0.03 0.03]
 [0.06 0.05 0.05 0.04 0.04 0.04 0.04]
 [0.07 0.06 0.06 0.05 0.05 0.04 0.04]
 [0.08 0.07 0.06 0.06 0.05 0.05 0.05]
 [0.09 0.08 0.07 0.06 0.06 0.05 0.05]
 [0.1  0.09 0.08 0.07 0.06 0.06 0.06]
 [0.1  0.09 0.08 0.08 0.07 0.07 0.06]
 [0.11 0.1  0.09 0.08 0.08 0.07 0.07]
 [0.12 0.11 0.1  0.09 0.08 0.08 0.07]
 [0.13 0.12 0.11 0.1  0.09 0.08 0.08]
 [0.14 0.13 0.11 0.1  0.09 0.09 0.08]
 [0.15 0.13 0.12 0.11 0.1  0.09 0.09]
 [0.16 0.14 0.13 0.12 0.11 0.1  0.09]
 [0.17 0.15 0.13 0.12 0.11 0.1  0.1 ]
 [0.18 0.16 0.14 0.13 0.12 0.11 0.1 ]
 [0.18 0.16 0.15 0.13 0.12 0.11 0.11]
 [0.19 0.17 0.16 0.14 0.13 0.12 0.11]
 [0.2  0.18 0.16 0.15 0.14 0.13 0.12]
 [0.21 0.19 0.17 0.15 0.14 0.13 0.12]
 [0.22 0.2  0.18 0.16 0.15 0.14 0.13]
 [0.23 0.2  0.18 0.17 0.15 0.14 0.13]
 [0.24 0.21 0.19 0.17 0.16 0.15 0.14]
 [0.25 0.22 0.2  0.18 0.17 0.15 0.14]]
```

[0.25 0.23 0.2 0.19 0.17 0.16 0.15]  
 [0.26 0.23 0.21 0.19 0.18 0.16 0.15]  
 [0.27 0.24 0.22 0.2 0.18 0.17 0.16]  
 [0.28 0.25 0.23 0.21 0.19 0.17 0.16]  
 [0.29 0.26 0.23 0.21 0.19 0.18 0.17]  
 [0.3 0.27 0.24 0.22 0.2 0.19 0.17]  
 [0.31 0.27 0.25 0.22 0.21 0.19 0.18]  
 [0.32 0.28 0.25 0.23 0.21 0.2 0.18]  
 [0.32 0.29 0.26 0.24 0.22 0.2 0.19]  
 [0.33 0.3 0.27 0.24 0.22 0.21 0.19]  
 [0.34 0.31 0.28 0.25 0.23 0.21 0.2 ]  
 [0.35 0.31 0.28 0.26 0.24 0.22 0.2 ]  
 [0.36 0.32 0.29 0.26 0.24 0.22 0.21]  
 [0.37 0.33 0.3 0.27 0.25 0.23 0.21]  
 [0.38 0.34 0.3 0.28 0.25 0.23 0.22]  
 [0.39 0.34 0.31 0.28 0.26 0.24 0.22]  
 [0.39 0.35 0.32 0.29 0.27 0.25 0.23]  
 [0.4 0.36 0.32 0.3 0.27 0.25 0.23]  
 [0.41 0.37 0.33 0.3 0.28 0.26 0.24]  
 [0.42 0.38 0.34 0.31 0.28 0.26 0.24]  
 [0.43 0.38 0.35 0.31 0.29 0.27 0.25]  
 [0.44 0.39 0.35 0.32 0.3 0.27 0.25]  
 [0.45 0.4 0.36 0.33 0.3 0.28 0.26]  
 [0.46 0.41 0.37 0.33 0.31 0.28 0.26]  
 [0.47 0.41 0.37 0.34 0.31 0.29 0.27]  
 [0.47 0.42 0.38 0.35 0.32 0.29 0.27]  
 [0.48 0.43 0.39 0.35 0.32 0.3 0.28]  
 [0.49 0.44 0.4 0.36 0.33 0.31 0.28]  
 [0.5 0.45 0.4 0.37 0.34 0.31 0.29]  
 [0.51 0.45 0.41 0.37 0.34 0.32 0.29]  
 [0.52 0.46 0.42 0.38 0.35 0.32 0.3 ]  
 [0.53 0.47 0.42 0.39 0.35 0.33 0.3 ]  
 [0.54 0.48 0.43 0.39 0.36 0.33 0.31]  
 [0.54 0.49 0.44 0.4 0.37 0.34 0.31]  
 [0.55 0.49 0.44 0.4 0.37 0.34 0.32]  
 [0.56 0.5 0.45 0.41 0.38 0.35 0.32]  
 [0.57 0.51 0.46 0.42 0.38 0.35 0.33]  
 [0.58 0.52 0.47 0.42 0.39 0.36 0.33]  
 [0.59 0.52 0.47 0.43 0.4 0.37 0.34]  
 [0.6 0.53 0.48 0.44 0.4 0.37 0.34]  
 [0.61 0.54 0.49 0.44 0.41 0.38 0.35]  
 [0.61 0.55 0.49 0.45 0.41 0.38 0.35]  
 [0.62 0.56 0.5 0.46 0.42 0.39 0.36]  
 [0.63 0.56 0.51 0.46 0.42 0.39 0.37]  
 [0.64 0.57 0.52 0.47 0.43 0.4 0.37]  
 [0.65 0.58 0.52 0.48 0.44 0.4 0.38]  
 [0.66 0.59 0.53 0.48 0.44 0.41 0.38]  
 [0.67 0.59 0.54 0.49 0.45 0.41 0.39]

```

[0.68 0.6 0.54 0.49 0.45 0.42 0.39]
[0.68 0.61 0.55 0.5 0.46 0.43 0.4 ]
[0.69 0.62 0.56 0.51 0.47 0.43 0.4 ]
[0.7 0.63 0.56 0.51 0.47 0.44 0.41]
[0.71 0.63 0.57 0.52 0.48 0.44 0.41]
[0.72 0.64 0.58 0.53 0.48 0.45 0.42]
[0.73 0.65 0.59 0.53 0.49 0.45 0.42]
[0.74 0.66 0.59 0.54 0.5 0.46 0.43]
[0.75 0.67 0.6 0.55 0.5 0.46 0.43]
[0.76 0.67 0.61 0.55 0.51 0.47 0.44]
[0.76 0.68 0.61 0.56 0.51 0.47 0.44]
[0.77 0.69 0.62 0.57 0.52 0.48 0.45]
[0.78 0.7 0.63 0.57 0.53 0.49 0.45]
[0.79 0.7 0.64 0.58 0.53 0.49 0.46]
[0.8 0.71 0.64 0.58 0.54 0.5 0.46]
[0.81 0.72 0.65 0.59 0.54 0.5 0.47]
[0.82 0.73 0.66 0.6 0.55 0.51 0.47]
[0.83 0.74 0.66 0.6 0.55 0.51 0.48]
[0.83 0.74 0.67 0.61 0.56 0.52 0.48]
[0.84 0.75 0.68 0.62 0.57 0.52 0.49]
[0.85 0.76 0.68 0.62 0.57 0.53 0.49]
[0.86 0.77 0.69 0.63 0.58 0.53 0.5 ]
[0.87 0.77 0.7 0.64 0.58 0.54 0.5 ]
[0.88 0.78 0.71 0.64 0.59 0.55 0.51]]

```

The value function is

```

[[-9.00000000e+04 -9.00000000e+04 -9.00000000e+04 -9.00000000e+04
-9.00000000e+04 -9.00000000e+04 -9.00000000e+04]
[-1.88767109e+01 -2.11792960e+01 -2.34818811e+01 -2.57844662e+01
-2.80870513e+01 -3.03896364e+01 -3.26922215e+01]
[-1.63820564e+01 -1.86846415e+01 -2.09872266e+01 -2.32898117e+01
-2.55923968e+01 -2.78949819e+01 -3.01975670e+01]
[-1.49227771e+01 -1.72253622e+01 -1.95279472e+01 -2.18305323e+01
-2.38595288e+01 -2.58155403e+01 -2.77715518e+01]
[-1.38874019e+01 -1.61899870e+01 -1.84882265e+01 -2.04442380e+01
-2.24002495e+01 -2.43562610e+01 -2.63122725e+01]
[-1.30843025e+01 -1.53868876e+01 -1.74528513e+01 -1.94088628e+01
-2.13648743e+01 -2.31398657e+01 -2.48931446e+01]
[-1.24281225e+01 -1.46937404e+01 -1.66497519e+01 -1.85979326e+01
-2.03512115e+01 -2.21044905e+01 -2.38577694e+01]
[-1.18733303e+01 -1.40375605e+01 -1.59935720e+01 -1.77948332e+01
-1.95481122e+01 -2.12414443e+01 -2.28508822e+01]
[-1.13927474e+01 -1.34827682e+01 -1.53853743e+01 -1.71386533e+01
-1.88289070e+01 -2.04383449e+01 -2.20477828e+01]
[-1.09688432e+01 -1.30021853e+01 -1.48305820e+01 -1.65632891e+01
-1.81727270e+01 -1.97689142e+01 -2.12667804e+01]
[-1.05896480e+01 -1.25782811e+01 -1.43499991e+01 -1.60084968e+01
-1.76148681e+01 -1.91127343e+01 -2.06106004e+01]
[-1.02430744e+01 -1.21728160e+01 -1.39184760e+01 -1.55279139e+01

```

-1.70600759e+01 -1.85579420e+01 -1.99724750e+01]  
 [-9.90005063e+00 -1.17936208e+01 -1.34945719e+01 -1.50816268e+01  
 -1.65794930e+01 -1.80109773e+01 -1.94176827e+01]  
 [-9.58689445e+00 -1.14505970e+01 -1.31153766e+01 -1.46577227e+01  
 -1.61236891e+01 -1.75303944e+01 -1.88781553e+01]  
 [-9.29881868e+00 -1.11374408e+01 -1.27723529e+01 -1.42785275e+01  
 -1.56997849e+01 -1.70679424e+01 -1.83975724e+01]  
 [-9.03210219e+00 -1.08493651e+01 -1.24376375e+01 -1.39138843e+01  
 -1.53144082e+01 -1.66440382e+01 -1.79302125e+01]  
 [-8.78379507e+00 -1.05616830e+01 -1.21244814e+01 -1.35708606e+01  
 -1.49352130e+01 -1.62434440e+01 -1.75063084e+01]  
 [-8.55151928e+00 -1.02949665e+01 -1.18364056e+01 -1.32577044e+01  
 -1.45921892e+01 -1.58642488e+01 -1.70940677e+01]  
 [-8.33332970e+00 -1.00466594e+01 -1.15629233e+01 -1.29494030e+01  
 -1.42583607e+01 -1.55108997e+01 -1.67148725e+01]  
 [-8.12761510e+00 -9.81438360e+00 -1.12962068e+01 -1.26613273e+01  
 -1.39452046e+01 -1.51678760e+01 -1.63461107e+01]  
 [-7.92488255e+00 -9.59124005e+00 -1.10478996e+01 -1.23942645e+01  
 -1.36507470e+01 -1.48517944e+01 -1.60030870e+01]  
 [-7.73029323e+00 -9.37305048e+00 -1.08156238e+01 -1.21275480e+01  
 -1.33626712e+01 -1.45386382e+01 -1.56695013e+01]  
 [-7.54568735e+00 -9.16733588e+00 -1.05843978e+01 -1.18792409e+01  
 -1.30959547e+01 -1.42505625e+01 -1.53563451e+01]  
 [-7.37009029e+00 -8.97274656e+00 -1.03662083e+01 -1.16436748e+01  
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 [-7.20266356e+00 -8.78814067e+00 -1.01604937e+01 -1.14113990e+01  
 -1.25842463e+01 -1.36979154e+01 -1.47637296e+01]  
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 -1.23459709e+01 -1.34368813e+01 -1.44835801e+01]  
 [-6.88950738e+00 -8.43022206e+00 -9.76560725e+00 -1.09824884e+01  
 -1.21136951e+01 -1.31885742e+01 -1.42168636e+01]  
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 [-6.59874694e+00 -8.10281229e+00 -9.40432681e+00 -1.05821845e+01  
 -1.16779770e+01 -1.27161703e+01 -1.37091786e+01]  
 [-6.45759058e+00 -7.94866161e+00 -9.22872976e+00 -1.03915641e+01  
 -1.14722624e+01 -1.24938463e+01 -1.34677035e+01]  
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 [-6.19087409e+00 -7.64856906e+00 -8.90131999e+00 -1.00313612e+01  
 -1.10775664e+01 -1.20686782e+01 -1.30095429e+01]  
 [-6.06457952e+00 -7.50741271e+00 -8.74327922e+00 -9.85733842e+00  
 -1.08922964e+01 -1.18629636e+01 -1.27913533e+01]  
 [-5.94256697e+00 -7.37158430e+00 -8.59010607e+00 -9.68991169e+00  
 -1.07076905e+01 -1.16683742e+01 -1.25791671e+01]  
 [-5.82455555e+00 -7.23805290e+00 -8.44318666e+00 -9.52982627e+00  
 -1.05320935e+01 -1.14747587e+01 -1.23734525e+01]  
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 [-5.59871940e+00 -6.98087026e+00 -8.15906504e+00 -9.21667009e+00  
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 -1.00308383e+01 -1.09326819e+01 -1.17895735e+01]  
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 -8.00043697e+00 -8.73649068e+00 -9.42708203e+00]  
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 -6.48016808e+00 -7.09188842e+00 -7.65849750e+00]  
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 -5.56593139e+00 -6.10242371e+00 -6.59338378e+00]  
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 -5.48900224e+00 -6.01968384e+00 -6.50451443e+00]  
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 [-2.28394460e+00 -3.26519721e+00 -4.03963833e+00 -4.69288882e+00  
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 [-2.23497751e+00 -3.21024919e+00 -3.97840534e+00 -4.62561536e+00

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-5.19167867e+00 -5.69749914e+00 -6.15806338e+00]
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-5.11920982e+00 -5.61957268e+00 -6.07372195e+00]
[-2.13899781e+00 -3.10206026e+00 -3.85842670e+00 -4.49399669e+00
-5.04793966e+00 -5.54217130e+00 -5.99098209e+00]
[-2.09134272e+00 -3.04874038e+00 -3.79959563e+00 -4.42914762e+00
-4.97751247e+00 -5.46621788e+00 -5.90862535e+00]
[-2.04429596e+00 -2.99619893e+00 -3.74107360e+00 -4.36544639e+00
-4.90762628e+00 -5.39044611e+00 -5.82774496e+00]
[-1.99785627e+00 -2.94441349e+00 -3.68348796e+00 -4.30194900e+00
-4.83907133e+00 -5.31623677e+00 -5.74728165e+00]
[-1.95200819e+00 -2.89312020e+00 -3.62680922e+00 -4.23935567e+00
-4.77057390e+00 -5.24215893e+00 -5.66817900e+00]
[-1.90673683e+00 -2.84206933e+00 -3.57019873e+00 -4.17781235e+00
-4.70330043e+00 -5.16944887e+00 -5.58952400e+00]
[-1.86202786e+00 -2.79173249e+00 -3.51439876e+00 -4.11628904e+00
-4.63662981e+00 -5.09715622e+00 -5.51212262e+00]
[-1.81786748e+00 -2.74208997e+00 -3.45945074e+00 -4.05579979e+00
-4.57059079e+00 -5.02588605e+00 -5.43519644e+00]
[-1.77424240e+00 -2.69312288e+00 -3.40489927e+00 -3.99609387e+00
-4.50565208e+00 -4.95529456e+00 -5.35942468e+00]
[-1.73073671e+00 -2.64433271e+00 -3.35077757e+00 -3.93660449e+00
-4.44080301e+00 -4.88540837e+00 -5.28415296e+00]
[-1.68763409e+00 -2.59602292e+00 -3.29745769e+00 -3.87808245e+00
-4.37710178e+00 -4.81643981e+00 -5.20994362e+00]
[-1.64504157e+00 -2.54835301e+00 -3.24482071e+00 -3.82010738e+00
-4.31380726e+00 -4.74788487e+00 -5.13625669e+00]
[-1.60294722e+00 -2.50130625e+00 -3.19227926e+00 -3.76252174e+00
-4.25121393e+00 -4.68046630e+00 -5.06354662e+00]
[-1.56133953e+00 -2.45478623e+00 -3.14049382e+00 -3.70584300e+00
-4.18948240e+00 -4.61319284e+00 -4.99137912e+00]
[-1.52020736e+00 -2.40834655e+00 -3.08944296e+00 -3.64950124e+00
-4.12795909e+00 -4.54715381e+00 -4.92010896e+00]]

```

The distance metric is 56688435504.040054

## 0.0.20 5.20

```

In [15]: new_Value_T_minus_2 = np.array(new_Value_T_minus_1)
        new_psi_T_minus_2 = np.array(new_psi_T_minus_1)
        for j in range(7):
            e = varepsilon[j]
            for i in range(100):
                w = W[i]
                new_Value_T_minus_2[i,j] = -9e+4
                for k in range(i):
                    v_value = e*u_mat[i,k] + beta*\
                        sum(Gamma[p]*new_Value_T_minus_1[k,p] for p in range(7))

```



```

        if v_value > new_Value_T_minus_2[i,j]:
            new_psi_T_minus_2[i,j] = W[k]
            new_Value_T_minus_2[i,j] = v_value
    new_delta_T_minus_2 = np.sum((new_Value_T_minus_1-new_Value_T_minus_2)**2)
    print("The policy function is\n", new_psi_T_minus_2)
    print("The value function is\n", new_Value_T_minus_2)
    print("The distance metric is", new_delta_T_minus_2)

```

The policy function is

```

[[0.  0.  0.  0.  0.  0.  0. ]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.04 0.04 0.04 0.04 0.04 0.04 0.04]
 [0.05 0.05 0.05 0.05 0.05 0.05 0.04]
 [0.06 0.06 0.06 0.06 0.06 0.05 0.05]
 [0.07 0.07 0.07 0.07 0.06 0.06 0.06]
 [0.08 0.08 0.08 0.07 0.07 0.07 0.07]
 [0.09 0.09 0.09 0.08 0.08 0.08 0.07]
 [0.1  0.1  0.1  0.09 0.09 0.08 0.08]
 [0.11 0.11 0.1  0.1  0.1  0.09 0.09]
 [0.12 0.12 0.11 0.11 0.1  0.1  0.1 ]
 [0.13 0.13 0.12 0.12 0.11 0.11 0.1 ]
 [0.14 0.14 0.13 0.13 0.12 0.12 0.11]
 [0.15 0.14 0.14 0.13 0.13 0.12 0.12]
 [0.16 0.15 0.15 0.14 0.14 0.13 0.13]
 [0.17 0.16 0.16 0.15 0.14 0.14 0.13]
 [0.18 0.17 0.17 0.16 0.15 0.15 0.14]
 [0.19 0.18 0.17 0.17 0.16 0.15 0.15]
 [0.2  0.19 0.18 0.17 0.17 0.16 0.16]
 [0.21 0.2  0.19 0.18 0.18 0.17 0.16]
 [0.22 0.21 0.2  0.19 0.18 0.18 0.17]
 [0.23 0.22 0.21 0.2  0.19 0.19 0.18]
 [0.24 0.23 0.22 0.21 0.2  0.19 0.19]
 [0.25 0.24 0.23 0.22 0.21 0.2  0.19]
 [0.26 0.25 0.23 0.23 0.22 0.21 0.2 ]
 [0.27 0.25 0.24 0.23 0.22 0.22 0.21]
 [0.28 0.26 0.25 0.24 0.23 0.22 0.22]
 [0.29 0.27 0.26 0.25 0.24 0.23 0.22]
 [0.29 0.28 0.27 0.26 0.25 0.24 0.23]
 [0.3  0.29 0.28 0.27 0.26 0.25 0.24]
 [0.31 0.3  0.29 0.28 0.26 0.25 0.24]
 [0.32 0.31 0.3  0.28 0.27 0.26 0.25]
 [0.33 0.32 0.3  0.29 0.28 0.27 0.26]
 [0.34 0.33 0.31 0.3  0.29 0.28 0.27]
 [0.35 0.34 0.32 0.31 0.3  0.28 0.27]
 [0.36 0.35 0.33 0.32 0.3  0.29 0.28]
 [0.37 0.35 0.34 0.33 0.31 0.3  0.29]

```

[0.38 0.36 0.35 0.33 0.32 0.31 0.3 ]  
 [0.39 0.37 0.36 0.34 0.33 0.32 0.3 ]  
 [0.4 0.38 0.37 0.35 0.34 0.32 0.31]  
 [0.41 0.39 0.37 0.36 0.34 0.33 0.32]  
 [0.42 0.4 0.38 0.37 0.35 0.34 0.33]  
 [0.43 0.41 0.39 0.38 0.36 0.35 0.33]  
 [0.44 0.42 0.4 0.38 0.37 0.35 0.34]  
 [0.45 0.43 0.41 0.39 0.38 0.36 0.35]  
 [0.46 0.44 0.42 0.4 0.38 0.37 0.36]  
 [0.47 0.45 0.43 0.41 0.39 0.38 0.36]  
 [0.48 0.45 0.44 0.42 0.4 0.39 0.37]  
 [0.49 0.46 0.44 0.43 0.41 0.39 0.38]  
 [0.5 0.47 0.45 0.43 0.42 0.4 0.39]  
 [0.5 0.48 0.46 0.44 0.42 0.41 0.39]  
 [0.51 0.49 0.47 0.45 0.43 0.42 0.4 ]  
 [0.52 0.5 0.48 0.46 0.44 0.42 0.41]  
 [0.53 0.51 0.49 0.47 0.45 0.43 0.42]  
 [0.54 0.52 0.5 0.48 0.46 0.44 0.42]  
 [0.55 0.53 0.5 0.48 0.47 0.45 0.43]  
 [0.56 0.54 0.51 0.49 0.47 0.45 0.44]  
 [0.57 0.55 0.52 0.5 0.48 0.46 0.45]  
 [0.58 0.55 0.53 0.51 0.49 0.47 0.45]  
 [0.59 0.56 0.54 0.52 0.5 0.48 0.46]  
 [0.6 0.57 0.55 0.53 0.51 0.48 0.47]  
 [0.61 0.58 0.56 0.53 0.51 0.49 0.47]  
 [0.62 0.59 0.57 0.54 0.52 0.5 0.48]  
 [0.63 0.6 0.57 0.55 0.53 0.51 0.49]  
 [0.64 0.61 0.58 0.56 0.54 0.52 0.5 ]  
 [0.65 0.62 0.59 0.57 0.54 0.52 0.51]  
 [0.66 0.63 0.6 0.58 0.55 0.53 0.51]  
 [0.67 0.64 0.61 0.58 0.56 0.54 0.52]  
 [0.68 0.65 0.62 0.59 0.57 0.55 0.53]  
 [0.69 0.65 0.63 0.6 0.58 0.55 0.53]  
 [0.7 0.66 0.64 0.61 0.58 0.56 0.54]  
 [0.7 0.67 0.64 0.62 0.59 0.57 0.55]  
 [0.71 0.68 0.65 0.63 0.6 0.58 0.56]  
 [0.72 0.69 0.66 0.63 0.61 0.59 0.56]  
 [0.73 0.7 0.67 0.64 0.62 0.59 0.57]  
 [0.74 0.71 0.68 0.65 0.62 0.6 0.58]  
 [0.75 0.72 0.69 0.66 0.63 0.61 0.59]  
 [0.76 0.73 0.7 0.67 0.64 0.62 0.59]  
 [0.77 0.74 0.7 0.68 0.65 0.62 0.6 ]  
 [0.78 0.75 0.71 0.68 0.66 0.63 0.61]  
 [0.79 0.75 0.72 0.69 0.67 0.64 0.62]  
 [0.8 0.76 0.73 0.7 0.67 0.65 0.62]  
 [0.81 0.77 0.74 0.71 0.68 0.65 0.63]  
 [0.82 0.78 0.75 0.72 0.69 0.66 0.64]  
 [0.83 0.79 0.76 0.73 0.7 0.67 0.65]

```

[0.84 0.8 0.77 0.73 0.71 0.68 0.65]
[0.85 0.81 0.78 0.74 0.71 0.69 0.66]
[0.86 0.82 0.78 0.75 0.72 0.69 0.67]
[0.87 0.83 0.79 0.76 0.73 0.7 0.68]
[0.88 0.84 0.8 0.77 0.74 0.71 0.68]
[0.89 0.85 0.81 0.78 0.75 0.72 0.69]
[0.9 0.86 0.82 0.78 0.75 0.72 0.7 ]
[0.9 0.86 0.83 0.79 0.76 0.73 0.71]
[0.91 0.87 0.84 0.8 0.77 0.74 0.71]
[0.92 0.88 0.84 0.81 0.78 0.75 0.72]
[0.93 0.89 0.85 0.82 0.78 0.75 0.73]
[0.94 0.9 0.86 0.83 0.79 0.76 0.73]
[0.95 0.91 0.87 0.83 0.8 0.77 0.74]]

```

The value function is

```

[[-9.00000000e+04 -9.00000000e+04 -9.00000000e+04 -9.00000000e+04
-9.00000000e+04 -9.00000000e+04 -9.00000000e+04]
[-9.00000000e+04 -9.00000000e+04 -9.00000000e+04 -9.00000000e+04
-9.00000000e+04 -9.00000000e+04 -9.00000000e+04]
[-4.87020633e+01 -5.10046483e+01 -5.33072334e+01 -5.56098185e+01
-5.79124036e+01 -6.02149887e+01 -6.25175738e+01]
[-4.42129004e+01 -4.65154855e+01 -4.88180706e+01 -5.11206557e+01
-5.34232408e+01 -5.57258259e+01 -5.80284110e+01]
[-4.13997661e+01 -4.37023512e+01 -4.60049362e+01 -4.83075213e+01
-5.06101064e+01 -5.29126915e+01 -5.52152766e+01]
[-3.90535002e+01 -4.13560853e+01 -4.36586704e+01 -4.59612555e+01
-4.82638406e+01 -5.05664257e+01 -5.27892615e+01]
[-3.71941021e+01 -3.94966872e+01 -4.17992723e+01 -4.41018574e+01
-4.64044425e+01 -4.84869841e+01 -5.04429956e+01]
[-3.56390121e+01 -3.79415972e+01 -4.02441822e+01 -4.25467673e+01
-4.46715745e+01 -4.66275860e+01 -4.85835975e+01]
[-3.42666222e+01 -3.65692073e+01 -3.88717924e+01 -4.11604730e+01
-4.31164845e+01 -4.50724960e+01 -4.70285075e+01]
[-3.30750699e+01 -3.53776550e+01 -3.76802401e+01 -3.97880832e+01
-4.17440947e+01 -4.37001062e+01 -4.56093796e+01]
[-3.20130929e+01 -3.43156780e+01 -3.66182631e+01 -3.85965308e+01
-4.05525423e+01 -4.24837108e+01 -4.42369898e+01]
[-3.10491821e+01 -3.33517672e+01 -3.55785423e+01 -3.75345538e+01
-3.94905653e+01 -4.12921585e+01 -4.30454374e+01]
[-3.01733137e+01 -3.24758988e+01 -3.46146315e+01 -3.65706430e+01
-3.84769025e+01 -4.02301815e+01 -4.19834604e+01]
[-2.93617199e+01 -3.16643050e+01 -3.37387631e+01 -3.56947746e+01
-3.75129917e+01 -3.92662707e+01 -4.09765732e+01]
[-2.86067910e+01 -3.09093761e+01 -3.29271693e+01 -3.48831808e+01
-3.66371233e+01 -3.83904023e+01 -4.00126624e+01]
[-2.79189507e+01 -3.02162289e+01 -3.21722405e+01 -3.40722506e+01
-3.58255295e+01 -3.75273561e+01 -3.91367940e+01]
[-2.72684189e+01 -2.95283886e+01 -3.14844001e+01 -3.33173217e+01
-3.50706007e+01 -3.67157623e+01 -3.83252002e+01]

```

[-2.66482914e+01 -2.88778568e+01 -3.08338683e+01 -3.26294814e+01  
 -3.43513955e+01 -3.59608334e+01 -3.75441978e+01]  
 [-2.60806278e+01 -2.82577293e+01 -3.02137408e+01 -3.19789496e+01  
 -3.36635552e+01 -3.52729931e+01 -3.67892689e+01]  
 [-2.55314885e+01 -2.76900657e+01 -2.96055431e+01 -3.13588221e+01  
 -3.30130234e+01 -3.46035624e+01 -3.61014285e+01]  
 [-2.50115679e+01 -2.71409264e+01 -2.90378796e+01 -3.07834579e+01  
 -3.23928958e+01 -3.39530306e+01 -3.54508968e+01]  
 [-2.45266338e+01 -2.66210058e+01 -2.84887402e+01 -3.02157944e+01  
 -3.18252323e+01 -3.33329031e+01 -3.48127713e+01]  
 [-2.40526164e+01 -2.61360717e+01 -2.79688196e+01 -2.96666550e+01  
 -3.12673734e+01 -3.27652395e+01 -3.41926438e+01]  
 [-2.36054713e+01 -2.56620543e+01 -2.74838856e+01 -2.91467344e+01  
 -3.07182341e+01 -3.22161002e+01 -3.36249802e+01]  
 [-2.31760277e+01 -2.52149092e+01 -2.70098682e+01 -2.86618004e+01  
 -3.01983134e+01 -3.16691355e+01 -3.30758409e+01]  
 [-2.27639886e+01 -2.47854656e+01 -2.65627231e+01 -2.81877830e+01  
 -2.97133794e+01 -3.11492149e+01 -3.25363135e+01]  
 [-2.23696903e+01 -2.43734265e+01 -2.61312000e+01 -2.77406379e+01  
 -2.92393620e+01 -3.06642809e+01 -3.20163929e+01]  
 [-2.19879063e+01 -2.39679614e+01 -2.57017564e+01 -2.72943508e+01  
 -2.87835581e+01 -3.01902635e+01 -3.15314589e+01]  
 [-2.16218981e+01 -2.35736631e+01 -2.52897172e+01 -2.68649072e+01  
 -2.83364130e+01 -2.97278114e+01 -3.10574415e+01]  
 [-2.12692207e+01 -2.31918791e+01 -2.48954190e+01 -2.64528680e+01  
 -2.79069694e+01 -2.92806663e+01 -3.05900816e+01]  
 [-2.09226471e+01 -2.28258709e+01 -2.45136349e+01 -2.60585698e+01  
 -2.74949303e+01 -2.88512227e+01 -3.01429365e+01]  
 [-2.05800445e+01 -2.24731935e+01 -2.41476267e+01 -2.56767857e+01  
 -2.71006320e+01 -2.84391836e+01 -2.97134929e+01]  
 [-2.02483746e+01 -2.21305909e+01 -2.37949493e+01 -2.53107775e+01  
 -2.67152553e+01 -2.80385894e+01 -2.93012523e+01]  
 [-1.99306068e+01 -2.17989210e+01 -2.34523468e+01 -2.49461344e+01  
 -2.63334713e+01 -2.76442912e+01 -2.88892131e+01]  
 [-1.96196554e+01 -2.14811532e+01 -2.31176314e+01 -2.45934570e+01  
 -2.59674631e+01 -2.72625071e+01 -2.84949149e+01]  
 [-1.93175309e+01 -2.11702018e+01 -2.27859615e+01 -2.42508545e+01  
 -2.56147857e+01 -2.68964989e+01 -2.81131308e+01]  
 [-1.90277272e+01 -2.08680773e+01 -2.24681937e+01 -2.39191846e+01  
 -2.52721831e+01 -2.65431498e+01 -2.77443690e+01]  
 [-1.87422834e+01 -2.05782736e+01 -2.21572423e+01 -2.36014167e+01  
 -2.49383546e+01 -2.61904724e+01 -2.73783608e+01]  
 [-1.84654965e+01 -2.02905915e+01 -2.18551178e+01 -2.32904653e+01  
 -2.46066847e+01 -2.58478698e+01 -2.70256834e+01]  
 [-1.81983137e+01 -2.00051477e+01 -2.15653141e+01 -2.29821640e+01  
 -2.42889169e+01 -2.55161999e+01 -2.66830808e+01]  
 [-1.79356602e+01 -1.97283609e+01 -2.12798703e+01 -2.26800395e+01  
 -2.39779655e+01 -2.51984321e+01 -2.63494952e+01]

[-1.76807537e+01 -1.94611781e+01 -2.10030835e+01 -2.23902358e+01  
 -2.36758410e+01 -2.48823506e+01 -2.60178253e+01]  
 [-1.74315696e+01 -1.91985245e+01 -2.07296011e+01 -2.21047920e+01  
 -2.33813835e+01 -2.45713992e+01 -2.57000575e+01]  
 [-1.71884396e+01 -1.89436180e+01 -2.04624183e+01 -2.18280051e+01  
 -2.30915797e+01 -2.42692747e+01 -2.53891061e+01]  
 [-1.69524203e+01 -1.86944339e+01 -2.01997648e+01 -2.15608223e+01  
 -2.28061359e+01 -2.39794710e+01 -2.50845663e+01]  
 [-1.67196809e+01 -1.84513039e+01 -1.99448583e+01 -2.12937595e+01  
 -2.25293491e+01 -2.36935405e+01 -2.47824418e+01]  
 [-1.64936415e+01 -1.82152847e+01 -1.96956742e+01 -2.10311060e+01  
 -2.22621662e+01 -2.34080967e+01 -2.44926381e+01]  
 [-1.62726903e+01 -1.79825452e+01 -1.94525442e+01 -2.07761995e+01  
 -2.19987650e+01 -2.31313098e+01 -2.42071943e+01]  
 [-1.60551456e+01 -1.77565058e+01 -1.92165249e+01 -2.05270154e+01  
 -2.17361114e+01 -2.28641270e+01 -2.39270448e+01]  
 [-1.58429837e+01 -1.75333623e+01 -1.89837855e+01 -2.02838854e+01  
 -2.14812049e+01 -2.26014734e+01 -2.36502579e+01]  
 [-1.56361362e+01 -1.73124111e+01 -1.87525594e+01 -2.00478661e+01  
 -2.12320208e+01 -2.23404393e+01 -2.33830751e+01]  
 [-1.54320215e+01 -1.70948663e+01 -1.85265201e+01 -1.98123001e+01  
 -2.09888908e+01 -2.20855328e+01 -2.31204216e+01]  
 [-1.52292889e+01 -1.68827045e+01 -1.83055689e+01 -1.95795606e+01  
 -2.07506154e+01 -2.18363487e+01 -2.28610437e+01]  
 [-1.50291293e+01 -1.66758570e+01 -1.80880241e+01 -1.93535212e+01  
 -2.05145961e+01 -2.15932187e+01 -2.26061372e+01]  
 [-1.48343586e+01 -1.64717423e+01 -1.78758623e+01 -1.91325700e+01  
 -2.02818567e+01 -2.13530906e+01 -2.23569531e+01]  
 [-1.46419558e+01 -1.62715826e+01 -1.76690147e+01 -1.89150253e+01  
 -2.00558173e+01 -2.11170713e+01 -2.21138231e+01]  
 [-1.44534690e+01 -1.60768119e+01 -1.74649001e+01 -1.87028634e+01  
 -1.98348661e+01 -2.08843319e+01 -2.18723481e+01]  
 [-1.42689457e+01 -1.58844091e+01 -1.72646030e+01 -1.84921424e+01  
 -1.96173213e+01 -2.06582925e+01 -2.16363288e+01]  
 [-1.40870996e+01 -1.56959223e+01 -1.70644433e+01 -1.82852949e+01  
 -1.93997929e+01 -2.04359686e+01 -2.14035893e+01]  
 [-1.39090025e+01 -1.55113990e+01 -1.68696726e+01 -1.80811802e+01  
 -1.91876310e+01 -2.02150174e+01 -2.11775500e+01]  
 [-1.37334501e+01 -1.53290774e+01 -1.66772698e+01 -1.78810205e+01  
 -1.89807835e+01 -1.99974726e+01 -2.09516651e+01]  
 [-1.35610925e+01 -1.51472313e+01 -1.64887830e+01 -1.76862498e+01  
 -1.87766688e+01 -1.97853108e+01 -2.07307139e+01]  
 [-1.33923892e+01 -1.49691343e+01 -1.63042597e+01 -1.74938470e+01  
 -1.85765091e+01 -1.95783322e+01 -2.05131692e+01]  
 [-1.32250883e+01 -1.47935818e+01 -1.61224136e+01 -1.73032267e+01  
 -1.83764024e+01 -1.93714846e+01 -2.03009830e+01]  
 [-1.30615679e+01 -1.46212243e+01 -1.59443165e+01 -1.71147399e+01  
 -1.81816317e+01 -1.91673699e+01 -2.00888212e+01]

[-1.29008259e+01 -1.44525210e+01 -1.57676420e+01 -1.69302165e+01  
 -1.79892289e+01 -1.89672103e+01 -1.98819736e+01]  
 [-1.27413693e+01 -1.42852201e+01 -1.55920895e+01 -1.67483704e+01  
 -1.78007421e+01 -1.87724396e+01 -1.96778589e+01]  
 [-1.25853413e+01 -1.41216997e+01 -1.54197320e+01 -1.65702734e+01  
 -1.76154722e+01 -1.85788240e+01 -1.94776993e+01]  
 [-1.24317843e+01 -1.39609576e+01 -1.52510287e+01 -1.63947209e+01  
 -1.74309489e+01 -1.83864212e+01 -1.92776448e+01]  
 [-1.22801370e+01 -1.38015011e+01 -1.50837278e+01 -1.62206982e+01  
 -1.72491028e+01 -1.81979344e+01 -1.90828741e+01]  
 [-1.21305817e+01 -1.36454731e+01 -1.49202074e+01 -1.60483406e+01  
 -1.70710057e+01 -1.80134111e+01 -1.88904713e+01]  
 [-1.19837526e+01 -1.34913224e+01 -1.47594653e+01 -1.58796373e+01  
 -1.68954532e+01 -1.78315372e+01 -1.87012361e+01]  
 [-1.18386338e+01 -1.33377653e+01 -1.46000088e+01 -1.57123365e+01  
 -1.67229711e+01 -1.76496911e+01 -1.85127493e+01]  
 [-1.16947928e+01 -1.31861181e+01 -1.44419680e+01 -1.55488160e+01  
 -1.65506135e+01 -1.74715941e+01 -1.83282259e+01]  
 [-1.15518835e+01 -1.30365628e+01 -1.42859400e+01 -1.53880740e+01  
 -1.63819102e+01 -1.72960416e+01 -1.81463798e+01]  
 [-1.14111095e+01 -1.28897337e+01 -1.41323829e+01 -1.52279886e+01  
 -1.62146094e+01 -1.71236841e+01 -1.79668533e+01]  
 [-1.12719193e+01 -1.27446149e+01 -1.39807357e+01 -1.50685320e+01  
 -1.60510889e+01 -1.69522088e+01 -1.77887563e+01]  
 [-1.11350807e+01 -1.26017056e+01 -1.38311804e+01 -1.49125040e+01  
 -1.58897426e+01 -1.67835055e+01 -1.76132038e+01]  
 [-1.09994966e+01 -1.24609316e+01 -1.36843513e+01 -1.47589470e+01  
 -1.57290006e+01 -1.66162047e+01 -1.74408463e+01]  
 [-1.08661585e+01 -1.23217414e+01 -1.35392325e+01 -1.46072997e+01  
 -1.55695440e+01 -1.64526842e+01 -1.72700807e+01]  
 [-1.07348391e+01 -1.21849028e+01 -1.33962672e+01 -1.44577444e+01  
 -1.54135160e+01 -1.62904826e+01 -1.71013774e+01]  
 [-1.06042515e+01 -1.20493187e+01 -1.32533579e+01 -1.43095284e+01  
 -1.52599590e+01 -1.61297405e+01 -1.69340765e+01]  
 [-1.04760709e+01 -1.19157873e+01 -1.31125840e+01 -1.41626994e+01  
 -1.51083117e+01 -1.59702840e+01 -1.67705561e+01]  
 [-1.03496022e+01 -1.17824493e+01 -1.29733937e+01 -1.40175806e+01  
 -1.49567502e+01 -1.58142560e+01 -1.66077360e+01]  
 [-1.02240129e+01 -1.16511299e+01 -1.28365552e+01 -1.38746713e+01  
 -1.48071948e+01 -1.56603761e+01 -1.64469940e+01]  
 [-1.01005537e+01 -1.15205423e+01 -1.27009710e+01 -1.37338973e+01  
 -1.46603658e+01 -1.55068190e+01 -1.62875374e+01]  
 [-9.97846307e+00 -1.13923616e+01 -1.25676330e+01 -1.35947071e+01  
 -1.45152470e+01 -1.53551718e+01 -1.61315094e+01]  
 [-9.85772198e+00 -1.12658930e+01 -1.24363136e+01 -1.34567213e+01  
 -1.43723377e+01 -1.52056165e+01 -1.59759283e+01]  
 [-9.73849894e+00 -1.11403037e+01 -1.23057260e+01 -1.33198828e+01  
 -1.42294416e+01 -1.50587874e+01 -1.58223712e+01]

```

[-9.62063755e+00 -1.10168444e+01 -1.21752089e+01 -1.31842986e+01
 -1.40886677e+01 -1.49124169e+01 -1.56707240e+01]
[-9.50417119e+00 -1.08947538e+01 -1.20470283e+01 -1.30509606e+01
 -1.39494774e+01 -1.47672981e+01 -1.55211686e+01]
[-9.38900925e+00 -1.07740127e+01 -1.19205596e+01 -1.29196412e+01
 -1.38126389e+01 -1.46243888e+01 -1.53722100e+01]
[-9.27521484e+00 -1.06547897e+01 -1.17949703e+01 -1.27890536e+01
 -1.36770547e+01 -1.44836149e+01 -1.52253809e+01]
[-9.16256586e+00 -1.05369283e+01 -1.16715111e+01 -1.26599766e+01
 -1.35418867e+01 -1.43440548e+01 -1.50802621e+01]
[-9.05099408e+00 -1.04191452e+01 -1.15494205e+01 -1.25317959e+01
 -1.34085486e+01 -1.42048646e+01 -1.49373528e+01]
[-8.93982659e+00 -1.03026789e+01 -1.14286794e+01 -1.24053272e+01
 -1.32772292e+01 -1.40680260e+01 -1.47944758e+01]
[-8.82941351e+00 -1.01875169e+01 -1.13086153e+01 -1.22797379e+01
 -1.31466416e+01 -1.39324419e+01 -1.46537019e+01]
[-8.72056413e+00 -1.00737225e+01 -1.11893923e+01 -1.21562787e+01
 -1.30184084e+01 -1.37990866e+01 -1.45145116e+01]
[-8.61307430e+00 -9.96107355e+00 -1.10715309e+01 -1.20341881e+01
 -1.28902277e+01 -1.36657485e+01 -1.43772391e+01]
[-8.50605687e+00 -9.84990606e+00 -1.09550645e+01 -1.19129388e+01
 -1.27637591e+01 -1.35344291e+01 -1.42404006e+01]]
The distance metric is 56667671972.15527

```

## 0.0.21 5.21

```

In [16]: def new_optimize(init=W, E = varepsilon, P = Gamma, u = lambda x: np.log(x),
                        beta = 0.9, error = 1e-9, maxiter = 1000):
    W = init
    Gamma = P
    varepsilon = E
    V = np.zeros((W.size, Gamma.size))
    for i in range(Gamma.size):
        V[:,i] = varepsilon[i]*np.log(W)
    c_mat = W.reshape(-1,1)-W
    c_mat[c_mat<=0] = 1e-7
    u_mat = u(c_mat)
    Error = 1
    count = 0
    while Error>error and count <= maxiter:
        new_W = np.tile(W.reshape(-1,1),(1,7))
        new_V = np.array(V)
        for j in range(7):
            e = varepsilon[j]
            for i in range(100):
                w = W[i]
                new_V[i,j] = -np.inf

```

```

        for k in range(i):
            v_value = e*u_mat[i,k] + beta*\
                sum(Gamma[p]*V[k,p] for p in range(7))
            if v_value > new_V[i,j]:
                new_W[i,j] = W[k]
                new_V[i,j] = v_value
        Error = ((V-new_V)**2).sum()
        V = new_V
    return new_V,new_W

V, new_W = new_optimize()
print("The policy function is\n", new_W)
print("The value function is\n", V)

```

The policy function is

```

[[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.04 0.04 0.04 0.04 0.04 0.04 0.04]
 [0.05 0.05 0.05 0.05 0.05 0.05 0.04]
 [0.06 0.06 0.06 0.06 0.06 0.05 0.05]
 [0.07 0.07 0.07 0.07 0.06 0.06 0.06]
 [0.08 0.08 0.08 0.07 0.07 0.07 0.07]
 [0.09 0.09 0.09 0.08 0.08 0.08 0.07]
 [0.1  0.1  0.1  0.09 0.09 0.08 0.08]
 [0.11 0.11 0.1  0.1  0.1  0.09 0.09]
 [0.12 0.12 0.11 0.11 0.1  0.1  0.1 ]
 [0.13 0.13 0.12 0.12 0.11 0.11 0.1 ]
 [0.14 0.14 0.13 0.13 0.12 0.12 0.11]
 [0.15 0.14 0.14 0.13 0.13 0.12 0.12]
 [0.16 0.15 0.15 0.14 0.14 0.13 0.13]
 [0.17 0.16 0.16 0.15 0.14 0.14 0.13]
 [0.18 0.17 0.17 0.16 0.15 0.15 0.14]
 [0.19 0.18 0.17 0.17 0.16 0.15 0.15]
 [0.2  0.19 0.18 0.17 0.17 0.16 0.16]
 [0.21 0.2  0.19 0.18 0.18 0.17 0.16]
 [0.22 0.21 0.2  0.19 0.18 0.18 0.17]
 [0.23 0.22 0.21 0.2  0.19 0.19 0.18]
 [0.24 0.23 0.22 0.21 0.2  0.19 0.19]
 [0.25 0.24 0.23 0.22 0.21 0.2  0.19]
 [0.26 0.25 0.23 0.23 0.22 0.21 0.2 ]
 [0.27 0.25 0.24 0.23 0.22 0.22 0.21]
 [0.28 0.26 0.25 0.24 0.23 0.22 0.22]
 [0.29 0.27 0.26 0.25 0.24 0.23 0.22]
 [0.29 0.28 0.27 0.26 0.25 0.24 0.23]
 [0.3  0.29 0.28 0.27 0.26 0.25 0.24]
 [0.31 0.3  0.29 0.28 0.26 0.25 0.24]

```



[0.32 0.31 0.3 0.28 0.27 0.26 0.25]  
[0.33 0.32 0.3 0.29 0.28 0.27 0.26]  
[0.34 0.33 0.31 0.3 0.29 0.28 0.27]  
[0.35 0.34 0.32 0.31 0.3 0.28 0.27]  
[0.36 0.35 0.33 0.32 0.3 0.29 0.28]  
[0.37 0.35 0.34 0.33 0.31 0.3 0.29]  
[0.38 0.36 0.35 0.33 0.32 0.31 0.3 ]  
[0.39 0.37 0.36 0.34 0.33 0.32 0.3 ]  
[0.4 0.38 0.37 0.35 0.34 0.32 0.31]  
[0.41 0.39 0.37 0.36 0.34 0.33 0.32]  
[0.42 0.4 0.38 0.37 0.35 0.34 0.33]  
[0.43 0.41 0.39 0.38 0.36 0.35 0.33]  
[0.44 0.42 0.4 0.38 0.37 0.35 0.34]  
[0.45 0.43 0.41 0.39 0.38 0.36 0.35]  
[0.46 0.44 0.42 0.4 0.38 0.37 0.36]  
[0.47 0.45 0.43 0.41 0.39 0.38 0.36]  
[0.48 0.45 0.44 0.42 0.4 0.39 0.37]  
[0.49 0.46 0.44 0.43 0.41 0.39 0.38]  
[0.5 0.47 0.45 0.43 0.42 0.4 0.39]  
[0.5 0.48 0.46 0.44 0.42 0.41 0.39]  
[0.51 0.49 0.47 0.45 0.43 0.42 0.4 ]  
[0.52 0.5 0.48 0.46 0.44 0.42 0.41]  
[0.53 0.51 0.49 0.47 0.45 0.43 0.42]  
[0.54 0.52 0.5 0.48 0.46 0.44 0.42]  
[0.55 0.53 0.5 0.48 0.47 0.45 0.43]  
[0.56 0.54 0.51 0.49 0.47 0.45 0.44]  
[0.57 0.55 0.52 0.5 0.48 0.46 0.45]  
[0.58 0.55 0.53 0.51 0.49 0.47 0.45]  
[0.59 0.56 0.54 0.52 0.5 0.48 0.46]  
[0.6 0.57 0.55 0.53 0.51 0.48 0.47]  
[0.61 0.58 0.56 0.53 0.51 0.49 0.47]  
[0.62 0.59 0.57 0.54 0.52 0.5 0.48]  
[0.63 0.6 0.57 0.55 0.53 0.51 0.49]  
[0.64 0.61 0.58 0.56 0.54 0.52 0.5 ]  
[0.65 0.62 0.59 0.57 0.54 0.52 0.51]  
[0.66 0.63 0.6 0.58 0.55 0.53 0.51]  
[0.67 0.64 0.61 0.58 0.56 0.54 0.52]  
[0.68 0.65 0.62 0.59 0.57 0.55 0.53]  
[0.69 0.65 0.63 0.6 0.58 0.55 0.53]  
[0.7 0.66 0.64 0.61 0.58 0.56 0.54]  
[0.7 0.67 0.64 0.62 0.59 0.57 0.55]  
[0.71 0.68 0.65 0.63 0.6 0.58 0.56]  
[0.72 0.69 0.66 0.63 0.61 0.59 0.56]  
[0.73 0.7 0.67 0.64 0.62 0.59 0.57]  
[0.74 0.71 0.68 0.65 0.62 0.6 0.58]  
[0.75 0.72 0.69 0.66 0.63 0.61 0.59]  
[0.76 0.73 0.7 0.67 0.64 0.62 0.59]  
[0.77 0.74 0.7 0.68 0.65 0.62 0.6 ]

```

[0.78 0.75 0.71 0.68 0.66 0.63 0.61]
[0.79 0.75 0.72 0.69 0.67 0.64 0.62]
[0.8 0.76 0.73 0.7 0.67 0.65 0.62]
[0.81 0.77 0.74 0.71 0.68 0.65 0.63]
[0.82 0.78 0.75 0.72 0.69 0.66 0.64]
[0.83 0.79 0.76 0.73 0.7 0.67 0.65]
[0.84 0.8 0.77 0.73 0.71 0.68 0.65]
[0.85 0.81 0.78 0.74 0.71 0.69 0.66]
[0.86 0.82 0.78 0.75 0.72 0.69 0.67]
[0.87 0.83 0.79 0.76 0.73 0.7 0.68]
[0.88 0.84 0.8 0.77 0.74 0.71 0.68]
[0.89 0.85 0.81 0.78 0.75 0.72 0.69]
[0.9 0.86 0.82 0.78 0.75 0.72 0.7 ]
[0.9 0.86 0.83 0.79 0.76 0.73 0.71]
[0.91 0.87 0.84 0.8 0.77 0.74 0.71]
[0.92 0.88 0.84 0.81 0.78 0.75 0.72]
[0.93 0.89 0.85 0.82 0.78 0.75 0.73]
[0.94 0.9 0.86 0.83 0.79 0.76 0.73]
[0.95 0.91 0.87 0.83 0.8 0.77 0.74]]

```

The value function is

```

[[      -inf      -inf      -inf      -inf      -inf
      -inf      -inf]
 [      -inf      -inf      -inf      -inf      -inf
      -inf      -inf]
 [-48.70206326 -51.00464835 -53.30723344 -55.60981853 -57.91240363
 -60.21498872 -62.51757381]
 [-44.21290041 -46.51548551 -48.8180706  -51.12065569 -53.42324078
 -55.72582588 -58.02841097]
 [-41.39976606 -43.70235115 -46.00493624 -48.30752134 -50.61010643
 -52.91269152 -55.21527662]
 [-39.05350019 -41.35608528 -43.65867037 -45.96125546 -48.26384056
 -50.56642565 -52.78926148]
 [-37.1941021  -39.49668719 -41.79927229 -44.10185738 -46.40444247
 -48.48698411 -50.44299561]
 [-35.63901206 -37.94159715 -40.24418225 -42.54676734 -44.67157452
 -46.62758602 -48.58359753]
 [-34.26662223 -36.56920733 -38.87179242 -41.16047298 -43.11648448
 -45.07249598 -47.02850749]
 [-33.07506989 -35.37765498 -37.68024007 -39.78808315 -41.74409466
 -43.70010616 -45.60937961]
 [-32.01309289 -34.31567799 -36.61826308 -38.5965308  -40.55254231
 -42.48371083 -44.23698978]
 [-31.04918207 -33.35176716 -35.57854231 -37.53455381 -39.49056531
 -41.29215848 -43.04543743]
 [-30.17331368 -32.47589878 -34.61463148 -36.57064299 -38.47690254
 -40.23018149 -41.98346044]
 [-29.36171989 -31.66430498 -33.7387631  -35.6947746  -37.51299172
 -39.26627067 -40.97657319]

```

[-28.60679104 -30.90937613 -32.92716931 -34.88318081 -36.63712334  
 -38.39040228 -40.01266236]  
 [-27.91895069 -30.21622895 -32.17224045 -34.07225059 -35.82552954  
 -37.52735607 -39.13679398]  
 [-27.26841891 -29.5283886 -31.4844001 -33.31732174 -35.07060069  
 -36.71576227 -38.32520019]  
 [-26.64829137 -28.87785682 -30.83386832 -32.62948139 -34.35139551  
 -35.96083342 -37.54419776]  
 [-26.0806278 -28.25772928 -30.21374079 -31.97894961 -33.66355515  
 -35.27299307 -36.7892689 ]  
 [-25.53148847 -27.69006571 -29.60554313 -31.35882207 -33.01302337  
 -34.60356241 -36.10142855]  
 [-25.01156785 -27.14092638 -29.03787955 -30.78345793 -32.39289584  
 -33.95303063 -35.45089677]  
 [-24.52663382 -26.62100577 -28.48874022 -30.21579436 -31.82523227  
 -33.3329031 -34.81277132]  
 [-24.05261643 -26.13607173 -27.96881961 -29.66665503 -31.26737339  
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 [-22.76398856 -24.78546564 -26.56272308 -28.18778299 -29.71337941  
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 [-21.98790625 -23.96796136 -25.70175637 -27.29435078 -28.78355813  
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 -28.85122274 -30.14293649]  
 [-20.58004452 -22.4731935 -24.1476267 -25.67678571 -27.10063204  
 -28.43918358 -29.71349289]  
 [-20.24837462 -22.13059091 -23.79494934 -25.31077751 -26.71525534  
 -28.0385894 -29.30125226]  
 [-19.9306068 -21.79892102 -23.45234675 -24.94613439 -26.33347125  
 -27.64429118 -28.88921309]  
 [-19.61965541 -21.4811532 -23.11763143 -24.59345704 -25.96746305  
 -27.26250709 -28.49491488]  
 [-19.31753094 -21.1702018 -22.78596153 -24.25085445 -25.61478569  
 -26.89649889 -28.11313079]  
 [-19.02772721 -20.86807733 -22.46819371 -23.91918455 -25.27218311  
 -26.54314979 -27.74436899]  
 [-18.74228343 -20.57827361 -22.15724232 -23.60141673 -24.93835463  
 -26.19047243 -27.37836079]

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C:\ProgramData\Anaconda3\lib\site-packages\ipykernel\_launcher.py:27: RuntimeWarning: invalid v

## 0.0.22 5.22

```
In [17]: X, Y = np.meshgrid(W, varepsilon)
new_fig = plt.figure(figsize=(8, 6))
new_ax = new_fig.add_subplot(111, projection='3d')
new_ax.plot_surface(X.T, Y.T, new_W)
new_ax.set_xlabel('w_vec')
new_ax.set_ylabel('varepsilon')
new_ax.set_title("Converged Policy Function")
new_ax.view_init(elev=60,azim=30)
plt.show()
```

