hw4

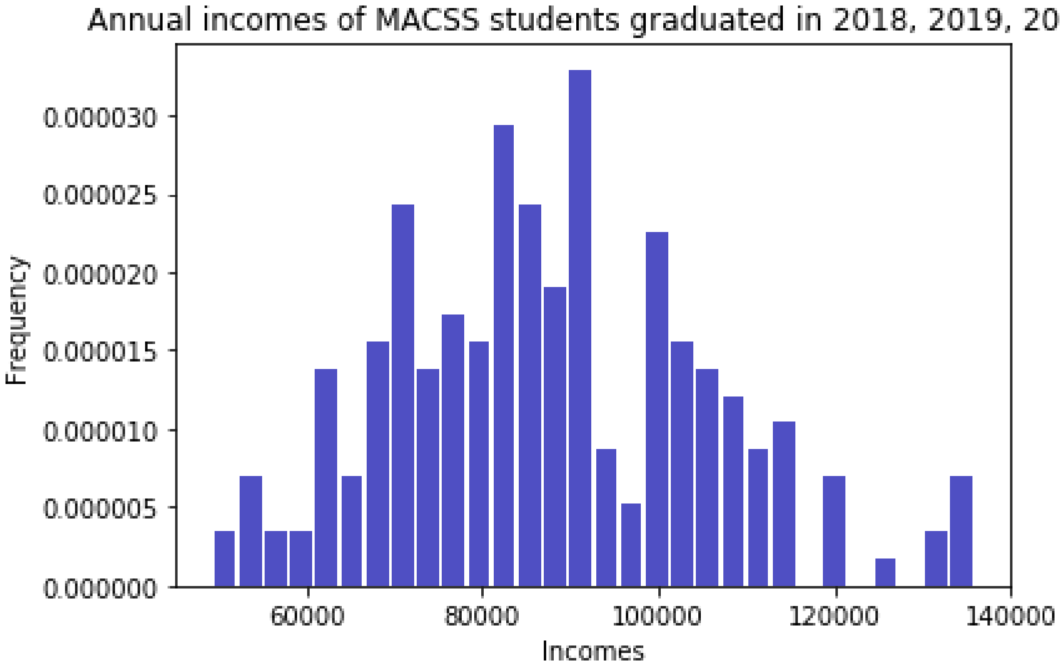
February 4, 2019

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import scipy.stats as sts
        from scipy.integrate import quad
```

0.1 1

0.1.1 a

```
In [2]: Incomes = np.loadtxt('incomes.txt')
        fig,ax = plt.subplots()
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('Annual incomes of MACSS students graduated in 2018, 2019, 2020')
        ax.hist(x=Incomes, bins=30, color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
        plt.show()
```



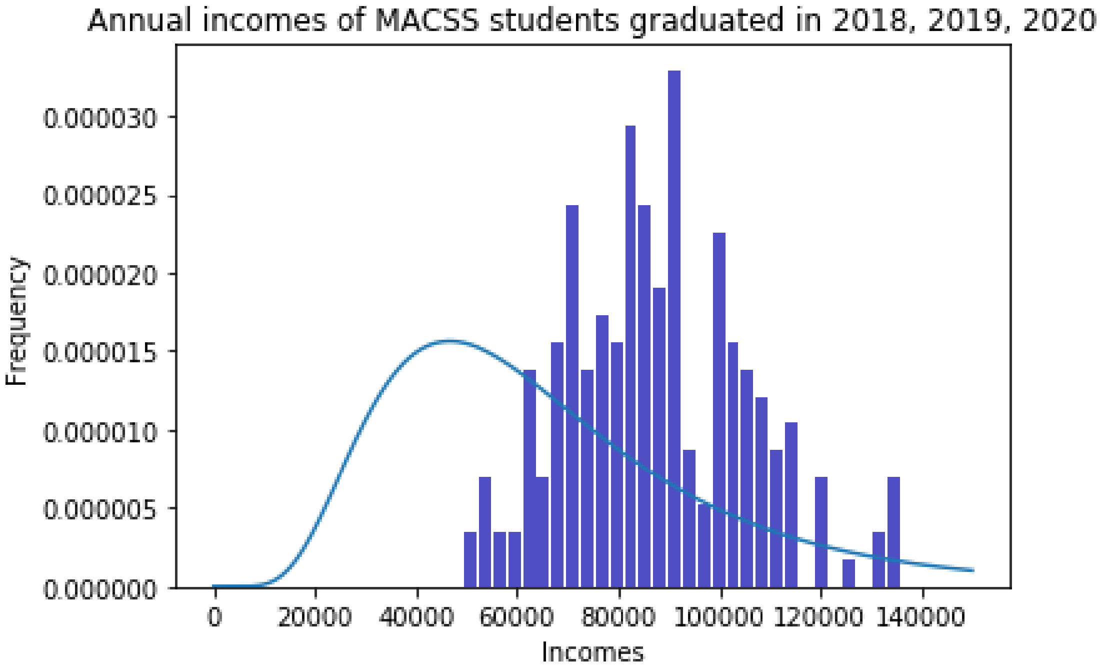
0.1.2 b

```
In [3]: def lognorm(x, mu=11, sigma=0.5):
            return 1/(x*sigma * np.sqrt(2 * np.pi))*np.e**(-(np.log(x) - mu)**2 / (2 * sigma**2
        def log_lik_truncnorm(x, mu=11, sigma=0.5, cut_lb=0, cut_ub=150000):
            pdf_vals = lognorm(x, mu, sigma)
            if cut_lb!=None and cut_ub!=None:
                total = quad(lambda x: lognorm(x, mu,sigma), cut_lb, cut_ub)[0]
            elif cut_lb!=None and cut_ub==None:
                total = 1-quad(lambda x: lognorm(x, mu,sigma), cut_lb, np.inf)[0]
            elif cut_lb==None and cut_ub!=None:
                total = quad(lambda x: lognorm(x, mu,sigma), -np.inf, cut_ub)[0]
            else:
                total = 1
            polishedpdf = pdf_vals/total
            ln_pdf_vals = np.log(polishedpdf)
            log_lik_val = ln_pdf_vals.sum()
            return log_lik_val
        total = quad(lognorm, 0, 150000)[0]
        X = np.linspace(0,150000, 150001)
        fig,ax = plt.subplots()
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('Annual incomes of MACSS students graduated in 2018, 2019, 2020')
        ax.plot(X, lognorm(X)/total)
        ax.hist(x=Incomes, bins=30, color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
        plt.show()
        LLV = log_lik_truncnorm(Incomes)
        print('The value of the log likelihood value is', LLV)
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: invalid val
```



```
The value of the log likelihood value is -2379.120591931827
```

0.1.3 c

```
In [4]: def crit(params, *args):
            mu, sigma = params
            x, cut_lb, cut_ub = args
            log_lik_val = log_lik_truncnorm(x, mu, sigma, cut_lb, cut_ub)
            neg_log_lik_val = -log_lik_val
            return neg_log_lik_val
```

```
In [5]: import scipy.optimize as opt
        mu_init = 11
        sig_init = 0.5
        params_init = np.array([mu_init, sig_init])
        mle_args = (Incomes, 0, 150000)
        results = opt.minimize(crit, params_init, args=(mle_args))
        mu_MLE, sig_MLE = results.x
        LLV_MLE = log_lik_truncnorm(Incomes, mu_MLE, sig_MLE)
        VCV_MLE = results.hess_inv
        print('mu_MLE=', mu_MLE, ' sig_MLE=', sig_MLE)
        print('The value of the log likelihood value is', LLV_MLE)
        print('The variance-covariance matrix is', VCV_MLE)
```

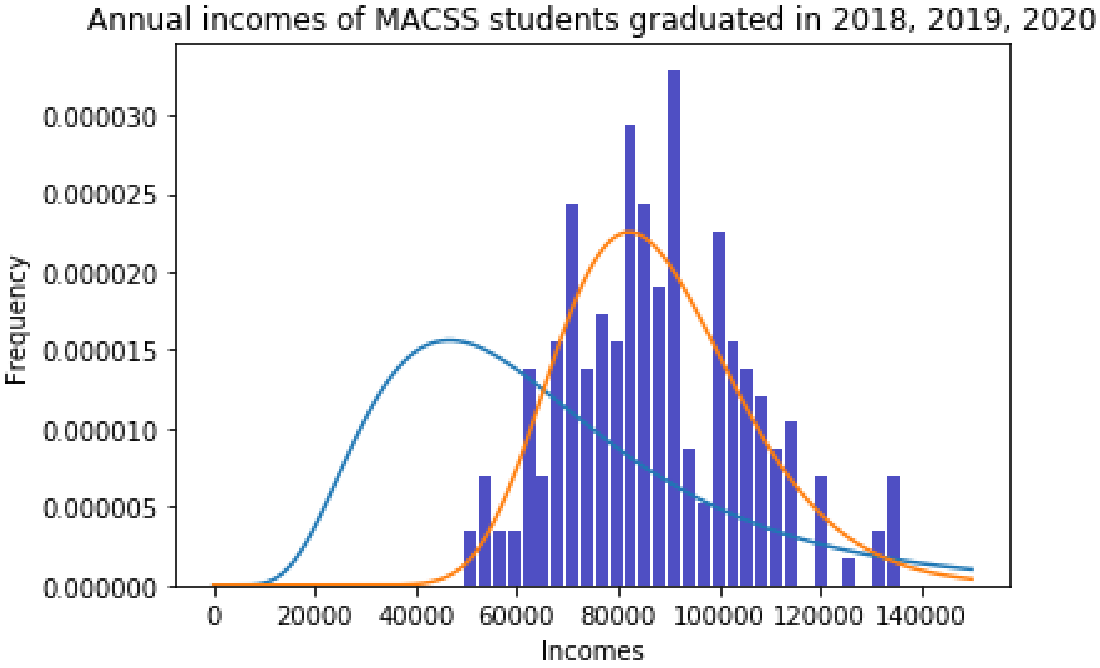
```
mu_MLE= 11.361699971261004  sig_MLE= 0.2117432623314924
The value of the log likelihood value is -2240.9343375116364
The variance-covariance matrix is [[8.67691939e-05 3.92325340e-05]
 [3.92325340e-05 1.16938120e-04]]
```

```
In [6]: total1 = quad(lognorm, 0, 150000)[0]
        total2 = quad(lambda x: lognorm(x, mu=mu_MLE,sigma=sig_MLE), 0, 150000)[0]
        X = np.linspace(0,150000, 150001)
        fig,ax = plt.subplots()
        ax.set_xlabel('Incomes')
        ax.set_ylabel('Frequency')
        ax.set_title('Annual incomes of MACSS students graduated in 2018, 2019, 2020')
        ax.plot(X, lognorm(X)/total1, label = 'initial guess')
        ax.plot(X, lognorm(X, mu=mu_MLE,sigma=sig_MLE)/total2, label = 'MLE result')
        ax.hist(x=Incomes, bins=30, color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
        plt.show()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: divide by z

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: invalid val
```



0.1.4 d

```
In [7]: mu_new, sig_new = np.array([11, 0.5])
        log_lik_h0 = log_lik_truncnorm(Incomes, mu_new, sig_new, 0, 150000)
        print('hypothesis value log likelihood', log_lik_h0)
        log_lik_mle = log_lik_truncnorm(Incomes, mu_MLE, sig_MLE, 0, 150000)
        print('MLE log likelihood', log_lik_mle)
        LR_val = 2 * (log_lik_mle - log_lik_h0)
        print('likelihood ratio value', LR_val)
        pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
        print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2379.120591931827
MLE log likelihood -2240.9343375116364
likelihood ratio value 276.37250884038167
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

0.1.5 e

```
In [8]: print("The probability that you will earn more than $100,000 is",
              quad(lambda x: lognorm(x, mu=mu_MLE,sigma=sig_MLE), 100000, 150000)[0]/total2)
        print("The probability that you will earn less than $75,000 is",
              quad(lambda x: lognorm(x, mu=mu_MLE,sigma=sig_MLE), 0, 75000)[0]/total2)
```

```
The probability that you will earn more than $100,000 is 0.23427612236731754
The probability that you will earn less than $75,000 is 0.26076018268903833
```

0.2 2

0.2.1 a

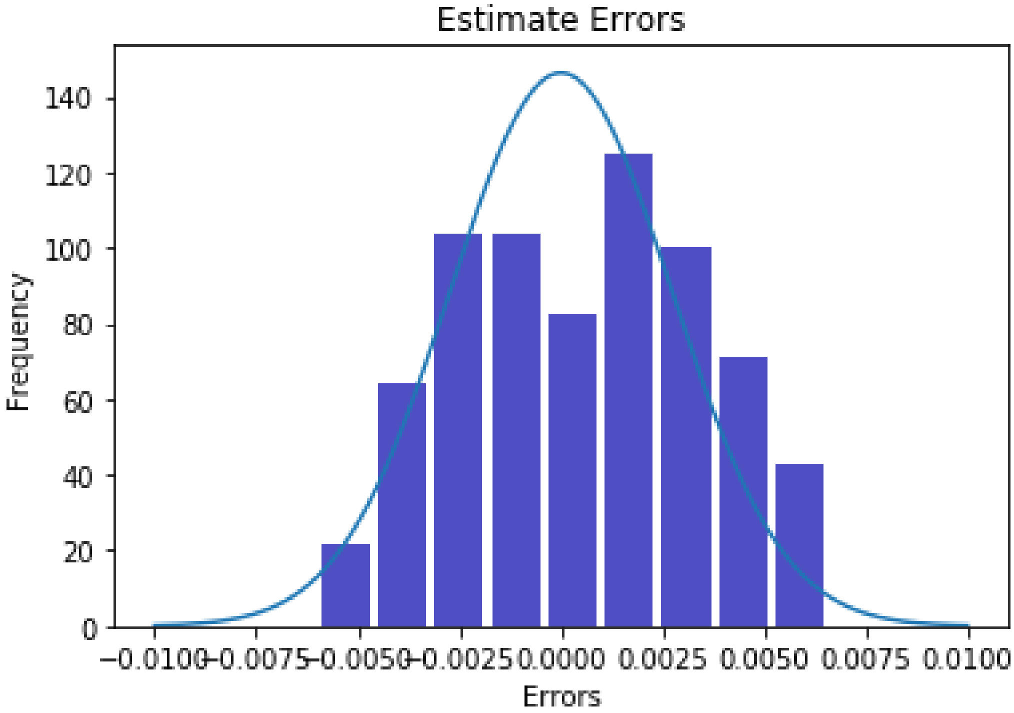
```
In [9]: def norm_pdf(x, sigma):
            sigma = np.abs(sigma)
            pdf_vals = (1/(sigma * np.sqrt(2 * np.pi)) *
                            np.exp( - x**2 / (2 * sigma**2)))
            return pdf_vals
        def log_lik_norm(x, beta0, beta1, beta2, beta3, sigma):
            error = x[:,0] - beta0 - beta1*x[:,1] - beta2*x[:,2] - beta3*x[:,3]
            pdf_vals = norm_pdf(error, sigma)
            ln_pdf_vals = np.log(pdf_vals)
            log_lik_val = ln_pdf_vals.sum()
            return log_lik_val
        def new_crit(params, *args):
            beta0, beta1, beta2, beta3, sigma = params
            x = args[0]
            log_lik_val = log_lik_norm(x,beta0, beta1, beta2, beta3, sigma)
            neg_log_lik_val = -log_lik_val
```

```
    return neg_log_lik_val
Data = []
with open('sick.txt') as f:
    for line in f:
        line = line.strip()
        data = line.split(',')
        Data.append(data)
Data.pop(0)
for i in range(len(Data)):
    Data[i] = [float(x) for x in Data[i]]
Data = np.array(Data)
new_Data = np.ones_like(Data)
new_Data[:,1:] = Data[:,1:]
beta0_init, beta1_init, beta2_init, beta3_init = np.linalg.lstsq(new_Data,Data[:,0])[0]
sigma_init = 0.01
parameters_init = np.array([beta0_init, beta1_init, beta2_init, beta3_init, sigma_init]
results2 = opt.minimize(new_crit, parameters_init, args=(Data))
beta0_MLE, beta1_MLE, beta2_MLE, beta3_MLE, sigma_MLE = results2.x
new_LLV_MLE = log_lik_norm(Data, beta0_MLE, beta1_MLE, beta2_MLE, beta3_MLE, sigma_MLE)
new_VCV_MLE = results2.hess_inv
print('beta0_MLE =',beta0_MLE,
      'beta1_MLE =',beta1_MLE,
      'beta2_MLE =',beta2_MLE,
      'beta3_MLE =',beta3_MLE,
      'sigma_MLE =',sigma_MLE)
print('The value of the log likelihood value is', new_LLV_MLE)
print('The variance-covariance matrix is', new_VCV_MLE)
Error = Data[:,0] - beta0_MLE - beta1_MLE*Data[:,1] - beta2_MLE*Data[:,2] - beta3_MLE*D
X2 = np.linspace(-0.01,0.01,10001)
fig,ax = plt.subplots()
ax.set_xlabel('Errors')
ax.set_ylabel('Frequency')
ax.set_title('Estimate Errors')
ax.plot(X2, norm_pdf(X2,sigma_MLE))
ax.hist(x=Error, bins='auto', color='#0504aa', alpha=0.7, rwidth=0.85, normed = True)
plt.show()
```

```
beta0_MLE = 0.2516415006612955 beta1_MLE = 0.012928701541639099 beta2_MLE = 0.40049960366812937
The value of the log likelihood value is 871.487646107643
The variance-covariance matrix is [[ 9.99749814e-01 -1.07472308e-02 -4.38873773e-04 -1.15948335
  -1.55722702e-04]
 [-1.07472308e-02  5.38385787e-01 -1.88499775e-02 -4.98050664e-01
   1.13266069e-02]
 [-4.38873773e-04 -1.88499775e-02  9.99230140e-01 -2.03364503e-02
   1.57898527e-03]
 [-1.15948335e-02 -4.98050664e-01 -2.03364503e-02  4.62631712e-01
  -1.05543181e-02]
 [-1.55722702e-04  1.13266069e-02  1.57898527e-03 -1.05543181e-02
```

```
   2.42229668e-04]]
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:30: FutureWarning: `rcond` pa
To use the future default and silence this warning we advise to pass `rcond=None`, to keep usi
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:9: RuntimeWarning: divide by 
  if __name__ == '__main__':
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:643: RuntimeWarning: inva
  grad[k] = (f(*((xk + d,) + args)) - f0) / d[k]
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:9: RuntimeWarning: divide by 
  if __name__ == '__main__':
```



0.2.2 b

```
In [10]: parameters_new = np.array([1,0,0,0,0.1])
         log_lik_h0 = log_lik_norm(Data, *parameters_new)
         print('hypothesis value log likelihood', log_lik_h0)
         log_lik_mle = log_lik_norm(Data, *results2.x)
         print('MLE log likelihood', log_lik_mle)
         LR_val = 2 * (log_lik_mle - log_lik_h0)
         print('likelihood ratio value', LR_val)
         pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 5)
         print('chi squared of H0 with 5 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2253.700688042125
MLE log likelihood 871.487646107643
likelihood ratio value 6250.376668299536
chi squared of H0 with 5 degrees of freedom p-value =  0.0
```

The results show that it's not likely that age, number of children, and average winter temperature have no effect on the number of sick days.