hw3

January 28, 2019

Import Packages

```
In [1]: import numpy as np
        import scipy.optimize as opt
        import sympy as sp
        import matplotlib.pyplot as plt
        from scipy.stats import norm
        from mpl_toolkits.mplot3d import Axes3D
```

0.0.1 5.1

The problem is:

$$\max_{W_2 \in [0, W_1]} u(W_1 - W_2)$$

0.0.2 5.2

The condition that characterizes the optimal amount of cake to leave for the next period W_3 in period 2 is:

$$\max_{W_3 \in [0, W_2]} u(W_2 - W_3)$$

The condition that characterizes the optimal amount of cake to leave for the next period W_2 in period 1 is:

$$\max_{W_2 \in [0, W_1]} [u(W_1 - W_2) + \max_{W_3 \in [0, W_2]} \beta u(W_2 - W_3)]$$

0.0.3 5.3

The condition that characterizes the optimal amount of cake to leave for the next period W_2, W_3, W_4 in period 1, 2, 3 are respectively:

$$\max_{W_2 \in [0, W_1]} \{u(W_1 - W_2) + \max_{W_3 \in [0, W_2]} \beta [u(W_2 - W_3) + \max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4)]\}$$

$$\max_{W_3 \in [0, W_2]} \beta [u(W_2 - W_3) + \max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4)]$$

$$\max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4)$$

From the 3rd condition, we know that $W_4 = 0$, from the 1st and 2nd condition, we know that if we differentiate the left hand side of 1st condition with respect to W_2 and W_3, every derivatives should equal to 0 to maximise it, i.e.,

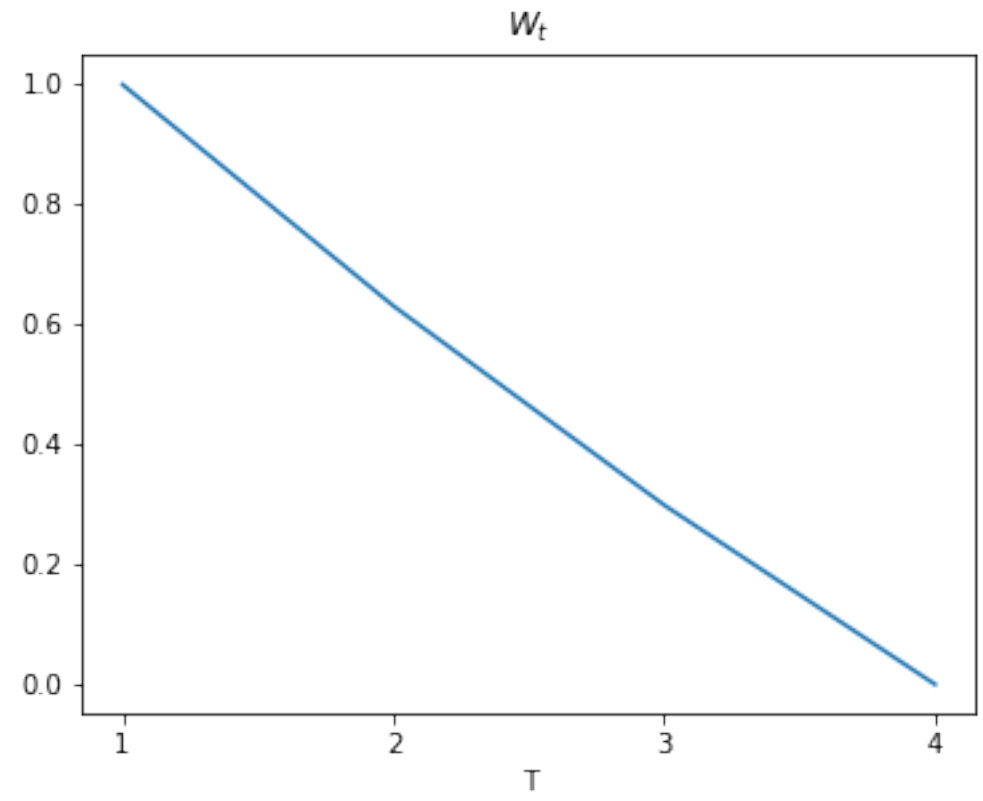
$$-u'(W_1 - W_2) + \beta u'(W_2 - W_3) = 0$$

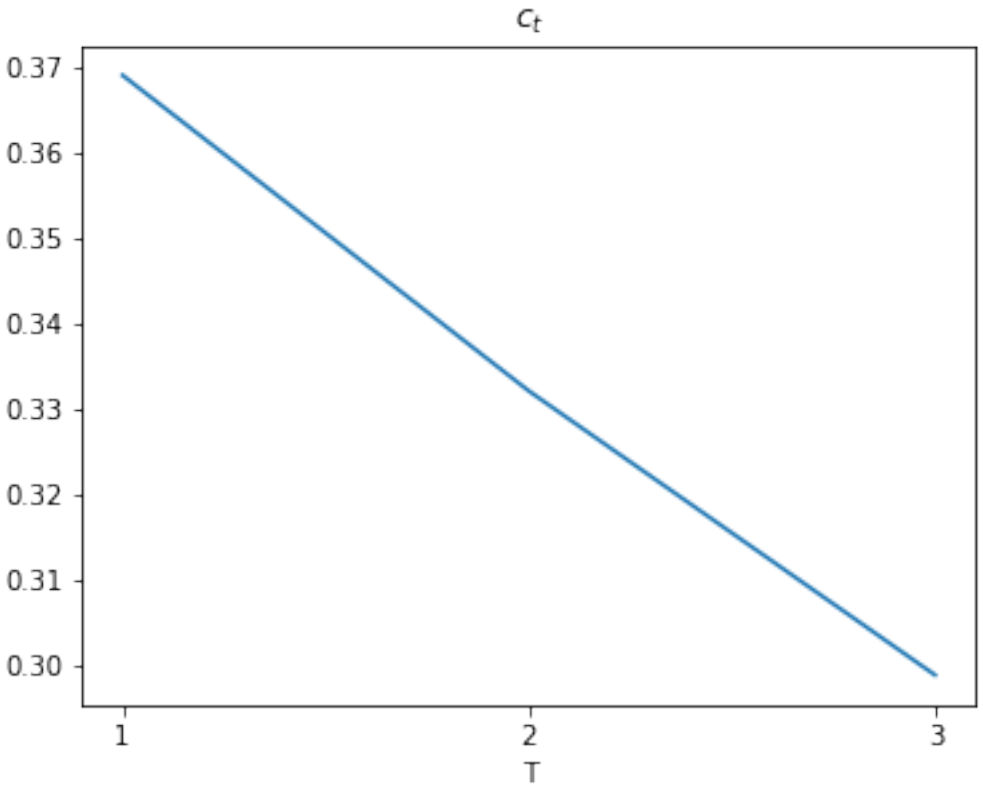
$$-\beta u'(W_2 - W_3) + \beta^2 u'(W_3 - W_4) = 0$$

We also know that $u(x) = ln(x)$,$W_1 = 1$ and $W_4 = 0$, so we can solve the equations to get $W_2 = 0.631$, $W_3 = 0.299$, we can then find out $c_1 = W_1 - W_2 = 0.369$, $c_2 = W_2 - W_3 = 0.332$, $c_3 = W_3 - W_4 = 0.299$ The evolve of $\{c_t\}_{t=1}^{3}$ and $\{W_t\}_{t=1}^{4}$ is as follows:

```
In [2]: W = [1, 1-1/(1+0.9+0.81), 1-1.9/(1+0.9+0.81), 0]
        c = [1/(1+0.9+0.81),0.9/(1+0.9+0.81),0.81/(1+0.9+0.81)]
        T = [1,2,3,4]
        fig, ax=plt.subplots(2,1,figsize=(6,10))
        ax[0].plot(T, W)
        ax[1].plot(T[:-1], c)
        ax[0].set_title(r"$W_t$")
        ax[0].set_xlabel('T')
        ax[0].set_xticks([1,2,3,4])
        ax[1].set_title(r"$c_t$")
        ax[1].set_xlabel('T')
        ax[1].set_xticks([1,2,3])

Out[2]: [<matplotlib.axis.XTick at 0x12056b3feb8>,
         <matplotlib.axis.XTick at 0x12056b3fd68>,
         <matplotlib.axis.XTick at 0x12056b2a630>]
```


W_t
1.0
0.8
0.6
0.4
0.2
0.0
1
2
3
4
T


c_t
0.37
0.36
0.35
0.34
0.33
0.32
0.31
0.30
1
2
3
T

0.0.4 5.4

The condition that characterizes the optimal choice in period T-1 is:

$$-u'(W_{T-1} - \psi_{T-1}(W_{T-1})) + \beta u'(\psi_{T-1}(W_{T-1})) = 0$$

Then we can express V_{T-1} as follows:

$$V_{T-1}(W_{T-1}) = u(W_{T-1} - \psi_{T-1}(W_{T-1})) + \beta u(\psi_{T-1}(W_{T-1}))$$

0.0.5 5.5

According to the former question, we know $V_T(\bar{W}) = u(\bar{W})$, suppose $V_{T-1}(\bar{W}) = V_T(\bar{W})$, that is:

$$u(\bar{W}) = u(\bar{W} - \psi_{T-1}(\bar{W})) + \beta u(\psi_{T-1}(\bar{W}))$$

$$-u'(\bar{W} - \psi_{T-1}(\bar{W})) + \beta u'(\psi_{T-1}(\bar{W})) = 0$$

Since $u(x) = ln(x)$ and we can solve former equations:

$$\psi_{T-1}(\bar{W}) = \frac{\beta}{1+\beta}\bar{W}$$

$$\psi_T(\bar{W}) = \bar{W}$$

$$V_{T-1}(\bar{W}) = ln(\frac{\bar{W}}{1+\beta}) + \beta ln(\frac{\beta\bar{W}}{1+\beta})$$

$$V_T(\bar{W}) = ln(\bar{W})$$

0.0.6 5.6

The finite horizon Bellman equation for the value function at time T-2 is:

$$V_{T-2}(W_{T-2}) = \max_{W_{T-1}} ln(W_{T-2} - W_{T-1})) + \beta ln(\frac{W_{T-1}}{1+\beta}) + \beta^2 ln(\frac{\beta W_{T-1}}{1+\beta})$$

The condition that characterizes the optimal choice in period T-2 is:

$$-\frac{1}{(W_{T-2} - \psi_{T-2}(W_{T-2}))} + (\beta + \beta^2)\frac{1}{\psi_{T-2}(W_{T-2})} = 0$$

The analytical solution for $\psi_{T-2}(W_{T-2})$ and $V_{T-2}(W_{T-2})$ is:

$$\psi_{T-2}(W_{T-2}) = \frac{\beta + \beta^2}{1 + \beta + \beta^2}W_{T-2}$$

$$V_{T-2}(W_{T-2}) = ln(\frac{W_{T-2}}{1+\beta+\beta^2}) + \beta ln(\frac{\beta W_{T-2}}{1+\beta+\beta^2}) + \beta^2 ln(\frac{\beta^2 W_{T-2}}{1+\beta+\beta^2})$$

0.0.7 5.7

By induction, the analytical solution for $\psi_{T-s}(W_{T-s})$ and $V_{T-s}(W_{T-s})$ is:

$$\psi_{T-s}(W_{T-s}) = \frac{\sum_{i=1}^{s} \beta^i}{1 + \sum_{i=1}^{s} \beta^i} W_{T-s}$$

$$V_{T-s}(W_{T-s}) = \left[\sum_{i=0}^{s-1} \beta^i ln \left(\frac{\beta^i W_{T-s}}{1 + \sum_{i=1}^{s} \beta^i} \right) \right] + \beta^s ln \left(\frac{\beta^s W_{T-s}}{1 + \sum_{i=1}^{s} \beta^i} \right)$$

Take limits of s tend to infinite, we have:

$$\psi(W_{T-s}) = \beta W_{T-s}$$

$$V(W_{T-s}) = \left(\frac{1}{1-\beta} \right) ln((1-\beta)W_{T-s}) + \frac{\beta}{(1-\beta)^2} ln(\beta)$$

0.0.8 5.8

$$V(W) = \max_{w \in [0,W]} u(W-w)) + \beta V(w)$$

0.0.9 5.9

The code is as follows:

```
In [3]: W = np.linspace(0.01, 1, 100)
```

0.0.10 5.10

```
In [4]: c_mat = W.reshape(-1,1)-W
        c_mat[c_mat<=0] = 1e-7
        u_mat = np.log(c_mat)
        N = 100
        beta = 0.9

In [5]: psi_T = np.empty(100)
        Value_T = np.empty(100)
        for i in range(100):
            w = W[i]
            value_func = lambda x: -np.log(w-x)
            psi_T[i] = max(float(opt.fmin(value_func, 0, disp = 0)),0)
            Value_T[i] = np.log(w-psi_T[i])
        print("The policy function W' is\n", psi_T)
        print("The value function V_T is\n", Value_T)
```

```
The policy function W' is
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0.]
The value function V_T is
 [-4.60517019 -3.91202301 -3.5065579  -3.21887582 -2.99573227 -2.81341072
 -2.65926004 -2.52572864 -2.40794561 -2.30258509 -2.20727491 -2.12026354
 -2.04022083 -1.96611286 -1.89711998 -1.83258146 -1.77195684 -1.71479843
 -1.66073121 -1.60943791 -1.56064775 -1.51412773 -1.46967597 -1.42711636
 -1.38629436 -1.34707365 -1.30933332 -1.27296568 -1.23787436 -1.2039728
 -1.17118298 -1.13943428 -1.10866262 -1.07880966 -1.04982212 -1.02165125
 -0.99425227 -0.96758403 -0.94160854 -0.91629073 -0.89159812 -0.86750057
 -0.84397007 -0.82098055 -0.7985077  -0.77652879 -0.75502258 -0.73396918
 -0.71334989 -0.69314718 -0.67334455 -0.65392647 -0.63487827 -0.61618614
 -0.597837   -0.5798185  -0.56211892 -0.54472718 -0.52763274 -0.51082562
 -0.49429632 -0.4780358  -0.46203546 -0.4462871  -0.43078292 -0.41551544
 -0.40047757 -0.38566248 -0.37106368 -0.35667494 -0.34249031 -0.32850407
 -0.31471074 -0.30110509 -0.28768207 -0.27443685 -0.26136476 -0.24846136
 -0.23572233 -0.22314355 -0.21072103 -0.19845094 -0.18632958 -0.17435339
 -0.16251893 -0.15082289 -0.13926207 -0.12783337 -0.11653382 -0.10536052
 -0.09431068 -0.08338161 -0.07257069 -0.0618754  -0.05129329 -0.04082199
 -0.03045921 -0.02020271 -0.01005034  0.        ]
```

0.0.11 5.11

```
In [6]: Value_T_plus_1 = np.zeros(100)
        delta_T = np.sum((Value_T-Value_T_plus_1)**2)
        print("The distance metric is", delta_T)
```

```
The distance metric is 178.92611065972804
```

0.0.12 5.12

```
In [7]: Value_T_minus_1 = np.array(Value_T)
        psi_T_minus_1 = np.array(psi_T)
        VT1_matrix = np.tile(Value_T.reshape(1,100),(100,1))
        VT1_matrix[c_mat<0] = -9e+4
        for i in range(N):
            Value_T_minus_1[i] = -9e+4
            for j in range(100):
                if u_mat[i,j]+beta*VT1_matrix[i,j] > Value_T_minus_1[i]:
                    psi_T_minus_1[i] = W[j]
                    Value_T_minus_1[i] = u_mat[i,j]+beta*VT1_matrix[i,j]
            for i in range(100):
```

```
                if psi_T_minus_1[i] >= W[i]:
                    psi_T_minus_1[i] = W[i]-0.01
        delta_T_minus_1 = np.sum((Value_T-Value_T_minus_1)**2)
        print("The policy function is\n", psi_T_minus_1)
        print("The value function is\n", Value_T_minus_1)
        print("The distance metric is", delta_T_minus_1)
```

```
The policy function is
 [0.   0.01 0.01 0.02 0.02 0.03 0.03 0.04 0.04 0.05 0.05 0.06 0.06 0.07
 0.07 0.08 0.08 0.09 0.09 0.09 0.1  0.1  0.11 0.11 0.12 0.12 0.13 0.13
 0.14 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.18 0.18 0.19 0.19 0.2
 0.2  0.21 0.21 0.22 0.22 0.23 0.23 0.24 0.24 0.25 0.25 0.26 0.26 0.27
 0.27 0.27 0.28 0.28 0.29 0.29 0.3  0.3  0.31 0.31 0.32 0.32 0.33 0.33
 0.34 0.34 0.35 0.35 0.36 0.36 0.36 0.37 0.37 0.38 0.38 0.39 0.39 0.4
 0.4  0.41 0.41 0.42 0.42 0.43 0.43 0.44 0.44 0.45 0.45 0.45 0.46 0.46
 0.47 0.47]
The value function is
 [-16.11809565  -8.74982335  -8.05667617  -7.43284371  -7.0273786
  -6.66246     -6.37477793  -6.11586407  -5.89272052  -5.69189132
  -5.50956976  -5.34548036  -5.19132968  -5.05259407  -4.91906268
  -4.79888442  -4.68110139  -4.57509666  -4.46973614  -4.37442596
  -4.2796015   -4.19259012  -4.10681096  -4.02676825  -3.94845801
  -3.87435004  -3.8023116   -3.73331873  -3.66662156  -3.60208303
  -3.53998945  -3.47936483  -3.42128016  -3.36412175  -3.30955959
  -3.25549236  -3.20404979  -3.1527565   -3.10396633  -3.05530583
  -3.00878582  -2.96262185  -2.91817009  -2.87425894  -2.83169933
  -2.78983132  -2.74900932  -2.70900273  -2.66978202  -2.63147837
  -2.59373804  -2.55699824  -2.5206306   -2.48533196  -2.45024064
  -2.41627434  -2.38237279  -2.34958297  -2.31685209  -2.28510339
  -2.2535212   -2.22274954  -2.19223815  -2.16238519  -2.13287434
  -2.10388681  -2.07531298  -2.0471421   -2.01944761  -1.99204864
  -1.96518097  -1.93851272  -1.91242394  -1.88644845  -1.86109466
  -1.83577685  -1.81108424  -1.78642517  -1.76232761  -1.73832619
  -1.71479569  -1.69141776  -1.66842824  -1.64564221  -1.62316935
  -1.600946    -1.5789671   -1.5572793   -1.5357731   -1.51459565
  -1.49354224  -1.47285167  -1.45223238  -1.43200681  -1.41180411
  -1.39200148  -1.37222046  -1.35280238  -1.33344679  -1.3143986 ]
The distance metric is 650.6032364682843
```

0.0.13 5.13

In [8]:
```
Value_T_minus_2=np.array(Value_T_minus_1)
psi_T_minus_2 = np.array(psi_T_minus_1)
VT2_matrix = np.tile(Value_T_minus_1.reshape(1,100),(100,1))
VT2_matrix[c_mat<0] = -9e+4
for i in range(N):
    Value_T_minus_2[i] = -9e+4
```

```
            for j in range(100):
                if u_mat[i,j]+beta*VT2_matrix[i,j] > Value_T_minus_2[i]:
                    psi_T_minus_2[i] = W[j]
                    Value_T_minus_2[i] = u_mat[i,j]+beta*VT2_matrix[i,j]
            for i in range(100):
                if psi_T_minus_2[i] >= W[i]:
                    psi_T_minus_2[i] = W[i]-0.01
        delta_T_minus_2 = np.sum((Value_T_minus_1-Value_T_minus_2)**2)
        print("The policy function is\n", psi_T_minus_2)
        print("The value function is\n", Value_T_minus_2)
        print("The distance metric is", delta_T_minus_2)
```

```
The policy function is
 [0.   0.01 0.02 0.02 0.03 0.04 0.04 0.05 0.06 0.06 0.07 0.08 0.08 0.09
 0.09 0.1  0.11 0.11 0.12 0.13 0.13 0.14 0.15 0.15 0.16 0.17 0.17 0.18
 0.18 0.19 0.19 0.2  0.21 0.21 0.22 0.23 0.23 0.24 0.25 0.25 0.26 0.27
 0.27 0.28 0.28 0.29 0.3  0.3  0.31 0.32 0.32 0.33 0.34 0.34 0.35 0.35
 0.36 0.36 0.37 0.38 0.38 0.39 0.4  0.4  0.41 0.42 0.42 0.43 0.44 0.44
 0.45 0.45 0.46 0.47 0.47 0.48 0.49 0.49 0.5  0.51 0.51 0.52 0.52 0.53
 0.54 0.54 0.55 0.55 0.56 0.57 0.57 0.58 0.59 0.59 0.6  0.61 0.61 0.62
 0.63 0.63]
The value function is
 [-17.30105439 -17.30105439 -12.4800112  -11.78686402 -11.16303156
 -10.60158234 -10.19611724  -9.83119864  -9.5027719   -9.21508983
  -8.95617596  -8.72315349  -8.50000993  -8.29918074  -8.11685918
  -7.9361129   -7.7720235   -7.61787282  -7.47019236  -7.33145675
  -7.19792536  -7.07306331  -6.95288505  -6.83510202  -6.72694159
  -6.62093686  -6.51557634  -6.42017208  -6.3248619   -6.23003744
  -6.14302606  -6.0572469   -5.97190488  -5.89186218  -5.81355194
  -5.73635069  -5.66224272  -5.59020428  -5.51972507  -5.45073219
  -5.38403502  -5.31920043  -5.25466191  -5.19256832  -5.1319437
  -5.07191624  -5.01383157  -4.95667316  -4.90078893  -4.84622677
  -4.79215955  -4.73988335  -4.68844078  -4.63714748  -4.58804154
  -4.53925138  -4.49059088  -4.44407086  -4.39777255  -4.35160858
  -4.30715682  -4.26324567  -4.21945122  -4.17689161  -4.13502359
  -4.09347602  -4.05265403  -4.01264744  -3.97312741  -3.9339067
  -3.89560304  -3.85786272  -3.8201815   -3.78344171  -3.74707406
  -3.71106814  -3.67576949  -3.64067817  -3.60620489  -3.57223859
  -3.53833704  -3.50527122  -3.4724814   -3.43975052  -3.40798174
  -3.37623305  -3.34465086  -3.3138792   -3.28330953  -3.25279814
  -3.22294517  -3.19343433  -3.16397654  -3.13498901  -3.10641518
  -3.07799121  -3.04982033  -3.02212584  -2.99466558  -2.96726661]
The distance metric is 673.4368745458985
```

0.0.14 5.14

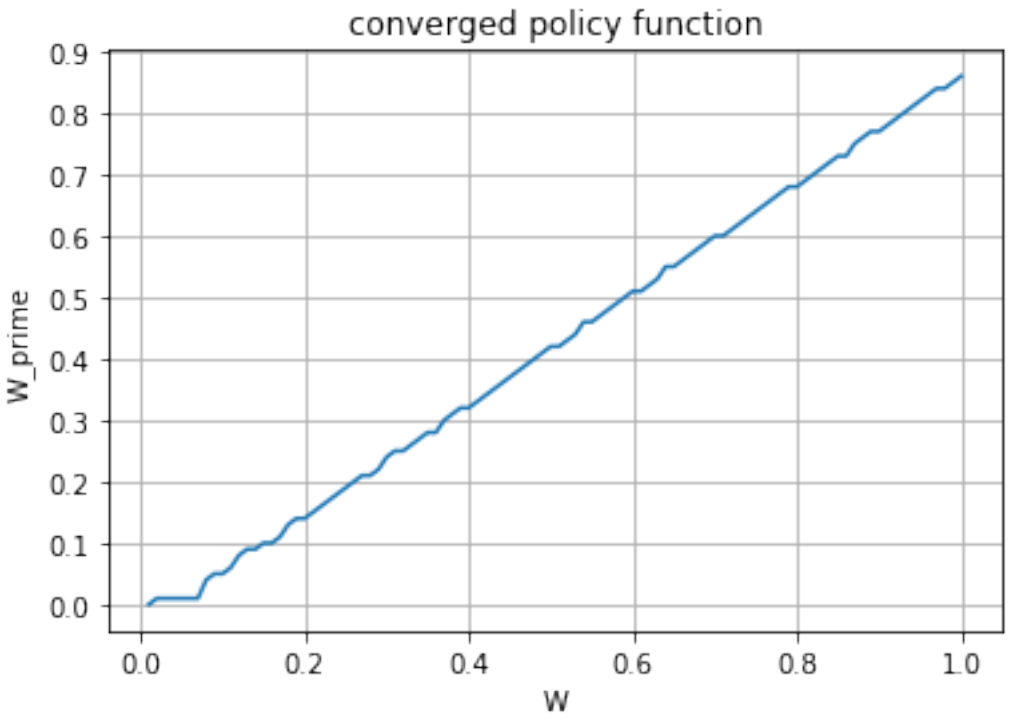
```
In [9]: def optimize(init=W, u = lambda x: np.log(x),
                         beta = 0.9, error = 1e-9, maxiter = 1000):
            W = init
            V = np.log(W)
            N = 100
            c_mat = W.reshape(-1,1)-W
            c_mat[c_mat<=0] = 1e-7
            u_mat = u(c_mat)
            Error = 1
            count = 0
            while Error>error and count <= maxiter:
                count += 1
                new_W = np.array(W)
                new_V = np.array(V)
                V_matrix = np.tile(V.reshape(1,100),(100,1))
                V_matrix[c_mat<=0] = -9e+4
                for i in range(N):
                    new_V[i] = -9e+4
                    for j in range(N):
                        if u_mat[i,j]+beta*V_matrix[i,j] > new_V[i]:
                            new_W[i] = W[j]
                            new_V[i] = u_mat[i,j]+beta*V_matrix[i,j]
                    for i in range(N):
                        if new_W[i] > W[i]:
                            new_W[i] = W[i]-0.01
                Error = ((V-new_V)**2).sum()
                V = new_V
            return new_V,new_W
        V, new_W = optimize()
        print("The value function is\n", V)
        print("The policy function is\n", new_W)
```

```
The value function is
 [-42.67084199 -42.67084199 -42.31577781 -41.91031271 -41.62263063
 -41.39948708 -41.21716553 -40.93815432 -40.67924046 -40.45609691
 -40.25526771 -40.06321183 -39.83018935 -39.6070458  -39.40621661
 -39.22389505 -39.04314877 -38.84289983 -38.64207063 -38.45974907
 -38.2790028  -38.1149134  -37.95224175 -37.77201774 -37.59127146
 -37.42718206 -37.26451041 -37.11035973 -36.96267927 -36.80822395
 -36.6455523  -36.49140162 -36.34372116 -36.19731667 -36.05858106
 -35.92504967 -35.78665896 -35.64025447 -35.50151886 -35.36798747
 -35.23507505 -35.10331102 -34.97844897 -34.85827071 -34.73371918
 -34.60195515 -34.4770931  -34.35691484 -34.23729367 -34.11870604
 -34.000923   -33.88854716 -33.78038673 -33.66748577 -33.54970274
 -33.43732689 -33.32916646 -33.22150741 -33.11477854 -33.00877381
 -32.90341329 -32.80227503 -32.70493065 -32.60267565 -32.49731514
 -32.39617688 -32.29883249 -32.20193934 -32.10588336 -32.0104791
```

```
 -31.91516892 -31.82034446 -31.72932002 -31.64171007 -31.54967981
 -31.45485534 -31.36383091 -31.27622096 -31.18901713 -31.10200575
 -31.01555536 -30.92969153 -30.84391237 -30.75857035 -30.67664836
 -30.59660565 -30.51497226 -30.42963024 -30.34770825 -30.26766554
 -30.18881659 -30.11033313 -30.0320229  -29.95421755 -29.8769401
 -29.79973885 -29.72293104 -29.64882307 -29.57509327 -29.50305484]
The policy function is
 [0.   0.01 0.01 0.01 0.01 0.01 0.01 0.04 0.05 0.05 0.06 0.08 0.09 0.09
 0.1  0.1  0.11 0.13 0.14 0.14 0.15 0.16 0.17 0.18 0.19 0.2  0.21 0.21
 0.22 0.24 0.25 0.25 0.26 0.27 0.28 0.28 0.3  0.31 0.32 0.32 0.33 0.34
 0.35 0.36 0.37 0.38 0.39 0.4  0.41 0.42 0.42 0.43 0.44 0.46 0.46 0.47
 0.48 0.49 0.5  0.51 0.51 0.52 0.53 0.55 0.55 0.56 0.57 0.58 0.59 0.6
 0.6  0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.68 0.69 0.7  0.71 0.72
 0.73 0.73 0.75 0.76 0.77 0.77 0.78 0.79 0.8  0.81 0.82 0.83 0.84 0.84
 0.85 0.86]
```

0.0.15 5.15

```
In [10]: fig,ax = plt.subplots()
         ax.plot(W,new_W)
         ax.set_xlabel("W")
         ax.set_ylabel("W_prime")
         ax.set_yticks([0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9])
         ax.set_title("converged policy function")
         ax.grid()
```



0.0.16 5.16

```
In [11]: sigma = 0.5
         varepsilon = np.array([i*sigma for i in range(1,8)])
         Gamma_func = lambda x: norm(loc = 4*sigma, scale = sigma).pdf(x)
         Gamma = Gamma_func(varepsilon)
```

0.0.17 5.17

```
In [12]: new_psi_T = np.zeros((100,7))
         new_Value_T = np.empty((100,7))
         for j in range(7):
             e = varepsilon[j]
             for i in range(100):
                 w = W[i]
                 new_value_func = lambda x: -e*np.log(w-x)
                 new_psi_T[i,j] = max(float(opt.fmin(new_value_func, 0, disp = 0)),0)
                 new_Value_T[i,j] = e*np.log(w-psi_T[i])
         print("The policy function W' is\n", new_psi_T)
         print("The value function V_T is\n", new_Value_T)
```

```
The policy function W' is
 [[0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0.]
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 [0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0.]]
The value function V_T is
 [[-2.30258509e+00 -4.60517019e+00 -6.90775528e+00 -9.21034037e+00
  -1.15129255e+01 -1.38155106e+01 -1.61180957e+01]
 [-1.95601150e+00 -3.91202301e+00 -5.86803451e+00 -7.82404601e+00
  -9.78005751e+00 -1.17360690e+01 -1.36920805e+01]
 [-1.75327895e+00 -3.50655790e+00 -5.25983685e+00 -7.01311579e+00
  -8.76639474e+00 -1.05196737e+01 -1.22729526e+01]
 [-1.60943791e+00 -3.21887582e+00 -4.82831374e+00 -6.43775165e+00
  -8.04718956e+00 -9.65662747e+00 -1.12660654e+01]
 [-1.49786614e+00 -2.99573227e+00 -4.49359841e+00 -5.99146455e+00
  -7.48933068e+00 -8.98719682e+00 -1.04850630e+01]
 [-1.40670536e+00 -2.81341072e+00 -4.22011608e+00 -5.62682143e+00
  -7.03352679e+00 -8.44023215e+00 -9.84693751e+00]
 [-1.32963002e+00 -2.65926004e+00 -3.98889006e+00 -5.31852007e+00
  -6.64815009e+00 -7.97778011e+00 -9.30741013e+00]
 [-1.26286432e+00 -2.52572864e+00 -3.78859297e+00 -5.05145729e+00
  -6.31432161e+00 -7.57718593e+00 -8.84005026e+00]
 [-1.20397280e+00 -2.40794561e+00 -3.61191841e+00 -4.81589122e+00
  -6.01986402e+00 -7.22383683e+00 -8.42780963e+00]
```

```
[-1.15129255e+00 -2.30258509e+00 -3.45387764e+00 -4.60517019e+00
 -5.75646273e+00 -6.90775528e+00 -8.05904783e+00]
[-1.10363746e+00 -2.20727491e+00 -3.31091237e+00 -4.41454983e+00
 -5.51818728e+00 -6.62182474e+00 -7.72546220e+00]
[-1.06013177e+00 -2.12026354e+00 -3.18039530e+00 -4.24052707e+00
 -5.30065884e+00 -6.36079061e+00 -7.42092238e+00]
[-1.02011041e+00 -2.04022083e+00 -3.06033124e+00 -4.08044166e+00
 -5.10055207e+00 -6.12066249e+00 -7.14077290e+00]
[-9.83056428e-01 -1.96611286e+00 -2.94916928e+00 -3.93222571e+00
 -4.91528214e+00 -5.89833857e+00 -6.88139500e+00]
[-9.48559992e-01 -1.89711998e+00 -2.84567998e+00 -3.79423997e+00
 -4.74279996e+00 -5.69135995e+00 -6.63991995e+00]
[-9.16290732e-01 -1.83258146e+00 -2.74887220e+00 -3.66516293e+00
 -4.58145366e+00 -5.49774439e+00 -6.41403512e+00]
[-8.85978421e-01 -1.77195684e+00 -2.65793526e+00 -3.54391368e+00
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[-8.30365603e-01 -1.66073121e+00 -2.49109681e+00 -3.32146241e+00
 -4.15182802e+00 -4.98219362e+00 -5.81255922e+00]
[-8.04718956e-01 -1.60943791e+00 -2.41415687e+00 -3.21887582e+00
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[-6.18937178e-01 -1.23787436e+00 -1.85681153e+00 -2.47574871e+00
 -3.09468589e+00 -3.71362307e+00 -4.33256025e+00]
[-6.01986402e-01 -1.20397280e+00 -1.80595921e+00 -2.40794561e+00
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 -2.84858571e+00 -3.41830285e+00 -3.98801999e+00]
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 -2.77165656e+00 -3.32598787e+00 -3.88031919e+00]
```

```
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[-3.56674944e-01 -7.13349888e-01 -1.07002483e+00 -1.42669978e+00
 -1.78337472e+00 -2.14004966e+00 -2.49672461e+00]
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 -1.68336138e+00 -2.02003366e+00 -2.35670594e+00]
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 -1.44954624e+00 -1.73945549e+00 -2.02936473e+00]
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 -1.31908186e+00 -1.58289823e+00 -1.84671460e+00]
[-2.55412812e-01 -5.10825624e-01 -7.66238436e-01 -1.02165125e+00
 -1.27706406e+00 -1.53247687e+00 -1.78788968e+00]
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[-1.43841036e-01 -2.87682072e-01 -4.31523109e-01 -5.75364145e-01
 -7.19205181e-01 -8.63046217e-01 -1.00688725e+00]
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 -6.86092114e-01 -8.23310537e-01 -9.60528960e-01]
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 -6.53411910e-01 -7.84094292e-01 -9.14776674e-01]
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 -6.21153398e-01 -7.45384078e-01 -8.69614758e-01]
[-1.17861167e-01 -2.35722334e-01 -3.53583500e-01 -4.71444667e-01
 -5.89305834e-01 -7.07167001e-01 -8.25028167e-01]
[-1.11571776e-01 -2.23143551e-01 -3.34715327e-01 -4.46287103e-01
 -5.57858878e-01 -6.69430654e-01 -7.81002430e-01]
[-1.05360516e-01 -2.10721031e-01 -3.16081547e-01 -4.21442063e-01
 -5.26802578e-01 -6.32163094e-01 -7.37523610e-01]
```

```
 [-9.92254694e-02 -1.98450939e-01 -2.97676408e-01 -3.96901877e-01
  -4.96127347e-01 -5.95352816e-01 -6.94578286e-01]
 [-9.31647891e-02 -1.86329578e-01 -2.79494367e-01 -3.72659156e-01
  -4.65823945e-01 -5.58988735e-01 -6.52153524e-01]
 [-8.71766936e-02 -1.74353387e-01 -2.61530081e-01 -3.48706774e-01
  -4.35883468e-01 -5.23060161e-01 -6.10236855e-01]
 [-8.12594647e-02 -1.62518929e-01 -2.43778394e-01 -3.25037859e-01
  -4.06297324e-01 -4.87556788e-01 -5.68816253e-01]
 [-7.54114449e-02 -1.50822890e-01 -2.26234335e-01 -3.01645779e-01
  -3.77057224e-01 -4.52468669e-01 -5.27880114e-01]
 [-6.96310337e-02 -1.39262067e-01 -2.08893101e-01 -2.78524135e-01
  -3.48155168e-01 -4.17786202e-01 -4.87417236e-01]
 [-6.39166858e-02 -1.27833372e-01 -1.91750057e-01 -2.55666743e-01
  -3.19583429e-01 -3.83500115e-01 -4.47416800e-01]
 [-5.82669081e-02 -1.16533816e-01 -1.74800724e-01 -2.33067633e-01
  -2.91334541e-01 -3.49601449e-01 -4.07868357e-01]
 [-5.26802578e-02 -1.05360516e-01 -1.58040773e-01 -2.10721031e-01
  -2.63401289e-01 -3.16081547e-01 -3.68761805e-01]
 [-4.71553397e-02 -9.43106795e-02 -1.41466019e-01 -1.88621359e-01
  -2.35776699e-01 -2.82932038e-01 -3.30087378e-01]
 [-4.16908045e-02 -8.33816089e-02 -1.25072413e-01 -1.66763218e-01
  -2.08454022e-01 -2.50144827e-01 -2.91835631e-01]
 [-3.62853464e-02 -7.25706928e-02 -1.08856039e-01 -1.45141386e-01
  -1.81426732e-01 -2.17712079e-01 -2.53997425e-01]
 [-3.09377019e-02 -6.18754037e-02 -9.28131056e-02 -1.23750807e-01
  -1.54688509e-01 -1.85626211e-01 -2.16563913e-01]
 [-2.56466472e-02 -5.12932944e-02 -7.69399416e-02 -1.02586589e-01
  -1.28233236e-01 -1.53879883e-01 -1.79526530e-01]
 [-2.04109973e-02 -4.08219945e-02 -6.12329918e-02 -8.16439890e-02
  -1.02054986e-01 -1.22465984e-01 -1.42876981e-01]
 [-1.52296037e-02 -3.04592075e-02 -4.56888112e-02 -6.09184150e-02
  -7.61480187e-02 -9.13776225e-02 -1.06607226e-01]
 [-1.01013537e-02 -2.02027073e-02 -3.03040610e-02 -4.04054146e-02
  -5.05067683e-02 -6.06081220e-02 -7.07094756e-02]
 [-5.02516793e-03 -1.00503359e-02 -1.50755038e-02 -2.01006717e-02
  -2.51258396e-02 -3.01510076e-02 -3.51761755e-02]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]]
```

0.0.18 5.18

In [13]:
```
new_Value_T_plus_1 = np.zeros((100,7))
new_delta_T = np.sum((new_Value_T-new_Value_T_plus_1)**2)
print("The distance metric is", new_delta_T)
```

```
The distance metric is 6262.4138730904815
```

0.0.19 5.19

```
In [14]: new_Value_T_minus_1 = np.array(new_Value_T)
         new_psi_T_minus_1 = np.array(new_psi_T)
         for j in range(7):
             e = varepsilon[j]
             for i in range(100):
                 w = W[i]
                 new_Value_T_minus_1[i,j] = -9e+4
                 for k in range(i):
                     v_value = e*u_mat[i,k] + beta*\
                               sum(Gamma[p]*new_Value_T[k,p] for p in range(7))
                     if v_value > new_Value_T_minus_1[i,j]:
                         new_psi_T_minus_1[i,j] = W[k]
                         new_Value_T_minus_1[i,j] = v_value
         new_delta_T_minus_1 = np.sum((new_Value_T-new_Value_T_minus_1)**2)
         print("The policy function is\n", new_psi_T_minus_1)
         print("The value function is\n", new_Value_T_minus_1)
         print("The distance metric is", new_delta_T_minus_1)
```

```
The policy function is
 [[0.   0.   0.   0.   0.   0.   0.  ]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.02 0.02 0.02]
 [0.04 0.04 0.03 0.03 0.03 0.03 0.03]
 [0.05 0.05 0.04 0.04 0.04 0.03 0.03]
 [0.06 0.05 0.05 0.04 0.04 0.04 0.04]
 [0.07 0.06 0.06 0.05 0.05 0.04 0.04]
 [0.08 0.07 0.06 0.06 0.05 0.05 0.05]
 [0.09 0.08 0.07 0.06 0.06 0.05 0.05]
 [0.1  0.09 0.08 0.07 0.06 0.06 0.06]
 [0.1  0.09 0.08 0.08 0.07 0.07 0.06]
 [0.11 0.1  0.09 0.08 0.08 0.07 0.07]
 [0.12 0.11 0.1  0.09 0.08 0.08 0.07]
 [0.13 0.12 0.11 0.1  0.09 0.08 0.08]
 [0.14 0.13 0.11 0.1  0.09 0.09 0.08]
 [0.15 0.13 0.12 0.11 0.1  0.09 0.09]
 [0.16 0.14 0.13 0.12 0.11 0.1  0.09]
 [0.17 0.15 0.13 0.12 0.11 0.1  0.1 ]
 [0.18 0.16 0.14 0.13 0.12 0.11 0.1 ]
 [0.18 0.16 0.15 0.13 0.12 0.11 0.11]
 [0.19 0.17 0.16 0.14 0.13 0.12 0.11]
 [0.2  0.18 0.16 0.15 0.14 0.13 0.12]
 [0.21 0.19 0.17 0.15 0.14 0.13 0.12]
 [0.22 0.2  0.18 0.16 0.15 0.14 0.13]
 [0.23 0.2  0.18 0.17 0.15 0.14 0.13]
 [0.24 0.21 0.19 0.17 0.16 0.15 0.14]
 [0.25 0.22 0.2  0.18 0.17 0.15 0.14]
```

```
[0.25 0.23 0.2  0.19 0.17 0.16 0.15]
[0.26 0.23 0.21 0.19 0.18 0.16 0.15]
[0.27 0.24 0.22 0.2  0.18 0.17 0.16]
[0.28 0.25 0.23 0.21 0.19 0.17 0.16]
[0.29 0.26 0.23 0.21 0.19 0.18 0.17]
[0.3  0.27 0.24 0.22 0.2  0.19 0.17]
[0.31 0.27 0.25 0.22 0.21 0.19 0.18]
[0.32 0.28 0.25 0.23 0.21 0.2  0.18]
[0.32 0.29 0.26 0.24 0.22 0.2  0.19]
[0.33 0.3  0.27 0.24 0.22 0.21 0.19]
[0.34 0.31 0.28 0.25 0.23 0.21 0.2 ]
[0.35 0.31 0.28 0.26 0.24 0.22 0.2 ]
[0.36 0.32 0.29 0.26 0.24 0.22 0.21]
[0.37 0.33 0.3  0.27 0.25 0.23 0.21]
[0.38 0.34 0.3  0.28 0.25 0.23 0.22]
[0.39 0.34 0.31 0.28 0.26 0.24 0.22]
[0.39 0.35 0.32 0.29 0.27 0.25 0.23]
[0.4  0.36 0.32 0.3  0.27 0.25 0.23]
[0.41 0.37 0.33 0.3  0.28 0.26 0.24]
[0.42 0.38 0.34 0.31 0.28 0.26 0.24]
[0.43 0.38 0.35 0.31 0.29 0.27 0.25]
[0.44 0.39 0.35 0.32 0.3  0.27 0.25]
[0.45 0.4  0.36 0.33 0.3  0.28 0.26]
[0.46 0.41 0.37 0.33 0.31 0.28 0.26]
[0.47 0.41 0.37 0.34 0.31 0.29 0.27]
[0.47 0.42 0.38 0.35 0.32 0.29 0.27]
[0.48 0.43 0.39 0.35 0.32 0.3  0.28]
[0.49 0.44 0.4  0.36 0.33 0.31 0.28]
[0.5  0.45 0.4  0.37 0.34 0.31 0.29]
[0.51 0.45 0.41 0.37 0.34 0.32 0.29]
[0.52 0.46 0.42 0.38 0.35 0.32 0.3 ]
[0.53 0.47 0.42 0.39 0.35 0.33 0.3 ]
[0.54 0.48 0.43 0.39 0.36 0.33 0.31]
[0.54 0.49 0.44 0.4  0.37 0.34 0.31]
[0.55 0.49 0.44 0.4  0.37 0.34 0.32]
[0.56 0.5  0.45 0.41 0.38 0.35 0.32]
[0.57 0.51 0.46 0.42 0.38 0.35 0.33]
[0.58 0.52 0.47 0.42 0.39 0.36 0.33]
[0.59 0.52 0.47 0.43 0.4  0.37 0.34]
[0.6  0.53 0.48 0.44 0.4  0.37 0.34]
[0.61 0.54 0.49 0.44 0.41 0.38 0.35]
[0.61 0.55 0.49 0.45 0.41 0.38 0.35]
[0.62 0.56 0.5  0.46 0.42 0.39 0.36]
[0.63 0.56 0.51 0.46 0.42 0.39 0.37]
[0.64 0.57 0.52 0.47 0.43 0.4  0.37]
[0.65 0.58 0.52 0.48 0.44 0.4  0.38]
[0.66 0.59 0.53 0.48 0.44 0.41 0.38]
[0.67 0.59 0.54 0.49 0.45 0.41 0.39]
```

```
 [0.68 0.6  0.54 0.49 0.45 0.42 0.39]
 [0.68 0.61 0.55 0.5  0.46 0.43 0.4 ]
 [0.69 0.62 0.56 0.51 0.47 0.43 0.4 ]
 [0.7  0.63 0.56 0.51 0.47 0.44 0.41]
 [0.71 0.63 0.57 0.52 0.48 0.44 0.41]
 [0.72 0.64 0.58 0.53 0.48 0.45 0.42]
 [0.73 0.65 0.59 0.53 0.49 0.45 0.42]
 [0.74 0.66 0.59 0.54 0.5  0.46 0.43]
 [0.75 0.67 0.6  0.55 0.5  0.46 0.43]
 [0.76 0.67 0.61 0.55 0.51 0.47 0.44]
 [0.76 0.68 0.61 0.56 0.51 0.47 0.44]
 [0.77 0.69 0.62 0.57 0.52 0.48 0.45]
 [0.78 0.7  0.63 0.57 0.53 0.49 0.45]
 [0.79 0.7  0.64 0.58 0.53 0.49 0.46]
 [0.8  0.71 0.64 0.58 0.54 0.5  0.46]
 [0.81 0.72 0.65 0.59 0.54 0.5  0.47]
 [0.82 0.73 0.66 0.6  0.55 0.51 0.47]
 [0.83 0.74 0.66 0.6  0.55 0.51 0.48]
 [0.83 0.74 0.67 0.61 0.56 0.52 0.48]
 [0.84 0.75 0.68 0.62 0.57 0.52 0.49]
 [0.85 0.76 0.68 0.62 0.57 0.53 0.49]
 [0.86 0.77 0.69 0.63 0.58 0.53 0.5 ]
 [0.87 0.77 0.7  0.64 0.58 0.54 0.5 ]
 [0.88 0.78 0.71 0.64 0.59 0.55 0.51]]
The value function is
 [[-9.00000000e+04 -9.00000000e+04 -9.00000000e+04 -9.00000000e+04
  -9.00000000e+04 -9.00000000e+04 -9.00000000e+04]
 [-1.88767109e+01 -2.11792960e+01 -2.34818811e+01 -2.57844662e+01
  -2.80870513e+01 -3.03896364e+01 -3.26922215e+01]
 [-1.63820564e+01 -1.86846415e+01 -2.09872266e+01 -2.32898117e+01
  -2.55923968e+01 -2.78949819e+01 -3.01975670e+01]
 [-1.49227771e+01 -1.72253622e+01 -1.95279472e+01 -2.18305323e+01
  -2.38595288e+01 -2.58155403e+01 -2.77715518e+01]
 [-1.38874019e+01 -1.61899870e+01 -1.84882265e+01 -2.04442380e+01
  -2.24002495e+01 -2.43562610e+01 -2.63122725e+01]
 [-1.30843025e+01 -1.53868876e+01 -1.74528513e+01 -1.94088628e+01
  -2.13648743e+01 -2.31398657e+01 -2.48931446e+01]
 [-1.24281225e+01 -1.46937404e+01 -1.66497519e+01 -1.85979326e+01
  -2.03512115e+01 -2.21044905e+01 -2.38577694e+01]
 [-1.18733303e+01 -1.40375605e+01 -1.59935720e+01 -1.77948332e+01
  -1.95481122e+01 -2.12414443e+01 -2.28508822e+01]
 [-1.13927474e+01 -1.34827682e+01 -1.53853743e+01 -1.71386533e+01
  -1.88289070e+01 -2.04383449e+01 -2.20477828e+01]
 [-1.09688432e+01 -1.30021853e+01 -1.48305820e+01 -1.65632891e+01
  -1.81727270e+01 -1.97689142e+01 -2.12667804e+01]
 [-1.05896480e+01 -1.25782811e+01 -1.43499991e+01 -1.60084968e+01
  -1.76148681e+01 -1.91127343e+01 -2.06106004e+01]
 [-1.02430744e+01 -1.21728160e+01 -1.39184760e+01 -1.55279139e+01
```

```
 -1.70600759e+01 -1.85579420e+01 -1.99724750e+01]
[-9.90005063e+00 -1.17936208e+01 -1.34945719e+01 -1.50816268e+01
 -1.65794930e+01 -1.80109773e+01 -1.94176827e+01]
[-9.58689445e+00 -1.14505970e+01 -1.31153766e+01 -1.46577227e+01
 -1.61236891e+01 -1.75303944e+01 -1.88781553e+01]
[-9.29881868e+00 -1.11374408e+01 -1.27723529e+01 -1.42785275e+01
 -1.56997849e+01 -1.70679424e+01 -1.83975724e+01]
[-9.03210219e+00 -1.08493651e+01 -1.24376375e+01 -1.39138843e+01
 -1.53144082e+01 -1.66440382e+01 -1.79302125e+01]
[-8.78379507e+00 -1.05616830e+01 -1.21244814e+01 -1.35708606e+01
 -1.49352130e+01 -1.62434440e+01 -1.75063084e+01]
[-8.55151928e+00 -1.02949665e+01 -1.18364056e+01 -1.32577044e+01
 -1.45921892e+01 -1.58642488e+01 -1.70940677e+01]
[-8.33332970e+00 -1.00466594e+01 -1.15629233e+01 -1.29494030e+01
 -1.42583607e+01 -1.55108997e+01 -1.67148725e+01]
[-8.12761510e+00 -9.81438360e+00 -1.12962068e+01 -1.26613273e+01
 -1.39452046e+01 -1.51678760e+01 -1.63461107e+01]
[-7.92488255e+00 -9.59124005e+00 -1.10478996e+01 -1.23942645e+01
 -1.36507470e+01 -1.48517944e+01 -1.60030870e+01]
[-7.73029323e+00 -9.37305048e+00 -1.08156238e+01 -1.21275480e+01
 -1.33626712e+01 -1.45386382e+01 -1.56695013e+01]
[-7.54568735e+00 -9.16733588e+00 -1.05843978e+01 -1.18792409e+01
 -1.30959547e+01 -1.42505625e+01 -1.53563451e+01]
[-7.37009029e+00 -8.97274656e+00 -1.03662083e+01 -1.16436748e+01
 -1.28325534e+01 -1.39646319e+01 -1.50518053e+01]
[-7.20266356e+00 -8.78814067e+00 -1.01604937e+01 -1.14113990e+01
 -1.25842463e+01 -1.36979154e+01 -1.47637296e+01]
[-7.04268053e+00 -8.60581912e+00 -9.96019657e+00 -1.11932094e+01
 -1.23459709e+01 -1.34368813e+01 -1.44835801e+01]
[-6.88950738e+00 -8.43022206e+00 -9.76560725e+00 -1.09824884e+01
 -1.21136951e+01 -1.31885742e+01 -1.42168636e+01]
[-6.74258797e+00 -8.26279533e+00 -9.58100137e+00 -1.07767738e+01
 -1.18955055e+01 -1.29484461e+01 -1.39574857e+01]
[-6.59874694e+00 -8.10281229e+00 -9.40432681e+00 -1.05821845e+01
 -1.16779770e+01 -1.27161703e+01 -1.37091786e+01]
[-6.45759058e+00 -7.94866161e+00 -9.22872976e+00 -1.03915641e+01
 -1.14722624e+01 -1.24938463e+01 -1.34677035e+01]
[-6.32176217e+00 -7.79548847e+00 -9.06130303e+00 -1.02069582e+01
 -1.12721557e+01 -1.22756568e+01 -1.32354277e+01]
[-6.19087409e+00 -7.64856906e+00 -8.90131999e+00 -1.00313612e+01
 -1.10775664e+01 -1.20686782e+01 -1.30095429e+01]
[-6.06457952e+00 -7.50741271e+00 -8.74327922e+00 -9.85733842e+00
 -1.08922964e+01 -1.18629636e+01 -1.27913533e+01]
[-5.94256697e+00 -7.37158430e+00 -8.59010607e+00 -9.68991169e+00
 -1.07076905e+01 -1.16683742e+01 -1.25791671e+01]
[-5.82455555e+00 -7.23805290e+00 -8.44318666e+00 -9.52982627e+00
 -1.05320935e+01 -1.14747587e+01 -1.23734525e+01]
[-5.71029117e+00 -7.10716482e+00 -8.30022139e+00 -9.36984323e+00
```

 -1.03596113e+01 -1.12901528e+01 -1.21733981e+01]
[-5.59871940e+00 -6.98087026e+00 -8.15906504e+00 -9.21667009e+00
 -1.01921846e+01 -1.11082789e+01 -1.19788088e+01]
[-5.48797141e+00 -6.85885770e+00 -8.02323663e+00 -9.06845414e+00
 -1.00308383e+01 -1.09326819e+01 -1.17895735e+01]
[-5.38052983e+00 -6.74084629e+00 -7.89234854e+00 -8.92153474e+00
 -9.87085524e+00 -1.07612066e+01 -1.16049676e+01]
[-5.27620294e+00 -6.62306325e+00 -7.76183148e+00 -8.78037838e+00
 -9.71768209e+00 -1.05937799e+01 -1.14254411e+01]
[-5.17481523e+00 -6.50879887e+00 -7.63553692e+00 -8.64239264e+00
 -9.56612054e+00 -1.04315782e+01 -1.12498440e+01]
[-5.07620561e+00 -6.39805088e+00 -7.51352436e+00 -8.50656423e+00
 -9.41920113e+00 -1.02715952e+01 -1.10790785e+01]
[-4.98022591e+00 -6.29060930e+00 -7.39346030e+00 -8.37567614e+00
 -9.27630509e+00 -1.01177153e+01 -1.09116517e+01]
[-4.88673946e+00 -6.18524878e+00 -7.27544888e+00 -8.24659910e+00
 -9.13514874e+00 -9.96454217e+00 -1.07488317e+01]
[-4.79557869e+00 -6.08092189e+00 -7.16118450e+00 -8.12030454e+00
 -8.99932033e+00 -9.81762276e+00 -1.05888486e+01]
[-4.70445925e+00 -5.97953418e+00 -7.05002255e+00 -7.99829198e+00
 -8.86415228e+00 -9.67125227e+00 -1.04332675e+01]
[-4.61558990e+00 -5.88092457e+00 -6.93927456e+00 -7.87704274e+00
 -8.73326419e+00 -9.53009591e+00 -1.02800943e+01]
[-4.52886219e+00 -5.78494486e+00 -6.83183297e+00 -7.75903133e+00
 -8.60503096e+00 -9.39053587e+00 -1.01311357e+01]
[-4.44417533e+00 -5.68963468e+00 -6.72750608e+00 -7.64471450e+00
 -8.47873639e+00 -9.25470746e+00 -9.98421627e+00]
[-4.36143546e+00 -5.59614824e+00 -6.62401678e+00 -7.53045012e+00
 -8.35672384e+00 -9.12135217e+00 -9.84133929e+00]
[-4.28055507e+00 -5.50502880e+00 -6.52262907e+00 -7.41970213e+00
 -8.23474843e+00 -8.99046408e+00 -9.70018293e+00]
[-4.20145242e+00 -5.41615945e+00 -6.42401945e+00 -7.31156768e+00
 -8.11673701e+00 -8.86278524e+00 -9.56291044e+00]
[-4.12405104e+00 -5.32914808e+00 -6.32721167e+00 -7.20412610e+00
 -8.00043697e+00 -8.73649068e+00 -9.42708203e+00]
[-4.04697570e+00 -5.24242037e+00 -6.23123197e+00 -7.09979921e+00
 -7.88617259e+00 -8.61402470e+00 -9.29499088e+00]
[-3.97120394e+00 -5.15773350e+00 -6.13774552e+00 -6.99721262e+00
 -7.77504319e+00 -8.49201214e+00 -9.16410279e+00]
[-3.89699459e+00 -5.07499364e+00 -6.04662608e+00 -6.89582491e+00
 -7.66429520e+00 -8.37400073e+00 -9.03681604e+00]
[-3.82428453e+00 -4.99411325e+00 -5.95568915e+00 -6.79721530e+00
 -7.55685361e+00 -8.25633859e+00 -8.91052148e+00]
[-3.75301437e+00 -4.91407054e+00 -5.86681980e+00 -6.69963497e+00
 -7.45045458e+00 -8.14207421e+00 -8.78770186e+00]
[-3.68312818e+00 -4.83496789e+00 -5.78009210e+00 -6.60365527e+00
 -7.34612769e+00 -8.02885322e+00 -8.66568930e+00]
[-3.61457323e+00 -4.75756651e+00 -5.69435447e+00 -6.51016882e+00

-7.24407270e+00 -7.91810523e+00 -8.54703387e+00]
[-3.54729977e+00 -4.68179475e+00 -5.60966761e+00 -6.41712879e+00
-7.14268499e+00 -7.80900230e+00 -8.42902246e+00]
[-3.48053407e+00 -4.60758540e+00 -5.52692774e+00 -6.32600935e+00
-7.04407538e+00 -7.70156072e+00 -8.31425808e+00]
[-3.41449505e+00 -4.53347743e+00 -5.44582691e+00 -6.23710583e+00
-6.94602360e+00 -7.59628676e+00 -8.19999370e+00]
[-3.34964598e+00 -4.46076737e+00 -5.36494653e+00 -6.14823648e+00
-6.85004389e+00 -7.49195987e+00 -8.08887325e+00]
[-3.28594475e+00 -4.38949720e+00 -5.28584388e+00 -6.06150877e+00
-6.75569307e+00 -7.39025521e+00 -7.97812526e+00]
[-3.22335142e+00 -4.31961101e+00 -5.20844249e+00 -5.97638954e+00
-6.66220663e+00 -7.28886750e+00 -7.87042446e+00]
[-3.16182812e+00 -4.25061814e+00 -5.13150255e+00 -5.89170268e+00
-6.57108719e+00 -7.19025789e+00 -7.76298288e+00]
[-3.10133887e+00 -4.18206320e+00 -5.05573079e+00 -5.80896281e+00
-6.48016808e+00 -7.09188842e+00 -7.65849750e+00]
[-3.04184948e+00 -4.11478973e+00 -4.98152144e+00 -5.72731882e+00
-6.39129873e+00 -6.99590872e+00 -7.55417061e+00]
[-2.98295797e+00 -4.04875071e+00 -4.90833620e+00 -5.64643844e+00
-6.30357043e+00 -6.90066262e+00 -7.45271424e+00]
[-2.92443593e+00 -3.98390165e+00 -4.83562613e+00 -5.56733579e+00
-6.21684272e+00 -6.80717618e+00 -7.35132653e+00]
[-2.86685029e+00 -3.91936313e+00 -4.76435597e+00 -5.48889436e+00
-6.13208884e+00 -6.71486120e+00 -7.25271691e+00]
[-2.81017155e+00 -3.85566189e+00 -4.69446978e+00 -5.41149298e+00
-6.04740198e+00 -6.62374176e+00 -7.15411884e+00]
[-2.75437159e+00 -3.79306856e+00 -4.62468976e+00 -5.33572122e+00
-5.96466211e+00 -6.53418287e+00 -7.05813914e+00]
[-2.69942356e+00 -3.73154526e+00 -4.55613481e+00 -5.26024056e+00
-5.88268755e+00 -6.44531352e+00 -6.96224273e+00]
[-2.64530186e+00 -3.67092064e+00 -4.48886135e+00 -5.18603121e+00
-5.80180717e+00 -6.35835091e+00 -6.86875628e+00]
[-2.59198198e+00 -3.61043139e+00 -4.42218370e+00 -5.11329593e+00
-5.72243542e+00 -6.27162321e+00 -6.77541742e+00]
[-2.53930172e+00 -3.55094200e+00 -4.35614468e+00 -5.04058586e+00
-5.64333277e+00 -6.18693634e+00 -6.68429798e+00]
[-2.48676027e+00 -3.49241997e+00 -4.29129562e+00 -4.96931570e+00
-5.56593139e+00 -6.10242371e+00 -6.59338378e+00]
[-2.43497483e+00 -3.43483433e+00 -4.22745620e+00 -4.89913306e+00
-5.48900224e+00 -6.01968384e+00 -6.50451443e+00]
[-2.38392397e+00 -3.37767592e+00 -4.16375496e+00 -4.82924687e+00
-5.41323048e+00 -5.93748692e+00 -6.41590210e+00]
[-2.33358712e+00 -3.32099718e+00 -4.10116163e+00 -4.76069193e+00
-5.33859807e+00 -5.85660654e+00 -6.32917439e+00]
[-2.28394460e+00 -3.26519721e+00 -4.03963833e+00 -4.69288882e+00
-5.26438873e+00 -5.77660179e+00 -6.24275025e+00]
[-2.23497751e+00 -3.21024919e+00 -3.97840534e+00 -4.62561536e+00

```
  -5.19167867e+00 -5.69749914e+00 -6.15806338e+00]
 [-2.18666772e+00 -3.15612748e+00 -3.91791609e+00 -4.55957633e+00
  -5.11920982e+00 -5.61957268e+00 -6.07372195e+00]
 [-2.13899781e+00 -3.10206026e+00 -3.85842670e+00 -4.49399669e+00
  -5.04793966e+00 -5.54217130e+00 -5.99098209e+00]
 [-2.09134272e+00 -3.04874038e+00 -3.79959563e+00 -4.42914762e+00
  -4.97751247e+00 -5.46621788e+00 -5.90862535e+00]
 [-2.04429596e+00 -2.99619893e+00 -3.74107360e+00 -4.36544639e+00
  -4.90762628e+00 -5.39044611e+00 -5.82774496e+00]
 [-1.99785627e+00 -2.94441349e+00 -3.68348796e+00 -4.30194900e+00
  -4.83907133e+00 -5.31623677e+00 -5.74728165e+00]
 [-1.95200819e+00 -2.89312020e+00 -3.62680922e+00 -4.23935567e+00
  -4.77057390e+00 -5.24215893e+00 -5.66817900e+00]
 [-1.90673683e+00 -2.84206933e+00 -3.57019873e+00 -4.17781235e+00
  -4.70330043e+00 -5.16944887e+00 -5.58952400e+00]
 [-1.86202786e+00 -2.79173249e+00 -3.51439876e+00 -4.11628904e+00
  -4.63662981e+00 -5.09715622e+00 -5.51212262e+00]
 [-1.81786748e+00 -2.74208997e+00 -3.45945074e+00 -4.05579979e+00
  -4.57059079e+00 -5.02588605e+00 -5.43519644e+00]
 [-1.77424240e+00 -2.69312288e+00 -3.40489927e+00 -3.99609387e+00
  -4.50565208e+00 -4.95529456e+00 -5.35942468e+00]
 [-1.73073671e+00 -2.64433271e+00 -3.35077757e+00 -3.93660449e+00
  -4.44080301e+00 -4.88540837e+00 -5.28415296e+00]
 [-1.68763409e+00 -2.59602292e+00 -3.29745769e+00 -3.87808245e+00
  -4.37710178e+00 -4.81643981e+00 -5.20994362e+00]
 [-1.64504157e+00 -2.54835301e+00 -3.24482071e+00 -3.82010738e+00
  -4.31380726e+00 -4.74788487e+00 -5.13625669e+00]
 [-1.60294722e+00 -2.50130625e+00 -3.19227926e+00 -3.76252174e+00
  -4.25121393e+00 -4.68046630e+00 -5.06354662e+00]
 [-1.56133953e+00 -2.45478623e+00 -3.14049382e+00 -3.70584300e+00
  -4.18948240e+00 -4.61319284e+00 -4.99137912e+00]
 [-1.52020736e+00 -2.40834655e+00 -3.08944296e+00 -3.64950124e+00
  -4.12795909e+00 -4.54715381e+00 -4.92010896e+00]]
The distance metric is 56688435504.040054
```

0.0.20 5.20

```
In [15]: new_Value_T_minus_2 = np.array(new_Value_T_minus_1)
         new_psi_T_minus_2 = np.array(new_psi_T_minus_1)
         for j in range(7):
             e = varepsilon[j]
             for i in range(100):
                 w = W[i]
                 new_Value_T_minus_2[i,j] = -9e+4
                 for k in range(i):
                     v_value = e*u_mat[i,k] + beta*\
                               sum(Gamma[p]*new_Value_T_minus_1[k,p] for p in range(7))
```

```
                    if v_value > new_Value_T_minus_2[i,j]:
                        new_psi_T_minus_2[i,j] = W[k]
                        new_Value_T_minus_2[i,j] = v_value
        new_delta_T_minus_2 = np.sum((new_Value_T_minus_1-new_Value_T_minus_2)**2)
        print("The policy function is\n", new_psi_T_minus_2)
        print("The value function is\n", new_Value_T_minus_2)
        print("The distance metric is", new_delta_T_minus_2)
```

```
The policy function is
 [[0.   0.   0.   0.   0.   0.   0.  ]
 [0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.04 0.04 0.04 0.04 0.04 0.04 0.04]
 [0.05 0.05 0.05 0.05 0.05 0.05 0.04]
 [0.06 0.06 0.06 0.06 0.06 0.05 0.05]
 [0.07 0.07 0.07 0.07 0.06 0.06 0.06]
 [0.08 0.08 0.08 0.07 0.07 0.07 0.07]
 [0.09 0.09 0.09 0.08 0.08 0.08 0.07]
 [0.1  0.1  0.1  0.09 0.09 0.08 0.08]
 [0.11 0.11 0.1  0.1  0.1  0.09 0.09]
 [0.12 0.12 0.11 0.11 0.1  0.1  0.1 ]
 [0.13 0.13 0.12 0.12 0.11 0.11 0.1 ]
 [0.14 0.14 0.13 0.13 0.12 0.12 0.11]
 [0.15 0.14 0.14 0.13 0.13 0.12 0.12]
 [0.16 0.15 0.15 0.14 0.14 0.13 0.13]
 [0.17 0.16 0.16 0.15 0.14 0.14 0.13]
 [0.18 0.17 0.17 0.16 0.15 0.15 0.14]
 [0.19 0.18 0.17 0.17 0.16 0.15 0.15]
 [0.2  0.19 0.18 0.17 0.17 0.16 0.16]
 [0.21 0.2  0.19 0.18 0.18 0.17 0.16]
 [0.22 0.21 0.2  0.19 0.18 0.18 0.17]
 [0.23 0.22 0.21 0.2  0.19 0.19 0.18]
 [0.24 0.23 0.22 0.21 0.2  0.19 0.19]
 [0.25 0.24 0.23 0.22 0.21 0.2  0.19]
 [0.26 0.25 0.23 0.23 0.22 0.21 0.2 ]
 [0.27 0.25 0.24 0.23 0.22 0.22 0.21]
 [0.28 0.26 0.25 0.24 0.23 0.22 0.22]
 [0.29 0.27 0.26 0.25 0.24 0.23 0.22]
 [0.29 0.28 0.27 0.26 0.25 0.24 0.23]
 [0.3  0.29 0.28 0.27 0.26 0.25 0.24]
 [0.31 0.3  0.29 0.28 0.26 0.25 0.24]
 [0.32 0.31 0.3  0.28 0.27 0.26 0.25]
 [0.33 0.32 0.3  0.29 0.28 0.27 0.26]
 [0.34 0.33 0.31 0.3  0.29 0.28 0.27]
 [0.35 0.34 0.32 0.31 0.3  0.28 0.27]
 [0.36 0.35 0.33 0.32 0.3  0.29 0.28]
 [0.37 0.35 0.34 0.33 0.31 0.3  0.29]
```

```
[0.38 0.36 0.35 0.33 0.32 0.31 0.3 ]
[0.39 0.37 0.36 0.34 0.33 0.32 0.3 ]
[0.4  0.38 0.37 0.35 0.34 0.32 0.31]
[0.41 0.39 0.37 0.36 0.34 0.33 0.32]
[0.42 0.4  0.38 0.37 0.35 0.34 0.33]
[0.43 0.41 0.39 0.38 0.36 0.35 0.33]
[0.44 0.42 0.4  0.38 0.37 0.35 0.34]
[0.45 0.43 0.41 0.39 0.38 0.36 0.35]
[0.46 0.44 0.42 0.4  0.38 0.37 0.36]
[0.47 0.45 0.43 0.41 0.39 0.38 0.36]
[0.48 0.45 0.44 0.42 0.4  0.39 0.37]
[0.49 0.46 0.44 0.43 0.41 0.39 0.38]
[0.5  0.47 0.45 0.43 0.42 0.4  0.39]
[0.5  0.48 0.46 0.44 0.42 0.41 0.39]
[0.51 0.49 0.47 0.45 0.43 0.42 0.4 ]
[0.52 0.5  0.48 0.46 0.44 0.42 0.41]
[0.53 0.51 0.49 0.47 0.45 0.43 0.42]
[0.54 0.52 0.5  0.48 0.46 0.44 0.42]
[0.55 0.53 0.5  0.48 0.47 0.45 0.43]
[0.56 0.54 0.51 0.49 0.47 0.45 0.44]
[0.57 0.55 0.52 0.5  0.48 0.46 0.45]
[0.58 0.55 0.53 0.51 0.49 0.47 0.45]
[0.59 0.56 0.54 0.52 0.5  0.48 0.46]
[0.6  0.57 0.55 0.53 0.51 0.48 0.47]
[0.61 0.58 0.56 0.53 0.51 0.49 0.47]
[0.62 0.59 0.57 0.54 0.52 0.5  0.48]
[0.63 0.6  0.57 0.55 0.53 0.51 0.49]
[0.64 0.61 0.58 0.56 0.54 0.52 0.5 ]
[0.65 0.62 0.59 0.57 0.54 0.52 0.51]
[0.66 0.63 0.6  0.58 0.55 0.53 0.51]
[0.67 0.64 0.61 0.58 0.56 0.54 0.52]
[0.68 0.65 0.62 0.59 0.57 0.55 0.53]
[0.69 0.65 0.63 0.6  0.58 0.55 0.53]
[0.7  0.66 0.64 0.61 0.58 0.56 0.54]
[0.7  0.67 0.64 0.62 0.59 0.57 0.55]
[0.71 0.68 0.65 0.63 0.6  0.58 0.56]
[0.72 0.69 0.66 0.63 0.61 0.59 0.56]
[0.73 0.7  0.67 0.64 0.62 0.59 0.57]
[0.74 0.71 0.68 0.65 0.62 0.6  0.58]
[0.75 0.72 0.69 0.66 0.63 0.61 0.59]
[0.76 0.73 0.7  0.67 0.64 0.62 0.59]
[0.77 0.74 0.7  0.68 0.65 0.62 0.6 ]
[0.78 0.75 0.71 0.68 0.66 0.63 0.61]
[0.79 0.75 0.72 0.69 0.67 0.64 0.62]
[0.8  0.76 0.73 0.7  0.67 0.65 0.62]
[0.81 0.77 0.74 0.71 0.68 0.65 0.63]
[0.82 0.78 0.75 0.72 0.69 0.66 0.64]
[0.83 0.79 0.76 0.73 0.7  0.67 0.65]
```

```
 [0.84 0.8  0.77 0.73 0.71 0.68 0.65]
 [0.85 0.81 0.78 0.74 0.71 0.69 0.66]
 [0.86 0.82 0.78 0.75 0.72 0.69 0.67]
 [0.87 0.83 0.79 0.76 0.73 0.7  0.68]
 [0.88 0.84 0.8  0.77 0.74 0.71 0.68]
 [0.89 0.85 0.81 0.78 0.75 0.72 0.69]
 [0.9  0.86 0.82 0.78 0.75 0.72 0.7 ]
 [0.9  0.86 0.83 0.79 0.76 0.73 0.71]
 [0.91 0.87 0.84 0.8  0.77 0.74 0.71]
 [0.92 0.88 0.84 0.81 0.78 0.75 0.72]
 [0.93 0.89 0.85 0.82 0.78 0.75 0.73]
 [0.94 0.9  0.86 0.83 0.79 0.76 0.73]
 [0.95 0.91 0.87 0.83 0.8  0.77 0.74]]
The value function is
 [[-9.00000000e+04 -9.00000000e+04 -9.00000000e+04 -9.00000000e+04
  -9.00000000e+04 -9.00000000e+04 -9.00000000e+04]
 [-9.00000000e+04 -9.00000000e+04 -9.00000000e+04 -9.00000000e+04
  -9.00000000e+04 -9.00000000e+04 -9.00000000e+04]
 [-4.87020633e+01 -5.10046483e+01 -5.33072334e+01 -5.56098185e+01
  -5.79124036e+01 -6.02149887e+01 -6.25175738e+01]
 [-4.42129004e+01 -4.65154855e+01 -4.88180706e+01 -5.11206557e+01
  -5.34232408e+01 -5.57258259e+01 -5.80284110e+01]
 [-4.13997661e+01 -4.37023512e+01 -4.60049362e+01 -4.83075213e+01
  -5.06101064e+01 -5.29126915e+01 -5.52152766e+01]
 [-3.90535002e+01 -4.13560853e+01 -4.36586704e+01 -4.59612555e+01
  -4.82638406e+01 -5.05664257e+01 -5.27892615e+01]
 [-3.71941021e+01 -3.94966872e+01 -4.17992723e+01 -4.41018574e+01
  -4.64044425e+01 -4.84869841e+01 -5.04429956e+01]
 [-3.56390121e+01 -3.79415972e+01 -4.02441822e+01 -4.25467673e+01
  -4.46715745e+01 -4.66275860e+01 -4.85835975e+01]
 [-3.42666222e+01 -3.65692073e+01 -3.88717924e+01 -4.11604730e+01
  -4.31164845e+01 -4.50724960e+01 -4.70285075e+01]
 [-3.30750699e+01 -3.53776550e+01 -3.76802401e+01 -3.97880832e+01
  -4.17440947e+01 -4.37001062e+01 -4.56093796e+01]
 [-3.20130929e+01 -3.43156780e+01 -3.66182631e+01 -3.85965308e+01
  -4.05525423e+01 -4.24837108e+01 -4.42369898e+01]
 [-3.10491821e+01 -3.33517672e+01 -3.55785423e+01 -3.75345538e+01
  -3.94905653e+01 -4.12921585e+01 -4.30454374e+01]
 [-3.01733137e+01 -3.24758988e+01 -3.46146315e+01 -3.65706430e+01
  -3.84769025e+01 -4.02301815e+01 -4.19834604e+01]
 [-2.93617199e+01 -3.16643050e+01 -3.37387631e+01 -3.56947746e+01
  -3.75129917e+01 -3.92662707e+01 -4.09765732e+01]
 [-2.86067910e+01 -3.09093761e+01 -3.29271693e+01 -3.48831808e+01
  -3.66371233e+01 -3.83904023e+01 -4.00126624e+01]
 [-2.79189507e+01 -3.02162289e+01 -3.21722405e+01 -3.40722506e+01
  -3.58255295e+01 -3.75273561e+01 -3.91367940e+01]
 [-2.72684189e+01 -2.95283886e+01 -3.14844001e+01 -3.33173217e+01
  -3.50706007e+01 -3.67157623e+01 -3.83252002e+01]
```

```
[-2.66482914e+01 -2.88778568e+01 -3.08338683e+01 -3.26294814e+01
 -3.43513955e+01 -3.59608334e+01 -3.75441978e+01]
[-2.60806278e+01 -2.82577293e+01 -3.02137408e+01 -3.19789496e+01
 -3.36635552e+01 -3.52729931e+01 -3.67892689e+01]
[-2.55314885e+01 -2.76900657e+01 -2.96055431e+01 -3.13588221e+01
 -3.30130234e+01 -3.46035624e+01 -3.61014285e+01]
[-2.50115679e+01 -2.71409264e+01 -2.90378796e+01 -3.07834579e+01
 -3.23928958e+01 -3.39530306e+01 -3.54508968e+01]
[-2.45266338e+01 -2.66210058e+01 -2.84887402e+01 -3.02157944e+01
 -3.18252323e+01 -3.33329031e+01 -3.48127713e+01]
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 -3.12673734e+01 -3.27652395e+01 -3.41926438e+01]
[-2.36054713e+01 -2.56620543e+01 -2.74838856e+01 -2.91467344e+01
 -3.07182341e+01 -3.22161002e+01 -3.36249802e+01]
[-2.31760277e+01 -2.52149092e+01 -2.70098682e+01 -2.86618004e+01
 -3.01983134e+01 -3.16691355e+01 -3.30758409e+01]
[-2.27639886e+01 -2.47854656e+01 -2.65627231e+01 -2.81877830e+01
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[-2.23696903e+01 -2.43734265e+01 -2.61312000e+01 -2.77406379e+01
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[-2.19879063e+01 -2.39679614e+01 -2.57017564e+01 -2.72943508e+01
 -2.87835581e+01 -3.01902635e+01 -3.15314589e+01]
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[-2.12692207e+01 -2.31918791e+01 -2.48954190e+01 -2.64528680e+01
 -2.79069694e+01 -2.92806663e+01 -3.05900816e+01]
[-2.09226471e+01 -2.28258709e+01 -2.45136349e+01 -2.60585698e+01
 -2.74949303e+01 -2.88512227e+01 -3.01429365e+01]
[-2.05800445e+01 -2.24731935e+01 -2.41476267e+01 -2.56767857e+01
 -2.71006320e+01 -2.84391836e+01 -2.97134929e+01]
[-2.02483746e+01 -2.21305909e+01 -2.37949493e+01 -2.53107775e+01
 -2.67152553e+01 -2.80385894e+01 -2.93012523e+01]
[-1.99306068e+01 -2.17989210e+01 -2.34523468e+01 -2.49461344e+01
 -2.63334713e+01 -2.76442912e+01 -2.88892131e+01]
[-1.96196554e+01 -2.14811532e+01 -2.31176314e+01 -2.45934570e+01
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[-1.93175309e+01 -2.11702018e+01 -2.27859615e+01 -2.42508545e+01
 -2.56147857e+01 -2.68964989e+01 -2.81131308e+01]
[-1.90277272e+01 -2.08680773e+01 -2.24681937e+01 -2.39191846e+01
 -2.52721831e+01 -2.65431498e+01 -2.77443690e+01]
[-1.87422834e+01 -2.05782736e+01 -2.21572423e+01 -2.36014167e+01
 -2.49383546e+01 -2.61904724e+01 -2.73783608e+01]
[-1.84654965e+01 -2.02905915e+01 -2.18551178e+01 -2.32904653e+01
 -2.46066847e+01 -2.58478698e+01 -2.70256834e+01]
[-1.81983137e+01 -2.00051477e+01 -2.15653141e+01 -2.29821640e+01
 -2.42889169e+01 -2.55161999e+01 -2.66830808e+01]
[-1.79356602e+01 -1.97283609e+01 -2.12798703e+01 -2.26800395e+01
 -2.39779655e+01 -2.51984321e+01 -2.63494952e+01]
```

```
[-1.76807537e+01 -1.94611781e+01 -2.10030835e+01 -2.23902358e+01
 -2.36758410e+01 -2.48823506e+01 -2.60178253e+01]
[-1.74315696e+01 -1.91985245e+01 -2.07296011e+01 -2.21047920e+01
 -2.33813835e+01 -2.45713992e+01 -2.57000575e+01]
[-1.71884396e+01 -1.89436180e+01 -2.04624183e+01 -2.18280051e+01
 -2.30915797e+01 -2.42692747e+01 -2.53891061e+01]
[-1.69524203e+01 -1.86944339e+01 -2.01997648e+01 -2.15608223e+01
 -2.28061359e+01 -2.39794710e+01 -2.50845663e+01]
[-1.67196809e+01 -1.84513039e+01 -1.99448583e+01 -2.12937595e+01
 -2.25293491e+01 -2.36935405e+01 -2.47824418e+01]
[-1.64936415e+01 -1.82152847e+01 -1.96956742e+01 -2.10311060e+01
 -2.22621662e+01 -2.34080967e+01 -2.44926381e+01]
[-1.62726903e+01 -1.79825452e+01 -1.94525442e+01 -2.07761995e+01
 -2.19987650e+01 -2.31313098e+01 -2.42071943e+01]
[-1.60551456e+01 -1.77565058e+01 -1.92165249e+01 -2.05270154e+01
 -2.17361114e+01 -2.28641270e+01 -2.39270448e+01]
[-1.58429837e+01 -1.75333623e+01 -1.89837855e+01 -2.02838854e+01
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[-1.56361362e+01 -1.73124111e+01 -1.87525594e+01 -2.00478661e+01
 -2.12320208e+01 -2.23404393e+01 -2.33830751e+01]
[-1.54320215e+01 -1.70948663e+01 -1.85265201e+01 -1.98123001e+01
 -2.09888908e+01 -2.20855328e+01 -2.31204216e+01]
[-1.52292889e+01 -1.68827045e+01 -1.83055689e+01 -1.95795606e+01
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[-1.50291293e+01 -1.66758570e+01 -1.80880241e+01 -1.93535212e+01
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[-1.48343586e+01 -1.64717423e+01 -1.78758623e+01 -1.91325700e+01
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[-1.46419558e+01 -1.62715826e+01 -1.76690147e+01 -1.89150253e+01
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[-1.44534690e+01 -1.60768119e+01 -1.74649001e+01 -1.87028634e+01
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[-1.42689457e+01 -1.58844091e+01 -1.72646030e+01 -1.84921424e+01
 -1.96173213e+01 -2.06582925e+01 -2.16363288e+01]
[-1.40870996e+01 -1.56959223e+01 -1.70644433e+01 -1.82852949e+01
 -1.93997929e+01 -2.04359686e+01 -2.14035893e+01]
[-1.39090025e+01 -1.55113990e+01 -1.68696726e+01 -1.80811802e+01
 -1.91876310e+01 -2.02150174e+01 -2.11775500e+01]
[-1.37334501e+01 -1.53290774e+01 -1.66772698e+01 -1.78810205e+01
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[-1.35610925e+01 -1.51472313e+01 -1.64887830e+01 -1.76862498e+01
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[-1.33923892e+01 -1.49691343e+01 -1.63042597e+01 -1.74938470e+01
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[-1.32250883e+01 -1.47935818e+01 -1.61224136e+01 -1.73032267e+01
 -1.83764024e+01 -1.93714846e+01 -2.03009830e+01]
[-1.30615679e+01 -1.46212243e+01 -1.59443165e+01 -1.71147399e+01
 -1.81816317e+01 -1.91673699e+01 -2.00888212e+01]
```

```
[-1.29008259e+01 -1.44525210e+01 -1.57676420e+01 -1.69302165e+01
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[-1.27413693e+01 -1.42852201e+01 -1.55920895e+01 -1.67483704e+01
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[-1.25853413e+01 -1.41216997e+01 -1.54197320e+01 -1.65702734e+01
 -1.76154722e+01 -1.85788240e+01 -1.94776993e+01]
[-1.24317843e+01 -1.39609576e+01 -1.52510287e+01 -1.63947209e+01
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[-1.22801370e+01 -1.38015011e+01 -1.50837278e+01 -1.62206982e+01
 -1.72491028e+01 -1.81979344e+01 -1.90828741e+01]
[-1.21305817e+01 -1.36454731e+01 -1.49202074e+01 -1.60483406e+01
 -1.70710057e+01 -1.80134111e+01 -1.88904713e+01]
[-1.19837526e+01 -1.34913224e+01 -1.47594653e+01 -1.58796373e+01
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[-1.18386338e+01 -1.33377653e+01 -1.46000088e+01 -1.57123365e+01
 -1.67229711e+01 -1.76496911e+01 -1.85127493e+01]
[-1.16947928e+01 -1.31861181e+01 -1.44419680e+01 -1.55488160e+01
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[-1.15518835e+01 -1.30365628e+01 -1.42859400e+01 -1.53880740e+01
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[-1.12719193e+01 -1.27446149e+01 -1.39807357e+01 -1.50685320e+01
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[-1.11350807e+01 -1.26017056e+01 -1.38311804e+01 -1.49125040e+01
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[-1.09994966e+01 -1.24609316e+01 -1.36843513e+01 -1.47589470e+01
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[-1.08661585e+01 -1.23217414e+01 -1.35392325e+01 -1.46072997e+01
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[-1.07348391e+01 -1.21849028e+01 -1.33962672e+01 -1.44577444e+01
 -1.54135160e+01 -1.62904826e+01 -1.71013774e+01]
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 -1.51083117e+01 -1.59702840e+01 -1.67705561e+01]
[-1.03496022e+01 -1.17824493e+01 -1.29733937e+01 -1.40175806e+01
 -1.49567502e+01 -1.58142560e+01 -1.66077360e+01]
[-1.02240129e+01 -1.16511299e+01 -1.28365552e+01 -1.38746713e+01
 -1.48071948e+01 -1.56603761e+01 -1.64469940e+01]
[-1.01005537e+01 -1.15205423e+01 -1.27009710e+01 -1.37338973e+01
 -1.46603658e+01 -1.55068190e+01 -1.62875374e+01]
[-9.97846307e+00 -1.13923616e+01 -1.25676330e+01 -1.35947071e+01
 -1.45152470e+01 -1.53551718e+01 -1.61315094e+01]
[-9.85772198e+00 -1.12658930e+01 -1.24363136e+01 -1.34567213e+01
 -1.43723377e+01 -1.52056165e+01 -1.59759283e+01]
[-9.73849894e+00 -1.11403037e+01 -1.23057260e+01 -1.33198828e+01
 -1.42294416e+01 -1.50587874e+01 -1.58223712e+01]
```

```
 [-9.62063755e+00 -1.10168444e+01 -1.21752089e+01 -1.31842986e+01
  -1.40886677e+01 -1.49124169e+01 -1.56707240e+01]
 [-9.50417119e+00 -1.08947538e+01 -1.20470283e+01 -1.30509606e+01
  -1.39494774e+01 -1.47672981e+01 -1.55211686e+01]
 [-9.38900925e+00 -1.07740127e+01 -1.19205596e+01 -1.29196412e+01
  -1.38126389e+01 -1.46243888e+01 -1.53722100e+01]
 [-9.27521484e+00 -1.06547897e+01 -1.17949703e+01 -1.27890536e+01
  -1.36770547e+01 -1.44836149e+01 -1.52253809e+01]
 [-9.16256586e+00 -1.05369283e+01 -1.16715111e+01 -1.26599766e+01
  -1.35418867e+01 -1.43440548e+01 -1.50802621e+01]
 [-9.05099408e+00 -1.04191452e+01 -1.15494205e+01 -1.25317959e+01
  -1.34085486e+01 -1.42048646e+01 -1.49373528e+01]
 [-8.93982659e+00 -1.03026789e+01 -1.14286794e+01 -1.24053272e+01
  -1.32772292e+01 -1.40680260e+01 -1.47944758e+01]
 [-8.82941351e+00 -1.01875169e+01 -1.13086153e+01 -1.22797379e+01
  -1.31466416e+01 -1.39324419e+01 -1.46537019e+01]
 [-8.72056413e+00 -1.00737225e+01 -1.11893923e+01 -1.21562787e+01
  -1.30184084e+01 -1.37990866e+01 -1.45145116e+01]
 [-8.61307430e+00 -9.96107355e+00 -1.10715309e+01 -1.20341881e+01
  -1.28902277e+01 -1.36657485e+01 -1.43772391e+01]
 [-8.50605687e+00 -9.84990606e+00 -1.09550645e+01 -1.19129388e+01
  -1.27637591e+01 -1.35344291e+01 -1.42404006e+01]]
The distance metric is 56667671972.15527
```

0.0.21 5.21

```
In [16]: def new_optimize(init=W, E = varepsilon, P = Gamma, u = lambda x: np.log(x),
                          beta = 0.9, error = 1e-9, maxiter = 1000):
             W = init
             Gamma = P
             varepsilon = E
             V = np.zeros((W.size, Gamma.size))
             for i in range(Gamma.size):
                 V[:,i] = varepsilon[i]*np.log(W)
             c_mat = W.reshape(-1,1)-W
             c_mat[c_mat<=0] = 1e-7
             u_mat = u(c_mat)
             Error = 1
             count = 0
             while Error>error and count <= maxiter:
                 new_W = np.tile(W.reshape(-1,1),(1,7))
                 new_V = np.array(V)
                 for j in range(7):
                     e = varepsilon[j]
                     for i in range(100):
                         w = W[i]
                         new_V[i,j] = -np.inf
```

```
                        for k in range(i):
                            v_value = e*u_mat[i,k] + beta*\
                                      sum(Gamma[p]*V[k,p] for p in range(7))
                            if v_value > new_V[i,j]:
                                new_W[i,j] = W[k]
                                new_V[i,j] = v_value
                Error = ((V-new_V)**2).sum()
                V = new_V
            return new_V,new_W

        V, new_W = new_optimize()
        print("The policy function is\n", new_W)
        print("The value function is\n", V)
```

```
The policy function is
 [[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.04 0.04 0.04 0.04 0.04 0.04 0.04]
 [0.05 0.05 0.05 0.05 0.05 0.05 0.04]
 [0.06 0.06 0.06 0.06 0.06 0.05 0.05]
 [0.07 0.07 0.07 0.07 0.06 0.06 0.06]
 [0.08 0.08 0.08 0.07 0.07 0.07 0.07]
 [0.09 0.09 0.09 0.08 0.08 0.08 0.07]
 [0.1  0.1  0.1  0.09 0.09 0.08 0.08]
 [0.11 0.11 0.1  0.1  0.1  0.09 0.09]
 [0.12 0.12 0.11 0.11 0.1  0.1  0.1 ]
 [0.13 0.13 0.12 0.12 0.11 0.11 0.1 ]
 [0.14 0.14 0.13 0.13 0.12 0.12 0.11]
 [0.15 0.14 0.14 0.13 0.13 0.12 0.12]
 [0.16 0.15 0.15 0.14 0.14 0.13 0.13]
 [0.17 0.16 0.16 0.15 0.14 0.14 0.13]
 [0.18 0.17 0.17 0.16 0.15 0.15 0.14]
 [0.19 0.18 0.17 0.17 0.16 0.15 0.15]
 [0.2  0.19 0.18 0.17 0.17 0.16 0.16]
 [0.21 0.2  0.19 0.18 0.18 0.17 0.16]
 [0.22 0.21 0.2  0.19 0.18 0.18 0.17]
 [0.23 0.22 0.21 0.2  0.19 0.19 0.18]
 [0.24 0.23 0.22 0.21 0.2  0.19 0.19]
 [0.25 0.24 0.23 0.22 0.21 0.2  0.19]
 [0.26 0.25 0.23 0.23 0.22 0.21 0.2 ]
 [0.27 0.25 0.24 0.23 0.22 0.22 0.21]
 [0.28 0.26 0.25 0.24 0.23 0.22 0.22]
 [0.29 0.27 0.26 0.25 0.24 0.23 0.22]
 [0.29 0.28 0.27 0.26 0.25 0.24 0.23]
 [0.3  0.29 0.28 0.27 0.26 0.25 0.24]
 [0.31 0.3  0.29 0.28 0.26 0.25 0.24]
```

```
[0.32 0.31 0.3  0.28 0.27 0.26 0.25]
[0.33 0.32 0.3  0.29 0.28 0.27 0.26]
[0.34 0.33 0.31 0.3  0.29 0.28 0.27]
[0.35 0.34 0.32 0.31 0.3  0.28 0.27]
[0.36 0.35 0.33 0.32 0.3  0.29 0.28]
[0.37 0.35 0.34 0.33 0.31 0.3  0.29]
[0.38 0.36 0.35 0.33 0.32 0.31 0.3 ]
[0.39 0.37 0.36 0.34 0.33 0.32 0.3 ]
[0.4  0.38 0.37 0.35 0.34 0.32 0.31]
[0.41 0.39 0.37 0.36 0.34 0.33 0.32]
[0.42 0.4  0.38 0.37 0.35 0.34 0.33]
[0.43 0.41 0.39 0.38 0.36 0.35 0.33]
[0.44 0.42 0.4  0.38 0.37 0.35 0.34]
[0.45 0.43 0.41 0.39 0.38 0.36 0.35]
[0.46 0.44 0.42 0.4  0.38 0.37 0.36]
[0.47 0.45 0.43 0.41 0.39 0.38 0.36]
[0.48 0.45 0.44 0.42 0.4  0.39 0.37]
[0.49 0.46 0.44 0.43 0.41 0.39 0.38]
[0.5  0.47 0.45 0.43 0.42 0.4  0.39]
[0.5  0.48 0.46 0.44 0.42 0.41 0.39]
[0.51 0.49 0.47 0.45 0.43 0.42 0.4 ]
[0.52 0.5  0.48 0.46 0.44 0.42 0.41]
[0.53 0.51 0.49 0.47 0.45 0.43 0.42]
[0.54 0.52 0.5  0.48 0.46 0.44 0.42]
[0.55 0.53 0.5  0.48 0.47 0.45 0.43]
[0.56 0.54 0.51 0.49 0.47 0.45 0.44]
[0.57 0.55 0.52 0.5  0.48 0.46 0.45]
[0.58 0.55 0.53 0.51 0.49 0.47 0.45]
[0.59 0.56 0.54 0.52 0.5  0.48 0.46]
[0.6  0.57 0.55 0.53 0.51 0.48 0.47]
[0.61 0.58 0.56 0.53 0.51 0.49 0.47]
[0.62 0.59 0.57 0.54 0.52 0.5  0.48]
[0.63 0.6  0.57 0.55 0.53 0.51 0.49]
[0.64 0.61 0.58 0.56 0.54 0.52 0.5 ]
[0.65 0.62 0.59 0.57 0.54 0.52 0.51]
[0.66 0.63 0.6  0.58 0.55 0.53 0.51]
[0.67 0.64 0.61 0.58 0.56 0.54 0.52]
[0.68 0.65 0.62 0.59 0.57 0.55 0.53]
[0.69 0.65 0.63 0.6  0.58 0.55 0.53]
[0.7  0.66 0.64 0.61 0.58 0.56 0.54]
[0.7  0.67 0.64 0.62 0.59 0.57 0.55]
[0.71 0.68 0.65 0.63 0.6  0.58 0.56]
[0.72 0.69 0.66 0.63 0.61 0.59 0.56]
[0.73 0.7  0.67 0.64 0.62 0.59 0.57]
[0.74 0.71 0.68 0.65 0.62 0.6  0.58]
[0.75 0.72 0.69 0.66 0.63 0.61 0.59]
[0.76 0.73 0.7  0.67 0.64 0.62 0.59]
[0.77 0.74 0.7  0.68 0.65 0.62 0.6 ]
```

```
 [0.78 0.75 0.71 0.68 0.66 0.63 0.61]
 [0.79 0.75 0.72 0.69 0.67 0.64 0.62]
 [0.8  0.76 0.73 0.7  0.67 0.65 0.62]
 [0.81 0.77 0.74 0.71 0.68 0.65 0.63]
 [0.82 0.78 0.75 0.72 0.69 0.66 0.64]
 [0.83 0.79 0.76 0.73 0.7  0.67 0.65]
 [0.84 0.8  0.77 0.73 0.71 0.68 0.65]
 [0.85 0.81 0.78 0.74 0.71 0.69 0.66]
 [0.86 0.82 0.78 0.75 0.72 0.69 0.67]
 [0.87 0.83 0.79 0.76 0.73 0.7  0.68]
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 [0.89 0.85 0.81 0.78 0.75 0.72 0.69]
 [0.9  0.86 0.82 0.78 0.75 0.72 0.7 ]
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C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:27: RuntimeWarning: invalid va
```
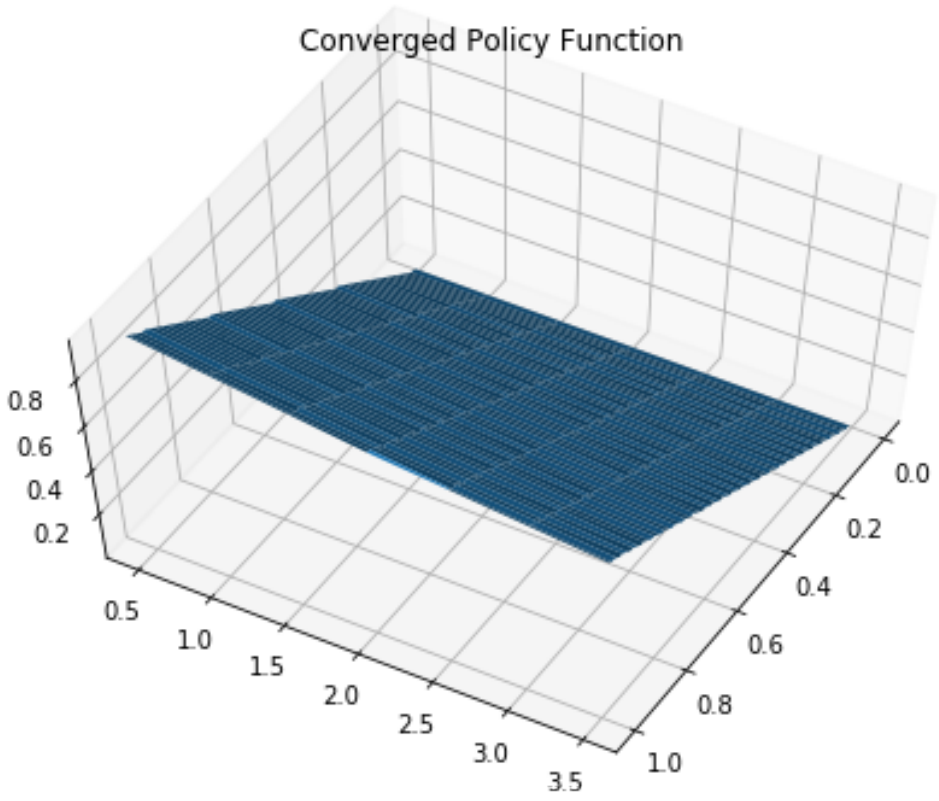

0.0.22 5.22

```
In [17]: X, Y = np.meshgrid(W, varepsilon)
         new_fig = plt.figure(figsize=(8, 6))
         new_ax = new_fig.add_subplot(111, projection='3d')
         new_ax.plot_surface(X.T, Y.T, new_W)
         new_ax.set_title("Converged Policy Function")
         new_ax.view_init(elev=60,azim=30)
         plt.show()
```


Converged Policy Function
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0.6
0.4
0.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0.0
0.2
0.4
0.6
0.8
1.0