hw2 Problem1

January 18, 2019

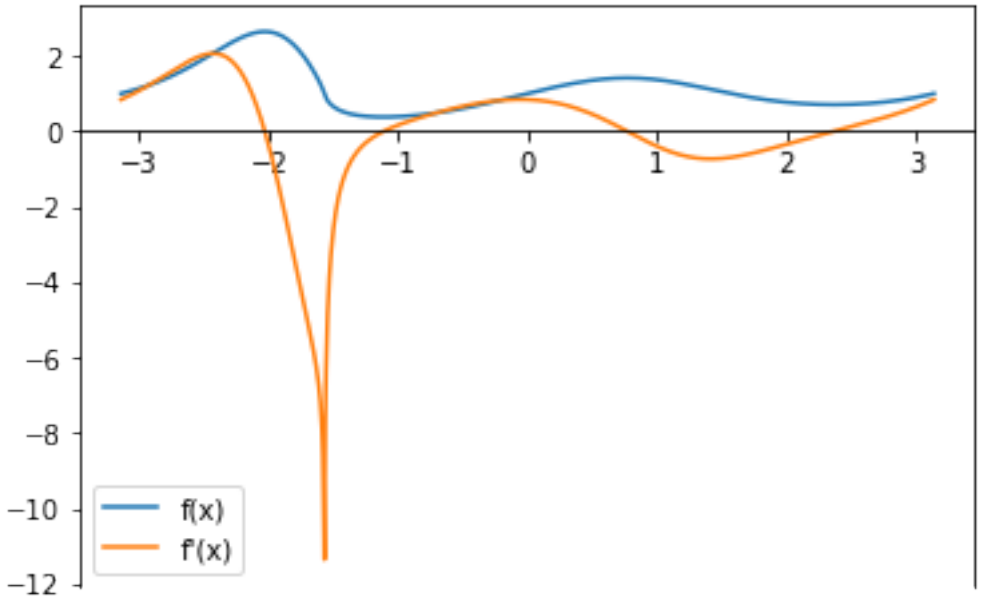
0.0.1 Import Packages

```
In [1]: import sympy as sp
        import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib import ticker
        import time
        from autograd import numpy as anp
        from autograd import grad
```

0.0.2 Problem 1

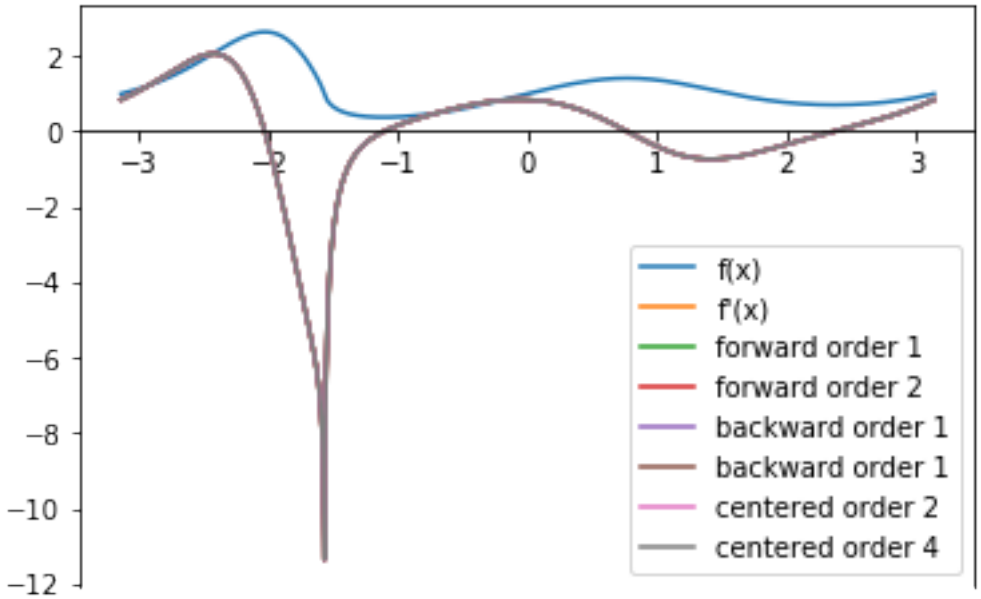
```
In [2]: def f(x):
            return (np.sin(x)+1)**(np.sin(np.cos(x)))
        def f_prime(X):
            def inner():
                x = sp.symbols('x')
                f = (sp.sin(x)+1) ** (sp.sin(sp.cos(x)))
                f_prime = sp.diff(f, x)
                return sp.lambdify(x, f_prime)
            return inner()(X)
        x = np.linspace(-np.pi,np.pi,1000)
        ax = plt.gca()
        ax.spines['bottom'].set_position("zero")
        ax.plot(x,f(x),label='f(x)')
        ax.plot(x,f_prime(x),label='f\'(x)')
        ax.legend()
```

```
Out[2]: <matplotlib.legend.Legend at 0x2a0ee636ba8>
```



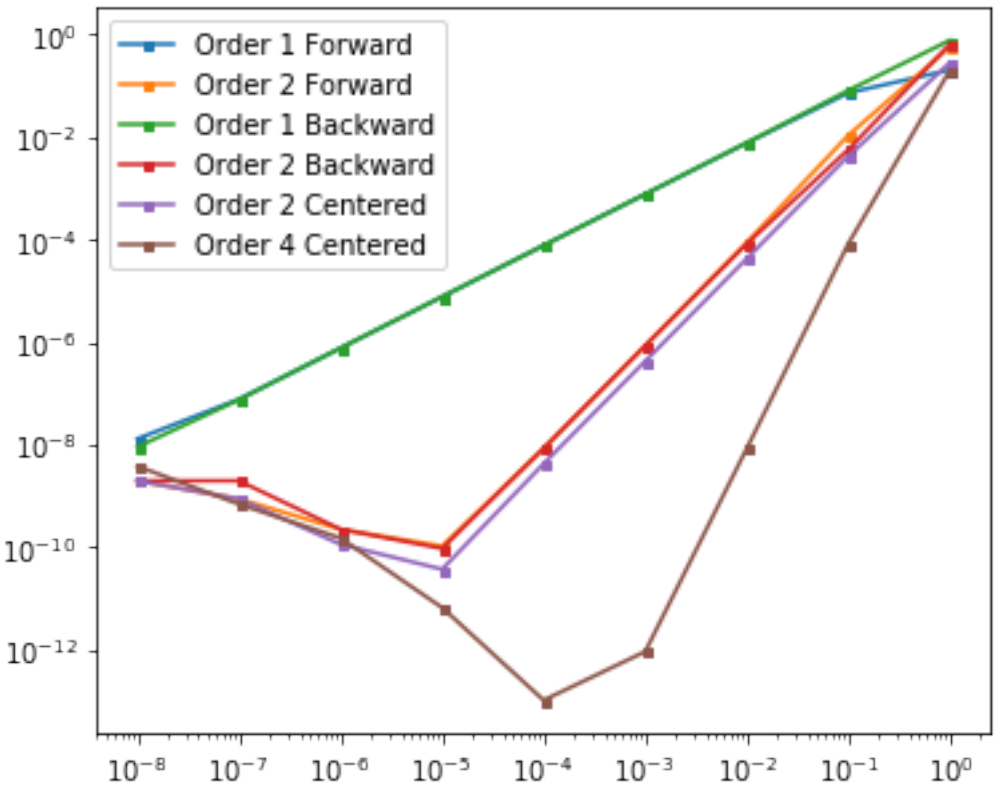
0.0.3 Problem 2

```
In [3]: def forward1(x,h=1e-5):
            return (f(x+h)-f(x))/h
        def forward2(x,h=1e-5):
            return (4*f(x+h)-3*f(x)-f(x+2*h))/(2*h)
        def backward1(x,h=1e-5):
            return (f(x)-f(x-h))/h
        def backward2(x,h=1e-5):
            return (3*f(x)-4*f(x-h)+f(x-2*h))/(2*h)
        def centered2(x,h=1e-5):
            return (f(x+h)-f(x-h))/(2*h)
        def centered4(x,h=1e-5):
            return (f(x-2*h)-8*f(x-h)+8*f(x+h)-f(x+2*h))/(12*h)
        ax2 = plt.gca()
        ax2.spines['bottom'].set_position("zero")
        ax2.plot(x,f(x),label='f(x)')
        ax2.plot(x,f_prime(x),label='f\'(x)')
        ax2.plot(x,forward1(x),label='forward order 1')
        ax2.plot(x,forward2(x),label='forward order 2')
        ax2.plot(x,backward1(x),label='backward order 1')
        ax2.plot(x,backward2(x),label='backward order 1')
        ax2.plot(x,centered2(x),label='centered order 2')
        ax2.plot(x,centered4(x),label='centered order 4')
        ax2.legend()

Out[3]: <matplotlib.legend.Legend at 0x2a0ef935b00>
```



0.0.4 Problem 3

```
In [4]: h = np.logspace(-8,0,9)
        ax3 = plt.figure(figsize=(6,5)).gca()
        ax3.loglog(h,np.abs(forward1(1,h)-f_prime(1)),label='Order 1 Forward',marker='s',ms=3)
        ax3.loglog(h,np.abs(forward2(1,h)-f_prime(1)),label='Order 2 Forward',marker='s',ms=3)
        ax3.loglog(h,np.abs(backward1(1,h)-f_prime(1)),label='Order 1 Backward',marker='s',ms=3
        ax3.loglog(h,np.abs(backward2(1,h)-f_prime(1)),label='Order 2 Backward',marker='s',ms=3
        ax3.loglog(h,np.abs(centered2(1,h)-f_prime(1)),label='Order 2 Centered',marker='s',ms=3
        ax3.loglog(h,np.abs(centered4(1,h)-f_prime(1)),label='Order 4 Centered',marker='s',ms=3
        ax3.legend()

Out[4]: <matplotlib.legend.Legend at 0x2a0ef9ef8d0>
```



0.0.5 Problem 4

```
In [5]: Data = np.load('plane.npy')
        X = np.zeros(8)
        Y = np.zeros(8)
        a = 500
        for i in range(8):
            alpha = np.pi*Data[i,1]/180
            beta = np.pi*Data[i,2]/180
            X[i] = a*np.tan(beta)/(np.tan(beta)-np.tan(alpha))
            Y[i] = a*np.tan(beta)*np.tan(alpha)/(np.tan(beta)-np.tan(alpha))
        X_prime = np.zeros(8)
        Y_prime = np.zeros(8)
        X_prime[0] = X[1]-X[0]
        Y_prime[0] = Y[1]-Y[0]
        for i in range(1,7):
            X_prime[i] = (X[i+1]-X[i-1])/(2*1)
            Y_prime[i] = (Y[i+1]-Y[i-1])/(2*1)
        X_prime[7] = X[7]-X[6]
        Y_prime[7] = Y[7]-Y[6]
        speed = np.sqrt(X_prime**2+Y_prime**2)
```

```
        for i in range(7, 15):
            print("t =", i, "speed =", speed[i-7])
```

```
t = 7 speed = 46.424200622134585
t = 8 speed = 47.00103938095249
t = 9 speed = 48.998805140367324
t = 10 speed = 50.09944162965336
t = 11 speed = 48.290350838205164
t = 12 speed = 51.56455904927243
t = 13 speed = 53.923033545053535
t = 14 speed = 51.51480056963612
```

0.0.6 Problem 5

```
In [6]: def J(f,x,h=1e-5):
            try:
                m = np.array(f(*x)).size
                n = np.array(x).size
                Matrix = np.zeros((m,n))
                for i in range(m):
                    for j in range(n):
                        new_x1 = np.array(x,dtype='float')
                        new_x2 = np.array(x,dtype='float')
                        new_x1[j]+=h
                        new_x2[j]-=h
                        Matrix[i,j] = (f(*new_x1)[i]-f(*new_x2)[i])/(2*h)
            except:
                try:
                    n = np.array(x).size
                    Matrix = np.zeros((1,n))
                    for i in range(n):
                        new_x1 = np.array(x,dtype='float')
                        new_x2 = np.array(x,dtype='float')
                        new_x1[i]+=h
                        new_x2[i]-=h
                        Matrix[1,i] = (f(*new_x1)-f(*new_x2))/(2*h)
                except:
                    try:
                        m = np.array(f(*x)).size
                        Matrix = np.zeros((m,1))
                        for i in range(m):
                            Matrix[i,0] = (f(*(x+h))[i]-f(*(x-h))[i])/(2*h)
                    except:
                        try:
                            Matrix = np.zeros((1,1))
                            Matrix[0,0] = (f(*(x+h))-f(*(x-h)))/(2*h)
                        except:
```

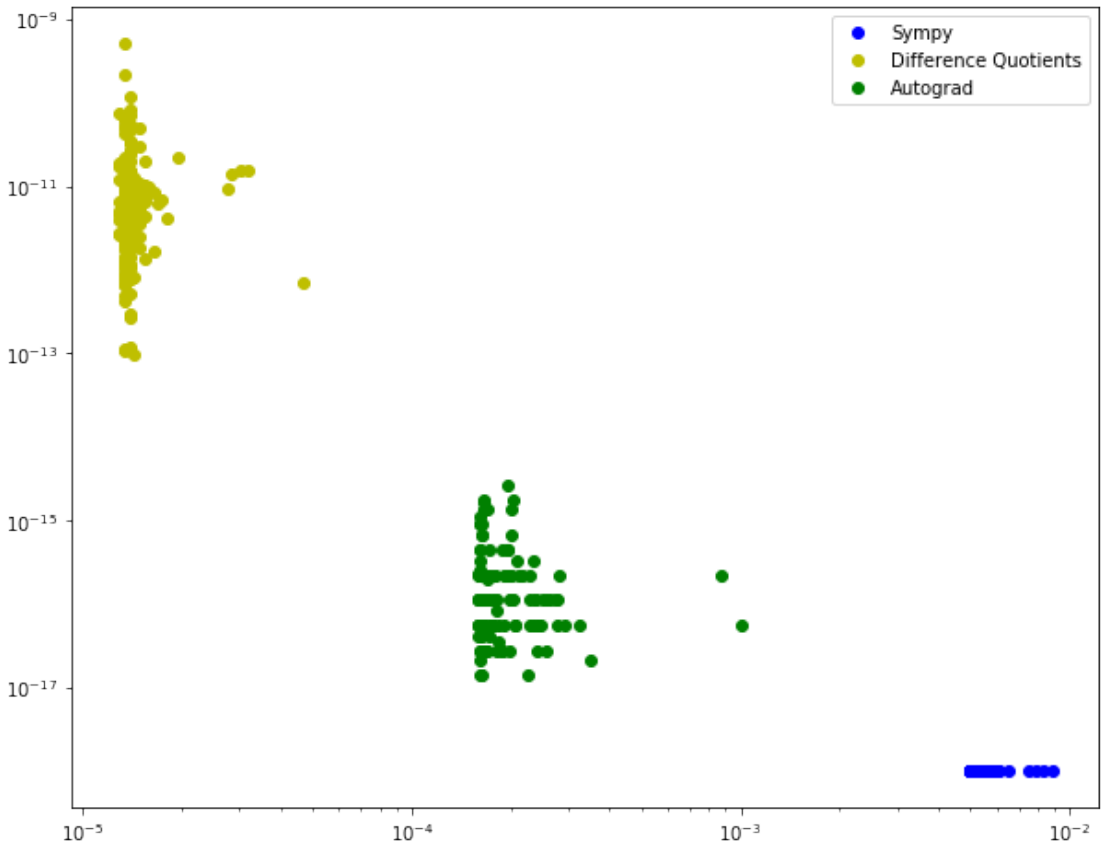
```
                        raise
            return Matrix
        def F(x,y):
            return x**2, x**3-y
        x = (1,2)
        print(J(F,x))
```

```
[[ 2.  0.]
 [ 3. -1.]]
```

0.0.7 Problem 7

```
In [7]: def Timer(N):
            Time1=np.zeros(N,dtype='float')
            Time2=np.zeros(N,dtype='float')
            Time3=np.zeros(N,dtype='float')
            Error1=1e-18*np.ones(N,dtype='float')
            Error2=np.zeros(N,dtype='float')
            Error3=np.zeros(N,dtype='float')
            g = lambda x: (anp.sin(x)+1)**(anp.sin(anp.cos(x)))
            dg = grad(g)
            for i in range(N):
                x = np.random.uniform(low=-np.pi, high=np.pi)
                time11 = time.clock()
                result1 = f_prime(x)
                time12 = time.clock()
                Time1[i] = time12-time11
                time21 = time.clock()
                result2 = centered4(x)
                time22 = time.clock()
                Time2[i] = time22-time21
                Error2[i] = abs(result2-result1)
                time31 = time.clock()
                result3 = dg(x)
                time32 = time.clock()
                Time3[i] = time32-time31
                Error3[i] = abs(result3-result1)
            return Time1,Time2,Time3,Error1,Error2,Error3
        Time1,Time2,Time3,Error1,Error2,Error3 = Timer(200)
        ax5 = plt.figure(figsize=(10,8)).gca()
        ax5.loglog(Time1,Error1,'ro',label='Sympy',color='b')
        ax5.loglog(Time2,Error2,'ro',label='Difference Quotients',color='y')
        ax5.loglog(Time3,Error3,'ro',label='Autograd',color='g')
        ax5.legend()
```

```
Out[7]: <matplotlib.legend.Legend at 0x2a0efe8e3c8>
```


Sympy
Difference Quotients
Autograd
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-5}
10^{-4}
10^{-3}
10^{-2}