Self Segregation caused by homophily preference

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May 2019

Abstract

Based on Schelling's Segregation model, people's preference for neighbors of the same race will lead to segregation. But do people really consider population composition while making decisions on moving? What's the impact of homophily preference on people's moving decision within the city? This paper addresses those problems using a discrete choice model. The model makes prediction on the probability of moving between blocks based on the income level and population composition of blocks. Using the data of 62 blocks in Chicago from 2011 to 2017, the paper finds people are more willing to live with those of the same race. Besides, the paper also finds interesting difference between whites and blacks. While whites are not sensitive to the income level of blocks, blacks care about both the general income level and the income structure of blocks.

Keywords: Segregation, Homophily Preference, Race, Chicago

1 Introduction

Segregation is a common problem in many cities in United States. Although racial equality has become social consensus and is protected by the law, the segregated community pattern still widely exists in cities. It seems that people segregate themselves by choosing residence. A famous model studying segregation was first proposed by Schelling (1971) [11]. The core idea is that individual residual preference correlated to rate of race in the community will lead to macro segregated community pattern. Schelling used graphs to test the equilibrium under different tolerance of different race and different initial population composition, and found that the mixed equilibrium only exists under extreme tolerance and limited range of ratio of different races, which means even slightly preference for neighbors of the same race can lead to stable segregation.

However, there was no mathematical model in that paper. The idea was tested and studied using computer simulations (Schelling (1972)[12], Schelling (1975)[13], Jones (1985)[5], Clark (1991)[14]), but the theory was not analyzed thoroughly. Young (1998) [15] was the first to connect the theory of stochastic dynamical systems with Schelling's segregation model. His major contribution is to make the conclusion that segregated patterns are "stochastically stable", providing support to Schelling's conclusion based on mathematical theories.

Based on Young's work, Zhang (2004)[6] extended the model to a two-dimensional one and incorporated an endogenous housing price. He proved that preference for similar neighbors quickly lead to clusters of same race and highly segregated residual pattern with both theoretical deduction and computer simulation. In another paper, Zhang (2011)[16] found that people get stuck with segregation even they are all integrationists as long as they prefer extreme high percentage of same race to extreme low percentage of same race. There are also researches using demographic data in different cities to test the model (Benenson, Omer & Hatna, 2002[2]; Yin, 2009[7]), the conclusions are similar to that of Zhang's works.

In contrast to negative conclusions of papers mentioned before, Card, Mas & Rothstein (2008)[4] found that neighborhoods consist of various races can be stable, since the estimated tipping points, i.e., break point of minority share when whites start to flow out, range from 5% to more than 20%.

Paolillo and Lorenz (2018)[8] extended the model to the case that agents with two overlapping characters, race and values. Agents are divided into ethnicity-oriented that are tolerant to values but intolerant to different ethnicity, and value-oriented that are tolerant to different ethnicity but intolerant to intolerance (i.e. ethnicity-oriented agents). The result of the paper suggested that the best way to reduce segregation is not to reject those with homophily preference on ethnicity, but to be more tolerant to them.

There are also rich literatures discussing different topics based on Schelling's segregation model. Bayer, Fang & McMillan (2005) [1] built a mechanism to explain how equality leads to segregation. Radi and Gardini (2015)[9] extended Schelling's model to the case of a game between local residents and newcomers. Caetano and Maheshri (2017)[3] applied Schelling's model to the study of segregation caused by parents' preference for children's peers. Sahasranaman and Jensen (2018)[10] studied dual segregation of ethnicity and wealth. As shown above, the theoretical models develop rapidly with the progress both in computer science and mathematical theories, and empirical studies keep finding interesting conclusions with richer and bigger data. However, the major model present researches used to simulate is the checkerboard model. My research will try to simulate with a more abstract model that simulates and makes prediction with numerical vectors of states. Besides, I will try to build a rule to compare model predictions with real-world data and judge the quality of models with different parameters. With the block-level panel data of Chicago from 2011 to 2017, the purpose of my research is to find best model to predict future population movement within this city and propose a method to find such models for different cities.

2 Theoretical Model

Based on the assumption that people care about the percentage of their ethnic group in the neighborhood they choose, I build a discrete choice model to describe the probability of an agent to move from one block to another:

$$
\begin{aligned}
\triangle_{i,j,t} &= \beta_1 \times (log(income_{j,t}) - log(income_{i,t})) \\
&\quad + \beta_2 \times (homophily_{j,t} - homophily_{i,t}) \qquad (1) \\
p_{i,j,t} &= \frac{1}{1+e^{-\triangle_{i,j,t}}} \qquad (2) \\
\hat{p}_{i,j,t} &= (1-r) \times \frac{p_{i,j,t}}{\sum_{kei} p_{i,k,t}} \qquad (3) \\
P_t &= \begin{bmatrix} r & \hat{p}_{1,2,t} & \hat{p}_{1,3,t} & \cdots & \hat{p}_{1,n,t} \\ \hat{p}_{2,1,t} & r & \hat{p}_{2,3,t} & \cdots & \hat{p}_{2,n,t} \\ \vdots & \vdots & \vdots & & \vdots \\ \hat{p}_{n-1,1,t} & \hat{p}_{n-1,2,t} & \hat{p}_{n-1,3,t} & \cdots & \hat{p}_{n-1,n,t} \\ \hat{p}_{n,1,t} & \hat{p}_{n,2,t} & \hat{p}_{n,3,t} & \cdots & r \end{bmatrix} \qquad (4)
\end{aligned}
$$

In this model, P is the transition matrix of the population in blocks. $p_{i,j,t}$ is the rough probability that the agent moves from block i to block j at period t. Since we only care about the relative size of probability to move to different blocks, so we can scaled all p to $(1-r)$ to ensure every row of P, which means the fraction of people moving to other block from one block, sums to 1. The parameter r represents the ratio of people staying in the same block, which may result from unable to find satisfying house, unable to afford moving or other reasons. With the transition matrix, it's easy to make prediction as follows:

$$\begin{aligned}\widehat{population}_{i,t+1} &= population_{i,t} + \sum_{jei} population_{j,t} \times \hat{p}_{j,i,t} \\ &\quad - \sum_{jei} population_{i,t} \times \hat{p}_{i,j,t} onumber \qquad (5) \\ &= \sum_{j} population_{j,t} \times \hat{p}_{j,i,t} \qquad (6)\end{aligned}$$

The interpretation of the equation is straightforward. The kth row of the transition matrix depicts the flow out of population in block k while the kth column of the transition matrix depicts the flow in of population from all blocks to block k. So one method to calculate the population in block k is to take the difference between inflow and outflow population and add the population at the beginning. Another understanding of the matrix is the kth column of the matrix depicts the source of the population in block k in next period, thus just multiply each probability by population of each block at the beginning.

The target of my model is to predict the migration within the city as accurately as possible. Since I don't have the data of migration, the loss function should be based on the transition of population in each block to judge the quality of my model. If the population in the block is large enough, the transition matrix can make accurate prediction. If the population in the block is small, the actual moving population may differ to the prediction greatly since random factors affecting people's migration have significant impact on blocks of small population. To avoid the noise of those blocks of small population, I use the weighted prediction error rate rather than average prediction error rate as the judgment criterion. Formally, the loss function is as follows:

$$
\begin{aligned}
Loss_{t+1} &= \frac{1}{n}\sum_i \frac{|population_{i,t+1} - \widehat{population}_{i,t+1}|}{population_{i,t+1}} \times \frac{population_{i,t+1}}{\sum_j population_{j,t+1}} \\
&= \frac{1}{n}\sum_i \frac{|population_{i,t+1} - \widehat{population}_{i,t+1}|}{\sum_j population_{j,t+1}} \qquad (7)
\end{aligned}
$$

By minimizing loss function, I can get best parameters of my model with programming. Another advantage of that loss function is that we avoid infinite error rate caused by a block with 0 people of specific ethnic group because now I take the sum of population of the whole city at period t as the denominator.

3 Data Analysis

This paper use American Community Survey Database to train the model. This database includes income and population composition statistics at different level. The database is public and can be accessed through the website of US Census Bureau.

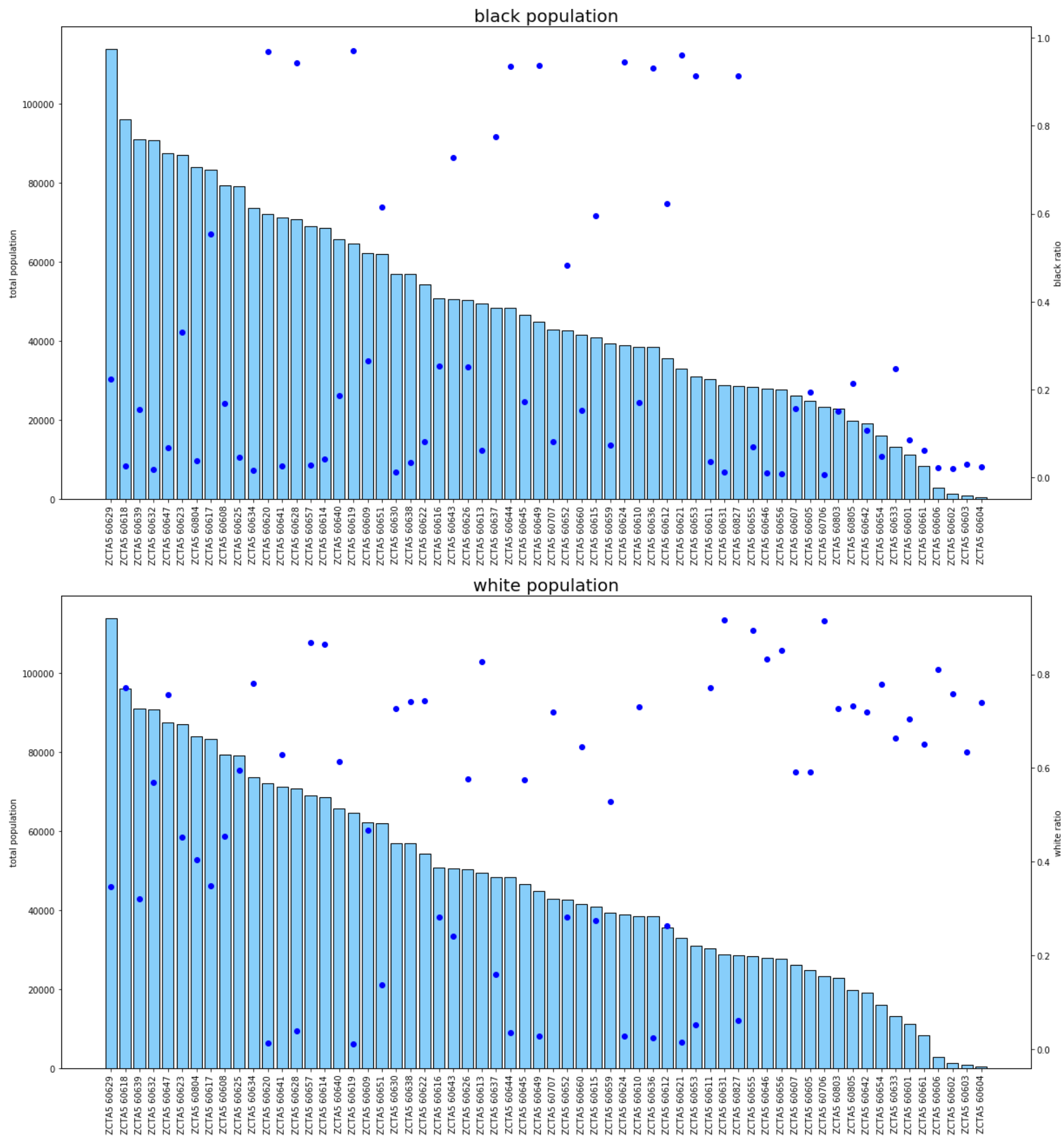
This paper aims to study people's migration within city, so I take Chicago as the target city and collect and analyze data at block level from 2011 to 2017. Blocks are represented by their zip codes. Since blacks and whites account for about 80 percent of total population in all blocks, I focus on those 2 ethnic groups and apply different parameters on different groups to see if they have significant difference that may lead to interesting inference.

For the summary of all variables grouped by years, please see the appendix.

Table 1 summarizes population data grouped by year and Table 2 summarizes income data grouped by year. There is a significant character of population data, which is discussed below.

Graph 1 depicts the average black and white population in each block of 7 years. The heterogeneity of blocks is noteworthy here. Some blocks have less than 100 or even 0 blacks while some have more than 5 blacks. As for percentage, blacks account for less than 5% in more than 1/4 of all blocks while account for over 90% in some blocks. The statistics of whites suggests similar heterogeneity. Based on the theory, we should put more attention on those blocks with large enough population of specific ethnic groups since those with small population are much more noisy. The great heterogeneity supports the use of weighted average error rate.

Graph 1. Average black and white population in each block



4 Model Application & Result

4.1 Result of Basic Model

This part analyzes the prediction based on the basic model. Chicago has 62 blocks with nonzero population. In this part, I use mean income and exact percentage of specific ethnic group as features of the block. Given parameter, I can make prediction on population of each block based on their features last year and calculate the loss function. Researches have shown that blacks and whites have different tolerance to ethnic groups other than themselves (Charles, 2003[17]; Zhang, 2011[16]), so it's better to set different parameters for them. Since there are other minorities and the model doesn't include transition matrix for income, it's better to make one-period prediction every year rather than simulate a series of data. After training, I get parameters as follows:

$$r_{black} = 0.98, \beta_{1,black} = -5.05, \beta_{2,black} = 88.21$$

$$r_{white} = 0.98, \beta_{1,white} = -7.51, \beta_{2,white} = 14.95$$

The average prediction error rate for each block is depicted in Graph 2. I divided those blocks into 3 groups, small blocks, medium blocks and large blocks. Since the population of blacks and whites are different, I set different threshold for those groups. For both ethnic groups, the threshold of medium group is 2,500, but the threshold of large group is 15,000 for blacks and 25,000 for whites. As is shown in the graph. The prediction is accurate in medium and large blocks but is really unstable for small blocks, the reason is the relatively huge impact of random factors on small blocks.

Table 3 summarizes weighted average error rate and average error rate for each of the 3 block groups from 2012 to 2017. The weighted average error rate is about 3% for both ethnic group every year, suggesting the prediction is accurate.

The coefficient of income, i.e., β_1 is noteworthy. It's negative, suggesting that the higher the mean income of the block, the less likely that people move there. It is inconsistent with the intuition that people want to live where the income is higher. However, a possible reason for the negative coefficient is that a block with higher mean income may have higher housing price, which reduces the attraction of the block. We will discuss the role of income in the model in detail in next part.

Graph 2. Average prediction error rate for each block

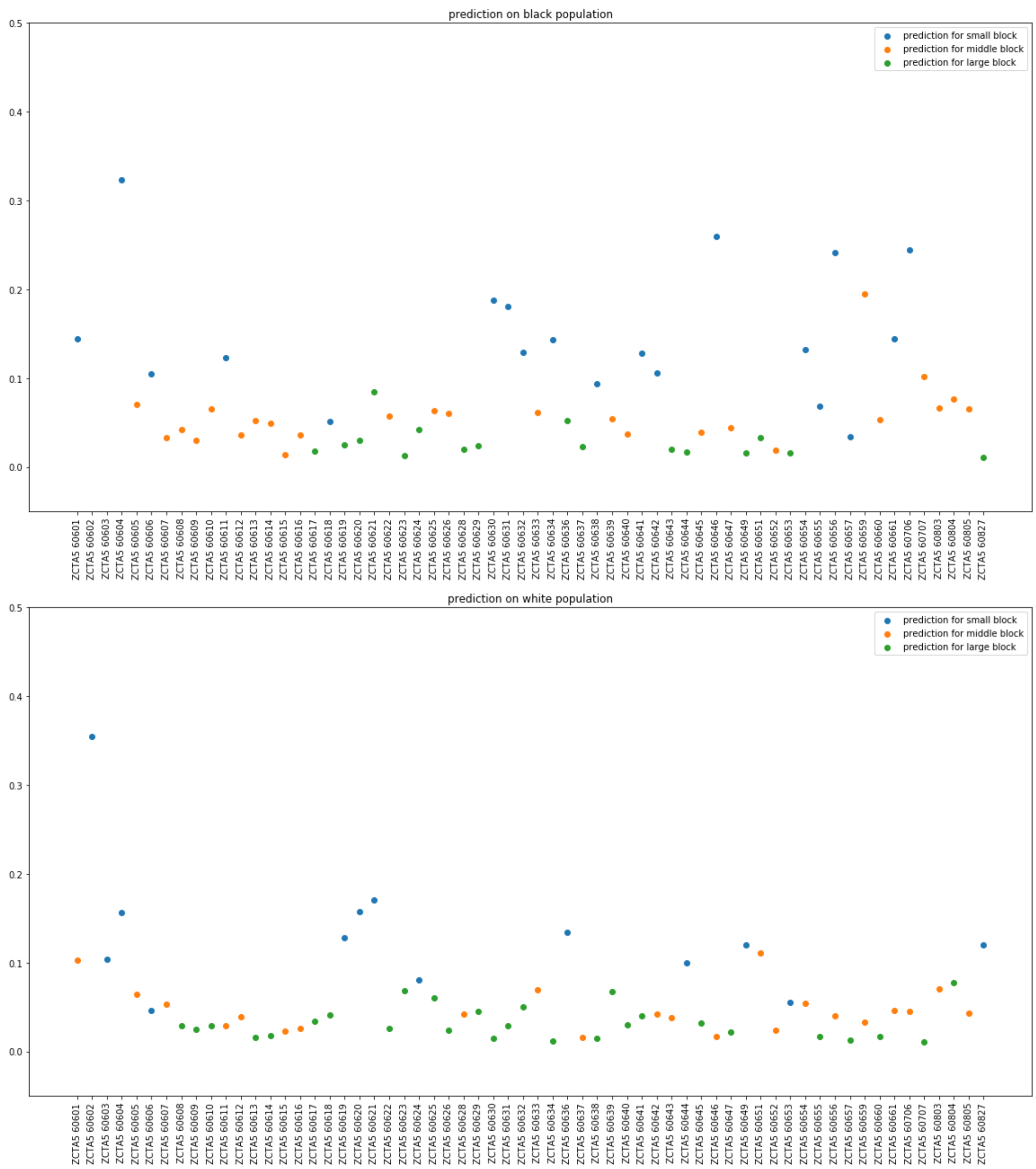


Table 3. Error rate for blacks and whites grouped by year

error rate for blacks	2012	2013	2014	2015	2016	2017
small block	0.125388	0.161906	0.359161	0.145573	0.184917	0.198986
medium block	0.056114	0.057022	0.071867	0.053815	0.072219	0.055366
large block	0.028683	0.027910	0.022013	0.036036	0.024018	0.025191
weighted average	0.034291	0.032680	0.030119	0.039930	0.030736	0.028752
error rate for whites	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**
small block	0.116622	0.150013	0.085454	0.168157	0.169404	0.107041
medium block	0.048756	0.046920	0.044531	0.051271	0.049408	0.037956
large block	0.055365	0.031297	0.032803	0.024092	0.020783	0.026411
weighted average	0.053757	0.036604	0.032497	0.030143	0.027401	0.028901

4.2 Discussion on Income

This section focus on the impact of income factor on people's migration decision and its significance to the model.

4.2.1 The choice of income

In the previous part, we use mean income to represent the *income* variable in the model. However, according to the statistics of income data (see Appendix Table 2), the values of 25, 50, 75 percentile and the mean of median income for 62 blocks in Chicago are significantly lower than those of mean income, suggesting that there may exist non-negligible income inequality. If that's the case, mean income may not be a good representation for the income variable when people consider migration. Table 4 compares the prediction error rate using different definition of income.

Table 4. Weighted average error rate using different definition of income

error rate for blacks	2012	2013	2014	2015	2016	2017
mean income	0.034291	0.032680	0.030119	0.039930	0.030736	0.028752
median income	0.029682	0.029998	0.027340	0.037251	0.028384	0.026461

error rate for whites	2012	2013	2014	2015	2016	2017
mean income	0.053757	0.036604	0.032497	0.030143	0.027401	0.028901
median income	0.053647	0.035663	0.031120	0.030095	0.026871	0.029055

As is shown in the table, the weighted average error rate for prediction on black population is reduced by about 0.3% using median income, which is a 10% reduction to previous results. Meanwhile, the weighted average error rates for prediction on white population are similar using different definitions. A possible reason is that the general income level of blacks is relatively lower and thus they pay more attention to the structure of income in the block. If that's the case, the model matches people's way of thinking better by switching to median income. On the other hand, whites may be more concerned about general income level of the block. As a result, mean income and median income have similar explanatory power.

4.2.2 The significance of income

Although using median income can improve the performance of the model, the significance of including income in the model hasn't been shown. This part will test the improvement caused by including income in the model. I train the model again while fixing β_1 at zero, and compare the prediction with previous one using median income. Table 5 summarizes the weighted average error rate of those two predictions.

Table 5. Weighted average error rate excluding and including income

error rate for blacks	2012	2013	2014	2015	2016	2017
without income	0.032677	0.031590	0.030051	0.039740	0.028725	0.027953
median income	0.029682	0.029998	0.027340	0.037251	0.028384	0.026461

error rate for whites	2012	2013	2014	2015	2016	2017
without income	0.032677	0.031590	0.030051	0.039740	0.028725	0.027953
median income	0.053647	0.035663	0.031120	0.030095	0.026871	0.029055

As is shown in the table, including median income improves the performance of prediction on black population but makes similar or even worse prediction on white population. The reason we mentioned last part might also explain the phenomenon here. Since the income of whites is high, the marginal utility of wealth is low and has little impact on their decision to move. The major factor affecting their migration decision is the environment of new neighborhood, which is percentage of whites in our model.

4.3 Discussion on homophily

In section 2.1, I use exact percentage of specific ethnic group to represent the *homophily* variable in the model. In this part, I will use a dummy variable to represent homophily variable. That is:

$$homophily = \begin{cases} 1\ if\ percentage > \phi \\ \\ 0\ otherwise \end{cases} \tag{8}$$

ϕ is a new parameter needs to be found by training. Table 6 compares the prediction error rate using exact percentage and dummy variable.

Table 6. Weighted average error rate using different definition of homophily

error rate for blacks	2012	2013	2014	2015	2016	2017
dummy variable	0.031155	0.030966	0.027868	0.038100	0.030891	0.030227
percentage	0.029682	0.029998	0.027340	0.037251	0.028384	0.026461
error rate for whites	2012	2013	2014	2015	2016	2017
dummy variable	0.053519	0.035693	0.033935	0.030332	0.026238	0.030386
percentage	0.053647	0.035663	0.031120	0.030095	0.026871	0.029055

As is shown in the table, the weighted average error rates for prediction on both blacks and whites are similar using different definitions. This finding suggests people might be not so sensitive to the population composition. They may only care about the general population composition of the block. It is reasonable because people have limited access to accurate statistics of population composition of every block.

5 Conclusion

This paper tries to explain people's moving decision within a city with a discrete choice model. With the data of Chicago from 2011 to 2017, I get the best parameters to describe residence-choosing behaviors of blacks and whites in Chicago based on the model. Blacks have higher coefficient for homophily, which suggests that blacks may be more fond of a segregated community pattern than whites. That is an interesting inference that supports the view that people segregate themselves. Besides, whites and blacks show different attitudes to income of a block. While whites are not sensitive about income of the block, blacks show great concern for both general income level and income structure of the block. Those conclusions provide reference for predicting population flow and making related policy.

There is still ample room for future research. In this paper, I use only income and population composition to predict people's moving decision. There are many other factors can be used, like housing price, public goods, wealth of the decision maker and so on. Besides, the model can be applied to population flow between cities or states. Since larger population leads to more accurate prediction, it should have a better performance on those higher-level researches.

6 Appendix

	Year	2011	2012	2013	2014
black population	count	62	62	62	62
	mean	15064.016129	14865.258065	14617.032258	14478.758065
	std	19163.612657	18702.157889	18425.133138	18180.946499
	min	0	0	0	10
	25%	1827.75	1859.75	1700.75	1669.5
	50%	4427.5	4494	4347.5	4233
	75%	25403.5	25655.25	25904.25	25079.25
	max	75869	72649	71002	69652
black rate	count	62	62	62	62
	mean	0.300237	0.299945	0.295957	0.293773
	std	0.342763	0.344000	0.342762	0.341311
	min	0	0	0	0.004815
	25%	0.040515	0.038981	0.037856	0.039869
	50%	0.165595	0.164146	0.156558	0.153084
	75%	0.533097	0.536640	0.535809	0.526432
	max	0.979562	0.978504	0.974338	0.971858
white population	count	62	62	62	62
	mean	21267.048387	22291.967742	22753.032258	23071.645161
	std	17689.779441	18676.730468	19134.622313	19356.340396
	min	321	351	339	326
	25%	6317	6894.5	6852.75	7276
	50%	19992.5	21155.5	21010.5	20897
	75%	32268.25	34559.25	35065.5	36921.25
	max	63391	70518	75860	76907
white rate	count	62	62	62	62
	mean	0.507465	0.518405	0.523778	0.526007
	std	0.291174	0.291052	0.293417	0.292847
	min	0.006839	0.007517	0.008857	0.008296
	25%	0.274583	0.290387	0.287005	0.287622
	50%	0.583600	0.591050	0.609351	0.614811
	75%	0.738630	0.749449	0.756871	0.758110
	max	0.905394	0.922888	0.923553	0.931595
total population	count	62	62	62	62
	mean	46861.516129	46900.354839	46969.967742	47084.951613
	std	27147.057295	26847.490848	26712.831372	26696.120889
	min	393	428	415	419
	25%	27865.25	27853.5	28208.25	28108.25
	50%	43883.5	43566	44078	44079.5
	75%	67363.75	66965.75	67296.5	67579.5
	max	112376	111893	113833	115013

	Year	2015	2016	2017
black population	count	62	62	62
	mean	14262.048387	14113.467742	13965.903226
	std	17814.784187	17585.214503	17390.753978
	min	12	28	11
	25%	1642	1740	1702.75
	50%	4372.5	4540.5	4435
	75%	24041.25	23277.5	22807.5
	max	67353	66942	66537
black rate	count	62	62	62
	mean	0.292148	0.291418	0.289013
	std	0.340509	0.338358	0.336210
	min	0.006652	0.005296	0.006414
	25%	0.036672	0.040344	0.037911
	50%	0.145420	0.139971	0.131808
	75%	0.540221	0.542120	0.515845
	max	0.969154	0.967368	0.963408
white population	count	62	62	62
	mean	23253.419355	23186.838710	23405.629032
	std	19442.414740	19317.192039	19378.972083
	min	380	381	418
	25%	7770.5	7838.5	7998.25
	50%	21440	21431.5	21683.5
	75%	37566.25	37377	35479.75
	max	78374	77910	76981
white rate	count	62	62	62
	mean	0.525297	0.520550	0.522409
	std	0.291600	0.288395	0.287907
	min	0.012033	0.013292	0.015161
	25%	0.275991	0.278185	0.282083
	50%	0.622566	0.613404	0.603769
	75%	0.747336	0.741069	0.753429
	max	0.924655	0.913662	0.903827
total population	count	62	62	62
	mean	47171.822581	47098.580645	47222.306452
	std	26688.734509	26491.581774	26485.770365
	min	545	619	668
	25%	28311	28272.75	28874
	50%	44155	44334.5	43902.5
	75%	67448	67829.75	68179
	max	114982	115104	114129

Table 1. Statistics of population data (grouped by year)

	Year	2011	2012	2013	2014
mean income	count	62	62	62	62
	mean	75693.758065	75549.274194	77636.403226	77727.790323
	std	34019.002717	33151.315275	38144.700123	38884.730418
	min	33534	33049	31965	31288
	25%	50327.5	49622.25	48580	49505.25
	50%	67383.5	67002	65965	67309
	75%	91505.25	94433.25	97909	97961.75
	max	169005	166752	214918	232790
median income	count	62	62	62	62
	mean	55054.096774	54807.516129	55478.274194	56351.161290
	std	22353.940314	22353.640437	24266.105957	26054.531192
	min	19692	19623	19548	19190
	25%	38611.25	38117.75	37740.25	37055.75
	50%	50999	50499.5	50383.5	51486.5
	75%	69348.25	70755	71707.75	71819.25
	max	109375	107056	132188	155750

	Year	2015	2016	2017
mean_income	count	62	62	62
	mean	79947.290323	82670.741935	88015.967742
	std	43090.234148	41553.540018	45318.732623
	min	30458	31845	32722
	25%	50204.75	52622.25	54778.25
	50%	68753.5	72261	74314
	75%	100561.5	104907.75	114223.5
	max	261215	224642	221184
median_income	count	62	62	62
	mean	56931.064516	59591.951613	62405.145161
	std	26768.025370	28867.416900	29962.273691
	min	19832	20150	19845
	25%	36291.25	37166.25	38857.75
	50%	52264.5	53835.5	56027
	75%	74683.75	79083.5	85457
	max	151731	150125	140114

Table 2. Statistics of income data (grouped by year)

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