

A multi-scaled agent-based model of residential segregation applied to a real metropolitan area

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ABSTRACT

Residential segregation influences many aspects of urban life. It affects people's access to centres of education, healthcare, business and determines the composition of our neighbourhoods, thereby impacting our social network and urban structure. In order to understand the potential impact of policies on residential segregation and complex urban system, a dynamic modelling support tool would be essential. This research article presents a multi-scaled agent-based model capable of simulating the relocation of residents of a representative population of a large urban area in a realistic environment for investigating the dynamics of residential segregation. Using an experiment, we show that this data-driven model can replicate plausible residential distribution and segregation patterns observed in the Auckland region (New Zealand's metropolis). Simulation outcomes are promising, demonstrating the potential of the model for investigating practical policy-relevant questions and acquiring valuable insights into the future state of the urban mosaic landscape and causes behind residential segregation dynamics.

1. Introduction

As segregation has become an important feature of a modern city (Batty, 2010), our understanding of its causes, role and impact on the social and urban fabric of our societies remain relatively limited (Bruch & Mare, 2006). Yet, our ability to accurately model, measure, understand and anticipate segregation would be essential in having a more equitable distribution of public services and better social cohesion in the society.

The pioneering work of Thomas Schelling (1969, 1971) was an important milestone in investigating this multifaceted phenomenon (Clark, 1991). It built the foundation for an individually-based modelling (Crooks & Heppenstall, 2012) investigation focusing on the actions of agents (persons/ household embodiments) who made choices regarding where to relocate and live in the simulated world. Although many of these abstract models help us think about the "real world" (Fossett, 2011), the unrealistic and simplistic nature of artificial worlds in Schelling-style models has prompted "questions about how well they portray the neighbourhood dynamics of real cities" (O'Sullivan, 2009, p. 507). As a result, there are indications in the recent years that residential mobility and segregation modelling development point towards more realistic trends and their applicability to real urban areas.

This more realistic modelling approach is often comprised of four key dimensions: 1) more representative spatial characteristics; 2) use of real/empirical data; 3) more consistent and reliable evaluation (e.g. calibration and validation) against empirical benchmarks; 4) broader explanatory factors (determinants).

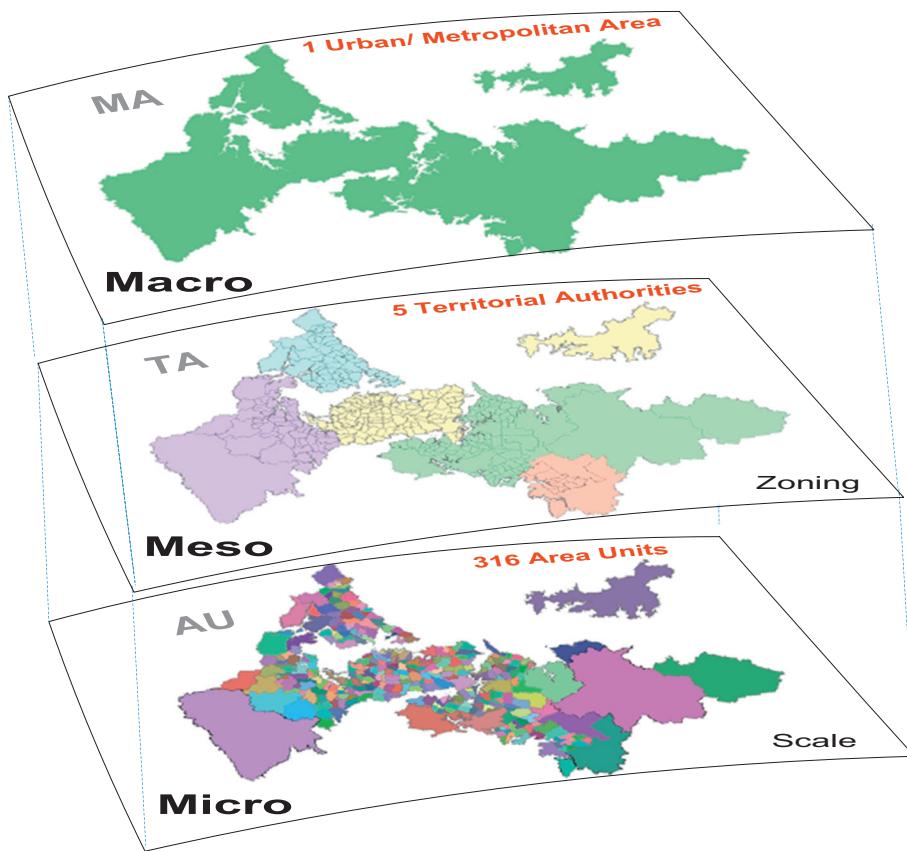
A model with more realistic characteristics has several advantages. Since "the outcomes of residential segregation models may strongly depend on the way that neighbourhoods are conceptualized and represented" (O'Sullivan, 2009, p. 508), it would be more consistent to use/ integrate real-world data which correspond to the same administrative spatial boundaries based on which data are collected (Rolle, 2014).

Subsequently, the combination of "real data along their spatial characteristics is the ultimate form of model validation" (Stanilov, 2012, p. 258), as the evaluation of the model would naturally become more intuitive and reliable. Furthermore, inclusion of more explanatory factors in the model would enhance the overall realistic trait of the model, not the least because of the possibility of comparing the effects of the implemented mechanisms on the empirical and historical benchmarks. Overall, a more realistic model has higher potential to communicate its insights more effectively and engage easier with policy-makers (Stanilov, 2012).

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Fig. 1. Multi-scaled modelling approach



Among agent-based models (ABM) in the residential segregation sphere, the pioneering work of Benenson, Omer, and Hatna (2002) has set a high standard for subsequent work (Bruch, 2014; Crooks, 2010; Feitosa, Le, & Vlek, 2011; Yin, 2009).

However, a comprehensive assessment of changing patterns of residential segregation should preferably allow a thorough examination at different *inter* and *intra* levels of spatially nested entities (Parisi, Licher, & Taquino, 2011). The geographical scale (e.g. divisions of a subdivided metropolitan area) can portray distinct dimensions of residential segregation (Reardon et al., 2009). A model with multi-scaled capability (illustrated in Fig. 1) will allow the investigation of segregation patterns on both *macro-segregation* (e.g. Metropolitan Area) and *meso-segregation* (e.g. Territorial Authority), based on their encompassed micro-spatial units' subdivisions (e.g. Area Units).

Similarly, the interpretation of shifting patterns of residential segregation and its social implications based on a single measure can be considered incomplete. Since there exist various paradigms and interpretations of segregation (Simpson, 2006), it is desirable to measure several dimensions of segregation (Massey & Denton, 1988; Reardon & O'Sullivan, 2004) in order to acquire more comprehensive portraits of the ethnic mosaic state in the *meso* and *macro* geographical entities of the urban area.

This research article presents an agent-based model of residential segregation which contributes to the same realistic modelling direction for analysing the effect of residential location decision of individual residents (agents) on the spatial ethnic mosaic pattern of the central Auckland region (New Zealand metropolis).

The following lists original features of the model. Firstly, the model deals with the entire population sizes based on census values, although only the relocating agents (informed by census mobility values for each ethnic group) are stochastically instantiated and make decisions about their residential location. Secondly, the model dynamic of residential location choice comprise of the main contextual mechanisms, including

group and personal preferences (e.g. behaviours conditioned by *bounded rationality*), empirical vacancy rates (as proxy for combination of real estate market condition and (local) government policies related to new housing development), as well as economic conditions (by empirically informed proxy of residents' economic circumstances to relocate locally or globally). Thirdly, while *intra*-urban migration (movements by existing population within the boundaries of the urban/metropolitan area and evidently its smaller spatial nested entities) takes place indigenously, *inter*-urban migration (movements between population of an external urban area and the simulated metropolitan area) has exogenous effect on the simulation dynamic, exhibiting an open urban system. Lastly, the effects of simulating residential decision-making of four major ethnic groups on various dimensions of segregation are measured and calibrated against their equivalent census-based benchmarks, before the simulations are projected into the future using Statistics New Zealand population growth projection estimates at mesogeographical scale as the base of segregation forecasting scenarios.

Thereby, the model is able to simulate future scenarios depending on changes in overall and ethnic-based population growth conditions and their distributions, including factors which are more susceptible to be influenced by macro (state, institutional) actors (such as control of international immigration, population birth rate, housing development/vacancy rates), as well as micro (individual) actors (such as changing preferences of relocating residents).

In this article, we focus on presenting the following experiments and results. First, we show that the model is capable of generating patterns that are fairly comparable to the empirical benchmarks built from the application of multiple measures of residential segregation on several quinquennial periods of census data, notwithstanding detailed mechanisms regarding residential decision making are not fully present (implemented). Then, we use various experiments with the model to show that 1) higher population growth (and immigration) does not necessarily (automatically) exacerbate the intensity of residential

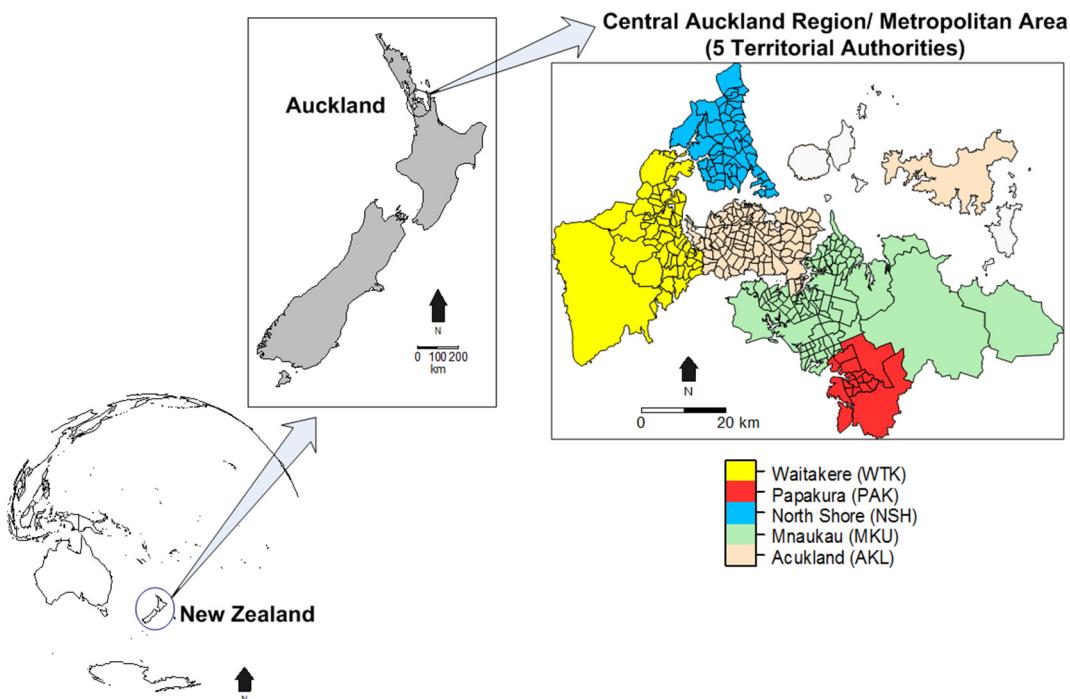


Fig. 2. Left map shows the location of New Zealand; Centre map identifies the location of Auckland in North Island of New Zealand; Right map illustrates the five territorial authorities of the central Auckland region (metropolitan area) and their area units. Note that only Waiheke Island has a significant population and so the other gulf islands (in white) are omitted in other figures because they were not used in the model, and to avoid visual ‘clutter’

segregation 2) segregation is not necessarily greater at the mesoscale than at the macroscale, 3) change of housing vacancy rate (in this case, tightening) in one of the meso-geographical units has an impact on the level of segregation in other meso-geographical units, as well as on the at the macro spatial unit (in this case, increase of segregation measured by the entropy-based information theory index).

2. Study area and its characteristics

The Auckland region is the largest and most populous urban area in New Zealand and is located in the North Island (see Fig. 2). However, this study focuses on five central territorial authorities (TA) of *Auckland City* (AKL), *Manukau* (MKU), *North Shore* (NSH), *Waitakere* (WTK) and *Papakura* (PAK). In this article, we identify this macro geographical entity as a Metropolitan Area (MA), which encompasses a total of 316 selected area units (AU). The census data has been adjusted for multiple-ethnicity counts. The original Statistics New Zealand (SNZ) sizes of the ethnic population for each TA and their yearly average change can be compared to those used in this study in the tables included in Appendix A. The census administrative boundaries for the metropolitan area, territorial authorities and their encompassed area units used in this study remained unchanged from 1991 to 2006 census periods.

Between 1991 and 2006, the population of this designated metropolitan area increased by about 29%, from less than 850,000 to about 1.1 million inhabitants (which roughly represented 26% of the entire New Zealand population in 2006). On the other hand, the changes in population proportion of each territorial authority (TA) have been moderate: in the most populated territorial authority of *Auckland* from 35.5% to 34.3%, *Manukau* from 26.4% to 28%, *North Shore* from 18.2% to 18%, *Waitakere* from 16.2% to 15.8% and in the least populated territorial authority of *Papakura* from 4.1% to 3.9%.

While the ethnic makeup of the central Auckland region is diverse, for the purpose of this study we focus primarily on the four major ethnic groups of Europeans, Asians, Pacific people and Maori. The analysis of quinquennial censuses (1991, 1996, 2001 and 2006) reveals significant changes in the structure of the ethnic population of this urban area.

While the population of all the ethnic groups in the designated area of study inflated in absolute terms between 1991 and 2006, the population of Asian residents had the highest growth by about 332%, whereas the increase was about 49% and 22% for the other two minority groups (Pacific people and Maori respectively) and only 2.4% for Europeans.

In proportional terms, the continuous growth of the Asian population and decline of the European population in the macro-geographical entity (designated metropolitan area) and all the five meso-geographical entities (TAs), as well as increasing share of Pacific people in *Manukau* (MKU) are particularly notable, as indicated in Table 1. By 2001 Asians had already outranked Pacific people to become the second largest ethnic group at the metropolitan area scale (after the European group). Ultimately, by 2006 their proportions were about 19% and 14% respectively.

The sequence-bars in Fig. 3 illustrate the proportional ethnic population change at the micro-geographical scale (316 AUs) between the years of 1991 and 2006. The proportional increases of Asian and Pacific people populations in 2006 are particularly notable, although for Pacific people, the increases are mostly concentrated in fewer specific area units (mostly encompassed in *Manukau*).

3. Model implementation

Fig. 4 summarises the operation of the model along with scheduled processes. The following sections outline the details of how these are implemented and operationalized. Most of the scheduled processes are applied at micro-geographical level (often with consideration of empirical values available at mesoscale). However, the relocations are individually based and the moving agents will finally resettle in nearby or more distant micro-geographical locations with consideration of their contextual and individual characteristics, preferences and economic freedom to relocate locally or globally.

Similar to Fossett (2006), each cycle in our model is considered equivalent to one year and the scenarios are simulated for 30 cycles, of which the last 15 are projected years for which census data are not available.

Table 1

Proportional population sizes of four ethnic groups in the metropolitan area (highlighted in grey) and each of five territorial authorities for censuses 1991, 1996, 2001 and 2006.

Year	Area	Euro	Asian	Pacific	Maori
1991	Metro	71.5%	5.7%	12.2%	10.6%
	Auckland	70.3%	8.0%	13.2%	8.5%
	Manukau	59.0%	5.8%	19.8%	15.4%
	North Shore	88.7%	3.5%	2.3%	5.4%
	Waitakere	72.5%	2.8%	5.9%	18.8%
	Papakura	75.6%	3.8%	9.9%	10.7%
1996	Metro	66.4%	10.0%	12.4%	11.2%
	Auckland	65.5%	13.0%	12.7%	8.8%
	Manukau	54.1%	10.1%	20.5%	15.3%
	North Shore	81.8%	8.5%	2.9%	6.8%
	Waitakere	69.2%	4.0%	6.4%	20.5%
	Papakura	70.2%	6.7%	11.0%	12.1%
2001	Metro	61.7%	14.0%	13.8%	10.5%
	Auckland	62.0%	17.7%	12.6%	7.7%
	Manukau	47.6%	14.0%	23.8%	14.5%
	North Shore	78.4%	12.2%	3.1%	6.3%
	Waitakere	66.3%	5.5%	6.9%	21.4%
	Papakura	65.1%	9.9%	13.0%	12.0%
2006	Metro	56.8%	19.2%	14.1%	10.0%
	Auckland	57.9%	23.0%	12.0%	7.1%
	Manukau	42.2%	19.7%	24.7%	13.4%
	North Shore	73.4%	17.7%	3.0%	5.9%
	Waitakere	61.3%	7.1%	8.8%	22.8%
	Papakura	60.3%	14.5%	13.6%	11.6%

3.1. Model initialisation

The model can be initialized by choosing one of the pre-set census year populations from 1991, 1996, 2001 or 2006. Here, we choose the *seed year* (the start of the simulations) by initializing the model with the 1991 census year where the population of the metropolitan area is 847,113. By doing so, the population of each of the four major ethnic groups in 316 area units in the model will correspond with AU-based population sizes in the 1991 census.

3.2. Mobility turnover

Many models of residential segregation or mobility are based on satisfaction/ dissonance mechanism (Benenson et al., 2002; Gaube & Remesch, 2013; Portugal, Benenson, & Omer, 1997; Yin, 2009) or “stress-resistance” (Benenson, 2004) in determining when a resident agent decides to relocate.

Instead, similar to hybrid models (Birkin, 2008; Mahdavi Ardestani, 2013; Mahdavi, O’Sullivan, & Davis, 2007; Wu, Birkin, & Rees, 2008), our model creates synthetic individuals (agents) by disaggregating data based on *ethnicity* and *location* characteristics. In addition, the mobility turnovers are generated by a probability-based process applied to each unit of ethnic group population in area units. This way, we deal with the aggregate real-world population, where only a fraction of the population relocates (which are created on-the-fly as decision making agents) at each cycle.

The rationale is that the practical abstraction of the details of the causes and motivations of individual people behind their movements allows the model to pay particular attention to the role of homophily rather than xenophobic behaviours on the formation of segregation. People decide to relocate for various reasons (Gilbert, 2006) such as the cost of housing and accommodation, proximity to family, workplace or catchment area of a high-ranking school, changes in employment location or dissatisfaction with neighbours or neighbourhood (environment). What is essential for the model is to roughly capture a

representative number of yearly relocations (which are seen as *intra-urban migrations*). The size of relocating population φ is binomially estimated as:

$$\varphi = \mathbf{B}(t_{jm}, p_{Mm}) \quad (1)$$

where t_{jm} is the population size of the ethnic group m in area unit j , and p_{Mm} is the probability of mobility for group m in each trial, set as the average values of rounded mobility percentages calculated for the Auckland region based on 1991 to 2006 census periods (see Table 2). For the projection years, we use 2006 values which indicate slightly higher mobility trends for Asians and Pacific people.

A fraction of these moving ethnic agents is further binomially designated as *global movers* – a sort of *economic freedom* proxy which enables them to relocate anywhere in the metropolitan area – as follows:

$$\varphi' = \mathbf{B}(\varphi, p_{Gm}) \quad (2)$$

where p_{Gm} is the probability of global location mobility for group m in each trial.

These probabilities are partly informed by the census median annual income trends statistics for the Auckland region. Through the years from 1991 to 2006 Europeans had constantly the highest median annual income followed by Maori, Pacific people and then Asians. However, we consider the higher percentage of homeownership by Asians than those of Maori and Pacific people and therefore adjusted their probability of global location mobility, so that it is not lower than the other two minority groups. By presuming that 40% of moving residents of European ethnicity have enough *economic freedom* to relocate to any part of the urban area during the period of 1991 to 1996, we set proportional values (roughly rounded) for other ethnic groups, as shown in Table 3. The increase of median income, as well as reasonable growth and progression in *economic freedom* over time are captured by increasing p_{Gm} probabilities by 5% in each census period from 1996 to 2006. As the median annual income trends have been also edging towards narrower gaps among different ethnic groups (particularly for Asians and Maori) for the projection years, we adjusted these values to reflect these realities.

3.3. Population growth/ decline/ migrations & housing vacancy

Often, changes in population occur as a result of the lifecycle processes of birth and death, as well as migration. However, due to the nature of the available data, the abstraction made in the model only deals with population growth (which implicitly includes the echelons of birth and *inter-urban immigration*) and decline (which implicitly includes deaths and *inter-urban emigration*).

During the years, for which census data exist, the population growth, decline (particularly for Europeans in certain areas), and the migration rate are calculated by the model for each area unit, using differences in the population numbers for each area unit, when compared with the information available for previous census periods. The proportional change in population at the metropolitan area and territorial authority levels would roughly correspond to the values presented in Table 1. For the years after 2006, the population change ensues by applying SNZ ethnic-based population change projections at territorial authority levels (as presented in Table 7) to the ethnic population of each area unit living in the corresponding TA.

At this stage, the provision of vacant space (including projection years) is applied as the percentage product of estimated population growth by stochastically calculated empirical vacancy rates of territorial authorities, using the average of vacancy rates from the two last statistical periods, as indicated in Table 4.

The vacancy rate v for area unit j situated in a meso territorial authority r is stochastically calculated using a uniform distribution plus and minus a constant value (C) which is set to 1 for the scenarios presented in this article:

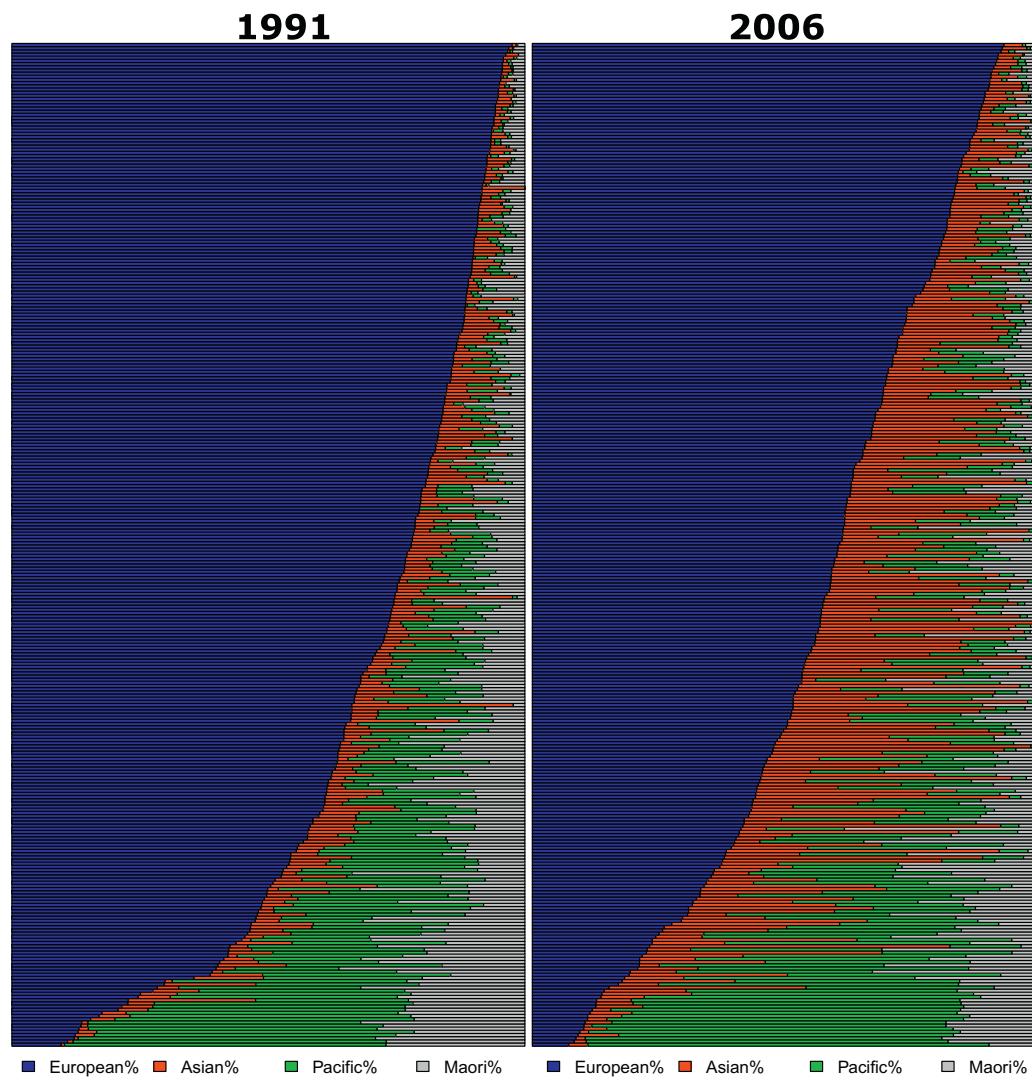


Fig. 3. Sequence-bars of ethnic population proportion changes at micro area unit scale based on census for years of 1991 and 2006

$$v_j = U(V_r - C, V_r + C) \quad (3)$$

Population decline (and growth) are estimated at the smallest geographical units and counted (aggregated) at *meso* levels. Their individual removal (from the aggregate population) will take place randomly, at each cycle, among the area units contained in the territorial authority for which the population decline has been calculated, as long as the population of randomly selected area units is more than 10 (as a precaution to avoid the possibility of ending up with implausible vacated area units).

3.4. Agent resettlement

Following the processes of *mobility turnover* and *population growth*, individual agents are instantiated. These agents are classified into three different types (see Table 5). Those generated by the *mobility turnover* process, are classified into *local* and *global* movers. The latter group benefits from a larger *vision* (Laurie & Jaggi, 2003) (a determining factor for the extent and distance that an agent considers the sphere of a potential search area for relocation) so that these agents can expand their search parameters within the entire metropolitan area. Conversely, *local* movers' *visions* are limited to the size of their (immediate) *neighbourhoods* (defined as a set of area units adjacent to the area unit where the agent resides prior to its relocation). Furthermore, the *global* movers have a more expansive *search limit* (a cap on the number of visited locations before deciding to resettle), while for *local* movers,

their *search limits* are set to the size of their immediate neighbourhoods. As a product of the *mobility turnover* which processes existing population, the migrations (movements) of both *local* and *global* movers are considered *intra-urban*, taking place indigenously within the boundaries of the urban/ metropolitan area (and encompassing nested spatial entities).

For the population growth, instantiated agents are classified as *growth & immigrants*. They will only search for suitable locations within the territorial authority for which the growth has been calculated, hence having TA-wide *vision*. As a product of the *population growth* which processes the inbound non-resident population, the migration of these agents is considered *inter-urban*, taking place exogenously between an external (urban) area(s) and the (simulated) metropolitan area.

The *search limit* is assigned to each *global mover* and *growth & immigrant* agent in a probabilistic way, using Poisson distribution (P). The average number of searches (λ) for each agent type used in simulations is given in Table 6. Although there is a correlation between the number of area units contained in the geographical area under the *vision* of an agent and its maximum *search limit* (λ -value), we set a plafond to recognize the empirical evidence that the majority of people visit an average of about 8 houses before making a decision to resettle.

Agents are also endowed with an ethnocentric/homophilic behaviour, by means of an assigned threshold which indicates the preference for choosing a location that has equal or higher (than the threshold)

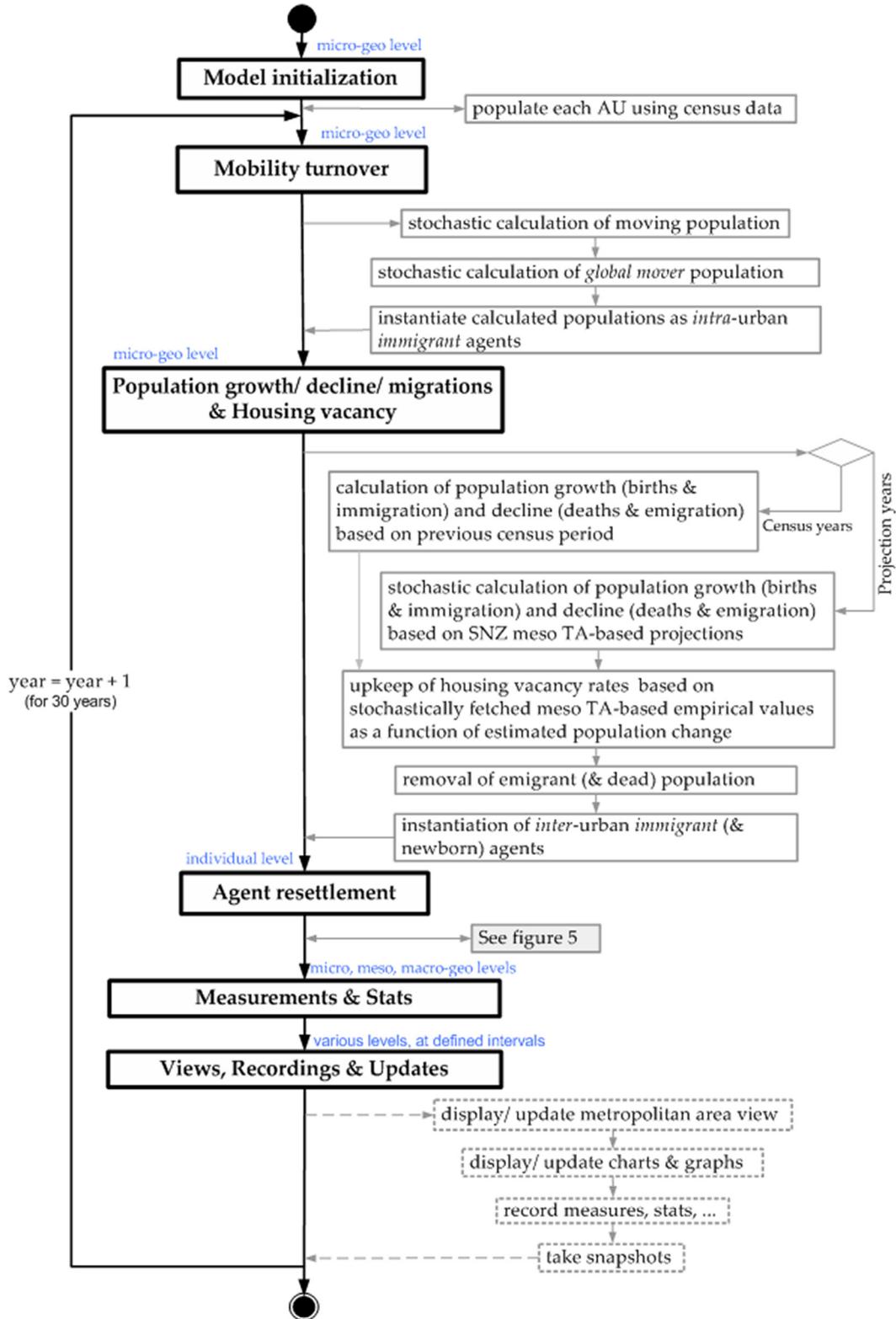


Fig. 4. Major scheduled processes (dotted lines can be optionally operationalized)

percentage of *co-ethnic* (Fossett & Warren, 2005) population. This threshold factor which influences the decision-making of agents is often referred to as *tolerance* (Θ , which varies on $[0,1]$) although sometimes other terms such as *preference* or *choice* are interchangeably used (Clark, 1991; Schelling, 1978). If t_j denotes the total population in area unit j and t_{jm} denotes the population of ethnic group m in the same area unit, a given agent belonging to the same ethnic group would relocate to j if

$\Theta \geq t_{jm} / t_j$; or in other words, when $\Theta \geq \pi_{jm}$ (where π_{jm} represents the proportion of group m in area unit j). Since we do not have the exact empirical values for this tolerance, we will approximate them during the calibration process (Section 4).

An agent resettlement (Fig. 5) starts by getting an estimation of co-ethnic compositions in its current area unit location. The rationale for using an estimation (rather than a precise calculation) stems from the

Table 2

Census-based rounded mobility percentages (p_{Mm}) for each ethnic groups in the Auckland region.

Ethnic group	1991	1996	2001	2006	p_{Mm} (1991–2006)	p_{Mm} (after 1996)
European	10%	10%	10%	10%	10%	10%
Asian	10%	11%	11%	12%	11%	12%
Pacific	9%	9%	10%	10%	9%	10%
Maori	11%	11%	12%	12%	12%	12%

Table 3

p_{Gm} values partly informed by census values of median incomes.

Ethnic group	[1991–1996]	[1996–2001]	[2001–2006]	after 2006
European	40%	45%	50%	50%
Asian	20%	25%	30%	40%
Pacific	20%	25%	30%	30%
Maori	20%	25%	30%	40%

Table 4

Empirical vacancy rates for each territorial authority.

Territorial authority (r)	[1991–1996]	[1996–2001]	[2001–2006]	V_r (used by model)
North Shore (NSH)	4.1	5.5	5.2	5.4
Waitakere (WTK)	4.1	6.1	5.5	5.8
Auckland city (AKL)	5.9	7.3	8.4	7.8
Manukau (MKU)	3.6	5.1	4.6	4.9
Papakura (PAK)	3.0	5.8	5.1	5.4

need for creating a more realistic situation using *bounded rationality*. This stipulates that relevant information on the exact ethnic composition of a neighbourhood is often inaccessible to those people who attempt to obtain it indirectly from sources such as realtors, family, friends, or by their own personal observations during visits. This estimation is performed by using a Gaussian distribution with the mean that represents the real proportion of co-ethnic group m in area unit j (noted as π_{jm}) and a bias standard deviation σ set to 0.07 in order to keep the dispersion of probability values from the exact values relatively small.

$$\pi_{jm}^* = N(\pi_{jm}, \sigma), \text{ when } \pi_{jm} = 0 \Rightarrow \pi_{jm}^* = 0 \quad (4)$$

If the estimated composition of the co-ethnic population of a visited location is equal or above the tolerance threshold, the agent resettle there, otherwise he lists this location as the ‘best so far’ (if it has a higher value closer to his *tolerance*) and continues his search (after reducing his *search limit* by one); Whether the search is conducted in the same location, or in another, it is determined by the individual’s *persistent* ($p_{persist}$) behaviour. The stochastic evaluation of whether an agent *persists* or not is performed using a continuous uniform distribution with minimum and maximum values between 0 and 1. If $p_{persist} > U(0,1)$, then the agent persists in continuing to search in the same location, otherwise, it moves to another one. Whether or not the next randomly selected location would be in the immediate neighbourhood, territorial authority or metropolitan area will be determined by the *type* and *vision* of the agent (described in Table 5). Without any

Table 5

Classification of agents.

Agent type	Generating process	Migration type	Migration dynamism	Vision	Search limit
Local mover	Mobility turnover	Intra-urban	Endogenous	Neighbourhood (N)	size(N)
Global mover	Mobility turnover	Intra-urban	Endogenous	Macro-Geo (MA)	$P(\lambda_{MA})$
Growth & Immigrant	Population growth	Inter-urban	Exogenous	Meso-Geo (TA)	$P(\lambda_{TA})$

Table 6

λ -values used for different geographical scales.

Geographical scale	# of AUs	λ
MA	316	12
TA (AKL)	104	10
TA (MKU)	87	8
TA (WTK)	55	7
TA (NSH)	53	7
TA (PAK)	17	5

empirical information currently available on the *persistent* attitudes of ethnic groups regarding relocation, the probability-based values set for each ethnic group will be done during calibration process (Section 4). However, it is assumed that when agents are forced to move (for various reasons), the emotional attachment to the existing location (where all the moving initially originate from, in the model) and the desire and perseverance to relocate in nearby or certain areas is somewhat higher among Europeans, followed by Pacific people and Maori, while it is lesser among Asians.

If an agent visits enough locations by reaching the end of his *search limit* goal without finding a place that matches his *tolerance*, he simply compromises and resettles in the *best* location found thus far. This includes the current location in which he resides (which can be seen as moving to a nearby area). In this sense, the agent exhibits “satisficer” (Benenson & Hatna, 2009) behaviour.

3.5. Measurements

In each cycle, multiple segregation measures and other related statistics (such as population numbers, proportions, vacancy numbers) are calculated for macro and *meso* geographical scales, using micro-spatial units (that is; the units of computation are always the AUs regardless of the measure being calculated for the metropolitan area or a territorial authority). These include *evenness* (multi-group Information Theory Index, H), *clustering* (Moran’s I, GMI), *concentration/ isolation* (Plurality Index, PI).

The following notation describes the measures: consider a macro spatial region (metropolitan area) R populated by M mutually exclusive ethnic population subgroups, indexed by m ; and let r index five *meso* sub-regions (territorial authorities). Let j index area units encompassed within the macro region R (or sub-regions r). Let π represent population proportion. Thus, we have:

M : number of ethnic groups

J : number of area units in R (or territorial authority r)

t_j : total population in area unit j

t_{jm} : total population of group m in area unit j

T : total population in R (or territorial authority r)

T_m : total population of group m in R (or territorial authority r)

π_m : the proportion of group m in R (or territorial authority r)

π_{jm} : the proportion of group m in area unit j

The information theory index (H) (Massey & Denton, 1988; Reardon & Firebaugh, 2002) is defined as

$$H = \sum_{j=1}^J \frac{t_j (E - E_j)}{T E} \quad (5)$$

where E is the entropy or the extent of ethnic diversity in R (or

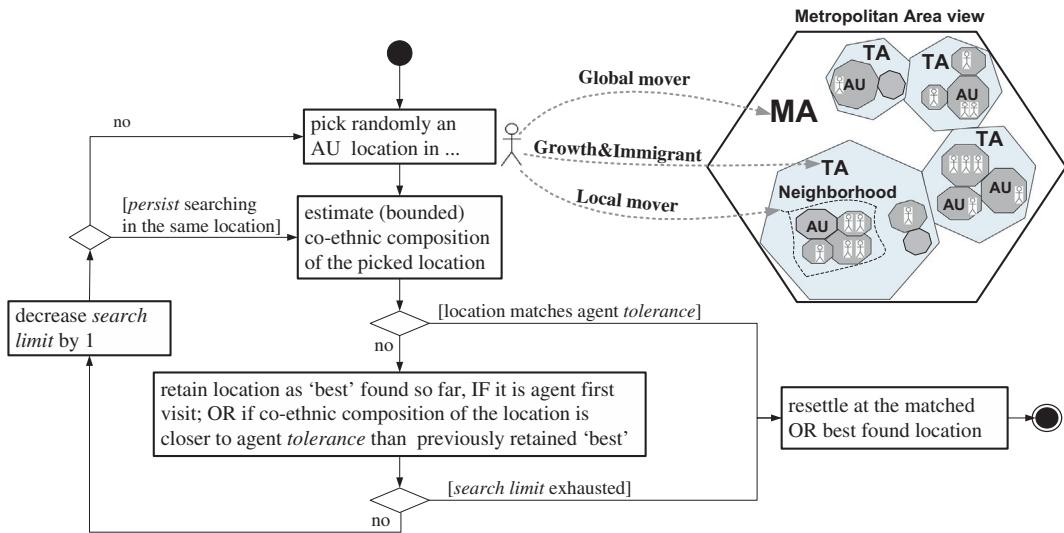


Fig. 5. Agent settlement process

territorial authority r), and E_j at area unit level, defined as

$$E = \sum_{m=1}^M \pi_m \ln \frac{1}{\pi_m} \quad (6)$$

$$E_j = \sum_{m=1}^M \pi_{jm} \ln \frac{1}{\pi_{jm}} \quad (7)$$

The plurality index (PI) of an ethnic group m at R (or territorial authority r) level is defined as

$$PI_m = \sum_{j=1}^J S_{jm} \quad (8)$$

where S_{jm} is the plural status of group m in area unit j , which is equal to 1 if the group is in plurality, or 0 if not.

Ultimately, the local (LMI) and global (GMI) Moran's I index (Anselin, 1995) for ethnic group m in R (or territorial authority r) are defined as:

$$LMI_{jm} = I_{jm} = \frac{(\pi_{jm} - \bar{\pi}_{jm}) \sum_{j=1}^J w_{jk} (\pi_{km} - \bar{\pi}_{km})}{\sum_{j=1}^J (\pi_{jm} - \bar{\pi}_{jm})^2} \quad (9)$$

$$GMI_m = \frac{\sum_{j=1}^J I_{jm}}{\sum_j \sum_k w_{jk}} \quad (10)$$

where (here) the variable of interest is the proportion of group m in area units j , and $\bar{\pi}_{jm}$ is the mean of π_{jm} values over contained area units ($\bar{\pi}_{jm} = [\sum_{j=1}^J \pi_{jm}] / J$). Furthermore, w_{jk} is the element of the spatial weight matrix that indicates the spatial link between area units of j and k . If the observation j is neighbour of (adjacent to) observation k , then $w_{jk} = 1$, otherwise $w_{jk} = 0$.

3.6. Views, recordings, and updates

The model can be used to perform a series of optional displays, updates, snapshots of the world view, charts/graphs, as well as recordings of various measures and statistics, at configurable (user-defined) intervals for advance analysis of simulation outputs.

4. Model calibration

The parameters involved in the calibration of a given model are normally those for which the exact empirical values are not available/

known or that (empirically informed) presupposed values are not considered fixed in specific scenarios and therefore open to further tuning (adjustment).

However, for the scenarios presented in this article, we (empirically or hypothetically) assigned specific values to the majority of the model parameters with the exception of *tolerance* (Θ) and *p_{persist}* parameters. To calibrate the model using these parameters, we use an approach similar to *pattern-oriented modelling* (Grimm et al., 2005) method. Our approach involves instructing the model (using a parameter file) to generate (in batch mode) simulation outputs by sweeping through an exhaustive combination of plausible input values for these parameters (with different values for each ethnic group). This stipulates searching for a model instance (a combination of parametric values) among a large parameter space that matches as close as possible with the empirically obtained segregation pattern(s). This process is vulnerable to the *equifinality* (Oreskes, Shrader-Frechette, & Belitz, 1994; O'Sullivan & Perry, 2013) problem (the possibility of having numerous instances of the model that can produce the same pattern). To overcome the *equifinality* problem, we use multiple measures of segregation (H, GMI and PI) at metropolitan area scale as "a series of filters through which [an instance of the] model should pass" (O'Sullivan & Perry, 2013, p. 223) until the most satisfactory instance is found. Although the found model instance may not produce precise (or even very closely) comparable patterns to the empirical benchmarks, it would be a reasonably good compromise on multiple patterns.

In residential segregation models, the *tolerance* (Θ) value plays an important role in the dynamic of the simulation and is sensitive to population proportion. It is difficult (and to a certain level, almost impossible) for a smaller minority group with high Θ -value to find locations with a matching Θ -level of concentration. In most residential segregation or Schelling-based models, the population size is fixed and Θ -values are constant. However, our model needs to adapt itself to a new set of conditions (Silva & Clarke, 2002), as the population increases and proportional size of the groups' populations subsequently changes. Even without projection years, the proportional sizes of populations (particularly of Europeans and Asians) are quite different at the beginning, middle and end of the simulations. With proportional sizes of about 57% for Europeans, 19% for Asians, 14% for Pacific people and 10% for Maori people in 2006, the found calibrated Θ -values of 0.37, 0.14, 0.12, and 0.08 approximately brings a 65%, 73%, 85% and 80% tolerance impact for each of these groups, respectively. Thus, the desire to be surrounded by own ethnic group is stronger for Pacific people, followed by Maori, Asians, and Europeans.

As tolerance preference threshold has a prominent role in the

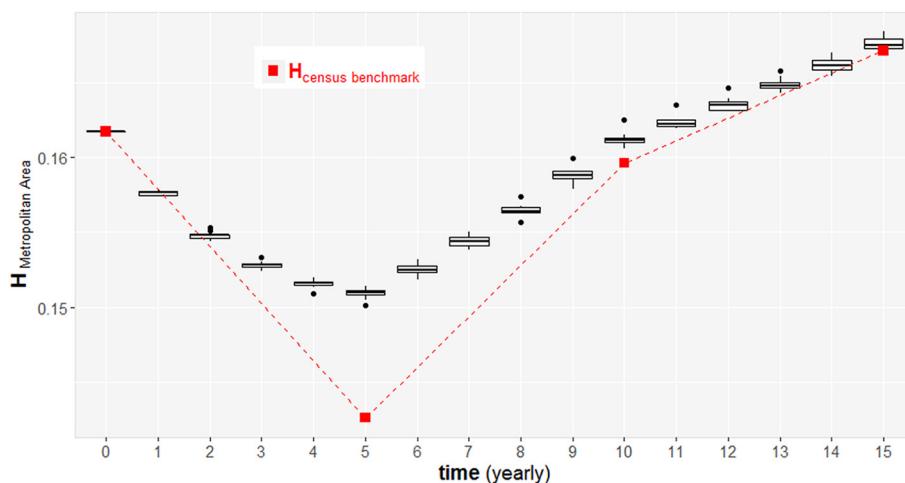


Fig. 6. H-values (simulated 10 times) for metropolitan area over 15 years after model calibration and their census-based equivalent benchmarks (in red) (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

formation and intensity of residential segregation, it is promising to note the achieved calibrated values correspond with some of the previous studies that indicate Pacific people are more likely to reside in “neighbourhoods where their co-ethnics predominated than was the case with Maori” (Johnston, Poulsen, & Forrest, 2009, p. 20) and that the Asians “are more widely dispersed” (Johnston, Poulsen, & Forrest, 2008, p. 238), sharing areas with Europeans to much greater extent than Pacific people and Maori.

Furthermore, the calibrated $p_{persist}$ values of 0.42, 0.22, 0.36 and 0.35 for Europeans, Asians, Pacific people and Maori respectively correspond with our presumption that the emotional attachment to the existing location and perseverance to relocate in nearby or certain areas are stronger in Europeans, followed by Pacific people and Maori, and lower among Asians.

Fig. 6 shows the H-values for the metropolitan area over 15 years compare to their equivalent census-based benchmarks after the model calibration. The H-values are the results of 10 simulation runs using different random seeds. Unlike the common practice in social simulation, we do use an average of 10 simulation runs, since communicating only summary statistics, “can hide important episodes of volatility” (Moss & Edmonds, 2005, p. 21).

Fig. 7 demonstrates how the replication of ethnic mosaic compares with its data-driven portrait. However, as Brown, Page, Riolo, Zellner, and Rand (2005) found, simulations generate some “invariant” area units (or “regions”) where the concentrations or plurality of a specific group is more frequent, or occur virtually all the time, whereas in other area units they are “path dependent”. For example, the calibration of the model did relatively well in replicating areas where Pacific people (in green) are in plurality, and less well in generating the (exact location of) Asian plurality cluster (in red).

The simulations generated plurality index (Fig. 8) and global Moran's I (Fig. 9) values for each ethnic group, which are fairly comparable to the census-based benchmarks – given that precise regeneration of several empirical patterns at the same time would be exceedingly difficult, if not impossible (unless all the agents in the model will ultimately make the exact same resettlement movements as their embodied people in real world).

Nevertheless, these few examples illustrate the capability of the model to replicate plausible historical residential distribution and segregation in the selected urban area for the study, although empirically-based details of the decision-making process are not fully present (implemented).

5. Model experimentation

Considering the lack of detailed information (data) on the process of residential location decision making and the subsequent limited

realistic explanatory factors implemented in the model and the lack of feasibility in assigning empirical values with certainty to few parameters, in this article, we concentrate on presenting the outcomes of selected experimentations which are not vulnerable to potential uncertainties in the model parameters.

Moreover, like many social phenomena, the residential segregation is produced by individual actions (of causal agents) that are partly (at the least), determined by the constraints presented by the urban environment and situations in which resettlement actions take place. The intent of this article is not to expound various causal chains linking all the elements involved in constituting this social and geographical phenomenon, but rather, in the following experimentations, our primary focus is to describe the observed (manifested) patterns of population shift and segregation.

5.1. Population change

States can impact population growth in various ways such as changes to the immigration rules, free access to birth control/ sterilisation or providing incentives to promote natural population growth by offering ‘baby-bonus’ programmes and the like.

We use Statistics New Zealand TA-based population change projection estimates (Table 7) for the major ethnic groups as average annual rates to assess the ethnic population distribution and segregation of the urban area for 2021, based on the assumptions about their mechanisms of settlement. These are based on three alternative series designated as *low*, *medium* and *high* which do not correspond with probabilities, but indicate the combination of certain assumptions. For instance, the *low* projection assumes low fertility, high mortality, low migration, and high inter-ethnic mobility. Conversely, the *high* projection assumes high fertility, low mortality, high migration, and low inter-ethnic mobility.

Fig. 10 shows the H-value outcomes of the simulation of scenarios for the entire urban area.

The simulation outcomes indicate that weighted average deviation of each area unit's population entropy from the ethnic entropy of the macro spatial container (whether this is the entire metropolitan area, as shown in Fig. 10, or a territorial authority, as shown in Fig. 11) is usually greater in the *low* context scenario, followed by *medium* and *high*, notwithstanding that this is less prevalent in *medium* and *high* cases for *meso* geographical cases of Papakura, Waitakere and, to a lesser extent, Auckland.

This indicates that the context of higher fertility and migration will not necessarily lead to higher levels of residential segregation measured by H-index at the urban area scale. In other words, higher population growth (and immigration) does not automatically exacerbate the intensity of ethnic population entropy.

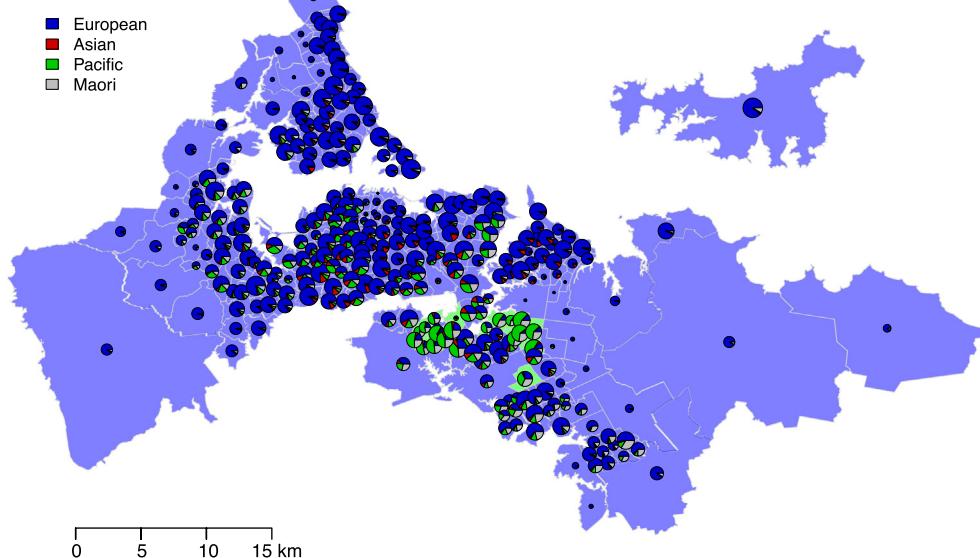
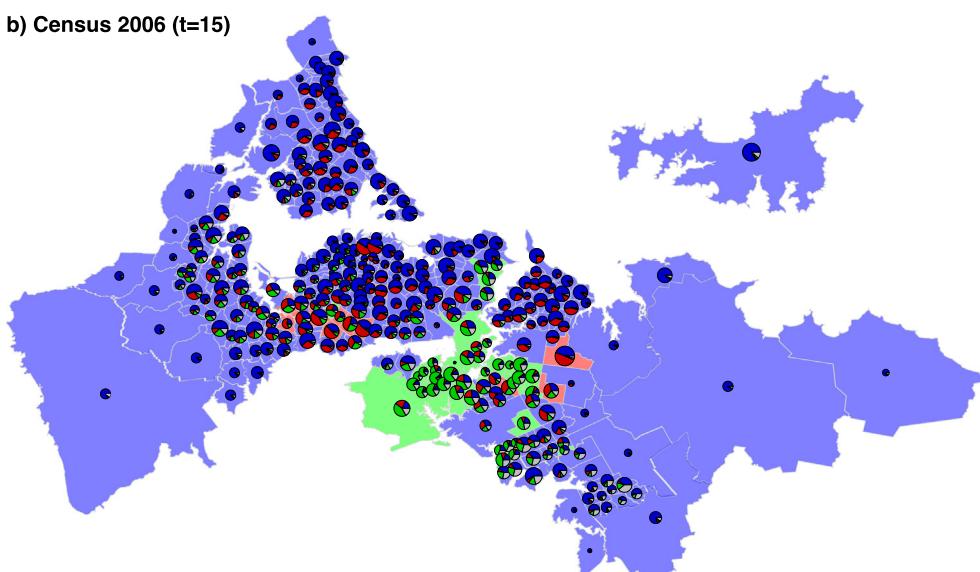
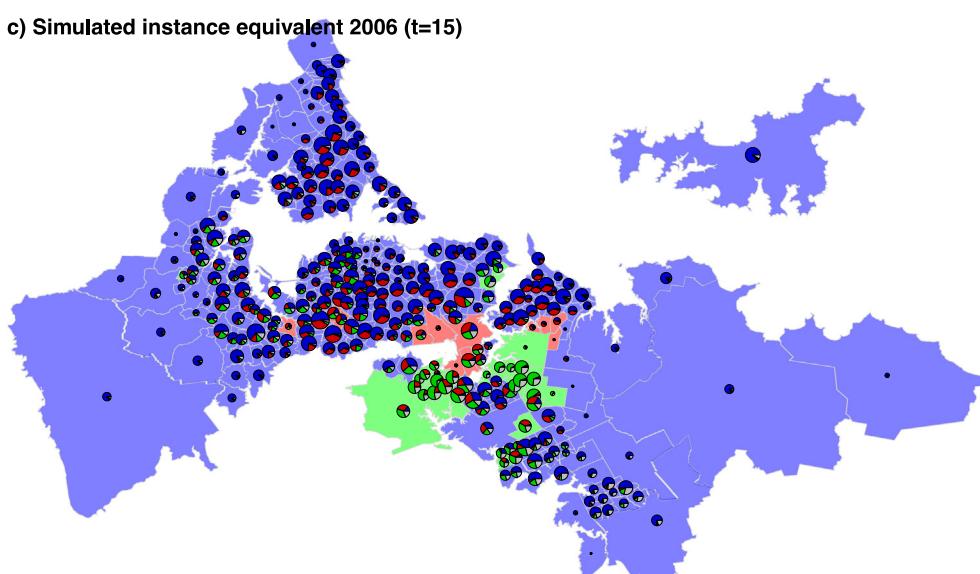
a) Census 1991 (t=0)**b) Census 2006 (t=15)****c) Simulated instance equivalent 2006 (t=15)**

Fig. 7. Maps of the metropolitan area where the colour of area units indicates the plurality status of the ethnic groups: a) initial state census 1991, b) census 2006, c) a simulated model instance, equivalent 2006

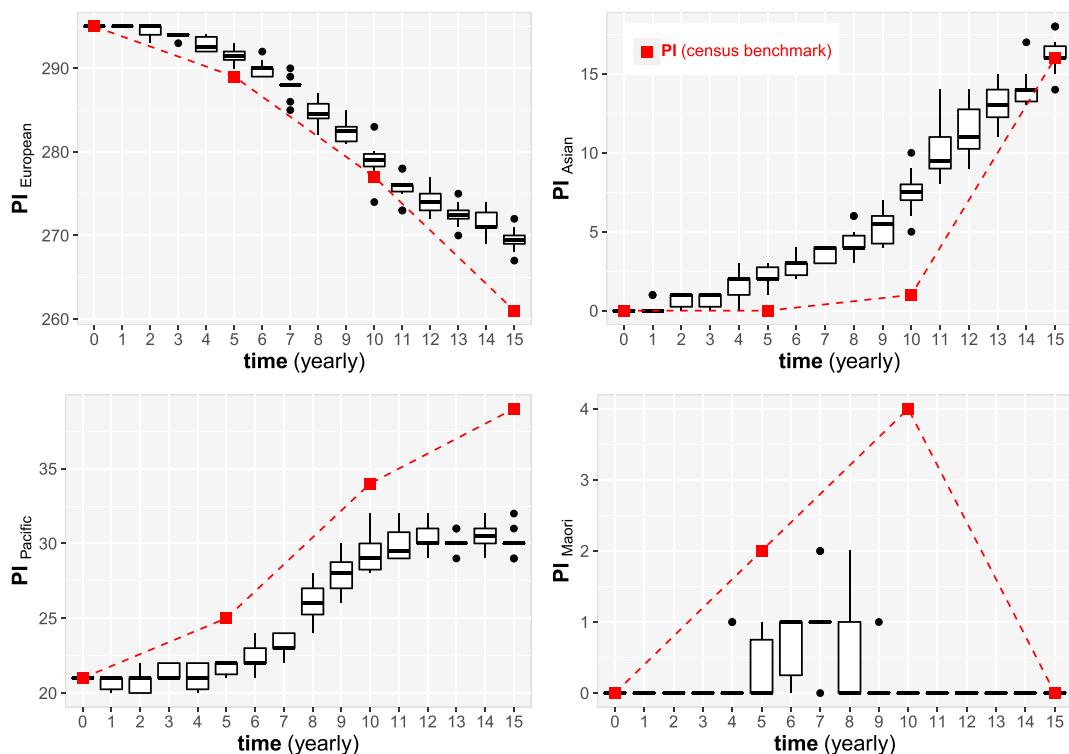


Fig. 8. Plurality Index/PI-values (simulated 10 times) for metropolitan area over 15 years after model calibration and their census-based equitant benchmarks (in red) (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

However, it is generally possible to detect different observations across numerous ranges of indices which measure various dimensions of segregation. For example, the simulations of *low* context scenario resulted in having less similar proportional values in the area units which cluster together (lower GMI-values) for Europeans, Maori,

Asians, and less evidently Pacific people. On the other hand, PI-values were clearly greater for Pacific people and Maori. For Asians, the PI-values were also greater, but more than *medium* context and to some extent comparable to *high* context scenario. So, Europeans were the only group where *low* context scenario produced significantly lower PI-

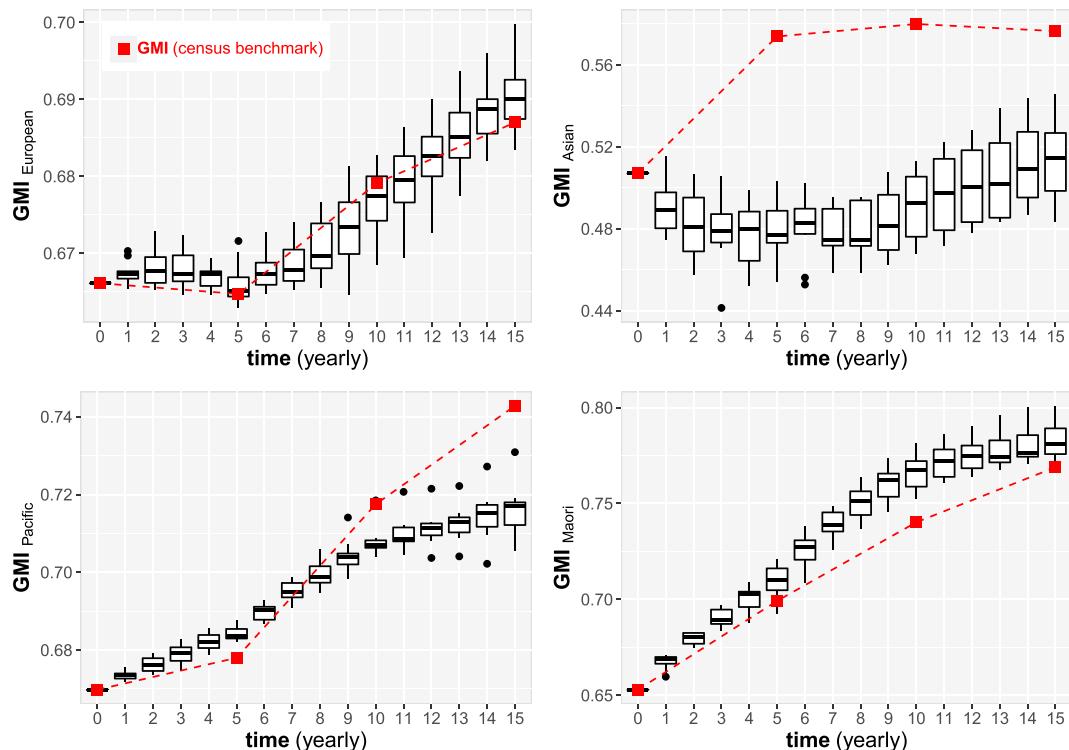


Fig. 9. Global Moran's I/ GMI-values (simulated 10 times) for metropolitan area over 15 years after model calibration and their census-based equitant benchmarks (in red) (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 7

Low, medium and high SNZ estimated values used for simulation of scenarios from 2006 to 2021.

	Low				Medium				High			
	Eur	Asi	Pac	Mao	Eur	Asi	Pac	Mao	Eur	Asi	Pac	Mao
NSH	-0.6	2.2	0.8	0.2	0.4	3.9	1.8	1.4	1.3	5.3	2.7	2.5
WTK	-0.8	2.6	2.0	0.9	0.3	4.2	2.9	1.9	1.3	5.6	3.8	2.8
AKL	-0.5	1.6	-0.8	-0.3	0.6	3.3	0.6	0.8	1.5	4.8	1.8	1.9
MKU	-2.0	2.6	2.0	0.7	-0.6	4.2	3.0	1.7	0.7	5.6	3.8	2.6
PAK	-0.8	2.2	4.3	1.1	0.3	4.0	5.3	2.3	1.3	5.5	6.1	3.3

values, although the PI-values for this group were also lower under *medium* and *high* context scenarios.

Figs. 10 and 11 also reveal that the levels of segregation at mesoscale spatial entities are not necessarily more intense than those at the macroscale urban area. In this case, only H-values at mesoscale of *Manukau* are actually greater than those at the metropolitan area.

5.2. Vacancy change

Governments (State or local) also have considerable power to influence the level of housing vacancy, for example, by promoting and financing (social) housing development projects. In the sphere of residential segregation modelling, it appears that little attention has been paid to the impact of vacancy rate on the dynamics of residential segregation. In fact, vacancy rates are often arbitrary and applied uniformly throughout a simulated city, although in reality, they vary considerably from city to city and from one area to another within the same city. To examine its potential effects, we reduced the vacancy rate for the *Auckland* TA from its original empirical value of 7.8% (see Table 4) to 2%, so that it can stochastically vary between 1% and 3%, although this change was only applied for the projection years (after 2006). The simulations for this *reduced vacancy* scenario are conducted where the population growth is set to the *medium* context and the outcomes are compared with the outcomes obtained in the *medium* context simulations described in the previous section where the *original* vacancy rate was used.

Fig. 12 presents proportional population change of simulation instances in 2021 from 2006 for each ethnic group at each territorial authority under two simulated scenarios. For example, it shows that while the proportion of Asians in *Auckland* TA increased since 2006 in the original scenario (original vacancy rate), it occurred less intensely (indicated by paler colour) when the vacancy was reduced. In case of *reduced vacancy* scenario, darker shades of green in other TAs also indicate where, in neighbouring territorial authorities, some of those

agents who were not able to resettle in *Auckland* TA (because of lack of vacancy) wound up. In the case of Asians, their proportions have increased expressively in *Manukau* and *Waitakere* when the vacancy was reduced. For Maori, the proportional increase occurred in *Waitakere* and *Papakura*.

In residential segregation terms measured by H-index, the reduced vacancy in one of the five territorial authorities has led to higher H-value at the metropolitan area scale (See Fig. 13.). This is also true for *Auckland* and *Waitakere*, while in *Manukau* the increase was actually lower comparing to the original case. In *Papakura*, the drop in H-values was less when the vacancy was reduced.

In terms of clustering dimension of residential segregation measured by GMI, the spatial autocorrelation for Asians was more in the negative zone for *Auckland*, when its vacancy was reduced, indicating higher dissimilar values clustering together. The reverse was true in *Manukau* (although less intensely) where the outcome of the reduced vacancy caused slightly greater positive GMI-values, indicating that shift of Asian population to *Manukau* created greater similar proportional values in the area units clustering together.

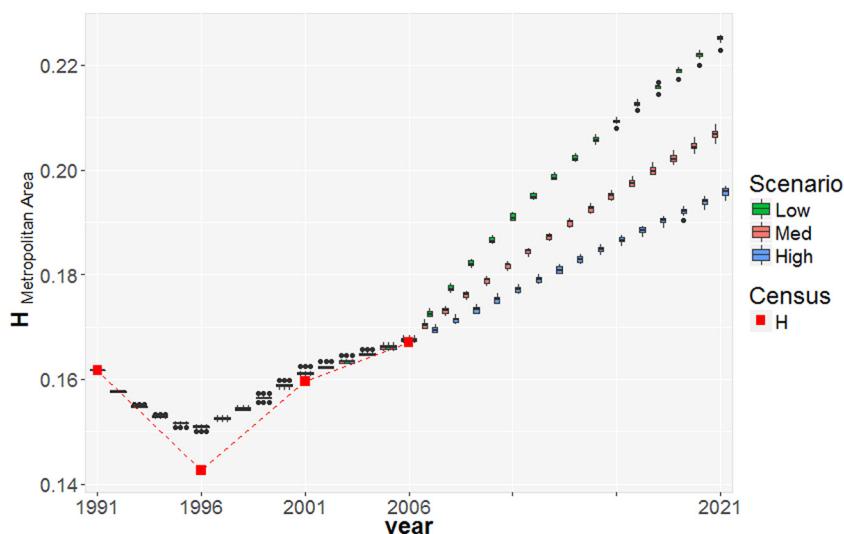
On the other hand, tighter vacant housing in *Auckland* contributed to significantly lower numbers of area units in the same TA where Asians were in plurality, while the opposite occurred in *Manukau*, where PI-values were significantly higher for Asians.

Ultimately, through this experiment, it is imperative to recognize the role and impact of housing vacancy in the residential segregation dynamic.

6. Conclusion

This research article introduces a new multi-scaled spatiotemporal agent-based model of residential segregation. The new methodological contribution of the model can be described as a combined set of characteristics which are put into operation to investigate residential segregation dynamics with specific reference to empirical (census) data.

Fig. 10. Simulated H-values (10 times) for metropolitan area under low (green), medium (coral) and high (blue) scenarios (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



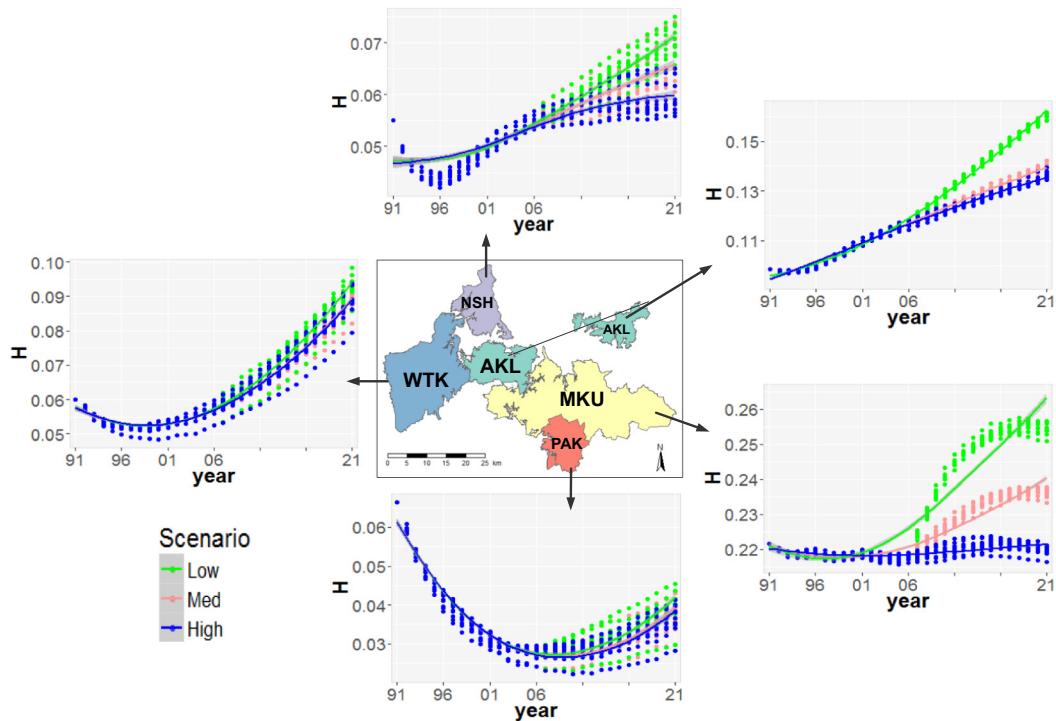


Fig. 11. Simulated H-values for five meso territorial authorities of Auckland City (AKL), Manukau (MKU), North Shore (NSH), Waitakere (WTK) and Papakura (PAK), under low, medium and high scenarios

The model can deal with entire population sizes corresponding to census values. Similar to microsimulation models, the probability based mobility turnover process allows focusing on the fraction of the population that relocate each year for various reasons. The choice mechanism comprises of individual preferences, economic circumstances, and behaviours conditioned by *bounded rationality* and other urban environment factors such as empirical housing vacancy. Both endogenous *intra-urban* and exogenous *inter-urban* migrations take place in realistic space consisting of multiple bounded-neighbourhood, recognising the fact that for most policy-relevant questions regarding urban spatial phenomenon, “space does matter” (Banos, 2012, p. 404).

We argued that the realistic spatial representation of a model is an important step aimed at building a “real-world situation” (Benenson, 1998, p. 40), as it creates a natural environment for using and integrating empirical data in a more consistent way. In turn, this leads to a more intuitive, reliable and robust evaluation of the model. The

realism of a model can also be increased by expanding on *explanatory factors*, such as using additional behavioural, urban (environmental), social and economic factors.

By taking advantage of the realistic feature of the model, we used a *pattern-oriented* approach to calibrate the model and demonstrated that the model is capable of generating patterns that are fairly comparable to the empirical benchmarks built from the application of multiple measures of residential segregation on several quinquennial periods of census data, although detailed mechanisms regarding residential decision making were not fully implemented.

The outcome of the experiment with the model where simulations were projected into the future using SNZ population growth estimates proved that higher population growth (and immigration) does not necessarily exacerbate the intensity of residential segregation. Moreover, it confirmed that segregation is not necessarily greater at the mesoscale than at the macroscale. Also, the result of an additional model

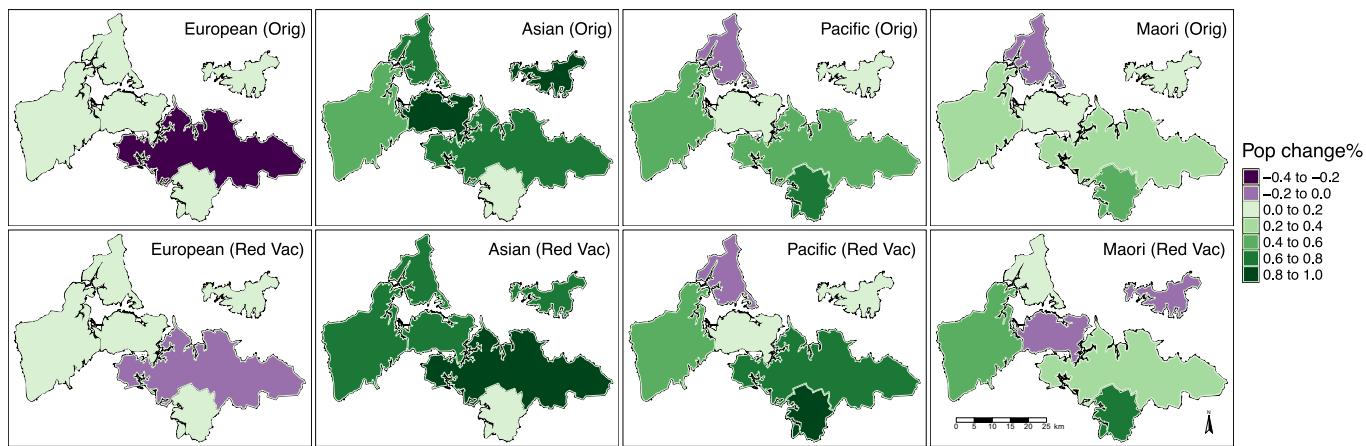


Fig. 12. Spatial representation of simulation outcomes: the percentage of population change for each ethnic group under two scenarios (*original* vs *reduced vacancy*) in each of the five territorial authorities. The gradients from lighter green (or purple) to darker indicate the amount of increased (or declined) proportion of ethnic group in 2021 from 2006 (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

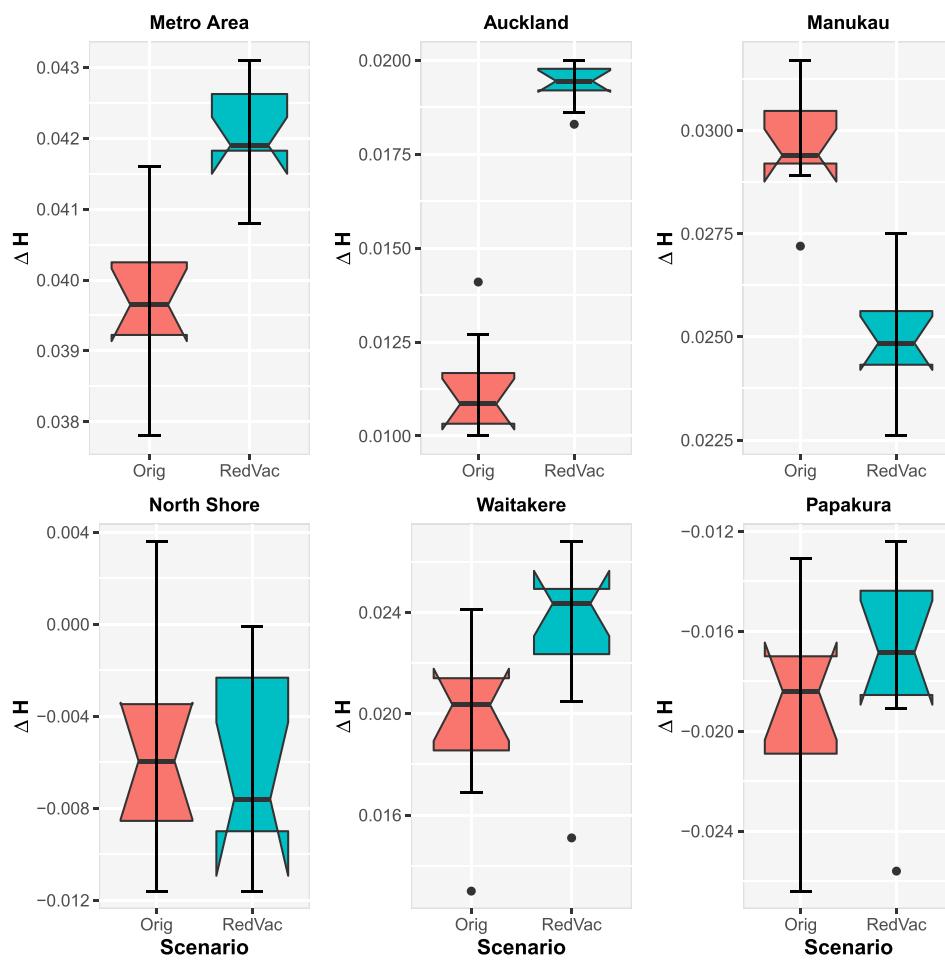


Fig. 13. H-values change in 2021 from 2006 under two scenarios (original vs reduced vacancy) for metropolitan area and five TAs

experiment demonstrated the important impact of housing vacancy effects on the dynamic and intensity of segregation – a topic which, thus far, has attracted little attention in the sphere of modelling of residential segregation.

There are a wide range of experiments that can be explored with this model. Furthermore, improvement (extension) of the model will provide possible avenues for future research. For example, expansion of more realistic *explanatory factors* to include decision-making based on the cost of housing located in different income zone neighbourhoods will provide a new direction for a set of scenarios for investigating the effects of housing cost on the residential segregation dynamic.

From an urban planning and policy related point of view, we argue that a realistic spatiotemporal and multi-scaled agent-based model capable of simulating large-scale population mobility dynamic along with the related social mechanisms is a powerful tool for visualisation and analysis of a multidimensional phenomenon nonlinearly emerging in a complex urban system. In addition, it assists us to compare and understand the effects of various factors on the current and future state of ethno-demographic distribution, structure and segregation mosaic of

a real metropolitan area and its smaller spatial constituents. Therefore, because of the embedded realism that is used in this model, it is suitable to be used as a more effective simulation tool for communicating the acquired insights in residential segregation dynamics, contribute to policy debates, and to help in planning ahead the adequate level of public services and infrastructure needed in the areas most largely affected by forecasted changes.

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Appendix A. TA-based ethnic population and yearly average change: SNZ vs study

TA	Data (selected AUs)	1991	1996	2001	2006	Change: %AVG/1Y
Europeans						
North Shore	SNZ (All)	N/A	154,600	157,100	163,500	0.83
	Study (53)	134,122	134,870	138,229	144,291	0.85
Waitakere	SNZ (All)	N/A	126,100	125,400	128,000	0.65
	Study (55)	102,244	102,937	102,797	103,969	0.64

Auckland	SNZ (All)	N/A	253,100	250,500	256,200	0.71
	Study (104)	211,177	209,236	212,010	217,250	0.79
Manukau	SNZ (All)	N/A	155,800	150,600	153,400	– 2.68
	Study (87)	131,832	129,550	126,841	129,145	– 2.69
Papakura	SNZ (All)	N/A	31,500	30,900	32,100	0.53
	Study (17)	26,578	25,866	25,546	26,056	– 2.52
Asians						
North Shore	SNZ (All)	N/A	16,800	26,100	41,900	0.96
	Study (53)	5322	13,951	21,500	34,852	0.99
Waitakere	SNZ (All)	N/A	12,600	20,000	32,400	0.93
	Study (55)	5200	9847	15,668	25,090	0.93
Auckland	SNZ (All)	N/A	52,200	76,600	109,100	1.07
	Study (104)	24,067	41,564	60,358	86,320	1.09
Manukau	SNZ (All)	N/A	30,000	46,000	75,700	1.04
	Study (87)	12,894	24,070	37,287	60,400	1.05
Papakura	SNZ (All)	N/A	1900	2700	4000	0.67
	Study (17)	1014	1494	2111	3033	0.66
Pacific people						
North Shore	SNZ (All)	N/A	5800	6600	7300	0.63
	Study (53)	3532	4743	5502	5994	0.68
Waitakere	SNZ (All)	N/A	20,600	26,300	30,500	0.85
	Study (55)	13,423	16,066	20,558	23,431	0.85
Auckland	SNZ (All)	N/A	52,400	54,400	56,600	0.74
	Study (104)	39,702	40,421	43,045	45,013	0.77
Manukau	SNZ (All)	N/A	66,500	82,300	99,800	1.00
	Study (87)	44,335	49,169	63,473	75,625	0.99
Papakura	SNZ (All)	N/A	3100	3500	4900	0.65
	Study (17)	2154	2375	2658	3719	0.63
Maori						
North Shore	SNZ (All)	N/A	13,300	13,300	14,200	0.57
	Study (53)	8162	11,278	11,177	11,561	0.72
Waitakere	SNZ (All)	N/A	22,200	24,000	26,200	0.74
	Study (55)	14,459	17,719	18,978	20,038	0.78
Auckland	SNZ (All)	N/A	35,800	33,000	34,900	– 2.57
	Study (104)	25,604	28,115	26,363	26,808	0.60
Manukau	SNZ (All)	N/A	47,800	50,000	53,900	0.79
	Study (87)	34,386	36,523	38,665	41,127	0.80
Papakura	SNZ (All)	N/A	9800	10,600	12,500	0.69
	Study (17)	6906	7648	8236	9675	0.70

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