P9120 - Homework # 3

Assigned: October 31st, 2024 Due at 9pm on November 16th, 2024

Maximum points that you can score in this Homework is 20.

1. (5 points) Consider a binary classification problem, where the input $X \in \mathbb{R}^4$ and the output $Y \in \{0, 1\}$. We build classification model using a neural network with two hidden layers. The first hidden layer contains 3 units and the second hidden layer contains 2 units. tanh activation function is used in both hidden layers. The output layer contains a single unit with sigmoid activation function.

   (a) Draw a picture of the network, similar to the figure on page 22 of lecture 6 notes.

   (b) Write out an expression for $f(X)$. Be as explicit as you can. How many parameters are there?

   (c) Consider binomial deviance loss on page 20 of lecture 6 notes. Calculate gradients of the loss w.r.t. parameters in the NN model using chain rule.

2. (5 points) Python Lab 3 exercise. (P9120_Lab3_Exercise.ipynb) (The extra exercise at the end of the notebook is optional and will not be graded. All other exercises, including the backward propagation derivation, are required and will be graded.)

3. (5 points) Exercise 8 (Section 10.10) of [ISL Python version] (page 466).

   8. From your collection of personal photographs, pick 10 images of animals (such as dogs, cats, birds, farm animals, etc.). If the subject does not occupy a reasonable part of the image, then crop the image. Now use a pretrained image classification CNN as in Lab 10.9.4 to predict the class of each of your images, and report the probabilities for the top five predicted classes for each image.

4. (5 points) Exercise 13 (Section 10.10) of [ISL Python version] (page 467).

13. Repeat the analysis of Lab 10.9.5 on the `IMDb` data using a similarly structured neural network. We used 16 hidden units at each of two hidden layers. Explore the effect of increasing this to 32 and 64 units per layer, with and without 30% dropout regularization.

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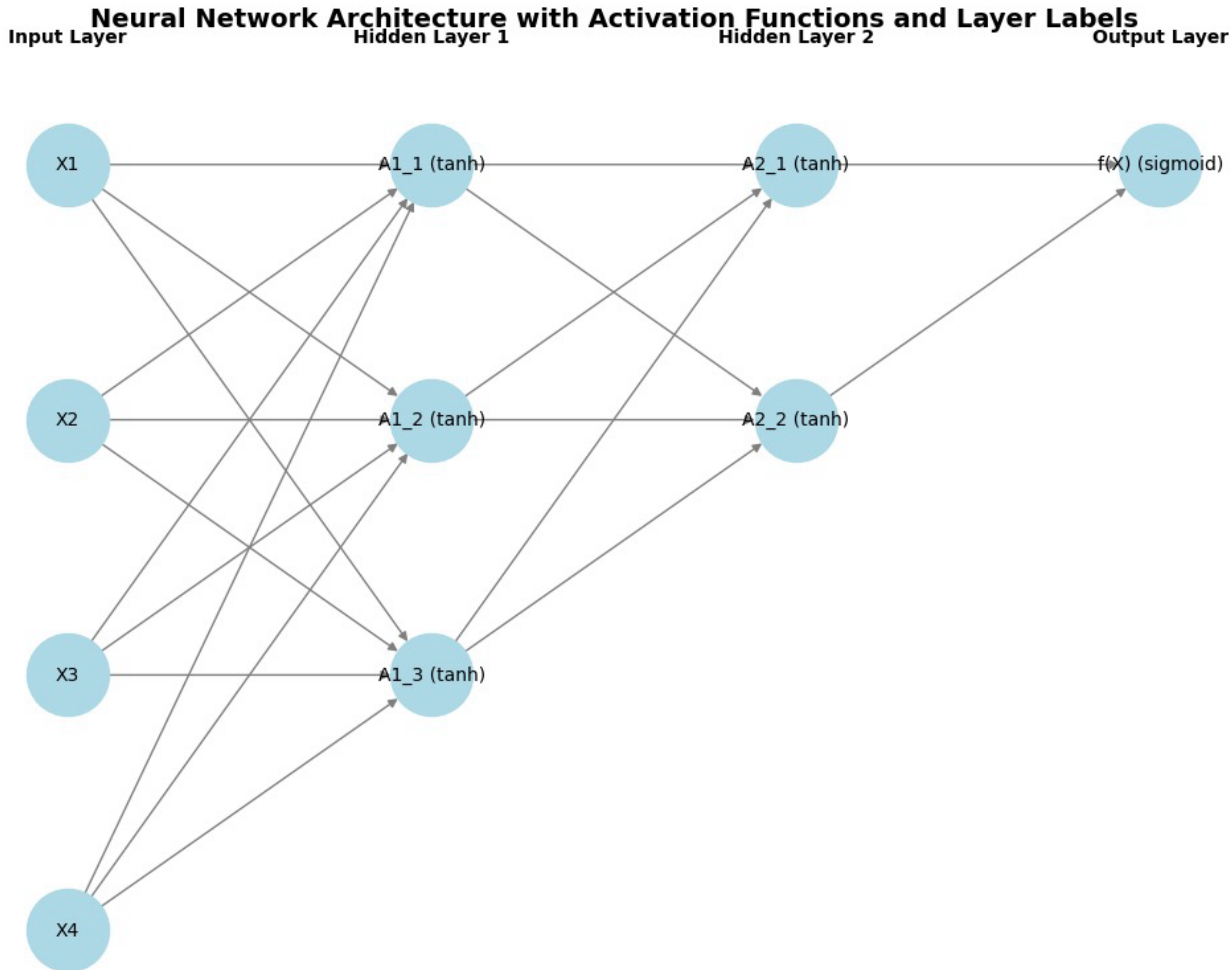
   (a) Draw a picture of the network, similar to the figure on page 22 of lecture 6 notes.



(b) Write out an expression for $f(X)$. Be as explicit as you can. How many parameters are there?

1) 1st hidden layer: $z_1^{(1)} = X^T w_1^{(1)} + b_1^{(1)}$ / $A_1 = \tanh(z_1^{(1)})$

$z_2^{(1)} = X^T w_2^{(1)} + b_2^{(1)}$ / $A_2 = \tanh(z_2^{(1)})$

$z_3^{(1)} = X^T w_3^{(1)} + b_2^{(1)}$ / $A_3 = \tanh(z_3^{(1)})$

$\# W = 4$ feature $\times 3$ unit$(A_i) = 12$ $\quad \# b = 3$

2) 2nd hidden layer: $z_1^{(2)} = X^T w_1^{(2)} + b_1^{(2)}$ / $A_1 = \tanh(z_1^{(2)})$

$z_2^{(2)} = X^T w_2^{(2)} + b_2^{(2)}$ / $A_2 = \tanh(z_2^{(2)})$

$\# W = 3$ unit in L1 $\times 2$ unit in L2 $= 6$ $\quad \# b = 2$

3) output layer: $z^{(3)} = A^T w^{(3)} + b^{(3)}$ / $f = \mathrm{sigmoid}(z^{(3)})$

$\# W = 2$ unit in L2 $\times 1$ in output $= 2$ $\quad \# b = 1$

$12 + 3 + 6 + 2 + 2 + 1 = 26$ parameter

$\therefore$ Total 26 parameters.

(c) Consider binomial deviance loss on page 20 of lecture 6 notes. Calculate gradients of the loss w.r.t. parameters in the NN model using chain rule.

$$l(\theta) = -[Y \log(f(X)) + (1-Y)\log(1-f(X))]$$

$\frac{dL}{dz^{(3)}} = f(X) - Y$ $\qquad \frac{dL}{dz^{(2)}} = \left(\frac{dL}{dz^{(3)}} \cdot W^{(3)}\right) \odot (1-(A^{(2)})^2)$ $\qquad \frac{dL}{dz^{(1)}} = \left(\frac{dL}{dz^{(2)}} \cdot W^{(2)}\right) \odot (1-(A^{(1)})^2)$

$\frac{dL}{dW^{(3)}} = \frac{dL}{dz^{(3)}} \cdot A^{(2)}$ $\qquad \frac{dL}{dW^{(2)}} = \frac{dL}{dz^{(2)}} \cdot A^{(1)}$ $\qquad \frac{dL}{dW^{(1)}} = \frac{dL}{dz^{(1)}} \cdot X$

$\frac{dL}{db^{(3)}} = \frac{dL}{dz^{(3)}}$ $\qquad \frac{dL}{db^{(2)}} = \frac{dL}{dz^{(2)}}$ $\qquad \frac{dL}{db^{(1)}} = \frac{dL}{dz^{(1)}}$