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Optimization Techniques

Extended Capacitated Warehouse Location Problem

Intermediate Report

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Abstract

This document presents a detailed overview of the current state of the project developed in the context of the curricular unit Optimization Techniques. We address an extension of the capacitated warehouse location problem, which includes different types of products and the existence of intermediate distributors. We introduced the designed Mathematical Programming model, along with a detailed analysis of the tests performed, through the IBM ILOG CPLEX Optimization Studio software.

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1 Introduction

The capacitated warehouse location problem is a topic vastly addressed in literature [1, 2], having several optimization algorithms been proposed for solving this problem. In its simplest formulation, this problem focuses on the selection of a number of warehouse locations which are required to service a set of customers at minimum cost. Each customer has an associated demand and there are constraints to the total demand that each warehouse meet.

Under the scope of the curricular unit Optimization Techniques, we have addressed an extended version of this problem, built upon the formulation presented by [2], with the aim of designing and testing a Mathematical Programming (MIP) model for it. The initial work considered includes the following considerations:

1. Limits to the number of warehouses: a lower and upper bound to the number of warehouses that may be opened are considered;

2. Minimum supplied demand: for a warehouse to be opened, it must supply at least a given amount of product;

3. Variable and fixed costs: variable costs are associated to supplying a given customer, while a fixed cost is associated to opening a specific warehouse.

We have extended this past formulation through the inclusion of different types of products that must be supplied, along with mandatory intermediate distributors that will be responsible for the delivery of the products directly to the customer. These distributors are already open and it is only required to select which to use. We considered the following types if distributors:

1. First level distributor: a large distributor, with capacity far greater than that of a warehouse. These present higher operational costs than second level distributor.

2. Second level distributor: a smaller distributor, with capacity smaller than that of a warehouse. Presents a reduced operational cost.

We further work under the assumption that warehouses are unable to access every distributor, which further limits the selection procedure.

For the testing of the developed MIP model, we have relied on the IBM ILOG CPLEX Optimization Studio software and associated tools, along with the datasets presented in [3]. Since these datasets are referent to the model formulation originally introduced in [1], further effort was put on the extension of these according to the formulation described in this section.

In what follows, section 2 introduces the MIP model developed for solving the problem, while section 3 presents an analysis of the results obtained. Section 5 summarizes the main conclusions to be drawn at this stage of the project.

2 Mathematical Programming Model

In the present section, we introduce the MIP model developed for the analysis of the considered extension of the capacitated warehouse location problem. In this context, Table 1 and Table 2 summarize the parameters and decision variables considered, respectively. The final problem formulation is presented immediately afterwards.

Parameter	Description
m	Number of potential warehouse locations
n	Number of customers
p	Number of products
d	Number of distributors
q_{jk}	Demand of customer j for product k
c_{ijk}	Cost of supplying all of the demand of customer j for product k from warehouse i
C_{jkz}	Cost of supplying all the demand of customer j for product k through distributor z
F_i	Fixed cost of opening warehouse i
Q_{ik}	Total capacity of warehouse i for product k
D_{kz}	Total capacity of distributor z for product k
L_i	Lower limit on the demand supplied by warehouse i, if it is opened
P_l	Lower limit on the number of warehouses that may be opened
P_u	Upper limit on the number of warehouses that may be opened
v_{iz}	$\begin{cases} 1, & \text{is warehouse } i \text{ is supported by distributor } z \\ 0, & \text{otherwise} \end{cases}$

Table 1: Parameters considered for the MIP model.

Decision Variable	Description
x_{ijkz}	Fraction of the demand of customer j for product k supplied from warehouse i through distributor z
y_i	$\begin{cases} 1, & \text{is warehouse } i \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$

Table 2: Decision variables considered for the MIP model.

Since the problem goal is to minimize the overall cost associated to the operation of the new warehouses, the objective function takes the following form:

$$\min \sum_{j=1}^{n} \sum_{k=1}^{p} \left(\sum_{i=1}^{m} c_{ijk} \cdot \sum_{z=1}^{d} x_{ijkz} + \sum_{z=1}^{d} C_{jkz} \cdot \sum_{i=1}^{m} x_{ijkz} \right) + \sum_{i=1}^{m} F_i \cdot y_i, \tag{1}$$

subjected to:

$$\sum_{i=1}^{m}\sum_{z=1}^{d} x_{ijkz} \geq 1, \qquad \forall_{j,k} \tag{2}$$

$$\sum_{n=1}^{j}\sum_{z=1}^{d} q_{jk} \cdot x_{ijkz} \leq Q_{ik} \cdot y_i, \qquad \forall_{i,k} \tag{3}$$

$$\sum_{m=1}^{i}\sum_{n=1}^{j} x_{ijkz} \cdot q_{jk} \leq D_{kz}, \qquad \forall_{k,z} \tag{4}$$

$$\sum_{n=1}^{j}\sum_{k=1}^{p}\sum_{z=1}^{d} q_j k \cdot x_{ijkz}, \geq L_i \cdot y_i \qquad \forall_{i} \tag{5}$$

$$P_l \leq \sum_{m=1}^{i} y_i \leq P_u \tag{6}$$

$$x_{ijkz} \leq \min(v_{iz}, \frac{Q_{ik}}{q_{jk}}) \cdot y_i, \qquad \forall_{i,j,k,z} \tag{7}$$

$$y_i \in \{0, 1\} \qquad \forall_{i} \tag{8}$$

$$x_{ijkz} \geq 0 \qquad \forall_{i,j,k,z} \tag{9}$$

Equation 2 ensures that the demand of every costumer for every product is satisfied. Equations 3 and 5 ensure that no customer is supplied from a closed warehouse and that the demand of each product supplied by a warehouse lies within the imposed limits (minimum demand supplied and warehouse capacity). Equation 4 guarantees that the capacity of each distributor is not exceeded. Ensuring that the number of open warehouses is within the predefined limits is performed through Equation 6. Equation 7 relates the amount supplied to an individual customer, through a given distributor, to the capacity of a warehouse. Moreover, it enables to prevent a warehouse to resort to a distributor to which it does not have access, providing bounds on the decision variable x_{ijkz}. Equation 8 is the integrality constraint and Equation 9 the non-negativity constraint.

3 Results and Discussion

In this section, we present the most relevant simulation outcomes that originated from a defined set of configurations. Table 4 summarizes the obtained results for the defined configurations. The following subsections are dedicated to further explaining the evaluated parameters and providing an interpretive description of the results.

Considering a total of 8 configurations, 8 different tests were performed with the model, using the parameters defined in Table 3.

Parameter	Config1	Config2	Config3	Config4	Config5	Config6	Config7	Config8
m	16	25	16	25	16	25	16	25
n	50	50	50	50	50	50	50	50
p	1	1	5	5	1	5	1	5
d	10	10	10	10	10	10	10	10
P_l	1	1	1	1	1	1	5	5
P_u	16	25	16	25	16	25	12	12
Coverage	Full	Full	Full	Full	Random	Random	Random	Random

Table 3: Parameter configuration for the simulations.

Results	Config1	Config2	Config3	Config4	Config5	Config6	Config7	Config8
Optimal Solution	1.06e+6	8.56e+5	4.87e+6	3.68e+6	1.06e+6	3.67e+6	1.07e+6	4.40e+6
Number of Open Warehouses	13	17	16	25	13	25	12	12
Gap (%)	0.00	0.29	0.00	0.00	0.00	0.00	0.00	0.83
Total Time (s)	1.08	5.97	3.50	3.94	0.52	2.32	0.5	120.62

Table 4: Simulation results obtained for each parameter configuration.

As the problem was extended in comparison to the original proposal, the testing datasets that were provided in [3] were not sufficient to simulate the model. Therefore, in order to obtain the additional values, a C++ data generator was formulated. All the generated data was created on the basis of the given original values and subjected to a proportional transformation using random ratios that are logical to the real problem.

- The minimum demand that a warehouse needs to supply in order to be worth to open the location is a percentage of the total capacity of each warehouse;

- The first level distributors have a capacity that is a ratio higher than the one of the original warehouse capacities, while the second level distributors' ratio is lower than the original warehouse capacities;

- The cost of supplying all the demand of a customer for a certain product through a distributor is given by a ratio of the given warehouse supply cost.

3.1 Varying number of warehouses

In the stated problem, the number of available warehouses is a main component in the objective function, as it adds a fixed cost per warehouse and supply costs for each product to each client. Therefore, it is expected that an increased number of warehouses results in a higher cost.

In order to validate this hypothesis, two simulations were performed using Config1 and Config2 which only differ in the number of available warehouses. Additionally, it is considered that all the distributors support all the warehouses, i.e. all warehouses can use any of the total number of distributors to store their products when supplying a client.

As declared in the first and second columns of Table 4, the optimal solution improves when fixing all parameters and only increasing the number of warehouses. This is the expected output since, for the same number of customers and intermediate distributors, there is a higher number of possible warehouses to supply the product. Therefore it is possible to narrow them to the lowest combination of costs.

A second part of the analysis to the number of warehouses was made using Config7 and Config8, with the goal of deriving what is the most limiting range of warehouses (higher minimum and lower maximum) that should be opened to still be possible to achieve a feasible solution. These configurations are similar to the previous configurations, however they assign randomly the distributors that are able to support each of the open warehouses. It should be noticed that in the present case, if the lower bound of open warehouses, P_l, is increased the objective function value will remain the same or necessarily increase if it is higher than the minimum number of open warehouses that satisfies all the constraints. For that reason, this parameter is no further explored. In the case of the maximum number of open warehouses, P_u, both simulations showed that a minimum of 12 locations should be open. Although the value is the same in both simulations, the cost increases for Config8 as the number of products is higher.

3.2 Varying number of products

The number of products is a limiting aspect of the problem as it is intrinsically linked to the solution space.

Config3 and Config4 were used in simulations dedicated to testing the effect of increasing the number of products, as it is the only differing aspect from the first two configurations. In Table 4 it is possible to observe that in both cases the objective function value increases since there are more costs associated with having a higher number of products being supplied.

Using Config8 the number of products was further increased in order to defy the model. Using 10 products, the optimal solution results in a further increased cost of 1.04e+7, with a gap to the optimal solution of 0.56% and a total processing time of 515.61 seconds. By increasing the number of used products to 50, the problem became much more computationally heavy and thus a limited simulation of 2 hours and 35 minutes was only able to achieve a gap of 74%.

3.3 Limited access and varying number of distributors

In the extended problem, one of the most restricting aspects is that the warehouses cannot supply products directly to the client as all have to go through a distributor. In a real scenario it is expected that not all warehouses are located conveniently in relation to the distributors, as geographical location can result in an unprofitable partnership. Alternatively it should not be expected that all warehouses establish contracts with all distributors. Therefore, we created a random attribution of partnerships between warehouses and distributors through an integer variable, v_{iz}. Therefore, it is expected that, since distributors influence the whole products' stream, their quantity, and consequent associated capacity, this will be the most limiting aspect of the extended model.

Simulations were performed using Config5 and Config6, which are similar to Config1 and Config2 respectively but with the mentioned random selection of distributors to support specific warehouses. Since the total demand capacity of the distributors is much larger than the total warehouse capacity, then the results do not differ much from the original tests.

Additionally, due to this requirement being associated with a limitation in capacity, it is expected that a minimum number of necessary distributors for each specific number of products will be required so that the model has a feasible solution. Therefore, using Config3 there is a minimum of 1 distributor that needs to be available and when increasing the number of product to 10, the number of necessary distributors has already increased to 3.

Additionally, the evolution of the available distributors is represented in relation to the objective function value and simulation time in Figure 1.

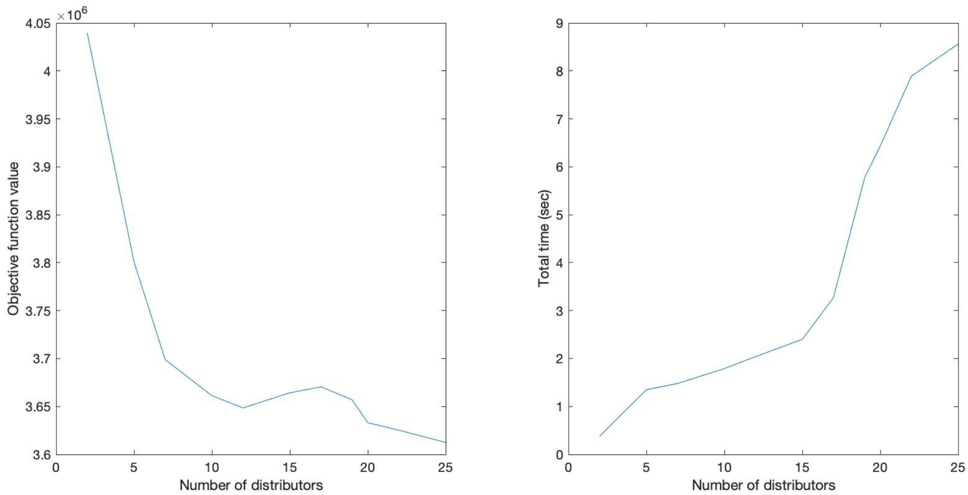

Figure 1: Study on the variation of the number of distributors in the objective function value and the total time.

Lastly, when varying the global capacity of the available distributors, it was noticed that the evaluation parameters change drastically comparatively with the remaining simulations. As point of comparison, the performed simulations show that the model cannot tolerate a decrease of more than 1/3 of the capacity used in the results given by Config6 since the problem becomes unfeasible due to the lack of total capacity in the distributors.

4 Conclusion

In conclusion, the formulated extended capacitated warehouse location model is an interesting approach to the classic problem that allows to analyse diverse aspects involving mixed integer programming and its sensitivity analysis. From the results obtained through simulations, we could confirm the expected logical outcomes and further evaluate specific cases of the extended model. Particularly, we could verify that the number of available warehouses inversely influences the objective function value, since the higher number of possibilities allows to choose the minimum total costs. Additionally, the constraint that defined the maximum number of possible open warehouses limits the solution space and depending on the remaining parameters, such as the number of products, it can even lead to unfeasible results. Regarding the number of products, the tests showed that a higher value leads to a more computationally heavy simulation and the objective value will increase. Lastly, the number of available distributors, with dedicated capacities, and the option to limit the support to specific warehouse makes this the most limiting restriction of the model.

All the simulations and results that are presented in this report will serve as comparison to the following Optimization course assignment in which the same model will be formulated using Constraint Programming.

References

[1] N. Christofides and J. E. Beasley, "Extensions to a Lagrangean relaxation approach for the capacitated warehouse location problem," *European Journal of Operational Research*, vol. 12, no. 1, pp. 19–28, 1983. 4

[2] J. Beasley, "An algorithm for solving large capacitated warehouse location problems," *European Journal of Operational Research*, vol. 33, pp. 314–325, feb 1988. 4

[3] J. B Easley, "Or-library: Distributing test problems by electronic mail," *Journal of the Operational Research Society*, vol. 41, no. 11, pp. 1069–1072, 1990. 4, 7