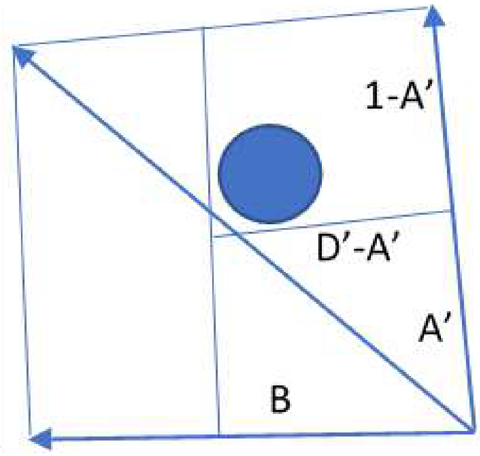

Figure 1. Variant 1 illustration.

$$\begin{cases} u = (A*(B-A))*B^2/\Big((A*B)^2 - A^2*B^2\Big) \\ v = ((A-B)*B)*A^2/\Big((A*B)^2 - B^2*A^2\Big) \end{cases}$$

Let the following:

$D = H_{AB}$

$D^{'} = H_{A'B}$

$1_{A'} = -1_A$

$1_D = H_{1_A 1_B}$

$1_{D'} = H_{1_{A'} 1_B}$

Variant 1: The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector $A^{'}$ intersects the perpendicular to B before B $||B|| \leq ||A'||/cos(\langle A', B\rangle)$ (see Fig.1) then the length of the result is equal to the ratio of areas:

$$\frac{(2*\|D'-A'\|-\|1_{A'}-A'\|)*\cot(\langle A',B\rangle)*\|1_{A'}-A'\|/2}{(1_{A'}*(1_{D'}-1_{A'})+1_B*(1_{D'}-1_B))/2}$$

Variant 2. The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector A' intersects B before the perpendicular to it $\|B\| > \|A'\|/\cos(\langle A', B\rangle)$ (see Fig.2) then the length of the result is equal to the ratio of areas:

$$\frac{\frac{(\|1_{A'}\|^2-\|A'\|^2)}{\cot(\langle A',B\rangle)}-\left(\frac{\|1_{A'}\|}{\cos(\langle A',B\rangle)}-\|B\|\right)^2*\cot(\langle A',B\rangle)}{2*(1_{A'}*(1_{D'}-1_{A'})+1_B*(1_{D'}-1_B))/2}$$

Variant 3. The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector A’ intersects the perpendicular to B before B $\|B\| \leq \|A'\|\cos(\langle A', B\rangle)$ (see Fig.3) then the length of the result is equal to the ratio of areas:

$$\frac{\|D'-A'\|*\|D'-A'\|*\tan(\langle A',B\rangle)/2}{1_{A'}*(1_{D'}-1_{A'})+1_B*(1_{D'}-1_B)/2}$$

Variant 4. The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector A’ intersects B before the perpendicular to it

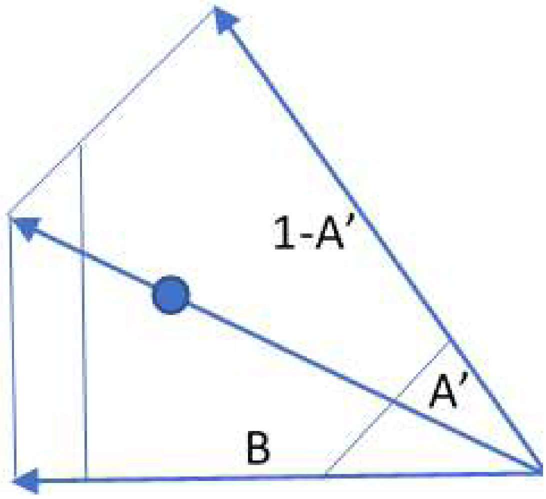

Figure 2. Variant 2 illustration.

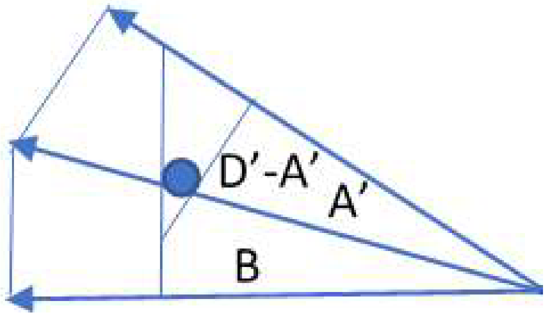

Figure 3. Variant 3 illustration.

$\|B\| > \|A'\|/cos(\langle A', B\rangle)$ (see Fig.4) then the length of the result is equal to the ratio of areas:

$$\frac{\|B\|*\|B\|-\|A'\|*\|A'\|*\tan(\langle A',B\rangle)/2}{1_{A'}*(1_{D'}-1_{A'})+1_B*(1_{D'}-1_B)/2}$$

Variant 5. Vectors A and B are co-oriented then the length of the result is: $\min(\{\|A\|\} \cup \{\|B\|\})$.

Variant 6. The angle between vectors A and B is acute ($A*B>0$), the perpendicular to vector A intersects B before the perpendicular to it $\|B\| > \|A\|/\cos(\langle A, B\rangle)$ (see Fig.5) then the length of the result is equal to the

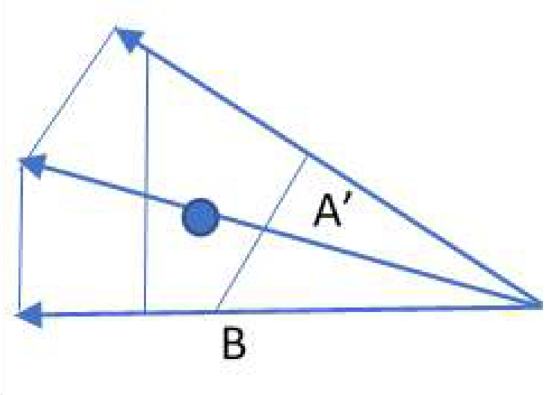

Figure 4. Variant 4 illustration.

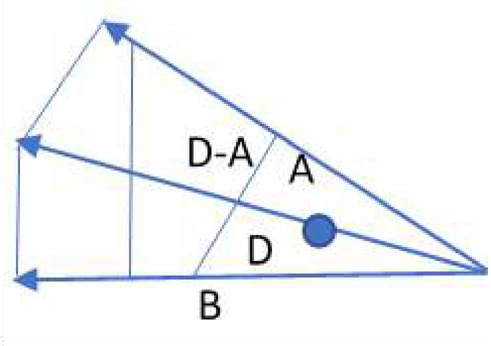

Figure 5. Variant 6 illustration.

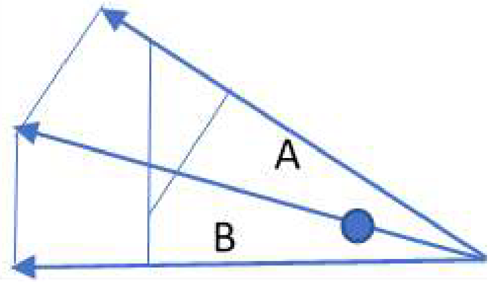

Figure 6. Variant 7 illustration.

ratio of areas:

$$\frac{A*(D-A)/2}{(1_A*(1_D-1_A)+1_B*(1_D-1_B))/2}$$

Variant 7. The angle between vectors A and B is acute ($A * B > 0$), the perpendicular to vector A intersects the perpendicular to B before B $\|B\| \leq \|A\|/\cos(\langle A, B\rangle)$ (see Fig.6) then the length of the result is equal to the ratio of areas:

$$\frac{((A+B)*D-A*A-B*B)/2}{(1_A*(1_D-1_A)+1_B*(1_D-1_B))/2}$$

Variant 8. Vectors A and B are orthogonal $((A * B = 0) \wedge (\|A\| + \|B\| > 0))$ (see Fig.7) then the length of the result is equal to the ratio of areas:

$\|A\| * \|B\|/1$.

Variant 9. Vectors A and B are differently directed, then the length of the result is equal to:

$$\max(\{0\} \bigcup \{A + B - 1\})$$

Properties of negation:

$$A =\sim (\sim A)$$

$$0 =\sim 1$$

$$1 =\sim 0$$

Properties of conjunction:

- zero element

$$A \tilde{\wedge} 0 = 0$$

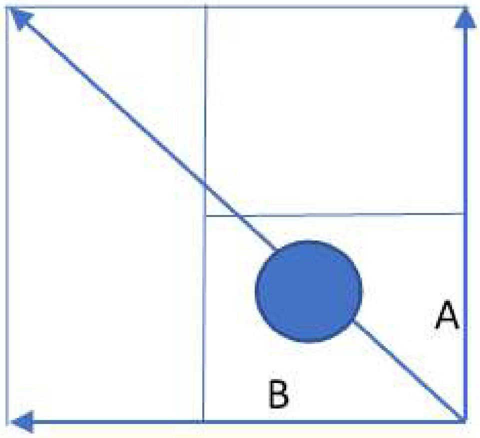

Figure 7. Variant 8 illustration.

- neutral element

$$A \tilde{\wedge} 1 = A$$

- idempotency

$$A \tilde{\wedge} A = A$$

- commutativity

$$A \tilde{\wedge} B = B \tilde{\wedge} A$$

- non-associativity

$$\neg(A \tilde{\wedge} (B \tilde{\wedge} C) = (A \tilde{\wedge} B) \tilde{\wedge} C)$$

- non-monotonicity

$$\neg(A \leq B \to A \tilde{\wedge} C \leq B \tilde{\wedge} C)$$

- monotonicity in direction

$$A \overset{\leftarrow}{\leq} B \to A \tilde{\wedge} C \overset{\leftarrow}{\leq} B \tilde{\wedge} C$$

Properties of disjunction:

$$A \tilde{\vee} B =\sim ((\sim A) \tilde{\wedge} (\sim B))$$

Properties of implication:

$$A \sim> B = ((\sim A) \tilde{\vee} B)$$

Properties of a fuzzy measure [16]:

$$A \tilde{\wedge} B \leq A$$

$$A \leq A \tilde{\vee} B$$

As concrete structural-static models we can consider finite models or simplicial complexes [1], and also we can consider their generalizations, for example, as a residual simplicial complex. Let us consider variants with simplicial complexes. An important question of such consideration is the canonical form of the corresponding simplicial complex.

For simplicial complexes and the corresponding sets defined by them, the (generalized) operations of union $\hat{\cup}$ and intersection $\hat{\cap}$ are naturally defined.

Each argument of the parameterized fuzzy expression can be matched with a residual simplicial complex as one of the parameters. We will also use the notion of residual simplicial complex to represent the results of parameterized fuzzy logics.

The residual simplicial complex can be given by $2*n$-simplicial complexes$\langle C_1, C_2, ..., C_{2*n}\rangle$ through an expression of the form:

$$C_1/(C_2/(.../C_{2*n}))$$

For a residual simplicial complex the following is true:

$$C_{i+1} \subset C_i$$

$$C_{i+2} \subseteq \partial C_i$$

$$(X \in C_i \cap C_{i+1}) \rightarrow \exists Y(Y \in C_i/C_{i+1}) \wedge (\emptyset \subset Y \cap X)$$

$$\emptyset \subset C_{2*n}$$

$$\partial C = \bigcup_{X \in C} 2^X/\{X\}$$

The height of the residual complex is $2*n$. We will consider simplicial complexes covering points of subsets of the set of points of the space spanned by the universal simplicial complex U whose residual simplicial complex is $\langle U, \emptyset, ..., \emptyset\rangle$.

The complement of $U\hat{/}C$ of the residual simplicial complex $\langle C_1, C_2, ..., C_{2*n}\rangle$ will be the height complex $2*m(m \leq n)$:

$$D_1/(D_2/(.../D_{2*m}))$$

with such smallest $T_1, T_k(2 \leq k \leq n)$:

$$U/\left(\bigcup_{i=1}^{n} C_{2*i-1}/C_{2*i}\right) \subseteq T_1 \subseteq U$$

$$C_{k-1}/\left(\bigcup_{i=1}^{n-k+1} C_{2*i-1+k}/C_{2*i+k}\right) \subseteq T_k \subseteq C_{k-1}$$

$$D_1/(\bigcup_{i=1}^{m} D_{2*i}/D_{2*i+1}) = T_1/(\bigcup_{i=1}^{n} T_{2*i}/T_{2*i+1})$$

$$((m < i) \wedge (i \leq n)) \rightarrow (T_i = \emptyset)$$

The intersection $O = I\hat{\cap}E$ of the two residual simplicial complexes I and E ($n \leq m$) is the height complex $2*l(l \leq n*(2*m-n+1))$:

$$O_1/(O_2/(.../O_{2*l}))$$

The tiers of the residual simplicial complex are filled in according to the tables Table III ($E_0 = E_{10}$), Table IV) in accordance with the order:

The difference $O = I\hat{/}E$ of the two residual simplicial complexes I and E ($n \leq m$) is the height complex $2*l(l \leq n*(2*m-n+1)$:

$$I\hat{/}E = I\hat{\cap}(U\hat{/}E)$$

Table III
Computable operations for calculating the intersection of two residual simplicial complexes

$I_1 \cap E_1$	$I_2 \cup E_2$	$I_3 \cap E_1$	$I_4 \cup E_4$	$I_5 \cap E_1$	$I_6 \cup E_0$
$I_2 \cup E_2$	$I_2 \cap E_2$	$I_2 \cap E_2$	$I_4 \cup E_4$	$I_5 \cap E_2$	$I_6 \cup E_0$
$I_1 \cap E_3$	$I_2 \cap E_2$	$I_3 \cap E_3$	$I_4 \cup E_4$	$I_5 \cap E_3$	$I_6 \cup E_0$
$I_4 \cup E_4$	$I_4 \cup E_4$	$I_4 \cup E_4$	$I_4 \cap E_4$	$I_4 \cap E_4$	$I_6 \cup E_0$
$I_1 \cap E_5$	$I_2 \cup E_5$	$I_3 \cap E_5$	$I_4 \cap E_4$	$I_5 \cap E_5$	$I_6 \cup E_0$
$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_0$
$I_1 \cap E_7$	$I_2 \cap E_7$	$I_3 \cap E_7$	$I_4 \cap E_7$	$I_5 \cap E_7$	$I_6 \cup E_0$
$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_0$
$I_1 \cap E_9$	$I_2 \cap E_9$	$I_3 \cap E_9$	$I_4 \cap E_9$	$I_5 \cap E_9$	$I_6 \cup E_0$
$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$

Table IV
Sequence (transposed) of computable operations to compute the intersection of two residual simplicial complexes

1	2	3	6	7	12	13	18	19	24
2	4	4	6	8	12	14	18	20	24
3	4	5	6	9	12	15	18	21	24
6	6	6	10	10	12	16	18	22	24
7	8	9	10	11	12	17	18	23	24
24	24	24	24	24	24	24	24	24	24

The residuum $O = I\hat{\rightarrow}E$ of the two residual simplicial complexes I and E ($n \leq m$) will be the height complex $2*l(l \leq n*(2*m-n+1))$:

$$I\hat{\rightarrow}E = U\hat{/}(I\hat{/}E)$$

The union of $O = I\hat{\cup}E$ of the two residual simplicial complexes I and E ($n \leq m$) is the height complex $2*l$ ($l \leq n*(2*m-n+1)$):

$$I\hat{\cup}E = U\hat{/}((U\hat{/}I)\hat{/}E)$$

The value of a fuzzy expression (predicate) in parametric fuzzy logic can be calculated as the length $hvol_1(C)$, area $hvol_2(C)$, volume $hvol_3(C)$ or hypervolume $hvol_{dim(C)}(C)$ of a simplicial complex. For each simplicial complex with a basis in linear vector space, a minimal covering simplex can be given, and its dimension $dim(C)$, equal to the dimension of the maximal simplex in the complex, can also be computed. If the space is n-dimensional, the value of the fuzzy expression can be computed:

$$1 + 2^{-dim(C)} * (hvol_{dim(C)}(C) - 2)$$

For the corresponding fuzzy operations, the properties of the fuzzy measure will also be fulfilled:

$$A\tilde{\wedge}B \leq A$$

$$A \leq A\tilde{\vee}B$$

Another kind of non-classical logics [4], [5] are substructural logics in which (structural) properties of deducibility such as monotonicity, contraction (absorption)