DYNAMICS

$$\dots a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = \dots d\frac{d^2x}{dt^2} + e\frac{dx}{dt} + fx$$

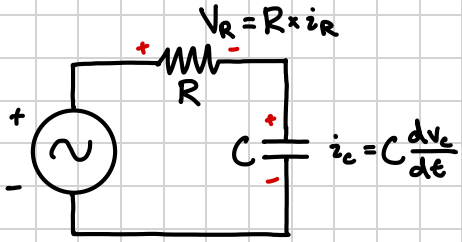
$(ay = bx)$
Algebraic
No dynamics

x (input) → System → y (output)

What if $x(t) = X\cos(\omega t)$
$\Rightarrow RHS = A\cos(\omega t) + B\sin(\omega t)$
$= C\cos(\omega t + \phi)$
$\Rightarrow y(t) = Y\cos(\omega t + \phi)$
(Y: unknown)

RC CIRCUIT

$V_{in}(t)$ → RC Circuit → $V_o(t)$



$$\frac{V_{in} - V_c}{R} = C\frac{dV_c}{dt}$$

$$V_{in} - RC\frac{dV_c}{dt} - V_c = 0$$

$$RC\frac{dV_c}{dt} + V_c = V_{in}$$

Let $V_{in} = \overline{V}_{in}\cos(\omega t)$

$$-RC\omega\overline{V}_c\sin(\omega t + \phi) + \overline{V}_c\cos(\omega t + \phi) = \overline{V}_{in}\cos(\omega t)$$

Solve for $\overline{V}_c$ and ϕ

Magnitude Gain: $M = \frac{\overline{V}_c}{\overline{V}_{in}}$

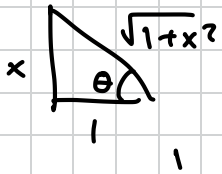
$$M = \frac{\overline{V}_c}{\overline{V}_{in}} = \frac{\cos(\omega t)}{\cos(\omega t + \phi) - RC\omega\sin(\omega t + \phi)}$$

$$M|_{t=0} = \frac{1}{\cos(\phi) - RC\omega\sin(\phi)}$$

$$\phi = -\cos^{-1}\left(\frac{1}{\sqrt{1 + R^2C^2\omega^2}}\right)$$

$$M|_{t=-\frac{\phi}{\omega}} = \frac{\cos(-\phi)}{1 - 0}$$

$$M = \frac{1}{M - RC\sqrt{1 - M^2}}$$



$$\cos\theta = \frac{1}{\sqrt{1+x^2}}$$

$$\tan\theta = \frac{x}{1}$$

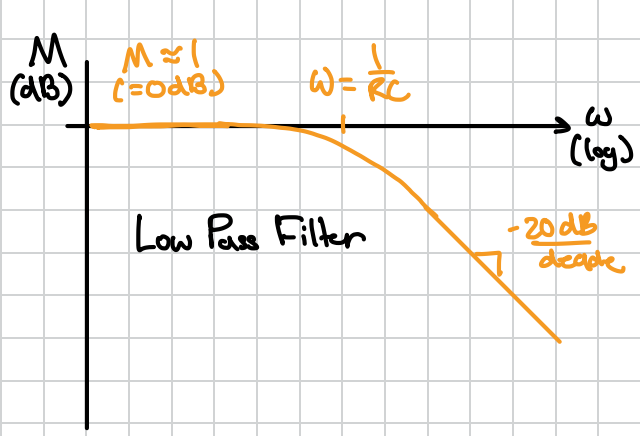
$$-RC\omega\sqrt{1 - M^2} = \frac{1 - M^2}{M}$$

$$R^2C^2\omega^2(1 - M^2) = \frac{(1 - M^2)^2}{M^2}$$

$$R^2C^2\omega^2 = \frac{1 - M^2}{M^2}$$

$$M^2(R^2C^2\omega^2 + 1) = 1$$

$$M = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$


M
(dB)
M ≈ 1
(=0dB)
ω = 1/RC
ω
(log)
Low Pass Filter
-20dB/decade


ϕ
(lin)
ϕ ≈ 0°
ω = 1/RC
ω (log)
ϕ ≈ 90°