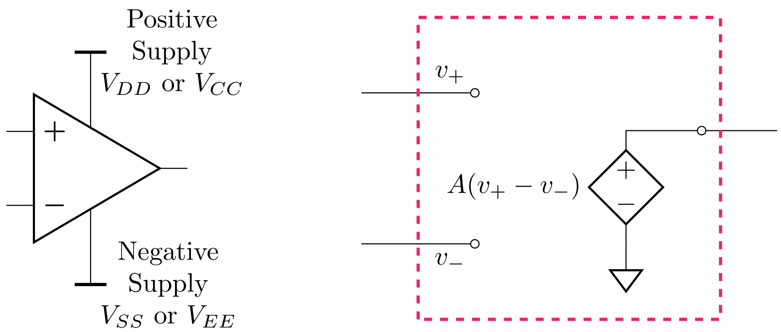
Lecture Notes 10: Op Amp Circuits

Recall that operational amplifiers or op amps behave as dependent voltage sources, amplifying the difference between their input terminals by a large amount, $v_o = A(v_+ + v_-)$.



The model above is for an ideal op amp. A real op amp has several limitations:

1. Output voltage range: The output voltage of an op-amp is usually limited to a certain range. For example, even if A were 10,000 and you applied $v_+ - v_- = 1$ V to the input, you would not see 10,000 V at the output. The output voltage range is usually limited by the supplies. Some op amps have output voltage range specifications like "$v_{ss} + 0.7$ to $v_{DD} - 0.7$," meaning that if the output voltage merely approaches the rails, the op amps' behavior will break down. Many modern amplifiers have **rail-to-rail output (RRO)** capability, meaning the the output can go all the way to down the negative supply and all the way up to the positive supply without issue. Some can even exceed the rails by perhaps 0.2 V.

When an op amp tries to exceed its output voltage, in reality the output tends to saturate. For example, consider an RRO amplifier with a gain of 10,000 and supplies at +3.3V and -3.3V. In an amplifier with a gain of 10,000, an input of $v_+ - v_- = 1\mu$V will produce an output of 0.01V. An output of 330μV will produce an output of 3.3V (proportionally larger, as expected). However, any input greater than 330μV will simply produce an output of 3.3V. The output has "hit the rail" or "saturated."

2. Input voltage range: Op amps generally don't work well if the inputs are near the rails, either. Op amps are generally symmetric, so there is no need to specify the range for the + input separately from the $-$ input. In addition, op amps are usually used in feedback, so $v_+ \approx v_-$ during operation anyway. The average of the inputs $(v_+ + v_-)/2$ is called the **common mode voltage** (as opposed to the **differential mode voltage** $v_+ - v_-$, which in feedback ends up being close to zero). The acceptable differential mode range is essentially the same information as the output voltage range (related by the gain). A specification on common mode voltage introduces new information. For example, in an op amp with rails at $\pm$3.3V and **rail-to-rail input (RRI)**, an input $v_+ - v_- = 1\mu\text{V} \rightarrow v_o = 0.01V$ as long as v_+ and v_- do not exceed the rails. However, the same difference (1μV) won't produce a 0.01V output if v_+ and v_- are both 4V, for example.

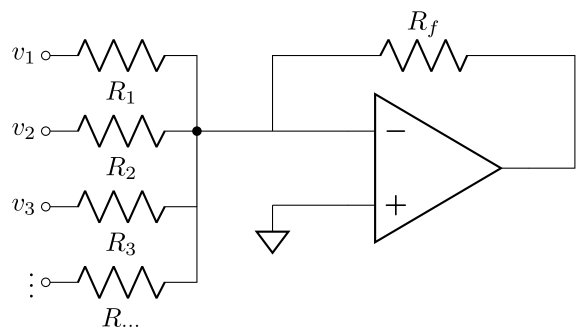
When the inputs to the op amp are beyond the common-mode input voltage range, the gain tends to drop off precipitously. This wreaks havoc on most op amp circuits which are relying on feedback and the approximations $A \gg 1$, $v_+ \approx v_-$, and so forth to perform their function.

3. Limited gain: Op amps do not have $A = \infty$. While most op amps' gain is sufficiently large that it might as well be infinity, this is not always the case. When A is not sufficiently large, one cannot use the Golden Rule of Negative Feedback anymore, and the circuit's behavior must be solved without assuming $v_+ = v_-$ and the answer will include A.

Where the amplifier's gain is sufficiently large is entirely application dependent. In high-precision circuits, it may not be enough to say that the output voltage is *close to* $v_{in} \times (1 + R_1/R_2)$ (in the non-inverting op amp configuration, for example). If we need the output to be within a few millivolts of our expectation, then any old op amp will probably do. But if we need the output to within a few microvolts of our expectation, then we may need ultra high gain.

As a practical matter, most op amps have sufficient gain for most applications and we can apply the Golden Rule of Negative Feedback liberally. We most often see the effects of low A at high frequencies, but we won't have the background or vocabulary to deal with that until later in the course.

Let us solve a couple more op amp circuits, which will give us practice with the Golden Rule of Negative Feedback, let us see the impact of non-ideal op amps, and showcase some more things that op amp circuits can do. First, consider an **inverting summing amplifier**:



This is a perfect circuit to solve using superposition. Apply v_1 alone, letting all other inputs be zero. Since the other inputs are voltages, they become "zero volts" or shorts to ground. By the Golden Rule of Negative Feedback, v_- is also zero volts, meaning that no current flows through R_2, R_3, etc. This leaves only R_1 and R_f, and therefore this circuit's response to a single input is that of an inverting amplifier,

$$\underbrace{v_o}_{\text{due to } v_1 \text{ alone}} = -\frac{R_f}{R_1} v_1$$

Due to the symmetry of the circuit, we will get the same answer for every other input. Therefore,

$$v_o = \left(-\frac{R_f}{R_1}\right) v_1 + \left(-\frac{R_f}{R_2}\right) v_2 + \left(-\frac{R_f}{R_3}\right) v_3 + \cdots = -R_f \sum_k \frac{1}{R_k} v_k$$

If all of the input-side resistors are equal (a common design choice for this circuit), namely $R_1 = R_2 = R_3 = \cdots = R$, then

$$v_o = -\frac{R_f}{R} \sum_k v_k$$

We now see where this circuit gets its name. It is a summer, in that it sums the input voltages. It is inverting, in that the output is negative if the inputs are positive. Finally, it is an amplifier, in that the sum is multiplied by R_f/R which we could choose to be > 1, < 1, or $= 1$ as we wish.

Now let us analyze this circuit if it is not ideal. First, consider the input voltage range. If the op amp is operating properly, both of its inputs will be at 0V. Even if the gain is not infinite, the v_- terminal will be a fraction of a fraction of a fraction of a volt away from 0. Therefore, this circuit runs no risk of exceeding the op amp's input voltage range.

Second, let us consider the output voltage range with all of the input-side resistors equal to R. In this case, we can conclude that the sum of the input voltages must be less than $R/R_f \times v_{o,limit}$ in order for the circuit to work properly. In other words, the larger the gain of the circuit, R_f/R, the less input voltage it can handle. For a given amplifier, this represents a significant tradeoff – large gains help us accurately measure small signals, but they make it hard to measure large signals and vice versa. The amplifier therefore has limited *dynamic range*, i.e. the ratio between the largest signal it can measure (determined by the amplifier limits) and the smallest signal it can measure (which, when multiplied by the gain, is still too small to overcome noise, for example).

Finally, consider the response with limited gain, $A < \infty$. Fortunately, superposition still applies, so we can treat the total amplifier response as the sum of ordinary inverting-amplifier responses. In an inverting amplifier,

$$\frac{v_- - v_1}{R_1} = \frac{v_o - v_-}{R_f} \Rightarrow v_- = \frac{R_f}{R+R_f}v_1 + \frac{R}{R+R_f}v_o$$

$$v_o = A(v_+ - v_-) = A\left(0 - \left(\frac{R_f}{R+R_f}v_1 + \frac{R}{R+R_f}v_o\right)\right)$$

$$v_o\left(1 + A\frac{R}{R+R_f}\right) = -A\frac{R_f}{R+R_f}v_1$$

$$v_o = -\frac{A\frac{R_f}{R+R_f}}{1+A\frac{R}{R+R_f}}v_1 = -\frac{AR_f}{R+R_f+AR}v_1$$

We can double check that *if* A were infinite, then $v_o \to -(R_f/R)v_1$, as expected. Now, for many inputs,

$$v_o = \left(-\frac{AR_f}{R+R_f+AR}v_1\right) + \left(-\frac{AR_f}{R+R_f+AR}v_2\right) + \left(-\frac{AR_f}{R+R_f+AR}v_3\right) + \cdots = -\frac{AR_f}{R+R_f+AR}\sum_k v_k$$

Great, the summer still functions as a summer; it's just that its gain isn't quite equal to $-R_f/R$. Note that we could have reached this qualitative conclusion (if not the quantitative equation) by noting that the amplifier responds the same way to every input, so $v_o = Cv_1 + Cv_2 + Cv_3 + \cdots = C\sum v_k$ is guaranteed by linearity and symmetry.

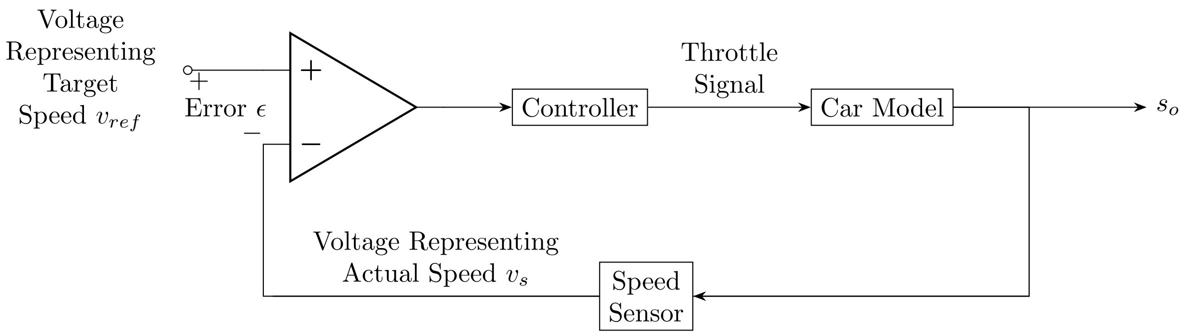
Control Systems In an amplifier, the objective is to put in an input signal an get out a particular output signal. The Golden Rule of Negative Feedback, i.e. the fact that $v_- = v_+$, is not the *purpose* of the amplifier, it's mainly a consequence of how the amplifier works and a useful analysis tool.

Consider now another class of systems where the purpose is actually to make one signal equal to another. An easily accessible example is cruise control in your car. You (the user) provide a target speed, s_{ref}, while the car measures its actual speed, s_o (o for output). The goal is to make the car's actual speed equal the target speed., $s_o \to s_{ref}$. The Golden Rule of Negative Feedback can helps us do this!

Consider the block diagram of a cruise control system below. The car's speed is sensed, which produces a voltage that represents (or encodes the information about) the car's speed. This is then compared against the reference "speed" (really a voltage representing the target speed), yielding the **error** ϵ. The amplifier amplifies this difference and passes its output, $A\epsilon$ to the **controller**, which converters $A\epsilon$ (a voltage) into a throttle signal (for example, a gas pedal position). As the throttle changes, the engine or motor generates more power which causes the car to move faster or slower.

The Golden Rule of Negative Feedback guarantees that $v_- = v_+$. Therefore, the voltage representing the actual speed v_s must equal the voltage representing the target speed, v_{ref}. In other words, the car's speed is being controlled to be equal to its target.

What is the output of the amplifier, $A\epsilon$? It's *whatever it has to be* to make $v_s = v_{ref}$. This is an important point, because *whatever it has to be* will change as the car goes up or down hill, goes with or against the wind, experiences aging tires and differing tire pressure, and so forth. Therefore, $A\epsilon$ and the throttle signal



will be changing all the time, but $v_s = v_{ref}$ should be rock solid. In other words, the purpose of this circuit is not to make a particular $A\epsilon$ (as it was in the prior amplifiers we have studied), but rather to make $v_s = v_{ref}$.

Recall that the Golden Rule has conditions, namely,

- The system must be in *negative* feedback
- The gain must be ∞ or at least very large
- The system must be stable

The system is in negative feedback – if the output s_o were too high, then v_s would be too high, which would cause the output of the amplifier, $A\epsilon$ to become lower, which would cause the Throttle Signal to become lower, which would reduce speed s_o. And the total gain around the loop is very large because of the op amp gain A.

Finally, we need to know if the system is **stable**. Stability is easily understood if you imagine standing on one foot. A mannequin standing on one foot might be stable if she is standing *exactly* right, but if she is bumped at all, she will fall over. This is instability. A person standing on one foot makes use of a negative feedback system: position is sensed with the eyes, the sensation of the ground beneath one's foot, the inner ear, and then the foot muscles are actuated to restore balance. This negative feedback system can be stable (if the person continues standing), but it can also become unstable – imagine tipping to the right, overcorrecting to the left, over-over-correcting to the right, and so forth until you fall. Determining whether a system will actually be stable or not is beyond the scope of ECE 302 and will be the overriding focus of a class on **automatic control**. We will always assume that negative feedback is stable in ECE 302.