

Introduction to Electrical Engineering (ECE 302H) –

Homework 2

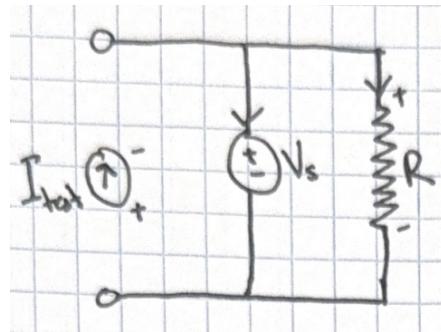
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Fall 2025

Problem 2.1. Equivalent Circuits*Solution.*

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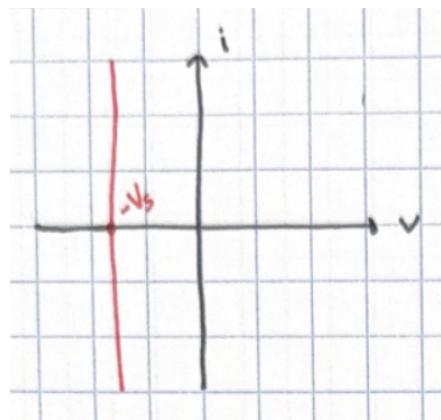
a) Sketch:



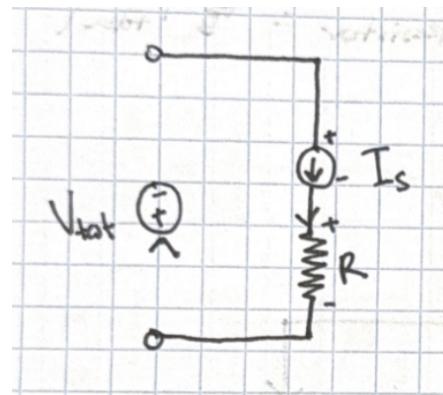
KVL:

$$\sum V = -V_{\text{test}} - V_s = 0$$
$$V_{\text{test}} = \boxed{-V_s}$$

i-v Characteristic:



b) Sketch:

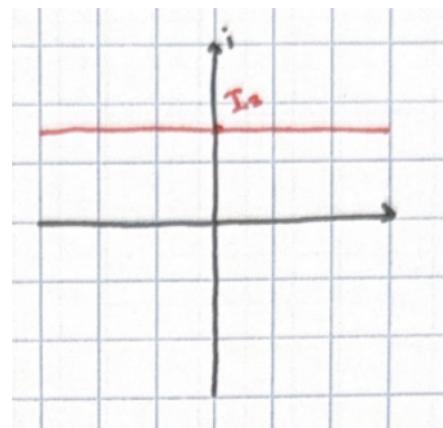


KCL:

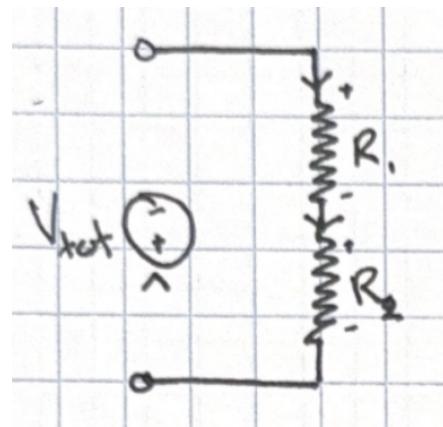
$$\sum I = I_{\text{test}} - I_s = 0$$

$$I_{\text{test}} = \boxed{I_s}$$

i-v Characteristic:

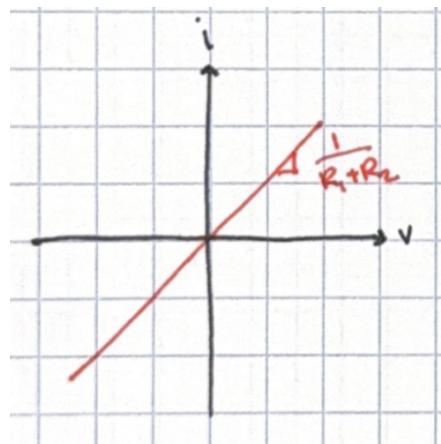


c) Sketch:



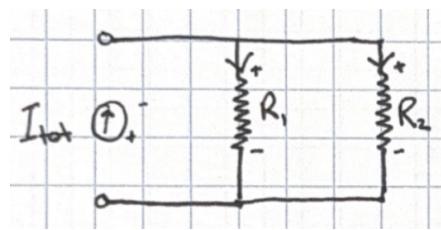
i-v Characteristic:

$$\begin{aligned}\sum V &= V_{\text{test}} - V_{R_1} - V_{R_2} = 0 \\ V_{\text{test}} &= V_{R_1} + V_{R_2} \\ \sum I &= I_{\text{test}} - I_{R_1} = 0 \\ \sum I &= I_{R_1} - I_{R_2} = 0 \\ I_{\text{test}} &= I_{R_1} = I_{R_2} \\ V_{\text{test}} &= I_{\text{test}} R_1 + I_{\text{test}} R_2 \\ &= I_{\text{test}} (R_1 + R_2) \\ &= I_{\text{test}} R_{\text{eq}} \\ R_{\text{eq}} &= R_1 + R_2 \\ I_{\text{test}} &= \frac{1}{R_1 + R_2} V_{\text{test}}\end{aligned}$$



$R_{\text{eq}} = R_1 + R_2$. If $R_1 \gg R_2$, then $R_{\text{eq}} = R_1$.

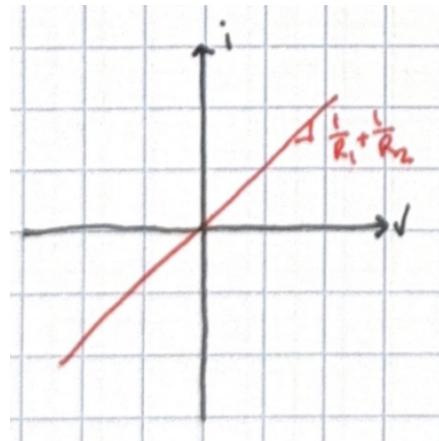
d) Sketch:



i-v Characteristic:

$$\begin{aligned}\sum V &= V_{\text{test}} - V_{R_1} = 0 \\ \sum V &= V_{\text{test}} - V_{R_2} = 0\end{aligned}$$

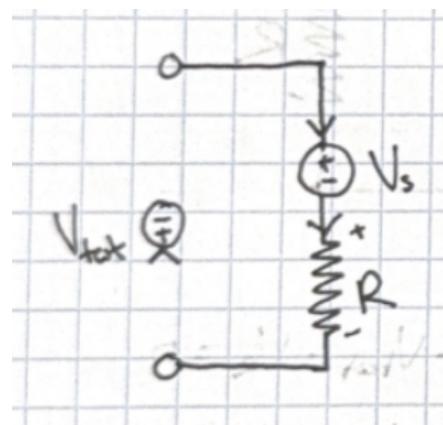
$$\begin{aligned}
 V_{\text{test}} &= V_{R_1} = V_{R_2} \\
 \sum I &= I_{\text{test}} - I_{R_1} - I_{R_2} = 0 \\
 I_{\text{test}} &= I_{R_1} + I_{R_2} \\
 V_{\text{test}} &= I_{R_1} R_1 = I_{R_2} R_2 \\
 I_{\text{test}} &= \frac{V_{\text{test}}}{R_1} + \frac{V_{\text{test}}}{R_2} \\
 &= \left(\frac{1}{R_1} + \frac{1}{R_2} \right) V_{\text{test}} \\
 I_{\text{test}} &= \frac{V_{\text{test}}}{R_{\text{eq}}} \\
 \frac{1}{R_{\text{eq}}} &= \frac{1}{R_1} + \frac{1}{R_2}
 \end{aligned}$$



$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}, \text{ If } R_1 \gg R_2, \text{ then } \frac{1}{R_{\text{eq}}} = \frac{1}{R_2} \text{ so } R_{\text{eq}} = R_2$$

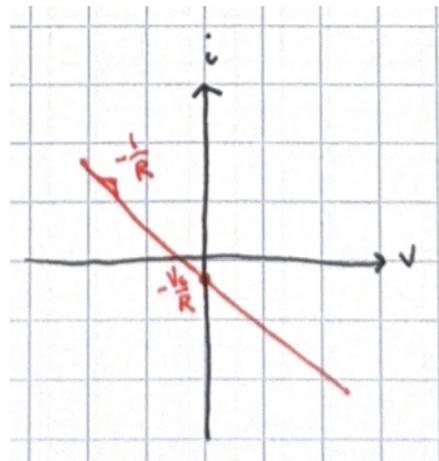
Remark. Logically this makes a lot of sense because if you have a path of higher and lower resistance in parallel, the current will flow through the one with lower resistance, R_2 .

e) Sketch:



i-v Characteristic:

$$\begin{aligned}\sum V &= -V_{\text{test}} - V_s - V_R = 0 \\ V_{\text{test}} &= -V_s - V_R \\ \sum I &= I_{\text{test}} - I_s = 0 \\ \sum I &= I_s - I_R = 0 \\ I_{\text{test}} &= I_s = I_R \\ V_R &= I_R R \\ &= I_{\text{test}} R \\ V_{\text{test}} &= -V_s - I_{\text{test}} R \\ I_{\text{test}} &= \frac{-V_s - V_{\text{test}}}{R} \\ &= -\frac{V_s}{R} - \frac{V_{\text{test}}}{R}\end{aligned}$$



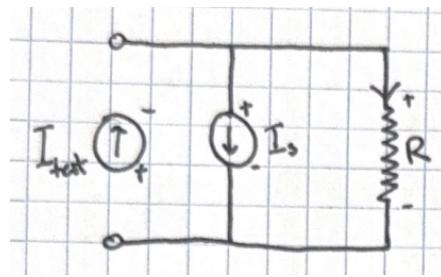
Open circuit when $I_{\text{test}} = 0$:

$$\begin{aligned}V_{\text{test}} &= -V_s - I_{\text{test}} R \\ &= \boxed{-V_s}\end{aligned}$$

Short circuit when $V_{\text{test}} = 0$:

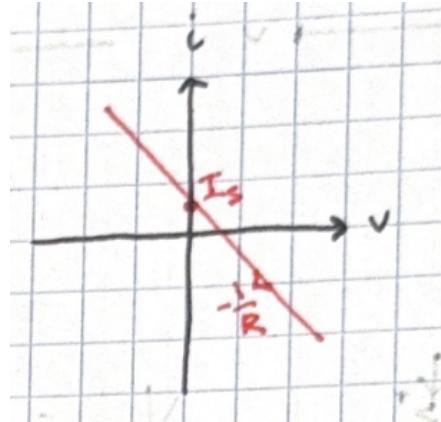
$$\begin{aligned}I_{\text{test}} &= \frac{-V_s - V_{\text{test}}}{R} \\ &= \boxed{\frac{-V_s}{R}}\end{aligned}$$

f) Sketch:



i-v Characteristic:

$$\begin{aligned}\sum V &= -V_{\text{test}} - V_s = 0 \\ \sum V &= -V_{\text{test}} - V_R = 0 \\ -V_{\text{test}} &= V_s = V_R \\ \sum I &= I_{\text{test}} - I_s - I_R = 0 \\ I_{\text{test}} &= I_s + I_R \\ V_R &= I_R R \\ I_{\text{test}} &= I_s + \frac{V_R}{R} \\ &= I_s - \frac{V_{\text{test}}}{R}\end{aligned}$$



Open circuit when $I_{\text{test}} = 0$:

$$\begin{aligned}I_{\text{test}} &= I_s - \frac{V_{\text{test}}}{R} \\ 0 &= I_s - \frac{V_{\text{test}}}{R} \\ V_{\text{test}} &= \boxed{I_s R}\end{aligned}$$

Short circuit when $V_{\text{test}} = 0$:

$$I_{\text{test}} = I_s - \frac{V_{\text{test}}}{R}$$

$$I_{\text{test}} = \boxed{I_s}$$

Now that we have both i-v equations for e) and f), lets see what happens when we combine them.

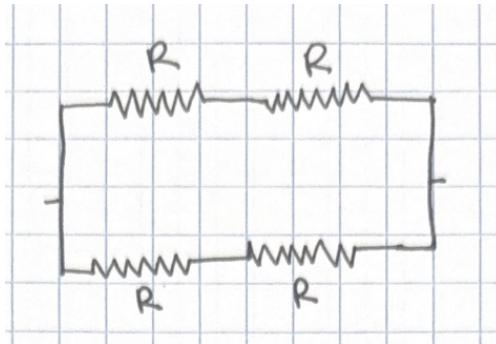
$$\begin{aligned} I_{\text{test}} &= \frac{-V_s - V_{\text{test}}}{R} \\ I_{\text{test}} &= I_s - \frac{V_{\text{test}}}{R} \\ \frac{-V_s - V_{\text{test}}}{R} &= I_s - \frac{V_{\text{test}}}{R} \\ -V_s - V_{\text{test}} &= I_s R - V_{\text{test}} \\ -V_s &= I_s R \\ V_s &= \boxed{-I_s R} \\ I_s &= \boxed{-V_s R} \end{aligned}$$

Problem 2.2. Series and parallel; voltage, current, and power*Solution.*

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We're trying to create an object that quadruples the power rating $P_{\text{total}} = 4P$ while keeping the resistance R the same.

1. Parallel-Series



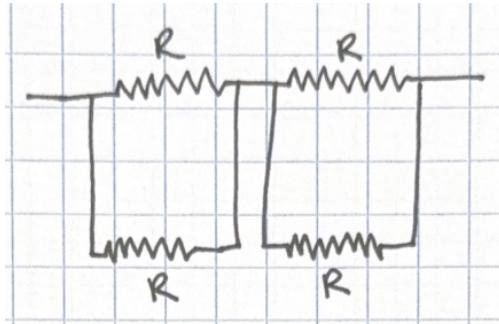
Check equivalency

$$\begin{aligned} R_{\text{eq}} &= \frac{1}{\frac{1}{R+R} + \frac{1}{R+R}} \\ &= \frac{1}{\frac{1}{2R} + \frac{1}{2R}} \\ &= \frac{1}{\frac{1}{R}} \\ &= R \end{aligned}$$

Solve for P_{total} :

$$\begin{aligned} I_{\text{branch}} &= \frac{I}{2} \\ P &= I_{\text{branch}}^2 R \\ P &= \left(\frac{I}{2}\right)^2 R \\ &= \frac{1}{4} I^2 R \\ &= \frac{1}{4} P_{\text{total}} \\ P_{\text{total}} &= 4P \end{aligned}$$

2. Series-Parallel



Check equivalency

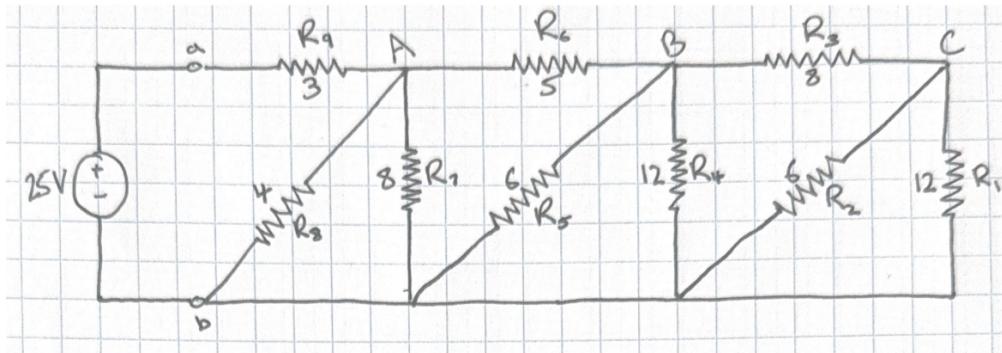
$$\begin{aligned}
 R_{\text{eq}} &= \frac{1}{\frac{1}{R} + \frac{1}{R}} + \frac{1}{\frac{1}{R} + \frac{1}{R}} \\
 &= \frac{1}{\frac{2}{R}} + \frac{1}{\frac{2}{R}} \\
 &= \frac{R}{2} + \frac{R}{2} \\
 &= R
 \end{aligned}$$

Solve for P_{total} :

$$\begin{aligned}
 I_{\text{branch}} &= \frac{I}{2} \\
 P &= I_{\text{branch}}^2 R \\
 &= \left(\frac{I}{2}\right)^2 R \\
 &= \frac{1}{4} I^2 R \\
 &= \frac{1}{4} P_{\text{total}} \\
 P_{\text{total}} &= 4P
 \end{aligned}$$

Problem 2.3. Resistor equivalent circuit exercise*Solution.*

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Use series and parallel resistor equivalent equations to solve for R_{eq}

$$\frac{1}{R_{(1,2)}} = \frac{1}{12} + \frac{1}{6} = \frac{1}{6.5}$$

$$R_{(1,3)} = 6.5 + 8 = 14.5$$

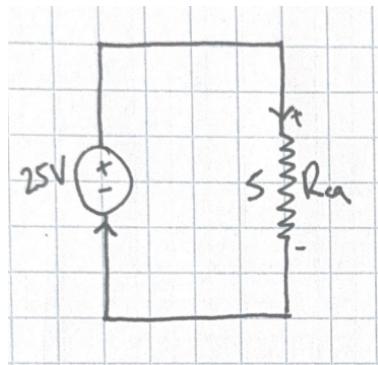
$$\frac{1}{R_{(1,5)}} = \frac{1}{14.5} + \frac{1}{12} + \frac{1}{6} = \frac{1}{3.14}$$

$$R_{(1,6)} = 3.14 + 5 = 8.14$$

$$\frac{1}{R_{(1,8)}} = \frac{1}{8.14} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$$

$$R_{(1,9)} = R_{eq} = 2 + 3 = \boxed{5\Omega}$$

Redraw equivalent circuit

Use KVL to solve for $I_{R_{eq}}$:

$$\sum V = 25 - I_{R_{eq}} R_{eq} = 0$$

$$25 = I_{R_{eq}} (5)$$

$$I_{R_{eq}} = I_{R_9} = 5$$

Remark. In this case, $I_{R_{eq}} = I_{R_9}$ because the current going into R_{eq} is the same as R_9 .

$$V_{R_9} = I_{R_9} R_9 = (5)(3) = \boxed{15 \text{ V}}$$

Use KVL to solve for V_{R_8} and V_{R_7} :

$$\begin{aligned}\sum V &= 25 - V_{R_9} - V_{R_8} = 0 \\ \sum V &= 25 - V_{R_9} - V_{R_7} = 0 \\ 25 - 15 &= V_{R_8} = V_{R_7} \\ V_{R_8} &= V_{R_7} = \boxed{10 \text{ V}}\end{aligned}$$

Use KCL on node A to solve for V_{R_6} :

$$\begin{aligned}\sum I &= I_{R_9} - I_{R_8} - I_{R_7} - I_{R_6} = 0 \\ I_{R_6} &= 5 - \frac{10}{4} - \frac{10}{8} = 1.25 \\ V_{R_6} &= I_{R_6} R_6 \\ &= (1.25)(5) = \boxed{6.25 \text{ V}}\end{aligned}$$

Use KVL to solve for V_{R_5} and V_{R_4} :

$$\begin{aligned}\sum V &= 25 - V_{R_9} - V_{R_6} - V_{R_5} = 0 \\ \sum V &= 25 - V_{R_9} - V_{R_6} - V_{R_4} = 0 \\ 25 - 15 - 6.25 &= V_{R_5} = V_{R_4} \\ V_{R_5} &= V_{R_4} = \boxed{3.75 \text{ V}}\end{aligned}$$

Use KCL on node B to solve for V_{R_3} :

$$\begin{aligned}\sum I &= I_{R_6} - I_{R_5} - I_{R_4} - I_{R_3} = 0 \\ I_{R_3} &= 1.25 - \frac{3.75}{6} - \frac{3.75}{12} = 0.3125 \\ V_{R_3} &= I_{R_3} R_3 \\ &= (0.3125)(8) = \boxed{2.5 \text{ V}}\end{aligned}$$

Use KVL to solve for V_{R_2} and V_{R_1} :

$$\begin{aligned}\sum V &= 25 - V_{R_9} - V_{R_6} - V_{R_3} - V_{R_2} = 0 \\ \sum V &= 25 - V_{R_9} - V_{R_6} - V_{R_3} - V_{R_1} = 0 \\ 25 - 15 - 6.25 - 2.5 &= V_{R_2} = V_{R_1} \\ V_{R_2} &= V_{R_1} = \boxed{1.25 \text{ V}}\end{aligned}$$

Problem 2.4. Solving Single-Loop Circuits*Solution.*

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a)

$$\sum V = V_{\text{in}} - V_{\text{diode}} - V_R = 0$$

$$V_R = V_{\text{in}} - V_{\text{diode}}$$

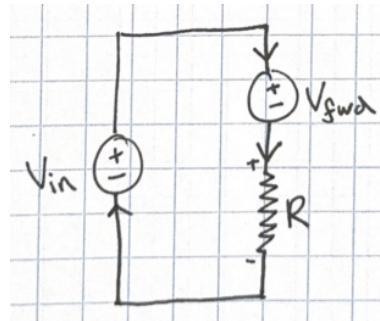
$$I_R = I = \frac{V_R}{R} = \frac{V_{\text{in}} - V_{\text{diode}}}{R}$$

$$I \propto \frac{1}{R}$$

Remark. Here $I = I_R$ because of KCL. There is only one path current flows in so it must be the same current that flows through all the components.

As R approaches 0, current approaches infinity. In real life, this would cause a short circuit because current will continuously ramp up without resistance.

b) Label voltage drops and current direction



Use KVL and KCL to solve for I

$$\sum V = V_{\text{in}} - V_{\text{fwd}} - V_R = 0$$

$$V_R = V_{\text{in}} - V_{\text{fwd}}$$

$$I_{\text{in}} = I_{\text{fwd}} = I_R = I$$

$$IR = V_{\text{in}} - V_{\text{fwd}}$$

$$I = \boxed{\frac{V_{\text{in}} - V_{\text{fwd}}}{R}}$$

c) Typical intensity = 35 mcd @ 20 mA

d) From the graph, we need approximately 27 mA of current to supply 45 mcd of luminosity.

$$R = \frac{V_{\text{in}} - V_{\text{fwd}}}{I}$$

$$\begin{aligned} &= \frac{5 - 2.2}{0.027} \\ &= \boxed{103.7 \Omega} \end{aligned}$$

Problem 2.5. Circuit Tradeoffs*Solution.*

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a) KVLs:

$$\begin{aligned}\sum V &= V_{\text{in}} - V_{\text{gs}} = 0 \\ V_{\text{in}} &= V_{\text{gs}} \\ \sum V &= V_m - V_{\text{out}} = 0 \\ V_m &= V_{\text{out}} \\ \sum V &= V_{\text{out}} - V_D = 0 \\ V_{\text{out}} &= V_D \\ V_{\text{out}} &= V_D = V_m\end{aligned}$$

KCLs:

$$\begin{aligned}\sum I &= -I_{\text{in}} - I_{\text{gs}} = 0 \\ I_{\text{gs}} &= -I_{\text{in}} \\ \sum I &= -I_m - I_D - I_{\text{out}} = 0 \\ I_D &= -I_m - I_{\text{out}}\end{aligned}$$

i-v's:

$$\begin{aligned}V_D &= I_D R_D \\ I_m &= V_{\text{gs}} G_m\end{aligned}$$

Solve for V_{out} :

$$\begin{aligned}V_{\text{out}} &= V_D \\ &= I_D R_D \\ &= (-I_m - I_{\text{out}}) R_D \\ &= (-V_{\text{gs}} G_m - 0) R_D \\ &= -V_{\text{gs}} G_m R_D\end{aligned}$$

Remark. It says that in the problem statement that you can treat open circuit elements like I_{out} as $i = 0$.

Solve for $V_{\text{out}}/V_{\text{in}}$:

$$\begin{aligned}\frac{V_{\text{out}}}{V_{\text{in}}} &= \frac{-V_{\text{gs}} G_m R_D}{V_{\text{gs}}} \\ &= \boxed{-G_m R_D}\end{aligned}$$

b) If G_m changes by a factor of 2, the gain would also change by a factor of 2.

c) KVLs:

$$\begin{aligned}\sum V &= V_{\text{in}} - V_{\text{gs}} - V_S = 0 \\ V_{\text{in}} &= V_{\text{gs}} + V_S \\ \sum V &= V_S + V_m - V_{\text{out}} = 0 \\ \sum V &= V_{\text{out}} - V_D = 0 \\ V_{\text{out}} &= V_D\end{aligned}$$

KCLs:

$$\begin{aligned}\sum I &= -I_{\text{in}} - I_{\text{gs}} = 0 \\ \sum I &= I_{\text{gs}} + I_m - I_S = 0 \\ I_S &= I_{\text{gs}} + I_m \\ \sum I &= -I_m - I_D - I_{\text{out}} \\ I_D &= -I_m - I_{\text{out}}\end{aligned}$$

i-v's:

$$\begin{aligned}V_S &= I_S R_S \\ V_D &= I_D R_D \\ I_m &= V_{\text{gs}} G_m\end{aligned}$$

Solve for V_{out} :

$$\begin{aligned}V_{\text{out}} &= V_D \\ &= I_D R_D \\ &= (-I_m - I_{\text{out}}) R_D \\ &= (-V_{\text{gs}} G_m - 0) R_D \\ &= -V_{\text{gs}} G_m R_D\end{aligned}$$

Solve for V_{in} :

$$\begin{aligned}V_{\text{in}} &= V_{\text{gs}} + V_S \\ &= V_{\text{gs}} + I_S R_S \\ &= V_{\text{gs}} + (I_{\text{gs}} + I_m) R_S \\ &= V_{\text{gs}} + (0 + I_m) R_S \\ &= V_{\text{gs}} + I_m R_S \\ &= V_{\text{gs}} + V_{\text{gs}} G_m R_S\end{aligned}$$

Solve for $V_{\text{out}}/V_{\text{in}}$:

$$\begin{aligned}\frac{V_{\text{out}}}{V_{\text{in}}} &= \frac{-V_{\text{gs}}G_mR_D}{V_{\text{gs}}(1 + G_mR_S)} \\ &= \boxed{-\frac{G_mR_D}{1 + G_mR_S}}\end{aligned}$$

The magnitude of the gain of this amplifier would be smaller because there is $1 + G_mR_S$ on the denominator.

- d) We can approximate the magnitude of the gain to be,

$$\begin{aligned}\frac{V_{\text{out}}}{V_{\text{in}}} &= \frac{G_mR_D}{1 + G_mR_S} \\ &= \frac{G_mR_D}{G_mR_S} \\ &= \frac{R_D}{R_S}\end{aligned}$$

This means that even for very large G_m , the gain would remain relatively similar.

- e) Gain would not change with our approximation if it varied by a factor of 2.
- f) In the first circuit, you can achieve more gain because it is a function of G_m , but in the second circuit, you can control the exact value of what gain can be.