Introduction to Computing (ECE 306H) – Homework 1

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1 Problems

Problem 1.1. Say we had a "black box," which takes two numbers as input and outputs their sum. See Figure 1.7a in the Textbook. Say we had another box capable of multiplying two numbers together. See figure 1.7b. We can connect these boxes together to calculate $p * (m + n)$. See Figure 1.7c. Assume we have an unlimited number of these boxes. Show how to connect them together to calculate:

a) $ax - by$

b) ijk

c) $a^2 - b^2$

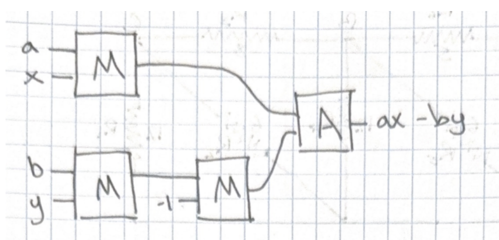
d) $a^3 + 3a^2b + 3ab^2 + b^3$ Try to do it with only one add box and two multiply boxes

e) b^4

Solution. For the purposes of this problem, I will represent an addition black box with $A(x, y)$ and multiplication black box as $M(x, y)$ where x and y are the two inputs.

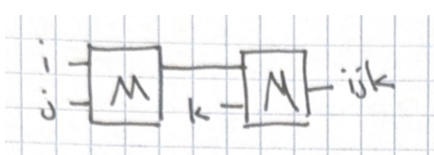
a)

$$ax - by = A(M(a, x), M(-1, M(b, y)))$$



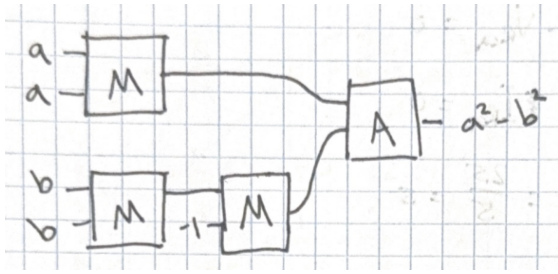
b)

$$ijk = M(i, M(j, k))$$



c)

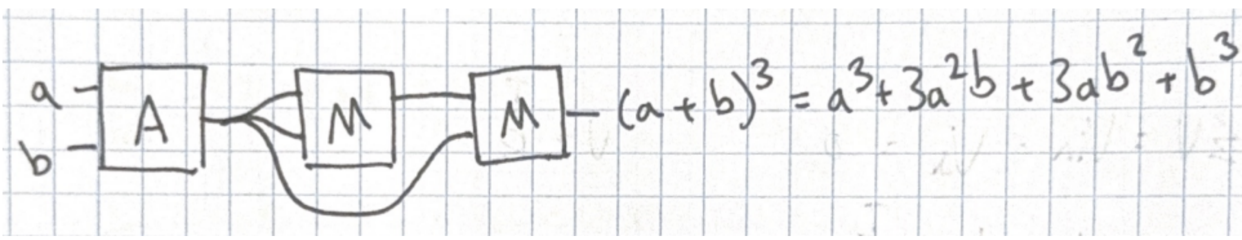
$$a^2 - b^2 = A(M(a, a), M(-1, M(b, b)))$$



d)

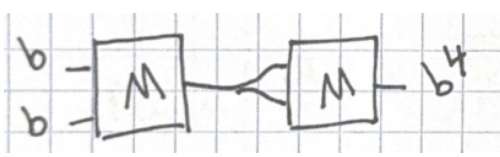
$$
\begin{aligned}
a^3 + 3a^2b + 3ab^2 + b^3 &= (a+b)^3 \\
&= M(A_1(a,b), M(A_1, A_1))
\end{aligned}
$$

> **Remark.** In this scenario, $A(a,b)$ is represented as A_1 because the black box's output is used multiple times which on paper is just represented as a connection and not a new black box.



e)

$$b^4 = M(M_1(b,b), M_1)$$



$\square$

Problem 1.2. Perform the following conversions, assuming unsigned numbers.

a) $(1432)_{16} = (\quad)_2$

b) $(110100100101)_2 = (\quad)_{16}$

c) $(1010001011010011)_2 = (\quad)_8$

d) $(1100)_8 = (\quad)_2$

e) $(1432)_8 = (\quad)_{16}$

f) $(1432)_{16} = (\quad)_8$

g) $(0100010101)_2 = (\quad)_{12}$

h) $(1234)_5 = (\quad)_7$

Solution. □

a) $(1010000110010)_2$

b) $(D25)_{16}$

c) $(121323)_8$

d) $(1001000000)_2$

e) First convert it to base-2 before converting to base-16
base-2: $(1100011010)_2$
base-16: $(31A)_{16}$

f) Same thing again
base-2: $(1010000110010)_2$
base-8: $(12062)_8$

g) Convert to base-10 then to base-12
base-10: $256 + 16 + 4 + 1 = (277)_{10}$
base-12: $(1B1)_{12}$

h) Convert to base-10 then to base-7
base-10: $4 + 15 + 50 + 125 = (194)_{10}$
base-7: $(365)_7$

Problem 1.3. Show the binary representation of the following signed decimal numbers in 8-bit 2's complement. If they cannot be represented in 8 bits, write Overflow.

a) 100

b) 128

c) -101

d) -128

e) -1

f) 42

Solution. □

a) $(01100100)_{\text{2's comp}}$

b) Overflow

c) $(10011011)_{\text{2's comp}}$

d) $(10000000)_{\text{2's comp}}$

e) $(11111111)_{\text{2's comp}}$

f) $(00101010)_{\text{2's comp}}$

Problem 1.4. Given a number in base 16, find the corresponding decimal value by interpreting it as an unsigned number, signed magnitude, one's complement, and two's complement.

a) $(22)_{16}$

b) $(4B)_{16}$

c) $(7F)_{16}$

Solution. $\square$

a) $2 + 2(16) = (34)_{10}$
$(100010)_2$
$(0100010)_{\text{signmag}}$
$(00100010)_{\text{1's comp}}$
$(00100010)_{\text{2's comp}}$

b) $11 + 4(16) = (75)_{10}$
$(1001011)_2$
$(01001011)_{\text{signmag}}$
$(01001011)_{\text{1's comp}}$
$(01001011)_{\text{2's comp}}$

c) $15 + 7(16) = (127)_{10}$
$(1111111)_2$
$(01111111)_{\text{signmag}}$
$(01111111)_{\text{1's comp}}$
$(01111111)_{\text{2's comp}}$

Problem 1.5. Without changing their values, convert the following signed numbers given in 2's complement representation into 8-bit signed numbers in 2's complement representation, if possible. Give their decimal values and 8-bit 2's complement representation. If not possible, write the decimal value.

a) 1

b) 101

c) 1011101010

d) 111110000000

e) 00101

f) 011111111

Solution. □

a) $(11111111)_{\text{2's comp}} = (-1)_{\text{decimal}}$

b) $(11111101)_{\text{2's comp}} = (-3)_{\text{decimal}}$

c) $(-278)_{\text{decimal}}$

d) $(10000000)_{\text{2's comp}} = (-128)_{\text{decimal}}$

e) $(00000101)_{\text{2's comp}} = (5)_{\text{decimal}}$

f) $(255)_{\text{decimal}}$

Problem 1.6. Add the following signed numbers given in 2's complement representation. Express your final answer in 8 bits and in decimal. If the result of the addition cannot fit inside 8 bits, write Overflow.

a) $1 + 101$

b) $01101111 + 01$

c) $0010 + 1100$

d) $0101 + 001001$

Solution. ☐

a)
$$\begin{cases} 11111111 \\ 11111101 \end{cases} \Rightarrow \quad (11111100)_{\text{2's comp}} = (-4)_{\text{decimal}}$$

b)
$$\begin{cases} 01101111 \\ 00000001 \end{cases} \Rightarrow \quad (01110000)_{\text{2's comp}} = (112)_{\text{decimal}}$$

c)
$$\begin{cases} 00000010 \\ 11111100 \end{cases} \Rightarrow \quad (11111110)_{\text{2's comp}} = (-2)_{\text{decimal}}$$

d)
$$\begin{cases} 00000101 \\ 00001001 \end{cases} \Rightarrow \quad (00001110)_{\text{2's comp}} = (14)_{\text{decimal}}$$

Problem 1.7. You wish to express -128 as a 2's complement number.

a) How many bits do you need? (the minimum number)

b) With this number of bits, what is the largest positive number you can represent? (Please give answers both in decimal and binary.)

c) With this number of bits, what is the largest unsigned number you can represent? (Please give answers both in decimal and binary.)

Solution. □

a) 8 bits

b) 127
$(1111111)_2$

c) 255
$(11111111)_2$

Problem 1.8. We have represented numbers in base-2 (binary) and in base-16 (hex). We are now ready for unsigned base-4, which we will call quad numbers. A quad digit can be 0, 1, 2, or 3.

a) What is the maximum unsigned decimal value that one can represent with 5 quad digits?

b) What is the maximum unsigned decimal value that one can represent with n quad digits? (Hint: your answer should be a function of n.)

c) Add the two unsigned quad numbers: 123 and 321.

d) What is the quad representation of the decimal number 333 using 5 quad digits?

Solution. □

a) $3(4)^0 + 3(4)^1 + 3(4)^2 + 3(4)^3 + 3(4)^4 = \boxed{1023}$

b) $4^n - 1$

c)

$$\begin{cases} 123 \\ 321 \end{cases} \Rightarrow \boxed{(1110)_2}$$

d) $(11031)_4$