**Q1 Solutions**

a)

NMOS:
Vg=0.4V
Vs=0V
Vgs=0.4V
Vt=0.1V

$$I_n = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{gs} - V_t)^2 = 40\mu A$$

PMOS:
Vg=0.4
Vs=Vdd=2V
Vgs=-1.6V
Vt=-0.3V

$$I_p = \mu_p C_{ox}\frac{W}{L} * \left((|V_{gs}| - |V_t|)^2 * |Vds| - \frac{1}{2}V_{ds}^2\right) = I_n = 40\mu A$$
$$V_{ds} = -0.00237V$$
$$V_{out} = 1.99763V \approx 2V$$

b)
i
Vgs=-2V,
Vt=0.1V
|Vgs|>|Vt|, on

ii
Vgs=0V
Vt=0.1V
Vgs<Vt, off

iii

| Va | Vb | Vout |
|----|----|------|
| 0 | 0 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

NOR gate

**Q2 solutions**

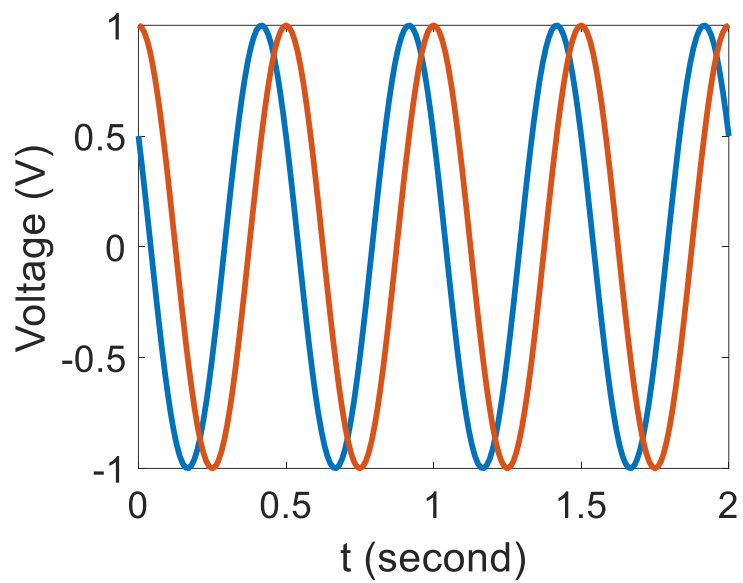a. $z_1 = \frac{2x^2 - y^2}{4x^2 + y^2} + \frac{3xy}{4x^2 + y^2} j$

$z_2 = 3 \cos \theta - j \sin \theta$

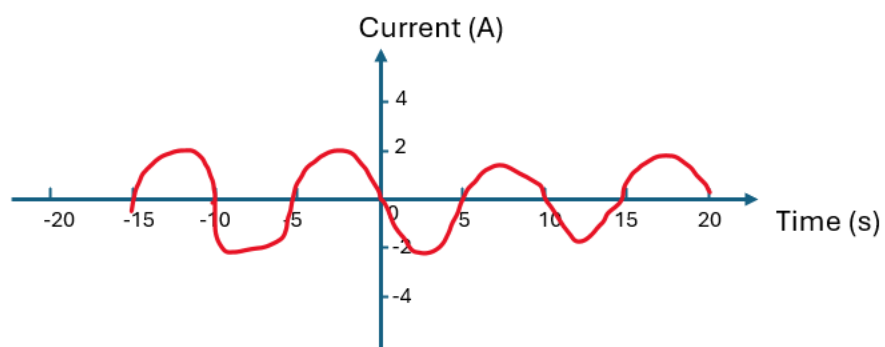$z_3 = -(\sin \theta + 2 \cos \theta) + j(\cos \theta - 2 \sin \theta)$

$z_4 = \cos \frac{n\pi}{4} - j \sin \frac{n\pi}{4}$

b. $V = \cos\left(4\pi t + \frac{\pi}{3}\right)$, Amplitude: 1 V, Frequency: 2 Hz, Phase $\frac{\pi}{3}$

The new voltage signal $V' = \cos(4\pi t)$



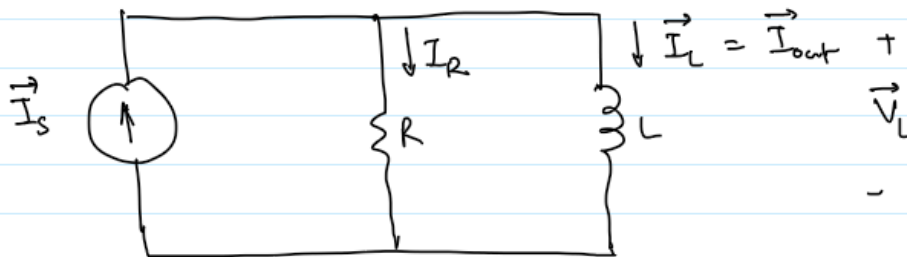c.  $I = 2\cos\left(0.2\pi t + \frac{\pi}{2}\right)$

**Q3 (25 points)**

(a) $\quad Z_R = R = 1\Omega$

$\quad Z_L = jwL = j(2\pi \times 100 \times 10\times10^{-6}) = j0.002\pi = j6.28\times10^{-3}\,\Omega$

(b) $\quad Z_R = R = 1\Omega$

$\quad Z_L = jwL = j(2\pi\times 10^6 \times 10\times10^{-6}) = j20\pi = j62.8\,\Omega$

(c)



$$\frac{\vec{V_L}}{R} + \frac{\vec{V_L}}{jwL} - \vec{I_s} = 0 \quad\Rightarrow\quad \vec{V_L}\left(\frac{jwL+R}{jwLR}\right) = \vec{I_s}$$

$$\Rightarrow\quad \vec{V_L} = \frac{jwLR}{R+jwL}\,\vec{I_s}$$

$$\vec{I_L} = \frac{\vec{V_L}}{jwL} = \vec{I_s}\,\frac{jwLR}{R+jwL}\cdot\frac{1}{jwL}$$

$$= \vec{I_s}\,\frac{R}{R+jwL}$$

$$= \vec{I_s}\,\frac{1}{1+j\frac{wL}{R}}$$

$$\frac{\vec{I_L}}{\vec{I_s}} = \frac{1}{1+j\frac{wL}{R}} = \frac{1}{\sqrt{1+\left(\frac{wL}{R}\right)^2}}\,e^{-j\tan^{-1}\left(\frac{wL}{R}\right)}$$

(d) corner frequency $\omega_c = \dfrac{R}{L} = 10^5 \text{ rad/s}$

$$f_c = \dfrac{\omega_c}{2\pi} = 15.9 \text{ kHz}$$

Low-pass filter since low frequency signals are passed and high frequency signals are suppressed.

(f) At $f = 1MHz$, $\omega = 2\pi f = 2\times10^6 \pi = 6.28\times10^6$ rad/s

$$\left|\frac{I_L}{I_s}\right| = \frac{1}{\sqrt{1 + \left(\frac{6.28\times10^6}{10^5}\right)^2}} = 0.0159$$
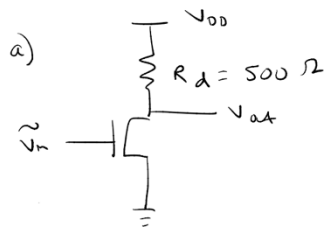
$$\angle\left(\frac{I_L}{I_s}\right) = -\tan^{-1}\left(\frac{6.28\times10^6}{10^5}\right) = -1.55 \text{ rad} = -89.1°$$

(g) From part (f), at $f = 1MHz$, $\left|\frac{I_L}{I_s}\right| = 0.0159$, $\angle\left(\frac{I_L}{I_s}\right) = -89.1°$

magnitude of $I_L(t) = 2\times0.0159 = 0.0318$ mA

phase of $I_L(t) = 60° - 89.1° = -29.1°$

$\approx -30°$

$$\therefore I_L(t) = 0.0318 \cos(2\pi10^6 t - 29.1°) \text{ mA}$$
$$= 0.0318 \cos(6.28\times10^6 t - 29.1°) \text{ mA}$$
$$= 0.0318 \cos(6.28\times10^6 t - 0.508) \text{ mA}$$

**Q4**

a)

$V_{DD}$

$R_d = 500 \ \Omega$

$V_{out}$

$\tilde{V}_{in}$

$$g_m = \frac{2 I_D}{(V_{GS} - V_T)} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$I_D = 5 \ mA = 5 \ e^{-3} \ A$

$V_{GS} = (2.5V - 0) = 2.5 V$

$V_T = 2V$

$\Rightarrow V_{GS} - V_T = 0.5 V$

$$\Rightarrow \boxed{g_n = \frac{2(5 e^{-3})}{0.5} = 0.02 \ \frac{A}{V}}$$

b)  Constant  voltage  gain :  can
     use  the  small  signal  model,

$$\frac{V_{out}}{V_{GS}} = -g_m R_L' \qquad g_m = 0.02 \ \frac{A}{V}$$

$$R_L' = \frac{R_d R_2}{R_d + R_L} = \frac{(500)(2 e 3)}{8 e 3} = 400 \ \Omega$$

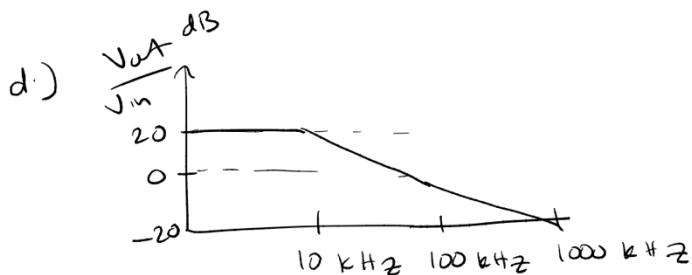$$\Rightarrow \frac{V_{out}}{V_{GS}} = -(0.02)(400) = \boxed{-8}$$

gain  is  8,  $\boxed{\text{inverting}}$

c)

$$\# \, dB \;=\; 20 \, \log \, (\#)$$

$$=\; 20 \, \log \, (8)$$

$$=\; 20 \, (0.9031)$$

$$\boxed{\# \, dB \;=\; 18.06} \quad \approx \; 20 \; dB.$$

d) $\dfrac{V_{out}}{V_{in}}$ dB



$\boxed{\text{Low pass filter}}$ since lower frequencies see high gain, higher frequencies are suppressed.

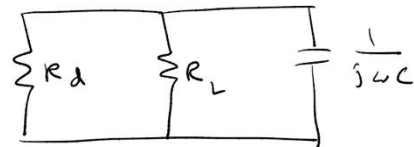e) Rolls off at $-20 \, dB / decade$.

So at $1 \, MHz \, (= 1000 \, kHz)$

at $\boxed{-20 \, dB}$

$-20 \, dB = 20 \, \log \, (x)$

$-1 = \log \, (x)$

$10^{-1} = x \;\Rightarrow\; \boxed{\text{gain is } \dfrac{1}{10}.}$

f)



$$\boxed{\dfrac{1}{Z_{total}} \;=\; \dfrac{1}{R_d} + \dfrac{1}{R_L} + j\omega C}$$