Q1. 30 Points total. This question has four parts (Part I-Part IV),

Part I: 6 Points. Consider the following components (a)-(c). In each case, state if the component is linear or non-linear and briefly explain your answer:

(a) A voltage source of magnitude $3I_0$, where I_0 is the current flowing through the source.

This is Linear. It is equivalent to a resistor of value = 3Ω.

(b) A resistor whose value is given by $R = 3I_0$, where I_o is the current flowing through the resistor.

Resistance value is not a constant, but depends on current through component. Hence it is Non-Linear.

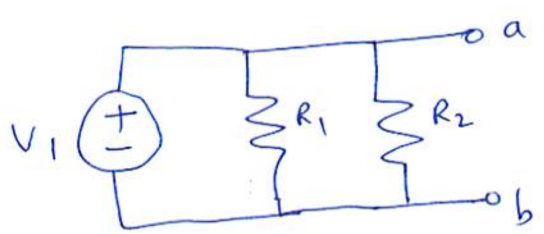
(c) A current source which provides a current of $5V_0$ when the voltage applied across it is V_0.

This is linear. It is equivalent to a resistor of value = $\frac{1}{5}\Omega$. (Using an external source).

Part II: 6 Points: Provide examples of circuits with (i) $V_{Th} = 0$ and $R_{Th} \neq 0$; and (ii) $V_{Th} \neq 0$; $R_{Th} = 0$. In each case, the circuit must have at least one source and two resistors.

(i) Any circuit without at least one independent source

(ii) When you deactivate sources, you should get a short circuit. Then $R_{Th} = 0$. Examples below.



R_{Th} (between a and b) is zero.

V_{Th} (between a and b) = V_1

Part III: 8 Points: The figure below, shows the ionic current (consisting of positive potassium ions) as a function of time flowing through a single biological component (called an ion channel) when a potential difference of 60 mV is applied across it. When the ion channel is open, the current is 4 pA (1 pA = 10^{-12} A) and when the ion channel is closed, the current is zero.

(i) Draw a simple linear circuit model (or an equivalent circuit) for the ion channel. Make sure that your equivalent circuit captures all the electrical characteristics described above.

switch in series with ion-channel.

Note: I will give full credit if the ion-channel is represented by a resistor of value = $R = 60mV/4pA = 1.5 \times 10^{10}\ \Omega$. +10

I will give bonus points for the foll. answer:
The ion channel cannot be represented by a ~~resistor~~ resistor, since we do not know it is linear. We have only one data point. We need at least 2 to say if it is linear.

(ii) For the time interval between t = 550 ms and t = 650 ms, calculate the total charge flowing through the ion channel, assuming it is open over the entire time interval.

$\Delta t = 100\,ms = 0.1\,s.$

$i = 4 \times 10^{-12}\,A$

$\Delta q = i \Delta t = 4 \times 10^{-13}$ Coulomb.

(iii) Scorpion venom blocks potassium ion channels and prevents any ions from flowing. Using your equivalent circuit proposed in (i), describe what a scorpion sting does to the ion channel. It opens the switch or closes the ion channel.

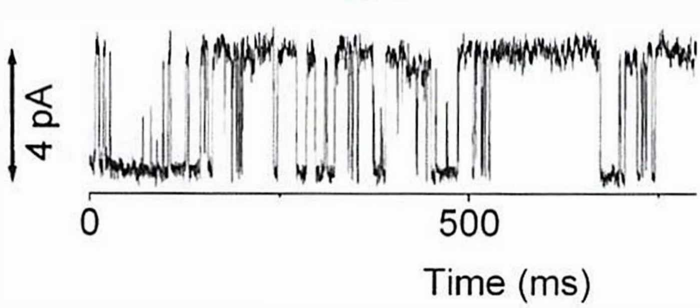


Figure for Q1 Part III

Part IV: 10 Points A diode is a circuit component with the following I-V characteristic:

$$i = I_0\left(\exp^{\frac{q}{nkT}v} - 1\right)$$

where i is the current through the diode, v is the voltage across the diode, and I_0, n, k, and T are all constants.

Non-linear.

a) Is the diode a linear component?

b) Can the following techniques be used to analyze a circuit containing diodes?

i) KVL Yes.

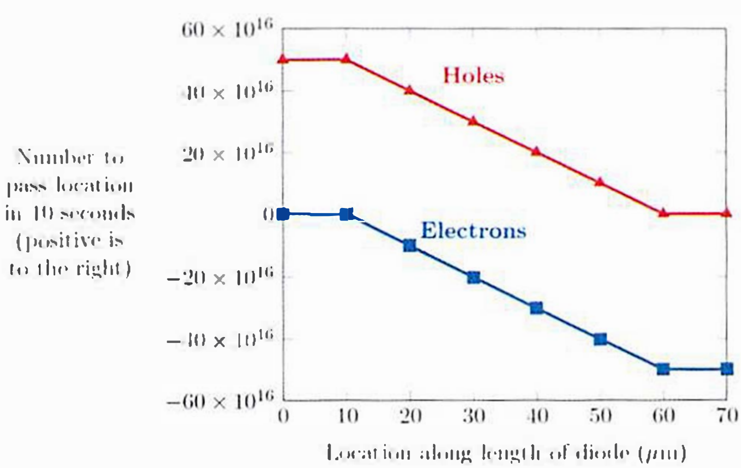
ii) Node Voltage Analysis Yes.

iii) Superposition No

iv) Norton Equivalent Circuits No.

Valid only for linear circuits.

c) The charge flow in a diode is due to a combination of electrons and holes. Electrons have −1 unit of charge (−1e−) and "holes" are positive charges which have +1 unit of charge (+1e−). An experiment was performed on a diode which measured how many electrons and holes crossed certain locations along the diode's length over a time of 10 seconds (see the Figure below). Explain how KCL is obeyed in the diode even though different numbers of charges pass each location in the diode.



The diode is 70 μm long. In this figure, positive numbers indicate that the charges are flowing from left to right; negative numbers indicate that charges are flowing from right to left.

What is the current in the diode described above? Express your answer in Amperes with an appropriate SI prefix (μ, m, k, M). Recall that 1 Coulomb has the same amount of charge as 6.24 × 10^{18} electrons.

Charge flowing in 1s $= \dfrac{50\times10^{16}}{10} = 5\times10^{16}$ charges/sec.

$= 5\times10^{16}\times1.6\times10^{-19}$ $\frac{\text{Coul}}{\text{s}}$

$= 8\times10^{-3}$ A ~~0.8 mA~~.

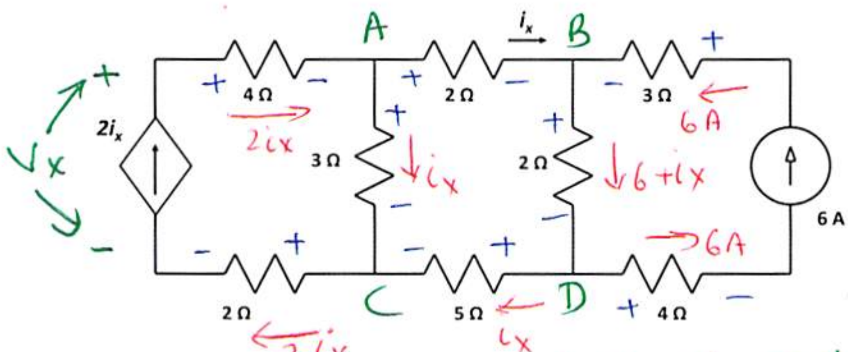
$= 8$ mA.

Q2. 15 Points. Consider the circuit shown below.

(i) Calculate i_x

(ii) Is the dependent source supplying or receiving power? Briefly justify your answer.

(iii) Calculate the power received/supplied by the dependent source.



(i) Mark currents using KCL at nodes A, B, C, D. (red)
Mark signs of potential drops (blue).

Go around loop ABCD (KVL)

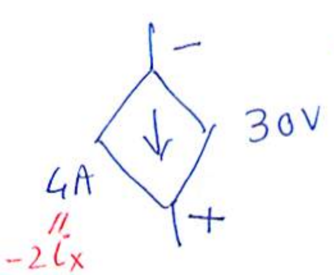
$$5i_x - 3i_x + 2i_x + 2(6+i_x) = 0. \Rightarrow i_x = -2A.$$

(ii)(iii). Calculate potential drop across $2i_x$ current source. Using KVL in Left loop. Let this drop be V_x (see fig).

$$2(2i_x) - V_x + 8i_x + 3i_x = 0.$$

$$4i_x + 8i_x + 3i_x = V_x = 15i_x = -30V.$$

Orientation of voltage and current in dependent source



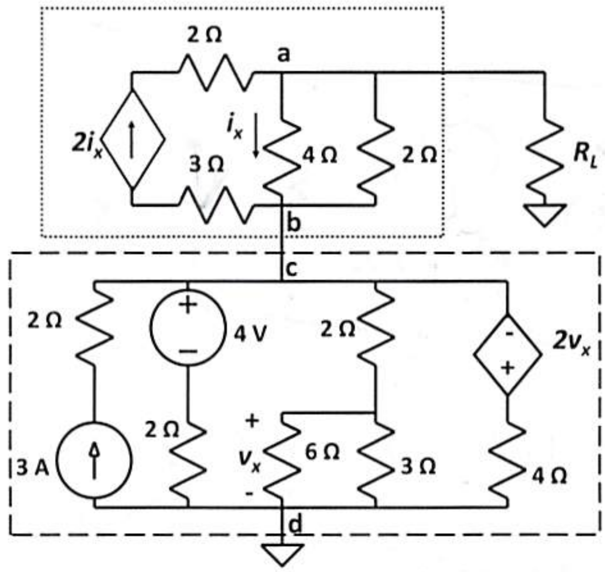
By passive sign conv, Power is generated and is equal to ~~2×60W~~ 30V×4A $\boxed{= 120W}$ Generated.

Q4. 20 Points. Consider the figure in the circuit shown below:

i) Calculate the Thevenin Equivalent circuit between a and b of the part of the whole circuit that is enclosed within the box with **dotted** lines.

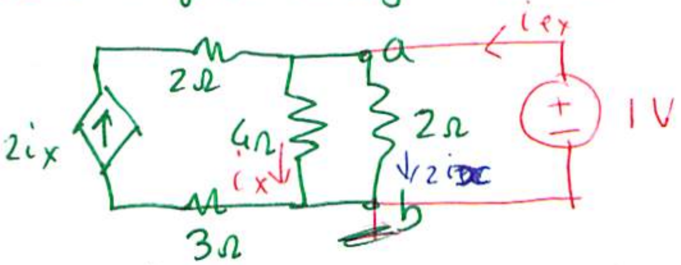
ii) Calculate the Thevenin Equivalent circuit between c and d of the part of the whole circuit that is enclosed within box with **dashed** lines.

iii) For what value of R_L is the power transferred from the rest of the circuit to R_L a maximum? Calculate the power dissipated in R_L.



Use Thevenin Eqs.

(i) This circuit has <u>no</u> independent source. Hence $V_{Th} = 0$.
We need to find only R_{Th}. (Use external source method.

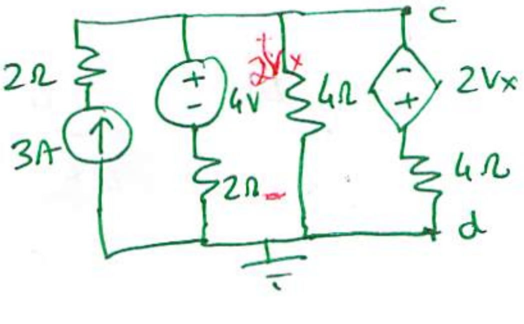


$R_{Th} = \frac{1V}{i_{ex}}$

At node a (KCL).
$i_x + 2i_x - 2i_x = i_{ex}$. or $i_{ex} = i_x$.

KVL at right loop:
$1V = 2 \times 2i_x$. $\Rightarrow$ $i_x = \frac{1}{4}A$. or $R_{Th} = 4\Omega$.

(ii) Simplify.



At node c

$$\frac{V_c - 4}{2} + \frac{V_c}{4} - 3 + \frac{V_c + 2V_x}{4} = 0 \quad \text{(1)}$$

$V_c = 2V_x$ or $V_x = \frac{V_c}{2}$

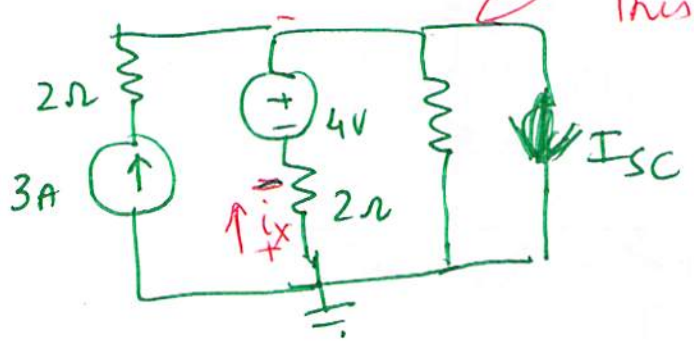
Eq 1 becomes

$$\frac{V_c - 4}{2} + \frac{V_c}{4} - 3 + \frac{V_c + V_c}{4} = 0$$

$$\Rightarrow V_c = V_c = 4V = V_{Th}$$

(ii) To find R_{Th}, use V_{OC}, I_{SC} method.

Note: $V_x = 0$, since 4Ω resistor is shorted.



This is also at 0V (Short circuit to ground)

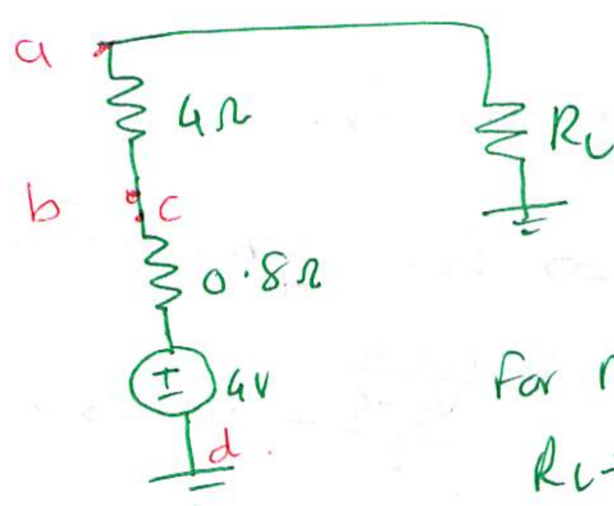
Hence $2i_x - 4 = 0$
or $i_x = 2A$.

Use KCL at top node.

$3A + 2A = I_{SC} = 5A$.

Hence $R_{Th} = \frac{4V}{5A} = 0.8V$.

(iii) Circuit becomes.



For max power to R_L

$R_L = 4 + 0.8\Omega = 4.8\Omega$.

$\text{Power} = \frac{V^2}{4R_L} = \frac{4^2}{4 \times 4.8} = \frac{4}{4.8} W$

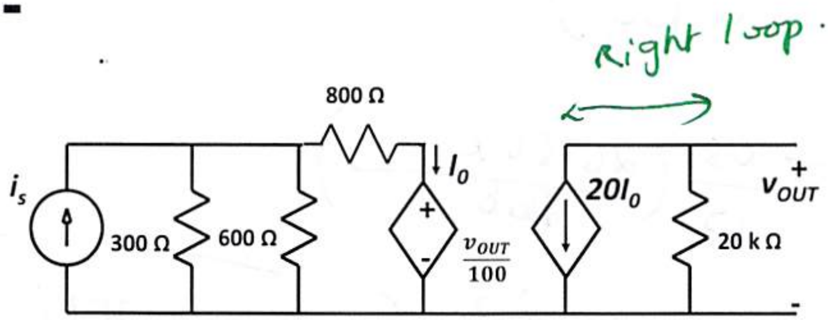
$= 0.833W$

Q5. 20 Points.

Consider the circuit below.

(i) Calculate v_{OUT} in terms of i_s

(ii) Calculate the power associated with the dependent voltage source, clearly indicating if power is generated or dissipated.



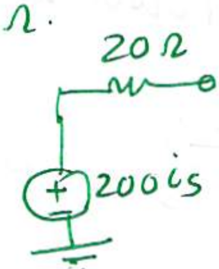
(i) From right loop

$V_{OUT} = -20I_o(20k\Omega) = -400k\,I_o$.

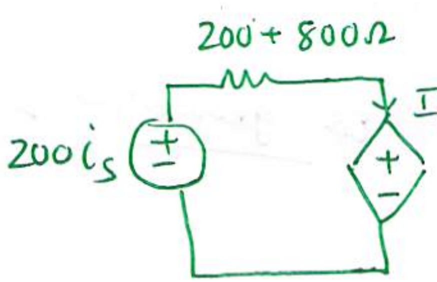
simplify left side. $300\Omega \parallel 600\Omega = 200\Omega$.

source transform:

$\Rightarrow$



Circuit becomes.



$\frac{V_{OUT}}{100} = -4k\,I_o$

use KVL

$-200 i_s + 1000 I_o - 4000 I_o = 0$

$i_s = -\frac{3000 I_o}{200} = -15 I_o$.

$V_{OUT} = \cancel{-400 I_o}$

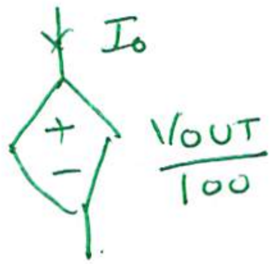
$-400k\,I_o$

$= -400k\left(-\frac{i_s}{15}\right)$

$= \frac{400k}{15} i_s$

$= 26.66k\,i_s$.

ii)



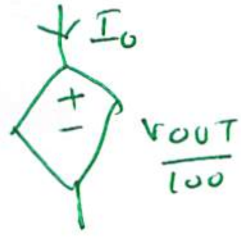
$$\text{Power} \equiv I_o\left(\frac{V_{OUT}}{100}\right)$$

$$\equiv -\frac{i_s}{15}\left(\frac{26.66k\, i_s}{100}\right)$$

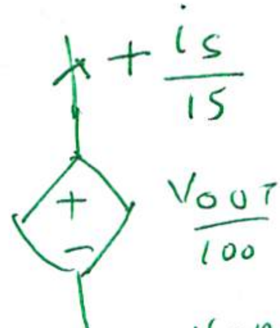
$$\equiv -\frac{26.66k\, i_s^2}{1500} \quad \boxed{= -17.77\, i_s^2}$$

Power is generated in dependent voltage source

Sign Check:
Let i_s be positive.
then



$\Rightarrow$



$\frac{V_{OUT}}{100} = 26.6k\, i_s$.

Hence power is generated ✓

Also for i_s negative.
same result.