Lecture Notes 2: Passive Sign Convention

Recall the following:

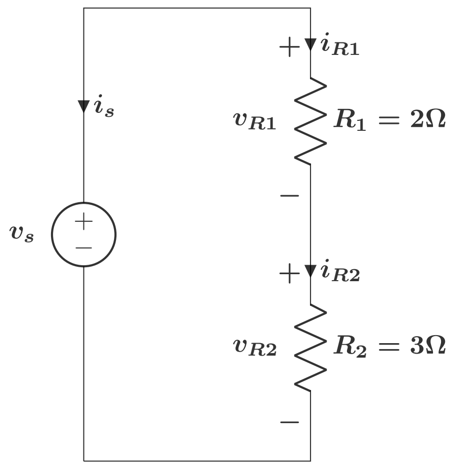
- Circuits consist of components interconnected by wires (nodes).
- Current flows *through* components and is measured in Amps = Coulombs/second.
- Voltage appears *across* components and is measured in Volts = Joules/Coulomb. Voltage is a measure of how much energy a charge gains or loses as it passes through a potential difference.
- Voltage across a component times current through the same component is the power being supplied or absorbed by that component.
- Conservation of energy leads to KVL $\sum_{loop} v = 0$.
- Conservation of charge leads to KCL $\sum_{node} i = 0$.
- Components enforce a relationship between their voltage and current, called the i-v relationship. Voltage sources obey $v(i) = \text{Const}$ while resistors obey $v(i) = \text{Const} \times i$.
- Solving a circuit involves (0) labeling the voltage across and current through each component, (1) writing the laws of physics as equations (KVL, KCL, component laws), and (2) solving the resulting system of equations.

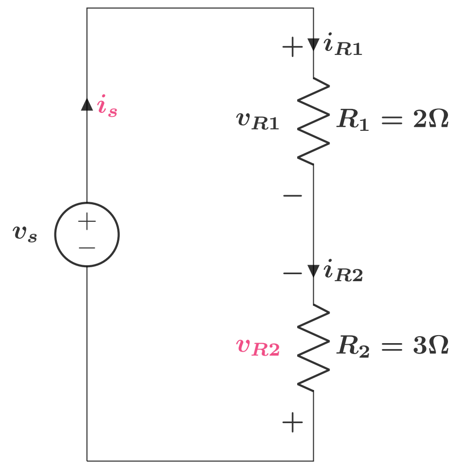
In the previous lecture, we discussed labeling the unknown voltages and currents, i.e. choosing what direction you will call "positive." We claimed that the orientation of the voltage and current labels is arbitrary, the same way that your choice of positive or negative directions in a free body diagram is arbitrary. In particular,

> The orientations of the voltage and current labels are *not* a claim about what direction you think current actually flows or what polarity you expect voltage to actually have. It is simply a declaration that "this direction shall count as positive" and is therefore arbitrary.

Let us see how that plays out:

(0) Label all component voltages and currents





(1) Write the laws of physics as equations

KVL: $v_s - v_{R1} - v_{R2} = 0$
KCL: $i_s + i_{R1} = 0$
KCL: $i_{R1} - i_{R2} = 0$
V Source: $v_s = V_1$
R1: $v_{R1} = R_1 i_{R1}$
R2: $v_{R2} = R_2 i_{R2}$
6 Equations 6 Unknowns

$v_s - v_{R1} + v_{R2} = 0$
KCL: $i_s - i_{R1} = 0$
$i_{R1} - i_{R2} = 0$
V Source: $v_s = V_1$
R1: $v_{R1} = R_1 i_{R1}$
R2: $v_{R2} = R_2 \times (-i_{R2})$
6 Equations 6 Unknowns

(1) Solve the system of equations

$$v_1 - i_{R1}R_1 - i_{R2}R_2 = 0$$
$$v_1 - i_{R1}R_1 - i_{R1}R_2 = 0$$
$$\Rightarrow \boxed{i_{R1} = \frac{v_1}{R_1 + R_2}}$$
$$\Rightarrow \boxed{i_{R2} = \frac{v_1}{R_1 + R_2}}$$
$$\Rightarrow \boxed{i_s = -\frac{v_1}{R_1 + R_2}}$$
$$v_{R1} = i_{R1}R_1$$
$$\Rightarrow \boxed{v_{R1} = \frac{R_1}{R_1 + R_2} V_1}$$
$$v_{R2} = i_{R2}R_2$$
$$\Rightarrow \boxed{v_{R2} = \frac{R_2}{R_1 + R_2} V_1}$$

$$v_1 - i_{R1}R_1 + (-i_{R2})R_2 = 0$$
$$v_1 - i_{R1}R_1 - i_{R1}R_2 = 0$$
$$\Rightarrow \boxed{i_{R1} = \frac{v_1}{R_1 + R_2}}$$
$$\Rightarrow \boxed{i_{R2} = \frac{v_1}{R_1 + R_2}}$$
$$\Rightarrow \boxed{i_s = +\frac{v_1}{R_1 + R_2}}$$
$$v_{R1} = i_{R1}R_1$$
$$\Rightarrow \boxed{v_{R1} = \frac{R_1}{R_1 + R_2} V_1}$$
$$v_{R2} = -i_{R2}R_2$$
$$\Rightarrow \boxed{v_{R2} = -\frac{R_2}{R_1 + R_2} V_1}$$

Now the results might look different, but they are identical in physical interpretation.

- i_s has an opposite sign in the second solution, but its positive direction is also defined oppositely $\Rightarrow$ both methods predict current going “up.”
- v_{R2} has an opposite sign in the second solution, but its positive orientation is also defined oppositely $\Rightarrow$ both methods predict higher potential above R_2 relative to below R_2.

Thus, the choice of polarity for the unknown voltages and currents is completely arbitrary. (These are nothing more than the direction you’re choosing to call “positive”).

Nevertheless, there is a commonly used convention that most electrical engineers use most of the time: If you use the **Passive Sign Convention**, you always label the unknown component current as “entering” the (+) end of the unknown component voltage. There are two advantages to this convention:

Follows Passive Sign Convention

Does Not Follow Passive Sign Convention
(but still gives correct answers!)

1. Component laws are *always* defined using the passive sign convention. If you use the PSC in circuit analysis, you never have to think about the component laws (e.g., $v = iR$ vs $v = (-i)R$)
2. When you use the passive sign convention, then power always follows this rule:
   - $v \times i > 0 \Rightarrow$ power is *entering* the component (absorbing or sinking power)
   - $v \times i < 0 \Rightarrow$ power is *leaving* the component (supplying or sourcing power)

   If you choose to not use the passive sign convention, then you must think hard about what direction current is *actually* flowing and what polarity the voltage *actually* has in order to determine if the charges are gaining energy from the component or giving energy to the component.

The passive sign convention still leaves a arbitrary choices (which direction do you want to call “positive” voltage?), but half as many as before.

The circuit above is an important circuit that we will return to again and again. It is called a **voltage divider**. Its result is important enough to memorize:

In a voltage divider, the voltage across any component

$$\text{This Component's Voltage} = \frac{\text{This Component's Value}}{\text{Sum of Component Values}} \times V_{in}$$

Stated this way, the voltage divider equation is easily extended to three or more components.