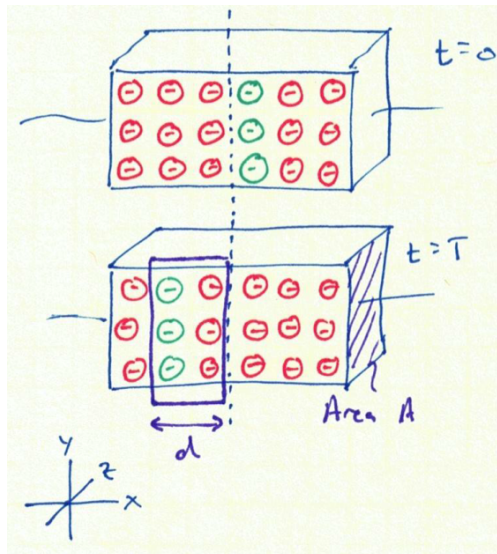


Lecture Notes 11: Conductors and Semiconductors

We have now finished our unit on circuits, and you are well prepared to analyze and understand a great many circuits. We now embark on a new unit to understand the all-important **transistor**, our first controllable device. In particular, we seek to understand the metal-oxide-semiconductor field-effect transistor (**MOSFET**), by far the most commonly used transistor in the modern world. It will take several lectures for us to build up to the actual MOSFET, beginning with an exploration of how current actually flows in materials.

Microscopic Current - Current Density

Let us develop a more sophisticated vocabulary for current. Consider a block of material with electrons flowing to the left, as shown below. It has a cross sectional area of A and the density of electrons is n (with units of electrons per cm^3 , or simply cm^{-3}).



The number of electrons that cross the dotted line in T seconds we will call N , and the distance that any electron travels in that time is d . The volume that contains the electrons that crossed the dotted line is $\text{VOL} = Ad$. The charge on each electron is $q = -1e^-$. Therefore, the current is

$$I = \frac{Q}{T} = \frac{q \times N}{T} = \frac{q \times (n \times \text{VOL})}{T} = \frac{qnAd}{T}$$

where n is the **number density** of electrons ($\#/\text{cm}^3$). By noting that d/T is the velocity of the electrons v (and with apologies for briefly using the letter v for something other than voltage) we find that

$$I = qnvA$$

The quantity I represents the total current – it applies to the whole device. If current varies at different points in the cross section, we need a *microscopic* quantity to describe the current *at each point*. This is called the **current density** (A/cm^2):

$$J = I/A = \vec{J}(x, y, z)$$

Combining this new terminology with our description of current above, we find that

$$J = qnv$$

Microscopic Voltage - Electric Field

We now have a good vocabulary to talk about current at the microscopic (i.e., point-by-point) level. We now need a microscopic description of voltage. We have previously discussed voltage almost as a “force” that pushes charge around a circuit, and charges moving with or against this “force” gain or lose energy. Nevertheless, voltage is taken between two points in space (i.e., across entire devices) – is there a *microscopic* quantity that can tell us how charges get pushed *at each point in space*?

This is particularly important because electrons don’t know about voltage – they can’t see the voltage where they came from or the voltage where they’re going. They only know information at their own location. Electrons are like blindfolded hikers trying to climb up a mountain. They can’t see which direction is up or down, but they can sense the “slope” under their feet, and will always travel “uphill.”

Is there any physical truth to this mountain climbing analogy?

⇒ Yes! When there is a voltage between two points, an **electric field** develops in the space between and near those points. The electric field is essentially the slope in voltage at any point in space, and it points downhill (from high voltage to low voltage).

Mathematically, slopes are expressed as derivatives, so

$$E = -\frac{dV}{dl} \quad \Delta V = -\int E \cdot dl \quad (1)$$

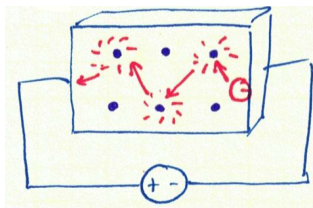
The electric field tends to push (+) charges downhill, causing them to lose energy. It tends to pull (-) charges uphill, causing them to lose energy. If a charge travels opposite the force of the electric field, then the charge is gaining energy (and something else must be doing the pushing).

Like current density, the electric field is a vector (meaning it has three components) and it is different at each point in space (meaning it varies with all three spacial dimensions), $\vec{E}(x, y, z) = \begin{bmatrix} E_x(x, y, z) \\ E_y(x, y, z) \\ E_z(x, y, z) \end{bmatrix}$.

We can think of the electric field as the microscopic (local to a location (x, y, z)) version of voltage.

Microscopic Resistance – Resistivity

Many materials are resistive: as electrons flow through the material, they bounce off of stationary atoms, losing energy each time.



Using the microscopic definitions of voltage (electric field) and current (current density), we can characterize a resistive material as having

$$R = \frac{V}{I} = \frac{El}{qn v A} = \underbrace{\frac{1}{qn \frac{v}{E}}}_{\text{Material Properties}} \times \underbrace{\frac{l}{A}}_{\text{Size/Geometry}} \quad (2)$$

We can further simplify this by assuming that velocity-per-electric-field, v/E , is a material property. Why should this be? v/E is how fast a charge moves for a given applied electric field (force). This ought to be related to how many atoms there are to get in the way, how many electrons there are, how densely packed the atoms are, and so forth, but it should be fairly linear (meaning that double E should cause double v). Let us therefore call this quantity the **mobility**,

$$\mu \triangleq \frac{v}{E}$$

Using this nomenclature, the resistance becomes a product of material constants alone times length over area. We lump all of the material constants together into a single material property called **resistivity**, $\rho = 1/qn\mu$.

$$R = \frac{1}{qn\mu} \frac{l}{A} = \rho \frac{l}{A}$$

Alternatively, we often speak of conductance instead of resistance,

$$G = \frac{1}{R} = qn\mu \frac{A}{l} = \sigma \frac{A}{l}$$

where $\sigma = qn\mu = 1/\rho$ is called the **conductivity** of the material.

Using microscopic quantities, we find that

$$J = \frac{I}{A} = qnv = qn\mu E$$

$$\boxed{J = \sigma E}$$

This equation (or its equivalent, $E = \rho J$) is the **microscopic version of Ohm's law**. It's the same idea as Ohm's Law – that current is linearly proportional to voltage – but applied to a single point in space as opposed to a whole device.

Conductors and Semiconductors

A lot clearly depends on n , the concentration of *mobile* charges (as opposed to stationary charges like atoms and electrons that are tightly bound to atoms). Where do *mobile* charges come from?

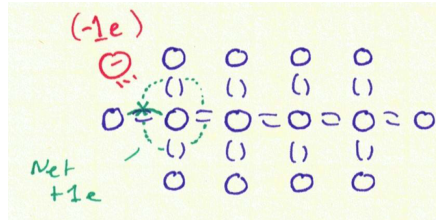
In solids, the number of protons equals the number of electrons in every atom, so the bulk solid is neutral. In addition, neutrality is maintained *everywhere* in the solid unless some charges become mobile.

Mechanism 1 – Thermal Generation: Thermal energy is random energy – atoms and electrons are all buzzing about and bumping into each other randomly. Sometimes (not very often), a confluence of random events will cause a single electron to gain a *lot* of energy, sufficient to break free from its atom and become mobile. This is a thermally-generated mobile electron.

This actually creates two charges. The electron ($-1e$) is a negative mobile charge. But the atom that it left behind is now short one electron, so that atom has a net charge of $+1e$.

This positive charge imbalance left behind is called a **hole**, and we think of it as being mobile. How is that possible? If a neighboring electron jumps into the hole, it will leave behind a hole \Rightarrow a **“bound” electron moving into a neighboring hole is equivalent to the hole moving to the neighboring atom**.

Therefore, thermal generation creates **electron-hole pairs**, a negative mobile electron and a positive “mobile” hole.



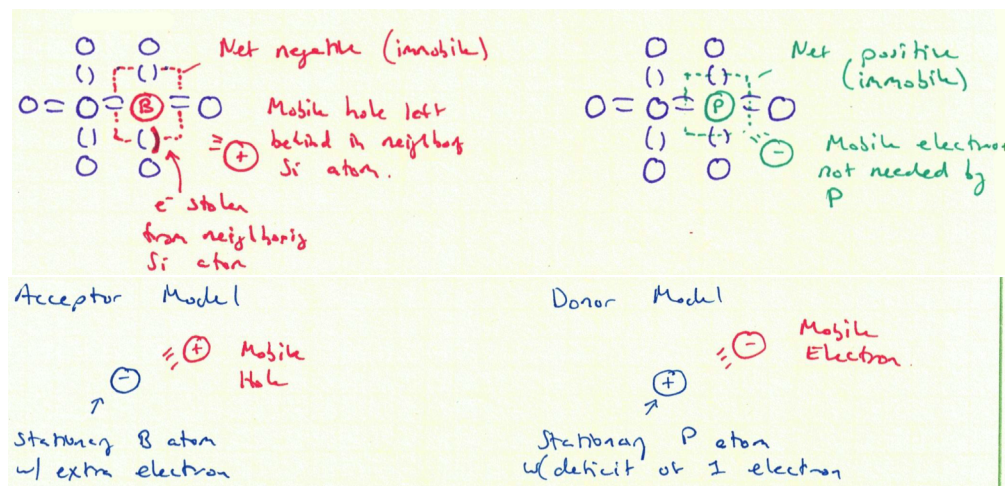
Mechanism 2 – Doping: All atoms want to have 8 electrons in their valence shell, and they can share electrons with neighboring atoms (**covalent bonding**) to make this happen. Silicon happens to have 4 electrons in its valence shell, and a Si crystal sees each Si atom bond to its 4 nearest neighbors. Thus every Si atom thinks it has 8 valence electrons and the whole configuration is quite stable.

Impurities in a silicon crystal may have more protons and electrons than Si (**donors**) or fewer (**acceptors**). Nevertheless, *all* atoms want to have 8 electrons in their valence shell. Therefore, if an impurity atom replaces a Si atom in the crystal, it will also bond with the nearest neighbors until the impurity has 8 valence electrons.

Donors: If an impurity has more than 4 valence electrons, the extra electron(s) are not needed for every atom to happily have a full valence shell, so they become mobile. They leave behind an atom that is short one electron, which becomes a *stationary* positive charge.

Acceptors: If an impurity has fewer than 4 valence electrons, it will steal a nearby electron to fill its valence shell. This leaves behind a mobile hole. The atom itself now has one electron beyond neutrality, so it becomes a *stationary* negative charge.

This is diagrammed in the figure below, with specific examples of Boron (an acceptor) and Phosphorus (a donor). Boron is Group III and therefore has 3 valence electrons. It “accepts” a nearby electron, becoming a stationary negative ion and leaving a mobile hole to wander the lattice. Phosphorus is Group V and has 5 valence electrons. It “donates” its extra electron to be mobile and wander the lattice, while the phosphorus atom itself becomes a stationary positive ion.



Technically, not every dopant becomes ionized. Nevertheless, the dopants *really* want to become ionized, so it only takes a little bit of thermal energy to get them there. In other words, extremely few of the atoms are cold enough to hang on to their hole/electron. It is a very good approximation at room temperature and above to say that every dopant is ionized. In 302, we will always use this approximation.