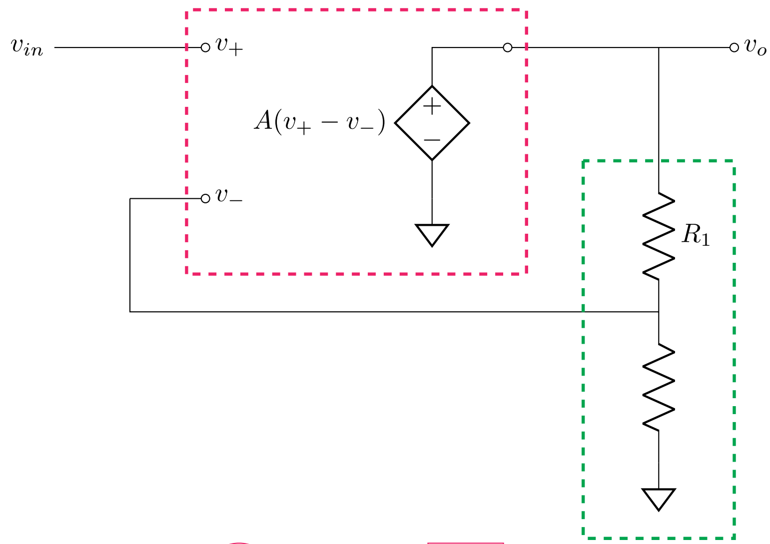
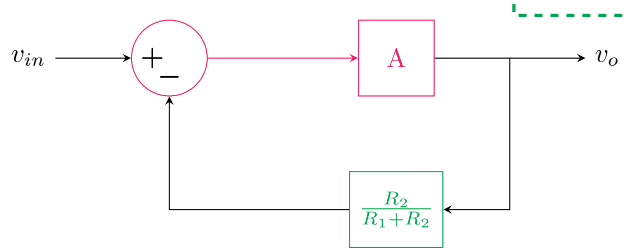
Lecture Notes 9: Feedback

A concept that is ubiquitous across all of engineering is **feedback**, wherein the output of a system affects its input. An example is shown below both as a circuit and as a block diagram. [1] Where the resistor because v_o does not depend on R_1 and R_2 and because v_- is open circuited, so the output of the resistor divider is also undisturbed):





The key features of feedback systems are

- **Amplification** or **gain** - a large multiplication of voltage or current (the dependent source)
- Feedback - the output must somehow influence the input (v_o influences v_-, for example)
- Specifically, **negative feedback**, wherein an increase in v_o would cause a decrease in v_o. (In the circuit above, if v_o were to be a little bigger, it would cause v_- to be a little bigger, which would cause the dependent source to output a little less voltage). Negative feedback has the possibility to be **stable**, wherein the output settles to a constant value. (By contrast, positive feedback tends to cause the output to go to infinity.)

In the circuit above, the equation for v_o clearly shows the negative feedback:

$$v_o = A(v_+ - v_-) = A\left(v_+ - \underbrace{\frac{R_2}{R_1 + R_2} v_o}_{\text{negative feedback}}\right)$$

[1] Recall that, when voltage carries information, using a block diagram is allowed when the output resistance of one stage is much less than the input resistance of the next stage. Where the amplifier output meets the resistor divider, this condition is satisfied because the amplifier has zero output resistance. Where the resistor divider output meets the amplifier input, the condition is also satisfied because the amplifier input has infinite resistance.

The final solution is

$$v_o = \frac{A}{1 + A\frac{R_2}{R_1+R_2}} v_{in} \underset{A \gg \frac{R_1+R_2}{R_2}}{\underset{\uparrow}{\approx}} \frac{R_1+R_2}{R_2} v_{in} = \left(1 + \frac{R_1}{R_2}\right) v_{in}$$

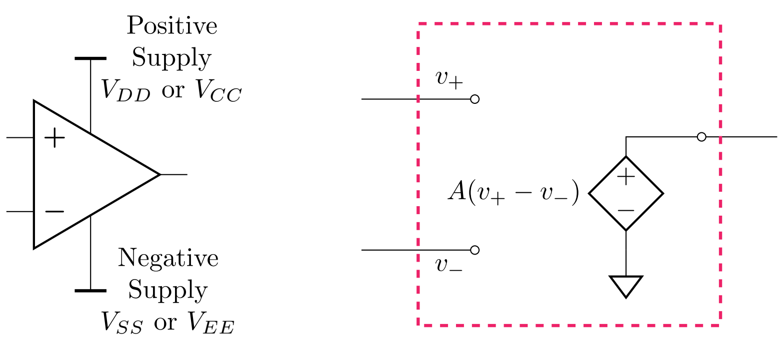
where we have assumed that $A \gg (R_1 + R_2)/R_2$, which means that $AR_2/(R_1 + R_2)$ dominates over the 1 in the denominator and then the A's cancel.

Note also that $v_- = \frac{R_2}{R_1+R_2} v_o \approx v_{in} = v_+$. This turns out to be a general property of negative feedback:

> The Golden Rule of Negative Feedback: In a stable negative feedback system with infinite gain, the negative amplifier input will be equal to the positive amplifier input.

Proof: if the negative input were not equal to the positive input, the output would be infinite. This is a possible outcome (unstable), but not for a stable system. We will not consider stability in ECE 302, and you can assume that any system with *negative* feedback is stable.

The real-life component that behaves as a voltage-controlled voltage source is the **Operational Amplifier** or **op-amp**.



An op amp needs power to provide the amplification, but the details of the op amp internals do not need to be known. We just know that charge goes from the positive supply to the negative supply, the op amp absorbs power, and somehow turns that into amplification.. While we do not know exactly *how* the power consumed provides amplification, we just need to know that it does.

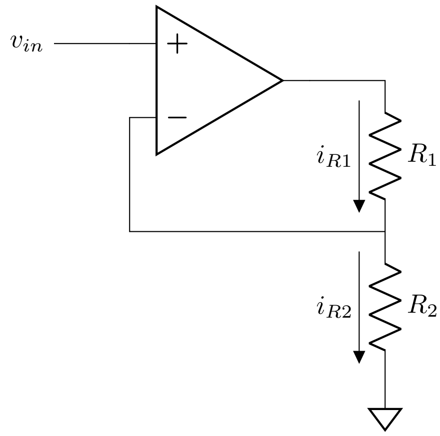
The power **rails** do limit the op amp. Op amps generall won't work properly if v_+, v_- and/or v_o are above the (+) rail or below the (-) rails. If those three voltages are "between the rails," the op amp is said to operate in the **linear region**. If the inputs are such that v_o tries to exceed one of the rails, then in practice v_o tends to saturate. In any case, outside of the linear region, it's sufficient to know that the op amp won't work correctly.

The Golden Rule of Negative Feedback provides us a rapid way to analyze the **non-inverting op amp configuration** from before:

Assume stable operation and infinite gain. The Golden Rule tells us that $v_- = v_+ = v_{in}$. Therefore $i_{R2} = v_{in}/R_2$. By KCL (and remembering that no current can flow into the (+) or (-) terminal of the op amp), $i_{R1} = i_{R2}$ and therefore $v_{R1} = R_1 i_{R1} = \frac{R_1}{R_2} v_{in}$. Finally, v_o is v_- plus the voltage across R_1, so

$$v_o = v_{in} + \frac{R_1}{R_2} v_{in} = \left(1 + \frac{R_1}{R_2}\right) v_{in}$$

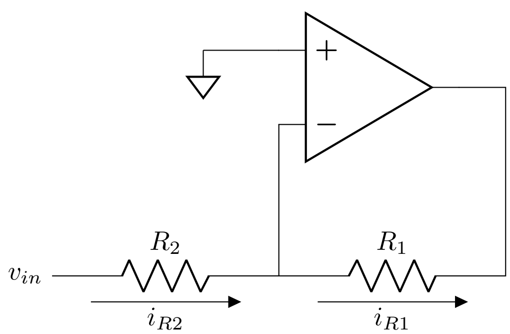
Using the Golden Rule to solve feedback problems is generally faster and more straightforward than the brute-force approach. You should know both, however, because (1) sometimes the amplifier gain A is not



infinity, and (2) when you solve the problem with a general value for A, you want to check your answer by making sure that it devolves to the correct expression as $A \to \infty$.

A point of terminology: we call A the gain *of the op amp*, while $(1 + R_1/R_2)$ is the *gain of the amplifier circuit*. This can be confusing, since the op amp is an amplifier and the entire circuit is an amplifier. When we need to be absolutely clear, we call A the **open-loop gain** of the op amp (the gain it would have by itself) and $(1 + R_1/R_2)$ the **closed-loop gain** of the entire circuit (with the feedback loop completed or "closed"). The same op amp always has the same open-loop gain, but putting that op amp in different configurations results in different closed-loop gain.

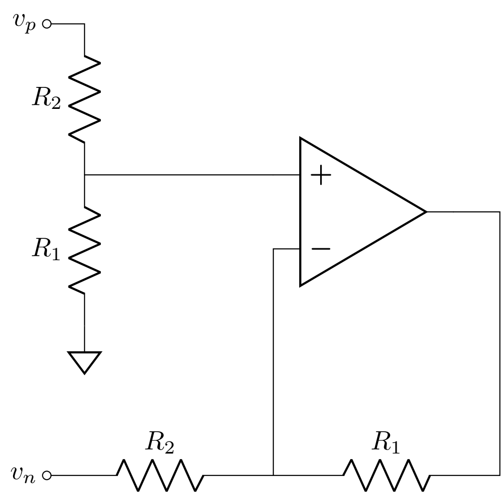
There are *many* op-amp configurations to do useful things. For example, consider the **inverting op amp configuration**:



$$v_- = v_+ = 0 \Rightarrow i_{R2} = \frac{v_{in}}{R_2} \Rightarrow i_{R1} = \frac{v_{in}}{R_2} \Rightarrow v_o = 0 - v_{R1} = -R_2\frac{v_{in}}{R_2} \Rightarrow \boxed{v_o = -\frac{R_1}{R_2}v_{in}}$$

This configuration is interesting, as the output is always **inverted** (negative) from the input. The magnitude of the gain can be > 1 (amplification) or < 1 (**attenuation**), neither of which was possible with the non-inverting amplifier.

Using superposition, we can create a **difference amplifier**:



Don't solve this from scratch – use the results that we already know! Through superposition and applying v_p alone (meaning we set v_n to zero), we have a resistor divider feeding a non-inverting amplifier, so

$$\underbrace{v_o}_{\text{due to v}_\text{p}\text{ alone}} = \frac{R_1}{R_1 + R_2} \times \left(1 + \frac{R_1}{R_2}\right) v_p = +\frac{R_1}{R_2} v_p$$

Applying v_n alone, the circuit behaves as an inverting amplifier:

$$\underbrace{v_o}_{\text{due to v}_\text{n}\text{ alone}} = -\frac{R_1}{R_2} v_n$$

Therefore, by superposition, the "true" output with both inputs applied simultaneously is:

$$v_o = +\frac{R_1}{R_2} v_P - \frac{R_1}{R_2} v_n = \frac{R_1}{R_2}(v_p - v_n)$$

This circuit allows us to calculate the difference between v_p and v_n, multiplied by an optional gain term (which we can make $= 1$ by setting $R_1 = R_2$).