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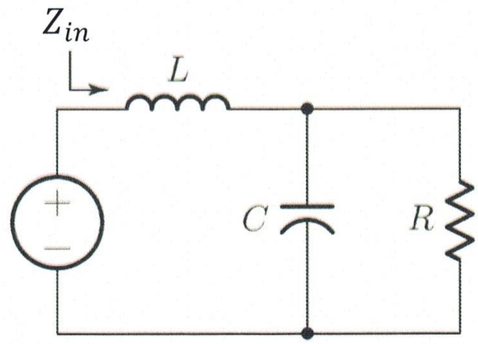
Q4 An RF signal generator is represented by a voltage source. It produces a sinusoidal voltage at angular frequency ω. One peculiarity of RF signal generators is that they work best when driving a particular load resistance, R_S. Nevertheless, they are often called upon to drive loads with resistance $R \neq R_S$ (as in the Figure). One way to make the net impedance seen by signal source, Z_{in}, be equal to R_S is with an RF matching network, consisting of L and C in the figure. To design this network, the actual load resistance R, the target apparent value R_S, and the frequency ω_{op} are known.



a) (2 points) In preparation for starting the problem, what is $(A + jB) \times (A - jB)$, where A and B are arbitrary real constants? Is the result real, imaginary, or complex?

$$(A + jB)(A - jB) = A^2 - j^2 B + AjB - AjB$$

$$= A^2 + B^2$$

Real.

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b) (8 points) Calculate Z_{in} as a function of L, C, R, and ω. Express your answer as a real part plus an imaginary part. (Hint: if you have $\frac{X}{A+jB}$, one way you can make the denominator real is by multiplying by $\frac{A-jB}{A-jB}$)

$$Z_{in} = j\omega L + \frac{R/j\omega C}{R + 1/j\omega C} = j\omega L + \frac{R}{1 + j\omega RC}$$

$$= j\omega L + \frac{R - j\omega R^2 C}{1 + \omega^2 R^2 C^2}$$

$$= \underbrace{\left(\frac{R}{1+\omega^2 R^2 C^2}\right)}_{Re\{Z_{in}\}} + j\underbrace{\left(\omega L - \frac{\omega R^2 C}{1+\omega^2 R^2 C^2}\right)}_{Im\{Z_{in}\}}$$

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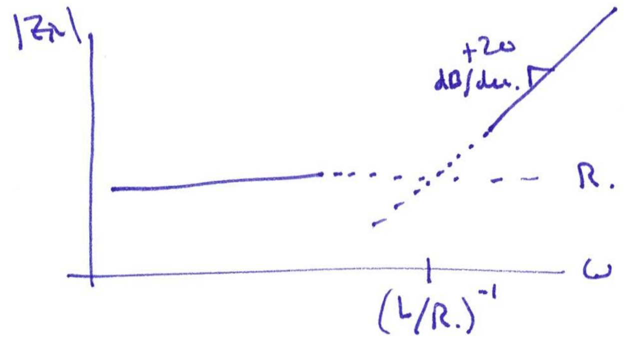
c) (6 points) What is the slope of $|Z_{in}|$ in for low ω? What is the slope of $|Z_{in}|$ for high ω? Extending these asymptotes into the intermediate-ω region, at what ω and $|Z_{in}|$ would they cross? Sketch these asymptotes on a log-log plot (magnitude in dBΩ, frequency on log scale), using solid lines where the asymptotes are good approximations and dashed lines in the intermediate-ω region. You do not need to assess the actual behavior of $|Z_{in}|$ in the intermediate region. [Hint: You can tell from the result of (a) or from the circuit directly.]

Low ω: 0 dB/dec $|Z_{in}| \approx R$.

High ω: +20 dB/dec $|Z_{in}| \approx \omega L$.

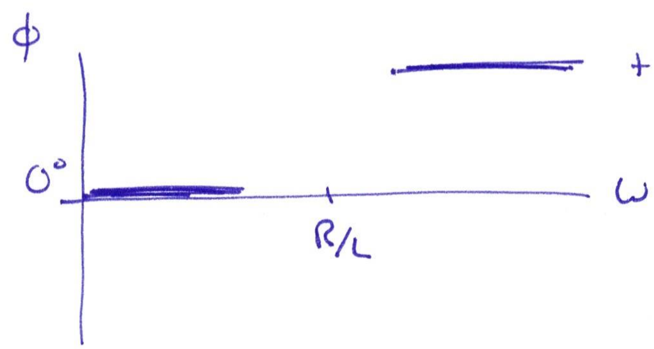
Crossover at $R = \omega L$ $\omega = \frac{R}{L}$, $|Z_{in}| = R$.



d) (4 points) What is the value of $\phi = \arg(Z_{in})$ for low ω? What about high ω? Sketch these asymptotes on a linear-log plot (phase linear, frequency log). You do not need to connect them. [Hint: You can tell from the result of (a) or from the circuit directly.]

Low ω: 0 $(Z_{in} \approx R + 0j)$

High ω: +90° $(Z_{in} \approx 0 + j\omega L)$



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e) (3 points) If we want to make Z_{in} look like R_s at the frequency of interest ω_{op}, then the real part of Z_{in} should equal R_s. What capacitor value causes $\mathrm{Re}\{Z_{in}\} = R_s$? Your answer will contain a factor $\sqrt{\frac{R}{R_s} - 1}$, which is known as Q_T. You can leave your answer in terms of Q_T. Partial credit may be granted if based upon good-faith (though incorrect) answers for (a).

$$\frac{R}{1+\omega^2 R^2 C^2} = R_s \Rightarrow C = \frac{\sqrt{\frac{R}{R_s} - 1}}{\omega R} = \frac{Q_T}{\omega_{op} R}$$

f) (2 points) If we want to make Z_{in} look like $R_s + 0j$ at the frequency of interest ω_{op}, then the imaginary part of Z_{in} should equal zero. Using the selected C from the last part, calculate what inductor value L causes $\mathrm{Im}\{Z_{in}\} = 0$. You can leave your answer in terms of Q_T. Partial credit may be granted if based upon good-faith (though incorrect) answers for (a).

$$\omega L - \frac{\omega R^2 C}{1+\omega^2 R^2 C^2} = 0 \quad \text{with} \quad C = \frac{Q_T}{\omega R}.$$

$$\omega L - \frac{R Q_T}{1+ Q_T^2} = 0 \qquad L = \frac{R Q_T}{(1+Q_T^2)\,\omega_{op}}.$$