Introduction to Electrical Engineering (ECE 302H) – Homework 5

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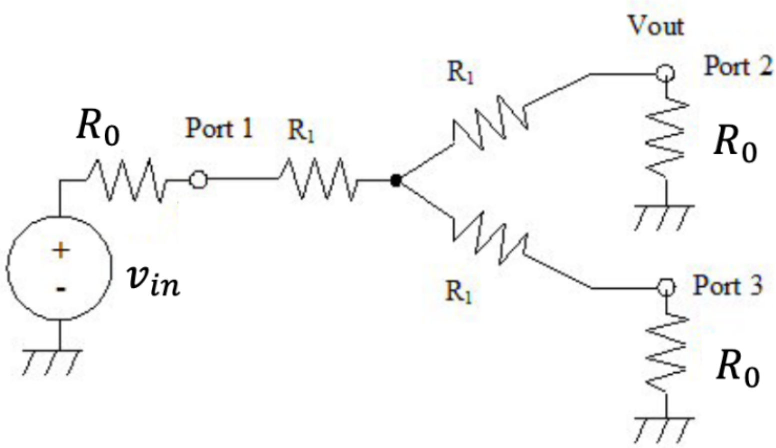
Problem 5.1. In rf circuits, it is very important to "match" the resistance of loads (the things that receive rf power) with signal sources. When we speak of the "resistance" of the signal source, we mean its Thevenin equivalent resistance. Let's examine an rf "power splitter" which has one signal source v_{in} with output resistance R_0 and it sends a signals to two loads, each a resistor of value R_0. A power splitting circuit composed of three resistors of value R_1 may be included. The three ports labeled are understood to be from the labeled point to ground.

a) Explain why the splitting circuit is necessary. In other words, imagine the two loads simply connected in parallel to Port 1. What equivalent resistance would the source see, i.e. what is the equivalent resistance to the right of "Port 1"? Does it match the source's output resistance?

b) Now analyze the full circuit. In particular, find the Thevenin Equivalent resistance from the perspective of each of the three ports. For Port 1, this includes everything but the signal source and its output impedance. For Port 2, this includes everything except Load 2. For Port 3, this includes everything except Load 3. Show that if $R_1 = R_0/3$, then the source and each load "sees" a matched Thevenin resistance looking into the rest of the circuit.

   (if you take ECE 363M Microwave and Radio Frequency Engineering, you'll learn more about why this matching is so important)



Solution. □

a) If we imagine the two loads simply being in parallel to Port 1 without the power splitter, the equivalent resistance to the right of Port 1 would just be two resistors of R_0 in parallel.

$$R_{eq} = \frac{R_0 R_0}{R_0 + R_0}$$
$$R_{eq} = \frac{R_0^2}{2R_0}$$
$$R_{eq} = \boxed{\frac{R_0}{2}}$$

The output Thevenin resistance of the source R_0 does not equal $R_0/2$ so the it's not matched. Hence why the circuit splitting is necessary.

b) The Thevenin Equivalent resistance from the perspective of Port 1 would be,

$$R_{eq} = R_1 + \frac{(R_1 + R_0)(R_1 + R_0)}{R_1 + R_0 + R_1 + R_0}$$
$$R_{eq} = R_1 + \frac{(R_1 + R_0)^2}{2(R_1 + R_0)}$$
$$R_{eq} = R_1 + \frac{R_1 + R_0}{2}$$
$$R_{eq} = \boxed{\frac{3}{2}R_1 + \frac{1}{2}R_0}$$

The Thevenin Equivalent resistance from the perspective of Port 2 and 3 would be,

$$R_{eq} = R_1 + \frac{(R_1 + R_0)(R_1 + R_0)}{R_1 + R_0 + R_1 + R_0}$$
$$R_{eq} = R_1 + \frac{(R_1 + R_0)^2}{2(R_1 + R_0)}$$
$$R_{eq} = R_1 + \frac{R_1 + R_0}{2}$$
$$R_{eq} = \boxed{\frac{3}{2}R_1 + \frac{1}{2}R_0}$$

> **Remark.** Notice that they are the same.

If we plug in $R_1 = \frac{R_0}{3}$,

$$R_{eq} = \frac{3}{2}R_1 + \frac{1}{2}R_0$$

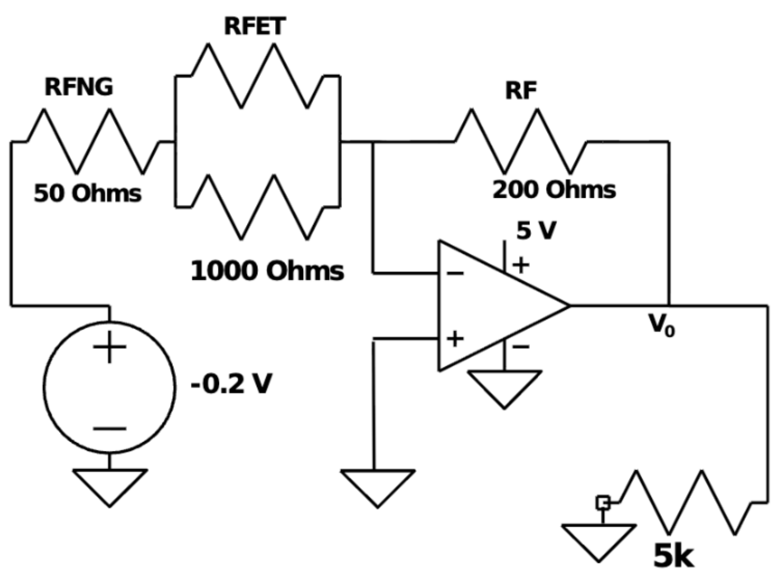
$$R_{eq} = \frac{3}{2}\frac{1}{3}R_1 + \frac{1}{2}R_0$$
$$R_{eq} = \boxed{R_0}$$

Problem 5.2. The circuit for this problem is similar to what you will use in Lab 7 to study Op Amp gain and also FETs. The circuit consists of a function generator with an output impedance of $50\,\Omega$ (the source plus R_{FNG}) feeding an amplifier based on an op-amp. In the amplifier, one of the resistors ($1000\,\Omega$) has a MOSFET in parallel with it. R_{FNG}, R_{FET}, and the $1000\,\Omega$ resistor (R_1) collectively have a value of R_s. We will learn in a couple of weeks that a MOSFET, in certain operating conditions, can be modeled as a variable resistor R_{FET}. The amplifier supplies a load whose equivalent resistance is $5\,\text{k}\Omega$. The Op Amp supply voltage is $5\,\text{V}$.

a) Calculate the value of the equivalent resistance R_s (which is a combination of R_{FNG}, R_{FET} and $1000\,\Omega$, as shown in the figure) and the output voltage of the Op Amp v_0 for $R_f = 200\,\Omega$ and $R_1 = 1000\,\Omega$. Assume $R_{FNG} = 50\,\Omega$ and take R_{FET} to be $60\,\Omega$.

b) Repeat with $R_{FET} = 2\,\text{k}\Omega$ and all other resistors and v_s being the same.

c) For $R_{FET} = 60\,\Omega$, the input voltage (which is initially at $-0.2\,\text{V}$) is increased in magnitude. What is the maximum input voltage magnitude for which the Op Amp still operates in the linear region?



Solution. □

a) Solve for the equivalent resistance R_s,

$$R_s = R_{FNG} + \frac{1000R_{FET}}{1000 + R_{FET}}$$
$$R_s = 50 + \frac{1000(60)}{1000 + 60}$$
$$R_s = \frac{5650}{53} = \boxed{106.604\,\Omega}$$

Solve for V_o using node analysis,

$$\frac{-0.2 - 0}{R_s} = \frac{0 - V_o}{R_f}$$
$$\frac{-0.2}{R_s} = \frac{-V_o}{R_f}$$
$$V_o = 0.2\frac{R_f}{R_s}$$
$$V_o = 0.2\frac{200}{106.604} = \boxed{375.221\,\mathrm{mV}}$$

b) Find new R_s value for $R_{FET} = 2\,\mathrm{k}\Omega$,

$$R_s = R_{FNG} + \frac{1000R_{FET}}{1000 + R_{FET}}$$
$$R_s = 50 + \frac{1000(2000)}{1000 + 2000}$$
$$R_s = \frac{2150}{3} = \boxed{716.667\,\Omega}$$

Plug in R_s to find new V_o,

$$V_o = 0.2\frac{R_f}{R_s}$$
$$V_o = 0.2\frac{200}{716.667} = \boxed{55.814\,\mathrm{mV}}$$

c) The Op Amp can output a maximum voltage of 5 V across it's output terminal. We can rewrite the earlier equation to solve for V_{in} from the source instead of V_o,

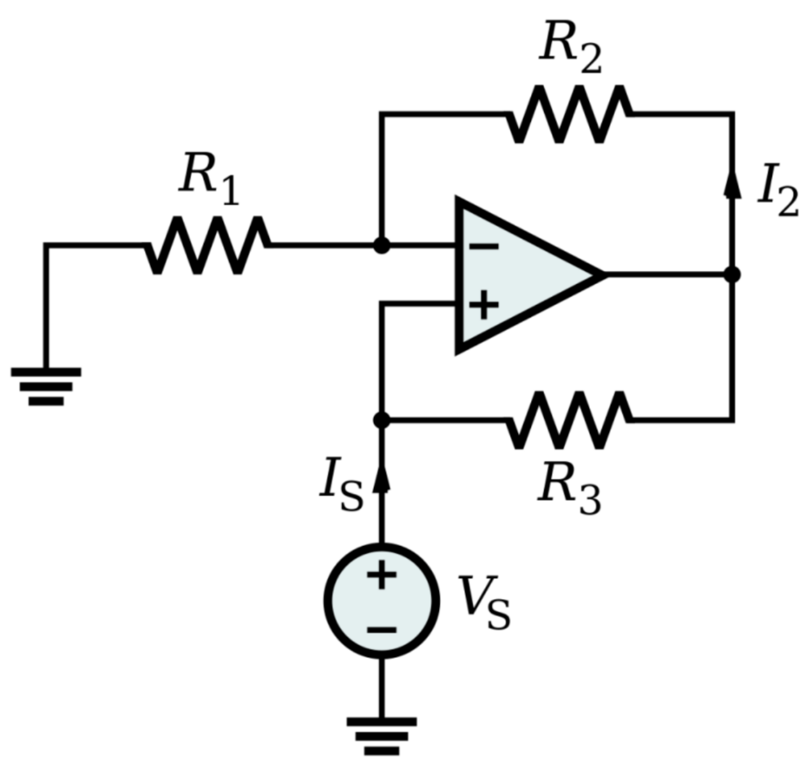
$$V_o = 0.2\frac{200}{106.604}$$
$$5 = V_{in}\frac{200}{106.604}$$
$$V_{in} = 5\frac{106.604}{200} = \boxed{2.665\,\mathrm{V}}$$

Problem 5.3. A negative resistor is an object whose current is proportional to its voltage $i = -v/R$ where the constant of proportionality $-R$ is a negative number.

a) Calculate the power in a negative resistor of value $-R$ (where R itself is positive, such that $-R$ is negative) as a function of the voltage applied to it and use it to show that a negative resistor always *supplies* power.

b) It is impossible for a passive component to supply net power, so true negative resistors do not exist in real life. However, it is possible to synthesize circuits that behave as negative resistors. Consider the circuit in the Figure. Calculate its input current I_S as a function of its input voltage V_S and show that the circuit emulates a negative resistor of value $-R$. Calculate the value of R assuming an ideal op amp.

c) Suppose that this circuit (the op amp and the three resistors) were placed in series with a real resistor of value $+R$ – what would the net resistance of the series combination be? How about if they were placed in parallel?



Solution. □

a) Recall that we know power to be $P = i \times V$. Subtitute in the current of the negative resistor to solve for it's power as a function of the voltage applied to it,

$$P(V) = i \times V$$
$$P(V) = \frac{V}{-R}V$$
$$P(V) = \boxed{-\frac{V^2}{R}}$$

From this equation we can see that a negative resistory always *supplies* power because P remains negative regardless of voltage polraity since it is squared.

b) Use node analysis to solve for I_S,

$$\frac{V_o - V_S}{R_2} = \frac{V_S - 0}{R_1}$$
$$\frac{V_o}{R_2} - \frac{V_S}{R_2} = \frac{V_S}{R_1}$$
$$V_o = V_S\frac{R_2}{R_1} + V_S$$

$$I_S = \frac{V_S - V_o}{R_3}$$
$$I_S = \frac{V_S}{R_3} - \frac{V_o}{R_3}$$
$$I_S = \frac{V_S}{R_3} - V_S\frac{R_2}{R_1 R_3} - \frac{V_S}{R_3}$$
$$I_S = \boxed{-V_S\frac{R_2}{R_1 R_3}}$$

From the equation $i = -v/R$,

$$\frac{1}{R} = \frac{R_2}{R_1 R_3}$$
$$R = \boxed{\frac{R_1 R_3}{R_2}}$$

c) In series, we know $R_{eq} = R_1 + R_2$,

$$R_{eq} = R - R = \boxed{0\,\Omega}$$

In parallel, we know $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$,

$$\frac{1}{R_{eq}} = \frac{1}{R} - \frac{1}{R}$$
$$\frac{1}{R_{eq}} = 0$$
$$R_{eq} \to \boxed{\infty}$$

Problem 5.4. Feedback can be used to create a circuit that behaves as a current source. This is known as a transconductance amplifier, i.e. one that converts a voltage in one place into a current (”conductance”) in another place (”trans”). Consider the circuit below with an op-amp arranged in feedback with a MOSFET. Assume that the drain (top) of the MOSFET is connected to some circuit (not open-circuited), but that we don’t know what that circuit is. The output current of the circuit i_o is the drain-source current of the MOSFET, indicated by an arrow near the top of the diagram.

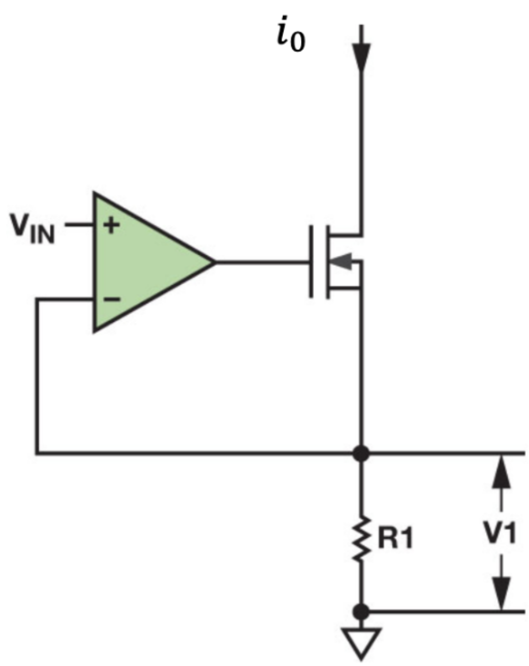
a) Assume that the MOSFET behaves such that the higher the voltage applied to its gate (on the left), the higher the current that will flow from its drain (top) to its source (bottom). Does the configuration below qualify as negative feedback? Explain.

b) If the amplifier gain is infinite, what will the voltage v_1 be? Other than the assumption above, do you need to know the IV characteristics of the MOSFET or of the load circuit (not shown) in order to solve for v_1?

c) What is i_o? Other than the assumption above, do you need to know the IV characteristics of the MOSFET or of the load circuit not shown in order to solve for i_o?

d) What accounts for the circuit’s remarkable lack of dependence on the details of its components or the load connected to it? In other words, explain how the circuit can maintain constant i_o even if you replaced the MOSFET with a different MOSFET with a different IV characteristic?

e) What are the input resistance and the Thevenin equivalent output resistance of the circuit?



Solution. □

a) Yes, this does qualify as negative feedback.

1. Consider the case where V_o is higher than expected, this means a higher voltage applied on the MOSFET gate.
2. A higher voltage applied on the MOSFET gate results in more current i_0, flowing out of the MOSFET.
3. A higher current flowing out of the MOSFET results in a higher current flowing into R_1.
4. A higher current flowing into R_1 results in an increase in V_1 because $V_1 = R_1 \times i_0$
5. A higher voltage V_1 results in an increase in the voltage of the negative terminal of the Op Amp.
6. An increase in the negative terminal voltage of the Op Amp will decrease V_o.

Notice, an increase in V_o led to a decrease in V_o. Negative feedback.

b) If the amplifier gain is infinite, we can apply the golden rule of negative feedback and set $V_+ = V_-$,

$$\begin{aligned} V_+ &= V_- \\ V_+ &= V_{in} \\ V_- &= V_1 \\ V_1 &= \boxed{V_{in}} \end{aligned}$$

In this scenario, we no longer need to know the IV characteristics of neither the MOSFET nor the load circuit.

c)

$$\begin{aligned} i_o &= \frac{V_{in} - 0}{R_1} \\ i_o &= \boxed{\frac{V_{in}}{R_1}} \end{aligned}$$

In this scenario, we no longer need to know the IV characteristics of neither the MOSFET nor the load circuit.

d) Because of the negative feedback i_0 is fixed since $V_1 = V_{in}$ causes the resistor to experience a fixed current. In other words, because of the resistor and negative feedback, this circuit overrides any MOSFET / output current variations.

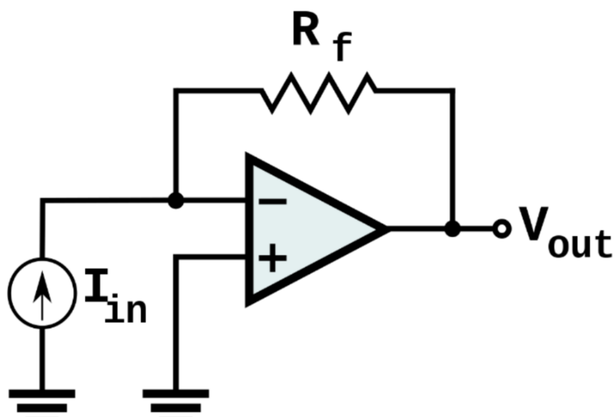
e) Under ideal conditions, the input resistance of the circuit would be infinite since the Op Amp doesn't draw any current. Likewise, the Thevenin equivalent output resistance would also be infinite since small changes in V_o are rejected by the loop, and the gate behaves like an ideal current source.

Problem 5.5. A transimpedance (or transresistance) amplifier is shown in the Figure. The name comes from ”resistance,” which turns a current into a voltage, and ”trans,” which means that the current in one place gets converted into a voltage in another place.

a) Calculate the transimpedance of the circuit, $r = V_o/I_{in}$, assuming that the op-amp gain is A. Then let $A \to \infty$. Use the $A \to \infty$ result for the rest of the problem.

b) Often I_{in} is provided by a relatively poor current source. When learning about Thevenin and Norton equivalents, we learned that a good current source has near-infinite output resistance, while a poor current source has an output resistance that is large but not negligibly so. We also learned that an object acts most like a good current source when it the load resistance is much lower than the circuit's output resistance. What is the equivalent input resistance of the op amp circuit, i.e. everything to the right of I_{in}? Is this a good circuit to load an imperfect current source?

c) This circuit might appear to be overkill. One might wonder why not just run I_{in} into a grounded resistor of value r to get the same current-to-voltage conversion factor. Explain why this would not be as good of an idea.



Solution. □

a) Perform node analysis on V_-,

$$\begin{aligned}
I_{in} &= \frac{V_- - V_o}{R_f} \\
I_{in} &= \frac{V_-}{R_f} - \frac{V_o}{R_f} \\
V_o &= A(V_+ - V_-) \\
V_+ &= 0 \\
V_o &= -AV_- \\
V_- &= -\frac{V_o}{A} \\
I_{in} &= -\frac{V_o}{AR_f} - \frac{V_o}{R_f} \\
I_{in} &= -V_o\left(\frac{1}{AR_f} + \frac{1}{R_f}\right) \\
I_{in} &= -V_o\left(\frac{1+A}{AR_f}\right) \\
\frac{V_o}{I_{in}} &= \boxed{r = -\frac{AR_f}{1+A}}
\end{aligned}$$

Take $A \to \infty$,

$$\begin{aligned}
r &= \lim_{A\to\infty} -\frac{AR_f}{1+A} \\
r &= \lim_{A\to\infty} -\frac{R_f}{\frac{1}{A}+1} \\
r &= \boxed{-R_f}
\end{aligned}$$

b) Perform node analysis on V_-,

$$\begin{aligned}
I_{in} &= \frac{V_- - V_o}{R_f} \\
V_o &= -AV_- \\
I_{in} &= \frac{V_- + AV_-}{R_f} \\
I_{in} &= \frac{V_-(1+A)}{R_f} \\
\frac{V_-}{I_{in}} &= \frac{R_f}{1+A}
\end{aligned}$$

As $A \to \infty$,

$$\frac{V_-}{I_{in}} = \boxed{R_{in} = 0}$$

In this case, the load or input resistance is 0, which quite literaly is the best case scenario for loading an imperfect current source.

c) If we just run I_{in} into a grounded resistor, this would not be a good idea because then we would be creating a non-zero input resistance R_{in}. In real life where we do not have access to ideal current sources, this non-zero input resistance would interfere with our expected behavior.

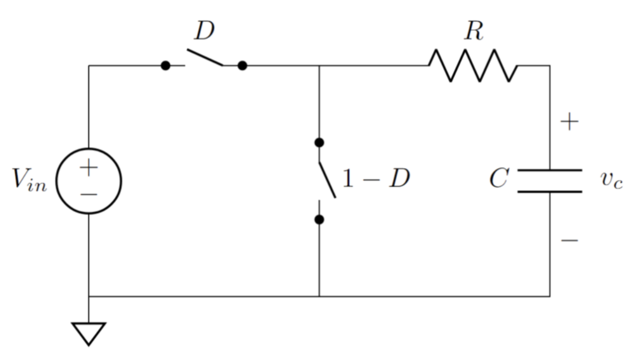
Problem 5.6. Duality

a) An op amp is a 3-terminal object: two input terminals (+) and (-) and one output terminal. Consider the following statements about op amps: *An op amp measures the voltage across the two input terminals* v_d *and produces an output voltage that is proportional to the input,* $v_o = Av_d$ *Real op amps are modeled as having some input resistance* R_i *(between + and −) and some output resistance* R_o *(in series with the output voltage* Av_d*).* Ideal op amps have $R_i \to \infty$, $R_o \to 0$, and $A \to \infty$. Based on this description, what would the dual of an op amp be? Copy these statements, replacing words as needed to exchange the role of voltage and current. Draw a circuit model to represent the op amp's dual (both non-ideal and ideal).

b) What is the dual of an open switch? What is the dual of a closed switch? Suppose a switch is turned on and off at a frequency f (period T). Further suppose that the switch spends a fraction of the period DT ($D < 1$) in the on position and the other fraction $(1 - D)T$ in the off position. (The on-fraction D is called the "duty cycle"). If you took the dual of this switch, what would the dual's duty cycle D' be?

c) A poor man's Digital to Analog converter is shown in the figure, in which two switches are alternately turned on such that only one switch is closed at a time. The top switch is closed with a duty cycle of D and the lower switch is closed whenever the top switch is open (i.e., a duty cycle of $(1 - D)$). This circuit involves a component we have not yet studied (but that we mentioned in lecture) called a *capacitor*, which has a value of C, units of *Farads* $= \frac{C}{V} = \frac{A \times s}{V}$, and a component law $i_c = C\frac{dv_c}{dt}$. You're not prepared to derive this circuit's operation yet, but the circuit's function is to cause v_c to be equal to DV_{in}. By adjusting the duty cycle of the top switch (and the corresponding duty cycle of the lower switch), the output voltage can be made to be any voltage less than V_{in}. There's a good chance you will implement this circuit in ECE 319K.

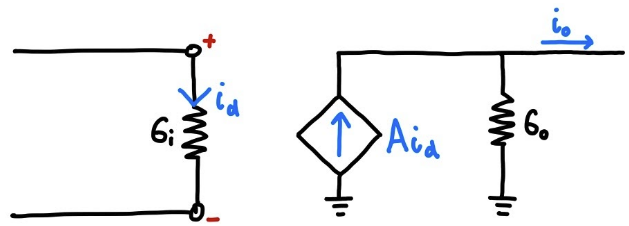
What would the component law be for the dual of a capacitor? Research what kind of component has that law. What would the dual of this circuit be? What would the dual's function be? What units would all of the components have, and what would the duty cycles of the switches be?



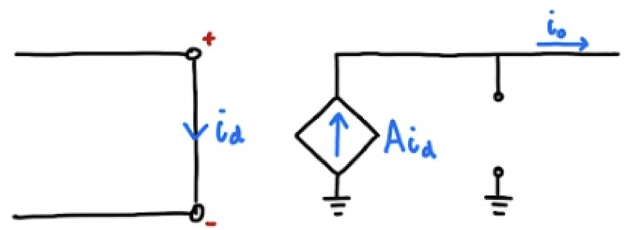
Solution. □

a) A *something* measures the **current through** the two input terminals i_d and produces an output **current** that is proportional to the input, $i_o = Ai_d$. Real *somethings* are modeled as having some input **conductance** G_i (between + and -) and some output **conductance** G_o (in **parallel** with the output **current** Ai_d). Ideal *somethings* have $G_i \to \infty$, $G_o \to 0$, and $A \to \infty$.

Non-ideal something,



Ideal something,



b) The dual of an open switch would be a closed switch and the dual of a closed switch would be an open switch, since open and closed circuits are duals of each other.

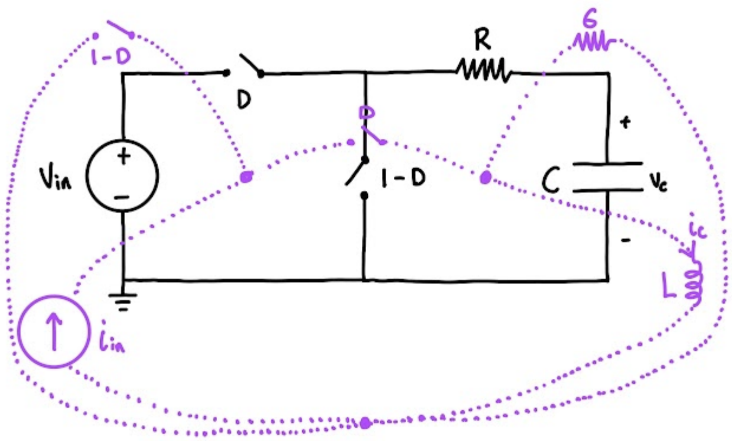
If you took the dual of this switch, the dual's duty cycle D' would just be $D' = 1 - D$ since the dual relationship flips the two states of on and off.

c) Given that a capacitor has units of farads which is $\frac{A \times s}{V}$, the dual unkown unit would have $\frac{V \times s}{A}$. A quick google search lets us find that this unit is called henries with symbol L. From this,

$$i_c = C\frac{dv_c}{dt}$$
$$v_c = L\frac{di_c}{dt}$$

Also known as an inductor.

By taking the dual of the circuit,



In the dual circuit, the current source would have units of amps, the conductance of units siemens, and inductor of units henries. The two switches would then have their respective duty cycles swapped.