

# COMPLEX NUMBERS

$\sqrt{x} \triangleq$  the number which when squared, equals  $x$

$$\sqrt{-1} = j \quad \text{Imaginary}$$

$$j^2 = -1$$

$$\frac{1}{j} = \frac{j}{j^2} = -j$$

$$3j + 2j = 5j$$

$$5j \times 2j = -10$$

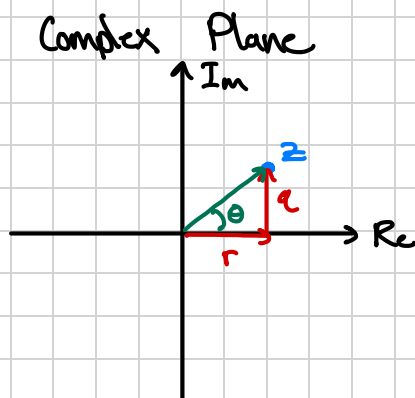
$$\boxed{3 + 2j} = ?$$

Complex #

$$Z = r + jq$$

Real part of  $Z$   
 $\text{Re}\{Z\}$

Im part of  $Z$   
 $\text{Im}\{Z\} = q$   
is Real



$$Z_1 \pm Z_2 = (r_1 \pm r_2) + (q_1 \pm q_2)j$$

$$Z_1 \times Z_2 = |Z_1| \times |Z_2| \angle (Q_1 + Q_2)$$

$$\frac{Z_1}{Z_2} = \frac{|Z_1|}{|Z_2|} \angle (Q_1 - Q_2)$$

## WHERE DOES $e$ COME IN

$$e^z ?$$

$$e^{r+jq} = e^r \times e^{jq}$$

$$\frac{d}{dq} e^{jq} = j e^{jq}$$

$$\boxed{e^{jq} = \cos(q) + j \sin(q)}$$

Euler's Formula

$$\frac{d}{dq} e^{jq} = -\sin(q) + j \cos(q)$$

$$= j \times [\cos(q) + j \sin(q)]$$

$$= j e^{jq}$$

Instead of  $z = |z| \angle \theta$

$$\Rightarrow z = |z| e^{jq}$$

$$z = |z| \cos \theta + j |z| \sin \theta$$

$$A \cos(\omega t + \phi) = \operatorname{Re} \{ A e^{j(\omega t + \phi)} \}$$

$$RC \frac{dv_c}{dt} + v_c = v_{in}$$

$$RC \frac{d}{dt} \operatorname{Re} \{ V_c e^{j(\omega t + \phi)} \} + \operatorname{Re} \{ V_c e^{j(\omega t + \phi)} \} = \operatorname{Re} \{ V_{in} e^{j\omega t} \}$$