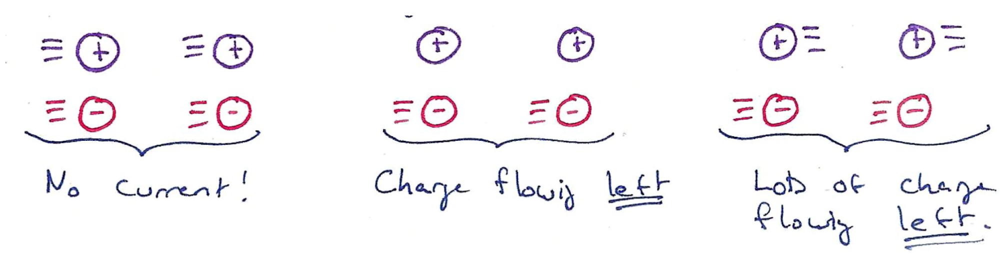
Lecture Notes 1: Terminology and Solving Circuits

What is electricity?

- It involves **charge**, such as the charge on an electron ($-1e$) or proton ($+1e$).
- There are *lots* of charges around us:
  Example:
  10^{25} water molecules in a glass
  10 electrons and protons per molecule
  $\Rightarrow 10^{26}$ electrons and protons in a glass of water
  $10^{26} = 100,000,000,000,000,000,000,000,000$!
- Because everyday quantities of charge are so large, it is inconvenient to talk about e. Instead, we use the unit of **Coulombs**$= 6.24 \times 10^{18}e$.

So is *everything* electricity? No! Positive and negative charges balance (cancel) in most matter. Most materials are electrically "neutral." We observe electrical phenomena when there is an *imbalance* of charge (static electricity) or an *imbalance* of the motion of charge. We will focus on moving charges (current).



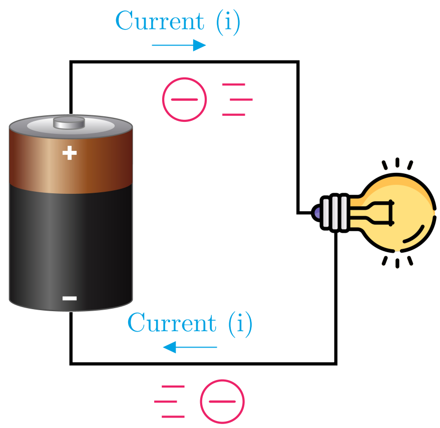
Note that all three cases above are electrically neutral – there is no imbalance in charge. There is only an imbalance in the *flow* of charge.

Current

Current is a measure of the *rate* and *direction* of the flow of charge (not of charge carriers). Electrons moving to the right count as charge flowing to the left.

The units of current are Coulombs/second = **Amperes or "Amps" (A)**.

Current tends not to flow unless the charges are "pushed" by some source of energy, like a battery:



In most circuits, electrons are the things that actually move, but we're concerned about current which flows in the opposite direction. The choice to talk about current instead of electron flow is called **conventional current** – we know that negative charges are flowing backwards, but we pretend as if positive charges are flowing forward.

Using the concept of conventional current, we would say that current flows clockwise in the circuit above. This is correct. We might imagine positive charges flowing clockwise – we know this is not actually happening, but the concept of "current" doesn't care.

Circuit Graph Terminology

We have now seen our first circuit and should use the right terminology to describe it.

The elements that make up a circuit are called **components** or, in graph theory terms, **branches**. The wires that connect components are called **nodes**. A node does not end until it hits a component – in other words, a bunch of interconnected wires all constitute a single node. All circuits involve **loops**, or pathways through wires and components that connect back to themselves. The term **mesh** describes minimum-sized loops, i.e., loops that do not encircle any other loops.

The circuit above may be described as having two nodes, two components or branches that connect those nodes, and one loop (which is also a mesh). We will most often keep track of the number of nodes, meshes, and components.

Voltage

In the circuit above, the imaginary positive charges that circulate clockwise flow from (-) to (+) in the battery, *gaining* energy. This energy enters the circuit from another domain – chemical in this case, but possibly mechanical, magnetic, or other domains – and is imparted on the imaginary positive charges.

The same charges then flow through the light bulb, *dropping off* their energy. The imaginary charges lose energy and that energy leaves the circuit to another domain – heat and light in this case, but possibly mechanical motion, chemical energy, or others.

The amount of **energy** that gets picked up or dropped off as an imaginary positive charge passes through a circuit component is called the component's **voltage** or **(electric) potential difference**.

The units of voltage are Joules/Coulomb = **Volts (V)**.

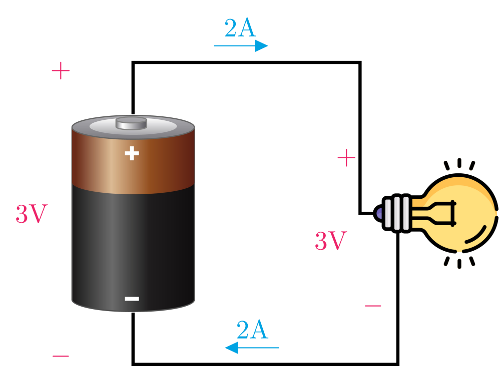
It makes no sense to speak of the voltage "at" at node. Voltage is only defined *between* two nodes. When people speak of the voltage "at" a node, they have implicitly assigned one node to count as "0 Volts" or **ground** and are implicitly measuring every other voltage relative to that node.

Power

Voltage is a measure of the energy gained or lost per unit charge. Current is a measure of charge flow per unit time. Therefore, their product gives power:

$$v \times i = \frac{\text{Energy}}{\text{Time}} = \text{Power}\left[\frac{J}{s} = \text{Watts}\right]$$

Expressed in sentences, we would say that the voltage *across a component* times *the current through that component* tells us the power (the rate of energy flow) into or out of that component.



- There are 2A flowing (2 C/s) *through* the battery and *through* the light bulb.
- There are 3V *across* the battery ⇒ if 1C flows from (-) to (+), the charge will gain 3V×1C=3J of energy from the battery.
- THere are 3V *across* the light bulb. ⇒ if 1C flows from (+) to (-), the charge will lose 3V×1C=3J of energy to the light bulb.
- With a continuous 2A current flow, the battery is continuously sourcing 3V×2A=6W of power. The bulb is continuously sinking 3V×2A=6W of power.

Solving Circuits

In the above example, we learned how to talk about circuits, but the voltages and currents were already given. How do we solve for what the voltages and currents will be?

Solving a physical system always involves the same process:

1. Write down all physical laws (as equations) (**just physics!**)
2. Solve the resulting system of equations (**just math!**)

99% of student challenges with circuits come from trying to take shortcuts instead of executing the two-step process above.

We are already well prepared to solve systems of equations – we know how to substitute one equation into another, how to add and subtract equations, perhaps even how to cast a system of equations in matrix form, and so forth. All we need to learn are the physical laws so we can do step (1) and generate the system of equations in the first place.

Here they are:

1. Conservation of energy. If a charge were to flow in a loop, it should end up with the same energy it started with. (This is analogous to hiking all over a mountain range. No matter how many ups and downs you traverse, if you return to the same point, then you end up with the same potential energy as you started with). In other words, if we add up all of the energy changes experienced by a charge, gains (positive voltages) and losses (negative voltages), they should sum to zero.

   ⇒ **Kirchoff's Voltage Law (KVL)** $\sum_{\text{around loop}} v = 0$

   (Note that we're not claiming that a particular charge actually traverses whichever loop you're looking at. We're just saying that *if* a charge did traverse that loop, then it must return to the same energy, and therefore the sum of voltages around that loop must be zero regardless of whether any individual charge actually goes around that particular loop).

2. Conservation of charge. Charge cannot be created or destroyed, so it cannot continuously pile up or be depleted from any one location. This means that if we add all of the current entering any region (often, but not necssarily, a node): current into (positive current) and current out of (negative current), they should sum to zero.

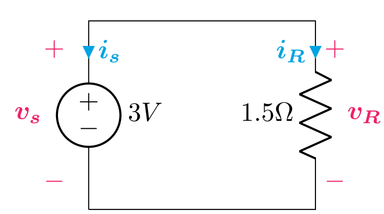
   ⇒ **Kirchoff's Current Law (KCL)** $\sum_{\text{entering region or node}} i = 0$

3. Component i-v relationships. A component does not control the entire circuit. All it controls is how its own voltage relates to its own current. This **component i-v relationship** is, mathematically, the definition of the component.
   Examples:

   - **Voltage Sources** have constant voltage for any current $v(i) = V_{source}$. Batteries behave much like voltage sources. Note that voltage sources can (and will) have current flowing through them, it's just that the voltage source doesn't care what it is and therefore it does not appear in the i-v equation. The current is determined by how the voltage source interacts with the rest of the circuit.
   - **Resistors** have a linear voltage-current characteristic, $v(i) = Ri$, where the coefficient is called the **resistance**. Resistance has units of $V/A = \Omega$ or **Ohms**. Old-fashioned incandescent light bulbs behave very much like resistors.

   Note that there are two pieces of information associated with every component. There's the component's behavior, which is captured by the component's name or symbol (is it a voltage source? a resistor? something else?), and then there is the component's value (a voltage source's voltage or a resistor's resistance).

Suppose that we are given a circuit consisting of a $3V$ voltage source and a 1.5Ω resistor.



Let's solve it:

Step 0: Label the voltages and currents for every component. These are the "unknowns" or "variables" that will appear in the system of equations. It turns out that the choice of orientation for voltages and currents is *completely arbitrary*, as long as you then write down the laws of physics *using the orientations chosen here*.

Step 1: Write down all relevant physical laws. This will include one KVL expression for every **mesh** (minimum-sized loops, i.e. loops that enclose no other loops). It will include one KCL expression for every node *except one* (you can see in the example that KCL at both nodes gives us the same equation – in general, we can always choose exactly one node to not apply KCL to). Finally, it will include all of the component laws. Based on the previous statement, we expect to generate a system of 4 equations (1 KVL, 1 KCL, and 2 component laws) and 4 unknowns (v_s, i_s, v_R, i_R). Writing down the equations:

$$\begin{aligned} &\text{KVL} & v_s - v_R &= 0 && (1)\\ &\text{KCL} & i_s + i_R &= 0 && (2)\\ &\text{Component Laws} & v_s &= 3V && (3)\\ & & v_R &= (3/2\Omega) \times i_R && (4) \end{aligned}$$

This is a system of equations with 4 equations and 4 variables. The arrangement of the components creates the structure of the equations, the voltages and currents are the variables or unknowns, and the component values are the coefficients.

Step 2: Math. If we've done Step 1 correctly, then Step 2 should just be doing math to solve the system of equations. We see directly that $v_s = 3V$. KVL then tells us that $v_R = 3V$. The resistor's circuit law then tells us that $i_R = 3V/1.5\Omega = 2A$. KCL then tells us that $i_s = -2A$. Note that this problem lent itself to a "follow-the-breadcrumbs" approach to solving the system of equations. We won't be so lucky in general and will need to leverage substitution or the other techniques mentioned earlier.

★ You can now solve *any* circuit! A larger circuit will simply have more KVL, KCL, and/or component law equations and more unknowns, which is mathematically annoying but not conceptually any harder. You are guaranteed to have $N_{equations} = N_{unknowns}$.

Note: You might wonder whether it's necessary to assign an "unknown" like v_s to a voltage source, since we already know its voltage. If we said that v_s was "known" from the beginning, then we would have

$$\begin{aligned} &\text{KVL} & \cancel{v_s}\, 3V - v_R &= 0 && (5)\\ &\text{KCL} & i_s + i_R &= 0 && (6)\\ &\text{Component Laws} & \cancel{v_s = 3V} & && (7)\\ & & v_R &= (1.5\Omega) \times i_R && (8) \end{aligned}$$

This is the same as casting the problem in the formal way and then substituting the voltage source component law equation into the other equations. In other words, it's a shortcut, which I will again encourage you not to use until you're much more comfortable solving circuits.