Sequences, Series, and Multivariable Calculus (M 408D) – Homework 1

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Fall 2025

1 Problems

Problem 1.1.

$$\int_0^1 x(1-x^2)^4 dx$$

Solution.

$$\begin{aligned} u &= 1-x^2 \\ \frac{du}{dx} &= -2x \\ du &= -2xdx \end{aligned}$$

$$\begin{aligned} \int_0^1 x(1-x^2)^4 dx &= -\frac{1}{2}\int_0^1 -2x(1-x^2)^4 dx \\ &= -\frac{1}{2}\int_1^0 u^4 du \\ &= \frac{1}{2}\left|\frac{1}{5}u^5\right|_0^1 \\ &= \boxed{\frac{1}{10}} \end{aligned}$$

□

Problem 1.2.

$$\int_0^{\pi/2} \sin x(2f'(\cos x)-1)dx$$

when $f(0)=2$ and $f(1)=4$

Solution.

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \\ \int_0^{\pi/2} \sin x(2f'(\cos x)-1)dx &= -\int_0^{\pi/2} 2f'(u)-1du \\ &= -2f(u)\Big| + u\Big| \\ &= -2f(\cos x)\Big|_0^{\pi/2} + \cos x\Big|_0^{\pi/2} \\ &= -2f(\cos(\pi/2)) + 2f(\cos(0)) + \cos(\pi/2) - \cos(0) \\ &= -2(2) + 2(4) + 0 - 1 \\ &= \boxed{3} \end{aligned}$$

□

Problem 1.3.

$$\int_0^{\pi/4} \left(\frac{1}{\cos^2\theta} + 2\cos 2\theta\right) d\theta$$

Solution.

$$\begin{aligned}
\int_0^{\pi/4} \left(\frac{1}{\cos^2\theta} + 2\cos 2\theta\right) d\theta &= \int_0^{\pi/4} (\sec^2\theta + 2\cos 2\theta) d\theta \\
&= \int_0^{\pi/4} \sec^2\theta d\theta + \int_0^{\pi/4} 2\cos 2\theta d\theta \\
&= \tan\theta\Big|_0^{\pi/4} + \sin 2\theta\Big|_0^{\pi/4} \\
&= 1 - 0 + 1 - 0 = \boxed{2}
\end{aligned}$$

□

Problem 1.4.

$$\int_1^2 6\ln(3x) dx$$

Solution.

$$\begin{aligned}
u &= 6\ln(3x) \\
du &= \frac{6}{x} dx \\
dv &= dx \\
v &= x \\
\int_1^2 6\ln(3x) dx = \int_1^2 u dv &= uv - \int v du \\
&= 6x\ln(3x)\Big|_1^2 - \int_1^2 6 dx \\
&= (6x\ln(3x) - 6x)\Big|_1^2 \\
&= 12\ln 6 - 12 - 6\ln 3 + 6 \\
&= 12\ln 6 - 6\ln 3 - 6 \\
&= 6(2\ln 6 - \ln 3 - 1) \\
&= 6\left(\ln\frac{36}{3} - 1\right) \\
&= \boxed{6(\ln 12 - 1)}
\end{aligned}$$

□

Problem 1.5.

$$\int \cos(\ln(x))dx$$

Solution.

$$\begin{aligned}
u &= \cos(\ln(x)) \\
du &= -\frac{1}{x}\sin(\ln(x))dx \\
dv &= dx \\
v &= x
\end{aligned}$$

$$\begin{aligned}
\int \cos(\ln(x))dx &= uv - \int vdu \\
&= x\cos(\ln(x)) + \int \sin(\ln(x))dx
\end{aligned}$$

$$\begin{aligned}
u &= \sin(\ln(x)) \\
du &= \frac{1}{x}\cos(\ln(x))dx \\
dv &= dx \\
v &= x
\end{aligned}$$

$$\begin{aligned}
&= x\cos(\ln(x)) + uv - \int vdu \\
&= x\cos(\ln(x)) + x\sin(\ln(x)) - \int \cos(\ln(x))dx
\end{aligned}$$

$$\begin{aligned}
2\int \cos(\ln(x)) &= x\cos(\ln(x)) + x\sin(\ln(x)) \\
\int \cos(\ln(x)) &= \boxed{\frac{1}{2}x(\cos(\ln(x)) + \sin(\ln(x)))}
\end{aligned}$$

$\square$

Problem 1.6. If f is a continuous function such that

$$\int_0^{16} f(x)dx = 16$$

determine the value of the integral

$$\int_0^4 6f(4x)dx$$

Solution.

$$\begin{aligned}
u &= 4x \\
du &= 4dx \\
\int_0^4 6f(4x)dx &= \int_0^{16} 6f(u)\frac{du}{4} \\
&= \frac{3}{2}\int_0^{16} f(u)du \\
&= \frac{3}{2}(16) = \boxed{24}
\end{aligned}$$

□

Problem 1.7.

$$\int_0^{\pi/4} 6x\sec^2 x dx$$

Solution.

$$\begin{aligned}
u &= 6x \\
du &= 6dx \\
dv &= \sec^2 x dx \\
v &= \tan x \\
\int_0^{\pi/4} 6x\sec^2 x dx &= uv - \int vdu \\
&= 6x\tan x\Big|_0^{\pi/4} - \int_0^{\pi/4} 6\tan x dx \\
&= 6x\tan x\Big|_0^{\pi/4} - 6\int_0^{\pi/4} \frac{\sin x}{\cos x} dx \\
u &= \cos x \\
du &= -\sin x dx \\
&= 6x\tan x\Big|_0^{\pi/4} + 6\int_1^{\sqrt{2}/2} \frac{1}{u} du \\
&= 6x\tan x\Big|_0^{\pi/4} + 6\ln(u)\Big|_1^{\sqrt{2}/2} \\
&= \frac{3\pi}{2}(1) - 0 + 6\ln(\sqrt{2}/2) - 0 \\
&= \frac{3\pi}{2} + 6\ln(\sqrt{2}) - 6\ln(2) \\
&= \frac{3\pi}{2} + 3\ln(2) - 6\ln(2) \\
&= \boxed{\frac{3\pi}{2} - 3\ln(2)}
\end{aligned}$$

□

Problem 1.8.

$$\int_0^1 4\sin(5\sqrt{x})dx$$

Solution.

$$\begin{aligned}
u &= \sqrt{x} \\
u^2 &= x \\
2udu &= dx \\
\int_0^1 4\sin(5\sqrt{x})dx &= \int_0^1 8u\sin(5u)du \\
w &= 8u \\
dw &= 8du \\
dv &= \sin(5u)du \\
v &= -\frac{1}{5}\cos(5u) \\
&= -\frac{8}{5}u\cos(5u) + \int \frac{8}{5}\cos(5u)du \\
&= -\frac{8}{5}u\cos(5u)\Big|_0^1 + \frac{8}{25}\sin(5u)\Big|_0^1 \\
&= \boxed{-\frac{8}{5}\cos(5) + \frac{8}{25}\sin(5)}
\end{aligned}$$

□

Problem 1.9.

$$\int_{\pi/6}^{\pi/3} = (9\sin 2x + 2\cos 2x)dx$$

Solution.

$$\begin{aligned}
&= \left(-\frac{9}{2}\cos 2x + \sin 2x\right)\Bigg|_{\pi/6}^{\pi/3} \\
&= -\frac{9}{2}(-\frac{1}{2}) + \frac{\sqrt{3}}{2} + \frac{9}{2}(\frac{1}{2}) - \frac{\sqrt{3}}{2} \\
&= \boxed{\frac{9}{2}}
\end{aligned}$$

□

Problem 1.10.

$$\int t^2 \cos(2 - t^3) dt$$

Solution.

$$\begin{aligned}
u &= 2 - t^3 \\
du &= -3t^2 dt \\
&= -\frac{1}{3}\int \cos u du \\
&= -\frac{1}{3}\sin u + C \\
&= \boxed{-\frac{1}{3}\sin(2 - t^3) + C}
\end{aligned}$$

□

Problem 1.11. Evaluate the integral

$$\int_1^2 x^2 f''(x) dx$$

when $f(1) = 9,\ f(2) = 7,\ f'(1) = 8,\ f'(2) = 6$

Solution.

$$\begin{aligned}
u &= x^2 \\
du &= 2x dx \\
dv &= f''(x) dx \\
v &= f'(x) \\
\int_1^2 x^2 f''(x) dx &= x^2 f'(x) - \int 2x f'(x) dx \\
u &= 2x \\
du &= 2dx \\
dv &= f'(x) dx \\
v &= f(x) \\
&= x^2 f'(x) - 2x f(x) + \int 2f(x) dx \\
&= (x^2 f'(x) - 2x f(x))\Big|_1^2 + \int_1^2 2f(x) dx \\
&= 4(6) - 4(7) - 1(8) - 2(9) + \int_1^2 2f(x) dx \\
&= \boxed{6 + 2\int_1^2 f(x) dx}
\end{aligned}$$

□

Problem 1.12.

$$\int_e^3 \frac{\ln x}{x^2} dx$$

Solution.

$$\begin{aligned} u &= \ln x \\ du &= \frac{1}{x} dx \\ dv &= \frac{1}{x^2} dx \\ v &= -\frac{1}{x} \\ \int_e^3 \frac{\ln x}{x^2} &= -\frac{\ln x}{x}\bigg| + \int \frac{1}{x^2} dx \\ &= \left(-\frac{\ln x}{x} - \frac{1}{x}\right)\bigg|_e^3 \\ &= -\frac{\ln 3}{3} - \frac{1}{3} + \frac{1}{e} + \frac{1}{e} \\ &= \boxed{\frac{2}{e} - \frac{1}{3}(\ln 3 + 1)} \end{aligned}$$

□

Problem 1.13.

$$\int (x^2 - 1)\cos(2x) dx \tag{1}$$

Solution.

$$\begin{aligned} \int (x^2 - 1)\cos(2x) dx &= \int x^2 \cos(2x) dx - \int \cos(2x) dx \\ u &= 2x \\ du &= 2dx \\ &= \int \frac{1}{8} u^2 du - \frac{1}{2}\sin(2x) + C \end{aligned}$$

> **Remark.** Integrate by part the first term and do it using the table method. I'm not LaTeX enoughed to do represent table method.

$$\boxed{\frac{1}{2}x^2 \sin(2x) + \frac{1}{2}x\cos(2x) - \frac{3}{4}\sin(2x) + C}$$

□