Lecture Notes 12: Diodes, Capacitors, and MIS

With a few facts, we can do quite a bit with semiconductors:

Fact 1: $J = (qn\mu)E = \sigma E$, where n is the density of mobile charge carriers. (n for mobile electronics, p for mobile holes)

Fact 2: Materials at $T > 0\text{K}$ will thermally generate a few electron-hole pairs. In an undoped semiconductor,

$$\underbrace{n_o}_{e^-/\text{cm}^3} = \underbrace{p_o}_{\text{holes/cm}^3} = n_i \quad \text{intrinsic carrier concentration} \tag{3}$$

For silicon, the **intrinsic carrier concentration** $n_i \approx 10^{10}/\text{cm}^3$ at room temperature. It gets higher/lower as temperature goes up/down.

Fact 3: Doping with acceptors (group III) creates mobile holes and stationary negative ions. We call this **P-type** material. Doping with donors (group V) creates mobile electrons and stationary positive ions. We call this **N-type** material. In the simple 302 model, all dopants are always ionized.

If the concentration of acceptors $N_a \gg n_i$, then the concentration of mobile holes $p \approx N_a$.

If the concentration of donors $N_d \gg n_i$, then the concentration of mobile electrons $n \approx N_d$.

Fact 4: In homework you will derive the **Mass Action Law**, which states that $n \times p = n_i^2$, even for doped silicon. This implies that a piece of silicon heavily doped with donors will have many electrons and very few holes. This makes sense – an ocean of mobile electrons will have much greater probability of meeting up with mobile holes and filling them, with plenty of electrons left over.

Example: Consider a piece of Si doped with $N_d = 10^{16}/\text{cm}^3$. Since $N_d \gg n_i$, $n \approx 10^{16}$. By the mass action law, $p = n_i^2/n = 10^{20}/10^{16} = 10^4/\text{cm}^3$. The conductivity of this piece of silicon will be dominated by electrons, since conductivity is proportional to the concentration of mobile charges and there is a high concentration of electrons and a low concentration of holes.

The current density due to electrons and holes are:

$$J_e = |q|_e \mu_e n E \qquad J_h = |q|_h \mu_h p E$$

Therefore, total current density is

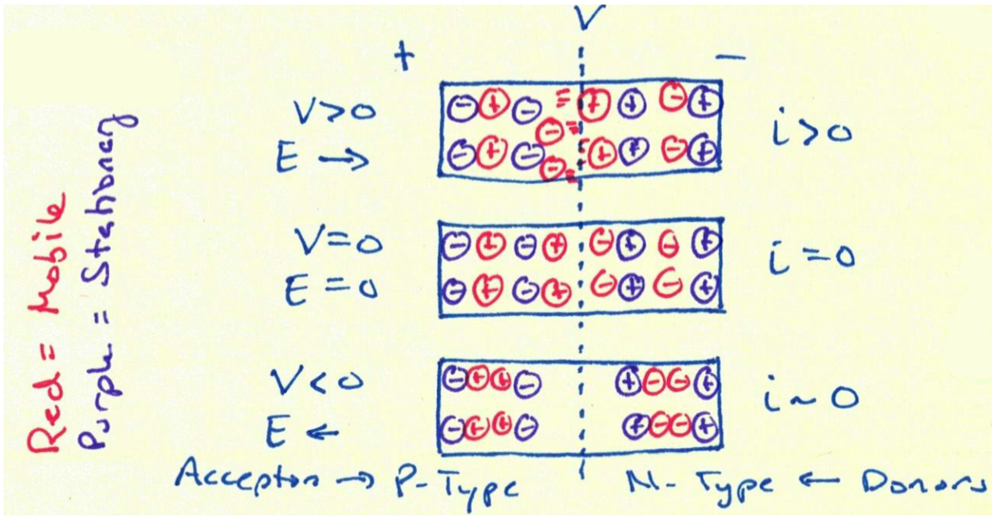
$$J = J_e + J_h = \underbrace{(|q|_e \mu_e n + |q|_h \mu_h p)}_{\sigma} E = \sigma E$$

Electrons and holes have the same amount of charge. Electrons have a bit higher mobility than holes, but only a factor of two or so. Therefore, conductivity all comes down to concentrations. N-type silicon will have conductivity dominated by electrons; P-type silicon will have conductivity dominated by holes.

Diodes

Now consider mating a P-type material to an N-type material, creating a **PN junction** as diagrammed below.

When no voltage is applied across the PN junction, no charges flow and the current is zero. The P-type material has lots of mobile holes and stationary negative ions, while the N-type material has lots of mobile electrons and stationary positive ions.

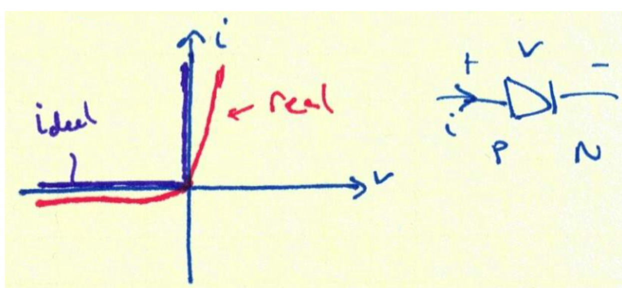


When a positive voltage is applied from P→N, an electric field points from P →N which pushes holes into the N region and pulls electrons into the P region. This represents current – we needn't calculate how much, we only need to know that the PN junction allows current to flow from P→N.

When a negative voltage is applied, the pressure on holes is, more than usual, to stay on the P side. The pressure on electrons is, more than usual, to stay on the N side. No charges cross the junction itself, and therefore there is no current.

This object is a **diode**.

PN junctions or diodes allow current to flow from P→N but not backwards.

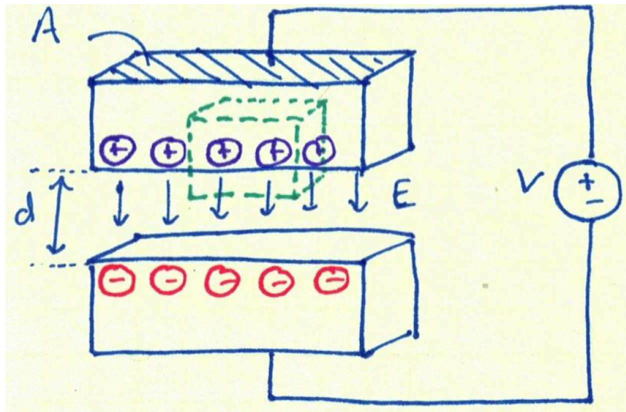


[In ECE 339, you will learn a lot more about diodes. In particular, the amount of current that flows is a balance between diffusion (which makes mobile charges want to move from high to low concentration) and drift (the force of the electric field). At zero voltage, there is a lot of diffusion which creates a so-called space-charge region with an electric field that precisely counters diffusion. With positive voltage, diffusion and drift work together to produce current. At negative voltage, bunched-up holes and electrons want to diffuse *even more*, but the electric field is also stronger, so the current continues to be close to zero.]

Capacitors

Another device we need to understand is a **capacitor** which often consists of two plates that are parallel but not touching.

When a positive voltage is applied between the top plate and the bottom plate, electrons are pulled from the



top plate to the bottom plate, leaving behind atoms on the top plate with fewer-than-usual electrons. Hence, there is an accumulation of positive charge on the bottom surface of the top plate, and an accumulation of negative charge on the top surface of the bottom plate. An electric field points from the top plate to the bottom plate.

Inside the conductors, in steady state, the electric potential is the same everywhere and the electric field is zero. Why? If $J = \sigma E$ and $J = 0$ everywhere (as must be the case in steady state), then $E = 0$ everywhere and the voltage, $\int E \cdot dl$, between any two points within the metal must be zero. If this condition were violated, there would be current flowing somewhere, which can't be. Thus voltage is the same everywhere in a conductor and the electric field is zero inside the conductor.

We want to figure out how much charge accumulates on each plate. We need *new physics* to figure out how much charge there will be. The new physics that we need is one of Maxwell's equations, called **Gauss's Law**.

> Guass' Law: For any closed three dimensional shape (like a box), $\oint \epsilon E \cdot dA = Q_{enclosed}$ where $\oint E \cdot dA$ is the integral of the electric field that points perpendicularly out of the box over the entire surface of the box.

The quantity ϵ is called the **permittivity** of the material. It is an intensive property like mass density or resistivity. Permittivity is usually expressed as $\epsilon = \epsilon_0 \epsilon_r$, where $\epsilon_0 = 8.85 \times 10^-12\, C/Vm$ is a universal constant called the **permittivity of free space** and ϵ_r is the dimensionless **relative permittivity**. When people speak of the permittivity of materials, they are usually referring to the relative permittivity. Air has a relative permittivity of approximately 1. Often, the material between capacitor plates will be SiO_2, which has a relative permittivity of 3.9.

Let us apply Gauss's law to green box in the figure. [Consider the green box to extend to the edges of the plate; it is drawn smaller for visualization purposes]:

1. Integrating along the bottom surface of the box gives $\int \epsilon E \cdot dA = \epsilon_{gap} E \times A$. We don't yet know what E is, but that's okay. ϵ_{gap} is the permittivity of the spacer material between the plates.

2. Integrating along the top surface of the box gives $\int \epsilon E \cdot dA = 0$ because $E = 0$ inside of a conductor and the top surface of the box is inside the top plate.

3. Integrating along the side surfaces of the box gives $\int \epsilon E \cdot dA = 0$ because E only points in the vertical direction. Therefore, even though there is non-zero E over parts of the side-surfaces, none of it actually exits or enters the box.

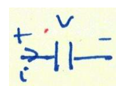
Therefore, $\oint \epsilon E \cdot dA = \epsilon_{gap} E \times A_{plate} = Q$, where Q is the total net charge that lives on the bottom surface of the top plate.

What is E? Notice that Gauss' law yields the same answer no matter where the bottom surface of the green box is – close to the top plate, close to the bottom plate, or more in the middle. This means that E is constant as a function of vertical position. Therefore, $\int E \cdot dl$ from one plate to the other can be simplified to $E \times d$ (because E does not vary along d), so $E = V/d$.

Putting these two conclusions together yields

$$\boxed{Q = \frac{\epsilon A}{d} V = CV} \tag{4}$$

Because the charge is equal to a constant (material parameters and geometry) times voltage, we give that constant its own name: **capacitance**. Capacitance tells you how much charge will develop on two electrodes if a voltage difference is applied to them. The units of capacitance are Coulombs/Volt = Ampere-seconds/Volt = **Farads**.

In circuits, we draw the capacitor as two separated parallel plates, . The capacitor's $i-v$ relationship is found by differentiating $Q = CV$ (since current is the derivative of charge),

$$\boxed{i = C\frac{dv}{dt}} \tag{5}$$

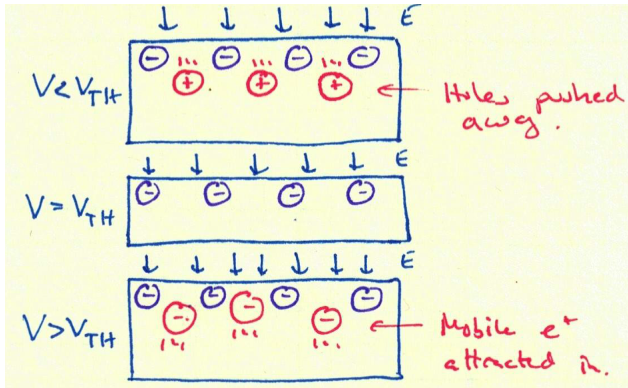
Circuits with capacitors end up with **differential equations** (equations involving derivatives and integrals), as opposed to algebraic equations (which only have multiplication by constants as we've seen so far). However, if the inputs are all constant or slowly varying, then $dv/dt \approx 0$ and the capacitor acts as an open circuit. We will treat capacitors as open circuits (but we'll want to know how much charge is on them) until the third and final unit of the course.

MIS Capacitors

If the bottom capacitor plate is actually a semiconductor, we call it a **metal-insulator-semiconductor (MIS)** capacitor, and we observe an interesting phenomenon. Gauss' law is still absolutely true, so the charge we calculated before is still correct. However, on the bottom plate, we now have to distinguish how much charge is attributable to stationary ions versus mobile charges.

For example, suppose the bottom plate is made of P-type material, with lots of stationary negative ions and lots of mobile holes. As voltage is applied across the capacitor plates and an electric field develops, it is initially sufficient to drive holes away from the surface. The surface then has a negative charge due to the negative ions, which satisfies Gauss's law. With stronger voltages and stronger electric fields, eventually virtually all of the holes will already be gone, and it will not be possible to build more negative charge on the surface simply by pushing positive charge away. If the voltage and electric field are strengthened further, satisfying Gauss' law requires the semiconductor to attract mobile electrons to the surface. (This is hard to do, since there aren't that many mobile electrons in P-type material, but Guass's law must be satisfied!).

This is a coarse explanation of what happens – you'll learn more in detail in a semiconductor devices course. For now, we can use the term **threshold voltage V_{th}** to describe the voltage that is enough to drive away all the mobile holes but not enough to start bringing in electrons. For voltages below V_{th}, the negative charge on the surface is almost entirely composed of stationary ions. For voltages above V_{th}, the negative charge on the surface is composed of *both* stationary ions and mobile electrons.



When $V > V_{th}$, we say that the surface of the semiconductor has undergone **inversion**, since it used to be P-type (lots of mobile holes) but now has been electrically converted to N-type (lots of mobile electrons)! This is the key to making a MOSFET transistor – the ability to change a material from P type to N type will give us control of the device.