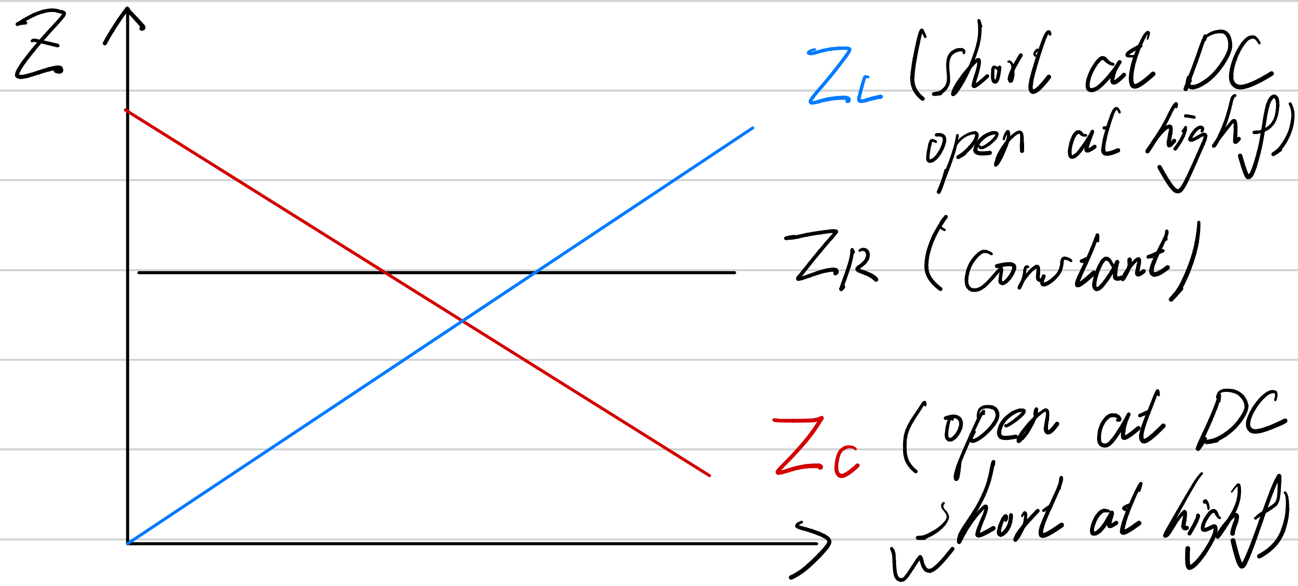
$Z_R = R$ $\quad\quad$ $Z_C = \frac{1}{j\omega C}$ $\quad\quad$ $Z_L = j\omega L$



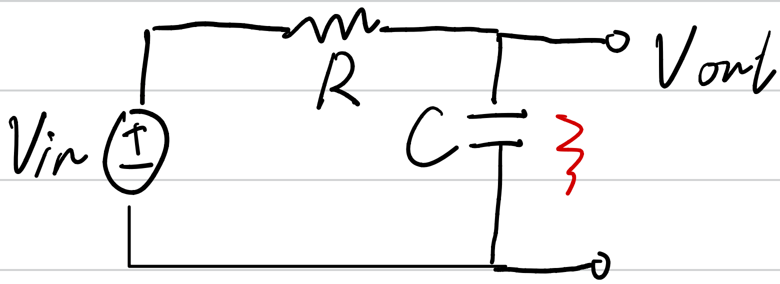
Why frequency domain?

1. physical insight

2. Math simple

TF: $$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

1. RC low-pass filter



$$V_{out} = V_{in} \times \frac{Z_C}{\underbrace{Z_R}_{R} + \underbrace{Z_C}_{\frac{1}{j\omega C}}}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= -\tan^{-1}(\omega RC)$$

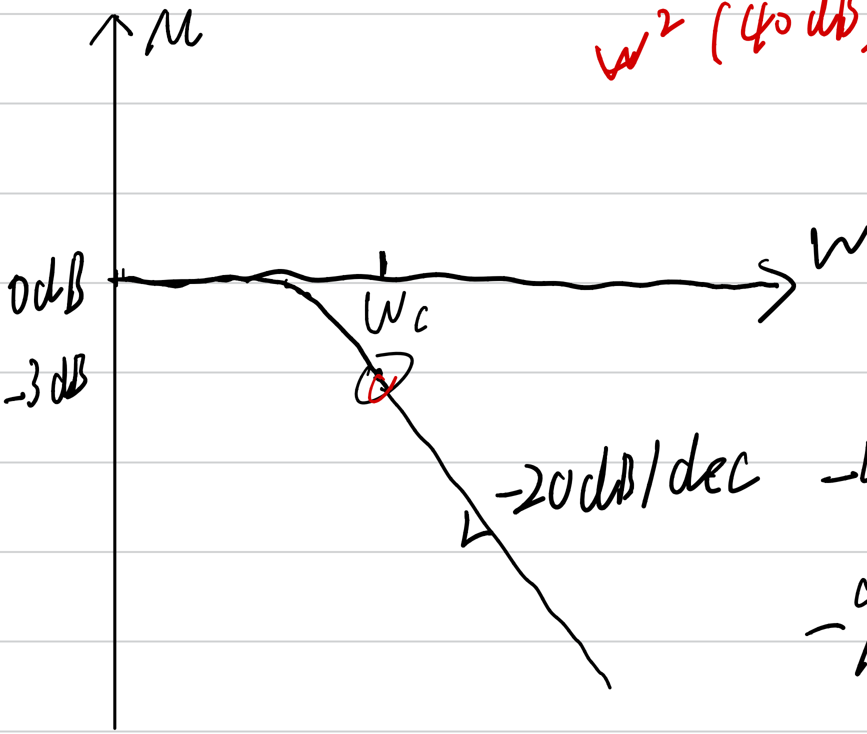
1

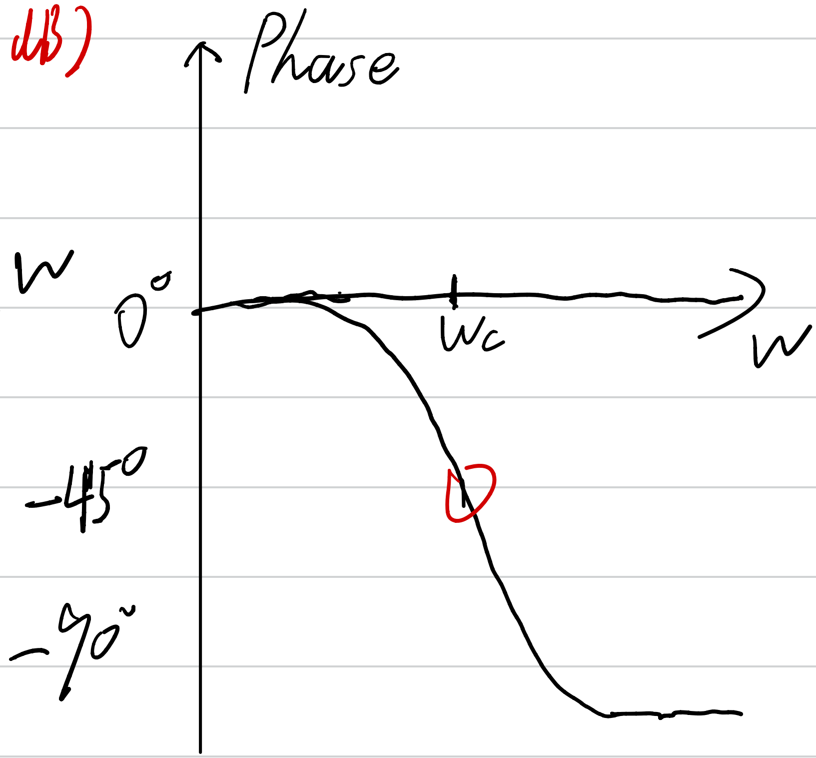
f	M	Phase	Behavior
$w \ll \frac{1}{RC}$	≈ 1 (0dB)	0°	passband
$w = \frac{1}{RC}$	$\frac{1}{\sqrt{2}}$ (-3dB)	-45°	cutoff
$w \gg \frac{1}{RC}$	$\frac{1}{wRC}$ (-20dB/dec)	-90°	stopband

$20\log_{10}\left(\frac{1}{w}\right) = -20$

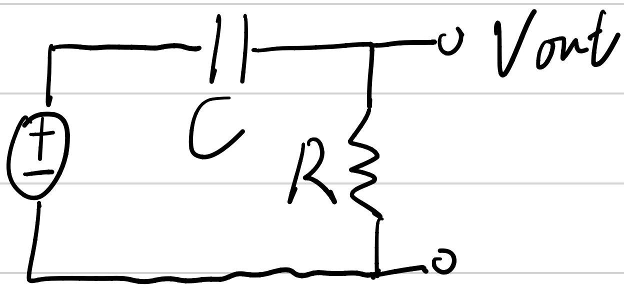
wRC (20dB)

w^2 (40dB)





RC high-pass filter



$$H(j\omega) = \frac{Z_R}{Z_R + Z_C}$$

$$= \frac{R}{R + \frac{1}{j\omega C}}$$

$$\frac{\omega RC}{1} = \omega RC$$

$$= \frac{j\omega RC}{1 + j\omega RC}$$

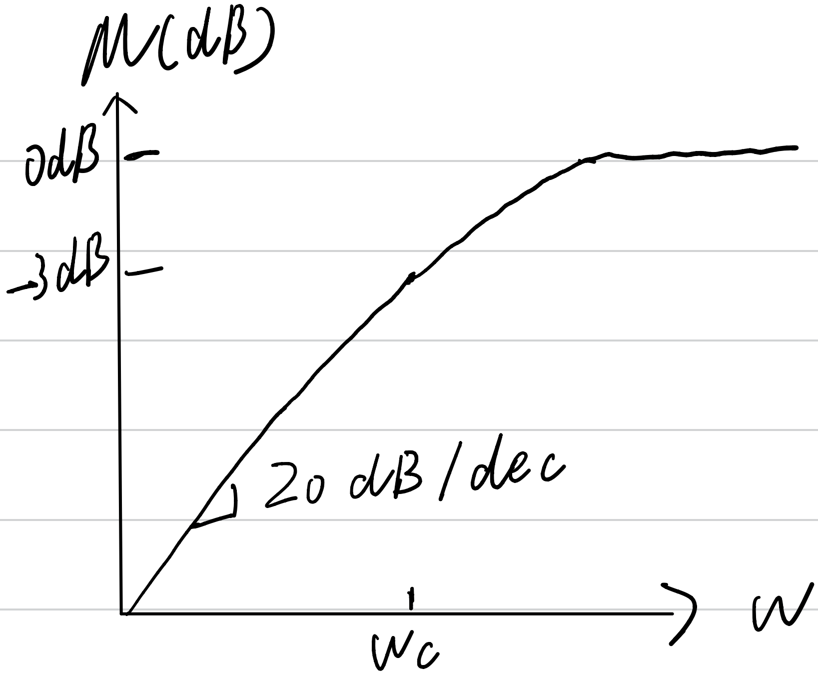
$$|H(j\omega)| = \boxed{\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}}$$

$$\angle H(j\omega) = 90^\circ - \tan^{-1}(\omega RC)$$

$$\frac{0}{}$$

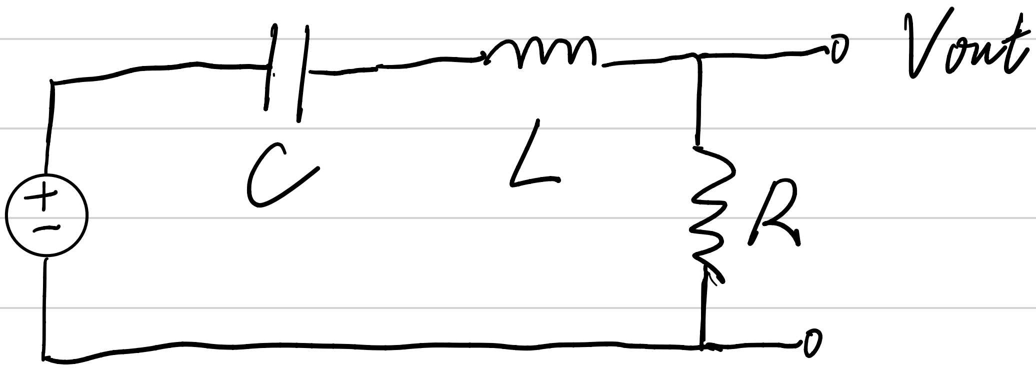
$$\tan^{-1}\left(\frac{\omega RC}{0}\right)$$

f	M	Phase	Behavior
$\omega \ll \omega_c$	ωRC (+20dB/dec)	$+90^\circ$	stopband
$\omega = \omega_c$	$\frac{1}{\sqrt{2}}$ (−3 dB)	$+45^\circ$	cutoff
$\omega \gg \omega_c$	1 (0dB)	0°	passband



RLC (high / low pass)

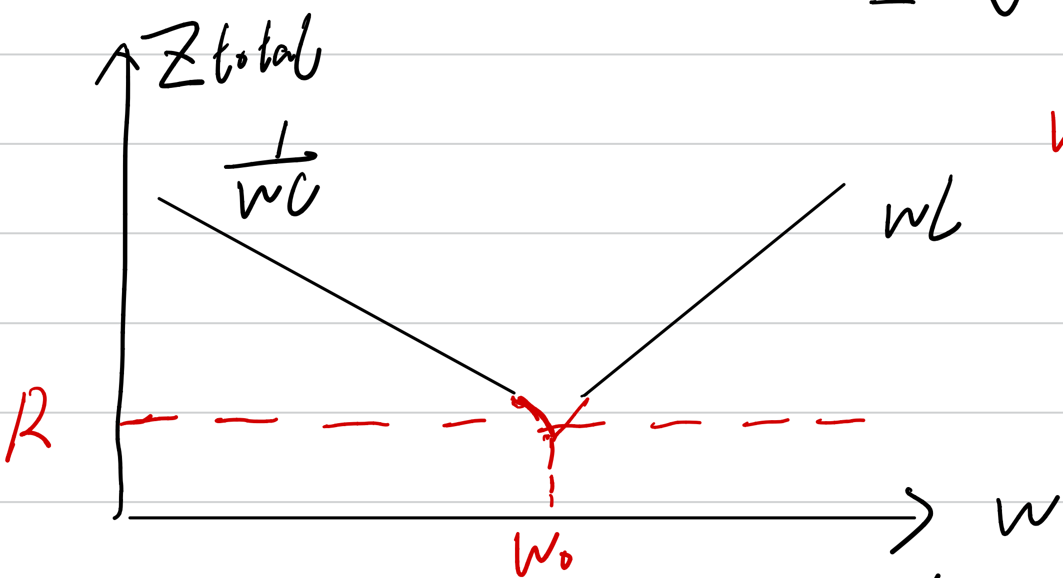
RLC circuit (series)



$$Z = Z_C + Z_L + Z_R = R + j\omega L + \frac{1}{j\omega C}$$

$$= \underline{R} + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega L = \frac{1}{\omega C}$$



$$I = \frac{V_{in}}{Z_{total}}$$

ω_0 resonant frequency $\omega = \frac{1}{\sqrt{LC}}$

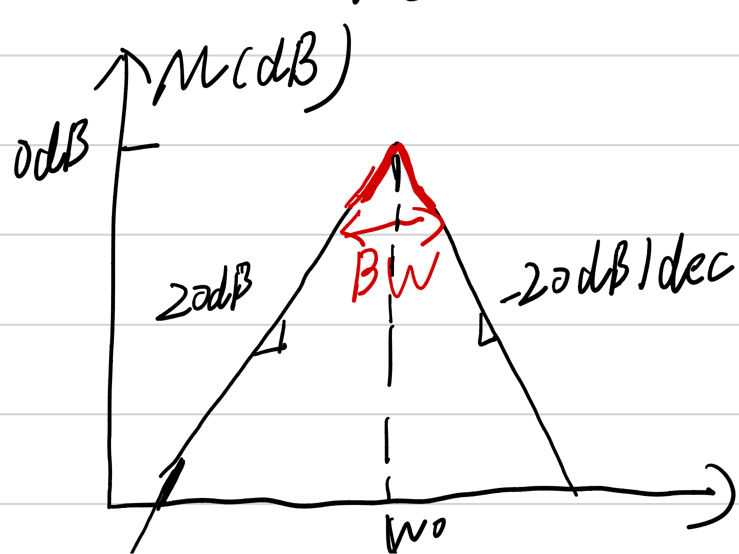
$$H(jw) = \frac{Z_R}{Z_{total}} = \frac{R}{R + jwL + \frac{1}{jwC}} = \frac{R}{R + j(wL - \frac{1}{wC})}$$

$$|H(jw)| = \frac{R}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

$$\angle H(jw) = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{wL - \frac{1}{wC}}{R}\right)$$

$$= -\tan^{-1}\left(\frac{wL - \frac{1}{wC}}{R}\right)$$

f	M	Phase	Behavior
$w \ll w_0$	$\frac{R}{\frac{1}{wC}} = wCR$ (+20dB/dec)	$+90°$	Capacitive
$w = w_0$	1 (0dB)	$0°$	resistive
$w \gg w_0$	$\frac{R}{wL}$ (−20dB/dec)	$-90°$	Inductive



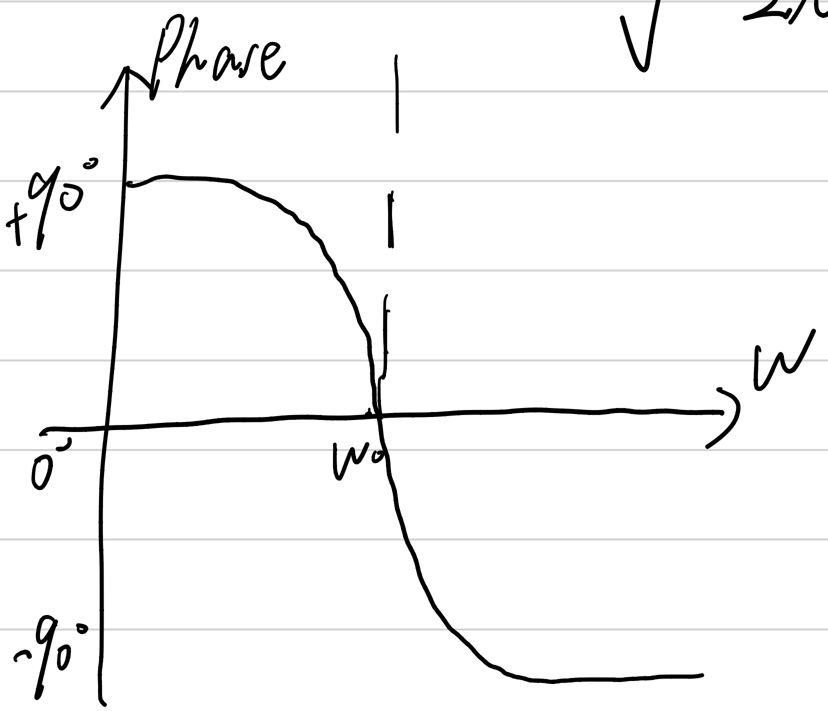
band-pass filter

w_0

$$w_0 L - \frac{1}{w_0 C} = 0$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$



RLC parallel

$$H(jw) = \frac{V_{out}}{V_{in}} = 1$$

$$H(jw) = \frac{I_{in}}{V_{in}} = \frac{1}{Z_{total}}$$

$$\frac{I_{in}}{V_{in}} = \frac{1}{Z_{total}} \qquad \frac{V_{in}}{I_{in}} = Z_{total}$$

$$\frac{1}{Z_{total}} = \frac{1}{Z_R} + \frac{1}{Z_C} + \frac{1}{Z_L}$$

Admittance

$$= \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}} + \frac{1}{j\omega L}$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$|H(j\omega)| = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\omega C - \frac{1}{\omega L}}{\frac{1}{R}}\right) = \tan^{-1}\left(R\left(\omega C - \frac{1}{\omega L}\right)\right)$$

f	M	Phase	Behavior
$\omega \ll \omega_0$	$\frac{1}{\omega L}$ (-20 /dec)	$-90°$	Inductive
$\omega = \omega_0$	$\frac{1}{R}$	$0°$	resistive
$\omega \gg \omega_0$	ωC (+20 dB/dec)	$+90°$	Capacitive

band-stop filter