Introduction to Electrical Engineering (ECE 302H) – Linearity

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1   System

Everything can be a system. A system just needs to have a form of input, whether that is doing something or not, and an output, which is just some reaction that we care about.

In a circuit specifically, inputs are usually independent sources, and outputs are any v, or i of interest anywhere in the circuit.

Remark 1.1. "System" is a very broad category and the one form of system that we are particularly interested in right now is "Linear Systems" or linearity.

2   Linearity

Definition 2.1. An equation is linear if it takes the follonwg form: $C_y y^1 = C_u u^1$

Definition 2.2. A SISO (single input single output) system is defined as:

$$y = f(u)$$
$$u \Rightarrow y$$

For some specific input u_1, $u_1 \Rightarrow y_1$.

$y = f(u)$ is linear if, for any constants a, b

$$f(au_1 + bu_2) = af(u_1) + bf(u_2)$$

Going back to the first definition,

$$C_y y = C_u u$$
$$y = \frac{C_u}{C_y} u$$

$$f(au_1 + bu_2) = \frac{C_u}{C_y}(au_1 + bu_2)$$
$$= a\left(\frac{C_u}{C_y}u_1\right) + b\left(\frac{C_u}{C_y}u_2\right)$$
$$= af(u_1) + bf(u_2)$$