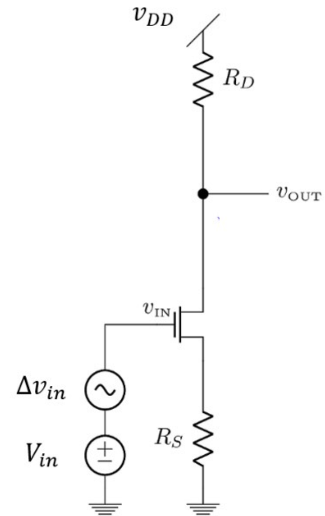
M3 Numerical Solutions
ECE 302H Fall 2023

Part I

Q1



Consider the circuit at right, consisting of an NMOS transistor and two resistors. The supply voltage v_{DD}, shown as a bar at the top of the circuit, is supplied by a constant voltage source from ground (not shown). The input has a constant voltage $V_¿$ and is to be perturbed by an amount $\Delta v_¿$; this is shown pictorially as two voltage sources in series. You may use the transconductance, $g_m = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_T)$, as a known quantity. Assume that the MOSFET is in the saturation region.

[a=6pt] Consider the circuit with $R_S \to 0$ and calculate the small-signal gain of the amplifier, $\Delta v_{out} / \Delta v_¿$.

$$V_o = -R_D g_m \Delta V_{gs} = \underline{-g_m R_D V_{in}}.$$

[b=3pt] The transconductance is very large but poorly controlled and may vary by up to a factor of 2 from circuit to circuit. If this were to happen, by what factor would the small-signal gain vary when $R_s \to 0$ as in (a)?

x 2

[c=7pt] Consider the circuit again with $R_S \neq 0$ and calculate the small-signal gain of the amplifier, $\Delta v_{out} / \Delta v_¿$.

$$\Delta V_o = -\Delta i R_D = \frac{-g_m R_D}{1 + g_m R_s} \Delta V_{in}$$

[d=3pt] Consider the transconductance to be very large. Using the result from (c) where $R_S \neq 0$, by what factor would the small-signal gain vary if g_m varied by a factor of 2?

Even if g_m varies by x2

$\frac{\Delta V_o}{\Delta V_i}$ won't change.

[e=6pt] Finally, consider that the power supply may have some perturbation such that $v_{DD} = V_{DD} + \Delta v_{DD}$. With no other perturbations in the circuit, calculate the Power Supply Rejection Ratio, i.e. $\Delta v_{DD} / \Delta v_{out}$.

$$\boxed{\frac{\Delta V_{dd}}{\Delta V_o} = 1}$$

This circuit provides no rejection of power supply disturbances.

Q2

[a=5pt] Prove de Moivre's formula, which states that $(\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$

$$(\cos\theta + j\sin\theta)^n = (e^{j\theta})^n = e^{jn\theta} = \cos n\theta + j\sin n\theta$$

[b=5pt] Use de Moivre's formula from part (a) to prove the trigonometric "double-angle" identities, namely

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\cos\theta\sin\theta$$

[Note that you may use de Moivre's formula for part (b) whether you proved it in part (a) or not]

$$(\cos\theta + j\sin\theta)^2 = (\cos^2\theta - \sin^2\theta) + j\,2\cos\theta\sin\theta$$
$$= \cos 2\theta + j\sin 2\theta$$
$$\Rightarrow \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\cos\theta\sin\theta$$

[c=5pt] Consider a particular transfer function T below. Express T in polar **and** rectangular coordinates.

$$T = \frac{3+4j}{12-5j}$$

$0.385e^{j75.75°}$

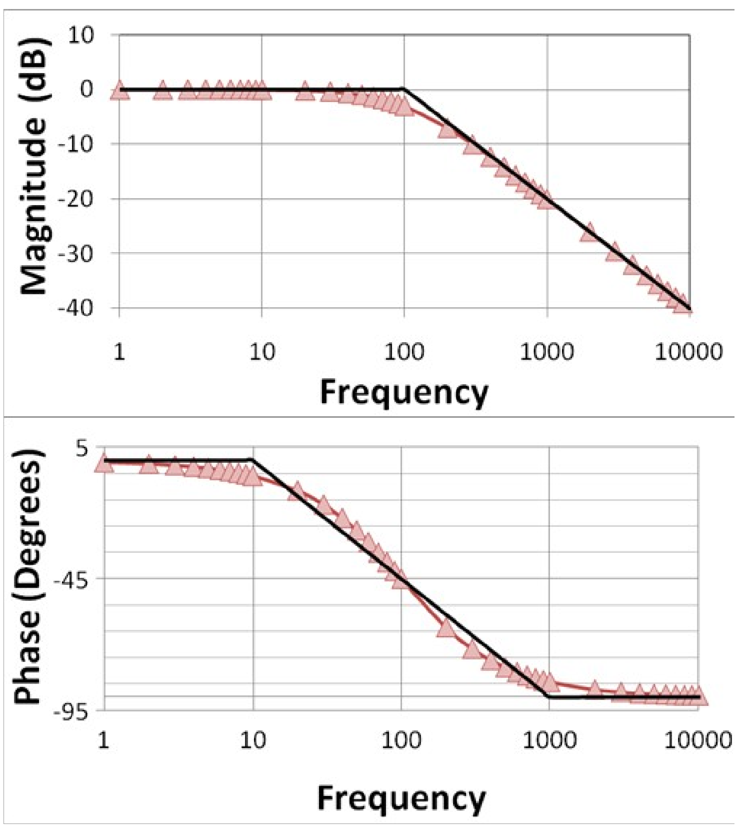
$0.0947 + 0.373j$

[d=5pt] On a logarithmic number line, what number falls exactly halfway between 1 and 10?

$\sqrt{10} = 3.162$

[e=5pt] A filter has a transfer function $\frac{V_o}{V_i}$ as given in the figure (the frequency axis is in Hz).

The input voltage is a cosine wave at 1 kHz with amplitude 3 and a phase of 20 degrees. What will the frequency, amplitude, and phase of the output voltage be?



$\boxed{Amp = 0.3}$

Output phase $= 20 - 85$

$\boxed{= -65°}$

Part II

Q3

(a) $V_{in}(t) = 10\cos(2\times10^6 t + 45^\circ)$ V

$V_{in} = 10e^{j45^\circ}$ V

(b) $Z_R = R = 1\Omega$

$Z_C = \frac{1}{j\omega C} = \frac{1}{j\times2\times10^6\times0.25\times e^{-6}} = -2j\Omega$

(c) $\frac{V_{out1}}{V_{in}} = \frac{Z_R}{2Z_R+Z_C} = \frac{1}{2-2j} = \frac{1}{4}(1+j)$

$\frac{V_{out2}}{V_{in}} = \frac{Z_C}{2Z_R+Z_C} = \frac{-2j}{2-2j} = \frac{1}{2}(1-j)$

(d) $I = \frac{V}{Z} = \frac{10e^{j45^\circ}}{2-2j} = \frac{10e^{j45^\circ}}{2\sqrt{2}\cdot e^{j-45^\circ}}$

$= \frac{5}{2}\sqrt{2}\,e^{j90^\circ}$

$I(t) = 3.535\cos(2\times10^6 t + 90^\circ)$

(e) $\omega \to \infty$. $Z_C \to 0$.

$|V_{out1}| = \frac{1}{2}|V_{in}| = 5V$

$|V_{out2}| = 0V$

Part 2

Q1

Answer to parts (i), (ii), (iii)

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Cutoff frequency : 159154.94 Hz
Input signal period : 0.001 s
Input DC component : 0.2 V
Input phasor component 1 magnitude: 0.37419571351545566 V
Input phasor component 1 phase: 0 degrees
Input phasor component 2 magnitude: 0.3027306914562628 V
Input phasor component 2 phase: 0 degrees
Input phasor component 3 magnitude: 0.2018204609708419 V
Input phasor component 3 phase: 0 degrees
Input phasor component 4 magnitude: 0.09354892837886393 V
Input phasor component 4 phase: 0 degrees
Output DC component : 0.2 V
Output phasor component 1 magnitude: 0.37418832740682695 V
Output phase component 1 phase: -0.3599952627020996 degrees
Output phasor component 2 magnitude: 0.30270679162948966 V
Output phase component 2 phase: -0.7199621043095835 degrees
Output phasor component 3 magnitude: 0.2017846165363827 V
Output phase component 3 phase: -1.0798721171883547 degrees
Output phasor component 4 magnitude: 0.09351939705903721 V
Output phase component 4 phase: -1.4396969206094188 degrees
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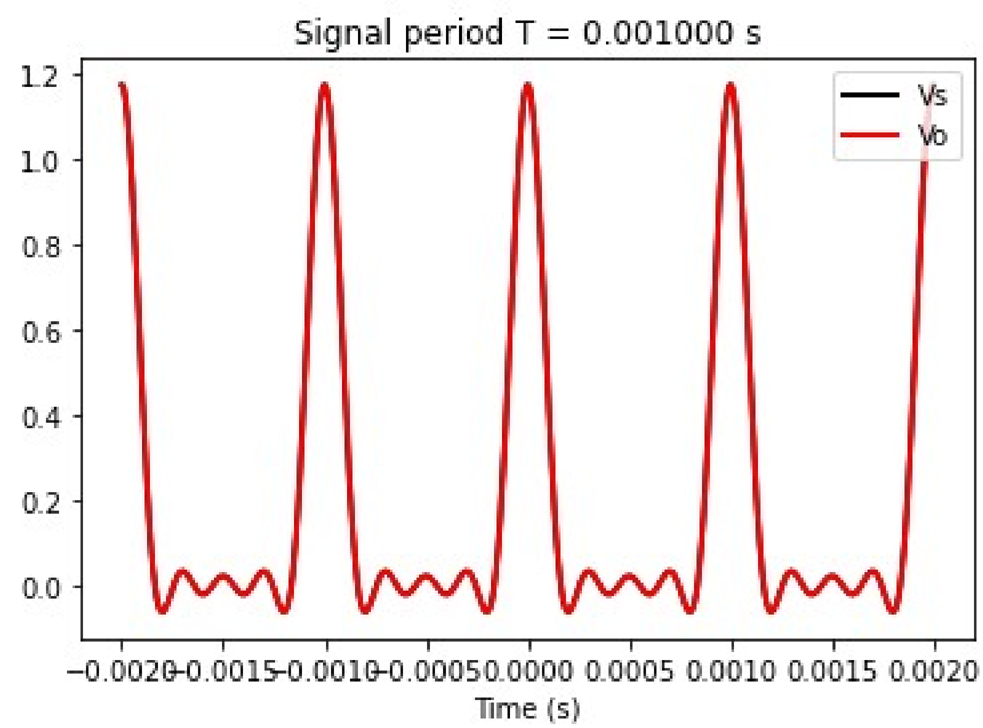
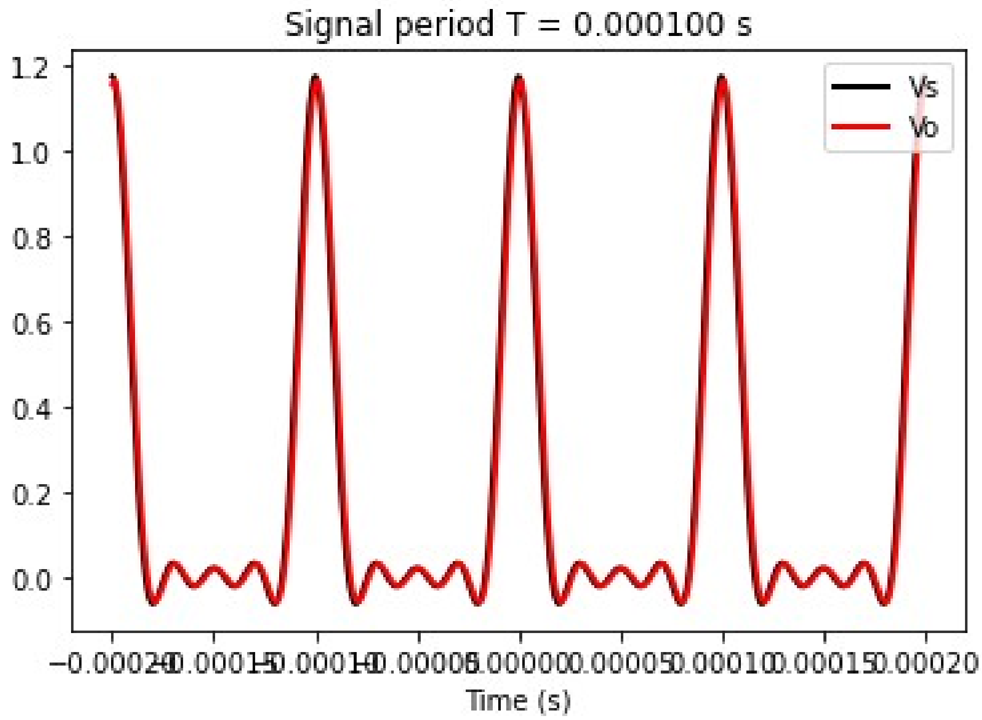
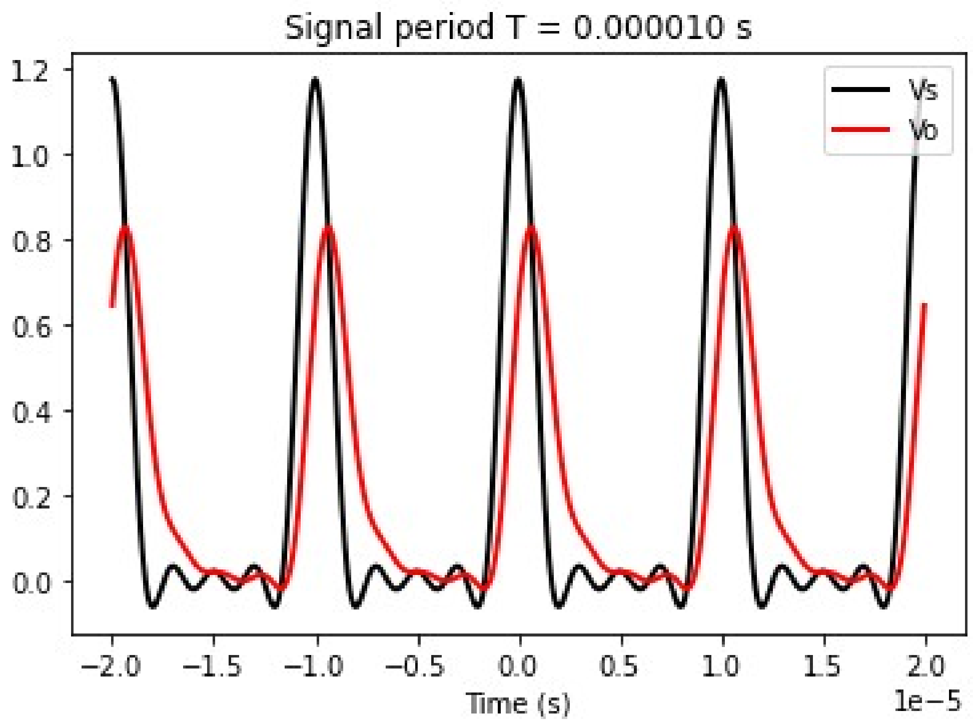
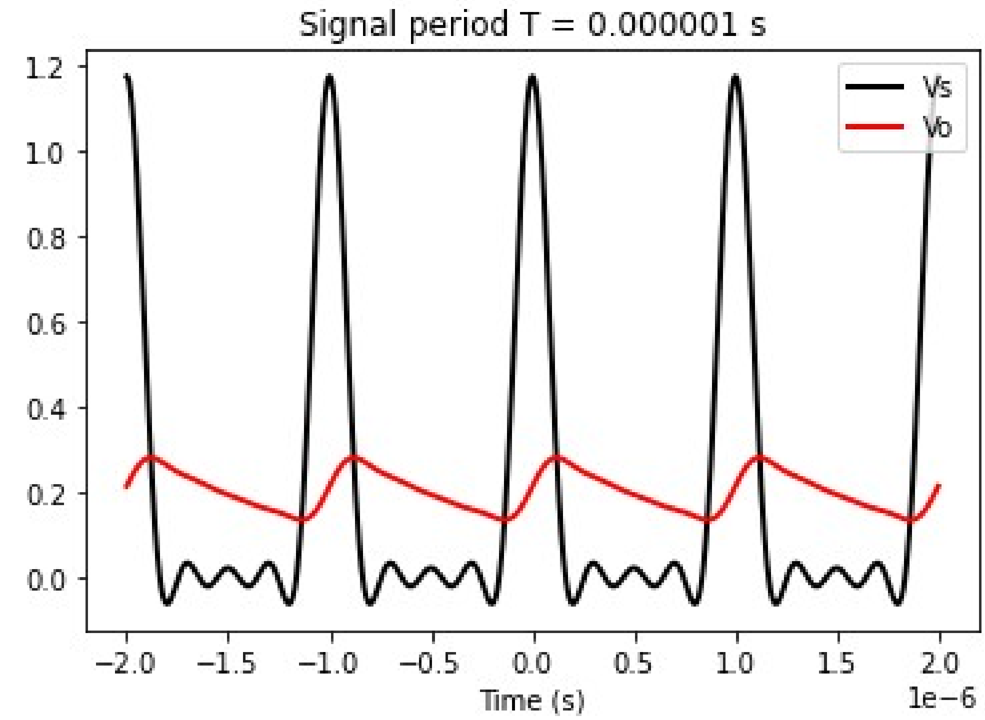
Plots for part (iv)




Signal period T = 0.000100 s
Vs
Vo
Time (s)


Signal period T = 0.000010 s
Vs
Vo
Time (s)
1e−5

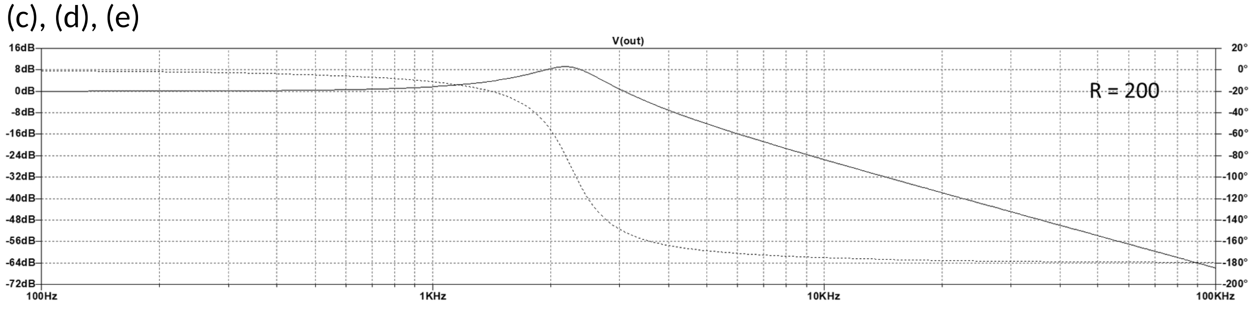


Q2

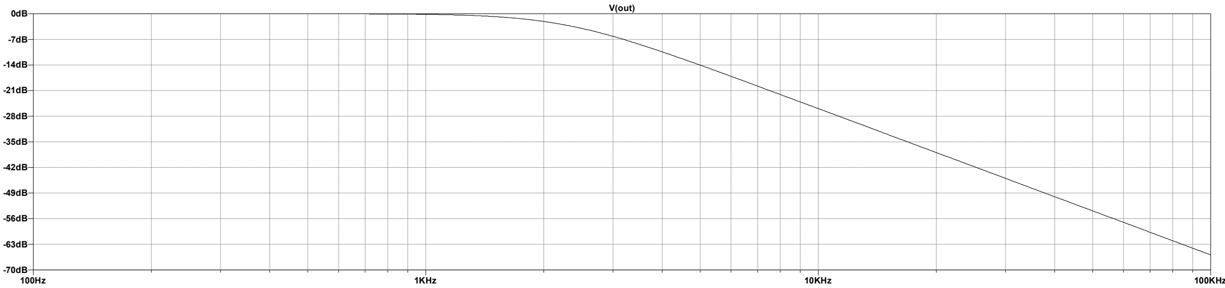
(a)
$$\frac{R + j\omega[L - R^2C + \omega^2R^2LC^2]}{1 + \omega^2R^2C^2}$$

(b)
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\left[(1-\omega^2LC)^2 + \left(\frac{\omega L}{R}\right)^2\right]^{1/2}}$$

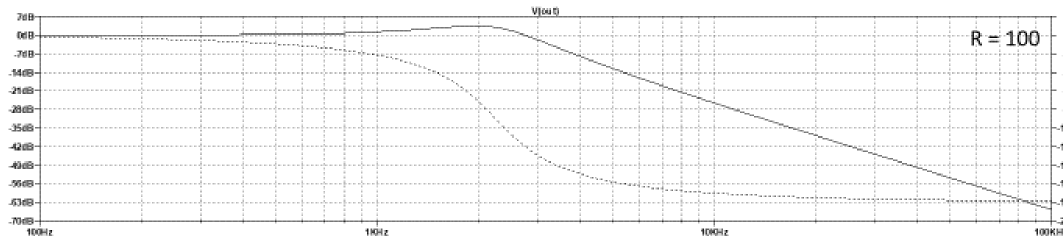
(c), (d), (e)



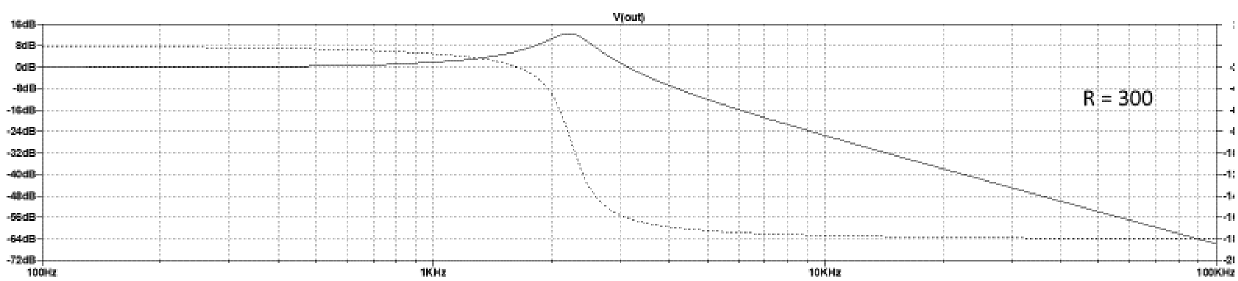
R = 50



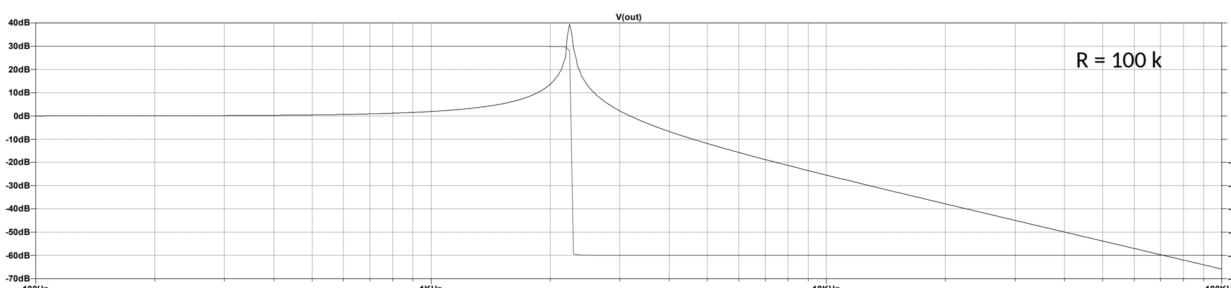
R = 100



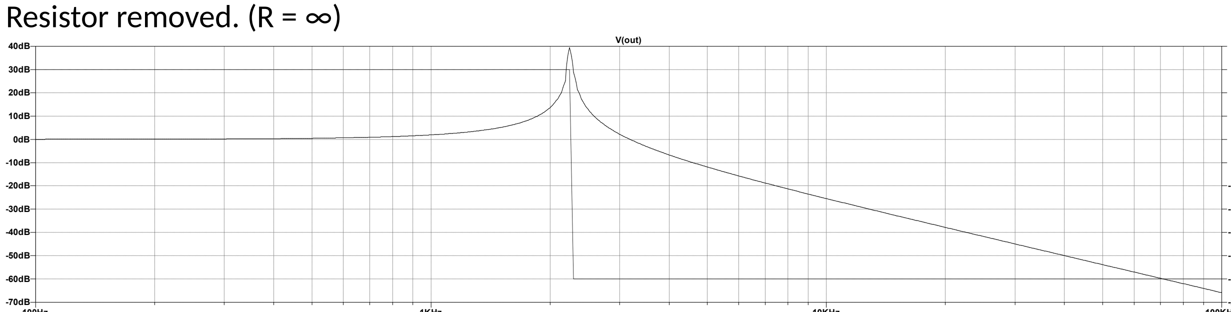
R = 300



R = 100 k



Resistor removed. (R = ∞)



R (Ohms)	f0 (kHz)	VMAX from LTSPICE (dB)
100k or ∞	2.23	39.39
300	2.23	12.59
200	2.18	9.16
100	1.99	3.57
50	-	0

(f) If the resistor value is below about 300 Ohms, the resonance frequency (peak VOUT response) starts to reduce; simultaneously, the peak value of VOUT drops. When R = 50 Ohms, there is no observable resonance and we can conclude the circuit is no longer resonant. As R increases above 100 Ohms, the peak voltage VOUT increases and saturates at ~ 40 dB, which is also the peak value for the LC circuit (with R removed).