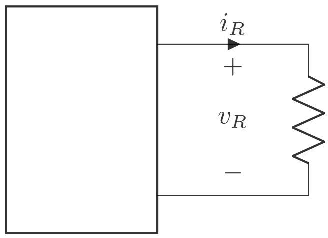
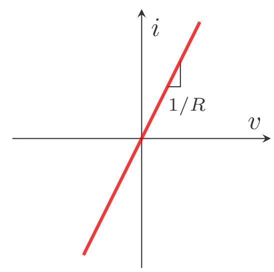
Lecture Notes 3: Components, One-Ports, and Equivalents

A circuit element's **component law** or **i-v characteristic** isolates the behavior of the component itself from its interaction with the rest of the circuit. In other words, the component *plus* the circuit will determine what v and i will be, but *the relationship between v and i is set by the component itself.*

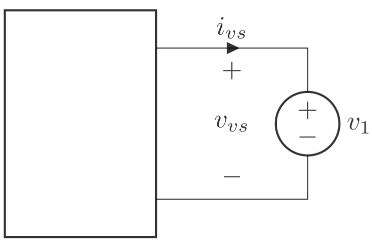
The circuit element's symbol, quation, and i-v curve all convey the same information.

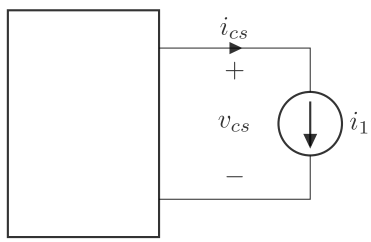


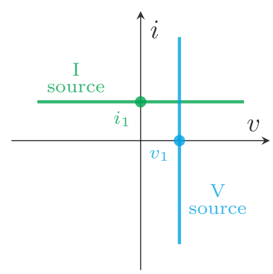
$v = Ri$



Two other common circuit elements are the voltage source and current source.



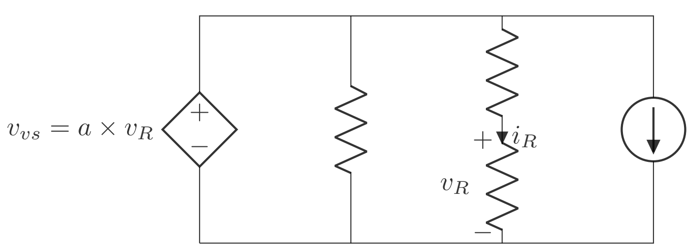




A **voltage source** always has its labeled voltage. Its voltage is not a function of its current, i.e., its voltage will be the same whether $1\mu A$ or $1kA$ of current flows through it. How much current ends up flowing depends on the solution of the whole circuit.

A **current source** always has its labeled current. Its current is not a function of its voltage, i.e., its current will be the same whether it blocks $1\mu V$ or $1kV$. How much voltage actually appears depends on the solution of the whole circuit.

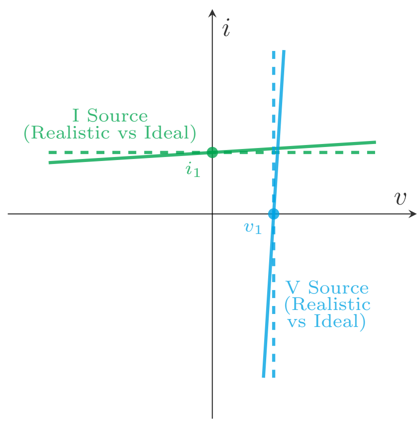
It is also common to have "dependent" sources whose voltage or current is not predetermined, but is a function of some other voltage or current in the system.



In the example above, the dependent voltage source (in a diamond) produces a voltage that is $a \times v_R$. Solving this circuit definitely requires solving a system of simultaneous equations, since v_s depends on v_R *and* v_R depends on v_s!

The components we have studied so far are idealized. There is no such thing as an ideal voltage source, an ideal current source, or an ideal resistor in a real life. These models are therefore always approximations of real-life components. We must carefully distinguish between real-life components (like batteries) and their idealized circuit models (like voltage sources) and be able to tell when the idealized approximation breaks down. Batteries, for instance, do behave like good voltage sources; however, a true voltage source will maintain its voltage even if a hundred bazillion amps flows through it, while a battery will not be able to support an arbitrarily high current (hopefully this is obvious).

As an example, consider that if you draw a lot of current out of a voltage source, the voltage it provides may droop (like the water pressure in your house – the pressure is nominally constant, but if someone turns on all the faucets at once, the water flow may be high enough to see a noticeable pressure drop). Conversely, if current were pumped back into a voltage source, the voltage at its terminals may rise. This behavior is still like a voltage source – a more-or-less constant voltage – but with a bit of a droop or a bump as current increases in the negative or positive direction.



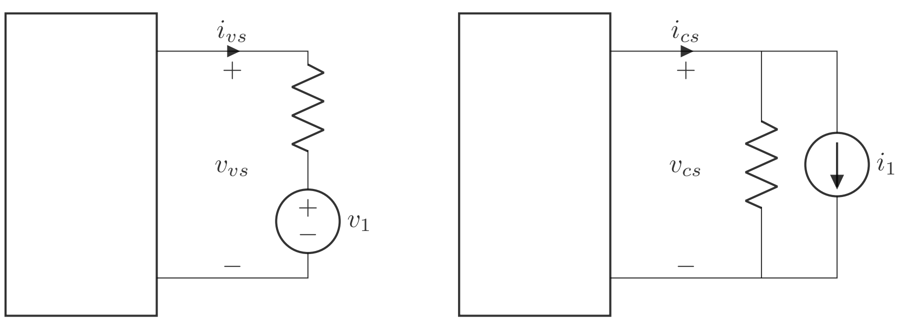
If the non-ideality is linear, then we may say that

$$v_{vs} = V_{nominal} + i_{vs} \times b$$

Think about this equation: if we move all the terms to one side, it's a sum of voltages that adds to zero. That's a KVL expression! Therefore, we ought to be able to *model* the real voltage source as a series connection of something that provides $V_{nominal}$ at all times and something that provides $i_{vs} \times b$.

What kind of component provides $V_{nominal}$ at all times? A voltage source of value $V_{nominal}$! What kind of component has voltage $i_{vs} \times b$? A resistor of values $R = b$!

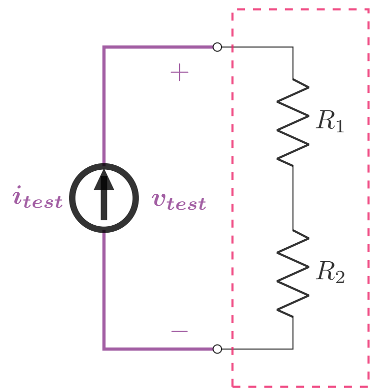
The analogous thing for a realistic current source would be to say that $i_{cs} = I_{nominal} + v_{vs} \times c$, which looks like a KCL expression for a current source $I_{nominal}$ in parallel with a resistor of value $1/c$.



What have we done? In real life, we only have one component (like the battery) with two terminals. We say that the battery has **one port**, or one voltage v_{vs} and one current i_{vs}. Nevertheless, we have used *multiple* ideal components to create a sort of compound component that behaves like the battery does. The battery doesn't really have a voltage source and a resistor inside of it, but if you did have a box with a voltage source and resistor inside of it, it would behave just like the battery. In fact, using only v_{vs} and i_{vs} measurements (i.e., if you can't look inside the box), there's no way you could even tell the difference.

Two one-port circuits that have the same $i - v$ relationship at the port are called **equivalent circuits**.

1. Resistors in series: Any two components are said to be in **series** if they have the same current by virtue of KCL (i.e., they *must* have the same current, not that they *happen to* have the same current).



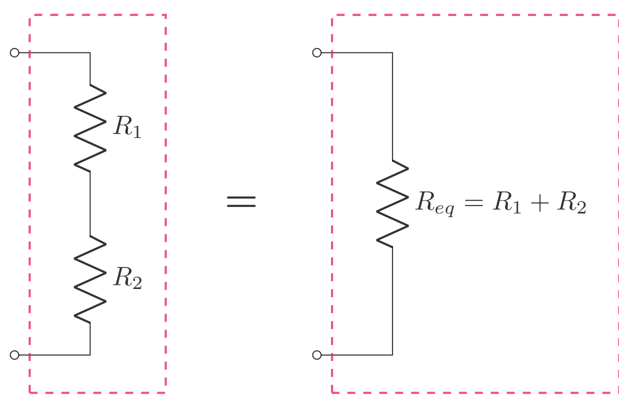
How do we find an equivalent circuit for two resistors in series? Well, an equivalent circuit has the same $i - v$ relationship, so the best place to start is to find the $i - v$ relationship for this circuit! Finding any $i - v$ relationship involves applying a test current to the port and solving for the resulting test voltage (or vice versa). In this case, we find that

$$v_{R1} + v_{R2} = v_{test} \qquad v_{R1} = R_1 i_{R1} \quad v_{R2} = R_2 i_{R2} \tag{9}$$

$$i_{test} - i_{R1} = 0 \qquad i_{R1} - i_{R2} = 0 \tag{10}$$

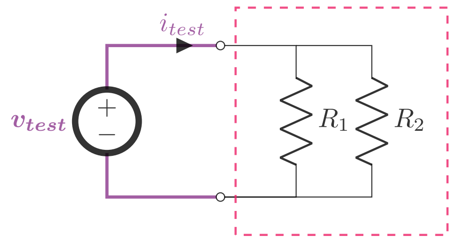
The two components being in series results in $i_{R1} = i_{R2}$. Solving, we find that $i_{test}R_1 + i_{test}R_2 = \boxed{v_{test} = (R_1 + R_2)i_{test}}$.

Now, we need an equivalent circuit that behaves like $v_{port} = \text{Const} \times i_{port}$. A simple resistor should suffice! And its value should be $R_{equiv} = R_1 + R_2$.



If we were solving a circuit, making this substitution would make our lives much easier, but it would obscure information about what's going on inside the box. Thus, before using an equivalent circuit to simplify a solution, first ask whether you care what's going on inside the box. If the answer is no, proceed! But if there's a dependent source inside the box that depends on something outside the box, or if there's a depeendnt source outside the box that depends on a value in side the box (like the voltage across R_2 specifically), then an equivalent circuit might not be the way to go.

2. Resistors in parallel Any two components are said to be in **parallel** if they have the same voltage by virtue of KVL (i.e., they *must* have the same voltage, not that they *happen to* have the same voltage). This is equivalent to saying that the components are connected between the same two nodes. We could find an equivalent circuit by first finding the circuit's $i - v$ relationship by applying a test current and solving for the resulting test voltage. It's easier in this case to apply a test voltage and solve for the resutling test current.



This gives us

$$i_{test} = \frac{v_{test}}{R_1} + \frac{v_{test}}{R_2} \Rightarrow \boxed{v_{test} = \frac{R_1 R_2}{R_1 + R_2} i_{test}}$$

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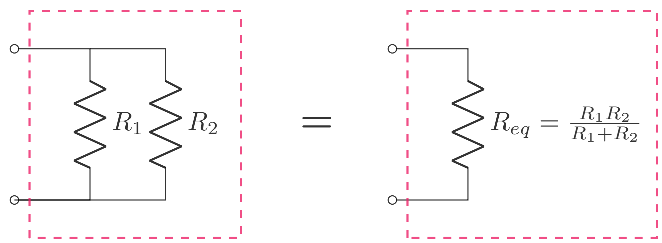
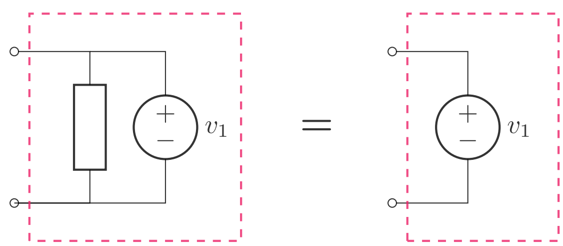
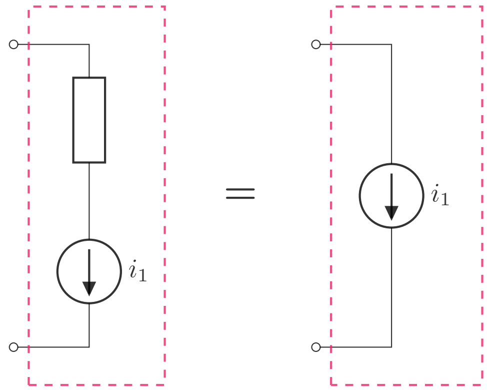


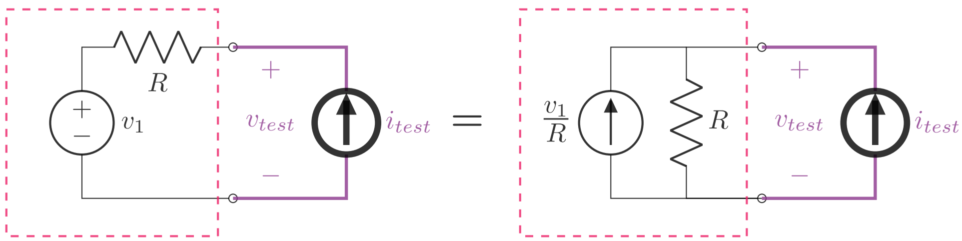
3. Voltage sources in parallel with anything behave as the voltage source alone because the voltage across the port terminals is v_{source} regardless of what the "anything" component is. And no matter how much current the "anything" component may draw, the ideal voltage source will maintain its voltage.



4. Current sources in series with anything behave as a current source alone because the current through the port terminals is i_{source} regardless of what the "anything" component is. And no matter how much voltage the "anything" component may develop, the ideal current source will maintain its current.



5. Source Transformations I claim the following two circuits in red boxes as equivalents:



This is proved by proving that the $i - v$ relationships are the same. The circuit on the left obeys $v_{test} = v_1 + Ri_{test}$. The circuit on the right obeys $v_{test} = (v_1/R + i_{test}) \times R = v_1 + Ri_{test}$, which is the same.

The reverse transform also works: