Homework 4 **ECE 302H: Introduction to Electrical Engineering**

I include time estimates for each problem. A problem may take you more or less time, but if you spend more than half of the allotted time on a problem and aren't making progress, it's time to ask for help!

Problem 4.1 – Linearity and Superposition with MOSFETs (0.5h)

The most commonly-used nonlinear circuit component is the MOSFET. The I-V characteristic of the MOSFET (ignoring channel length modulation) is given by

$$i = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left(v_{gs} - V_{TH}\right)^2$$

a) Formally show that this equation is nonlinear by using the definition of linearity.
b) It is common for v_{gs} to be the sum of multiple sine waves. Let vgs = V1 sin(ωt)+V3 sin(3ωt), which can be modeled as two voltage sources in series, one of value V1 sin(ωt) and the other of value V3 sin(3ωt). With two sources, it may be tempting to apply superposition to the circuit to solve for the current, i.e. to solve for the current at 1ω and the current at 3ω separately. Show that this can not be done by calculating the current that you would get if you applied superposition and comparing it to the actual current. The fact that superposition does not work for nonlinear circuits will seriously hamper us later on. (You will learn later in the course that *any* periodic function can be represented by a sum of sine waves, one of the most impactful and remarkable observations of the 19th century)
c) In HW3, we linearized the MOSFET (aka, we found the small-signal model for the MOSFET) by saying that each variable could be represented by an operating point plus a deviation. For example, $i \rightarrow I + \Delta i$. We then saw that the operating point terms on both sides cancelled and then we approximated any terms with a product of two deviations (e.g., Δv_{gs}^2) as zero. This time, linearize the equation above by taking the derivative of i with respect to v_{gs} and constructing a line that is tangent to the true $i\left(v_{gs}\right)$ at an operating point $\left(V_{gs}, I\right)$.

Problem 4.2 – Superposition R-2R DAC (1h)

Consider the circuit in the Figure composed of some resistors of value R and others with value 2R.

a) Use superposition to calculate the output voltage as a function of the inputs.
b) Suppose that the inputs VA-VC are digital values, i.e. they can only be equal to 0 or VDD. This circuit is then a digital-to-analog converter (DAC). Explain how the DAC works in conceptual terms. How many different voltages can be produced in this DAC?
c) Calculate the output if the digital input [VC VB VA] = [101].
d) How would you create a similar DAC with twice as much resolution? (you do not need to show that it works; just draw the circuit)

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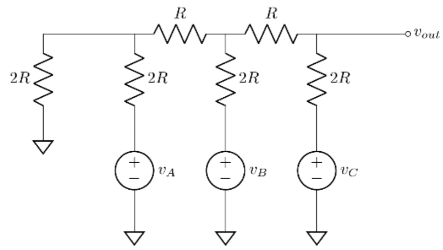
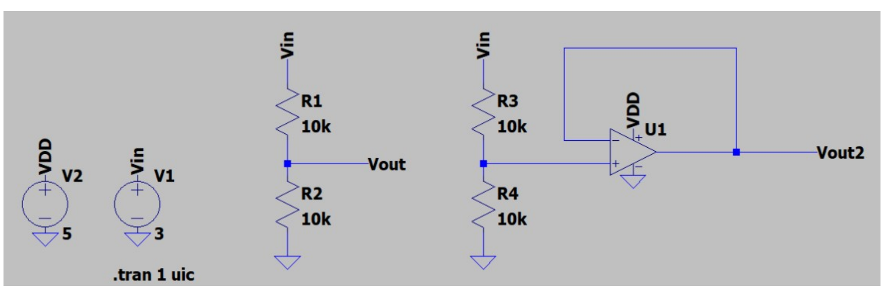
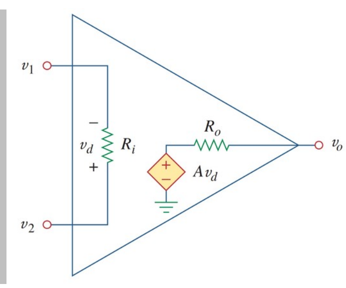


Problem 4.3 – Thevenin Equivalents of Voltage Dividers (1h)

Recall the two circuits that we simulated in HW1 and built in Lab 1: the resistive voltage divider, and the voltage divider plus op-amp, as recreated in the Figure.

a) Find the Thevenin equivalent circuit for the voltage divider. Then, imagine that Vout is loaded with a current source of value Iout and plot Vout vs Iout using the Thevenin equivalent.
b) Find the Thevenin equivalent circuit for the voltage divider plus op-amp circuit (the equivalent circuit model for the op-amp is given on the right; you may assume an ideal op-amp, namely $R_i = \infty$ and $R_o = 0$ and when you have the final answers, take the limit as $A \rightarrow \infty$). Then, imagine that Vout2 is loaded with a current source of value Iout and plot Vout vs Iout using the Thevenin equivalent.
c) Show that your results match what you found in HW1 and Lab 1.
d) What advantage does the second circuit provide over the first circuit? State your answer in terms of Thevenin Equivalent Circuit parameters.





Problem 4.4 – Thevenin Equivalents of Test Equipment (0.5h)

The Moku:Go has a signal generator that you want to use to excite a resistive circuit. The signal generator's Thevenin equivalent circuit is a voltage source with an output impedance of 200 ohm.

a) Use the concept of Thevenin/Norton equivalent circuits to explain why the resistive circuit can be modeled as a single resistor.
b) You program the signal generator to output 5V. What voltage will you see if the resistive circuit's equivalent resistance is the following? Req << 200Ω, Req >> 200Ω, and Req = 200Ω
c) What resistive circuit equivalent resistance is required so that the actual voltage it experiences is within 1% of the voltage you programmed? Is that resistance a minimum or a maximum?

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d) A competing product, the Analog Discovery 2, has an output impedance of 20 ohms. Is this better or worse than the Moku:Go? Explain your answer, including what you mean by "better."

e) In Lab 1, you programmed the signal generator to produce 2V but when we measured the output, it really produced 4V. If you asked an experienced engineer what was happening, they might say "The signal generator's output resistance in 50 ohms and it's expecting to drive a matched load." Explain how this statement justifies why you saw 4V in Lab 1 instead of 2V.

Problem 4.5 – System Identification (0.5h)

Suppose you have an unknown system (e.g., in a box). It is meant to take two voltages and one current as inputs v_1, v_2, i_3 and produce a single voltage as output v_o. All you know about the box is that the system inside is linear and the results of the following experiments, with voltages in volts and currents in amps.

	v_1	v_2	i_3	v_o
Experiment 1	3	2	1	37
Experiment 2	2	2	2	36
Experiment 3	1	5	3	44

Find the system equation, i.e. the equation that relates the inputs and the output. Be sure to include the units for each constant.

Problem 4.6 – Thevenin Equivalent of Complex Circuit (1h) (adapted from Ulaby 3.71)

Find the Thevenin equivalent for the circuit below at the port consisting of terminals a and b. (Hint: as always, apply v_{test} and solve for i_{test} or apply i_{test} and solve for v_{test}. When solving, you may want/need to apply other techniques, such as node analysis or equivalent circuits.)