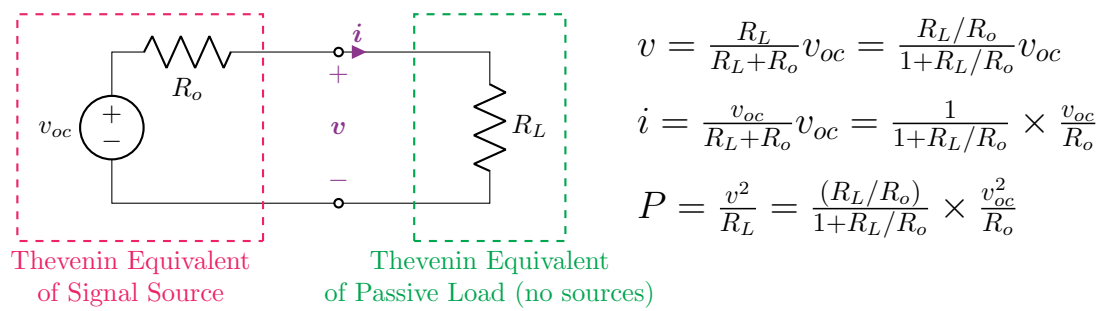# Lecture Notes 7: Applications of Thevenin Equivalent Circuits

Designing large systems does not mean that you have to solve super large circuits – nobody does this. Instead, the secret is to break the large system up into manageable building blocks. Then you don't need to remember (or even know) the internal workings of each block – you just need to know the block's behavior and, particularly, how it interacts with the rest of the circuit.

> The reason Thevenin (and Norton) Equivalents are so important is because they give us a nearly-universal way to describe how linear circuits interact.

In fact, most circuit interactions involve one signal source (the thing generating the signal) and one signal receiver, both of which are one-port circuits. The signal source can be represented by its Thevenin Equivalent (v_{oc} in series with R_o). The signal receiver can also be represented by its Thevenin Equivalent, which won't have any v_{oc} because the receiver is generally not designed to generate signals. This means that the majority of circuit interactions boil down to the following:

$$v = \frac{R_L}{R_L+R_o}v_{oc} = \frac{R_L/R_o}{1+R_L/R_o}v_{oc}$$

$$i = \frac{v_{oc}}{R_L+R_o}v_{oc} = \frac{1}{1+R_L/R_o} \times \frac{v_{oc}}{R_o}$$

$$P = \frac{v^2}{R_L} = \frac{(R_L/R_o)}{1+R_L/R_o} \times \frac{v_{oc}^2}{R_o}$$

Thevenin Equivalent of Signal Source

Thevenin Equivalent of Passive Load (no sources)

Clearly, the **interaction between the source and the load is primarily governed by the relative size of the Thevenin resistances of the receiver and generator, R_L/R_o.** There are three main scenarios that concern us:

Scenario 1: If $R_L \gg R_o$, then $v \approx v_{oc}$. The voltage that the generator actually produces *at its terminals* is constant and independent of R_L as long as R_L is "big enough." If we wanted information to be represented by *voltage*, this is the scenario we would want. If this condition is satisfied, we can further simplify the signal generator to an equivalent circuit of just the v_{oc} voltage source alone.

If R_L is made smaller, it will draw more current, which will cause v to droop a bit below v_{oc}. We say that the receiver is **loading** the generator. (A good visual for this is a person standing up straight and then being given a heavy box to carry. They're likely to slump a little bit because they're carrying a heavy *load*).

Scenario 2: If $R_L \ll R_o$, then $i \approx v_{oc}/R_o = i_N$. In these cases, the *current* is approximately constant and independent of R_L as long as R_L is "small enough." If we wanted information to be represented by *current*, this is the scneario we would want. In this case, it makes more sense to use the Norton Equivalent Circuit to describe the generator. $R_o \gg R_L$ so $R_o||R_L \approx R_L$ and we can approximate the signal generator as just the i_N current source alone.

If R_L is made bigger, it will develop more voltage across it, which will cause some of i_N to be shunted away through R_o. The current that actually makes it to the receiver i will thus droop a bit below i_N. In this case, we also say that the receiver is **loading** the generator. It's precisely analogous to Scenario 1.

Scenario 3: If $R_L = R_o$, then $v = v_{oc}/2$, $i = v_{oc}/(2R_o) = i_N/2$, and $P = v_{oc}^2/(4R_o)$. This scenario is said to be **impedance matched** and is commonly used for high-frequency circuits. What is so special about impedance matching? It turns out that the impedance matched case corresponds to maximum power.

The Maximum Power Transfer Theorem states that choosing $R_L = R_o$ draws the most possible power out of a signal generator with a given R_o.
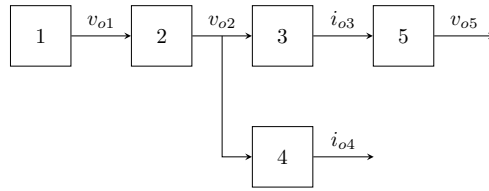
Proof:

$$P = \frac{v^2}{R_L} = \left(\frac{v_{oc}}{R_o + R_L}\right)^2 \frac{1}{R_L} \quad \Rightarrow \quad \frac{dP}{dR_L} = v_{oc}^2 \frac{(R_o + R_L)^2 - 2R_L(R_o + R_L)}{(R_o + R_L)^2} = v_{oc}^2 \frac{R_o - R_L}{(R_o + R_L)^2}$$

and therefore $dP/dR_L = 0$ when $R_L = R_o$.

Each of the scenarios above has merit. Most circuits use voltage to encode information. We don't want the presence of the receiver to corrupt the information, so we want $R_L \gg R_o$. Some other circuits use current to encode information (it turns out that current-mode circuits are often faster than voltage-mode circuits, such as ECL, CML, transconductance amplifiers, current feedback, ...). In these cases, we still don't want the presence of the receiver to corrupt the information, but in this case that implies that we want $R_L \ll R_o$, thus making i independent of the R_L. Finally, the impedance-matched cases yields maximum power – this turns out to be important in RF applications where the signal is very weak in the first place; it turns out that getting as much *energy* to the receiver as possible is the best way to overcome noise.

When Scenario 1 or Scenario 2 is satisfied, we can simplify a large circuit using a **block diagram**:
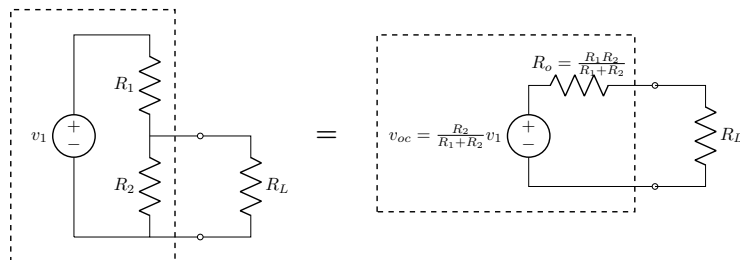

The information being sent from [1] to [2] is supposed to be a voltage. By drawing a block diagram, we are assuming that [1] will generate v_{o1} regardless of how much current [2] draws. In other words, we are assuming that the equivalent resistance that [2] presents to [1] is much greater than the output resistance of [1]. If this assumption is violated, we may mislead others (and ourselves!) by drawing the block diagram at all, since [2] does not simply receive v_{o1} but plays an active role in determining what v_{o1} will be.

Similarly, the information being sent from [3] to [5] is supposed to be a current. By drawing a block diagram, we are assuming that [3] will generate i_{o3} regardless of how much voltage develops across [5]. In other words, we are assuming that the equivalent resistance that [5] presents to [3] is much less than the output resistance of [3]. If this assumption is violated, we may mislead others and ourselves by drawing the block diagram at all, since [5] does not simply receive i_{o3} but plays an active role in determining what i_{o3} will be.
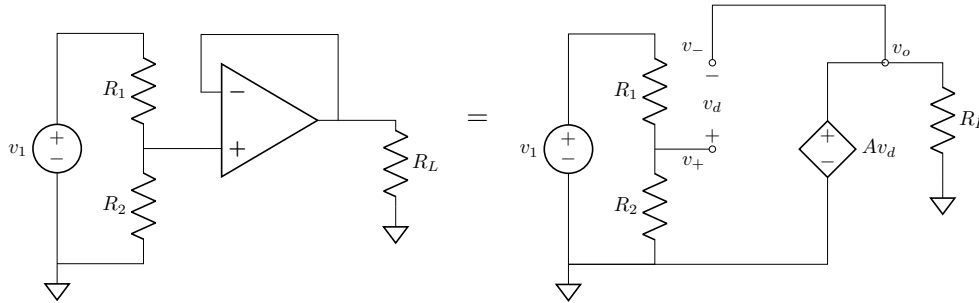
For this circuit to work as intended, note that the output of [5] should be terminated with an open circuit, while the output of [4] should be terminated with a short circuit.

Consider a voltage divider example:

If you are expecting the output of the voltage divider to be $v_{oc}\frac{R_2}{R_1+R_2}$, you might be sorely mistaken if R_L is not much greater than R_o which, in this case, is $R_1||R_2$. If R_L is no much greater than R_o, then v will droop from its expected value as R_L draws non-negligible current.

Some circuits are very good at helping with this problem. Consider the following circuit which makes use of a new component, an **operational amplifier** or **op amp**. You don't need to know much about the op amp at this point; you just need to know that its equivalent circuit is the dependent voltage source shown in the diagram.

Let's see what this circuit does. The op amp's output voltage v_o is a gain term A (usually very large) times the difference between its input terminals, v_+ and v_-. Because we have shorted the output to the negative input, we can say that $v_- = v_o$.

We also know that $v_+ = v_1 \times R_2/(R_1 + R_2)$ per the usual voltage divider equation. **How do we know? Because the op amp looks like an open circuit, i.e. it draws no current from the voltage divider.** Therefore,

$$v_o = A \times \left(\frac{R_2}{R_1 + R_2} v_1 - v_o \right) \tag{19}$$

$$(1 + A)v_o = \frac{R_2}{R_1 + R_2} A v_1 \tag{20}$$

$$v_o = \frac{A}{1 + A} \frac{R_2}{R_1 + R_2} v_1 \tag{21}$$

$$v_o \approx \frac{R_2}{R_1 + R_2} v_1 \quad (\text{if } A \gg 1) \tag{22}$$

What an amazing little circuit. The op amp presents infinite resistance to the prior circuit, meaning that it can receive the output voltage of the voltage divider without corrupting it at all. The op amp also presents zero output resistance to any downstream circuits (you can see this from the equation where v_o does not depend on R_L; you can also notice that the op amp behaves as a perfect voltage source in the circuit diagram).

This means the op amp (in this configuration) can *load* whatever we want and it can *be loaded by* whatever we want without concern that it will corrupt its own input voltage or that its output voltage will be corrupted. This circuit is often added just to ensure that Scenario 1 is actually satisfied (when information is encoded as voltage). Because it acts as a "perfect" receiver and a "perfect" generator but otherwise simply replicates its input signal, this circuit is called a **buffer**.