# RC CIRCUIT

$$RC\frac{dv_c}{dt} + V_c = V_{in}$$

$$V_{in}(t) = \vec{V}_{in}\cos(\omega t) = Re\{\vec{V}_{in}e^{j\omega t}\}$$

$$V_c(t) = \underbrace{\vec{V}_c \cos(\omega t + \phi)}_{\text{unknown}} = Re\{\vec{V}_c e^{j(\omega t+\phi)}\} = Re\{\vec{V}_c e^{j\omega t}e^{j\phi}\}$$

$Re\{\ \}$ disappear

$$\left(\frac{d}{dt}\right)^n \Longrightarrow \times (j\omega)^n$$

$$RC(j\omega)\vec{V}_c e^{j\phi} + \vec{V}_c e^{j\phi} = \vec{V}_{in}e^{j0}$$

$$\boxed{\frac{\vec{V}_c}{\vec{V}_{in}}} = \frac{\vec{V}_c e^{j\phi}}{\vec{V}_{in}e^{j0}} = \boxed{\frac{1}{1+j\omega RC}}$$ Transfer Function

$$RC\,\vec{V}_c \cancel{e^{j\omega t}} e^{j\phi} j\omega + \vec{V}_c \cancel{e^{j\omega t}} e^{j\phi} = \vec{V}_{in}\cancel{e^{j\omega t}}$$

$$\vec{V}_c e^{j\phi}(RCj\omega + 1) = \vec{V}_{in}e^{j0}$$

$$\frac{\vec{V}_c e^{j\phi}}{\vec{V}_{in}e^{j0}} = \frac{1}{1+RCj\omega}$$

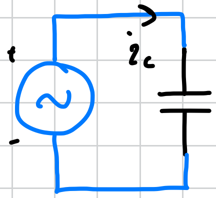
find $\frac{\vec{V}_c}{\vec{V}_{in}} = |TF| = \left|\frac{\vec{V}_c}{\vec{V}_{in}}e^{j\phi}\right| = \left|\frac{\vec{V}_c}{\vec{V}_{in}}\right|\left|1e^{j\phi}\right| = \frac{\vec{V}_c}{\vec{V}_{in}}$

$$\left|\frac{1}{1+RCj}\right| = \frac{|1|}{|1+RCj\omega|} = \frac{1}{\sqrt{1+(\omega RC)^2}}$$ ☆ Gain

find $\phi = \angle TF$

$$\angle\frac{1}{1+RCj\omega} = \frac{1}{Me^{j\arctan(\omega RC/1)}} = Me^{j\underbrace{[-\arctan(\omega RC)]}_{\phi}}$$

# CAPACITOR FREQUENCY DOMAIN



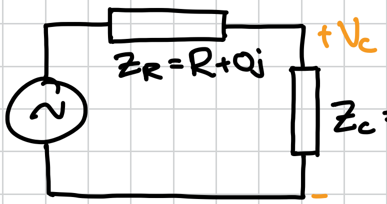

$$i_c = C\frac{dv_c}{dt}$$

$$I_c e^{j\phi} = C j\omega V_c e^{j0}$$

$$\vec{i}_c = j\omega C \vec{V}_c$$

$$\vec{V}_c = \underbrace{\left(\frac{1}{j\omega C}\right)}_{\text{impedance } Z_c}\vec{i}_c$$

# IMPEDANCE SOLUTION



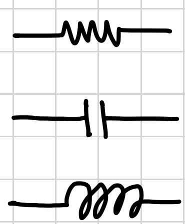

$$\vec{V}_c = \frac{Z_c}{Z_R + Z_c}\vec{V}_{in} = \frac{1/j\omega C}{R + 1/j\omega C}\vec{V}_{in} = \frac{1}{j\omega RC + 1}\vec{V}_{in}$$

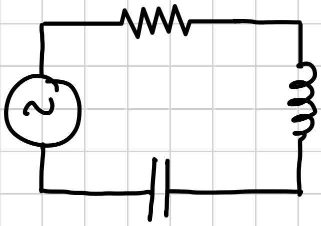
# RLC

Resistors: $v = Ri$ $\quad \vec{v} = R\vec{i}$ $\quad Z_R = R$

Capacitors: $i = C\frac{dv}{dt}$ $\quad \vec{v} = \frac{1}{j\omega C}\vec{i}$ $\quad Z_c = \frac{1}{j\omega C}$

Inductor: $v = L\frac{di}{dt}$ $\quad \vec{v} = j\omega L\vec{i}$ $\quad Z_L = j\omega L$





$$\vec{i} = \frac{1}{Z_R + Z_L + Z_C}\vec{V_{in}}$$

$$= \frac{1}{R + j\omega L + \frac{1}{j\omega C}}\vec{V_{in}}$$

$$= \frac{j\omega C}{j\omega RC + (1 - \omega^2 LC)}\vec{V_{in}}$$

$$\vec{V_c} = Z_c \times i = \frac{1}{j\omega C} \times \frac{j\omega C}{(1-\omega^2 LC) + j\omega RC} = \frac{1}{(1-\omega^2 LC) + j\omega RC}\vec{V_{in}}$$

$$\left|\frac{\vec{V_c}}{\vec{V_{in}}}\right| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$