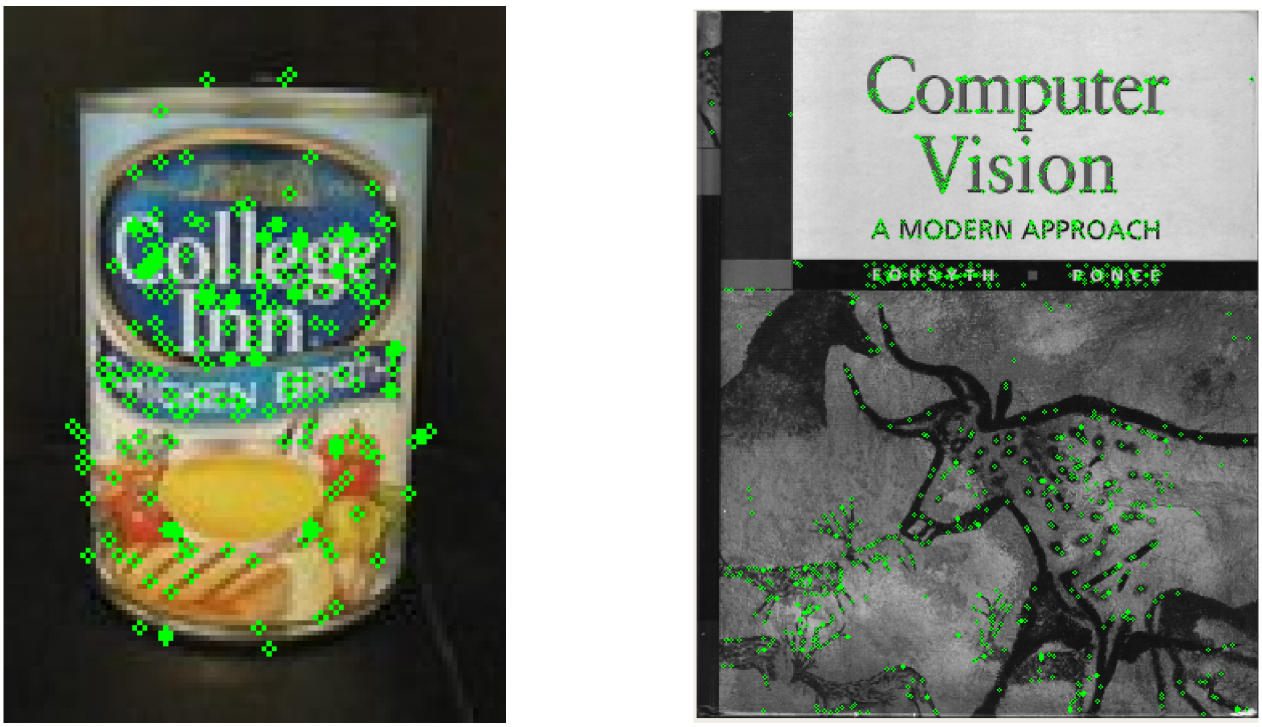
Q1.2

Include the DoG pyramid figure for `model_chickenbroth.jpg` in your writeup. Answer:

Figure 1: Difference of Gaussian Pyramid for `model_chickenbroth.jpg`

Q1.5

Include the image with the detected keypoints in your report for atleast one image (similar to the one shown in Fig. 3 (b)). You can use any of the provided images. (hint: for plotting keypoints you can use cv2.circle(...))



(a) Detected keypoints for `model_chickenbroth.jpg`

(b) Detected keypoints for `pf_scan_scaled.jpg`

Figure 2: Detected keypoints after removing points with high principal curvature

Q2.4

Include the resulting figure of the brief matches in your PDF report. Also, include results with the two incline images and with the computer vision textbook cover page (template is in file pf scan scaled.jpg) against the other pf_* images. Briefly discuss any cases that perform worse or better.

Answer: We observe that the BRIEF matches are particularly not invariant to rotations as can be observed from match comparison between 7 and 8. Further there are many spurious matches in the presence of multiple similar objects as in the matches in 7.

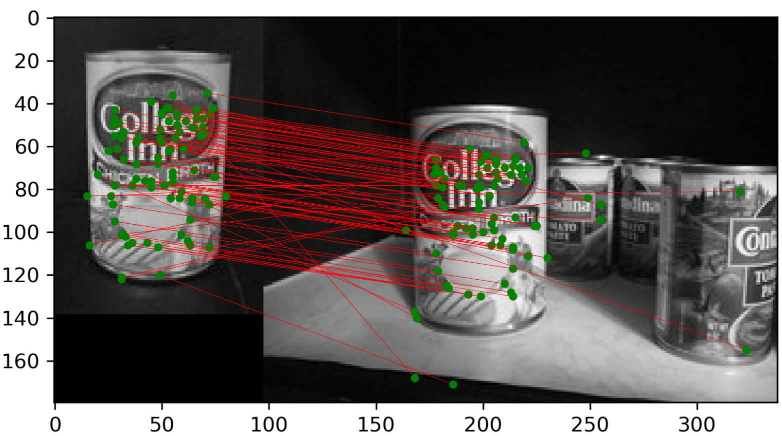


Figure 3: BRIEF descriptor matches between `model_chickenbroth.jpg` and `chickenbroth_1.jpg`

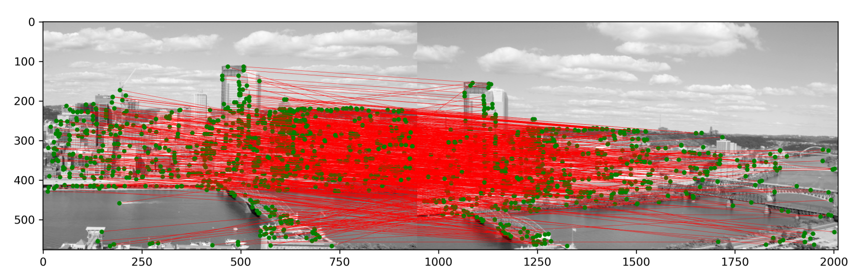


Figure 4: BRIEF descriptor matches between `incline_L` and `incline_R`

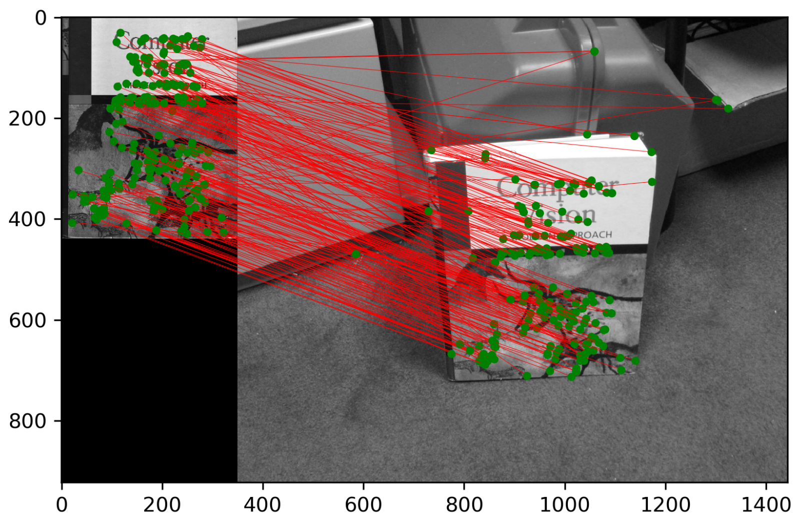


Figure 5: BRIEF descriptor matches between `pf_scan_scaled.jpg` and `pf_stand.jpg`

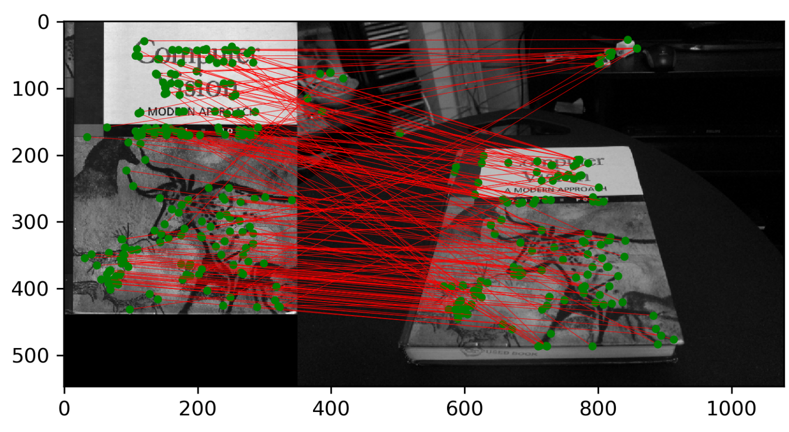


Figure 6: BRIEF descriptor matches between `pf_scan_scaled.jpg` and `pf_desk.jpg`

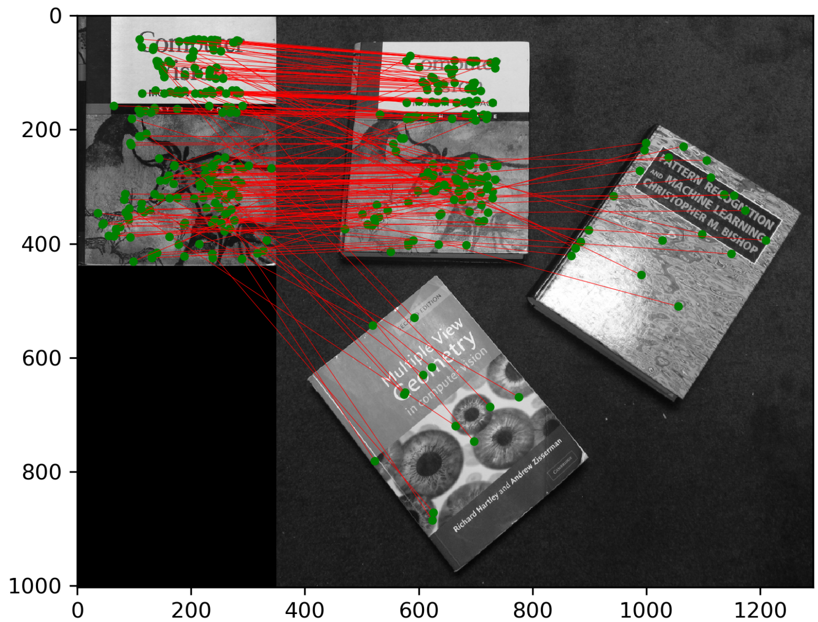


Figure 7: BRIEF descriptor matches between `pf_scan_scaled.jpg` and `pf_floor.jpg`

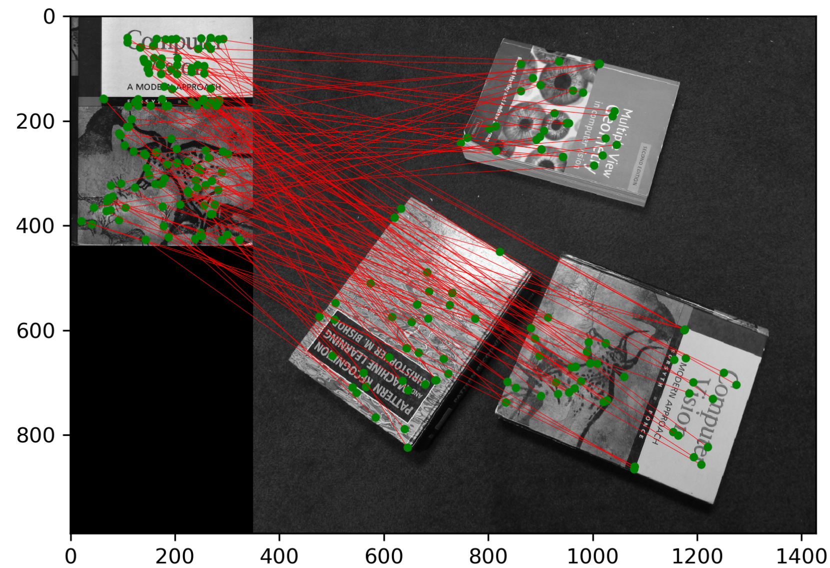


Figure 8: BRIEF descriptor matches between `pf_scan_scaled.jpg` and `pf_floor_rot.jpg`

Q2.5

Take the model chickenbroth.jpg test image and match it to itself while rotat- ing the second image (hint: cv2.getRotationMatrix2D(...), cv2.warpAffine(...)) in increments of 10 degrees. Count the number of correct matches at each rotation and construct a bar graph showing rotation angle vs the number of correct matches. Include the graph in your PDF and explain why you think the descriptor behaves this way.

Answer: As was observed in the previous question, the number of matches drastically reduces as the rotation angle is increased. This is the case with the BRIEF descriptors, because it constructs a bitset descriptor for each patch (size 9x9) by comparing randomly chosen pixel locations. On rotating the image the keypoints are bound to give significantly different descriptors for the same patch, as the comparison would occur at different pixel intensities owing to patch itself to be rotated. However when the image is transformed by out of plane transformations, it is generally invariant, as the intensities do not change enough, and the randomness of the comparison and the distance metric helps to encapsulate these perspective transforms.

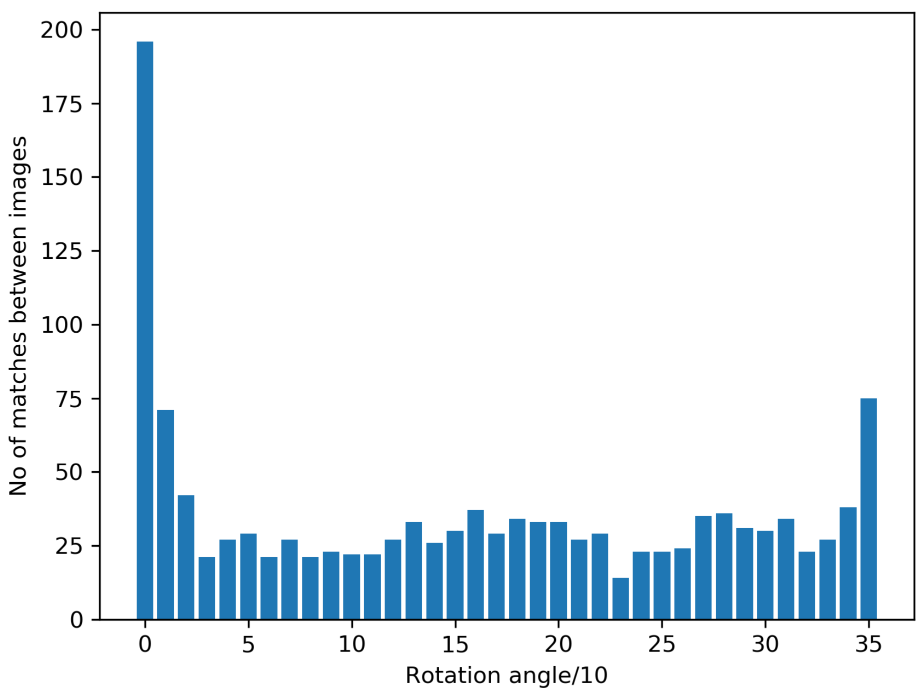


Figure 9: Bar graph showing the number of matches vs rotation angle of the image

Q3.1

1. Given the N correspondences across the two views and using Equation 8, derive a set of 2N independent linear equations in the form

$$\mathbf{Ah} = \mathbf{0}$$

We have a set of N 2D homogeneous coordinates $\{x_1, x_2, \dots x_n\}$ at a camera view, and a set $\{u_1, u_2, \dots u_n\}$ at another. We also know that these points are confined to a plane, and thus we have the relation:

$$\lambda\tilde{\mathbf{x}} = \mathbf{H}\tilde{\mathbf{u}}$$

For one set of homogeneous coordinates, we have the following relation:

$$\lambda \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}$$

Writing as equations we have:

$$\begin{aligned} \lambda x_1 &= h_{11}u_1 + h_{12}v_1 + h_{13} \\ \lambda y_1 &= h_{21}u_1 + h_{22}v_1 + h_{23} \\ \lambda &= h_{31}u_1 + h_{32}v_1 + h_{33} \end{aligned}$$

Now, to obtain x_1, and y_1, we normalize by λ as below:

$$\begin{aligned} x_1 &= \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + h_{33}} \\ &\implies h_{31}u_1x_1 + h_{32}v_1x_1 + h_{33}x_1 - h_{11}u_1 - h_{12}v_1 - h_{13} = 0 \\ y_1 &= \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + h_{33}} \\ &\implies h_{31}u_1y_1 + h_{32}v_1y_1 + h_{33}y_1 - h_{21}u_1 - h_{22}v_1 - h_{23} = 0 \end{aligned}$$

This can be written in matrix form as follows:

$$\overset{A}{\begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1x_1 & v_1x_1 & x_1 \\ 0 & 0 & 0 & -u_2 & -v_2 & -1 & u_1y_1 & v_1y_1 & y_1 \end{bmatrix}} \overset{h}{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}} = 0$$

We can then stack other instances of N-1 remaining points to form a matrix $\mathbf{A}$ with size 2Nx9 and solve for $\mathbf{h}$ in a least squares sense to obtain $\mathbf{h}$. Therefore we have $\mathbf{Ah} = 0$ as required

2. How many elements are there in $\mathbf{h}$?
   There are a total of 9 elements in $\mathbf{h}$ as shown in 1.

$$\mathbf{h} = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}$$

3. How many point pairs (correspondences) are required to solve this system? Hint: How many degrees of freedom are in H? How much information does each point correspondence give?

   We require a total of 4 pairs of points to solve this system.

   Even though there are 9 variables in $\mathbf{h}$, we have 8 degrees of freedom when we normalize by h_{33} owing to scale ambiguity in the planar homography. As seen in part 1, every pair of corresponding points, generates 2 equations, therefore we need 4 points to obtain 8 linear equations to solve the system.

4. Show how to estimate the elements in h to find a solution to minimize this homogeneous linear least squares system. Step us through this procedure. Hint: Use the Rayleigh quotient theorem (homogeneous least squares)

   We know that for this problem $\mathbf{h}$ needs to be in the null-space of A. This can be solved in the least squares fashion, by finding the eigenvector of A, corresponding to the last (smallest) eigenvalue of $\mathbf{A}$.

   We could find a least squares solution to the system as follows:

$$\mathbf{Ah} = 0$$

   We use the Rayleigh Quotient theorem [1] to obtain the least squares solution to the given system. The Rayleigh Quotient is defined as:

$$r(x) = \frac{\mathbf{x}^\top \mathbf{A} \mathbf{x}}{\mathbf{x}^\top \mathbf{x}}$$

A key property of the Rayleigh quotient is that, if $\mathbf{x}$ is the eigenvector of $\mathbf{A}$ then $r(x)$ gives the corresponding eigenvalue. Also for $\|\mathbf{x}\|_2 = 1$, as is the case with the required vector $\mathbf{h}$ we have:

$$r(x) = \mathbf{h}^\top \mathbf{A}^\top \mathbf{A}\mathbf{h}$$

The solution of the system reduces now to minimizing $r(x)$ that is:

$$h = \underset{h}{\operatorname{argmin}} \left(\mathbf{h}^\top \mathbf{A}^\top \mathbf{A}\mathbf{h}\right)$$

Taking the SVD of $\mathbf{A}$ to obtain the orthonormal basis, we have:

$$\begin{aligned}
h &= \underset{h}{\operatorname{argmin}} \left(\mathbf{h}^\top (\mathbf{U\Sigma V}^\top)^\top (\mathbf{U\Sigma V}^\top)\mathbf{h}\right) \\
&= \underset{h}{\operatorname{argmin}} \left(\mathbf{h}^\top (\mathbf{V\Sigma U}^\top)(\mathbf{U\Sigma V}^\top)\mathbf{h}\right) \\
&= \underset{h}{\operatorname{argmin}} \left((\mathbf{h}^\top \mathbf{V})\mathbf{\Sigma^2}(\mathbf{V}^\top \mathbf{h})\right)
\end{aligned}$$

Taking $\mathbf{M} = \mathbf{V}^\top \mathbf{h}$ we have

$$M = \underset{M}{\operatorname{argmin}} \left(\mathbf{M}^\top \mathbf{\Sigma^2} \mathbf{M}\right)$$

Here we will obtain $\mathbf{h} = \mathbf{VM}$, and the row vector $\|\mathbf{M}\|_2 = 1$, since $\mathbf{V}^\top$ is orthonormal and $\|\mathbf{h}\| = 1$ Now we have:

$$\begin{aligned}
M &= \underset{M}{\operatorname{argmin}} \left(\mathbf{M}^\top \begin{pmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_9^2 \end{pmatrix} \mathbf{M}\right) \\
&= \underset{M}{\operatorname{argmin}} \left(\sum_{i=0}^{9} \sigma_i^2 M_i^2\right)
\end{aligned}$$

Now since we have $\sigma_1 \geq \sigma_2 \geq \ldots \sigma_9$, we know that:

$$\begin{aligned}
\sum_{i=0}^{9} \sigma_i^2 M_i^2 &\geq \sum_{i=0}^{9} \sigma_9^2 M_i^2 \\
&\geq \sigma_9^2 \sum_{i=0}^{9} M_i^2
\end{aligned}$$

Since we require $\|\mathbf{M}\|_2 = 1$, we can observe that setting $\mathbf{M} = [0 \ldots 0, 1]^\top$ minimizes $\mathbf{M}$ Since we know that $\mathbf{h} = \mathbf{VM}$, and M is of the above form. $\mathbf{h} =$ **last column of V**

Q6.1

Visualize the warped image (similar to Figure 5(c)) and the generated panorama and include both in the writeup and save only H2to1 as results/q6 1.npy using Numpy's np.save() function.

Answer:

Figure 10: `incline_r.jpg` image warped according to the homography obtained after RANSAC

Figure 11: Panorama generated by overlapping `incline_l.png` on warped `incline_r.png`

Q6.2

Visualize the resulting panaroma with adjusted size and blending and include it in the writeup.

Answer:

Figure 12: Panorama generated from `incline_l.png` and `inline_r.png` after blending the images and projecting them to a common frame of reference

Q6.3

Run your code on the image pair data/incline L.jpg, data/incline R.jpg. However during debugging, try on scaled down versions of the images to keep running time low. Visualize the resulting panaroma on the full sized images and include it in the writeup. Answer: Output

of Q6.3 is the same as Q6.2 and is visualized as below:

Figure 13: Panorama generated from `incline_l.png` and `inline_r.png` after blending the images and projecting them to a common frame of reference

Q7.2

Use matplotlib's plot function to display the points of the sphere in yellow. For this question disregard self-occlusion (i.e. points on the projected sphere occluding other points). Visualize the image and include it in your written response.

Answer:

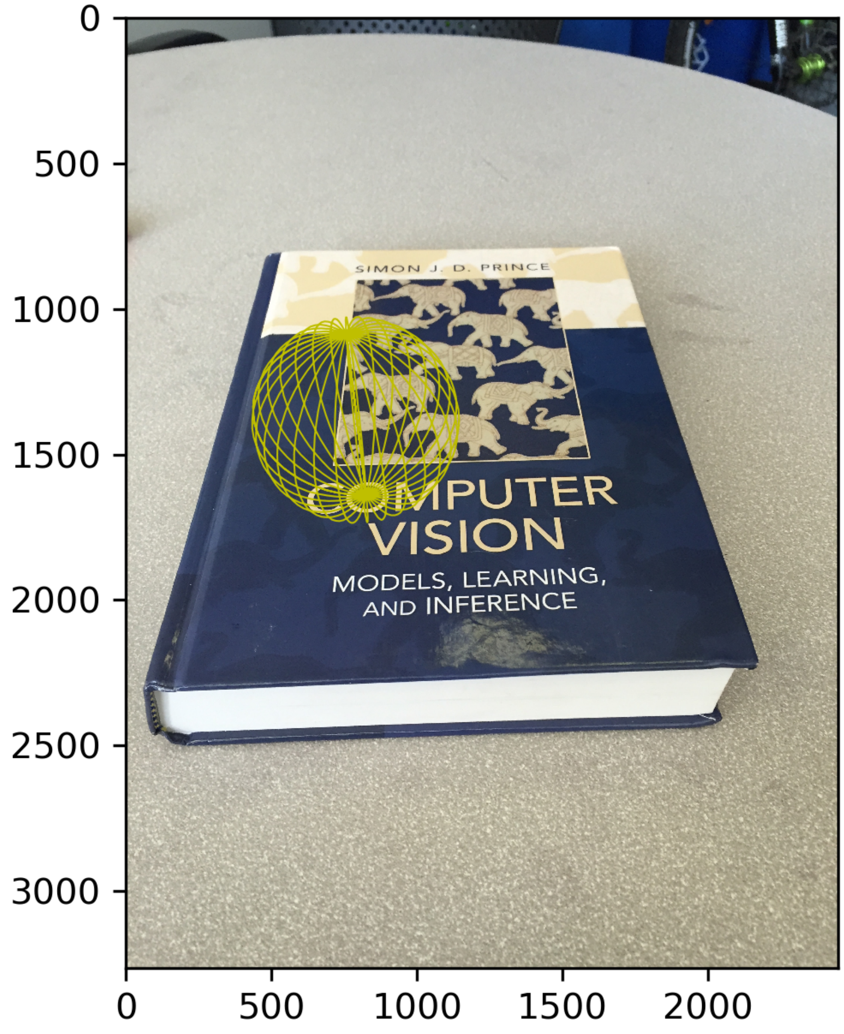


Figure 14: 2D projection of the Sphere 3D coordinates using the Rotation and translation estimated from the given planar point correspondences

Discussion citation

For this homework, I have discussed with *Tarasha Khurana* andrewID: `tkhurana@andrew.cmu.edu`, *Viraj Parimi* andrewID: `vparimi@andrew.cmu.edu` and *Rohit Jena* andrewID: `rjena@andrew.cmu.edu`

References

[1] Rayleigh Quotient Theorem,
https://en.wikipedia.org/wiki/Rayleigh_quotient