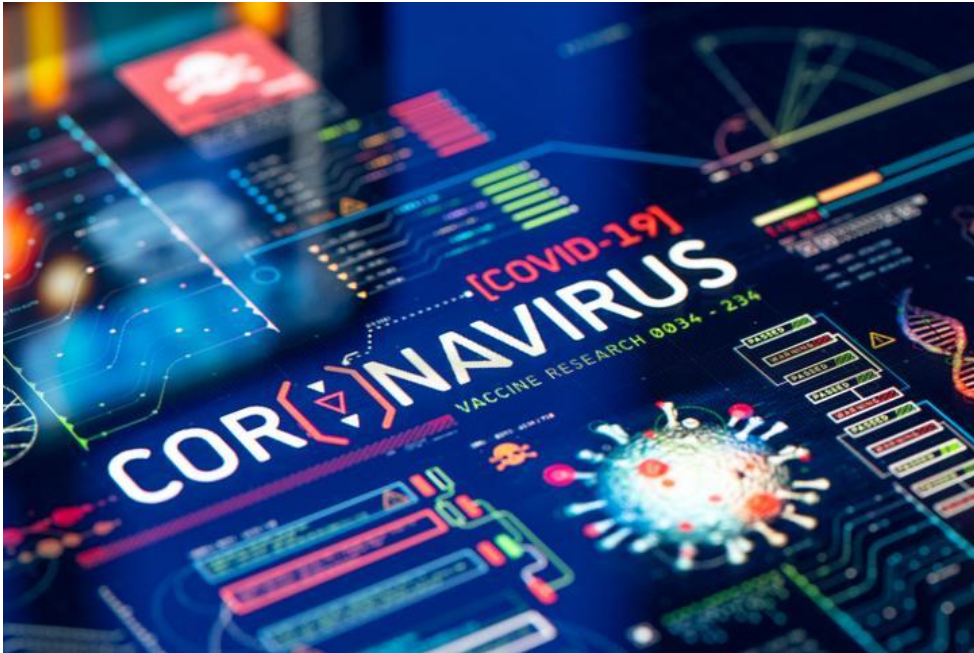# Utilizing Time Series Models to Capture and Forecast Different Markets' Volatility

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Introduction

- Unprecedented changes posted on the financial market due to Covid 19
- Irregular & abnormal volatility
- Different time series model and their adaptability to incorporate the abnormal market behaviors

Project Plan

1. Selecting financial data from different industries & sectors
2. Fitting proper ARMA-GARCH model for different data
3. Validating the model
   a. using tests, Interval coverage and VaR coverage
4. Forecasting future volatility (future studies)
5. Drawing conclusions based on model and forecasts

Data Selection: Sector Funds(ETFs) Log Returns

- Exchange Traded Funds for different sectors:
  - **XLV - Health Care Select Sector SPDR Fund**
  - XLE - Energy Select Sector SPDR Fund,
  - XLF - Finance Select Sector SPDR Fund,
  - XLK - Technology Select Sector SPDR Fund,
  - XLI - Industrial Select Sector SPDR Fund,
- Data Resource: Yahoo Finance
  - https://finance.yahoo.com/quote/XLV?p=XLV&.tsrc=fin-srch
- Time range: from 2011.04.05 - 2021.04.01 (10 years)

Model Selection

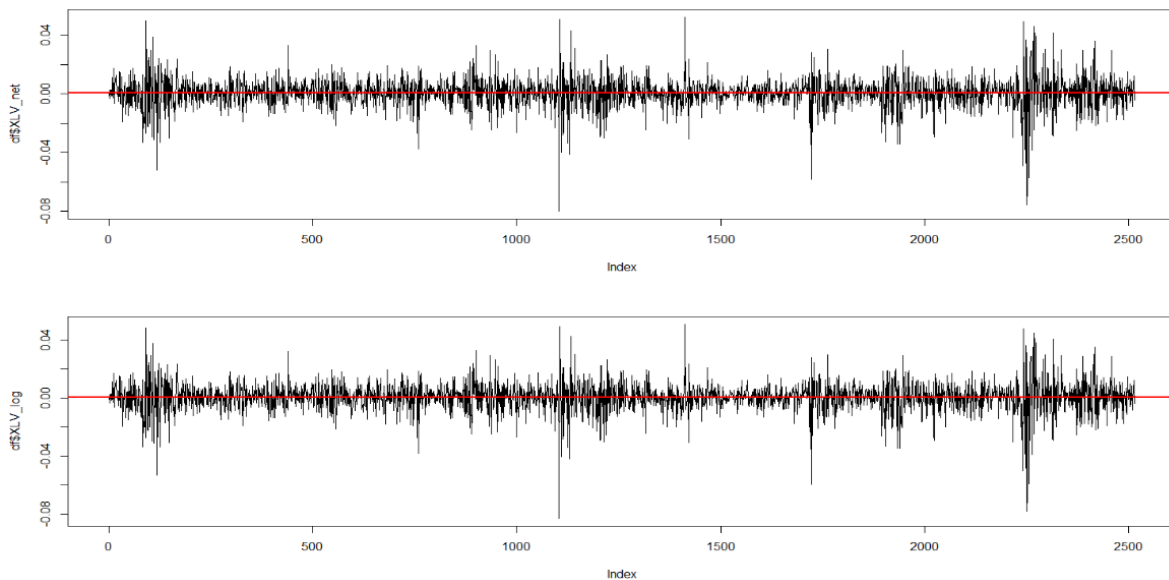Health Care Select Sector SPDR Fund
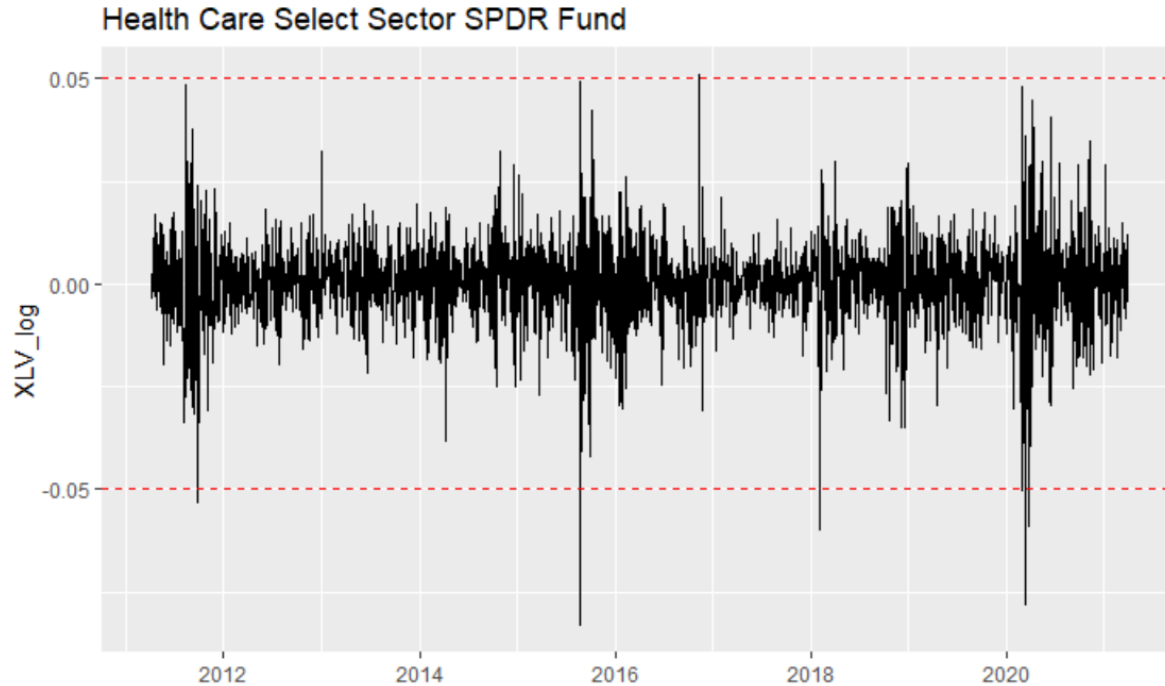
Plot log returns:

- Having irregular pattern of variation
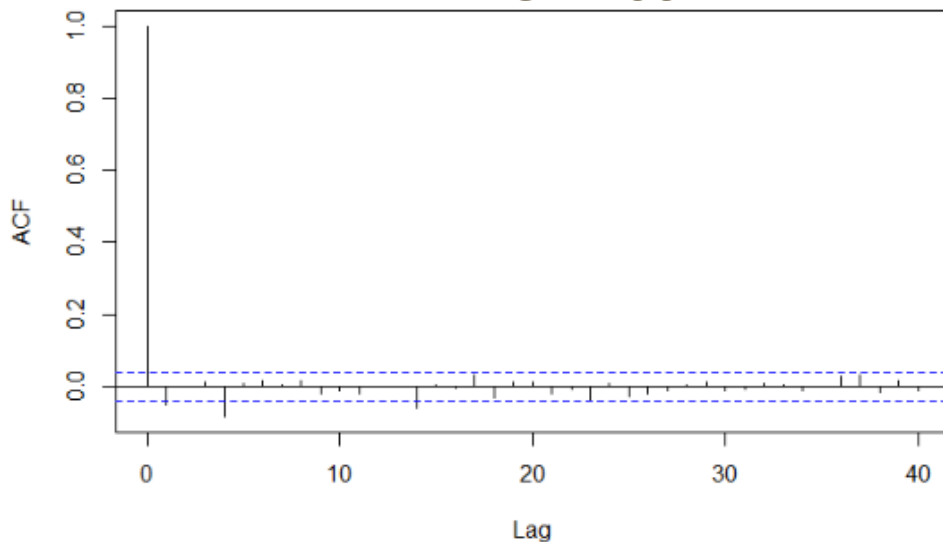
Model Selection

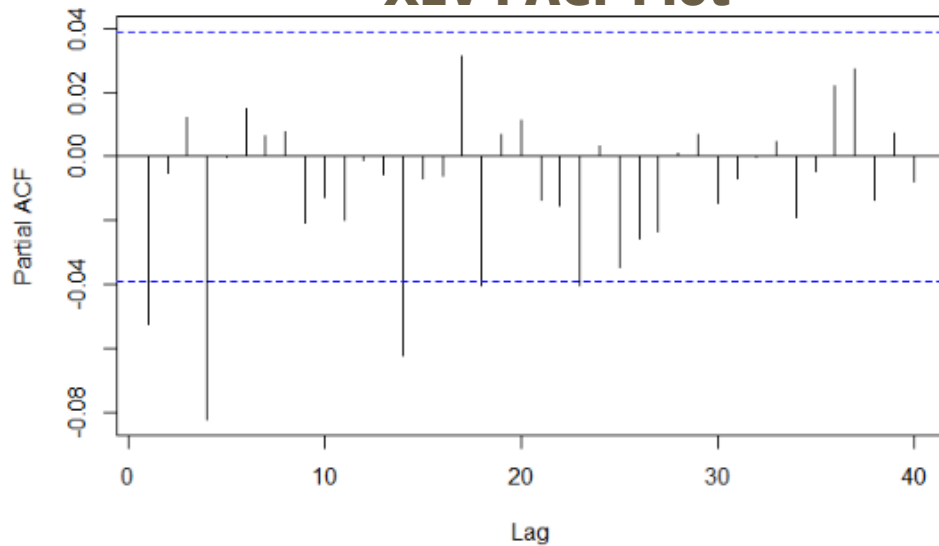XLV Net Return versus Log Return Plot

- Net return and log return have similar distribution patterns.

- Choose log return since it has a well-defined domain while net return is lower bounded by -1, which may cause trouble.

Model Selection: ARIMA Model

XLV ACF Plot

XLV PACF Plot

- Find possible orders for AR & MA models
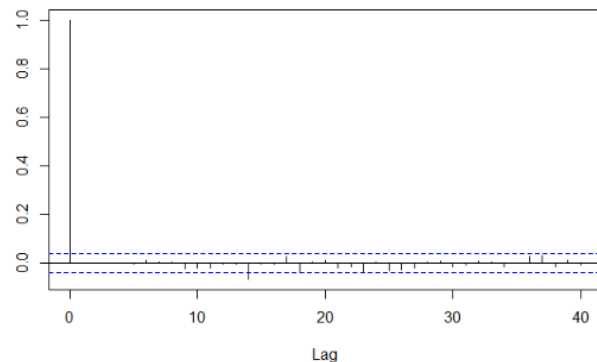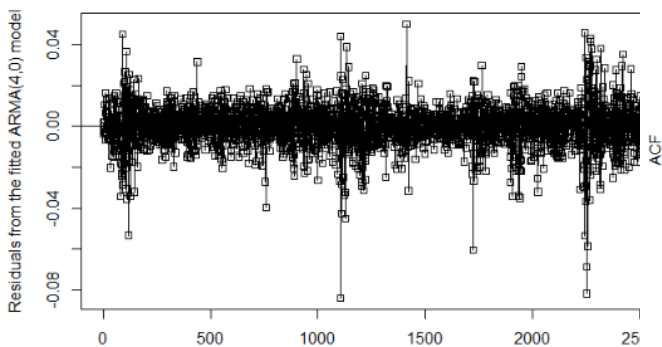
MA(1) or MA(4); AR(1), AR(4) or AR(14)

- Fit different combinations of p、q values and then choose the one with minimal AIC

Best Model: ARIMA(4,0,0)

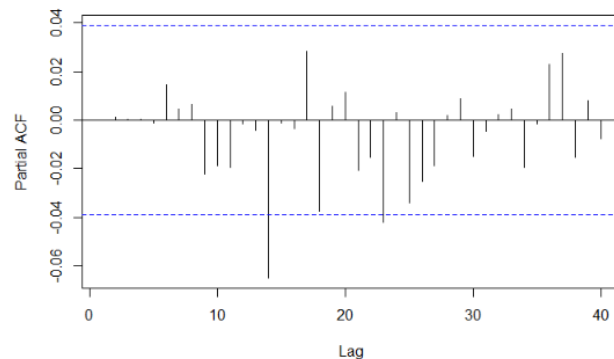# Model Selection: ARIMA Model

- Fitting ARIMA(4,0,0) model to mean-corrected data

- Gain residuals from the fitted ARIMA(4,0,0) model

- Check if the fitted model is appropriate

    1. Test for Randomness:

```
            Box-Pierce test

data:  y.res_XLV
X-squared = 19.989, df = 20, p-value = 0.4586


            Box-Ljung test

data:  y.res_XLV
X-squared = 20.119, df = 20, p-value = 0.4505
```

Model Selection: ARIMA Model

2. Test for Normality:

Normal Q-Q Plot for XLV
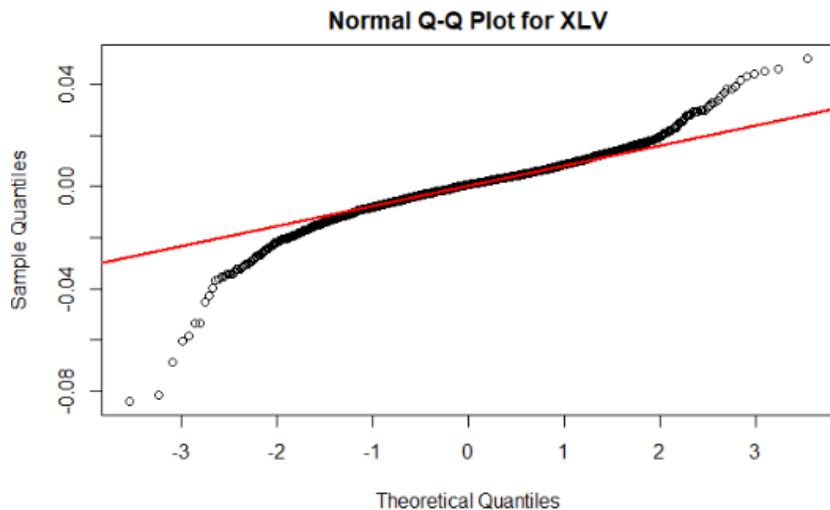

```
          Shapiro-Wilk normality test

data:  y.res_XLV
W = 0.93367, p-value < 2.2e-16


          Jarque Bera Test

data:  y.res_XLV
X-squared = 5021.1, df = 2, p-value < 2.2e-16
```

Satisfy Randomness Assumption
Not Satisfy Normality Assumption

Model Selection: ARIMA Model

- Summary

Funds Sector	Best Model	Test for Randomness (p-value)		Test for Normality (p-value)	
		Box-Pierce Test	Box-Ljung Test	Shapiro-Wilk Normality Test	Jarque Bera Test
XLV	ARIMA(4,0,0)	0.4586	0.4505	< 2.2e-16	< 2.2e-16
XLI	ARIMA(10,0,4)	0.7135	0.7066	< 2.2e-16	< 2.2e-16
XLK	ARIMA(4,0,0)	0.7114	0.705	< 2.2e-16	< 2.2e-16
XLF	ARIMA(4,0,4)	0.4993	0.4916	< 2.2e-16	< 2.2e-16
XLE	ARIMA(5,0,4)	0.6507	0.6435	< 2.2e-16	< 2.2e-16

Model Selection: Distribution Model

→ Check the error term normality using histogram
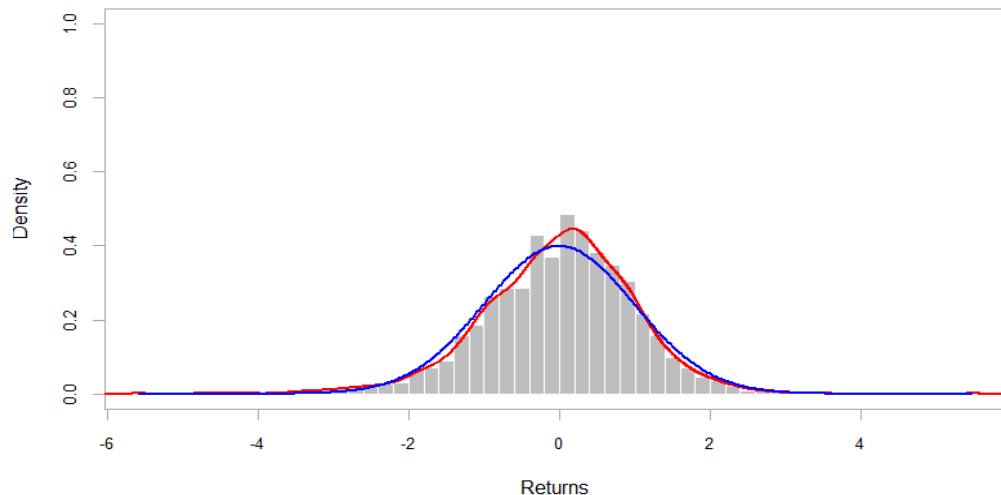
This is the histogram that we use for XLV standardized residuals. From the graph, we can see the red line (i.e. std residuals) is pretty close to normal distribution, which is shown in blue, except for nuances in peak and tail. So for the XLV dataset, we choose t distribution for further analysis.

Model Selection: Variance Model (sGARCH v.s. gjrGARCH)

→ Use information criteria (AIC and BIC)

|	Standard GARCH (normalized)	GJR GARCH (normalized)
AIC	-6.6314	-6.5894
BIC	-6.6105	-6.5708

→ GJR GARCH model

$$R_t = \sigma_t \varepsilon_t, \ \varepsilon_t \sim N(0,1)$$
$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})R_{t-1}^2 + \beta \sigma_{t-1}^2,$$
$$I_{t-1} = \begin{cases} 0 \text{ if } R_{t-1} \geq 0, & \text{(good news)} \\ 1 \text{ if } R_{t-1} < 0. & \text{(bad news)} \end{cases}$$

For XLV data, standard GARCH model gives us smaller IC values, so we choose standard GARCH for further analysis. However, after doing the same process for other data, we realized GJR model is relatively more proper to use for them based on the results we got from information criteria analysis.

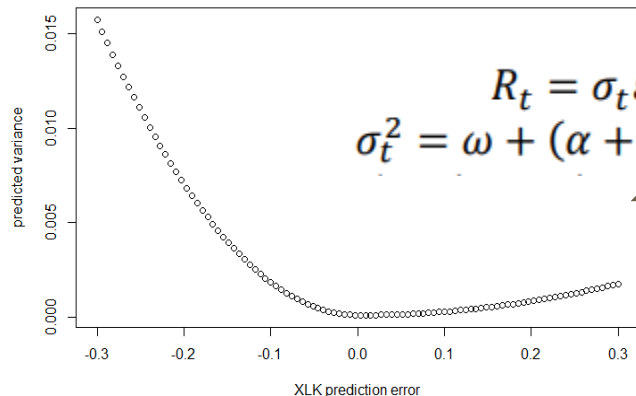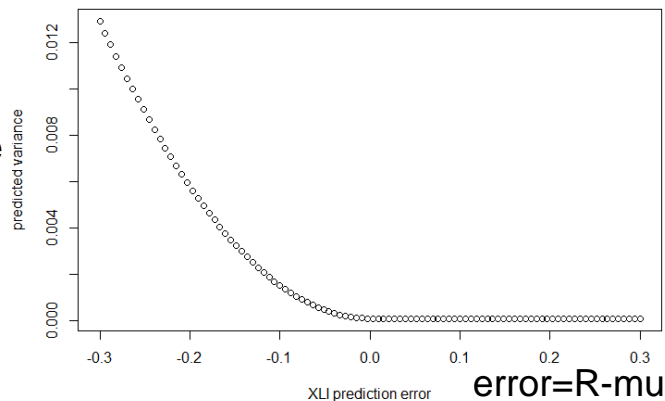# Model Selection: Variance Model (Leverage Effect)

Variance

error=R-mu

$$R_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$
$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})R_{t-1}^2 + \beta \sigma_{t-1}^2,$$
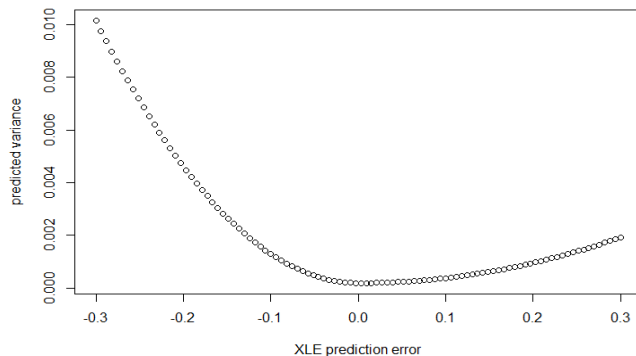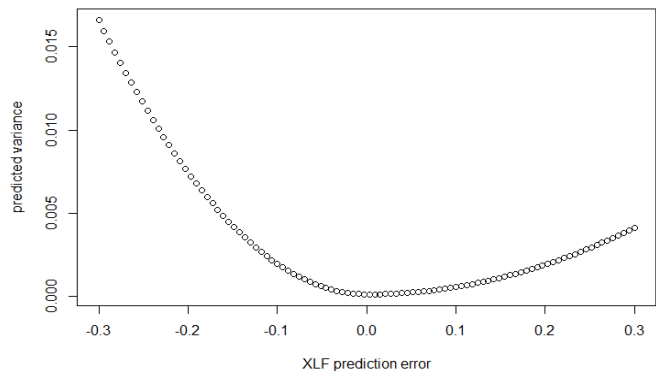
This may indicate that the leverage effect is prevalent in financial returns, or gamma is statistically significant in most of the stock data.

Model Selection: Summary

The GARCH specifications for sector funds data are as follows:

	ARMA (p,q)	Variance model	Distribution model
XLV	p=4, q=0	s-GARCH	T distribution
XLI	p=10, q=4	GJR-GARCH	T distribution
XLK	p=4, q=0	GJR-GARCH	Skewed t distribution
XLF	p=4, q=4	GJR-GARCH	Skewed t distribution
XLE	p=5, q=4	GJR-GARCH	T distribution

Model Validation: Autocorrelation

→ Autocorrelation: Ljung-Box Test

```
Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------
                                statistic p-value
Lag[1]                              0.297  0.5858
Lag[2*(p+q)+(p+q)-1][11]            2.292  1.0000
Lag[4*(p+q)+(p+q)-1][19]            6.721  0.9307
d.o.f=4
H0 : No serial correlation
```

For XLV returns, p-values: fail to reject H0.

No serial correlation in this model's residuals.

Does not exhibit significant lack of fit

For other data, also find no serial correlation.

Model Validation: ARCH Effect

Check for possible conditional heteroscedasticity

→ ARCH LM Test

```
Weighted ARCH LM Tests
------------------------------------------
              Statistic Shape Scale P-Value
ARCH Lag[3]     0.04391 0.500 2.000  0.8340
ARCH Lag[5]     0.87242 1.440 1.667  0.7711
ARCH Lag[7]     1.11197 2.315 1.543  0.8947
```

For XLV returns, p-values: fail to reject H0.

- There is no ARCH effect in this model.

- For other data, there is no ARCH effect in their models.

Model Validation: Backtesting (CI Coverage)

→ Use first 1948 samples (8-year data) to estimate the GARCH model on rolling basis (refit the model after each day)

	Mu <dbl>	Sigma <dbl>	Skew <dbl>	Shape <dbl>	Shape(GIG) <dbl>	Realized <dbl>
2019-01-02	0.0014239575	0.01848623	0	0	0	-0.006415525
2019-01-03	0.0004919056	0.01733767	0	0	0	0.001643578
2019-01-04	-0.0001436210	0.01604103	0	0	0	-0.010196941

→ Use the estimated mean and standard deviation to fit 95% confidence intervals for the fund returns beginning from 2019-01-02:
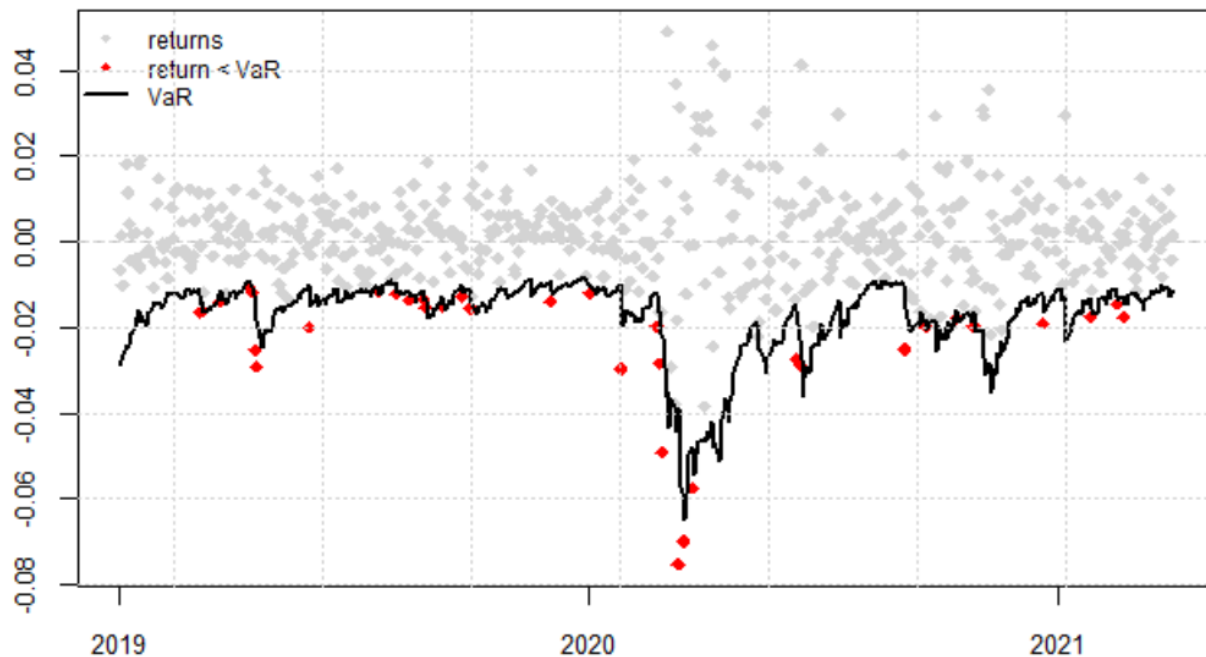
(plot attached on the next page)

CIs for predicted returns
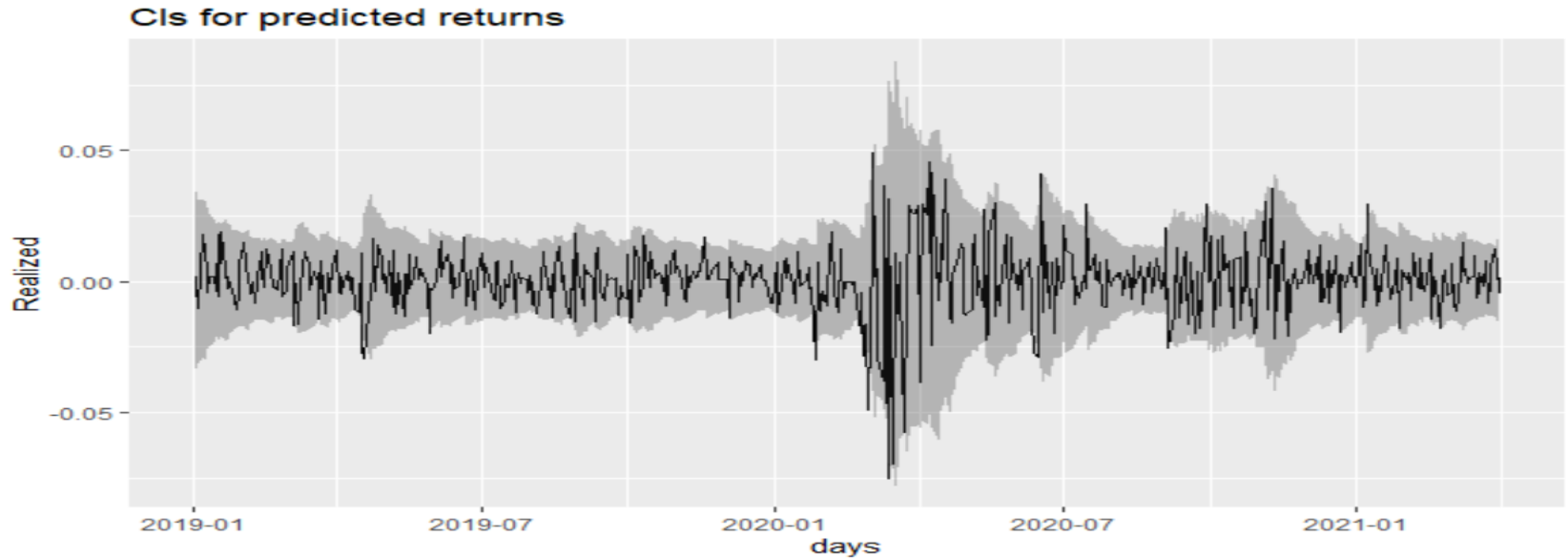
→ Calculate the coverage rate (percentage of realized values that fall into the 95% CI) for each sectors

Sector Funds	XLV	XLE	XLF	XLK	XLI
CI Coverage	0.9470	0.9531	0.9428	0.9349	0.9329

Model Validation: Backtesting (VaR Coverage)

For XLV returns, under 5% alpha

the the frequency of actual return being less than predicted VaR is 6.17%
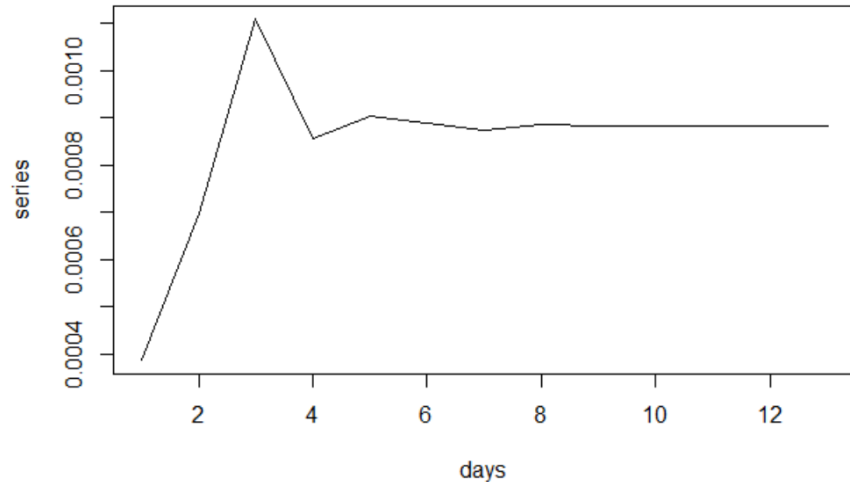
Model Validation: Backtesting (VaR Coverage)

For other sectors' data, the VaR coverage under 5% alpha is:

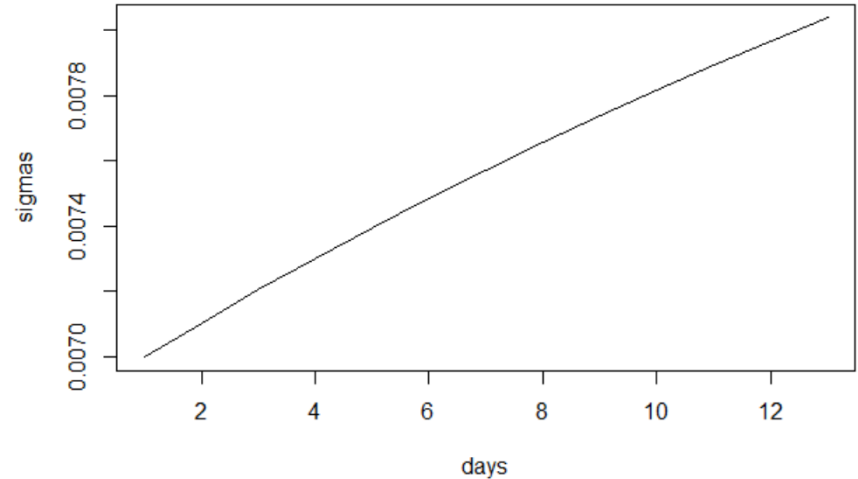|	XLV	XLI	XLK	XLF	XLE
VaR Coverage	6.17%	5.29%	5.84%	5.52%	5.12%

All the coverage ratios are quite close to 5%, so we conclude that the models are valid.

Out-of-Sample Forecast: for the Next 13 Days in XLV Fund

Returns

Volatility


```
# GARCH forecast
vgarchfit <- ugarchfit(data=xlv, spec=vgarchspec, out.sample = 13)
vgarchforecast <- ugarchforecast(fitORspec=vgarchfit, n.ahead=13, n.roll=13)
```

Conclusions

1.We choose different sector funds to see how COVID have posted significant market volatility and the garch model captures the sudden changes in the market well;

2.ARMA-GARCH model is better to use for our data of interest, different datasets use different ARMA order;

3.GJR-GARCH captures leverage effect and is a better choice for financial returns;

4.For the forecast, the volatility will increase as time passes, and the returns tend to be steady after 6 days.

Potential Next Steps

- Investigate the forecasting feature of garch model
- Exclude extreme values (outliers) in 2020 in prediction
- Construct detailed analysis on the market behaviors according to specific industry features
- Conclude on overall findings, especially for the comparisons between different sectors
- Try on potential machine learning models, such as LSTM for data forecasting

Thank You