自动控制原理上机

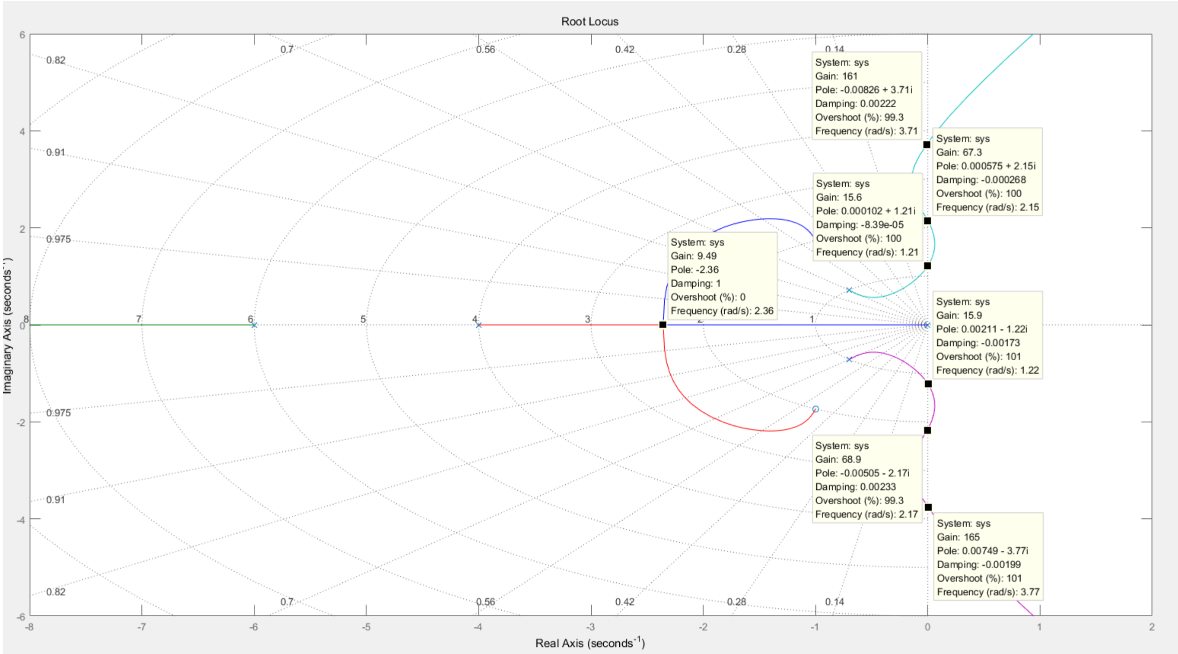
根轨迹分析和Bode图分析

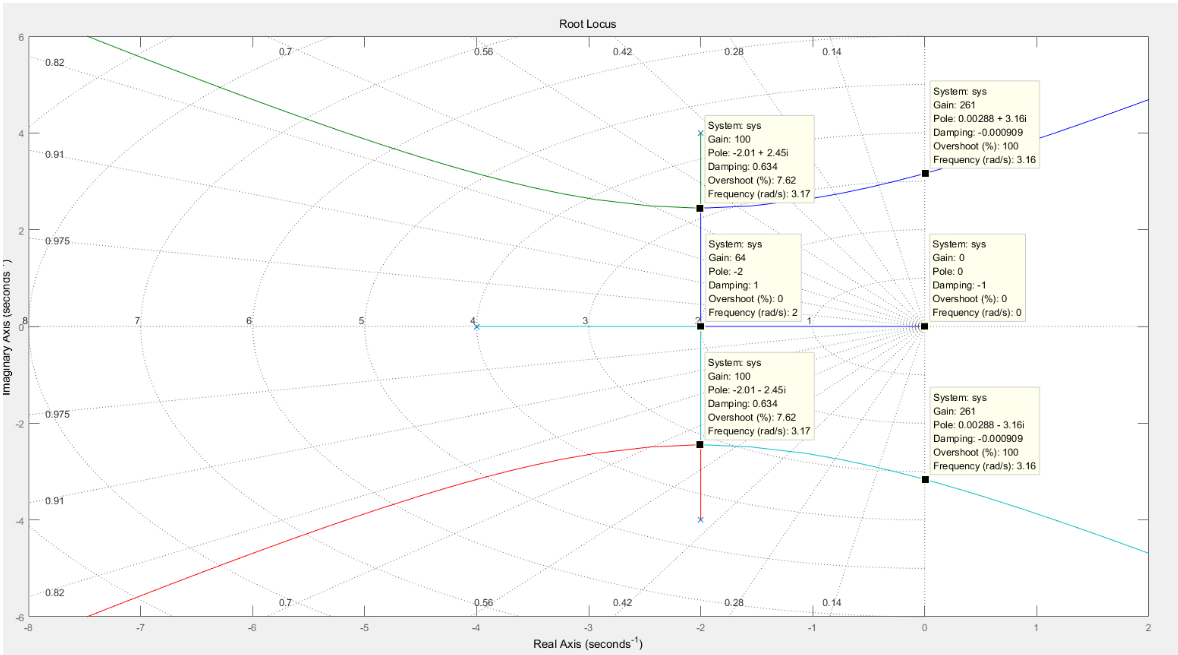
一、基于根轨迹的性能分析

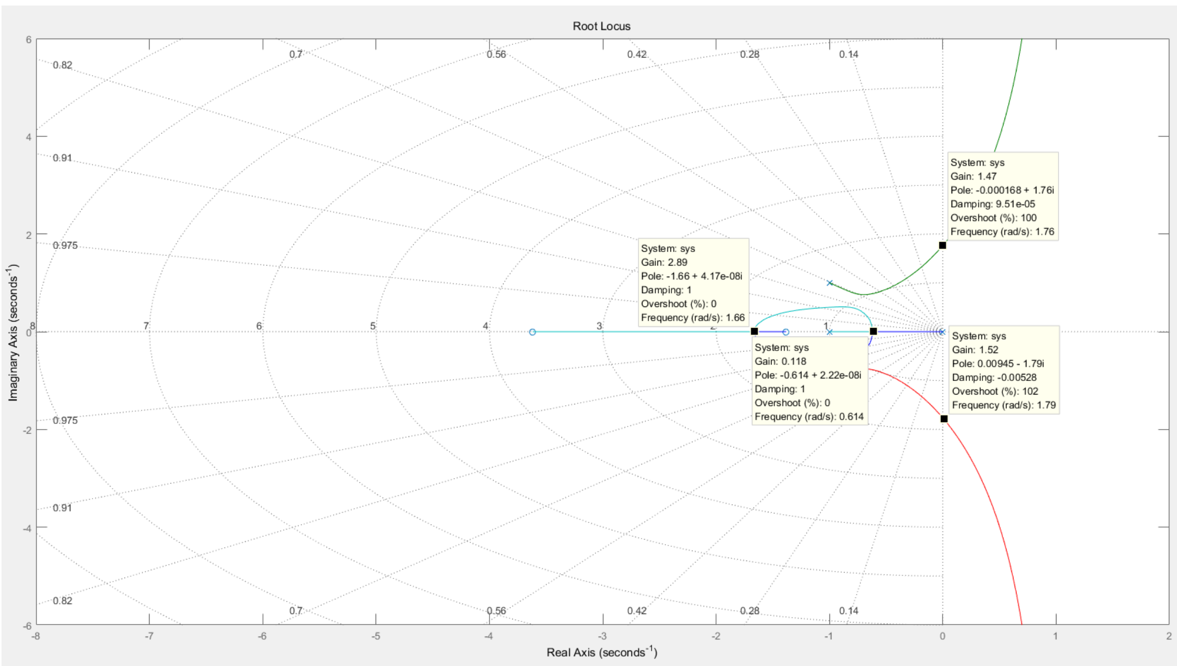
1. 对开环传递函数 $G(s)$、$G_1(s)$和 $G_2(s)$分别画出关于根轨迹增益 k 的闭环根轨迹图，给出根轨迹的分离点、与虚轴的交点，给出使闭环系统稳定的参数 k 的范围。
2. 对开环传递函数 $G(s)$、$G_1(s)$和 $G_2(s)$，借助等阻尼比射线，找出使闭环主导极点的阻尼比在 0.3～0.8 之间的某一根轨迹增益，画出在该增益下单位反馈闭环系统的阶跃响应。比较从阶跃响应上得到超调与从根轨迹信息框里的超调，进而给出简单的结论。
3. 对开环传递函数 $G_3(s)$画出不同零点时的根轨迹，并与不含零点时的根轨迹进行比较，给出简单的结论。

问题1

```
clc
clear all
close all
num=[1 2 4];
den=conv(conv([1 4 0],[1 6]),[1 1.4 1]);
sys=tf(num,den);
figure(1);
rlocus(sys)
grid
%编写限制根轨迹实轴和虚轴范围的 m 文件
axis([-8 2 -6 6])
%确定某闭环极点对应的开环增益，以及该增益对应的其它闭环极点
%[k,p]=rlocfind(sys)

num=[1];
den=conv([1 4 0],[1 4 20]);
sys=tf(num,den);
figure(3);
rlocus(sys)
grid
%编写限制根轨迹实轴和虚轴范围的 m 文件
axis([-8 2 -6 6])
%确定某闭环极点对应的开环增益，以及该增益对应的其它闭环极点
%[k,p]=rlocfind(sys).

num=[1 5 5];
den=conv([1 1 0],[1 2 2]);
sys=tf(num,den);
figure(5);
rlocus(sys)
grid
axis([-8 2 -6 6])
%确定某闭环极点对应的开环增益，以及该增益对应的其它闭环极点
%[k,p]=rlocfind(sys).

```


Root Locus
System: sys
Gain: 161
Pole: -0.00826 + 3.71i
Damping: 0.00222
Overshoot (%): 99.3
Frequency (rad/s): 3.71
System: sys
Gain: 67.3
Pole: 0.000575 + 2.15i
Damping: -0.000268
Overshoot (%): 100
Frequency (rad/s): 2.15
System: sys
Gain: 15.6
Pole: 0.000102 + 1.21i
Damping: -8.39e-05
Overshoot (%): 100
Frequency (rad/s): 1.21
System: sys
Gain: 9.49
Pole: -2.36
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 2.36
System: sys
Gain: 15.9
Pole: 0.00211 - 1.22i
Damping: -0.00173
Overshoot (%): 101
Frequency (rad/s): 1.22
System: sys
Gain: 68.9
Pole: -0.00505 - 2.17i
Damping: 0.00233
Overshoot (%): 99.3
Frequency (rad/s): 2.17
System: sys
Gain: 165
Pole: 0.00749 - 3.77i
Damping: -0.00199
Overshoot (%): 101
Frequency (rad/s): 3.77
Imaginary Axis (seconds⁻¹)
Real Axis (seconds⁻¹)

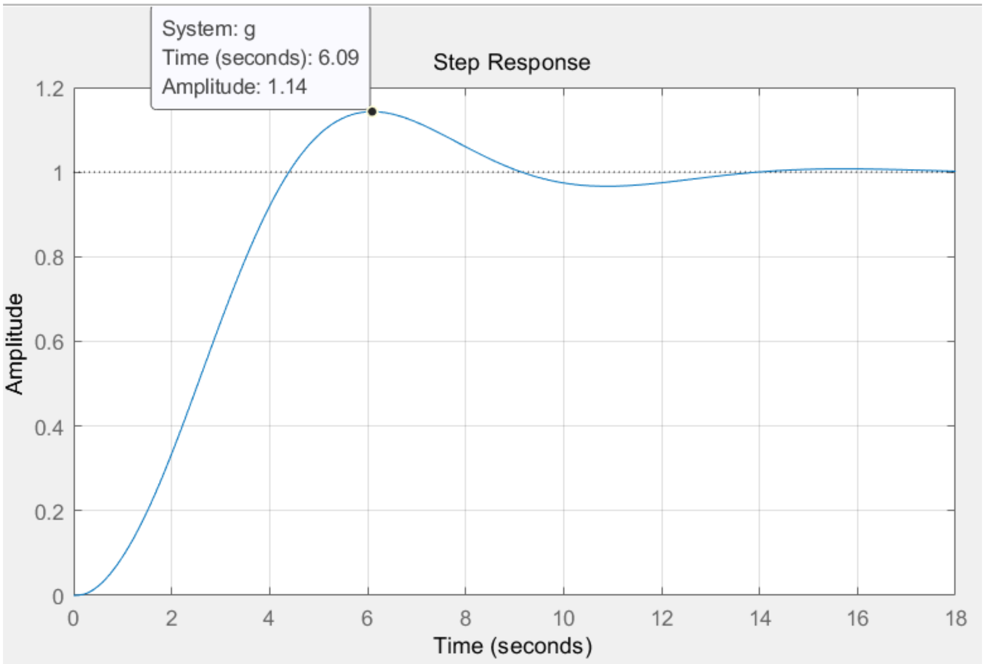

Root Locus
System: sys
Gain: 261
Pole: 0.00288 + 3.16i
Damping: -0.000909
Overshoot (%): 100
Frequency (rad/s): 3.16
System: sys
Gain: 100
Pole: -2.01 + 2.45i
Damping: 0.634
Overshoot (%): 7.62
Frequency (rad/s): 3.17
System: sys
Gain: 64
Pole: -2
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 2
System: sys
Gain: 0
Pole: 0
Damping: -1
Overshoot (%): 0
Frequency (rad/s): 0
System: sys
Gain: 100
Pole: -2.01 - 2.45i
Damping: 0.634
Overshoot (%): 7.62
Frequency (rad/s): 3.17
System: sys
Gain: 261
Pole: 0.00288 - 3.16i
Damping: -0.000909
Overshoot (%): 100
Frequency (rad/s): 3.16
Imaginary Axis (seconds⁻¹)
Real Axis (seconds⁻¹)

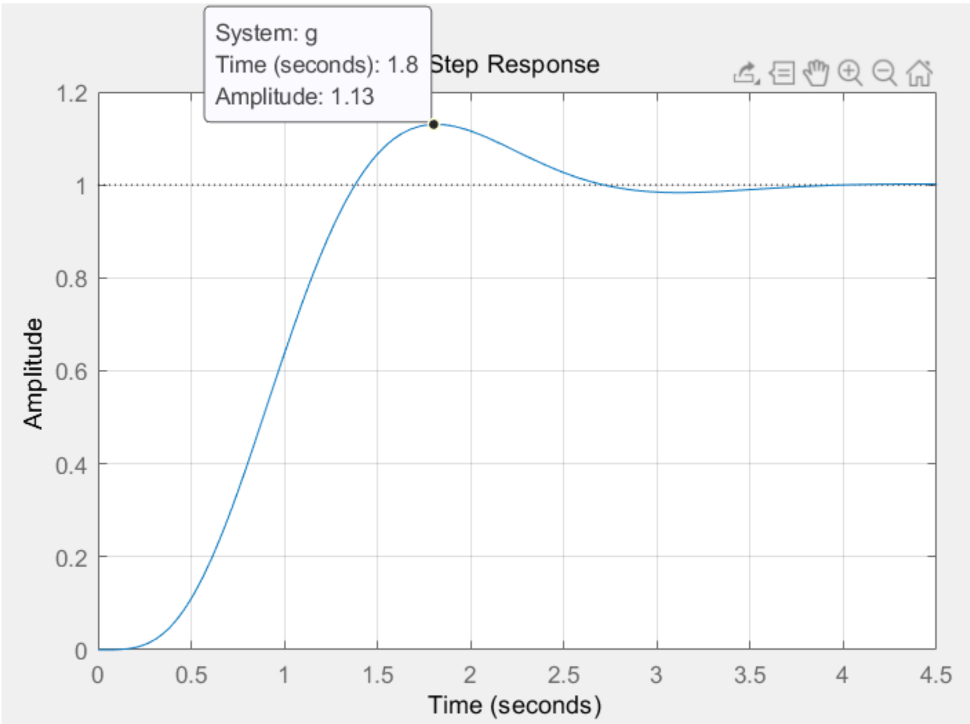

Root Locus
System: sys
Gain: 1.47
Pole: -0.000168 + 1.76i
Damping: 9.51e-05
Overshoot (%): 100
Frequency (rad/s): 1.76
System: sys
Gain: 2.89
Pole: -1.66 + 4.17e-08i
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 1.66
System: sys
Gain: 0.118
Pole: -0.614 + 2.22e-08i
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 0.614
System: sys
Gain: 1.52
Pole: 0.00945 - 1.79i
Damping: -0.00528
Overshoot (%): 102
Frequency (rad/s): 1.79
Imaginary Axis (seconds⁻¹)
Real Axis (seconds⁻¹)

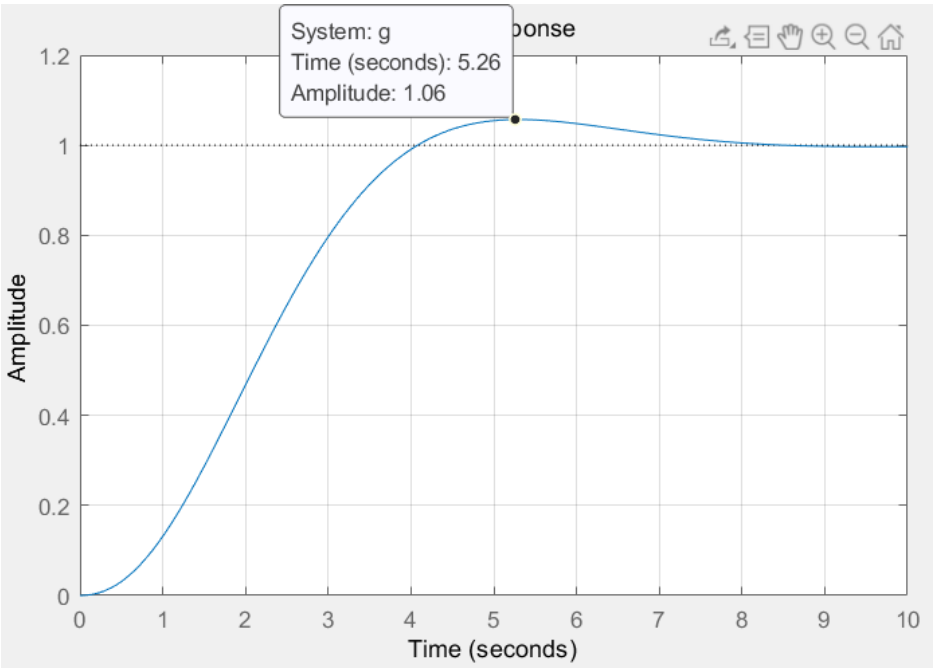
	分离点	与虚轴交点	使闭环系统稳定的k的范围
$G(s)$	$(-2.36, j0)$	$(0,3.71j),(0,2.15j),(0,1.21j),$ $(0,-1.21j),(0,-2.15j),(0,-3.71j)$	$0<k<15.6, 67.9<k<161$
$G_1(s)$	$(-2,j0),$ $(-2,2.45j),$ $(-2,-2.45j)$	$(0,3.16j),(0,0j),(0,-3.16j)$	$0<k<261$
$G_2(s)$	$(-0.614,0j),$ $(-1.66,0j)$	$(0,1.76j),(0,-1.76j)$	$0<k<1.5$

问题2

```
clc
clear all
close all
num1=[1 2 4];
den1=conv(conv([1 4 0],[1 6]),[1 1.4 1]);
sys=tf(num1,den1);
figure(1);
rlocus(sys)
grid
axis([-8 2 -6 6])
%增益K=2.76，超调22.8%,阻尼比：0.426
figure(4)
K=2.76;
numa=K.*num1;
g=feedback(tf(numa,den1),1);
step(g)

num2=[1];
den2=conv([1 4 0],[1 4 20]);
sys=tf(num2,den2);
figure(2);
rlocus(sys)
grid
axis([-8 2 -6 6])
%增益K=102，超调11%
figure(5)
K=102;
numb=K.*num2;
g=feedback(tf(numb,den2),1);
step(g)

num3=[1 5 5];
den3=conv([1 1 0],[1 2 2]);
sys=tf(num3,den3);
figure(3);
rlocus(sys)
grid
axis([-8 2 -6 6])
%增益K=0.201，超调7.58%,阻尼比0.635
figure(6)
K=0.201;
numc=K.*num3;
g=feedback(tf(numc,den3),1);
step(g)
```


System: g
Time (seconds): 6.09
Amplitude: 1.14
Step Response
Amplitude
Time (seconds)


System: g
Time (seconds): 1.8
Amplitude: 1.13
Step Response
Amplitude
Time (seconds)



	根轨迹信息框根轨迹增益	信息框超调量	单位反馈闭环系统的阶跃响应超调量	相对误差
$G(s)$	2.76	22.8%	14%	38.56%
$G_1(s)$	102	11%	13%	18%
$G_2(s)$	0.201	7.58%	6%	22%

- **结论**

  在误差允许范围内，从根轨迹信息框得到的超调和阶跃响应得到的超调量基本一致。造成差异的原因可能是Matlab在做根轨迹分析时，是当作二阶系统来进行，而实际的系统常为更高阶的系统。本身的拟合算法存在一定缺陷。

问题3

```
function rootzero(z)
num=[1 -z];
den=[1 2 2 0];
sys=tf(num,den);
rlocus(sys)
axis([-8 2 -6 6])
```

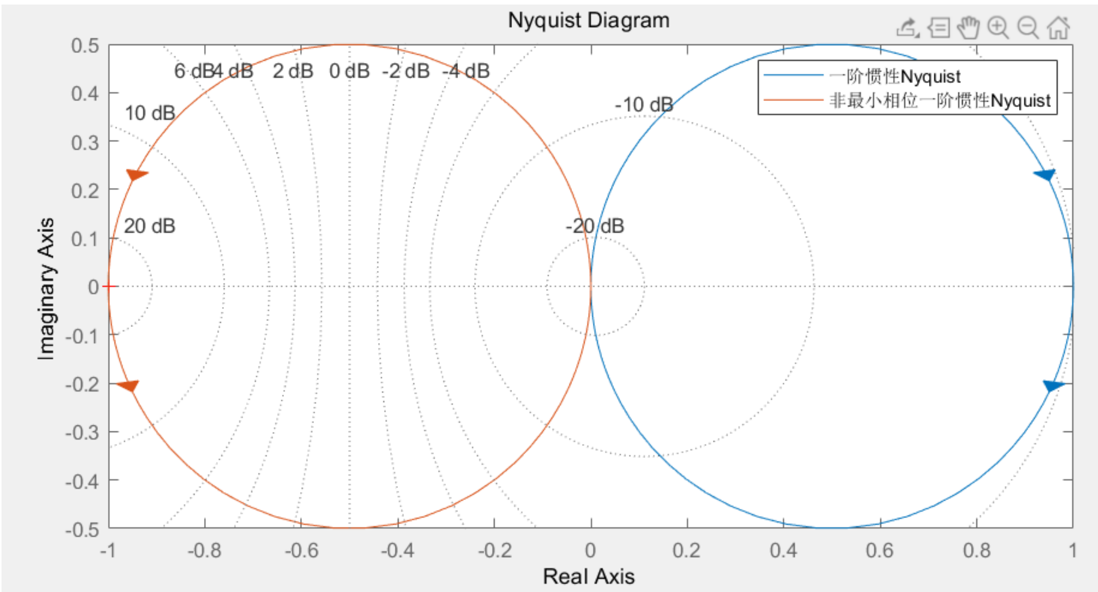
```
figure(1)
subplot(2,3,1);rootzero(-100)
subplot(2,3,2);rootzero(-90)
subplot(2,3,3);rootzero(-80)
subplot(2,3,4);rootzero(-70)
subplot(2,3,5);rootzero(-60)
subplot(2,3,6);rootzero(-50)
figure(2)
subplot(2,3,1);rootzero(-40)
subplot(2,3,2);rootzero(-30)
subplot(2,3,3);rootzero(-20)
subplot(2,3,4);rootzero(-10)
subplot(2,3,5);rootzero(0)
```

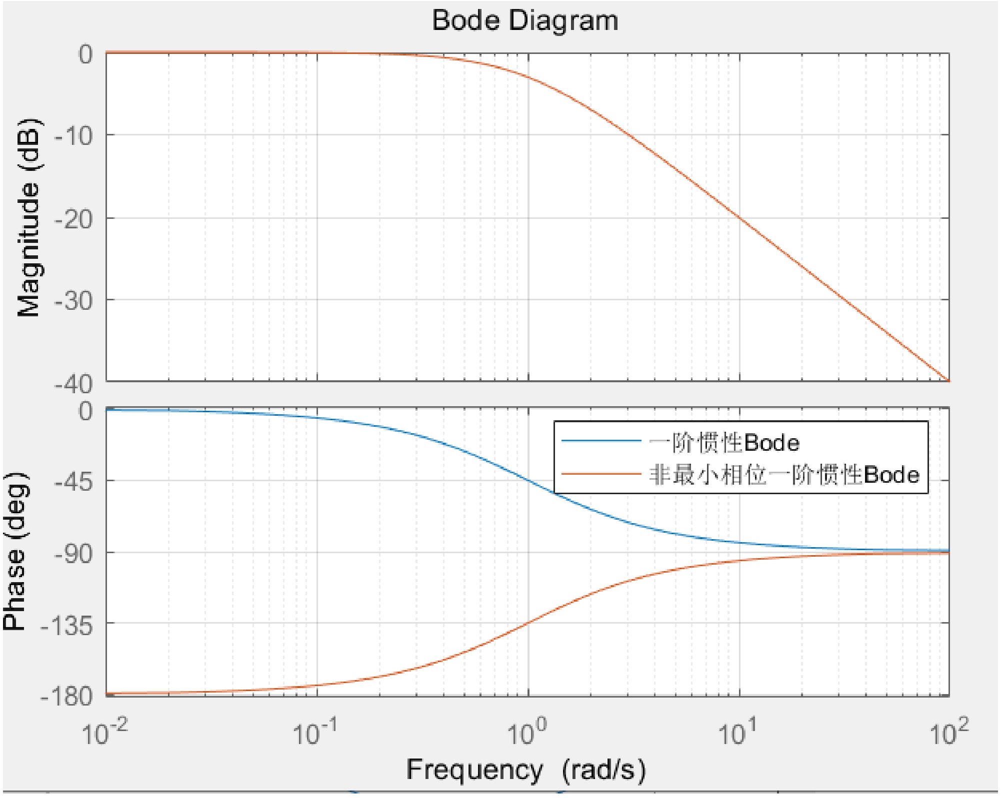
结论：开环零点对根轨迹的绘制有影响。-------------------

二、线性系统的频率特性分析

问题1&2

```
clc
clear all
close all
%Question1
num=[1];
den1=[1 1];
den2=[1 -1];
sys1=tf(num,den1);
sys2=tf(num,den2);
figure(1);
hold on
nyquist(sys1)
nyquist(sys2)
hold off
grid
legend('一阶惯性Nyquist','非最小相位一阶惯性Nyquist');

figure(2);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('一阶惯性Bode','非最小相位一阶惯性Bode');

```

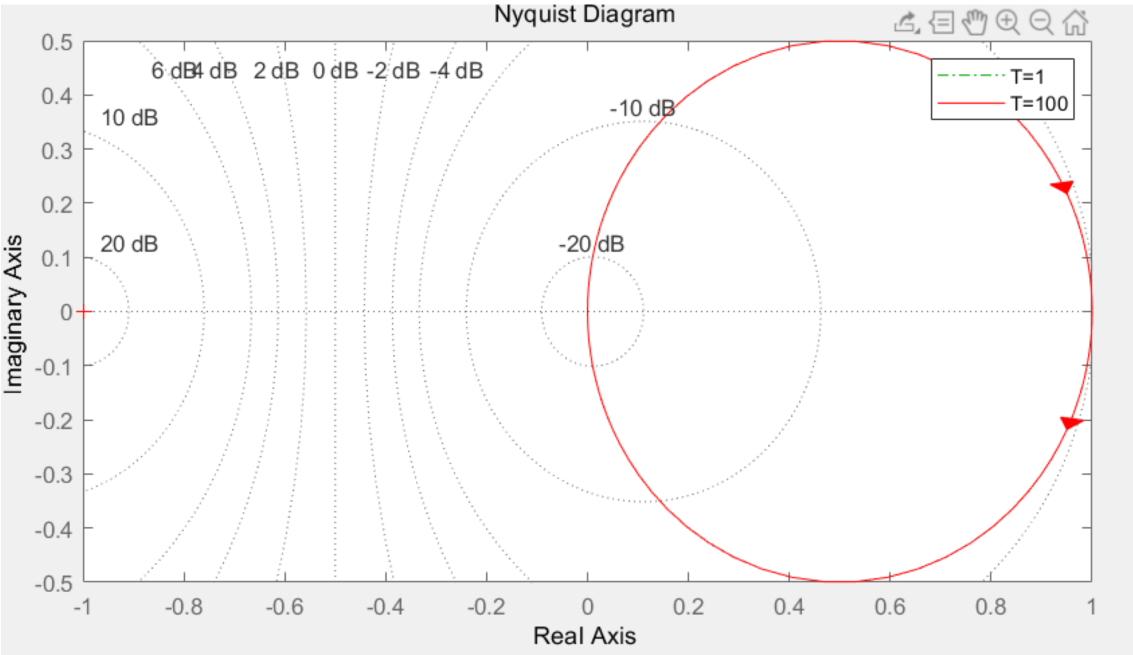


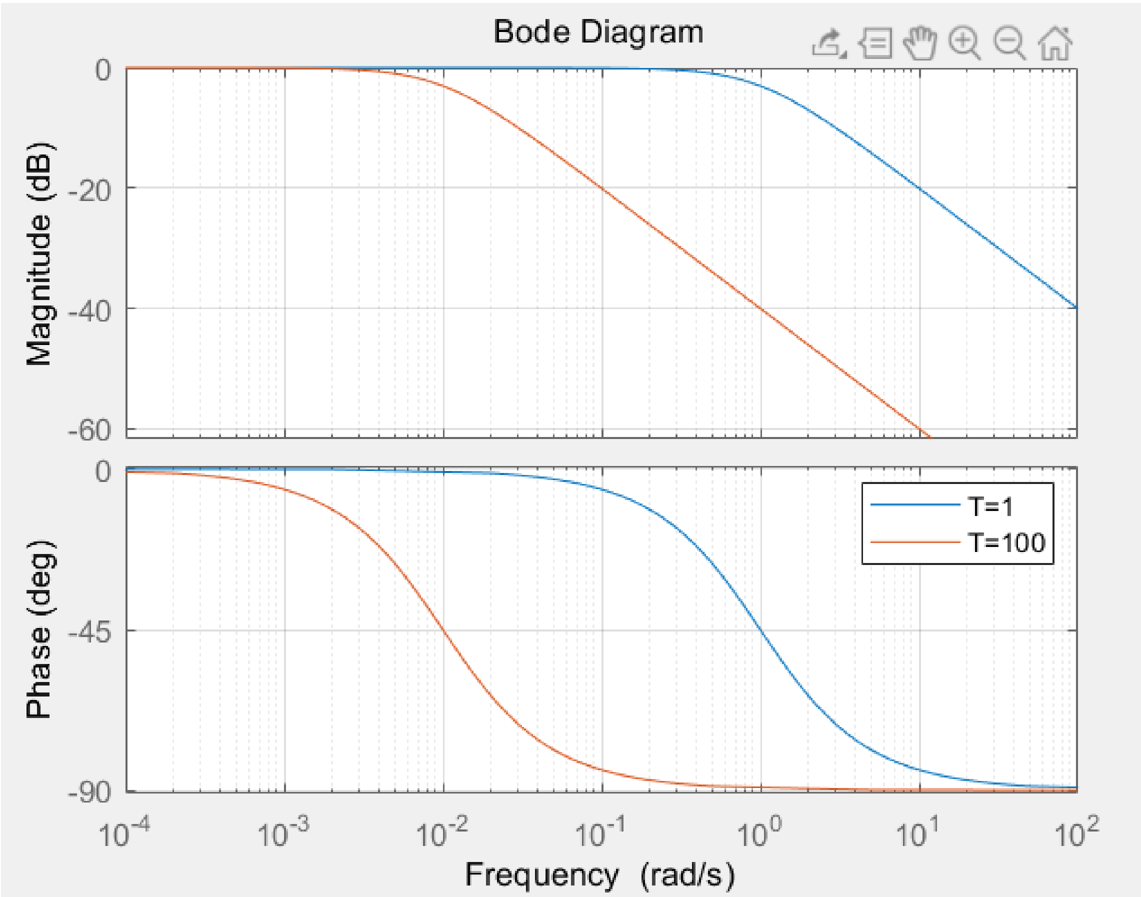


结论：Nyquist图关于虚轴对称。Bode图幅频特性相同，相频特性关于0°线对称

问题3

```
clc
clear all
close all
num=[1];
den1=[1 1];
den2=[100 1];
sys1=tf(num,den1);
sys2=tf(num,den2);
figure(1);
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('T=1','T=100');

figure(2);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('T=1','T=100');
```





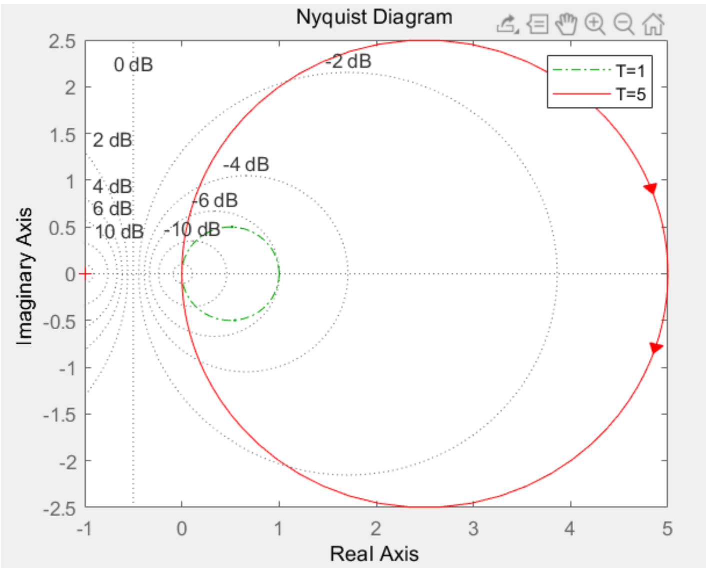
结论：对于惯性环节，选取K=1,T=1和100，得到的Nyquist曲线重合。这是因为惯性环节 $\frac{K}{1+j\omega T}$ 的幅相特性是以（$\frac{K}{2},j0$）为圆心的半圆。复平面上相同的点，对于不同的T，有不同的 ω 值。

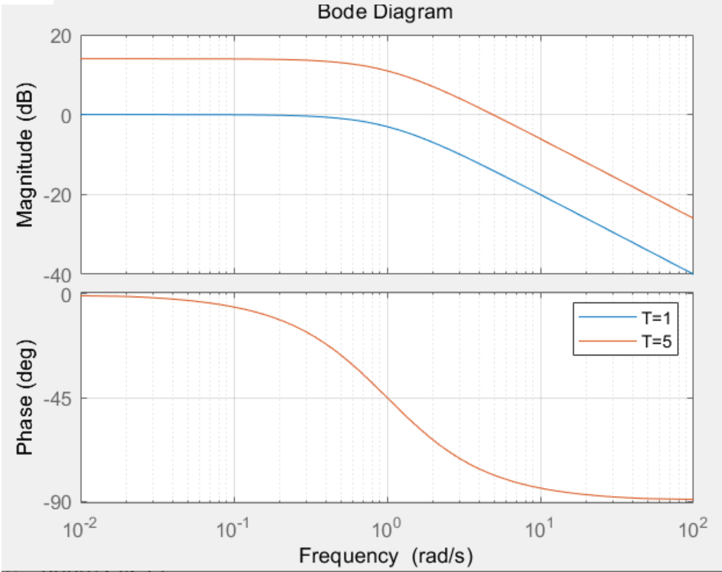
惯性环节转折频率 $\omega=\frac{1}{T}$，故T=100的幅频特性转折频率小。两系统的幅频特性斜率相同，相频特性有差异。

问题4

```
clc
```

```
clear all
close all
num1=[1];
num2=[5];
den=[1 1];
sys1=tf(num1,den);
sys2=tf(num2,den);
figure(1);
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('T=1','T=5');

figure(2);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('T=1','T=5');

```

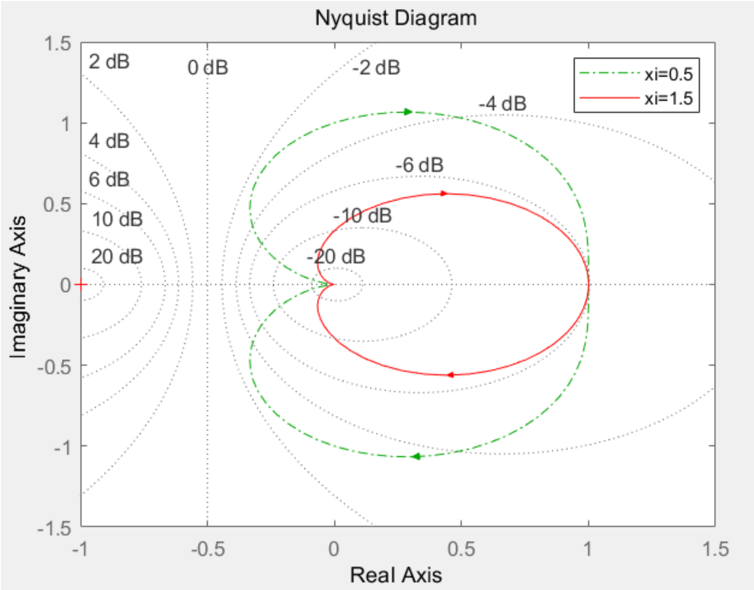


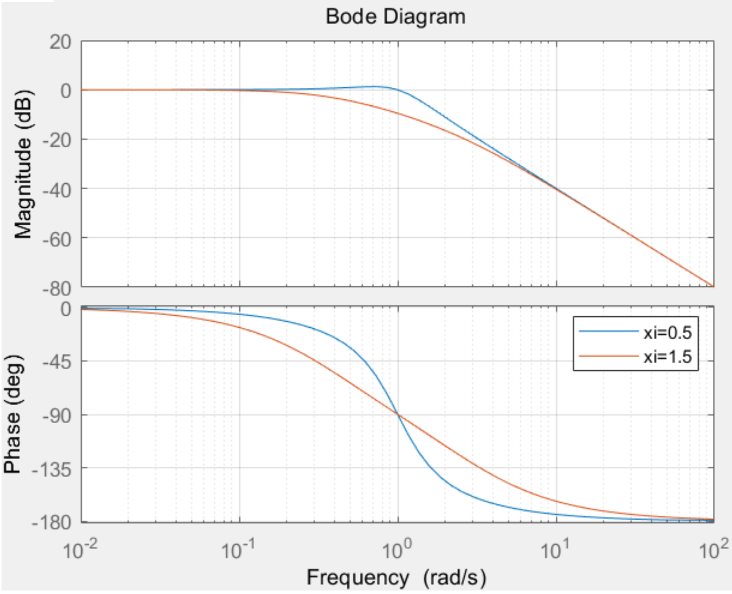


结论：取T=1,K=1和5。由上题分析知，惯性环节$\frac{K}{1+j\omega T}$ 的幅相特性是以 ($\frac{K}{2}, j0$)为圆心的半圆。增大K会使Nyquist图半径增大，圆心右移。同时，Bode图的基准线为 $\omega=1, L(\omega)=20lgK$，K越大，与$L(\omega)$ 轴的交点坐标值越大。

问题5

```
clc
clear all
close all
num1=[1];
num2=[1];
T1=1;xi1=0.5;
den1=[T1^2 2*T1*xi1 1];
T2=1;xi2=1.5;
den2=[T2^2 2*T2*xi2 1];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
figure(1);
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('xi=0.5','xi=1.5');

figure(2);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('xi=0.5','xi=1.5');

```



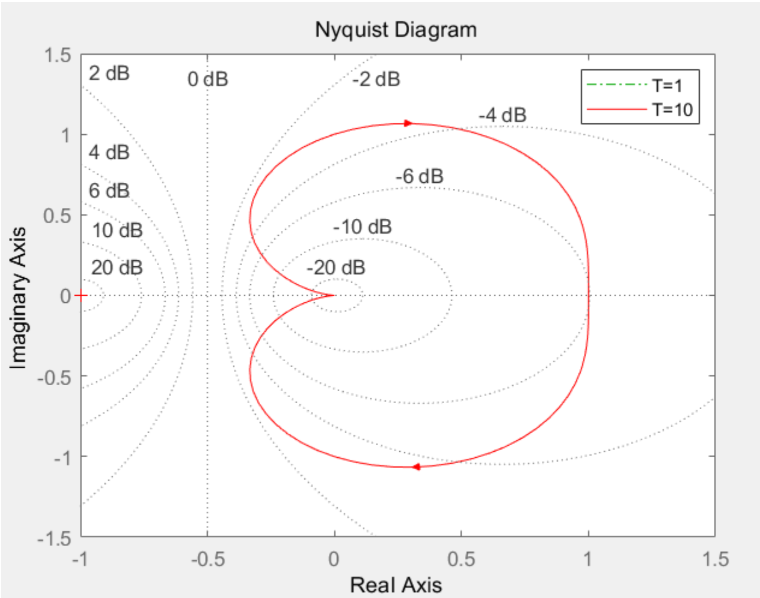


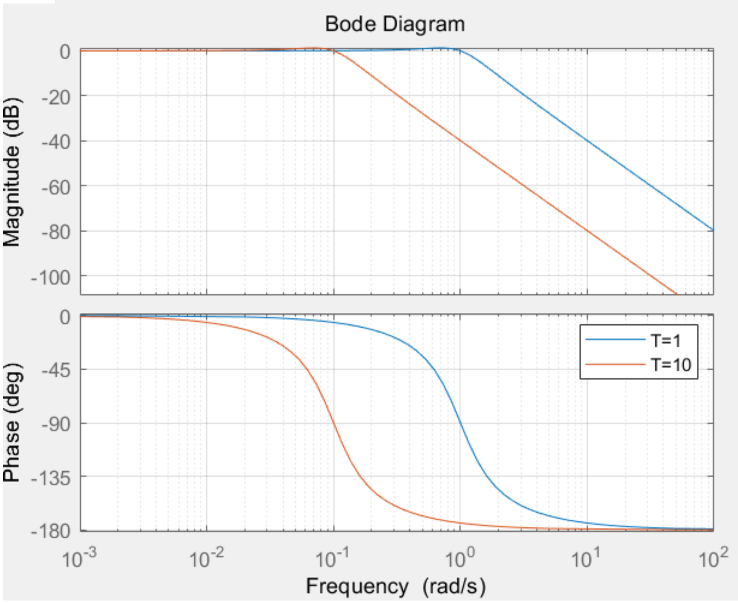
结论：取T=1，ξ分别等于0.5和1.5。阻尼比越小，对应的Nyquist图所围成的面积越大。阻尼比小于0.707，则会出现谐振峰值，可在Bode图上观测得到。对应的Nyquist图上也存在到原点距离最大的一点，该点即为谐振点。

问题6

```
clc
clear all
close all
num1=[1];
num2=[1];
T1=1;xi1=0.5;
den1=[T1^2 2*T1*xi1 1];
T2=10;xi2=0.5;
den2=[T2^2 2*T2*xi2 1];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
figure(1);
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('T=1','T=10');
```

```

figure(2);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('T=1','T=10');
```



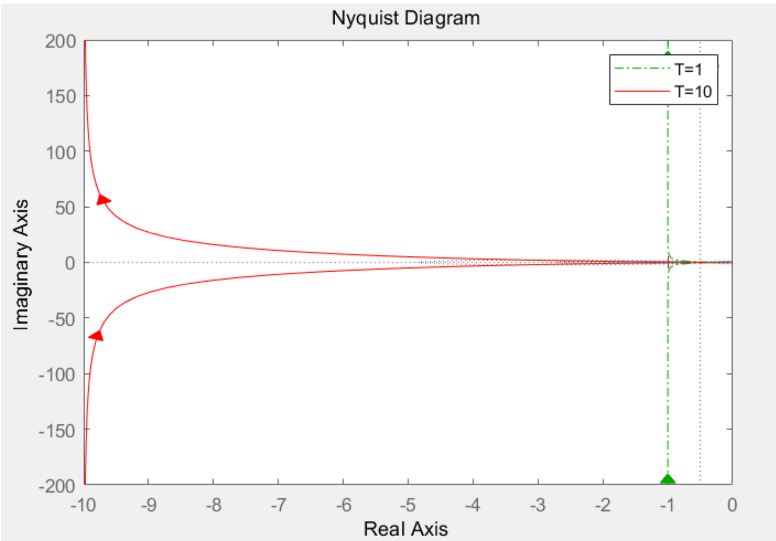


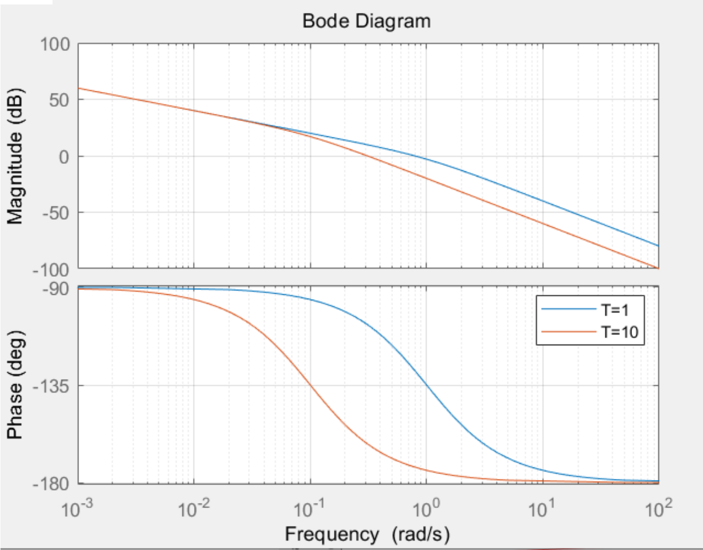
分析：取 ξ=0.5, T分别1和10。 Nyquist图重合，但转折频率为 $\omega=\frac{1}{T}$，故T越大，转折频率越小（从Bode图中可以看出）。

问题7

```
clc
clear all
close all
num1=[1];
num2=[1];
T1=1;
den1=[T1 1 0];
T2=10;
```

```
den2=[T2 1 0];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
figure(1);
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('T=1','T=10');

figure(2);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('T=1','T=10');

```



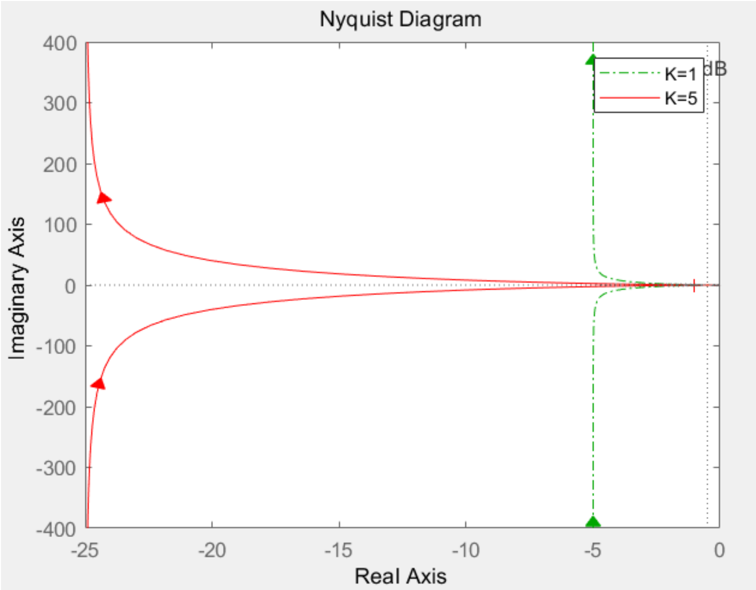


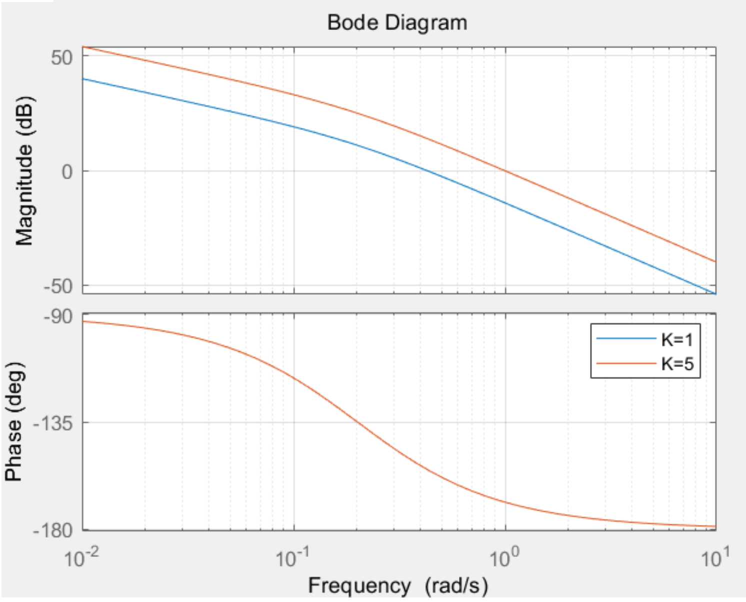
分析：取K=1, T分别等于1和10。T的大小影响Nyquist图的起点，也影响Bode图的相频特性，以及幅频特性的转折频率。

问题8

```
clc
```

```
clear all
close all
num1=[1];
num2=[5];
T1=5;
den1=[T1 1 0];
T2=5;
den2=[T2 1 0];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
figure(1);
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('K=1','K=5');

figure(2);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('K=1','K=5');

```



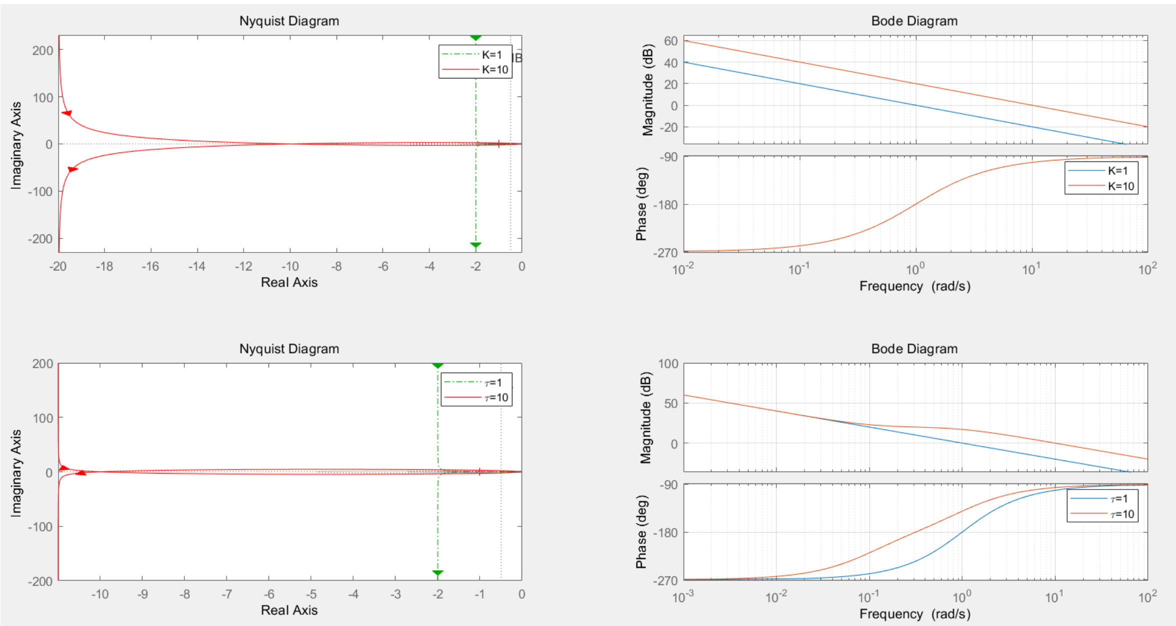


对于K=1或5，T=5，由Nyquist稳定判据 Z=P-R，P=0，R=0知系统稳定。

问题9

```
clc
clear all
close all
%\tau=1,T=1
num1=[1 1];%K=1
num2=[10 10];%K=10
den1=[1 -1 0];
den2=[1 -1 0];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
figure(1);
subplot(2,2,1)
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('K=1','K=10');
%figure(2);
subplot(2,2,2)
```

```
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('K=1','K=10');

%T=1,K=1
num1=[1 1];%\tau=1
num2=[10 1];%\tau=10
den1=[1 -1 0];
den2=[1 -1 0];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
%figure(3);
subplot(2,2,3);
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('\tau=1','\tau=10');
%figure(4);
subplot(2,2,4);
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('\tau=1','\tau=10');

```



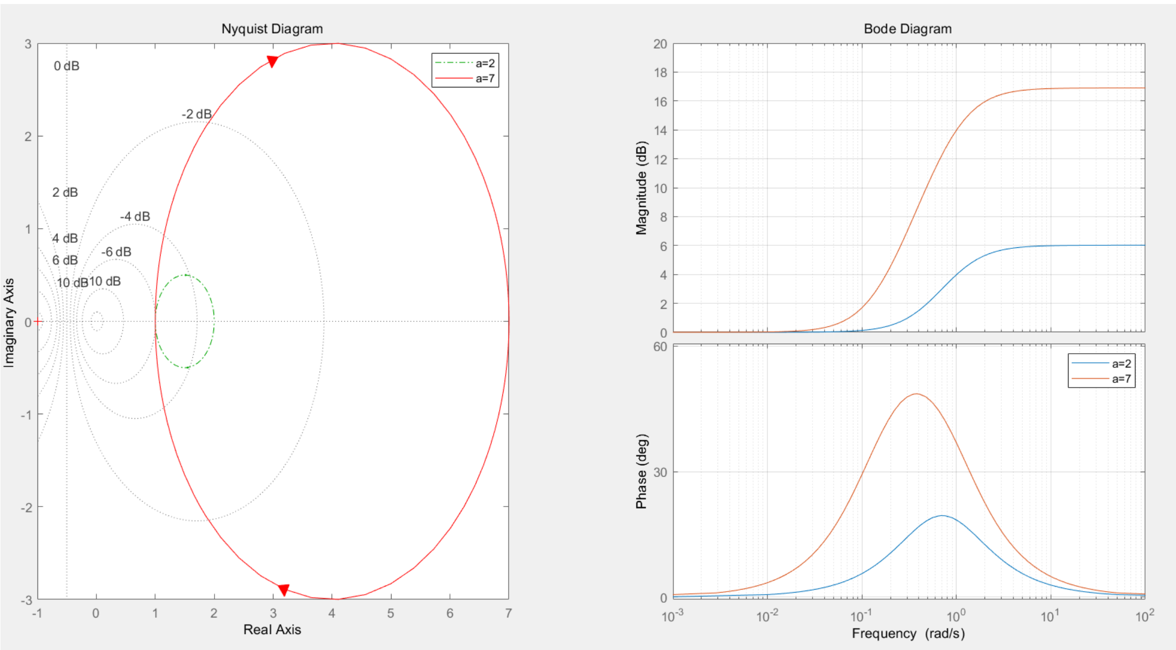
结论：取T=1, $\tau=1$ K分别等于1和10。由于有串联积分环节，需画出增补的Nyquist图，由增补Nyquist图可知，当K=1时，曲线恰经过(-1,j0)点，闭环系统临界稳定。当K=10，围绕$(-1,j0)$恰好绕过角度π，又P=1，由Nyquist判据知，系统稳定。当K较大时，系统稳定；当K较小时，系统不稳定。

取T=1, k=1,τ 分别等于1和10。由于有串联积分环节，需画出增补的Nyquist图，由增补Nyquist图可知，当τ =1时，曲线恰经过(-1,j0)点，闭环系统临界稳定。当τ =10，围绕$(-1,j0)$恰好绕过角度π，又P=1，由Nyquist判据知，系统稳定。当τ较大时，系统稳定；当τ较小时，系统不稳定。

由Routh判据知，当 $K\tau > 1$，系统稳定。

问题10

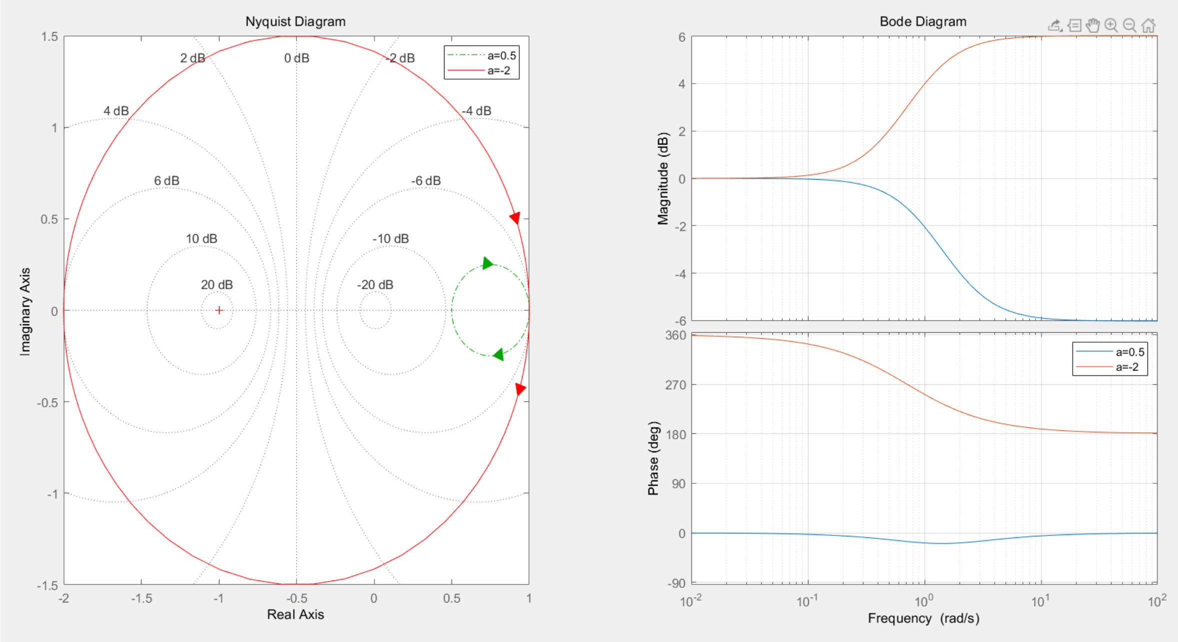
```
clc
clear all
close all
% T=1
num1=[2 1];
num2=[7 1];
den1=[1 1];
den2=[1 1];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
figure(1);
subplot(1,2,1)
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('a=2','a=7');
subplot(1,2,2)
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('a=2','a=7');
```



结论：取T=1, a分别等于2和7。由Nyquist图可知，a越大，半径越大。由Bode图知，a越大，幅频特性和相角特性能达到的最大值也增大。只要a>1，闭环系统就稳定。

问题11

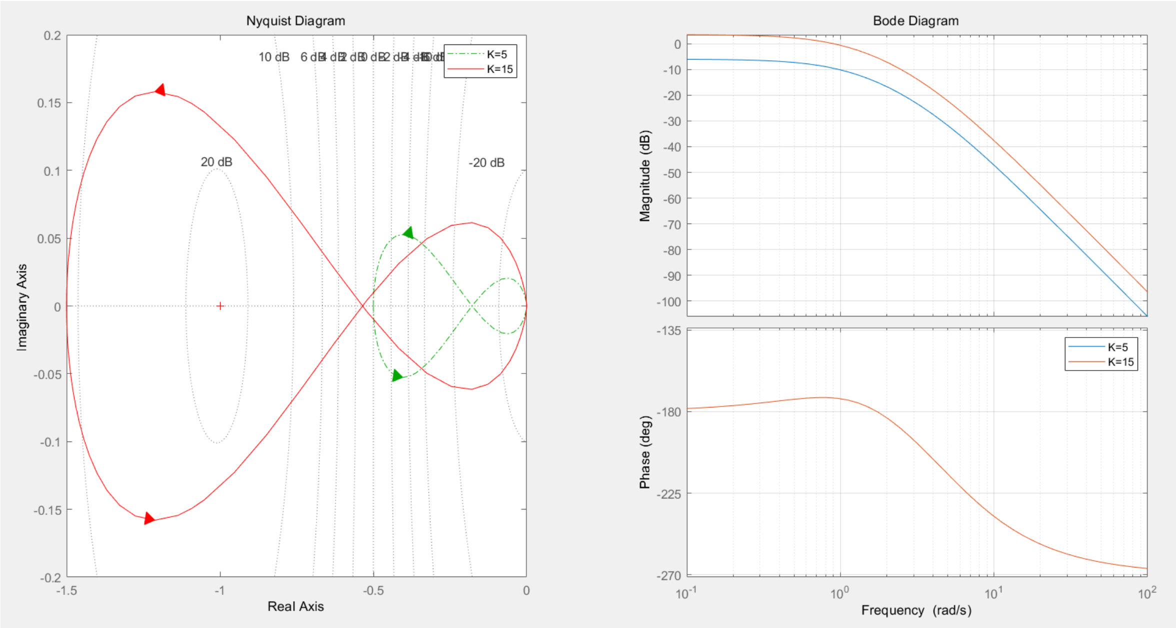
代码主体框架同上，取a=0.5和-2



当a<-1，闭环系统不稳定；当a>-1，系统稳定。

问题12

```
clc
clear all
close all
num1=[5];
num2=[15];
den1=[1 6 3 -10];
den2=[1 6 3 -10];
sys1=tf(num1,den1);
sys2=tf(num2,den2);
figure(1);
subplot(1,2,1)
hold on
nyquist(sys1,'g-.')
nyquist(sys2,'r')
hold off
grid
legend('K=5','K=15');
subplot(1,2,2)
bode(sys1)
hold on
bode(sys2)
hold off
grid
legend('K=5','K=15');
```



取K分别为5和15，由Nyquist图知，K=15时，$\omega = 0 \to +\infty$ 曲线绕$(-1, j0)$ 的角度为π，又因为开环传函在右半平面的极点数为1，满足Nyquist稳定判据，闭环系统稳定。同理，K=5时闭环系统不稳定。

可以得出结论当K>10时，闭环系统稳定；否则不稳定。