For office use only	Team Control Number	For office use only
T1 _____	**93641**	F1 _____
T2 _____		F2 _____
T3 _____	Problem Chosen	F3 _____
T4 _____	**B**	F4 _____

2018
MCM/ICM
Summary Sheet

Modeling of the development trend of language

Summary

The task of our group is to select the new work location and the working language for a multinational company based on predicting the future development trend of language. (We assume that all of the employees have mastered English.)

According to the assumption, we developed an index to evaluate the value of a language. Therefore, we analyzed the factors that influence the development of language from two aspects: external and internal.

We think that the difficulty of the language-self and the language family are the internal factors which affect the development of the language. Furthermore, by looking at a large number of literature, we found out a report about clear language classification submitted by Government Accountability Office (GAO) to United States Department of State.

Simultaneously, in the literature, we realized that many languages are from the same family. Hence, to some extent, it will be easier for a foreign language user to learn a language from the same language family. Given that our service object is the staff who has mastered English, so we have quantified this standard through other languages in conjunction with the English family.

Moreover, there are various kinds of the external factors which may affect the development of language, such as policy, social, business and so on. However, due to the complexity of practical problems, we cannot fully take all the factors that may affect the development of language in the future into account, so we assumed that all the previous influences of external factors have reflected into the number of the language user. Afterwards, we established the Grey Prediction Model with previous data to estimate the number of language users in the future.

Finally, we simulated the weight ratio between the three factors mentioned above through the analytic hierarchy process.

At the same time, we think that the development trend of population distribution has no correlation with language. Additionally, in consideration of providing a relatively beneficial economic development environment for the company in future, we determined to choose the local accumulation of GDP (by accumulative error) as the criteria.

To summary, we can make a conclusion that the optimal of six locations are United Kingdom, Canada, Spain, India, Russia and Mexico.

Memorandum

To: Chief Operation Officer

From: Team 93641

Date: 12 February 2018

Subject: Investigation result of the trend of language and recommendations of location options for new offices.

This memo is a brief report about our investigation and analysis of the influence of language development on the selection of office space. The memo will discuss the analysis process and then provide the recommendation of location options for new based on our investigation result.

According to the statistics provided by Ethnologue in 2017, there are approximately 7099 languages spoken on Earth currently. The prevailed rate of language is not only relative to the number of native speakers, but also the foreigners who master a second or third language. The total amount of the language speakers may have a dynamic change caused by various kinds of elements include the local population, the trend of immigration, national culture inclusiveness, national diplomatic policy, Gross Domestic Product and so on in the context of globalization currently. With the development of modern linguistics recently years, language and its analysis application have involved in various academic areas includes multinational business.

Due to the complexity of practical problems, we cannot fully take all the factors that may affect the development of language in the future into account.

Therefore, in this survey, we only selected three main factors that have a great impact: the number of language users, the difficulty of language, and the language family.

Furthermore, the language users are mainly divided into two categories: native speakers and foreign language users. All native speakers attach great importance to their mother tongue and hope that it will continue to be prevail, and foreign language acquisition was mainly affected by the spread of culture, so we believe that the native speaker is the basic condition to guarantee the existing of language, while the number of foreign language users influences the trend of the development of the language. In addition, language difficulty will also influence the interest and confidence of foreign language users in learning the language. At the same time, because of the development of human society, many languages are from the same family. Hence, to some extent, it will be easier for a foreign language user to learn a language from the same language family. To summary, we use these three factors as variables to establish a mathematical model and obtain the top 14 languages with better development trend in the future.

On the other hand, in consideration of providing a relatively beneficial economic development environment for the company in future, we collect and rank all countriesâĂŹ GDP in the world, and then corresponding to their official languages simultaneously. Afterwards, with the language mentioned above ranking as the major premise, after combining the two groups of rankings, the top six countries will be selected as the optimal solution for future office location as well as the official language. (Notes: We exclude America and China where have set the company already.) They are United Kingdom, Canada, Spain, India, Russia and Mexico.

Furthermore, through our analysis of the distribution trend of immigrant population,

we can exclude its influence on language development.

To sum up, there is too much uncertainty in the prediction of language development, so we suggest that this evaluation is supposed to be used only as a short-term prediction model.

Nevertheless, with the rapid development of modern science and technology, the company's business model are then update to reform. In recent years, the popularity of network sales channels has been able to gradually replace bricks-and-mortar business extent and also save the resources for our company.

As we know, the net profit can affect the statement of financial position of a company, and it equals to the sales revenue minus expense. If we can get the support of reliable information that the increase of network sales is conducive to the growth of the total sales volume of the company, and can save offline entity company cost and reduces the total cost at the same time, we strongly suggest to reduce the number of the new company and expand more network sales channels, so that the company can maximize the interest of shareholders and keep up with the development of The Times

Let me know if you have any further questions about the location options for new offices.

1 Introduction

According to the statistics provided by Ethnologue [1] in 2017 that there are approximately 7099 languages spoken on Earth currently [2]. Furthermore, Ethnologue claimed that Mandarin Chinese, Spanish, English, Hindustani, Arabic, Bengali, Portuguese, Russian, Punjabi and Japanese are the ten of most prevailed languages [2]. The prevailed rate of language is not only relative with the number of native speaker, but also the foreigners who master a second or third language. The total amount of the language speakers may has a dynamic change caused by various kinds of elements include the local population, the trend of immigration, national culture inclusiveness, national diplomatic policy, Gross Domestic Product etc., in the context of globalization today specially. With the development of modern linguistics recently years, language and its analysis application have involved in divers academic area includes multinational business.

1.1 Restatement of the Problem

Based on the core vision concept, the multinational service company decided to explore additional business service in other place apart from New York and Shanghai. Simultaneously, for successfully operation its business, the company requires their employee to master both English and an additional language and location the place of the new branch company. Hence, the company hired our team to investigate and recommend the optimal solution of the new working language and location.

2 Analysis of the Problem

We will consider both language and location two aspects to determine the new office location and working language.

1. In terms of language, we take the number of language users, the difficulty of the language itself and the impact of the language family in to account.

    - An assessment of Language Vitality and Language Endangered stated that once users no longer use the language, or the language communication occasions are dwindling, and no longer will it on to the next generation which means there are no new users, the language will be endangered [3]. Therefore, we can infer that the trend of language development should be related to the number of language users.
    - On the other hand, the learning difficulty level of the language itself and its language family are also the factor for foreign language users. Moreover, as humans in learning a second language, the native language and understanding of language universals from his native language will be consciously or unconsciously applied to language acquisition, so as to deepen the understanding of language and improve the efficiency of language learning. In this sense, the learner's native language background has laid a certain foundation for his foreign language learning, thus, the universal phenomenon of language which regarded as language family can promote the acquisition of second language [4].

2. We consider that the fundamental purpose of an enterprise is to maximize its benefits.
    - Hence, we can respectively choose a representative country as location for each language based on its cradle and the countriesâĂŹ Gross Domestic Product (GDP) which suggests the future economic development level.
3. By collecting statistics on the global migration population and distribution in recent years, we can roughly infer and simulate the distribution map of future immigrant population.

3 Model

3.1 Model A: Evaluate whether a language is worth learning.

In view of the multiple factors of language development trend, we decide to utilize Analytic Hierarchy Process (AHP) propounded by T.L.Satty in the 1970s.

3.1.1 Analysis

We believe that there are two main reasons for the development of language. One is the external influence factors, and another one is the internal factor.

Following, we regard the change of total number of language users to assess the effects of external factors, and with the difficulty of the language-self and the language family to assess the effects of the internal factor.

So, we have three factors in this model:

- The development trend of the number of people using the language.
- Language difficulty.
- Language family.

3.1.2 Assumption

- In short term, the difficulty of the language will keep a constant level.
- Any external factors which may affect the language development will also affect the number of language users, such as national policy, war or other anthropic factors and natural factors.

3.1.3 Definitions and symbols

The definitions and symbols are shown in table 1.

C_1	The number of language users
C_2	Language difficulty rating
C_3	Classification of languages family
L	Language development trend coefficient

Table 1: symbols

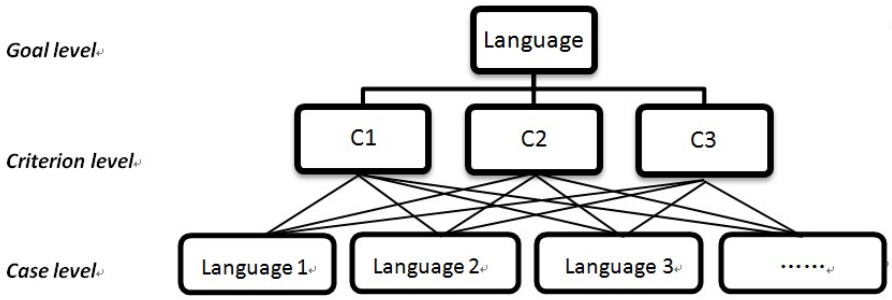

Figure 1: Modeling procedure

3.1.4 Modeling

According to Analytic Hierarchy Process (AHP), we use a_{ij} to represent the influence of $\frac{C_i}{C_j}$ to upper level. After comparing between C_1, C_2, C_3 we can get the Paired Comparison Matrix $A = (a_{ij})_{n\times n}$, and $a_{ij} > 0$, $a_{ij} = \frac{1}{a_{ji}}$

Based on the experience, we can predicate the weight between C_1, C_2, C_3 is $C_1 : C_2 = 3 : 1$, $C_2 : C_3 = 2 : 1$ and $C_1 : C_3 = 2 : 1$, then:

$$A = \begin{bmatrix} 1 & 3 & 4 \\ \frac{1}{3} & 1 & \frac{1}{2} \\ \frac{1}{4} & 2 & 1 \end{bmatrix}$$

Because $a_{12} \cdot a_{23} \neq a_{13}$, the paired comparison is not completely consistent, and we need to verify the consistency of this matrix.

Firstly, calculating the eigenvalues and corresponding eigenvectors of matrix A by Matlab: $\lambda = 3.1078$, $\vec{u} = (0.9214, 0.2215, 0.3194)^T$

Then, calculating the Consistency Indicators (CI) :

$$CI = \frac{\lambda - n}{n - 1} = \frac{3.1078 - 3}{3 - 1} = 0.0539$$

And find out Random Consistency Indicators (RI) through the table that RI=0.58.

n	2	3	4	5	6	7	8	9	10
RI	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Table 2: The value of RI

Finally, calculating Random Consistency Ratio (CR):

$$CR = \frac{CI}{RI} = \frac{0}{0.58} = 0.0929$$

Due to $CR < 0.1$, we deem matrix A has a satisfied consistency. That is, the eigenvector corresponding to the maximum characteristic root can be used as the weight vector of the comparison factor.

We get the maximum eigenvalue $\lambda = 3$ and corresponding eigenvectors $\vec{u} = (0.9214, 0.2215, 0.3194)^T$. Then we obtain weight vector $(0.6301, 0.1515, 0.2184)^T$.

Therefore, language development trend coefficient:

$$L = 0.6301C_1 + 0.1515C_2 + 0.2184C_3$$

3.2 Model B: The number of language speakers in the future prediction model

Based on the data collected, we decided to use the grey prediction to predict the number of future speakers.

3.2.1 Analysis

We believe that there are many reasons for changing the number of speakers in a language. It contains the war, culture communication, international trade and other difficult to standard quantitative indicators, also these indicators have a certain extent of randomness. However, it is clear that these cases effected on past data, in other words, the past data contains all the indicators that affect the number of speakers. Thus, we chose the Grey Model (GM) which can predict the data from less past data with a high precision.

3.2.2 Assumption

- There will be no mutation in the short term.

3.2.3 Definition and symbols

Grey system theory is based on the concept of correlation space and smooth discrete function to define gray derivative and gray differential equation. The dynamic model of differential equation is set up with discrete data column, which is the basic model of the gray system. And the Model is similar and not unique, so this Model is a Grey Model, which is called Grey Model (GM), which means Grey Model is made by using discrete random Numbers to become random and the generation number which is significantly weakened and more regular and then establish the model of differential equation form, so as to facilitate the study and description of its change process [5].

Grey prediction refers to the estimation and prediction of the development and change of system behavior characteristics using GM model.

Simultaneously, it is also possible to estimate the occurrence of abnormal situations of behavior characteristics, as well as the future time distribution of the occurrence of events in a specific time zone and so on.

$x^{(0)}$	The sample sequences. (Existing data)
$x^{(1)}$	Make an Accumulation Generation Operator on $x^{(0)}$
C	Posteriori error

Table 3: symbols

3.2.4 Modeling

Assuming the number of language user over the years is

$$x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)$$

then do an Accumulation Generation Operator (*AGO*), getting a sequence

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)) = (x^{(1)}(1), x^{(1)}(1) + x^{(0)}(2), ..., x^{(1)}(n-1) + x^{(0)}(n))$$

Among that,

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \qquad (k = 1, 2, ..., n)$$

Obtaining the Mean sequence that,

$$z^{(0)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \qquad (k = 2, 3, ..., n)$$

Then,

$$z^{(1)} = (z^{(1)}(2), z^{(1)}(3), ..., z^{(1)}(n))$$

So we set up the grey differential equation

$$x^{(0)}(k) + az^{(1)}(k) = b, \qquad (k = 2, 3, ..., n)$$

The corresponding albino differential equation is

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b$$

Let,

$$\vec{u} = (a, b)^T, Y = (x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n))^T$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$$

Based on Least squares method, we led the $J(\vec{u}) = (Y - B\vec{u})^T(Y - B\vec{u})$to further getting the minimum value

$$\vec{u} = (a, b)^T = (B^T B)^{(-1)} B^T Y$$

Then we solve for the equation that

$$\vec{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{(-ak)} + \frac{a}{b}, \qquad (k = 0, 1, ..., n-1, ...)$$

Then we obtain the predict data

$$\vec{x}^{(1)}(k+1) = x^{(1)}(k+1) - x^{(1)}(k), \qquad (k = 0, 1, ..., n-1, ...)$$

Finally, we utilize After Test Rule to analyze the prediction error and determine the accuracy of the data

$$C = \frac{var(|x^{(1)} - x^{(0)}|)}{var(x^{(0)})}$$

Level	posteriori error C	Error of frequency
Good	C < 0.35	P > 0.95
Qualified	C < 0.45	P > 0.80
Pass	C < 0.50	P > 0.70
Fail	C < 0.65	P $\leq$ 0.70

Table 4: Accuracy standard of posteriori error [6]

3.3 Model C: Future population of China prediction model

3.3.1 Analysis

China's future population has a huge relationship with China's current birth rate and death rate. Thus, we fitting the difference between birth rate and death rate with the year by the least square method, then solve the differential equation.

3.3.2 Assumption

China has had zero net migration in recent years.

3.3.3 Definition and symbols

P(x)	The population of China
x	The year

Table 5: symbols

	1972	1977	1982	1987	1992	1997	2002	2007	2012
Mortality (per thousand people)	7.61	6.87	6.6	6.72	6.64	6.51	6.41	6.93	7.15
Birth rate (per thousand people)	29.77	18.93	22.28	23.33	18.27	16.57	12.86	12.1	12.1
Difference value	22.16	12.06	15.68	16.61	11.63	10.06	6.45	5.17	4.95

Table 6: 1972-2012 Mortality and Birth rate in China

3.3.4 Modeling

We found the following data from the world bank.

Then, we use Matlab to fitting the difference value with years by the least square method,and get the result:

$$Linear\ model\ Poly_1 : f(x) = p_1 \times x + p_2$$
$$Cofficients(with\ 95\%\ confidence\ bound)$$
$$p_1 = -0.3827(-0.5431, -0.2224)$$
$$p_2 = 774.1(454.6, 1093)$$

Integrating both sides of this equation

$$\int f(x)dx = \int -0.3827x + 774.1dx$$

Then,

$$P(x) = -0.19135x^2 + 774.1x + C$$

The real population in 1972 is 862030000, plug this number in above function and figure out the C equal to 861247593.62.

Therefore,

$$P(x) = -0.19135x^2 + 774.1x + 861247593.62$$

4 Application

We will apply the model mentioned above to solve the practical problem. The process will be divided into two steps: Data processing and Problem solving.

4.1 Data processing

4.1.1 Predict: the number of users C_1

Firstly, we utilize Model B to predict the number of language users in 50 years. We selected 14 languages with the largest number of people in the previous years, and obtained the previous year's data by searching.

Subsequently, we plotted the data into a line chart, and simply observed the trend of the number of language users.

	2017 [7]	2014 [8]	2011 [9]	2009 [10]	2005 [11]
Mandarin Chinese	1090	1197	1284	1213	1051
English	983	335	372	328	510
Hindustani	544	260	260	182	490
Spanish	527	414	437	329	420
Arabic	422	237	295	221	230
Malay	281		60.8	39.1	
Russian	267	167	154	144	255
Bengali	261	193	242	181	215
Portuguess	229	203	219	178	213
French	229	75	76.1	67.8	130
Japanese	129	122	128	122	127
German	129	78.2	76.8	90.3	229
Korean	77	77.2	77.2	66.3	71
Panjabi	148			62.6	88

Table 7: Total number of language users(million)

We plug the data into model b, take the state as a unit, and calculated the result by Matlab. Then we sorted the data in descending order according to the data size of âĂIJAfter 50 yearsâĂİ, and got the following table.

The posterior difference ratio of each prediction data is less than 0.35, so we believe that the prediction accuracy is relatively high.

Therefore, we can conclude that compared to the total number of language users in 2017 the member of top 10 in next 50 years do not have any change. However, the order has a dramatic change. For instance, English will become the most spoken language.

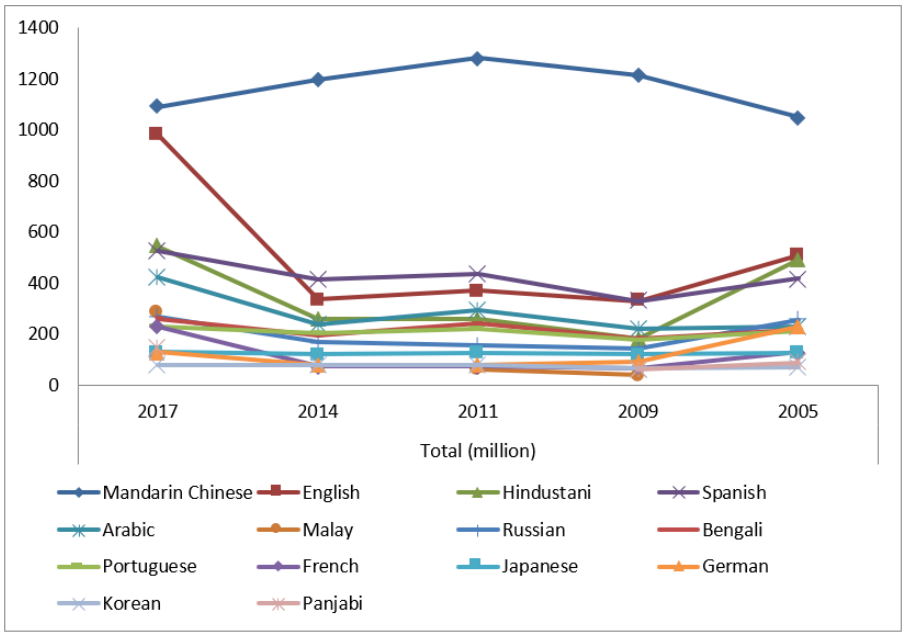

Figure 2: Results

Language	After 30 years	After 50 years	C
English	1190.2410	1922.2897	0.3246
Mandarin Chinese	1089.3346	1049.7645	0.2687
Hindustani	525.9848	707.5967	0.1670
Spanish	588.6219	672.9143	0.2526
Arabic	475.1541	586.7825	0.3402
French	265.8385	459.6159	0.3219
Malay	350.2279	440.1033	0.3057
Russian	309.2814	388.8702	0.2172
Portuguese	243.1803	259.5467	0.3482
Bengali	271.4378	248.6587	0.3494
German	175.9098	203.3289	0.1253
Japanese	127.5109	135.3985	0.3444
Panjabi	127.3669	110.0944	0.3239
Korean	82.6025	86.2441	0.3343

Table 8: the number of language speakers After 30 or 50 years for each languages

4.1.2 Language difficulty rating C_2

We find out the following official report published by Government Accountability Office (GAO) as a reference criterion for rating.

Note: In order to facilitate the view, we marked the selected language.

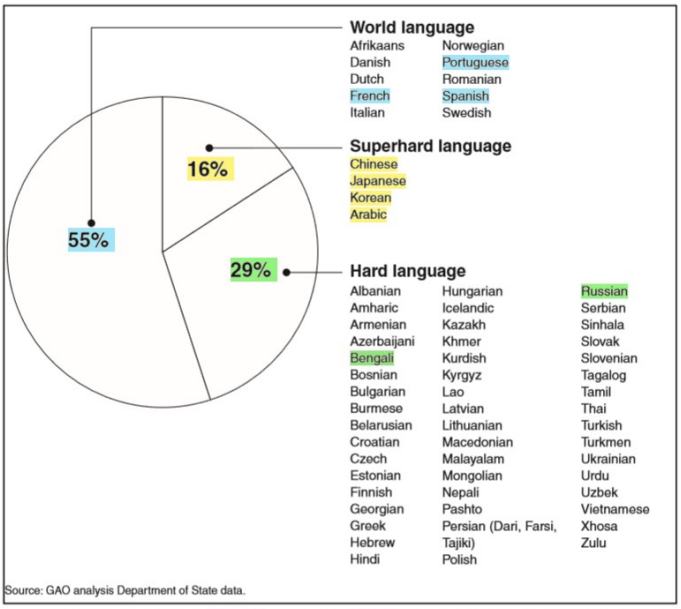

Figure 3: Language difficulty rating standard [12]

According to the figure, we select the proportion as the score to evaluate the difficulty of language. Based on the precondition of problem that all of the employee have master English, we set the score as 100.

(Note: as the difficulty increasing, the score decreasing)

Language	Mandarin Chinese	English	Hindustani	Spanish	Arabic	Malay	Russian
Score	16	100	29	55	16	35.5	29
Language	Bengali	Portuguese	French	Japanese	German	Korean	Panjabi
Score	29	55	55	16	35.5	16	35.5

Table 9: Language difficulty rating scale

4.1.3 Language family C_3

Following is the integration information from Ethnologue [1].

Mandarin Chinese	Sino-Tibaten, Sinitic
English	Indo-European, Germanic
Hindustani	Indo-European, Indo-Aryan
Spanish	Indo-European, Romance
Arabic	Afro-Asiatic, Semitic
Malay	Austronesian, Malayo-Polynesian
Russian	Indo-European, Slavic
Bengali	Indo-European, Indo-Aryan
Portuguese	Indo-European, Romance
French	Indo-European, Romance
Japanese	Japonic
German	Indo-European, Germanic
Korean	Koreanic
Panjabi	Indo-European, Indo-Aryan

Table 10: Language Family

According to the analysis of the previous analysis (the language of similar languages family is more acceptable), it is reasonable to assume that the 14 languages are graded according to the language family.

(Note: Assuming that English is the native language, the following data can be obtained based on the similarity ratio of language family.)

Language	Mandarin Chinese	English	Hindustani	Spanish	Arabic	Malay	Russia
Percentage(%)	0	100	50	50	0	0	50
Language	Bengali	Portuguese	French	Japanese	German	Korean	Panjab
Percentage(%)	50	50	50	0	100	0	50

Table 11: Language difficulty rating scale

4.2 Problem solving

In order to provide the best working language choice for the client company, we plug the data obtained above into Model A, and obtained the corresponding scores of each

language, and sorted the scores from large to small, and reached the following table.

Score	language
English	765.3393
Mandarin Chinese	688.8137
Spanish	379.3324
Hindustani	335.9257
Arabic	301.8186
Malay	226.0568
Russian	199.3809
French	175.9465
Bengali	175.5357
Portuguese	161.6696
German	116.4374
Panjabi	85.74133
Japanese	82.76862
Korean	54.47184

Table 12: The order of prediction result

We use model C to predict the population of China over the next 50 years.

$$P(x) = -0.19135x^2 + 774.1x + 861247593.62$$

$$P(2068) = -0.19135 \times 2068^2 + 774.1 \times 2068 + 861247593.62 = 862030100.42$$

As can be seen from the comparison, the change of population has no effect on the geographical distribution of language users.

Therefore, when choosing an office location for the client company, our main concern is the local economy.

The Gross Domestic Product (GDP) is the total value of the various sectors of the national economy that a country or region added in a certain period of time [14], so we choose GDP as the basis for judging the economic level of each country.

In order to reduce the error, we accumulate the GDP index from 1960 to 2016 and rank the country according to it.

Following table shows the data of top 14 countries.

We combine the official language used by the countries in figure 9 with figure 8 to take the intersection. (Note: set language as major premise, and the company already has offices in New York and Shanghai.)

Following are our final result.

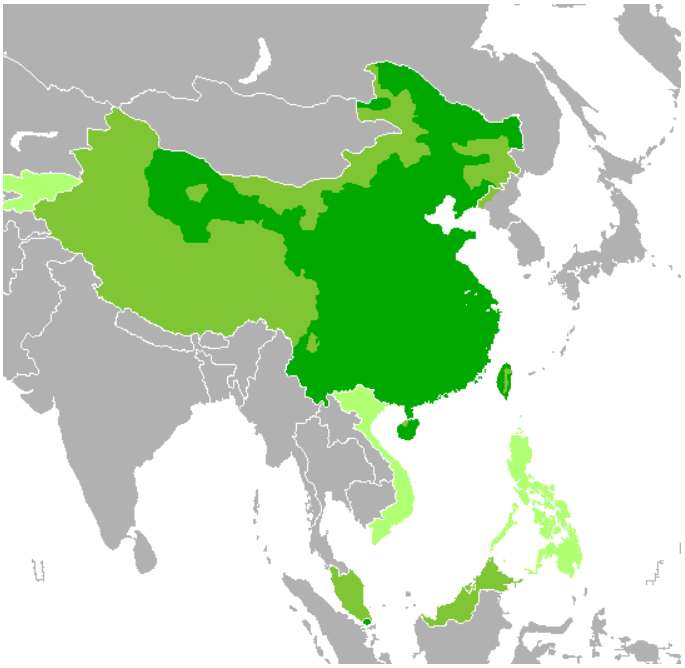

Figure 4: the geographical distribution of language users

Accumulation of GDP(1960-2016)($)	Country
America	38.6083×10^{13}
Japan	15.1888×10^{13}
China	10.2059×10^{13}
German	8.94624×10^{13}
France	6.58257×10^{13}
United Kingdom	6.48233×10^{13}
Italy	5.28358×10^{13}
Brazil	3.61641×10^{13}
Canada	3.51382×10^{13}
Spain	3.00586×10^{13}
India	2.90331×10^{13}
Russian	2.51573×10^{13}
Mexico	2.37087×10^{13}
Korea	2.29995×10^{13}

Table 13: The order of prediction result

①	②	③	④	⑤	⑥
English		Spanish	Hindustani	Russian	Spanish
Unite Kingdom	Canada	Spain	India	Russia	Mexico

Figure 5: final result

5 Weakness

5.1 Model A

- The setting of indicators and weights is subjective and lacks the theoretical support of professional systems.
- Restricted by the selected index, the influence of other aspects cannot be taken into account, lead to resulting in a certain deviation from the true weight vector.

5.2 Model B

- The long-term value cannot be accurately predicted.Because the sample size is small, the prediction data can only reach an ideal accuracy in the short term. With the increase of time, the predicted data will deviate significantly.
- Selected data – the total number of people using a certain language is not necessarily accurate due to different statistical time, which leads to certain errors in the predicted data.

5.3 Model C

- Factors that affect the population in addition to birth and death are not considered.

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