

MATH 446: Project 02

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Questions

1. $f(x) = 3x^3 - 7x^2 + 3x - e^x + 2 = 0$

$$\begin{aligned}g1(x) &= \frac{e^x - 2}{3x^2 - 7x + 3} = x \\r1 &= -0.24789639 \\x_0 &= -1 \\numberofsteps &= 17\end{aligned}\tag{1}$$

$$\begin{aligned}g2(x) &= \frac{3x^4 - 7x^3 + 3x^2 + 2x}{e^x} = x \\r2 &= 0.62616943 \\x_0 &= 1 \\numberofsteps &= 22\end{aligned}\tag{2}$$

$$\begin{aligned}g3(x) &= \left(\frac{-7x^2 + 3x - e^x + 2}{-3.0} \right)^{\frac{1}{3}} = x \\r3 &= 2.46222248 \\x_0 &= 2 \\numberofsteps &= 84\end{aligned}\tag{3}$$

$$\begin{aligned}g2(x) &= \frac{3x^4 - 7x^3 + 3x^2 + 2x}{e^x} = x \\r4 &= 6.07305409 \\x_0 &= 6 \\numberofsteps &= 34\end{aligned}\tag{4}$$

2. $S = |g'(r)|$

$$\begin{aligned}\frac{d}{dx}g1(x) &= \frac{e^x(3x^2 - 13x + 10) + 2(6x - 7)}{(3x^2 - 7x + 3)^2} \\|\frac{d}{dx}g1(r1)| &= 0.269034\end{aligned}\tag{5}$$

$$\begin{aligned}\frac{d}{dx}g2(x) &= \frac{-3x^4 + 19x^3 - 24x^2 + 4x + 2}{e^x} \\|\frac{d}{dx}g2(r2)| &= 0.375249\end{aligned}\tag{6}$$

$$\frac{d}{dx}g_3(x) = \frac{1}{3} \left(\frac{-7x^2+3x-e^x+2}{3} \right)^{\frac{-2}{3}} \left(\frac{-14x+3-e^x}{-3} \right) \quad (7)$$

$$\left| \frac{d}{dx}g_3(r_3) \right| = 0.791783$$

$$\frac{d}{dx}g_4(x) = \frac{9x^2-14x+3}{3x^3-7x^2+3x+2}$$

$$\left| \frac{d}{dx}g_4(r_4) \right| = 0.575836 \quad (8)$$

3. $\lim_{k \rightarrow \infty} \frac{e_{k+1}}{e_k} = S$
 1. For r1: $\lim_{k \rightarrow \infty} \frac{e_{k+1}}{e_k} \approx 0.2690342181$
 2. For r2: $\lim_{k \rightarrow \infty} \frac{e_{k+1}}{e_k} \approx 0.3752496237$
 3. For r3: $\lim_{k \rightarrow \infty} \frac{e_{k+1}}{e_k} \approx 0.7917836336$
 4. For r4: $\lim_{k \rightarrow \infty} \frac{e_{k+1}}{e_k} \approx 0.5758353631$

Code

*% Computes the fixed point of a function using the FPI.
% Written by Zachary Ferguson*

```
function fixed_point_iteration
    fprintf('Fixed Point Iteration\nWritten by Zachary Ferguson\n\n');

    fprintf('f(x) = 3*x^3 - 7*x^2 + 3*x - e^x + 2 = 0\n\n');
    f = @(x) 3*x^3 - 7*x^2 + 3*x - exp(x) + 2;

    fprintf('g1(x) = (e^x - 2) / (3x^2 - 7x + 3) = x\n');
    g1 = @(x) (exp(x) - 2) / (3*x^2 - 7*x + 3);
    fprintf('r1 = %.10f\n\n', compute_fixed_point(g1, -1, f));

    fprintf('g2(x) = (3*x^4 - 7*x^3 + 3*x^2 + 2*x) / e^x = x\n');
    g2 = @(x) (3*x^4 - 7*x^3 + 3*x^2 + 2*x) / exp(x);
    fprintf('r2 = %.10f\n\n', compute_fixed_point(g2, 1, f));

    fprintf('g3(x) = ((-7*x^2 + 3*x - e^x + 2) / -3.0)^(1/3) = x\n');
    g3 = @(x) ((-7*x^2 + 3*x - exp(x) + 2) / -3.0)^(1/3);
    fprintf('r3 = %.10f\n\n', compute_fixed_point(g3, 2, f));

    fprintf('g4(x) = ln(3*x^3 - 7*x^2 + 3*x + 2) = x\n');
    g4 = @(x) log(3*x^3 - 7*x^2 + 3*x + 2);
    fprintf('r4 = %.10f\n', compute_fixed_point(g4, 6, f));
end
```

% Compute the fixed point of g(x).

```

function xc = compute_fixed_point(g, x0, f, tol)
    if nargin < 4
        tol = 1e-9;
    end

    r = fzero(f, x0);
    fprintf('r = %f\n', r);
    ei = 0;

    prev_x = x0;
    x = g(x0);
    n = 1;
    while (abs(prev_x - x) > 0.5 * tol)
        prev_x = x;
        x = g(x);
        n = n + 1;
        ei1 = abs(x - r);
        if (abs(prev_x - x) <= 0.5 * tol)
            fprintf('e_(i+1)/e_i = %.10f\n', ei1 / ei);
        end
        ei = ei1;
    end
    fprintf('n = %d\n', n);
    xc = x;
end

```

Output

Fixed Point Iteration
 Wrtten by Zachary Ferguson

$$f(x) = 3x^3 - 7x^2 + 3x - e^x + 2 = 0$$

$$g1(x) = (e^x - 2) / (3x^2 - 7x + 3) = x$$

$$r = -0.247896$$

$$e_{(i+1)}/e_i = 0.2690342181$$

$$n = 17$$

$$r1 = -0.2478963963$$

$$g2(x) = (3x^4 - 7x^3 + 3x^2 + 2x) / e^x = x$$

$$r = 0.626169$$

$$e_{(i+1)}/e_i = 0.3752496237$$

$$n = 22$$

$$r2 = 0.6261694387$$

```

g3(x) = ((-7*x^2 + 3*x - e^x + 2) / -3.0)^(1/3) = x
r = 2.462222
e_(i+1)/e_i = 0.7917836336
n = 84
r3 = 2.4622224868

```

```

g4(x) = ln(3*x^3 - 7*x^2 + 3*x + 2) = x
r = 6.073054
e_(i+1)/e_i = 0.5758353631
n = 34
r4 = 6.0730540924

```