

MATH 446: Project 02

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Questions

Q1.

a.

$$\begin{aligned} f(x) &= x^3 - 2x - 2 = 0 \\ r &= 1.76929235 \end{aligned} \tag{1}$$

b.

$$\begin{aligned} f(x) &= e^x + x - 7 = 0 \\ r &= 1.67282169 \end{aligned} \tag{2}$$

c.

$$\begin{aligned} f(x) &= e^x + \sin(x) - 4 = 0 \\ r &= 1.12998049 \end{aligned} \tag{3}$$

Q3.

a.

$$f(x) = 27x^3 + 54x^2 + 36x + 8 = 0 \tag{4}$$

$$f'(x) = 81x^2 + 108x + 36 \tag{5}$$

$$f''(x) = 162x + 108 \tag{6}$$

$$f'''(x) = 162 \neq 0 \tag{7}$$

$$\begin{aligned} r &= -\frac{2}{3} \\ f(r) = f'(r) = f''(r) &= 0 \rightarrow \text{multiplicity of } r \text{ is } 3 \end{aligned} \tag{8}$$

Q9.

$$f(x) = 14xe^{x-2} - 12e^{x-2} - 7x^3 + 20x^2 - 26x + 12 \quad (9)$$

$$f'(x) = 14xe^{x-2} + 2e^{x-2} - 21x^2 + 40x - 26 \quad (10)$$

$$f''(x) = 14xe^{x-2} + 16e^{x-2} - 42x + 40 \quad (11)$$

$$f'''(x) = 14xe^{x-2} + 30e^{x-2} - 42 \quad (12)$$

$$r_1 = 0.85714285$$

$$f'(r_1) \approx -2.67817 \neq 0 \rightarrow M = \lim_{i \rightarrow \infty} \frac{e_{i+1}}{e_i^2} = \left| \frac{f''(r_1)}{2f'(r_1)} \right| \approx 1.69939188 \quad (13)$$

$$r_2 = 2.0$$

$$f(r_2) = f'(r_2) = f''(r_2) = 0 \rightarrow \text{multiplicity of } r_2 \text{ is } 3$$

$$\therefore S = \lim_{i \rightarrow \infty} \frac{e_{i+1}}{e_i} = \frac{m-1}{m} = \frac{2}{3} \quad (14)$$

Code

% Computes the roots of a function using the Newton's Method.

% Written by Zachary Ferguson

% Solve the project questions

function newtons_method

% Q1a

fprintf('Q1a:\n\tf(x) = x^3 - 2x - 2 = 0\n');

f = @(x) x^3 - 2*x - 2;

fp = @(x) 3*x^2 - 2;

x0 = 2;

fprintf('\tx0 = %g\n', x0);

fprintf('\tr = %.10f\n', compute_root(f, fp, x0));

% Q1b

fprintf('Q1b:\n\tf(x) = e^x + x - 7 = 0\n');

f = @(x) exp(x) + x - 7;

fp = @(x) exp(x) + 1;

x0 = 0;

fprintf('\tx0 = %g\n', x0);

fprintf('\tr = %.10f\n', compute_root(f, fp, x0));

% Q1c

fprintf('Q1c:\n\tf(x) = e^x + sin(x) - 4 = 0\n');

f = @(x) exp(x) + sin(x) - 4;

fp = @(x) exp(x) + cos(x);

x0 = 2;

fprintf('\tx0 = %g\n', x0);

fprintf('\tr = %.10f\n', compute_root(f, fp, x0));

% Q3a

fprintf('Q3a:\n\tf(x) = 27x^3 + 54x^2 + 36x + 8 = 0\n');

f = @(x) 27*x^3 + 54*x^2 + 36*x + 8;

fp = @(x) 81*x^2 + 108*x + 36;

x0 = 0.0;

```

r = -2/3;
fprintf('\tx0 = %g\n', x0);
xc = compute_root(f, fp, x0, 1e-16);
fprintf('\txc = %.16f\n', xc);
fprintf('\tForward Error = |r - xc| = %.16f\n', abs(r-xc));
fprintf('\tBackward Error = f(xc) = %.16f\n', f(xc));
fprintf('\tmultiplicity of r is 3\n');
xc = compute_root(f, fp, x0, 1e-16, 3);
fprintf('\txc = %.16f\n', xc);
fprintf('\tForward Error = |r - xc| = %.16f\n', abs(r-xc));
fprintf('\tBackward Error = f(xc) = %.16f\n', f(xc));

% Q9
fprintf('Q9:\n\tf(x) = 14xe^(x-2) - 12e^(x-2) - 7x^3 + 20x^2 - 26x + 12\n');
f = @(x) 14*x*exp(x-2) - 12*exp(x-2) - 7*x^3 + 20*x^2 - 26*x + 12;
fp = @(x) 14*x*exp(x-2) + 2*exp(x-2) - 21*x^2 + 40*x - 26;
fpp = @(x) 14*x*exp(x-2) + 4*exp(x-2) - 42*x + 40;
x0 = 0;
r = compute_root(f, fp, x0, 1e-9, 1, 2);
fprintf('\tr1 = %.10f\n', r);
fprintf('\tM = lim i->inf (e_(i+1)/(e_i)^2) = %.10f\n\n', ...
    abs(fpp(r)/(2*fp(r))));

x0 = 3.0;
r = compute_root(f, fp, x0, 1e-9, 1, 1);
fprintf('\tr2 = %.10f\n', r);
fprintf('\tmultiplicity of r2 is 3 -> ');
fprintf('S = lim i->inf (e_(i+1)/e_i) = %.10f\n', 2/3);
end

% Compute the foot to f(x)
function r = compute_root(f, fp, x0, tol, m, print_ei)
    if nargin < 4
        tol = 1e-9;
    end
    if nargin < 5
        m = 1;
    end
    if nargin < 6
        print_ei = 0;
    end

    r = fzero(f, x0);

    n = 0;
    x = x0;
    ei = 1;
    ei_1 = 1;
    while (abs(f(x)) >= 0.5 * tol)
        x = x - m * f(x) / fp(x);
        n = n + 1;
        ei_1 = ei;
        ei = abs(r - x);
        if print_ei == 1

```

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        fprintf('\te_i = %.8f; e_(i+1)/e_i = %.8f\n', ei, ei/ei_1);
    elseif print_ei == 2
        fprintf('\te_i = %.8f; e_(i+1)/(e_i)^2 = %.8f\n', ei, ei/(ei_1^2));
    end
end
fprintf('\tn = %d\n', n)
r = x;
end

```

Output

Q1a:

```

f(x) = x^3 - 2x - 2 = 0
x0 = 2
n = 4
r = 1.7692923542

```

Q1b:

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f(x) = e^x + x - 7 = 0
x0 = 0
n = 7
r = 1.6728216986

```

Q1c:

```

f(x) = e^x + sin(x) - 4 = 0
x0 = 2
n = 5
r = 1.1299804987

```

Q3a:

```

f(x) = 27x^3 + 54x^2 + 36x + 8 = 0
x0 = 0
n = 31
xc = -0.6666638081419596
Forward Error = |r - xc| = 0.0000028585247071
Backward Error = f(xc) = 0.0000000000000000
multiplicity of r is 3
n = 1
xc = -0.6666666666666666
Forward Error = |r - xc| = 0.0000000000000000
Backward Error = f(xc) = 0.0000000000000000

```

Q9:

```

f(x) = 14xe^(x-2) - 12e^(x-2) - 7x^3 + 20x^2 - 26x + 12
e_i = 0.45386858; e_(i+1)/(e_i)^2 = 0.45386858
e_i = 0.19642053; e_(i+1)/(e_i)^2 = 0.95351302
e_i = 0.05608698; e_(i+1)/(e_i)^2 = 1.45374528
e_i = 0.00639065; e_(i+1)/(e_i)^2 = 2.03151817
e_i = 0.00009651; e_(i+1)/(e_i)^2 = 2.36321178
e_i = 0.00000002; e_(i+1)/(e_i)^2 = 2.41307008
e_i = 0.00000000; e_(i+1)/(e_i)^2 = 1.75788724
n = 7
r1 = 0.8571428571
M = lim i->inf (e_(i+1)/(e_i)^2) = 1.6993918897

e_i = 0.73383688; e_(i+1)/e_i = 0.73383688
e_i = 0.52975850; e_(i+1)/e_i = 0.72190225

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e_i = 0.37665244; e_(i+1)/e_i = 0.71098896
e_i = 0.26413220; e_(i+1)/e_i = 0.70126241
e_i = 0.18301754; e_(i+1)/e_i = 0.69290129
e_i = 0.12555240; e_(i+1)/e_i = 0.68601292
e_i = 0.08544813; e_(i+1)/e_i = 0.68057740
e_i = 0.05780157; e_(i+1)/e_i = 0.67645219
e_i = 0.03892465; e_(i+1)/e_i = 0.67341853
e_i = 0.02612762; e_(i+1)/e_i = 0.67123586
e_i = 0.01749716; e_(i+1)/e_i = 0.66968057
e_i = 0.01169797; e_(i+1)/e_i = 0.66856388
e_i = 0.00781113; e_(i+1)/e_i = 0.66773373
e_i = 0.00521055; e_(i+1)/e_i = 0.66706772
e_i = 0.00347263; e_(i+1)/e_i = 0.66646198
e_i = 0.00231214; e_(i+1)/e_i = 0.66581839
e_i = 0.00153764; e_(i+1)/e_i = 0.66502955
e_i = 0.00102094; e_(i+1)/e_i = 0.66396188
e_i = 0.00067630; e_(i+1)/e_i = 0.66242890
e_i = 0.00044646; e_(i+1)/e_i = 0.66015724
n = 20
r2 = 2.0004598418
multiplicity of r2 is 3 -> S = lim i->inf (e_(i+1)/e_i) = 0.666666667

```