MATH 446: Project 02

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Contents

Questions

1. $f(x) = 3x^3 - 7x^2 + 3x - e^x + 2 = 0$

$$\begin{gathered} g1(x) = \frac{e^x - 2}{3x^2 - 7x + 3} = x \\ r1 = -0.24789639 \\ x_0 = -1 \\ number of steps = 17 \end{gathered} \tag{1}$$

$$\begin{gathered} g_2(x) = \frac{3x^4 - 7x^3 + 3x^2 + 2x}{e^x} = x \\ r2 = 0.62616943 \\ x_0 = 1 \\ number of steps = 22 \end{gathered} \tag{2}$$

$$\begin{gathered} g_3(x) = \left(\frac{-7x^2 + 3x - e^x + 2}{-3.0}\right)^{\frac{1}{3}} = x \\ r3 = 2.46222248 \\ x_0 = 2 \\ number of steps = 84 \end{gathered} \tag{3}$$

$$\begin{gathered} g_4(x) = \ln(3x^3 - 7x^2 + 3x + 2) = x \\ r4 = 6.07305409 \\ x_0 = 6 \\ number of steps = 34 \end{gathered} \tag{4}$$

2. $S = |g'(r)|$

$$\begin{gathered} \frac{d}{dx} g_1(x) = \frac{e^x(3x^2 - 13x + 10) + 2(6x - 7)}{(3x^2 - 7x + 3)^2} \\ |\frac{d}{dx} g_1(r1)| = 0.269034 \end{gathered} \tag{5}$$

$$\begin{gathered} \frac{d}{dx} g_2(x) = \frac{-3x^4 + 19x^3 - 24x^2 + 4x + 2}{e^x} \\ |\frac{d}{dx} g_2(r2)| = 0.375249 \end{gathered} \tag{6}$$

$$\begin{gathered} \frac{d}{dx} g_3(x) = \frac{1}{3}\left(\frac{-7x^2 + 3x - e^x + 2}{3}\right)^{\frac{-2}{3}}\left(\frac{-14x + 3 - e^x}{-3}\right) \\ |\frac{d}{dx} g_3(r3)| = 0.791783 \end{gathered} \tag{7}$$

$$\begin{gathered} \frac{d}{dx} g_4(x) = \frac{9x^2 - 14x + 3}{3x^3 - 7x^2 + 3x + 2} \\ |\frac{d}{dx} g_4(r4)| = 0.575836 \end{gathered} \tag{8}$$

3. $\lim_{k\to\infty} \frac{e_{k+1}}{e_k} = S$
   1. For r1: $\lim_{k\to\infty} \frac{e_{k+1}}{e_k} \approx 0.2690342181$
   2. For r2: $\lim_{k\to\infty} \frac{e_{k+1}}{e_k} \approx 0.3752496237$
   3. For r3: $\lim_{k\to\infty} \frac{e_{k+1}}{e_k} \approx 0.7917836336$
   4. For r4: $\lim_{k\to\infty} \frac{e_{k+1}}{e_k} \approx 0.5758353631$

Code

Fixed Point Iterative Method

```
% Computes the fixed point of a function using the FPI.
% Written by Zachary Ferguson

function xc = fixed_point_iteration(g, x0, f, tol)
    % Compute the fixed point of g(x).
    % Input:
    %   g   - function to solve for the fixed point.
    %   x0  - initial guess
    %   f   - f(x) = g(x) - x
    %   tol - solution tolerance
    % Output:
    %   xc - computed root to the function g(x) = x.
    if nargin < 4
        tol = 1e-9;
    end

    r = fzero(f, x0);
    fprintf('r = %f\n', r);
    ei = 0;

    prev_x = x0;
    x = g(x0);
    n = 1;
    while (abs(prev_x - x) > 0.5 * tol)
        prev_x = x;
        x = g(x);
        n = n + 1;
        ei1 = abs(x - r);
        if (abs(prev_x - x) <= 0.5 * tol)
             fprintf('e_(i+1)/e_i = %.10f\n', ei1 / ei);
        end
        ei = ei1;
    end
    fprintf('n = %d\n', n);
    xc = x;
end
```

Main

```
% MATH 446: Project 02
% Written by Zachary Ferguson
```

```
function main()
    fprintf('MATH 446: Project 02\nWritten by Zachary Ferguson\n\n');

    fprintf('f(x) = 3*x^3 - 7*x^2 + 3*x - e^x + 2 = 0\n\n');
    f = @(x) 3*x^3 - 7*x^2 + 3*x - exp(x) + 2;

    fprintf('g1(x) = (e^x - 2) / (3x^2 - 7x + 3) = x\n')
    g1 = @(x) (exp(x) - 2) / (3*x^2 - 7*x + 3);
    fprintf('r1 = %.10f\n\n', fixed_point_iteration(g1, -1, f));

    fprintf('g2(x) = (3*x^4 - 7*x^3 + 3*x^2 + 2*x) / e^x = x\n');
    g2 = @(x) (3*x^4 - 7*x^3 + 3*x^2 + 2*x) / exp(x);
    fprintf('r2 = %.10f\n\n', fixed_point_iteration(g2, 1, f));

    fprintf('g3(x) = ((-7*x^2 + 3*x - e^x + 2) / -3.0)^(1/3) = x\n');
    g3 = @(x) ((-7*x^2 + 3*x - exp(x) + 2) / -3.0)^(1/3);
    fprintf('r3 = %0.10f\n\n', fixed_point_iteration(g3, 2, f));

    fprintf('g4(x) = ln(3*x^3 - 7*x^2 + 3*x + 2) = x\n');
    g4 = @(x) log(3*x^3 - 7*x^2 + 3*x + 2);
    fprintf('r4 = %.10f\n', fixed_point_iteration(g4, 6, f));
end
```

Output

```
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f(x) = 3*x^3 - 7*x^2 + 3*x - e^x + 2 = 0

g1(x) = (e^x - 2) / (3x^2 - 7x + 3) = x
r = -0.247896
e_(i+1)/e_i = 0.2690342181
n = 17
r1 = -0.2478963963

g2(x) = (3*x^4 - 7*x^3 + 3*x^2 + 2*x) / e^x = x
r = 0.626169
e_(i+1)/e_i = 0.3752496237
n = 22
r2 = 0.6261694387

g3(x) = ((-7*x^2 + 3*x - e^x + 2) / -3.0)^(1/3) = x
r = 2.462222
e_(i+1)/e_i = 0.7917836336
n = 84
r3 = 2.4622224868

g4(x) = ln(3*x^3 - 7*x^2 + 3*x + 2) = x
r = 6.073054
e_(i+1)/e_i = 0.5758353631
n = 34
r4 = 6.0730540924
```