For reference, we list here the formulas for the 8 posterior probabilities.

1.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (F, A, YC), D_0, I) &= \sum_{N^{s|f,a,yc}} p(S_k = 1 | x_k = (F, A, YC), N^{s|f,a,yc}, D_0, I) \\
&\times \frac{p(N^{s|f,a,yc}|I) * p(D_0|N^{s|f,a,yc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|f,a,yc}} \frac{N^{s|f,a,yc} - N_0^{s|f,a,yc}}{N^{f,a,yc} - N_0^{f,a,yc}} \\
&\times \left(\begin{array}{c} N^{f,a,yc} \\ N_0^{f,a,yc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|f,a,yc} \\ N_0^{s|f,a,yc} \end{array} \right) * \left(\begin{array}{c} N^{f,a,yc} - N^{s|f,a,yc} \\ N_0^{f,a,yc} - N_0^{s|f,a,yc} \end{array} \right) \\
&\times \sum_{N^{s|f,a}} p(N^{s|f,a} - N^{s|f,a,yc} | N^{s|f,a}, I) * \sum_{N^{s|f}} p(N^{s|f,a} | N^{s|f}, I) * p(N^s - N^{s|f} | I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|f,a,nc}$, $N^{s|f,a}$ and $N^{s|m}$.

2.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (F, A, NC), D_0, I) &= \sum_{N^{s|f,a,nc}} p(S_k = 1 | x_k = (F, A, NC), N^{s|f,a,nc}, D_0, I) \\
&\times \frac{p(N^{s|f,a,nc}|I) * p(D_0|N^{s|f,a,nc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|f,a,nc}} \frac{N^{s|f,a,nc} - N_0^{s|f,a,nc}}{N^{f,a,nc} - N_0^{f,a,nc}} \\
&\times \left(\begin{array}{c} N^{f,a,nc} \\ N_0^{f,a,nc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|f,a,nc} \\ N_0^{s|f,a,nc} \end{array} \right) * \left(\begin{array}{c} N^{f,a,nc} - N^{s|f,a,nc} \\ N_0^{f,a,,nc} - N_0^{s|f,a,nc} \end{array} \right) \\
&\times \sum_{N^{s|f,a}} p(N^{s|f,a,nc} | N^{s|f,a}, I) * \sum_{N^{s|f}} p(N^{s|f,a} | N^{s|f}, I) * p(N^s - N^{s|f} | I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|f,a,nc}$, $N^{s|f,a}$ and $N^{s|m}$

3.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (F, C, YC), D_0, I) &= \sum_{N^{s|f,c,yc}} p(S_k = 1 | x_k = (F, C, YC), N^{s|f,c,yc}, D_0, I) \\
&\times \frac{p(N^{s|f,c,yc}|I) * p(D_0|N^{s|f,c,yc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|f,c,yc}} \frac{N^{s|f,c,yc} - N_0^{s|f,c,yc}}{N^{f,c,yc} - N_0^{f,c,yc}} \\
&\times \left(\begin{array}{c} N^{f,c,yc} \\ N_0^{f,c,yc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|f,c,yc} \\ N_0^{s|f,c,yc} \end{array} \right) * \left(\begin{array}{c} N^{f,c,yc} - N^{s|f,c,yc} \\ N_0^{f,c,yc} - N_0^{s|f,c,yc} \end{array} \right) \\
&\times \sum_{N^{s|f,c}} p(N^{s|f,c} - N^{s|f,c,yc} | N^{s|f,c}, I) * \sum_{N^{s|f}} p(N^{s|f} - N^{s|f,c} | N^{s|f}, I) * p(N^s - N^{s|f} | I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|f,c,nc}$, $N^{s|f,a}$ and $N^{s|m}$

4.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (F, C, NC), D_0, I) &= \sum_{N^{s|f,c,nc}} p(S_k = 1 | x_k = (F, C, NC), N^{s|f,c,nc}, D_0, I) \\
&\times \frac{p(N^{s|f,c,nc}|I) * p(D_0|N^{s|f,c,nc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|f,c,nc}} \frac{N^{s|f,c,nc} - N_0^{s|f,c,nc}}{N^{f,c,nc} - N_0^{f,c,nc}} \\
&\times \left(\begin{array}{c} N^{f,c,nc} \\ N_0^{f,c,nc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|f,c,nc} \\ N_0^{s|f,c,nc} \end{array} \right) * \left(\begin{array}{c} N^{f,c,nc} - N^{s|f,c,nc} \\ N_0^{f,c,nc} - N_0^{s|f,c,nc} \end{array} \right) \\
&\times \sum_{N^{s|f,c}} p(N^{s|f,c,nc}|N^{s|f,c}, I) * \sum_{N^{s|f}} p(N^{s|f} - N^{s|f,c}|N^{s|f}, I) * p(N^s - N^{s|f}|I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|f,c,nc}$, $N^{s|f,a}$ and $N^{s|m}$

5.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (M, A, YC), D_0, I) &= \sum_{N^{s|m,a,yc}} p(S_k = 1 | x_k = (M, A, YC), N^{s|m,a,yc}, D_0, I) \\
&\times \frac{p(N^{s|m,a,yc}|I) * p(D_0|N^{s|m,a,yc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|m,a,yc}} \frac{N^{s|m,a,yc} - N_0^{s|m,a,yc}}{N^{m,a,yc} - N_0^{m,a,yc}} \\
&\times \left(\begin{array}{c} N^{m,a,yc} \\ N_0^{m,a,yc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|m,a,yc} \\ N_0^{s|m,a,yc} \end{array} \right) * \left(\begin{array}{c} N^{m,a,yc} - N^{s|m,a,yc} \\ N_0^{m,a,yc} - N_0^{s|m,a,yc} \end{array} \right) \\
&\times \sum_{N^{s|m,a}} p(N^{s|m,a} - N^{s|m,a,yc}|N^{s|m,a}, I) * \sum_{N^{s|m}} p(N^{s|m,a}|N^{s|m}, I) * p(N^{s|m}|I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|m,a,nc}$, $N^{s|m,a}$ and $N^{s|m}$

6.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (M, A, NC), D_0, I) &= \sum_{N^{s|m,a,nc}} p(S_k = 1 | x_k = (M, A, NC), N^{s|m,a,nc}, D_0, I) \\
&\times \frac{p(N^{s|m,a,nc}|I) * p(D_0|N^{s|m,a,nc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|m,a,nc}} \frac{N^{s|m,a,nc} - N_0^{s|m,a,nc}}{N^{m,a,nc} - N_0^{m,a,nc}} \\
&\times \left(\begin{array}{c} N^{m,a,nc} \\ N_0^{m,a,nc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|m,a,nc} \\ N_0^{s|m,a,nc} \end{array} \right) * \left(\begin{array}{c} N^{m,a,nc} - N^{s|m,a,nc} \\ N_0^{m,a,nc} - N_0^{s|m,a,nc} \end{array} \right) \\
&\times \sum_{N^{s|m,a}} p(N^{s|m,a,nc}|N^{s|m,a}, I) * \sum_{N^{s|m}} p(N^{s|m,a}|N^{s|m}, I) * p(N^{s|m}|I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|m,a,nc}$, $N^{s|m,a}$ and $N^{s|m}$

7.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (M, C, YC), D_0, I) &= \sum_{N^{s|m,c,yc}} p(S_k = 1 | x_k = (M, C, YC), N^{s|m,c,yc}, D_0, I) \\
&\times \frac{p(N^{s|m,c,yc}|I) * p(D_0|N^{s|m,c,yc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|m,c,yc}} \frac{N^{s|m,c,yc} - N_0^{s|m,c,yc}}{N^{m,c,yc} - N_0^{m,c,yc}} \\
&\times \left(\begin{array}{c} N^{m,c,yc} \\ N_0^{m,c,yc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|m,c,yc} \\ N_0^{s|m,c,yc} \end{array} \right) * \left(\begin{array}{c} N^{m,c,yc} - N^{s|m,c,yc} \\ N_0^{m,c,yc} - N_0^{s|m,c,yc} \end{array} \right) \\
&\times \sum_{N^{s|m,c}} p(N^{s|m,c} - N^{s|m,c,yc} | N^{s|m,c}, I) * \sum_{N^{s|m}} p(N^{s|m} - N^{s|m,c} | N^{s|m}, I) * p(N^{s|m}|I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|m,c,nc}$, $N^{s|m,a}$ and $N^{s|m}$

8.

$$
\begin{aligned}
p(S_k = 1 \mid x_k = (M, C, NC), D_0, I) &= \sum_{N^{s|m,c,nc}} p(S_k = 1 | x_k = (M, C, NC), N^{s|m,c,nc}, D_0, I) \\
&\times \frac{p(N^{s|m,c,nc}|I) * p(D_0|N^{s|m,c,nc}, I)}{p(D_0|I)} \\
&= \frac{1}{p(D_0|I)} * \sum_{N^{s|m,c,nc}} \frac{N^{s|m,c,nc} - N_0^{s|m,c,nc}}{N^{m,c,nc} - N_0^{m,c,nc}} \\
&\times \left(\begin{array}{c} N^{m,c,nc} \\ N_0^{m,c,nc} \end{array} \right)^{-1} * \left(\begin{array}{c} N^{s|m,c,nc} \\ N_0^{s|m,c,nc} \end{array} \right) * \left(\begin{array}{c} N^{m,c,nc} - N^{s|m,c,nc} \\ N_0^{m,c,nc} - N_0^{s|m,c,nc} \end{array} \right) \\
&\times \sum_{N^{s|m,c}} p(N^{s|m,c,nc} | N^{s|m,c}, I) * \sum_{N^{s|m}} p(N^{s|m} - N^{s|m,c} | N^{s|m}, I) * p(N^{s|m}|I),
\end{aligned}
$$

where the three probabilities in the last line are respectively the prior distribution of $N^{s|m,c,nc}$, $N^{s|m,a}$ and $N^{s|m}$