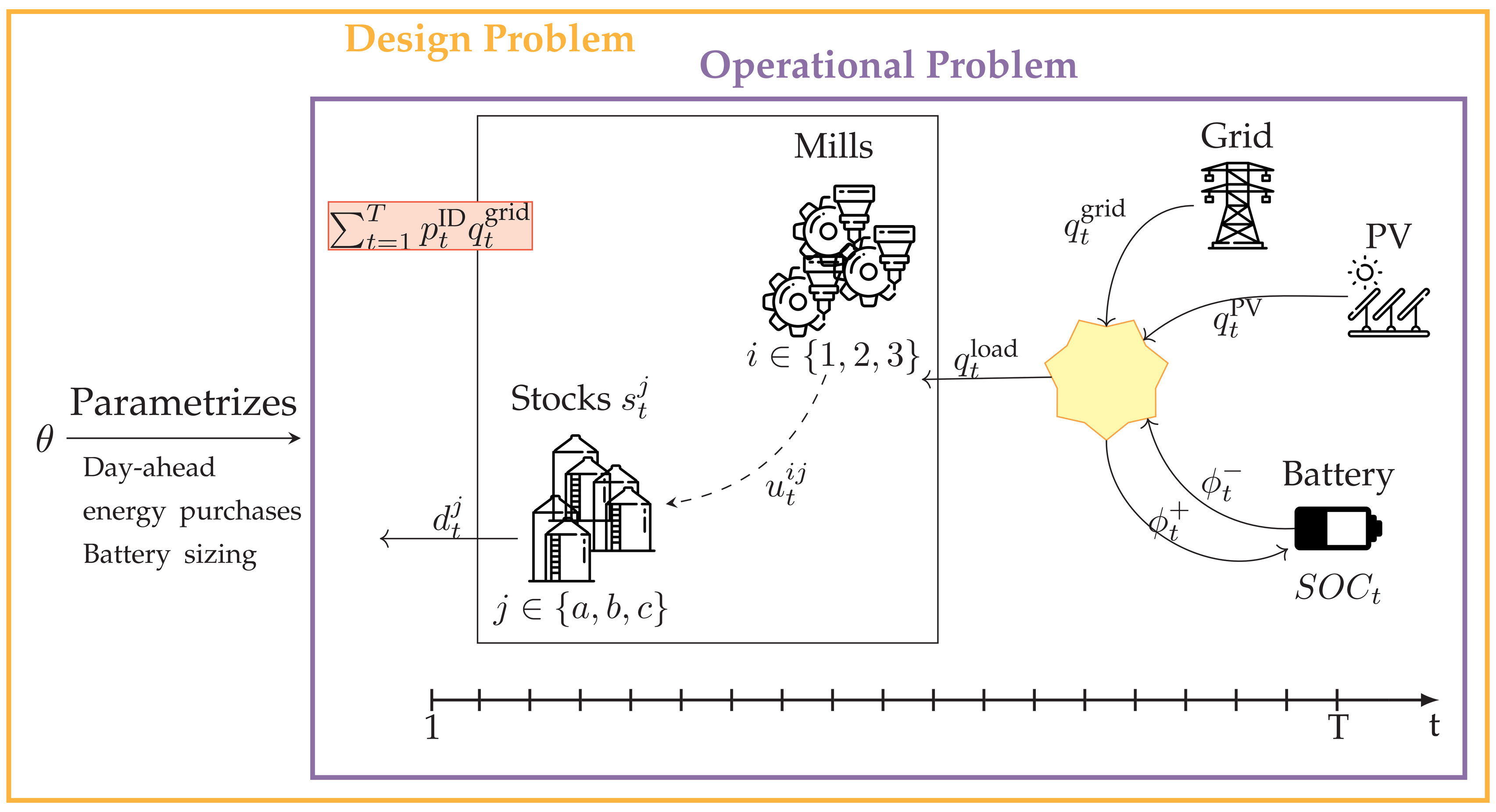
Factory Production and Energy Supply Planning with Multistage Stochastic Optimization

Zoé Fornier[1,2], Vincent Leclère[2]

[1] METRON, [2] CERMICS (Ecole des Ponts)

Motivations: study case



Problem Find a production and energy supply plan minimizing expected daily energy costs ($T = 24$). Energy is bought in real time or in advance.

Decision Variables

Continuous: production, stocks, energy purchases, charge and stocks;
Binary: $b_t^{ij} = \mathbb{1}_{\{j \text{ produced by } i \text{ at } t\}}$

Constraints

Dynamic Equations for energy and product stocks;
Shared resources: hard constraints (binary)
Demand: by product.

Data

Deterministic energy prices and demand;
Stochastic: solar energy q_t^{PV}.

Mathematical Model

We consider the strategic problem:

$$\boxed{(\mathrm{P}): \quad \min_{\theta \in \Theta} \quad I(\theta) + V(x_0; \theta)}$$

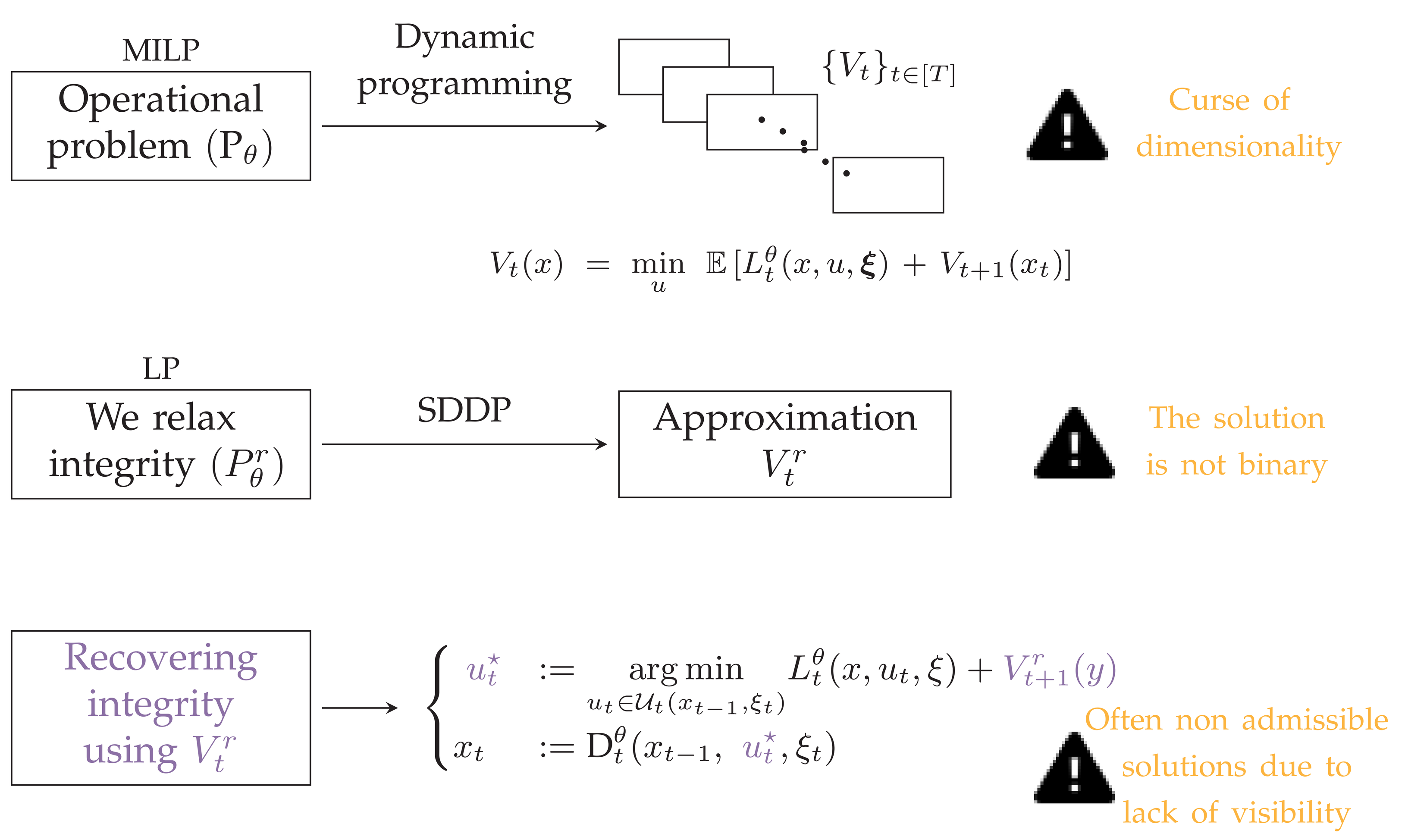
where $V(x_0;\theta)$ is the optimal value of a **multistage mixed-integer stochastic problem:**

$$
\begin{aligned}
(\mathrm{P}_\theta) \qquad V(x_0;\theta) := \min_{(\boldsymbol{u_t}, \boldsymbol{x_t})_{t\in[T]}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} L_t^\theta(\boldsymbol{x_{t-1}}, \boldsymbol{u_t}, \boldsymbol{\xi_t})\right] \\
\text{s.t.} \quad & \boldsymbol{x_t} = \mathrm{D}_t^\theta(\boldsymbol{x_{t-1}}, \boldsymbol{u_t}, \boldsymbol{\xi_t}) && \forall t \in [T], \\
& \boldsymbol{x_t} \in X_t^\theta && \forall t \in [T], \\
& \boldsymbol{u_t} \in \mathcal{U}_t^\theta(\boldsymbol{x_{t-1}}, \boldsymbol{\xi_t}) \subset U_t^\theta && \forall t \in [T], \\
& \sigma(\boldsymbol{u_t}) \subset \sigma(\boldsymbol{\xi_1}, \dots, \boldsymbol{\xi_t}) && \forall t \in [T].
\end{aligned}
$$

State of the art

- **Expected Value (EV) Strategy**
  Principle: replace every random variable by its expected value and solve a deterministic program.
  Pros: use of deterministic solvers, no stagewise independence.
  Cons: doesn't consider uncertainties.
- **Model Predictive Control (MPC)**
  Principle: solve deterministic problems, adjusting trajectory as random realizations are revealed.
  Pros: use of deterministic solvers, no stagewise independence.
  Cons: no solution quality guarantee, slow online running time.
- **Stochastic Dynamic Programming (SDP)**
  Principle: with stagewise independence, we solve the problem with dynamic equations.
  Pros: few assumptions, easily implemented
  Cons: curse of dimensionality.
- **Stochastic Dual Dynamic Programming (SDDP)**
  Principle: solves continuous multistage linear stochastic problems with Benders-like cuts.
  Pros: fast in practice, and theoretical guarantee.
  Cons: cannot handle integer variables (without heavy computational burden, see SDDiP).

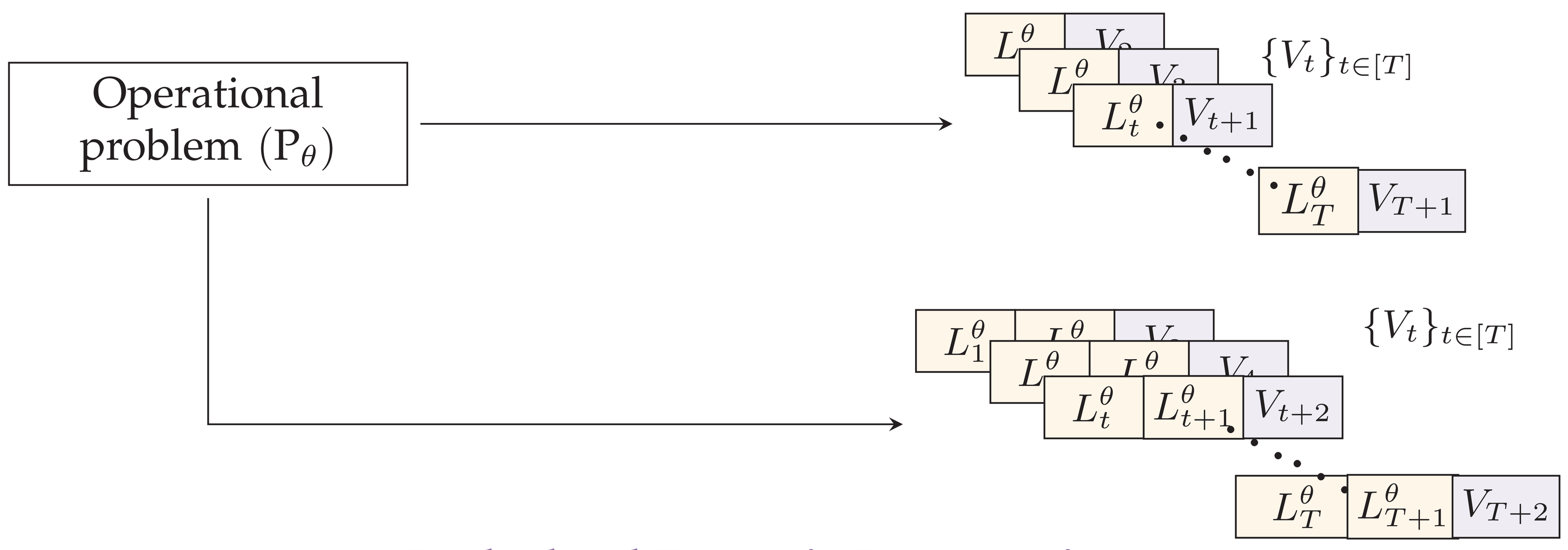
Solving the operational problem



Look-ahead strategy

Dynamic Programming

Reduces a $T-$stage problem to T consecutive $1-$stage problems.



Look-ahead Dynamic Programming

Reduces (P_θ) to $T+1$ consecutive $2-$stage problems.

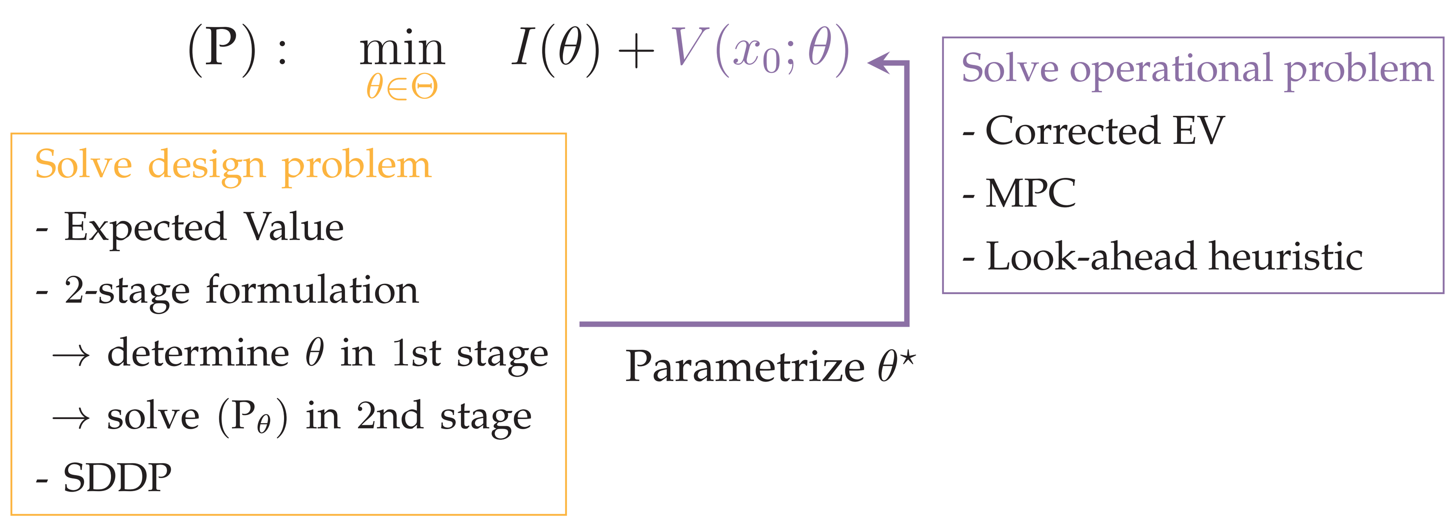
$$
\begin{aligned}
V_t(x) = \min_{\boldsymbol{u} \in \mathcal{U}_t(x,\xi), x_t} \mathbb{E}\Big[L_t^\theta(x, \boldsymbol{u}, \boldsymbol{\xi}_t) + & \min_{\boldsymbol{x_{t+1}}, \boldsymbol{u_{t+1}}} \mathbb{E}\big[L_{t+1}^\theta(\boldsymbol{x_t}, \boldsymbol{u_{t+1}}, \boldsymbol{\xi_{t+1}}) + V_{t+2}(\boldsymbol{x}_{t+1})\big]\Big] \\
& \boldsymbol{x_t} = \mathrm{D}_t^\theta(x, \boldsymbol{u}, \boldsymbol{\xi}), \\
& \boldsymbol{x_{t+1}} = \mathrm{D}_{t+1}^\theta(\boldsymbol{x_t}, \boldsymbol{u_{t+1}}, \boldsymbol{\xi_{t+1}}), \\
& \boldsymbol{u_{t+1}} \in \mathcal{U}_{t+1}(\boldsymbol{x_t}, \boldsymbol{\xi_{t+1}}), \\
& \sigma(\boldsymbol{u_{t+1}}) \subset \sigma(\boldsymbol{\xi}_{t+1}).
\end{aligned}
$$

By keeping integrity constraints at $t+1$, we less often get non-admissible solutions.

Solving the design problem

The problem (P) can be decomposed in two parts:

1. a design problem with variable θ
2. an operational sub-problem (P_θ) parametrized by θ.



Numerical Results

Evaluation Criterion

For a state-based feedback ψ, and a scenario $\xi_{[T]}$, we define the **Anticipative Regret (AR)**:

$$AR^\psi(\xi_{[T]}) = \frac{\hat{V}^\psi(x_0, \xi_{[T]}) - \hat{V}^{\psi_{ant}}(x_0, \xi_{[T]})}{|\hat{V}^{\psi_{ant}}(x_0, \xi_{[T]})|},$$

where $\hat{V}^{\psi_{ant}}$ is the value obtained knowing the full scenario from the beginning.

Operational problem results

SOC_{max}	0.5h			3h			6h		
Solar factor	L-A	MPC	EV	L-A	MPC	EV	L-A	MPC	EV
0.5	4.9	**0.5**	1.0	6.1	**0.5**	2.4	5.4	**0.5**	3.2
1.0	6.1	**1.3**	4.6	3.9	**0.9**	6.3	2.4	**0.6**	6.4
2.0	8.7	**3.9**	14	4.5	**1.5**	15	4.0	**1.4**	15
3.0	11	**5.6**	27	9.1	**3.6**	28	8.2	**3.5**	28

Table 1: Anticipative Regret (AR) in % for different methods (EV strategy, MPC, Look-ahead) for the operational problem: MPC yields the most satisfactory results.

Design problem results

Solar Factor	OPT MPC	OPT 2stage	OPT SDDP	AR (in %) MPC	AR (in %) 2stage	AR (in %) SDDP
0.5	6067	**6023**	6038	1.6	**0.9**	1.1
1.0	5471	5483	**5451**	2.1	2.3	**1.7**
2.0	4552	4553	**4481**	4.2	4.2	**2.5**
3.0	3714	3691	**3641**	8.7	7.9	**6.7**

Table 2: Expected Cost (Opt) and Anticipative Regret (AR) for different methods (EV, 2−stage, SDDP) determining θ and then MPC.

References

[1] Z. Fornier, D. Grosso, and V. Leclere. Joint production and energy supply planning of an industrial microgrid. https://hal.science/hal-03927692, Jan. 2023. working paper or preprint.