

Heap

heapsort

1. build max heap
2. Exchange the root node with the last node
3. Heapify again
4. Repeat the above until the heap is empty

priority queue using heap

Maximum $O(1)$ Insert $O(\log n)$
Extract_Maximum $O(\log n)$ Increase-Key $O(\log n)$

Hash Table

Open addressing

- linear probing
- quadratic probing
- double hashing \rightarrow close to uniform hashing

Chaining

better implementation: BST

Hash function example

1. Division: $h(k) = k \% m$
2. Mid-square: $h(k) = \text{bits}(i, i+r-1)(k^2)$ 改成二進位，取任意段
3. shift folding
4. digit analysis

Dynamic hashing

ex. using directories:

directory depth = number of bits of the index of the hash table

only processing the overflow box, other noew boxes just point to the same address

ex. directoryless dynamic hashing

Disjoint Set

Implementation

1. Array: $O(1)$ find_set
union by size
2. Tree: $O(1)$ union use array: store the address of the node
union by height
path compression
3. Linklist representation: head, tail, data
union by size

Linear Sorting

Break even point is about $k \cdot 10^6$

Counting Sort:

given k = number of possible input element $\rightarrow O(n+k)$

after $C[]$ is completed

for ($j=n; j \geq 1; j--$) { \rightarrow stable

$B[C[A[j]]] = A[j];$

$C[A[j]]--;$

}

Radix Sort:

LSD first

time complexity of RadixSort(implemented by counting sort) : $O((n + r) \log k / \log r)$

when $r=10$, $\log k / \log r = d$ = number of digits

RB Tree

Rule:

- 1.root is black
- 2.leaf(nil) is black
- 3.children of a red node are black
- 4.same black height

$nil.left = nil.right = root$

$n \leq 2^{(h/2)} - 1$

Insert:

let z be red, insert like BST.

//possibly violated rule: 2 & 3

if z is root \rightarrow z be black

if z.p is red

//assume $z.p = z.p.p.left$, else:change all left&right

if uncle is red $\rightarrow z.p.p = red$, $z.p.children = black$, check $z.p.p.p$ (let $z = z.p.p$)

if uncle is black

if $z = z.p.right$

$z = z.p$, left_rotate(z)

right_rotate(z.p.p) //note: wont have to check again

Delete:

\leftarrow ????

define x,y,z as in the slide

possibly violated rule: 2 & 3 & 4, but only when y is black

[draw]

Graph

$\langle u, v \rangle$ leaves u and enters v in digraph

simple path : vertice distinct

strongly connected: for every distinct (u,v),

there is a directed path from v to u and another from u to v

representation:

adjacency matrix: $O(V^2)$ space

adjacency list: $O(V+E)$ space

*inverse adjacency list

DFS

$O(V+E)$ using adjacency list

mark v.pi (its parent) to build a DFtree/forest

Edge

tree edge: in the tree

back edge: to a black vertex

forward edge & cross edge: only in digraph

B-tree

[draw]

keys:non-increasing order

Minimum degree of B-tree: $t \geq 2$

Every node other than root have at least $t-1$ keys \rightarrow t children

Every node can have at most $2t-1$ keys \rightarrow $2t$ children (In this case, this node is full)

height $\leq \log \frac{(n+1)/2}{t}$

FFT

pointwise multiplication: find $2n$ points $\rightarrow O(n)$

Evaluation & Interpolation

Assumption: n is a power of 2.