

Problem 1

1.hash table

$S = \{18, 34, 9, 37, 40, 32, 89\}$

$h_1(S) = \{7, 1, 9, 4, 7, 10, 1\}$

$h_2(S) = \{9, 5, 10, 8, 1, 3, 10\}$

double hashing

round 1	0	0	0	0	0	0	0	18	0	0	0
round 2	0	34	0	0	0	0	0	18	0	0	0
round 3	0	34	0	0	0	0	0	18	0	9	0
round 4	0	34	0	0	37	0	0	18	0	9	0
round 5	0	34	0	0	37	0	0	18	40	9	0
round 6	0	34	0	0	37	0	0	18	40	9	32
round 7	89	34	0	0	37	0	0	18	40	9	32

Linear probing

round 1	0	0	0	0	0	0	0	18	0	0	0
round 2	0	34	0	0	0	0	0	18	0	0	0
round 3	0	34	0	0	0	0	0	18	0	9	0
round 4	0	34	0	0	37	0	0	18	0	9	0
round 5	0	34	0	0	37	0	0	18	40	9	0
round 6	0	34	0	0	37	0	0	18	40	9	32
round 7	0	34	89	0	37	0	0	18	40	9	32

2.(a)

According to the graph, $n=2$.

11不會指到10指到的那欄

$[00] = [4k]$, $[01] = [4k+1]$, $[10] = [4k+2]$, $[11] = [4k+3]$. thus,

2 in $[01]$ is wrong -> 2 belongs to $[10]$

13 in $[10]$ is wrong -> 13 belongs to $[01]$

Graph				
0 0	128	64	32	16
0 1	25	1	17	13
1 0	2	10	14	
1 1	31	7	3	

2.(b)

Index	Data				
0 0	0 0 0	128	64	32	16
	1 0 0	4			
0 1	0 0 1	25	1	17	49
	1 0 1	13			
1 0		2	10	14	30
1 1		31	7	3	

3.

兩個玩家分別先將自己要出的拳輸入hash function, 然後在FB訊息貼上hash值, 雙方在互相確認對方出拳.但為了避免hash值被認出, 需要為每次出拳配置一個亂碼.

Algorithm:

Let (rock, paper, scissors)=(1,2,3)

要求每位玩家出拳的同時,自己附上3碼數字亂碼abc,

hash function= $7^a + 11^b + 13^c + 17^{\text{出拳}}$

FB訊息上雙方同時打出 (hash值 , abc)

一眼看不出來對方出什麼就解決了有人慢出的問題

雙方都出拳後再互相由hash值及亂碼反推出拳即可得到結果

4.(a)

[6, 31, 2, 41, 30, 45, 44]

T1 index	Data
0	
1	
2	44
3	31
4	
5	
6	6

T2 index	Data
0	2
1	
2	
3	
4	30
5	41
6	45

4.(b)

取 T1 三格與 T2 兩格 ($h1(k)=x1,x2,x3$ $h2(k)=y1,y2,y3$) , 總共只能填 6 筆資料 ,

但(x_i,y_i) 共有 9 種組合。當 7 種組合出現時, 會出現infinite sequence of displacement

Let $x1=2,x2=3,x3=6$, $y1=4,y2=0,y3=6$

2 -> (2,0)

6 -> (6,0)

30 -> (2,4)

31 -> (3,4)

44 -> (2,6)

45 -> (3,6)

34 -> (6,4)

Insert 34-> move 6 -> 2-> 44-> 45-> 31-> 30-> 2 -> 6 -> 34.....

infinite sequence of displacement occur when inserting 34

Problem 2

1. Max Heap: [10, 9, 8, 6, 7, 2, 1, 5, 4, 3]
In-order: [5, 6, 4, 9, 7, 3, 10, 2, 8, 1]

2. Min Heap: [1, 4, 2, 5, 7, 8, 3, 10, 6, 9]
In-order: [10, 5, 6, 4, 7, 9, 1, 8, 2, 3]

3.

```
int q, index=0;
```

```
int arr[q];
```

```
void find_smaller(struct Node *t){
    arr[index]=t->data;
    index++;
    if(t->left != NULL)
        if(t->left->data < q)
            find_smaller(t->left);
    if(t->right != NULL)
        if(t->right->data < q)
            find_smaller(t->right);
    return;
}
int main(){
    #gain q and min tree
    find_smaller(root);
    print arr, k=strlen(arr)
}
```

The function find_smaller will be called k times,
Each time has a complexity of $O(1)$,
so total complexity is $O(k)$.

4.

(Reference: heap 上課ppt)

如果有個資料在 $a[i]$, (index從1開始)

他會在第 $\lceil \log i \rceil$ 層, 他的parent是 $a[\lceil i/2 \rceil]$

他的children是 $a[2i]$, $a[2i+1]$

#array index start from 1

```
void heapify( int arr[]){
    L=strlen(arr)
    for(i=L/2;i>=0;i--){
        j=i
        while(2j<=L){ //is not leaf
            child_1=arr[2j]
            child_2=arr[2j+1]
            if(arr[j]<child_1)
                swap(arr[j],child_1)
                j=2j
            else if(arr[j]<child_2)
                swap(arr[j],child_2)
                j=2j+1
            else
                break
        }
    }
}
```

$$\begin{aligned} &= h + 2(h-1) + 2^2(h-2) + \dots + \\ &\quad + 2^{h-1} \cdot 1 \\ &= \sum_{i=0}^{h-1} 2^i (h-i) \\ &= h \sum_{i=0}^{h-1} 2^i - \sum_{i=0}^{h-1} 2^i (h-i) \\ &= 2h = 2h + 4(h-1) + \dots + 2^h \\ &= 2h - h = -h + 2 + 1 + \dots + 2^h \\ &= S - 2^{h+1} = h - 2 \\ &\Rightarrow \mathcal{A}(h) = O(\log n) \\ &= 2^h n_2 n_1 = O(\log n) - 2 \\ &= \leq 2n - 6(\log n) \\ &= O(n) \end{aligned}$$

}

according to [heap 上課ppt page10] \rightarrow
Time complexity analysis $= O(N)$

Problem 3

1. $\langle x \rangle$ represent an end of a word(double circle)
graph—→

2.

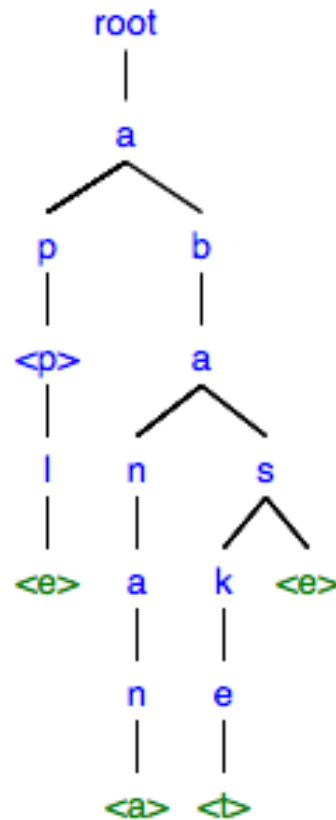
```

struct Node *root;
void insert(char word[],int N){
    struct Node *temp=root;
    for(int i=0;i<N;i++){
        temp=temp->children[word[i]-'a'];
        temp->is_word=1;
        temp->tag++;
    }
}

void delete(char word[],int N){
    struct Node *temp=root;
    for(int i=0;i<N;i++){
        temp=temp->children[word[i]-'a'];
        temp->is_word=0;
        if(temp->tag > 0) temp->tag--;
    }
}

int query(char word[], int N){
    struct Node *temp=root;
    for(int i=0;i<N;i++){
        if(temp->children[word[i]-'a']->is_word==0)
            return 0;
        temp=temp->children[word[i]-'a'];
    }
    return 1;
}

```



each function contain a single for loop, which will run N times, and each time takes $O(1)$.

3.

建兩棵樹，一顆是原本的trie (trie1)，另一顆先把輸入字串(Wi)反過來再建trie (trie2)。

如此做，則，若S'是S反過來的字串，找

“S is a suffix of how many words in trie1”的作法等同於 “S’ is a prefix of how many words in trie2”.

另外，當 insert & delete 的時候改變tag值 (見上題code)，

tag值即是從root到該字的字串是多少個字的prefix。

```

int how_many_prefix(char word[], int N){
    struct Node *temp=root;
    for(int i=0;i<N;i++){
        if(temp->children[word[i]-'a']->is_word==0)
            return 0;
        temp=temp->children[word[i]-'a'];
    }
    return temp->tag;
}

```

4.

```

struct Node *root;
//initialize: level=0,temp=root;
int is_leaf(struct Node *temp){
    int k=1;
    for(int i=0;i<26;i++){
        if(temp->children[i]->is_word!=0)
            k=0;
    }
    return k;
}
int first_player_strategy(struct Node *temp, int level){
    if (is_leaf) //when is_leaf and level is odd, first player win, return true
        return (level%2);

    //when level is odd, second player's turn
    //if all choices lead to first player's win, return true
    if(level%2){
        int now=1;
        for(int i=0;i<26;i++){
            if(!first_player_strategy(temp->children[i],level+1))
                now=0;
        }
        return now;
    }

    //when level is even, first player's turn
    //if any choice lead to first player's win, return true
    else{
        int now=0;
        for(int i=0;i<26;i++){
            if(first_player_strategy(temp->children[i],level+1))
                now=1;
        }
        return now;
    }
}
int second_player_strategy(struct Node *root){
    int ans=1; //if all choices lead to second player's win,return true
    for(int i=0;i<26;i++){
        if(!first_player_strategy(root->children[i],0))
            ans=0;
    }
    return ans;
}
int neither_has(struct Node *root){
    if(!first_player_strategy(root,0) && !second_player_strategy(root))
        return 1;
    return 0;
}

```