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- [1]三角不等式 $||a|-|b||\leq|a+b|\leq|a|+|b|$
- [2]平均值不等式 $a_1,a_2,a_3\ldots\ldots a_n$(n个正数)
  $\frac{a_1+a_2+a_3\ldots+a_n}{n}\geq\sqrt[n]{a_1a_2a_3\ldots a_n}\geq\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_n}}$

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算术平均            几何平均            调和平均
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证[1]:

$-2|a||b|\leq2ab\leq2|a||b|$,同时加上$a^2+b^2$

$(|a|-|b|)^2\leq(a+b)^2\leq(|a|+|b|)^2$

开方得证.

证[2]:

- 左侧:
  - 当n为2^t时

    知$\frac{a_1+a_2}{2}\geq\sqrt{a_1a_2}$

    $\frac{a_1+a_2+a_3+a_4}{4}\geq\frac{2\sqrt{a_1a_2}+2\sqrt{a_3a_4}}{4}\geq\frac{2\sqrt{4\sqrt{a_1a_2a_3a_4}}}{4}=\sqrt[4]{a_1a_2a_3a_4}$

    ......

    第t个等式即为$\frac{a_1+a_2+a_3\ldots+a_n}{n}\geq\sqrt[n]{a_1a_2a_3\ldots a_n}$
  - 当n不为 2^t时

    则必存在一个l使得$2^l\leq n<2^{l+1}$

    记$a^*=\sqrt[n]{a_1a_2a_3\ldots a_n}$

    则在原列后补充$2^{l+1}-n$个a^*,即为

    $a_1,a_2,a_3\ldots a_n,a^*,a^*\ldots$(共$2^{l+1}$个)

    由上知$\frac{a_1+a_2+a_3\ldots+a_n+a^*+a^*+\ldots}{2^{l+1}}\geq\sqrt[2^{l+1}]{a_1a_2a_3\ldots a_na^{*2^{l+1}-n}}$

    而$a_1a_2a_3\ldots a_n=a^{*n}$

    故$\sqrt[2^{l+1}]{a_1a_2a_3\ldots a_na^{*2^{l+1}-n}}=\sqrt[2^{l+1}]{a^{*n}a^{*2^{l+1}-n}}=a^*$

    $\frac{a_1+a_2+a_3\ldots+a_n+a^*+a^*+\ldots}{2^{l+1}}\geq a^*$

    则$a_1+a_2+a_3+\ldots a_n+(2^{l+1}-n)a^*\geq2^{l+1}a^*$

    即$\frac{a_1+a_2+a_3\ldots+a_n}{n}\geq a^*=\sqrt[n]{a_1a_2a_3\ldots a_n}$

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左侧得证
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- 右侧：由左侧不等式$\frac{\frac{1}{a_1}+\frac{1}{a_2}...+\frac{1}{a_n}}{n} \geq \sqrt[n]{\frac{1}{a_1}\frac{1}{a_2}\dots\frac{1}{a_n}} = \frac{1}{\sqrt[n]{a_1a_2...a_n}}$

  由于两侧都大于0,故

  $\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}...+\frac{1}{a_n}} \leq \sqrt[n]{a_1a_2\dots a_n}$

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右侧得证
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