IDENTIFICATION IN DIFFERENCE-IN-DIFFERENCES MODELS WITH ROY-LIKE SELF-SELECTION

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INTRODUCTION

- Difference-in-differences: Quasi-experimental variation to estimate causal effects
- Identifying assumption: Parallel trends
- Justification: "Quasi-random" treatment assignment
- Plausibility with rational agents: reforms, physical mobility etc.
- How does self-selection interact with the parallel trends assumption?

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  - Fuzzy Design
  - Necessary and sufficient conditions
  - **Roy models**, dynamic choices and learning

WHAT I DO

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- Roy-like selection mechanisms
- Repeated static designs
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- Goals
    - Gain better understanding of when self-selection is (not) a problem
    - Theoretical conditions and justifications for parallel trends

SETUP I

- Two periods: 0,1
- Potential outcomes $(Y_t(0), Y_t(1))$
- Fuzzy design
- 4 Groups
  - Always-treated
  - Never-treated
  - Switchers-in
  - Switchers out

SETUP II

- ATE on the Switchers into treatment
  $ATS = \mathbb{E}[Y_1(1) - Y_1(0)|D_0 = 0, D_1 = 1]$

  = ATT in design with pre-treatment period

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- Parallel trends (PT)
  $\mathbb{E}[Y_1(0) - Y_1(0) | D_0 = d_0, D_1 = d_1] = \tau$

- for constant τ and all (d_0, d_1)

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- $(U_0, Y_0(0), Y_0(1)) \perp (U_1, Y_1(0), Y_1(1))$

PREVIOUS RESULT ON ROY-MODELS

$$\begin{tikzcd}
\boxed{U_0, Y_0(0), Y_0(1)} \arrow[d] & \boxed{U_1, Y_1(0), Y_1(1)} \arrow[d] \\
D_0 = f_0(U_0) & D_1 = f_1(U_1)
\end{tikzcd}$$

Result:

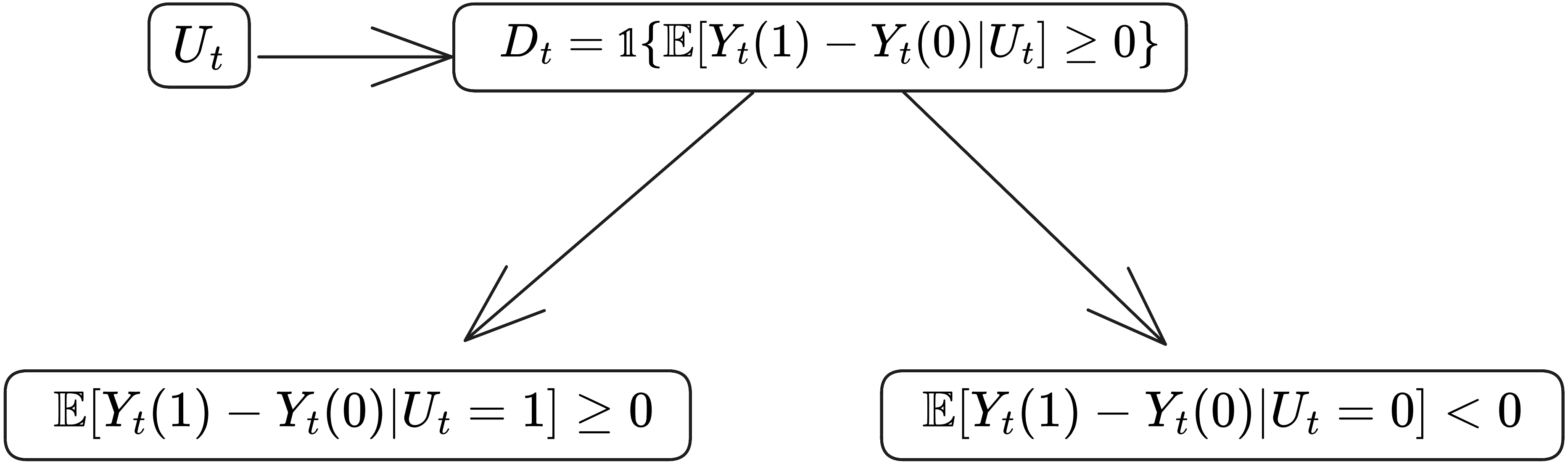
$$\mathbf{PT} \Leftrightarrow \mathbb{E}[Y_t(d)|D_0 = d_0, D_1 = d_1] = \mu_t$$

For some constant μ_t

ALTERNATIVE SETUPS I

- $Y_t(.)$ still unknown
- Roy-style selection:
  $D_t = \mathbb{1}\{\mathbb{E}[Y_t(1) - Y_t(0)|U_t] \geq 0\}$
- Suppose information U_t is a binary signal
- Suppose signal is informative
- Rational agent acts according to information

ALTERNATIVE SETUPS I



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- Model the *treated* potential outcomes
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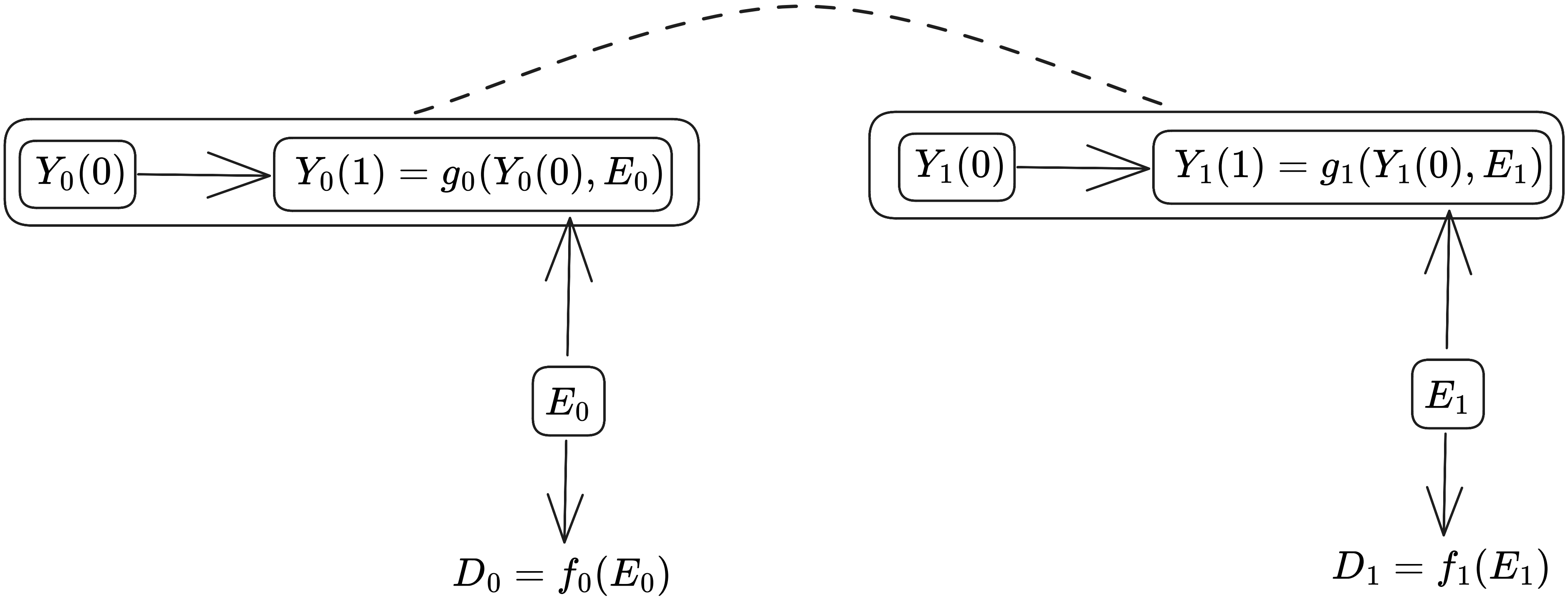

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A SIMPLE EXAMPLE, TWFE

- $Y_t(d_t) = \alpha_i + \lambda_t + E_{it} d_{it} + \varepsilon_{it}$
- Roy-selection
  - $D_{it} = \mathbb{1}\{Y_{it}(1) - Y_{it}(0) \geq 0\}$
  - $\Leftrightarrow D_{it} = \mathbb{1}\{E_{it} \geq 0\}$

EXTENSIONS AND OTHER RESULTS

- Binary potential outcomes
- Link to ignorability and lagged-dependent variable adjustment
- Covariates

A MAJOR LIMITATION

- All results imply $\mathbb{E}[Y_1(0)|D_0 = d_0, D_1 = d_1] = \mu_1$
- But then we do not need to do DiD!
- Consider ATS: $\mathbb{E}[Y_1(1) - Y_1(0)|D_0 = 0, D_1 = 1]$

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SUMMARY, DISCUSSION AND OPEN QUESTIONS

- Relationship of self-selection in Roy-style model and parallel trends
- Different setups lead to identical necessary and sufficient conditions
  - Restrict dependence over time
  - Strong assumptions on info structure
  - Model potential outcomes and treatment effect
- Mean independence likely too restrictive in most applied settings