Modeling a Bank Customer Queue

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November 9, 2013

1 Assumptions

1.1 Exponential-family service time distribution

We note that the arrival time and service time distributions we were given are both generated from a small sample size, as shown by the frequencies being multiples of 0.05, and not very precise, as they are given in discrete minutes. We chose to use the **exponential family of distributions ?** to initially parametrize the continuous-time real distributions of both the arrival and service times.

Suppose that X denotes a random variable. We first define a **sufficient statistic** to be a vector-valued function $\boldsymbol{T}(X)$ such that no other function $T_1(X)$ that is not a member of $\boldsymbol{T}$ provides any additional information about X. In short, $\boldsymbol{T}$ is sufficient in estimating $f_X(x)$.

The exponential family has the convenient property, by the **Pitman-Koopman-Darmois theorem**, that it is the only family of distributions that has a sufficient statistic $\boldsymbol{T}$ whose dimension does not increase as the sample size n increases. In other words, if there is a finite set of things that represents all we know about X, then the only reasonable estimate for the distribution of X is an exponential-family distribution.

Members of the exponential family take on the form

$$f_X(x|\theta) = h(x)g(\boldsymbol{\eta})\exp\left(\boldsymbol{\eta} \cdot \mathbf{T}(x)\right),$$

where $\boldsymbol{\eta}$ is a normalization factor to ensure the distribution integrates to 1.

1.2 Arrival time

1.2.1 Traffic waves

1.3 Service time

1.4 Improvements in service

2 Model

2.1 Mean waiting time

2.2 Mean length of queue

3 Simulation approach

4 Results

4.1 Distributions

4.2 Current quality of service

4.3 Simulation

4.4 Recommendations

5 Conclusion