On-Line Bayesian Parameter Estimation in Electrocardiogram State Space Models

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Introduction

We propose a **Bayesian** approach to **parameter estimation** in **electrocardiogram (ECG) state space models (SSMs)**. The contribution of the work is to show how

- a **Kalman filter (KF)** can be used to estimate the unknown parameters of an ECG SSM in an **on-line** manner
- the proposed method can be used to **reduce noise** and to **find the wave boundaries** in ECG signals

The method can be applied to on-line ECG measurements with **varying beat morphology, heart rate, and noise**.

Materials and methods

The ECG signal is characterized using the **discretized Wiener process acceleration (DWPA)** model [1]

$$\mathbf{x}_k = \begin{bmatrix} s_k \\ \dot{s}_k \\ \ddot{s}_k \end{bmatrix}, \quad \mathbf{F}_k = \begin{bmatrix} 1 & \Delta t & \frac{(\Delta t)^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{x}_k = \mathbf{F}_{k-1}\,\mathbf{x}_{k-1} + \mathbf{q}_{k-1},$$

where s_k is the signal, $\mathbf{q}_{k-1} \sim \mathrm{N}(\mathbf{0}, \mathbf{Q}_{k-1})$ with

$$\mathbf{Q}_{k-1} = \begin{bmatrix} \frac{1}{20}(\Delta t)^5 & \frac{1}{8}(\Delta t)^4 & \frac{1}{6}(\Delta t)^3 \\ \frac{1}{8}(\Delta t)^4 & \frac{1}{3}(\Delta t)^3 & \frac{1}{2}(\Delta t)^2 \\ \frac{1}{6}(\Delta t)^3 & \frac{1}{2}(\Delta t)^2 & \Delta t \end{bmatrix} q,$$

and q is the power spectral density of the zero-mean white noise in the continuous model. The measurement model is

$$y_k = \mathbf{H}_k\,\mathbf{x}_k + r_k,$$

where $\mathbf{H} = [1\ \ 0\ \ 0]$ and $r_k \sim \mathrm{N}(\mathbf{0}, R_k)$. The unknown parameters to be estimated are $\boldsymbol{\theta} = [q,\ R]^\top$ where $R \triangleq R_k$.

In order to estimate $\boldsymbol{\theta}$, we use a KF that yields the following recursion [2] for the **energy function** φ:

$$\varphi_k(\boldsymbol{\theta}) = \begin{cases} -\log p(\boldsymbol{\theta}), & k = 0, \\ \varphi_{k-1}(\boldsymbol{\theta}) + \frac{1}{2}\log 2\pi S_k(\boldsymbol{\theta}) + \frac{1}{2} v_k(\boldsymbol{\theta})^2 / S_k(\boldsymbol{\theta}), & k \geq 1. \end{cases}$$

Minimizing φ_k gives the **MAP estimate** of $\boldsymbol{\theta}$ at time k.

The proposed method is applied to noise reduction and wave delineation in ECGs, using a similar **fixed-lag Rauch-Tung-Striebel smoother** as in [1]. The method produces estimates of the ECG signal and its **first two derivatives**. Noise reduction follows from the smoothness of the signal estimate. Wave delineation is performed using a modified version of [3]: wave boundaries are found by looking for **small absolute values of the first derivative**. The method is tested on real ECG data obtained from the **QT Database** [4].

Results

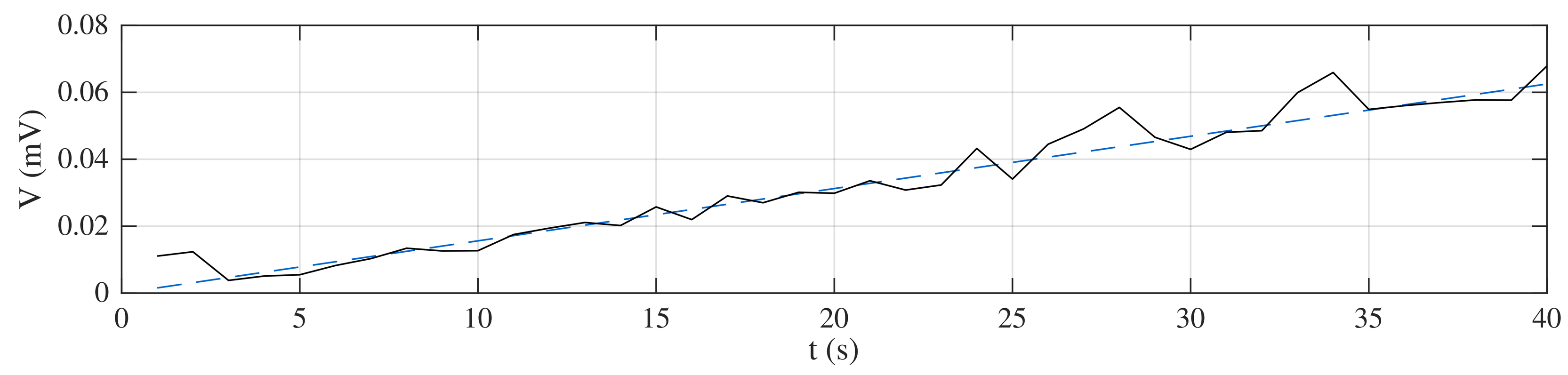

Fig. 1. The true measurement noise variance $R \triangleq R_k$ (the blue dashed line) and its estimate $\hat{R}$ (the black solid line) at each full second during a 40 s estimation period.

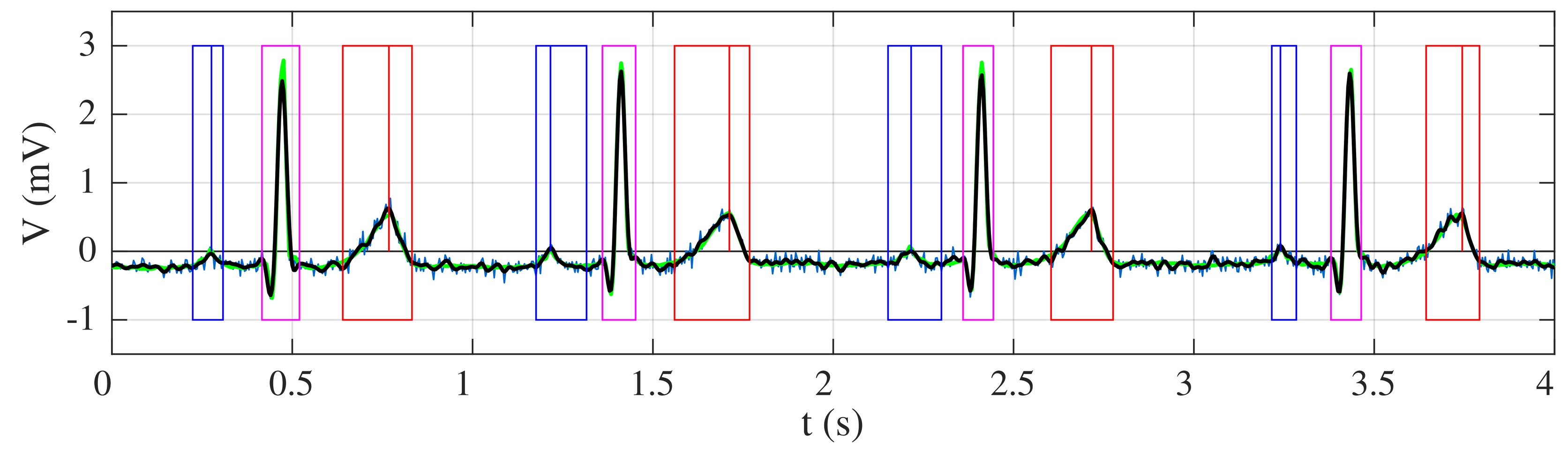

Fig. 2. An example of delineation results with input SNR 15 dB. The original signal is shown in green, the noisy measurements in blue, and the denoised signal in black. The blue, magenta, and red rectangles (and stems) denote the estimated onset and offset (and apex) of the P waves, the QRS complexes, and the T waves, respectively.

Conclusion

The DWPA model offers several benefits over the conventional Gaussian kernel approach [5] with respect to parameter estimation:

- **only two unknown parameters** instead of dozens
- **requires less inputs** from the user
- **computationally cheaper**
- less likely to suffer from problems due to saddle points
- enables **on-line parameter updates**

The experimental results indicate that the proposed method provides a promising framework for noise reduction and wave delineation in ECGs.

References

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