Empirical Analysis of Asset Pricing Models: The Fama-French Model & Multi-Beta Ross Model

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1 Introduction to the Fama-French Model

The Fama-French model, a brainchild of Eugene F. Fama and Kenneth R. French, represents a significant advancement in the field of asset pricing. Historically emerging as an extension of the Capital Asset Pricing Model (CAPM), it addresses some of the shortcomings found in the CAPM by introducing multiple factors to explain stock returns. This introduction section provides an overview of the model's inception, theoretical foundation, and its pivotal role in modern financial analysis.

1.1 Historical Background

The inception of the Fama-French model in the early 1990s marked a paradigm shift in asset pricing theory. Its predecessors, primarily the CAPM, proposed by Sharpe, Lintner, and Mossin independently in the 1960s, suggested that the market risk (beta) was the sole explanatory variable for stock returns. However, empirical inconsistencies with CAPM's predictions led to the development of the Fama-French model.

1.2 Theoretical Framework

The model is grounded in the efficient market hypothesis, asserting that stock prices reflect all available information. The Fama-French model expands the CAPM's uni-dimensional risk assessment, introducing size and value factors, alongside market risk, as pivotal determinants of stock returns. Specifically, it posits that smaller firms (small-cap stocks) and companies with high book-to-market ratios (value stocks) tend to outperform the market on a risk-adjusted basis.

2 Methodology Overview

The empirical analysis conducted in this study aims to investigate the performance of two distinct portfolios in the context of the Fama-French three-factor model. The methodology encompasses the calculation of weekly returns for the selected portfolios and the incorporation of Fama-French factors to assess the impact of market, size, and value risks on portfolio returns.

2.1 Choice of Period

The analysis covers the period from January 1, 2015, to the most recent week. This period was specifically chosen for several reasons:

- It represents a significant timeframe in the financial markets, encompassing various market cycles, including bullish and bearish phases.
- This duration provides a comprehensive view of the stocks' behavior in different economic conditions, enhancing the robustness of the analysis.
- The period is recent enough to ensure the relevance and applicability of the findings in the current market context.

2.2 Selection of Stocks

Two portfolios were constructed for the analysis:

1. **Portfolio 1:** A mix of S&P 500 and Russell 2000 companies. This portfolio combines large-cap stocks (S&P 500) known for their stability and small-cap stocks (Russell 2000) recognized for their potential higher growth. The blend aims to capture a broad spectrum of the market, providing insights into different market segments.

2. **Portfolio 2:** Composed primarily of large-cap stocks. This portfolio is designed to reflect the performance of well-established companies with significant market capitalization, which are often considered safer investments.

The choice of stocks in these portfolios is intended to provide a diverse range of insights into market behavior, encompassing different sectors and company sizes.

2.3 Data Collection and Analysis

Weekly stock price data were collected for each stock in the portfolios. The weekly returns were then calculated to measure the performance of each stock and the overall portfolios. In addition to portfolio returns, Fama-French three-factor model data were sourced from the renowned database at Dartmouth's Tuck School of Business.

2.4 Regression Analysis

The core of the empirical analysis involves conducting regression analysis to explore the relationship between the portfolio returns and the Fama-French factors: market risk (Mkt-RF), size (SMB), and value (HML). This approach allows for an assessment of how different risk factors influence the returns of the selected portfolios.

3 Portfolio Constituents

3.1 Portfolio 1: S&P 500 and Russell 2000 Mix

S&P 500 Companies:

- Apple Inc. (AAPL)
- Microsoft Corporation (MSFT)
- Amazon.com, Inc. (AMZN)
- Alphabet Inc. (GOOG)
- Berkshire Hathaway Inc. (BRK.B)
- Tesla, Inc. (TSLA)
- Meta Platforms, Inc. (META)
- JPMorgan Chase & Co. (JPM)
- Bank of America Corporation (BAC)
- Wells Fargo & Company (WFC)
- Citigroup Inc. (C)
- Exxon Mobil Corporation (XOM)
- Chevron Corporation (CVX)
- UnitedHealth Group Incorporated (UNH)
- The Home Depot, Inc. (HD)

Russell 2000 Companies:

- Accenture plc (ACN)
- Alpha Pro Tech, Ltd. (APT)
- AutoNation, Inc. (AN)
- Booz Allen Hamilton Holding Corporation (BAH)
- Cadence Design Systems, Inc. (CDNS)
- Ciena Corporation (CIEN)
- CTS Corporation (CTS)
- Constellation Brands, Inc. (STZ)
- Cintas Corporation (CTAS)
- Cummins Inc. (CMI)
- Denny's Corporation (DENN)
- Diamondback Energy, Inc. (FANG)
- Expedia Group, Inc. (EXPE)
- First Horizon Corporation (FHN)
- FleetCor Technologies, Inc. (FLT)

3.2 Portfolio 2: Large-Cap Stock Portfolio

- Apple Inc. (AAPL)
- Microsoft Corporation (MSFT)
- Amazon.com, Inc. (AMZN)
- Alphabet Inc. (GOOG)
- PepsiCo, Inc. (PEP)
- Tesla, Inc. (TSLA)
- Meta Platforms, Inc. (META)
- JPMorgan Chase & Co. (JPM)
- Bank of America Corporation (BAC)
- Wells Fargo & Company (WFC)
- Citigroup Inc. (C)
- Exxon Mobil Corporation (XOM)
- Chevron Corporation (CVX)
- UnitedHealth Group Incorporated (UNH)
- The Home Depot, Inc. (HD)
- Costco Wholesale Corporation (COST)
- The Coca-Cola Company (KO)
- Nike, Inc. (NKE)

- The Walt Disney Company (DIS)
- Visa Inc. (V)
- Mastercard Incorporated (MA)
- Coca-Cola Consolidated, Inc. (COKE)
- Salesforce, Inc. (CRM)
- Adobe Inc. (ADBE)
- Netflix, Inc. (NFLX)
- Intel Corporation (INTC)
- AT&T Inc. (T)
- Abbott Laboratories (ABT)
- Merck & Co., Inc. (MRK)
- McDonald's Corporation (MCD)
- Procter & Gamble Co. (PG)
- Verizon Communications Inc. (VZ)
- Eli Lilly and Company (LLY)
- Walmart Inc. (WMT)
- International Business Machines Corporation (IBM)
- Honeywell International Inc. (HON)
- Caterpillar Inc. (CAT)

4 Portfolios summaries

Metric	**Portfolio 1**	**Portfolio 2**
Returns	0.3512	0.2981
Excess Return	0.3014	0.2446
Returns Standard Deviation	2.8114	2.2351
Excess Returns Standard Deviation	2.8419	2.2656

Table 1: Descriptive Statistics of Portfolios 1 and 2

5 Graphical Analysis

Graphical representations of the portfolios' excess performance are provided below. These graphs include the trend of returns over the analyzed period and other relevant visualizations.

5.1

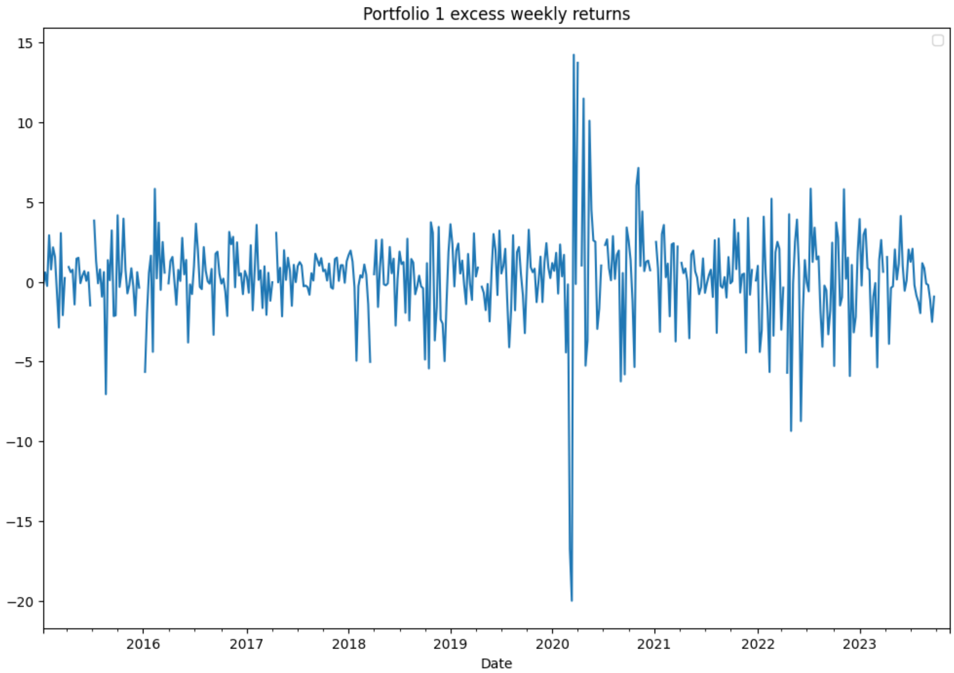

Figure 1: Weekly Excess Returns of Portfolio 1 from 2015 to Present

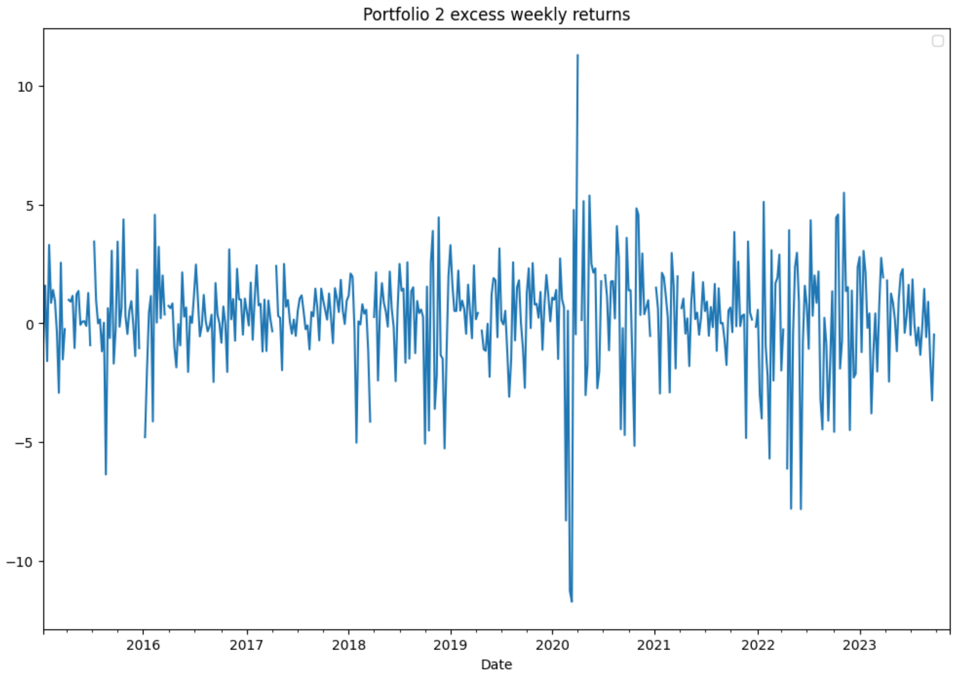

Figure 2: Weekly Excess Returns of Portfolio 2 from 2015 to Present

6 Fama-French Model

The Fama-French model is represented by the following equation:

$$R_{it} - R_{ft} = \alpha_i + \beta_{i,Mkt-RF}(R_{Mt} - R_{ft}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \epsilon_{it} \quad (1)$$

Where:

- $R_{it} - R_{ft}$: Excess return of portfolio i at time t
- $R_{Mt} - R_{ft}$: Market excess return at time t
- SMB_t: Size premium at time t
- HML_t: Value premium at time t
- ϵ_{it}: Error term

7 Portfolio 1: OLS Regression Results

```
Portfolio 1: OLS Regression Results
==============================================================================
Dep. Variable:         Excess_Returns   R-squared:                       0.076
Model:                            OLS   Adj. R-squared:                  0.070
Method:                 Least Squares   F-statistic:                     12.03
Date:                Sun, 26 Nov 2023   Prob (F-statistic):           1.39e-07
Time:                        17:27:34   Log-Likelihood:                -1065.9
No. Observations:                 440   AIC:                             2140.
Df Residuals:                     436   BIC:                             2156.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.2628      0.131      2.005      0.046       0.005       0.520
Mkt-RF         0.2254      0.055      4.063      0.000       0.116       0.334
SMB            0.2680      0.099      2.699      0.007       0.073       0.463
HML           -0.1229      0.063     -1.966      0.050      -0.246   -4.72e-05
==============================================================================
Omnibus:                      104.573   Durbin-Watson:                   2.390
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2218.496
Skew:                          -0.389   Prob(JB):                         0.00
Kurtosis:                      13.973   Cond. No.                         2.48
==============================================================================
```

Interpretation for Portfolio 1:

- The model explains about 7.6% of the variability in the excess returns (R-squared: 0.076).
- The positive coefficient for Mkt-RF (0.2254) suggests a positive relationship with market excess returns.
- SMB has a positive coefficient (0.2680), indicating a tilt towards smaller companies.
- HML has a negative coefficient (-0.1229), suggesting a tilt away from value stocks.

8 Portfolio 2: OLS Regression Results

```
==============================================================================
Dep. Variable:         Excess_Returns   R-squared:                       0.081
Model:                            OLS   Adj. R-squared:                  0.075
Method:                 Least Squares   F-statistic:                     12.85
Date:                Sun, 26 Nov 2023   Prob (F-statistic):           4.65e-08
Time:                        17:25:27   Log-Likelihood:                -965.05
No. Observations:                 440   AIC:                             1938.
Df Residuals:                     436   BIC:                             1954.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.2011      0.104      1.930      0.054      -0.004       0.406
Mkt-RF         0.2374      0.044      5.381      0.000*      0.151        0.324
SMB            0.0552      0.079      0.699      0.485*     -0.100        0.210
HML           -0.1024      0.050     -2.060      0.040*     -0.200       -0.005
==============================================================================
Omnibus:                       72.785   Durbin-Watson:                   2.503
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              470.416
Skew:                          -0.498   Prob(JB):                    7.09e-103
Kurtosis:                       7.966   Cond. No.                         2.48
==============================================================================
```

Interpretation for Portfolio 2:

- The model explains approximately 8.1% of the variability in the excess returns (R-squared: 0.081).
- A significant positive coefficient for Mkt-RF (0.2374) implies sensitivity to market movements.
- The smaller coefficient for SMB (0.0552) suggests a lesser emphasis on smaller companies.
- HML's negative coefficient (-0.1024) indicates a preference away from value-oriented stocks.

9 Analysis of the Multi-Beta Ross Model

The Multi-Beta Ross Model, also known as the Arbitrage Pricing Theory (APT) model proposed by Stephen Ross, is an asset pricing model that extends the CAPM by incorporating multiple factors in determining asset returns. Unlike CAPM, which relies on a single market factor, the APT model allows for a broader set of factors, such as GDP growth, inflation rates, and others, that could potentially affect asset prices.

9.1 Theoretical Background

The APT posits that asset returns can be explained by their sensitivities to several macroeconomic factors. These sensitivities, known as factor loadings or betas, vary across assets. The return on an asset is modeled as a linear function of these factors, plus an idiosyncratic error term. The model is grounded in the concept of no-arbitrage and relies on the law of one price.

9.2 Model Specification

In the context of the Multi-Beta Ross Model, we consider the following linear equation:

$$R_i = \alpha_i + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{in}F_n + \epsilon_i \tag{2}$$

Where:

- R_i: Return of asset i

- α_i: Asset-specific expected return
- β_{ij}: Sensitivity of asset i to factor j
- F_j: Factor j
- ϵ_i: Idiosyncratic error term

9.3 Cross-Sectional Regression Analysis

To empirically test the Multi-Beta Ross Model, we perform a cross-sectional regression analysis. This analysis involves regressing the returns of a set of assets against their respective factor loadings to estimate the risk premia associated with each factor. The regression equation is given by:

$$R_{it} = \gamma_0 + \gamma_1 \beta_{i1} + \gamma_2 \beta_{i2} + \ldots + \gamma_n \beta_{in} + \eta_{it} \tag{3}$$

Where γ_j represents the risk premium for factor j and η_{it} is the error term.

9.4 Data and Methodology

For the analysis, we use historical data on asset returns and the corresponding factor loadings. The factor loadings are estimated through time-series regressions of asset returns on factor returns. The cross-sectional regression is then conducted to estimate the risk premia and assess the model's explanatory power.

We apply the Ross Multi-Beta Model to a range of financial indices using quarterly data from 2000 to October 1, 2023.

For our analysis, we first process our dataset by taking the first difference of level data for all variables. This step is crucial for ensuring the stationarity of the time series, a prerequisite for the effective application of the Ross Model. The transformation involves differencing each time series data point from its previous value, thus focusing on the change rather than the absolute level.

9.5 Macroeconomic Variables

We use the following macroeconomic indicators in our analysis:

- 10-Year Treasury Yield
- Real GDP US
- Industrial Production
- Consumer Price Index
- Personal Consumption Expenditures
- Unemployment Rate
- Nonfarm Payrolls
- Consumer Sentiment
- Housing Starts
- Crude Oil Prices WTI
- US Euro Exchange Rate
- M2 Money Stock
- Dow Jones Industrial Average
- Manufacturing PMI
- Producer Price Index
- Business Inventories

9.6 Financial Market Indices

Our analysis focuses on the following financial market indices:

- NASDAQ Composite Index
- S&P 500 Energy
- S&P 500 Materials
- S&P 500 Consumer Discretionary
- S&P 500 Real Estate
- S&P 500 Finance
- S&P 500 Healthcare
- S&P 500 EQW
- S&P Small-Cap
- S&P Mid-Cap
- S&P Utilities

9.7 Estimation of Factor Loadings

The first step in our analysis is the estimation of factor loadings, or betas, for each financial index with respect to the macroeconomic variables. This is achieved through time-series regression analysis. For each financial index, we regress its differenced returns on the differenced values of the macroeconomic variables over the specified period. The regression model is formulated as follows:

$$\Delta R_{it} = \alpha_i + \beta_{i1}\Delta F_{1t} + \beta_{i2}\Delta F_{2t} + \ldots + \beta_{in}\Delta F_{nt} + \epsilon_{it} \tag{4}$$

where ΔR_{it} represents the first difference of returns for index i at time t, ΔF_{jt} denotes the first difference of the j-th macroeconomic factor, and β_{ij} is the sensitivity of the index to the factor.

9.8 Cross-Sectional Regression Analysis

After obtaining the factor loadings, we perform a cross-sectional regression analysis to estimate the risk premia associated with each macroeconomic factor. The excess returns of each index are regressed against their respective factor loadings:

$$\Delta R_{it} = \gamma_0 + \gamma_1\beta_{i1} + \gamma_2\beta_{i2} + \ldots + \gamma_n\beta_{in} + \eta_{it} \tag{5}$$

In this model, γ_j represents the risk premium for factor j, and η_{it} is the error term. The coefficients γ_j are of particular interest as they indicate the additional return investors require for bearing the risk associated with each macroeconomic factor.

9.9 Interpretation of Results

The interpretation of the regression results involves analyzing the significance and magnitude of the estimated risk premia (γ_j coefficients). A significant and positive γ_j suggests that the corresponding factor is priced in the market, and assets sensitive to this factor are expected to earn higher returns. Conversely, a non-significant γ_j implies that the factor does not play a significant role in explaining the returns of the indices.

9.10 Model Assessment

The efficacy of the Ross Multi-Beta Model in explaining the returns of the financial indices is assessed through various statistical measures, including the R-squared and F-statistic of the cross-sectional regressions. A higher R-squared value indicates that the model explains a greater portion of the variance in the returns, thus confirming its relevance in asset pricing.

10 Analysis of Estimated Betas from Ross Multi-Beta Model

The Ross Multi-Beta Model has been applied to a range of financial indices using a variety of macroeconomic variables for the period 2000 to October 1, 2023. The estimated betas, representing the sensitivity of each index to the respective macroeconomic factors, reveal interesting insights into the dynamics of financial markets.

10.1 Interpretation of Betas

The estimated betas show the following key observations:

- **Interest Rates and Bond Yields:** Most indices have negative betas with respect to the 10-Year Treasury Yield, indicating an inverse relationship with interest rates. This is in line with traditional market dynamics where higher interest rates generally lead to lower stock prices.
- **Economic Growth:** Real GDP growth in the US shows positive betas for almost all indices, suggesting that economic growth positively impacts stock market performance.
- **Industrial Activity:** Industrial Production has mixed impacts, with some sectors like Energy and Real Estate showing negative betas, indicating their sensitivity to industrial cycles.
- **Inflation:** The Consumer Price Index and Producer Price Index have varying effects, but generally, higher inflation seems to have a positive impact on sectors like Materials and Consumer Discretionary.
- **Consumer Sentiment and Housing:** Positive betas for Consumer Sentiment and Housing Starts for most indices indicate the importance of consumer confidence and housing market health in influencing stock prices.
- **Oil Prices:** The Energy sector, as expected, has a strong positive beta with Crude Oil Prices WTI, while other sectors show lesser sensitivity.
- **Exchange Rates:** The US Euro Exchange Rate has minimal impact on most indices, with betas close to zero.
- **Monetary Supply:** The M2 Money Stock shows significant negative betas for several indices, particularly in sectors like Energy and Real Estate.
- **Dow Jones Industrial Average:** Negative betas with respect to the Dow Jones Industrial Average for several indices suggest diverse market movements compared to this traditional market benchmark.
- **Manufacturing and Business Inventories:** Manufacturing PMI and Business Inventories have varied impacts across sectors, reflecting the complex interactions between manufacturing activity, stock levels, and stock market performance.

10.2 Implications for Investors

These findings offer valuable insights for investors, highlighting the importance of macroeconomic factors in asset pricing and portfolio management. Understanding these sensitivities can help investors make informed decisions based on economic indicators and sector-specific trends.

11 Risk Premias from Cross-Sectional Regression

The following table presents the estimated risk premias for various macroeconomic factors based on the cross-sectional regression analysis. These premias indicate the additional expected return per unit of risk associated with each factor.

Macroeconomic Factor	Risk Premium
Constant	0.000015
10-Year Treasury Yield	-0.000312
Real GDP US	0.000203
Industrial Production	0.001284
Consumer Price Index	-0.000123
Personal Consumption Expenditures	0.000017
Unemployment Rate	-0.001612
Nonfarm Payrolls	0.000175
Consumer Sentiment	0.003697
Housing Starts	0.005101
Crude Oil Prices (WTI)	-0.000180
US Euro Exchange Rate	-0.000001
M2 Money Stock	-0.001524
Dow Jones Industrial Average	-0.000842
Manufacturing PMI	0.000198
Producer Price Index	0.000054
Business Inventories	-0.000036

Table 2: Estimated Risk Premias for Macroeconomic Factors

11.1 Interpretation of Risk Premias

- **Interest Rates and Treasury Yield:** A negative risk premium for the 10-Year Treasury Yield suggests that higher yields are associated with lower returns, reflecting typical risk-averse investor behavior.
- **Economic Indicators:** Positive risk premias for Real GDP and Industrial Production indicate that stronger economic growth and industrial activity are generally associated with higher stock returns.
- **Consumer Metrics:** Consumer Sentiment and Housing Starts show notably high positive risk premias, highlighting the significant impact of consumer confidence and housing market health on market returns.
- **Inflation and Prices:** The Consumer Price Index and Producer Price Index have negative risk premias, implying that higher inflation rates may lead to lower stock returns.
- **Unemployment and Payrolls:** The negative premium for the Unemployment Rate underscores the market's adverse reaction to higher unemployment, while Nonfarm Payrolls have a positive impact.
- **Oil Prices:** The negative premium for Crude Oil Prices (WTI) suggests that higher oil prices may not necessarily translate into higher overall market returns.
- **Exchange Rates and Money Stock:** The negative premias for the US Euro Exchange Rate and M2 Money Stock indicate their inverse relationship with market returns.
- **Market Indices:** The negative premium for the Dow Jones Industrial Average suggests a potential divergence in the movement of this index compared to other market indices.
- **Manufacturing and Business Inventories:** Manufacturing PMI shows a positive premium, indicating that increased manufacturing activity is favorable for returns, while Business Inventories have a negligible negative impact.

These risk premias offer insights into how various macroeconomic factors influence expected returns in financial markets, guiding investors in making informed decisions.

12 Conclusion

This empirical analysis of the Fama-French and Ross Multi-Beta models has provided significant insights into the dynamics of asset pricing and the influence of various macroeconomic factors on financial markets.

Through a meticulous examination of two diverse portfolios and a range of financial indices, we have uncovered intriguing patterns and relationships that hold profound implications for investors and market participants.

12.1 Key Takeaways

- **Relevance of Multiple Factors:** The Fama-French and Ross models affirm the complexity of the financial markets, where multiple factors, rather than just market risk, influence asset returns. These models extend beyond the CAPM, offering a more nuanced view of asset pricing.
- **Macroeconomic Sensitivities:** Our analysis highlights the varying degrees of sensitivity different assets have to specific macroeconomic factors. This understanding is crucial for investors seeking to optimize their portfolios based on economic forecasts and trends.
- **Sector-Specific Insights:** The study reveals how different sectors respond to economic indicators, providing valuable information for sector-based investment strategies and risk management.
- **Model Limitations and Considerations:** While the models provide significant insights, they also come with limitations. The modest R-squared values suggest that there are other unaccounted factors or market inefficiencies affecting asset returns.
- **Practical Implications for Investors:** The estimated risk premias and factor sensitivities offer practical guidance for investment decisions, portfolio diversification, and risk assessment.

12.2 Future Research Directions

Future research could explore additional factors, including global economic indicators, geopolitical risks, and technological advancements, to further enhance the understanding of asset pricing dynamics. Additionally, incorporating machine learning and advanced statistical techniques could refine the models and improve their predictive capabilities.

12.3 Concluding Thoughts

In conclusion, our study underscores the importance of a multifaceted approach in understanding asset pricing. The Fama-French and Ross Multi-Beta models, with their emphasis on multiple risk factors, provide a more comprehensive framework than traditional models, capturing the complex interplay of various market forces. As the financial landscape continues to evolve, these models will remain indispensable tools for investors and researchers alike in navigating the intricacies of the financial markets.