Empirical Application 3

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1 Choice of Variables

For our project, we selected key economic indicators based on their usage in previous empirical studies. These include the US Gross Domestic Product (GDP), which reflects the economic health and growth of the United States. From the United Kingdom's perspective, we chose the overall performance of UK shares as an indicator of stock market dynamics, the UK Consumer Price Index (CPI) to track inflation and cost of living changes, and the UK GDP to assess the country's economic status. Additionally, the USD/GBP exchange rate is included to understand the currency value fluctuations between the US dollar and the British pound, an important aspect in international finance. These variables collectively offer a robust framework for analyzing economic trends and interactions between the two countries.

2 Test for Cointegration

2.1 ADF tests

The Augmented Dickey-Fuller (ADF) test is used to determine the stationarity of a time series. The hypotheses for the ADF test are defined as follows:

- Null Hypothesis (H0): The time series has a unit root, indicating it is non-stationary.

$$H0 : \delta = 0 \quad (unitroot) \tag{1}$$

- Alternative Hypothesis (H1): The time series does not have a unit root, suggesting it is stationary.

$$H1 : \delta < 0 \quad (nounitroot) \tag{2}$$

In our empirical analysis, the Augmented Dickey-Fuller (ADF) test was applied to assess the stationarity of the selected economic series. The initial ADF test results for the levels of the series indicated non-stationarity for all variables: US GDP, ALL UK Shares, USD/GBP exchange rate, UK CPI, and UK GDP (see Table 1). Specifically, all the p-values are significantly above the conventional threshold of 0.05. This implies that the null hypothesis of a unit root could not be rejected for any of these series in their level form.

However, upon differencing the series once and reapplying the ADF test, the results dramatically shifted, indicating stationarity. The p-values are hence dropping well below the 0.05 threshold. These results strongly reject the null hypothesis of a unit root in the first-differenced series, suggesting that each series is integrated of order one, I(1) (see Table 2).

This change in stationarity status after differencing is a critical finding, as it sets the stage for subsequent cointegration analysis to uncover long-term equilibrium relationships among these variables.

2.2 Trace test and Max Eigenvalues test

Before discussing the results, it is important to understand the hypotheses tested in the Johansen cointegration Trace and Max Eigenvalue tests:

- **Trace Test:**
  - Null Hypothesis (H0): The number of cointegration relationships is less than or equal to r.
  - Alternative Hypothesis (H1): The number of cointegration relationships is greater than r.
- **Max Eigenvalue Test:**
  - Null Hypothesis (H0): The number of cointegration relationships is exactly r.
  - Alternative Hypothesis (H1): The number of cointegration relationships is $r + 1$.

In our empirical analysis, after establishing the stationarity of the first-differenced series through the Augmented Dickey-Fuller test, we next sought to determine the optimal lag length for the Johansen cointegration test. This was done using a Vector Autoregression (VAR) model on the differenced data. The selection of the optimal lag length was guided by various information criteria: As per Table 3, the Akaike Information Criterion (AIC) suggested 5 lags, the Bayesian Information Criterion (BIC) indicated 0 lags, and the Hannan-Quinn Information Criterion (HQIC) proposed 1 lag. We opted to follow the AIC recommendation, choosing 5 lags, as AIC is known for its tendency to favor more comprehensive models, potentially capturing more complex dynamics in the data, which is crucial in cointegration analysis.

In table 4, We have the results of The Johansen cointegration Trace test, conducted with the lag order we found, and yielded the following eigenvalues: [0.18, 0.12, 0.11, 0.07, 0.01]. The trace statistics for these eigenvalues were [66.96, 41.07, 24.70, 9.95, 0.80], with corresponding critical values for the 90%, 95%, and 99% confidence levels. The trace statistic for the first eigenvalue (66.96) did not exceed the critical values at the 95% and 99% confidence levels, suggesting that we cannot confirm the presence of cointegration at these more stringent levels.

Similarly, the Max Eigenvalue test results in table 5, which were [25.89, 16.37, 14.74, 9.15, 0.80], also did not surpass their respective critical values at the higher confidence levels. This outcome further suggests that while there might be some indication of cointegration at a less strict confidence level (90%), there is no conclusive evidence of cointegration at the 95% and 99% levels. Therefore, based on these results, the presence of cointegrating relationships among the variables at higher confidence levels is not strongly supported.

This leads us to consider alternative modeling approaches, such as a Vector Autoregression (VAR) model in first differences, to better understand the dynamics of the variables in question.

2.3 VAR Model

2.3.1 VAR model's results

In our empirical study, the Vector Autoregression (VAR) model, estimated using Ordinary Least Squares (OLS), intricately captures the dynamics among key economic indicators. This model, incorporating five equations for US GDP, ALL UK Shares, USD/GBP, UK CPI, and UK GDP, demonstrates robust statistical measures. The log likelihood of 1894.00, alongside a low Final Prediction Error (FPE) of approximately 9.38e-19 and a determinant of the estimated variance-covariance matrix (Det(Omegamle)) of around 3.75e-19, signifies a model with excellent predictive capabilities and a well-specified structure. The Akaike Information Criterion (AIC) at -41.5383, Bayesian Information

Criterion (BIC) at -38.6563, and Hannan-Quinn Information Criterion (HQIC) at -40.3672 all suggest a well-fitted model that balances model complexity with the goodness of fit, capturing the relationships among variables effectively (see Table 6.

Upon examining the VAR model's results, we observe a pattern of relatively few significant coefficients across the equations, each representing the dynamics of US GDP, ALL UK Shares, USD/GBP, UK CPI, and UK GDP. This pattern indicates that while there are instances of notable influences between variables at certain lags, such as the positive impact of ALL UK Shares on US GDP at lag 1, and the persistence effect in the USD/GBP exchange rate, these instances are relatively sparse. For Example, in the ALL UK Shares equation, while the lagged value of itself at lag 1 shows a significant positive impact (coefficient: 0.337483, p-value: 0.001), several other coefficients are not statistically significant. Similarly, in the USD/GBP equation, its own first lag shows a robust positive influence (coefficient: 0.388839, p-value: 0.000), but many other coefficients do not reach statistical significance. The UK CPI and UK GDP equations also exhibit a combination of significant and non-significant coefficients. Moreover, the magnitudes of these significant coefficients are generally modest, suggesting that while certain economic indicators do exert influence on others, these effects are not overwhelmingly strong (see Tables 8 to 11).

2.3.2 Correlation matrix

Furthermore, the correlation matrix of residuals displayed in table 12 reveals additional insights. Notably, there is a significant correlation between US GDP and UK GDP (0.881), indicating a strong association between these two economies. Other correlations, such as between US GDP and ALL UK Shares (0.414), suggest interconnectedness, albeit to a lesser degree. These correlations imply that the residuals, while unexplained by the model, still carry systematic information that may reflect underlying economic linkages or omitted variables.

Overall, the low values of AIC, BIC, and HQIC indicate that the model does an excellent job in capturing the relationships without overfitting, despite its complexity. The low FPE and Det(Omegamle) values further affirm the model's accuracy in forecasting. The correlations among residuals, while not necessarily indicative of causation, point towards potential areas of interconnectedness and mutual influence among the economic variables, warranting further exploration to understand these relationships fully.

2.3.3 Granger Causality

Granger causality tests are predicated on the following hypotheses:

- Null Hypothesis (H0): Variable X does not Granger-cause Variable Y. Formally, the past values of X do not provide any statistically significant information for predicting Y.
- Alternative Hypothesis (H1): Variable X Granger-causes Variable Y. This means the past values of X do contribute significantly to the prediction of Y.

An integral part of our analysis involved conducting Granger causality tests to explore potential causal relationships between the variables. The essence of Granger causality is to test whether past values of one variable can predict the current value of another, beyond what could be predicted by the past values of the latter alone. Our results predominantly indicate a lack of Granger causality between most pairs of variables at a 5% significance level. For instance, tests such as those from ALLUKSHARES to USGDP, USD/GBP to USGDP, and UKCPI to USGDP, among others, failed to reject the null hypothesis, suggesting that these variables do not Granger-cause USGDP. This pattern was consistent across most pairs, with notable exceptions like ALLUKSHARES Granger-causing UKCPI, where the

null hypothesis was rejected (Table 13).

The general lack of Granger causality in our results suggests that, in most cases, past values of one variable do not significantly contribute to predicting the future values of another within the context of our model. The exception where Granger causality is found (ALLUKSHARES to UKCPI) indicates a specific area where one variable's past values provide significant information about the future values of another, hinting at a potentially more direct or linear relationship in this particular case

2.3.4 Robustness Tests

In addition to the coefficient analysis, we conducted further diagnostic tests to ensure the robustness of the VAR model. A key aspect of this was checking the stability of the model, which is crucial for its reliability in forecasting. The stability test confirmed that the VAR model is stable, indicating that it is suitable for capturing the dynamics of the time series data without leading to explosive behaviors.

Furthermore, we performed the Ljung-Box test on the residuals of each variable in the model to check for autocorrelation. The Ljung-Box test results for USGDP, ALL UK Shares, USD/GBP, UK CPI, and UK GDP showed lbpvalues of 0.967, 0.905, 0.996, 0.907, and 0.999 respectively for 10 lags. These high p-values indicate a lack of significant autocorrelation in the residuals, suggesting that the model does a good job of capturing the underlying patterns in the data without leaving unexplained serial correlations (Table 14).

Moreover, We conducted a Forecast Error Variance Decomposition (FEVD) for a horizon of 10 periods. FEVD is a crucial analytical tool in multivariate time series analysis, as it decomposes the variance of the forecast error of each variable into portions that can be attributed to shocks to itself and to other variables in the model. This decomposition allows us to understand the relative importance of each variable within the system in explaining the movements in all other variables, both in the short and long term (Figure 1).

APPENDIX

Table 1: ADF Test Results (Level)

Variable	**ADF Statistic**	**p-value**	**Critical Values (1%, 5%, 10%)**
USGDP	-1.1338	0.7014	-3.4805, -2.8835, -2.5785
ALLUKSHARES	-2.2552	0.1868	-3.4805, -2.8835, -2.5785
USD/GBP	-1.9053	0.3296	-3.4813, -2.8839, -2.5787
UKCPI	1.3068	0.9966	-3.4851, -2.8855, -2.5796
UKGDP	-0.7544	0.8321	-3.4805, -2.8835, -2.5785

Table 2: ADF Test Results (First Difference)

Variable	**ADF Statistic**	**p-value**	**Critical Values (1%, 5%, 10%)**
USGDP	-13.1112	1.63e-24	-3.4805, -2.8835, -2.5785
ALLUKSHARES	-8.4777	1.42e-13	-3.4805, -2.8835, -2.5785
USD/GBP	-9.7067	1.04e-16	-3.4809, -2.8837, -2.5786
UKCPI	-2.9237	0.0427	-3.4851, -2.8855, -2.5796
UKGDP	-14.4663	6.72e-27	-3.4805, -2.8835, -2.5785

Table 3: VAR Model Order Selection

Model Selection Criteria	AIC	BIC	HQIC
Selected Order	5	0	1

Table 4: Johansen Cointegration Trace Test Results

Test Statistic	**Eigenvalue**	**Trace Statistic**	**Critical Values**		
			90%	95%	99%
Trace Statistic ($r \leq 0$)	0.1818	66.9629	65.8202	69.8189	77.8202
Trace Statistic ($r \leq 1$)	0.1192	41.0729	44.4929	47.8545	54.6815
Trace Statistic ($r \leq 2$)	0.1080	24.6983	27.0669	29.7961	35.4628
Trace Statistic ($r \leq 3$)	0.0685	9.9535	13.4294	15.4943	19.9349
Trace Statistic ($r \leq 4$)	0.0062	0.8009	2.7055	3.8415	6.6349

Table 5: Johansen Cointegration Max Eigenvalue Test Results

Test Statistic	**Eigenvalue**	**Max Eigenvalue Statistic**	**Critical Values**		
			90%	95%	99%
Trace Statistic ($r \leq 0$)	0.1818	25.88990702	31.2379	33.8777	39.3693
Trace Statistic ($r \leq 1$)	0.1192	16.37465913	25.1236	27.5858	32.7172
Trace Statistic ($r \leq 2$)	0.1080	14.74483429	18.8928	21.1314	25.865
Trace Statistic ($r \leq 3$)	0.0685	9.15257132	12.2971	14.2639	18.52
Trace Statistic ($r \leq 4$)	0.0062	0.80088152	2.7055	3.8415	6.6349

Table 6: VAR Model Fit Statistics

Fit Statistic	**Value**
AIC (Akaike Information Criterion)	-41.5383
FPE (Final Prediction Error)	9.38242×10^{-19}
Det(Omega_mle)	3.74632×10^{-19}
BIC (Bayesian Information Criterion)	-38.6563
HQIC (Hannan-Quinn Information Criterion)	-40.3672

Table 7: VAR Equation Results for USGDP

Variable	**Coefficient**	**Std. Error**	**t-stat**	**Prob**
const	0.009707	0.002955	3.285	0.001
L1.USGDP	-0.189841	0.207822	-0.913	0.361
L1.ALLUKSHARES	0.051854	0.022147	2.341	0.019
L1.USD/GBP	0.049306	0.030236	1.631	0.103
L1.UKCPI	-0.315904	0.298287	-1.059	0.290
L1.UKGDP	-0.090852	0.119778	-0.759	0.448
L2.USGDP	-0.092987	0.220031	-0.423	0.673
L2.ALLUKSHARES	0.003892	0.023989	0.162	0.871
L2.USD/GBP	-0.020938	0.031827	-0.658	0.511
L2.UKCPI	0.220780	0.242484	0.910	0.363
L2.UKGDP	-0.061144	0.125330	-0.488	0.626
L3.USGDP	-0.015584	0.205258	-0.076	0.939
L3.ALLUKSHARES	0.023097	0.024152	0.956	0.339
L3.USD/GBP	0.085531	0.033963	2.518	0.012
L3.UKCPI	-0.169934	0.248296	-0.684	0.494
L3.UKGDP	-0.078393	0.121879	-0.643	0.520
L4.USGDP	0.045833	0.201291	0.228	0.820
L4.ALLUKSHARES	0.002530	0.024323	0.104	0.917
L4.USD/GBP	-0.022158	0.031261	-0.709	0.478
L4.UKCPI	-0.164035	0.248359	-0.660	0.509
L4.UKGDP	-0.040496	0.122606	-0.330	0.741
L5.USGDP	0.032593	0.200645	0.162	0.871
L5.ALLUKSHARES	0.017827	0.022122	0.806	0.420
L5.USD/GBP	0.011992	0.029370	0.408	0.683
L5.UKCPI	0.494110	0.266591	1.853	0.064
L5.UKGDP	-0.023099	0.119165	-0.194	0.846

Table 8: VAR Equation Results for ALLUKSHARES

Variable	**Coefficient**	**Std. Error**	**t-stat**	**Prob**
const	0.009999	0.014180	0.705	0.481
L1.USGDP	0.560749	0.997416	0.562	0.574
L1.ALLUKSHARES	0.337483	0.106290	3.175	0.001
L1.USD/GBP	0.122000	0.145115	0.841	0.401
L1.UKCPI	-3.595649	1.431591	-2.512	0.012
L1.UKGDP	-0.276066	0.574861	-0.480	0.631
L2.USGDP	-1.288824	1.056013	-1.220	0.222
L2.ALLUKSHARES	0.081364	0.115134	0.707	0.480
L2.USD/GBP	-0.045178	0.152751	-0.296	0.767
L2.UKCPI	0.805931	1.163775	0.693	0.489
L2.UKGDP	0.563980	0.601504	0.938	0.348
L3.USGDP	-0.400285	0.985113	-0.406	0.684
L3.ALLUKSHARES	-0.009057	0.115914	-0.078	0.938
L3.USD/GBP	0.098711	0.162999	0.606	0.545
L3.UKCPI	2.375585	1.191668	1.993	0.046
L3.UKGDP	0.000523	0.584945	0.001	0.999
L4.USGDP	0.556197	0.966073	0.576	0.565
L4.ALLUKSHARES	-0.093892	0.116734	-0.804	0.421
L4.USD/GBP	-0.155892	0.150031	-1.039	0.299
L4.UKCPI	0.116392	1.191967	0.098	0.922
L4.UKGDP	-0.414604	0.588434	-0.705	0.481
L5.USGDP	1.597139	0.962971	1.659	0.097
L5.ALLUKSHARES	0.073814	0.106173	0.695	0.487
L5.USD/GBP	0.192938	0.140958	1.369	0.171
L5.UKCPI	0.794148	1.279472	0.621	0.535
L5.UKGDP	-1.186734	0.571918	-2.075	0.038

Table 9: VAR Equation Results for USD/GBP

Variable	**Coefficient**	**Std. Error**	**t-stat**	**Prob**
const	0.010142	0.009279	1.093	0.274
L1.USGDP	-0.053989	0.652653	-0.083	0.934
L1.ALLUKSHARES	0.112877	0.069550	1.623	0.105
L1.USD/GBP	0.388839	0.094955	4.095	0.000
L1.UKCPI	-0.958530	0.936753	-1.023	0.306
L1.UKGDP	0.072440	0.376157	0.193	0.847
L2.USGDP	-0.824296	0.690995	-1.193	0.233
L2.ALLUKSHARES	-0.061721	0.075337	-0.819	0.413
L2.USD/GBP	-0.384466	0.099952	-3.847	0.000
L2.UKCPI	-1.690073	0.761509	-2.219	0.026
L2.UKGDP	0.544589	0.393591	1.384	0.166
L3.USGDP	-0.833997	0.644602	-1.294	0.196
L3.ALLUKSHARES	0.146851	0.075847	1.936	0.053
L3.USD/GBP	0.241995	0.106658	2.269	0.023
L3.UKCPI	-0.809308	0.779761	-1.038	0.299
L3.UKGDP	0.141163	0.382755	0.369	0.712
L4.USGDP	-0.630105	0.632143	-0.997	0.319
L4.ALLUKSHARES	0.037253	0.076384	0.488	0.626
L4.USD/GBP	-0.159961	0.098172	-1.629	0.103
L4.UKCPI	0.943528	0.779956	1.210	0.226
L4.UKGDP	0.287111	0.385038	0.746	0.456
L5.USGDP	-0.901874	0.630114	-1.431	0.152
L5.ALLUKSHARES	0.029995	0.069474	0.432	0.666
L5.USD/GBP	0.092136	0.092235	0.999	0.318
L5.UKCPI	0.851848	0.837214	1.017	0.309
L5.UKGDP	0.444463	0.374231	1.188	0.235

Table 10: VAR Equation Results for UKCPI

Variable	**Coefficient**	**Std. Error**	**t-stat**	**Prob**
const	0.001847	0.000982	1.882	0.060
L1.USGDP	-0.159871	0.069050	-2.315	0.021
L1.ALLUKSHARES	0.024236	0.007358	3.294	0.001
L1.USD/GBP	0.015313	0.010046	1.524	0.127
L1.UKCPI	0.325285	0.099108	3.282	0.001
L1.UKGDP	0.069270	0.039797	1.741	0.082
L2.USGDP	-0.089410	0.073107	-1.223	0.221
L2.ALLUKSHARES	-0.007773	0.007971	-0.975	0.329
L2.USD/GBP	-0.001052	0.010575	-0.100	0.921
L2.UKCPI	0.200056	0.080567	2.483	0.013
L2.UKGDP	0.064749	0.041642	1.555	0.120
L3.USGDP	0.019541	0.068199	0.287	0.774
L3.ALLUKSHARES	0.007181	0.008025	0.895	0.371
L3.USD/GBP	0.004578	0.011284	0.406	0.685
L3.UKCPI	-0.039519	0.082498	-0.479	0.632
L3.UKGDP	0.013540	0.040495	0.334	0.738
L4.USGDP	0.020307	0.066881	0.304	0.761
L4.ALLUKSHARES	-0.008331	0.008081	-1.031	0.303
L4.USD/GBP	0.003920	0.010387	0.377	0.706
L4.UKCPI	0.584771	0.082519	7.086	0.000
L4.UKGDP	-0.025514	0.040737	-0.626	0.531
L5.USGDP	0.084593	0.066666	1.269	0.204
L5.ALLUKSHARES	0.001276	0.007350	0.174	0.862
L5.USD/GBP	-0.005231	0.009758	-0.536	0.592
L5.UKCPI	-0.406988	0.088577	-4.595	0.000
L5.UKGDP	-0.039316	0.039593	-0.993	0.321

Table 11: VAR Equation Results for UKGDP

Variable	**Coefficient**	**Std. Error**	**t-stat**	**Prob**
const	0.010977	0.005072	2.164	0.030
L1.USGDP	0.103997	0.356773	0.291	0.771
L1.ALLUKSHARES	0.076993	0.038020	2.025	0.043
L1.USD/GBP	0.085171	0.051907	1.641	0.101
L1.UKCPI	0.180396	0.512076	0.352	0.725
L1.UKGDP	-0.375242	0.205626	-1.825	0.068
L2.USGDP	-0.247379	0.377733	-0.655	0.513
L2.ALLUKSHARES	-0.013481	0.041183	-0.327	0.743
L2.USD/GBP	-0.061981	0.054639	-1.134	0.257
L2.UKCPI	0.539511	0.416279	1.296	0.195
L2.UKGDP	0.002260	0.215156	0.011	0.992
L3.USGDP	0.035787	0.352372	0.102	0.919
L3.ALLUKSHARES	0.026906	0.041462	0.649	0.516
L3.USD/GBP	0.141615	0.058304	2.429	0.015
L3.UKCPI	0.111149	0.426256	0.261	0.794
L3.UKGDP	0.014568	0.209233	0.070	0.944
L4.USGDP	0.118610	0.345561	0.343	0.731
L4.ALLUKSHARES	-0.022976	0.041755	-0.550	0.582
L4.USD/GBP	-0.045357	0.053666	-0.845	0.398
L4.UKCPI	-0.212528	0.426363	-0.498	0.618
L4.UKGDP	-0.084103	0.210481	-0.400	0.689
L5.USGDP	0.395556	0.344452	1.148	0.251
L5.ALLUKSHARES	-0.003294	0.037978	-0.087	0.931
L5.USD/GBP	0.036439	0.050420	0.723	0.470
L5.UKCPI	0.146187	0.457663	0.319	0.749
L5.UKGDP	-0.294913	0.204573	-1.442	0.149

Table 12: Correlation Matrix of Residuals

	USGDP	ALLUKSHARES	USD/GBP	UKCPI	UKGDP
USGDP	1.000000	0.413663	0.149576	0.201482	0.881182
ALLUKSHARES	0.413663	1.000000	0.017591	0.200922	0.311174
USD/GBP	0.149576	0.017591	1.000000	-0.036312	0.087719
UKCPI	0.201482	0.200922	-0.036312	1.000000	0.310695
UKGDP	0.881182	0.311174	0.087719	0.310695	1.000000

Table 13: Granger Causality Test Results

From	**To**	**p-value**
ALLUKSHARES	USGDP	0.178
USD/GBP	USGDP	0.178
UKCPI	USGDP	0.567
UKGDP	USGDP	0.949
USGDP	ALLUKSHARES	0.415
USD/GBP	ALLUKSHARES	0.739
UKCPI	ALLUKSHARES	0.079
UKGDP	ALLUKSHARES	0.368
USGDP	USD/GBP	0.381
ALLUKSHARES	USD/GBP	0.183
UKCPI	USD/GBP	0.155
UKGDP	USD/GBP	0.514
USGDP	UKCPI	0.193
ALLUKSHARES	UKCPI	0.022
USD/GBP	UKCPI	0.674
UKGDP	UKCPI	0.440
USGDP	UKGDP	0.837
ALLUKSHARES	UKGDP	0.402
USD/GBP	UKGDP	0.250
UKCPI	UKGDP	0.756

Table 14: Ljung-Box Test Results

Variable	**Ljung-Box Statistic**	**p-value**
USGDP	3.502307	0.96702
ALLUKSHARES	4.792812	0.904582
USD/GBP	1.989273	0.996422
UKCPI	4.75363	0.907016
UKGDP	0.638795	0.999979

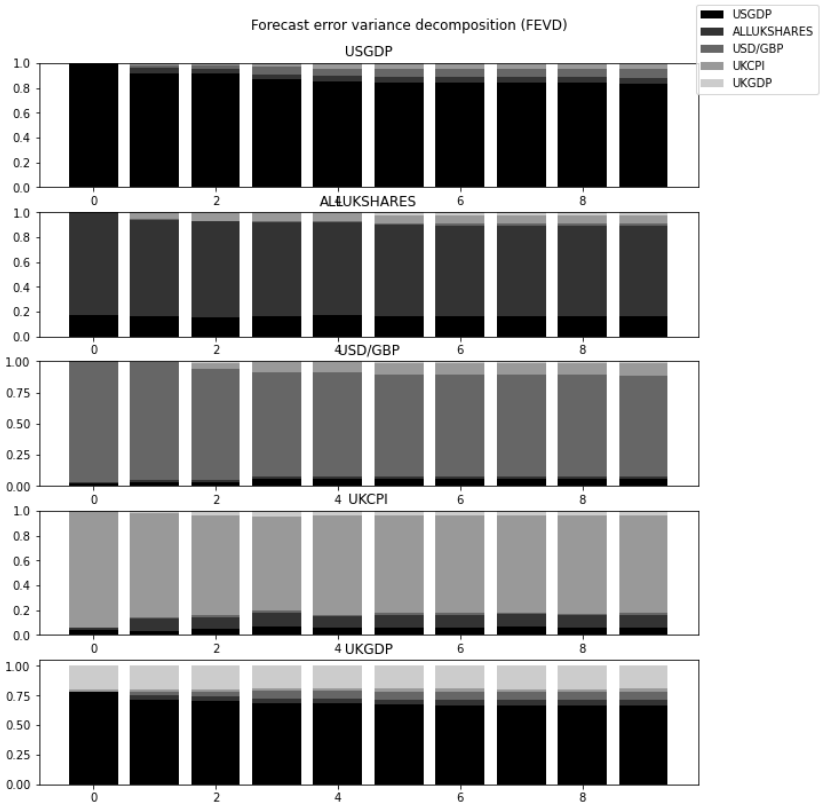

Figure 1: Forecast Error Variance Decomposition (FEVD) Plot