# Quantitative Trading Puzzles

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## 1 Even Before Odd

To find the probability that we do not see an odd-numbered face until we have seen all even-numbered faces on a fair 6-sided die, we need to consider the sequence in which the faces appear. Let's break down the problem step-by-step.

1. **Label the faces:**
   - Even faces: $2, 4, 6$
   - Odd faces: $1, 3, 5$
2. **Sequence requirements:**
   - We need to see all even faces $(2, 4, 6)$ before we see any odd faces $(1, 3, 5)$.
3. **Total permutations:**
   - The total number of ways to arrange all six faces is 6!.
4. **Valid permutations:**
   - We need to find the number of ways to arrange the sequence such that all even faces appear before all odd faces.
   - Treat the event of seeing all even faces before odd faces as dividing the sequence into two blocks: the first block consists of the even faces (2, 4, 6), and the second block consists of the odd faces (1, 3, 5).
5. **Number of valid permutations:**
   - The number of ways to arrange the even faces (2, 4, 6) among themselves is 3!.
   - The number of ways to arrange the odd faces (1, 3, 5) among themselves is 3!.
   - Therefore, the total number of valid permutations where all even faces come before all odd faces is $3! \times 3!$.

6. **Calculate probabilities:**
   - The probability that the even faces come before the odd faces is the ratio of the valid permutations to the total permutations.

So, let's compute this explicitly:

$$\text{Total number of permutations} = 6!$$

$$\text{Number of valid permutations} = 3! \times 3!$$

Calculating the factorials:

$$6! = 720$$

$$3! = 6$$

$$\text{Number of valid permutations} = 6 \times 6 = 36$$

Now, the probability $P$ is:

$$P = \frac{\text{Number of valid permutations}}{\text{Total number of permutations}} = \frac{36}{720} = \frac{1}{20}$$

Therefore, the probability that we will not see an odd-numbered face until we have seen all even-numbered faces is:

$$\boxed{\frac{1}{20}}$$

## 2 Number of Draws Before Seeing an Ace

To find the expected number of cards drawn from a 52-card deck before seeing the first ace, we can consider the structure of the deck and the distribution of the cards. Let's break down the problem:

We can think of the deck as divided into 5 segments by the 4 aces:

1. Before the first ace
2. Between the first and second ace
3. Between the second and third ace
4. Between the third and fourth ace
5. After the fourth ace

Given there are 48 non-ace cards and 4 aces, we can distribute the non-ace cards into the 5 segments.

1. **Average Cards per Segment:**

- The total number of non-ace cards is 48.
- These cards are distributed across 5 segments (gaps).

$$\text{Average number of non-ace cards per segment} = \frac{48}{5} = 9.6$$

2. **Expected Number of Cards Until First Ace:**

   - Since each segment, on average, contains 9.6 non-ace cards, the expected number of cards drawn before encountering the first ace includes the cards from one of these segments plus one more card (the ace itself).
   - Therefore, the expected number of draws before seeing the first ace is:

$$9.6 + 1 = 10.6$$

By considering the deck as divided into segments and using the average number of non-ace cards per segment, the expected number of cards drawn before seeing the first ace is indeed approximately:

$$\boxed{10.6}$$

This approach accounts for the fact that the deck is finite and the draws are without replacement, leading to a more accurate expectation value.

# 3 Max number before 4

Jim will roll a fair six-sided die until he gets a 4. We need to find the expected value of the highest number he rolls through this process. Given that Jim will eventually roll a 4, the possible highest numbers are 4, 5, and 6.

Analysis

Let's denote $k$ as the highest number rolled before Jim rolls a 4. The possible values for $k$ are 4, 5, and 6. We will determine the probability for each case and use the concept of expected value.

1. **Probability that the highest number is 4**:

The highest number will be 4 if Jim rolls 4 before rolling 5 or 6. Since 4, 5, and 6 are equally likely to be the first occurrence among the rolls, the probability $P(k = 4)$ is:

$$P(k = 4) = \frac{1}{3}$$

2. **Probability that the highest number is 6**:

The highest number will be 6 if Jim rolls 6 before rolling 4. Since there are only two numbers to consider (6 and 4), the probability $P(k = 6)$ is:

$$P(k = 6) = \frac{1}{2}$$

3. **Probability that the highest number is 5**:

The highest number will be 5 if Jim rolls 5 before rolling 4 and 4 before rolling 6. There are 3! (6) possible arrangements of 4, 5, and 6. Only 1 of these arrangements (5 before 4, and 4 before 6) fits the criteria. Hence, the probability $P(k = 5)$ is:

$$P(k = 5) = \frac{1}{6}$$

Expected Value Calculation

Now, we calculate the expected value $E[k]$ using the probabilities and the values of $k$:

$$E[k] = P(k = 4) \times 4 + P(k = 5) \times 5 + P(k = 6) \times 6$$

Substituting the probabilities:

$$E[k] = \left(\frac{1}{3} \times 4\right) + \left(\frac{1}{6} \times 5\right) + \left(\frac{1}{2} \times 6\right)$$

Calculating each term:

$$E[k] = \frac{4}{3} + \frac{5}{6} + \frac{6}{2}$$

Combining the terms over a common denominator:

$$E[k] = \frac{8}{6} + \frac{5}{6} + \frac{18}{6} = \frac{31}{6}$$

Thus, the expected value of the highest number Jim rolls before rolling a 4 is:

$$\boxed{\frac{31}{6}}$$

# 4 Probability that Player A Wins

Two players take turns rolling two six-sided dice. Player A goes first, followed by Player B. If Player A rolls a sum of 6, they win. If Player B rolls a sum of 7, they win. If neither rolls their desired value, the game continues until someone wins. What is the probability that Player A wins?

Analysis

Let's denote $P_A$ as the probability that Player A wins and $P_B$ as the probability that Player B wins. Since one of the players must eventually win:

$$P_A + P_B = 1$$

We can set up the probability for Player A winning based on the possible outcomes of the first roll.

1. **Probability that Player A wins on their first roll**:

$$P(\text{A wins on the first roll}) = \frac{5}{36}$$

2. **Probability that Player B wins on their first roll**:

$$P(\text{B wins on the first roll}) = \frac{6}{36} = \frac{1}{6}$$

3. **Probability that neither wins on the first round**:

$$P(\text{No win on the first round}) = 1 - P(\text{A wins}) - P(\text{B wins}) = 1 - \frac{5}{36} - \frac{6}{36} = \frac{25}{36}$$

Recursive Equation for $P_A$

If Player A does not win on their first roll and Player B does not win on their first roll, the game effectively restarts with Player A's turn. Thus, we can express $P_A$ recursively:

$$P_A = \frac{5}{36} + \frac{25}{36}(1 - P_A)$$

Simplify the equation:

$$P_A = \frac{5}{36} + \frac{25}{36}(1 - P_A)$$

$$P_A = \frac{5}{36} + \frac{25}{36} - \frac{25}{36}P_A$$

$$P_A = \frac{30}{36} - \frac{25}{36}P_A$$

$$P_A + \frac{25}{36}P_A = \frac{30}{36}$$

$$\left(1 + \frac{25}{36}\right)P_A = \frac{30}{36}$$

$$\frac{61}{36}P_A = \frac{30}{36}$$

$$P_A = \frac{30}{61}$$

Thus, the probability that Player A wins is:

$$\boxed{\frac{30}{61}}$$