

A Multi-objective PSO with Pareto Archive for Personalized E-course Composition in Moodle Learning System

Ying Gao, Lingxi Peng, Fufang Li, MiaoLiu, Waixi Li

Department of Computer Science and Technology, Guangzhou University

Guangzhou, 510006, P.R. of China

falcongao@sina.com.cn

Abstract—A velocity-free fully informed particle swarm optimization algorithm is firstly proposed for multi-objective optimization problems in this paper. It finds the non-dominated solutions along the search process using the concept of Pareto dominance and uses an external archive for storing them. Distinct from other multi-objective PSO, particles in swarm only have position without velocity and all personal best positions are considered to update particle position in the algorithm. The theoretical analysis implies that the algorithm will cause the swarm mean converge to the center of the Pareto optimal solution set in a multi-objective search space. Then, the algorithm is applied to the personalized e-course composition in Moodle learning system. The relative experimental results show that the algorithm has better performance and is effective.

Keywords- Multi-objective optimization; PSO; Pareto dominance ; E-learning; Moodle

I. INTRODUCTION

Problems with multiple objectives are present in a lot of real-life optimization problems which called multi-objective optimization problems (MOOPs). In MOOPs there are several conflicting objectives to be optimized and it is difficult to identify what the best solution is. A MOOP can be formulated as finding the best possible solutions that satisfy these objectives under different tradeoff situations. A family of solutions in the feasible solution space forms a Pareto-optimal front, which describes the tradeoff among several contradictory objectives of MOOP. PSO is a branch of evolutionary algorithms for optimization problems. A few researchers have studied PSO to solve MOOPs. In[1], the concepts of evolutionary techniques are combined with PSO and Pareto dominance is used. Li[2] proposed sorting population into non-domination levels such that individuals from better fronts can be selected. Coello[3] incorporated the concept of Pareto dominance into PSO. In[4], an archive interacted with the primary population is used to store the non-dominated solutions and define the local best positions. In [5], the crowding distance was incorporated into PSO. Mostaghim[6] proposed the strategies for finding good local guides in multi-objective PSO.

With the growth of Internet and computer technology, E-learning can provide a convenient and efficient learning environment and practical utilities anytime and anywhere. E-learning not only realizes the concept of classroom and platform independence, but also provides an interactive

learning environment. Since learners' characteristics differ, the conventional common E-course does not always meet all learners' expectations. Some research works emphasized that personalized learning can offer efficient and effective learning for individual learners in E-learning[7]. To effectively compose E-learning materials for different learners in E-learning system, some factors need to be considered. These factors include the difficulty of E-learning material, the ability level of the learners, the required time for reading the E-learning material, lower and upper bounds on the expected learning time for individual learners, the learning concepts of the E-learning material, and the expected learning concepts of the learners. The problem is one of the most challenging problems that has to meet many requires. In general, it aims to optimize a set of objectives simultaneously by adjusting the control variables. As a result, the problem is a high-dimensional non linear multi-objective optimization problem.

In this paper, a new multi-objective optimization algorithm, called the velocity-free fully informed PSO for MOOPs is firstly proposed. Different from other multi-objective PSO, particles in swarm only have position without velocity, and all personal best positions are considered to update particle position in the algorithm. The theoretical analysis implies that in a multi-objective search space, the algorithm will cause the swarm mean converge to the center of the Pareto optimal solution set. Then, the algorithm is applied to the personalized E-course composition in Moodle learning system. The experimental results show that the algorithm is effective.

II. SCHEME OF ALGORITHM

In our proposed multi-objective algorithm with Pareto archive, the swarm with size N is initialized in the search space; the personal best position of each particle is set as the particle itself; and the archive $A(0)$ with maximum capacity M stores all the non-dominated particles with respect to the swarm. After that, the same iteration steps are run circularly to find the optimal set of the optimization problem, until the maximum iteration number T is reached. Within the iteration, each particle updates its personal best position by using the functions $GET_PBEST()$. Based on all personal best positions obtained, the position of each particle gets then an update by using the function $UP_PARTICLE()$. The function $EVALUATE()$ is used to calculate the fitness value of each new particle. The archive $A(t)$ is updated and pruned to store the best non-dominated solutions found up to the

current iteration time. The steps of the algorithm as follows:

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Begin
 $t=0$ , Initialize the swarm  $S(0)$  with size  $N$ ;
1) For  $i=1:N$ 
     $INITIALIZE(\mathbf{x}_i(0));$ 
     $\mathbf{p}_i(0) \leftarrow \mathbf{x}_i(0);$ 
End
2)  $EVALUATE(S(0));$ 
3)  $A(0) \leftarrow NON\_DOMINATED(S(0));$ 
While  $t < T$  Do
1) For  $i=1:N$ 
     $\mathbf{p}_i(t) \leftarrow GET\_PBEST();$ 
     $\mathbf{x}_i(t+1) \leftarrow UP\_PARTICLE();$ 
End
2)  $EVALUATE(S(t+1));$ 
3)  $A(t+1) \leftarrow NON\_DOMINATED(S(t+1) \setminus A(t));$ 
4)  $PRUNE\_ARCHIVE(A(t+1));$ 
5)  $t \leftarrow t+1;$ 
End While
Output obtained Pareto optimal set;
End

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In the initialization phase, the swarm is randomly generated. Each particle is assigned random values for each dimension from the respective domain. The initial value for the personal best position of each particle is set to be the particle itself, $\mathbf{p}_i(0) = \mathbf{x}_i(0)$, where $\mathbf{x}_i(0)$ is the position of the i th particle. The algorithm maintains an archive for storing the non-dominated solutions found during the entire search process. The archive is initialized to contain the non-dominated solutions from the swarm $S(0)$. Function $NON_DOMINATED(0)$ returns the non-dominated solutions from the swarm $S(0)$.

The personal best position $\mathbf{p}_i(t)$ is the best position achieved by the particle itself so far. If the current position of a particle is dominated by the position contained in its memory, then we keep the position in memory; otherwise, the current position of this particle replaces it. Function $GET_PBEST()$ returns the personal best position. The update equation of $\mathbf{p}_i(t)$ is shown as follows:

$$\mathbf{p}_i(t+1) = \begin{cases} \mathbf{p}_i(t) & \text{if } \mathbf{f}(\mathbf{p}_i(t)) < \mathbf{f}(\mathbf{x}_i(t+1)) \\ \mathbf{x}_i(t+1) & \text{otherwise} \end{cases} \quad (1)$$

The function $UP_PARTICLE()$ returns the update of the particle's positions based on all personal best positions obtained. The update equation is shown as follows:

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \sum_{k=1}^N c_k r_k (\mathbf{p}_k(t) - \mathbf{x}_i(t)) \quad (2)$$

$\mathbf{x}_i (i=1,2,\dots,N)$ are position vector, $c_k (k=1,\dots,N)$ are acceleration coefficients. $r_k (k=1,\dots,N)$ are independent uniform random sequences distributed in the range $[0,1]$.

It is important to retain non-dominated solutions found during entire search process. An external archive with maximal capacity M is adopted to retain non-dominated

solutions. When the archive has reached its maximal capacity, an approach based upon the crowding distance is adopted to reduce the archive size without damaging its distribution characteristics. All of the non-dominated solutions from both the current swarm and the archive are stored first in the archive by function $NON_DOMINATED()$. If the archive has reached its maximal capacity M , then the most sparsely spread M solutions, i.e., M solutions with the largest crowding distance values, are retained in the archive.

III. MATHEMATICAL ANALYSIS ON THE CONVERGENCE

Let Δ represents the Pareto-optimal set, and let \mathbf{X}^* be a randomly chosen solution from Δ . Then without any loss of generality, in a particular iteration the personal best position of any particle of the swarm may be expressed as:

$$\mathbf{p}_i(t) = \mathbf{X}^* + \mathbf{D}_i(t), \quad (i = 1, 2, \dots, N) \quad (3)$$

where the difference vectors $\mathbf{D}_i(t)$ measure the distance of the personal best position found by i th particle from \mathbf{X}^* at time-step iteration t . Eq. (2) may be written as:

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \sum_{k=1}^N c_k r_k (\mathbf{X}^* + \mathbf{D}_k(t) - \mathbf{x}_i(t)) \quad (4)$$

Eq. (4) may be modified as below:

$$\mathbf{x}_i(t+1) = (1 - \sum_{k=1}^N c_k r_k) \mathbf{x}_i(t) + \sum_{k=1}^N c_k r_k \mathbf{X}^* + \sum_{k=1}^N c_k r_k \mathbf{D}_k(t) \quad (5)$$

Then, taking expectations on both sides of Eq. (5) we get,

$$E(\mathbf{x}_i(t+1)) = (1 - \sum_{k=1}^N c_k E(r_k)) E(\mathbf{x}_i(t)) + \sum_{k=1}^N c_k E(r_k) E(\mathbf{X}^*) + \sum_{k=1}^N c_k E(r_k) E(\mathbf{D}_k(t)) \quad (6)$$

Let us denote the mean value of the position of the i th particle at t as $\mathbf{a}_i(t)$. Also, let us define $E(\mathbf{D}_k(t)) = \bar{\mathbf{D}}_k(t)$; $E(\mathbf{X}^*) = \bar{\mathbf{X}}$. Now for uniform random numbers $E(r_k) = 0.5$. From these statements Eq. (6) may be rearranged as:

$$\mathbf{a}_i(t+1) = (1 - 0.5 \sum_{k=1}^N c_k) \mathbf{a}_i(t) + 0.5 \sum_{k=1}^N c_k \bar{\mathbf{X}} + 0.5 \sum_{k=1}^N c_k \bar{\mathbf{D}}_k(t) \quad (7)$$

Take Z-transform [8] of both side of Eq.(7). Dropping the subscript for each such particle i , we get:

$$Z(\mathbf{a}(t+1)) = (1 - 0.5 \sum_{k=1}^N c_k) Z(\mathbf{a}(t)) + 0.5 \sum_{k=1}^N c_k Z(\bar{\mathbf{X}}) + 0.5 \sum_{k=1}^N c_k Z(\bar{\mathbf{D}}_k(t))$$

\Rightarrow

$$Z(\mathbf{a}(t)) = (1 - 0.5 \sum_{k=1}^N c_k) Z(\mathbf{a}(t-1)) + 0.5 \sum_{k=1}^N c_k Z(\bar{\mathbf{X}}) + 0.5 \sum_{k=1}^N c_k Z(\bar{\mathbf{D}}_k(t-1))$$

Assume the system to be a causal one and hence we can take the one sided Z-transform. Since c_k , and $\bar{\mathbf{X}}$ are constants, the above equation may be simplified as follows:

$$\mathbf{a}(z) (1 - (1 - 0.5 \sum_{k=1}^N c_k) z^{-1}) = 0.5 \sum_{k=1}^N c_k \bar{\mathbf{X}} Z(U(t)) + 0.5 \sum_{k=1}^N c_k z^{-1} \bar{\mathbf{D}}_k(z)$$

where $U(t)$ denotes the unit step function.

\Rightarrow

$$\mathbf{a}(z) = 0.5 \sum_{k=1}^N c_k \bar{\mathbf{X}} \cdot \frac{1}{(1 - (1 - 0.5 \sum_{k=1}^N c_k) z^{-1})} \cdot \frac{z}{z-1}$$

$$\begin{aligned}
& + 0.5 \cdot \frac{z^{-1}}{(1 - (1 - 0.5 \sum_{k=1}^N c_k) z^{-1})} \cdot \sum_{k=1}^N c_k \bar{\mathbf{D}}_k(z) \\
\Rightarrow \\
\mathbf{a}(z) &= 0.5 \sum_{k=1}^N c_k \bar{\mathbf{X}}^* \cdot \frac{z^2}{(z - (1 - 0.5 \sum_{k=1}^N c_k))} \cdot \frac{1}{z-1} \\
& + 0.5 \cdot \frac{1}{(z - (1 - 0.5 \sum_{k=1}^N c_k))} \cdot \sum_{k=1}^N c_k \bar{\mathbf{D}}_k(z)
\end{aligned}$$

Now, final value theorem[8] states that $\lim_{t \rightarrow \infty} \mathbf{a}(t) = \lim_{z \rightarrow 1} (z-1) \mathbf{a}(z)$, provided the limit exists. Then,

$$\begin{aligned}
\lim_{t \rightarrow \infty} \mathbf{a}(t) &= 0.5 \sum_{k=1}^N c_k \bar{\mathbf{X}}^* \cdot \lim_{z \rightarrow 1} \frac{z^2}{(z - (1 - 0.5 \sum_{k=1}^N c_k))} \\
& + 0.5 \cdot \lim_{z \rightarrow 1} \frac{z-1}{(z - (1 - 0.5 \sum_{k=1}^N c_k))} \sum_{k=1}^N c_k \bar{\mathbf{D}}_k(z)
\end{aligned}$$

Considering first term of the expansion,

$$\lim_{t \rightarrow \infty} \mathbf{a}(t) = 0.5 \sum_{k=1}^N c_k \bar{\mathbf{X}}^* \cdot \lim_{z \rightarrow 1} \frac{1}{(1 - (1 - 0.5 \sum_{k=1}^N c_k))} = \bar{\mathbf{X}}^*$$

For another components, limiting value depends upon the nature of $\bar{\mathbf{D}}_k(z)$, i.e. the function to be optimized. If the poles of these terms are within unit circle, then these difference vectors may be expanded as:

$$\bar{\mathbf{D}}_k(z) = \sum_{z_j \in \text{poles of } \bar{\mathbf{D}}(z)} \frac{C_j z}{z - z_j}, \text{ such that } |z_j| < 1$$

Therefore, if $|z_j| < 1$ is satisfied, then this limit exists and the limiting value can be given as:

$$\lim_{z \rightarrow 1} \frac{z-1}{(z - (1 - 0.5 \sum_{k=1}^N c_k))} \sum_{k=1}^N c_k \bar{\mathbf{D}}_k(z) = 0$$

Thus, in a multi-objective search space, the algorithm will cause the swarm mean converge to the center of the Pareto optimal solution set.

IV. PERSONALIZED E-COURSE COMPOSITION MODEL

To effectively compose E-learning materials for different learners in Moodle learning system, some factors that affect learning efficiency and performance need to be considered. The individual characteristics of the learner can be described with parameters as follows:

- ◆ $\{S_1, S_2, \dots, S_K\}$ denotes K learners.
- ◆ $\{LC_1, LC_2, \dots, LC_K\}$ denotes N materials in database.
- ◆ $A = \{a_1, a_2, \dots, a_K\}$ denotes the ability levels of K learners, where a_i is the ability level of learner S_i .
- ◆ $D = \{d_1, d_2, \dots, d_N\}$ denotes the difficulty levels of N materials, where d_i is the difficulty level of material LC_i .

- ◆ $\{C_1, C_2, \dots, C_M\}$ denotes M learning concepts.
- ◆ $R = \{r_1, r_2, \dots, r_N\}$ represents the covered learning concepts of N materials. Each r_i for the materials covered includes many learning concepts, i.e., $r_i = \{r_{i1}, r_{i2}, \dots, r_{iM}\}$, where r_{ij} is a binary value, if $r_{ij} = 1$ this denotes that the material LC_i covers the learning concept C_j . Otherwise, $r_{ij} = 0$.
- ◆ $H = \{h_1, h_2, \dots, h_K\}$ denotes the expected learning targets of K learners. Each h_i for the learners covered includes many learning concepts, i.e., $h_i = \{h_{i1}, h_{i2}, \dots, h_{iM}\}$, where h_{ij} is a binary value, if $h_{ij} = 1$ this denotes that the material S_i covers the learning concept C_j . Otherwise, $h_{ij} = 0$.
- ◆ $x_{ij}, 1 \leq i \leq N, 1 \leq j \leq K$ denotes a decision variable, if the learning material LC_i is composed into a personalized e-course for the learner S_j , $x_{ij} = 1$. Otherwise, $x_{ij} = 0$.
- ◆ $t_i, 1 \leq i \leq N$ denotes the required time for reading the material LC_i .
- ◆ $t_{l_j}, 1 \leq j \leq K$ denotes the lower bound on the expected learning time of an e-course for the learner S_j .
- ◆ $t_{u_j}, 1 \leq j \leq K$ denotes the upper bound on the expected learning time of an e-course for the learner S_j .

The number of genes of a chromosome represents the number of materials in E-learning database encoded as $\mathbf{x} = \{x_{1k}, x_{2k}, \dots, x_{Nk}\}$, $1 \leq k \leq K$. It represents a series of materials' combination of N materials in E-learning database. Each gene in the chromosome is a binary value; if the material LC_j in the database is selected into personalized adaptive materials for the learner S_k , $x_{jk} = 1$. Otherwise, $x_{jk} = 0$. That is, $\mathbf{x} = \{x_{1k}, x_{2k}, \dots, x_{Nk}\}$ denotes the combination of materials' for the learner S_k . The model involves the four objective functions as follows:

The average difference between the covered learning concept of the E-learning material and the expected learning target of a learner can be defined as:

$$f_1(\mathbf{x}) = \sum_{i=1}^M \sum_{j=1}^N x_{jk} |r_{ji} - h_{ki}| / \sum_{j=1}^N x_{jk}, 1 \leq k \leq K \quad (8)$$

The average difference between the difficulty level of e-learning materials and the learner's ability level can be defined as:

$$f_2(\mathbf{x}) = \sum_{j=1}^N x_{jk} |D_j - A_k| / \sum_{j=1}^N x_{jk}, 1 \leq k \leq K \quad (9)$$

The total required learning time between the lower bound and upper bound of the expected time for the learner can be derived using

$$f_3(\mathbf{x}) = \max \left(t_{l-k} - \sum_{i=1}^N t_i x_{ik}, 0 \right) + \max \left(0, \sum_{i=1}^N t_i x_{ik} - t_{u-k} \right), \quad 1 \leq k \leq K \quad (10)$$

To avoid the situation where the learning concepts covered in a personalized e-course are not balance, $f_4(\mathbf{x})$ is used to balance the weight of learning concepts, which can be derived using

$$f_4(\mathbf{x}) = \sum_{i=1}^M h_{ki} \left| \sum_{j=1}^N x_{jk} r_{ji} - \frac{\sum_{j=1}^N \sum_{i=1}^M x_{jk} r_{ji}}{\sum_{i=1}^M h_{ki}} \right|, 1 \leq k \leq K \quad (11)$$

The smaller value which is derived from four objective functions mean that composed personalized e-course is closer to the demands of learners. As a result, the problem of personalized e-course composition is defined as finding an optimal solution that simultaneously meets four objective functions. The overall optimal solution is represented by $\min \mathbf{f}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), f_4(\mathbf{x})\}$. The multi-objective PSO with Pareto archive is used to solve the MOOP.

V. EXPERIMENTAL RESULTS

The effectiveness of the algorithm has been demonstrated on various standard test problems. These problems have a known set of Pareto-optimal solutions and are characterized to test the algorithms on different aspects of their performance. The two common metrics used to compare are convergence metric γ [9] and divergence metric Δ [9]. Ten test problems are used to test the performance of the proposed algorithm. They include ZDT1-ZDT5[10], SCH1 and SCH2 [11], FON [12] and KUR[13]. The initial population was generated from a uniform distribution. Population size $N=100$. All experiments were repeated for 50 runs. The maximum number of iterations is set to 5000. The solutions accepted after iteration processes are used to calculate the metrics. The results are shown in Tables 1. It can be known that the proposed algorithm is effective.

TABLE 1 MEAN AND VARIANCES OF THE CONVERGENCE AND DIVERSITY METRICS FOR THE MULTI-OBJECTIVE BENCHMARK FUNCTIONS

Problem	Diversity Δ (mean \pm variances)	Convergence γ (mean \pm variances)
ZDT1	0.2413 \pm 0.0129	0.0032 \pm 0.0004
ZDT2	0.2634 \pm 0.0378	0.0041 \pm 0.0011
ZDT3	0.3914 \pm 0.0622	0.0057 \pm 0.0021
ZDT4	0.3167 \pm 0.0173	0.1016 \pm 0.0093
ZDT5	0.3142 \pm 0.0435	0.0318 \pm 0.0164
SCH1	0.2167 \pm 0.0178	0.0041 \pm 0.0003
SCH2	0.3719 \pm 0.0532	0.0032 \pm 0.0027
FON	0.2613 \pm 0.0541	0.0452 \pm 0.0512
KUR	0.0879 \pm 0.0324	0.3125 \pm 0.0361

The algorithm is also applied to personalized E-course composition, and was run for 100, 150 and 200 E-learning materials, respectively, spanning 20 concepts and the e-learning materials were divided into 5 difficulty levels in Moodle learning system. The values for the covered learning concepts and the time required for each e-learning material were assigned randomly. The algorithms were tested for 150 learners who were divided into 5 ability levels. The values for the expected learning targets for the learners and the lower and upper bounds on learning time for each learner were set randomly. The experimental show that the

algorithm has better results than other algorithms. Due to limited length, they were omitted here in the paper.

VI. CONCLUSIONS

We proposed a multi-objective velocity-free fully informed PSO in this paper. The algorithm finds the non-dominated solutions along the search process using the concept of Pareto dominance and an external archive. And particles in swarm are velocity-free and all personal best positions are considered to update particle position. The proposed algorithm is tested and applied to the personalized e-course composition in Moodle learning system. The experimental results show that the algorithm is effective.

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