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Sensor Fusion for Ultrasonic and Laser Arrays in Mobile Robotics: A Comparative Study of Fuzzy, Dempster and Bayesian Approaches

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Abstract

In any autonomous mobile robot, if not the most, one important issue to be designed and implemented on the robot, is environment perception and its role in autonomous navigation. There are many grid-based and topological methods for environment mapping. Among the grid-based methods the main difference is about the method of data integration that is applied to mapping. In this paper, three different approaches are formulated to perform sensor data integration in the process of generation of a generalized version of occupancy grids map of the environment. The methods are formulated based on Bayesian, Fuzzy and Dempster-Shafer approaches to data fusion/integration. Although, they are famous for data fusion applications, in this research work they have been applied, formulated and simulated to solve a unique problem: map building for the same mobile robot, equipped with 8 Polaroid ultrasonic range finder sensors and operating in the same environment. The simulation results are applied for comparative study of the merits of the methods and their applicability in the map building and environment perception for autonomous mobile robots. They show that the Bayesian approach gives more appropriate maps, by which, A* path planning algorithm leads to shorter and safer routes for the mobile robot to navigate.

Keywords

map building, multi-sensor data fusion, Bayesian rule of Probability Combination, Dempster's rule of combination, fuzzy integration, path planning (A* algorithm)

INTRODUCTION

Autonomous mobile robots must be able to acquire and maintain models of their environments to carry out complex missions in indoor environments, efficiently. Recent research has produced two fundamental paradigms for modeling indoor robot environments: the grid-based (metric) paradigm, and the topological paradigm. Grid-based approaches, such as those proposed by Moravec [1] and Elfes and many others, represent environment

by evenly-spaced grids. Each grid cell indicates the presence of an obstacle in the corresponding region of the environment. Topological approaches represent the environment by graph. Nodes in such graphs correspond to distinct situations, places or landmarks (such as doorways). They are connected by arcs if a direct path exists between them. Occupancy grids are considerably easy to construct and maintain even in large-scale environments. Since the intrinsic geometry of a grid corresponds directly to the geometry of the environment, the robot's position within its model can be determined by its position and orientation in the real world. We have selected grid-based environment representation for mapping. In this paper, several approaches to sensor data fusion are formulated and simulated for a unique test-bed including the same robot and two different environments. The resulting maps are compared and also applied to path planning simulation, so as to gain reliable comparative results.

GRID-BASED MAPPING AS INTERNAL REPRESENTATION

All sensors should first convert their measurements to a common internal representation before the actual fusion is performed. As an internal representation, grid-based representation has been selected. Elfes [2] introduced the occupancy grids concept, as a framework for efficient map building in mobile robots indoor navigation applications. In this framework, the environment area is divided into many tiny square cells. Each cell in the map has two states. It is either occupied or empty. The map is generated by measuring probability values for each cell in it i.e. by judging about the state of the cell. Actually, Elfes proposed to define and measure $P_{occ}(C_j)$ as the probability of being occupied for the cell C_j . Nevertheless, in occupancy grids structure for mapping, there is no measure for the ignorance of a sensor about the state of a cell. Thus it is suggested to measure two values corresponding to each cell in the map. We nominate them $m_O(C_j)$ and $m_E(C_j)$ defined as the measures of the occupancy and emptiness for the state of the cell C_j , respectively. On the contrary of the

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evident equation:

$$P_{occ}(C_j) + P_{emp}(C_j) = 1 \quad (1)$$

the sum of m_O and m_E values is not necessarily equal to 1. Indeed, while they are computed in our formulation, always the inequality:

$$m_O(C_j) + m_E(C_j) \leq 1 \quad (2)$$

is true and the value of

$$m_I(C_j) = 1 - m_O(C_j) - m_E(C_j) \quad (3)$$

can be interpreted as a suitable measure for the existing measured ignorance about the state of the cell C_j .

FORMULATION OF MULTI-SENSOR DATA FUSION METHODS

While the multi-sensor mobile robot is moving around an environment, a lot of m_O and m_E measure values are derived for every cell¹. The question, mentioned to answer in this section is: How the two couples of values for some cell can be integrated, so that the resulting values are more informative, with a higher level of certainty and less redundancy?

Fuzzy Method

Many authors have tried to apply fuzzy logic theory, for information integration. In an approach, a fuzzy rule-base is utilized. In this method, fusion of the sensory data actually takes place while combining the rules. Indeed, in this approach, by each of the rules, some pre-processing is done for each sensory data and then, the processed sensory information are integrated while the combination process of the rules existing in the rule-base is being executed [4]. The approach that is applied to the comparative study in this paper, is the process of generating a fuzzy map[3]. In this method, two fuzzy sets named E and O are defined as the set of Empty and Occupied cells of the map. Each cell C_j in the map, belongs to E and O with the membership values $\mu_E(C_j)$ and $\mu_O(C_j)$ respectively. When the sensor S_i gives a measurement, it is assumed to generate two new fuzzy sets E_i and O_i . Membership functions are defined for these sets, by the following equations:

$$\begin{aligned} \mu_{E_i}(C_j) &= m_E(C_j) \\ \mu_{O_i}(C_j) &= m_O(C_j) \end{aligned} \quad (4)$$

After each measurement, the fuzzy sets E and O are improved by:

$$E \cup E_i \longrightarrow E \quad ; \quad O \cup O_i \longrightarrow O \quad (5)$$

where \cup stands for a fuzzy union operator. Since there are many possible choices for the union operator, there are many possibilities for doing the fusion and gradual improvement of the map. For example, we can use the well-known MAX operator. Oriolo et.al in [3] have suggested to apply Dombi union operator. It is defined

¹See the work of Oriolo et.al [3] for derivation of m_O and m_E measures, for each cell in the map, from the range data that are obtained by an ultrasonic sensor.

by:

$$\begin{aligned} \mu_{A \cup B}(x) &= u_\lambda(\mu_A(x), \mu_B(x)) \\ &= \frac{1}{1 + \left[\left(\frac{1}{\mu_A(x)} - 1 \right)^{-\lambda} + \left(\frac{1}{\mu_B(x)} - 1 \right)^{-\lambda} \right]^{-\frac{1}{\lambda}}} \end{aligned} \quad (6)$$

with $\lambda \in (0, \infty)$. The Dombi union operator is chosen for aggregation of fuzzy data, due to its flexibility. By choosing λ in (6), the aggregation strength can be tuned. The fuzzy sets E and O can be combined to generate a new fuzzy set of unsafe cells which is called a fuzzy map ([3]). We have used the simple combination rule:

$$M = O \cup \bar{E} \quad (7)$$

where for \cup we used the simple MAX operator and for complementing, the general rule:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

was applied. The resulting fuzzy map can be used as an input to either a grid-search algorithm such as A* or a potential-based path planning algorithm.

Bayesian Method

Elfes, when introduced the occupancy grids, suggested to use Bayesian formula of probability combination, for integration of the range data to build the map [2]. In his approach, the range finder sensor is modeled by some conditional distribution $P[r|z]$ where z & r are the actual and the measured distances, respectively. Then by Bayes's rule in the probability theory, having one sensed range value r , the state of each cell C_i in the map, can be calculated by:

$$\begin{aligned} P[s(C_i) = OCC. | r] &= \\ \frac{P[r|s(C_i)=OCC.] \times P[s(C_i)=OCC.]}{\sum_{s(C_j)} P[r|s(C_j)] \times P[s(C_j)]} \end{aligned} \quad (8)$$

where $P[r|s(C_i) = OCC.]$ can be calculated from:

$$P[r|z] = P[r|(s(C_i) = OCC.) \wedge (s(C_k) = EMP., k < i)] \quad (9)$$

If multiple measurements are available (obtained by several sensors or by a single sensor but in several time instances), the map can be improved by the following equation:

$$\begin{aligned} P[s(C_i) = OCC. | \{r_{t+1}\}] &= \\ \frac{P[r_{t+1}|s(C_i)=OCC.] \times P[s(C_i)=OCC. | \{r_t\}]}{\sum_{s(C_j)} P[r_{t+1}|s(C_j)] \times P[s(C_j) | \{r_t\}]} \end{aligned} \quad (10)$$

Actually (10) is an updating formula. In (8) and (10), the conditional probability $P[r|s(C_i)]$ must be calculated using (9) and this is the difficult point. In this paper, we applied the values of m_O and m_E for calculation of probability values. Actually, it is formulated as:

$$P[s(C_i) = OCC.] = (1 + m_O(C_i) - m_E(C_i)) / 2 \quad (11)$$

Assuming that the sensors data are independent, the formula (10) is simplified to the maximum entropy formula as follows:

$$P[s(C_i) = OCC. | \{r_{t+1}\}] = \frac{P_1 \times P_2}{P_1 \times P_2 + (1 - P_1) \times (1 - P_2)} \quad (12)$$

where P_1 and P_2 are defined by:

$$P_1 = P[s(C_i) = OCC. | \{r_t\}]$$

$$P_2 = P[s(C_i) = OCC. | r_{t+1}]$$

Of course, the independence assumption is not a valid assumption but not so far from reality and as the results show, it does not make trouble in the map building process and after enough instances of sensing, its drawbacks are relatively compensated.

Dempster-Shafer Reasoning Method

In Bayesian approach, all propositions for which there is no information, are assigned an equal a priori probability. In evidential reasoning approach, some ignorance measure values are assigned to unknown propositions. This ignorance is reduced only when supporting information becomes available. Many authors have used the Dempster-Shafer evidential reasoning theory in sensor data fusion applications [5], but mostly, in object recognition or classification and not in map building problems. Here we give a new formulation for fusion of range data in occupancy grids map, using evidential theory. Dempster's Rule of Combination is used to fuse the propositions X_1 and X_2 from the two sensors S_1 and S_2 by the following formulas:

$$m_{12}(X) = K \times \sum_{X_1 \cap X_2 = X} (m_1(X_1) \times m_2(X_2)) \quad (13)$$

$$K = \left(1 - \sum_{X_1 \cap X_2 = \emptyset} (m_1(X_1) \times m_2(X_2)) \right)^{-1} \quad (14)$$

where X is a non-empty subset of 2^Θ and m_{12} is the orthogonal sum of the two bpa's. The denominator in (14) is a normalization factor. Assume that there is an occupancy grids map, including two maps, i.e. there are two values $m_O(C_i)$ and $m_E(C_i)$ for each cell C_i in the map. They are initialized to zero for all of the cells in the map (complete ignorance). Also assume that a new sensed range is given and after processing, two values $m'_E(C_i)$ and $m'_O(C_i)$ are calculated for the same cell. By the discussion given about the ignorance measurement, current value and the new sensed value of ignorance about the state of the cell C_i can be measured using (3). In the nomenclature of this paper, they are called $m_I(C_i)$ and $m'_I(C_i)$ respectively. Fortunately, for each cell C_i in the current map and the new map, there is the same FOD. This common Θ has only two singletons, named as:

$$X_1 = \{C_i \text{ is occupied}\}$$

$$X_2 = \{C_i \text{ is empty}\}$$

and there are 4 focal elements. The first is the empty set. The next two are the singletons themselves with the mass values:

$$\begin{aligned} m(X_1) &= m_O(C_i) & m'(X_1) &= m'_O(C_i) \\ m(X_2) &= m_E(C_i) & m'(X_2) &= m'_E(C_i) \end{aligned} \quad (15)$$

and the last focal element is Θ with bpa measure values:

$$m(\Theta) = m_I(C_i) \quad ; \quad m'(\Theta) = m'_I(C_i)$$

Actually, the current and the sensory knowledge are represented by the function values of m and m' respectively. By the above definitions, the normalization factor in (14) is equal to:

$$\begin{aligned} K &= (1 - m(X_1) \times m'(X_2) - m(X_2) \times m'(X_1))^{-1} \\ &= (1 - m_O \times m'_E - m_E \times m'_O)^{-1} \end{aligned} \quad (16)$$

and the resulting measure values can be calculated based on (14) by the following equation:

$$\begin{aligned} m''_O &= K \times (m_O \times m'_O + m_O \times m'_I + m_I \times m'_O) \\ m''_E &= K \times (m_E \times m'_E + m_E \times m'_I + m_I \times m'_E) \end{aligned} \quad (17)$$

where the argument C_i has been omitted for abstraction. Equation (17) shows that how m_O and m_E values for each cell are improved in every sensing instance.

SIMULATION

A mobile robot, similar to Nomad 200TM is simulated. It has a cylindrical shape, equipped with a ring of 8 ultrasonic sensors around its perimeter. The simulated environment is manually mapped to an ideal (true) map. Actually, this ideal map is an occupancy grids map. The cells that are located on free (occupied) areas, are associated with zero (one) occupancy probability values. In our work, the range finding operation of the ultrasonic sensors was simulated by ray-tracing technique. In this technique, the sensed distance is derived by tracing a ray that is initiated from the sensor and directed toward the central radiation axis of the sensor and finally it is ended to the nearest occupied cell in its way. To simulate the sensory noise, two kinds of noises have been exerted to the ray tracing mechanism. The radiation direction is the central radiation direction corrupted by a Gaussian white noise. Also the distance that is derived from ray-tracing is corrupted itself by a white noise. The two environments that were selected for exploration, map building and path planning are a rectangular room. Their dimensions are 15 meters by 20 meters. They contain 4 walls on their perimeter and some obstacles inside. These objects are supposed to be modeled during the map building process. Figures 1 and 2 show the true map of the environments that have been divided into many small 10cm by 10cm square cells. The first environment shown in figure 1 is the model of a laboratory with some desks and chairs inside. The second environment is the map of a small section of a hospital, including a corridor and some neighboring rooms containing two beds or one bed and one table or only one table.

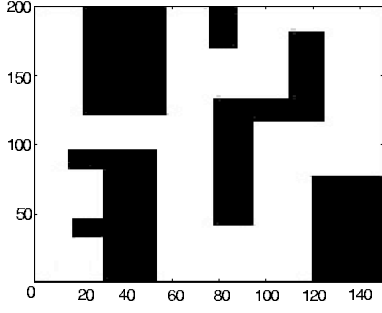


Figure 1. The true map of the first simulated environment to be generated in the robot's mind.

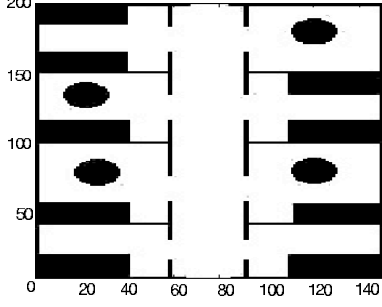


Figure 2. The true map of the second simulated environment to be generated in the robot's mind.

The map building algorithm is as follows: Firstly, there is no information about the location of the walls (dimensions of the room) and the obstacles. There is an initially blank map in the robot's memory with 150×200 cells. In an ideal case, this map converges to the map depicted in figures 1 or 2 gradually. The robot begins to navigate inside the room either by human aid or by reactive obstacle avoidance. During this phase, each of the ultrasonic sensors gives a distance measurement in every sensing instance. These values give m_O and m_E values for some cells in the map and these measure values are combined through the fusion formulas. At this point, the fusion method is considered as the opening key in the problem. In the case of the fuzzy fusion, we have used MAX and Dombi union operators in two different explorations, so as to compare the results. For each operator, (5) is used to fuse the measure values and the final map used for path planning is the fuzzy map with values given by (7). In the case of the Bayesian fusion, in each iteration m_O and m_E values are transformed to probability values by (11) and are combined with the corresponding values in the occupancy grids map, using (12). In this case, the occupancy grids map is directly fed to the input of the path planning algorithm. Finally in the case of using Dempster-Shafer evidential reasoning theory for fusion, measure values are integrated by (16) and (17) and the final map used for path planning is given by:

$$\text{MAP}(C_i) = \frac{(1 + m_O(C_i) - m_E(C_i))}{2} \quad (18)$$

In figures 3 and 4 the resulting maps for fuzzy fusion,

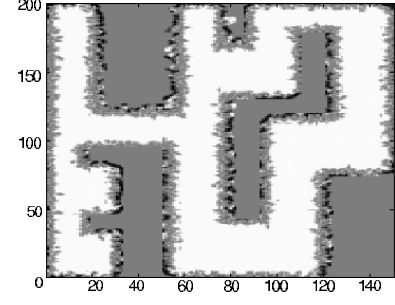


Figure 3. The map of the first simulated environment, generated by fuzzy fusion (MAX operator)

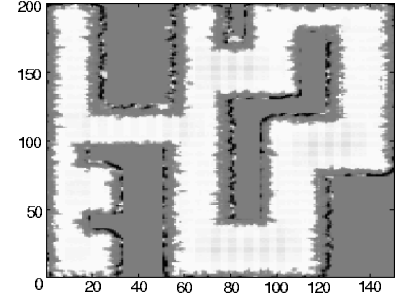


Figure 4. The map of the first simulated environment, generated by fuzzy fusion (Dombi operator)

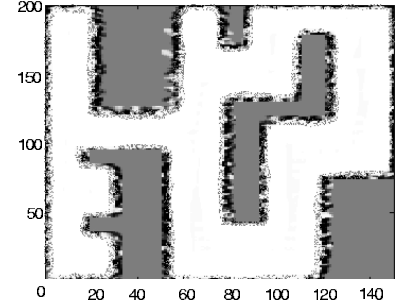


Figure 5. The map of the first simulated environment, generated by Bayesian fusion

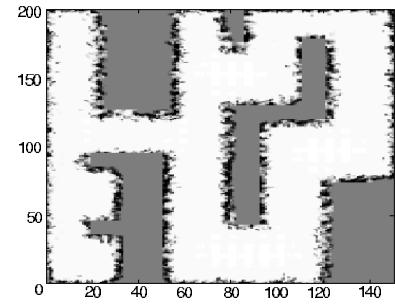


Figure 6. The map of the first simulated environment, generated by Dempster's rule of combination

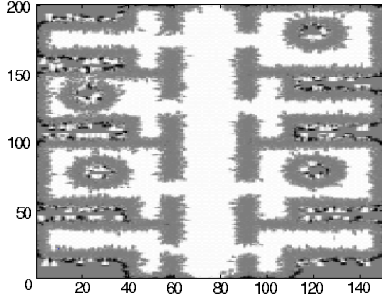


Figure 7. The map of the second simulated environment, generated by fuzzy fusion (MAX operator)

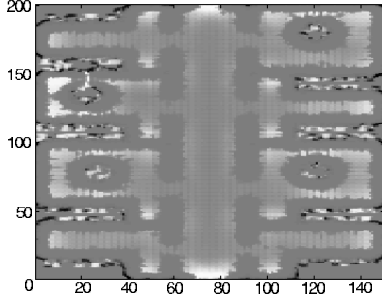


Figure 8. The map of the second simulated environment, generated by fuzzy fusion (Dombi operator)

using MAX and Dombi's operators and in figure 5 the resulting map for Bayesian fusion and in figure 6 the map resulted from fusion by using Dempster's rule of combination, are depicted. Of course these maps are associated with the first simulation (environment depicted in figure 1). Figures 7, 8, 9 and 10 show the resulting maps for the second environment (depicted in figure 2), that were generated by using fuzzy fusion with MAX operator, fuzzy fusion with Dombi operator, Bayesian fusion and Dempster's rule of combination, respectively. All of the maps are gradually generated during the activation of the sensor ring at 4500 different random points in the environment. In order to compare the maps, a fitness factor is defined and calculated for each map. This factor represents the similarity of the generated map to the corresponding true map of the simulated environment. Generally, the existing fitness between an

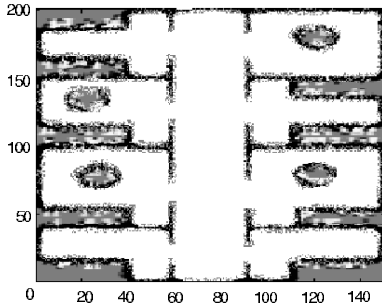


Figure 9. The map of the second simulated environment, generated by Bayesian fusion

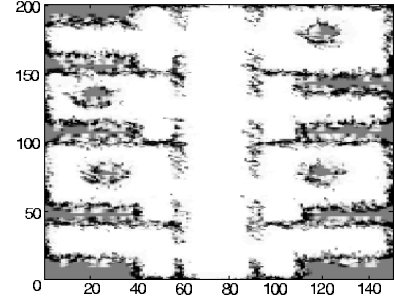


Figure 10. The map of the second simulated environment, generated by Dempster's rule of combination

occupancy grids map M and an ideal map M_I is defined by the following equation:

$$f(M, M_I) = 1 - \frac{\sum_{c \in \mathcal{B} \cup \mathcal{P}} |M(c) - M_I(c)|^2}{\text{Total number of the cells in } \mathcal{B} \cup \mathcal{P}} \quad (19)$$

where \mathcal{B} is the set of the cells which fall on the blank areas in the ideal map, \mathcal{P} is the set of the cells, falling on the perimeters of the obstacles in the ideal map and $M(c)$ is the occupancy probability of the cell c in the map M . In the case of perfect fitness, the above factor will be equal to 1. In the worst case of complete mismatch between M and M_I , we have $M_I(c) = 1$ and $M(c) = 0$ in the occupied area and vice versa. Thus, the numerator and denominator will be equal in this case and the fitness factor will be zero. It is evident that higher value for the factor, means that the maps are matched better. The cells that fall inside the obstacles in the real map, have not been considered in this formulation, because it is not important what the created maps judge about the state of such cells. Only the blank areas and the perimeter of the obstacles are significant to be identified as free and occupied cells. In order to gain more comparable results to analyze the performance of the fusion methods, we have also used these maps as inputs to A* path planning algorithm. There are two inputs in this algorithm. The first is a cellular map of the environment and the second is the coordinates of some START and GOAL points in that map. Since we decide to achieve comparative results, 30 difficult paths have been chosen for path planning. Two quantities have been calculated for each path, the length of the path and the safety measure of the path. A suitable path must be not only as short as possible, but also as far as possible from the obstacles. For a path P , its safety measure, $\alpha_d(P)$ was defined and calculated by:

$$\alpha_d(P) = \sum_{C \in P} \gamma^{-d_{min}(C)} \quad (20)$$

where $d_{min}(C)$ is the minimum of the distance between the cell C and the cells on the perimeter of an obstacle or a wall in the environment. It is calculated with respect to the true map of the environment. $\gamma > 1$ is a constant that controls the variations of $\alpha_d(P)$, with $d_{min}(C)$. In this simulation it was equal to 1.2. It is ap-

Table 1. Summary of the simulated mapping and planning results: For each map of each environment, the sum of two factors for 30 paths are calculated as useful criteria for comparison. The factors are their length and safety measure. By the fitness measures for each map, it is also shown that how close each map is to the ideal map.

Fusion Method	$\sum \text{Len}(P)$	$\sum \alpha_d(P)$	$f(M_{\text{fusion method}}, M_I)$
Fuzzy (MAX operator) for the 1 st environment	472(m)	4253	0.8223
Fuzzy (Dombi operator) for the 1 st environment	523(m)	3954	0.7349
Dempster-Shafer for the 1 st environment	465(m)	4385	0.8529
Bayesian for the 1 st environment	458(m)	3591	0.8753
Fuzzy (MAX operator) for the 2 nd environment	592(m)	3568	0.7257
Fuzzy (Dombi operator) for the 2 nd environment	-	-	0.2374
Dempster-Shafer for the 2 nd environment	552(m)	3492	0.6239
Bayesian for the 2 nd environment	500.5(m)	3224	0.8125

parent that a path P is safer for lower values of $\alpha_d(P)$. The sum of the above two quantities, associated with the 30 paths are displayed in two columns in table 1. In each column, the results of summing over the paths which are generated by using different maps, are presented in different rows. Also fitness calculation results are abstracted in this table. Path planning experiments have not been tried and abstracted in this table for the second environment map that is generated by Dombi' union operator, because this map (shown in figure 8) is quite noisy and inappropriate. Its low fitness measure affirms this fact.

CONCLUSIONS AND FURTHER RESEARCH

Fuzzy, Dempster-Shafer and Bayesian methods for sensor data fusion were formulated in a unique framework (occupancy grids map building) for a unique application (environment mapping in mobile robotic) in this paper. Thus, a reliable comparison was achieved for the performance of fusion methods in this application. Since the environment mapping application is a popularly important application in mobile robotic domain, such a comparison will be quite useful for robotic researcher. The simulation results, abstracted in table 1, apparently show that the map created by using Bayesian fusion is more appropriate for navigation and path planning in occupancy grids maps. Bayesian map has both more fitness to the ideal map and more appropriate generated routes. The maps that are generated by fuzzy and Dempster-Shafer fusion methods are less fitted to the true map and also they lead to more lengthy and more dangerous paths, while applied as an input map in A* path planning algorithm. In spite of the fact that there is naturally a trade-off between the length and the safety of the routes, it is seen that other maps generate both more lengthy and more dangerous routes with respect to Bayesian map. That is because A* path planning generates more wavy routes when its input map is not appropriate. Thus, the wavy routes are both more lengthy and more dangerous. In future research, formulating some new and more robust and certain methods for fusion in map building process can be considered.

Since the Bayesian method has given the best result in this research, we have tried to improve its performance. In this improvement, we have generalized the concept of entropy and a new concept named as pseudo information measure has been introduced and formulated to give different alternative formulations for Bayesian fusion [6].

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