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Reference Input Wheel Slip Tracking Using Sliding Mode Control

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ABSTRACT

The design of a sliding mode controller for the purpose of controlling wheel slip, λ, is described in this paper. The control objective is to track a reference input wheel slip, λ_r. Through the use of tire force measurements or observers, it is shown that the sliding mode controller can track any reference input wheel slip, λ_r. The design strategy investigated is based on a sliding surface that only contains the error between the reference input wheel slip, λ_r, and the actual wheel slip, λ. The design strategy uses continuous switching, in the form of a saturation function, to reduce chattering in the system response. Simulations are performed to demonstrate the effectiveness of the proposed sliding mode controller.

INTRODUCTION

The introduction of actively controlling the wheel dynamics of a vehicle first appeared in antilock braking systems (ABS). The primary function is to prevent the lock-up of the wheel during an emergency brake maneuver [13]. Ideally, it is desired for ABS to maintain the wheel slip, λ, at the peak of the μ-λ curve. By maintaining the wheel slip, λ, at the peak, maximum longitudinal (tractive) tire force, F_x, is generated. This allows for shorter stopping distances compared to other operating points on the μ-λ curve. However, with the introduction of vehicle dynamics control using active braking, an operating point may not necessarily be at the peak of the μ-λ curve. In fact, it could be at a point that is in the traditionally unstable region of the μ-λ curve [1], [2]. The main difficulty arising in the design of a wheel slip, λ, control is due to the strong nonlinearity and uncertainty in the problem [6]. Sliding mode control is a preferable approach for this kind of problem; due to changes which always happen in working conditions, the robustness of sliding mode control is an advantage [10]. Conventionally designed ABS controllers are implemented using a complex rule based controller. The complexity of the rule base means that it is improbable that all operating conditions are tested by conventional simulations and track tests [26]. Tao and Wang [22] investigate the inclusion of additional logic to a conventional ABS controller to improve robustness. Vantsevich, et. al. [25] devise a control based on combining the driving and braking modes of wheel loading. However, alternative, robust control strategies, such as sliding mode control, have been shown to generate satisfactory results in controlling the wheel slip, λ.

Sliding mode controllers have been designed previously for the purpose of controlling wheel slip, λ. Drakunov, et. al. [6] use sliding modes to optimally search for the maximum braking torque. The authors use a longitudinal tire force, F_x, observer to provide feedback information. Ünsal, et. al. [23] use a sliding observer for vehicle speed to help the control track a reference wheel slip, λ_r. Kazemi, et. al. [10] use sliding mode control to calculate a desired brake torque based on a reference wheel slip, λ_r. The authors' control also addresses split surfaces. Kueon, et. al. [12] incorporate some intelligence into their sliding mode controller. The intelligence portion of the authors' control compensates for the uncertainty in the system. El Hadri, et. al. [8] present a sliding mode control that uses online estimation of tire forces. Lee and Sin [15], and Landaluze, et. al. [14] also present their views on applying sliding mode control to the wheel dynamics control problem.

This paper also presents a sliding mode controller for tracking a reference wheel slip, λ_r. The reference wheel slip, λ_r, can be at any level, even in the traditionally unstable region of the μ-λ curve. In order to maintain a slip level in this region, information on the longitudinal (tractive) tire force, F_x, on each wheel has to be supplied to the control. With the development of skid detectors [20], tire-road friction monitors [9], and tire force observers/estimators [19], or measuring brake torque [4], the information is available. Additionally, with the introduction of "smart tires" to market, the longitudinal tire force, F_x, as well as the lateral tire force, F_y, will be available, allowing for more robust control schemes [5], [17], [18]. The control algorithm developed in this paper uses a nonlinear tire model to provide the tire forces for simulation purposes. The control is developed taking into account the vehicle lateral dynamics. The lateral dynamics are included as part of the longitudinal wheel speed calculation, w_x, which is used in the calculation of the wheel slip, λ. This is an important consideration with the increasing implementation of control algorithms for vehicle lateral dynamics.

PROBLEM SETUP

It is being assumed that the following sensor information is available: wheel speed, ω, at each corner, longitudinal accelerometer, a_x, per axle lateral accelerometers, a_y, and a yaw rate sensor, Ω, at the center of gravity of the vehicle. Also, tire force sensors or observers are assumed to be present. Figure 1 and Figure 2 show the layout of the variables for the vehicle and the wheel dynamics, respectively.

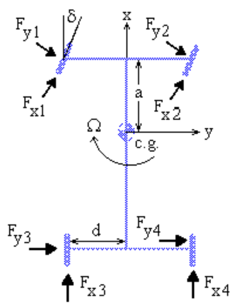

Figure 1. Vehicle dynamics layout (during braking)

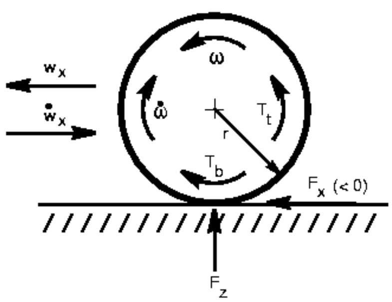

Figure 2. Wheel dynamics layout (during braking)

In formulating the wheel slip, λ, control problem, the wheel acceleration, $\dot{\omega}$, is needed. The wheel acceleration, $\dot{\omega}$, is calculated by

$$I_w \dot{\omega} = T_t - T_b \qquad (1)$$

where T_t is the tire tractive torque, and T_b is the brake torque applied to the wheel. Equation (1) can be expanded to

$$I_w \dot{\omega} = -F_x r - F_z f_r r - T_b \qquad (2)$$

where I_w is the mass moment inertia of the wheel about the axis of rotation, F_z is the wheel normal force, r is the wheel radius and f_r is the rolling resistance coefficient. During wheel deceleration, $F_x < 0$. For simulation purposes, the longitudinal (tractive) tire force, F_x, is modeled using a Dugoff tire model [16]. Figure 3 shows the longitudinal tire force, F_x, against wheel slip, λ, for various surface indexes, μ.

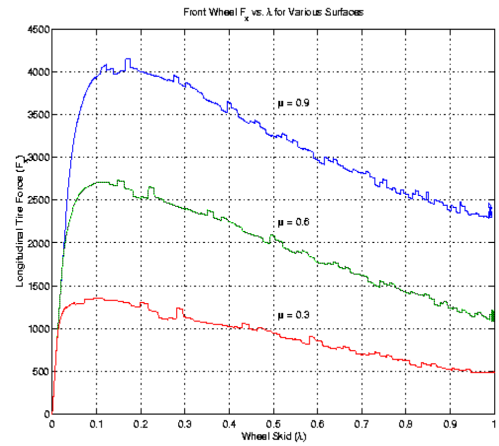

Figure 3. F_x (N) vs. λ on various surface indexes, μ, with noise

The wheel longitudinal velocity, w_x, is required for the calculation of wheel slip, λ. These wheel speeds take into account the lateral dynamics of the vehicle, and are calculated as

$$\begin{aligned}
w_{x1} &= (v_x + \Omega d)\cos\delta + (v_y + \Omega a)\sin\delta \\
w_{x2} &= (v_x - \Omega d)\cos\delta + (v_y + \Omega a)\sin\delta \\
w_{x3} &= v_x + \Omega d \\
w_{x4} &= v_x - \Omega d
\end{aligned} \qquad (3)$$

where w_{x1}, w_{x2}, w_{x3}, w_{x4} are the wheel longitudinal velocities for the left front, right front, left rear and right rear wheels, respectively. v_x is the vehicle longitudinal velocity, Ω is the vehicle yaw rate, v_y is the vehicle lateral velocity, δ is the front steerable road wheel angle, a is the distance from the front axle to the vehicle center of gravity and d is half of the vehicle trackwidth. Note, for a straight-line maneuver ($\delta = 0$) on a consistent surface ($\Omega = 0$), $w_x = v_x$, which is the typically used value in the wheel slip, λ, calculation. During the braking on a wheel, $w_x > \omega r$. Hence, the wheel slip, λ, is calculated as

$$\lambda = 1 - \frac{\omega r}{w_x} \qquad (4)$$

Differentiating equation (4) gives the derivative form

$$\dot{\lambda} = \frac{1}{w_x}\left[-\dot{\omega} r + \frac{\omega r}{w_x}\dot{w}_x \right] \qquad (5)$$

Using equation (1) and equation (4), and substituting into equation (5) gives

$$\dot{\lambda} = \frac{1}{w_x}\left[-\frac{r}{I_w}\left(-F_x r - F_z f_r r - T_b\right) + \frac{1-\lambda}{w_x}\dot{w}_x\right] \qquad (6)$$

The wheel normal force, F_z, can be determined from the vehicle acceleration sensors and the physical vehicle parameters.

CONTROL DESIGN

In this section, the sliding mode controller is described to allow the wheel slip, λ, to track a reference input wheel slip, λ_r.

CONTROL COMPONENTS

The required components for a sliding mode controller are a switching function, s, an equivalent control brake torque, $T_{b,eq}$, a striking control brake torque, $T_{b,h}$, and a continuous switching control, T_b.

Switching function, s

The control objective is to drive the system state $(\lambda, \dot{\lambda})$ to the desired values $(\lambda_r, \dot{\lambda}_r)$, where λ_r is the reference input wheel slip, and $\dot{\lambda}_r$ is the time derivative of the reference input wheel slip. Define a switching function, s, as

$$s = \lambda - \lambda_r \qquad (7)$$

The switching function, s, is used by the sliding mode control to change the structure of the control law. Based on equation (7), the most common way to adjust the structure of the control law is using the sign of the switching function, sgn(s). However, using sgn(s) to switch between control structures can result in chattering during the sliding motion.

Equivalent control brake torque, $T_{b,eq}$

The goal of sliding mode control is to achieve a reduction in the system order dynamics, called sliding motion. This sliding motion occurs when the state $(\lambda_r, \dot{\lambda}_r)$ reaches the sliding surface defined by $s = 0$. The control that moves the state along the sliding surface is called the equivalent control. In this problem, it is called the equivalent control brake torque, $T_{b,eq}$. The dynamics of sliding motion are governed by

$$\dot{s} = 0 \qquad (8)$$

Differentiating equation (7), and substituting into equation (8) gives

$$\dot{\lambda} = \dot{\lambda}_r \qquad (9)$$

Assuming that the reference input wheel skid, λ_r, is constant ($\dot{\lambda}_r = 0$),

$$\dot{\lambda} = 0 \qquad (10)$$

Substituting equation (6) into equation (10) gives

$$\frac{1}{w_x}\left[-\frac{r}{I_w}\left(-F_x r - F_z f_r r - T_{b,eq}\right) + \frac{1-\lambda}{w_x}\dot{w}_x\right] = 0 \qquad (11)$$

where the equivalent control brake torque, $T_{b,eq}$, is used. Solving for the equivalent brake torque, $T_{b,eq}$, gives

$$T_{b,eq} = -F_x r - F_z f_r r - (1-\lambda)\frac{\dot{w}_x I_w}{r} \qquad (12)$$

Approximate equivalent control brake torque, $\hat{T}_{b,eq}$

In analyzing equation (12), all variables and parameters are known, except for the wheel longitudinal acceleration, $\dot{w}_x$. A benefit to using sliding mode control is that it can handle uncertainty in the system [11], as long as the bounds of the uncertainty are known. Assume that the wheel longitudinal acceleration, $\dot{w}_x$, is not known, but can be bounded by

$$\left|\dot{w}_x\right| \le \overline{W}_x \qquad (13)$$

where $\overline{W}_x$ is the maximum wheel longitudinal acceleration. Since the wheel longitudinal acceleration, $\dot{w}_x$, is not known, it has to be replaced with its estimate, $\hat{\dot{w}}_x$, in the design. Since only an estimate is available, only an approximate equivalent brake torque, $\hat{T}_{b,eq}$, can be determined.

$$\hat{T}_{b,eq} = -F_x r - F_z f_r r - (1-\lambda)\frac{\hat{\dot{w}}_x I_w}{r} \qquad (14)$$

Hitting control brake torque, $T_{b,h}$

If the system state $(\lambda, \dot{\lambda})$ is not on the sliding surface, a control term has to be added to the overall brake torque control signal, T_b, to drive the system to the sliding surface. This additional term is called the hitting control brake torque, $T_{b,h}$, and is calculated by starting with the overall brake torque control, T_b, equation. Define the brake torque control, T_b as

$$T_b = \hat{T}_{b,eq} - T_{b,h}\,\mathrm{sgn}(s) \qquad (15)$$

Note, when on the sliding surface ($s = 0$), it is desired to have $T_{b,h} = 0$. So, when on the sliding surface, the hitting control has no affect. However, since equation

(15) contains the approximate equivalent brake torque, $\hat{T}_{b,eq}$, the hitting control brake torque, $T_{b,h}$, additionally effects the system to keep the state on the sliding surface. Due to the uncertainty in the system, the state $(\lambda, \dot{\lambda})$ could stray off of the sliding surface. The hitting control brake torque, $T_{b,h}$, acts to return the state back to the sliding surface.

The hitting control brake torque, $T_{b,h}$, is determined by using the following reaching condition [21],

$$s\dot{s} \le -\eta \,|\, s \,| \tag{16}$$

where $\eta > 0$ is a design parameter. Using (8) and (10), (16) can be expressed by

$$s\dot{\lambda} \le -\eta \,|\, s \,| \tag{17}$$

Using the definition of $\dot{\lambda}$ given in (6) and the definition of T_b given in (15), (17) becomes

$$\begin{aligned} &\frac{-sr}{w_x I_w}\left[-F_x r - F_z f_r r - \left(\hat{T}_{b,eq} - T_{b,h}\,\mathrm{sgn}(s)\right)\right] \\ &\qquad + \frac{s(1-\lambda)\dot{w}_x}{w_x} \le -\eta \,|\, s \,| \end{aligned} \tag{18}$$

Using the definition for the approximate equivalent control brake torque, $\hat{T}_{b,eq}$, (18) becomes

$$\frac{s(1-\lambda)}{w_x}\left(\dot{w}_x - \hat{\dot{w}}_x\right) - \frac{r}{w_x I_w} T_{b,h} \,|\, s \,| \le -\eta \,|\, s \,| \tag{19}$$

Defining the striking control brake torque, $T_{b,h}$, as

$$T_{b,h} = \frac{w_x I_w}{r}\left(F + \eta\right) \tag{20}$$

where F is defined shortly, and substituting (20) into (19)

$$\frac{s(1-\lambda)}{w_x}\left(\dot{w}_x - \hat{\dot{w}}_x\right) - F \,|\, s \,| \le 0 \tag{21}$$

$$\left| \frac{1-\lambda}{w_x}\left(\dot{w}_x - \hat{\dot{w}}_x\right) \right| |\, s \,| \le |\, F \,||\, s \,| \tag{22}$$

The function F has to be designed to ensure the reaching condition specified in (16). From inspection of (22), choosing

$$F \ge \frac{(1-\lambda)\left|\dot{w}_x - \hat{\dot{w}}_x\right|}{w_x} \tag{23}$$

ensures the reaching condition. (23) assumes that $\lambda \in [0,1]$ and $w_x > 0$.

<u>Brake torque control, T_b</u>

In the development of the hitting control brake torque, $T_{b,h}$, the brake torque control, T_b, was assumed to have the form

$$T_b = \hat{T}_{b,eq} - T_{b,h}\,\mathrm{sgn}(s) \tag{24}$$

With using the form specified in (24), the discontinuous switching, $\mathrm{sgn}(\cdot)$, causes chattering in the control. Chattering is high frequency, finite oscillations that can result from neglecting fast dynamics, such as in actuators and sensors, in the control design process [26]. Chattering is an undesirable phenomenon as excessive wear on the actuators can occur [7]. A solution to this issue is to replace the discontinuous switching with smooth, continuous switching, such as $\mathrm{sat}(\cdot)$. Thus, the brake torque control, T_b, has the form

$$T_b = \hat{T}_{b,eq} - T_{b,h}\,\mathrm{sat}\left(\frac{s}{\Phi}\right) \tag{25}$$

where $\Phi > 0$ is a design parameter representing the boundary layer width around the $s = 0$ sliding surface.

SIMULATIONS

The simulations performed are straight-line maneuvers ($\delta = 0$), where the reference input wheel slip, λ_r, on each wheel is constant. The following tables layout the simulation parameters.

Wheel	λ_r	Φ	η
Left Front	0.12	0.025	0.5
Right Front	0.12	0.025	0.5
Left Rear	0.06	0.015	0.3
Right Rear	0.06	0.015	0.3

Table 1. Simulation #1 parameters on 0.9 surface index

Wheel	λ_r	Φ	η
Left Front	0.12	0.025	0.5
Right Front	0.12	0.025	0.5
Left Rear	0.06	0.015	0.3
Right Rear	0.06	0.015	0.3

Table 2. Simulation #2 parameters on 0.6 surface index

Wheel	λ_r	Φ	η
Left Front	0.8	0.025	0.5
Right Front	0.7	0.025	0.5
Left Rear	0.6	0.015	0.3
Right Rear	0.5	0.015	0.3

Table 3. Simulation #3 parameters on 0.9 surface index

White noise is added to the surface index, μ, for all three simulations to generate unknown disturbance in the longitudinal tire force, F_x, signals.

SIMULATION #1

Figure 4 and Figure 5 show the results of this simulation. Figure 4 shows the wheel slip, λ, for the wheels as they track their reference input wheel slip, λ_r. It can be seen that the control tracks the reference input satisfactorily. The tracking errors can be attributed to using continuous switching to reduce chattering, and the white noise added to the surface index, μ. Figure 5 shows the brake torque control signal, T_b. The smooth control action is suitable for the brake actuators.

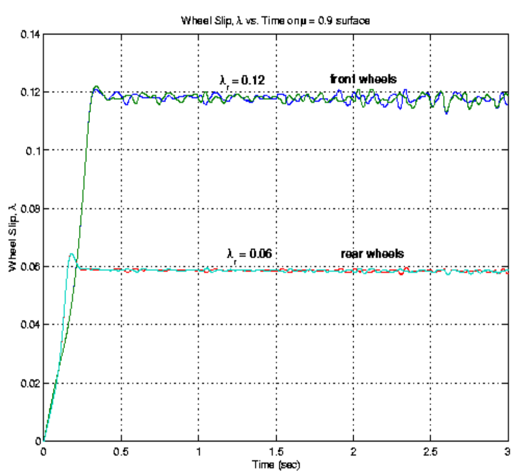

Figure 4. Wheel slip, λ, vs. Time on $\mu = 0.9$ surface

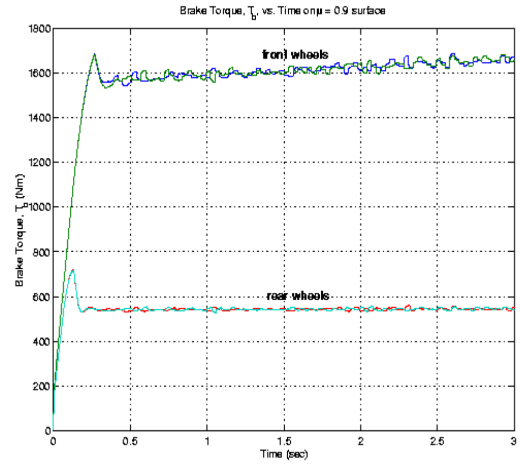

Figure 5. Brake torque, T_b, (Nm) vs. Time on $\mu = 0.9$ surface

SIMULATION #2

The values of the reference input wheel slip, λ_r, are just past the peak of the $\mu - \lambda$ curve. Figure 6 and Figure 7 show the results of this simulation. These results are similar to the results of Simulation #1. Figure 6 shows the wheel slip, λ, for the wheels as they track their reference input wheel slip, λ_r. It can be seen that the control tracks the reference input satisfactorily. The tracking errors can be attributed to using continuous switching to reduce chattering, and the white noise added to the surface index, μ. Figure 7 shows the brake torque control signal, T_b. The smooth control action is suitable for the brake actuators.

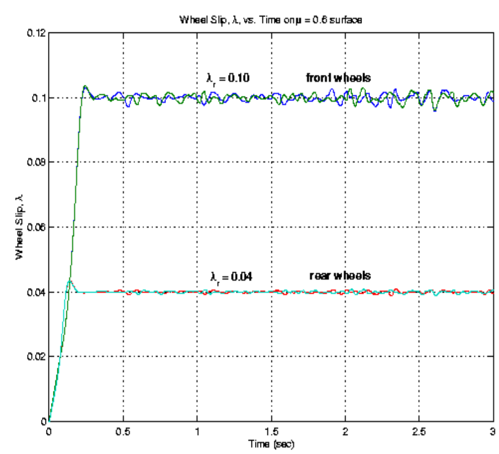

Figure 6. Wheel slip, λ, vs. Time on $\mu = 0.6$ surface

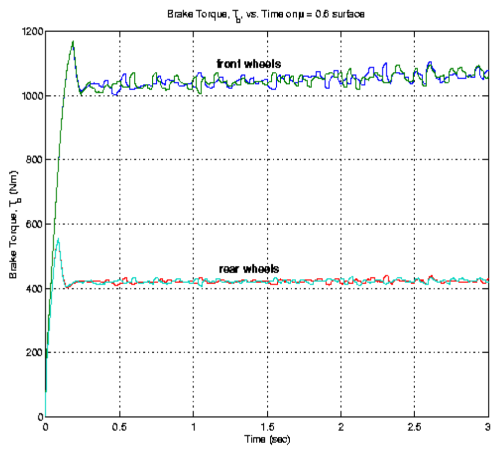

Figure 7. Brake torque, T_b, (Nm) vs. Time on $\mu = 0.6$ surface

SIMULATION #3

Figure 8 shows the result of the simulation. The reference input wheel slip, λ_r, for each wheel is in the traditionally unstable region of the $\mu - \lambda$ curve as seen in Figure 3. It is seen that by feeding back information on the longitudinal tire force, F_x, the control is capable of tracking the reference input wheel slip, λ_r, in the traditionally unstable region of the $\mu - \lambda$ curve.

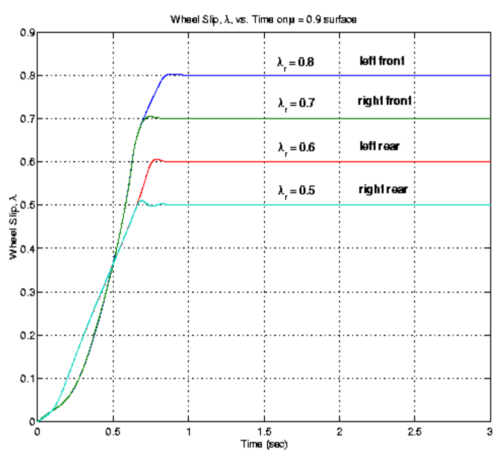

Figure 8. Wheel slip, λ, vs. Time on $\mu = 0.9$ surface

CONCLUSION

This paper investigated the use of sliding mode control for the purpose of controlling the wheel slip, λ. The control objective focused on tracking a reference input wheel slip, λ_r. The reference input wheel slip, λ_r, could be at any level, even in the traditionally unstable region of the μ-λ curve. To achieve this objective, the control required information on the longitudinal tire forces, F_x, which is obtainable through using actual sensor signals or observers. In the development of the control system, the proposed control system incorporates the vehicle lateral dynamics in the wheel longitudinal velocity, w_x, calculations. This is done in preparation of using this wheel slip controller in an overall vehicle dynamics controller (which will also require the lateral tire forces, F_y). Simulations showed the effectiveness of the proposed sliding mode controller.

Though the control presented shows good performance, further investigation is required. It was assumed during the development that the reference input wheel slip, λ_r, is constant. This may not be the case if this strategy is to be part of a vehicle dynamics stability controller where the reference input wheel slip, λ_r, could vary with time. The simulations performed use longitudinal tire force, F_x, feedback signals that contain white noise to check the disturbance rejection of the control. It would be beneficial to include road disturbance, outlined in [27], into the simulations to further verify the control. The continuous switching control used in this paper reduces system chattering; however, it cannot ensure that perfect tracking occurs. A more robust control design can easily be implemented into the structure that is presented in this paper [3].

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