



Turning Standard Line (TSL) Based Path Planning Algorithm for Narrow Parking Lots

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Abstract

Parking path planning is an essential technology for intelligent vehicles. Under a confined area, a parking path has to guide a vehicle into a parking space without collision. To realize this technology, circle-based planning algorithms have been studied. The main components of these algorithms are circles and straight lines; subsequently, the parking path of the algorithm is designed by the combination of these geometric lines. However, the circle-based algorithm was developed in an open space within an unlimited parking lot width, so a feasible path cannot always be guaranteed in a narrow parking lot. Therefore, we present a parking planning algorithm based on Turning Standard Line (TSL) that is a straight line segment. The algorithm uses the TSL lines to guide sequential quadratic Bézier curves. A set of these curves from parking start to goal position creates a continuous parking path. Although the size of free space in a parking lot is small, iterative TSL guides Bézier curves to draw a feasible parking path. We use a sampling technique to find the optimal path and select the minimum-cost path. The planning algorithm proposed in this paper is verified by simulation in various-size parking environments. The simulation results show that the generated path by this algorithm is adapted to the narrow width of a parking lot.

Introduction

Automatic parking system is an important area of study to enhance the safety and comfort of drivers. The main components of this system include space detection, forward-backward motion control, and parking path planning. In particular, parking path is various depending on the parking lot size and shape. The non-holonomic constraints of a vehicle also makes the parking planning process more complex due to the limited range of motion for vehicles.

In order to overcome this complexity, Dubins [1] used a finite sequence of two geometric components (circle of minimum turning radius and straight line) to develop minimal length paths for parking. It is easy to implement this method in practice with low computational time since simple geometric elements are used.

However, its scope of application is highly limited: different calculations have to be applied depending on the configurations of start and goal for parking, the number of straight lines and circles might be changed by the contour of a free space, and the potential for collision is excluded.

Dubin's paths have a minimum length; however, they consist of discontinuous curvatures at the connecting point between an arc of a circle and a straight line. The discontinuity limits the performance of tracking controls. In order to make up for this limitation, M. Tounsi and J. Le Corre in [2, 3] provided a continuous-curvature path generation method using clothoid curve. However, the process to find parameters for this curve in a narrow parking space requires highly complex computations.

A nonlinear control theory-based planning algorithm is discussed in [4, 5, 6, 7], where fuzzy and artificial neural network are used to generate control commands. Baturone, I. in [4] used fuzzy control system in various configurations, and he showed that this method was flexible to diverse parking configurations. Nevertheless, the algorithm requires more precise vehicle model than any other methods since the planning algorithm is tightly coupled with motion controller. The computational expense is more massive than any others because the accurate model generally requires the high degree of calculations.

The methods presented above generate paths for automatic parking in an open space which has an infinite width between parked vehicles. They also require sharply increasing computational resource to be implemented in narrow parking lots. However, free spaces in real parking lots are limited by other parked vehicles and parking lines; constantly, the size of the space may be inadequate for a vehicle to enter a parking space in one motion through a reverse driving.

In this paper, we propose a Turning Standard Line (TSL) based path planning algorithm using sequential Bézier curves. The curves are parametric curves within convex hull of the polygon made by control points. Our algorithm draws TSL which is a straight line segment connecting the start and end of the unit path to search for the required

control points. The number of unit paths is determined by parking zone size and vehicle size. By using a sampling method for selecting the optimal parking path, the TSL-based algorithm can provide a feasible parking path satisfying non-holonomic constraints and non-collision in a narrow parking zone.

This paper is organized as follows. The section of problem state defines parking problems as well as describes geometric and kinematic constraints. The sections of basic curve for path generation and parking path generation show the main concepts of TSL based path generation in parking constraints, and the next section presents the simulation results. Finally, we conclude TSL based-parking path planning algorithm.

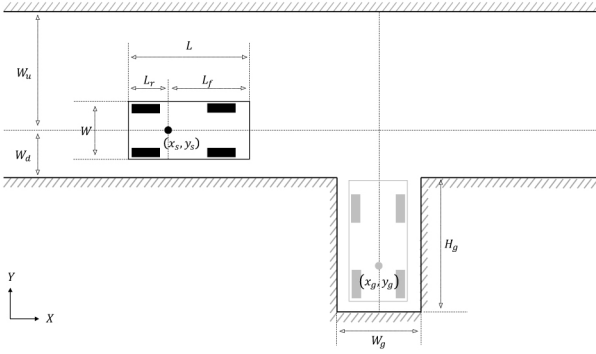


Figure 1. Perpendicular Parking Problem

Problem Statement

Geometric Constraints

Depending on a heading angle at the goal position of a parking lot, there are three types of parking geometry: perpendicular, parallel, angled parking. This paper focuses on the perpendicular parking problem because most buildings in cities provide narrow parking spaces. Fig.1 simplifies the parking problem if we can assume that the environments and the location of empty parking space are known. A vehicle starts from (x_s, y_s) to (x_g, y_g) without collision. The vehicle has width W and length L divided into L_f and L_r . W_g and H_g means the size of a parking space, and the minimum size of them is generally regulated by law. The geometric parameters W_u and W_d represents the width from the vehicle to the upward wall and width from the vehicle to the downward wall respectively. The complexity of parking increases as the sum of W_u and W_d decreases due to the shrinking free space for changing heading angle. This paper focuses on generating a parking path according to decreasing the parameters.

Kinematic Constraints

In order to simplify the model of a vehicle motion, a kinematic bicycle model is a common approximation in [8, 9]. This model assumes that the lateral slip of each wheel is zero and the two left and right wheels are depicted by one single wheel in Fig. 2. The front steering wheel angle is represented by δ . The model is represented by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ k \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \ddot{\psi} \quad (1)$$

Let L denote the vehicle wheelbase, the curvature is determined by the following relationship [1]:

$$k = \frac{\tan\delta}{L} \quad (2)$$

The steering angle, δ , is mechanically limited by δ_{max} , thus the curvature constraint is defined as

$$|k| < k_{max} = \frac{\tan \delta_{max}}{L} \quad (3)$$

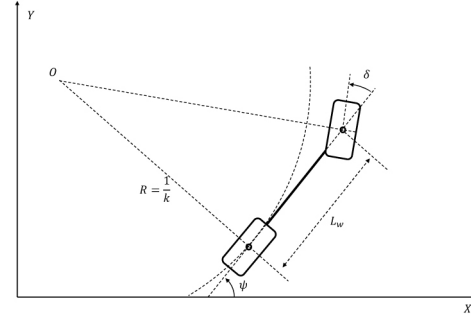


Figure 2. Kinematic Bicycle Model of a Front-steering Vehicle

Hence, a generated path for parking has to ensure that the max curvature of it is less than k_{max} .

Basic Curve for Path Generation

Bézier Curve

Created by the French engineer Pierre Bézier, a Bézier curve is a type of parametric curve defined by control points. Connecting these points can form a polygon which is called Bézier polygon; subsequently, the Bézier curve exists within the convex hull of this polygon. The first and last control points are the start and end points of the curve; however, this curve does not pass through the intermediate control points [10]. The slopes at the control points are the same as ones of segment lines of its polygon. A Bézier curve is mathematically defined by

$$P(t) = \sum_{k=0}^n B_{k,n}(t) P_i, \quad t \in [0, 1] \quad (4)$$

where t is a parameter, P_i is control point, and the polynomials

$$B_{k,n}(t) = \frac{n!}{k!(n-k)!} t^k (1-t)^{n-k} \quad (5)$$

is called as Bernstein basis polynomials of degree n . If a control point is $P_i(x_i, y_i)$ and $n = 2$, the positions of the curve are followed by

$$x(t) = (1-t)^2 x_0 + 2t(1-t)x_1 + t^2 x_2 \quad (6)$$

$$y(t) = (1-t)^2 y_0 + 2t(1-t)y_1 + t^2 y_2 \quad (7)$$

which is called quadratic Bézier curve.

Turning Standard Line (TSL)

In the process of parking driving, a vehicle stop is required to gear positions and allow driving in reverse. This allows the curvature of a driving path to be discontinuous. For this reason, the proposed paper generates a parking path with several unit curves based on a quadratic Bézier curve. Like the previous section, the curve requires three control points. The points are estimated by two Turning Standard Line (TSL) which are straight line segments. The slope of the first TSL is known since the heading angle at the point of a directional change is determined by the previous unit curve. The last TSL is selected by geometric and kinematic constraints as well as the influence of it on the overall parking path. More detailed description for computing TSL is provided in the next section.

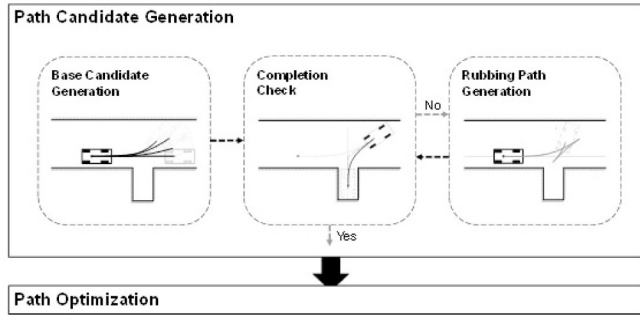


Figure 3. Overview of TSL parking path planning algorithm

Parking Path Generation

Drivers face various parking situations such as unexpected starting position, diverse parking space size and shrinking free space between parked vehicles. In particular, a narrow parking width obstructs a simple path generation for parking, so more than two forward-backward motion are required. The first forward path is important to determine the overall path; however, the analytical calculation of this effect has a high complexity. In this algorithm, the sampling technique is applied to select one path candidate. Fig. 3 provides an overview of the proposed algorithm.

Path Candidate Generation

This part includes three sub-part algorithm: base candidate generation, completion check, and rubbing path generation. Each step finds a feasible unit path until the vehicle can reach the parking goal position. All paths satisfy the curvature constraint and non-collision condition.

In base candidate generation, the candidates of the first forward path is produced by the TSL center point calculated from the start position and the goal position as:

$$x_c = \frac{x_s \sin \psi_s \cos \psi_g - x_g \cos \psi_s \sin \psi_g}{\sin(\psi_s - \psi_g)} + \frac{(y_s - y_g) \cos \psi_s \sin \psi_g}{\sin(\psi_s - \psi_g)} \quad (8)$$

$$y_c = \frac{-y_s \cos \psi_s \sin \psi_g - y_g \sin \psi_s \cos \psi_g}{\sin(\psi_s - \psi_g)} + \frac{(x_s - x_g) \sin \psi_s \cos \psi_g}{\sin(\psi_s - \psi_g)} \quad (9)$$

where ψ_s are ψ_g are heading angles at the start position and the goal position, respectively. TSLs for the candidates are drawn from (x_c, y_c) with the sampled angle:

$$\theta_{sample} = \varphi_s + t \cdot \Delta \theta_s, \quad \theta_{sample} \leq \psi_g$$

where $\Delta \theta_s$ is the unit step of angle sample. Besides the length of each TSL is the max length in collision-free space.

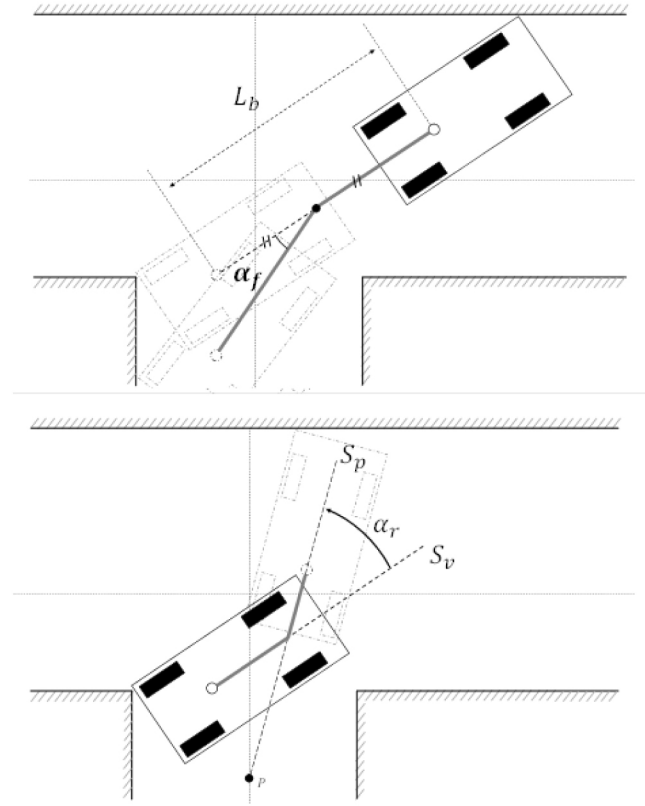


Figure 4. TSL candidate generation in rubbing path generation

The second step (completion check) generates a final path from each position of the previous results to the goal position that excludes the non-feasible candidates due to the probability of collision. The selected paths are sent to path optimization step if the number of the final path is more than zero. Otherwise, the proposed algorithm draws additional paths to change the vehicle heading in the rubbing path generation step.

A narrow width parking space makes it impossible for a vehicle to enter the goal position with a one single backward driving motion. Additional forward-backward paths for turning heading angle are required. For this path the rubbing path generation (the last step) has two unit curves: backward and forward paths. In order to produce the backward TSL, the max length L_b from the previous vehicle position to the wall through the straight line is calculated in Fig.4, and the center point is the middle of the straight line segment. From this point, the TSL is tilted by the angle α ; however, this angle is restricted by a parking contour or a curvature constraint of the vehicle. The max angle α_{max} is selected since the purpose of the rubbing path is to rotate a vehicle heading near to the heading at the parking goal position. Based on the result of the backward TSL, the tangential vector of the forward unit path at the starting point is determined. Unlike the backward path, the center point of the forward path has to exist on straight line that the vehicle at the goal position is to indicate because the complete path, which is described in previous step, is going to be generated after this process. The point is calculated by intersection between the straight lines S_v and S_p where the point P is on the direction of goal heading. If the max magnitude of the tilted angle α_r is found, the step generating the rubbing path is finished.

Table 1. Vehicle parameters

Vehicle parameters	Value	Unit
Length from CG to front bumper, L_f	3.5	m
Length from CG to rear bumper, L_r	1.5	m
Width, W	1.8	m
Wheel base, L_b	2.81	m
Max steering angle, δ_{max}	32.8	deg
Start position, (x_s, y_s, ψ_s)	$(-6, 0, 0)$	(m, m, deg)
Goal position, (x_g, y_g, ψ_g)	$(0, -5, 90)$	(m, m, deg)

Table 2. Geometric parameters of parking zone

Geometric parameters	Case 1	Case 2	Case 3	Case 4
Width of parking space, W_g	2.5 m	2.5 m	2.5 m	2.5 m
Height of parking space, H_g	5 m	5 m	5 m	5 m
Width between CG to lower wall, W_d	1 m	1 m	1 m	1 m
Width between CG to upper wall, W_u	7.0 m	6.3 m	5.2 m	4.8 m

The algorithm of completion check runs again after rubbing path generation. The rubbing path is reproduced until the path to the goal position is feasible or the repetition number is over a threshold value.

Path Optimization

The directional switch of the longitudinal driving is the biggest impact on the quality of a parking path because the time for this process is more than the driving time. However, the path candidates produced in this algorithm have the same number of iterations of direction change. The overall distance only have to be considered for the shortest time. Besides the parking path is used as the reference of tracking controller.

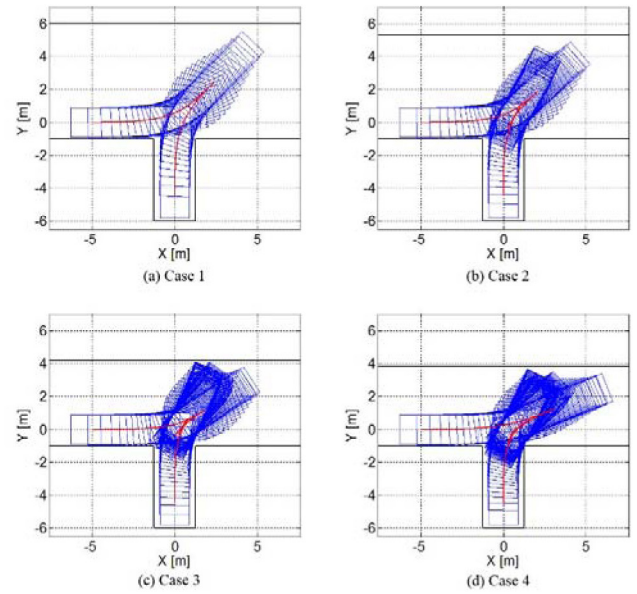


Figure 5. Parking path and vehicle trajectory

Table 3. The results of parking path for four cases

Characteristics	Case 1	Case 2	Case 3	Case 4
Number of a forward-backward set	1	2	3	4
Max curvature [1/m]	0.227	0.228	0.227	0.230
Arc length [m]	15.76	18.07	18.03	21.21

In order to enhance the performance of it, the derivative of curvature has to be the minimum. Thus the cost function is

$$f(x) = w_1 L_c + w_2 \dot{k}_{max} \quad (10)$$

where L_c is the total length of the candidate path and w_i is the weight of the total length and the max derivative of curvature, respectively.

Simulation Result

The TSL-based parking path algorithm was verified through simulation studies in various parking environments. The results of the four case studies are presented in this paper. The vehicle parameters used in this paper were from the real vehicle, as shown in Table 1. Also, the geometric parameters of four parking lots are shown in Table 2. In cases, the width from CG of the vehicle to the upper wall is decreasing in order to show the change of the resulted path in narrow parking environments.

Fig. 5 shows the parking path in each case. Case 1 is the general parking environment of which the width is over 7 m. The proposed algorithm generates the simple path that has one forward-backward iteration without collision as shown in Fig.5 (a). This path consists of two Bézier curves: the forward and the backward curves. The curvatures of the curves are limited to k_{max} which is 0.23 in this simulation. However, the width of the way is smaller, a vehicle cannot reach the final position through the one iteration parking path because of the constraints of geometry and curvature. To overcome this constraints, the TSL-based algorithm in Case 2 and Case 3 generates the additional iterations, as shown in Fig.1 (b) and (c). Although the width is same as the vehicle length, the feasible path is produced with four iteration like Fig.1 (d). All information of iteration, heading and curvature of the each case are presented in Fig.2. Also, Table 3 summary the simulation results.

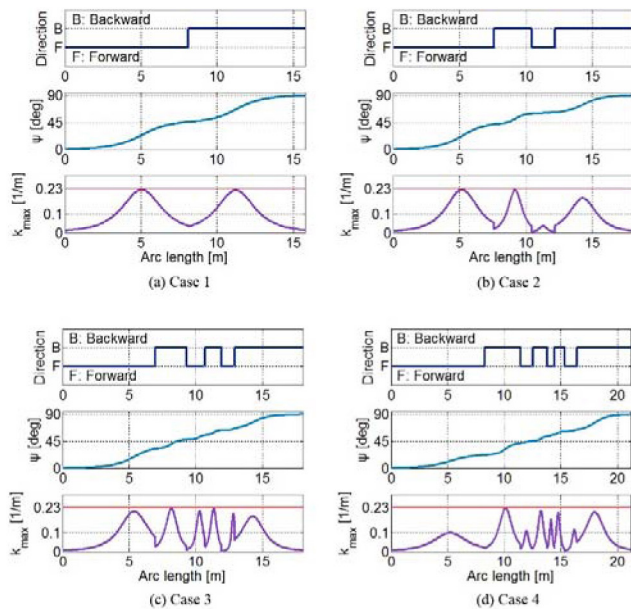


Figure 6. Heading angle and driving direction of the parking path

Conclusions

In this paper, we described the development of parking path planning algorithm based on TSL. The proposed planning algorithm uses Bézier curves designed by TSL in narrow parking zone to generate units for parking paths. The slope and length of this line are adapted to the parking environment and the configurations at the start and goal position. In order to estimate these parameters, this algorithm applies the sampling technique. As the reference of the TSL samples, unit paths are serially drawn within geometric and kinematic constraints until the paths reach the targeted parking position. In the path candidates, the TSL-based algorithm selects the optimal path that has the minimum cost of the overall length and the derivative of curvature. We verified this planning algorithm with narrow parking environment simulations. The simulation results showed that the generated path is adapted to the width of free space in a parking lot. This algorithm can also draw a feasible parking path in a narrow area.

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