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Geometric Path Planning for Automatic Parallel Parking in Tiny Spots

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Abstract: This paper deals with path planning for car-like vehicle in parallel parking problems. Our path planning method uses simple geometry of the vehicle kinematic model. The presented strategy consists in retrieving the vehicle from the parking spot and reversing the obtained path to park the vehicle. Two methods for parking in tiny spaces, where parking in one trial is not possible, are proposed. In these cases, the number of needed trials to park the vehicle can be calculated from a simple formula or from an iterative algorithm. The proposed planning methods are independent of the initial position and the orientation of the vehicle. Reference trajectories are generated so that the vehicle can park by following them. Simulations are provided for both methods.

Keywords: Automatic Parking System, Trajectory Planning, Path Planning, Parallel Parking, Geometric Approaches, Autonomous vehicles.

1. INTRODUCTION

Since parking spots have become very narrow in big cities, drivers need to be experimented and very attentive when maneuvering the vehicle. This often leads to minor scratches on the car and increases traffic jam by multiple repositioning. Therefore, automatic parking is a solution to reduce stress and increase comfort and security of the driver. Here we consider parallel parking problem, which is particularly demanding for the driver.

There are many methods to tackle the trajectory generation for the parking problem:

- Methods based on the use of reference functions. Lee et al. (1999) present a method using Lyapunov function to stabilize the vehicle to a line corresponding to the parking spot. Paromtchik et al. (1997) propose the optimization of two values to find steering angles and durations of commands to execute the parking maneuvers. These methods strongly depend on gains and parameters chosen for the functions, which can be difficult to adjust and not certainly leads to correct parking maneuvers.

- Methods based on fuzzy logic (for exemple Zhao and Collins (2005)) or neural network (for example Jenkins and Yuhas (1993)) to learn human technique. They are limited to human experts knowledge and are difficult to generalize.

- Methods based on two phases path planning, for example Jacobs et al. (1991) and Laumond et al. (1994): creation of collision-free path by a lower-level geometric planner that ignores the motion constraints and subdivisions of this path to create an admissible path. An optimization routine can reduce the length of the path.

- Geometric methods based on admissible collision-free circular arcs, which lead the vehicle in the parking spot in one trial (Lo et al. (2003), Gupta and Divekar (2010), Choi et al. (2011)). The trajectories created with these methods involve easy geometrical equations. Whereas in the method proposed by Gupta and Divekar (2010) the minimum parking spot depends on the initial position of the vehicle, Lo et al. (2003) and Choi et al. (2011) propose a minimal parking spot, which only depends on the characteristics of the vehicle. These methods propose only parking in one trial (without longitudinal velocity sign changing); possible only if the parking spot is sufficiently long.

Here a geometric approach is considered based on retrieving a vehicle from parking spot and reversing this procedure to solve the parking problem (Lo et al. (2003), Choi et al. (2011)). If the vehicle is parked and the parking spot is long enough to retrieving in one trial, a human driver steers the front wheel to maximum angle toward the outside of the parking spot and moves forward until the vehicle is retrieved. Then, he steers on the opposite side to position correctly the vehicle along the road. This procedure being reversible, we apply it in parallel parking problem to form a simple path composed by two circles connected by a tangent point.

Whereas in Lo et al. (2003) and Choi et al. (2011) the authors proposed two identical circles, here only the circle with the minimum radius for the second part of the maneuver is retained. This allows to have vehicles

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with a different maximal steering angle for the right direction than for the left direction. At the beginning of the maneuver, this also avoids steering at zero speed until the maximum steering angle is reached, which is demanding to the electric machine of the steering column.

In Choi et al. (2011), the initial position of the vehicle has to be parallel to the parking spot, on a minimum circle path. In the present solution, if the vehicle is sufficiently far from the spot, its initial position can be everywhere, with any orientation. In consequence, a new path can be calculated at every moment of the maneuver if the real position of the vehicle differs with respect to the planned path. Two generalizations are also proposed to park the vehicle in several trials when the parking spot is not long enough to park the vehicle in one trial. The parking maneuvers are presented assuming that the width of the parking is at least equal to the width of the vehicle. The study concerns the parking on the right side, but it can be easily generalized for the left side parking.

Reeds and Shepp (1990) showed that optimal paths for a vehicle going forwards and backwards can be obtained with circles of minimal radius. In consenquece, this paper puts forward paths constituated when possible by arcs of circle of minimal admissible radius.

In the next section, the path planning in one trial is depicted. Sections 3 and 4 are devoted to the generalization for parking in several trials. In section 5, the differences between these two generalization methods are discussed. In section 6, the generation of reference trajectories is outlined. Finally, section 7 contains the conclusion.

2. PATH PLANNING IN ONE TRIAL

2.1 Model of the vehicle

In this paper front wheels steering vehicles are studied. The model chosen for simulations is the Renault Fluence ZE. Table 1, Table 2, Fig. 1, and Fig. 2 show the notations and the values used here. The vehicle is represented by its bounding rectangle, including the outside rear-view mirrors. The front track is approximated to have the same length as the rear track.

Table 1. Model of used vehicle

Parameters	*Notation*	*Value*
Wheelbase	a	2701 mm
Track	2b	1537 mm
Front overhang	d_{front}	908 mm
Rear overhang	d_{rear}	1114 mm
Distance from the left, right wheel to the left, right side of the vehicle (exterior mirrors folded)	d_l,d_r	136 mm
Maximum left, right steering angle	δ_{lmax},δ_{rmax}	38 degree

2.2 Geometric properties

The parking maneuver is a low-speed movement. Consequently the Ackerman steering is considered with the four wheels rolling without slipping, around the instantaneous center of rotation. Different turning radius are calculated.

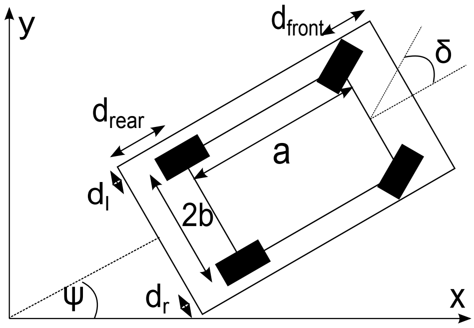

Fig. 1. Vehicle in global (x, y, ψ)-coordinates

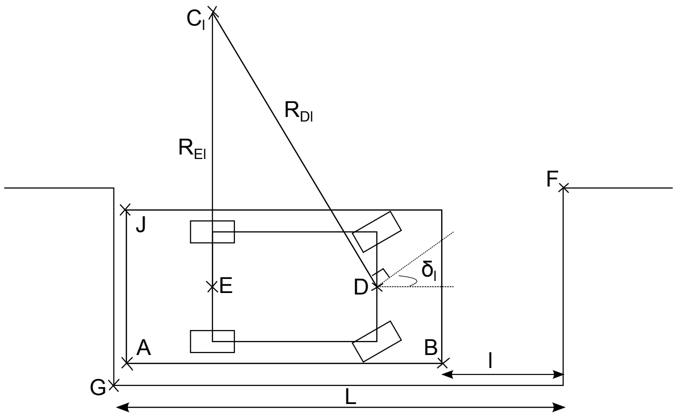

Fig. 2. Vehicle in a parking spot

With E and D being respectively the center of the rear and the front track, it yields:

$$R_E = a/\tan\delta\ ,\ R_D = a/\sin\delta \tag{1}$$

For example, to have R_{El}, we take δ_l. Minimum radius is obtained with the maximum steering angle. With B, A, J being respectively the right front, the right rear and the left rear extremities of the vehicle, applying the Pythagorean Theorem it results:

$$\begin{aligned} R_{Bl} &= \sqrt{(R_{El} + b + d_r)^2 + (a + d_{front})^2} \\ R_{Ar} &= \sqrt{(R_{Er} + b + d_l)^2 + {d_{rear}}^2} \\ R_{Jr} &= \sqrt{(R_{Er} - b - d_r)^2 + {d_{rear}}^2} \end{aligned} \tag{2}$$

2.3 Strategy

To park the vehicle in one trial, the problem is taken in the reversed way by retrieving the vehicle from the parking spot like in Lo et al. (2003) and Choi et al. (2011). A trial or a maneuver is defined as a sequence without velocity sign changing. It is assumed that the parking space is long enough for the one trial parking and that the distance between the rear vehicle and the used vehicle, when it is

Table 2. Notations

Meaning	*Notation*
Distance between 2 points (ex: A, B)	d_{AB}
Circle of center C and radius R	$C(C, R)$
Left, right instantaneous center of rotation	C_l, C_r
Distance between a point (ex: A) and C_l, C_r	R_{Al}, R_{Ar}
Length of the parking spot	L
Absolute value of the steering angle (right, left)	δ (δ_r, δ_l)
Orientation of the vehicle	ψ

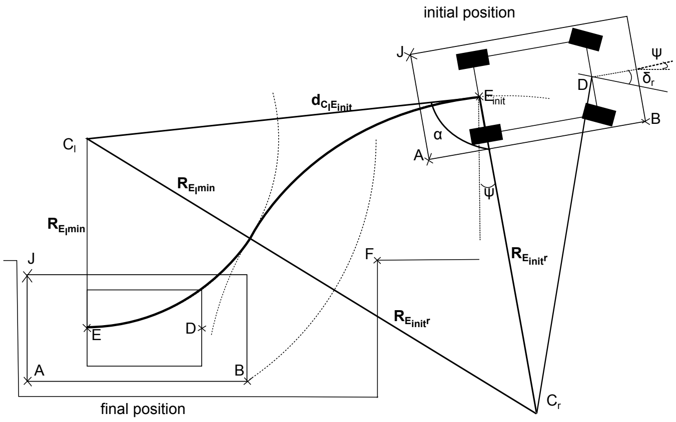

Fig. 3. Strategy for parallel parking in one trial

parked, is small. In these conditions, to exit the parking spot, a human driver steers the front wheels to maximum angle toward the outside of the parking spot and moves forward until the vehicle is retrieved. This creates a path in two arcs of circle, connected by a tangential point. Choi et al. (2011) propose to take all both circles of minimum radius. This means, that at the beginning of the maneuver the car has to be perfectly positioned and parallel to the parking spot.

One of the objectives in this study is to allow the vehicle any initial position and orientation and to give it the possibility to calculate a new maneuver at any moment without need to replace itself at initial position, for instance if the path of the vehicle differs from the calculated one. For that, the second arc of circle (of the retrieving) must have the possibility to be of higher radius than the minimal radius. See Fig. 3 for visualization of one trial path and its concerned values.

The first arc of circle (which is the second in the parking maneuver) is of minimum radius, but the second has to connect the first circle with the real initial point of the car. As the first arc of circle is of minimum radius and as the position where the car has to park is known, C_l and R_{E_lmin} can be deduced. During this arc of circle, the point E goes all over $C(C_l, R_{Elmin})$. For the second arc of circle, we search $C(C_r, R_{E_{init}r})$, which allows E to go from its initial position to $C(C_l, R_{E_lmin})$.

As C_l and the initial position of E are known, $d_{C_lE_{init}}$ is calculated. Having the initial orientation of the vehicle, the angle $\alpha = C_l\widehat{E_{init}}C_r$ is deduced from: $d_{C_lC_r} = R_{E_lmin} + R_{E_{init}r}$ and from the lines $(E_{init}C_r)$ and $(E_{init}D)$ beeing penpedicular. Applying the Al-Kashi Theorem to the triangle $E_{init}C_rC_l$ (see Fig. 3) it yields:

$$R_{E_{init}r} = \frac{{d_{C_lE_{init}}}^2 - {R_{E_lmin}}^2}{2R_{E_lmin} + 2d_{E_{init}C_l}\cos\alpha} \quad (3)$$

$$\delta_r = \arctan\left(a/R_{E_{init}r}\right) \quad (4)$$

2.4 Conditions for feasability

The one trial parking without collision is possible under two conditions: the length of the parking has to be bigger than a minimal length and the circle $C(C_r, R_{E_{init}r})$ has to be admissible for the vehicle.

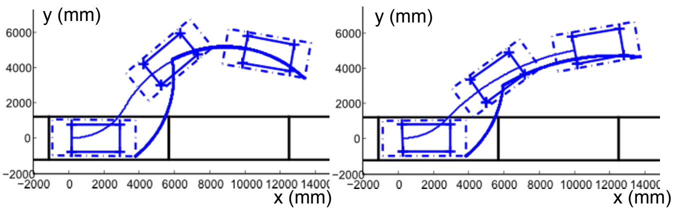

Fig. 4. Simulations of parking in 1 trial with different initial orientations of the vehicle

Like in Choi et al. (2011) the minimal length is:

$$L_{min} = d_{rear} + \sqrt{{R_{B_lmin}}^2 - (a/(\tan\delta_l) - b - d_l)^2} \quad (5)$$

Until the end of the section, $E = E_{init}$ is taken. This means, that the vehicle is considered at the start position. For $C(C_r, R_{Er})$ being admissible, $R_{Er} \geq R_{E_rmin}$ is necessary. Taking $R_{Er} = R_{E_rmin}$, the circle is admissible when $d_{EC_l} \geq d_{EC_lmin}$. From (3) it is deduced:

$$d_{EC_lmin} = R_{E_rmin}\cos\alpha + \sqrt{(R_{E_rmin}\cos\alpha)^2 + R_{E_lmin} + 2R_{E_rmin}R_{E_lmin}} \quad (6)$$

If $d_{EC_l} < d_{EC_lmin}$ the vehicle has to move forward until $d_{EC_l} \geq d_{EC_lmin}$.

2.5 Simulation

Simulation on Matlab were performed with the cinematic vehicle model with the parameters indicated in section 2.1. With this model the minimal length of the parking is 6320 mm. The Fig. 4 shows two simulations with different initial orientations. It can be noticed, that the second arc of circle of the path belongs to the same circle in both cases. Only the length of these arcs of circle differs. In these simulations the initial position of the vehicle is not parallel to the parking spot, in consequence these results cannot be obtained with the method of Choi et al. (2011).

3. PARKING IN N PARALLEL TRIALS

When the length of the parking spot is smaller than Lmin, but still bigger than the length of the vehicle, it is possible to park the vehicle in several trials. One solution is to go to the nearest parallel position from the parking spot using the one trial method. Then the vehicle executes series of forward and backward moves to park in the spot. At the end of each of these moves, the vehicle has to be parallel to the spot.

3.1 Position at the end of the first trial

During the first trial, the vehicle parks in the nearest parallel position to the parking spot. This means that for the second arc of circle of the parking maneuver we have $d_{C_lB_{parallel}} = d_{C_lF} = R_{B_lmin}$. Having this arc of circle, the first one is calculated like in section 2. However, to know the second arc of circle, the nearest parallel position has to be calculated, defined by the distance d between the parking spot and the vehicle in nearest parallel position (see Fig. 5 for visualization of the vehicle in the nearest parallel position).

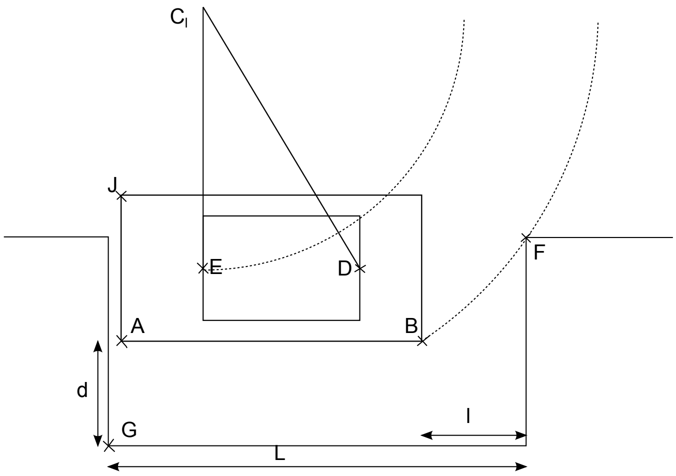

Fig. 5. Vehicle at the nearest parallel position to the parking spot after the first trial

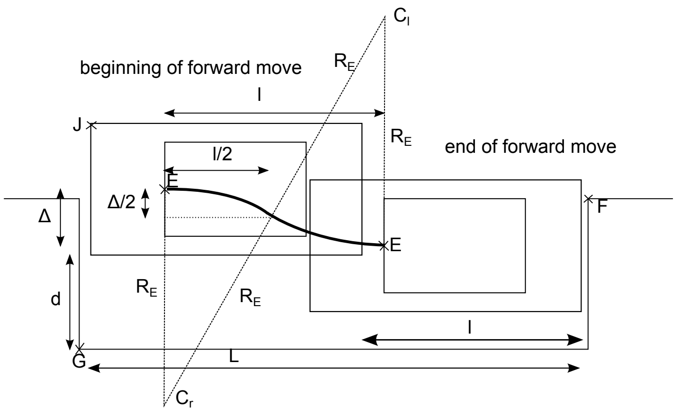

Fig. 6. Forward move during parking in n parallel trials

Calculation of d:
If the vehicle was parked in the spot and it tried to retrieve with maximum steering angle, there would have be the point F_1: intersection of the circle $C(C_l, R_{Blmin})$ and the line $x = x_F$ (see Fig. 5 and Fig. 8). In these conditions, y_{F_1} can be easily calculated. Then d is deduced: $d = y_F - y_{F_1}$.

3.2 Number of trials

During the series of forward and backward moves, the vehicle covers longitudinally the available distance l, where $l = L - (a + d_{front} + d_{rear})$. During the first l/2 distance, the steering is to left, and during the second l/2 distance, the steering is to right (see Fig. 6). To be always parallel to the parking spot at the end of each move and to maximize lateral displacement, left and right steering angles must be maximal and equal: $\delta = min(\delta_{lmax}, \delta_{rmax})$. $R_E = a/\tan\delta$ is deduced. The lateral displacement during a forward or backward move is:

$$\Delta = 2\left(R_E - \sqrt{{R_E}^2 - l^2/4}\right) \tag{7}$$

The total number of trials is deduced (first trial plus series of forward and backward moves):

$$nb_{trials} = IntegerPart(d/\Delta) + 2 \tag{8}$$

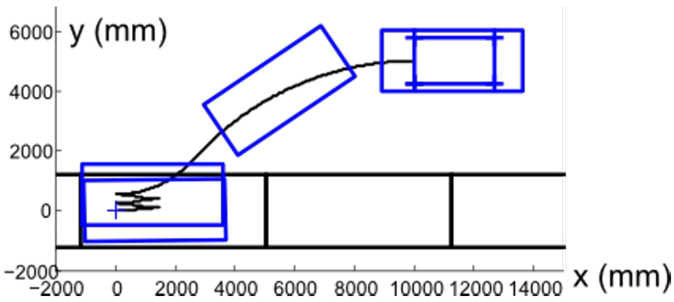

Fig. 7. Simulation of parking in 5 trials by parallel method

3.3 Simulation

Simulation on Matlab was performed with the same model as in section 2.5. The Fig. 7 shows a case with parking in 5 trials.

4. PARKING IN N OPTIMAL TRIALS

To find the parking path with fewest maneuvers, the retrieving path like a human driver would do is searched and then reversed. The retrieving by a human is composed by forward and backward moves until the vehicle can retrieve. A forward move is steering to maximum angle toward left and moving forward until approaching the front obstacle. A backward move is steering to maximum angle toward right and moving backward until approaching the rear or lateral obstacle. When a forward move allows the vehicle to retrieve without collision, the concerned arc of circle is considered, which will be the second arc of circle of the parking path. Then, the feasible arc of circle is searched, which connects by a tangential point the real initial position of the vehicle to the second arc of circle.

4.1 Algorithm for all trials except for the first one

Here is defined the algorithm, which finds the arcs of circle for forward and backward moves, excluding the first trial, by retrieving the vehicle. The first and second arcs of circle composing the first trial are determined in the next section.

(1) The last position of the vehicle when it is parked is known: it is the goal position. C_l, C_r, R_{B_lmin}, R_{B_rmin} for this position are calculated (see Fig. 8).
(2) If $R_{B_lmin} \leq d_{C_lF}$ go to (3), else go to (4).
(3) The vehicle can now retrieve in one trial. Having current C_l, the method detailed in next section is applied to find the path of the first trial.
(4) It is not possible to retrieve in one trial. The vehicle moves forward with maximum left steering angle. The created arc of circle $C(C_l, R_{B_lmin})$ cross the line $x = x_F$ to a point F_1 (see Fig. 8). At the end of this arc of circle, the point B of the vehicle is in F_1.
(5) Having C_l, F_1, and B before step (4), the rotation angle θ_l is deduced.
(6) C_r is the instantaneous center of rotation during the maximum steering; consequently, it can be considered that it belongs to the vehicle. When the vehicle moves along the arc of circle determined in steps (4) and (5), all points of the vehicle, including C_r, are transformed by the rotation of center C_l and angle θ_l:

$$\forall(x, y) \in vehicle \tag{9}$$

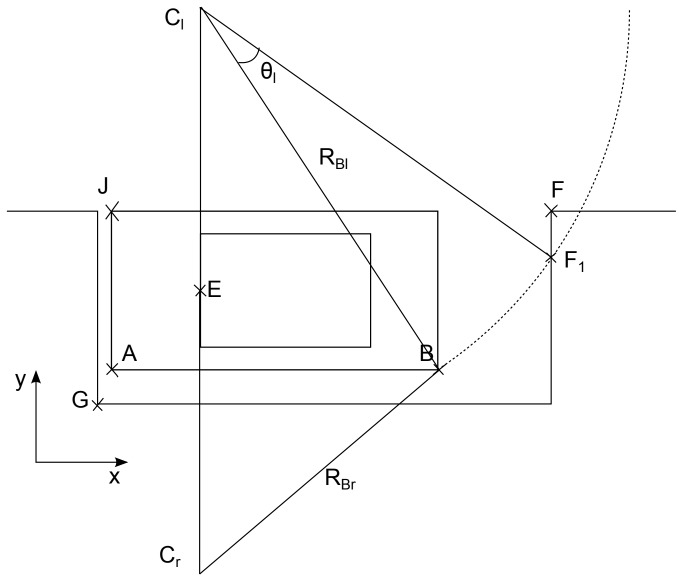

Fig. 8. Vehicle moving forward with maximum left steering angle

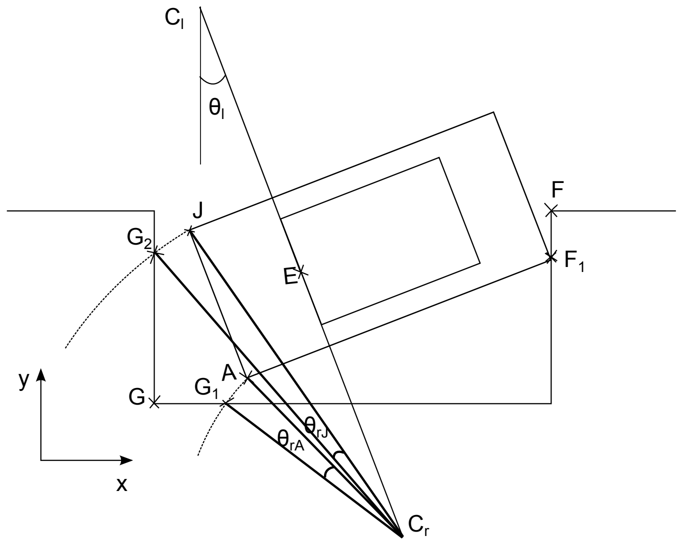

Fig. 9. Vehicle moving backward with maximum right steering angle

$$\begin{pmatrix} x_{new} \\ y_{new} \end{pmatrix} = C_l + \begin{pmatrix} \cos\theta_l & -\sin\theta_l \\ \sin\theta_l & \cos\theta_l \end{pmatrix} \cdot \left(\begin{pmatrix} x \\ y \end{pmatrix} - C_l \right)$$

From now on, only points after this rotation are considered.

(7) The vehicle moves backward with maximum right steering angle. The created arc of circle $C(C_r, R_{A_r min})$ crosses the line $y = y_G$ to a point G_1 (see Fig. 9). Moreover, the created arc of circle $C(C_r, R_{J_r min})$ crosses the line $x = x_G$ to a point G_2.

(8) Having current C_r, A, J, and calculated G_1 and G_2, the rotation angles θ_{rA} and θ_{rJ} are deduced.

(9) The vehicle must move backward as far as possible, but without collision. Consequently, it must stop when J is in G_2 or when A is in G_1. The corresponding arc of circle is defined by $\theta_r = min(\theta_{rA}, \theta_{rJ})$.

(10) C_l is the instantaneous center of rotation during the left maximum steering; consequently, it can be considered that it belongs to the vehicle. When the vehicle moves along the arc of circle determined in steps (7), (8), and (9), all the points of the vehicle, including C_l, are transformed by the rotation of center C_r and angle θ_r:

$$\forall (x, y) \in vehicle \tag{10}$$

$$\begin{pmatrix} x_{new} \\ y_{new} \end{pmatrix} = C_r + \begin{pmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{pmatrix} \cdot \left(\begin{pmatrix} x \\ y \end{pmatrix} - C_r \right)$$

From now on, only points after this rotation are considered.

(11) New $d_{C_l F}$ is calculated and then go to step (2).

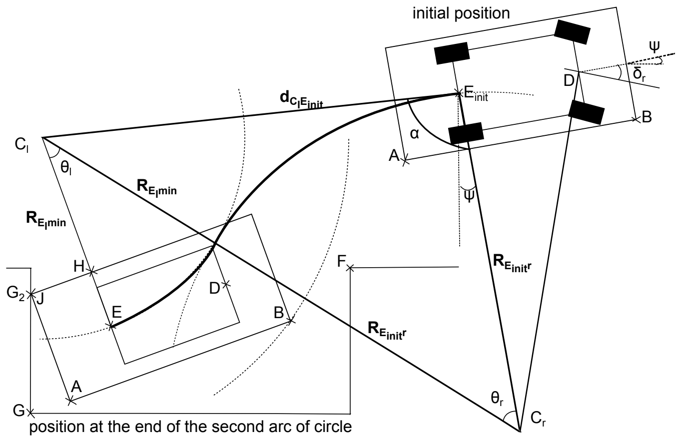

Fig. 10. The two arcs of circle of the first trial during parking in n optimal trials

4.2 First trial

At the end of the algorithm above, the current step is the step (3) and current position of the vehicle, including Cl, is known. This is the position, in which the vehicle will be at the end of the first trial (after the two arcs of circle of the first trial) in maneuver of entering the parking spot.

This means that during the second arc of circle, the point E goes all over $C(C_l, R_{E_l min})$. Like in section 2, $C(C_r, R_{E_{init}r})$, which allows E to go from its initial position to $C(C_l, R_{E_l min})$, is searched (see Fig. 10).

Like in section 2, $d_{C_l E_{init}}$, angle $\alpha = C_l\widehat{E_{init}}C_r$ are calculated and then by applying the Al-Kashi Theorem, $R_{E_{init}r}$ (3) and corresponding δ_r (4) are obtained.

Remark about the feasibility:
The condition of non-collision with the already parked vehicles is satisfied by positioning the vehicle on a non-collision path during the algorithm detailed in the previous section. The condition of feasibility of the first arc of circle is the same as in section 2.

The rotation angles θ_r and θ_l for the two arcs of circle of the first trial could be determined as follows:
As the initial positions of C_r, C_l and E and their positions at the end of the second arc of circle are known (Fig. 10), the angles $\theta_r = E_{init}\widehat{C_r}C_l$ and $\theta_l = E\widehat{C_l}C_r$ for the first and second arc of circle are deduced.

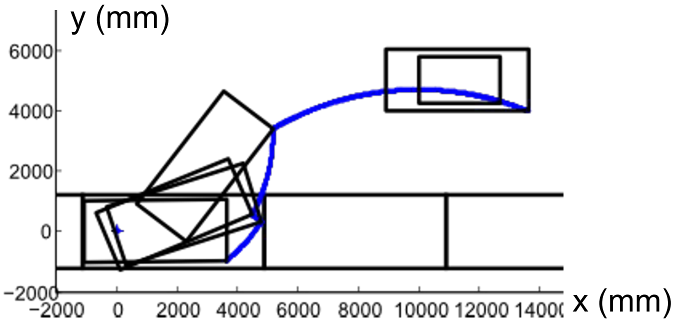

Fig. 11. Simulation of parking in 3 trials by otimal algorithm

4.3 Simulation

Simulation on Matlab was performed with the same model as in section 2.5. The Fig. 11 shows a case with parking in 3 trials.

5. DISCUSSION

Sections 3 and 4 presented two methods for parallel parking in n trials, when parking in one trial is not possible.

The method in n parallel trials is very simple to implement and the number of trials is easily calculated from the geometry of the vehicle and of the parking spot. Nevertheless, the condition of displacement with beginning and end of each move being parallel to the parking spot is very restraining: the total number of trials increases very fast when L decreases (see Table 3), what can be unsatisfactory if a passenger or a driver is present in the vehicle.

The method in n optimal trials is instinctive, when reversing the human retrieving maneuver. With this method, the number of trials can be calculated only from an iterative algorithm. Geometric transformations and formulas involved in the algorithm are more complex in comparison with the method in n parallel trials. It is not easy to visualize for a human which trajectory the vehicle will generate. Nevertheless the number of trials implied in this method is much less considerable than in the method with n parallel trials (see Table 3 for comparison of these methods using model of the section 2.1 for a spot of standard width 2,5 m). In consequence, this method is to be used in priority with passengers on board or to reduce the time of the parking maneuver.

Table 3. Comparison between the two methods in several trials

Length of the spot (cm)	617	616	597	575	567	543
Parallel method (trials)	1	3	5	11	14	32
Optimal method (trials)	1	3	3	3	5	7

6. GENERATION OF REFERENCE TRAJECTORIES

To make the vehicle follow the generated path, time control commands of the steering angle δ and longitudinal velocity v need to be built. As each path presented in this paper can be divided in arcs of circle, a general control approach used for each arc of circle is presented.

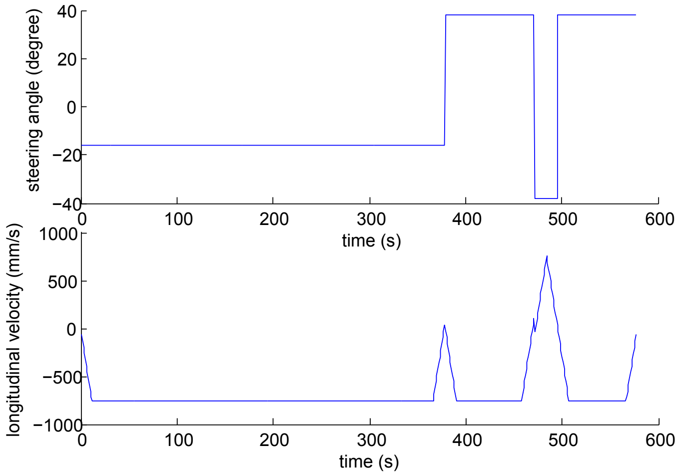

Fig. 12. Simulation of control commands for the parking in 3 optimal trials

Each arc of circle is defined by a radius R and an angle θ. The control commands are generated for the middle of front track, in consequence each radius concerned in the arcs of circles can be calculated by $R = a/\sin\delta$.

For each arc of circle of the path, the used θ and δ have been presented in sections 3 and 4. In consequence, δ_{known} for each arc of circle and the length of each arc of circle $l_{arc} = \theta R$ are deduced.

To generate longitudinal velocity, the same method as in Choi et al. (2011) is used. The needed time to reach a desired maximal velocity or to reach null velocity from the maximal velocity v_{max} is $t_1 = v_{max}/\gamma_{des}$, where γ_{des} is the desired acceleration for the parking maneuver. During t_1, the vehicle goes over the distance $d_1 = \gamma_{des}{t_1}^2/2$. Then, the vehicle stays at constant speed v_{max} during the time $t_2 = (l_{arc} - 2d_1)/v_{max}$.

The control commands are:
$\forall t \in (0, 2t_1 + t_2)$

$$\delta(t) = k_\delta \delta_{known} \qquad t \in [0, 2t_1 + t_2] \tag{11}$$

$$v(t) = \begin{cases} k_v \gamma_{des} t & t \in [0, t_1] \\ k_v v_{max} & t \in [t_1, t_1 + t_2] \\ -k_v \gamma_{des} t & t \in [t_1 + t_2, 2t_1 + t_2] \end{cases} \tag{12}$$

where $k_\delta = \pm 1$ corresponds to a left (+1) or right steering (-1), $k_v = \pm 1$ corresponds to forward (+1) or backward (-1) motion. A simulated example is shown in Fig. 12.

These commands are open-loop in the (x, y, ψ)-coordinates. The steering column and the engine are controlled to execute these commands to provide the desired path and orientation of the vehicle. Possible errors can be compensated by recalculating the geometric path at any moment.

7. CONCLUSION

Easy geometric path planning for parallel parking was proposed. Two methods for the parking in several trials, when the parking in one trial is not possible, were desribed. The methods are independent of initial position and orientation of the vehicle. The parking is possible as long as the length of the parking spot is longer than the length of the vehicle. The methods allow to generate a new path, without replacing the vehicle at any initial position, if the path of the vehicle differs from the firstly generated one.

Simulations were performed and in future, these methods will be tested on real vehicles. The methods will be generalized for the forward parallel parking and for the other parking configurations.

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