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Terrain and Traffic Optimized Vehicle Speed Control

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Abstract: The paper considers optimal control of vehicle speed when the vehicle is driven in a particular geographic region with specific terrain and traffic patterns. The vehicle route is assumed to be unknown in advance. The properties of the terrain and traffic flow are modeled stochastically. A method is proposed for constructing a control policy off-line to optimally prescribe vehicle speed set-point as a function of current driving conditions, for best on average fuel economy and travel speed performance. A related method is proposed to evaluate expected average fuel economy and travel speed performance of sub-optimal control policies, such as the policies which use constant speed offset relative to average traffic speed. The optimal control law which prescribes vehicle speed set-point can be deployed in advanced vehicle cruise control systems or incorporated into a driver advisory function. In addition, the value function of optimal or suboptimal control policies may be used as a terminal cost in a receding horizon optimization of vehicle speed over routes with known initial segments, or for fuel efficient vehicle routing.

Keywords: Automotive control, vehicle speed control, adaptive cruise control, stochastic control, stochastic dynamic programming, optimal control, fuel economy, vehicle routing.

1. INTRODUCTION

Traditionally powertrain control has been based on rigidly tying control decisions to instantaneous operating conditions of the vehicle and to driver requests. The growing interest in improving Real World fuel economy has more recently stimulated research on control policies that are optimized for a particular route being travelled (Woestman et al., 2002; Rajagopalan, 2002; Finkeldei and Back, 2004; Hellstrom et. al., 2006; Katsargyri et. al., 2009), or are customizable to particular driver preferences through vehicle HMI (Christoffel, 1999).

In this paper, we do not assume advanced knowledge of the route to be travelled, only that the vehicle is being driven in a specific geographic region. Our objective is to determine a control policy which, on one hand, is responsive only to current operating conditions, as for the conventional powertrain control strategy, but on the other hand, it is optimized for best on-average performance of a vehicle travelling frequently in a given region. We focus on a specific problem, motivated by vehicle speed control applications, where the objective is to prescribe a vehicle speed set-point in such a way as to achieve optimal trade-off between expected average fuel economy and expected average travel speed.

Our approach is based on aggregating region's terrain and traffic properties and reflecting them in the transition probabilities of a Markov Chain model. The stochastic dynamic programming is then applied (off-line) to generate control policies which perform *best on average*. After being proposed as a drive-cycle independent powertrain control strategy optimization technique in (Kolmanovsky et. al., 2002), stochastic dynamic programming has been used in several references to construct control policies for Hybrid Electric Vehicles, see e.g., (Lin et. al., 2004; Johannesson et. al., 2007).

The overall system for controlling vehicle speed is depicted in Figure 1. The instantaneous road grade, θ, and the *reference speed,* v_t, defined as the average traffic speed at the present location of the vehicle, are estimated on-board of the vehicle. Then the vehicle speed set-point, v^+, is computed according to the constructed off-line optimal control law. This set-point is passed to the vehicle Adaptive Cruise Controller (ACC). The ACC relies on radar to safely follow the set-point, whenever possible, while in traffic. The ACC may switch to fuel-efficient distance following ("distance hybrid"), see (Kolmanovsky and Filev, 2009), if other vehicles are encountered. Alternatively, the optimal set-point can be provided as an advisory to the driver.

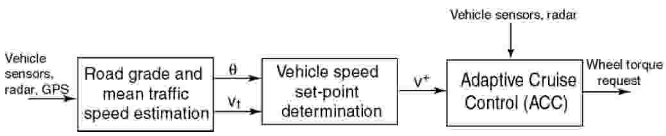

Fig. 1: Schematics of the system which computes optimal vehicle speed set-points to be used by vehicle adaptive cruise control system.

The vehicle fuel consumption is strongly influenced by vehicle speed, road grade and vehicle acceleration. Figure 2, for instance, illustrates the dependence of fuel economy on vehicle speed and road grade, assuming constant speed driving for a light truck type of a vehicle. The dependence on acceleration is exemplified by considering a light truck vehicle following a similar truck over last 100 sec of US06 cycle. By driving optimally and with smaller accelerations, the follower vehicle can increase fuel economy by 10.2%, 36.0%, and 70.6%, if relative distance between two vehicles is allowed to vary within 1 m, 5 m and 10 m range, respectively.

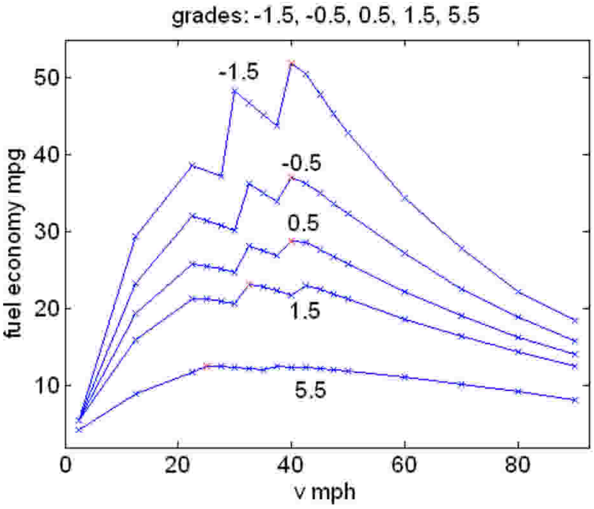

Fig. 2: Fuel economy (mpg) as a function of vehicle speed (mph) and road grade (deg) for constant speed driving. Discontinuities in the curves are associated with gear changes.

The paper is organized as follows. In Section 2 we review the model for estimating fuel consumption, and we discuss our approach to model stochastically the behaviour of road grade and reference speed. In Section 3, we consider the stochastic optimization of the vehicle speed set-point based on stochastic dynamic programming. We also present an approach to evaluate expected average fuel economy and travel speed of suboptimal control policies. Numerical results are reported in Section 4, and concluding remarks are made in Section 5.

2. VEHICLE AND OPERATING CONDITION MODELING

2.1 Vehicle modelling

The model for estimating fuel consumption is based on approximating a higher fidelity, experimentally validated vehicle model. The fuel flow, W_f (kg/sec), is modelled quasi-statically as a function of engine speed, ω_e (rpm), and engine brake torque, τ_e (Nm), i.e.,

$$W_f = W_f(\omega_e, \tau_e). \tag{1}$$

The engine brake torque is estimated assuming that the vehicle speed changes from v to v^+ over a time interval $[t, t^+]$ and vehicle acceleration is constant:

$$\tau_e = \tau_{rl} + \tau_{aux} + I(r) \cdot \frac{(v^+ - v)}{(t^+ - t)} \cdot \frac{1}{r} \tag{2}$$

In (2), $r = \frac{\pi}{30} \frac{r_t}{r_g}$, r_t is the tire radius (m), r_g is the overall gear ratio (including final drive ratio) from wheels to engine crankshaft, $I(r) = I_{eng} + mr^2$ is the equivalent total rotational inertia, I_{eng} is the engine (and FEAD) inertia, m is the vehicle mass (kg), and τ_{aux} is the loss torque (Nm) due to running auxiliaries. The τ_{rl} is the road load torque (Nm),

$$\tau_{rl} = \left(\frac{1}{2} C_D A v^2 + mC_{rr} \cos(\theta/180 \cdot \pi) + mg \sin(\theta/180 \cdot \pi) \right) \times \\ \times \frac{v}{\eta_{tot} \omega_e \frac{\pi}{30}}. \tag{3}$$

In (3), C_D is the coefficient of drag, A is the vehicle frontal area (m^2), C_{rr} is the (scaled) vehicle rolling resistance coefficient, θ is the road grade (deg), and η_{tot} is the efficiency accounting for losses in the driveline. Considering in this paper a vehicle with a converter-less transmission (such as a dual clutch transmission),

$$\omega_e = v / r. \tag{4}$$

A classifier for the gear as a function of vehicle speed, vehicle acceleration and road grade has been developed based on the data from a high fidelity vehicle model simulation, so that

$$r_g = r_g\left(v, \frac{v^+ - v}{t^+ - t}, \theta\right). \tag{5}$$

2.2 Operating conditions modelling

Two Markov Chains, each with a finite number of states, are employed to reflect patterns in road grade, θ, and reference vehicle speed, v_t. With this approach, stochastic dynamic programming can be applied to construct control policies which tend to perform well on average (Kolmanovsky et. al., 2002; Johannesson et. al., 2007; Lin et. al., 2004). The transition probabilities are denoted by $P(\theta^+, | \theta)$, $P(v_t^+ | v_t)$ where θ^+, v_t^+ designate road grade and reference vehicle speed one distance increment, Δs, ahead, respectively. Distance increments, as opposed to time increments, are used because we view road grade and reference speed as ``spatially'' prescribed quantities, reflecting terrain and traffic properties along roads in the given geographical region. The travel time over a single distance increment is approximated as

$$t^+ - t = \frac{2\Delta s}{v + v^+}. \tag{6}$$

In the subsequent numerical examples, transition probabilities were estimated from frequencies of transitions observed when driving an experimental vehicle with the traffic over eight typical routes, each of about 30 km, in and around Dearborn, Michigan. Figure 3 shows transition probabilities constructed assuming cells corresponding to Markov Chain states were [$-\infty$, -6[, [-6, -5[, … , [5, 6[, [6, $+\infty$ [for the grade (deg) and [0, 2[, [2,4[, … ,[28, 30[, [30, $+\infty$[for the reference speed (m/s).

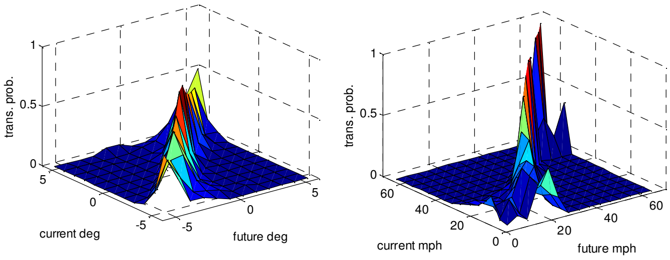

Fig. 3: Transition probabilities for road grade (left) and reference speed (right).

The Markov Chains for the grade and reference speed were combined assuming statistical independence

$$P(\theta^+, v_t^+ \mid \theta, v_t) = P(\theta^+, \mid \theta) \cdot P(v_t^+ \mid v_t). \tag{7}$$

Remark 1: The model can be extended with further details. For instance, transition probabilities can be conditioned on other explanatory variables (vehicle heading direction, time of the day, day of the week, road class, etc). Markov Chain states can be defined as clusters of road grade and reference speed values which have appeared in measured data. The Markov Chain model can reflect the behaviour of elevation, h, and reference vehicle acceleration, a_t in addition to the road grade and reference speed. These details may increase both model complexity and the computational complexity of generating optimal policies using stochastic dynamic programming. The strength of stochastic modelling in reflecting aggregate properties can, alternatively, be exploited to generate reasonably performing control policies based on not overly complicated models. The desired structure of the control law (which variables it can depend on due to practical constraints such as sensor availability) can provide guidance to the needed model structure.

3. OPTIMAL AND SUBOPTIMAL CONTROL POLICIES

3.1 Optimal control problem formulation

With the functional relationships outlined in Section 2, the mass flow rate of fuel, $W_f(\theta, v, v^+)$, can be estimated as a function of the grade, θ, and vehicle speeds, v and v^+ at the beginning and at the end of the road segment of length Δs, respectively. To simplify the computational treatment, the optimal control problem is formulated with respect to vehicle speed offset relative to reference speed, i.e., with respect to $u = v - v_t, u^+ = v^+ - v_t^+$, which vary over a smaller range as compared to v. The expected segment travel time is approximated as

$$\Delta\bar{t}(v_t, u, u^+) = E_{v_t^+}\left[\frac{2\Delta s}{v_t + u + v_t^+ + u^+}\right],$$

and the expected fuel consumption over the segment is approximated as

$$\overline{W}_f(\theta, v_t, u, u^+) = E_{v_t^+}\left[W_f(\theta, v_t + u, v_t^+ + u^+)\frac{2\Delta s}{v_t + u + v_t^+ + u^+}\right],$$

with $E_{v_t^+}[\cdot]$ denoting the expectation conditional to the given values of θ, v_t, u, u^+. This approximation is justified given sparse, close to diagonal structure of $P(v_t^+ \mid v_t)$ matrix, see Figure 3. The objective of minimizing the expected average fuel consumption can be stated as

$$E\sum_{k=0}^{\infty} q^k \overline{W}_f(\theta(k), v_t(k), u(k), u^+(k)) \to \min_{u^+(\cdot)}. \tag{8}$$

Here E denotes the expectation, k is the road segment number and $0 \le q < 1$ is a discount factor which ensures the convergence of the infinite sum in (8). The value $q = 0.96$ was used in the subsequent numerical examples. The minimization of (8) is performed subject to several constraints. The constraint (9) ensures that the vehicle speed does not deviate from the reference speed, v_t, by more than a pre-determined amount, while satisfying other constraints such as speed limits, acceleration limits, etc:

$$u^+(k) \in O(\theta(k), v_t(k), u(k)), \tag{9}$$

where $O(\theta, v_t, u)$ denotes the *set* of feasible vehicle speed offsets relative to reference speed. An interesting possibility is to identify $O(\theta, v_t, u)$ from values of $v^+ - v_t^+$ which were observed at different θ, v_t, u in the recorded vehicle trajectories of several vehicles with several drivers. The constraint (10) is imposed to encourage an average travel speed, v_{tgt}:

$$E\sum_{k=0}^{\infty} q^k \Delta\bar{t}(v_t(k), u(k), u^+(k)) \le \sum_{k=0}^{\infty} q^k \frac{\Delta s}{v_{tgt}} = \frac{\Delta s}{(1-q)v_{tgt}}. \tag{10}$$

Under appropriate assumptions, see (Altman, 1999; Wirth, 2001), as $q \to 1$ the optimal control policy and the value function (multiplied by $(1-q)$) of the discounted problem (8)-(10) may be shown to converge to the optimal control policy for the averaged cost control problem. Thus problem (8)-(10) has a straightforward interpretation: Given a vehicle travelling along various roads in a given geographical region we wish to prescribe the vehicle speed to maintain as a function of the current driving conditions (grade and reference speed), to optimize a trade-off between expected averages of fuel economy and travel time.

Based on the results in (Altman, 1999), the discounted optimal control problem (8)-(10) reduces to finding a value function, $V(\theta, v_t, u, \lambda)$, which satisfies the Bellman equation:

$$V(\theta, v_t, u, \lambda) = \min_{u^+} Q(\theta, v_t, u, u^+, \lambda),$$

where (11)

$$\begin{aligned} Q(\theta, v_t, u, u^+, \lambda) &= \left(\overline{W}_f(\theta, v_t, u, u^+) + \lambda \Delta \bar{t}(v_t, u, u^+)\right) \\ &+ \sum_{\theta^+, v_t^+} q \cdot V(\theta^+, v_t^+, u^+, \lambda) \cdot P(\theta^+, v_t^+ \mid \theta, v_t) \end{aligned},$$

for an appropriate value of the multiplier, $\lambda \geq 0$, and over the feasible domain of (θ, v_t, u). The optimal control policy is a minimizer in (11), i.e.,

$$U^*(\theta, v_t, u, \lambda) \in \underset{u^+ \in O(\theta, v_t, v_t + u)}{\arg\min} \; Q(\theta, v_t, u, u^+, \lambda). \tag{12}$$

With U^* computed off-line, the vehicle speed set-point one distance step ahead at the time instant k is computed on-line as a function of current road grade, $\theta(k)$, reference speed, $v_t(k)$, and vehicle speed, $v(k)$, as

$$v^+(k) = U^*(\theta(k), v_t(k), v(k) - v_t(k), \lambda) + v_t^+(k). \tag{13}$$

The value function iterations for $n \geq 0$ may be used to approximate V:

$$\begin{aligned} &V_{n+1}(\theta, v_t, u, \lambda) = \min_{u^+} Q_n(\theta, v_t, u, u^+, \lambda), \\ &Q_n(\theta, v_t, u, u^+, \lambda) = \left(\overline{W}_f(\theta, v_t, u, u^+) + \lambda \Delta \bar{t}(v_t, u, u^+)\right), \\ &+ \sum_{\theta^+, v_t^+} q \cdot V_n(\theta^+, v_t^+, u^+, \lambda) \cdot P(\theta^+, v_t^+ \mid \theta, v_t), \\ &u^+ \in O(\theta, v_t, u), \\ &V_0(\theta, v_t, u, \lambda) = 0. \end{aligned} \tag{14}$$

Under usual assumptions, the iterations (14) converge; computationally these iterations are performed until a stopping criterion, either in terms of maximum number of iterations being exceeded or in terms of $\|V_{n+1}(\cdot) - V_n(\cdot)\|$ becoming less than a threshold, is satisfied. An alternative approach to solve (11) is using linear programming; see (Altman, 1999).

Remark 2: On-board of the vehicle, the grade $\theta(k)$ in (13) may be obtained from GPS location and digital road maps, or based on on-board estimates (from longitudinal accelerometer, differential GPS or wheel speed and vehicle model). The reference speed, $v_t(k)$, can be generated from estimated traffic speed based on radar measurements or based on traffic information transmitted to the vehicle via wireless communications. The reference speed one step ahead, $v_t^+(k)$, can be estimated as the most likely next state from the Markov Chain transition probability matrix.

3.2 Determining the value of multiplier λ

Our approach to selecting λ is based on separately computing two functions related to expected cumulative fuel consumption and travel time. These functions are defined as

$$\begin{aligned} &F_{n+1}(\theta, v_t, u, \lambda) = \overline{W}_f(\theta, v_t, u, u^+) \\ &+ \sum_{\theta^+, v_t^+} q \cdot F_n(\theta^+, v_t^+, u^+, \lambda) \cdot P(\theta^+, v_t^+ \mid \theta, v_t), \\ &F_0 = 0, \\ &T_{n+1}(\theta, v_t, u, \lambda) = \Delta \bar{t}(v_t, u, u^+) + \\ &\sum_{\theta^+, v_t^+} q \cdot T_n(\theta^+, v_t^+, u^+, \lambda) \cdot P(\theta^+, v_t^+ \mid \theta, v_t), \\ &T_0 = 0, \\ &u^+ = U^*(\theta, v_t, u, \lambda). \end{aligned} \tag{15}$$

We note that $V_n = F_n + \lambda T_n$ and that F_n, T_n is a monotonically increasing with n sequence of functions. The functions $F^*(\theta, v_t, u, \lambda)$, $T^*(\theta, v_t, u, \lambda)$ in the limit of these iterations estimate expected discounted cumulative fuel consumption and expected discounted travel time for a vehicle trajectory starting in a location with specified θ, v_t, u.

If we assume that the vehicle can start its trip at a random location within a given geographical region, the distribution of initial grade and reference speed will be according to the *steady-state probability distribution* of the Markov Chain, $p^*(\theta, v_t)$. This steady-state probability distribution can be computed as a left eigenvector of unit magnitude of the transition probability matrix corresponding to the eigenvalue of 1. Hence, we propose to average $F^*(\theta, v_t, u, \lambda)$, $T^*(\theta, v_t, u, \lambda)$ according to the initial location by defining,

$$\begin{aligned} \overline{T}^*(\lambda) &= \sum_{\theta, v_t} T^*(\theta, v_t, 0, \lambda) \cdot p^*(\theta, v_t), \\ \overline{F}^*(\lambda) &= \sum_{\theta, v_t} F^*(\theta, v_t, 0, \lambda) \cdot p^*(\theta, v_t), \end{aligned} \tag{16}$$

and convert these to expected average speed and mile-per-gallon,

$$\begin{aligned} \bar{v}^*(\lambda) &= \frac{\Delta s}{(1-q)\overline{T}^*(\lambda)}, \\ MPG^*(\lambda) &= \frac{1}{\frac{(1-q)\overline{F}^*(\lambda)}{\Delta s} \cdot 1000 \cdot 1.6 \cdot \frac{1000}{773} \cdot \frac{1}{3.785}}. \end{aligned} \tag{17}$$

Note that in (16) we assumed $u = 0$ initially so that the initial speed of the vehicle matches reference speed. The quantities (17) are two scalars which can be plotted as a function of λ. See Figure 5. The selection of λ may be made by the driver of the vehicle through an HMI interface, based on a graphical display of such plots.

3.3 Evaluation of alternative control policies

Alternative, sub-optimal control policies can be evaluated using expressions (15)-(17). In this case, $u^+ = U^*(\theta, v_t, u, \lambda)$ is replaced by $u^+ = U^a(\theta, v_t, u, \alpha)$, where $U^a(\theta, v_t, u, \alpha)$ is a suboptimal control policy and where α is a control policy parameter (if any). Examples of such sub-optimal policies include a *constant speed offset policy*

$$u^+ \equiv \alpha,$$

and *a quasi-static policy,* $u^+ = U^{qs}(\theta, v_t, \alpha)$, defined as

$$u^+ \in arg \min_{u^+ \in O(\theta, v_t, u)} \left(\alpha + W_f^{ss}(\theta, v_t + u^+)\right) \frac{\Delta s}{v_t + u^+},$$

where $W_f^{ss}(\theta, v)$ is the steady-state fuel flow for a vehicle on a grade θ and moving with velocity v.

4. FUEL ECONOMY ANALYSIS

4.1 Constant and quasi-static vehicle speed offset policies

We first consider a sub-optimal, constant speed offset policy according to which the vehicle is driven with a constant offset with respect to reference speed. The offset resulting in infeasible negative vehicle speed set-point, i.e, u such that $v_t + u \leq 0$, was interpreted as giving vehicle the minimum speed, $v = v_{min} > 0$. The expected vehicle speed and fuel economy, averaged over the vehicle starting point, are illustrated in Figure 4. As the vehicle speed offset increases, the expected vehicle speed increases and the expected fuel economy *generally* decreases. This decrease is not monotonic given intricate dependence of fuel economy on gear shifts. There is about 1.56 mpg (6.65%) average benefit to drop the speed by 3 m/sec (6.75 mph) and the expected travel speed is reduced by 8 mph in this case.

The quasi-static policy has also been evaluated for different values of λ. Interestingly, it showed worse expected fuel economy at comparable average travel speed versus constant offset policy (e.g., about 0.39 mpg worse at 50 mph and about 0.84 mpg worse at 69.5 mph). This conclusion may be specific to our vehicle model and transition probabilities.

4.2 Evaluation of optimal policies

Optimal control policy has been constructed numerically for $O(\theta, v, v_t) = [-5,5]$ and several values of the multiplier, $\lambda \in \{4\times10^{-3}, 8\times10^{-3}, 12\times10^{-3}, 16\times10^{-3}\}$. Large speed offset range (+/-5 m/s = +/- 11.25 mph) was selected consistently with the range of constant offset policies to estimate potential benefits of dynamically adjusting vehicle speed relative to the reference speed. The results are reported for the case when an artificial constraint is added to ensure that the vehicle speed one step ahead does not leave the grid/cells used to perform value iterations (14):

$$1 \leq v_t^+ + u^+ \leq 29 \; if \; P(v_t^+ \mid v_t) > 0. \tag{19}$$

This constraint was not used for constant offset policies and it slightly eased the numerical implementation of value iterations (14). Figure 5 demonstrates that the expected vehicle speed and fuel economy are, as expected from (11), monotonic with λ. As has been verified by re-generating Figure 4 with constraint (19) added, for $\lambda = 16\times10^{-3}$ the expected travel speed and fuel economy of optimal policies are close to those of the constant vehicle speed offset policy with +5 m/s offset. This is expected since large λ emphasize the objective of travelling fast. The fuel economy is improved for smaller values of λ, with some degradation in travelling speed. For instance, comparing the results for $\lambda = 4\times10^{-3}$ optimal policy and for zero constant offset policy (i.e., if vehicle is travelling with the reference speed) suggests about 30% fuel economy improvement opportunity at a comparable average travel speed. Hence, the ability to adjust speed dynamically relative to traffic and depending on the grade can be beneficial for fuel economy. A cross-section of the optimal control policy for $\lambda = 4\times10^{-3}$ is illustrated in Figure 6. Figure 7 presents simulated vehicle speed set-point in response to step changes in reference speed and road grade. Such step changes are not produced by Markov Chain model, yet are useful for policy evaluation. Depending on the reference speed and grade, the optimal control can effect convergence to constant set-point or to periodically changing set-point. In the case of periodically changing set-point (the so called "pulse and glide"), within each period, initial speed increase is followed by a longer period of coasting.

6. CONCLUSIONS

Vehicle speed control can be *customized* to terrain and traffic patterns in the region where the vehicle is being repeatedly driven by using pre-recorded routes to induce Markov Chain transition probabilities and by applying off-line stochastic dynamic programming to construct an optimal control policy for vehicle speed set-point as a function of the current driving conditions (road grade, average traffic speed and vehicle speed). With this optimal control policy, the *expected average* fuel economy can be improved without significantly degrading travel time. For suboptimal control policies (constant speed offset, quasi-static minimization, etc.) a method has been proposed to determine expected average fuel economy and travel speed. The value function of optimal or suboptimal policies can be incorporated as a terminal cost in the receding horizon (MPC-based) optimization of vehicle speed over routes with known initial segments (Katsargyri et. al., 2009). Finally, the segment costs in routing problems, where the best route in terms of expected fuel economy and travel time is to be determined, can be based on the value function constructed for each segment using our approach.

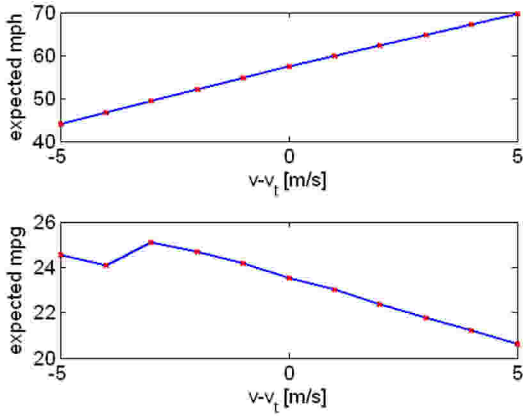

Fig. 4: Dependence of expected average vehicle speed (mph) and expected average fuel economy (mpg) on the vehicle speed offset (in m/s) relative to reference speed.

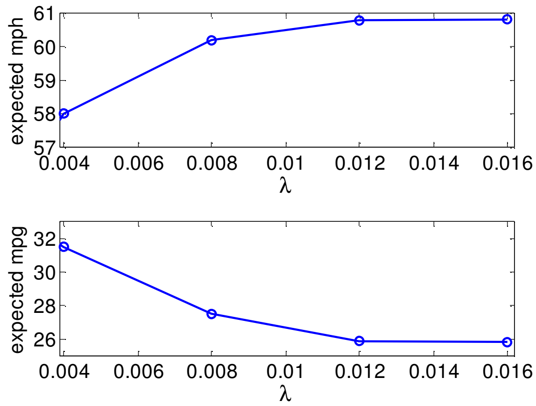

Fig. 5: Dependence of expected average vehicle speed (mph) and expected average fuel economy (mpg) on the weight λ for the optimal policies.

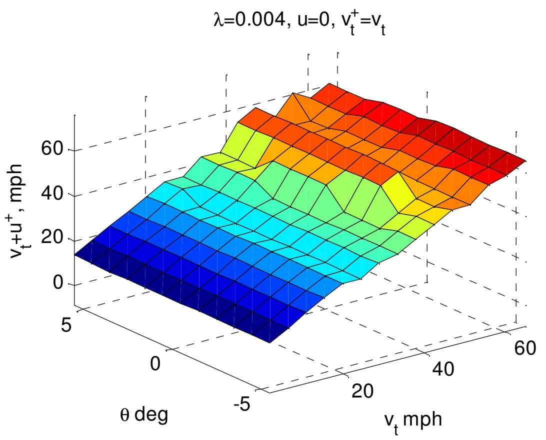

Fig. 6: $v_t + u^+$ as a function of road grade and reference speed if $v - v_t = 0, \lambda = 4\times10^{-3}$ with the optimal policy.

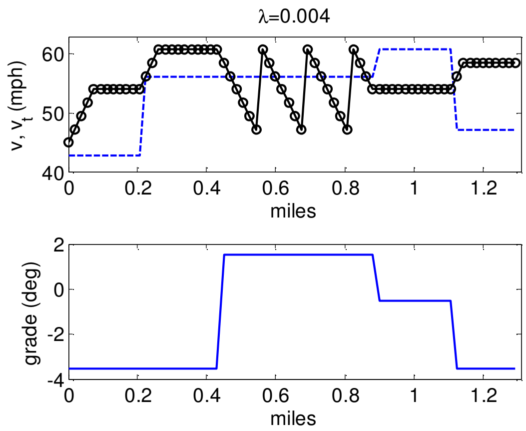

Fig. 7: Simulated vehicle speed set-point (top, solid) in response to steps in reference speed (top, dashed) and grade (bottom, solid).

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