Ultrasonic robot localization using Dempster-Shafer theory

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Abstract

In this paper we present a method for ultrasonic robot localization without *a priori* world models utilizing the ideas of distinctive places and open space attraction. This method was incorporated into a **move-to-station** behavior, which was demonstrated on the Georgia Tech mobile robot. The key aspect of our approach was to use Dempster-Shafer theory to overcome the problem of the uncertainty in the range measurements returned by the sensors. The state of the world was modeled as a two element frame of discernment Θ: *empty* and *occupied*. The world itself was represented as a grid, with the belief in whether a grid element was empty or occupied was set to total ignorance (*don't know*) at the beginning of the robot behavior. A belief model of the range readings was used to compute the belief of points in the environment being empty, occupied, or unknown. Belief from repeated measurements updated the world map according to Dempster's rule of combination. The current belief in the empty space was used to construct a weighted centroid of the empty space (or *station*) after each move of the robot. By moving toward this center of mass and continually adding to the beliefs of the points in the environment the robot iteratively moved to the center of the open space. Experiments demonstrated that the robot was able to localize itself with a repeatability of 1.5 feet in a 33 foot square room, regardless of the starting position within the open space. This method is contrasted with a technique which did not explicitly model the belief in the range readings; that technique was unable to consistently converge on the center of the room within ten moves.

1. INTRODUCTION

Localization of an autonomous mobile robot without the use of *a priori* world models is an important issue in navigation through unknown or partially known environments. Techniques such as the use of distinctive places in qualitative navigation [5, 4] have focused on control issues and mapping strategies for navigation which assume minimal perception. These methods require inexpensive, computationally tractable, and reliable perceptual algorithms to position a robot in an unknown environment at a location which can be accurately re-acquired each time the robot returns to that area.

This paper reports on one such localization algorithm. The algorithm uses range readings from inexpensive wide beam ultrasonic transducers to build a model of the empty regions in the room; the robot moves to the centroid of the empty space, weighted by the degree of belief in the empty regions. The algorithm starts with complete ignorance about the room. It assumes that the world is static and that the empty space is bounded (i.e., it is a room, not a long hallway or outdoor space); otherwise the algorithm will not converge on a location. Experiments performed on George, the Georgia Tech Denning DRV-1 mobile robot show for a cluttered tool room that if the room is unchanged between visits, the robot will position itself within a foot and a half radius of the same point in the room.

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The specific problem which motivated this work was to have a mobile robot locate itself reliably in a cluttered room which did not lend itself to *a priori* modeling. One view of the room is shown in Figure 1. Once at the distinctive location, or *station*, the robot was to employ more complex perceptual algorithms using computer vision. The better the repeatability of the localization routine, called the **move-to-station** behavior, the more reliable the complex perception would be.

Our approach was to use ultrasonic transducers to place the robot in the "center" of the room. The problem with spurious range readings, which typically make accurate localization with ultrasonics difficult or impossible, were overcome by explicitly modeling the belief in the transducers' observations. Our belief model differed from previous Bayesian uncertainty representations [6, 7] by using Dempster-Shafer theory.

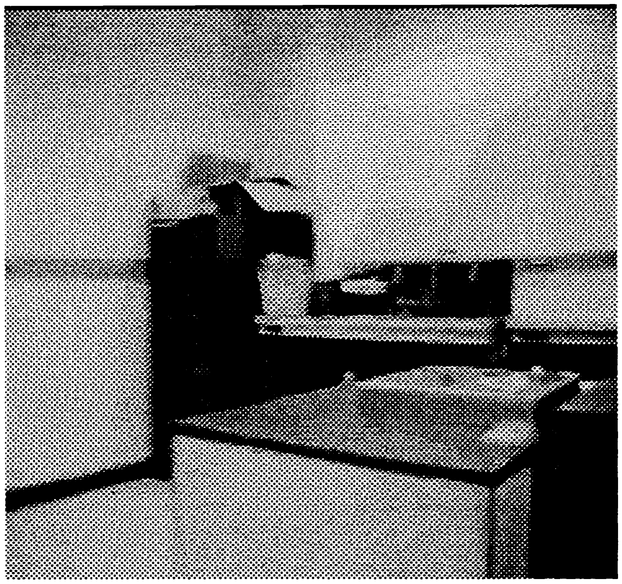


Figure 1: One view of the cluttered tool room.

2. RELATED WORK

Related work in this area falls into two categories: motor behaviors that can be extended for use in robotic localization and techniques for extracting reliable sensing from ultrasonic transducers. Our **move-to-station** motor behavior for localizing the robot builds on two previously reported concepts: *distinctive places* [5] and *wide open space attraction* [1]. Kuipers and Byun in [5] define a distinctive place as a location that a robot can achieve by performing a hill climbing search over a distinctiveness measure which has a local maximum. The primary advantage of distinctive places is that they can be used to construct a qualitative map of the world. Anderson and Donath in [1] discuss an attraction to open spaces as one type of robotic reflexive motor behavior for exploring the world. Neither approach relies on world models.

Unfortunately, these approaches are not concerned with the re-acquisition of a particular location with precision. Kuipers and Byun do not address the issue of repeatability since their efforts focus on *recognizing* distinctive places for qualitative map making. Wide open space attraction is not offered as a localization behavior but rather as an exploratory behavior where the robot continuously moves toward more open space. We can adapt these concepts for localization by noting that the centroid of the open space in a room can be considered a distinctive place, and that if sufficiently accurate sensing is used, a robot should be able to reliably re-acquire that place.

Wide beam ultrasonic transducers are a popular range sensor for robotic navigation because they are inexpensive and sense a large region. The operating range of a Polaroid lab grade ultrasonic transducer is typically modeled as a cone with a half angle β of 15° over a distance R of approximately 9 cm to 762 cm. However, sonar range readings are noisy and often produce false readings due to specular reflections; the reader is directed to [3] for a description of the behavior of ultrasonics. A variety of methods have been discussed for compensating for the poor performance. These methods include taking multiple readings and averaging them, interpreting the readings according to some algorithm such as line fitting [3], or explicitly modeling the uncertainty in the range reading. Methods in the second category either rely on previously acquired models of the environment, and so the solution is not applicable to our problem.

Our approach falls in the category of explicitly modeling the uncertainty associated with a range reading. This is similar to the method used by Matthies and Elfes in [6] for updating a grid based model of the world. The uncertainty in the sonar range readings is represented by a probability density function depending on the range reading and the angle. Each element in the sonar grid is assumed to have an equal likelihood of being occupied or empty ($P(occupied) = P(empty) = 0.5$). The probabilities for an element are updated with the uncertainty in the sonar range reading using Bayes' rule. Our approach differs in that it uses Shafer belief functions instead of probability density functions and updates the grid using Dempster's rule of combination from Dempster-Shafer theory [8]. Dempster-Shafer theory has a significant advantage for this application over Bayesian theory: it models the initial unsensed condition of the world as *ignorance* rather than as equally likely to be occupied or empty. Only sensing of the world causes a belief probability mass to be assigned to a point being occupied or empty. Therefore, a robot can distinguish between areas that have not been sensed (which would be represented as $P(occupied) = P(empty) = 0.5$ in [6]) and areas which have been sensed but the evidence is ambiguous (which would also be represented as $P(occupied) = P(empty) = 0.5$).

3. DEMPSTER-SHAFER MODEL OF ULTRASONICS

3.1. Review of Dempster-Shafer Theory

Dempster-Shafer theory can be viewed as a model of transferring belief from different sources of evidence into a total amount of belief about one or more propositions [9]. According to this model, a sensor is a source of evidence about a set of propositions, called *focal elements*. The set of focal elements which may be observed by a sensor is called its *frame of discernment* and is frequently denoted by Θ. The evidence from each observation is represented as a Shafer belief function [8]. The belief function allocates a total belief mass of 1.0 among each subset of Θ. One of the subsets which can receive belief mass is the set Θ itself; the belief assigned to this set is also called *ignorance* because the belief function does not know how to allocate a portion of the evidence to specific elements.

The belief from multiple observations can be combined into a single belief function using Dempster's rule of combination. This rule assumes that the observations are independent and have a non-empty set intersection. Any two belief functions Bel_1 and Bel_2, with elements A_i and B_j respectively, may be combined into a new belief function using Dempster's rule of combination. The rule specifies the combined belief mass assigned to each C_k, where C is the set of all subsets produced by $A \cap B$. The rule is:

$$m(C_k) = \frac{\sum_{A_i \cap B_j = C_k; C_k \neq \emptyset} m(A_i)m(B_j)}{1 - \sum_{A_i \cap B_j = \emptyset} m(A_i)m(B_j)} \tag{1}$$

where the focal elements of $Bel_1 = A = \{A_1, \ldots, A_i\}$ and $Bel_2 = B = \{B_1, \ldots, B_j\}$.

The combination of two belief functions is also known as taking the *orthogonal sum*, $\oplus$, and is written as:

$$Bel_3 = Bel_1 \oplus Bel_2 = (m(C_1), \ldots, m(C_k))$$

As a by-product, Dempster's rule of combination generates a measure of the disagreement between belief functions. Note that in Equation (1) the divisor reflects the portions of the belief functions which do not intersect. This belief mass is assigned to $\emptyset$; since the resulting belief function must have a total belief of 1.0, the combined evidence is normalized by 1 minus the mass in $\emptyset$. The larger the mass assigned to $\emptyset$, the more disagreement between the two beliefs about the propositions in the frame of discernment. Shafer formalized this measure of conflict in the *weight of conflict* metric, Con [8]:

$$Con(Bel_1, Bel_2) = \log(\frac{1}{1-\kappa}); \text{where } \kappa = \sum_{A_i \cap B_j = \emptyset} m_1(A_i)m_2(B_j) \tag{2}$$

Con takes a value between 0 and ∞; if $\kappa = 0.0$, $Con = 0.0$, and if $\kappa = 1.0$ (i.e., the belief functions have no agreement), $Con = \infty$. It is additive.

Dempster-Shafer theory has two differences from Bayesian theory which are noteworthy for this application. First, Dempster-Shafer theory does not require *a priori* knowledge about what is expected to be encountered in the world. Unlike the Bayesian approaches of [6, 7], a Dempster-Shafer application does not have to assume a likelihood of a space being occupied. Instead, the application can simply assume ignorance – that is, the space is either occupied or empty but no evidence is available at this time to indicate which. Second, Dempster-Shafer theory allows *ignorance* to be explicitly represented. As a result, the belief in regions of space which have not be sensed can be assigned to total ignorance.

3.2. Model of Ultrasonics

The behavior of an ultrasonic transducer can be modeled in terms of what it observes (the propositions) and the functions which quantify the belief in those observations (Shafer belief functions). An ultrasonic transducer reports one of two propositions about a point in space: whether it is *occupied* or *empty*. The frame of discernment (FOD) Θ where the focal elements are *occupied* or *empty*, and *don't know* is used to signify ignorance:

$$\Theta = \{occupied, empty\}$$

Figure 2 shows a bird's eye view of a a range reading from an ultrasonic transducer with a maximum range of R and angle width of β; the shaded area represents the operating range of the transducer. The reading indicates a surface has been detected along an arc of width a, reflecting the accuracy of the transducer. The points in the arc are assumed to be occupied, and are shown as Region I in Figure 2. The region of space between the transducer and the arc of the range reading is *empty* (Region II). Any other point of space is unknown because either it is "shadowed" by the arc of the range reading (Region III) or outside the transducer's operating range as given by R, β (Region IV).

The belief in a point in space (r, α) being occupied or empty is a function of three parameters:

1. *Range* – From [3, 6, 7], the closer the range reading is to the transducer, the more believable. This can be represented as a linear function: $\frac{R-r}{R}$.
2. *Angle* – Likewise the closer the point in space is to the axis of the beam, the more believable the observation. This is due to the assumption that surfaces are perpendicular to the beam. As a result, the "outside" portions of the beam are more likely to be reflected away from the transducer and not return. The decreasing belief in an proposition about a point in space as a function of angle is represented by: $\frac{\beta-|\alpha|}{\beta}$.
3. *Proposition* – It is also noted that a point in space reported as *empty* is more likely to be correct than a point reported as *occupied* [3, 6]. Spurious reflections are expected to result in false range readings while an empty region would not. Therefore the belief in a point in space being occupied can be discounted by $Max_{occupied}$, where $0.0 < Max_{occupied} < 1.0$.

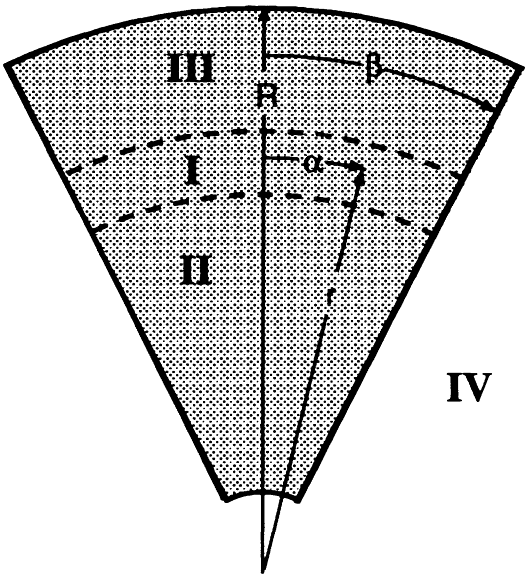


Figure 2: The regions of space observed by an ultrasonic transducer.

The range and angle of the reading are independent but both impact the belief in whether a point in space is either empty or occupied; therefore we average the influences to get a total effect. As a result the belief for a point (r, α) is captured as a belief function for each of the regions of space as given below.

Region I:

$$\begin{aligned} m(occupied) &= \frac{(\frac{R-r}{R}) + (\frac{\beta-\alpha}{\beta})}{2} \times Max_{occupied} \\ m(empty) &= 0.00 \\ m(don't\ know) &= 1.00 - m(occupied) \end{aligned}$$

Region II:

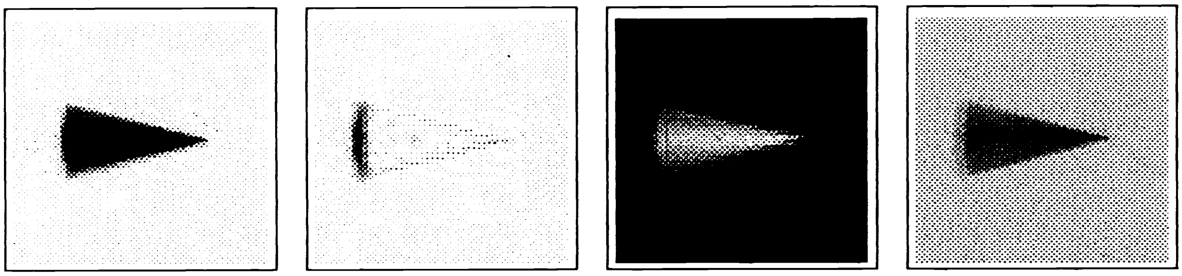
$$\begin{aligned} m(occupied) &= 0.00 \\ m(empty) &= \frac{(\frac{R-r}{R}) + (\frac{\beta-\alpha}{\beta})}{2} \\ m(don't\ know) &= 1.00 - m(empty) \end{aligned}$$

Region III, IV:

$$\begin{aligned} m(occupied) &= 0.00 \\ m(empty) &= 0.00 \\ m(don't\ know) &= 1.00 \end{aligned}$$

Figure 3 shows the distribution of belief for a single observation using the following parameters: maximum reading R= 762 cm, half angle of beam β= 15°, maximum belief in a point being occupied $Max_{occupied} = 0.98$. The darker shading of the first three panels represents the higher probability assigned to those points. The final panel shows a gray-scale composite view of the three beliefs combined; this is how we visually analyzed our results.

Multiple observations of a point in space from either repeated range readings from a single transducer or from several transducers can be combined using Dempster's rule of combination. Figure 4 shows range readings from two



$m(empty)$ $m(occupied)$ $m(don't\ know)$ Bel

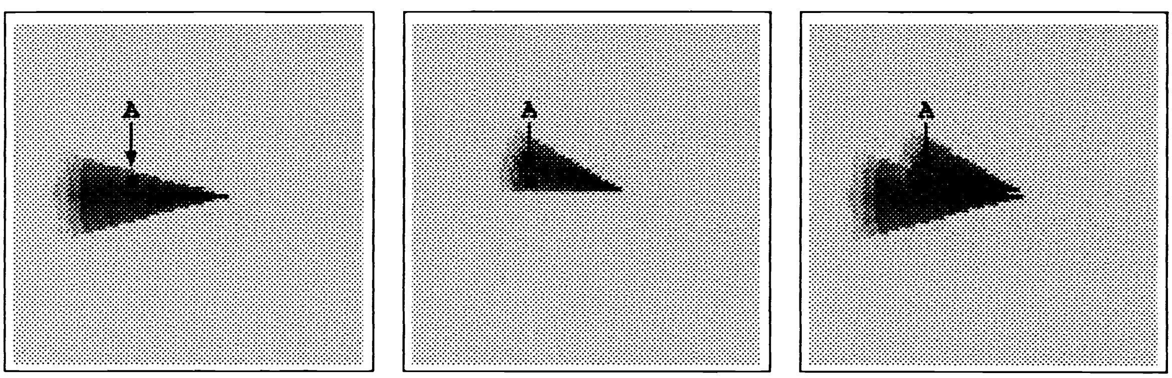
Figure 3: Distribution of belief for a single simulated range reading from a Polaroid ultrasonic transducer.

adjacent transducers mounted on a mobile robot. The operating range of the transducers overlap, leading to a higher belief in the propositions observed for those points in space. For example, the belief function for point A in Figure 4 from the two observations are based on $r_1 = 205$ cm, $\alpha_1 = 8.74°$ and $r_2 = 211$ cm, $\alpha_2 = -8.66°$:

$$Bel_1 : \quad m(occupied) = 0.00, m(empty) = 0.8113, m(don't\ know) = 0.1887$$
$$Bel_2 : \quad m(occupied) = 0.00, m(empty) = 0.8012, m(don't\ know) = 0.1988$$

The total belief for point A is:

$$\begin{aligned} Bel_3 &= Bel_1 \bigoplus Bel_2 \\ Bel_3 &: \quad m(occupied) = 0.00, m(empty) = 0.9625, m(don't\ know) = 0.0375 \end{aligned}$$



Bel_1 Bel_2 Bel_3

Figure 4: Distribution of belief for range readings from two Polaroid ultrasonic transducers.

4. APPLICATION OF THE MODEL TO THE move-to-station BEHAVIOR

The **move-to-station** motor behavior iteratively moves the robot to the station, which for this application is the center of a cluttered tool room shown in Figure 1. The robot begins with complete ignorance of the world or a blank world map. It then senses the surrounding regions in the room, noting the empty and occupied regions on the map. Given the position of the center of the empty regions, it then moves toward that center. Once at the center it senses the room and updates the world map. The robot may be able sense new regions of the world as well as update previously sensed areas due to its new positon and the position of center can be refined. The new center is computed and the robot moves again. This sense-update-move cycle is repeated until the robot is within 13 cm (5 inches) of the center. This is slightly more than the smallest movement our mobile robot can reliably make. If at any time the computed center is more than 50 cm (20 inches) away from the current position, the robot stops and samples again. This frequent sampling provides continuity in the map building.

The world map used by the **move-to-station** behavior is a 256 by 256 element grid following [6] reflecting the location of occupied and empty regions in the world. Each grid element in the world map contains a Shafer belief function on the FOD $\{occupied, empty\}$ and a Con value. The map is stationary; that is, each element represents the same location in the world. The robot's position in the map is recorded by shaft encoders. The robot begins in the center of the map where each element has a belief function of $m(don'tknow) = 1.0$ and $Con = 0.0$; these values change whenever the robot senses that region and the map is updated. Each element is a square roughly 7.8 cm (3.1 inches); the size of the map was determined by the size of the tool room. There is, of course, a trade-off between precision and speed depending on the grid size. The criteria for the grid size was to insure a reasonable number of elements would be contribute to the computation of the center of the room. If the grid size was too small, the center moved dramatically in response to updates made to a few grid elements.

The map is updated using Dempster's rule of combination. The ultrasonic sensing capabilities of the robot are modeled using the belief functions presented in Section 3.2. The robot has twenty four ultrasonics around its circumference covering a full 360°. The operating ranges of the transducers overlap so more than one sensor may sense the same element. Which grid elements get updated depends on which sensor i the element is relative to:

$$(x, y) = (r_{robot} \times \cos\alpha_i + r\cos(\alpha + \alpha_i), r_{robot} \times \sin\alpha_i + r\sin(\alpha + \alpha_i)) \tag{3}$$

where r_{robot}, the radius of the robot (or more specifically the radius of the ring of transducers), is 35.3 cm (13.9 inches). It should be noted that the ultrasonic belief model is in polar coordinates while the grid is in Cartesian coordinates.

We define the center of the room (C_x, C_y) based on all eligible elements (x_i, y_i) in the world map as the centroid of the elements' location weighted by the belief that they are empty:

$$(C_x, C_y) = (\frac{\sum x_i \times m_i(empty)}{\sum m_i(empty)}, \frac{\sum y_i \times m_i(empty)}{\sum m_i(empty)}) \tag{4}$$

There are two criteria a point in space (x_i, y_i) must meet to be included in Equation (4).

1. $m(empty) > 0.8$: The belief model assigns less belief to elements being empty as the distance from the sensor increases. However, impact on the relative distance of the grid element on the weighted centroid was much larger than the decrease in the belief. If every element was included in the computation, elements with little belief in being empty at large distances from the robot introduced noise and prevented reliable localization. We addressed this problem by setting a arbitrary threshold of $m(empty) > 0.8$ before considering a point in the calculations. Alternatively, the belief function could have been modified.

2. $Con > 1.0$: One side-effect of Dempster's rule of combination is that it is a "winner take all" rule. When a belief function with a high belief that an element is *empty* is combined with another belief function indicating that the element is *occupied*, the total belief reflects whichever belief was the highest. The Con metric for the element however increases.

   The winner-take-all phenomena is problematic since the ultrasonic sensors are very coarse and overlap. A sensor may detect a surface which only occupies the left side of the beam (see Figure 2) but the entire arc (Region I) must be reported as occupied. Therefore the robotic sensors will exhibit conflicts, which are indicative of areas that need to be resolved as to whether they are really *empty* or *occupied*. To keep these elements from adversely affecting the mass calculation we added one other test which disqualified points whose conflict was greater than 1.0 (or more than 60% disagreement) from being considered.

The net result of eliminating these elements was to make the center of mass less sensitive to minor changes in the mass distribution and to eliminate the consideration of points with considerable conflict. Interestingly enough in the experimental runs we made frequently as few as 1.6% of the points had sufficiently large probability to be considered but with our 256 by 256 sampling size this amounted to nearly 1,100 points and we noted that the center of mass remained sufficiently stable for our tests.

The complexity of the updating algorithm is $O(nm)$, where n is the vertical dimension of the discretization and m the horizontal. The computations at each element in the grid were done in constant time and consisted of the conversion of each Cartesian point (x, y) to a polar point (r, α), updating of belief and Con values using Dempster's rule of combination, and the addition of the relevant elements to the centroid "mass." Of these calculations the majority of the time is spent converting between the Cartesian and polar spaces since the correct sensor be determined for the point.

5. EXPERIMENTAL RESULTS

Experiments were conducted using the Georgia Tech Denning DRV-1 mobile robot to determine the repeatability of the **move-to-station** behavior and to compare the use of explicit belief models versus averaging readings. Sensing was done by twenty-four Polaroid lab grade II ultrasonic transducers mounted in a ring directly on the robot. Processing and motor control of the robot was performed on a micro-Vax II workstation, which serves as the robot's control computer. The expected location of the station was determined prior to the experiments as the locus of the final position of the robot using the **move-to-station** algorithm starting from various parts of the room. This locus was a circle approximately 1.5 feet in radius.

5.1. Repeatability

The twenty experiments in this set all followed the same procedure. In terms of motor behaviors, each test began with the robot returning to the expected location of the station using reactive **move-to-goal** and **avoid-obstacles** behaviors [2]. Once at the expected location of the station, the robot turned off the **move-to-goal** and **avoid-obstacles** behaviors and instantiated the **move-to-station** behavior. The **move-to-station** behavior continued until the robot had positioned itself at the centroid of the empty space. No single run took more than five moves before the robot had converged on the center. Physically, the robot was initially placed near a doorway of a cluttered tool room containing a variety of equipment, desks, and boxes. The goal for the **move-to-goal** behavior was the general direction and distance to the station within the room. In each test, both the starting position for the robot and the goal for **move-to-goal** behavior were altered as much as six feet to insure that the robot would be in a different location before the **move-to-station** behavior was instantiated. This would be consistent with the types of positioning errors that would be accumulated during robot navigation.

Figure 5 shows the process of building the evidence map using Dempster's rule of combination for a series of steps of the robot. These figures show the 128 by 128 sample size region surrounding the robot, which covers approximately a 5 by 5 meters (33 by 33 feet) area. The sequence begins at the point where the robot switches from the **move-to-goal** behavior to the **move-to-station** behavior. The starting point for the robot is represented by the intersecting dashed lines, the position of the robot before each movement by the intersecting solid lines, and the current centroid by the dark dot. Notice that in the first few moves new regions of empty space are sensed and the location of the centriod changes accordingly. The robot stopped within 12 cm (5 inches) of the predicted center of mass, over 1.13 meters (44 inches) from its initial position and after moving a total distance of over 1.23 meters (48 inches). The robot physically was within the station radius.

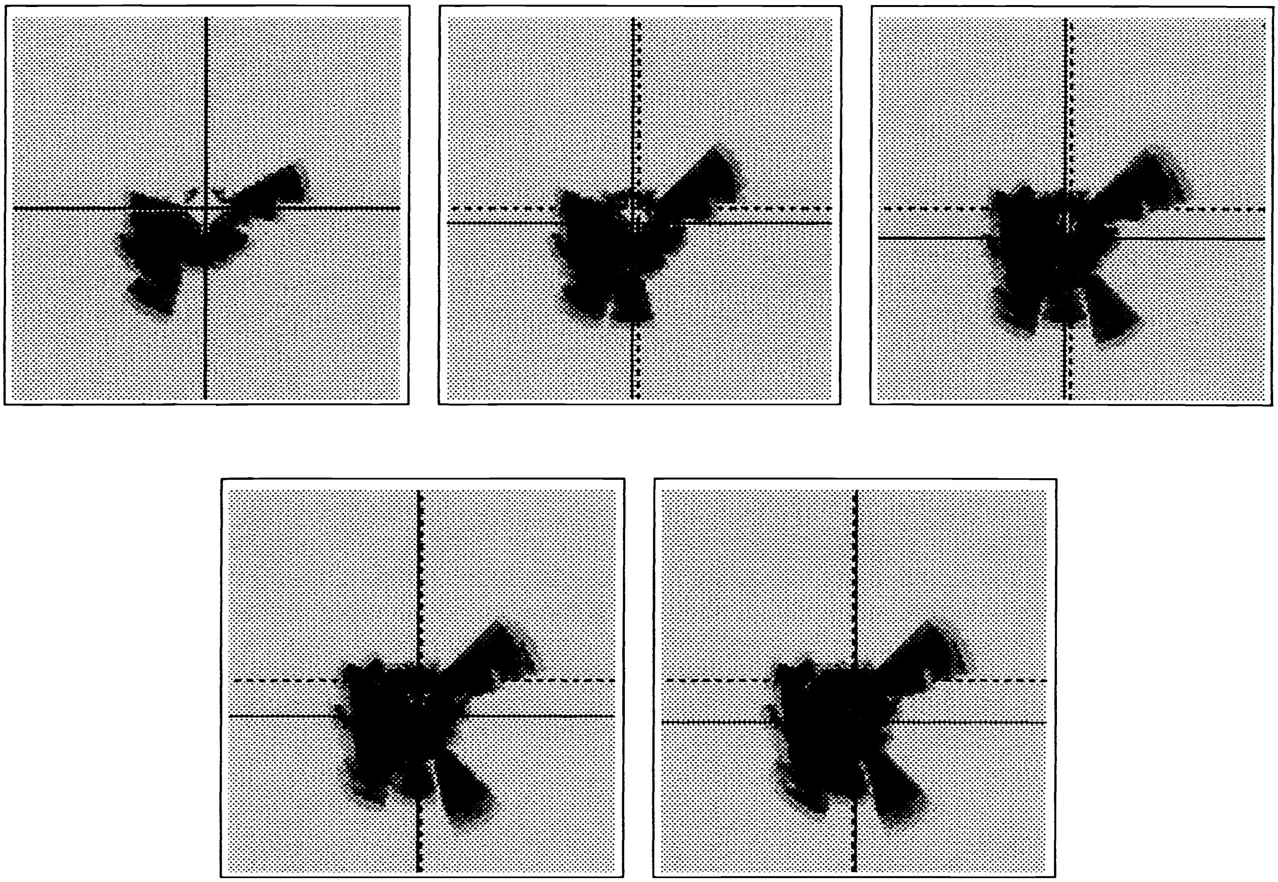


Figure 5: Sample **move-to-station** experiment: accumulation of belief for range readings and movement of the robot to the center of mass

5.2. Explicit Models vs. Averaging Readings

Additional experiments compared the results of overcoming spurious readings by explicitly modeling the belief with those from the common, but *ad hoc*, approach of averaging consecutive range readings in the hopes of smoothing out a "bad" reading. Under the *ad hoc* approach, the location of the station would be given by the vector sum of the ultrasonic readings. We implemented this approach; as expected, it did not perform reliably. The robot would not repeatedly move to the same location much less the center of the room, often stopping anywhere up to five feet from the correct space or stopping very close to it only to start moving away again (these runs were terminated after the robot had not reached the center in ten moves).

Two major reasons for the poor performance are cited below.

- Frequently, as noted before, the sensors readings in one location provide a vastly different view of the open space than reading from another location due to reflections and occluded open spaces. This makes it very unlikely that a readings from a single location followed by a single movement to the apparent center of open space will repeatedly converge to the same area.
- Measurement-movement iterations suffer from a similar problem. Once the robot moves the new sensor readings cause a new center of the open space to be located, often in a very different location from the previously-believed

center of open space. Because of this phenomena the robot often either centers itself at a "local center" or places itself near the true center of the open space only to be "bumped off" by spurious readings.

6. CONCLUSIONS AND DISCUSSION

Equation (4) worked well for the open space in the tool room we used in our tests. This does not mean it is without problems. In large open spaces it degenerates into the open space behaviors of [1], where the centroid of the empty space never converges and the robot keeps moving into new areas. Part of this problem results from uneven sampling of the world. Consider that the robot has entered a very large area such as a gymnasium from a doorway. The robot will be drawn to the open area and move forward. It then resamples and updates the grid. The elements in the direction that the robot has moved will now be sampled twice and will have accrued a higher belief that they are empty than the ones behind the robot. These higher belief elements will influence the robot to move forward again. Eventually the robot will stop when it encounters another wall and finally creates a local maxima, but it will not be in what we would call the "center" of the room. More importantly, that final position may not be repeatable because it will depend on the entry point. If the robot enters from another door and direction, it will stop at another wall. This problem can be overcome for large spaces by locating the boundaries of its surroundings and find the limits of the open space using a **avoid-past** behavior before determining the center of it.

7. ACKNOWLEDGEMENTS

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