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Automatic Parallel Parking with Geometric Continuous-Curvature Path Planning

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Abstract— **This paper presents the experimental results of our path planning for a passenger vehicle in parallel parking problems. The generation of the path planning consists in two parts: create a simple geometric path for the parallel parking in one or more maneuvers by circle arcs and then transform it into a continuous-curvature path with the use of clothoids. Experimental results of the parking and of the localization of the vehicle confirm the chosen approach.**

I. Introduction

A. Motivation in an industrial context

Since parking spots have become very narrow in big cities, maneuvering the vehicle becomes more and more difficult even for experimented drivers. Multiple repositioning often leads to minor damages on the car and increases traffic jam. Therefore, automatic parking is a solution to reduce stress and increase the comfort and the safety of the driver. To be commercialized, the automatic parking has to fulfill the expectations of the drivers with a fast, predictable and little demanding solution to the tires. Nevertheless, one requirement is to be affordable, which implies low computational cost and few sensors. In this paper, we consider parallel parking problem, which is particularly demanding for the driver.

B. State of the art for parallel parking and continuous-curvature paths

There are several methods to generate the trajectory for the parking problem. Methods based on fuzzy logic [1] or neural network [2] to learn human technique are often limited to the knowledge of the human experts and are difficult to generalize. Some methods use reference functions to stabilize the vehicle in the parking spot [3] and optimize the parameters of the commands [4]. However, the solution strongly depends on the initial parameters chosen for the functions, which are difficult to adjust and do not certainly lead to correct parking maneuvers. Consequently, these methods are not adapted to the industrial context.

In some studies there are two phases for the path planning, for example [5] and [6]: creation of collision-free path by a lower-level geometric planner that ignores the motion constraints and subdivision of this path to create an admissible path. An optimization routine can reduce the length of the path. These methods are more adapted for general path planning and become very complicated when applied to the parking problem.

Recent geometric methods involve easy geometrical equations and create trajectories based on admissible collision-free circular arcs, which lead the vehicle in the parking spot in one trial [7], [8], [9] or in several trials if the parking spot is too narrow [10]. The parking is possible independently of the initial position and the orientation of the vehicle and as long as its length and width are smaller than those of the parking spot. The geometric approach based on retrieving a vehicle from parking spot and reversing this procedure to solve the parking problem ([7], [9], [10]) is particularly instinctive and adapted to the parking problem. However, the curvature of this type of path is discontinuous: a vehicle tracking such a path has to stop at each circle arc to reorient its front wheels. This leads to an undesirable time delay. Moreover, steering at stop is particularly demanding to the steering column and induces a faster wear of the tires. Curvature continuity of the generated path is, therefore, a desirable property.

Continuous-curvature paths can be created from smooth curves, which can be divided into two categories: curves with coordinates that can be expressed in a closed-form (for example B-splines [11], polar splines [12] or quintic polynomials [13]) and parametric curves for which curvature is a function of their arc length, for example clothoids [14], cubic spirals [15] or intrinsic splines [16].

However, the path planning methods, which usually use these curves, often require optimization and a high computational cost. In the method we present in this article, in order to reduce the complexity, we first create a geometrical path with circle arcs and then transform it in a continuous-curvature path with a low computational cost by using clothoid curves. In [17] a method for a parking in one maneuver using circle arcs and Bezier curve fitting is presented. The calculation time does not allow the use of splines, so the authors use instead Bezier curves to smooth the path. Nevertheless, the admissibility of the created path is not guaranteed and a complex controller is needed. As shown in a recent work [18] we have already combined the simplicity of geometric path planning [10] with continuous-curvature path using clothoids. This curve answer the constraints of admissibility of the path for a real vehicle [19], [20] (constraint on the curvature, with a maximal steering angle, and on its derivative, with a maximal steering speed) and describe very well the behavior of car-like vehicles. In this method, a path composed of circle arcs is generated and

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each of these circle arcs is transformed into a sequence of clothoid, circle arc, clothoid in order to create a continuous-curvature path. If a vehicle has a constant velocity and steers its wheels with a constant angular velocity, the center of the rear track follows a clothoidal path. Consequently simple control signals suffice for the vehicle to execute the path.

C. Content of the article

In this work, we remind the geometric continuous-curvature path planning in the global architecture of the automatic parallel parking for an industrial context. Furthermore, we show by experimental results, that the precision of the parking maneuver and of a simple pose estimation are satisfactory for further integration of this path planning method in the automatic parking function.

In the next section, the global architecture of the automatic parallel parking is presented. The section 3 details the geometric continuous-curvature path planning presented first in [18]. In the section 4, the control of the steering angle and the pose estimation are outlined. New experimental results are presented in the section 5. Finally, the section 6 is devoted to the conclusion.

II. Global architecture of the automatic parking

A. Steps of the automatic parking

The first step for the automatic parking is the knowledge of the size and position of the parking spot and of the pose of the vehicle compared to this spot (Fig. 1). A common solution is to make the vehicle travel next to the spot and scan the spot for its length and width. Consequently, the pose of the vehicle is deduced. Intelligent infrastructures can also be considered with the detection of the vehicle pose and communication of the location of the free spots.

The second step is to calculate the trajectory needed to enter the parking spot (Fig. 1). The path planning section is dedicated to this step.

Two approaches can be considered for the third step, that is the vehicle control (Fig. 1). In both cases we suppose the presence of a sensor ordering an emergency stop to avoid collision if necessary. If the stop occurs, a pose estimation and trajectory regeneration algorithm decide to restart the maneuver or to definitely finish it.

• The first approach is the path following control. The control signals of the longitudinal speed and of the steering angle are adjusted to the pose estimation at each time step. This solution requires a good pose estimation and therefore precise exteroceptive sensors. Such control strategies are particularly well adapted for long trajectories with a considerable longitudinal speed. This solution can be considered if the coordinates of points on the path can be easily calculated. Another approach can be considered for the particular parking problem if the computation of all single point is not desired.

• The second approach is the open loop control with optional trajectory regeneration. In this approach, the control signals composed of the longitudinal speed and of the steering are deduced from the trajectory and depends only on the traveled distance. This distance is estimated using internal sensors of the vehicle. To have a more precise solution and introduce some loop closing, a trajectory regeneration can be performed. For example, each time the vehicle stops, the trajectory is recalculated based on the pose estimation from internal or exteroceptive sensors. As the initial pose of the vehicle and of the parking spot can be less precise, the regeneration can reduce the errors. This solution can be considered if the steering and speed control signals can be deduced without calculating the coordinates of the single points on the path.

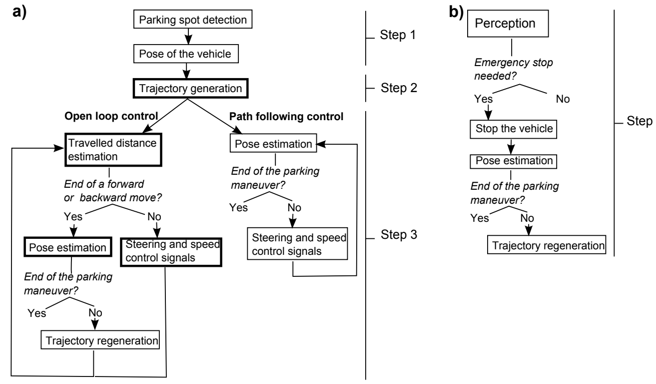

Fig. 1. Global architecture of the automatic parking. a) Steps of the parking routine b) Emergency stop routine, can occur at any moment during step 3

B. Sensors and perception

Modern cars are equipped with ABS systems that utilize angular encoders measuring the speed of the four wheels. The steering wheel angle is also commonly available in modern cars. Consequently this internal information can be used for all systems, even without the specific sensors that can be added on the vehicle and that increase the price of the automatic parking feature. Some exteroceptive sensors are also often added to help the driver: affordable ultrasound sensors, simple telemeters or cameras. Even if the provided information is not always accurate, the perception can be achieved using these low-cost sensors and help producing affordable automatic parking devices. The information from these sensors should allow to model the environment, detect the parking spot and localize the vehicle as it is done in hand-free parking devices. An autonomous navigation can also be added in semi-controlled environments [21].

C. Our positioning in the global parking problem

Our work presented here concerns the step 2 with the path planning section and some parts in the step 3 in the open loop control approach (bold boxes in Fig. 1). We present the control of the steering signal and the distance and the pose estimation using only internal vehicle sensors. The experimental results enlighten our choices.

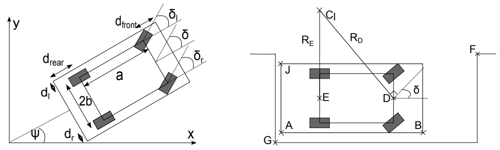

Fig. 2. Vehicle in global (x, y, ψ)-coordinates (left) and in a parking spot (right)

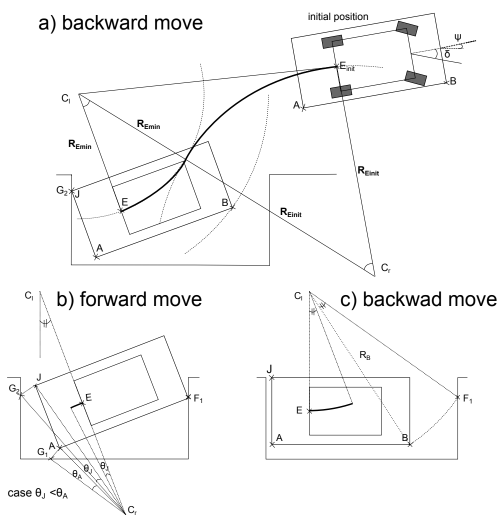

Fig. 3. Parking in three maneuvers with the "reversed" geometric method

III. PATH PLANNING

A. Geometric path planning with circle arcs

The parking maneuver is a low-speed movement. Consequently the Ackerman steering is considered with the four wheels rolling without slipping, around the instantaneous center of rotation. Different turning radius are calculated. For example, with E being respectively the center of the rear track, $R_E = a/\tan\delta$. The other radius can be deduced geometrically. We approximate maximum right and left steering angles to be equal and associate the different minimum radius, for example R_{Dmin}. The notations used for the vehicle are presented in Fig. 2 and Fig. 3 and we denote $R_{min} = R_{Emin}$.

The approach chosen for the first step of the continuous-curvature path planning is the generalized "reverse" method [10]. To create this path, the vehicle is considered in the parking spot and a retrieving path, like a human driver would do, is defined. Then, this path is reversed. This path is composed by circle arcs representing forward and backward moves of the vehicle. When a forward move allows the vehicle to retrieve without collision, the concerned circle arc is considered and a feasible circle arc connecting this arc by a tangential point to the real initial position is calculated. In Fig. 3, an example of a parking in three maneuvers is presented. The vehicle follows the path in the order a), b), c) but the path was created in the inverse order.

B. Clothoid turns

In this section, we briefly describe general properties of clothoids applied to path planning for vehicles [20]. A

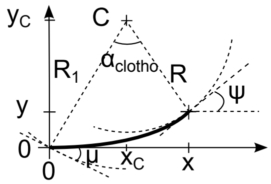

Fig. 4. Clothoid turns

clothoid is a curve whose curvature $\kappa = 1/R$ varies linearly with its arc length L: $\kappa(L) = \sigma L + \kappa(0)$, σ is the sharpness of the clothoid. It is also commonly used to define a clothoid by its parameter A with $A^2 = RL$, where $A = 1/\sqrt{\sigma}$.

Without loss of generality, consider the start configuration of the vehicle $q_i = (x_i, y_i, \psi_i, \delta_i) = (0, 0, 0, 0)$. The vehicle is moving with constant positive longitudinal velocity and with constant steering velocity to the left. The vehicle is then describing a clothoid whose parameter A depends on the longitudinal and steering velocities. The configuration for any position q of the vehicle at a distance L from the initial configuration is then (see Fig. 4):

$$q = \begin{cases} x = A\sqrt{\pi}C_f(\frac{L}{\pi A}) \\ y = A\sqrt{\pi}S_f(\frac{L}{\pi A}) \\ \psi = \frac{A^2}{2R^2} \\ \delta = 1/R \end{cases} \tag{1}$$

where C_f and S_f are the Fresnel integrals: $C_f(x) = \int_0^x \cos\frac{\pi}{2}u^2\,du$ and $S_f(x) = \int_0^x \sin\frac{\pi}{2}u^2\,du$.

The center of the circular arc C, located at a distance R from a configuration q, in the direction normal to ψ is:

$$x_C = x_R - R\sin\psi \quad , \quad y_C = y_R + R\cos\psi \tag{2}$$

In addition, we define the radius R_1 and the angle μ between the orientation of q_i and the tangent to the circle of center C and radius R_1:

$$R_1 = \sqrt{{x_C}^2 + {y_C}^2} \quad , \quad \mu = \arctan x_C/y_C \tag{3}$$

Then we define a clothoidal sequence $CAC(A, L, \theta)$ by: 1) clothoid of parameter A, of length L, of initial null curvature and of end curvature equal to $1/R$; 2) optional arc of circle of radius R and of angle θ; 3) clothoid of parameter A, of length L, of initial curvature equal to $1/R$ and of end null curvature.

C. Calculus of useful parameters

R_1 represents the distance from the point E at the beginning of a circle arc of the geometric path to the instantaneous center of rotation C. To make the most of the maximal steering angles of the vehicle, we want this radius R_1 to be as small as possible. The minimal time to steer from null steering to maximal steering is $t_{min} = \frac{\delta_{max}}{v_\delta}$, where v_δ is the maximal desired steering velocity. The minimal associated length of the path is then defined as $L_{min} = v_{longi}t_{min}$, where v_{longi} is the maximal desired longitudinal velocity. The parameter of the used clothoid is then ${A_{min}}^2 = R_{min}L_{min}$ and R_1 is calculated using Eq. 1 - 3. Let also call α_{clotho} the angle formed by a clothoid started with a null curvature and finished with a curvature $1/R_{min}$.

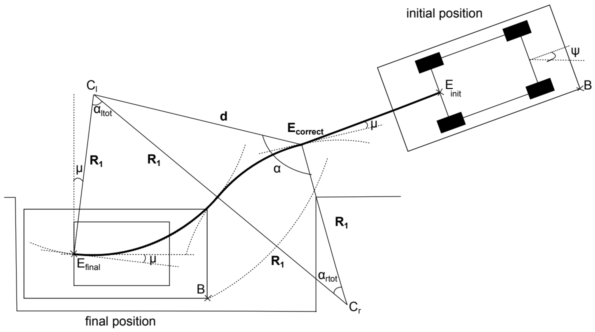

Fig. 5. Path for a parking in one maneuver

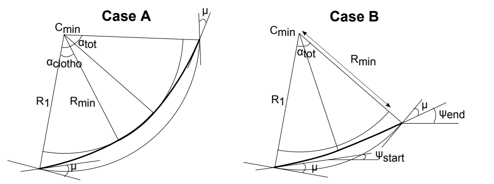

Fig. 6. Replacing of a circle arc by an A or B sequence

D. Strategy

The first step of the parking path generation is to create a geometric path composed of circle arcs, with a parking in a single or more maneuvers. The radius R_{min} used to create this path is replaced here by the radius R_1. Another modification concerns *C* the instantaneous center of rotation. In the geometrical methods, it is located on the same line than the rear track. This means, that the orientation of the vehicle is tangent in *E* to the circle arc of the geometric path. In our case, in order to use clothoids, the orientation of the vehicle situated on the circles of the geometric path presents always an angle μ with the tangent to the circle in *E*. This constraint is thus taken into account during the geometric path generation: *C* is transformed by the rotation of center *E* and angle μ. The figure 5 shows the path generated by a geometric method using the μ-tangency criteria and the radius R_1, for the parking in one maneuver.

The second step is to transform each circle arc of angle α_{tot} of the geometric method into a clothoidal sequence. Three cases are classically considered [19] (see Fig. 6 for the cases A and B).

Case A) $\alpha_{tot} \geq 2\alpha_{clotho}$: We use the clothoidal sequence $CAC(A_{min}, L_{min}, \alpha_{tot} - 2\alpha_{clotho})$.

Case B) $2\alpha_{clotho} > \alpha_{tot} \geq 2\mu$: We use the clothoidal sequence $CAC(A_{new}, L_{new}, 0)$ where A_{new} and L_{new} are:

$$A_{new} = \sqrt{\frac{{R_1}^2 \sin{(\frac{\beta}{2}+\mu)}^2}{\pi(\cos\frac{\beta}{2} C_f(\sqrt{\frac{\beta}{\pi}}) + \sin\frac{\beta}{2} S_f(\sqrt{\frac{\beta}{\pi}}))^2}} \quad (4)$$

$$L_{new} = A_{new}\sqrt{\beta} \quad (5)$$

where $\beta = (\psi_{end} - \psi_{start}) mod 2\pi$ is the deflection of the turn.

Case C) $\alpha_{tot} < 2\mu$: In this case it is no more possible to use the method of the case B and this case is not dealt with in [19]. Nevertheless, a left steering is still possible. A clothoid of parameter A_{min} and of length L_{caseC} is calculated to satisfy the following criteria: $\alpha_{tot} = 2\alpha_{clothocaseC}$ and in the end position it is no more possible to go forward (or backward, depending on the case) without a collision. As this calculus includes parametric curves with Fresnel integrals, an approached solution is considered. Finally, the sequence $CAC(A_{min}, L_{caseC}, 0)$ is used.

The last step is the calculation of the starting point of the first circle arc. To minimize lateral offset, only circles of radius R_1 are considered during the first step. In consequence, the vehicle has to go backward or forward without steering from its initial position E_{init} until it arrives at a point $E_{correct}$ on a circle of radius R_1 connected with a tangential point to the second circle (see Fig. 5). The coordinates of $E_{correct}$ can be calculated applying the Al-Kashi theorem to the triangle $C_l C_r E_{correct}$ [18].

IV. CONTROL SIGNALS AND POSE ESTIMATION

A. Control signals

To make the vehicle follow the generated path, control signals of the steering angle δ and longitudinal velocity v have to be built. As the trajectory is clothoidal, calculation of trajectory coordinates implies approximation of numerous Fresnel integrals, which requires a lot of calculation time. Moreover, all those points would require a large memory storage. Consequently, we propose to generate open loop control signals (Step 3 of Fig. 1), which depend only on the traveled distance and need few calculations and memory storage as we show below.

The speed control signal ensures only that the vehicle accelerates to a desired speed, then stays at this speed to finally decelerate and stop when the vehicle has traveled the total distance of a forward or a backward move.

For the steering control signal, the vehicle memorizes some information during the path planning routine: L_{line} the length of the first line with straight wheels, R_{min} and the parameters A, L, θ for each clothoidal sequence $CAC(A, L, \theta)$. The length of a circle arc on a clothoidal sequence is then $L_{arc} = \theta R_{min}$ and the total length of a clothoidal sequence is $L_{CAC} = 2L + L_{arc}$. Whatever the number of clothoidal sequences A) or C) (see Section III) is, the parameter *A* is to be calculated only once. For each sequence B), the parameter *A* has to be calculated every time. Finally, the distance control signal of the steering angle is:

- For the initial movement in a straight line:

$$\forall d \in (0, L_{line}) \qquad \delta(d) = 0 \quad (6)$$

- For a clothoidal sequence $CAC(A, L, \theta)$: $\forall d \in (0, L_{CAC})$

$$\delta(d) = \begin{cases} k_\delta |\arctan\frac{a}{R}| & d \in [0, L] \\ k_\delta |\arctan\frac{a}{R_{min}}| & d \in]L, L+L_{arc}[\\ k_\delta |\arctan\frac{a}{R_i}| & d \in [L+L_{arc}, L_{CAC}] \end{cases} \quad (7)$$

where $k_\delta = \pm 1$ corresponds to a left (+1) or right steering (-1), $R = \frac{A^2}{d}$ and $R_i = \frac{A^2}{L_{seq}-d}$.

B. Pose and distance estimation

The different internal sensors present on the vehicle allow to measure the steering wheel angle δ and the four wheel speed (ABS sensors present in modern cars). The elementary lateral displacements $\Delta_{RR}, \Delta_{RL}, \Delta_{FR}, \Delta_{FL}$ of respectively rear right, rear left, front right and front left wheels are deduced. Exteroceptive sensors for environment perception are not part of our study. Consequently, we propose odometric techniques for the estimation of the pose and of the traveled distance.

The most common technique uses only the speed of the two rear wheels. The elementary angular displacement ω and the elementary lateral displacement Δ are classically calculated at each time step k:

$$\Delta = \frac{\Delta_{RR}+\Delta_{RL}}{2} \quad , \quad \omega = \frac{\Delta_{RR}+\Delta_{RL}}{2b} \tag{8}$$

It is also possible to make a fusion of all available measurements using an odometric Extended Kalman Filter (EKF) as in [22] where the unknown quantities (Δ, ω) are linked to the measured variables $(\Delta_{FL}, \Delta_{FR}, \Delta_{RL}, \Delta_{RR}, \delta)$ by a redundant and non linear system:

$$\left\{ \begin{array}{ccc} \tan(\delta) & = & a\omega/\Delta \\ \Delta_{RL} & = & \Delta - b\omega \\ \Delta_{RR} & = & \Delta + b\omega \\ \Delta_{FL}\cos(\delta_l) & = & \Delta - b\omega \\ \Delta_{FR}\cos(\delta_r) & = & \Delta + b\omega \end{array} \right. \tag{9}$$

where $\delta_l = \arctan\left(\frac{\tan(\delta)a}{a-\tan(\delta)b}\right)$ and $\delta_r = \arctan\left(\frac{\tan(\delta)a}{a+\tan(\delta)b}\right)$ are the steerings of the front left and right wheels (see Fig. 2).

The distance estimation d and the pose of the vehicle (x, y, ψ) are then deduced for the next time step:

$$\begin{array}{l} d_{k+1} = d_k + \Delta \\ x_{k+1} = x_k + \Delta\cos(\psi_k + \omega/2) \\ y_{k+1} = y_k + \Delta\sin(\psi_k + \omega/2) \\ \psi_{k+1} = \psi_k + \omega \end{array} \tag{10}$$

V. Experimental results

A. Vehicle and ground truth sensors

The car used for our experiments is a Renault ZOE with automatic speed and steering control (see Fig. 10). The sources of errors that affect the pose estimation and the result of the parking maneuver are numerous. The first one is due to the resolution of the ABS encoders and of the steering wheel encoder. The other ones result from the approximate nature of the Ackerman model and from the performance of the speed and steering response to the control signals (the car was not especially prepared for low speed maneuvers).

All algorithms were integrated in a complete Matlab Simulink model allowing automatic control of the car. The implementation of this model in the vehicle was operated thanks to dSPACE MicroAutoBox, a real-time system for performing fast prototyping. The maximal longitudinal speed chosen for the presented experiments was 0.6 m.s^{-1}.

Two sensors allow the measurement of the ground truth. A GPS RTK produce true positions of the car with a precision of 2 centimeters and a Correvit is used to measure the odometry of the center of the rear track with a resolution of 1.9 millimeters and a measurement deviation less than $\pm 0.1\%$.

B. Pose and distance estimation

The experimental results presented here were obtained during automatized parking maneuvers using the path planning presented in section III and the control signals presented in the section IV, with the traveled distance measured by the Correvit. All the other measurements and estimations were also produced on-line, for further analizes. The techniques tested for pose estimation are the use of the two rear wheels sensors (classic odometry) and the use of the odometric EKF (section IV). In both techniques the calculated elementary longitudinal displacement is used for the distance estimation.

The pose and distance estimation were tested on different ground and conditions (asphalt and concrete slab as good ground conditions, gravel and wet concrete slab as difficult ground conditions) for parking in one or several maneuvers. Results are shown on Fig. 7 and Fig. 8 with a bird view representation, distance error (with respect to the Correvit), longitudinal and lateral errors (with respect to the GPS RTK) as functions of the real traveled distance (given by the Correvit). The distance estimation with the classic odometry method is correct for all ground conditions (generally less than 10cm of difference). However, for the distance calculated with the EKF method, the results are more mitigated: they are correct on good ground conditions (asphalt and concrete slab) but fall on wet ground and gravel (errors of 20 to 50cm at the end of the maneuver). Similarly, the pose estimation is always correct for the classic odometry and becomes degraded for the EKF method on difficult ground conditions (wet concrete slab and gravel).

It is to notice that, by changing the covariance matrices of the EFK, we can obtain the same performances than with the classic odometry, but never better. This could be due to the slipping of the front wheels during low speed movements with consequent steering angles. Another explanation is that during our experiment, the information provided by the sensors of the rear wheels was not corrupted. In [22], the EKF was tested with a maximum speed of 50 km/h and without maximum steering angles, during long travels on the road and in that situation the classic odometer alone was less efficient that the EKF estimation. To verify our EFK, we tested it in similar conditions and observed that the EKF performs much better than the classic odometry during these kind of travels (see Fig. 9). In the case of parking maneuvers, the classic odometer provide good results without large failures and consequently the benefits of the EKF (reduction of the error during failures or noisis of one or several sensors) can not be seen.

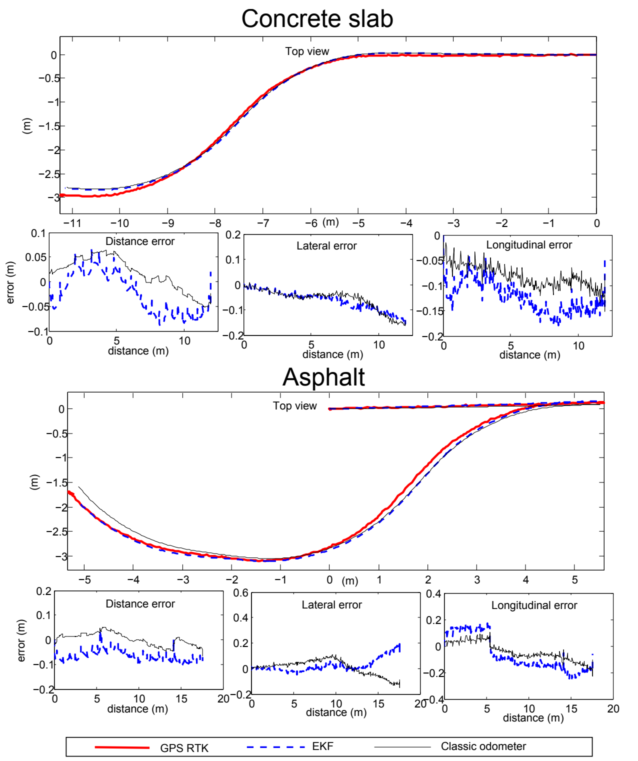

Fig. 7. Pose and distance estimation for the experiments on good ground conditions

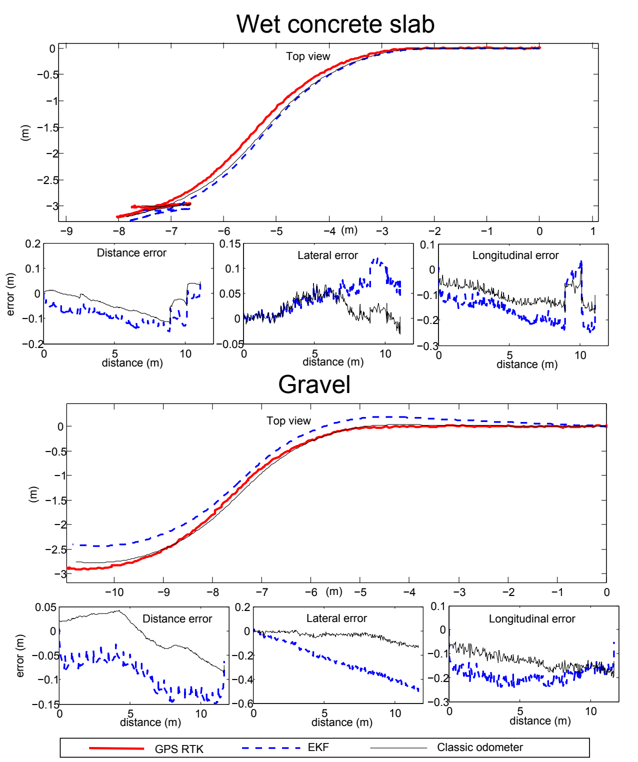

Fig. 8. Pose and distance estimation for the experiments on difficult ground conditions

C. Parking maneuvers

Regarding the results of pose and distance estimation, the automatic parking was tested with distance control issued from the classic odometry. We tested the parking in one or several maneuvers (different sizes of the parking spot) and with different initial positions and orientations. The Fig. 10 shows the different steps of a parking in four maneuvers. The picture A represents the initial position of the vehicle, B shows the end of the first maneuver, the picture C is taken during the entrance in the parking spot, D represents the end of the second maneuver, E shows the end of the third maneuver and finally the end of the parking is on the picture F. The Fig. 11 shows the real trajectories of the vehicle during automatic parking compared to the simulated ones.

The experimental results are quite good and the errors are mainly due to the execution delay of the control signals. This delay explains also the lateral error of the end position. Despite the differences between the simulations and the ground truth, the vehicle never exceeded the parking spot and the end position is suitable for the parking in two to four maneuvers. For the parking in one maneuver, the delay causes a little overstep, but that can be corrected using low-cost sensor, like ultrasounds. For smaller parking spots, where more than four maneuvers are needed (not shown here), the accumulation of errors becomes too big to correctly park the vehicle. Future work on the regeneration of the trajectory should solve this problem.

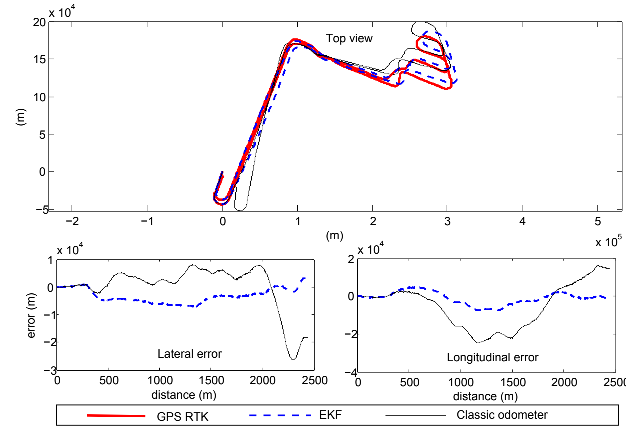

Fig. 9. Pose estimation for a long travel on the road

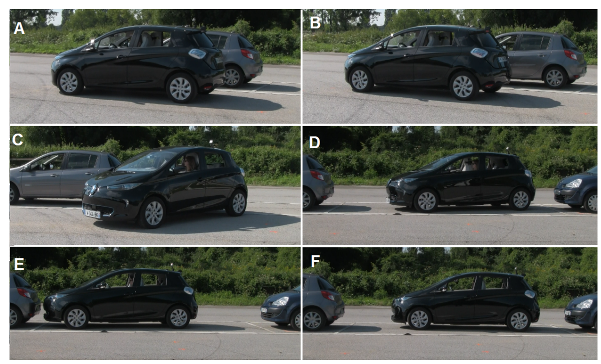

Fig. 10. Parking in 4 maneuvers

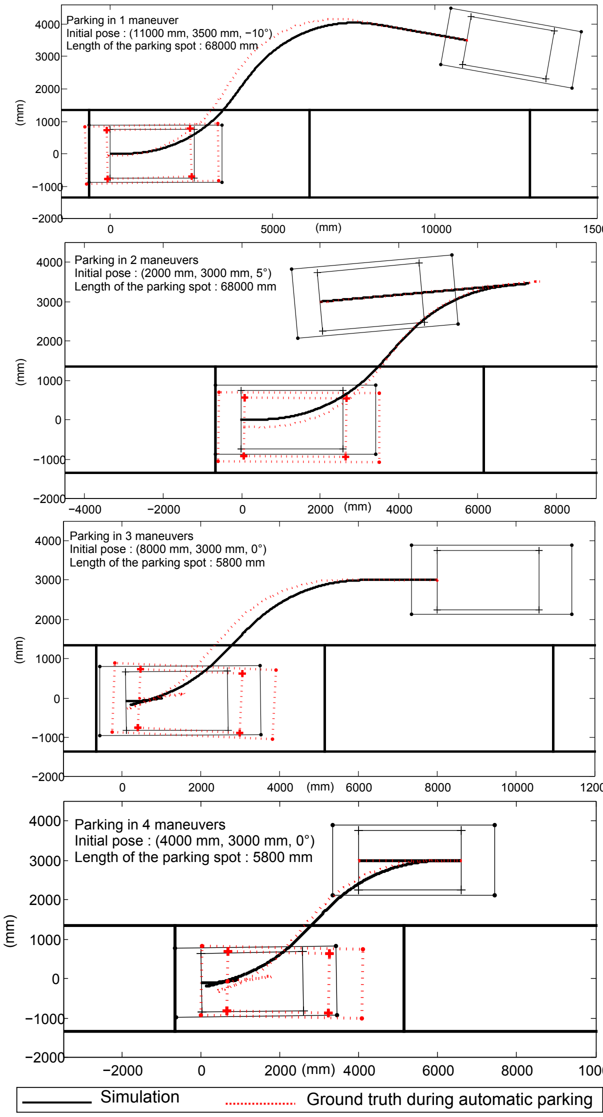

Fig. 11. Experiments of parking maneuvers

VI. Conclusion

An algorithm for the automatic parallel parking was proposed. Path planning, control signals, pose and distance estimation were detailed. Our path planning method creates continuous-curvature trajectories for the parking in one or several maneuvers for any initial position and orientation of the vehicle and as long as the length and width of the parking spot are bigger than those of the vehicle. Distance control signal allows to control the vehicle without calculation of points on the trajectory. Classic pose and distance estimation using the sensors of the two rear wheels allows a correct localization and produce precise distance information for the control signals. Experiments of parking in one or several maneuvers with a prototype confirm the suitability of the chosen strategies for the automatic parking. In the future, regeneration of the trajectory will allow a better precision during the execution of the maneuver. This method can also be generalized for other parking configurations.

Acknowledgment

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References

[1] Y. Zhao and E. G. Collins, “Robust automatic parallel parking in tight spaces via fuzzy logic,” *Robotics and Autonomous Systems*, vol. 51, no. 2-3, 2005.

[2] R. E. Jenkins and H. P. Yuhas, “A simplified neural network solution through problem decomposition: the case of the truck backer-upper,” *IEEE Trans. Neural Networks*, vol. 4, no. 4, 1993.

[3] I. E. Paromtchik, P. Garnier, and C. Laugier, “Autonomous maneuvers of a nonholonomic vehcicle,” in *Proc. Int. Symp. Experimental Robotics*, Barcelona, Spain, June 1997.

[4] S. Lee, M. Kim, Y. Youm, and W. Chung, “Control of a car-like mobile robot for parking problem,” in *Proc. IEEE Int. Conf. Robot. Automat.*, Detroit, Michigan, USA, May 1999.

[5] P. Jacobs, J. P. Laumond, and M. Taix, “Efficient motion planners for nonholonomic mobile robots,” in *Proc. IEEE/RJS Int. Work. Intelligent Robots and Systems*, Osaka, Japan, November 1991.

[6] J. P. Laumond, P. E. Jacobs, M. Taix, and R. M. Murray, “A motion planner for nonholonomic mobile robots,” *IEEE Trans. Robot. Automat.*, vol. 10, no. 5, 1994.

[7] Y. K. Lo, A. B. Rad, C. W. Wong, and M. L. Ho, “Automatic parallel parking,” in *Proc. IEEE Conf. Intelligent Transportation Systems*, Shanghai, China, October 2003.

[8] A. Gupta and R. Divekar, “Autonomous parallel parking methodology for Ackerman configured vehicles,” in *Proc. Int. Conf. Control, Communication and Power Engineering*, Chennai, India, July 2010.

[9] S. Choi, C. Boussard, and B. d’Andrea Novel, “Easy path planning and robust control for automatic parallel parking,” in *Proc. 18th IFAC World Congress*, Milano, Italy, August-September 2011.

[10] H. Vorobieva, S. Glaser, N. Minoiu-Enache, and S. Mammar, “Geometric path planning for automatic parallel parking in tiny spots,” in *Proc. 13th IFAC Symp. Control in Transportation Systems*, Sofia, Bulgaria, September 2012.

[11] K. Komoriya and K. Tanie, “Trajectory design and control of a wheel-type mobile robot using b-spline curve,” in *Proc. IEEE-RSJ Int. Conf. Intelligent Robots and Systems*, Tsukuba, Japan, September 1989.

[12] W. L. Nelson, “Continuous-curvature paths for autonomous vehicles,” in *Proc. IEEE Int. Conf. Robot. Automat.*, Scottsdale, AZ, May 1989.

[13] A. Takahashi, T. Hongo, and Y. Ninomiya, “Local path planning and control for agv in positioning,” in *Proc. IEEE-RSJ Int. Conf. Intelligent Robots and Systems*, Tsukuba, Japan, September 1989.

[14] R. Liscano and D. Green, “Design and implementation of a trajectory generator for an indoor mobile robot,” in *Proc. IEEE-RSJ Int. Conf. Intelligent Robots and Systems*, Tsukuba, Japan, September 1989.

[15] Y. Kanayama and B. I. Hartman, “Smooth local path planning for autonomous vehicles,” in *Proc. IEEE Int. Conf. Robot. Automat.*, Scottsdale, AZ, May 1989.

[16] A. Piazzi and C. G. L. Bianco, “Quintic g-splines for trajectory planning of autonomous vehicles,” in *Proc. IEEE Intelligent Vehicles Symp.*, Dearborn, MI, October 2000.

[17] Z. Liang, G. Zheng, and J. Li, “Automatic parking path optimization based on bezier curve fitting,” in *Proc. IEEE Int. Conf. Automation and Logistics*, Zhengzhou, China, August 2012.

[18] H. Vorobieva, N. Minoiu-Enache, S. Glaser, and S. Mammar, “Geometric continuous-curvature path planning for automatic parallel parking,” in *Proc. 10th IEEE Int. Conf. Networking, Sensing and Control*, Evry, France, April 2013.

[19] A. Scheuer and T. Fraichard, “Continuous- curvature path planning for car-like vehicles,” in *Proc. IEEE-RSJ Int. Conf. Intelligent Robots and Systems*, Grenoble, France, September 1997.

[20] T. Fraichard and A. Scheuer, “From Reeds and Shepp’s to continuous-curvature paths,” *IEEE Trans. Robotics*, vol. 20, no. 6, 2004.

[21] K. Sung, J. Choi, and D. Kwak, “Vehicle control system for automatic valet parking with infrastructure sensors,” in *Proc. IEEE Int. Conf. Consumer Electronics*, Las Vegas, NV, USA, January 2011.

[22] P. Bonnifait, P. Bouron, P. Crubillé, and D. Meizel, “Data fusion of four abs sensors and gps for an enhanced localization of car-like vehicles,” in *Proc. IEEE Int. Conf. Robot. Automat.*, Seoul, Korea, May 2001.