

# MOBILE ROBOT LOCALIZATION VIA FUSION OF ULTRASONIC AND INERTIAL SENSOR DATA

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## ABSTRACT

We describe two possible structures for a localization system which should exploit ultrasonic sensor measures as well as inertial and odometric data to maintain a correct estimate of the location of a mobile robot. The objective is to reduce the position and orientation error in the presence of slippage, and, at the same time, to identify the bias of the gyroscope. The two algorithms have the classical predictor-corrector structure of the Extended Kalman Filter, but they differ in the use of the angular velocity measure coming from the gyroscope. Experimental trials, performed on a robotic wheelchair equipped with five wide-beam sonars, allow a comparative assessment of the algorithms.

**KEYWORDS:** Localization, Kalman Filter, Inertial Navigation

## INTRODUCTION

Localization, i.e., estimating the location (position and orientation) of a mobile robot from sensory data, is an essential functionality for an autonomous mobile robot [1]. Despite the variety of sensors, models and techniques adopted for solving this problem, a common feature of mobile robots is a dead-reckoning system which computes incrementally the current robot location, given the previous location and a measure of the movement performed by the vehicle. If motion is achieved by means of wheels, the use of encoders represents the easiest and cheapest way to compute an odometric prediction. The accuracy of such prediction can be improved by using inertial sensors such as gyroscopes, that provide a better estimation of the robot orientation. Note that orientation estimation is critical for localization, as an orientation error generates a position error that increases as the robot moves.

It is well known that dead-reckoning is not enough for maintaining an accurate estimate of the robot location over long paths; position and orientation errors can diverge because of system perturbations and measurement noise. In particular, encoders introduce errors due to limited resolution, unequal wheel diameters and espe-

cially wheel slippage on the floor. On the other hand, the main source of error for a rate gyroscope is the bias on the angular velocity measure, that results in unbounded orientation errors when the latter is integrated. A heuristic technique that combines measurements from inertial and odometric sensors so as to correct errors due to slippage and bias was proposed in [2]. A more systematic approach to integrate inertial and odometric data is provided by the Extended Kalman Filter (EKF) paradigm, which results in a typical predictor-corrector structure [3, 4].

Enhanced localization modules can be realized if exteroceptive sensors (e.g., range finders, vision systems) are available in addition to proprioceptive sensors [5]. Essentially, these systems are based on the principle of performing a comparison between the actual exteroceptive measures and their value as predicted on the basis of the odometric estimate and of the a priori knowledge of the environment. In structured environments, beacons or other markers can be used; in unstructured environments, however, the localization algorithm must exploit natural features, e.g., workspace obstacles. This kind of approach is taken, e.g., in [6, 7].

In this paper, we extend our previous work [8] on map-based localization so as to include the information coming from inertial sensors. In particular, we use an EKF to combine odometric and gyroscopic data with range measurements provided by ultrasonic range transducers. Our objective is to reduce the position and orientation error and, at the same time, to identify the bias of the gyroscope. Two different algorithms are worked out and compared by experiments on a robotized wheelchair prototype. The obtained localization system proves to be robust with respect to both wheel slippage and gyroscope bias.

## THE LOCALIZATION SYSTEM

We consider a mobile robot with unicycle kinematics, in which motion is generated by two independently actuated wheels. The robot location is given by  $x = (p^x, p^y, \phi)$ , where  $p^x, p^y$  are the cartesian coordinates of the wheel axle midpoint and  $\phi$  is the vehicle orientation with respect to the horizontal axis. The distance between the midpoint and each wheel is denoted by  $a$ .

The on-board sensory system includes two incremental encoders measuring the rotation of each wheel, a gyroscope that provides a measure of the robot angular velocity, and a set of ultrasonic transducers which measure distances between fixed points on the robot body and obstacle surfaces in the environment.

We shall present two different localization algorithms, whose common structure proceeds directly from the EKF equations (e.g., see [9]). At the  $k$ -th sampling instant, the kinematic model of the robot is used to compute an odometric location prediction  $\hat{x}_{k/k-1}$  and an associated covariance matrix  $P_{k/k-1}$ ; an observation prediction  $\hat{z}_k$  is then formed and compared with the measures  $z_k$  provided by the sensory system. The results are the innovation term  $v_k$  and its covariance matrix  $S_k$ , that are used by the EKF to produce the state estimate  $\hat{x}_k$  and the associated covariance  $P_k$ .

The two proposed algorithms essentially differ in the use of the encoder and rate gyro outputs. In the first algorithm, the former are regarded as inputs to the kinematic model while the latter is included in the observation vector  $z$ . In the second algorithm, their roles are reversed: the orientation prediction is made on the basis of

the angular velocity measured by the gyro, while the encoder data are also used to form an observation variable. Since we are dealing with a strongly nonlinear problem, it is expected that the performance of the two filters can be remarkably different.

In the following, we describe each algorithm in some detail.

### Definitions for the first algorithm

Define the state vector as

$$x = \begin{pmatrix} p_x \\ p_y \\ \phi \\ b \end{pmatrix}, \quad (1)$$

where  $b$  denotes the rate gyro bias, and the inputs for the odometric model as

$$u_k = \begin{pmatrix} \delta s_k \\ \delta \phi_k \end{pmatrix} = \begin{pmatrix} \frac{\delta s_k^R + \delta s_k^L}{2} \\ \frac{\delta s_k^R - \delta s_k^L}{2a} \end{pmatrix},$$

where  $\delta s_k^L$  and  $\delta s_k^R$  are the distance traveled by the left and the right wheel, respectively, during the  $k$ -th sampling interval. The odometric prediction is then

$$\hat{x}_{k/k-1} = f(\hat{x}_{k-1}, u_k) = \hat{x}_{k-1} + \begin{bmatrix} \cos \tilde{\phi}_k & 0 \\ \sin \tilde{\phi}_k & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} u_k, \quad (2)$$

where  $\tilde{\phi}_k = \hat{\phi}_{k-1} + \delta \phi_k / 2$  is the average robot orientation during the sampling interval.

The observation vector  $z_k$  is computed from the available measures as

$$z_k = \begin{pmatrix} z_k^s \\ z_k^g \end{pmatrix},$$

where the subvector  $z_k^s$  contains the current range readings (one reading for each sonar) and the last observation variable represents the robot orientation reconstructed through the gyro output:

$$z_k^g = \hat{\phi}_{k-1} + \omega_k^g T_c - b_k T_c,$$

where  $\omega_k^g$  is the gyro angular velocity measure and  $T_c$  is the sampling time.

The observation prediction consists of subvectors  $\hat{z}_k^s$  (predicted range readings) and  $\hat{z}_k^g$  (predicted orientation). The first is computed on the basis of the given environment map  $\mathcal{M}$  and of the odometric prediction  $\hat{x}_{k/k-1}$

$$\hat{z}_k^s = \hat{z}_k^s(\hat{x}_{k/k-1}, \mathcal{M}).$$

While we do detail the above function, it should be noted that it depends on the way in which the environment map is represented—in our case, a list of segments—as well as on the model of interaction between the environment and the ultrasonic sensors.

As for  $\hat{z}_k^g$ , we simply let

$$\hat{z}_k^g = \hat{\phi}_{k/k-1}.$$

## Definitions for the second algorithm

In the second algorithm, the state vector  $x$  is again defined as in eq. (1), while the inputs for the odometric model are

$$u_k = \begin{pmatrix} \delta s_k \\ \delta \phi_k \end{pmatrix} = \begin{pmatrix} \frac{\delta s_k^R + \delta s_k^L}{2} \\ \omega_k^g T_c - b_k T_c \end{pmatrix}.$$

With this choice, the location prediction  $\hat{x}_{k/k-1}$  is computed through the same equation (2) of the previous case. Although obtained by inertial data too, we shall call also this prediction ‘odometric’ for simplicity.

The observation vector  $z_k$  is defined as

$$z_k = \begin{pmatrix} z_k^s \\ z_k^e \end{pmatrix}.$$

Subvector  $z_k^s$  and its predicted value  $\hat{z}_k^s$  are obtained as in the first algorithm. The value of the last observation variable is computed from the encoder outputs as

$$z_k^e = \hat{\phi}_{k-1} + \frac{\delta s_k^R - \delta s_k^L}{2a},$$

and predicted as

$$\hat{z}_k^e = \hat{\phi}_{k/k-1}.$$

## Odometric prediction uncertainty

Having clarified the different choice of input and observation variables in the two algorithm, we can now resume a unified treatment. The following equations are valid for both cases, provided the appropriate input  $u_k$  and observation  $z_k$  are used.

Denote by  $C_k$  the covariance matrix of the gaussian white noise which corrupts the input measure  $u_k$ . In our previous work [8], we have shown that it is convenient to introduce a dependence of  $C_k$  on  $u_k$ . In particular, larger corrections should be allowed at high angular velocities, when slippage typically occurs, while still rejecting inconsistent sensor data at low angular velocities. To this end, we set

$$C(u_k) = \begin{bmatrix} \sigma_{\delta s}^2 & 0 \\ 0 & \sigma_{\delta \phi}^2 \end{bmatrix} = \gamma_k \begin{bmatrix} \bar{\sigma}_{\delta s}^2 & 0 \\ 0 & \bar{\sigma}_{\delta \phi}^2 \end{bmatrix},$$

where  $\bar{\sigma}_{\delta s}^2$  and  $\bar{\sigma}_{\delta \phi}^2$  are constants and  $\gamma_k$  is a positive factor whose value is the result of a fuzzy computation based on the two inputs  $\delta s_k$  and  $\delta \phi_k$ . Roughly speaking, the set of fuzzy rules has been synthesized so as to increase  $\gamma_k$  when the robot is moving at high angular velocity and to decrease it when motion is slowing.

Since  $C_k = C(u_k)$ , the covariance matrix associated with the prediction error may be written as [10]

$$P_{k/k-1} = J_x^f(\hat{x}_{k-1})P_{k-1}(J_x^f(\hat{x}_{k-1}))^T + J_u^f(u_k)C(u_k)(J_u^f(u_k))^T + W. \quad (3)$$

Here,  $J_x^f(\cdot)$  and  $J_u^f(\cdot)$  are the Jacobian matrices of  $f$  with respect to  $x$  and  $u$ , respectively,  $P_{k-1}$  is the covariance matrix at the previous time instant  $t_{k-1}$ , and

$W = \text{diag}\{\sigma_x^2, \sigma_y^2, \sigma_\phi^2, \sigma_b^2\}$  is the covariance matrix of the gaussian white-noise which directly affects the state in the kinematic model (2).

The second term in the right-hand-side of eq.(3), which characterizes the influence of the odometric error on the prediction covariance matrix, may be expressed as

$$Q(u_k) = J_u^f(u_k)C_kJ_u^f(u_k)^T = \left[ \begin{array}{c|c} R_{\tilde{\phi}_k} \begin{pmatrix} \delta s_k^2 & 0 \\ 0 & \delta s_k^2 \sigma_{\delta\phi}^2/4 \end{pmatrix} R_{\tilde{\phi}_k}^T & Q_{12} \\ \hline Q_{12}^T & Q_{22} \end{array} \right], \quad (4)$$

with the rotation matrix  $R_{\tilde{\phi}_k}$  and the off-diagonal block  $Q_{12}$  given by

$$R_{\tilde{\phi}_k} = \begin{bmatrix} \cos \tilde{\phi}_k & -\sin \tilde{\phi}_k \\ \sin \tilde{\phi}_k & \cos \tilde{\phi}_k \end{bmatrix}, \quad Q_{12} = -\frac{\delta s_k \sigma_{\delta\phi}^2}{2} \begin{bmatrix} \sin \tilde{\phi}_k & 0 \\ \cos \tilde{\phi}_k & 0 \end{bmatrix}, \quad Q_{22} = \begin{bmatrix} \sigma_{\delta\phi}^2 & 0 \\ 0 & 0 \end{bmatrix}.$$

### EKF equations

The *innovation* term and the associated covariance are then computed as

$$v_k = z_k - \hat{z}_k, \quad S_k = J_x^h(\hat{x}_{k/k-1})P_{k/k-1}(J_x^h(\hat{x}_{k/k-1}))^T + R_k,$$

where  $J_x^h(\cdot)$  is the Jacobian matrix of  $h$  with respect to  $x$  and  $R_k$  is the covariance matrix of the observation gaussian white-noise. The first term of  $S_k$  represents the uncertainty on the observation due to the uncertainty on the odometric prediction.

At this stage, it is possible to correct the odometric state estimate on the basis of available observations. In particular, the final location and bias estimates are obtained as

$$\hat{x}_k = \hat{x}_{k/k-1} + K_k v_k,$$

where  $K_k$  is the Kalman gain matrix

$$K_k = P_{k/k-1}(J_x^h(\hat{x}_{k/k-1}))^T S_k^{-1}.$$

Finally, the covariance matrix associated with the final state estimate  $x_k$  is given by

$$P_k = P_{k/k-1} - K_k S_k K_k^T.$$

### EXPERIMENTAL RESULTS

Experiments trials in an office-like environment have been carried out using a robotized wheelchair prototype built at the robotics lab of the Università di Roma Tre. The vehicle has two driving wheels equipped with incremental encoders. Five ultrasonic sensors with a radiation cone of about  $90^\circ$  are available. The first sonar is oriented in the forward direction, while the other two couples make an angle of  $\pm 55^\circ$  and  $\pm 90^\circ$ , respectively, with the forward axis. The control software runs on a on-board notebook 486-PC equipped with a data acquisition card (DAQPad 1200 by National Instruments), and has been developed using the graphical language LabVIEW, while some of the most time-critical routines have been written in C.

We report the outcome of an experiment performed in a long corridor. The wheelchair starts approximately at point (31,18), and, after a right turn, follows

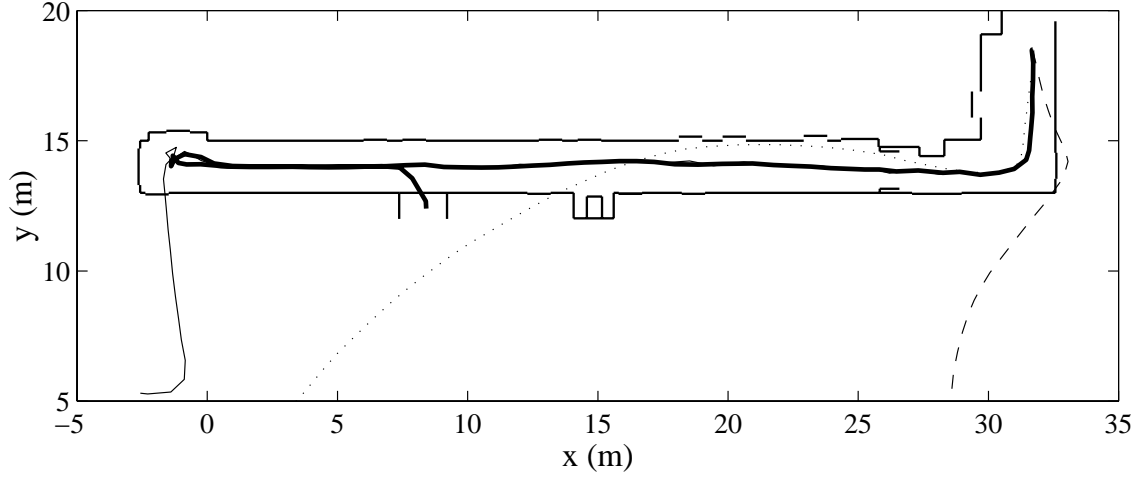


Figure 1: The wheelchair paths as reconstructed by: the first algorithm (thin), the second algorithm (thick), dead-reckoning using encoders and gyro (dashed), dead-reckoning using encoders only (dotted)

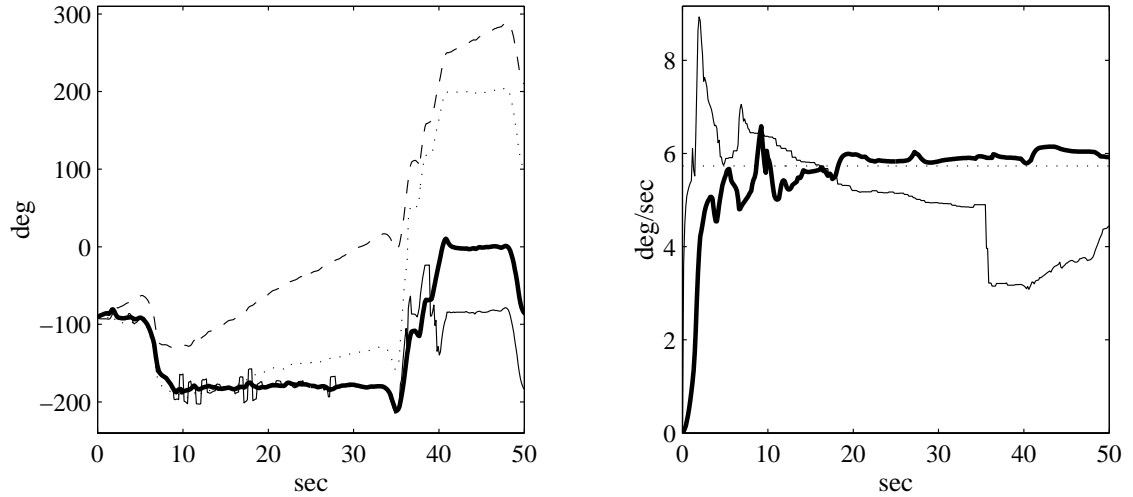


Figure 2: Orientation (left) and bias (right) estimated by: the first algorithm (thin), the second algorithm (thick), dead-reckoning using encoders and gyro (dashed), dead-reckoning using encoders only (dotted)

the corridor to its left end, where, after a quick maneuver, reverses its direction to conclude the path entering a door (see Fig. 1). The average speed of the wheelchair is 0.4 m/s, while sonar readings are collected every 0.3 s.

Figure 1 shows the wheelchair paths reconstructed by the two proposed algorithms. While the second behaves very well, the first ‘gets lost’ when the vehicle executes the maneuver to revert its direction, essentially due to the poor orientation prediction provided by encoders in the presence of slippage. In the same situation, the gyro gives a much better prediction, thus explaining the better performance of the second algorithm. For comparison, we also report the paths reconstructed by pure odometric and odometric+inertial dead-reckoning: note how the latter behaves even worse than the former due to the integration of the rate gyro bias. It should be

mentioned that the covariance matrices associated to the sensor (encoders, sonars, gyroscope) measures have been set to the same values for both algorithms.

The estimates of orientation and bias provided by the two algorithms, reported in Fig. 2, confirm the above remarks. Note in particular how the second algorithm correctly estimates a bias of about 6 deg/sec on the gyroscope reading.

## CONCLUSIONS

We have presented and compared two possible structures for a localization system which should exploit ultrasonic sensor measures as well as inertial and odometric data to maintain a correct estimate of the location of a mobile robot. While both algorithms have the classical predictor-corrector structure of the Extended Kalman Filter, and make use of an environment map during the prediction phase, they differ in the use of the angular velocity measure coming from the gyroscope. Experimental trials, performed on a robotic wheelchair equipped with five wide-beam sonars, have shown that the second structure—in which the gyro information is used for orientation prediction—provides robust performance under severe slipping conditions.

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