ODOMETRY AND SONAR DATA FUSION FOR MOBILE ROBOT NAVIGATION

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Abstract: To solve the problems in guidance, navigation, and control for an autonomous robot, its accurate positioning and localization are needed. Two or more different sensors are often used to obtain reliable data useful for control system. Extended Kalman Filter (EKF) is widely used to fuse those data to obtain one optimal result. The signals used during navigation cannot be always considered as white noise signals. On the other hand, colored signals will cause the EKF to diverge. This paper presents the data fusion system for mobile robot navigation. Odometry and sonar signals are fused using Extended Kalman Filter (EKF) and Adaptive Fuzzy Logic System (AFLS). The AFLS was used to adapt the gain and therefore prevent the Kalman filter divergence. The fused signal is more accurate than any of the original signals considered separately. The enhanced, more accurate signal is used to guide and navigate the robot. *Copyright © 2000 IFAC*

Keywords: Autonomous Robots, Guidance, Navigation and Control, Sensor fusion, Kalman Filter, Adaptive Fuzzy Logic System

1. INTRODUCTION

There are three systems required for the autonomous vehicle to follow the designed path. Those systems are navigation, guidance, and control system (Gai, 1996), (Kaminer, *et al.*, 1998).

In navigation problem, two basic position-estimation methods usually applied: absolute and relative positioning (Borenstein and Feng, 1996), (Shoval, *et al.*, 1998), (Jetto, *et al.*, 1999), (Jetto, *et al.*, 1999). Absolute positioning is based on navigation beacons, active or passive landmark, map matching, or satellite-based navigation signal, where absolute positioning sensors interact with dynamic environment. Relative positioning is usually based on odometry sensors, or inertial sensors.

For positioning, two types of sensors are available, internal and external sensors. Internal sensor will provide physical variables that can be measured on the vehicle. The examples of these sensors are encoders, gyroscopes, accelerometers and compasses. External sensors measure relationships between the robot and its environment, which can be natural or artificial objects. The examples of external sensors are sonars, radars and laser range finders.

Measurements of internal sensors are quite accurate for short period. However, for long-term estimation, the measurements usually produce a typical drift. External sensors do not produce the drift, however, because of their characteristics, the measurements are not always available (Santini, *et al.*, 1997).

This results to the idea of using multi sensors in navigation. Internal sensors can be used to estimate the position of the vehicle during a particular period. External sensors are then implemented to correct the errors that come from internal sensors. Both of those types of sensors have bound errors, and therefore a simple reset of internal errors is not sufficient. A better way is to fuse those two measurements in such a way so it will produce the best desire estimation. The common method is by using the Extended Kalman Filter (EKF), such as shown in the work by Jetto, *et al.*, (1997, 1999), Tham, *et al.*, (1999), and Sasiadek and Wang (1999).

Odometry sensor is commonly used as an internal sensor. It is mounted on the robot's driving wheels and register angular movements of the wheels, which are then translated into linear movements. This process has limited accuracy, for example, if slip has occurred on the wheel, the odometry would register the movement, but in fact, the vehicle may stay on its position. In the long period, the incremental motion of odometry will cause the accumulative error in positioning process. On the other hand, the advantage of using odometry is that the measurement signal is always available.

Other errors can also occur in odometry sensors. One is systematic error. This error causes the bias in one direction of the movement of the vehicle. Borenstein and Feng (1996) presented their method to correct this error. The method is based on a benchmark experiment performed prior to regular operation of the vehicle. The experiment can find the systematic error and, subsequently, the error is applied to correct the control system parameters. If the systematic errors occur frequently, this method may not be sufficient. For example, if the vehicle uses elastic tires, the benchmarking process has to be performed each time the unequal diameter occurs. It is beneficial that the error correction is done in real time operation.

It is widely known that poorly designed mathematical model for the EKF will lead to the divergence. Clearly, if the plant parameters are subject to perturbations and dynamics of the system are too complex to be characterized by an explicit mathematical model, an adaptive scheme is needed. Jetto, *et al*., (1999) used Fuzzy Logic Adapted Kalman Filter (FLAKF) to prevent from the filter from divergence when fusing measurement from odometry and sonar sensors. In this method, the ratio of innovation and covariance of innovation is used as input to the fuzzy logic, and the output is used to weight the process noise covariance in EKF. Sasiadek and Wang (1999) used exponential data weighting to prevent the divergence. Mean value and covariance of innovation are used as the input of the Fuzzy Logic Adaptive Controller (FLAC). The output is then used to weight process noise and measurement noise covariance in EKF. This FLAC is implemented on the flying vehicle navigating in three-dimensional space. Both those methods have shown improvement in the estimation of the vehicle position in comparison with the EKF only.

In this paper, the systematic error in odometry sensor is corrected during real-time operation of the vehicle by using measurements result from the sonar sensor. EKF is applied to fuse those two signals to find the best estimation of position. Adaptive Fuzzy Logic System (AFLS) is used to prevent the filter from divergence. The objective of this paper is to develop an efficient method for signal fusing to get accurate positioning.

2. MODEL

The model of the vehicle used in the simulation is based on a differential-drive. In this model, the vehicle can be steered by differentiating the wheels angular velocity. The kinematic model of this vehicle is described by the following equations:

$$\dot{x}(t) = v(t)\sin\theta(t) \tag{1}$$

$$\dot{y}(t) = v(t)\sin\theta(t) \tag{2}$$

$$\dot{\theta}(t) = \omega(t) \tag{3}$$

where $v(t)$ and $\omega(t)$ are, respectively, the linear and angular velocities of the robot, and are expressed by:

$$v(t) = \frac{\omega_r(t) + \omega_l(t)}{4} D \tag{4}$$

$$\omega(t) = \frac{\omega_r(t) - \omega_l(t)}{2d} D \tag{5}$$

where D and d are the wheel diameter and the distance between the odometry encoder respectively.

If we denote the state variable of the vehicle as $\mathbf{x}(t) = [x(t) \quad y(t) \quad \theta(t)]^T$, and the vehicle control input as $\mathbf{u}(t) = [v(t) \quad \omega(t)]^T$, the kinematic model in equations (1) - (3) can be written in the form of stochastic differential equation as:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) + \mathbf{w}(t) \tag{6}$$

where $\mathbf{w}(t)$ is a zero-mean Gaussian white noise with covariance matrix $\mathbf{Q}(t)$, which represents the model inaccuracies. This time equation is linearized and sampled in a small period $T = t_{k+1} - t_k$. Assuming that during this time interval, the linear and angular velocities are constant, and that the vehicle is following an arc path (see Wang (1988)), then, the equations for Extended Kalman Filter can be expressed by:

$$\mathbf{x}_{k+1}^{-} = \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \tag{7}$$

$$\mathbf{P}_{k+1}^{-} = \mathbf{A}_k \mathbf{P}_k \mathbf{A}_k^T + \mathbf{Q}_k \tag{8}$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^{-}\mathbf{C}_{k+1}^{T}[\mathbf{C}_{k+1}\mathbf{P}_{k+1}^{-}\mathbf{C}_{k+1}^{T} + \mathbf{R}_{k+1}]^{-1} \quad (9)$$

$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^{-} + \mathbf{K}_{k+1}[\mathbf{y}_{k+1} - \mathbf{C}_{k+1}\mathbf{x}_{k+1}^{-}] \quad (10)$$

$$\mathbf{P}_{k+1} = [\mathbf{I} - \mathbf{K}_{k+1}\mathbf{C}_{k+1}]\mathbf{P}_{k+1}^{-} \quad (11)$$

where:

$$\mathbf{x}_k = [x_k \quad y_k \quad \theta_k]^T \quad (12)$$

$$\mathbf{B}_k = \begin{bmatrix} T\cos\left(\theta_k + \frac{\Delta\theta_k}{2}\right) & 0 \\ T\sin\left(\theta_k + \frac{\Delta\theta_k}{2}\right) & 0 \\ 0 & 1 \end{bmatrix} \quad (13)$$

$$\mathbf{A}_k = \begin{bmatrix} 1 & 0 & -v_k T\sin\theta_k \\ 0 & 1 & v_k T\cos\theta_k \\ 0 & 0 & 1 \end{bmatrix} \quad (14)$$

$$\mathbf{Q}_k = [\mathbf{Q}_1 \quad \mathbf{Q}_2 \quad \mathbf{Q}_3] \quad (15)$$

$$\mathbf{Q}_1 = \begin{bmatrix} Q_{11}T + Q_{33}(T^3/3)v_k^2\sin^2\theta_k \\ -Q_{33}(T^3/3)v_k^2\sin\theta_k\cos\theta_k \\ -Q_{33}(T^2/2)v_k\sin\theta_k \end{bmatrix} \quad (16)$$

$$\mathbf{Q}_2 = \begin{bmatrix} -Q_{33}(T^3/3)v_k^2\sin\theta_k\cos\theta_k \\ Q_{22}T + Q_{33}(T^3/3)v_k^2\cos^2\theta_k \\ Q_{33}(T^2/2)v_k\cos\theta_k \end{bmatrix} \quad (17)$$

$$\mathbf{Q}_3 = \begin{bmatrix} -Q_{33}(T^2/2)v_k\sin\theta_k \\ Q_{33}(T^2/2)v_k\cos\theta_k \\ Q_{33}T \end{bmatrix} \quad (18)$$

and, $Q_{11} = \sigma_x^2$, $Q_{22} = \sigma_y^2$, and $Q_{33} = \sigma_z^2$ are diagonal elements of covariance matrix $\mathbf{Q}(t)$ of $\mathbf{w}(t)$ in Eq. (6).

The measurement, in this case, will consist of the measurement from odometry sensor and sonar sensor. The size of the measurement vector depends on the number of active sonar sensor. In general, this vector can be expressed as (See Jetto et. al. (1999)):

$$\mathbf{y}(\mathbf{x}_k, \Pi) = [x_k \quad y_k \quad \theta_k \quad d_{1k} \quad d_{2k} \quad \dots \quad d_{nk}]^T \quad (19)$$

where d_{nk} is the measurement of sonar nth at time k.

Many methods can be implemented to fuse the signals using EKF. Three me thods are described here. The first method is by filtering each measurement signal prior of comparison process. The error of those signals is then used to correct the measurement signal. This method was implemented by Green and Sasiadek (1998). The second method is by taking the error between the predicted position and measured position. The predicted position comes from control algorithm, and the measured position comes from the sensors (See Jetto, *et al.*, 1999). This error is then fed to the EKF. The advantage of using this method is that although only one measurement signal is available, the correction still can be done. The more the measurement signals become available, the estimation will be more accurate. The disadvantage of this method is that one has to know the dynamic model of the vehicle in order find the predicted position. This method is sometime called total state space EKF (Maybeck, 1979). The third method requires finding the best error estimation between two measurement signals. This is performed by taking the error between two measurement signals, which come from sensors, and this error is then fed to the EKF as presented by Sasiadek and Wang (1999). Using this method, the dynamic model of the vehicle is less important, but, when one measurement signal is not available, the correction cannot be performed. This method is sometime referred as indirect feedback EKF (Maybeck, 1979).

In this paper, the first method is used. The measurement signals from odometry and sonar sensors are combined into one measurement vector. This measurement result is then compared with the predicted position, and the error is fed to the EKF.

3. ADAPTIVE FUZZY LOGIC SYSTEM

In Kalman filter model, both process noise $\mathbf{w}_k$ and measurement noise $\mathbf{v}_k$ are assumed zero-mean white noise sequence with covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$. If the model of EKF is tuned perfectly, the residual between actual and predicted measurement should be a zero-mean white noise process.

Often, we do not know all parameters of the model or we want to reduce the complexity of modeling. Therefore, in real application, the exact value of $\mathbf{Q}_k$ and $\mathbf{R}_k$ is not known. If the actual process and measurement noises are not a zero-mean white noise, the residual in Kalman filter will also not be a white noise. If this is happened, the Kalman filter would diverge or at best converge to a large bound.

Jetto, *et al.* (1999) used fuzzy logic adapted Kalman filter to prevent the filter from divergence. The fuzzy logic controller uses one input and one output. The ratio between innovation and covariance of innovation process is used as an input. The output is a constant, which is used to weight the process noise covariance. The controller uses five fuzzy rules, and it is implemented in a wheeled mobile robot equipped with odometry and sonar sensors.

Sasiadek and Wang (1999) used fuzzy logic adapted controller (FLAC) to prevent the filter from divergence when fusing signals coming from INS and GPS on flying vehicle. Nine rules were used. There were two inputs, which are the mean value and covariance of innovation, and the output is a constant that is used to weight exponentially the model and measurement noise covariance.

In the case of fusing signals that come from odometry and sonar sensors, sometime only odometry measurements are available. The innovation will be a white noise as long as the process and measurement noises are assumed as a white noise. However, when the sonar measurements become available, and combined with the odometry measurement, the innovation might be not a white noise anymore. This will cause the filter to diverge.

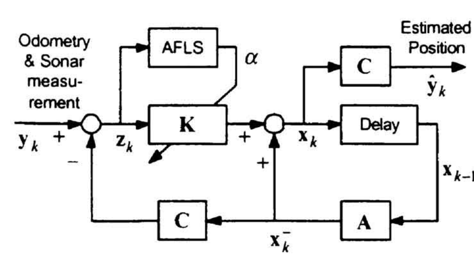

Fig. 1. Adaptive Fuzzy Logic System (AFLS) scheme

When systematic error occurs in the vehicle, the process and measurement noise actually are not a gaussian white noise, which causes divergence in EKF. AFLS can be used to adapt the filter gain so that the divergence can be avoided. The adaptation process used in this paper is based on exponential data weighting (Lewis, 1986). The scheme of the adaptation process is shown in Fig. 1.

The membership function used in this AFLS is displayed in Fig. 2 - Fig. 4. The AFLS uses nine rules, which are summarized in Table 1.

4. WEIGHTED EKF

Using exponential data weighting as an adaptation process, the equation for the EKF will be different. For exponential data weighting, the weighted process and measurement noise covariance can be written as:

$$\mathbf{R}_k = \mathbf{R}\alpha^{-2(k+1)} \tag{20}$$

$$\mathbf{Q}_k = \mathbf{Q}\alpha^{-2(k+1)} \tag{21}$$

where $\alpha \geq 1$. $\mathbf{Q}$ and $\mathbf{R}$ are constant matrices of process and measurement noise covariance. For $\alpha > 1$, as time k increases, $\mathbf{Q}_k$ and $\mathbf{R}_k$ will decrease, which means that the most recent measurement is given higher weighting.

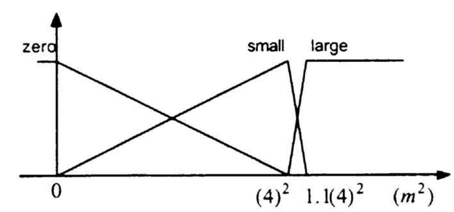

Fig. 2. MF of innovation process covariance

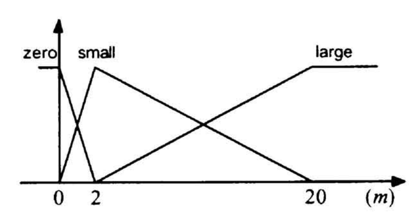

Fig. 3. MF of innovation process mean value

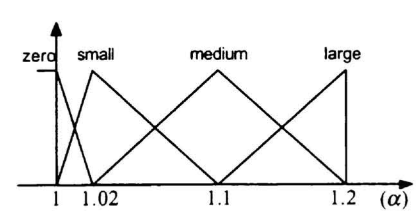

Fig. 4. MF of α

Table 1. Rulestable for AFLS

α		Innovation process mean value		
		Zero	Small	Large
Innovation process covariance	Zero	Small	Zero	Large
	Small	Zero	Large	Medium
	Large	Large	Medium	Zero

If the weighted estimation covariance is defined as:

$$\mathbf{P}_k^{\alpha-} = \mathbf{P}_k^{-}\alpha^{2k} \tag{22}$$

then the EKF equations become:

$$\mathbf{x}_{k+1}^{-} = \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \qquad (23)$$

$$\mathbf{P}_{k+1}^{\alpha-} = \alpha^2 \mathbf{A}_k \mathbf{P}_k^{\alpha} \mathbf{A}_k^T + \mathbf{Q}_k \qquad (24)$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^{\alpha-} \mathbf{C}_{k+1}^T [\mathbf{C}_{k+1} \mathbf{P}_{k+1}^{\alpha-} \mathbf{C}_{k+1}^T + \frac{\mathbf{R}_{k+1}}{\alpha^2}]^{-1} \qquad (25)$$

$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^{-} + \mathbf{K}_{k+1} [\mathbf{y}_{k+1} - \mathbf{C}_{k+1} \mathbf{x}_{k+1}^{-}] \qquad (26)$$

$$\mathbf{P}_{k+1}^{\alpha} = [\mathbf{I} - \mathbf{K}_{k+1} \mathbf{C}_{k+1}] \mathbf{P}_{k+1}^{\alpha-} \qquad (27)$$

5. EXPERIMENTS AND RESULTS

Simulation experiments have been conducted to show the implementation of AFLS when fusing the signals that come from odometry and sonar sensor. Systematic error in odometry measurement, which comes from unequal in wheel's diameter, is also considered. The vehicle is planned to follow sinus path in in-door environment. The map of the in-door environment along with the movement of the mobile vehicle that has systematic error is shown in Fig. 5.

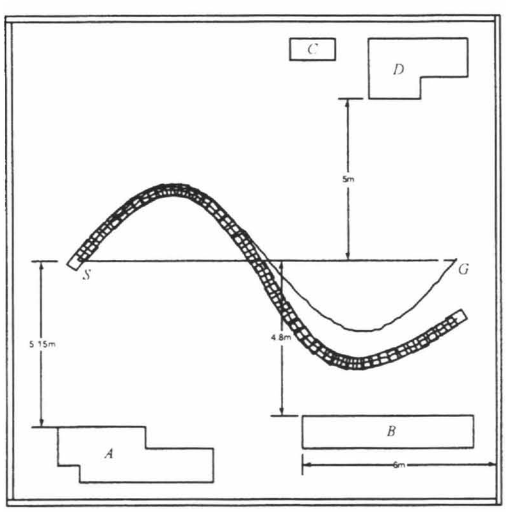

Fig. 5. Map of in-door environment

Three simulation experiments have been performed. The first experiment is to show the implementation of EKF in the mobile robot using odometry sensor, where the sensor has systematic error. The result of this experiment is shown in Fig. 6. In this experiment, it shows that the implementation of EKF with only one measurement signal is available, cannot be used to correct the systematic error. The EKF in this case only filters the Gaussian white noise of the odometry measurement error. The systematic error however, still present in the movement of the mobile vehicle.

The second experiment is to use the EKF to fuse measurement signals that come from odometry and sonar sensor without using AFLS. This experiment result is shown in Fig. 7. The present of sonar sensor, which measures the relation of the mobile vehicle and its environment, reduces the systematic error, and the mobile vehicle can follow the designed path. However, the movement of the mobile vehicle in this case is not smooth. The result of sonar measurement in this experiment is not used efficiently to improve the position estimation.

The third experiment is to use AFLS to adapt the gain of EKF to prevent the filter from divergence. In this experiment, when the sonar measurement becomes available, the EKF uses this signal to improve its estimation. AFLS makes the position estimation smoother than without AFLS. The result of this experiment is shown in Fig. 8.

6. CONCLUSIONS

In this paper, Extended Kalman Filter (EKF) has been used to estimate the position of the mobile vehicle. To prevent the filter from divergence, the innovation and covariance of innovation process are monitored by using Adaptive Fuzzy Logic System (AFLS). The result is an adaptation in the gain of EKF.

Odometry and sonar sensors have been used to simulate the method. From the simulation experiment, it shows that beside the improvement in the estimation of position, the method can also be used to correct the systematic error. Using this method, real-time operation of the vehicle can be reduced.

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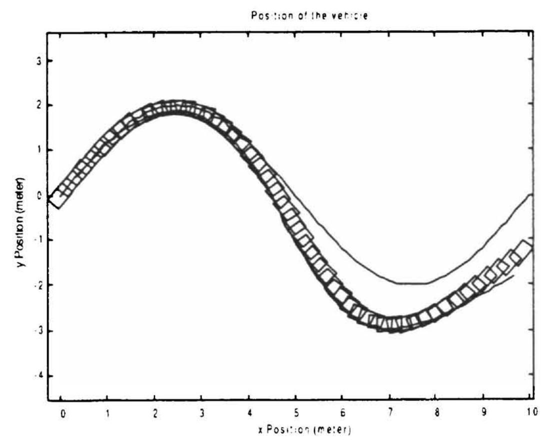

Fig. 6. Simulation experiment result using EKF with only odometry measurement.

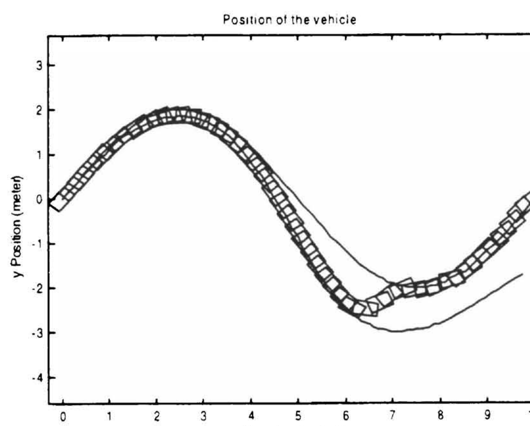

Fig. 7. Simulation experiment result using EKF with odometry and sonar measurement

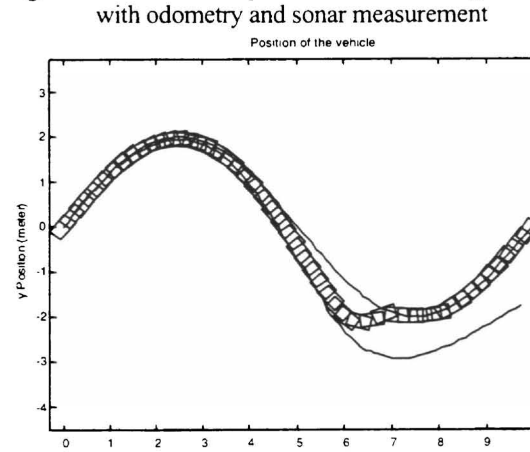

Fig. 8. Simulation experiment result using EKF with odometry and sonar measurement, adapted by AFLS