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Bi-level Modelling of Arterial Traffic Control

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Abstract: A bi-level optimization problem is defined for controlling an arterial corridor of traffic network. The hierarchical relations between two optimization problems allow optimizing two types of control variables: the split of green duration and the cycle time of the traffic lights. The benefit of the bi-level formulation has been identified and assessed.

Keywords: traffic light control, arterial crossroad control, non-linear optimization, bi-level optimization

1. INTRODUCTION

The simple store and forward model of urban network is accepted as a simple formal one, which is actively used for the developments of control strategies in urban traffic networks. The model is introduced by (Gazis, 1963) and it is actively exploited in previous and recent researches (Tamura, 19xx; Tettamanti, 2008; Aboudolas, 2009; de Oliveira, 2010). The engineering meaning of this model concerns the continuity of a flow, which allows estimating the dynamics of vehicle queue lengths in front of the crossroad traffic lights section. The model results in linear dynamical discrete type relations. This model leads to control policy which contains steps for definition and solution of appropriate optimization problem, which proves the optimal control policy of the management strategy. Following the procedures for solving the defined optimization problems, efforts have been done for simplifying the computational workload by means to perform on-line calculations and on-line control evaluations in closed and open loop control structures (Diakaki C., 2002; Barisone A., 2002; Dotoli M., 2006). The developments target decrease of the traffic congestions, which can be additionally accessed by performance function addressing level of emissions, noise, time travelling, fuel economy and others. An important overview for the different classes of optimization problems is provided in (Papageorgiou et al, 2003). The control policies are classified as fixed time control (Robertson, 1969) and traffic response control (Hunt, 1982). These models lead to linear time invariant state space problems, which do not present nonlinear events as congestion dynamics and it spills back in the traffic network. Examples for inclusion of additional non-linear constraints and deriving non-linear state space problem target more precisely to cope non-linear events to the traffic model (Papageorgiu et al, 2003). Particularly, these additional constrains has the form of inequalities, where the control or state space variables have to take extreme values between predefined limits. In general, the state variables x are assumed to be outflows or the queue lengths, the control variables are the split g_i of the green light durations and/or the total sum of the green $G=\sum g_i$ for different stages of the traffic lights. The time cycle c_1 for the traffic lights is assumed as parameter, not as unknown argument of the optimization problem.

The goal of the paper is the definition of optimization problem, which has extended dimension of the control variable both with the split and the cycle of the traffic lights. The case under consideration is an arterial corridor of a traffic network. A non-linear optimization problem is derived which have hierarchical relation between two optimization problems. The problem is defined as bi-level optimization and allows evaluation in optimal manner both types of control variables: the split of green duration and the cycle time of the traffic lights.

In general, the traffic control systems within arterial corridor are developed by means to attempt optimization of the traffic flow on urban road network. According to (Haj-Salem et al, 1995) the arterial control has strong impact to the adjacent road traffic. For the sake of simplicity the store-and-forward modelling is well suited for control of the dynamical changes of queues in the arterial corridor under congested and saturated traffic conditions. The importance of the arterial traffic management is the reason to apply bi-level modelling for such traffic system.

2. BI-LEVEL AND MULTILEVEL MODELLING

Multilevel theory makes use and develops decomposition approaches applied for solving both mathematical programming and variation problems. Such decomposition technique allows the original complex optimization problem to be reduced to a set of low order optimization subproblems. Then the solution of the complex problem is found as a vector of the subproblems solutions. The local subproblems are influenced (coordinated) by the coordination problem to generate the components of the global solution of the original

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problem. Many practical problems are formulated in terms of multilevel optimization models (Stackelberg, 1952)

$$\min_{x_k} f_k(x_1,...,x_k) \quad \text{(1a)}$$
$$g_k(x_1,...,x_k) \le 0,$$

where x_{k-1} is the solution of

$$\min_{x_{k-1}} f_{k-1}(x_1,...,x_k) \quad \text{(1b)}$$
$$g_{k-1}(x_1,...,x_k) \le 0,$$

x_1 is the solution of problem

$$\min_{x_1} f_1(x_1,...,x_k) \quad \text{(1c)}$$
$$g_1(x_1,...,x_k) \le 0.$$

Problem (1a) is solved at upper level, where the coordinator controls the variables of the solutions x_k for minimizing the f_k function. Similarly, problem (1c) is the first-level and corresponds to the lower hierarchical level. The multilevel optimization problem (1) is hard to be solved (Bard et al, 1998; Dempe, 2000). Even in the simplest version of two-level optimization it becomes non-convex and/or non-smooth and belongs to the class of global optimization (Dempe, 2000). The solution of the optimal design problems with non-smooth structure can be found by different manners - applying a penalty function method; Karush-Kuhn-Tucker type conditions; a pure non-differentiable optimization technique (bundle optimization algorithm).

The multilevel hierarchical system is an attempt for traffic control management modelling where the different traffic flows are the subsystems of the multilevel system. Each traffic flow (subsystem) has in some degree relation with others. The constraints of the subsystems' problems reflect the different kinds of resources which have to be taken into account (weather, traffic jam, incidents, etc). The goal of the whole traffic flow system is minimization of conjunctions of traffic light crossroads. The next research steps concern integration of different control loops in the hierarchy and identification of appropriate levels for autonomic behaviour.

2.1 Formal bi-level modelling

The bi-level model is developed for a simple case of arterial road network. This case is situated in the centre of Sofia (between Eagle Bridge and University of Sofia). This place is always under pressure of congestions because it is a highway termination and a transport bottleneck in the city centre.

The values $x_i(k)$ concerns the number of vehicles, which are waiting in queues in front of the traffic lights for the corresponding street junctions, for time interval k. The values u_j represent the relative duration of the green light as a part of the cycle time c_k of the cross road section. The saturation flows s_i are assumed to be constant. The values of q are the corresponding inflows of the sections.

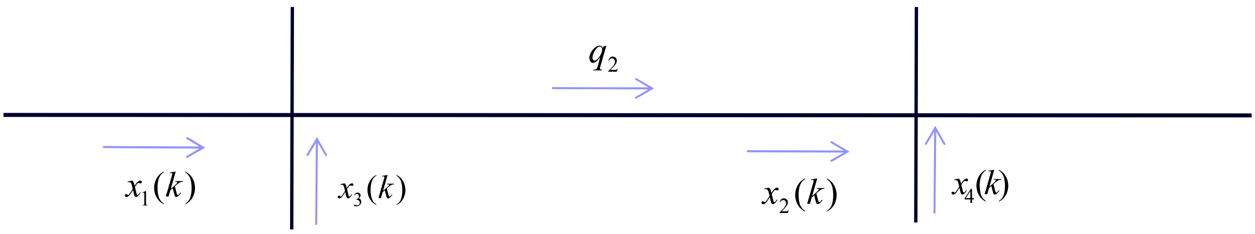

Fig.1. Arterial road network

The optimization problem defined according to the road network, given in fig. 1 can be assumed analytically like:

$$\min_{u_1,u_2}(a_1x_1^2 + a_2x_2^2 + a_3x_3^2 + a_4x_4^2 + r_1u_1^2 + r_2u_2^2) \quad (2)$$
$$x_1(k+1) = x_1(k) + q_{1in}(k) - s_1u_1c$$
$$x_2(k+1) = x_2(k) + s_1u_1c_1 - s_2u_2c_2$$
$$x_3(k+1) = x_3(k) + q_{3in}(k) - (L_1c_1 - u_1c_1)s_1$$
$$x_4(k+1) = x_4(k) + q_{4in}(k) - (L_2c_2 - u_2c_2)s_2,$$

where the optimization problem is defined and solved for each control period k;

$x_i,\ i = 1,4$ describe the state of the queues for the control period;

$x_{i0},\ i = 1,4$ are the initial values for each control period of vehicles which stay in a queue in front of each crossroad section;

$u_j,\ j = 1,2$ are the control variables which are evaluated according to the optimization problem;

k is assumed to be 1 as a control period;

$c_l, l = 1,2$ are the time cycles of the traffic lights and they are assumed to be constant values.

For this classical problem the values of the time cycle c_l are assumed to be known, according to reference plans or others off-line considerations.

The solutions $u_j(c_j)$, j=1,2 of this problem are inexplicit functions of the time cycles c_l. Their values can also be evaluated according to additional requirements, from which the traffic conditions in the road network can benefit.

These considerations lead to a bi-level formulation for an optimization problem which has to evaluate both the relative durations $u_j, j = 1,2$ of the green lights as values of the time cycles $c_l, l = 1,2$. The bi-level formulation of the global optimization problem is illustrated in fig.2.

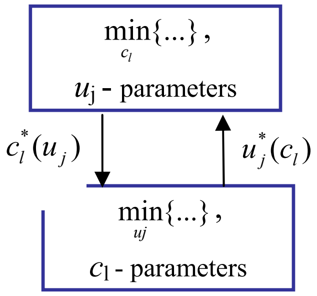

Fig.2. Bilevel formalization of the optimization problem

The lower level optimization problem is defined analytically according to (2) where c_l are given parameters. The solutions of (2) $u_j^*(c_l)$ are given as parameters to the upper level problem which solves and evaluate the optimal durations $c_l^*(u_j)$ for the given set u_j. Thus, an iterative sequence of calculations will result in finding global optimal solutions $c_l^{opt}(u_j^{opt})$, $j = 1,2$ The upper level optimization problem can be defined analytically taking into account additional requirements towards the traffic conditions.

2.2 Upper level conditions

For the case of arterial traffic network new optimization problem which targets the maximization of the traffic flow on the arterial road stretch is developed in the paper. For the case of fig.1, the upper level problem is defined as maximization of the traffic flows q_2 on the stretch between sections 1 and 2. The integral relation between the traffic flow and the traffic density is assumed as

$$q_2 = v\rho_2,$$

where v is the average speed of the traffic flow;
ρ_2 is the density of the flow in section 2.

However for the solution of this relation in differential form, $v(\rho)$ is assumed to be a function of the flow density ρ, which defines the functional dependence between q and ρ in the fundamental diagram. At the same time the flow speed v is also function of the density ρ. Due to several forms, available for the approximations $v(\rho)$, a simple linear approximation (Greenshield, 1935) is used here

$$v = v_{free}(1 - \rho_2 / \rho_{\max}),$$

which is numerically defined according to the values of the free speed v_{free} and the density $\rho_{\max}$, which prevents the traffic to move and the speed is zero, $v = 0$,

$$\rho_{\max} \longrightarrow v = 0$$

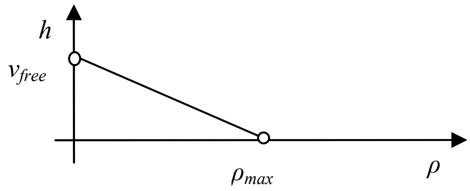

Fig.3. Linear approximation

Applying these simple integral dependances the relation of the traffic flow on the arterial stretch becomes analytically defined:

$$q_2 = v(\rho_2) =$$
$$v_{free}(1 - \rho_2 / \rho_{\max})\rho_2 = v_{free}\rho_2 - \rho_2^2 v_{free} / \rho_{\max}$$

The flow density ρ_2 which is the number of vehicles x_2 on the stretch with longitude L_2 can be represented linearly like

$$\rho_2(x_2) = x_2 / L_2 .$$

Thus, the traffic flow $q_2(x_2)$ as a function of the number of vehicles x_2 can be expressed like quadratic function

$$q_2(x_2) = \frac{v_{free}}{L_2} x_2 - \frac{v_{free}}{\rho_{\max} L^2} x_2^2 .$$

This relation can be used as a part of the performance function of the upper level optimization problem. Respectively, the maximization of $q_2(x_2)$

$$\max_{c_1,c_2}\{q_2(x_2(c_1,c_2)\} =$$
$$= \max_{c_1,c_2}\left\{\frac{v_{free}}{L_2} x_2(c_1,c_2) - \frac{v_{free}}{\rho_{\max} L^2} x_2^2(c_1,c_2)\right\}$$

by changing as independent arguments the time cycles $c_l, l = 1,2$ will increase the traffic flow on the arterial stretch. The upper level optimization problem is derived analytically here like:

$$\max_{c_l, l=1,2}\{H(c_l) = q_2(x_2(c_l)) - c_l^T h c_l\}, \qquad (3)$$

where $q_2(c_l)$ is analytically derived as quadratic function to $x_2(c_l)$; h is a weighted matrix in the goal function.

The engineering meaning of this goal function is a maximization of the arterial traffic flow with penalizing the durations of the time cycles $c_l, l = 1, m$ by means to constrain the waiting cars on sections 3 and 4 (fig.1).
After substitutions of relation $x_2(c_1, c_2)$ in (3) the upper level optimization problem becomes

$$\max_{c_1,c_2}\left\{x_2(c_1,c_2)-\frac{1}{\rho_{\max}L}x_2^2(c_1,c_2)-h(c_1^2-h_2c_2^2)\right\} \quad (4)$$

$$x_2=x_{20}+u_1s_1c_1-u_2s_2c_2$$

2.2 Analitical Solution of Bi-level Problem

For simplification of the notations the low level optimization problem is written like

$$\min_{u_1,u_2}\left\{J=x_1^2+x_2^2+x_3^2+x_4^2+\alpha_1u_1^2+\alpha_2u_2^2\right\} \quad (5)$$

where

$$x_1=x_{10}-s_1u_1c_1$$
$$x_2=x_{20}+s_1u_1c_1-s_2u_2c_2$$
$$x_3=x_{30}-s_1(c_1-u_1c_1)$$
$$x_4=x_{40}-s_2(c_2-u_2c_2)$$

The analytical solutions $u_1(.)$ and $u_2(.)$ are found according to the requirements for minimization of the goal function J or

$$\frac{dJ}{du_1}=0=2x_1\frac{dx_1}{du_1}+2x_2\frac{dx_2}{du_1}+2x_3\frac{dx_3}{du_1}+2\alpha_1u_1 \quad (6)$$

$$\frac{dJ}{du_2}=0=2x_2\frac{dx_2}{du_2}+2x_4\frac{dx_4}{du_2}+2\alpha_2u_2$$

Using the constraint relations, the derivatives are

$$\frac{dx_1}{du_1}=s_1u_1 \qquad \frac{dx_2}{du_2}=-s_2c_2$$
$$\frac{dx_2}{du_1}=s_1c_1 \qquad \frac{dx_4}{du_2}=s_2c_2$$
$$\frac{dx_3}{du_1}=s_1c$$

After substitution in (6) it holds

$$(3s_1c_1+\frac{\alpha_1}{s_1c_1})u_1-s_2c_2u_2=x_{10}-x_{20}-x_{30}+s_1c_1$$

$$-s_1c_1u_1+(2s_2c_2+\frac{\alpha_2}{s_2c_2})u_2=x_{20}-x_{40}+s_2c_2$$

The analytical solution of this linear system towards $u_1(c_1,c_2)$ and $u_2(c_1,c_2)$ gives

$$u_1(c_1,c_2)=\frac{\begin{vmatrix}b_1 & a_{12}\\ b_2 & a_{22}\end{vmatrix}}{\det|A|} \qquad u_2(c_1,c_2)=\frac{\begin{vmatrix}a_{11} & b_1\\ a_{21} & b_2\end{vmatrix}}{\det|A|},$$

where

$$a_{11}=3s_1c_1+\frac{\alpha_1}{s_1c_1} \quad a_{12}=-s_2c_2$$

$$a_{21}=-s_1c_1 \qquad a_{22}=2s_2c_2+\frac{\alpha_2}{s_2c_2}$$

$$A=\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}$$

$$b_1=x_{10}-x_{20}-x_{30}+s_1c_1 \qquad b_2=x_{20}-x_{40}+s_2c_2$$

For simplicity of the analytical relations it is assumed the particular case $\alpha_1=0, \quad \alpha_2=0$. Hence, the relations $u_i(c_l)$ are

$$u_1(c_1,c_2)=\frac{2x_{10}-x_{20}-2x_{30}-x_{40}}{5s_1c_1}+\frac{s_2c_2}{5s_1c_1}+\frac{2}{5} \quad (7)$$

$$u_2(c_1,c_2)=\frac{x_{10}+2x_{20}-x_{30}-3x_{40}}{5s_2c_2}+\frac{s_1c_1}{5s_2c_2}+\frac{2}{5}$$

where it is assumed $c_1\neq 0$, $c_2\neq 0$

Using relation (7) the upper level optimization problem can be derived analytically as it follows below. Substituting (7) in the description of x_2 it follows

$$x_2(c_1,c_2)=x_{20}+\frac{1}{5}(x_{10}+3x_{20}-x_{30}+2x_{40})+\frac{1}{5}s_1c_1-\frac{2}{5}s_2c_2$$

or

$$x_2(c_1,c_2)=\frac{1}{5}(x_{10}+2x_{20}-x_{30}+2x_{40})+\frac{1}{5}s_1c_1-\frac{2}{5}s_2c_2\,. \quad (8)$$

$$H(c_1,c_2)=q_2(c_1,c_2)-\beta_1c_1^2-\beta_2c_2^2$$

or

$$H(c_1,c_2)=\frac{v_{free}}{L}x_2(c_1,c_2)-\frac{v_{free}}{\rho_{\max}L^2}x_2^2(c_1,c_2)-\beta_1c_1^2-\beta_2c_2^2$$

For simplification of the relations, the weighted coefficients are assumed equal to $\beta_1=\beta_2=\frac{v_{free}}{L}$. Thus, the upper level problem is derived to

$$\max_{c_1,c_2}\left\{H(c_1,c_2)=x_2(c_1,c_2)-\frac{1}{\rho_{\max}L}x_2^2(c_1,c_2)-c_1^2-c_2^2\right\} \quad (9)$$

$$x_2=x_{20}+u_1s_1c_1-u_2s_2c_2,$$
$$c_1,c_2\geq 0\,.$$

The upper level problem due to its definition can be solved as unconstrained optimization problem:

$$\frac{dH}{dc_1}=\frac{dx_2}{dc_1}-\frac{2}{\rho_{\max}L}x_2(c_1,c_2)\frac{dx_2}{dc_1}-2c_1=0 \quad (10)$$

$$\frac{dH}{dc_2}=\frac{dx_2}{dc_2}-\frac{2}{\rho_{\max}L}x_2(c_1,c_2)\frac{dx_2}{dc_2}-2c_2=0\,.$$

The derivatives $\frac{dx}{dc}$ can be found by direct differentiation of relation (8) or

$$\frac{dx_2}{dc_1} = \frac{s_1}{5}, \qquad \frac{dx_2}{dc_2} = -\frac{2s_2}{5}.$$

After substitution in (10) and rearrangements having in mind that the requirements for no negativity of $c_l, l = 1,2$, it follows:

$$c_1^{opt} = \frac{s_1[5L\rho_{\max} - 2(x_{10} - x_{30} + 2x_{20} + 2x_{40})]}{50L\rho_{\max} + 2s_1^2 + 8s_2^2},$$

$$c_2^{opt} = \frac{s_2[-10L\rho_{\max} + 4(x_{10} - x_{30} + 2x_{20} + 2x_{40})]}{50L\rho_{\max} + 2s_1^2 + 8s_2^2},$$

$$\text{if} \quad c_1 > 0, \quad c_2 > 0.$$

From practical considerations the time cycles c_k could not be lower than lower values $c_{l\min}, l = 1,2$ because the traffic flow will saturate on the other sections, crossing the arterial stretch 2. These additional requirements transform the upper bound problem on quadratic programming one with constraints

$$\max_{c_1,c_2}\left\{ H(c_1,c_2) = x_2(c_1,c_2) - \frac{1}{\rho_{\max}L}x_2^2(c_1,c_2) - c_1^2 - c_2^2 \right\}$$

$$x_2 = x_{20} + u_1 s_1 c_1 - u_2 s_2 c_2,$$

$$0 \le c_1 \le c_{1\max} = \frac{1}{5}(x_{10} + 2x_{20} - x_{30} + 2x_{40}) + \frac{1}{5}s_1 c_1 - \frac{2}{5}s_2 c_2,$$

$$0 \le c_2 \le c_{2\max}.$$

The numerical values, used for the arterial road section of Fig.1 are estimated according to:

$x_{10} = 50$; $s_1 = 0.44 vehicles/s$

$x_{20} = 30$; $s_2 = 0.33 vehicles/s$

$x_{30} = 30$; $s_3 = 0.28 vehicles/s$

$x_{40} = 30$; $s_4 = 0.28 vehicles/s$

L=800 m; $\rho_{\max} = 0.175\ vehicles/m$

The lower and upper optimization problems have been modified with requirements for no negativity and bounded solutions of $u_j, j = 1,2$ and $c_l, l = 1,2$. The calculations were performed in a sequence, assuming constant new inflows to all crossroad sections. A comparison has been made between two control policies: using bi-level model (continuous blue lines in figures below) and optimization with constrained values $c_l, l = 1,2$ of the traffic light cycle (dashed red lines). The comparison has been performed for the queue lengths for the arterial direction x_1, x_2, x_3, x_4 in front of the crossroad section. It can be seen that the queue lengths for the arterial direction x_1 and x_2 decrease faster for the bi-level model in comparison with the optimization problem with constant time cycles c_l. This advantage is taken for x_1 and x_2 in return, fig.4,5 for the worst case of x_3 and x_4, fig.6,7, which are crossing the arterial line. The relative value of the green light u_1 towards the time cycle c_1 (for the first crossroad section) and the corresponding u_2 (for the second crossroad section) change, fig.8 and fig.9. The time cycles c_1, and c_2 are given in fig.10. For the case of bi-level control c_1 increases while c_2 is kept constant.

3. SIMULATION RESULTS

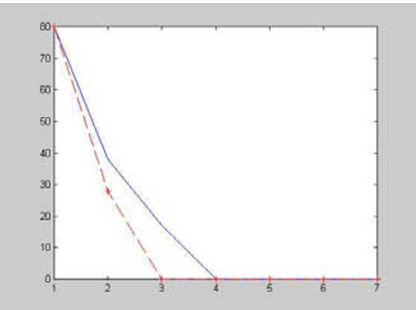

Fig.4 Queue length x_1 towards k

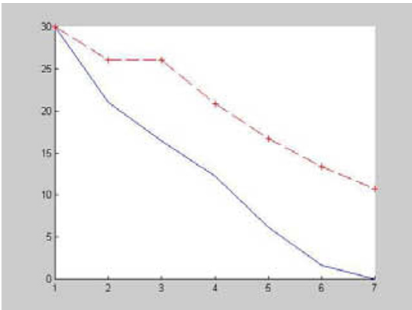

Fig. 5 Queue length x_2 towards k

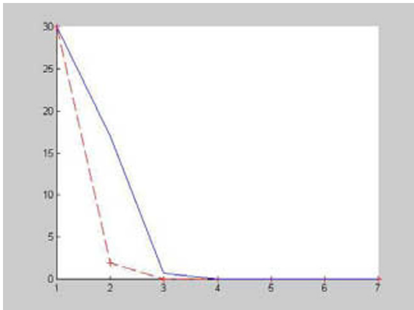

Fig.6 Queue length x_3 towards k

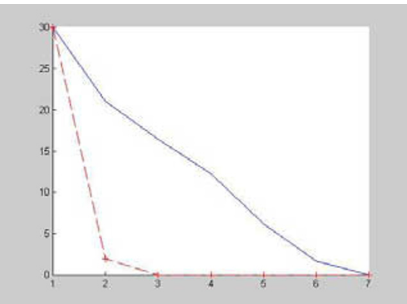

Fig. 7 Queue length x_4 towards k

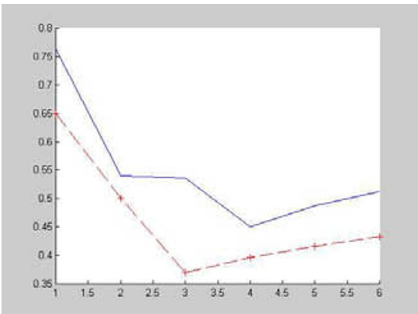

Fig.8 u_1 changes towards k

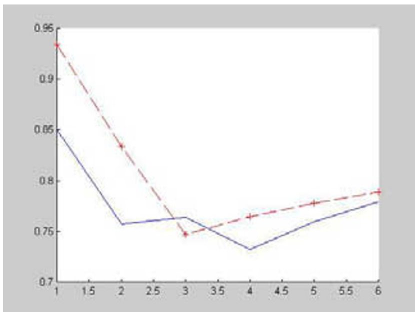

Fig.9. u_2 changes towards k

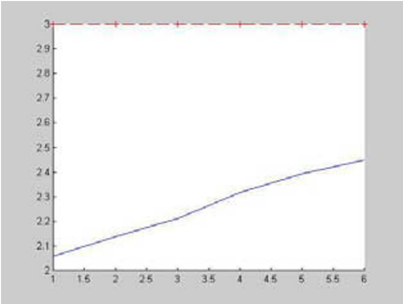

Fig.10 Time cycles c_1 and c_2 behaviour towards k

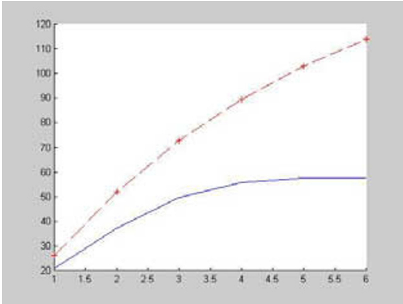

Fig.11. Integral queue length x_2 towards k

An integral assessment of the bi-level control policy is presented in fig.11 by evaluating the total queue length x_2 for the overall control horizon. The bi-level model decreases the total queue length in comparison with the case of one-level optimization with constant time cycles c_l=const, l=1,2. Thus, the bi-level modelling approach demonstrates a potential for improvement of the traffic conditions on an arterial transportation stretch.

4. CONCLUSIONS

A bi-level model for controlling a particular arterial transportation network is worked out in the paper. This new model allows the arguments of the optimization problem to be increased both with the relative duration of the green phases u_j and the time cycles of the traffic flows c_l as well. Thus, an integral increase of the arguments of the optimization problem is achieved, which benefit the optimization of the traffic behaviour. These results are simulated in a simple arterial network which allows the derivation of analytic relations for the solutions of the low level optimization problem. For more general case both low level and upper level optimization problems have to be solved simultaneously. This will generate delays for the evaluation of the problem solution and problems for numerical efficient methods for solving bi-level models have to be taken into consideration.

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