Labs
Machine Learning Course
Fall 2019

EPFL
School of Computer and Communication Sciences
Martin Jaggi & Rüdiger Urbanke
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Problem Set 9, Nov 14, 2019
(Theory Questions SVD).

Goals. The goal of this exercise is to

- familiarize yourself with the theory related to SVD.
- have time to discuss Project 2 with the assistants and teammates.

1 Theory Questions

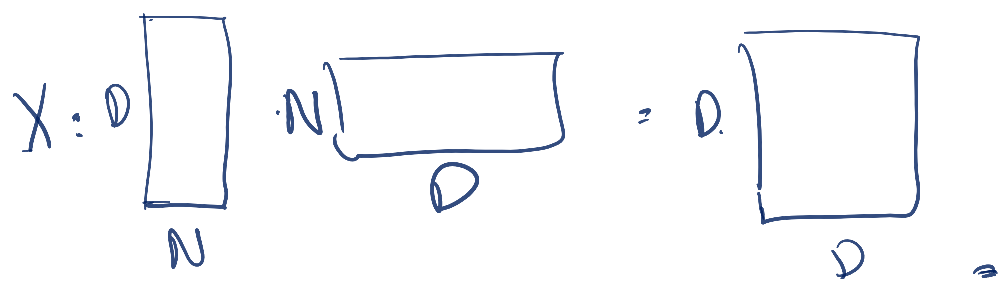
Problem 1 (How to compute U and S efficiently):
In class, we saw that solving the eigenvector/value problem for the matrix $XX^\top$ gives us a way to compute U and S. But in some instances $D \gg N$. In those cases, is there a way to accomplish this computation more efficiently?

Problem 2 (Positive semi-definite):
Show that if X is a $N \times N$ symmetric matrix then the SVD has the form $USU^\top$, where U is a $N \times N$ unitary matrix and S is a $N \times N$ diagonal matrix with non-necessarily positive entries. Show that if X is positive semi-definite, then all entries of S are non-negative.

P1:

$X = USV^T$

$XX^T = USV^T V S^T U^T = USS^T U^T = U S_D^2 U^T$



$X^T X = V S^T \underbrace{U^T U}_{I} S V^T = V S^T S V^T$

Let v_i denote the columns of V. Then:

$$X^T X v_j = s_j^2 v_j$$

The j-th column of V is an eigenvector of $X^T X$ with eigen value s_j^2.

$\Rightarrow$ solving the eigenvalue/vector problem for the matrix $X^T X$ gives us a way to compute V and S.

$$X X^T (X v_j) = s_j^2 X v_j$$

$$X X^T u_j = s_j^2 u_j$$

$\Rightarrow$ we can compute the desired eigenvectors u_j from v_j without solving the eigenvector/value problem for $X X^T$ $(D \times D)$.

P2-

$X = (N \times N)$

$X = U S V^T \Rightarrow$
$U: N \times N$
$V: N \times N$
$S: N \times N$.

* the columns of U are the eigen vectors of $A = X X^T$
* the columns of V are the eigen vectors of $B = X^T X$

Since X is $N \times N$ symmetric $XX^T = X^TX$

$\Rightarrow$ the eigenspace associated to each distinct eigenvalue of A is the same eigenspace associated to the same distinct eigenvalue of B.

$\underline{U = V}$.

$X = USV^T \Rightarrow U^TXV = UU^TSV^TV = S$