PCA in high dimensions

- We need to solve the eigenvector/eigenvalue equation

$$\frac{1}{N}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}}^\top \boldsymbol{c}_i = \lambda_i \boldsymbol{c}_i$$

  where $\boldsymbol{c}_i = \bar{\boldsymbol{X}}\boldsymbol{b}_i$
- We want to recover the original eigenvectors $\boldsymbol{b}_i$ of the data covariance matrix $\boldsymbol{S} = \frac{1}{N}\bar{\boldsymbol{X}}^\top\bar{\boldsymbol{X}}$
- Left-multiplying eigenvector equation by $\bar{\boldsymbol{X}}^\top$ yields

$$\underbrace{\frac{1}{N}\bar{\boldsymbol{X}}^\top\bar{\boldsymbol{X}}}_{=\boldsymbol{S}}\bar{\boldsymbol{X}}^\top\boldsymbol{c}_i = \lambda_i\bar{\boldsymbol{X}}^\top\boldsymbol{c}_i$$

  and we recover $\bar{\boldsymbol{X}}^\top\boldsymbol{c}_i$ as an eigenvector of $\boldsymbol{S}$ with (the same) eigenvalue λ_i

Note: To perform PCA as discussed in the lecture we need to make sure that $\|\bar{\boldsymbol{X}}^\top\boldsymbol{c}_i\| = 1$.