

Flow networks and the maximum flow problem

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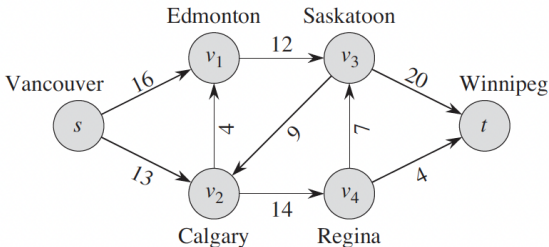
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Definition

A flow network $G = (V, E)$ is a directed graph in which each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v) \geq 0$. It also has a certain source s and sink t .

- If E contains an edge (u, v) , then there is no (v, u) . This is easy to bypass though.
- Each vertex lies on some path from s to t . As such, a flow network is also connected.



Definition

A flow in G is a real-valued function $f : V \times V \rightarrow \mathbb{R}$ that satisfies the following properties:

1. Capacity constraint: for all $u, v \in V$, we require $0 \leq f(u, v) \leq c(u, v)$.
2. Flow conservation: for all $u \in V \setminus \{s, t\}$, we require

$$\sum_{v \in V} f(v, u) = \sum_{v \in V} f(u, v). \quad (1)$$

If $(u, v) \notin E$, there can be no flow from u to v , so $f(u, v) = 0$.

Definition

A value $|f|$ of a flow f is defined as

$$|f| = \sum_{v \in V} f(s, v) - \sum_{v \in V} f(v, s). \quad (2)$$

