ECE6554 Final Project: Planar Ducted Fan

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I. Introduction

The objective of this project is to design and compare a set of controllers to regulate a planar ducted fan. The controllers include a static linear controller, an adaptive linear controller, and an adaptive controller with nonlinearities in the span of the control. We begin by defining and discussing the system, then cover each controller design, and finally examine the results of each controller.

Equations 1 and 2 define the dynamics of the system. The system is controlled by the thrust tau and flap deflection ψ which produce the body frame force defined in Equation 3. Additionally the flaps are restricted to motion between $-\pi/3$ and $\pi/3$. All controllers for this project are designed to output the force vector $\vec{f}$, since the the thrust and flap deflection can be easily recovered from these values. Gains and reference trajectories are set to keep f_x small and f_y positive so the flap limits are not hit.

$$m\ddot{q} = -d\dot{q} + R(\theta)\vec{f} - m\vec{g} \tag{1}$$

$$J\ddot{\theta} = -rf_x \tag{2}$$

$$\vec{f} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix} \tau \tag{3}$$

II. Controller Design

The following section describes the process of designing each controller. The results and performance of each controller is shown in Section III.

A. Linear Controller

The first controller examined is a linear controller designed around a linearized equilibrium state. The first step is to identify equilibrium conditions for the system. It is clear that the velocities must be zero for the system to be in equilibrium. Additionally $R(\theta)\vec{f}$ must cancel $m\vec{g}$, and f_x must be zero. This can only be achieved if $\theta = 0$ and $f_y = mg$. Therefore, the equilibrium conditions are:

$$\theta_{eq} = \dot{q}_{eq} = \dot{\theta}_{eq} = \psi_{eq} = 0 \tag{4}$$

$$\tau_{eq} = mg \tag{5}$$

Additionally, q_{eq} can be arbitrary since the dynamics are invariant to position. Next we linearize the dynamics around these conditions.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{\tau_{eq}}{m} & -\frac{d}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{d}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & \frac{1}{m} \\ -\frac{r}{J} & 0 \end{bmatrix} \tag{7}$$

Our control law for the linear system is given by Eqaution 8 where K is computed by MATLAB's `icare(A,B,Q)` function. We use $Q =$ `diag([.1, 1, 1, 1, 1, 1])`, so that horizontal position is tracked less aggressively than orientation. This helps keep the system near the equilibrium hover state so the linearized equations remain valid.

$$\vec{f} = r - Kx + \vec{f}_0 \tag{8}$$

The objective for the controller is to track some desired state $\vec{x}_d$. The reference signal is calculated as defined in Equation 9.

$$r = (B^\top B)^{-1} B^\top (\dot{\vec{x}}_d - A + BK)\vec{x}_d \tag{9}$$

B. Adaptive Linear Controller

The second controller builds on the first by adding adaptation. The adaptive controller adds a reference model state defined by Equation 10. The system matrices for the model are defined as $A_m = A - BK$ and $B_m = B$ where K is the gain matrix from the linear controller.

$$\dot{x}_m = A_m x_m + B_m r \tag{10}$$

The new control law is defined by Equation 11 and the gains adapt via the dynamics defined in Equations 12 and 13 where $e = x - x_m$. For both Γ_x and Γ_r we use an identity matrix of respective size and we calculate P with the with MATLAB's `icare(A,B,Q)` function using the same Q matrix as the linear controller. The static offset $\vec{f_0}$ has been replaced by $\hat{\alpha}$ to allow the controller to update the equilibrium thrust as described by Equation 14. Note that this is effectively a nonlinearity with $\Phi(x) = 1$. We have set $\Gamma_\alpha = 1$.

$$\vec{f} = K_x x + K_r r + \hat{\alpha} \tag{11}$$

$$\dot{K}_x^\top = -\Gamma_x x e^\top P B \tag{12}$$

$$\dot{K}_r^\top = -\Gamma_r r e^\top P B \tag{13}$$

$$\dot{\hat{\alpha}}^\top = -\Gamma_\alpha e^\top P B \tag{14}$$

The reference signal for the adaptive linear controller is given by Equation 15, where $x_{m,d}$ is the desired state of the system.

$$r = (B_m^\top B_m)^{-1} B_m^\top (\dot{\vec{x}}_{m,d} - A_m)\vec{x}_{m,d} \tag{15}$$

C. Adaptive Controller with Nonlinearities

For the final controller we re-examine the system dynamics to identify nonlinearities in the span of the control. Looking at the linear dynamics we can rearrange Equation 1 to Equation 16. From Equation 17 we can determine that the structure of the nonlinearity should be $\Phi(x) = [\sin(\theta), \cos(theta)]^\top$.

$$m\ddot{q} = -d\dot{q} + R(\theta)\left[\vec{f} - mR^\top(\theta)\vec{g}\right] \tag{16}$$

$$\alpha^\top \Phi(x) = -mR^\top(\theta)\vec{g} = -mR^\top(\theta)\begin{bmatrix} 0 \\ g \end{bmatrix} = -mg\begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix} \tag{17}$$

The adaptive nonlinear control law is defined in Equation 18. We note that this nonlinearity is the equivalent of changing from world coordinates to robot coordinates. For the adaptation laws we use a modified error $\tilde{e}$ with the terms corresponding to linear velocity rotated by $R^\top(\theta)$ such that B will be the same as the linear case. The adaptation laws are thus given by Equations 19-21. The adaptive gains Γ_x, Γ_r, and Γ_α are set to be identity matrices of respective size.

$$\vec{f} = K_x x + K_r r - \hat{\alpha}^\top \Phi(x) \tag{18}$$

$$\dot{K}_x^\top = -\Gamma_x x \tilde{e}^\top PB \tag{19}$$

$$\dot{K}_r^\top = -\Gamma_r r \tilde{e}^\top PB \tag{20}$$

$$\dot{\hat{\alpha}}^\top = -\Gamma_\alpha x \tilde{e}^\top PB \tag{21}$$

III. Results

The following section presents and discusses the performance of the controllers described previously. Each controller is tested on a step command $x_d(t) = [1, 1, 0, 0, 0, 0]$ and a sinusoid $x_d(t) = [\sin(t), \cos(t), 0, \cos(t), -\sin(t), 0]$. Additionally we compare the controllers based on positional error (euclidean distance to the desired position).

A. Linear Controller

First, we validate the linear controller by simulating the estimated system using the parameters provided in the project description. Figure 1 shows the step response of the system moving from the origin to the point $(x, y) = (1, 1)$. Figure 2 shows the response to a sinusoid signal which has the system follow a cicle around the origin in the XY plane. The plots show the linear controller tracks the estimated system reasonably well.

Now we add uncertainty to the system test the linear controller when the estimate does not match. The exact values added to the parameters are provided in Table 1. Figure 3 shows the step response to the system with uncertainty. It is clear the linear controller is no longer able to track the desired trajectory, in particular the y-position due to the uncertainty in the fan's weight. Figure 4 shows the sinusoid response to the system with uncertainty which has similar difficulty in tracking the desired trajectory.

Parameter	Value
δm	0.3571 kg
δd	0.0182 kg/sec
δr	-0.0318 m
δJ	-0.0082 kg m^2

Table 1 Perturbations added to the system parameters.

We can also see the euclidean distance error for the uncertain system in Figures 21 and 22. Since the the linear controller cannot account for the difference in the system's mass, it is the only controller which settles at a greater

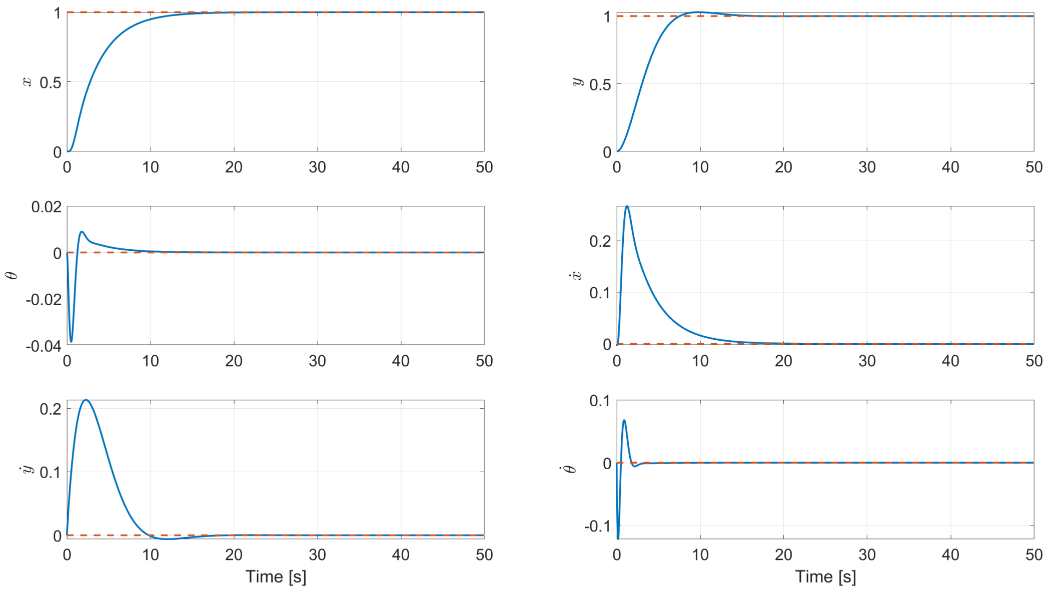


Fig. 1 Linear controller step response.

distance error than it started at.

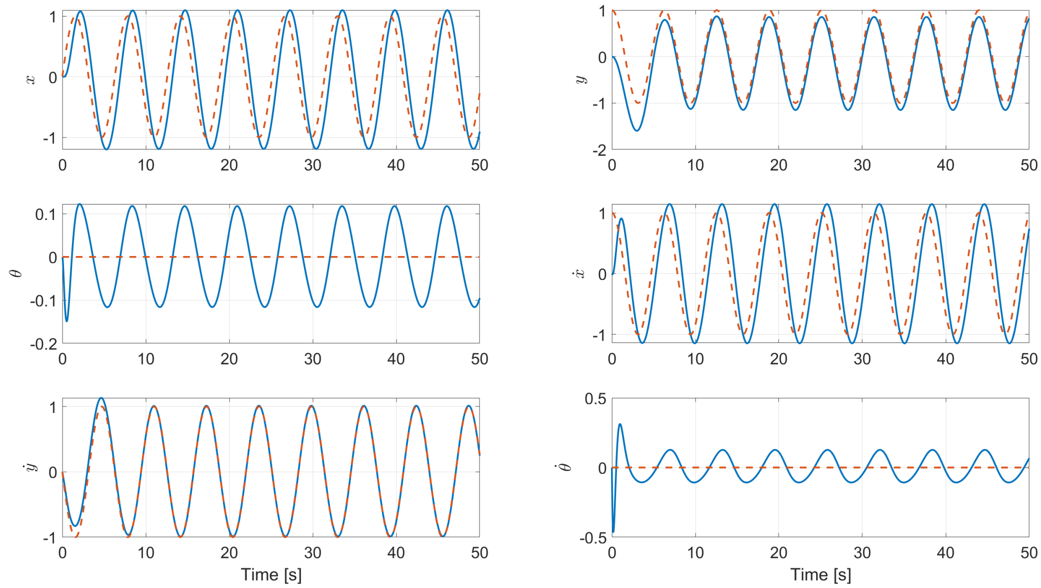


Fig. 2 Linear controller sinusoid response.

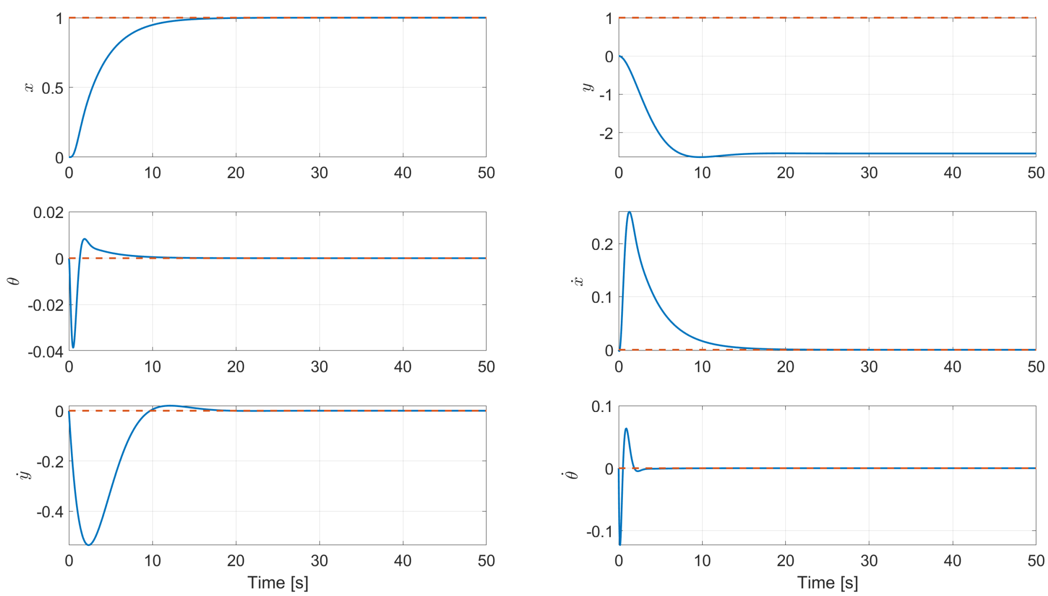


Fig. 3 Linear controller step response with uncertainty.

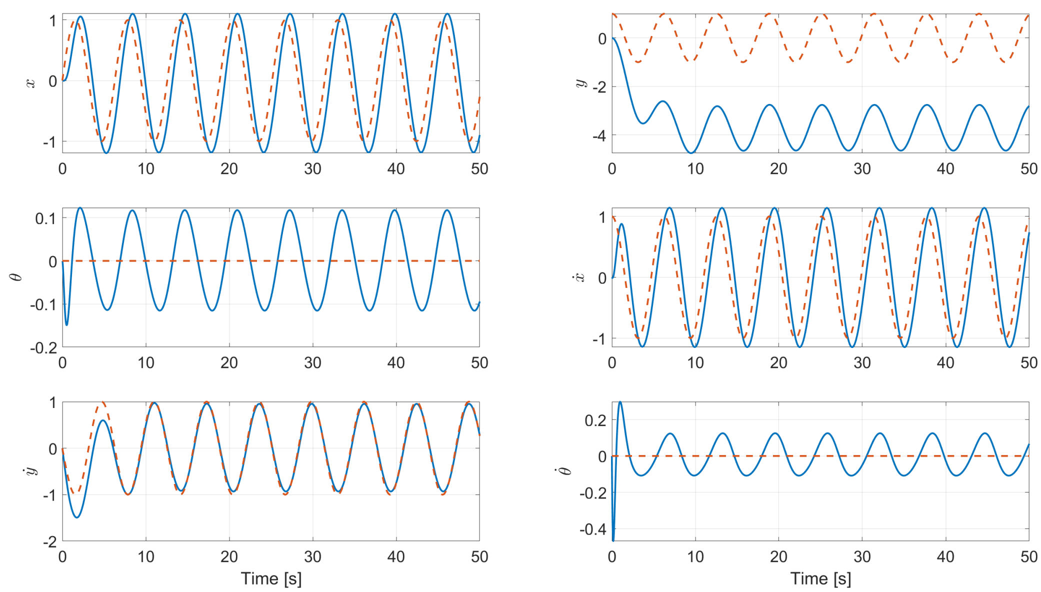


Fig. 4 Linear controller sinusoid response with uncertainty.

B. Adaptive Linear Controller

The adaptive controller attempts to address the uncertainties by updating the controller gains. The initial gains are set using the estimated parameters via the matching conditions in Equations 22 and 23. Additionally we set $\hat{\alpha}(0) = \vec{f}_0$.

$$A_m - A = BK_x(0) \tag{22}$$

$$B_m = BK_r(0) \tag{23}$$

The adaptive controller is simulated in response to the step and sinusoid signals on the uncertain system. The state responses are shown in Figures 5 and 7. Unlike the static linear controller, the adaptive controller is able reach the desired y-position in the for both the step and sinusoid responses. The controller gains are also plotted in Figures 6 and 8.

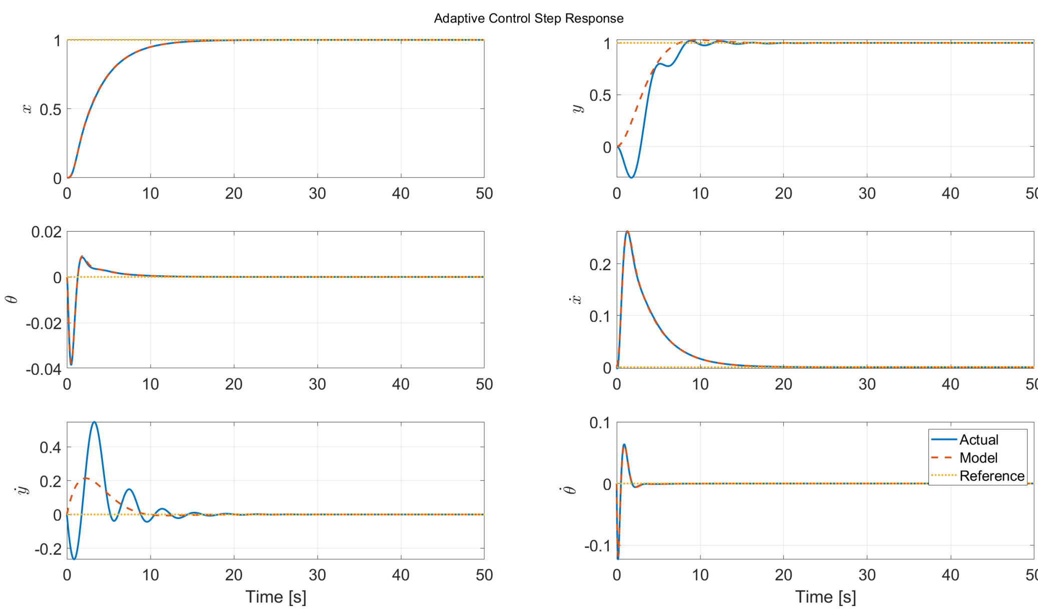
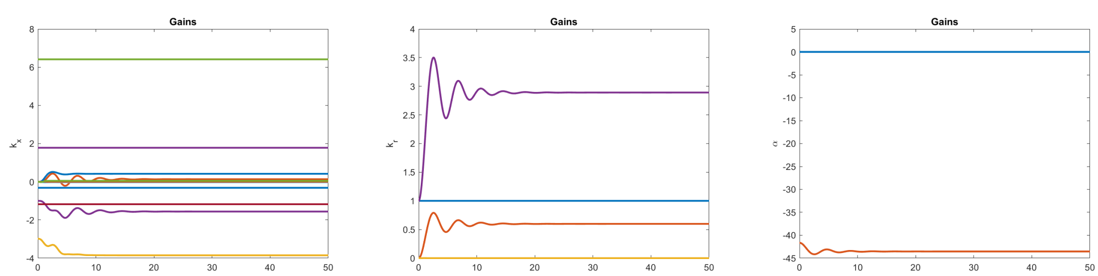


Fig. 5 Adaptive linear controller step response.



(a) K_x gains for adaptive linear system step response. **(b) K_r gains for adaptive linear system step response.** **(c) $\hat{\alpha}$ gains for adaptive linear system step response.**

Fig. 6 Adaptive gains step response.

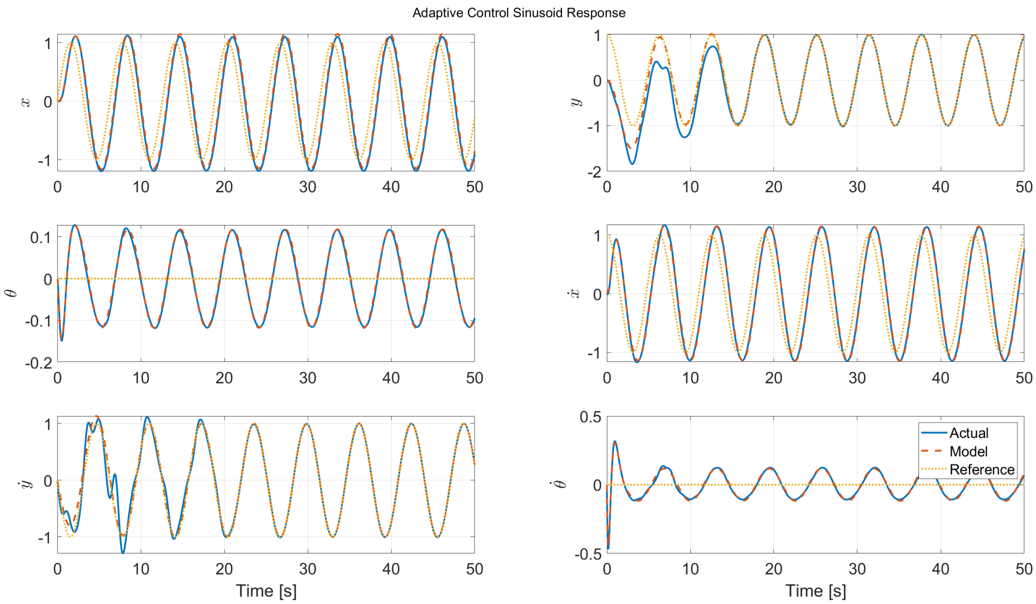
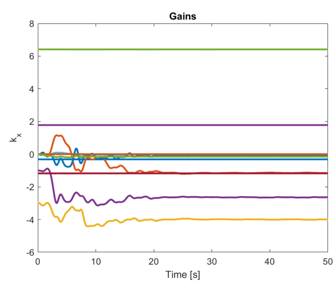
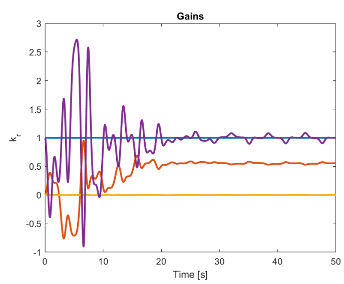


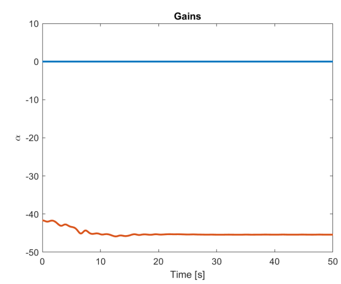
Fig. 7 Adaptive linear controller sinusoid response.



(a) $\mathbf{K}_x$ gains for adaptive linear system sinusoid response.



(b) $\mathbf{K}_r$ gains for adaptive linear system sinusoid response.



(c) $\hat{\alpha}$ gains for adaptive linear system sinusoid response.

Fig. 8 Adaptive gains sinusoid response.

We also examine the controller response starting from post-adaptation gains. After the previous simulations we record the final values of K_x, K_r, and $\hat{\alpha}$ and use them as initial conditions to simulate the step and sinusoid responses. Post-adaptation state responses are shown in Figures 9 and 11. In both cases, the post-adaptation controller tracks the model trajectory more closely and smoothly from the start. This makes sense since the gains should already be close to their true values.

The gains are plotted in Figures 10 and 12. For both cases there appears to be significantly less deviation from the initial guess compared to the original simulation. The step response gains still jump to slightly different values. This may be because the the step reference signal provides a less observable adaptive system.

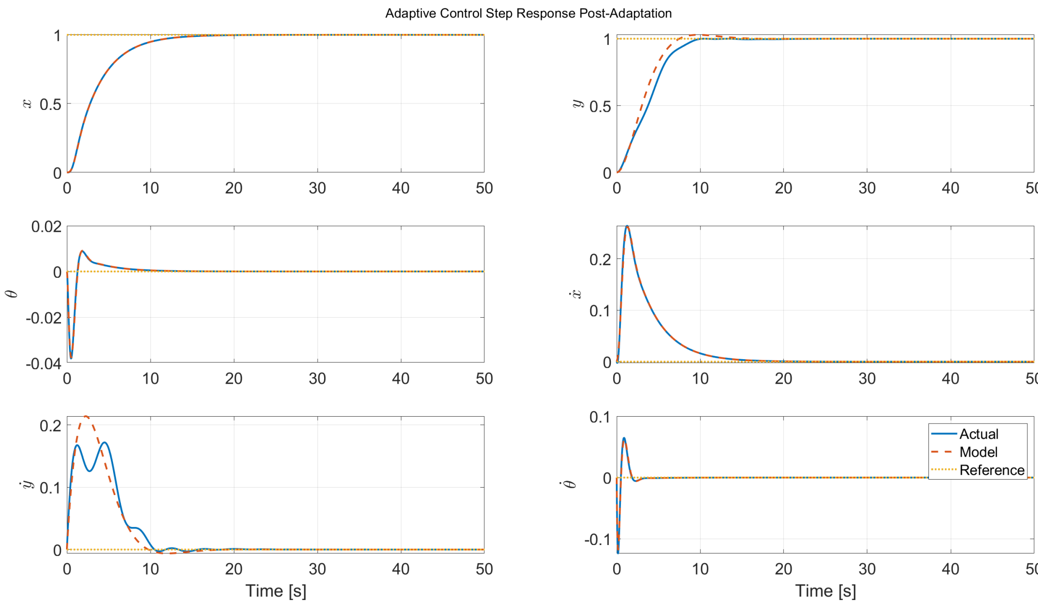
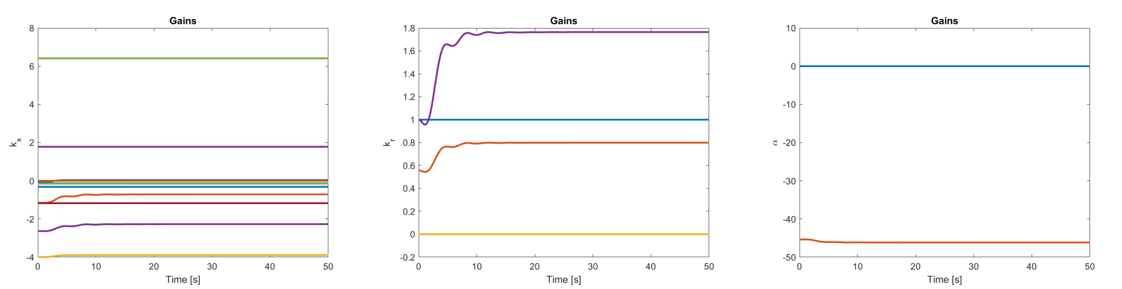


Fig. 9 Adaptive linear controller step response post-adaptation.



(a) $\mathbf{K}_x$ gains for adaptive linear system step response. **(b) $\mathbf{K}_r$ gains for adaptive linear system step response.** **(c) $\hat{\alpha}$ gains for adaptive linear system step response.**

Fig. 10 Adaptive gains step response post-adaptation.

Distance error for the adaptive linear controller is shown in Figures 21 and 22.For both responses, the post-adaptation

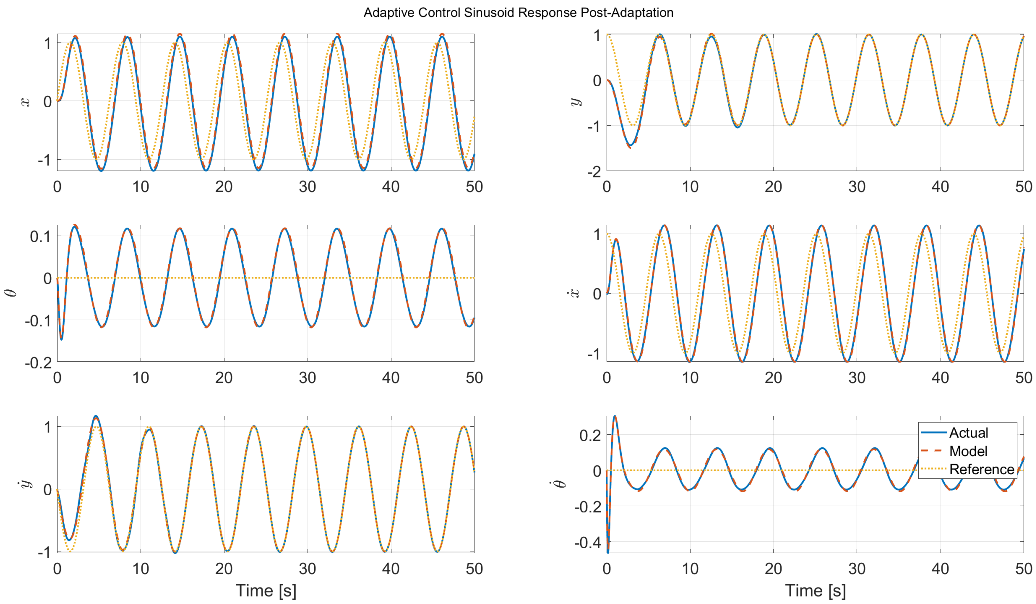
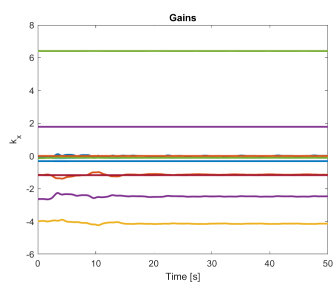
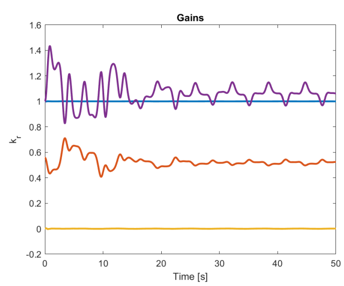


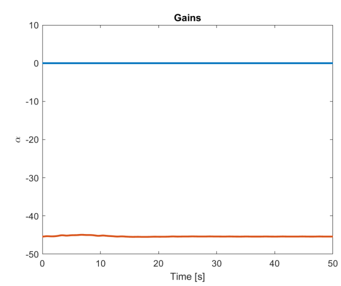
Fig. 11 Adaptive linear controller sinusoid response post-adaptation.



(a) K_x gains for adaptive linear system sinusoid response.



(b) K_r gains for adaptive linear system sinusoid response.



(c) $\hat{\alpha}$ gains for adaptive linear system sinusoid response.

Fig. 12 Adaptive gains sinusoid response post-adaptation.

controller reduces the error a little more quickly, but settles at about the same error as the original controller. Both adaptive controllers far out-perform the static linear controller.

C. Adaptive Controller with Nonlinearities

Lastly we test the nonlinear on the same scenarios. We use the same matching conditions as the linear adaptive case to set $K_x(0)$ and $K_r(0)$. Additionally, we set $\hat{\alpha}(0)$ according to Equation 24.

$$\hat{\alpha}(0) = \begin{bmatrix} -mg & 0 \\ 0 & -mg \end{bmatrix} \tag{24}$$

The step and sinusoid responses are shown in Figures 13 and 15, respectively. At a glance the response is similar to the linear adaptive controller, though the transient response tracks slightly better for the nonlinear controller. The nonlinear controller is ultimately able to track the model state. The gains are also plotted in Figures 14 and 16. The gain response is similar to the linear adaptive controller.

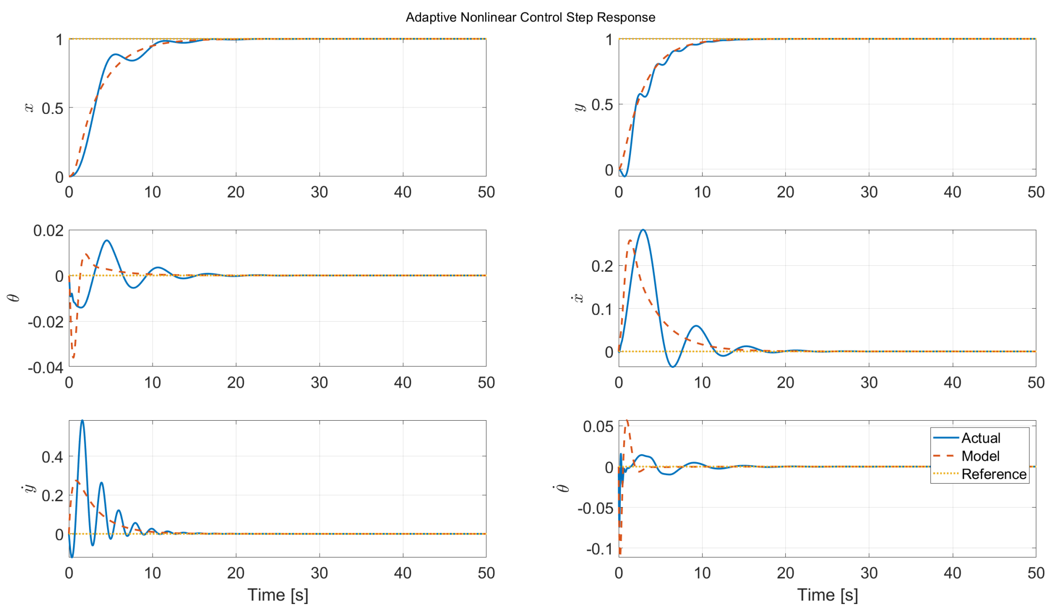
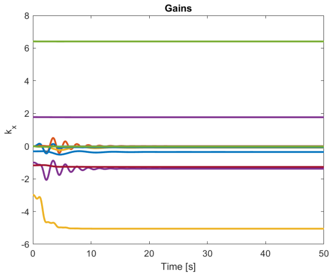
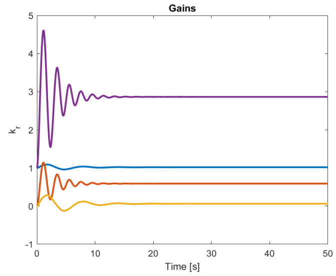


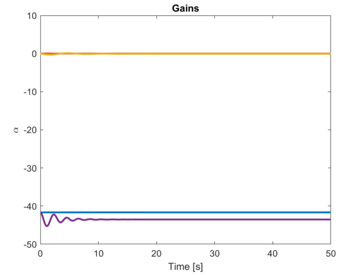
Fig. 13 Adaptive nonlinear controller step response.



(a) $\mathbf{K}_x$ gains for adaptive nonlinear system step response.



(b) $\mathbf{K}_r$ gains for adaptive nonlinear system step response.



(c) $\hat{\alpha}$ gains for adaptive nonlinear system step response.

Fig. 14 Nonlinear adaptive gains step response.

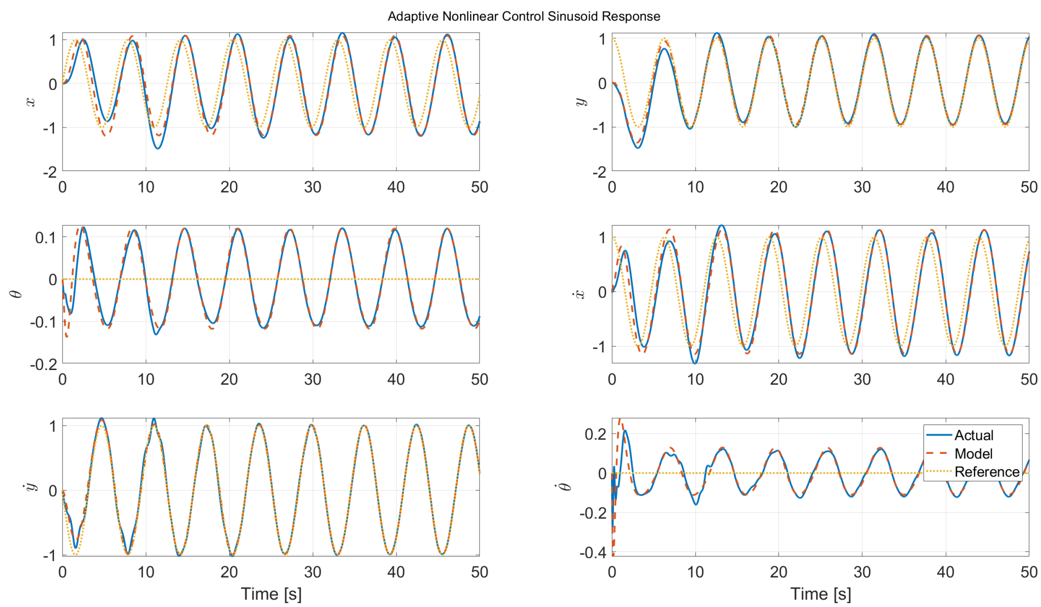
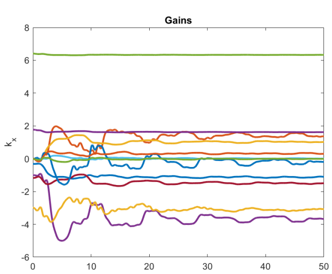
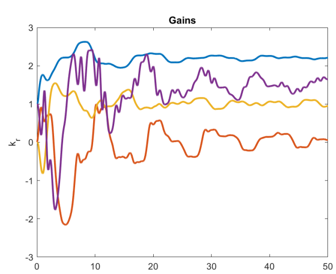


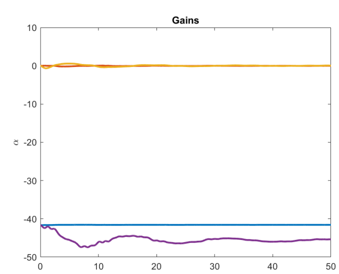
Fig. 15 Adaptive nonlinear controller sinusoid response.



(a) $\mathbf{K}_x$ gains for adaptive nonlinear system sinusoid response.



(b) $\mathbf{K}_r$ gains for adaptive nonlinear system sinusoid response.



(c) $\hat{\alpha}$ gains for adaptive nonlinear system sinusoid response.

Fig. 16 Nonlinear adaptive gains sinusoid response.

We repeat the post-adaptation tests with the nonlinear controller. The final values of the gains are recorded after the previous simulations and used as initial conditions. The state responses are shown in Figures 17 and 19. Similarly to the adaptive linear case, the post adaptation controller tracks the model state more closely from the start. The gains also change less dramatically over time as shown in Figures 18 and 20.

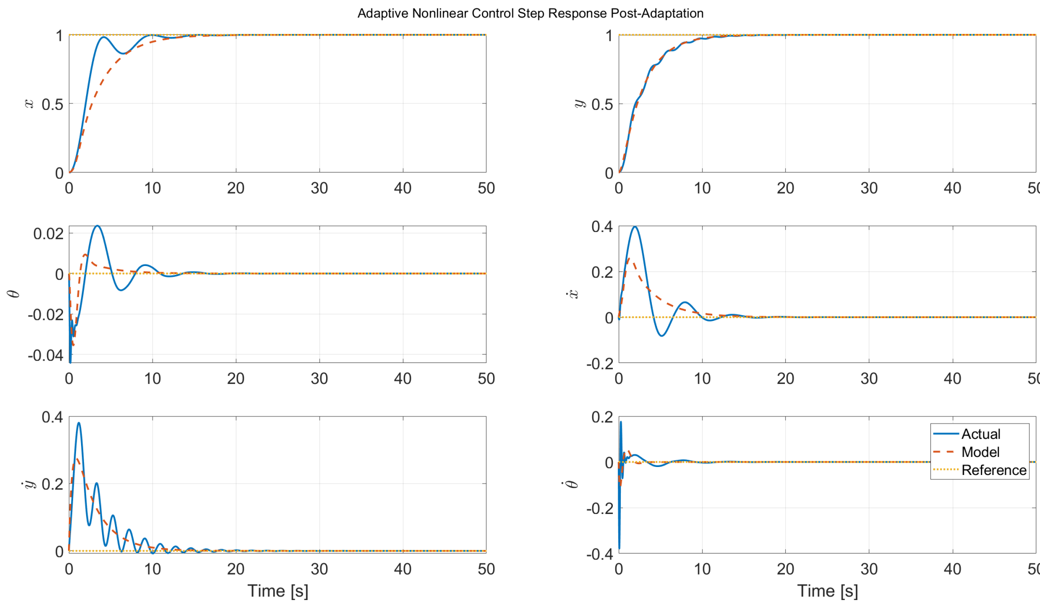
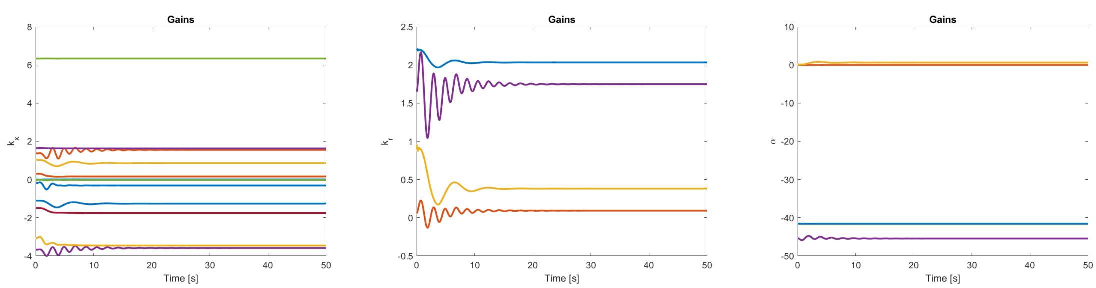


Fig. 17 Adaptive nonlinear controller step response post-adaptation.



(a) $\mathbf{K}_x$ gains for adaptive nonlinear system step response. **(b) $\mathbf{K}_r$ gains for adaptive nonlinear system step response.** **(c) $\hat{\alpha}$ gains for adaptive nonlinear system step response.**

Fig. 18 Nonlinear adaptive gains step response post-adaptation.

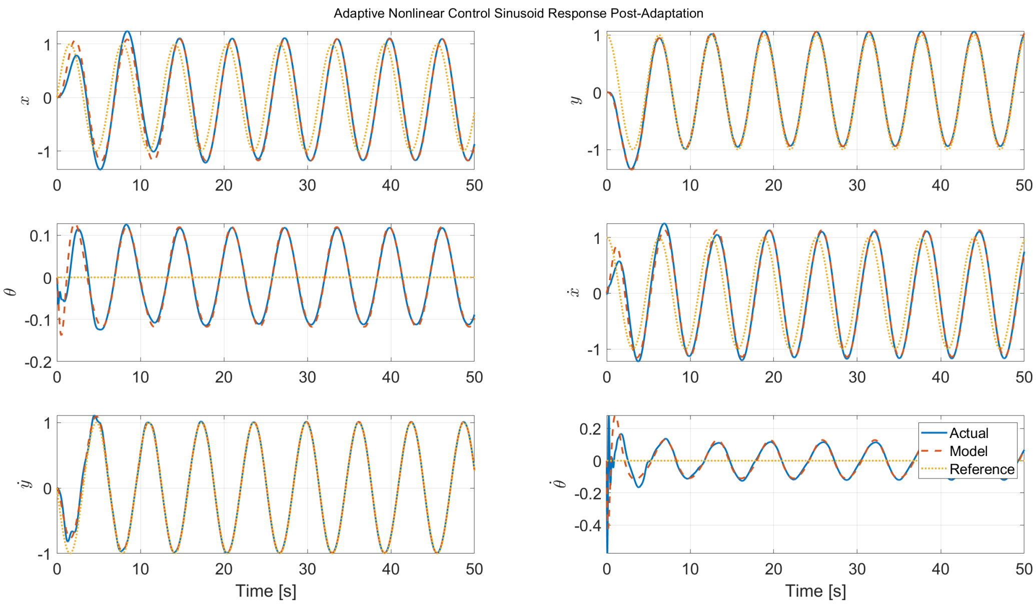
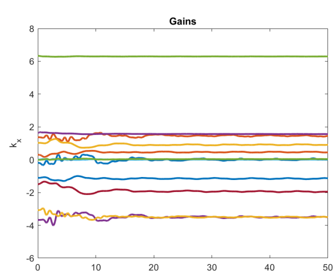
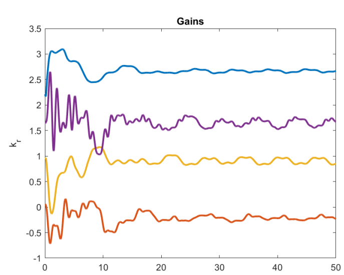


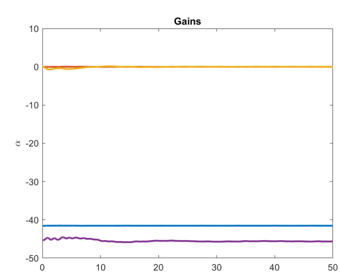
Fig. 19 Adaptive nonlinear controller sinusoid response post-adaptation.



(a) $\mathbf{K}_x$ gains for adaptive nonlinear system sinusoid response.



(b) $\mathbf{K}_r$ gains for adaptive nonlinear system sinusoid response.



(c) $\hat{\alpha}$ gains for adaptive nonlinear system sinusoid response.

Fig. 20 Nonlinear adaptive gains sinusoid response post-adaptation.

Examining the euclidean distance error in Figures 21 and 22, we see that the post-adaptation nonlinear controller closes the error more quickly, but settles to about the same error as the original nonlinear controller. Additionally, the adaptive nonlinear controller reduces error a little faster than adaptive linear controller; however, the improvement is not dramatic. Additionally the final tracking error is about the same for all controllers except the static linear controller.

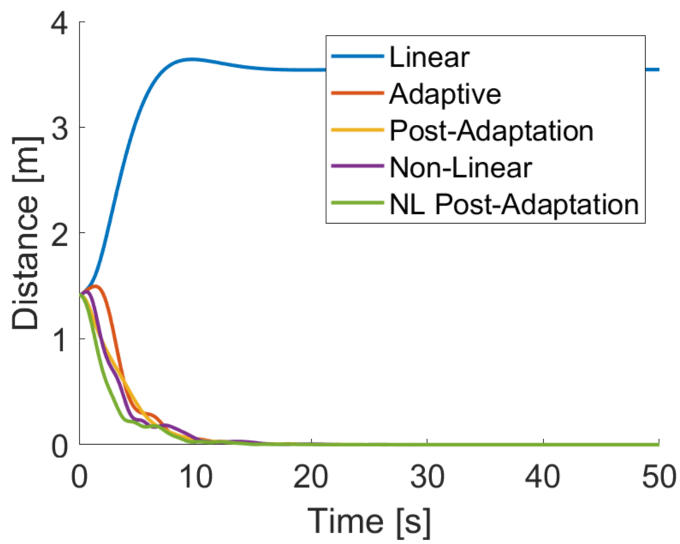


Fig. 21 Euclidean distance error for step response.

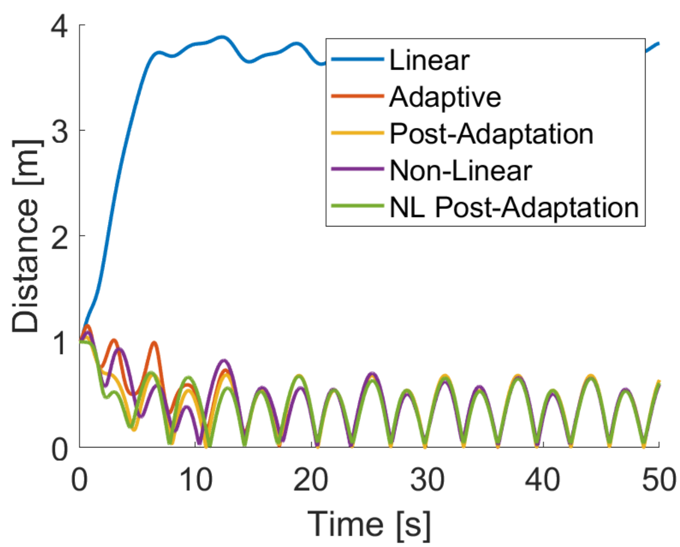


Fig. 22 Euclidean distance error for sinusoid response.

IV. Conclusion

This project has examined three different controllers for the planar ducted fan: static linear, adaptive linear, and adaptive nonlinear. Adaptation clearly provides a significant improvement when uncertainty is present in the system parameters. The nonlinear adaptation law offers some improvement over linear adaptation; however, for this system the improvement is quite small. This may be because the model system is set to keep the angle relatively small, meaning the linear approximation could already be fairly accurate. Further testing could be done to see if the nonlinear controller offers better performance when tracking a more aggressive nonlinear system.

Additionally, both adaptive controllers perform better earlier when using post adaptation gains. In a way, the post adaptation controller approach achieves something similar to system identification. The controller uses a test run to get the gains close to their true values, so that less adaptation is needed during later runs.