

#1-10:

Prove:

$$\sum_{i=1}^n i = n(n+1)/2 \text{ for } n \geq 0, \text{ by induction}$$

Figure 1: formula

$$\sum_{i=1}^n i = n(n+1)/2 \text{ for } n \geq 0, \text{ by induction}$$

Figure 2: formula

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Figure 3: formula

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Let $n = 1$. Then:

establishing our base case.

Assume that:

We must show:

QED

#1-10:

Prove:

$$n(n+1)/2 = 1 = \sum_{i=1}^1 i$$

Figure 4: formula

$$\sum_{i=1}^n i = n(n+1)/2 \quad \text{for } n \leq k,$$

Figure 5: formula

$$\sum_{i=1}^{n+1} i = (n+1)(n+2)/2$$

Figure 6: formula

$$\sum_{i=1}^{n+1} i = (n+1) + \sum_{i=1}^n i$$

Figure 7: formula

$$\begin{aligned} \sum_{i=1}^{n+1} i &= (n+1) + \sum_{i=1}^n i \\ &= (n+1) + n(n+1)/2 \\ &= [2(n+1) + n(n+1)]/2 \\ &= (n+1)(n+2)/2 \end{aligned}$$

Figure 8: formula

$$\sum_{i=1}^n i = n(n+1)/2 \text{ for } n \geq 0, \text{ by induction}$$

Figure 9: formula

Let $n = 1$. Then:

$$n(n+1)/2 = 1 = \sum_{i=1}^1 i$$

Figure 10: formula

establishing our base case.

Assume that:

$$\sum_{i=1}^n i = n(n+1)/2 \text{ for } n \leq k,$$

Figure 11: formula

We must show:

QED

#1-11:

Prove: $\text{SUM}\{i=1,n\}(i^2) = n(n+1)(2n+1)/6$ for $n \geq 0$, by induction

let $n = 1$. Then $n(n+1)(2n+1)/6 = 1 = \text{SUM}\{i=1,1\}(i^2)$

Assume that $\text{SUM}\{i=1,n\}(i^2) = n(n+1)(2n+1)/6$.

$$\begin{aligned} \text{SUM}\{i=1, n+1\}(i^2) &= (n+1)^2 + \text{SUM}\{i=1,n\}(i^2) \\ &= (n+1)^2 + n(n+1)(2n+1)/6 \\ &= (n+1)[6(n+1) + n(2n+1)] / 6 \\ &= (n+1)[2n^2 + 7n + 6] / 6 \\ &= (n+1)(n+2)(2n+3) / 6 \\ &= (n+1)(n+2)(2(n+1) + 1) / 6 \text{ which we needed to show :.} \end{aligned}$$

1-12: // $\text{SUM}\{i=1,n\}(i^3) = n^2(n+1)^2/4$ for $n \geq 0$, by induction

1-13: // Prove $\text{SUM}\{i=1,n\}((i)(i+1)(i+2)) = n(n+1)(n+2)(n+3)/4$ for $n \geq 0$, by induction

$$\sum_{i=1}^{n+1} i = (n+1)(n+2)/2$$

Figure 12: formula

$$\sum_{i=1}^{n+1} i = (n+1) + \sum_{i=1}^n i$$

Figure 13: formula

$$\begin{aligned} \sum_{i=1}^{n+1} i &= (n+1) + \sum_{i=1}^n i \\ &= (n+1) + n(n+1)/2 \\ &= [2(n+1) + n(n+1)]/2 \\ &= (n+1)(n+2)/2 \end{aligned}$$

Figure 14: formula

