CENG 2010 - Programming Language Concepts

Week 4: Context-Free Grammars and Syntactical Analysis

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March 27, 2023

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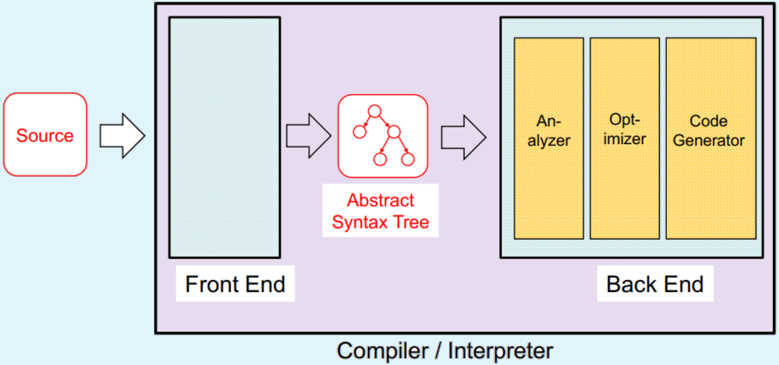
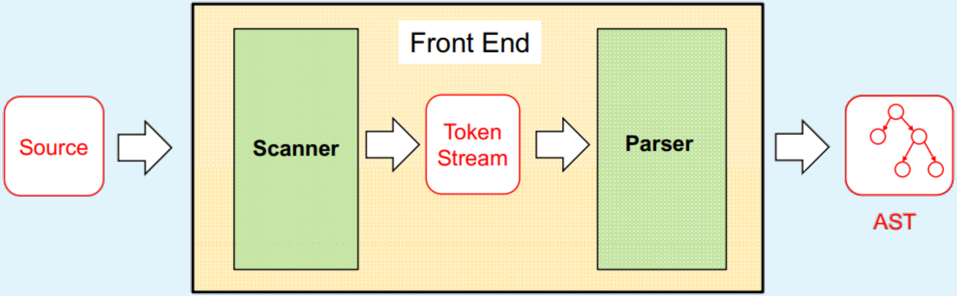
- regular expressions, context-free grammars, and parsing constitute the syntax of a language

Architecture of Compilers and Interpreters



Front-End: Lexer and Parser



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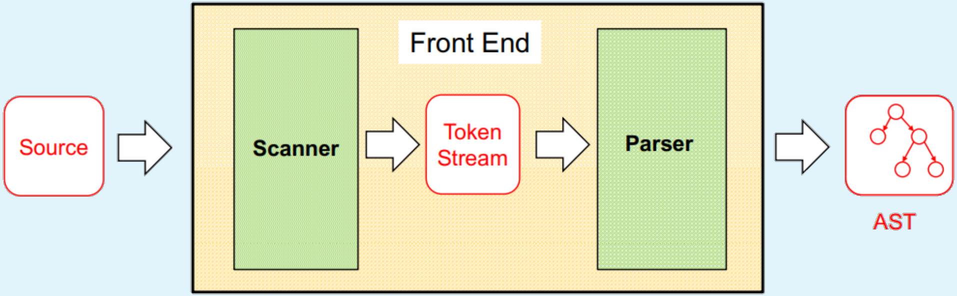

scanner/lexer	:	source code	$\mapsto$	tokens (keywords, variables, numbers, etc.)	regular expressions
parser	:	tokens	$\mapsto$	abstract syntax trees/parse trees	context free grammars

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- regular expressions for tokenizing (scanning/lexing), and context free grammars for parsing tokens

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Lemma

$L(G) = \{\mathrm{a}^n\mathrm{b}^n \mid n \geqslant 0\}$

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- set $B \subseteq \Sigma^*$ is context-free if $B = L(G)$ for some CFG G

Example

- grammar $G = \big(\{S, T, U\}, \{a, b, c\}, P, S\big)$ with P :

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- grammar $G' = \big(\{S, A, B, C\}, \{a, b, c\}, P, S\big)$ with P :

$$\begin{aligned} S &\rightarrow ABC \\ A &\rightarrow aA \mid \varepsilon \\ B &\rightarrow bB \mid \varepsilon \\ C &\rightarrow cC \mid \varepsilon \end{aligned}$$

  constructs the same language $\mathcal{L}$

Some Useful Tricks to Design Grammars

1. arbitrary number of symbols:

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   $S \to aSb \mid \varepsilon$

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4. matching *linearly* related symbols:

   $S \to aSbb \mid \varepsilon$

   generates $\mathcal{L} = \{a^n b^{2n} \mid n \geqslant 0\}$

Example

- $\{0^n1^m \mid m \leqslant n\}$ $G = (\{S\}, \{0, 1\}, P, S)$

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- $\{a^n (b^m \mid c^k) \mid n \geqslant 0, m, k > n\}$

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  where $\quad P = \quad \{S \quad \to \quad 0S1 \mid 0S \mid \varepsilon\}$

- $\{1^n 0 \mid n \geqslant 0\}$ $\quad G = \big(\{S\}, \{0,1\}, P, S\big)$

  where $\quad P = \quad \{S \quad \to \quad 0 \mid 1S\}$

- $\{a^n (b^m \mid c^k) \mid n \geqslant 0, m, k > n\} = \{a^n b^m \mid m > n \geqslant 0\} \cup \{a^n c^k \mid k > n \geqslant 0\}$ $\quad G = \big(\{S, T, U, V, Y\}, \{a, b, c\}, P, S\big)$

Example

- $\{0^n1^m \mid m \leqslant n\}$ $\quad G = (\{S\}, \{0,1\}, P, S)$

  where $P = \{S \rightarrow 0S1 \mid 0S \mid \varepsilon\}$

- $\{1^n0 \mid n \geqslant 0\}$ $\quad G = (\{S\}, \{0,1\}, P, S)$

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- $\{a^n(b^m \mid c^k) \mid n \geqslant 0, m, k > n\} = \{a^nb^m \mid m > n \geqslant 0\} \cup \{a^nc^k \mid k > n \geqslant 0\}$ $\quad G = (\{S, T, U, V, Y\}, \{a, b, c\}, P, S)$

  where $P = \{$
  $$\begin{aligned} T &\rightarrow aTb \mid U \\ U &\rightarrow Ub \mid b \end{aligned}$$

Example

- $\{0^n 1^m \mid m \leqslant n\}$ $\quad G = (\{S\}, \{0,1\}, P, S)$

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$$\text{where} \quad P = \{ \begin{array}{lcl} T & \rightarrow & aTb \mid U \\ U & \rightarrow & Ub \mid b \\ V & \rightarrow & aVc \mid Y \\ Y & \rightarrow & Yc \mid c \end{array}$$

Example

- $\{0^n 1^m \mid m \leqslant n\}$ $\quad G = (\{S\}, \{0,1\}, P, S)$

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Table of Contents

Example

CFG G: $S \rightarrow [S] \mid SS \mid \varepsilon$

Example

CFG G: $S \to [S] \mid SS \mid \varepsilon$ three derivations of $[[]]$:

1. $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$

Example

CFG G: $S \to [S] \mid SS \mid \varepsilon$ three derivations of [[]]:

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CFG G: $S \to [S] \mid SS \mid \varepsilon$ three derivations of $[[\,]]$:

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Definition

- in leftmost derivation always leftmost nonterminal is replaced ① ②

Example

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Definition

- in leftmost derivation always leftmost nonterminal is replaced 1 2
- in rightmost derivation always rightmost nonterminal is replaced 1 3

Example

CFG G: $S \to [S] \mid SS \mid \varepsilon$ three derivations of $[[]]$:

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Definition

- in leftmost derivation always leftmost nonterminal is replaced ① ②
- in rightmost derivation always rightmost nonterminal is replaced ① ③
- parse tree is representation of derivation in which replacement order is ignored

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Definition

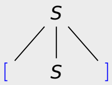
- in leftmost derivation always leftmost nonterminal is replaced 1 2
- in rightmost derivation always rightmost nonterminal is replaced 1 3
- parse tree is representation of derivation in which replacement order is ignored
- CFG is ambiguous if some string has different parse trees

Example

CFG G: $S \to [\,S\,] \mid S\,S \mid \varepsilon$

1. $S \xrightarrow[G]{1} [\,S\,]$

parse tree

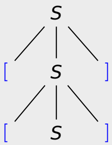


Example

CFG G: $S \to [\,S\,] \mid S\,S \mid \varepsilon$

1. $S \xrightarrow[G]{1} [\,S\,] \xrightarrow[G]{1} [\,[\,S\,]\,]$

parse tree

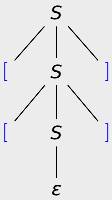


Example

CFG G: $S \to [\, S\,] \mid S\, S \mid \varepsilon$

➊ $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[\,]]$

parse tree



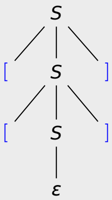
Example

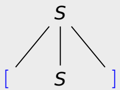
CFG G: $S \rightarrow [\,S\,] \mid S\,S \mid \varepsilon$

1. $S \xrightarrow[G]{1} [\,S\,] \xrightarrow[G]{1} [[\,S\,]] \xrightarrow[G]{1} [[\,]]$

2. $S \xrightarrow[G]{1} [\,S\,]$

parse trees





Example

CFG G: $S \to [\,S\,] \mid S\,S \mid \varepsilon$

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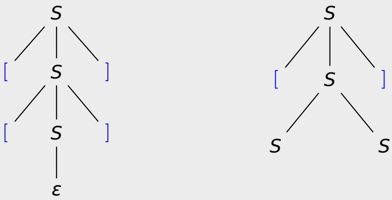
parse trees



Example

CFG G: $S \to [\,S\,] \mid S\,S \mid \varepsilon$

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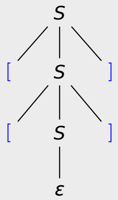
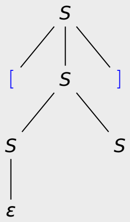
2. $S \xrightarrow[G]{1} [\,S\,] \xrightarrow[G]{1} [\,S\,S\,] \xrightarrow[G]{1} [\,S\,]$

parse trees





Example

CFG G: $S \rightarrow [\,S\,] \mid S\,S \mid \varepsilon$

➊ $S \xrightarrow[G]{1} [\,S\,] \xrightarrow[G]{1} [[\,S\,]] \xrightarrow[G]{1} [[\,]]$

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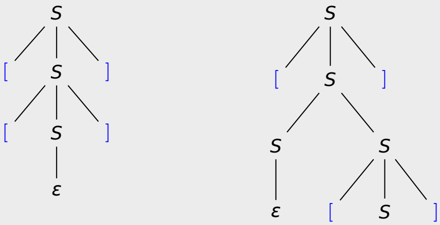
parse trees



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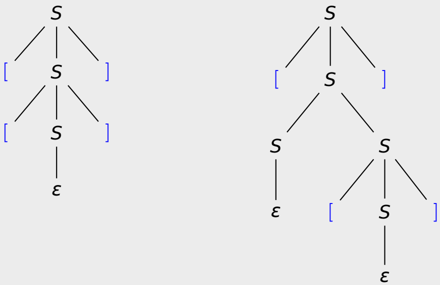

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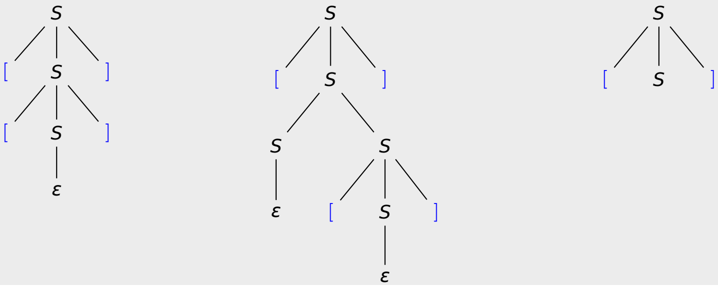
parse trees



Example

CFG G: $S \to [\,S\,] \mid S\,S \mid \varepsilon$

① $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$

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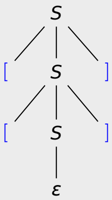
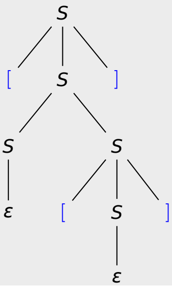
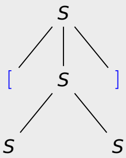
parse trees







Example

CFG G: $S \to [\,S\,] \mid S\,S \mid \varepsilon$

1. $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[\,]]$
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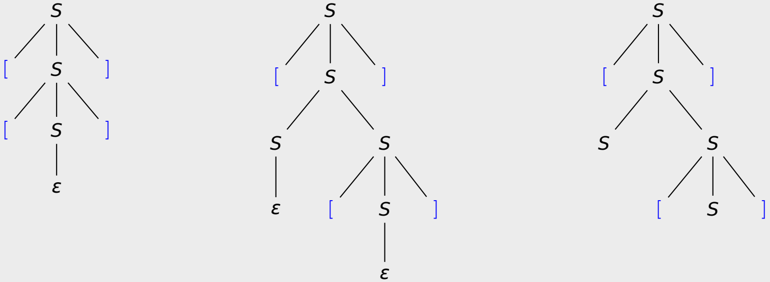
parse trees



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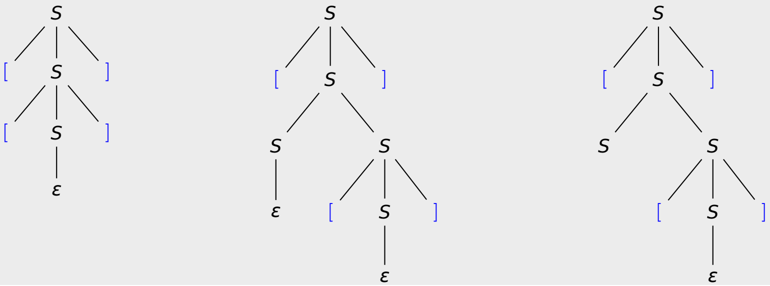

Example

CFG G: $S \to [S] \mid SS \mid \varepsilon$

1. $S \xrightarrow[G]{1} [S] \xrightarrow[G]{1} [[S]] \xrightarrow[G]{1} [[]]$
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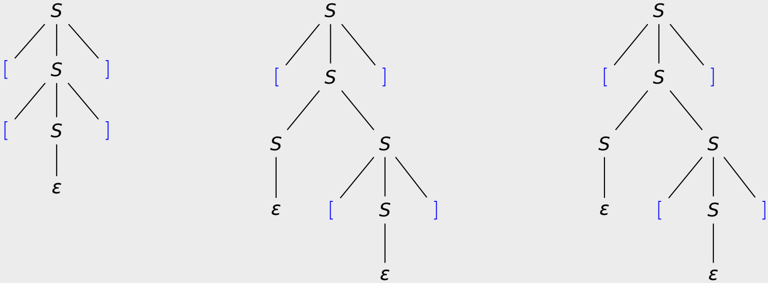

Example

- CFG G : $S \rightarrow [S] \mid S\,S \mid \varepsilon$

- G is ambiguous

Example

- CFG G : $S \rightarrow [S] \mid S\,S \mid \varepsilon$
- CFG G': $S \rightarrow \varepsilon \mid T$

  $T \rightarrow T\,U \mid U$

  $U \rightarrow [\,] \mid [T]$
- G is ambiguous

Example

- CFG G : $S \rightarrow [S] \mid S\,S \mid \varepsilon$
- CFG G': $S \rightarrow \varepsilon \mid T$
  $T \rightarrow T\,U \mid U$
  $U \rightarrow [\,] \mid [T]$
- G is ambiguous G' is unambiguous $L(G) = L(G')$

Example

- CFG G : $\quad S \to [S] \mid S\,S \mid \varepsilon$
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$$\begin{array}{c} S \\ | \\ T \\ | \\ U \\ | \\ [T] \\ | \\ [U] \\ | \\ [[\,]] \end{array}$$

Example

- CFG G : $S \rightarrow S - S \mid int$
- G is ambiguous

  with G $7-5-2$ could be parsed as $(7-5)-2$ and $7-(5-2)$

Example

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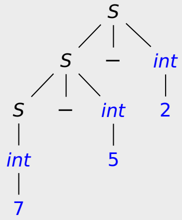
with G $7 - 5 - 2$ could be parsed as $(7 - 5) - 2$ and $7 - (5 - 2)$

Example

- CFG G : $S \rightarrow S - S \mid \textcolor{blue}{int}$
- CFG G': $S \rightarrow S - \textcolor{blue}{int} \mid \textcolor{blue}{int}$
- G is ambiguous $\quad G'$ is unambiguous $\quad L(G) = L(G')$

with G $\quad 7 - 5 - 2$ could be parsed as $\quad (7-5)-2$ and $7-(5-2)$
with G' $\quad 7 - 5 - 2$ could only be parsed as $\quad (7-5)-2$



Example

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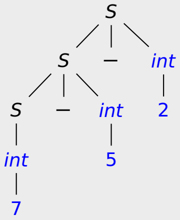
- (if applicable) one way to remove ambiguity is to benefit from associativity of binary operators



Example

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- G is ambiguous G' is unambiguous $L(G) = L(G')$

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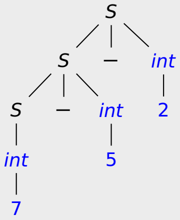


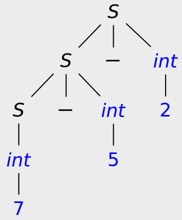
Example

- CFG G : $S \to S - S \mid int$
- CFG G': $S \to S - int \mid int$
- G is ambiguous G' is unambiguous $L(G) = L(G')$

  with G $7-5-2$ could be parsed as $(7-5)-2$ and $7-(5-2)$
  with G' $7-5-2$ could only be parsed as $(7-5)-2$

- (if applicable) one way to remove ambiguity is to benefit from associativity of binary operators
- G' enforces the "-" operator to be left associative
- CFG G'': $S \to int - S \mid int$

  turns the "-" operator into a right associative one



Example

- CFG G : $S \rightarrow S \times S \mid S + S \mid \mathit{int}$

- G is ambiguous

  with G $7 + 5 \times 2$ could be parsed as $7 + (5 \times 2)$ and $(7 + 5) \times 2$

Example

- CFG G : $S \to S \times S \mid S + S \mid int$
- CFG G': $S \to S + T \mid T$
  $T \to T \times U \mid U$
  $U \to int \mid (S)$
- G is ambiguous

  with G $7 + 5 \times 2$ could be parsed as $7 + (5 \times 2)$ and $(7 + 5) \times 2$

Example

- CFG G : $S \rightarrow S \times S \mid S + S \mid int$
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- G is ambiguous G' is unambiguous $L(G) = L(G')$

  with G $7 + 5 \times 2$ could be parsed as $7 + (5 \times 2)$ and $(7 + 5) \times 2$
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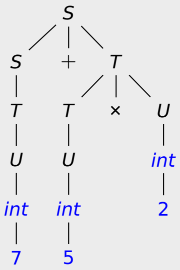

Example

- CFG G : $S \to S \times S \mid S + S \mid int$
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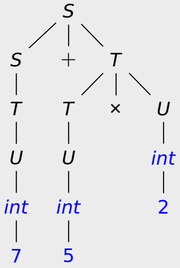
- (if applicable) one way to remove ambiguity is to benefit from precedence of operators



Parse Trees vs Abstract Syntax Trees

- parse trees (PT) are concrete syntax trees containing all derivation details

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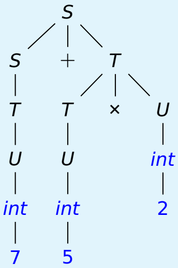
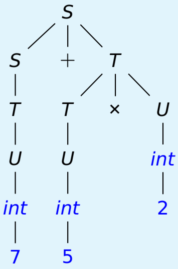


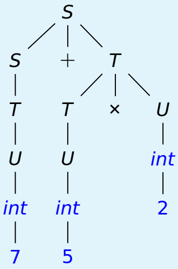
Parse Trees vs Abstract Syntax Trees

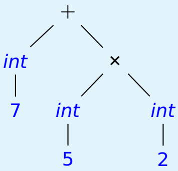
- parse trees (PT) are concrete syntax trees containing all derivation details
- abstract syntax trees (AST) are reduced PTs eliminating irrelevant information for the evaluation step



Parse Trees vs Abstract Syntax Trees

- parse trees (PT) are concrete syntax trees containing all derivation details
- abstract syntax trees (AST) are reduced PTs eliminating irrelevant information for the evaluation step





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there exist context-free sets without unambiguous grammars

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$$S \to T \mid W$$

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Lemma

there is no CFG G' such that $L(G')$ is unambiguous with $L(G') = A$

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1. given an ambiguous CFG G, the language $L(G)$ may or may not be ambiguous

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   - one can find an unambiguous CFG G' such that $L(G') = L(G)$
2. there is no algorithm to convert ambiguous CFG to unambiguous CFG
3. unambiguous context free languages can be parsed by deterministic push down automata

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- next configuration relation is binary relation $\xrightarrow[M]{1}$ defined as: $(p, ay, A\beta) \xrightarrow[M]{1} (q, y, \gamma\beta)$
  for all $((p, a, A), (q, \gamma)) \in \delta$ with $a \in \Sigma \cup \{\varepsilon\}$ and $y \in \Sigma^*$, $\beta \in \Gamma^*$

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$A = \{x \in \{[,]\}^* \mid x \text{ is balanced}\}$ is accepted by NPDA $M = (Q, \Sigma, \Gamma, \delta, 1, \bot, F)$ with

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input: [[ ] [[ ]]]
state: 1
stack: $\bot$

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input:		[	[	]	[	[	]	]	]
state:	1	1							
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input:		[		[		]		[		[		]		]		]	
state:	1		1		1		1		1								
stack:	$\perp$		$\perp$		$\perp$		$\perp$		$\perp$								
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input:	[	[	]	[	[	]	]	]
state:	1	1	1	1	1	1		
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input: $[\ [\ ]\ [\ [\ ]\ ]\ ]$

state: 1 1 1 1 1 1

stack:

$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
	[	[	[	[	[
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stack:	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	
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stack:	$\perp$		$\perp$		$\perp$		$\perp$		$\perp$		$\perp$		$\perp$		
			[		[		[		[		[		[		
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								]
state:	1	1	1	1	1	1	1	1
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state:	1	1	1	1	1	1	1	1	1
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input:	[	[	]	[	[	]	]	]	ε
state:	1	1	1	1	1	1	1	1	1
stack:	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
		[	[	[	[	[	[	[	
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input:		[	[	]	[	[	]	]	]	ε
state:	1	1	1	1	1	1	1	1	1	2
stack:	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	
		[	[	[	[	[	[	[		
			[		[	[	[			
						[				

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$A = \{x \in \{[,]\}^* \mid x \text{ is balanced}\}$ is accepted by NPDA $M = (Q, \Sigma, \Gamma, \delta, 1, \perp, F)$ with

1. $Q = \{1, 2\}$
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input:	[	[	]	[	[	]	]	]	ε		[[ ]]]
state:	1	1	1	1	1	1	1	1	1	2	
stack:	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$		
		[	[	[	[	[	[	[			
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stack:	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$			$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
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Theorem

CFGs and NPDAs are equivalent:

1. $A = L(G)$ for some CFG G $\iff$
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Determinism in PDAs (Informally)

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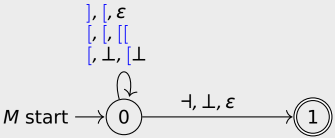
Definition

A deterministic pushdown automaton (DPDA) is an octuple $M = (Q, \Sigma, \Gamma, \delta, \bot, \dashv, s, F)$

1. $\dashv$ is a special symbol not in Σ, called the right end-marker
2. for any $p \in Q$, $a \in \Sigma \cup \{\varepsilon\}$, $A \in \Gamma$, the set $\delta \subseteq (Q \times (\Sigma \cup \{\dashv\} \cup \{\varepsilon\}) \times \Gamma) \times (Q \times \Gamma^*)$ contains
   - at most one element of the form $((p, a, A), (q, \beta))$
   - exactly one transition of the form $((p, a, A), (q, \beta))$ or $((p, \varepsilon, A), (q, \beta))$

Example

$A = \{x \in \{[,]\}^* \mid x \text{ is balanced}\}$ is accepted by DPDA $M = \big(\{0, 1\}, \{[,]\}, \{[, \bot\}, \delta, \bot, \dashv, 0, \{1\}\big)$ with



the final state acceptance criterion.

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  - *menhir* for OCaml

menhir

- generates parsers in compatible with OCaml programs

  input : list of *parsing rules* in the form of *context free grammars* with corresponding *actions*

  output : a parser as a DPDA accepting the language generated by the input grammar

- recognizes patterns specified by the rules, associates the corresponding actions

menhir (Specifications)

```
%{
        header
%}

%token tkn1
%left  tkn2
%start <ocaml-type> S
%%

S:
  | ...          { action }
  | production1  { action };
T:
  | production2  { action };
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left	defines the `tkn1` left associative
S, T, etc	CFG non-terminals such that *S* is the start symbol
production	the CFG production rules
actions	are OCaml expressions of the same type
%%	the obligatory sign separates the header from the production rules

Example (Balanced Parentheses – Abstract Syntax Tree)

```
(* ast.ml *)
open Printf

type bp =
  | Join   : (bp*bp) → bp
  | Single: bp       → bp
  | Const : bp
  | Str    : string  → bp
  | Int    : int     → bp

let rec bp2String (b: bp): string =
  match b with
  | Join(b1, b2) → bp2String b1 ^ "␣" ^ bp2String b2
  | Single b1    → "LPAREN␣" ^ bp2String b1 ^ "␣RPAREN"
  | Const        → "LPAREN␣RPAREN"
  | Str s        → "IDENT=[" ^ s ^ "]"
  | Int i        → "NUM=[" ^ (string_of_int i) ^ "]"

let printBp (b: bp): unit =
  printf "%s\n" (bp2String b)
```

Example (menhir: Balanced Parentheses – Header and Tokens)

```
(* parser.mly *)
%{
    open Ast
%}

%token BLANK LPAREN RPAREN EOL
%token <string> IDENT
%token <int> NUM
%start <bp> start

%%
```

Example (menhir: Balanced Parentheses – Production Rules)

```
(* parser.mly *)
start:
  | a = T; EOL                    { a };

T:
  | a = T; b = U                  { Join(a,b) }
  | b = U                         { b };

U:
  | LPAREN; RPAREN                { Const }
  | LPAREN; a = T; RPAREN         { Single a }
  | s = IDENT                     { Str s }
  | i = NUM                       { Int i }
  | BLANK                         { Str "" }
```

Example (menhir: Balanced Parentheses – Main)

```
(* main.ml *)
open Printf
open Lexing
open Lexer
open Ast
open Parser

let tokenSwitch (t: Lexer.token): Parser.token =
  match t with
  | BLANK    → BLANK
  | LPAREN   → LPAREN
  | RPAREN   → RPAREN
  | EOL      → EOL
  | IDENT s  → IDENT s
  | NUM i    → NUM i

let compose (f: α → β) (g: β → γ): α → γ = fun x → g (f x)
let astOfString (s: string): bp = start (compose tokenize tokenSwitch) (from_string s)
let main: unit = let ast = astOfString "[][][[[[]]]][ekici2023]" in printBp ast;
```

Remark (in "parser.mly")

where

start: (Lexing.lexbuf → Parser.token) → bp   function maps an input stream to parser tokens

Thanks! & Questions?