轨迹优化-闭式求解

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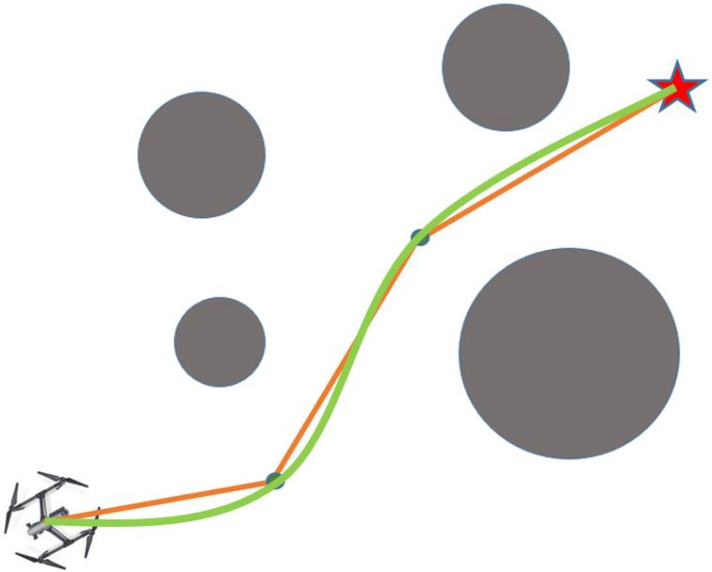

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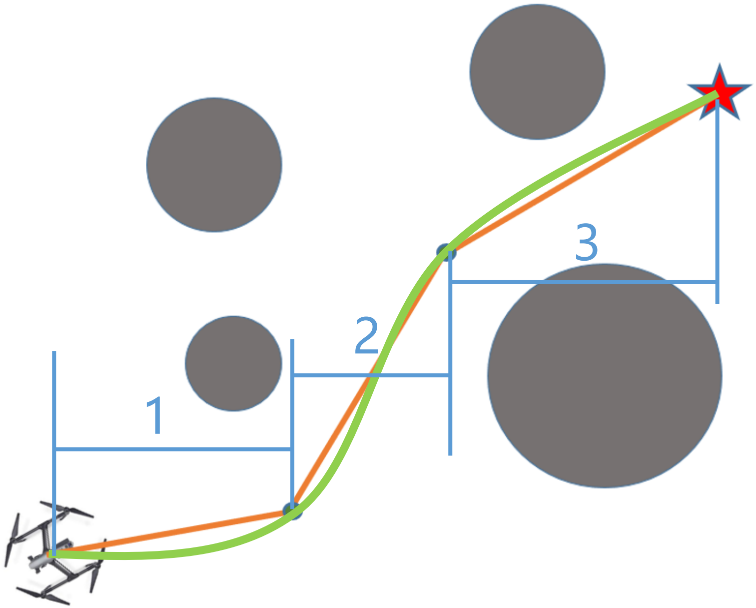
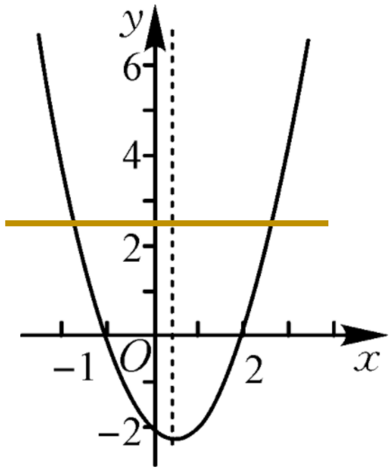
PART

闭式求解

闭式求解

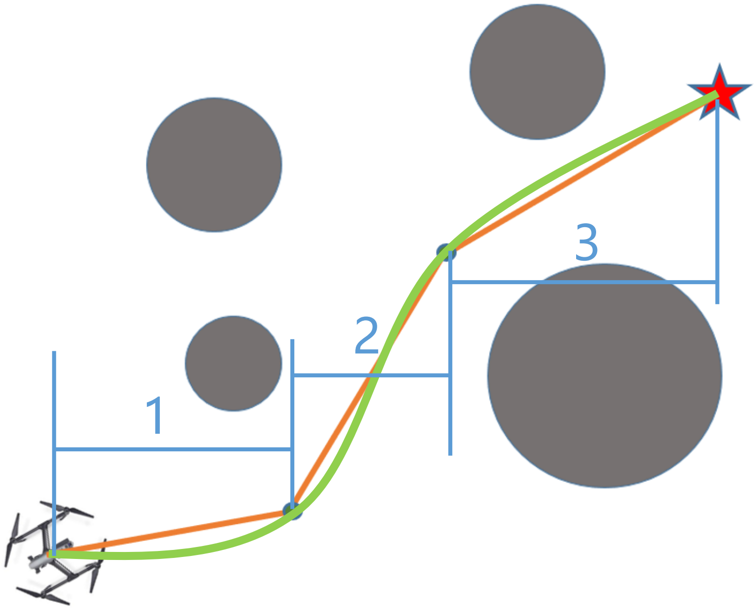


$$\min p^T Q p$$
$$s.t. \quad A_{eq} p = b_{eq},$$



Richter C, Bry A, Roy N. Polynomial trajectory planning for aggressive quadrotor flight in dense indoor environments

闭式求解



$$\min p^T Q p$$

$$\boxed{s.t.\quad A_{eq} p = b_{eq},}$$

$$A_i p_i = d_i,\ A_i = [A_0\ A_t]_i^T,\ d_i = [d_0, d_T]_i$$

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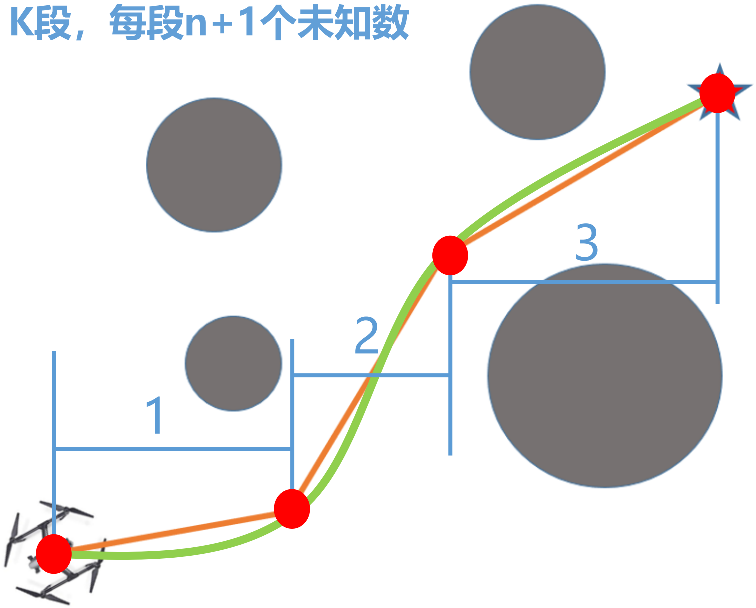
闭式求解

K段，每段n+1个未知数



$$\underbrace{A_{total}}_{k(n+1)\times 6k}\begin{bmatrix} p_1 \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} d_1 \\ \vdots \\ d_k \end{bmatrix} = \underbrace{\begin{bmatrix} p_1(t_0) \\ v_1(t_0) \\ a_1(t_0) \\ p_1(t_1) \\ v_1(t_1) \\ a_1(t_1) \\ \vdots \\ p_k(t_{k-1}) \\ v_k(t_{k-1}) \\ a_k(t_{k-1}) \\ p_k(t_k) \\ v_k(t_k) \\ a_k(t_k) \end{bmatrix}}_{6k\times 1}$$

$$p = A^{-1}\boxed{d}$$

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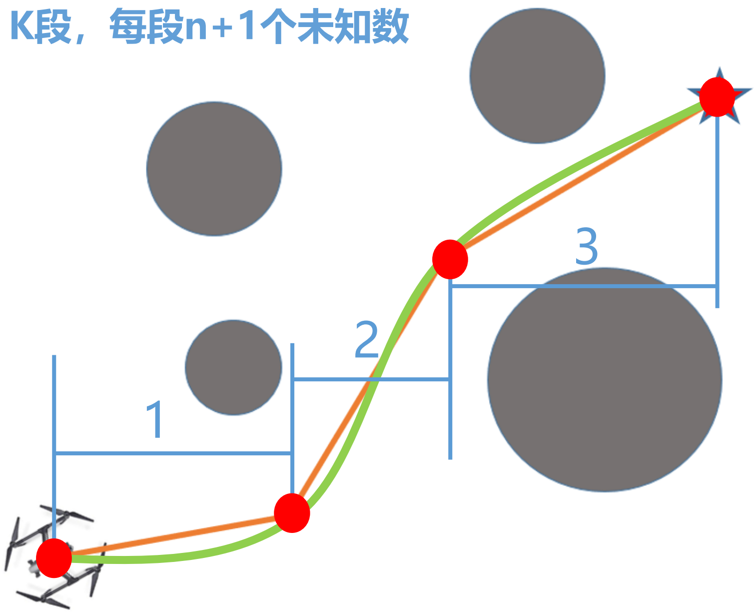
连续性约束



$$\underbrace{A_{total}}_{k(n+1)\times 6k}\begin{bmatrix} p_1 \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} d_1 \\ \vdots \\ d_k \end{bmatrix} = \underbrace{\begin{bmatrix} p_1(t_0) \\ v_1(t_0) \\ a_1(t_0) \\ p_1(t_1) \\ v_1(t_1) \\ a_1(t_1) \\ \vdots \\ p_k(t_{k-1}) \\ v_k(t_{k-1}) \\ a_k(t_{k-1}) \\ p_k(t_k) \\ v_k(t_k) \\ a_k(t_k) \end{bmatrix}}_{6k\times 1}$$

$$p_i(t_i) = p_{i+1}(t_i),\ \ v_i(t_i) = v_{i+1}(t_i),\ \ a_i(t_i) = a_{i+1}(t_i)$$

消除重复变量

$$\begin{bmatrix} a \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} a$$

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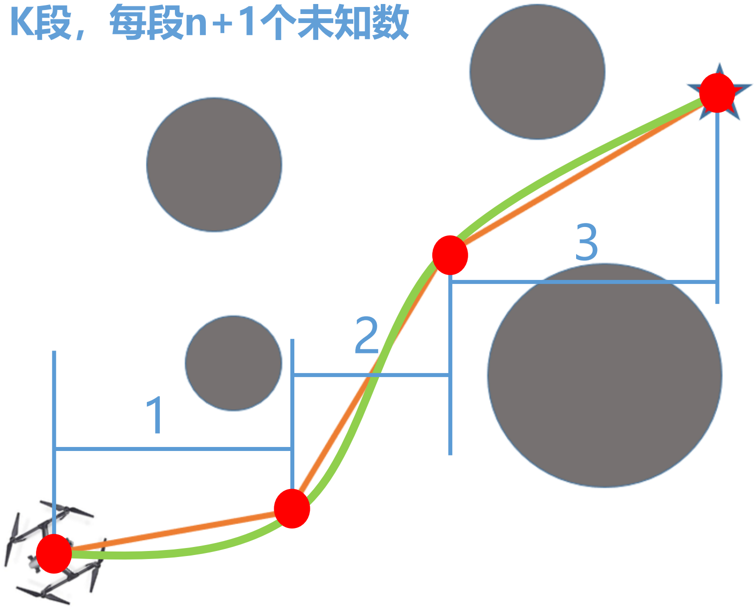
连续性约束

消除重复变量

K段，每段n+1个未知数



$$\underbrace{\begin{bmatrix} d_1 \\ \vdots \\ d_k \end{bmatrix}}_{6k\times 1} = \underbrace{\begin{bmatrix} 1 &&&&&&&&& \\ & 1 &&&&&&&& \\ && 1 &&&&&&& \\ &&& 1 &&&&&& \\ &&&& 1 &&&&& \\ &&&&& 1 &&&& \\ &&& 1 &&&&&& \\ &&&& 1 &&&&& \\ &&&&& 1 &&&& \\ &&&&&& 1 &&& \\ &&&&&&& 1 && \\ &&&&&&&& 1 & \\ &&&&&&&&& \ddots \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} p(t_0) \\ v(t_0) \\ a(t_0) \\ p(t_1) \\ v(t_1) \\ a(t_1) \\ p(t_2) \\ v(t_2) \\ a(t_2) \\ \vdots \\ p(t_k) \\ v(t_k) \\ a(t_k) \end{bmatrix}}_{3(k+1)\times 1}$$

$$d = Md'$$

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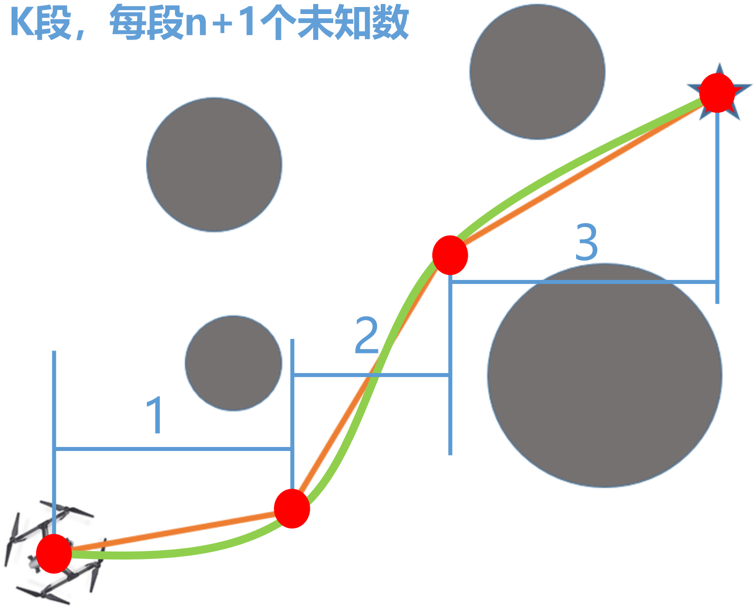
元素位置置换

K段，每段n+1个未知数



$$d' = C \begin{bmatrix} d_F \\ d_P \end{bmatrix}$$

例子：

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} a \\ c \\ d \\ b \end{bmatrix}$$

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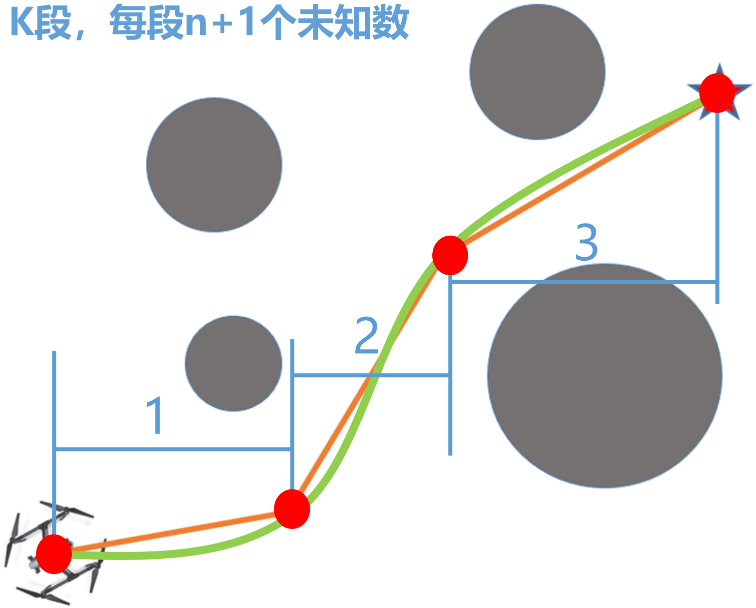
无约束优化问题

K段，每段n+1个未知数



$$d = MC \begin{bmatrix} d_F \\ d_P \end{bmatrix}$$

$$p = A^{-1}d = \underbrace{A^{-1}MC}_{K} \begin{bmatrix} d_F \\ d_P \end{bmatrix} = K \begin{bmatrix} d_F \\ d_P \end{bmatrix}$$

$$\min J = p^T Q p$$

$$J = \begin{bmatrix} d_F \\ d_P \end{bmatrix}^T \underbrace{K^T Q K}_{R} \begin{bmatrix} d_F \\ d_P \end{bmatrix}$$

$$= \begin{bmatrix} d_F \\ d_P \end{bmatrix}^T \begin{bmatrix} R_{FF} & R_{FP} \\ R_{PF} & R_{PP} \end{bmatrix} \begin{bmatrix} d_F \\ d_P \end{bmatrix}$$

$$= d_F^T R_{FF} d_F + d_F^T R_{FP} d_P + d_P^T R_{PF} d_F + d_P^T R_{PP} d_P$$

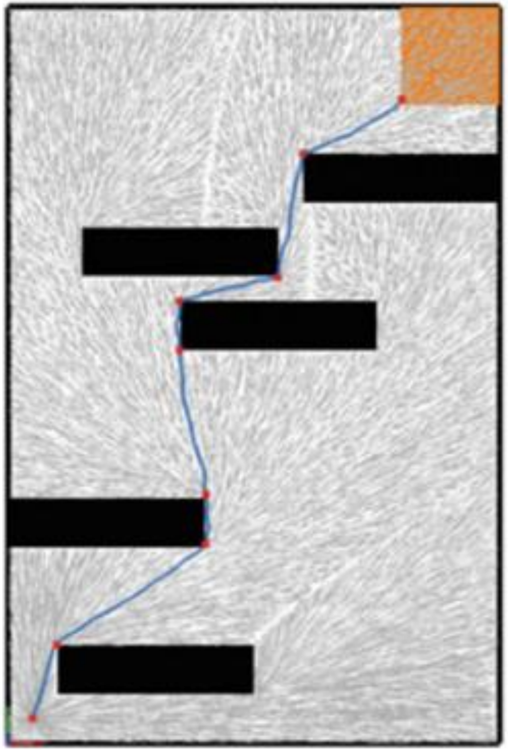
$$Q\text{对称} \Rightarrow R\text{对称} \Rightarrow = d_F^T R_{FF} d_F + 2 d_F^T R_{FP} d_P + d_P^T R_{PP} d_P$$

$$\Rightarrow 2R_{FP}^T d_F + 2R_{PP} d_P = 0 \quad (\text{注意} R_{PP}^T = R_{PP})$$

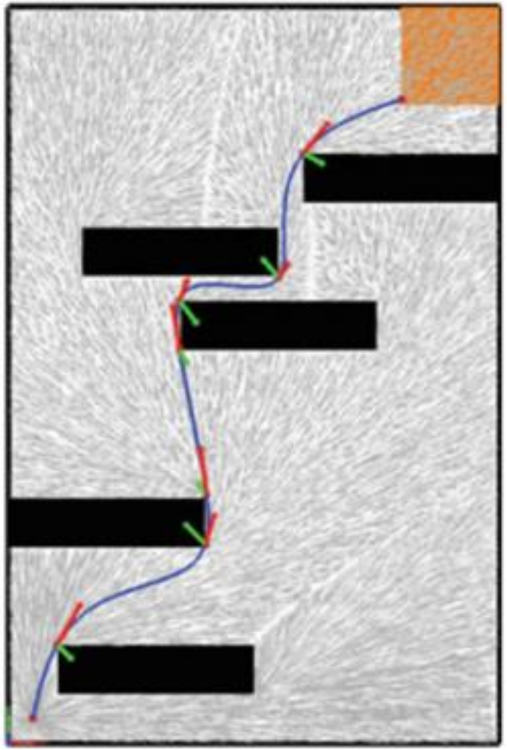
$$\Rightarrow d_p = -R_{PP}^{-1} R_{FP}^T d_F$$

$$p = K \begin{bmatrix} d_F \\ d_P \end{bmatrix}$$

闭式求解

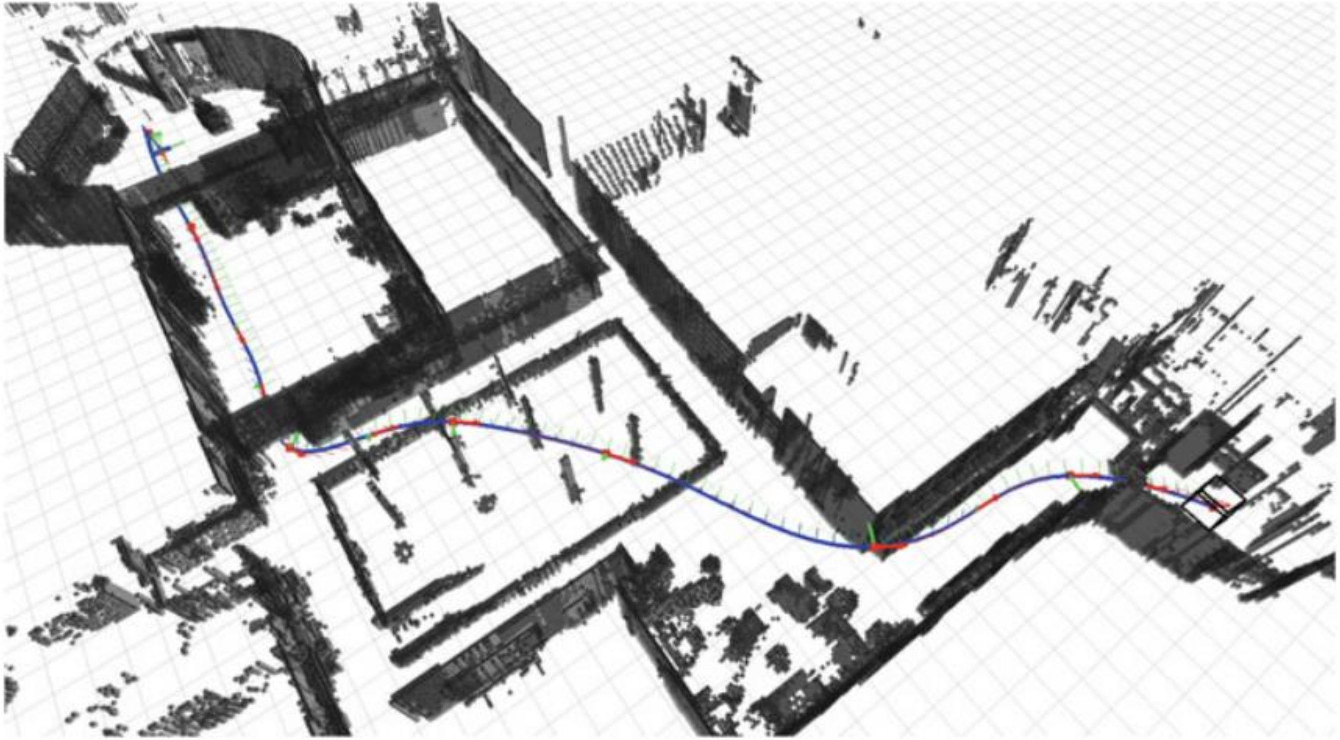


Pruned waypoints from straight-line RRT* become waypoints in 6c.

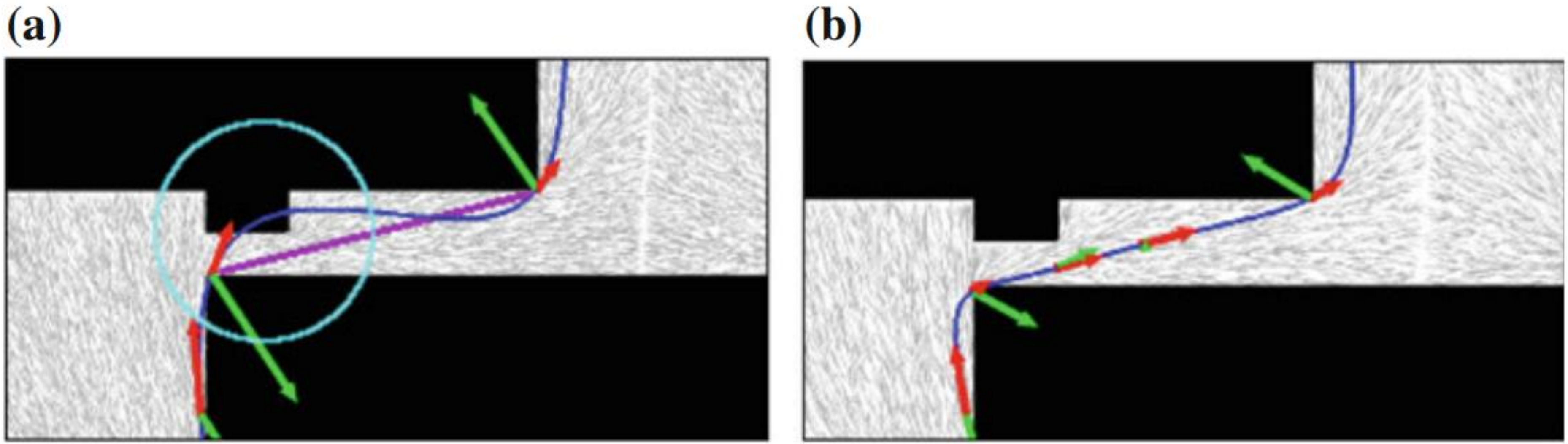


Solution by our algorithm after 3s running time, finds much lower cost than 6a.

闭式求解



闭式求解



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PART

闭式求解代码讲解

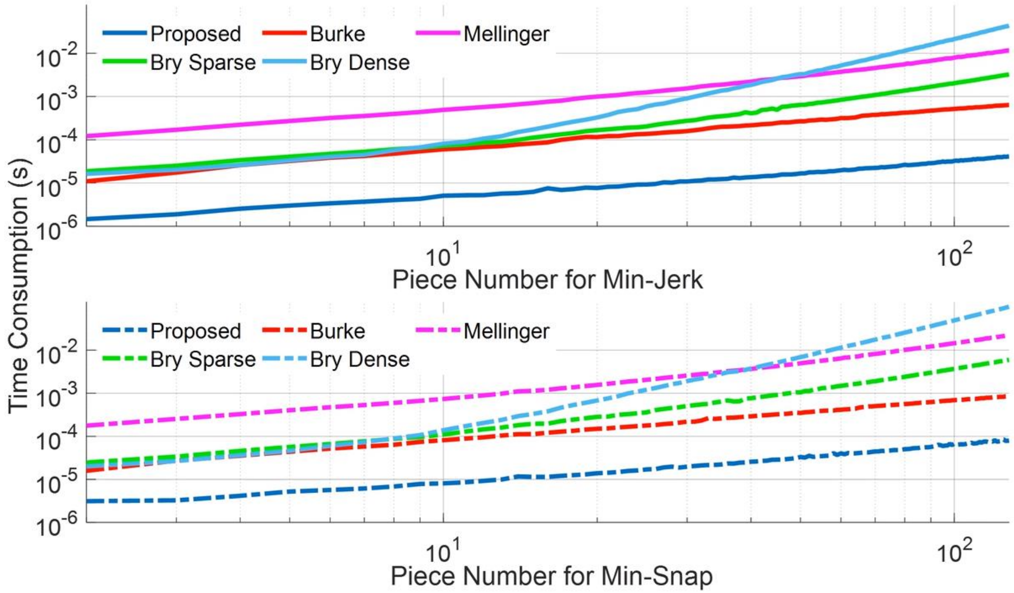
闭式求解

总结一下整个闭式法的步骤：

1. 先确定轨迹阶数（比如5阶），再确定d向量中的约束量（pva），进而根据各段的时间分配求得A_{total}。
2. 根据连续性约束构造映射矩阵M，并确定d向量中哪些量是Fix(比如起点终点pva，中间点的p等)，哪些量是Free，进而构造置换矩阵C，并求得$K = A^{-1}MC$。
3. 计算QP目标函数中的Q（$\min Jerk/Snap$）并计算$R = K^TQK$，根据fix变量的长度将R拆分成$R_{FF}, R_{FP}, R_{PF}, R_{PP}$四块。
4. 填入已知变量得到d_F，并根据$d_p = -R_{PP}^{-1}R_{FP}^T d_F$计算得到d_P。
5. 根据公式 $p = K \begin{bmatrix} d_F \\ d_P \end{bmatrix}$ 计算得到轨迹参数p。

闭式求解

Speeding Up Traditional Min-Jerk/Snap Trajectory Generation



D. Mellinger and V. Kumar, "Minimum snap trajectory generation and control for quadrotors," ICRA 2011.
A. Bry, C. Richter, A. Bachrach, and N. Roy, "Aggressive flight of fixed-wing and quadrotor aircraft in dense indoor environments," IJRR 2015.
D. Burke, A. Chapman, and I. Shames, "Generating minimum-snap quadrotor trajectories really fast," ArXiv, vol. abs/2008.00595, IROS 2020.

[Generating Large-Scale Trajectories Efficiently using Double Description]

THANKS

线性代数-求导

$$\frac{dx^T}{dx} = I \qquad \frac{dx}{dx^T} = I \qquad \frac{dx^T A}{dx} = A$$

A为$n \times n$的矩阵， x为$n \times 1$的列向量

$$\frac{dAx}{dx^T} = A \qquad \boxed{\frac{dAx}{dx} = A^T} \qquad \frac{dxA}{dx} = A^T$$

$$\frac{\partial u}{\partial x^T} = \left(\frac{\partial u^T}{\partial x}\right)^T \qquad \frac{\partial u^T v}{\partial x} = \frac{\partial u^T}{\partial x} v + \frac{\partial v^T}{\partial x} u^T \qquad \frac{\partial u v^T}{\partial x} = \frac{\partial u}{\partial x} v^T + u \frac{\partial v^T}{\partial x}$$

$$\frac{dx^T x}{dx} = 2x \qquad \frac{dx^T Ax}{dx} = (A + A^T)x$$

$$\frac{dX^T AX}{dX} = 2AX$$ （如果A为对称阵）

$$\frac{\partial AB}{\partial x} = \frac{\partial A}{\partial x} B + A \frac{\partial B}{\partial x} \qquad \frac{\partial u^T X v}{\partial X} = u v^T$$

$$\frac{\partial u^T X^T X u}{\partial X} = 2Xuu^T \qquad \frac{\partial [(Xu - v)^T (Xu - v)]}{\partial X} = 2(Xu - v)u^T$$