轨迹优化

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公众号：Joe学习笔记

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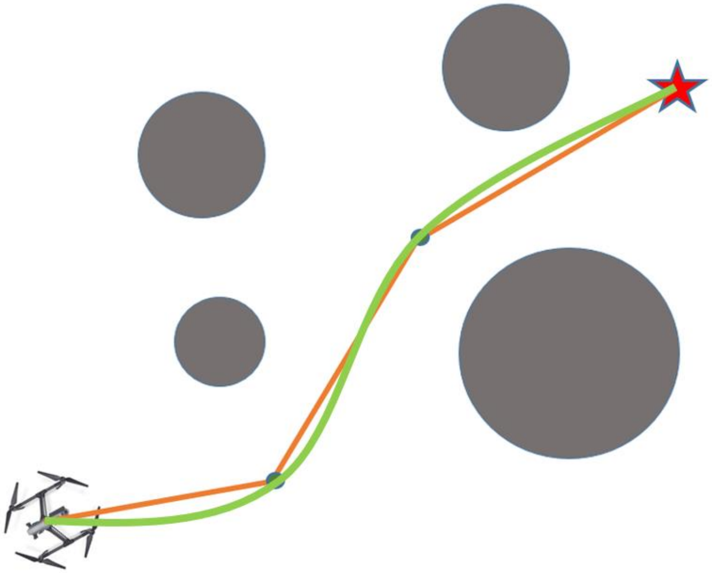

联系方式

关注公众号：Joe学习笔记，获取PPT和代码

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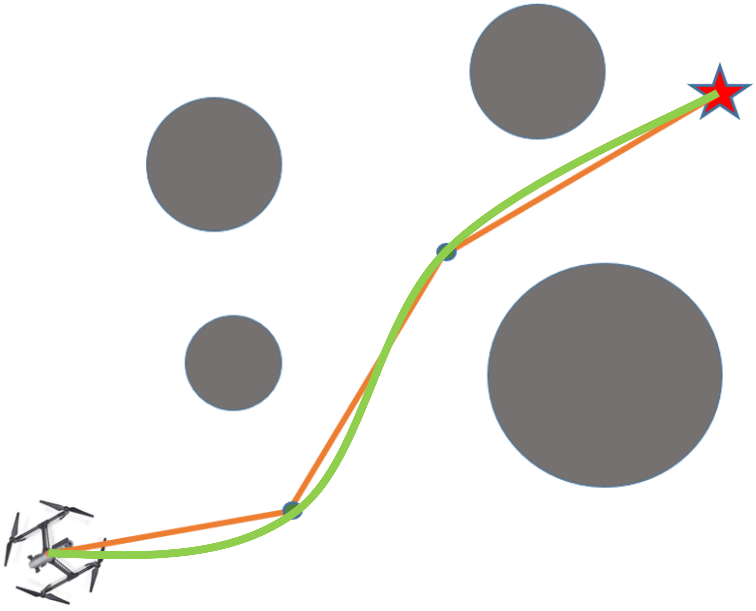
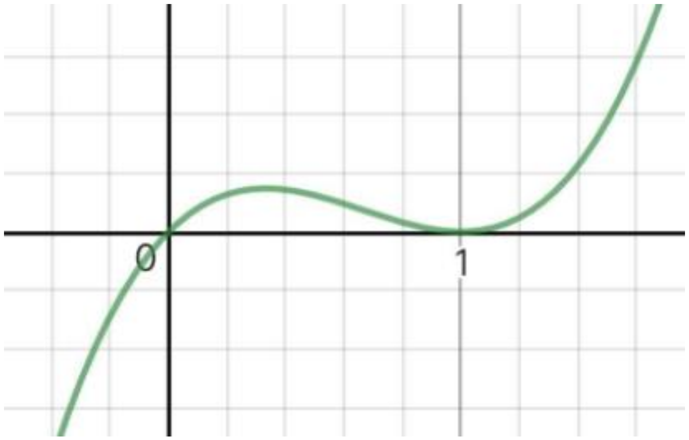
绪论



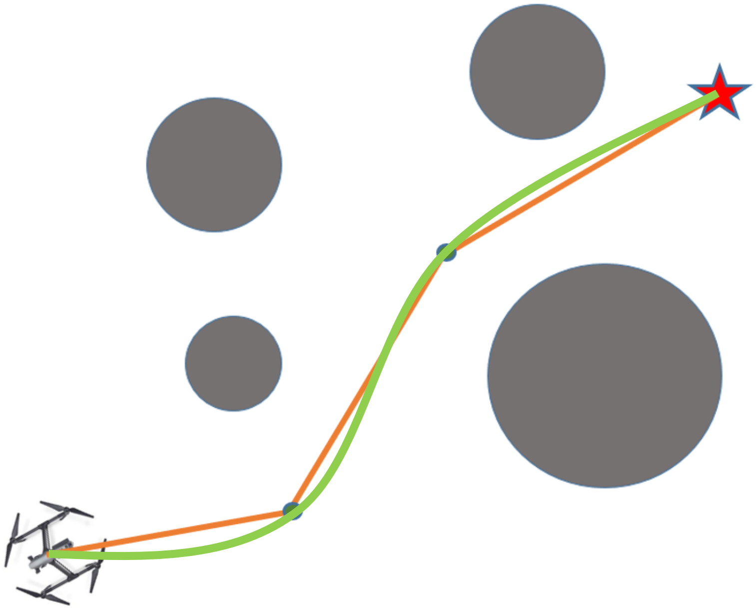
路径规划 + 轨迹优化

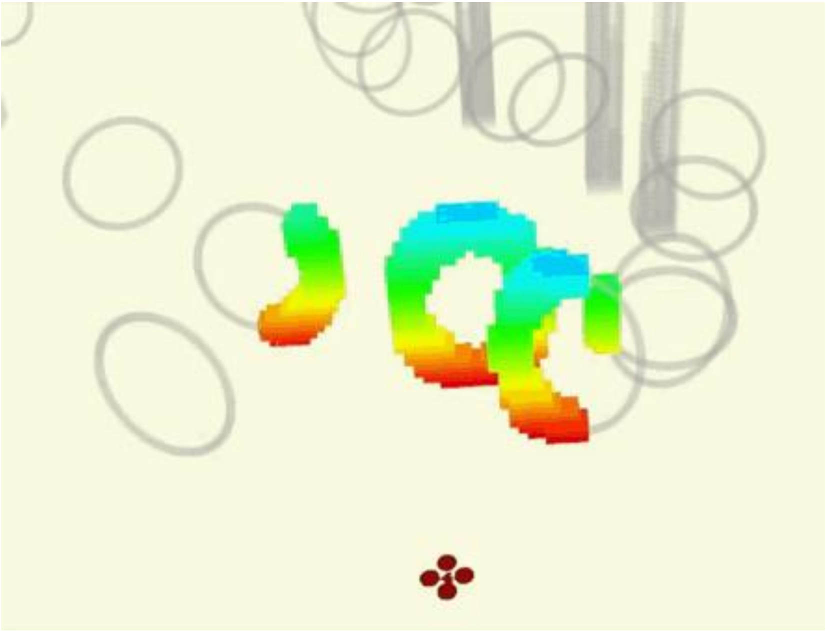
轨迹: 带时间参数的曲线



$$t^3 - 2t^2 + t$$

绪论





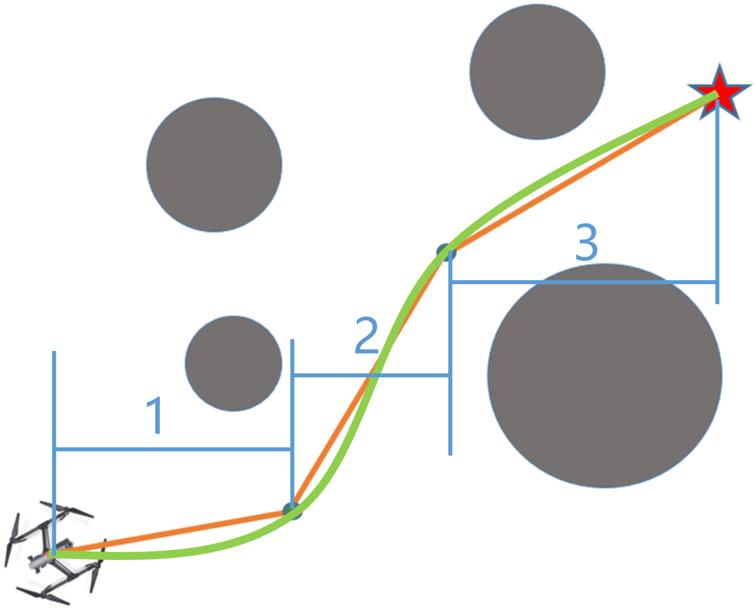
路径规划 + 轨迹优化

01

PART

多项式轨迹

多项式轨迹



时间t根据路径长度和平均速度分配

$$p(t) = p_0 + p_1 t + p_2 t^2 \ldots + p_n t^n = \sum_{i=0}^{n} p_i t^i$$

$$p(t) = [1, t, t^2, \ldots, t^n] \cdot p$$

$$p = [p_0, p_1, \ldots, p_n]^T$$

$$v(t) = p'(t) = [0, 1, 2t, 3t^2, 4t^3, \ldots, nt^{n-1}] \cdot p$$

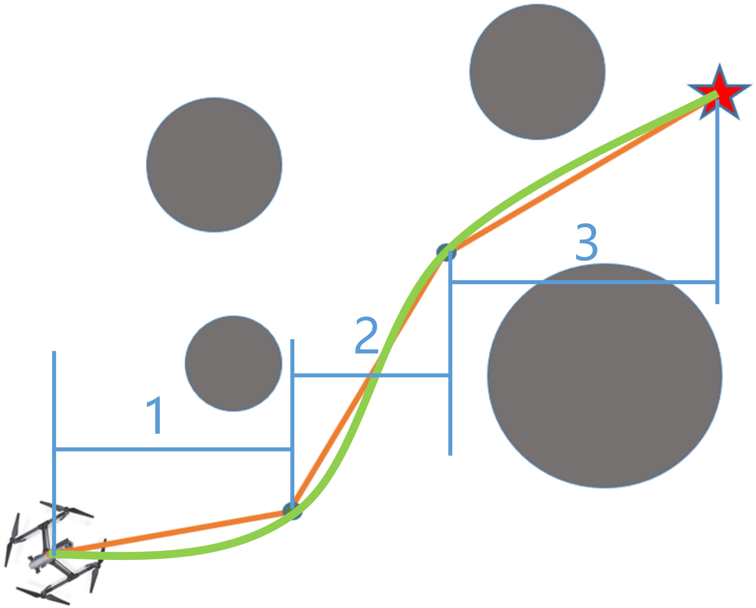
$$a(t) = p''(t) = [0, 0, 2, 6t, 12t^2, \ldots, n(n-1)t^{n-2}] \cdot p$$

$$jerk(t) = p^{(3)}(t) = [0, 0, 0, 6, 24t, \ldots, \frac{n!}{(n-3)!} t^{n-3}] \cdot p$$

$$snap(t) = p^{(4)}(t) = [0, 0, 0, 0, 24, \ldots, \frac{n!}{(n-4)!} t^{n-4}] \cdot p$$

https://blog.csdn.net/q597967420/article/details/76099491

多项式轨迹



时间t根据路径长度和平均速度分配

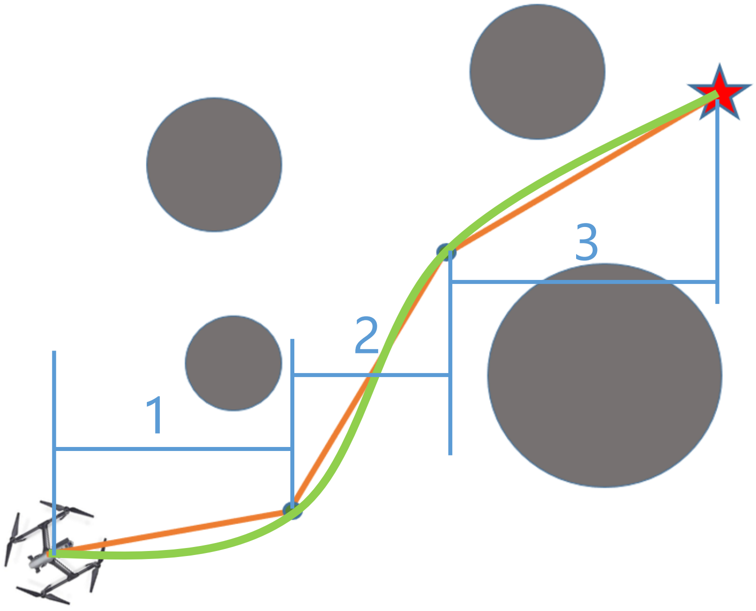
$$p(t) = \begin{cases} [1, t, t^2, \ldots, t^n] \cdot p_1 & t_0 \le t < t_1 \\ [1, t, t^2, \ldots, t^n] \cdot p_2 & t_1 \le t < t_2 \\ \ldots & \\ [1, t, t^2, \ldots, t^n] \cdot p_k & t_{k-1} \le t < t_k \end{cases}$$

$$p_i = [p_{i_0}, p_{i_1}, \ldots, p_{i_n}]^T$$

$$p = [p_1^T, p_2^T, \ldots, p_k^T]^T$$

求解参数p，确定轨迹

Minimum Snap



基本要求：

1. 两段轨迹之间连续
2. 轨迹经过固定点
3. 轨迹无碰撞

高级要求：

轨迹最顺滑、能量最优等

$$\min f(p)$$

$$s.t.\quad A_{eq}p = b_{eq},$$

$$A_{ieq}p \le b_{ieq}$$

https://blog.csdn.net/q597967420/article/details/76099491

Minimum Snap

$$\boxed{minimum\ snap: \quad \min f(p) = \min(p^{(4)}(t))^2}$$

$$minimum\ jerk: \quad \min f(p) = \min(p^{(3)}(t))^2$$

$$minimum\ acce: \quad \min f(p) = \min(p^{(2)}(t))^2$$

Minimum Snap Trajectory Generation and Control for Quadrotors (2011 ICRA)

Minimum Snap

！！索引从0开始

$$Q_i = \int_{t_{i-1}}^{t_i} [0,0,0,0,24,\ldots,\frac{n!}{(n-4)!}t^{n-4}]^T [0,0,0,0,24,\ldots,\frac{n!}{(n-4)!}t^{n-4}]\,\mathrm{d}t$$

$$= \begin{bmatrix} 0_{4\times 4} & 0_{4\times(n-3)} \\ 0_{(n-3)\times 4} & \frac{r!}{(r-4)!}\frac{c!}{(c-4)!}\frac{1}{(r-4)+(c-4)+1}\left(t_i^{(r+c-7)} - t_{i-1}^{(r+c-7)}\right) \end{bmatrix}_{(n+1)*(n+1)}$$

$$\min \int_0^T (p^{(4)}(t))^2\,\mathrm{d}t$$

$$= \min \sum_{i=1}^{k} \int_{t_{i-1}}^{t_i} (p^{(4)}(t))^2\,\mathrm{d}t$$

$$= \min \sum_{i=1}^{k} \int_{t_{i-1}}^{t_i} \left([0,0,0,0,24,\ldots,\frac{n!}{(n-4)!}t^{n-4}]\cdot p\right)^T [0,0,0,0,24,\ldots,\frac{n!}{(n-4)!}t^{n-4}]\cdot p\,\mathrm{d}t$$

$$= \min \sum_{i=1}^{k} p^T \boxed{\int_{t_{i-1}}^{t_i} [0,0,0,0,24,\ldots,\frac{n!}{(n-4)!}t^{n-4}]^T [0,0,0,0,24,\ldots,\frac{n!}{(n-4)!}t^{n-4}]\,\mathrm{d}t}\, p$$

$$= \min \sum_{i=1}^{k} p^T Q_i p$$

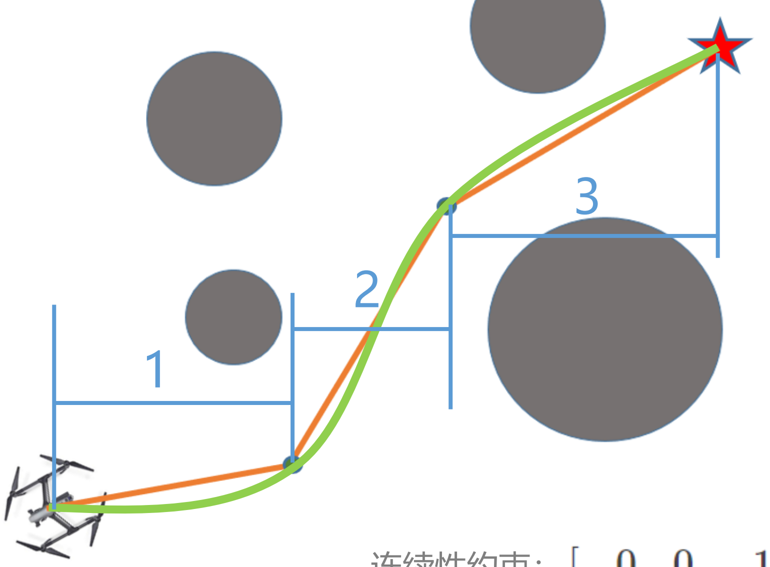
$$Q = \begin{bmatrix} Q_1 & & & \\ & Q_2 & & \\ & & \ddots & \\ & & & Q_k \end{bmatrix}$$

$$\min p^T Q p$$

Minimum Snap Trajectory Generation and Control for Quadrotors (2011 ICRA)

Minimum Snap

K段，每段n+1个未知数



$$\boxed{\min f(p)}$$

$$s.t.\quad A_{eq}p = b_{eq},$$

$$A_{ieq}p \le b_{ieq}$$

位置约束：$[1, t_0, t_0^2, \ldots, t_0^n, \underbrace{0 \ldots 0}_{(k-1)(n+1)}]p = p_0$

速度约束：$[0, 1, 2t_0, \ldots, nt_0^{n-1}, \underbrace{0 \ldots 0}_{(k-1)(n+1)}]p = v_0$

加速度约束：$[0, 0, 2, \ldots, n(n-1)t_0^{n-2}, \underbrace{0 \ldots 0}_{(k-1)(n+1)}]p = a_0$

连续性约束：$[\underbrace{0 \ldots 0}_{(i-1)(n+1)}, 1, t_i, t_i^2, \ldots, t_i^n, -1, -t_i, -t_i^2, \ldots, -t_i^n, \underbrace{0 \ldots 0}_{(k-i-1)(n+1)}]p = 0$

$$
\begin{bmatrix}
1, t_0, t_0^2, \ldots, t_0^n, \underbrace{0 \ldots 0}_{(k-1)(n+1)} \\
0, 1, 2t_0, \ldots, nt_0^{n-1}, \underbrace{0 \ldots 0}_{(k-1)(n+1)} \\
0, 0, 2, \ldots, n(n-1)t_0^{n-2}, \underbrace{0 \ldots 0}_{(k-1)(n+1)} \\
\vdots \\
\underbrace{0 \ldots 0}_{(i-1)(n+1)}, 1, t_i, t_i^2, \ldots, t_i^n, \underbrace{0 \ldots 0}_{(k-i)(n+1)} \\
\vdots \\
\underbrace{0 \ldots 0}_{(k-1)(n+1)}, 1, t_k, t_k^2, \ldots, t_k^n \\
\underbrace{0 \ldots 0}_{(k-1)(n+1)}, 0, 1, 2t_k, \ldots, nt_k^{n-1} \\
\underbrace{0 \ldots 0}_{(k-1)(n+1)}, 0, 0, 2, \ldots, n(n-1)t_k^{n-2} \\
\underbrace{0 \ldots 0}_{(i-1)(n+1)}, 1, t_i, t_i^2, \ldots, t_i^n, -1, -t_i, -t_i^2, \ldots, -t_i^n, \underbrace{0 \ldots 0}_{(k-i-1)(n+1)} \\
\underbrace{0 \ldots 0}_{(i-1)(n+1)}, 0, 1, 2t_i, \ldots, nt_i^{n-1}, -0, -1, -2t_i, \ldots, -nt_i^{n-1}, \underbrace{0 \ldots 0}_{(k-i-1)(n+1)} \\
\underbrace{0 \ldots 0}_{(i-1)(n+1)}, 0, 0, 2, \ldots, \frac{n!}{(n-2)!}t_i^{n-2}, -0, -0, -2, \ldots, -\frac{n!}{(n-2)!}t_i^{n-2}, \underbrace{0 \ldots 0}_{(k-i-1)(n+1)}
\end{bmatrix}_{(4k+2)\times(n+1)k}
$$

$$
p = \begin{bmatrix} p_0 \\ v_0 \\ a_0 \\ \vdots \\ p_i \\ \vdots \\ p_k \\ v_k \\ a_k \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

等式约束个数：

3 (起点PVA)

k-1 (中间固定点的P)

3 (终点PVA)

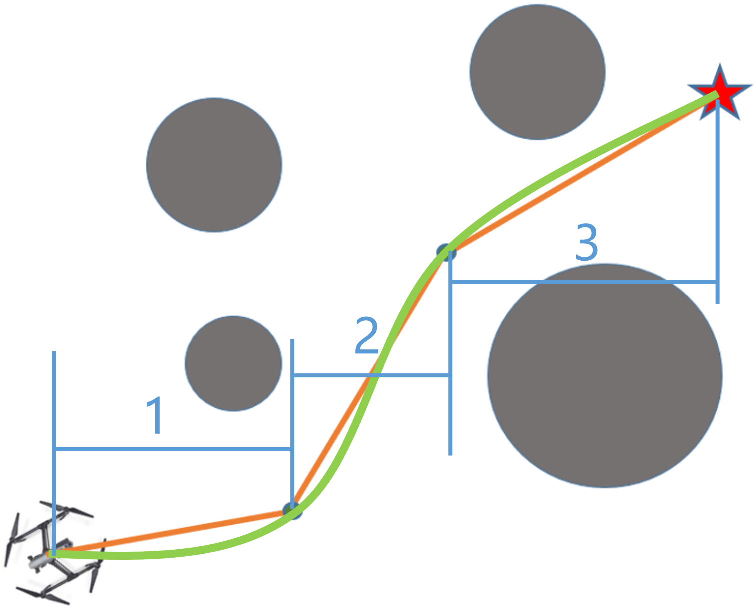
3(k-1) (中间点PVA连续)

4k+2

$$A_{eq}p = b_{eq}$$

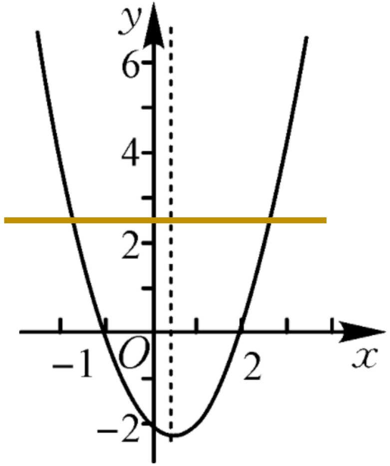
Minimum Snap Trajectory Generation and Control for Quadrotors (2011 ICRA)

从理论到代码



$$\min p^T Q p$$
$$s.t. \quad A_{eq} p = b_{eq},$$

二次规划问题



https://blog.csdn.net/u011231598/article/details/79615321

02

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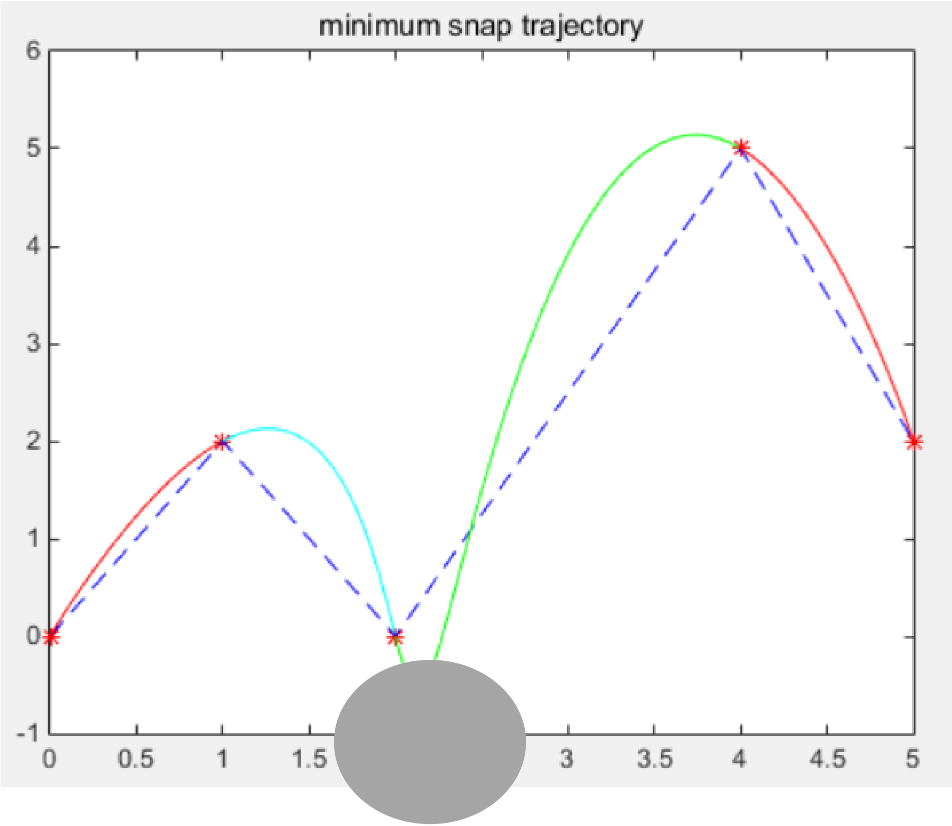
Minimum Snap代码讲解

03

PART

安全飞行走廊

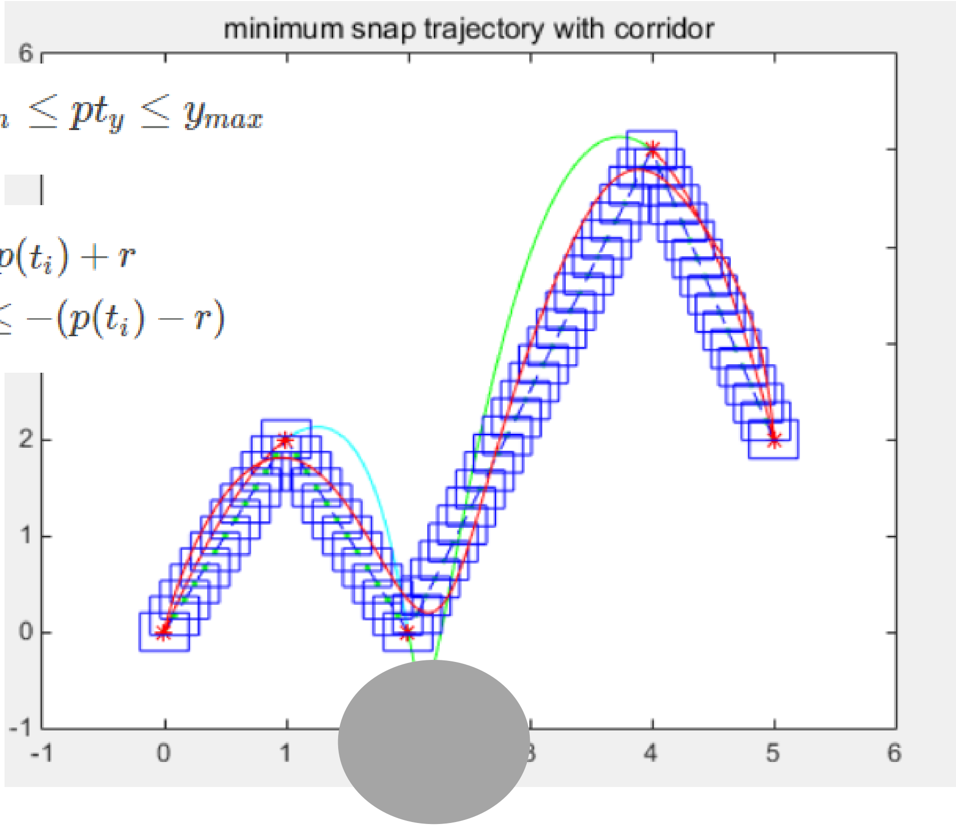
安全飞行走廊



安全飞行走廊

$$x_{min} \le pt_x \le x_{max},\ y_{min} \le pt_y \le y_{max}$$

$$[1, t_i, t_i^2, \ldots, t_i^n]p \le p(t_i) + r$$
$$[-1, -t_i, -t_i^2, \ldots, -t_i^n]p \le -(p(t_i) - r)$$



THANKS