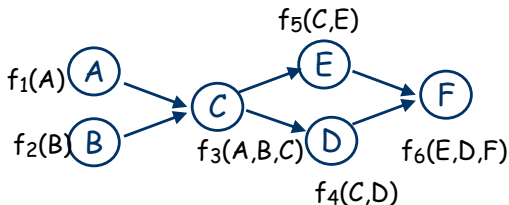# $P(D|h, -i)$的求解过程

$P(D|h, -i) = \Sigma_A P(A) \Sigma_B P(B) P(D|A, B) \Sigma_C P(C|A) \Sigma_E P(E|C)$
$\quad \Sigma_F P(F|D) P(h|E, F) \Sigma_G P(G) P(-i|F, G) \Sigma_J P(J|h, -i) \Sigma_K P(K|-i)$

- 求$c_1 = \Sigma_K P(K|-i)$, $c_2 = \Sigma_J P(J|h, -i)$
- 求$f_1(F) = \Sigma_G P(G) P(-i|F, G)$
- 求$f_2(D, E) = \Sigma_F P(F|D) P(h|E, F) f_1(F)$
- 求$f_3(C, D) = \Sigma_E P(E|C) f_2(D, E)$
- 求$f_4(A, D) = \Sigma_C P(C|A) f_3(C, D)$
- 求$f_5(A, D) = \Sigma_B P(B) P(D|A, B)$
- 求$f_6(D) = \Sigma_A P(A) f_5(A, D) f_4(A, D)$
- $Pr(d|h, -i) = \frac{f_6(d)}{f_6(d) + f_6(-d)}$, $Pr(-d|h, -i) = \frac{f_6(-d)}{f_6(d) + f_6(-d)}$

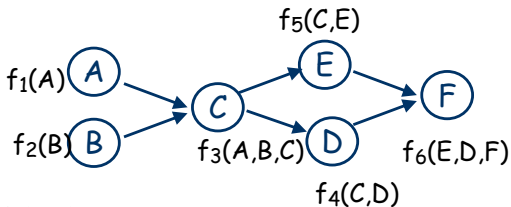**We will use buckets as a notational device to do Variable Elimination**

VE Ordering:
C,F,A,B,E,D

$f_5(C,E)$

$f_1(A)$ A

$f_2(B)$ B

C

E

D

F

$f_3(A,B,C)$

$f_6(E,D,F)$

$f_4(C,D)$

1. C:

2. F:

3. A:

4. B:

5. E:

6. D:

VE Ordering:
C,F,A,B,E,D

$f_1(A)$ A $f_5(C,E)$ E F
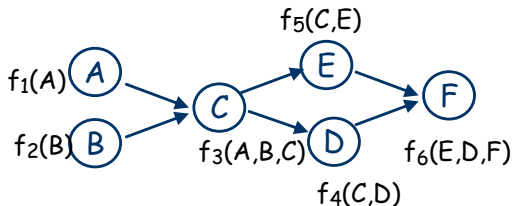
$f_2(B)$ B C $f_3(A,B,C)$ D $f_6(E,D,F)$

$f_4(C,D)$

1. C: $f_3(A,B,C)$, $f_4(C,D)$, $f_5(C,E)$

2. F: $f_6(E,D,F)$

3. A: $f_1(A)$

4. B: $f_2(B)$

5. E:

6. D:

VE Ordering:
C,F,A,B,E,D

1. ~~C: $f_3(A,B,C)$, $f_4(C,D)$, $f_5(C,E)$~~

2. F: $f_6(E,D,F)$
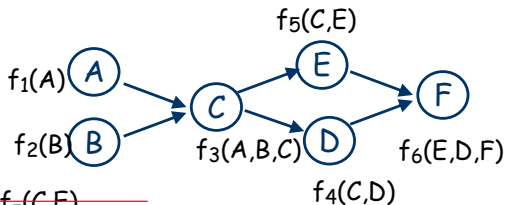
3. A: $f_1(A)$, $f_7(A,B,D,E)$

4. B: $f_2(B)$

5. E:

6. D:

1. Eliminating C:
   Σ_C $f_3(A,B,C)$, $f_4(C,D)$, $f_5(C,E)$
   $= f_7(A,B,D,E)$

VE Ordering:
C,F,A,B,E,D

$f_5(C,E)$

$f_1(A)$ (A)

(E)

(C)

(F)

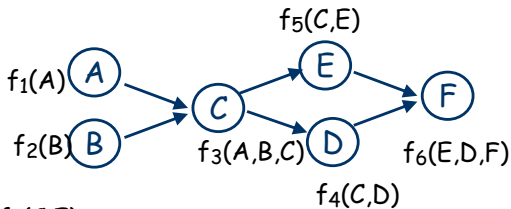$f_2(B)$ (B)

$f_3(A,B,C)$ (D)

$f_6(E,D,F)$

$f_4(C,D)$

1. ~~C: $f_3(A,B,C)$, $f_4(C,D)$, $f_5(C,E)$~~

2. ~~F: $f_6(E,D,F)$~~

3. A: $f_1(A)$, $f_7(A,B,D,E)$

4. B: $f_2(B)$

5. E: $f_8(E,D)$

6. D:

2. Eliminating F:

$\Sigma_F\, f_6(E,D,F)$ = $f_8(E,D)$

Eliminate A, placing new factor f9 in first applicable bucket.

VE Ordering:
C,F,A,B,E,D

1. ~~C: $f_3(A,B,C)$, $f_4(C,D)$, $f_5(C,E)$~~

2. ~~F: $f_6(E,D,F)$~~

3. ~~A: $f_1(A)$, $f_7(A,B,D,E)$~~

4. B: $f_2(B)$, $f_9(B,D,E)$

5. E: $f_8(E,D)$

6. D:

3. Eliminating A:
$\Sigma_A f_1(A)$, $f_7(A,B,D,E)$
   = $f_9(B,D,E)$

Eliminate B, placing new factor f10 in first applicable bucket.

VE Ordering:
C,F,A,B,E,D

$f_5(C,E)$

$f_1(A)$ A

$f_2(B)$ B

C

E

D

F

$f_3(A,B,C)$

$f_4(C,D)$

$f_6(E,D,F)$

1. ~~C: $f_3(A,B,C)$, $f_4(C,D)$, $f_5(C,E)$~~

2. ~~F: $f_6(E,D,F)$~~

3. ~~A: $f_1(A)$, $f_7(A,B,D,E)$~~

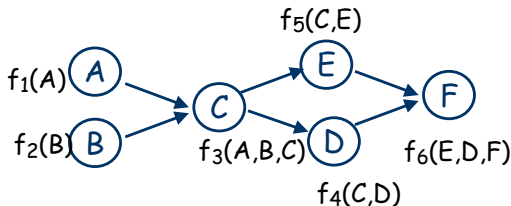4. ~~B: $f_2(B)$, $f_9(B,D,E)$~~

5. E: $f_8(E,D)$, $f_{10}(D,E)$

6. D:

4. Eliminating B:
Σ_B $f_2(B)$, $f_9(B,D,E)$
   = $f_{10}(D,E)$

Eliminate E, placing new factor f11 in first applicable bucket.

VE Ordering:
C,F,A,B,E,D

$f_1(A)$ A
$f_2(B)$ B
C
$f_3(A,B,C)$
D
$f_4(C,D)$
$f_5(C,E)$
E
F
$f_6(E,D,F)$

1. ~~C: $f_3(A,B,C)$, $f_4(C,D)$, $f_5(C,E)$~~

2. ~~F: $f_6(E,D,F)$~~

3. ~~A: $f_1(A)$, $f_7(A,B,D,E)$~~

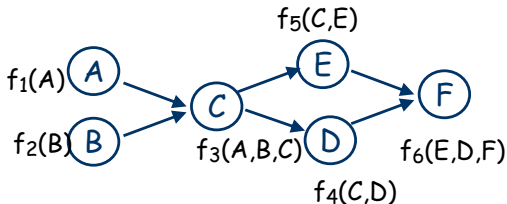4. ~~B: $f_2(B)$, $f_9(B,D,E)$~~

5. ~~E: $f_8(E,D)$, $f_{10}(D,E)$~~

6. D: $f_{11}(D)$

5. Eliminating E:

Σ_E $f_8(E,D)$, $f_{10}(D,E)$
  = $f_{11}(D)$

f_{11} is the final answer, <u>once we normalize it</u>.