Practice Olympiad 1

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Time: 60 minutes

The practice exam consists of four problems which you may solve in any order. Write all your solutions in the A4-sheets provided. Scratch paper that you wish not to submit, along with this piece of paper, you are free to keep. Usage of scratch paper is encouraged; it is an essential skill in competitions.

You may not turn this paper around until instructed by David. You are allowed to ask for clarifications, extra paper, access to the toilets, etc. by raising your hand. You are not allowed to exit the classroom during the last 15 minutes of exam time.

When I have Fears That I May Cease to Be
BY JOHN KEATS

When I have fears that I may cease to be
Before my pen has gleaned my teeming brain,
Before high-pilèd books, in charactery,
Hold like rich garners the full ripened grain;
When I behold, upon the night's starred face,
Huge cloudy symbols of a high romance,
And think that I may never live to trace
Their shadows with the magic hand of chance;
And when I feel, fair creature of an hour,
That I shall never look upon thee more,
Never have relish in the faery power
Of unreflecting love—then on the shore
Of the wide world I stand alone, and think
Till love and fame to nothingness do sink.

1. Given that a, b, c, g satisfy

$$(g-a)\cdot\frac{3}{2}+a=\frac{b+c}{2}$$

Find a value for $\dfrac{g-b}{\frac{a+c}{2}-g}$ (5)

2. Consider the following system of equations:

$$\begin{cases}3y-xy=7-3x\\5y+2xy=-7-5x\end{cases}$$

a) find $x+y$ (3)

b) find $\frac{1}{x}+\frac{1}{y}$ (2)

3. a, b, c, d, e, f are numbers that satisfy the equation

$$ax^2+bx+c=dx^2+ex+f$$

for <u>all values</u> of x.

a) Prove $a+b+c=d+e+f$. (1)

b) Prove that $a=d$, $b=e$, $c=f$. (4)

4. a) Expand $(a+b)(a^2-ab+b^2)$. (2)

b) Find a prime divisor of 8027. (1)