Investigating patterns in the product of Euler's totient function φ and the sum of divisors function σ

Research Question: How do the patterns and bounds in the graph of $y = \varphi(x) \cdot \sigma(x)$ relate to the Riemann zeta function and the Basel problem?

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1 Abstract

This paper aims to investigate patterns in the graph of $y = \varphi(x) \cdot \sigma(x)$.

Contents

2 Introduction

Broadly speaking, number theory is the branch of mathematics concerned with the properties of integers. In a school setting, this often includes topics such as divisibility rules, prime factorization, and, most importantly, the **Fundamental Theorem of Arithmetic**.

Theorem 1 (Fundamental Theorem of Arithmetic) *Every integer $n > 1$ can be represented as a product of prime factors in precisely one way, up to the order of the factors.*

This theorem asserts that for any integer n, there exists a unique set of distinct prime numbers $p_1, p_2, ..., p_k$, along with their respective exponents $\alpha_1, \alpha_2, ..., \alpha_k$, such that

$$n = p_1^{\alpha_1} p_2^{\alpha_2} ... p_k^{\alpha_k}.$$

This form of writing numbers as a product of primes is the key to the proofs delivered in this paper. For example, we can construct all the factors (positive divisors) of a number when given its prime factorization. Namely, they are precisely the numbers of the form

$$d = p_1^{\beta_1} p_2^{\beta_2} ... p_k^{\beta_k},$$

where $\beta_i \leq \alpha_i$ for all $1 \leq i \leq k$.

3 The zeta function

hello [**basel**] hello!!

4 An Explanation for the Patterns

5 Appendices