Subject: Mathematics

Investigating patterns in the product of Euler's totient function φ and the sum of divisors function σ

Research Question: What gives rise to the patterns and bounds in the graph of $y = \varphi(x) \cdot \sigma(x)$, and how does this relate to the Riemann zeta function and the Basel problem?

Word count: 237

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Abstract / Preliminary Research Plan

I will first define the functions φ and σ, and provide motivations and justifications to why they are important and an integral part of number theory.

Secondary source for this: **Introduction to Analytic Number Theory** by *Tom M. Apostol*

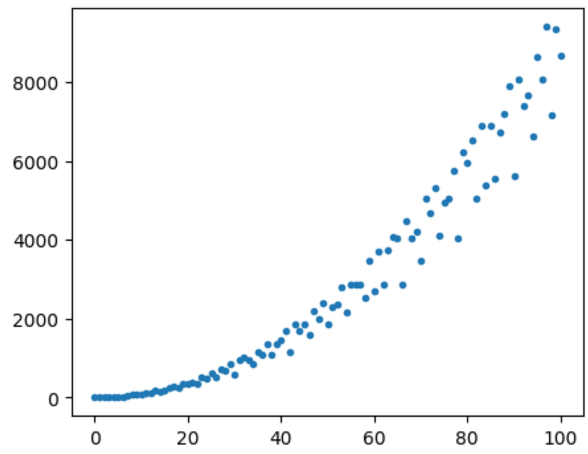

Figure 1: The graph I wish to investigate

Next I will present the graph I plan to study, $y = \varphi(x) \cdot \sigma(x)$, and suggest computing bounds for this function.

I might hint towards patterns in the graph already at this point, but I should leave the patterns until the end, for coherence. Instead, I introduce the upper bound of x^2 and lower bound of $\dfrac{x^2}{\zeta(2)} = x^2 \cdot \dfrac{6}{\pi^2}$

I should go on to prove these bounds, but perhaps before that I must allude to the famous Basel Problem.

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ... = \frac{\pi^2}{6}$$

I will discuss the relationship of this with the Riemann Zeta function, and perhaps see the need to refer to secondary sources here. **Modern Olympiad Number Theory** by *Aditya Khurmi* is an ok choice for this, but there may exist better options.

After that, the bounds will be established and we may move on to discussing the patterns in this graph.

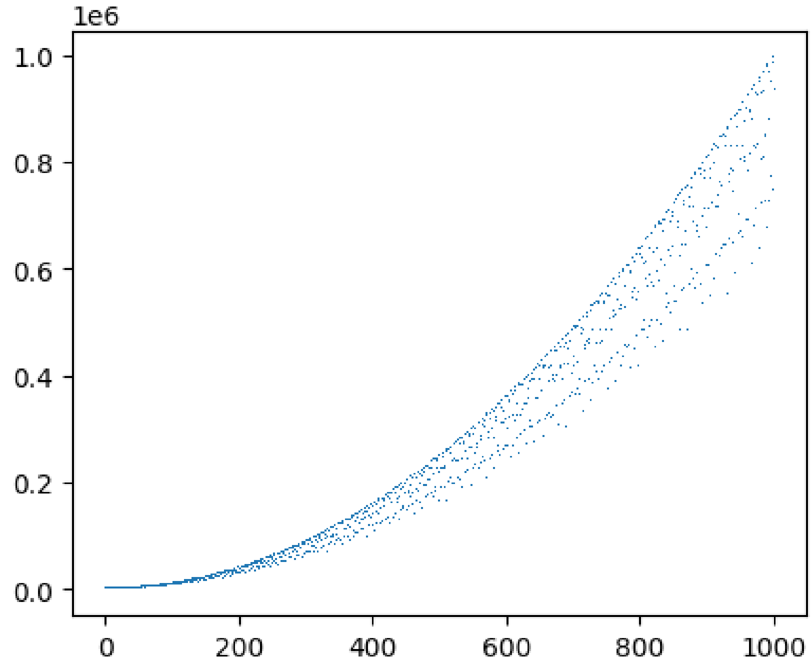

Figure 2: We see that the graph has denser regions when enough data is included

The patterns can be clearly seen and I will be showing why they are manifest. Maybe that requires me providing an exact definition on what a "dense sort of line thing" we see here is. That may require additional secondary sources.

I think this is enough content, and it will produce an analysis that I hope does not end up being too shallow nor too obstruse.