

**Computing the dynamic frictional coefficient μ_d
between wood and steel by measuring the
relationship between the force of friction and mass.**

Research Question: How does changing the mass of a box sliding against a plane affect the force of friction it experiences?

1 Introduction

The author finds it perplexing how the force of friction an object experiences in kinematics happens to be proportional to the size of the normal force of the surface. It is quite remarkable indeed how many physical phenomena may be described with a proportional model. Yet, attempting to answer *why* proportionality describes the behaviour of friction is far beyond the scope of this discussion. Instead, we shall merely satisfy our curiosity with examples that show proportionality is indeed valid.

My research uses a minimal yet clever setup to measure the size of friction in relation to the mass of the box, with the mass as the independent variable. Both the speed at which the dynamic friction is measured and the area of contact surface between the box and the plane

2 Background Information

Theory says dynamic friction is proportional to the normal force. This means that a frictional coefficient exists, which is F_μ/N where N is the normal force.

2.1 Relevant Theory

“The rate limiting reaction is believed to be the initial reaction between hydrogen peroxide and the iron oxides [Miller1995, ValentineWang1998, Miller1999]. Additionally, the results reveal that the decomposition of H_2O_2 follows pseudo-first order kinetics:

$$-\frac{d[\text{H}_2\text{O}_2]}{dt} = k_{\text{app}}[\text{H}_2\text{O}_2] \quad (2.1)$$

and thus:

$$\ln \left(\frac{[\text{H}_2\text{O}_2]}{[\text{H}_2\text{O}_2]_0} \right) = -k_{\text{app}}t \quad (2.2)$$

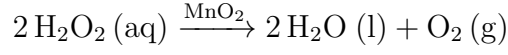
where k_{app} is the apparent first order rate constant, and $[\text{H}_2\text{O}_2]$ and $[\text{H}_2\text{O}_2]_0$ are the concentrations of H_2O_2 in the solution at any time t and time zero, respectively.[Huang]”

The data presented in **Section 4** agrees with theory, that the reaction is pseudo-first order. The aim of this investigation is to measure how k_{app} changes in relation to the amount of catalyst used.

Now a general expression for pressure as a function of time will be mathematically derived from (2.2).

Notation			
$n(\text{H}_2\text{O}_2)$	H_2O_2 in solution at time t	P	Pressure at time t
$n(\text{O}_2)$	O_2 released up to t	P_0	Initial pressure
n_0	Initial H_2O_2 in solution	P_∞	Final pressure
n_{air}	Initial air molecules	V_l	Solution volume
R	Ideal gas constant	V_f	Flask volume
T	Temperature	V_g	$= V_f - V_l$, Gas volume

The catalysed decomposition is



From stoichiometry,

$$n(\text{O}_2) = \frac{n_0 - n(\text{H}_2\text{O}_2)}{2} \implies n(\text{H}_2\text{O}_2) = n_0 - 2n(\text{O}_2).$$

With $[\text{H}_2\text{O}_2] = n(\text{H}_2\text{O}_2)/V_l$ and $PV_g = (n_{\text{air}} + n(\text{O}_2))RT$,

$$n(\text{O}_2) = \frac{PV_g}{RT} - n_{\text{air}}$$

so

$$[\text{H}_2\text{O}_2] = \frac{n_0 + 2n_{\text{air}} - 2PV_g/(RT)}{V_l}$$

At completion $n(\text{H}_2\text{O}_2) = 0$, hence $n_0 + 2n_{\text{air}} = 2P_{\infty}V_g/(RT)$ and therefore

$$[\text{H}_2\text{O}_2] = \frac{2(P_{\infty} - P)V_g}{RT V_l}$$

Combining this with the integrated pseudo-first order law

$$-k_{\text{app}}t = \ln \frac{[\text{H}_2\text{O}_2]}{[\text{H}_2\text{O}_2]_0}, \quad [\text{H}_2\text{O}_2]_0 = \frac{n_0}{V_l}$$

and simplifying yields

$$2(P_{\infty} - P) \frac{V_g}{RT} = n_0 e^{-k_{\text{app}}t}$$

and hence

$$P(t) = P_{\infty} - \frac{n_0 RT}{2V_g} e^{-k_{\text{app}}t}$$

For brevity define

$$c := \frac{n_0 RT}{2V_g},$$

giving the general form

$$P(t) = P_{\infty} - c e^{-k_{\text{app}} t}.$$

2.2 Existing Results

For pyrolusite (the primary manganese ore) powder, the highest reported specific rate constant is

$$k_{\text{pyro}} = 0.061 \text{ min}^{-1} \text{ mM}^{-1},$$

measured at room temperature in 200 mL of solution containing 29.4 mM H_2O_2 .**[Do]** This suggests that the rate constant is proportional to the concentration of MnO_2 in solution.

The results obtained in this investigation will be compared with this result. **Hypothesis.** Ideally a proportional relationship between the mass of MnO_2 and k_{app} is observed in this investigation, and the result does not deviate majorly from the above value.

3 Experimental

3.1 Materials & Apparatus

Materials	
33% (v/v) H ₂ O ₂ (aq), diluted to 3.3%	10 ± 0.5 mL per trial
Pure distilled water	20 ± 0.5 mL per trial
Fine MnO ₂ powder	≈ 1g total
Equipment	
220 mL Erlenmeyer flask	Thermometer
20 mL Medical syringe (±0.5 mL)	Electronic scale (±0.001 g)
Vernier Pressure sensor ¹	Rubber stopper
Magnetic stir bar and stir plate	Parafilm

This fit gives the specific rate constant $k_{\text{spec}} \approx 0.00009093 \text{ s}^{-1} \text{ mM}^{-1} = 0.0054558 \text{ min}^{-1} \text{ mM}^{-1}$, which is within an acceptable range of the known $k_{\text{pyro}} = 0.061 \text{ min}^{-1} \text{ mM}^{-1}$ from **Section 2.2**.

3.2 Conclusion

The data support a direct, near-proportional increase of the apparent first-order rate constant k_{app} with suspended MnO₂ concentration. This matches the general expectation that more active surface yields faster reaction, but the measured specific rate is lower than literature ($0.061 \text{ min}^{-1} \text{ mM}^{-1}$); plausible causes include incomplete O₂ capture (leakage, connector losses), variable effective surface area (agglomeration or settling of powder), the initial connection artifact, and instrument limits (pressure sensor accuracy, uncertain V_g). These uncertainties (notably $V_l \pm 3.3\%$ and sensor $\pm 2 \text{ kPa}$) moderately affect k_{app} and explain observed scatter and the anomalously low 0.05 g point, so quantitative agreement

with literature is tentative though the trend is robust.

Methodological limitations with meaningful impact: reliance on headspace pressure rather than direct concentration assays, imperfect seals, limited replicates per mass and a narrow mass range, and possible temperature/stirring inconsistencies. Strengths include a clear theoretical derivation linking $P(t)$ to $[\text{H}_2\text{O}_2]$ and continuous high-frequency data collection.

Realistic, relevant improvements: use pressure-rated fittings or water-displacement gas collection; immobilize catalyst or better disperse powder; increase replicate number and mass range; directly assay $[\text{H}_2\text{O}_2]$ (titration or spectrophotometry) for cross-validation; calibrate sensor and measure V_g precisely. With these changes the proportional relationship can be tested more reliably and quantitative agreement improved.