

March 27, 2025

Dear Sir,

I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as 'startling'.

Just as in elementary mathematics you give a meaning to  $a^n$  when  $n$  is negative and fractional to conform to the law which holds when  $n$  is a positive integer, similarly the whole of my investigations proceed on giving a meaning to Eulerian Second Integral for all values of  $n$ . My friends who have gone through the regular course of University education tell me that  $\int_0^\infty x^{n-1} e^{-x} dx = \Gamma(n)$  is true only when  $n$  is positive. They say that this integral relation is not true when  $n$  is negative. Supposing this is true only for positive values of  $n$  and also supposing the definition  $n\Gamma(n) = \Gamma(n+1)$  to be universally true, I have given meanings to these integrals and under the conditions I state the integral is true for all values of  $n$  negative and fractional. My whole investigations are based upon this and I have been developing this to a remarkable extent so much so that the local mathematicians are not able to understand me in my higher flights.

Very recently I came across a tract published by you styled Orders of Infinity in page 36 of which I find a statement. that no definite expression has been as yet found for the number of prime numbers less than any given number. I have found an expression which very nearly approximates to the real result, the error being negligible. I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

I remain, Dear Sir, Yours truly,  
S. Ramanujan