Activity 7

△ Item 1 a) $X = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ (x_1 x_2)

$u_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\tilde{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\left(\frac{1}{\sqrt{2}}[1\ 1\ 0]\right)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\left(\frac{1}{\sqrt{2}}\right) = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$

$u_2 = \frac{1}{\frac{\sqrt{6}}{2}}\begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix} = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$\sqrt{\frac{1}{4}+\frac{1}{4}+1}$ $\sqrt{\frac{6}{4}}$

b) ii) $u_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\tilde{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\left(\frac{1}{\sqrt{2}}[1\ 0\ 1]\right)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix}$

$u_2 = \frac{1}{\frac{\sqrt{2}}{2}}\begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

iii) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

△ Item 2 a) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$

b) $U^TU = \underset{2\times3}{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix}}\underset{3\times2}{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{2}{\sqrt{6}} \end{bmatrix}}$

$= \begin{bmatrix} 1 & \frac{1}{2\sqrt{3}} - \frac{1}{2\sqrt{3}} \\ \frac{1}{2\sqrt{3}} - \frac{1}{2\sqrt{3}} & \frac{1}{6}+\frac{1}{6}+\frac{4}{6} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $X = U[a_1\ a_2]$

$3\times2\quad 3\times2$ (a_1, a_2: 2×1 each)

$3\times2 \quad 2\times2$

d) $X = UA$, $U^TU = I$

$\rightarrow U^TX = A$

$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$

$= \begin{bmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} \\ 0 & \sqrt{\frac{4}{6}} \end{bmatrix}$

△ Item 3 a) $X = UT$

$n\times p$ ($n>p$) $\quad p\times p$ invertible

orthonormal basis

$X = UT$

$XG = U$

$\rightarrow G = T^{-1}$

$(AB)^{-1} = B^{-1}A^{-1}$

$AA^{-1} = I$

b) $P_X = X(X^TX)^{-1}X^T = UT(T^TU^TUT)^{-1}T^TU^T$

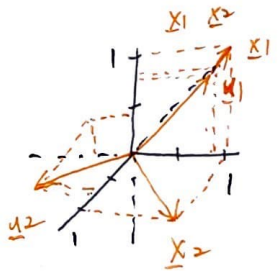
$= UU^T$

$= U(U^TU^T)U^T$

$(T^TT)^{-1} = T^{-1}T^{-T}$

△ Item 4 a) $X = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$ (0.7, 0.4)

(x_1 x_2; u_1 u_2)



b) $\hat{b} =$

△ Item 5 a) $Q = zz^T$, $z = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\rightarrow Q = \begin{bmatrix} 1 \\ 1 \end{bmatrix}[1\ 1] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

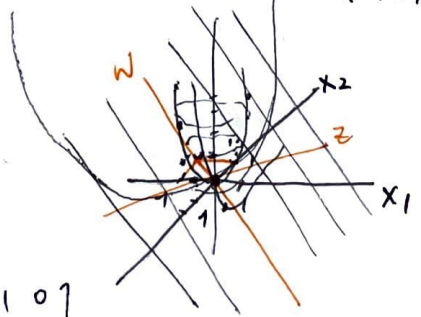
$y = x^TQx = \underset{1\times2}{[x_1\ x_2]}\underset{2\times2}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}\underset{2\times1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{1\times2}{[x_1\ x_2]}\underset{2\times1}{\begin{bmatrix} x_1+x_2 \\ x_1+x_2 \end{bmatrix}}$

$= x_1(x_1+x_2) + x_2(x_1+x_2)$

$= x_1^2 + 2x_1x_2 + x_2^2 = (x_1+x_2)^2$

$(0,0), (1,1), (-1,-1)$ (0, 4, 4)

$(0,-1), (-1,0)$ (1, 1)



b) y) lots of set (x_1, x_2) to make $y = 0$

△ Item 3 b) $P_X = X(X^TX)^{-1}X^T$ $P_U =$

$= X(I)^{-1}X^T$

$= XX^T$

d) $x^TQx \geq 0$

↓

positive definite for all $X \neq 0$

$\Rightarrow Q \geq 0$