

An aerial photograph of a city, likely Madison, Wisconsin, taken from a high vantage point looking down at the city and the surrounding water. The sun is setting behind a hill in the background, casting a warm, golden glow over the entire scene. The city is built on a peninsula, with numerous buildings and streets visible. The water is a deep blue, and several sailboats are scattered across it. A large, semi-transparent rectangular box is overlaid on the center of the image, containing the course title in large, bold, black text.

CS/ECE/ME 532 Matrix Methods in Machine Learning

Welcome!



Activity 11: K-means and SVD. Low-rank inversion via SVD



Low-rank decompositions:

$$A \approx TW^T = \begin{bmatrix} t_{1,1} & t_{1,2} \\ t_{2,1} & t_{2,2} \\ t_{3,1} & t_{3,2} \\ t_{4,1} & t_{4,2} \end{bmatrix} \begin{bmatrix} w_{1,1} & \dots & w_{1,6} \\ w_{2,1} & \dots & w_{2,6} \end{bmatrix}$$

taste vectors or patterns

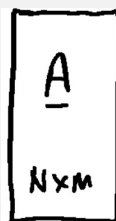
K-Means: cluster centers

SVD for least squares:

$$\min_w \|Aw - y\|^2$$

$$w^* = (A^T A)^{-1} A^T y \rightarrow w^* = V \Sigma^{-1} U^T y$$

economy SVD



Problems arise when the singular values are small!

- Fix by dropping corresponding singular vectors.

Hint for 1g: \tilde{T}

$$A \approx TW^T = \begin{bmatrix} t_{1,1} & t_{1,2} \\ t_{2,1} & t_{2,2} \\ t_{3,1} & t_{3,2} \\ t_{4,1} & t_{4,2} \end{bmatrix} \begin{bmatrix} w_{1,1} & \dots & w_{1,6} \\ w_{2,1} & \dots & w_{2,6} \end{bmatrix} \quad a_{\text{JON}} = \begin{bmatrix} 6 \\ 4 \\ ? \\ ? \end{bmatrix}$$

$$\min_{w_{\text{JON}}} \|\tilde{T} w_{\text{JON}} - y\|_2^2$$

$$\hat{a}_{\text{JON}} = T w_{\text{JON}}$$

The Singular Value Decomposition: $A = U \Sigma V^T$

- Singular values $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_N \geq 0$
- “Importance” of patterns in U, V ranked by σ_i

SVD gives the ‘best’ rank- r approximation:

$$\min_{\text{rank } \tilde{A} \leq r} \|A - \tilde{A}\|_F \rightarrow \tilde{A} = \sum_{i=1}^r \sigma_i u_i v_i^T$$

Frobenius norm

Eckart-Young, 1936