

Activity 4  $\Delta$  Item /  $\odot$  column

a)  $X = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$   
 $4 \times 5$   
 $a_1 \ a_2 \ a_3 \ a_4 \ a_5$

$$a_2 = a_3 + a_5$$

$$a_3 = a_1 + a_4$$

lín dep.

~~a1~~  $\rightarrow a_5 = -a_3$

left  $a_1, a_4$

⑦ row  $\left[ \begin{array}{ccccc|c} 1 & 0 & 1 & 0 & -1 & b_1 \\ 0 & 0 & 1 & 1 & -1 & b_2 \\ 0 & 0 & 0 & 0 & 0 & b_3 \\ 0 & 0 & 0 & 0 & 0 & b_4 \end{array} \right]$

$$b_3 = b_4$$

$$|b_3| = 0$$

let  $b_1, b_2$

(3)

b) set?  
lin-ind.

$a_1, a_2$   
 $a_2, a_3$   
 $a_3, a_4$   
 $a_4, a_5$

$$a_1, \underline{a_2}, a_3 - a_5$$

~~$a_1, a_3$~~

$a_2, a_3, a_4$

$a_4, a_5$

(4)  ~~$a_1, a_2, a_3, a_5$~~

~~$a_1, a_3, a_4, a_5$~~

if  $|a_2| = 0$   $\star$

$$a_1, (a_2, a_4, a_5)$$

$\frac{1}{2}, ($

$$a_3(a_4) \rightarrow 9$$

$a_4(a_5)$

c)  $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & b \\ 0 & 1 & -1 \end{bmatrix}$ ,  $\text{rank}(A) = 2$   
 $\rightarrow \text{row}[1]$

$$\rightarrow n \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + m \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ -1 \end{bmatrix}$$

$$a = n, b = n + m,$$

$$m = -1$$

$$\Delta \text{Item 2} \quad \begin{matrix} 3 \times 2 \\ a_1 \ a_2 \end{matrix} \quad \rightarrow b = n-1 \rightarrow \underline{b = a-1}$$

$$Ax=b, A=\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow b=\begin{bmatrix} 8 \\ 6 \\ -2 \end{bmatrix}$$

$$\text{rank}(A) = 2$$

$$\text{rank}([A \ b]) = 2$$

$$b = 8a_1 - 2a_2$$

b)  $\text{rank}(C \oplus b) = 3 \rightarrow X = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$$b = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \text{rank}(A) = 2$$

→ no solution

c)  $AX=b \rightarrow$  has a solution  
 $\text{rank}(b)$  is full rank (= number of columns)  
 $\Rightarrow \text{rank}(A) = \text{rank} \{[A \ b]\}$ , and rank is  
 condition the number of A's columns.

Δ Item 3 a)  $Ax=b$ ,  $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ -2 & 4 \end{bmatrix}$ ,  $b = \begin{bmatrix} 2 \\ -2 \\ -4 \end{bmatrix}$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{rank}(A) = 1$$

$$= \text{rank}([A \ b]) = 1$$

$$\text{rank}(A) = j < N_0, \text{ columns}$$

$\Rightarrow$  infinite sol.

$$\textcircled{1} \quad x_1 - 2x_2 = 2$$

$(0, 0)$

$$\rightarrow x_2 = \frac{x_1 - 2}{2}$$

$$(2, 0)$$

b)  $\text{rank}(A) < \text{number of columns}$