CS/ECE/ME 532
Matrix Methods in Machine Learning

Welcome!

Activity 7

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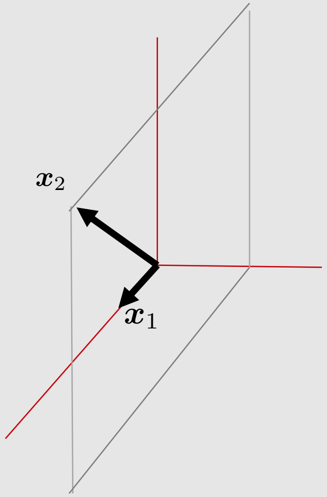
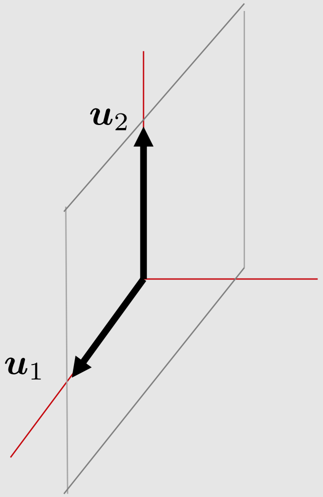
(lecture 2.9)



A basis

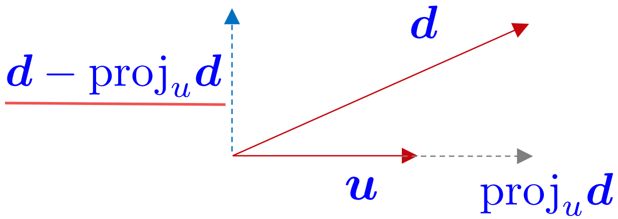


An orthonormal basis

Gram-Schmidt Orthogonalization

1. set $\tilde{u}_1 = x_1$
2. normalize: $u_1 = \frac{\tilde{u}_1}{||\tilde{u}_1||_2}$
3. set $\tilde{u}_2 = x_2 - \text{proj}_{u_1} x_2$
4. normalize: $u_2 = \frac{\tilde{u}_2}{||\tilde{u}_2||_2}$
5. set $\tilde{u}_3 = x_3 - \text{proj}_{u_1} x_3 - \text{proj}_{u_2} x_3$

$\vdots$

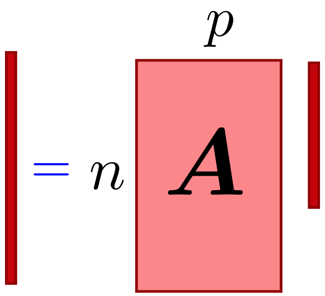


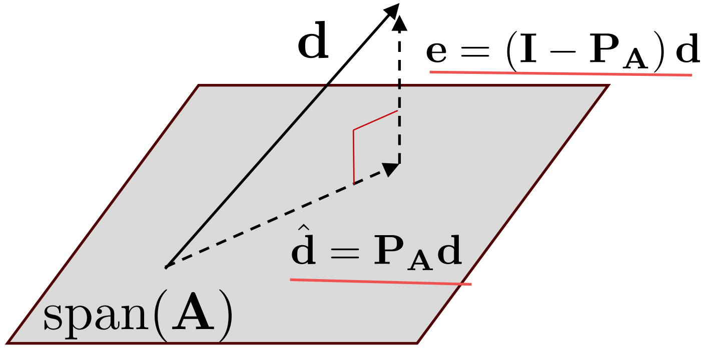
$$\text{proj}_u d = u(u^T d)$$

projection of d onto u

amount of d in the direction of u

$$\mathbf{d} = \mathbf{A}\mathbf{w}$$





U^T = U^(-1)/ ui^T @ ui = 1/ ui^T @ uj = 0

How can we leverage an orthonormal basis?:

$$\hat{\mathbf{d}} = \mathbf{A}\mathbf{w} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{d} = \mathbf{P_A}\mathbf{d}$$

Today

$$= \mathbf{U}\mathbf{U}^T\mathbf{d} = \mathbf{P_U}\mathbf{d}$$

Key fact: $\mathbf{U^T U} = \mathbf{I}$