In [1]:
```python
import numpy as np
from scipy.sparse import csc_matrix
from scipy.sparse.linalg import eigs
import pandas as pd

edges_file = open('wisconsin_edges.csv', "r")
nodes_file = open('wisconsin_nodes.csv', "r")
# print('edges_file:', edges_file)
# print('nodes_file:', nodes_file)

# create a dictionary where nodes_dict[i] = name of wikipedia page
nodes_dict = {}
for line in nodes_file:
    nodes_dict[int(line.split(',',1)[0].strip())] = line.split(',',1)[1].strip()

# print('nodes_file:', nodes_dict)
node_count = len(nodes_dict)

# create adjacency matrix
A = np.zeros((node_count, node_count))
for line in edges_file:
    from_node = int(line.split(',')[0].strip())
    to_node = int(line.split(',')[1].strip())
    A[to_node, from_node] = 1.0
```

1a)i. adding 0.001 to each entry of A

In [2]:
```python
A += A + 0.001
# print(A)
```

1a)ii. normalize A

In [3]:
```python
A = A/A.sum()
# print(A)
```

1a)iii. use an eigen decomposition to rank the importance of the Wikipedia pages.

In [4]:
```python
# Hint -- instead of computing the entire eigen-decomposition of a matrix X usin
# s, E = np.linalg.eig(A)
# you can compute just the first eigenvector with:
s, E = eigs(csc_matrix(A), k = 1)
print('s:', s)
print('E:', E)
```

```
s: [0.00045468+0.j]
E: [[0.00038314+0.j]
 [0.00194767+0.j]
 [0.00038314+0.j]
 ...
 [0.0354986 +0.j]
 [0.00284333+0.j]
 [0.00038314+0.j]]
```

1b) What is the title of the page ranked 1st (i.e, the most important page)?

Wisconsin.

In [5]:
```
page_rank_scores = np.abs(E).flatten().real
print(page_rank_scores)
```

```
[0.00038314 0.00194767 0.00038314 ... 0.0354986  0.00284333 0.00038314]
```

In [6]:
```
most_imp_ind = np.argmax(page_rank_scores)
print(most_imp_ind)
```

```
5089
```

In [7]:
```
node_title_data = pd.read_csv('wisconsin_nodes.csv', names = ['Index', 'Title',
node_title_data
row01 = node_title_data.iloc[most_imp_ind]
print(row01['Title'])
```

```
"Wisconsin"
```

1c) What is the title of the page ranked 3rd?

Madison

In [8]:
```
third_imp_ind = np.argsort(page_rank_scores)[-3]
print(third_imp_ind)
```

```
1345
```

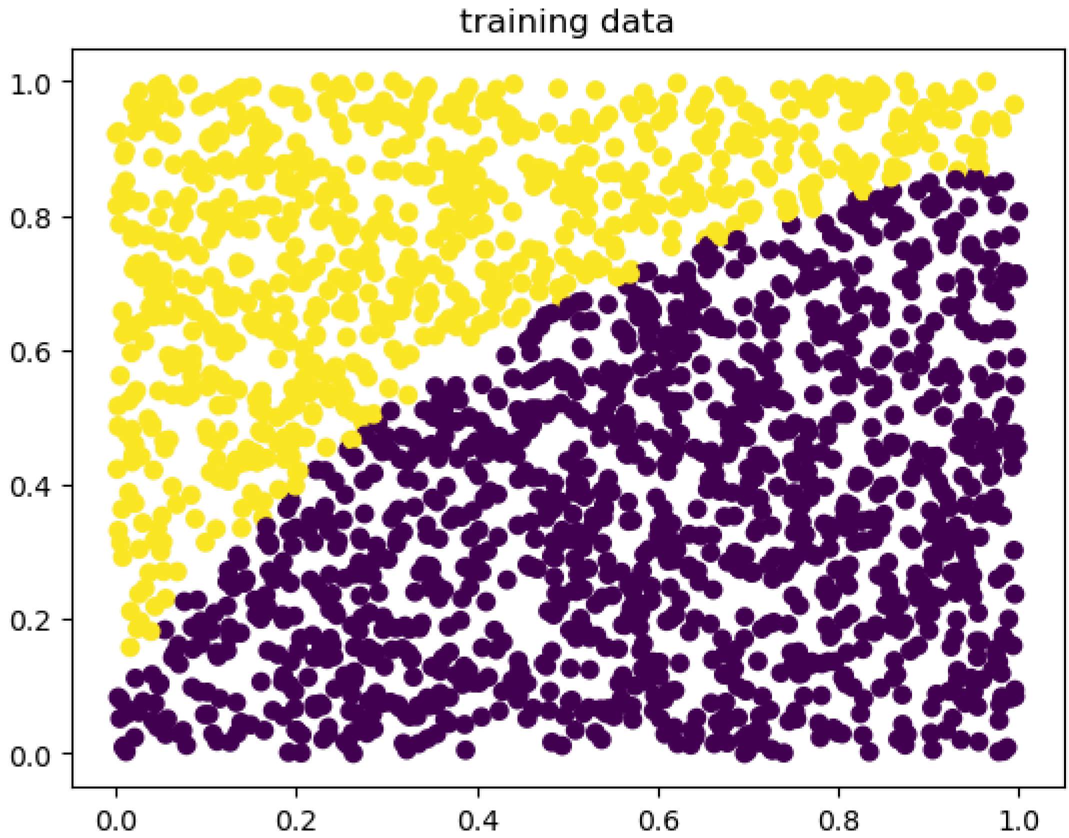
In [9]:
```
row02 = node_title_data.iloc[third_imp_ind]
print(row02['Title'])
```

```
"Madison
```

In [ ]:

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pickle

pkl_file = open('classifier_data.pkl', 'rb')
x_train, y_train = pickle.load(pkl_file)
print(x_train.shape)
print(y_train.shape)

# for idx in range(2000):
#     print(x_train[idx], y_train[idx])

n_train = np.size(y_train)

plt.scatter(x_train[:,0],x_train[:,1], c=y_train[:,0])
plt.title('training data')
plt.show()
```

```
(2000, 2)
(2000, 1)
```



2a) For a binary linear classifier, explain (mathematically) why the logistic loss function does not suffer from the same problem as the squared error loss on easy to classify point.

Unlike squared error loss, the logistic loss function uses a logarithmic scale to reduce penalties as predictions near the actual label, preventing disproportionate penalties for well-classified points.

2b) Compute an expression for the gradient (with respect to w) of the ℓ2 regularized logistic loss.

∇L(w)= ∑{i=1}^n -yixi.T/(1+e*(*yixi.T*w)) + 2λw

2c) Use the expression for the gradient that you derived to implement gradient descent and train a classifier on the provided dataset. For simplicity, you may assume λ = 1.

In [2]:
```
def logistic_graddescent(X, y, tau, w_init, it, lambda_val=1):
    n, d = X.shape
    W = np.zeros((d, it + 1))
    W[:, 0] = w_init.flatten()

    for k in range(it):
        total_grad = np.zeros(w_init.shape)
        for i in range(n):
            xi = X[i, :].reshape(1, 2)
            yi = y[i]
            den= 1 / (1 + np.exp(yi * (xi @ W[:, k])))
            grad = (-xi.T*yi * den).reshape(2, 1)
            total_grad += grad

        reg_grad = 2 * lambda_val * W[:, [k]]
        total_grad += reg_grad
        W[:, [k+1]] = W[:, [k]] - tau * total_grad

    return W
```

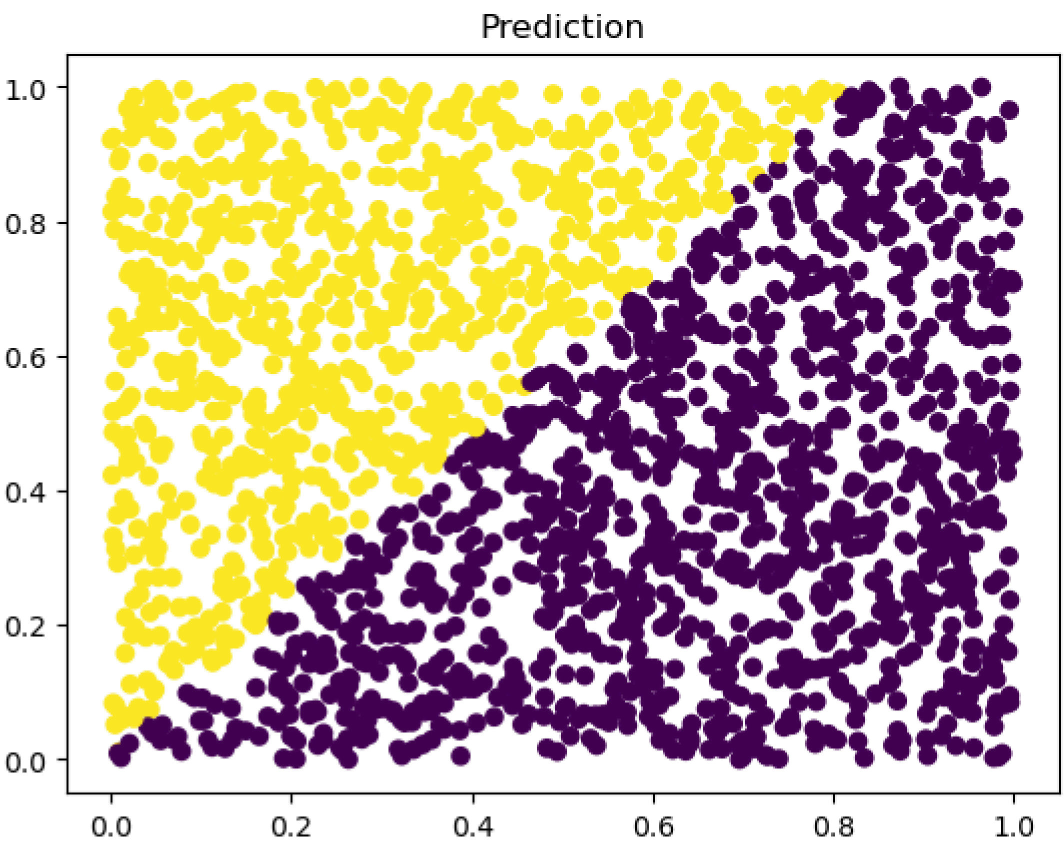
In [3]:
```
w_init = np.array([[1],[1]])
it = 200
tau = .0005
W = logistic_graddescent(x_train,y_train,tau,w_init,it)
W_op = W[:,-1]
W_op
```

Out[3]:
```
array([-5.50824156,  4.56250139])
```

d) Plot the data points (indicating their class with different colors) and plot the decision boundary. What is the error rate of your classifier on the training data?

Error Rate is 11.4%.

In [4]:
```
y_hat = np.sign(x_train@W_op)
y_hat = y_hat.reshape((2000,1))
# print(y_hat)
plt.scatter(x_train[:,0],x_train[:,1], c=y_hat[:,0])
plt.title('Prediction')
plt.show()
```



In [5]:
```
error_vec = [0 if i[0]==i[1] else 1 for i in np.hstack((y_hat, y_train))]
print('Errors: '+ str(sum(error_vec)))
print('Error Rate:' + str(sum(error_vec)*100/n_train) + '%')
```

```
Errors: 228
Error Rate:11.4%
```

e) Train a classifier using the squared error loss, and plot the decision boundary. How does this compare with a decision boundary when trained with logistic loss?

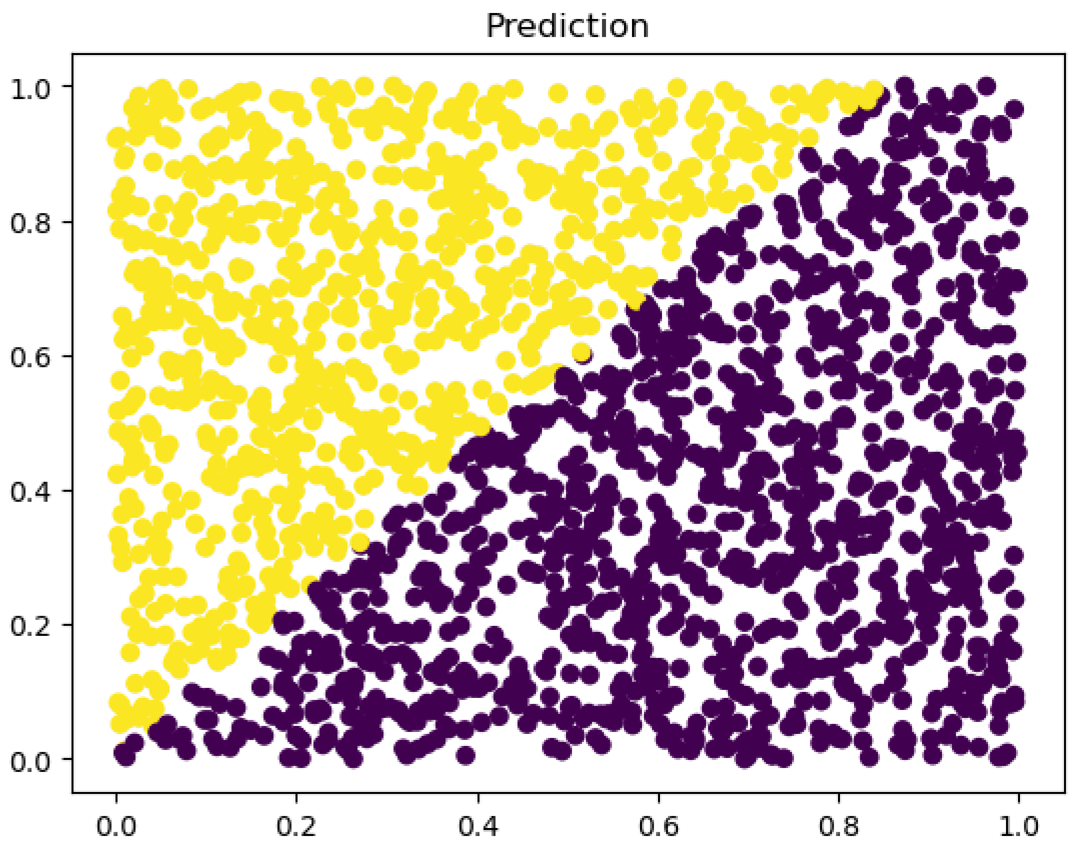
While both squared error and logistic loss yield classifiers with comparable error rates, the weights derived from each method differ.

In [6]:
```
def squared_error_graddescent(X,y,tau,w_init,it):
    W = np.zeros((w_init.shape[0], it+1))
    W[:, [0]] = w_init
    Z = np.zeros((w_init.shape[0], it+1))
    for k in range (it):
        Z[:, [k]] = W[:, [k]] - tau * ((X.T @ (X @ W[:, [k]] - y)))
        W[:, [k+1]] = 1/(1+tau) * Z[:, [k]]
    return W
```

In [7]:
```
w_init = np.array([[1],[1]])
it = 200
tau = .0005
W = squared_error_graddescent(x_train,y_train,tau,w_init,it)
W_op = W[:,-1]
W_op
```

Out[7]:
```
array([-1.98438138,  1.69197551])
```

In [8]:
```
y_hat = np.sign(x_train@W_op)
y_hat = y_hat.reshape((2000,1))
# print(y_hat)
plt.scatter(x_train[:,0],x_train[:,1], c=y_hat[:,0])
plt.title('Prediction')
plt.show()
```



In [9]:
```
error_vec = [0 if i[0]==i[1] else 1 for i in np.hstack((y_hat, y_train))]
print('Errors: '+ str(sum(error_vec)))
print('Error Rate:' + str(sum(error_vec)*100/n_train) + '%')
```

```
Errors: 229
Error Rate:11.45%
```

f) Add 1000 easy to classify data points to the training set: more specifically, 1000 points with yi = −1 and x = [10, 0]T . Re-train your classifiers and comment on the performance when trained with the logistic loss and the squared error.

The classifier trained with logistic loss improved, showing a reduced error rate of 7.5%, down from approximately 11.4% with the original 2000 data points. Conversely, the classifier using squared error loss experienced a substantial increase in error rate to 74.3%.

In [10]:
```
new_x = np.array([[10, 0]] * 1000)
new_y = np.array([-1] * 1000).reshape(-1, 1)

x_train = np.vstack((x_train, new_x))
y_train = np.concatenate((y_train, new_y))
n_train = np.size(y_train)
```

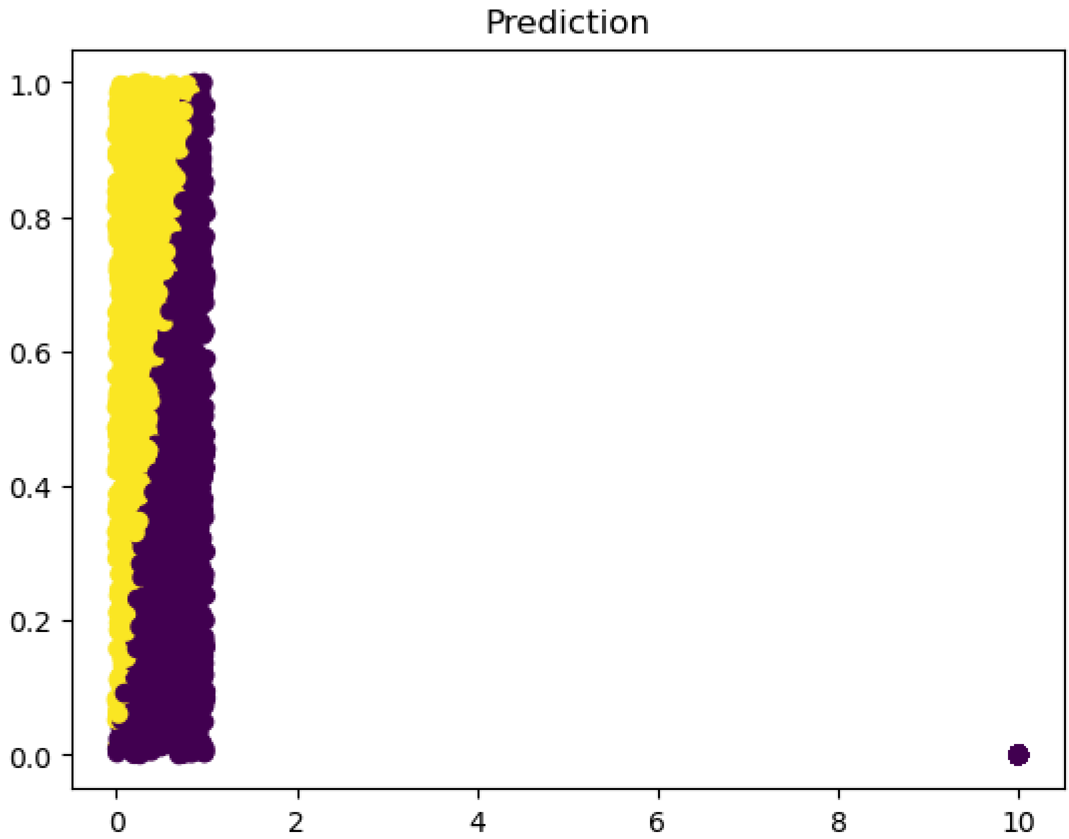
```python
print("New x_train shape:", x_train.shape)
print("New y_train shape:", y_train.shape)
```

```
New x_train shape: (3000, 2)
New y_train shape: (3000, 1)
```

In [11]:
```python
w_init = np.array([[1],[1]])
it = 200
tau = .0005
W = logistic_graddescent(x_train,y_train,tau,w_init,it)
W_op = W[:,-1]
W_op
```

Out[11]:
```
array([-5.96611322,  4.96768549])
```

In [12]:
```python
y_hat = np.sign(x_train@W_op)
y_hat = y_hat.reshape((3000,1))
# print(y_hat)
plt.scatter(x_train[:,0],x_train[:,1], c=y_hat[:,0])
plt.title('Prediction')
plt.show()
error_vec = [0 if i[0]==i[1] else 1 for i in np.hstack((y_hat, y_train))]
print('Errors: '+ str(sum(error_vec)))
print('Error Rate:' + str(sum(error_vec)*100/n_train) + '%')
```

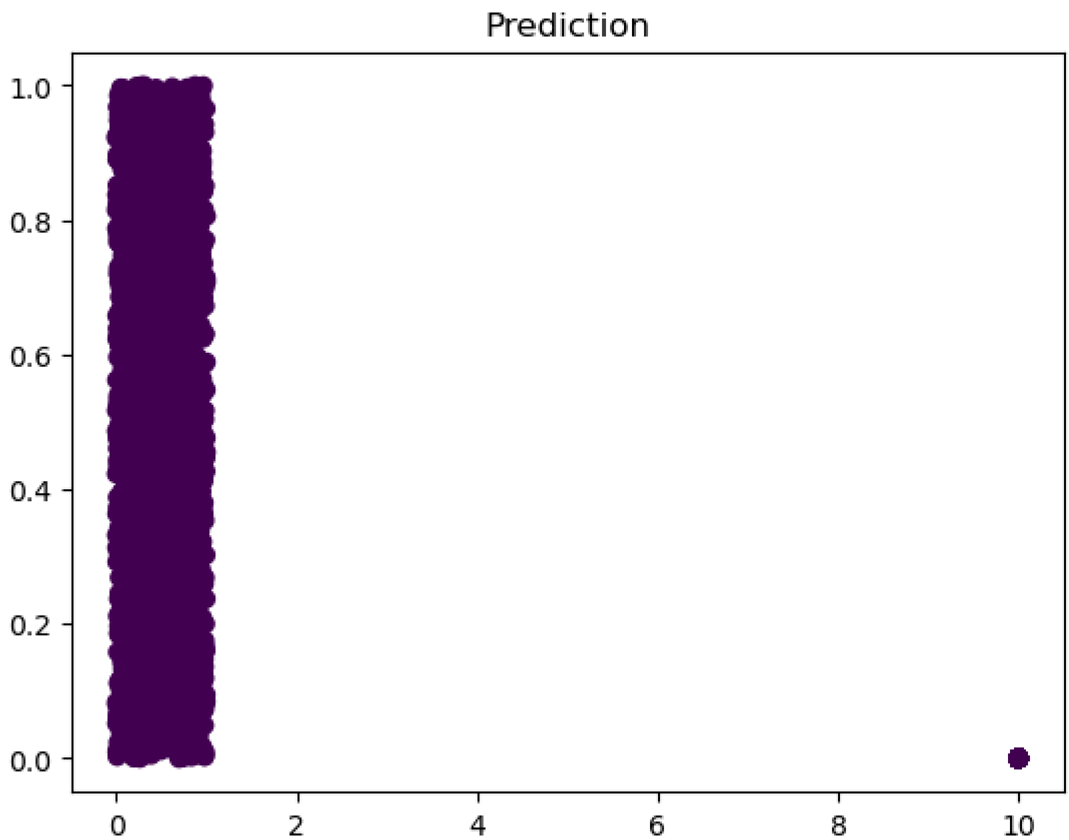



```
Errors: 226
Error Rate:7.533333333333333%
```

In [13]:
```python
w_init = np.array([[1],[1]])
it = 150
tau = .0005
W = squared_error_graddescent(x_train,y_train,tau,w_init,it)
W_op = W[:,-1]
W_op
```

Out[13]: array([9.43776652e+253, 4.59094739e+251])

In [14]:
```
y_hat = np.sign(x_train@W_op)
y_hat = y_hat.reshape((3000,1))
# print(y_hat)
plt.scatter(x_train[:,0],x_train[:,1], c=y_hat[:,0])
plt.title('Prediction')
plt.show()
error_vec = [0 if i[0]==i[1] else 1 for i in np.hstack((y_hat, y_train))]
print('Errors: '+ str(sum(error_vec)))
print('Error Rate:' + str(sum(error_vec)*100/n_train) + '%')
```




```
Errors: 2230
Error Rate:74.33333333333333%
```

In [ ]: