Assignment 1

a) $y = x_1 a_1 + x_2 a_2 - b$

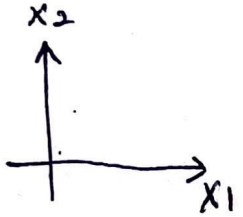
$\text{sign}\{y\} \begin{cases} 1, & y > 0 \\ -1, & y < 0 \end{cases}$

$\rightarrow y = x^T w = [x_1 \; x_2 \; +1] \begin{bmatrix} a_1 \\ a_2 \\ -b \end{bmatrix}$

$\rightarrow \begin{cases} x = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}, & w = \begin{bmatrix} a_1 \\ a_2 \\ -b \end{bmatrix} \end{cases}$

a_1, a_2, b real numbers

b)



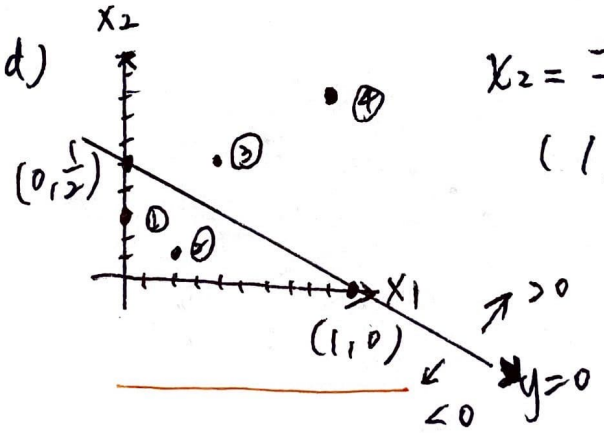
$y = 0 = x_1 a_1 + x_2 a_2 - b$

$\rightarrow x_2 = \frac{-a_1 x_1 + b}{a_2} \rightarrow \begin{cases} \text{slope} = \frac{-a_1}{a_2} \\ \text{intercept with } x_2\text{-axis} = \frac{b}{a_2} \end{cases}$

c)

$$X = \begin{bmatrix} 0 & 0.3 & 1 \\ 0.2 & 0.1 & -1 \\ 0.4 & 0.6 & 1 \\ 0.9 & 0.8 & 1 \end{bmatrix}$$

d)



$x_2 = \frac{-1 x_1 + 1}{2} = \frac{-1}{2} x_1 + \frac{1}{2}$

$(1, 0), (0, \frac{1}{2})$

$\begin{cases} ①, ② < 0 \\ \rightarrow \text{label } -1 \\ ③, ④ > 0 \\ \rightarrow \text{label } 1 \end{cases}$

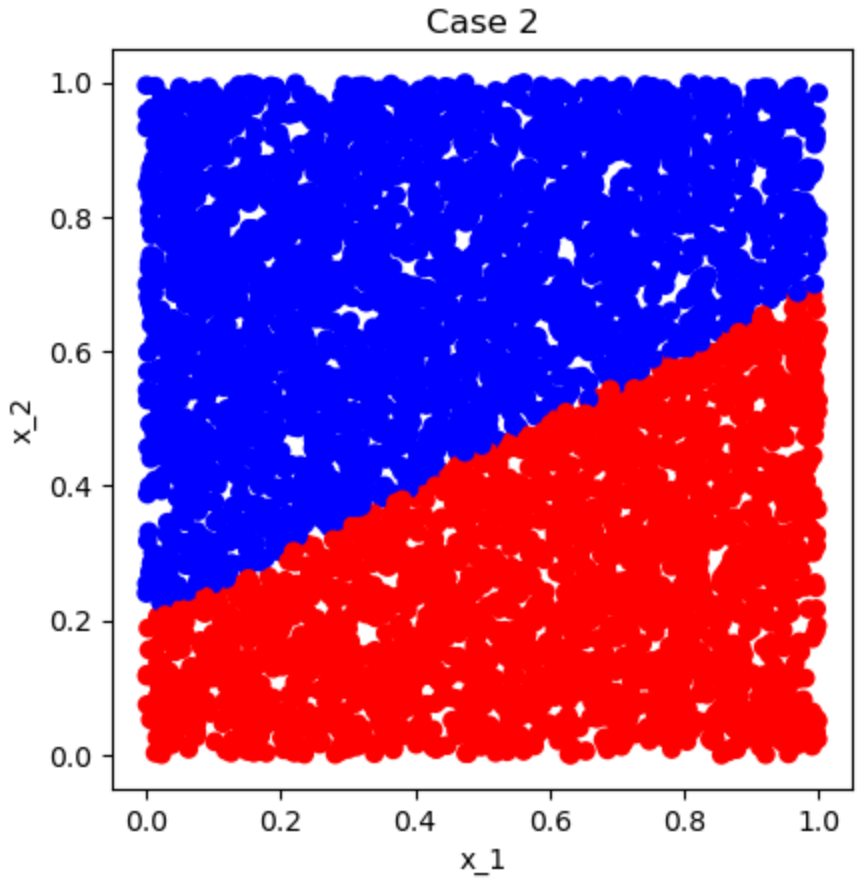
e) boundary seems to go through

$(0, 0.2)$ & $(1, 0.7) \rightarrow x_2 = a x_1 + b \rightarrow x_2 = 0.5 x_1 + 0.2$

$\begin{cases} 0.2 = b \\ 0.7 = a + 0.2 \end{cases}$

$\rightarrow a = 0.5$

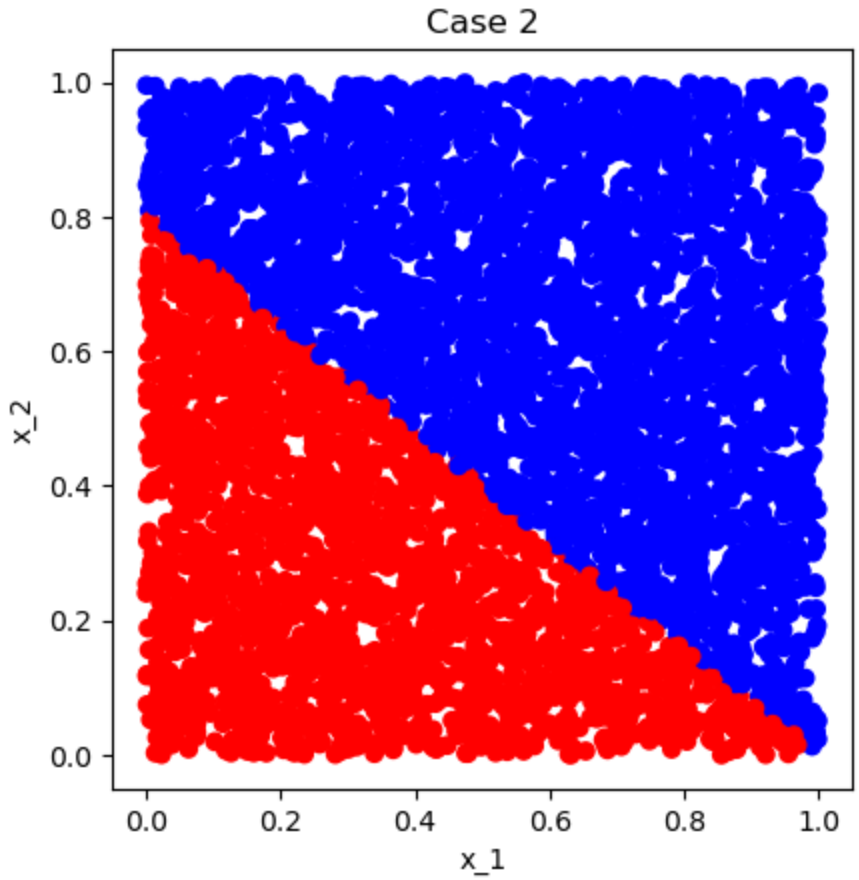
In [1]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt

# number of features
p = 2

# number of examples
n = 5000

# generate matrix of n (random) examples of p features with a column of all ones
X0 = np.random.rand(n,p)
onevec = np.ones(shape = (n,1))
X = np.concatenate((X0,onevec),axis=1)

# Classifier weights
w = [[-1], [2], [-0.4]]

# Multiply feature matrix with weights  yhat = X*w
yhat = X@w

# Decide which class based on whether yhat is >< 0
# sign function returns +1 when yhat(i)>0 and -1 when yhat(i)<0
pred_label = np.sign(yhat);

plt.scatter(X[:,[0]],X[:,[1]], color = ['r' if i==-1 else 'b' for i in pred_label])

plt.xlabel("x_1")
plt.ylabel("x_2")
plt.title("Case 2")
plt.axis('square')
plt.show()
```



e) Describe the decision boundary you observe using a sentence: The decision boundary appears to pass

through point1 at (0, 0.2) and point2 at (1, 0.7), suggesting that the boundary equation could be represented as x_2 = 0.5 * x_1 + 0.2.

In [2]:
```
# Classifier weights
w = [[1.6], [2], [-1.6]]

# Multiply feature matrix with weights  yhat = X*w
yhat = X@w

# Decide which class based on whether yhat is >< 0
# sign function returns +1 when yhat(i)>0 and -1 when yhat(i)<0
pred_label = np.sign(yhat);

plt.scatter(X[:,[0]],X[:,[1]], color = ['r' if i==-1 else 'b' for i in pred_label])

plt.xlabel("x_1")
plt.ylabel("x_2")
plt.title("Case 2")
plt.axis('square')
plt.show()
```



f) Briefly describe how the change in the weights changed the decision boundary: The slope and intercepts of the decision boundary are influenced by both the sign and magnitude of the weights.