CS/ECE/ME 532
Matrix Methods in Machine Learning

Welcome!

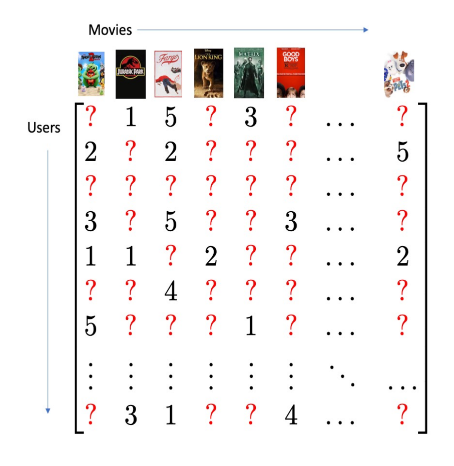
Announcements

- Mid Course Feedback Survey
- This week:
  - Today: finish Unit 4
  - Thursday: start Unit 5
- Next week: spring break
- After spring break:
  - Tuesday: Assessment 3 review
  - Thursday: Assessment 3
- Assignment 7 is due the week after Assessment 3, but:
  - Q1 on the Eigendecomposition (attempt before the assessment)

Activity 15

Matrix Completion

$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$$
$$\texttt{diag}(\sigma) = \Sigma$$



$$\begin{bmatrix} ? & 1 & 5 & ? & 3 & ? & \dots & ? \\ 2 & ? & 2 & ? & ? & ? & \dots & 5 \\ ? & ? & ? & ? & ? & ? & \dots & ? \\ 3 & ? & 5 & ? & ? & 3 & \dots & ? \\ 1 & 1 & ? & 2 & ? & ? & \dots & 2 \\ ? & ? & 4 & ? & ? & ? & \dots & ? \\ 5 & ? & ? & ? & 1 & ? & \dots & ? \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \dots \\ ? & 3 & 1 & ? & ? & 4 & \dots & ? \end{bmatrix}$$

Goal: find the matrix of minimum rank that agrees on all known entries.

Let $\mathcal{S} = \{\boldsymbol{A} : A_{i,j} = X_{i,j} \ \forall \ (i,j) \in \Omega\}$

$$\widehat{\boldsymbol{X}} = \arg\min_{\boldsymbol{A}\in\mathcal{S}} \text{rank}(\boldsymbol{A})$$ Hard!

relaxation

$$\widehat{\boldsymbol{X}} = \arg\min_{\boldsymbol{A}\in\mathcal{S}} ||\sigma||_1$$ Tractable!

nuclear norm $||\boldsymbol{A}||_*$

[1] Guaranteed Minimum-Rank Solutions of Linear Matrix Equations via Nuclear Norm Minimization, B. Recht et. al.

Iterative Singular Value Thresholding

0. Set unknown entries to zero.
1. find best rank r approximation — *take SVD and truncate*
2. reset known entries to original values
3. repeat steps 1-2 ...

[2] A Singular Value Thresholding Algorithm for Matrix Completion, Cai et. al.

Item 3 is an (optional) preview of Activity 16

3a) Hint: $\mathbf{X}^{\mathbf{T}}\mathbf{X}$ is positive definite…

3b, 3c) Equation of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$