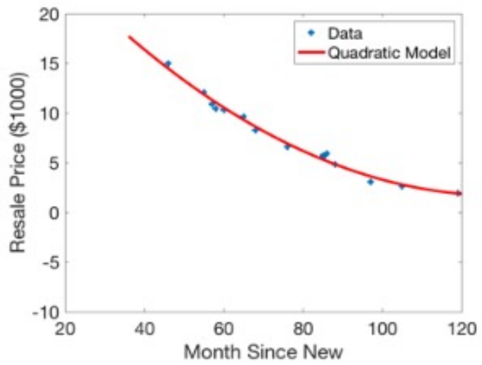
CS/ECE/ME 532
Matrix Methods in Machine Learning

Welcome!

This week and beyond

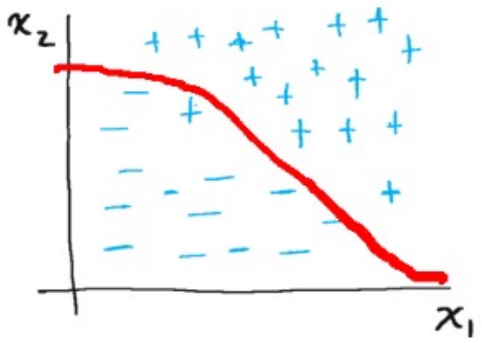
- **Unit 2:** Linear systems of equations in ML
  - Foundational Linear Algebra topics
  - Prediction and forecasting
  - Classifier design
  - Setting the stage for what's coming next: the SVD

Key idea: Find $\boldsymbol{w}$ such that $\boldsymbol{X}\boldsymbol{w} \approx \boldsymbol{y}$



$$\begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ & \vdots & \\ 1 & t_n & t_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$

Model fitting



$$\mathrm{sign}(\boldsymbol{x}_1^T \boldsymbol{w}) = -1$$
$$\mathrm{sign}(\boldsymbol{x}_2^T \boldsymbol{w}) = +1$$
$$\vdots$$

Classification

Activity 4

Definitions:

if a squared matrix, linear independent

- $\text{span}(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots \boldsymbol{a}_n)$ = all the vectors we can write as a weighted sum of $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots \boldsymbol{a}_n$
- $\boldsymbol{a}_1, \ldots, \boldsymbol{a}_n$ are linearly dependent if we can write $\sum_i \alpha_i \boldsymbol{a}_i = \boldsymbol{0}$ for α_i that aren't all zero
- $\text{rank}(\boldsymbol{A})$ = number of linearly independent columns (or rows) in $\boldsymbol{A}$

given given

$$\boldsymbol{A}\boldsymbol{w} = \boldsymbol{d}$$

Solve for $\boldsymbol{w}$

Three options:

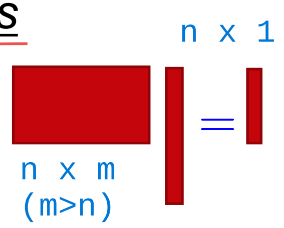
1. Unique solution
2. Infinite number of solutions
3. No solution

Option 2: *An infinite number of solutions*

- happens when:

  i) d is in the span of the columns of A and

  ii) columns of A are linearly dependent

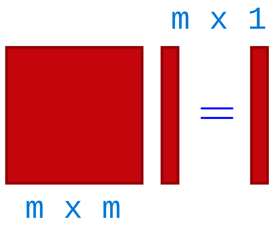


Option 1: *A unique solution*

- usually doesn't happen with real data
- happens when:

  i) d is in the span of the columns of A and

  ii) columns of A are linearly independent



Option 3: *No solution*

- Usually what happens with real data
- We can find approximate solution
- happens when:

  d is not in the span of the columns of A