

An aerial photograph of a city, likely Madison, Wisconsin, taken from a high vantage point looking down at the city and the surrounding water. The sun is setting behind a hill in the background, casting a warm, golden glow over the entire scene. The city is densely packed with buildings, and the water is filled with numerous sailboats. A large, semi-transparent rectangular box is overlaid on the center of the image, containing the course title in large, bold, black text.

CS/ECE/ME 532 Matrix Methods in Machine Learning

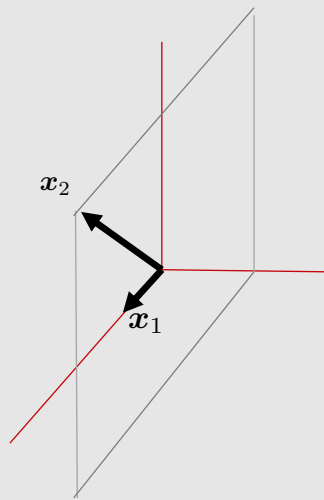
Welcome!



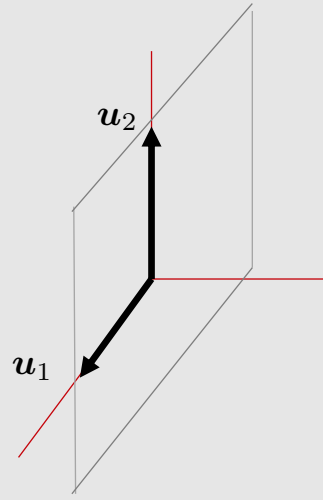
Activity 7



Matt Malloy
(lecture 2.9)



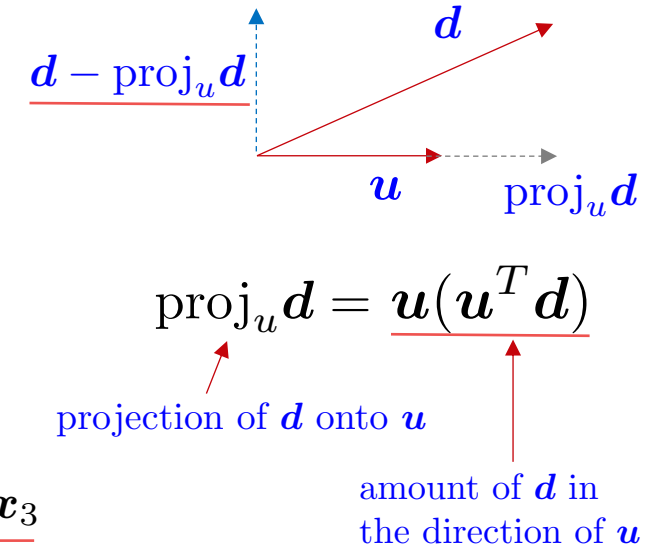
A basis



An orthonormal basis

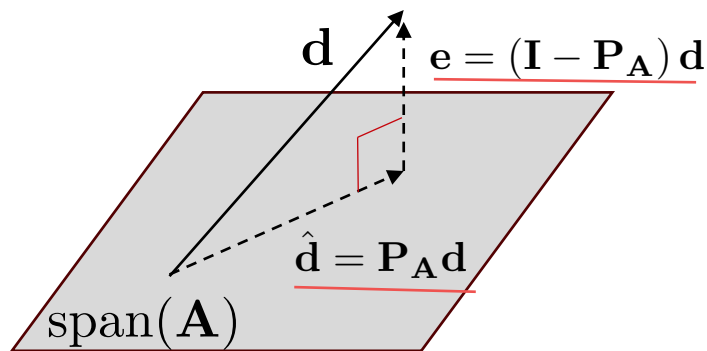
Gram-Schmidt Orthogonalization

1. set $\tilde{u}_1 = x_1$
2. normalize: $u_1 = \frac{\tilde{u}_1}{\|\tilde{u}_1\|_2}$
3. set $\tilde{u}_2 = x_2 - \text{proj}_{u_1} x_2$
4. normalize: $u_2 = \frac{\tilde{u}_2}{\|\tilde{u}_2\|_2}$
5. set $\tilde{u}_3 = x_3 - \text{proj}_{u_1} x_3 - \text{proj}_{u_2} x_3$
- \vdots



$$\underline{d = Aw}$$

$$\begin{matrix} | \\ = n \\ | \end{matrix} \begin{matrix} p \\ \boxed{A} \end{matrix} \begin{matrix} | \\ \\ | \end{matrix}$$



$U^T U = I$ / $U_i^T U_j = \delta_{ij}$
How can we leverage an orthonormal basis?:

$$\hat{d} = Aw = A(A^T A)^{-1} A^T d = P_A d$$

Today

$$= \underline{UU^T d = P_U d}$$

Key fact: $\underline{U^T U = I}$