In [123…
```python
import numpy as np
from scipy.io import loadmat
import matplotlib.pyplot as plt
```

In [124…
```python
def ista_solve_hot( A, d, la_array ):
    # ista_solve_hot: Iterative soft-thresholding for multiple values of
    # lambda with hot start for each case - the converged value for the previous
    # value of lambda is used as an initial condition for the current lambda.
    # this function solves the minimization problem
    # Minimize |Ax-d|_2^2 + lambda*|x|_1 (Lasso regression)
    # using iterative soft-thresholding.
    max_iter = 10**4
    tol = 10**(-3)
    tau = 1/np.linalg.norm(A,2)**2
    n = A.shape[1]
    w = np.zeros((n,1))
    num_lam = len(la_array)
    X = np.zeros((n, num_lam))
    for i, each_lambda in enumerate(la_array):
        for j in range(max_iter):
            z = w - tau*(A.T@(A@w-d))
            w_old = w
            w = np.sign(z) * np.clip(np.abs(z)-tau*each_lambda/2, 0, np.inf)
            X[:, i:i+1] = w
            if np.linalg.norm(w - w_old) < tol:
                break
    return X
```

1a)

As the value of lambda increases, the first norm typically decreases, while the second norm increases. This reflects a fundamental trade-off between bias and variance in the model.

In [125…
```python
X = loadmat("BreastCancer.mat")['X']
y = loadmat("BreastCancer.mat")['y']
print(X.shape)
```

```
(295, 8141)
```

In [126…
```python
X_100 = X[:100]
y_100 = y[:100]

lam = np.logspace(-6, np.log10(20), 10)
n_lam = len(lam)
print(lam)
```

```
[1.00000000e-06 6.47478803e-06 4.19228800e-05 2.71441762e-04
 1.75752787e-03 1.13796204e-02 7.36806300e-02 4.77066461e-01
 3.08890421e+00 2.00000000e+01]
```

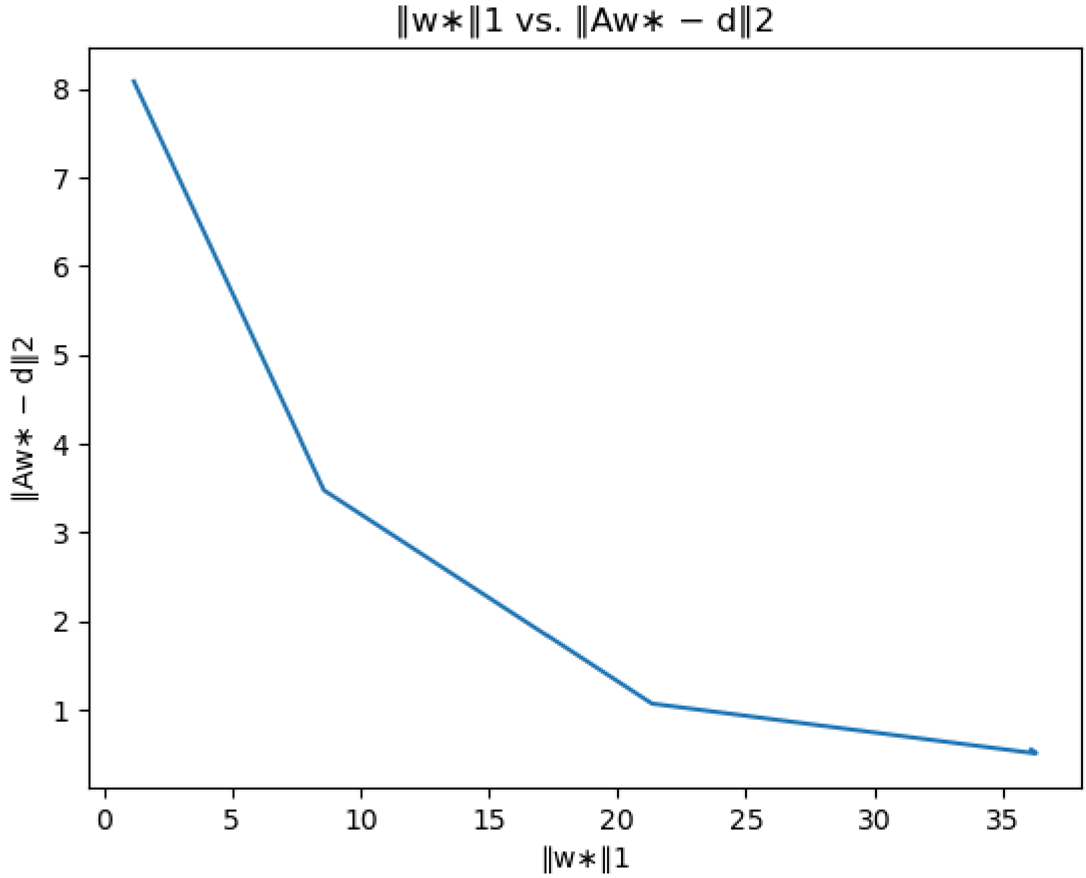
In [127…
```python
w = ista_solve_hot(X_100, y_100, lam)
print(w.shape)
```

```
(8141, 10)
```

In [145…

```
norm1 = np.linalg.norm(w, 1, axis = 0)
norm2 = np.zeros(n_lam)
for i, v in enumerate(lam):
    norm2[i] = np.linalg.norm(X_100 @ w[:, i:i+1] - y_100, 2)
    print('lambda:', lam[i], '\t norm1:', norm1[i], '\t norm2:', norm2[i])

plt.plot(norm1, norm2)
plt.xlabel('‖w∗‖1')
plt.ylabel('‖Aw∗ − d‖2')
plt.title('‖w∗‖1 vs. ‖Aw∗ − d‖2')
plt.show()
```

```
lambda: 1e-06    norm1: 36.10905595263049        norm2: 0.5517470751599178
lambda: 6.4747880286952536e-06   norm1: 36.151741471852745       norm2: 0.5436398
408034341
lambda: 4.192288001653537e-05    norm1: 36.193673714475096       norm2: 0.5356685
010959967
lambda: 0.00027144176165949066   norm1: 36.2346929549114         norm2: 0.5278377
493439237
lambda: 0.0017575278688808204    norm1: 36.27374547957   norm2: 0.520192903482240
5
lambda: 0.011379620405527818     norm1: 36.303817819750485       norm2: 0.5130421
807992127
lambda: 0.07368062997280773      norm1: 36.27970077895268        norm2: 0.5084634
133651202
lambda: 0.4770664608946602       norm1: 21.338759130907867       norm2: 1.0702501
054748743
lambda: 3.088904209892758        norm1: 8.57945814107803         norm2: 3.4769493
942094587
lambda: 20.000000000000004       norm1: 1.1791406752485474       norm2: 8.0830395
4614701
```



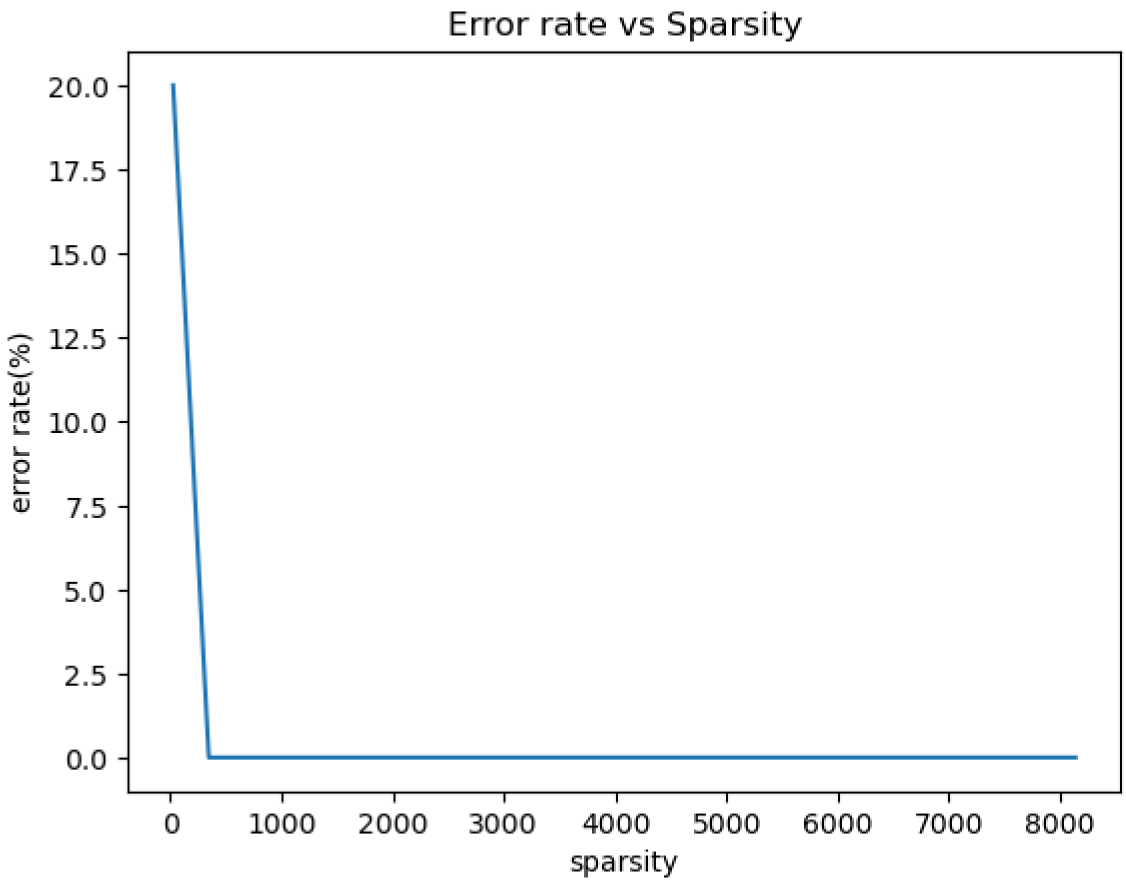
1b)

As lambda increases, the error rate tends to rise, whereas the sparsity of the model generally decreases.

In [147…

```
error_rate = np.zeros(n_lam)
sparsity = np.zeros(n_lam)
for i in range(n_lam):
    error_rate[i] = np.sum(np.sign(X_100 @ w[:, i:i+1]) != y_100)*100 / len(y_10
    sparsity[i] = np.count_nonzero(w[:, i:i+1])
    print('lambda:', lam[i], '\t error_rate:', error_rate[i], '\t sparsity:', sp
    # print('lambda:', lam[i], '\t error_rate:', error_rate[i], '\t sparsity:',


plt.plot(sparsity, error_rate)
plt.xlabel('sparsity')
plt.ylabel('error rate(%)')
plt.title('Error rate vs Sparsity')
plt.show()
```

```
lambda: 1e-06    error_rate: 0.0         sparsity: 8141.0
lambda: 6.4747880286952536e-06    error_rate: 0.0        sparsity: 8141.0
lambda: 4.192288001653537e-05     error_rate: 0.0        sparsity: 8141.0
lambda: 0.00027144176165949066    error_rate: 0.0        sparsity: 8141.0
lambda: 0.0017575278688808204     error_rate: 0.0        sparsity: 8141.0
lambda: 0.011379620405527818      error_rate: 0.0        sparsity: 8140.0
lambda: 0.07368062997280773       error_rate: 0.0        sparsity: 8129.0
lambda: 0.4770664608946602        error_rate: 0.0        sparsity: 2701.0
lambda: 3.088904209892758         error_rate: 0.0        sparsity: 346.0
lambda: 20.000000000000004        error_rate: 20.0       sparsity: 25.0
```



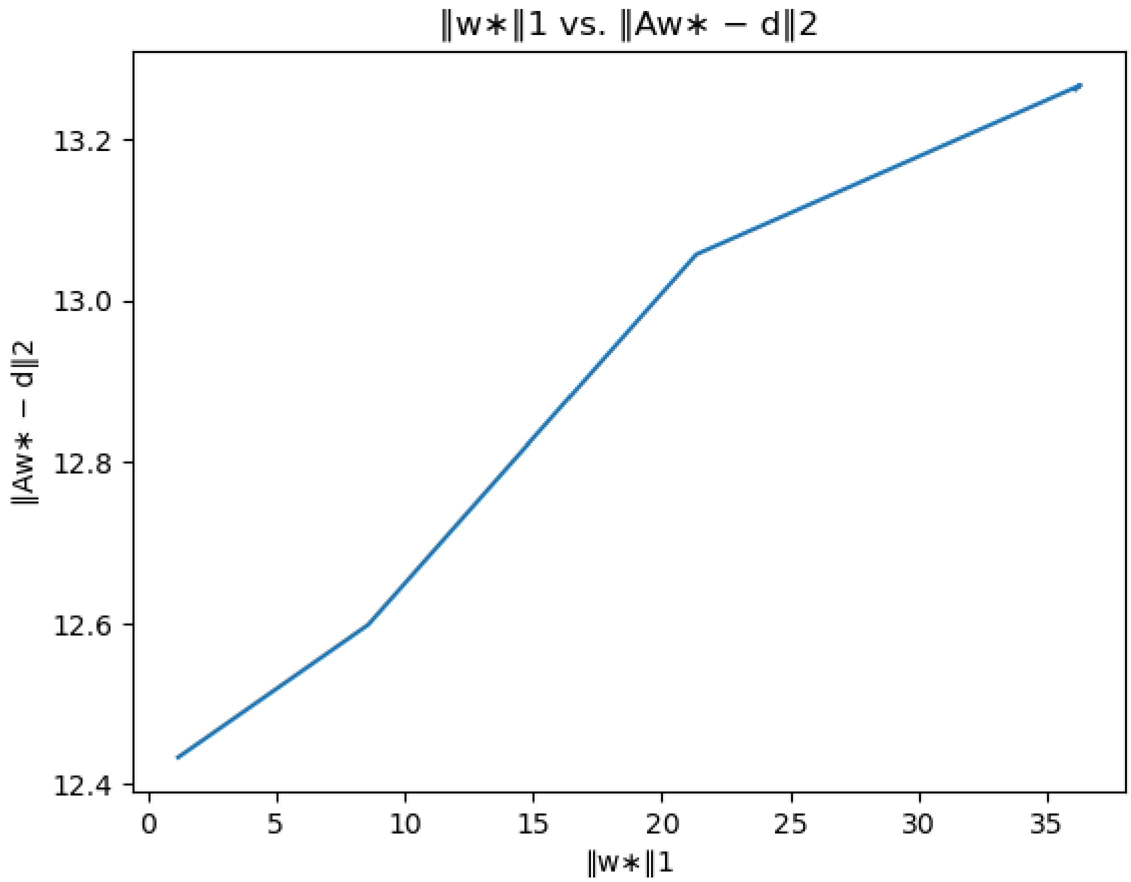
1c)

When the same classifier is applied to new data, a larger lambda value typically results in a slightly smaller second norm and a reduced error rate.

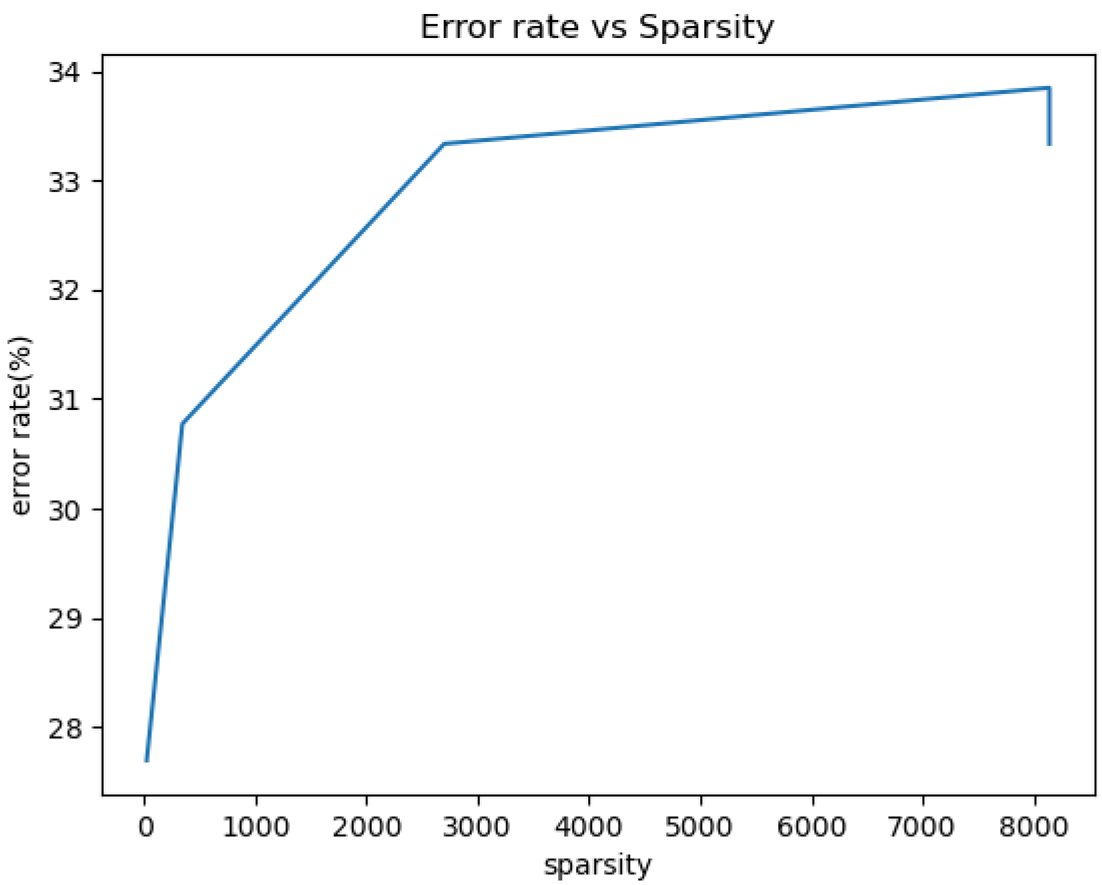
In [148…

```
X_rest = X[100:]
y_rest = y[100:]

norm1 = np.linalg.norm(w, 1, axis = 0)
norm2 = np.zeros(n_lam)
for i, v in enumerate(lam):
    norm2[i] = np.linalg.norm(X_rest @ w[:, i:i+1] - y_rest, 2)
    print('lambda:', lam[i], '\t norm1:', norm1[i], '\t norm2:', norm2[i])

plt.plot(norm1, norm2)
plt.xlabel('‖w∗‖1')
plt.ylabel('‖Aw∗ − d‖2')
plt.title('‖w∗‖1 vs. ‖Aw∗ − d‖2')
plt.show()

error_rate = np.zeros(n_lam)
sparsity = np.zeros(n_lam)
for i in range(n_lam):
    error_rate[i] = np.sum(np.sign(X_rest @ w[:, i:i+1]) != y_rest)*100 / len(y_
    sparsity[i] = np.count_nonzero(w[:, i:i+1])
    print('lambda:', lam[i], '\t error_rate:', error_rate[i], '\t sparsity:', sp

plt.plot(sparsity, error_rate)
plt.xlabel('sparsity')
plt.ylabel('error rate(%)')
plt.title('Error rate vs Sparsity')
plt.show()
```

```
lambda:  1e-06    norm1:  36.10905595263049        norm2:  13.261888968463277
lambda:  6.4747880286952536e-06    norm1:  36.151741471852745        norm2:  13.263176
846667902
lambda:  4.192288001653537e-05    norm1:  36.193673714475096        norm2:  13.264442
63283514
lambda:  0.00027144176165949066    norm1:  36.2346929549114         norm2:  13.265681
706787987
lambda:  0.0017575278688808204    norm1:  36.27374547957    norm2:  13.26686128787486
8
lambda:  0.011379620405527818      norm1:  36.303817819750485        norm2:  13.267767
429817468
lambda:  0.07368062997280773       norm1:  36.27970077895268         norm2:  13.267012
155120579
lambda:  0.4770664608946602        norm1:  21.338759130907867        norm2:  13.057214
165324124
lambda:  3.088904209892758         norm1:  8.57945814107803          norm2:  12.598122
884562544
lambda:  20.000000000000004        norm1:  1.1791406752485474        norm2:  12.433559
544541195
```




```
lambda: 1e-06    error_rate: 33.333333333333336          sparsity: 8141.0
lambda: 6.4747880286952536e-06   error_rate: 33.84615384615385   sparsity: 8141.0
lambda: 4.192288001653537e-05    error_rate: 33.84615384615385   sparsity: 8141.0
lambda: 0.00027144176165949066   error_rate: 33.84615384615385   sparsity: 8141.0
lambda: 0.0017575278688808204    error_rate: 33.84615384615385   sparsity: 8141.0
lambda: 0.011379620405527818     error_rate: 33.84615384615385   sparsity: 8140.0
lambda: 0.07368062997280773      error_rate: 33.84615384615385   sparsity: 8129.0
lambda: 0.4770664608946602       error_rate: 33.333333333333336          sparsit
y: 2701.0
lambda: 3.088904209892758        error_rate: 30.76923076923077   sparsity: 346.0
lambda: 20.000000000000004       error_rate: 27.692307692307693          sparsit
y: 25.0
```



2.

The performance of the LASSO and ridge regression is similar.

In [131…

```
np.random.seed(0)
random_indices = np.random.permutation(X.shape[0])
random_X = X[random_indices]

subset_size = [30]*5 + [29]*5
subset_X = [random_X[sum(subset_size[:i]):sum(subset_size[:i+1])] for i in range
```

In [132…

```
random_y = y[random_indices]
subset_y = [random_y[sum(subset_size[:i]):sum(subset_size[:i+1])] for i in range
```

In [133…

```
def lasso_model(i, j, lam):
    X_train = np.concatenate(subset_X[:i] + subset_X[i+1:j] + subset_X[j+1:])
    y_train = np.concatenate(subset_y[:i] + subset_y[i+1:j] + subset_y[j+1:])
    X_val = subset_X[i]
    y_val = subset_y[i]
    X_test = subset_X[j]
    y_test = subset_y[j]
    w = ista_solve_hot(X_train, y_train, lam)

    best_lam = 0
    lowest_error_rate = 1000
    best_w = np.zeros((X_train.shape[1],1))
    for k in range(len(lam)):
        error_rate = np.sum(np.sign(X_val @ w[:, k:k+1]) != y_val)*100 / len(y_v
        if error_rate < lowest_error_rate:
            lowest_error_rate = error_rate
```

```
            best_lam = lam[k]
            best_w = w[:, k:k+1]

    error_rate = np.sum(np.sign(X_test @ best_w) != y_test)*100 / len(y_test)
    squared_error = np.linalg.norm(X_test @ best_w - y_test, 2)
    return  error_rate, squared_error
```

In [134…

```
def ridge_reg(A, d, lam):
    tau = 1/np.linalg.norm(A, 2)**2
    r_num = A.shape[0]
    c_num = A.shape[1]
    w = np.zeros((c_num,1))
    lam_num = len(lam)
    w_set = np.zeros((c_num, lam_num))
    for i, v in enumerate(lam):
        w = A.T @ np.linalg.inv(A@A.T + v*np.eye(r_num)) @ d
        w_set[:, i:i+1] = w
    return w_set
```

In [138…

```
def ridge_model(i, j, lam):
    X_train = np.concatenate(subset_X[:i] + subset_X[i+1:j] + subset_X[j+1:])
    y_train = np.concatenate(subset_y[:i] + subset_y[i+1:j] + subset_y[j+1:])
    X_val = subset_X[i]
    y_val = subset_y[i]
    X_test = subset_X[j]
    y_test = subset_y[j]
    w = ridge_reg(X_train, y_train, lam)

    best_lam = 0
    lowest_error_rate = 1000
    best_w = np.zeros((X_train.shape[1],1))
    for k in range(len(lam)):
        error_rate = np.sum(np.sign(X_val @ w[:, k:k+1]) != y_val)*100 / len(y_v
        if error_rate < lowest_error_rate:
            lowest_error_rate = error_rate
            best_lam = lam[k]
            best_w = w[:, k:k+1]

    error_rate = np.sum(np.sign(X_test @ best_w) != y_test)*100 / len(y_test)
    squared_error = np.linalg.norm(X_test @ best_w - y_test, 2)
    return  error_rate, squared_error
```

In [136…

```
total_error_rate = 0
total_squared_error = 0
for i in range(10):
    for j in range(10):
        if i != j:
            # print('i:', i, 'j:', j)
            error_rate, squared_error = lasso_model(i, j, lam)
            total_error_rate += error_rate
            total_squared_error += squared_error
print('total error rate:', total_error_rate/90, '%')
print('total squared error:', total_squared_error/90)
```

```
total error rate: 13.87611749680715 %
total squared error: 2.7501117229014698
```

In [139…

```
total_error_rate = 0
total_squared_error = 0
```

```python
for i in range(10):
    for j in range(10):
        if i != j:
            # print('i:', i, 'j:', j)
            error_rate, squared_error = ridge_model(i, j, lam)
            total_error_rate += error_rate
            total_squared_error += squared_error
print('total error rate:', total_error_rate/90, '%')
print('total squared error:', total_squared_error/90)
```

```
total error rate: 13.38697318007663 %
total squared error: 2.4736496585048036
```

In [ ]:

```python

```