CS/ECE/ME 532
Matrix Methods in Machine Learning

Welcome!

Activity 9

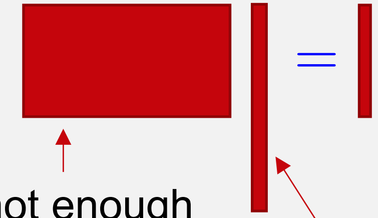
For 2b: $\boldsymbol{X} = \boldsymbol{U}\boldsymbol{\Sigma}$

Orthonormal columns

diagonal, depends on γ

(infinite solutions)

$\boldsymbol{X}\boldsymbol{w} = \boldsymbol{y}$



not enough equations

Too many unknowns

(X is lin dep)

$\boldsymbol{X}^T\boldsymbol{X}$ is not invertible!

```
np.linalg.inv(X.T@X)
---------------------------------------------------------------------------
LinAlgError: Singular matrix
```

Infinite number of solutions to both:

$$\boldsymbol{X}\boldsymbol{w} = \boldsymbol{y} \qquad \min_{\boldsymbol{w}} ||\boldsymbol{X}\boldsymbol{w} - \boldsymbol{y}||^2$$

Ridge Regression

→ Find an *interesting* solution -- small $||\boldsymbol{w}||^2$

$$\min_{\boldsymbol{w}} ||\boldsymbol{X}\boldsymbol{w} - \boldsymbol{y}||^2 + \lambda||\boldsymbol{w}||^2$$

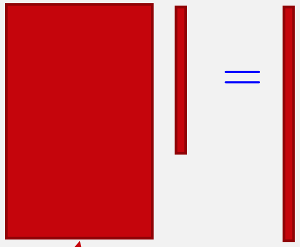
Regularizer!

Regularization parameter!

$$\boldsymbol{w}^* = (\boldsymbol{X}^T\boldsymbol{X} + \lambda\boldsymbol{I})^{-1}\boldsymbol{X}^T\boldsymbol{y}$$

(no solutions)

$\boldsymbol{X}\boldsymbol{w} = \boldsymbol{y}$



Enough equations, but columns are “barely” linearly independent

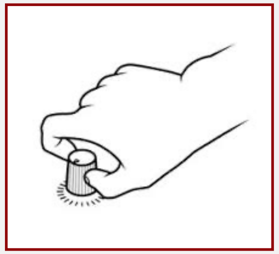
$\boldsymbol{X}^T\boldsymbol{X}$ is invertible (in theory!)

```
>> inv(X'*X)
Warning: Matrix is close to singular
or badly scaled. Results may be
inaccurate.
```

```
np.linalg.inv(X.T@X)
---------------------------------------------------------------------------
LinAlgError: Singular matrix
```

$\boldsymbol{X}^T\boldsymbol{X}$ is ill conditioned → **Ridge Regression**

$\boldsymbol{X}^T\boldsymbol{X} + \lambda\boldsymbol{I}$ is always invertible for positive λ



How can we find a good value for λ?

→ **Cross-validation**