

An aerial photograph of a city, likely Madison, Wisconsin, taken from a high vantage point looking down at the city and the surrounding water. The sun is setting behind a hill in the background, creating a warm, golden glow over the entire scene. The city is densely packed with buildings, and the water is filled with numerous sailboats. A large, semi-transparent rectangular box is overlaid on the center of the image, containing the course title in large, bold, black text.

CS/ECE/ME 532 Matrix Methods in Machine Learning

Welcome!



Activity 6



unknown

$$Ax = y$$

(usually) no solution!

$$n \quad \overset{p}{A} \quad = \quad \quad$$

- Usually what happens with real data
- The next best thing: find approximate solution

$$x^* = \arg \min_x \|Ax - y\|_2^2$$

this is a called a least squares solution

$$x^* = (A^T A)^{-1} A^T y$$

when does this inverse exist?

Note: X must be a square matrix!

$$X^{-1}(-1) @ X = I$$

$$XX^{-1} = I$$

if rank(A) = p, then $A^T A$ is invertible



The least squares solution is unique

Positive definiteness (P.D.)

$$Q \succ 0$$



$$x^T Q x > 0 \text{ for all } x \neq 0$$



Q is invertible

Invertible Matrix Theorem

The invertible matrix theorem is a theorem in linear algebra which gives a series of equivalent conditions for an $n \times n$ square matrix A to have an inverse. In particular, A is invertible if and only if any (and hence, all) of the following hold:

1. A is row-equivalent to the $n \times n$ identity matrix I_n .
2. A has n pivot positions.
3. The equation $Ax = 0$ has only the trivial solution $x = 0$.
4. The columns of A form a linearly independent set.
5. The linear transformation $x \mapsto Ax$ is one-to-one.
6. For each column vector $b \in \mathbb{R}^n$, the equation $Ax = b$ has a unique solution.
7. The columns of A span \mathbb{R}^n .
8. The linear transformation $x \mapsto Ax$ is a surjection.
9. There is an $n \times n$ matrix C such that $CA = I_n$.
10. There is an $n \times n$ matrix D such that $AD = I_n$.
11. The transpose matrix A^T is invertible.
12. The columns of A form a basis for \mathbb{R}^n .
13. The column space of A is equal to \mathbb{R}^n .
14. The dimension of the column space of A is n .
15. The rank of A is n .
16. The null space of A is $\{0\}$.
17. The dimension of the null space of A is 0.
18. 0 fails to be an eigenvalue of A .
19. The determinant of A is not zero.
20. The orthogonal complement of the column space of A is $\{0\}$.
21. The orthogonal complement of the null space of A is \mathbb{R}^n .
22. The row space of A is \mathbb{R}^n .
23. The matrix A has n non-zero singular values.