ass02-calculation

February 15, 2024

[11]:
```
import numpy as np
```

0.0.1 1. a) Are the columns of the following matrix linearly independent?

Yes, because the rank number is equal to the column number.

[12]:
```
A = np.array([[0.92, 0.92], [-0.92, 0.92], [0.92, -0.92], [-0.92, -0.92]])
np.linalg.matrix_rank(A)
```

[12]:
```
2
```

0.0.2 1. b) Are the columns of the following matrix linearly independent?

Yes, because the rank number is equal to the column number.

[13]:
```
A = np.array([[1, 1, 1], [-1, 1, -1], [1, -1, -1]])
np.linalg.matrix_rank(A)
```

[13]:
```
3
```

0.0.3 1. c) Are the columns of the following matrix linearly independent?

No, because the rank number is not equal to the column number.

[14]:
```
A = np.array([[1, 2, 2], [3, 4, 5], [5, 6, 8]])
np.linalg.matrix_rank(A)
```

[14]:
```
2
```

0.0.4 1. d) What is the rank of the following matrix?

2.

[15]:
```
A = np.array([[5, 2], [-5, 2], [5, -2]])
np.linalg.matrix_rank(A)
```

[15]:
```
2
```

0.0.5 1. e) Suppose the matrix in part d is used in to solve the system of linear equations A__T@A@w = d. Does a unique solution exist? Explain why.

Yes, a unique solution exists, because the the matrix, A__T@A, is invertible.

[16]:
```
A = np.array([[5, 2], [-5, 2], [5, -2]])
A_T = A.transpose()
M = A_T @ A
M_inv = np.linalg.inv(M)
```

[ ]:
```

```

[ ]:
```

```

Assignment 2

∵ a) $\|\cdot\|_a$ and $\|\cdot\|_b$ are norms on $\mathbb{R}^n$

∴ $f(x)=\|x\|_a+\|x\|_b$ a norm on $\mathbb{R}^n$

satisfies properties

(1) $\|x\|_a \geq 0$, $\|x\|_b \geq 0$, $\forall x$

$\rightarrow f(x)=\|x\|_a+\|x\|_b \geq 0$

(2) $\|x\|_a = 0$, $\|x\|_b = 0$, iff $x=0$

$\rightarrow f(x)=\|x\|_a+\|x\|_b=0$

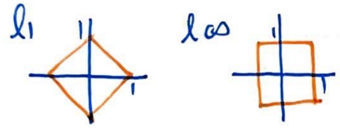
(3) $\cancel{\|bx\|_a+\|bx\|_b}$ $\forall \alpha \in \mathbb{R}$

$\|\alpha x\|_a+\|\alpha x\|_b = \alpha(\|x\|_a+\|x\|_b) = \alpha f(x)$

(4) $f(x+y) = \|x+y\|_a+\|x+y\|_b \leq \|x\|_a+\|x\|_b+\|y\|_a+\|y\|_b = f(x)+f(y)$

b) norm ball in $\mathbb{R}^2$

∴ $f(x)=\|x\|_1+\|x\|_\infty=1$

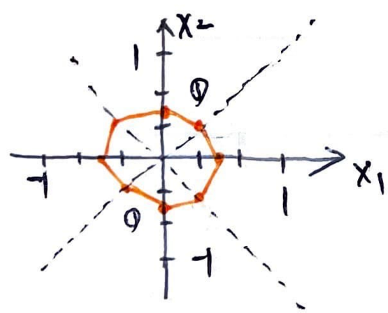


$\|x\|_1 = |x_1|+|x_2|$

$$\|x\|_\infty = \begin{cases} |x_2| \geq |x_1| \rightarrow |x_2| \\ |x_1| > |x_2| \rightarrow |x_1| \end{cases}$$

(max)

$$\Rightarrow \begin{cases} |x_2| > |x_1| \rightarrow 2|x_2|+|x_1|=1 \quad - ① \\ |x_1| > |x_2| \rightarrow 2|x_1|+|x_2|=1 \end{cases}$$



① $(0, \frac{1}{2})$

$(\frac{1}{3}, \frac{1}{3})$