

An aerial photograph of a city, likely Madison, Wisconsin, taken from a high vantage point. The city is situated on a peninsula, with a large body of water (Monona Lake) in the foreground and background. The sun is setting behind a hill in the distance, creating a warm, golden glow over the entire scene. Several sailboats are visible on the water. A large, semi-transparent rectangular box is overlaid on the center of the image, containing the course title in black text.

# CS/ECE/ME 532 Matrix Methods in Machine Learning

*Welcome!*





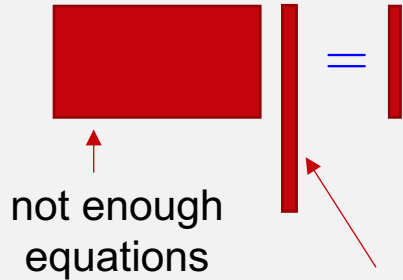
# Activity 9

For 2b:  $X = U\Sigma$  Orthonormal columns  
diagonal, depends on  $\gamma$



(infinite solutions)

$$Xw = y$$



Too many unknowns

(X is lin dep)

$X^T X$  is not invertible!

```
np.linalg.inv(X.T@X)
```

**LinAlgError: Singular matrix**

Infinite number of solutions to both:

$$Xw = y$$

$$\min_w ||Xw - y||^2$$

## Ridge Regression

→ Find an *interesting* solution -- small  $||w||^2$

$$\min_w ||Xw - y||^2 + \lambda ||w||^2$$

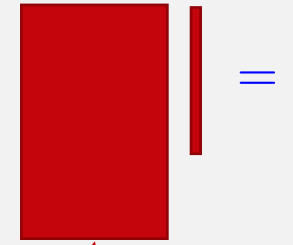
Regularizer!

Regularization parameter!

$$w^* = (X^T X + \lambda I)^{-1} X^T y$$

(no solutions)

$$Xw = y$$



Enough equations, but  
columns are “barely”  
linearly independent

$X^T X$  is invertible (in theory!)

```
>> inv(X'*X)
```

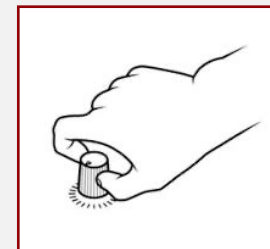
Warning: Matrix is close to singular  
or badly scaled. Results may be  
inaccurate.

```
np.linalg.inv(X.T@X)
```

**LinAlgError: Singular matrix**

$X^T X$  is ill conditioned → **Ridge Regression**

$X^T X + \lambda I$  is always invertible for positive  $\lambda$



How can we find a good value for  $\lambda$ ?

→ **Cross-validation**