

An aerial photograph of a city, likely Madison, Wisconsin, taken from a high vantage point looking down at the city and the surrounding water. The sun is setting behind a hill in the background, casting a warm, golden glow over the entire scene. The city is built on a peninsula, with numerous buildings and streets visible. The water is a deep blue, and several sailboats are scattered across it. A large, semi-transparent rectangular box is overlaid on the center of the image, containing the course title in large, bold, black text.

CS/ECE/ME 532 Matrix Methods in Machine Learning

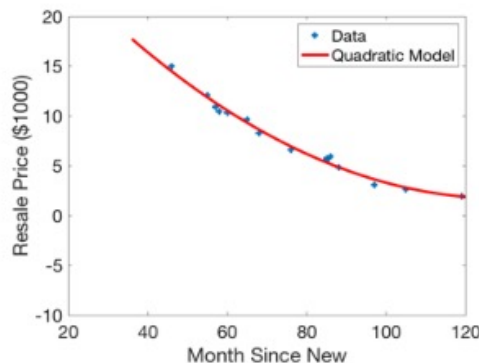
Welcome!



This week and beyond

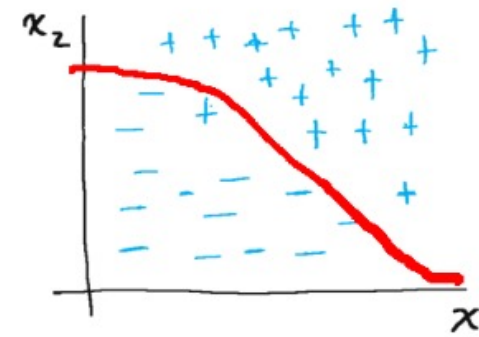
- **Unit 2:** Linear systems of equations in ML
 - Foundational Linear Algebra topics
 - Prediction and forecasting
 - Classifier design
 - Setting the stage for what's coming next: the SVD

Key idea: Find w such that $Xw \approx y$



Model fitting

$$\begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \vdots & \vdots & \vdots \\ 1 & t_n & t_n^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$



$$\begin{aligned} \text{sign}(x_1^T w) &= -1 \\ \text{sign}(x_2^T w) &= +1 \\ &\vdots \end{aligned}$$

Classification

Activity 4



Definitions:

- $\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$ = all the vectors we can write as a weighted sum of $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n$ if a squared matrix, linear independent
- $\mathbf{a}_1, \dots, \mathbf{a}_n$ are linearly dependent if we can write $\sum_i \alpha_i \mathbf{a}_i = \mathbf{0}$ for α_i that aren't all zero
- $\text{rank}(\mathbf{A})$ = number of linearly independent columns (or rows) in \mathbf{A}

given $\mathbf{A} \mathbf{w} = \mathbf{d}$ given

Solve for \mathbf{w}

Three options:

1. Unique solution
2. Infinite number of solutions
3. No solution

Option 1: A unique solution

- usually doesn't happen with real data
- happens when:

- \mathbf{d} is in the span of the columns of \mathbf{A} and
- columns of \mathbf{A} are linearly independent

$m \times m$ $m \times 1$

Option 2: An infinite number of solutions

- happens when:

- \mathbf{d} is in the span of the columns of \mathbf{A} and
- columns of \mathbf{A} are linearly dependent

$n \times m$ $n \times 1$
($m > n$)

Option 3: No solution

- Usually what happens with real data
- We can find approximate solution
- happens when:
 \mathbf{d} is not in the span of the columns of \mathbf{A}

$n \times m$ $n \times 1$
($m < n$)