Activity 21

1. $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\hat{y} = x_1^2 w_1 + x_2^2 w_2 + \sqrt{2} x_1 x_2 w_3 + \sqrt{2} x_1 w_4 + \sqrt{2} x_2 w_5 + w_6$

a) $\hat{y} = \phi^T(x) w$, $\phi$? $w$?

$\phi(x) = [x_1^2 \; x_2^2 \; \sqrt{2}x_1x_2 \; \sqrt{2}x_1 \; \sqrt{2}x_2 \; 1]$

$w = [w_1 \; w_2 \; w_3 \; w_4 \; w_5 \; w_6]$

b) $\underline{\phi^T(x_i)\phi(x_j)} = K(x_i, x_j)$

$[\underbrace{x_{i1}^2}_{1} \; \underbrace{x_{i2}^2}_{2} \; \underbrace{\sqrt{2}x_{i1}x_{i2}}_{3} \; \sqrt{2}x_{i1} \; \sqrt{2}x_{i2} \; 1] \begin{bmatrix} x_{j1}^2 \\ x_{j2}^2 \\ \sqrt{2}x_{j1}x_{j2} \\ \sqrt{2}x_{j1} \\ \sqrt{2}x_{j2} \\ 1 \end{bmatrix}$ 1 2 3

$= \overset{1}{(x_{i1}x_{j1})^2} + \overset{2}{(x_{i2}x_{j2})^2} + \overset{3}{(2 x_{i1}x_{i2}x_{j1}x_{j2})}$

$+ \overset{4}{2x_{i1}x_{j1}} + \overset{5}{2x_{i2}x_{j2}} + 1 \rightarrow$ don't cal ∵ always 1

$= \left( [x_{i1} \; x_{i2}] \begin{bmatrix} x_{j1} \\ x_{j2} \end{bmatrix} + 1 \right)^2$ (2)

$= (\overset{1}{x_{i1}x_{j1}} + \overset{2}{x_{i2}x_{j2}} + 1)(x_{i2}x_{j2}+1) + (x_{i2}x_{j2}+1)^2$

$= (x_{i1}x_{j1})^2 + 2(x_{i1}x_{j1})(x_{i2}x_{j2}+1) + (x_{i2}x_{j2})^2$

$= (x_{i1}x_{j1})^2 + 2x_{i1}x_{j1}x_{i2}x_{j2} + 2x_{i1}x_{j1} + \dots$

$+ 2x_{i2}x_{j2} + 1$

c) only think multiplication 3+3

$\phi^T(x_i)\phi(x_j)$ inner product of 6 variables

6 $[x_{i1}^2 \; x_{i2}^2 \; x_{i1}x_{i2} \; x_{i1} \; x_{i2} \; 1] \begin{bmatrix} x_{j1}^2 \\ x_{j2} \\ x_{j1}x_{j2} \\ x_{j1} \\ x_{j2} \\ 1 \end{bmatrix}$ 3+3+5

~~square~~

$\rightarrow \cancel{(x_{i1}x_{j1})^2 + (x_{i2}x_{j2})^2}$

$(x_i^T x_j + 1)^2$

$= \left( [\underset{1}{x_{i1}} \; \underset{2}{x_{i2}}] \begin{bmatrix} x_{j1} \\ x_{j2} \end{bmatrix} + 1 \right)^2$ 3

d)
2. $(\Phi^T\Phi + \lambda I)^{-1}\Phi^T = \cancel{\Phi^T(\Phi\Phi^T + \lambda I)^{-1}}$

$\cancel{= (\Phi^T\Phi)}$

$\Rightarrow (\Phi^T\Phi + \lambda I)^{-1}\Phi^T(\Phi\Phi^T + \lambda I)$

$\cancel{= \Phi^T}$

$\Rightarrow \Phi^T(\Phi\Phi^T + \lambda I)^{-1} = \cancel{(\Phi\Phi^T + \lambda I)\Phi^T}$

b) $\Rightarrow w = (\Phi^T\Phi + \lambda I)^{-1}\Phi^T y$

$= \Phi^T(\Phi\Phi^T + \lambda I)^{-1} y$

c) $[K]_{i,j} = \phi^T(x_i)\phi(x_j)$

d) $= (x_i^T x_j + 1)^2$

e) $\alpha = (\Phi\Phi^T + \lambda I)^{-1} y$

$K(x, x_j) = (x^T x_j + 1)^2$

$\cancel{\hat{y}(x) = \sum_{j=1}^{N} (x^T x_j + 1)^2 (\Phi\Phi^T + \lambda I)^{-1} y}$

$\downarrow$

$\phi^T(x)\phi(x_j)$

2.e) $\hat{y}(x) = \phi^T(x)\,\Phi^T\underbrace{(\Phi\Phi^T+\lambda I)^{-1}y}_{\alpha}$

$= [-\phi(x)-]\begin{bmatrix}| & | & & |\\ \phi_1 & \phi_2 & \cdots & \phi_N\\ | & | & & |\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_N\end{bmatrix}$

$= \sum_{j=1}^{N} K(x, x_j)\,\alpha_j$

$K(x_i, x_j) = \phi(x_i)\,\phi(x_j)$

3a) $K(x, x_j) = \phi^T(x)\,\phi(x_j) = x^T x_j$

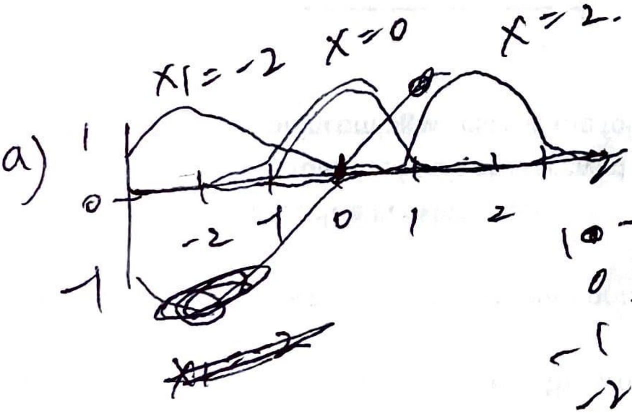
b) $\hat{y}(x) = \sum_{j=1}^{N} \cancel{K(x,} \; x^T x_j \alpha_j$

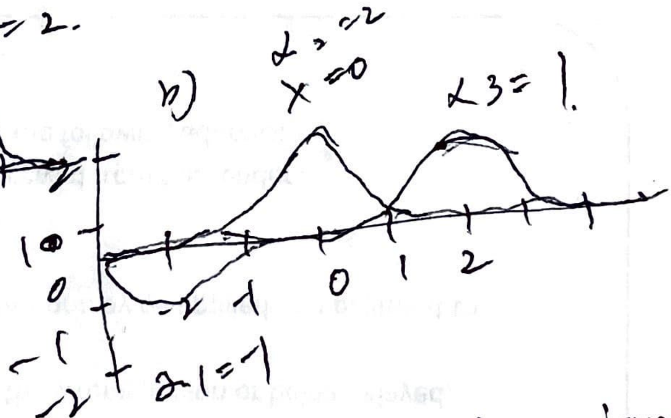
$x_j$ serves different weight to $x$.

$x =$

$e^{-\frac{(x-0.3)^2}{0.01}}$ = 1. (height)

$e^{-\frac{(x+2)^2}{2}}$ = -1 height




c) ~~normal,~~ weighted ($\alpha$), normal function (Gaussian,) each $x_j$