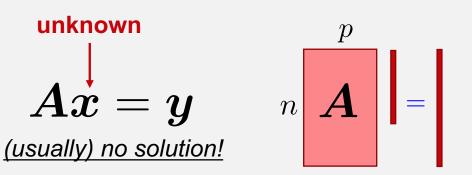
## CS/ECE/ME 532 Matrix Methods in Machine Learning



Welcome!

## Activity 6





- Usually what happens with real data
- The next best thing: find approximate solution

$$oldsymbol{x}^* = rg\min_{oldsymbol{x}} ||oldsymbol{A}oldsymbol{x} - oldsymbol{y}||_2^2$$

this is a called <u>a least squares solution</u>

$$\boldsymbol{x}^* = (\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{y}$$

when does this inverse exist?

Note: X must be a square matrix!

$$XX^{-1} = I$$

if rank $(\mathbf{A}) = p$ , then  $\mathbf{A}^T \mathbf{A}$  is invertible



The least squares solution is unique

## Positive definiteness (P.D.)

$$Q \succ 0$$

 $\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} > 0 \text{ for all } \boldsymbol{x} \neq 0$ 



Q is invertible

## The investible match theorem is a theorem in linear algebra which gives a series of equivalent conditions for an experiment of the horse matrix. A to have an inverse, in particular, A is invertible if and only if any (and hence, all) of the following 1. A is row-equivalent to the $a_2 \times a$ identity matrix. $a_1 \times a$ . 2. A has $a_1$ pivol positions. 3. The equation $A_1 = \emptyset$ has only the trivial solution $x = \emptyset$ . 4. The oclumes of A form all investigation of the $A_1 \times a$ is one-to-one. 5. The linear transformation $x \mapsto A_1$ is one-to-one. 6. For each column vector $b_1 \in \mathbb{R}^n$ , the equation $A_1 = b_1$ has a unique solution. 7. The columns of $A_1 \times a$ particles $A_1 \times a$ is surjection. 9. There is an $a \times a$ matrix $C_1 \times a$ but that $C_1 \times a$ is $C_1 \times a$ . 10. There is an $a \times a$ matrix $C_2 \times a$ but that $C_1 \times a$ . 11. The transpose matrix $A_1 \times a$ is inventible. 12. The columns appear of $A_1 \times a$ is inventible. 13. The olimns of $A_1 \times a$ is inventible. 14. The dimension of the column space of $A_1 \times a$ . 15. The matrix of $A_1 \times a$ . 16. The rutil space of $A_1 \times a$ . 17. The dimension of the null space of $A_1 \times a$ .

21. The orthogonal complement of the null space of A is R