Neural network example

In [110…

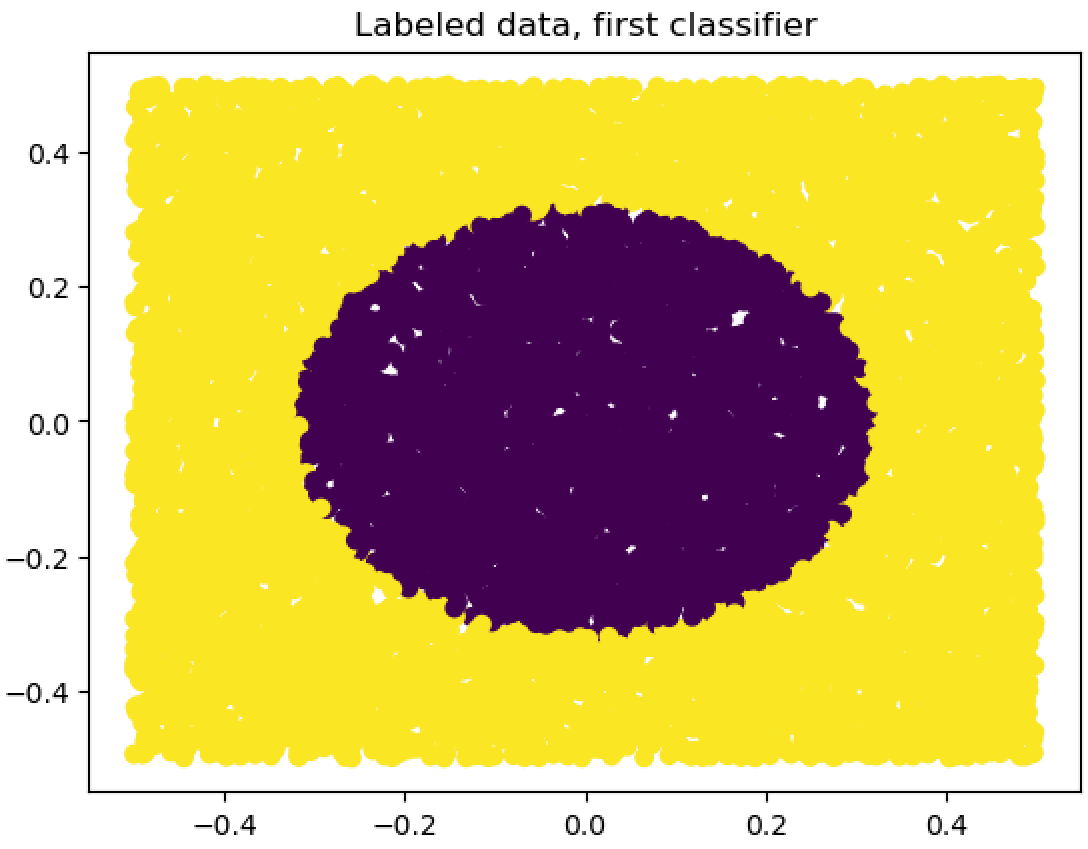
```
import numpy as np
import matplotlib.pyplot as plt

p = int(2) #features
n = int(10000) #examples

## generate training data
X = np.random.rand(n,p)-0.5
Y1 = np.sign(np.sum(X**2,1)-.1).reshape((-1, 1))/2+.5
Y2 = np.sign(5*X[:,[0]]**3-X[:,[1]])/2+.5
Y = np.hstack((Y1, Y2))
```

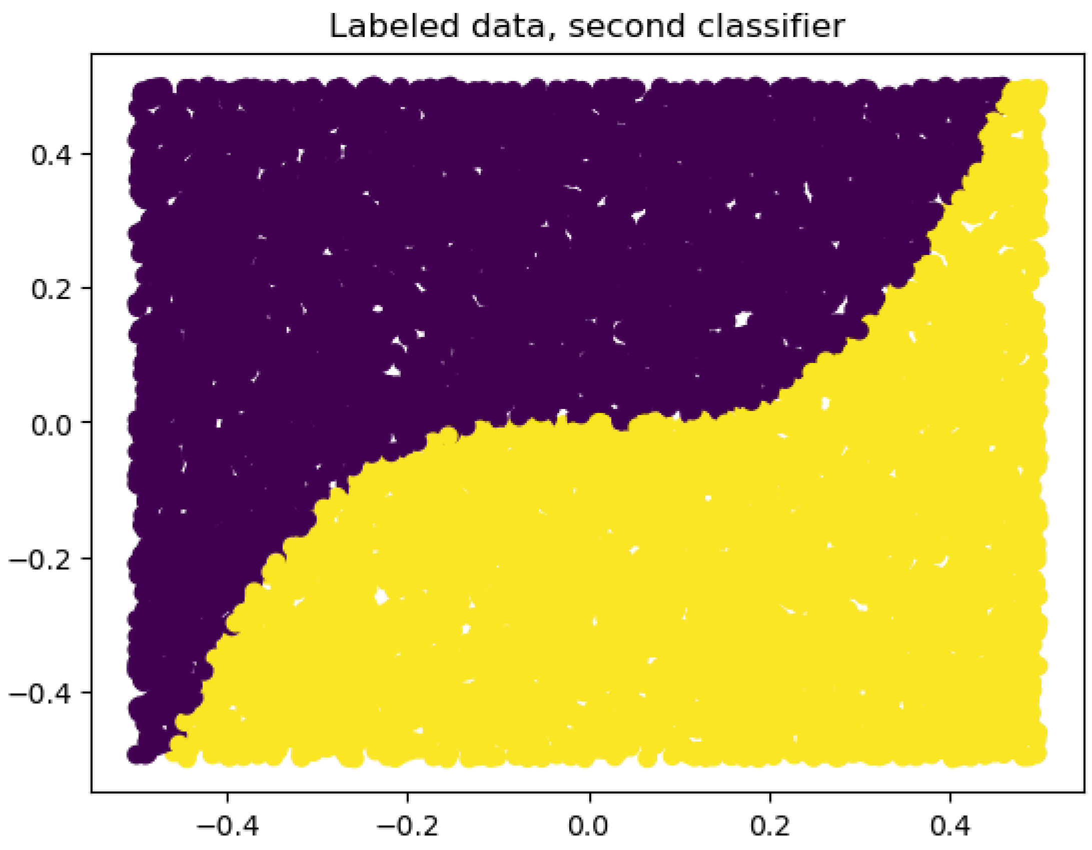
In [111…

```
# Plot training data for first classification problem
plt.scatter(X[:,0], X[:,1], c=Y1.flatten())
plt.title('Labeled data, first classifier')
plt.show()
```



In [112…

```
# Plot training data for second classification problem
plt.scatter(X[:,0], X[:,1], c=Y2.flatten())
plt.title('Labeled data, second classifier')
plt.show()
```



In [114…

```python
## Train NN
Xb = np.hstack((np.ones((n,1)), X))
q = np.shape(Y)[1] #number of classification problems
# M = 2 #number of hidden nodes
# M = 3
M = 4

## initial weights
V = np.random.randn(M+1, q);
W = np.random.randn(p+1, M);

alpha = 0.1 #step size
# L = 10 #number of epochs
L = 100

def logsig(_x):
    return 1/(1+np.exp(-_x))

for epoch in range(L):
    ind = np.random.permutation(n)
    for i in ind:
        # Forward-propagate
        H = logsig(np.hstack((np.ones((1,1)), Xb[[i],:]@W)))
        Yhat = logsig(H@V)
         # Backpropagate
        delta = (Yhat-Y[[i],:])*Yhat*(1-Yhat)
        Vnew = V-alpha*H.T@delta
        gamma = delta@V[1:,:].T*H[:,1:]*(1-H[:,1:])
        Wnew = W - alpha*Xb[[i],:].T@gamma
        V = Vnew
        W = Wnew
    print('epoch: ', epoch)
```

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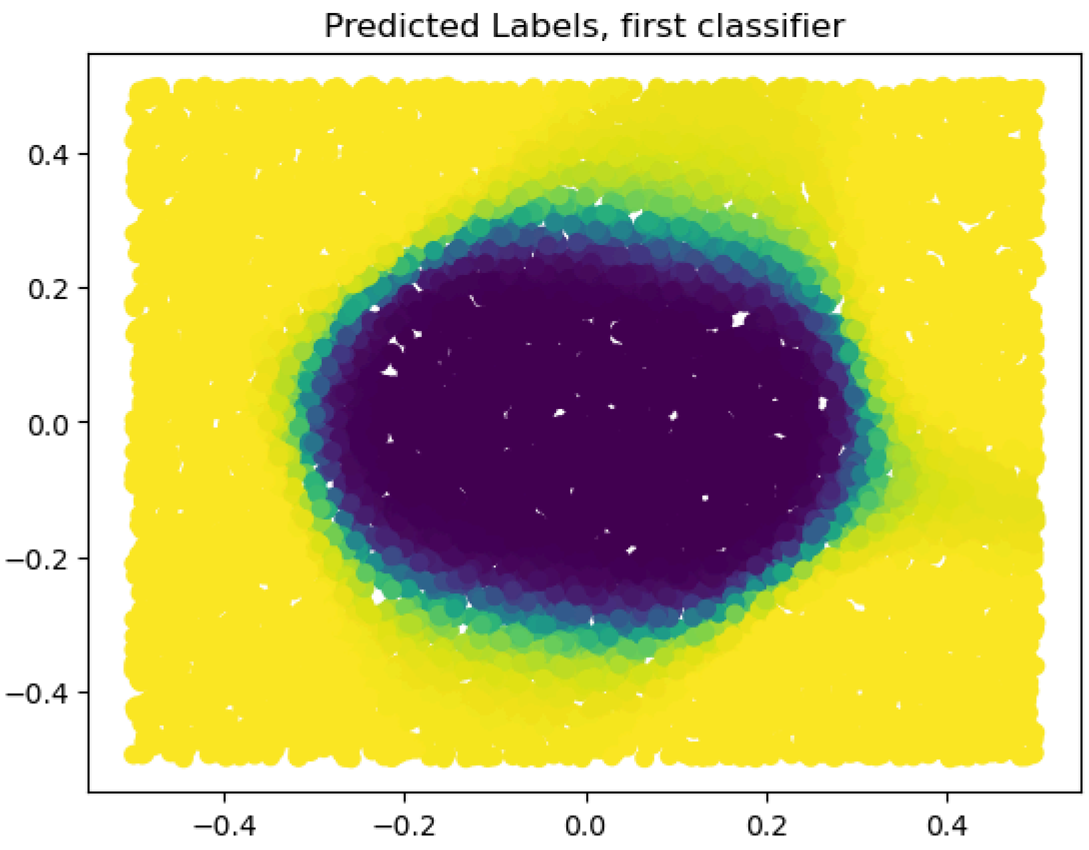
epoch:  98
epoch:  99
```

In [115…

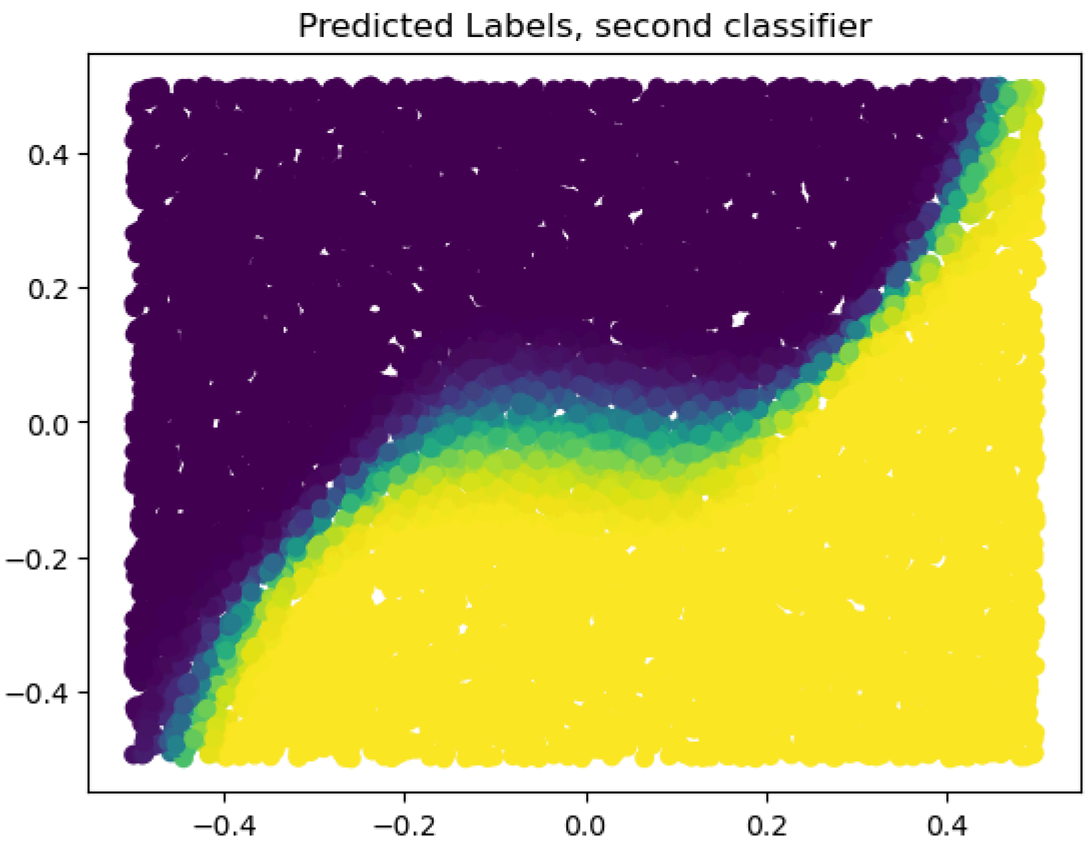
```python
## Final predicted labels (on training data)
H = logsig(np.hstack((np.ones((n,1)), Xb@W)))
Yhat = logsig(H@V)
```

In [116…

```python
plt.scatter(X[:,0], X[:,1], c=Yhat[:,0])
plt.title('Predicted Labels, first classifier')
plt.show()
```



In [117…
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,1])
plt.title('Predicted Labels, second classifier')
plt.show()
```



In [118…
```
err_c1 = np.sum(abs(np.round(Yhat[:,0])-Y[:,0]))
print('Errors, first classifier:', err_c1)
```

```
err_c2 = np.sum(abs(np.round(Yhat[:,1])-Y[:,1]))
print('Errors, second classifier:', err_c2)
```

```
Errors, first classifier: 170.0
Errors, second classifier: 100.0
```

a) Use M = 2 hidden nodes and ten epochs in SGD. Run this four or five times and comment on the performance of the two classifiers and whether it varies from run to run.

1st time case1=2653.0, case2= 791.0

2nd time case1=2731.0, case2=776.0

3rd time case1=2899.0, case2=695.0

4th time case1=2935.0, case2=709.0

5th time case1=2918.0, case2=727.0

The performance changes because of the initial weights change.

b) Repeat M = 2 but use 100 epochs in SGD. Run this several times and comment on the performance of the classifiers and whether it varies from run to run.

1st time case1=2558.0, case2=737.0

2nd time case1=2731.0, case2=776.0

3rd time case1=2899.0, case2=695.0

The performance changes because of the initial weights change.

c) Recall the two-layer network results from the previous problem. How do the possible decision boundaries change when you add a hidden layer?

The decision boundaries

d) Now use M = 3 hidden nodes and run 100 epochs of SGD (or as many as you can compute). Does going from two to three hidden nodes affect classifier performance?

The errors decrease a lot. (case1=435.0, case2=448.0)

e) Repeat the previous part for M = 4 hidden nodes and comment on classifier performance.

The errors decrease a lot. (case1=170.0, case2=100.0)

```
In [ ]:
```

Two neuron example

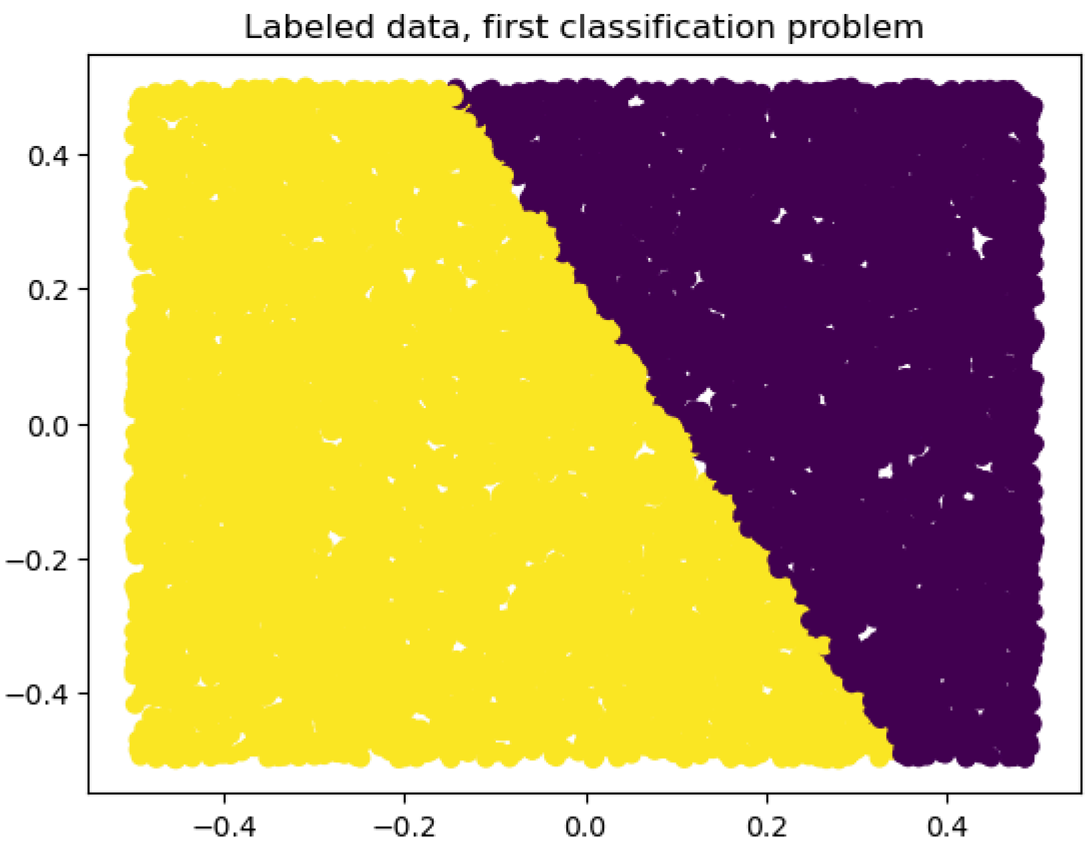
In [27]:
```
import numpy as np
import matplotlib.pyplot as plt

p = int(2) #features
n = int(10000) #examples

## generate some features for training data
X = np.random.rand(n,p)-0.5

## generate labels of the feature vectors with known functions
## Note that sign()/2+0.5 maps output to be 0 or 1,
## which is the range of the activation fuction
Y1 = np.sign(-2*X[:,[0]]+.2-X[:,[1]])/2+.5
Y2 = np.sign(5*X[:,[0]]**3-X[:,[1]])/2+.5
Y = np.hstack((Y1, Y2))
```

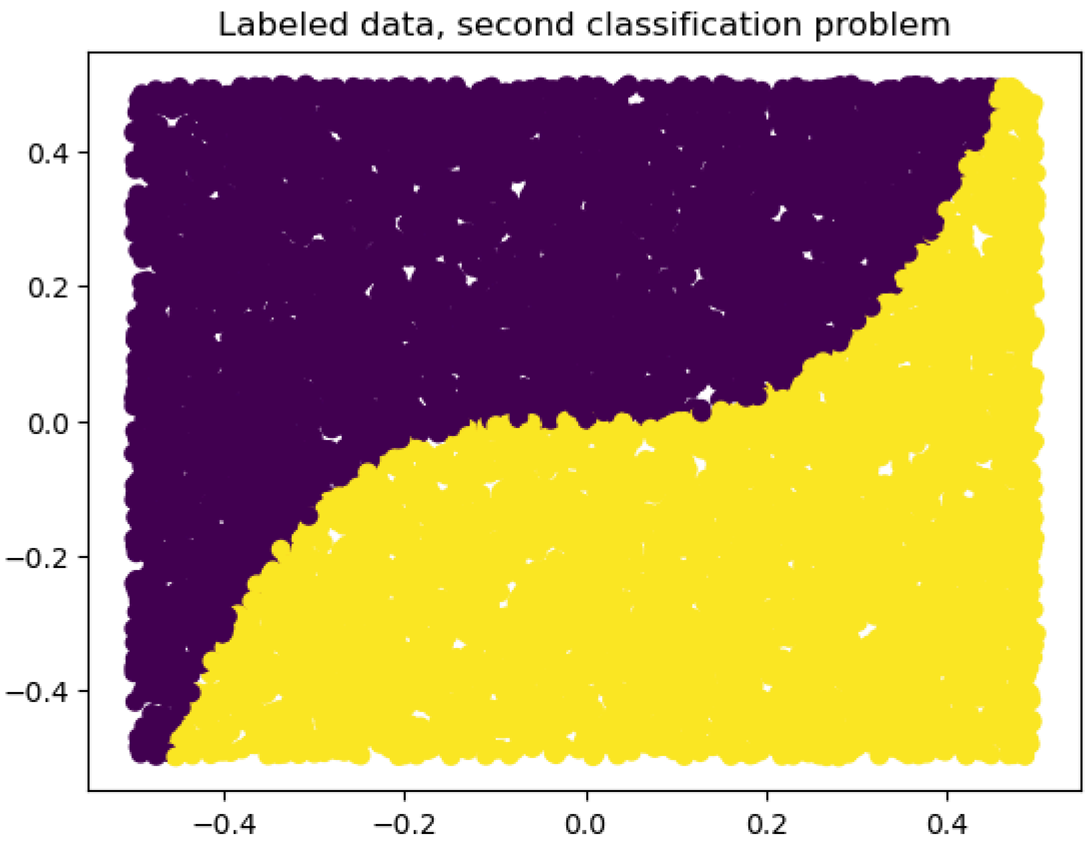
In [28]:
```
# Plot training data for first classification problem
plt.scatter(X[:,0], X[:,1], c=Y1.flatten())
plt.title('Labeled data, first classification problem')
plt.show()
```



a) Do you expect that a single neuron will be able to accurately classify data from case 1? Why or why not? Explain the impact of the bias term associated with w1,0.

Yes, a single neuron can create a linear function which can accurately classify data from case 1 with a simple line. With the constant and its weight, we can decide how far our boundary will be close to the origin.

```
In [29]: # Plot training data for second classification problem
plt.scatter(X[:,0], X[:,1], c=Y2.flatten())
plt.title('Labeled data, second classification problem')
plt.show()
```



b) Do you expect that a single neuron will be able to accurately classify data from case 2? Why or why not? Explain the impact of the bias term associated with w2,0.

No, a single neuron cannot create a curve to accuaratеly classify case 2 data. With the constant and its weight, we can decide how far our boundary will be close to the origin.

```
In [30]: ## Train NN
Xb = np.hstack((np.ones((n,1)), X))
q = np.shape(Y)[1] #number of classification problems
M = 3 #number of hidden nodes

## initial weights
W = np.random.randn(p+1, q);

alpha = 0.1 #step size
# L = 10 #number of epochs
# L = 5
L = 20

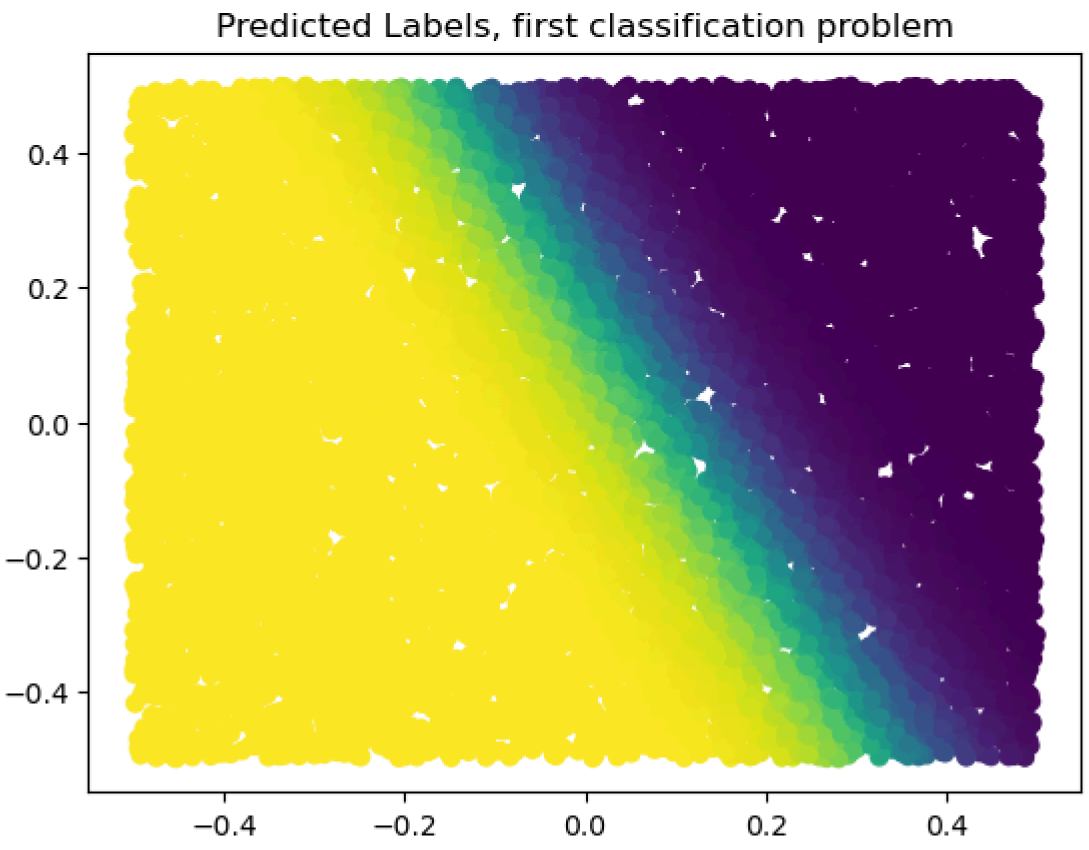
def logsig(_x):
    return 1/(1+np.exp(-_x))

for epoch in range(L):
    ind = np.random.permutation(n)
    for i in ind:
        # Forward-propagate
        Yhat = logsig(Xb[[i],:]@W)
         # Backpropagate
        delta = (Yhat-Y[[i],:])*Yhat*(1-Yhat)
        Wnew = W - alpha*Xb[[i],:].T@delta
        W = Wnew
    print('epoch: ', epoch)
```

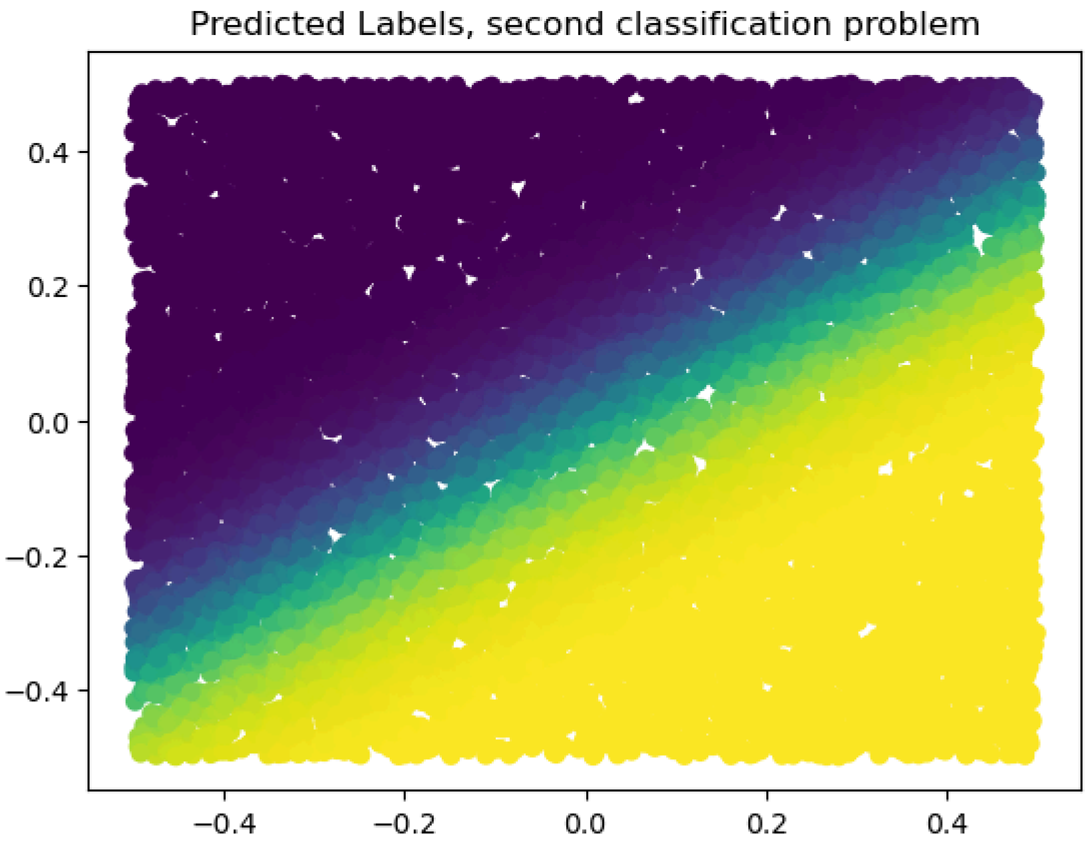
```
epoch:  0
epoch:  1
epoch:  2
epoch:  3
epoch:  4
epoch:  5
epoch:  6
epoch:  7
epoch:  8
epoch:  9
epoch:  10
epoch:  11
epoch:  12
epoch:  13
epoch:  14
epoch:  15
epoch:  16
epoch:  17
epoch:  18
epoch:  19
```

In [31]:
```
## Final predicted labels (on training data)
H = logsig(np.hstack((np.ones((n,1)), Xb@W)))
Yhat = logsig(Xb@W)
```

In [32]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,0])
plt.title('Predicted Labels, first classification problem')
plt.show()
```



In [33]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,1])
plt.title('Predicted Labels, second classification problem')
plt.show()
```



In [34]:
```
err_c1 = np.sum(abs(np.round(Yhat[:,0])-Y[:,0]))
print('Errors, first classification problem:', err_c1)
```

```
err_c2 = np.sum(abs(np.round(Yhat[:,1])-Y[:,1]))
print('Errors, second classification problem:', err_c2)
```

```
Errors, first classification problem: 64.0
Errors, second classification problem: 728.0
```

c) Run SGD for one epoch. This means you cycle through all the training data one time, in random order. Repeat this five times and find the average number of errors in cases 1 and 2.

first classification problem Errors: 53.0 second classification proble Errorsm: 772.0

d) Run SGD over twenty epochs. This means you cycle through all the training data twenty times, in random order. Repeat this five times and find the average number of errors in cases 1 and 2.

first classification problem Errors: 64.0 second classification problem Errors: 728.0

e) Explain the differences in classification performance for the two cases that result with both one and twenty epochs.

```
In [ ]:
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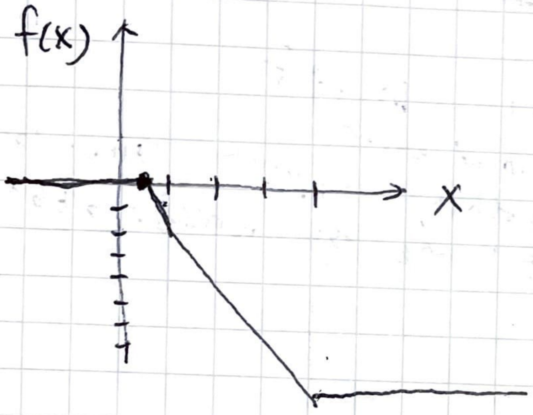
△ Activity 04

(a) $f(x) = 2(x-0.5)_+ - 2(2x-1)_+ + 4(0.5x-2)_+$

① $x \geq 0.5$ $\quad 2x-1-4x+2 = -2x+1 \quad (0,1)$
$4>$ $\quad (\frac{1}{2},0)$

② $x>4 \quad \cancel{2x}-1-\cancel{4x}+2+\cancel{2x}-8 = -7$

③ $x \leq 0.5 \quad 0$



2b) $v_1(w_1x+b_1)_+ + v_2(w_2x+b_2)_+$

① $x<-1, \; f(x)=0$

$v_1(x+1)_+ + v_2(x-1)_+$

② $1>x>-1$

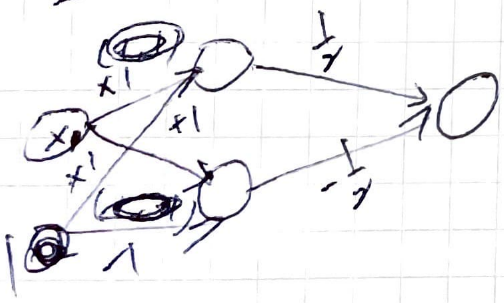
$v_1(x+1) = f(x)$

$(-1,0), (1,1)$ $\quad 1.$

$2v_1 = 1 \to v_1 = \frac{1}{2}$

$\frac{1}{2}(x+1)$

③ $\frac{1}{2}(x+1) + (-\frac{1}{2})(x-1) = 1$



+1
x
+1
+1
1
-1
$\frac{1}{2}$
$-\frac{1}{2}$