

Activity 4 Δ Item 1 \odot column

a) $X = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $a_2 = a_3 + a_5$
 4×5 $a_3 = a_1 + a_4$
 a_1, a_2, a_3, a_4, a_5 lin dep.
 $\rightarrow a_5 = -a_3$

let a_1, a_4

\odot row $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ b_1 $b_3 = b_4$
 b_2 $|b_3| = 0$
 b_3 let b_1, b_2 (\rightarrow)
 b_4 $a_1, a_2, a_3 - a_5$ 3

b) set? a_1, a_2 $a_1, a_2, a_3 - a_5$ 3
 $a_2 - a_5$ a_1, a_3 ~~a_4~~
 $a_3, a_4 - a_5$ a_2, a_3, a_4 1
 a_4, a_5 1

(4) ~~a_1, a_2, a_3, a_5~~ $\Rightarrow 1+1+1+1=4$
 ~~a_1, a_3, a_4, a_5~~ $a_1 \cdot 0$
 $a_1, (a_3, a_4, a_5)$ $+ a_2 \cdot 8 = 0$
 ~~a_1, a_2~~ $a_3(a_4) \rightarrow 5$
 $a_4(a_5)$

c) $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & b \\ 0 & 1 & -1 \end{bmatrix}$, $\text{rank}(A) = 2$
 $\rightarrow n \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + m \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ -1 \end{bmatrix}$
 $a = n, b = n + m,$
 $m = -1$

Δ Item 2 3×2 a_1, a_2 $\rightarrow b = n - 1 \rightarrow \underline{b = a - 1}$
 $Ax = b, A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ a) $b = \begin{bmatrix} 8 \\ 6 \\ -2 \end{bmatrix}$
 $\text{rank}(A) = 2$ $\text{rank}([A \ b]) = 2$
 $b = 8a_1 - 2a_2$

b) $\text{rank}([A \ b]) = 3$ $\rightarrow X = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$
 $b = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ $> \text{rank}(A) = 2$ $> X$
 \Rightarrow no solution

c) $Ax = b \rightarrow$ has a solution
 $\text{rank}(b)$ is full rank (number of columns)
 $\Rightarrow \text{rank}(A) = \text{rank}([A \ b])$, and rank is the number of A's columns.

Δ Item 3 a) $Ax = b, A = \begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix}, b = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\text{rank}(A) = 1$
 $= \text{rank}([A \ b]) = 1$

$\text{rank}(A) = 1 < \text{No. columns}$
 \Rightarrow infinite sol.

\odot $x_1 - 2x_2 = 2$ $(0, 0)$
 $\rightarrow x_2 = \frac{x_1 - 2}{2}$ $(2, 0)$

b) $\text{rank}(A) < \text{number of columns}$