

An aerial photograph of a city, likely Madison, Wisconsin, taken from a high vantage point looking down at the city and the surrounding water. The sun is setting behind a hill in the background, casting a warm, golden glow over the entire scene. The city's buildings are visible on the left, and the water is filled with several sailboats. A large, semi-transparent rectangular box is overlaid on the center of the image, containing the course title in large, bold, black text.

CS/ECE/ME 532 Matrix Methods in Machine Learning

Welcome!



Activity 14



Eigendecomposition

$$B e_i = \lambda_i e_i$$

↑ eigenvector ↑ eigenvalue

B (square) symmetric matrix:

$$B = E \Lambda E^T$$

↖ diagonal
↘ orthonormal rows, cols

Connection with SVD:

$$A = U \Sigma V^T$$

abuse of notation ↘

$$A A^T = U \Sigma V^T V \Sigma^T U^T = U \Sigma^2 U^T$$

→ Eigenvectors of $A A^T$ are left singular vectors of A

$$A^T A = V \Sigma^T U^T U \Sigma V^T = V \Sigma^2 V^T$$

→ Eigenvectors of $A^T A$ are right singular vectors of A

Eigenvalues: $\lambda_i = \sigma_i^2$

Power iteration (main idea)

$$\begin{aligned}
 (A A^T)^k &= U \Sigma^2 U^T U \Sigma^2 U^T \dots U \Sigma^2 U^T \\
 &= U \Sigma^{2k} U^T \\
 \lim_{k \rightarrow \infty} \Sigma^{2k} &= \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & & \vdots \\ \vdots & & \ddots & \\ 0 & \dots & & 0 \end{bmatrix} \quad \rightarrow \text{imagine that } \sigma_1 = 1, \text{ others less than 1} \\
 &= U \Sigma^{2k} U^T \rightarrow u_1 u_1^T
 \end{aligned}$$

Adjacency matrix and PageRank

- Graph: nodes with edges between them
- Adjacency matrix: non-zero entry \tilde{A}_{ij} if edge from j to i
- Transition probability matrix: normalize columns of \tilde{A} to 1

$$\|Q_{:,j}\|_1 = 1$$

$$\lambda_1 = 1$$



Graph

$$\tilde{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Adjacency matrix

$$Q = \begin{bmatrix} 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 1 \\ 1/2 & 1/3 & 1/2 & 1/2 & 0 \end{bmatrix}$$

Transition probability matrix
(Markov Chain)

$Q^k b \rightarrow$ direction of first eigenvector of Q .

The first eigenvector is the steady-state probability distribution