Week 6 Practice Problems

June 6, 2022

1 Practice Problems based on Week 6 content

1.1 General Course Logistics

- Work on your project! This week you should finish builiding your math model and begin implementing it. It's also a goode idea to work on the report – write as you go! This will save you a lot of time at the end of the project. Write up your introduction and background. Start filling in the details about your model.

1.2 Convex Programming

1.2.1 Convex Programming Theory

In Module 10, Lecture 7, we prove that the set of optimal solutions to a convex program is itself a convex set. Use that fact to prove that any locally optimal solution to a convex program is also a globally optimal solution.

1.2.2 Small circles

Suppose we are given a set of points $(x_i, y_i) \in \mathbb{R}^2$. We would like to find the smallest circle that contains all the points. How can we model this as an optimization problem with a convex objective function? You can test your model by generating n random points with

[4]:
```
X = 4 .+ randn(2,50);
```

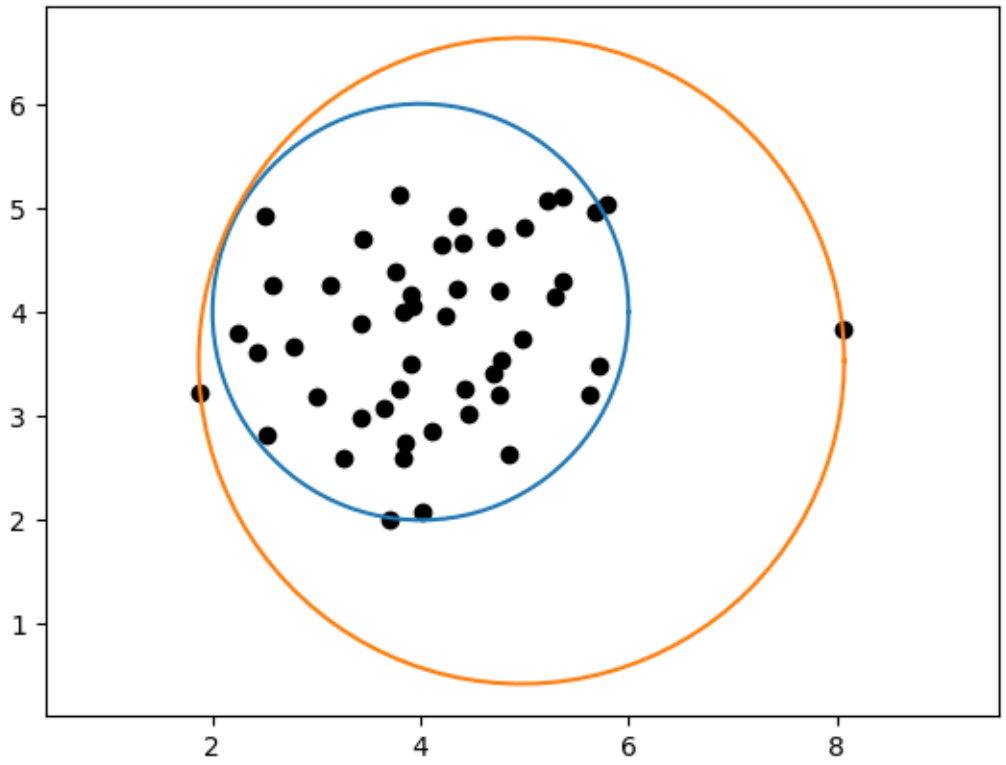
[4]:
```
2×50 Array{Float64,2}:
 2.52345  4.38101  3.53826  4.7792   ...  4.8775   4.87424  5.11443  3.48545
 6.17501  4.52571  4.79064  4.45131      5.72218  5.37879  3.56337  3.70853
```

This is a 2x50 matrix where each column is one of the points. Produce a plot of the randomly generated points and the enclosing sphere of smallest area.

[7]:
```
### Note: You can use the code below to get started by generating points and␣
 ↪plotting one possible circle

using PyPlot
X = 4 .+ randn(2,50) # generate 50 random points
t = range(0,stop=2*pi,length=100) # parameter that traverses the circle
r = 2; x = 4; y = 4 # radius and coordinates of the center
```

```
plot( x .+ r*cos.(t), y .+ r*sin.(t)) # plot circle radius r with center (x1,x2)
scatter( X[1,:], X[2,:], color="black") # plot the 50 points
axis("equal"); # make x and y scales equal
```



1.3 Integer Programming Basics

1.3.1 Basic IP Modeling

You're trying to pack as many souvenirs as possible to bring home from your trip, but your suitcase has a limited capacity. It can hold a maximum of 30 pounds of weight and 15 gallons of volume. Which souvenirs should you pack? The weights and volumes are as follows:

Souvenir Number	1	2	3	4	5	6	7	8	9	10
Weight	5	6	7	6	4	6	7	3	8	5
Volume	2	4	5	3	3	2	3	1	2	4

1.3.2 Fixed Cost Practice

Comquat owns four production plants at which personal computers are produced. Comquat can sell up to 20,000 computers per year at a price of $3,500 per computer. For each plant the production capacity, cost per computer, and fixed cost of operating the plant for a year are given below. Determine how Comquat can maximize its yearly profit from computer production.

Plant	Production capacity	Fixed Cost ($ Million)	Per computer cost ($)
1	10,000	9	1,000
2	8,000	5	1,700
3	9,000	3	2,300
4	6,000	1	2,900

- Find at least two different valid choices of your big-Ms. What's the smallest big-M you can find?