

Week 3 Practice Problems

June 29, 2023

Practice Problems based on Week 3 content

(1) General Course Logistics

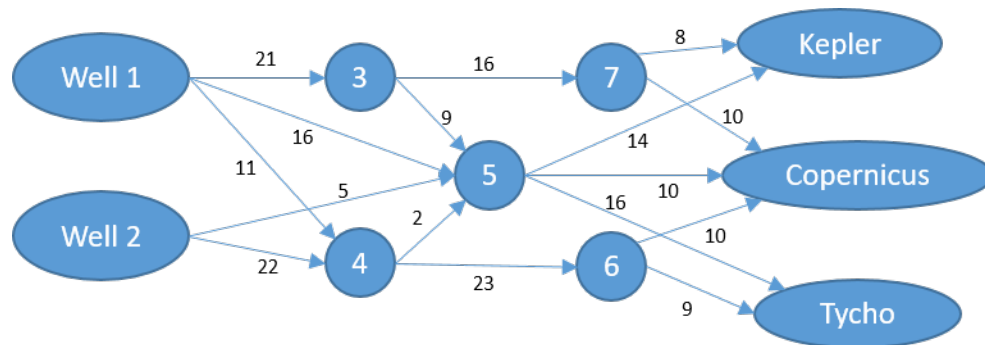
- Review the project assignment. Find a group and start brainstorming topics, even if it's a group of size 1!

(2) Min-Cost Network Flow Concepts and Definitions

- Find a better upper bound on the “Max Flow” longest path example given in Module 5 Lecture 8.

Max Flow Problem

Last week you figured out how to feed the Martian colonists. This week, your challenge is to provide them with water. There are three bases that will need water: Kepler, Copernicus, and Tycho. Two wells have been installed. There are five pipe intersections where water can be rerouted. The pipes have specified capacities (in gallons/day) as seen in the network below:



Well 1 can supply up to 30 gallons per day, and Well 2 can supply up to 27 gallons per day. Kepler needs 17 gallons/day, Copernicus needs 15 gallons/day, and Tycho needs 21 gallons/day. What is the maximum flow through the network? Can the daily demand be met?

Find a minimum cut in the network above.

(3) Duality and Sensitivity Analysis

- Take the dual of the dual. Convert the general-form dual to standard form, then take the dual.
- Prove the theorem of Weak Duality for standard form LPs.

- Pick your favorite example from class so far and take the dual of the problem.
- Recall the Stigler diet problem from Week 1's practice problems. Suppose you wanted to find an even cheaper solution and you found an online retailer that would sell you calcium supplements and protein powder. The calcium supplements contain 0.5 g of calcium each and the protein powder comes in packets of 10 g.
 - (a) How much would we be willing to pay for the calcium supplements? How much for the protein powder? Hint: Use duality!
 - (b) Suppose we could buy the calcium supplements for \$0.01 / pill. What is your new optimal diet? How much do you save per year, compared to your diet from the original problem (without the supplements)?
- Consider the following linear program:

$$\begin{aligned}
 &\max_{x_1, x_2} 3x_1 + x_2 \\
 &\text{s.t. } 2x_1 + x_2 \leq 4 \\
 &\quad 3x_1 + 2x_2 \geq 6 \\
 &\quad 4x_1 + 2x_2 = 7 \\
 &\quad x_1, x_2 \text{ free}
 \end{aligned}$$

- (a) Find a solution to this linear program.
- (b) Using the theorem of Complementary Slackness and the optimal primal solution, find the optimal dual solution.
- (c) How would our solution and objective value change if we increased the coefficient on x_1 in the objective to 5? What if we change the right-hand side on the 3rd constraint to 8?

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