Question 13.4

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt

datapath = "datasets/universal_autoencoder_samples.csv"

# Import data
X = np.loadtxt(datapath, delimiter=",")

X= X.T

# Normalize X to be between 0 and 1
X_min = X.min(axis=0)
X_max = X.max(axis=0)
X_normalized = (X - X_min) / (X_max - X_min)


# Print the shape of X
print("Shape of X:", X_normalized.shape)
```

```
Shape of X: (100, 2)
```

In [ ]:
```
def sigmoid(z, d=False):
    """Compute the sigmoid function or its derivative."""
    sig = 1 / (1 + np.exp(-z))
    if d:
        return (
            sig * (1 - sig) + 1e-12
        )  # Add epsilon to avoid division by zero in gradients
    return sig


def relu(z, d=False):
    """Compute the ReLU function or its derivative."""
    if d:
        return (
            np.where(z > 0, 1, 0) + 1e-12
        )  # Use np.where for better handling with arrays
```

```
    return np.maximum(0, z)  # np.maximum is more idiomatic and clear

layers = [
    # activation, shape:(out,in)
    {"act": relu, "shape": (1024, 2)},
    {"act": relu, "shape": (50, 1024)},
    {"act": relu, "shape": (1024, 50)},
    {"act": sigmoid, "shape": (2, 1024)},
]
```

In [ ]:

```
# Global properties
l, errors, epochs = len(layers), [], 1000
# Adam optimizer properties
lr, b1, b2 = 0.002, 0.9, 0.999
rw, mw, rb, mb = {}, {}, {}, {}

# Layer properties
a, w, b, f = {}, {}, {}, {}

# Initialize each layer's parameters
for i, layer in enumerate(layers, 1):
    n_out, n_in = layer["shape"]
    f[i] = layer["act"]

    # Xavier Initialization for weights
    w[i] = np.random.randn(n_out, n_in) / np.sqrt(n_in)

    # Initialize biases and Adam parameters (momentum and velocity) for weights and biases
    b[i], rb[i], mb[i] = (
        np.zeros((n_out, 1)),
        np.zeros((n_out, 1)),
        np.zeros((n_out, 1)),
    )
    rw[i], mw[i] = np.zeros((n_out, n_in)), np.zeros((n_out, n_in))
```

In [ ]:

```
def forward_pass(x):
    a[0] = x.T
    for i in range(1, l + 1):
        a[i] = f[i]((w[i] @ a[i - 1]) + b[i])
```

```python
def backward_pass():
    dz, dw, db = {}, {}, {}
    for i in range(l, 0, -1):
        if i == l:
            d = 0.5 * (a[l] - a[0])  # Loss derivative
        else:
            d = w[i + 1].T @ dz[i + 1]
        dz[i] = d * f[i](a[i], d=True)
        dw[i] = dz[i] @ a[i - 1].T
        db[i] = np.sum(dz[i], axis=1, keepdims=True)
    return dz, dw, db


def adam_update(m, r, z, dz):
    for i in range(1, l + 1):
        m[i] = b1 * m[i] + (1 - b1) * dz[i]
        r[i] = b2 * r[i] + (1 - b2) * (dz[i] ** 2)
        m_hat = m[i] / (1.0 - b1**t)
        r_hat = r[i] / (1.0 - b2**t)
        z[i] -= lr * m_hat / (np.sqrt(r_hat) + 1e-12)


# Main training loop
for t in range(1, epochs + 1):
    # Training
    for batch in np.split(X_normalized, 1):  # Assuming batching might need revision
        forward_pass(batch)
        dz, dw, db = backward_pass()
        adam_update(mw, rw, w, dw)
        adam_update(mb, rb, b, db)

    # Validation
    forward_pass(X_normalized)
    errors.append(np.mean((a[l] - a[0]) ** 2))
    print("Val loss - ", errors[-1])
```

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Val loss -  0.06624673810235165
Val loss -  0.04686015767558502
Val loss -  0.039029754231910824
Val loss -  0.03792728012404295
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Val loss -  3.34827368114176e-05
Val loss -  3.4567099308507465e-05
Val loss -  3.842577149557127e-05
Val loss -  3.8642572440349456e-05
Val loss -  4.012217544861699e-05
Val loss -  4.9054365541896884e-05
Val loss -  3.617317542843557e-05
Val loss -  3.4121091940882426e-05
Val loss -  3.818175812758455e-05
Val loss -  6.376006380580427e-05
Val loss -  5.1241747755765394e-05
Val loss -  6.44253561066312e-05
Val loss -  7.459278344060218e-05
Val loss -  0.00010905864530727761
Val loss -  0.00010342720302391427
Val loss -  0.00011246411481910357
Val loss -  0.00010517429459475149
Val loss -  0.0001323081745292796
Val loss -  8.158643379522332e-05
Val loss -  7.003947067678972e-05
Val loss -  3.52857134674292e-05
Val loss -  3.0747741955649374e-05
Val loss -  1.7565605077227165e-05
Val loss -  2.990346138948545e-05
Val loss -  3.160114442015781e-05
Val loss -  3.350087948843573e-05
Val loss -  3.573995302458915e-05
Val loss -  3.258887260097258e-05
Val loss -  2.744249011691433e-05
Val loss -  1.942858961315874e-05
Val loss -  2.133828530167572e-05
Val loss -  1.7664300942815927e-05
Val loss -  2.780931704738724e-05
Val loss -  3.83365530187068e-05
Val loss -  7.12550535855972e-05
Val loss -  7.416197922529999e-05
Val loss -  9.170150490410072e-05
Val loss -  0.00012818143850243762
```

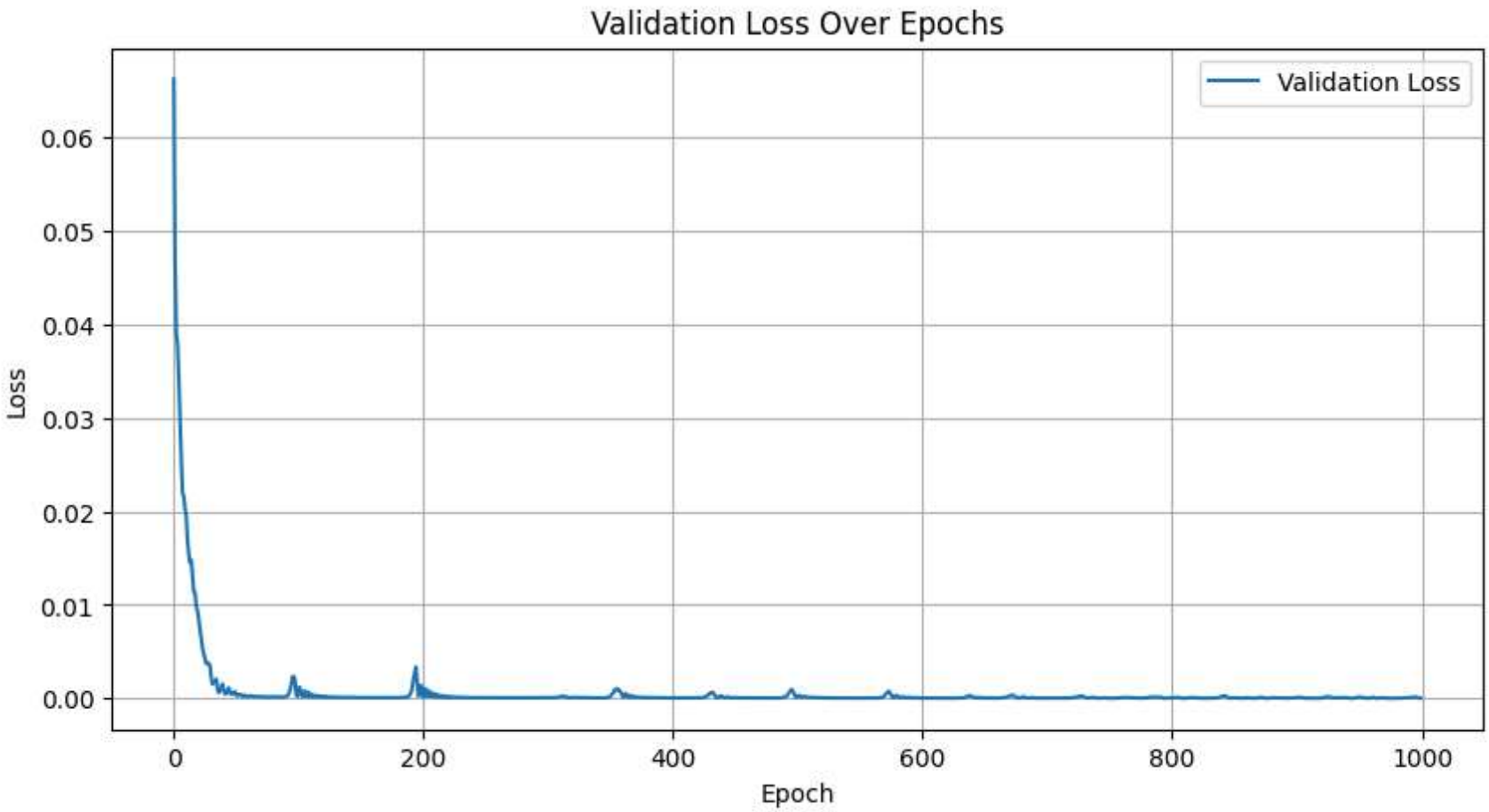
```
Val loss -  0.0002143783161501656
Val loss -  0.0001629983283502016
Val loss -  0.00014697435546482456
Val loss -  0.00011550498059848358
Val loss -  0.00010122980027564534
Val loss -  7.056835048746909e-05
Val loss -  7.675386135309068e-05
Val loss -  6.206123262095708e-05
Val loss -  5.9713399153866056e-05
Val loss -  5.38636264969005e-05
Val loss -  4.471971166711472e-05
Val loss -  5.6297609481280494e-05
Val loss -  6.0639847063552086e-05
Val loss -  0.00010569105971686966
Val loss -  8.091659624201469e-05
Val loss -  8.261437721386017e-05
Val loss -  6.913855560013526e-05
Val loss -  5.0087203531401914e-05
Val loss -  2.3294487982745833e-05
Val loss -  1.6164836444764393e-05
Val loss -  1.5716699783997462e-05
Val loss -  2.073848945088215e-05
Val loss -  4.408760695085919e-05
Val loss -  5.5892570100871316e-05
Val loss -  8.176561804650356e-05
Val loss -  0.00010191612494296565
Val loss -  0.00015555607997825717
Val loss -  0.00011232189271000531
Val loss -  8.958908394286973e-05
Val loss -  8.136928623139422e-05
Val loss -  7.702852621787138e-05
Val loss -  3.591112146848649e-05
Val loss -  2.073255596107852e-05
Val loss -  1.9888494390984623e-05
Val loss -  4.800133916941604e-05
Val loss -  9.184435350073516e-05
Val loss -  0.00010476588482708105
Val loss -  0.00011066718819492
Val loss -  6.427022837608866e-05
Val loss -  3.761242647195669e-05
Val loss -  1.5179532335813336e-05
Val loss -  2.3117595590859498e-05
```

```
Val loss -  4.445187763077148e-05
Val loss -  6.059814131483816e-05
Val loss -  8.466423313985946e-05
Val loss -  5.6389737187949924e-05
Val loss -  4.2757123178549863e-05
Val loss -  3.494187601062149e-05
Val loss -  3.6014450192332616e-05
Val loss -  2.4033238438684587e-05
Val loss -  2.7182027164829483e-05
Val loss -  1.833317815808084e-05
Val loss -  1.9205416608681147e-05
Val loss -  1.2377906086110696e-05
Val loss -  1.596751016448582e-05
Val loss -  1.3718939315196588e-05
Val loss -  2.1094898892234608e-05
Val loss -  1.9171943736443898e-05
Val loss -  2.4778141176067143e-05
Val loss -  3.164365112339812e-05
Val loss -  5.5897349141236976e-05
Val loss -  4.737950347488938e-05
Val loss -  5.048377757546158e-05
Val loss -  5.883718604343016e-05
Val loss -  8.179894916834453e-05
Val loss -  6.97441463847295e-05
Val loss -  8.315797889679288e-05
Val loss -  8.430169572705009e-05
Val loss -  0.00010256509640569136
Val loss -  0.00011640592758260835
Val loss -  0.00017773532722420603
Val loss -  0.0001320772133022645
Val loss -  0.00011319762560459284
Val loss -  7.467116416856815e-05
Val loss -  3.6221124518868074e-05
Val loss -  2.5490895537533644e-05
```

In [ ]:

```python
# Import matplotlib
import matplotlib.pyplot as plt

# Plot the validation errors
plt.figure(figsize=(10, 5))
plt.plot(errors, label="Validation Loss")
plt.title("Validation Loss Over Epochs")
```

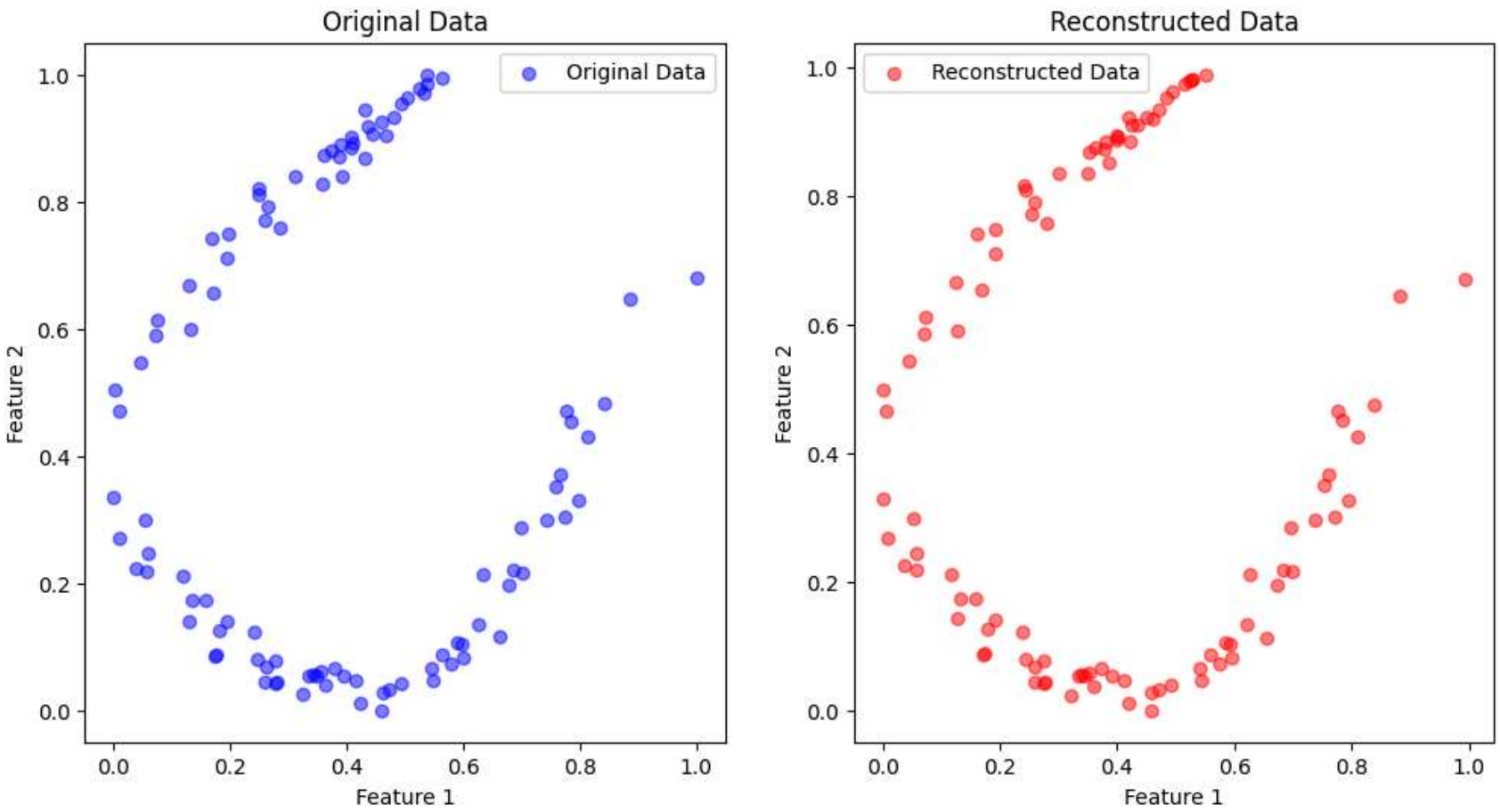
```
plt.xlabel("Epoch")
plt.ylabel("Loss")
plt.legend()
plt.grid(True)
plt.show()
```



In [ ]:
```
import matplotlib.pyplot as plt
print(a[l].shape)

# 'a[l]' contains the reconstructed output from the last layer
# Since 'a[l]' has shape (2, number of samples), we transpose it to align with the original data
reconstructed_X = a[l].T

# Plot original data
```

```python
plt.figure(figsize=(12, 6))
plt.subplot(1, 2, 1)
plt.scatter(
    X_normalized[:, 0], X_normalized[:, 1], c="blue", alpha=0.5, label="Original Data"
)
plt.title("Original Data")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")
plt.legend()

# Plot reconstructed data
plt.subplot(1, 2, 2)
plt.scatter(
    reconstructed_X[:, 0],
    reconstructed_X[:, 1],
    c="red",
    alpha=0.5,
    label="Reconstructed Data",
)
plt.title("Reconstructed Data")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")
plt.legend()

plt.show()
```

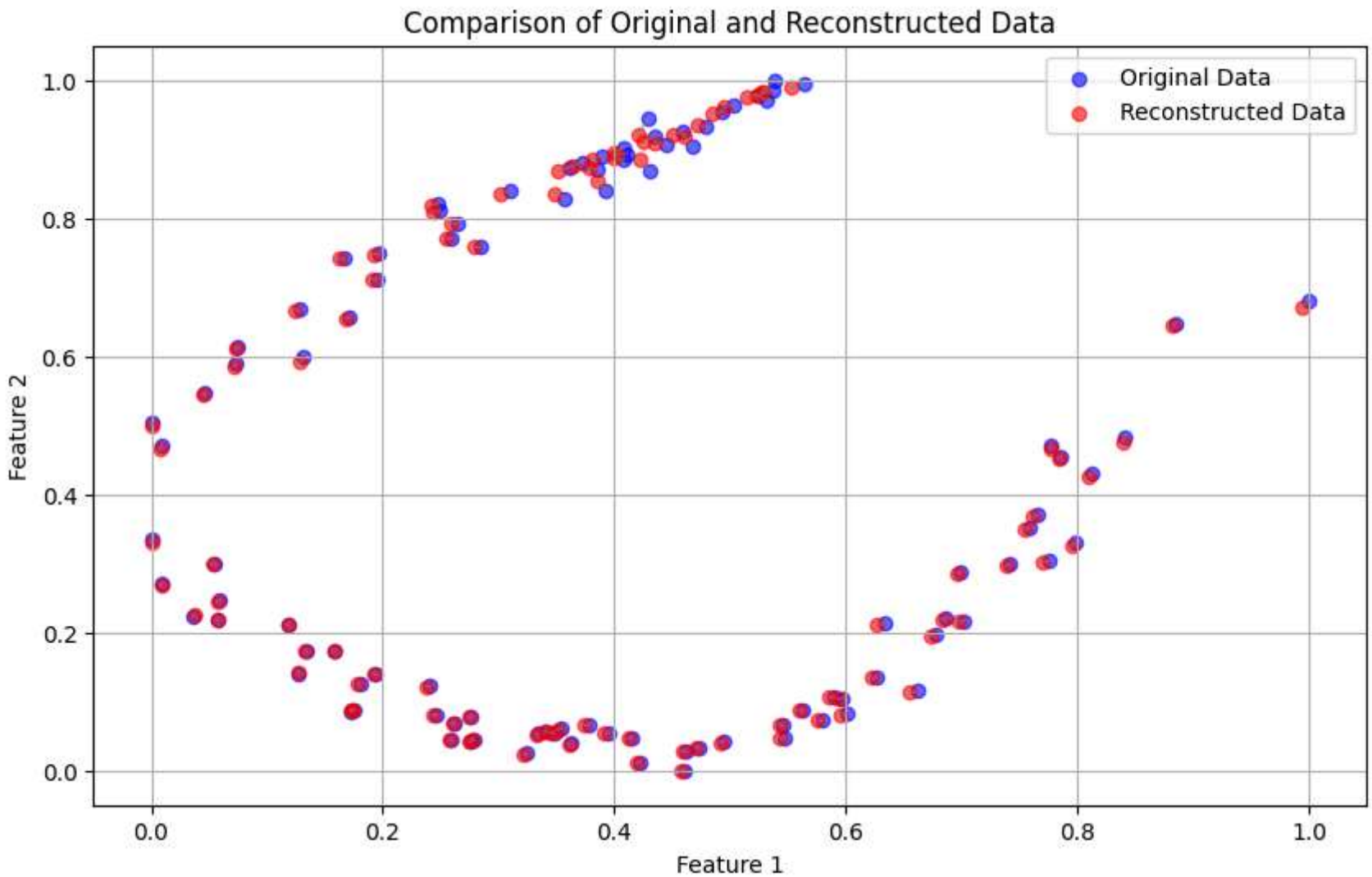
```
(2, 100)
```



In [ ]:

```
import matplotlib.pyplot as plt

# 计算重建数据
# 假设 a[L] 是网络的最后一层输出，形状为 (2, 样本数)，转置以便和原始数据对齐
reconstructed_X = a[l].T

# 创建一个图形和坐标轴
plt.figure(figsize=(10, 6))
plt.scatter(
    X_normalized[:, 0], X_normalized[:, 1], c="blue", alpha=0.6, label="Original Data"
)
plt.scatter(
    reconstructed_X[:, 0],
```

```python
    reconstructed_X[:, 1],
    c="red",
    alpha=0.6,
    label="Reconstructed Data",
)

# 为每个点绘制连接线
for i in range(len(X_normalized)):
    plt.plot(
        [X_normalized[i, 0], reconstructed_X[i, 0]],
        [X_normalized[i, 1], reconstructed_X[i, 1]],
        "gray",
        alpha=0.5,
    )

# 添加图例和标签
plt.title("Comparison of Original and Reconstructed Data")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")
plt.legend()
plt.grid(True)
plt.show()
```



Question 13.8

In [ ]:
```
# get MNIST data from online repository
from sklearn.datasets import fetch_openml

x, y = fetch_openml("mnist_784", version=1, return_X_y=True)
```

```
x = x.T

# convert string labels to integers
y = np.array([int(v) for v in y])[:, np.newaxis]
y = y.T

print("input shape = ", x.shape)
print("output shape = ", y.shape)
```

```
input shape =  (784, 70000)
output shape =  (1, 70000)
```

In [ ]:

```
# 设置抽样数量
num_samples = 50000

# 生成随机的样本索引
indices = np.random.choice(x.shape[1], num_samples, replace=False)

# 使用这些索引来选择样本
# Sample the data
x_sample = x[indices]
y_sample = y[:, indices]

# 输出抽样后的数据形状
print("Sampled input shape = ", x_sample.shape)
print("Sampled output shape = ", y_sample.shape)
```

```
Sampled input shape =  (784, 50000)
Sampled output shape =  (1, 50000)
```

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler


# Activation functions and their derivatives
def relu(x):
    return np.maximum(0, x)


def relu_derivative(x):
```

```python
    return (x > 0).astype(x.dtype)


def softmax(x):
    exp_x = np.exp(x - np.max(x, axis=1, keepdims=True))
    return exp_x / np.sum(exp_x, axis=1, keepdims=True)


# Loss function
def compute_loss(y, y_pred):
    m = y.shape[0]
    log_likelihood = -np.log(y_pred[range(m), y])
    loss = np.mean(log_likelihood)
    return loss


# Batch normalization forward and backward
def batch_norm_forward(x, gamma, beta, eps=1e-8):
    mean = np.mean(x, axis=0)
    variance = np.var(x, axis=0)
    x_norm = (x - mean) / np.sqrt(variance + eps)
    out = gamma * x_norm + beta
    cache = (x, x_norm, mean, variance, gamma, beta)
    return out, cache


def batch_norm_backward(dout, cache):
    x, x_norm, mean, variance, gamma, beta = cache
    N = x.shape[0]
    x_mu = x - mean
    std_inv = 1.0 / np.sqrt(variance + 1e-8)

    dx_norm = dout * gamma
    dvar = np.sum(dx_norm * x_mu, axis=0) * -0.5 * std_inv**3
    dmu = np.sum(dx_norm * -std_inv, axis=0) + dvar * np.mean(-2.0 * x_mu, axis=0)

    dx = (dx_norm * std_inv) + (dvar * 2 * x_mu / N) + (dmu / N)
    dgamma = np.sum(dout * x_norm, axis=0)
    dbeta = np.sum(dout, axis=0)

    return dx, dgamma, dbeta
```

```python
# Initialize parameters
def initialize_parameters(layer_sizes):
    parameters = []
    for i in range(len(layer_sizes) - 1):
        W = np.random.randn(layer_sizes[i], layer_sizes[i + 1]) * np.sqrt(
            2.0 / layer_sizes[i]
        )  # He initialization
        b = np.zeros((1, layer_sizes[i + 1]))
        gamma = np.ones((1, layer_sizes[i + 1]))
        beta = np.zeros((1, layer_sizes[i + 1]))
        parameters.append({"W": W, "b": b, "gamma": gamma, "beta": beta})
    return parameters


# Forward and backward passes
def forward_backward_propagation(X, y, parameters):
    caches = []
    activations = [X]
    A = X

    # Forward pass with batch normalization and ReLU
    for param in parameters[:-1]:  # Last layer without batch normalization
        Z = np.dot(A, param["W"]) + param["b"]
        BN, cache_bn = batch_norm_forward(Z, param["gamma"], param["beta"])
        A = relu(BN)
        activations.append(A)
        caches.append((param, cache_bn))

    # Output layer
    final_Z = np.dot(A, parameters[-1]["W"]) + parameters[-1]["b"]
    final_A = softmax(final_Z)
    activations.append(final_A)
    caches.append((parameters[-1], None))

    # Compute loss
    loss = compute_loss(y, final_A)

    # Backward pass
    grads = []
    dA = final_A
    dA[range(y.shape[0]), y] -= 1  # Derivative of cross-entropy loss with softmax
```

```python
    dA /= y.shape[0]

    for i in reversed(range(len(caches))):
        param, cache_bn = caches[i]
        if cache_bn:
            dZ = relu_derivative(cache_bn[0]) * dA  # Apply relu derivative
            dA, dgamma, dbeta = batch_norm_backward(dZ, cache_bn)
            grads.append(
                {
                    "dW": np.dot(activations[i].T, dA),
                    "db": np.sum(dA, axis=0),
                    "dgamma": dgamma,
                    "dbeta": dbeta,
                }
            )
        else:
            dW = np.dot(activations[i].T, dA)
            db = np.sum(dA, axis=0)
            grads.append({"dW": dW, "db": db})
        dA = np.dot(dA, param["W"].T)

    grads.reverse()
    return loss, grads


# Update parameters
def update_parameters(parameters, grads, learning_rate):
    for param, grad in zip(parameters, grads):
        param["W"] -= learning_rate * grad["dW"]
        param["b"] -= learning_rate * grad["db"]
        if "dgamma" in grad and "dbeta" in grad:
            param["gamma"] -= learning_rate * grad["dgamma"]
            param["beta"] -= learning_rate * grad["dbeta"]


# Main training function
def train_with_norm(X, y, layer_sizes, epochs, learning_rate, init_para):
    parameters = init_para
    losses = []  # 存储损失历史
    for epoch in range(epochs):
        loss, grads = forward_backward_propagation(X, y, parameters)
        update_parameters(parameters, grads, learning_rate)
```

```python
        losses.append(loss)
        if epoch % 10 == 0:
            print(f"Epoch {epoch}, Loss: {loss}")
    return parameters, losses


# Load data
X, y = fetch_openml("mnist_784", version=1, return_X_y=True, as_frame=False)
y = y.astype(int)

# Preprocess data
scaler = StandardScaler()
X_normalized = scaler.fit_transform(X)

# Split data into training and validation sets
X_train, X_val, y_train, y_val = train_test_split(
    X_normalized, y, test_size=0.2, random_state=42
)


# Define layer sizes and train the network
layer_sizes = [784, 128, 64, 10]  # Including input layer and output layer
init_parameter = initialize_parameters(layer_sizes)

import copy
init_parameter_wo_batch_norm = copy.deepcopy(init_parameter)

trained_parameters, lossess_with_batch_norm = train_with_norm(
    X_train,
    y_train,
    layer_sizes,
    epochs=10,
    learning_rate=0.01,
    init_para=init_parameter,
)
```

```
Epoch 0, Loss: 2.695709478303017
```

In [ ]:

```python
def no_batch_norm_forward(x, gamma, beta):
    out = gamma * x + beta
    cache = (x, gamma, beta)
    return out, cache
```

```python
def no_batch_norm_backward(dout, cache):
    x, gamma, beta = cache
    dx = dout * gamma
    dgamma = np.sum(dout * x, axis=0)
    dbeta = np.sum(dout, axis=0)
    return dx, dgamma, dbeta




# Forward and backward passes
def forward_backward_propagation_wo_norm(X, y, parameters):
    caches = []
    activations = [X]
    A = X

    # Forward pass with batch normalization and ReLU
    for param in parameters[:-1]:  # Last layer without batch normalization
        Z = np.dot(A, param["W"]) + param["b"]
        BN, cache_bn = no_batch_norm_forward(Z, param["gamma"], param["beta"])
        A = relu(BN)
        activations.append(A)
        caches.append((param, cache_bn))

    # Output layer
    final_Z = np.dot(A, parameters[-1]["W"]) + parameters[-1]["b"]
    final_A = softmax(final_Z)
    activations.append(final_A)
    caches.append((parameters[-1], None))

    # Compute loss
    loss = compute_loss(y, final_A)

    # Backward pass
    grads = []
    dA = final_A
    dA[range(y.shape[0]), y] -= 1  # Derivative of cross-entropy loss with softmax
    dA /= y.shape[0]

    for i in reversed(range(len(caches))):
```

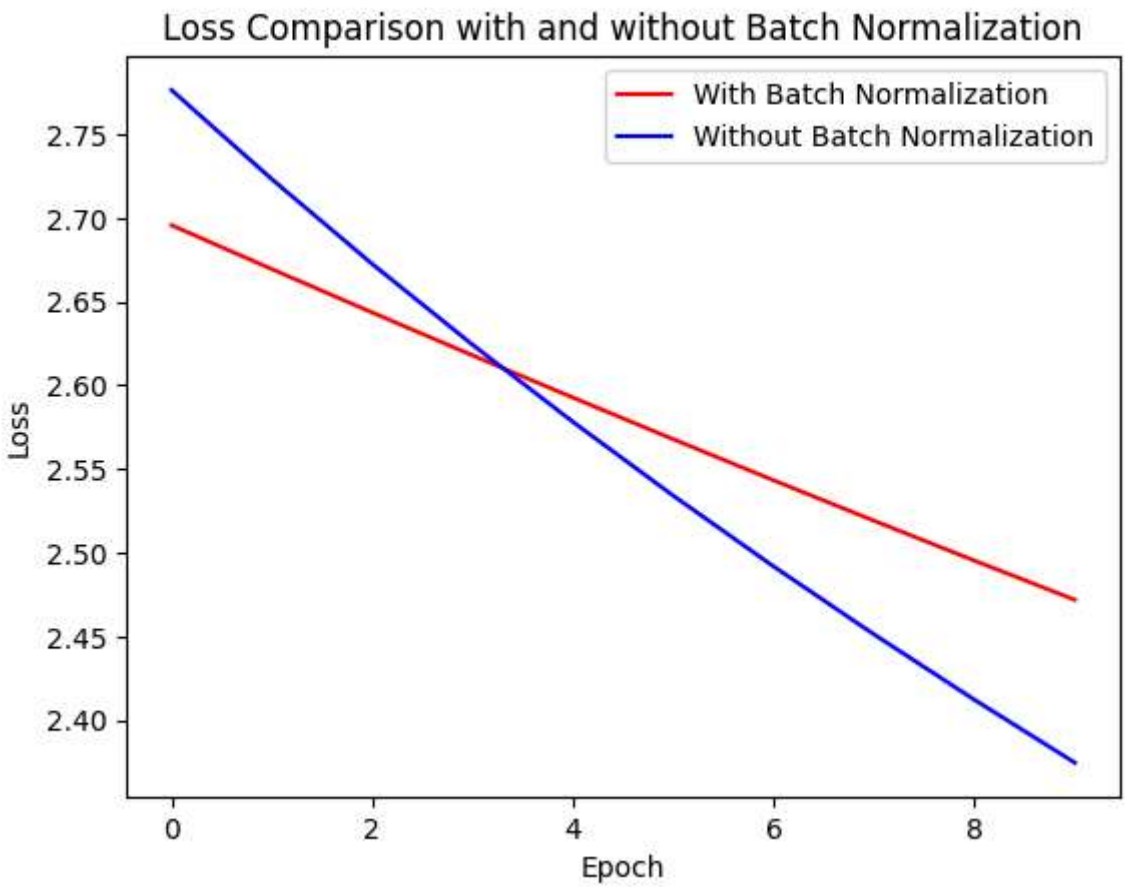
```python
        param, cache_bn = caches[i]
        if cache_bn:
            dZ = relu_derivative(cache_bn[0]) * dA  # Apply relu derivative
            dA, dgamma, dbeta = no_batch_norm_backward(dZ, cache_bn)
            grads.append(
                {
                    "dW": np.dot(activations[i].T, dA),
                    "db": np.sum(dA, axis=0),
                    "dgamma": dgamma,
                    "dbeta": dbeta,
                }
            )
        else:
            dW = np.dot(activations[i].T, dA)
            db = np.sum(dA, axis=0)
            grads.append({"dW": dW, "db": db})
        dA = np.dot(dA, param["W"].T)

    grads.reverse()
    return loss, grads


# Main training function
def train_without_norm(X, y, layer_sizes, epochs, learning_rate, init_para):
    parameters = init_para
    losses = []  # 存储损失历史
    for epoch in range(epochs):
        loss, grads = forward_backward_propagation_wo_norm(X, y, parameters)
        update_parameters(parameters, grads, learning_rate)
        losses.append(loss)
        if epoch % 10 == 0:
            print(f"Epoch {epoch}, Loss: {loss}")
    return parameters, losses


layer_sizes = [784, 128, 64, 10]  # Including input layer and output layer
trained_parameters, lossess_without_batch_norm = train_without_norm(
    X_train,
    y_train,
    layer_sizes,
    epochs=10,
    learning_rate=0.01,
```

```
    init_para=init_parameter_wo_batch_norm,
)
```

```
Epoch 0, Loss: 2.776456197184883
```

In [ ]:

```
plt.plot(
    range(10), lossess_with_batch_norm, label="Without Batch Normalization", color="red"
)
plt.plot(
    range(10),
    lossess_without_batch_norm,
    label="With Batch Normalization",
    color="blue",
)


plt.xlabel("Epoch")

plt.ylabel("Loss")

plt.title("Loss Comparison with and without Batch Normalization")

plt.legend()
plt.show()
```



Question 13.9

In [ ]:
```
import numpy as np

data_path = "./datasets/noisy_sin_sample.csv"

# Load the data using numpy
data = np.loadtxt(data_path, delimiter=",")

# Separate the features and target
X = data[:-1, :].T  # Transpose to make each row an observation
y = data[-1, :]
```

```python
# Split the data into training and validation sets (2/3 training, 1/3 validation)
num_samples = X.shape[0]
train_size = int(2 / 3 * num_samples)

# Training data
X_train = X[:train_size]
y_train = y[:train_size]

# Validation data
X_val = X[train_size:]
y_val = y[train_size:]

X_train.shape, y_train.shape, X_val.shape, y_val.shape
```

Out[ ]: ((14, 1), (14,), (7, 1), (7,))

In [ ]:

```python
import numpy as np


def initialize_weights(layers):
    weights = {}
    for i in range(1, len(layers)):
        weights[f"W{i}"] = np.random.randn(layers[i], layers[i - 1]) * np.sqrt(
            1.0 / layers[i - 1]
        )
        weights[f"b{i}"] = np.zeros((layers[i], 1))
    return weights


def tanh(x):
    return np.tanh(x)


def tanh_derivative(x):
    return 1 - np.tanh(x) ** 2


def forward_pass(X, weights, layers):
    activations = {"A0": X}
    for i in range(1, len(layers)):
        z = np.dot(weights[f"W{i}"], activations[f"A{i-1}"]) + weights[f"b{i}"]
        a = tanh(z)
```

```python
        activations[f"Z{i}"] = z
        activations[f"A{i}"] = a
    return activations


def compute_loss(y, y_hat):
    return np.mean((y - y_hat) ** 2)


def backward_pass(activations, weights, layers, X, y):
    m = X.shape[1]
    y = y.reshape(1, -1)  # Reshape y to match output dimensions
    gradients = {}
    # Output layer error
    delta = (activations[f"A{len(layers)-1}"] - y) * tanh_derivative(
        activations[f"Z{len(layers)-1}"]
    )
    gradients[f"W{len(layers)-1}"] = (
        1 / m * np.dot(delta, activations[f"A{len(layers)-2}"].T)
    )
    gradients[f"b{len(layers)-1}"] = 1 / m * np.sum(delta, axis=1, keepdims=True)

    # Hidden layers error
    for i in range(len(layers) - 2, 0, -1):
        delta = np.dot(weights[f"W{i+1}"].T, delta) * tanh_derivative(
            activations[f"Z{i}"]
        )
        gradients[f"W{i}"] = 1 / m * np.dot(delta, activations[f"A{i-1}"].T)
        gradients[f"b{i}"] = 1 / m * np.sum(delta, axis=1, keepdims=True)

    return gradients


def update_weights(weights, gradients, learning_rate):
    for key in weights.keys():
        weights[key] -= learning_rate * gradients[key]
    return weights


# Neural network architecture
layers = [
    1,
```

```
    10,
    10,
    10,
    1,
]  # Three hidden layers with 10 units each, 1 input and 1 output

# Initialize weights
weights = initialize_weights(layers)

# Learning rate and epochs
learning_rate = 0.01
epochs = 10000
patience = 10  # Patience for early stopping

# Early stopping variables
best_loss = np.inf
patience_counter = 0

train_losses, val_losses = [], []
bast_epoch = 0

# Training loop
for epoch in range(epochs):
    # Forward pass on training data
    activations_train = forward_pass(X_train.T, weights, layers)
    train_loss = compute_loss(y_train, activations_train[f"A{len(layers)-1}"])
    train_losses.append(train_loss)

    # Forward pass on validation data
    activations_val = forward_pass(X_val.T, weights, layers)
    val_loss = compute_loss(y_val, activations_val[f"A{len(layers)-1}"])
    val_losses.append(val_loss)

    # Backpropagation and weight update
    gradients = backward_pass(activations_train, weights, layers, X_train.T, y_train)
    weights = update_weights(weights, gradients, learning_rate)

    # Check for early stopping
    if val_loss < best_loss:
        best_loss = val_loss
        best_weights = weights
        patience_counter = 0
```

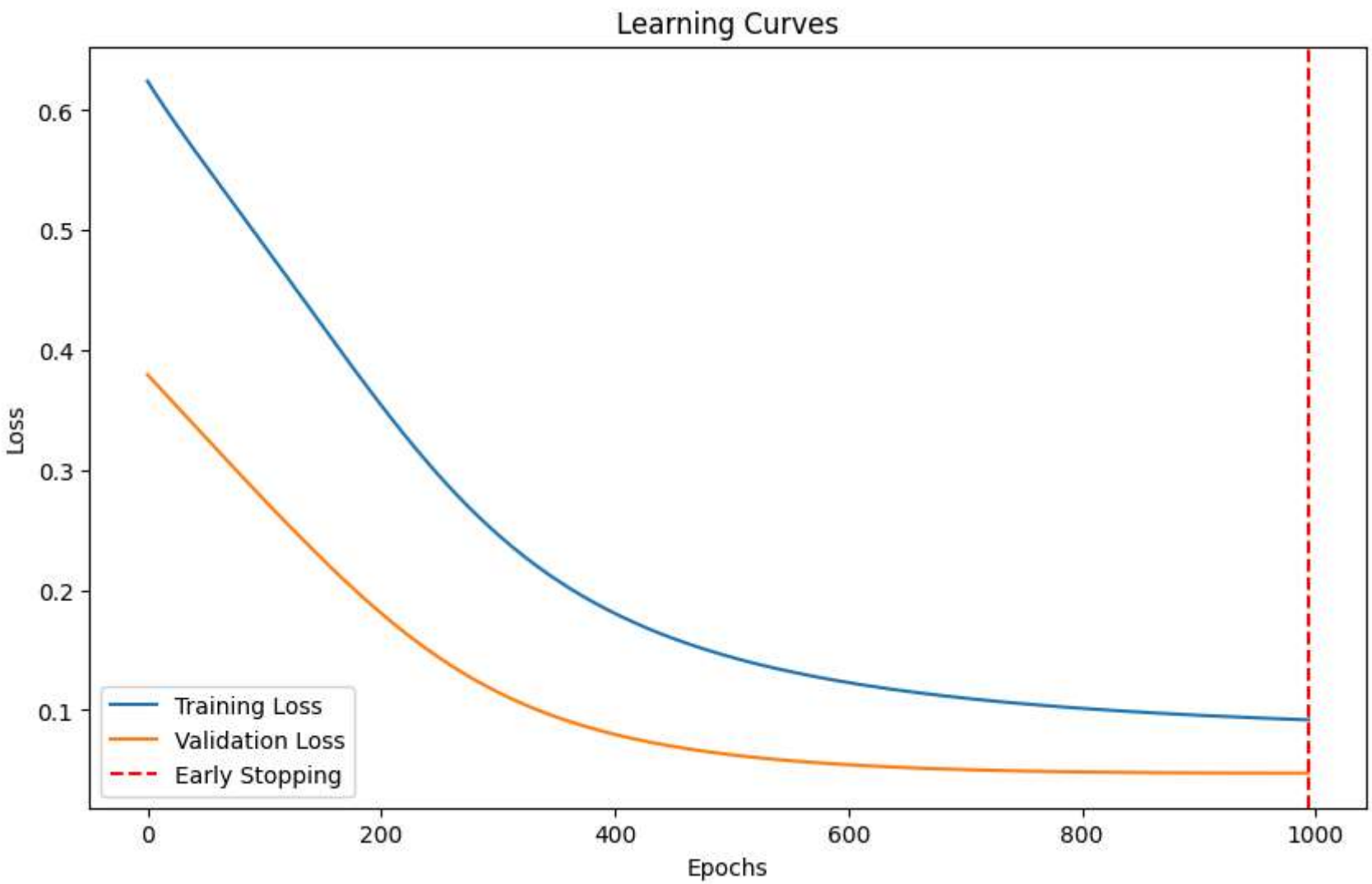
```
    else:
        patience_counter += 1
        if patience_counter >= patience:
            print(f"Early stopping at epoch {epoch} with validation loss: {best_loss}")
            best_epoch = epoch
            break

    # Print training information every 50 epochs
    if epoch % 50 == 0:
        print(
            f"Epoch {epoch}, Training Loss: {train_loss:.4f}, Validation Loss: {val_loss:.4f}"
        )

# The final best weights are stored in `best_weights`
```

```
Epoch 0, Training Loss: 0.6243, Validation Loss: 0.3796
Epoch 50, Training Loss: 0.5536, Validation Loss: 0.3270
Epoch 100, Training Loss: 0.4871, Validation Loss: 0.2751
Epoch 150, Training Loss: 0.4198, Validation Loss: 0.2255
Epoch 200, Training Loss: 0.3541, Validation Loss: 0.1806
Epoch 250, Training Loss: 0.2949, Validation Loss: 0.1433
Epoch 300, Training Loss: 0.2462, Validation Loss: 0.1148
Epoch 350, Training Loss: 0.2086, Validation Loss: 0.0941
Epoch 400, Training Loss: 0.1805, Validation Loss: 0.0796
Epoch 450, Training Loss: 0.1596, Validation Loss: 0.0696
Epoch 500, Training Loss: 0.1440, Validation Loss: 0.0626
Epoch 550, Training Loss: 0.1320, Validation Loss: 0.0577
Epoch 600, Training Loss: 0.1228, Validation Loss: 0.0543
Epoch 650, Training Loss: 0.1156, Validation Loss: 0.0519
Epoch 700, Training Loss: 0.1098, Validation Loss: 0.0502
Epoch 750, Training Loss: 0.1051, Validation Loss: 0.0491
Epoch 800, Training Loss: 0.1013, Validation Loss: 0.0483
Epoch 850, Training Loss: 0.0982, Validation Loss: 0.0478
Epoch 900, Training Loss: 0.0956, Validation Loss: 0.0475
Epoch 950, Training Loss: 0.0934, Validation Loss: 0.0474
Early stopping at epoch 994 with validation loss: 0.047342225003456286
```

In [ ]:

```
import matplotlib.pyplot as plt

# Plotting the learning curves
plt.figure(figsize=(10, 6))
plt.plot(train_losses, label="Training Loss")
```

```python
plt.plot(val_losses, label="Validation Loss")
plt.xlabel("Epochs")
plt.ylabel("Loss")
plt.title("Learning Curves")
plt.axvline(x=best_epoch, color="r", linestyle="--", label="Early Stopping")
plt.legend()
plt.show()
```



Question 13.10

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.preprocessing import OneHotEncoder
from sklearn.metrics import accuracy_score
import time


# Load data
x, y = fetch_openml("mnist_784", version=1, return_X_y=True)
x = x.astype(np.float32)
```

In [ ]:
```
# Standardize data
scaler = StandardScaler()
x = scaler.fit_transform(x)

# Convert labels to integers and then one-hot encode
y = np.array([int(v) for v in y])[:, np.newaxis]
encoder = OneHotEncoder()
y_onehot = encoder.fit_transform(y).toarray()

# Split the data
x_train, x_test, y_train, y_test = train_test_split(
    x, y_onehot, test_size=0.2, random_state=42
)

# Neural network parameters
input_size = 784  # Number of input neurons (28x28 pixels)
hidden_size = 50  # Number of hidden neurons
output_size = 10  # Number of output neurons (digits 0-9)
learning_rate = 0.1
epochs = 500

# Weight initialization
np.random.seed(42)
```

```python
W1 = np.random.randn(input_size, hidden_size) * 0.01
b1 = np.zeros((1, hidden_size))
W2 = np.random.randn(hidden_size, output_size) * 0.01
b2 = np.zeros((1, output_size))


# Activation functions
def sigmoid(x):
    return 1 / (1 + np.exp(-x))


def softmax(x):
    exp_x = np.exp(x - np.max(x, axis=1, keepdims=True))
    return exp_x / np.sum(exp_x, axis=1, keepdims=True)


# Storing loss and accuracy history
loss_history = []
train_accuracy_history = []
valid_accuracy_history = []

# Training the network
for epoch in range(epochs):
    start_time = time.time()

    # Forward pass
    z1 = np.dot(x_train, W1) + b1
    a1 = sigmoid(z1)
    z2 = np.dot(a1, W2) + b2
    a2 = softmax(z2)

    # Compute the loss
    loss = -np.mean(np.log(a2[np.arange(len(y_train)), y_train.argmax(axis=1)]))
    loss_history.append(loss)

    # Backpropagation
    dz2 = a2 - y_train
    dW2 = np.dot(a1.T, dz2)
    db2 = np.sum(dz2, axis=0, keepdims=True)
    dz1 = np.dot(dz2, W2.T) * a1 * (1 - a1)
    dW1 = np.dot(x_train.T, dz1)
    db1 = np.sum(dz1, axis=0, keepdims=True)
```

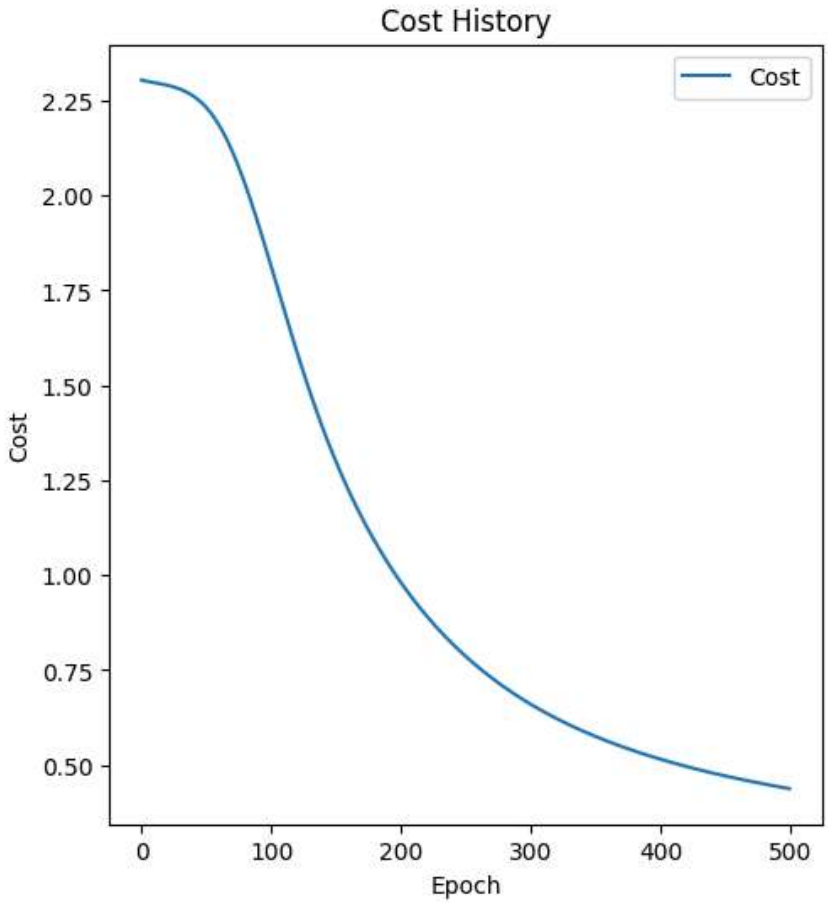
```
    # Update weights
    W2 -= learning_rate * dW2 / len(x_train)
    b2 -= learning_rate * db2 / len(x_train)
    W1 -= learning_rate * dW1 / len(x_train)
    b1 -= learning_rate * db1 / len(x_train)

    # Training accuracy
    train_predictions = np.argmax(a2, axis=1)
    train_accuracy = accuracy_score(y_train.argmax(axis=1), train_predictions)
    train_accuracy_history.append(train_accuracy)

    # Validation accuracy
    z1_valid = np.dot(x_test, W1) + b1
    a1_valid = sigmoid(z1_valid)
    z2_valid = np.dot(a1_valid, W2) + b2
    a2_valid = softmax(z2_valid)
    valid_predictions = np.argmax(a2_valid, axis=1)
    valid_accuracy = accuracy_score(y_test.argmax(axis=1), valid_predictions)
    valid_accuracy_history.append(valid_accuracy)

    # Timing
    end_time = time.time()
    elapsed_time = end_time - start_time

    # Print training progress
    if epoch % 10 == 0 or epoch == epochs - 1:
        print(
            f"step {epoch} done in {elapsed_time:.1f} secs, train acc = {train_accuracy:.4f}, valid acc = {valid_accu
        )
```

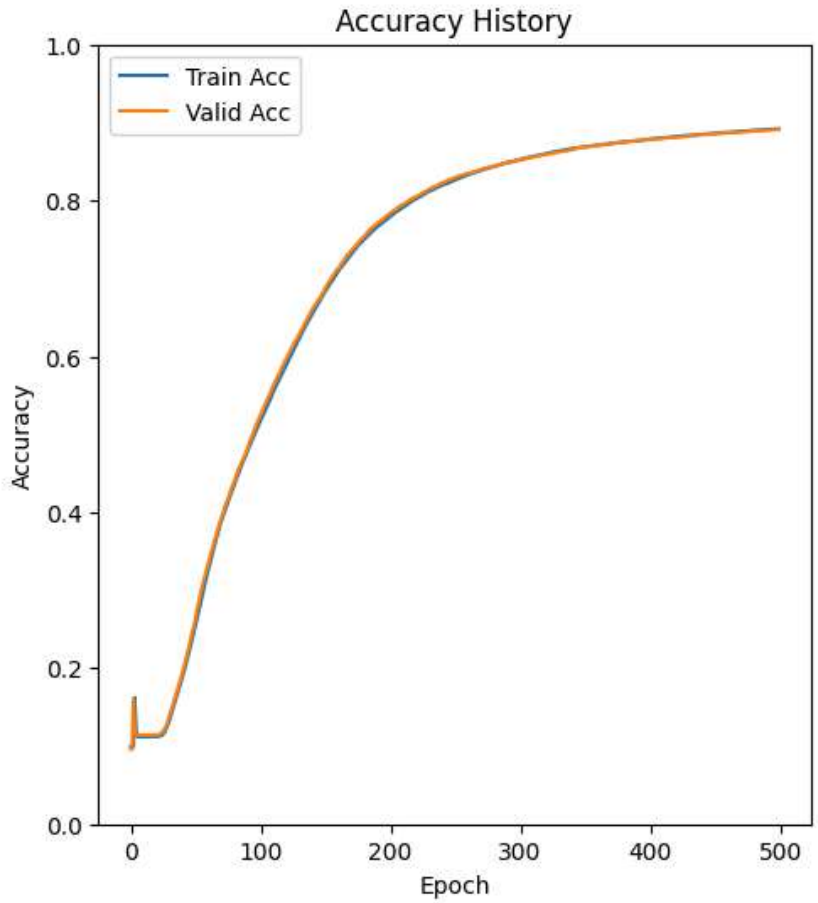
```
step 0 done in 0.4 secs, train acc = 0.0993, valid acc = 0.0959
step 10 done in 0.4 secs, train acc = 0.1121, valid acc = 0.1143
step 20 done in 0.4 secs, train acc = 0.1122, valid acc = 0.1145
step 30 done in 0.4 secs, train acc = 0.1348, valid acc = 0.1395
step 40 done in 0.4 secs, train acc = 0.1904, valid acc = 0.1960
step 50 done in 0.4 secs, train acc = 0.2564, valid acc = 0.2664
step 60 done in 0.4 secs, train acc = 0.3287, valid acc = 0.3360
step 70 done in 0.4 secs, train acc = 0.3902, valid acc = 0.3937
step 80 done in 0.4 secs, train acc = 0.4371, valid acc = 0.4411
step 90 done in 0.4 secs, train acc = 0.4792, valid acc = 0.4821
step 100 done in 0.4 secs, train acc = 0.5178, valid acc = 0.5249
step 110 done in 0.4 secs, train acc = 0.5556, valid acc = 0.5636
step 120 done in 0.4 secs, train acc = 0.5898, valid acc = 0.5977
step 130 done in 0.4 secs, train acc = 0.6240, valid acc = 0.6297
step 140 done in 0.4 secs, train acc = 0.6553, valid acc = 0.6611
step 150 done in 0.4 secs, train acc = 0.6844, valid acc = 0.6884
step 160 done in 0.4 secs, train acc = 0.7098, valid acc = 0.7129
step 170 done in 0.5 secs, train acc = 0.7319, valid acc = 0.7360
step 180 done in 0.4 secs, train acc = 0.7505, valid acc = 0.7541
step 190 done in 0.4 secs, train acc = 0.7664, valid acc = 0.7709
step 200 done in 0.4 secs, train acc = 0.7794, valid acc = 0.7836
step 210 done in 0.4 secs, train acc = 0.7916, valid acc = 0.7954
step 220 done in 0.4 secs, train acc = 0.8025, valid acc = 0.8052
step 230 done in 0.4 secs, train acc = 0.8118, valid acc = 0.8147
step 240 done in 0.4 secs, train acc = 0.8197, valid acc = 0.8224
step 250 done in 0.4 secs, train acc = 0.8263, valid acc = 0.8295
step 260 done in 0.4 secs, train acc = 0.8331, valid acc = 0.8349
step 270 done in 0.4 secs, train acc = 0.8389, valid acc = 0.8397
step 280 done in 0.4 secs, train acc = 0.8439, valid acc = 0.8439
step 290 done in 0.4 secs, train acc = 0.8485, valid acc = 0.8481
step 300 done in 0.4 secs, train acc = 0.8525, valid acc = 0.8519
step 310 done in 0.4 secs, train acc = 0.8566, valid acc = 0.8560
step 320 done in 0.4 secs, train acc = 0.8601, valid acc = 0.8591
step 330 done in 0.4 secs, train acc = 0.8635, valid acc = 0.8624
step 340 done in 0.4 secs, train acc = 0.8666, valid acc = 0.8655
step 350 done in 0.4 secs, train acc = 0.8689, valid acc = 0.8687
step 360 done in 0.4 secs, train acc = 0.8710, valid acc = 0.8706
step 370 done in 0.4 secs, train acc = 0.8730, valid acc = 0.8729
step 380 done in 0.4 secs, train acc = 0.8752, valid acc = 0.8749
step 390 done in 0.4 secs, train acc = 0.8766, valid acc = 0.8769
step 400 done in 0.4 secs, train acc = 0.8785, valid acc = 0.8784
step 410 done in 0.4 secs, train acc = 0.8805, valid acc = 0.8796
```

```
step 420 done in 0.4 secs, train acc = 0.8819, valid acc = 0.8813
step 430 done in 0.4 secs, train acc = 0.8834, valid acc = 0.8828
step 440 done in 0.5 secs, train acc = 0.8848, valid acc = 0.8844
step 450 done in 0.5 secs, train acc = 0.8863, valid acc = 0.8855
step 460 done in 0.4 secs, train acc = 0.8874, valid acc = 0.8868
step 470 done in 0.5 secs, train acc = 0.8888, valid acc = 0.8877
step 480 done in 0.5 secs, train acc = 0.8897, valid acc = 0.8889
step 490 done in 0.5 secs, train acc = 0.8911, valid acc = 0.8900
step 499 done in 0.4 secs, train acc = 0.8918, valid acc = 0.8910
```

In [ ]:

```python
# Plotting cost and accuracy history
plt.figure(figsize=(12, 6))

plt.subplot(1, 2, 1)
plt.plot(loss_history, label="Cost")
plt.title("Cost History")
plt.xlabel("Epoch")
plt.ylabel("Cost")
plt.legend()

plt.subplot(1, 2, 2)
plt.plot(train_accuracy_history, label="Train Acc")
plt.plot(valid_accuracy_history, label="Valid Acc")
plt.title("Accuracy History")
plt.xlabel("Epoch")
plt.ylabel("Accuracy")
plt.ylim(0, 1.0)  # Set the y-axis to be from 0 to 1.0
plt.legend()

plt.show()
```


Cost History
Cost
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0
100
200
300
400
500
Epoch


Accuracy History
Train Acc
Valid Acc
1.0
0.8
0.6
0.4
0.2
0.0
Accuracy
0
100
200
300
400
500
Epoch