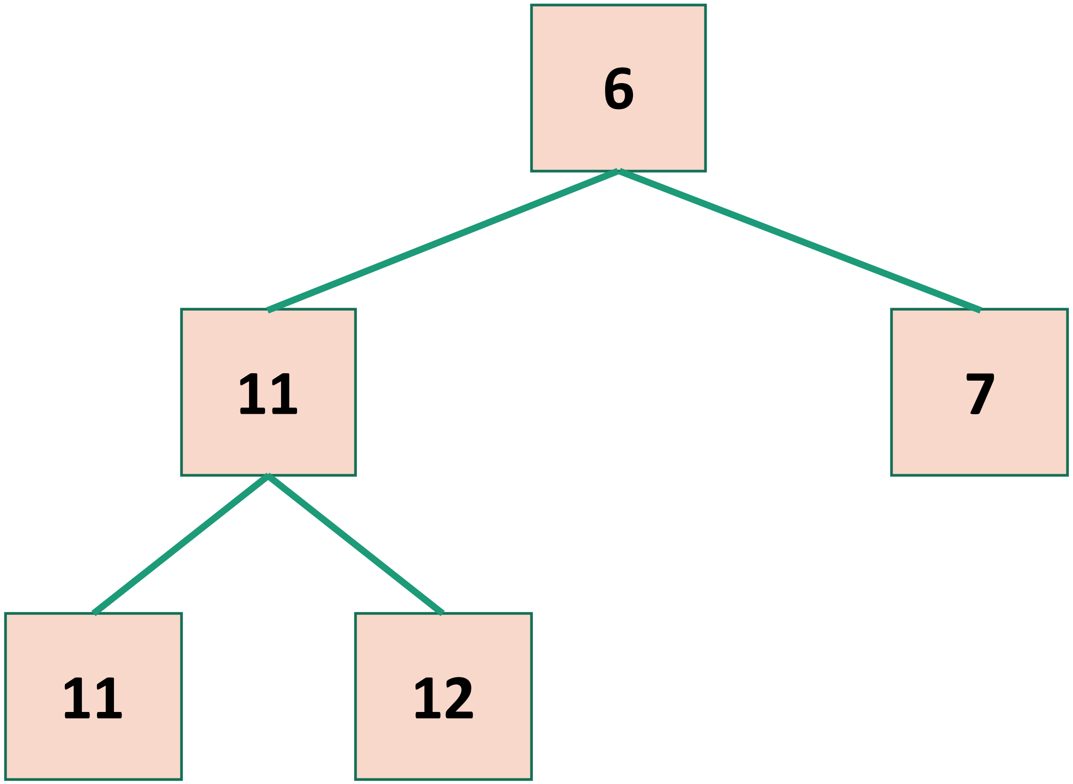
Priority Queue and Heap

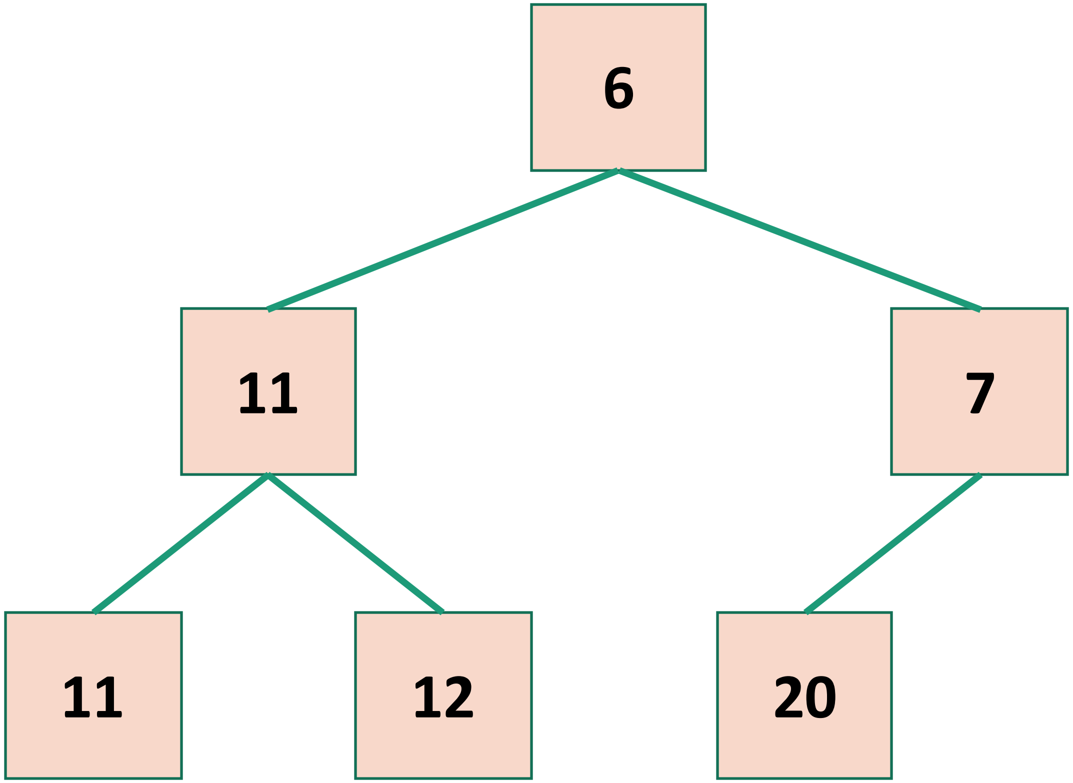
Some slides were created by Dr. Jianfeng Ren.

Edited by Heshan Du

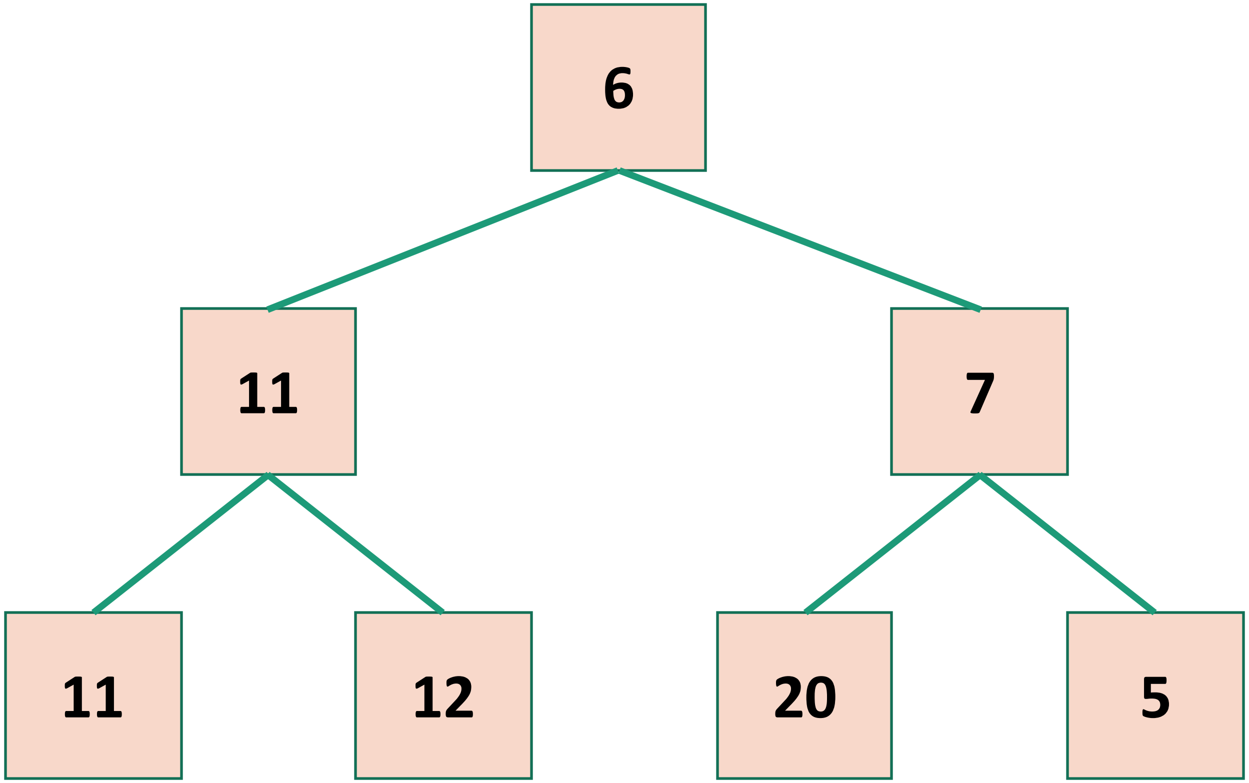
Exercise 1: insert 20, 5, 4



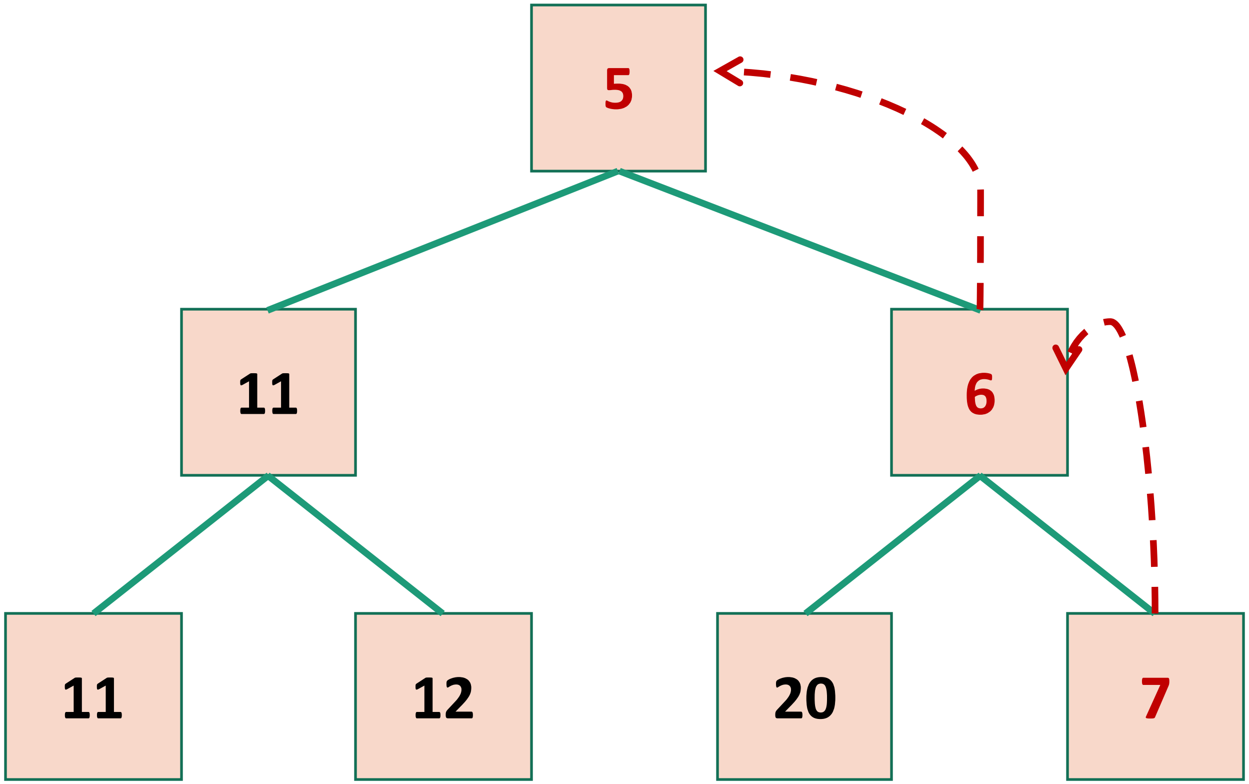
insert 20



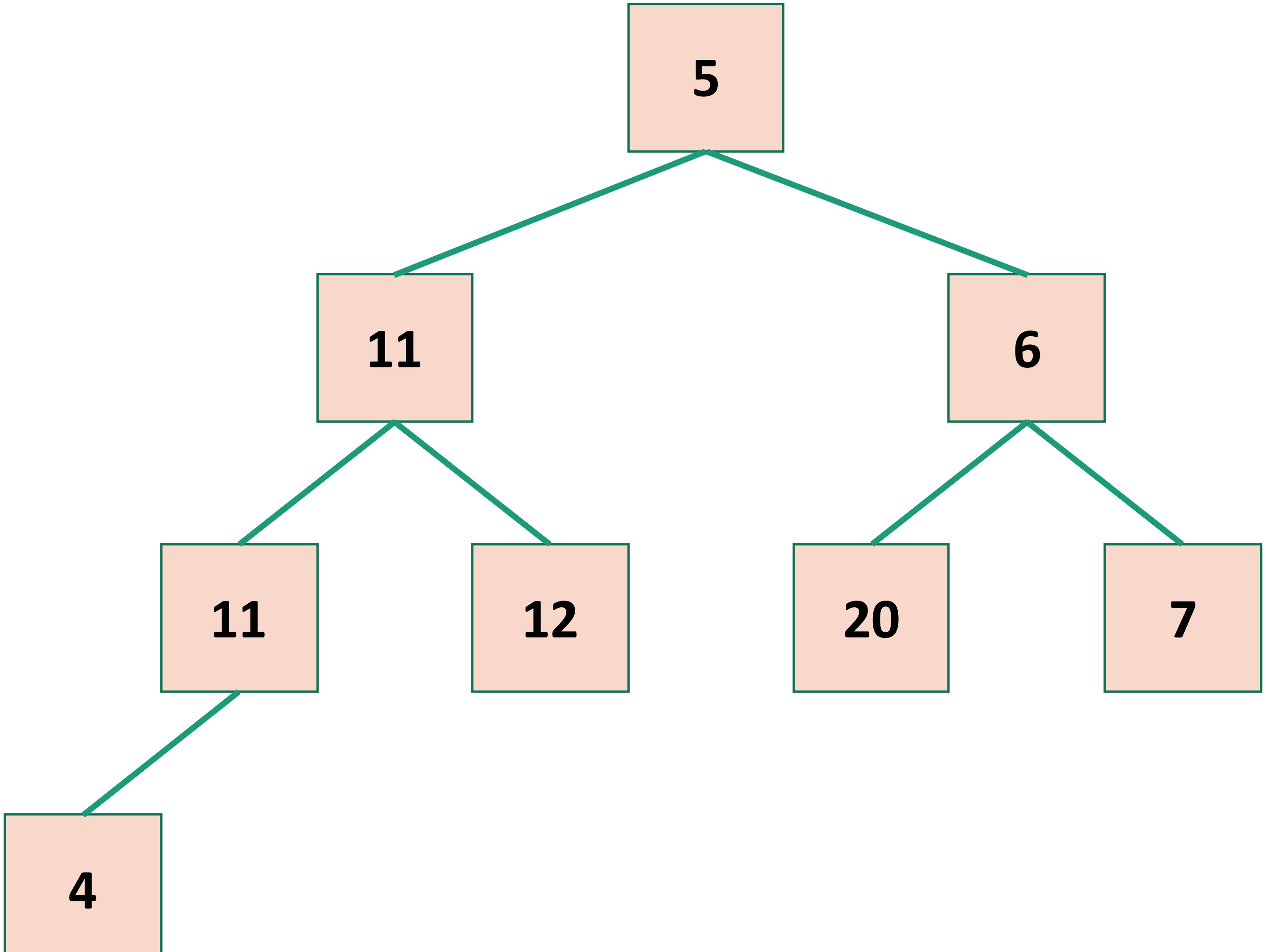
insert 5



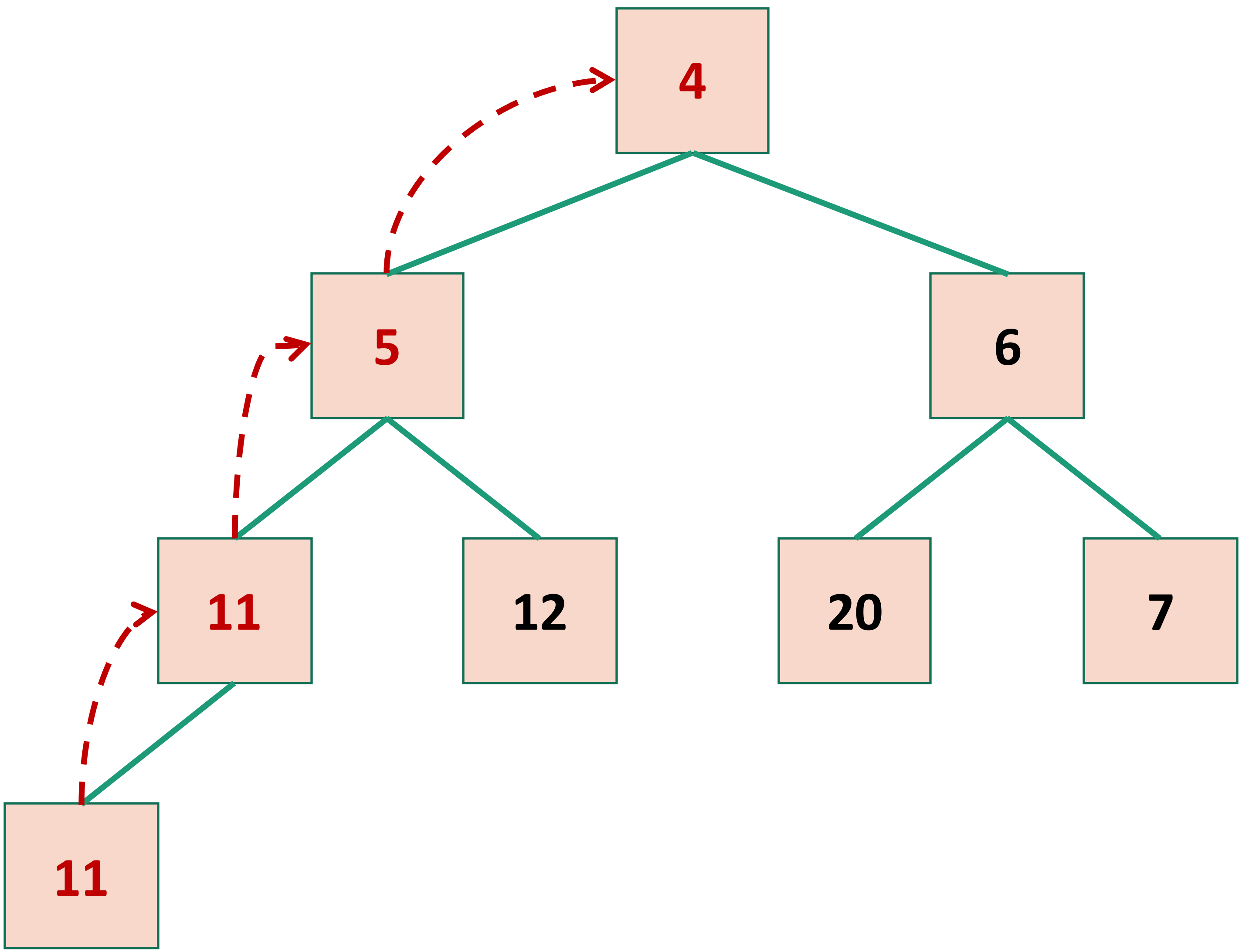
insert 5



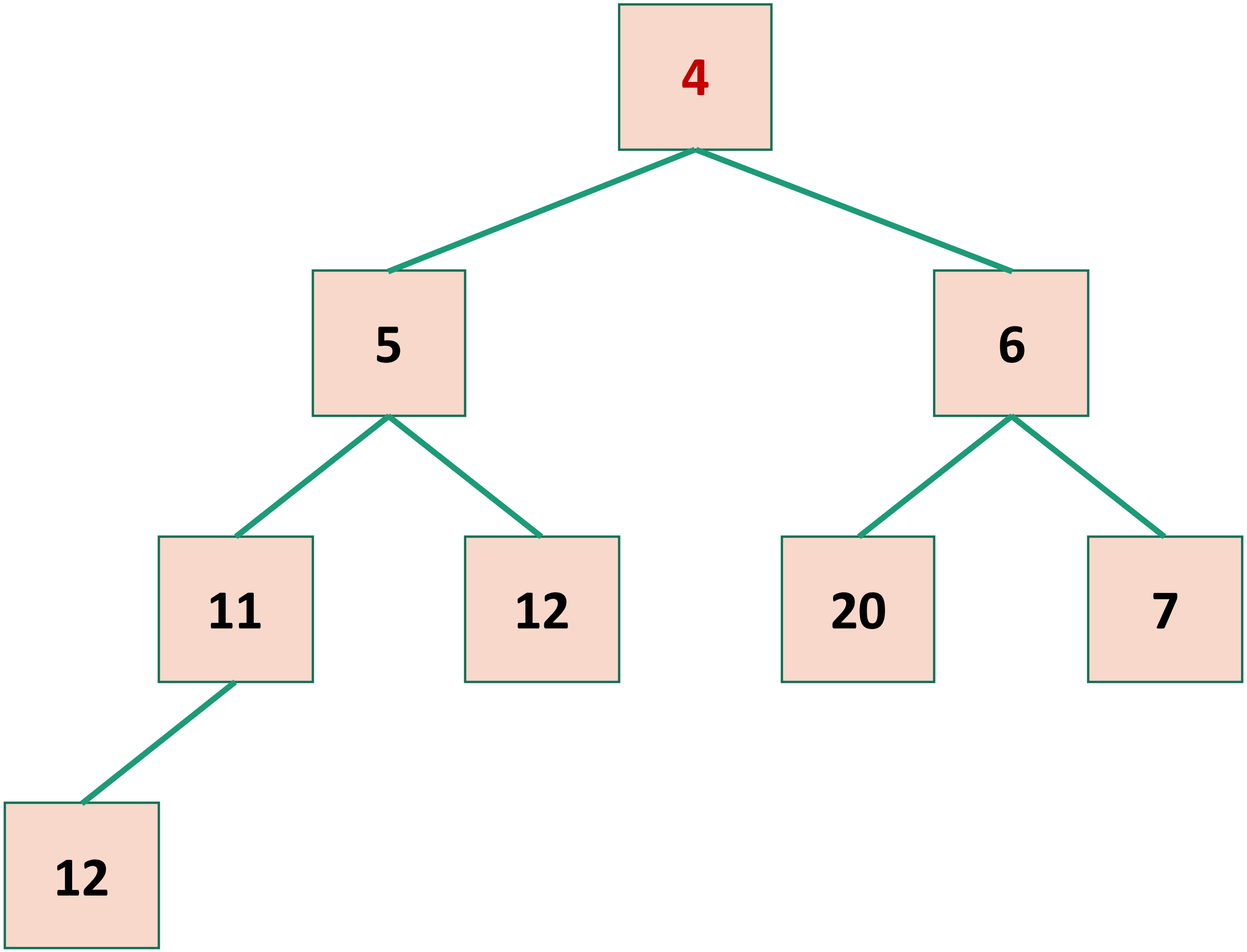
insert 4



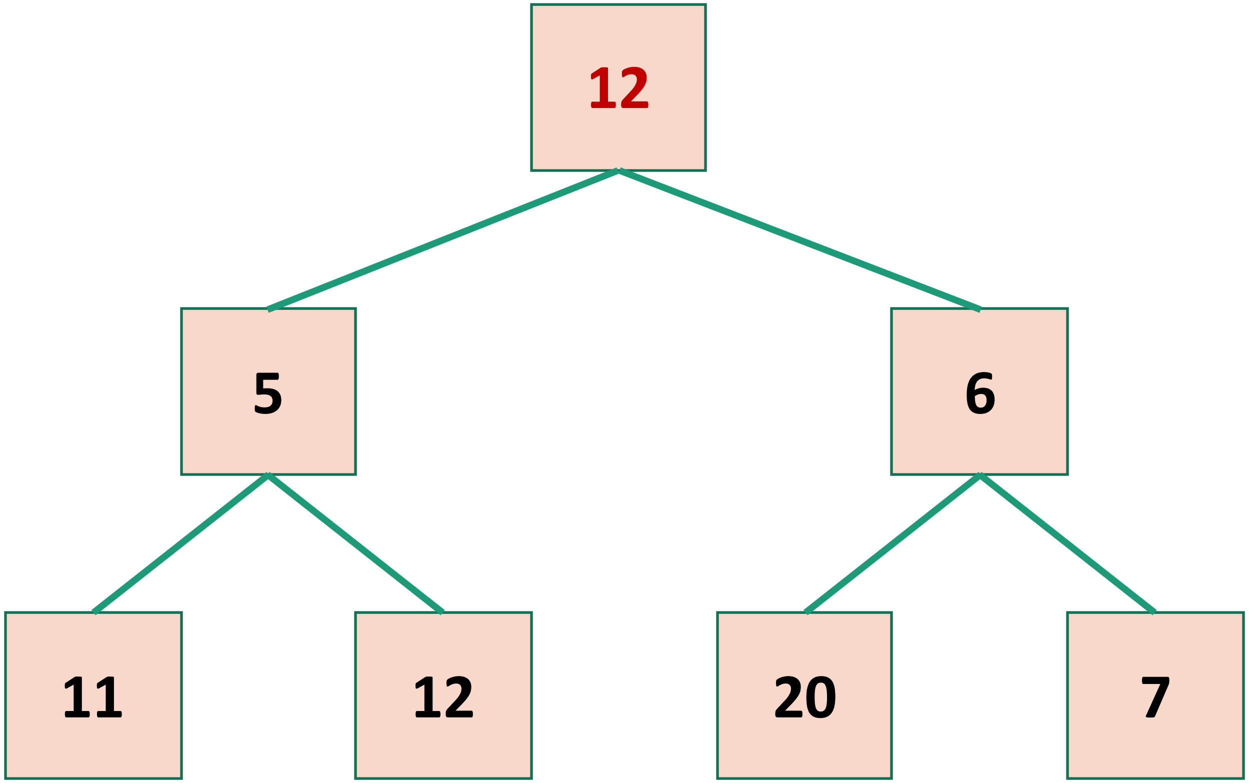
insert 4



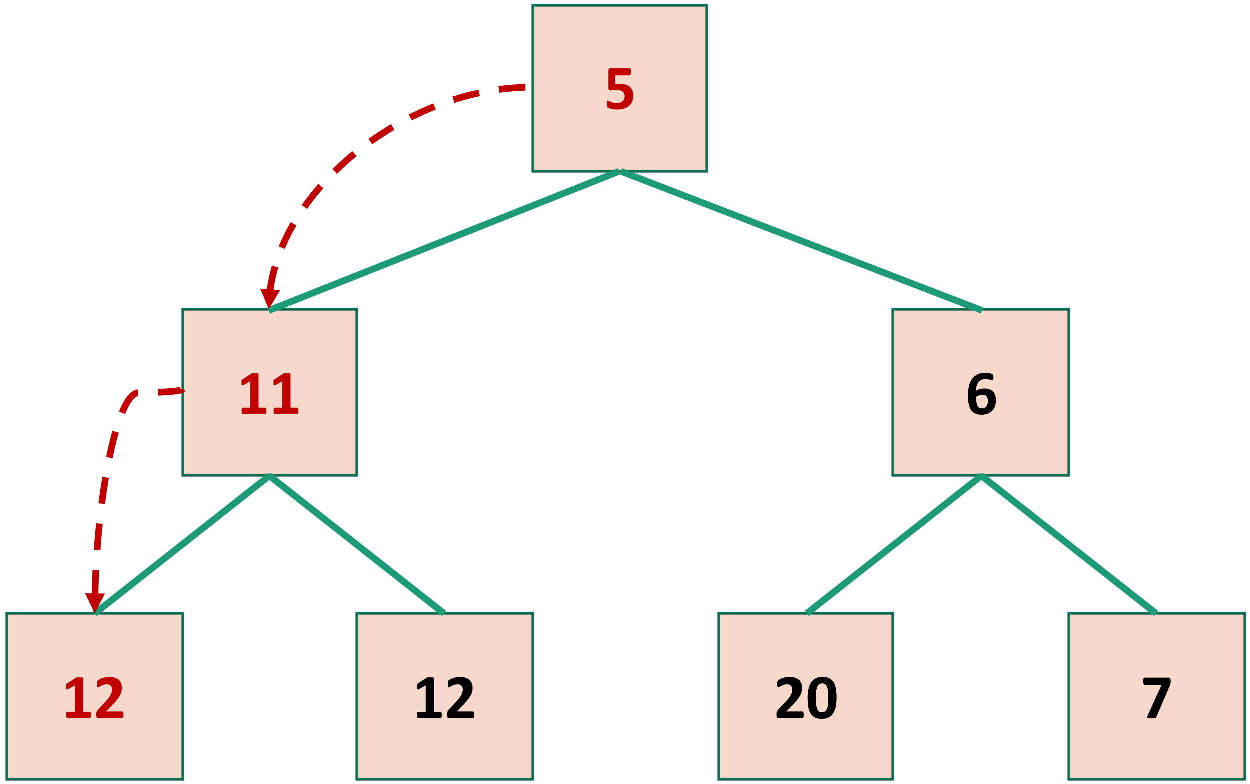
Exercise 2: remove 4



remove 4



remove 4



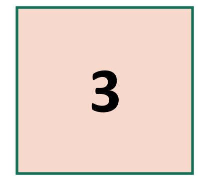
Exercise 3: heap sort

- Sort the following sequence in non-increasing order using in-place heap sort:

  [3 6 9 2 5 8]

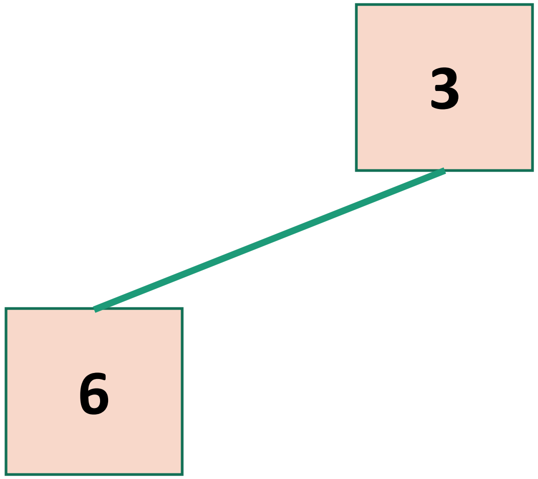
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**3** 6 9 2 5 8]



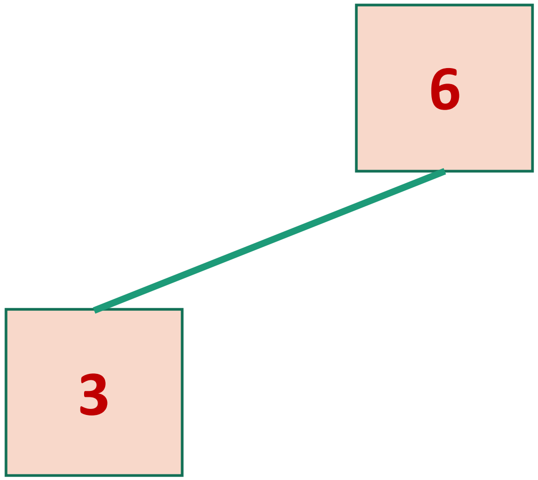
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**3 6** 9 2 5 8]



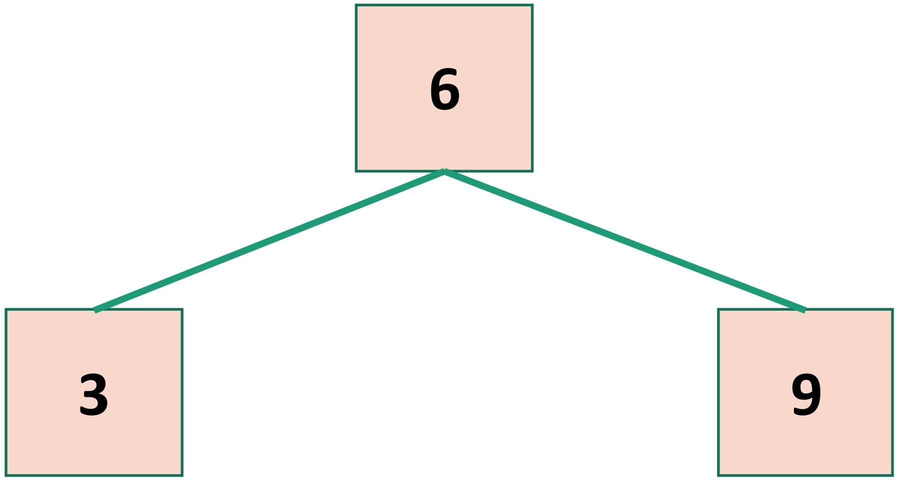
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**6 3** 9 2 5 8]



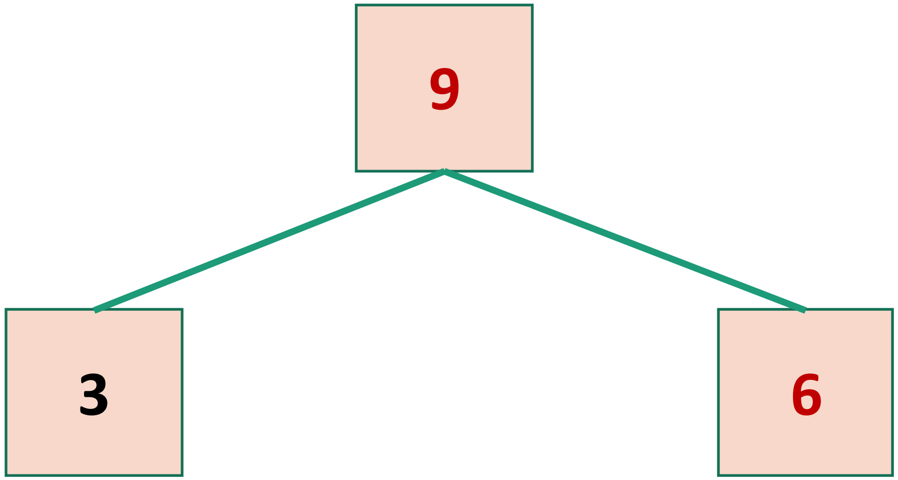
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**6 3 9** 2 5 8]



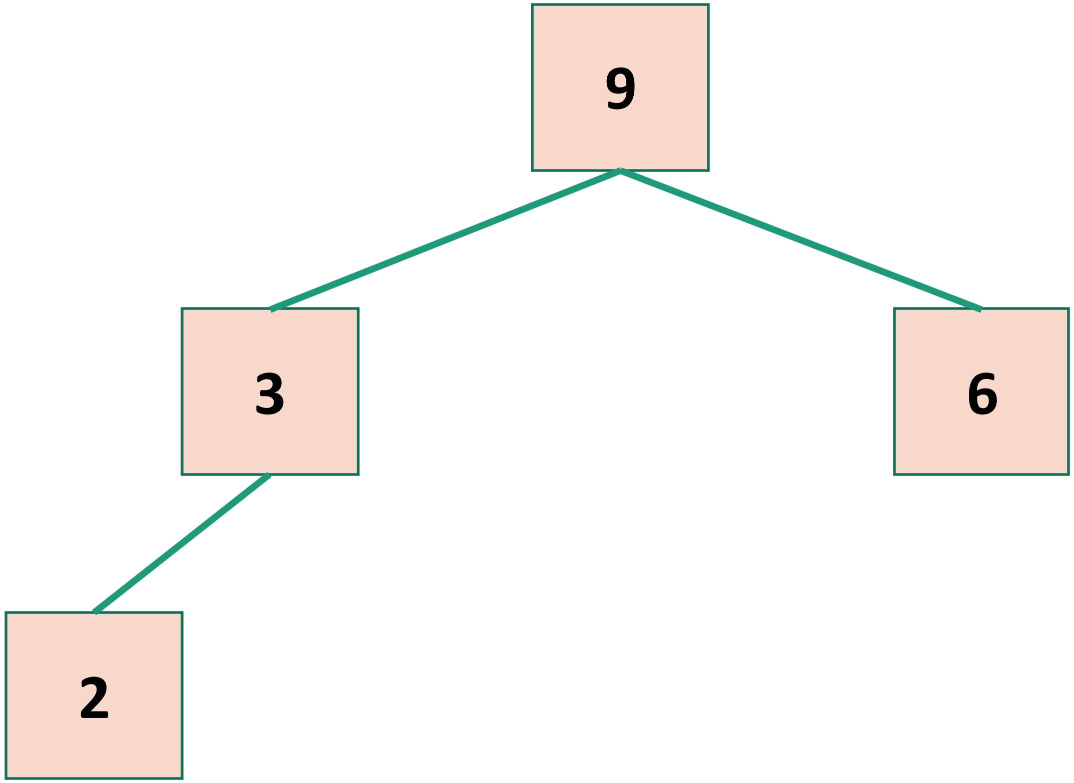
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**9 3 6** 2 5 8]



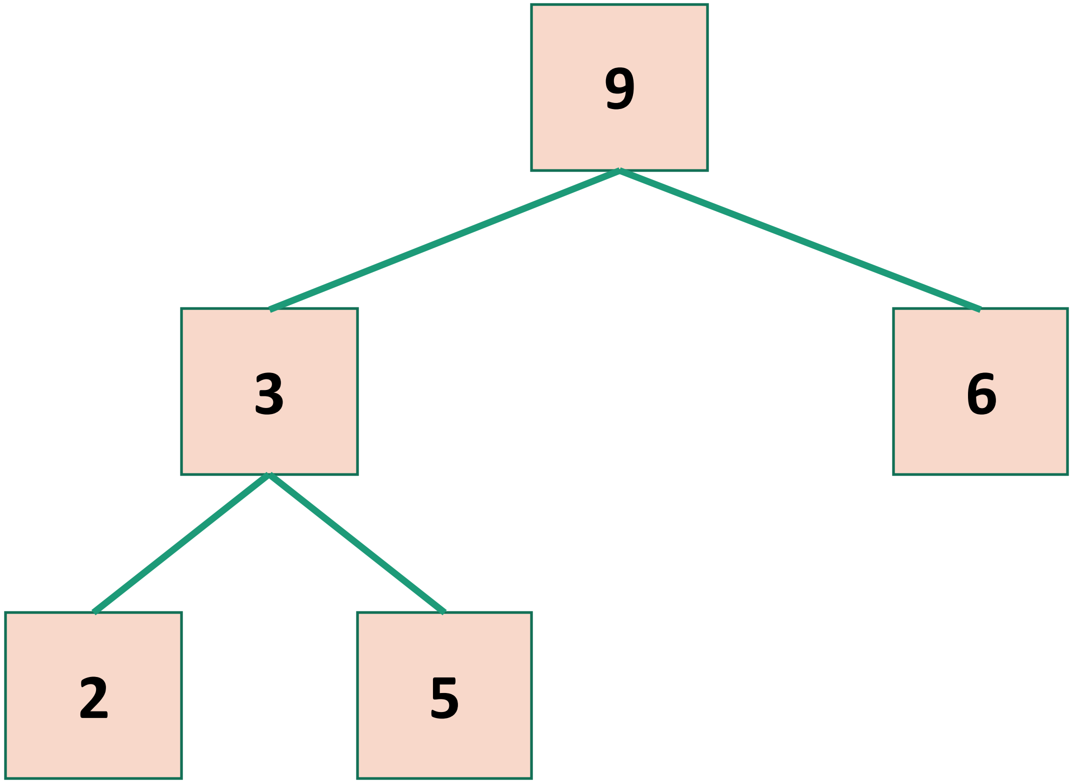
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**9 3 6 2** 5 8]



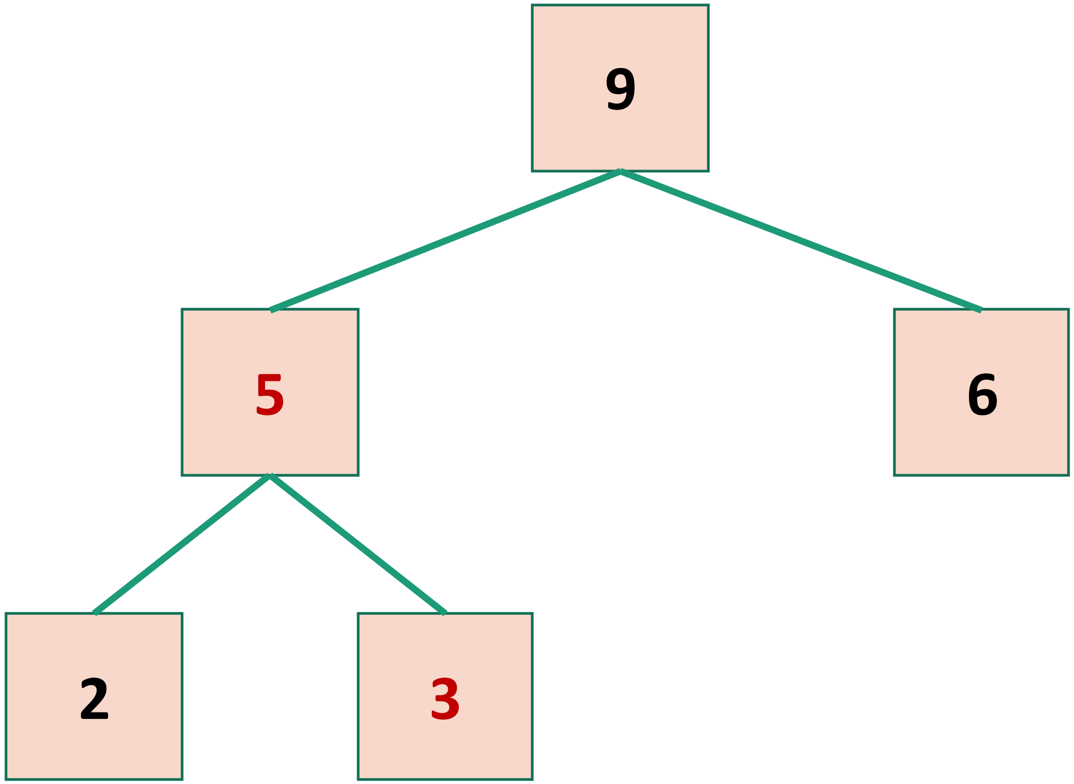
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**9 3 6 2 5** 8]



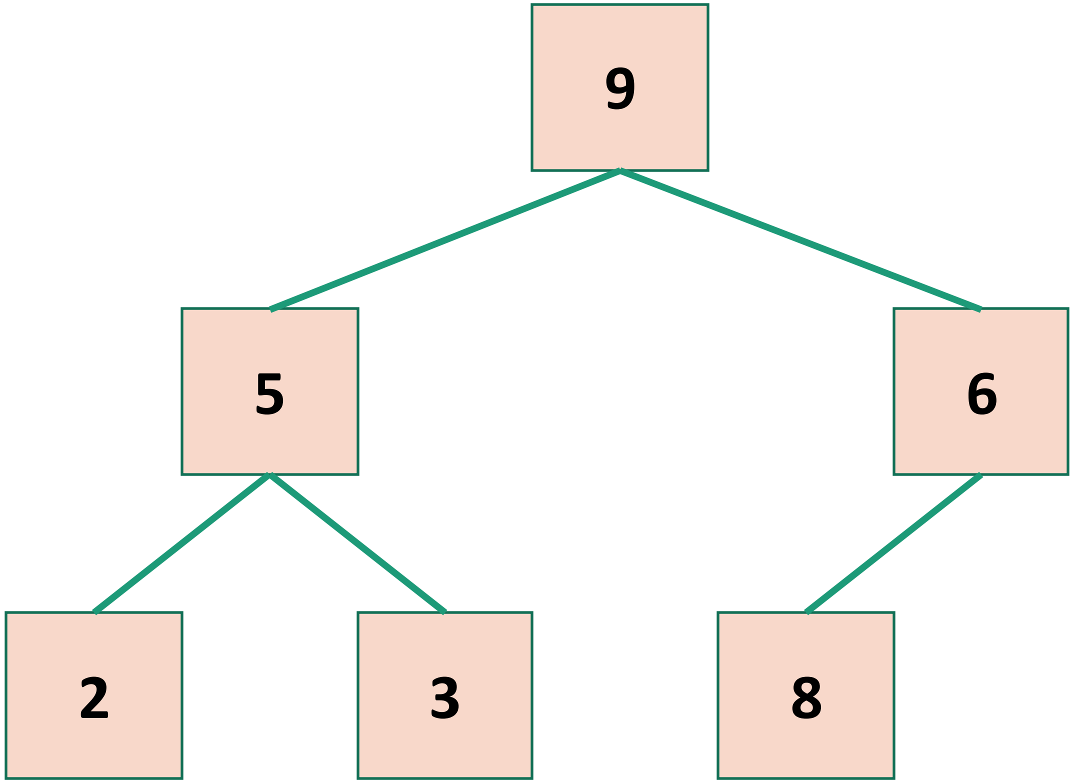
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**9 5 6 2 3** 8]



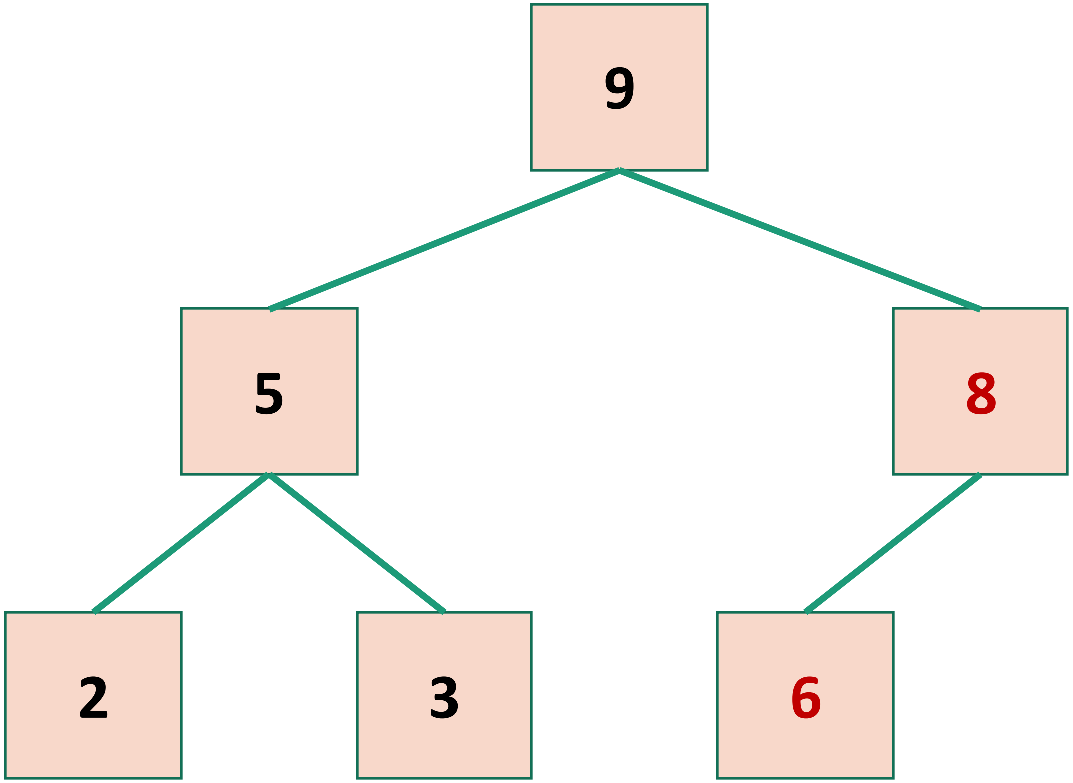
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**9 5 6 2 3 8**]



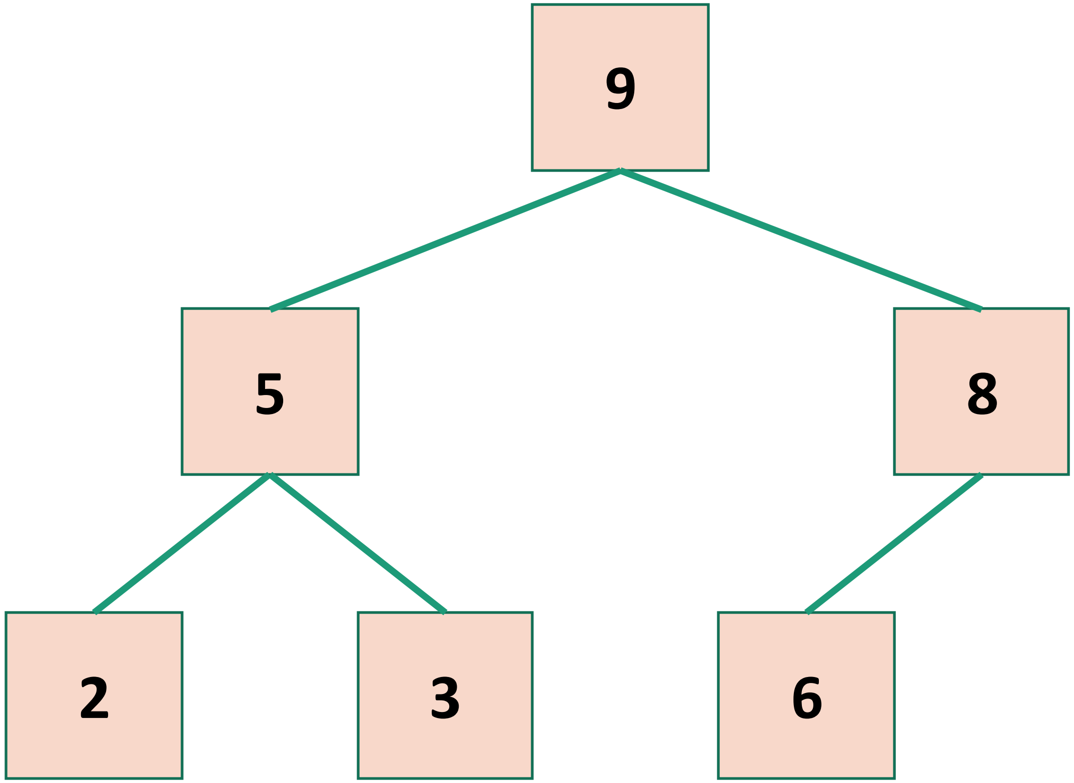
heap sort - phase 1

- Original sequence: [3 6 9 2 5 8]
- Array: [**9 5 8 2 3 6**]



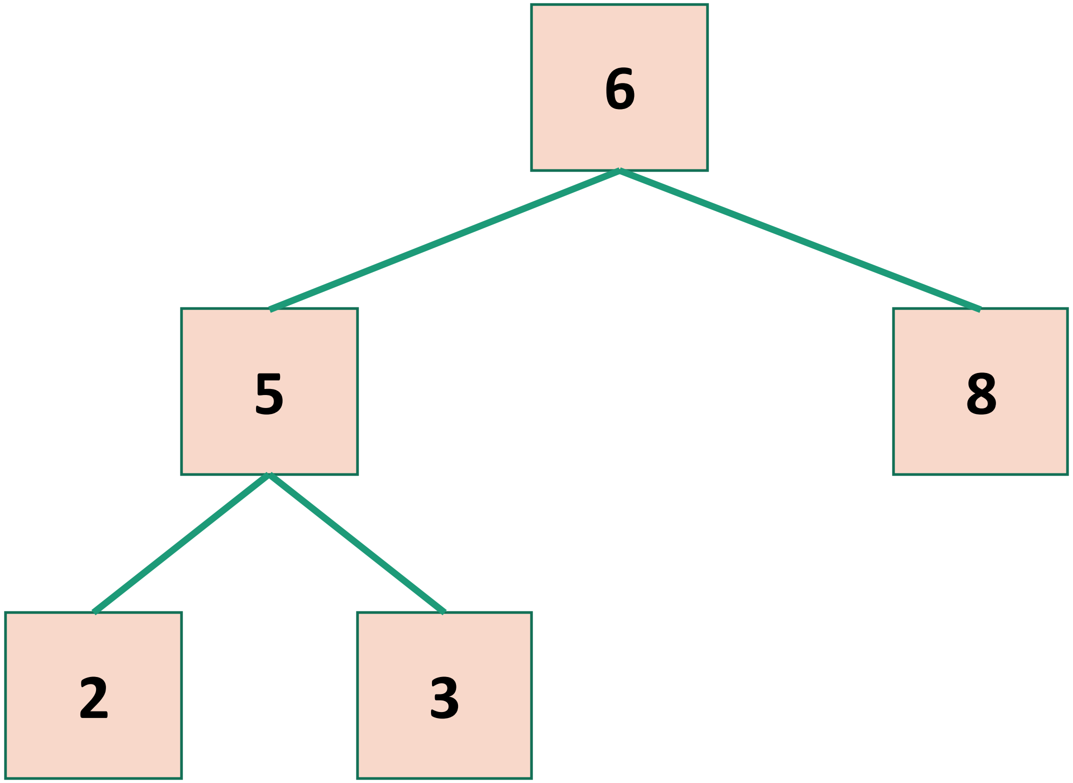
heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [**9 5 8 2 3 6**]



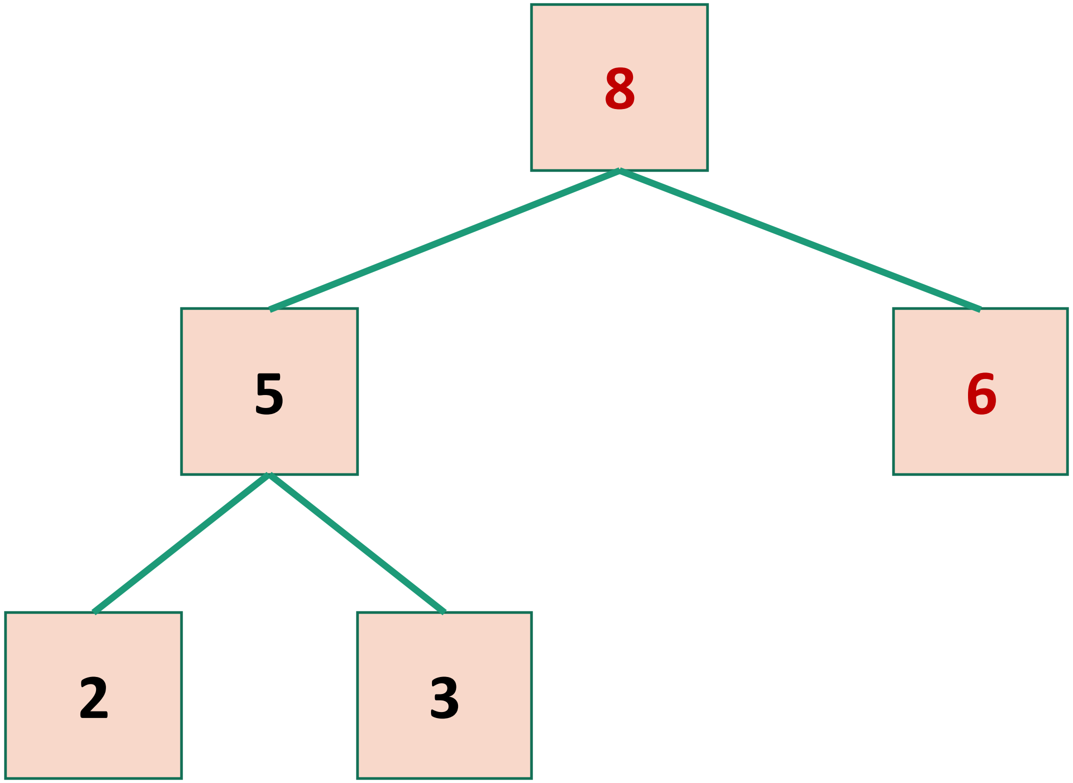
heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [***9*** **6 5 8 2 3**]



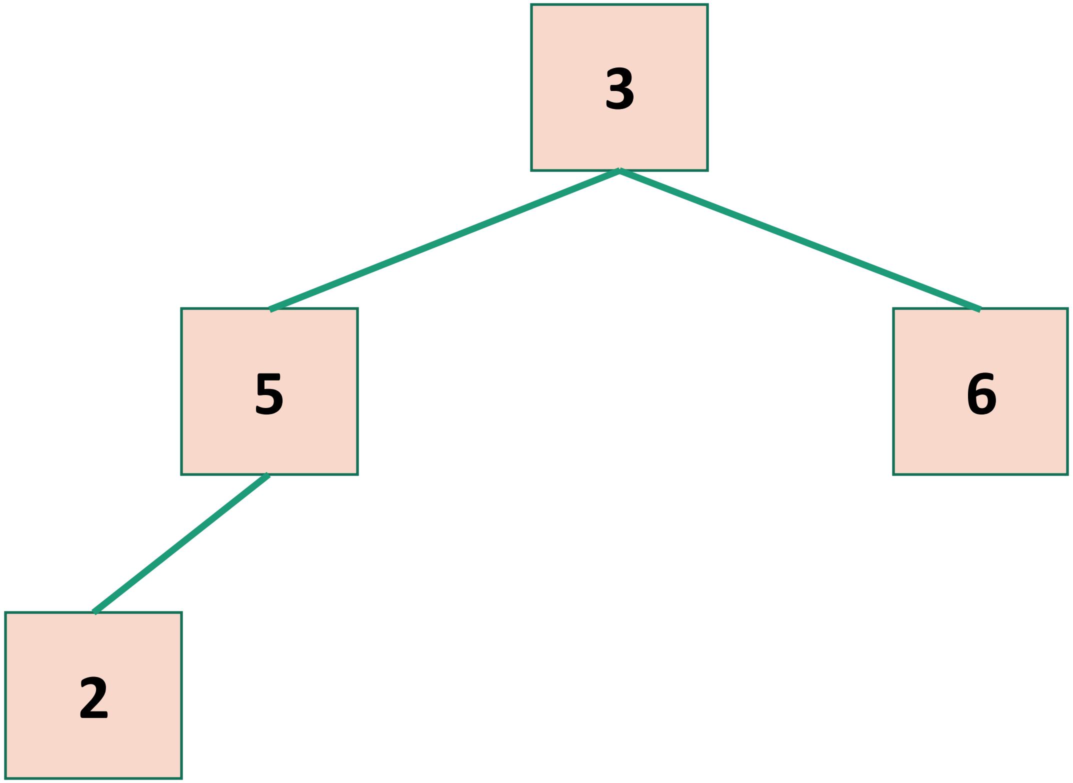
heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [***9*** **8 5 6 2 3**]



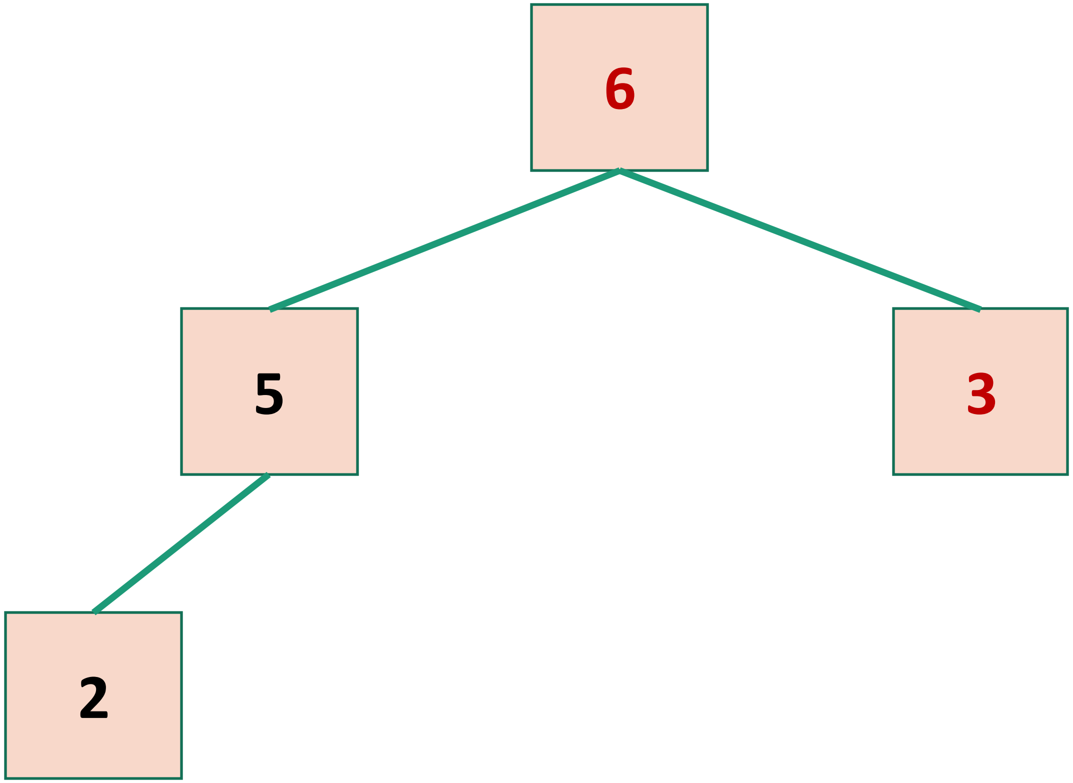
heap sort - phase 2

- Original heap: [9 5 8 2 3 6]
- Array: [*9 8* 3 5 6 2]



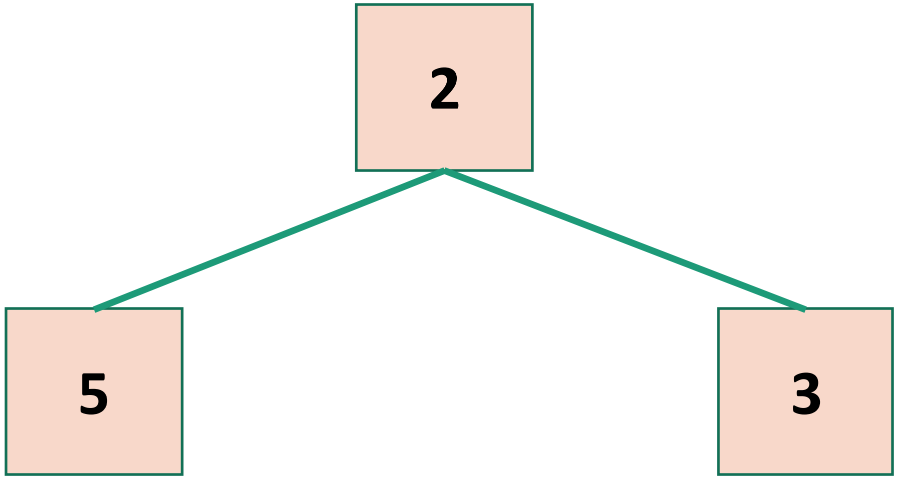
heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [***9 8*** **6 5 3 2**]



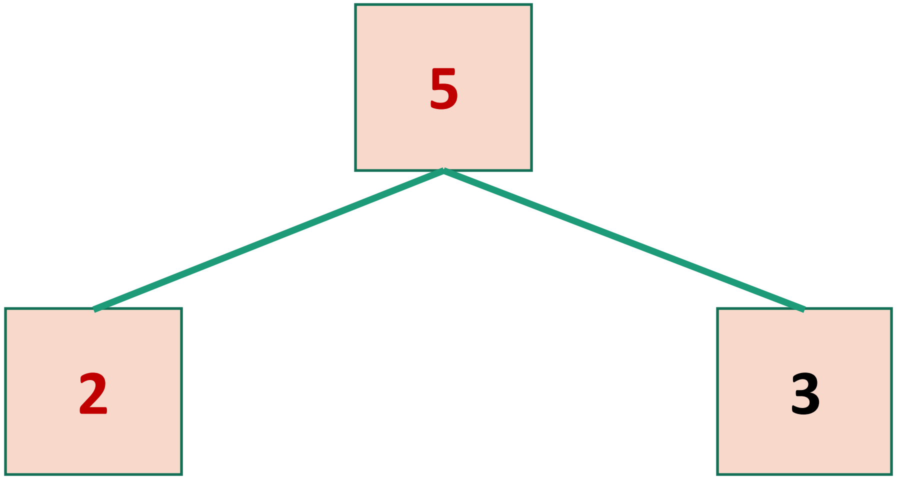
heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [***9 8 6*** **2 5 3**]



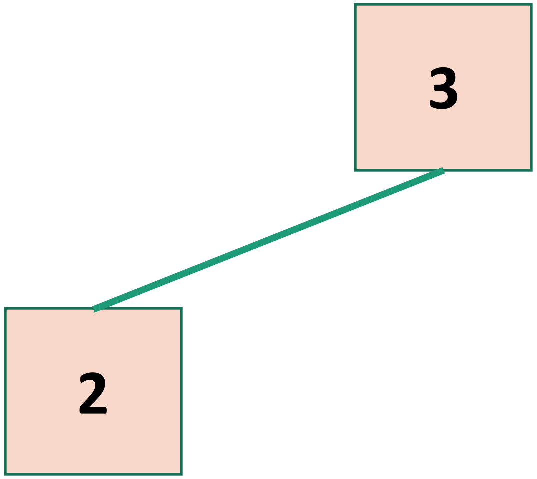
heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [***9 8 6*** **5 2 3**]



heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [***9 8 6 5*** **3 2**]



heap sort - phase 2

- Original heap: [**<u>9 5 8 2 3 6</u>**]
- Array: [***9 8 6 5 3*** **<u>2</u>**]

2

heap sort - phase 2

- Original heap: [**9 5 8 2 3 6**]
- Array: [***9 8 6 5 3 2***]

Exercise 4

Illustrate the execution of the selection-sort algorithm on the following input sequence:

(22, 15, 36, 44, 10, 3, 9, 13, 29, 25).

Exercise 4: Solution

22	15	36	44	10	3	9	13	29	25
3	15	36	44	10	22	9	13	29	25
3	9	36	44	10	22	15	13	29	25
3	9	10	44	36	22	15	13	29	25
3	9	10	13	36	22	15	44	29	25
3	9	10	13	15	22	36	44	29	25
3	9	10	13	15	22	36	44	29	25
3	9	10	13	15	22	25	44	29	36
3	9	10	13	15	22	25	29	44	36
3	9	10	13	15	22	25	29	36	44

Exercise 5

Illustrate the execution of the insertion-sort algorithm on the following input sequence:

(22, 15, 36, 44, 10, 3, 9, 13, 29, 25).

Exercise 5: Solution

22	15	36	44	10	3	9	13	29	25
15	22	36	44	10	3	9	13	29	25
15	22	36	44	10	3	9	13	29	25
10	15	22	36	44	3	9	13	29	25
3	10	15	22	36	44	9	13	29	25
3	9	10	15	22	36	44	13	29	25
3	9	10	13	15	22	36	44	29	25
3	9	10	13	15	22	29	36	44	25
3	9	10	13	15	22	25	29	36	44

Exercise 6

Show that the sum $\sum_{i=1}^{n} \log i$, appearing in the analysis of heap-sort, is $\mathrm{O}(n \log n)$.

Exercise 6: Solution

Proof: To show $\sum_{i=1}^{n} \log i$ is $\mathrm{O}(n \log n)$, by the definition of O, we need to show that there exist a positive real constant c and a positive integer n_0 such that for all $n \geq n_0$, $\sum_{i=1}^{n} \log i \ \leq cn \log n$.

$\sum_{i=1}^{n} \log i = \log 1 + \log 2 + \cdots + \log n$

$\leq n \log n$

for every $n \geq 1$. Let $c = 1$, $n_0 = 1$. By the definition of O, we have $\sum_{i=1}^{n} \log i$ is $\mathrm{O}(n \log n)$.

Exercise 7

Show that the sum $\sum_{i=1}^{n} \log i$, appearing in the analysis of heap-sort, is $\Omega(n \log n)$.

Exercise 7: Solution

Proof: To show $\sum_{i=1}^{n} \log i$ is $\Omega(n \log n)$, by the definition of Ω, we need to show that there exist a positive real constant c and a positive integer n_0 such that for all $n \geq n_0$, $\sum_{i=1}^{n} \log i \ \geq cn \log n$.

$\sum_{i=1}^{n} \log i = \log 1 + \log 2 + \cdots + \log n$

$\geq \frac{n}{2} \log \frac{n}{2}$ (consider the last $\frac{n}{2}$ terms)

$= \frac{n}{2} \log n \ - \frac{n}{2}$

Let $c = \frac{1}{4}$. By solving $\frac{n}{2} \log n \ - \frac{n}{2} \geq cn \log n$, we have $n \geq 4$. So we let $n_0 = 4$. By the definition of Ω, we have $\sum_{i=1}^{n} \log i$ is $\Omega(n \log n)$.