AE2ADS: Algorithms Data Structures and Efficiency

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Some slides are from Dr. Brian Logan in the University of Nottingham.

Aim and Learning Objectives

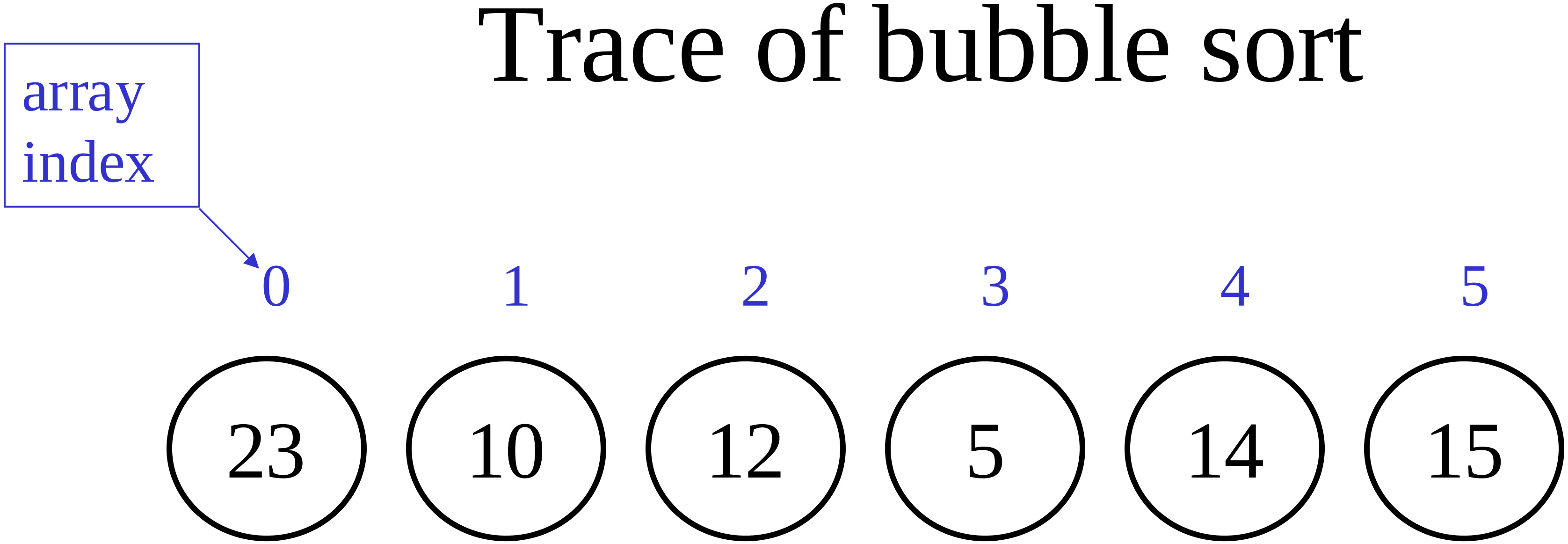
- To be able to *understand* and *describe* the three simple sorting algorithms: bubble sort, selection sort and insertion sort;
- To be able to *analyze* the complexity of the three simple sorting algorithms;
- To be able to *implement* the three simple sorting algorithms.

Bubble sort

```
void  bubbleSort(int[] arr){
   int i;
   int j;
   int temp;
   for(i = arr.length-1; i > 0; i--){
      for(j = 0; j < i; j++){
         if(arr[j] > arr[j+1]){
            temp = arr[j];
            arr[j] = arr[j+1];
            arr[j+1] = temp;
         }//
      }// end inner loop
   }//end outer loop}// end bubble sort
```

swap adjacent elements, if in the wrong order

Trace of bubble sort



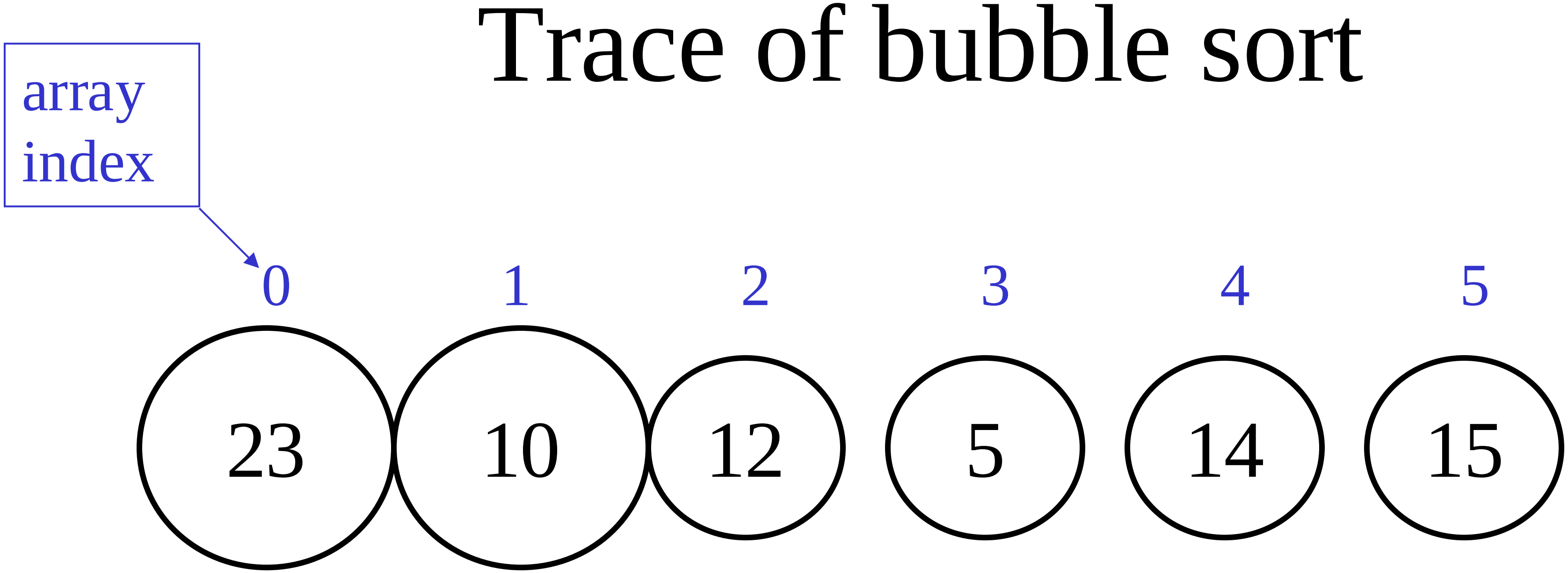
array index

0	1	2	3	4	5
23	10	12	5	14	15

We have an unsorted input array, positions 0-5 are unsorted.

i = 5, first iteration of the outer loop

Trace of bubble sort



array index

0 1 2 3 4 5

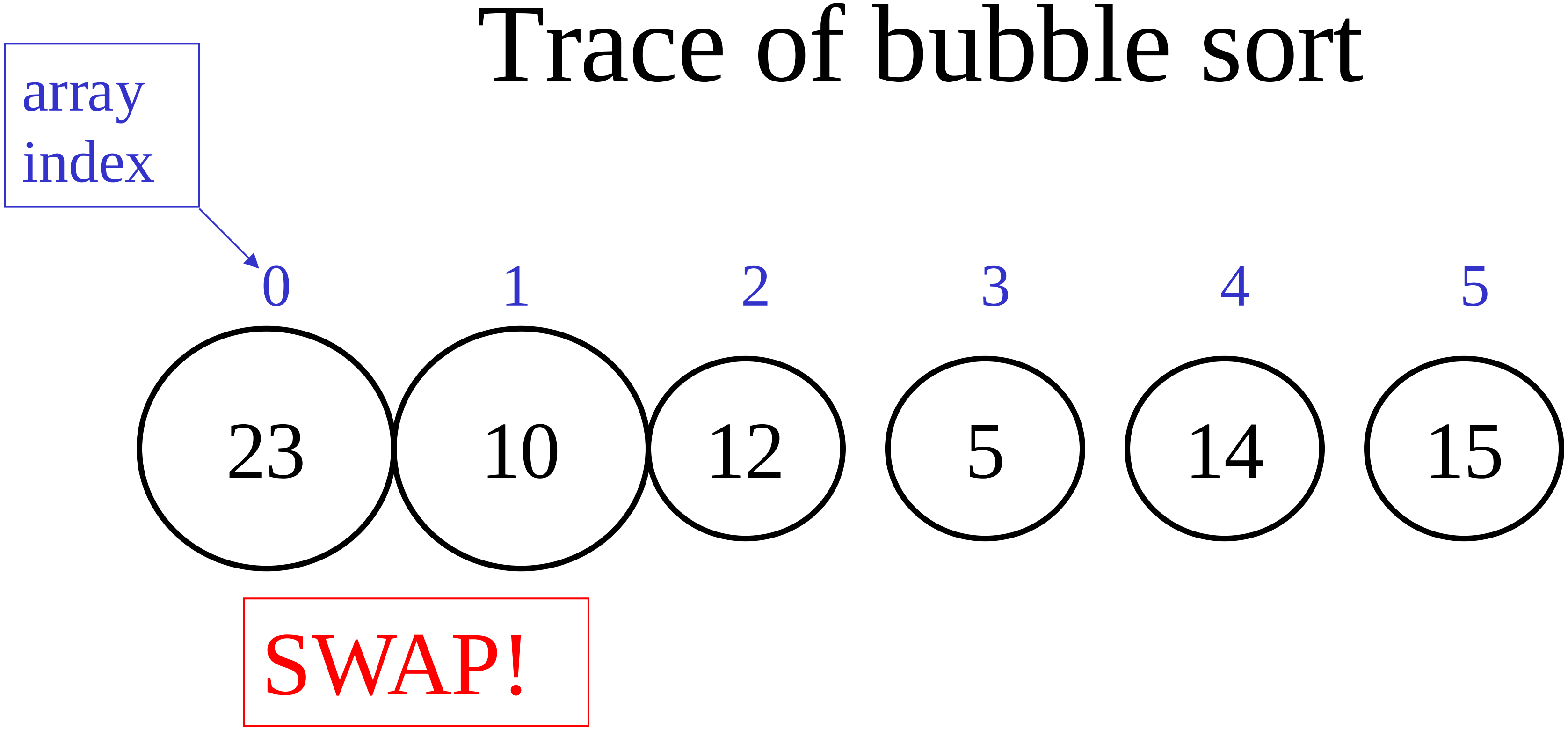
23 10 12 5 14 15

i = 5, first iteration of the outer loop

j = 0, arr[0]…arr[j] are all less than or equal to arr[j]

j = 0, comparing arr[0] and arr[1]

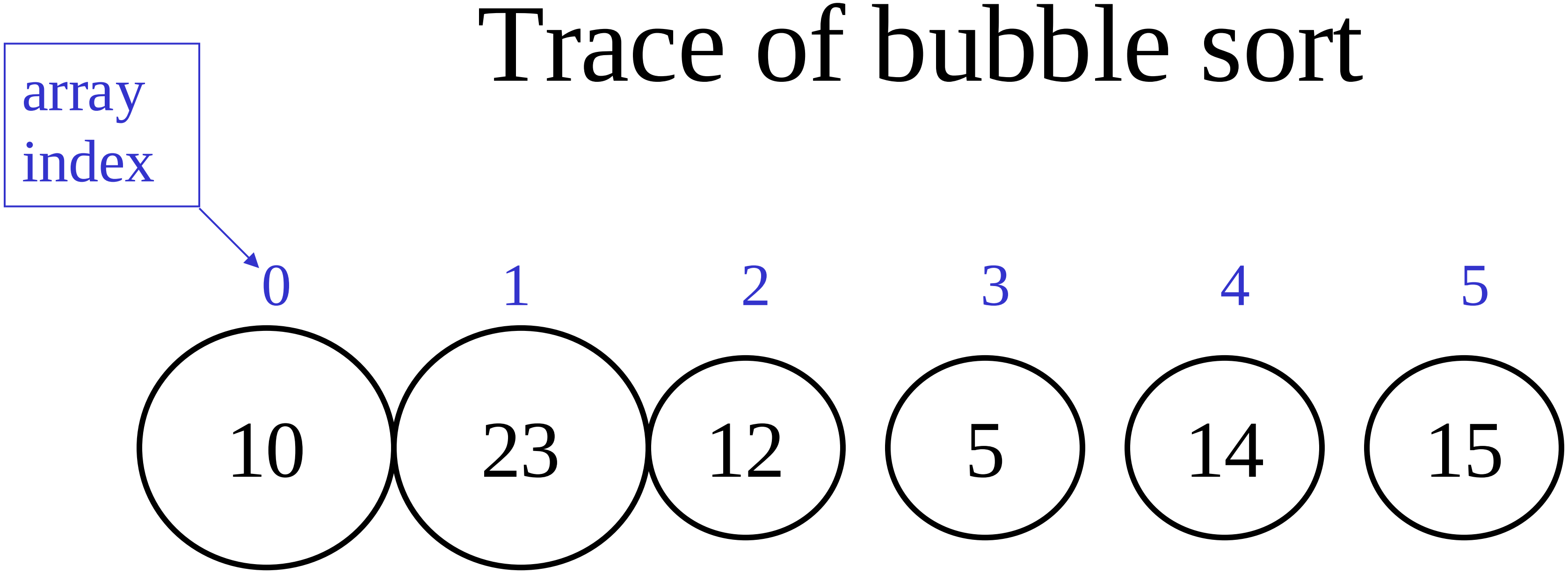
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 0, comparing arr[0] and arr[1]

Trace of bubble sort



array index

0	1	2	3	4	5
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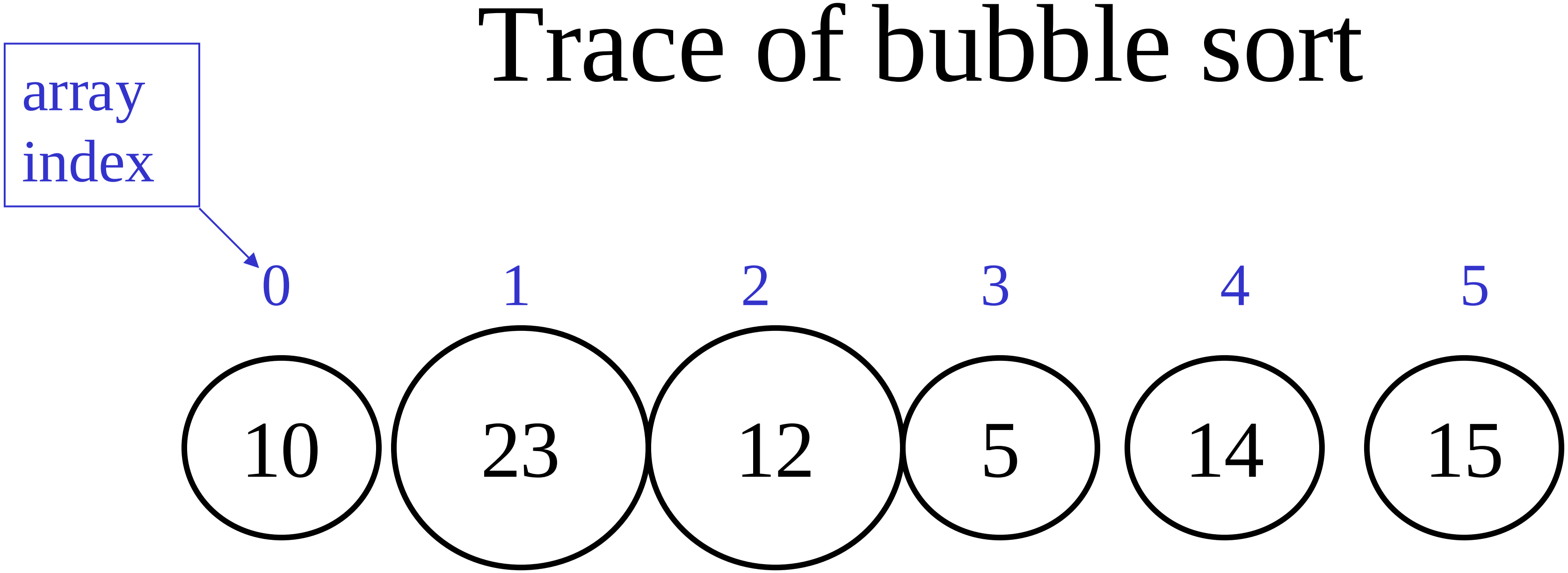
SWAP!

i = 5, first iteration of the outer loop

j = 0, comparing arr[0] and arr[1]

Trace of bubble sort



array index

0	1	2	3	4	5
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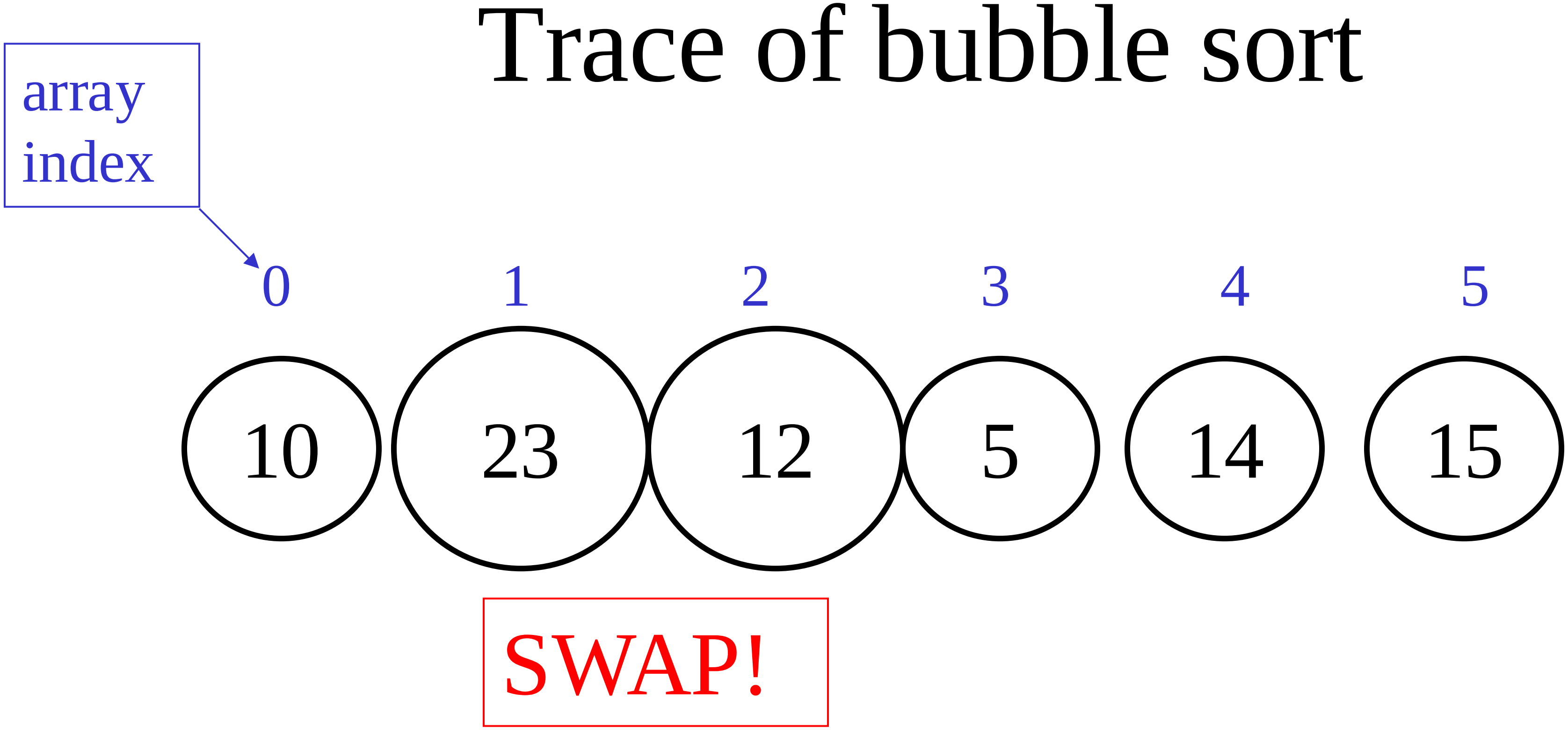
i = 5, first iteration of the outer loop

j = 1, arr[0]…arr[j] are all less than or equal to arr[j]

j = 1, comparing arr[1] and arr[2]

Trace of bubble sort



array index

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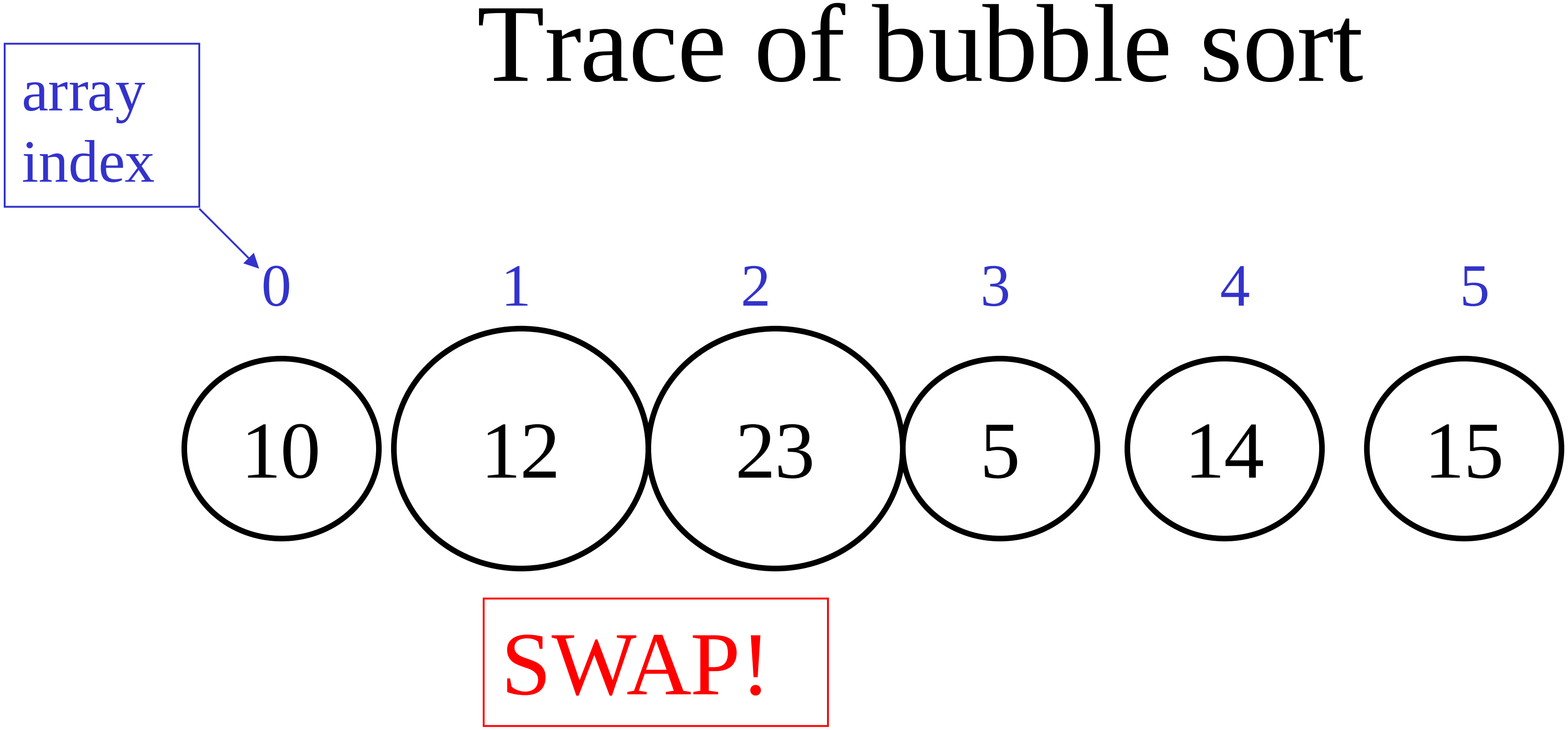
SWAP!

i = 5, first iteration of the outer loop

j = 1, comparing arr[1] and arr[2]

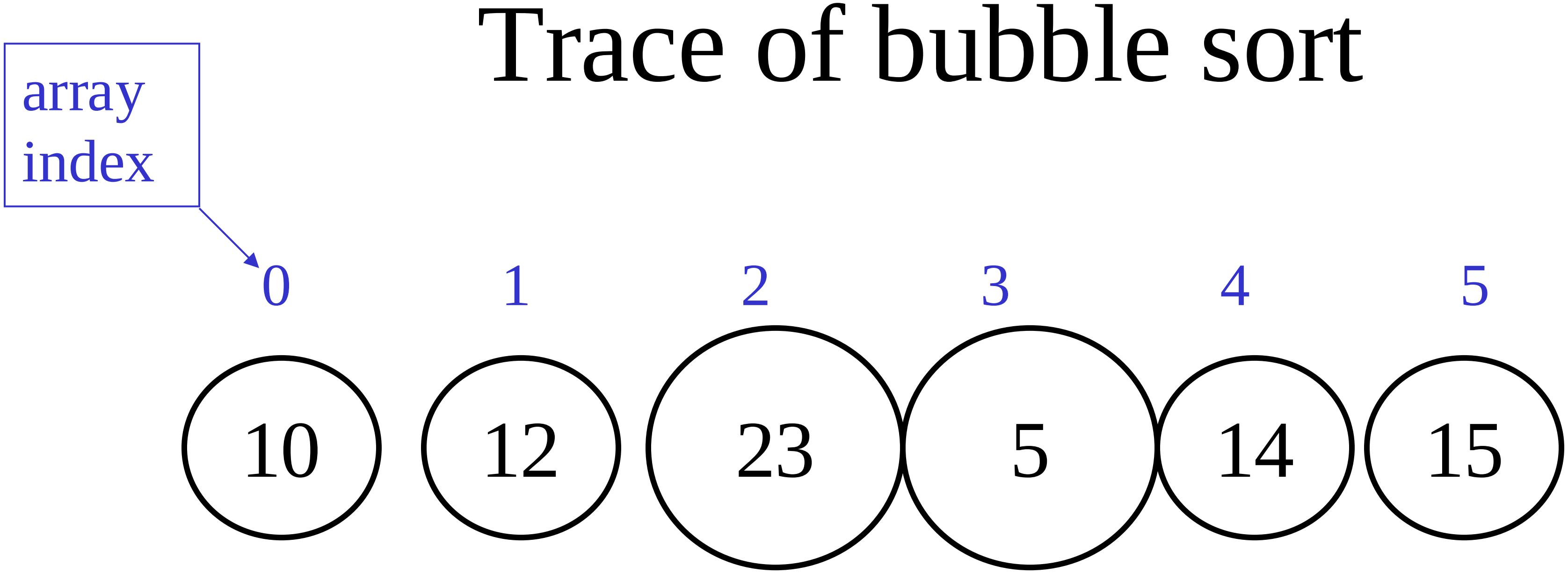
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 1, comparing arr[1] and arr[2]

Trace of bubble sort

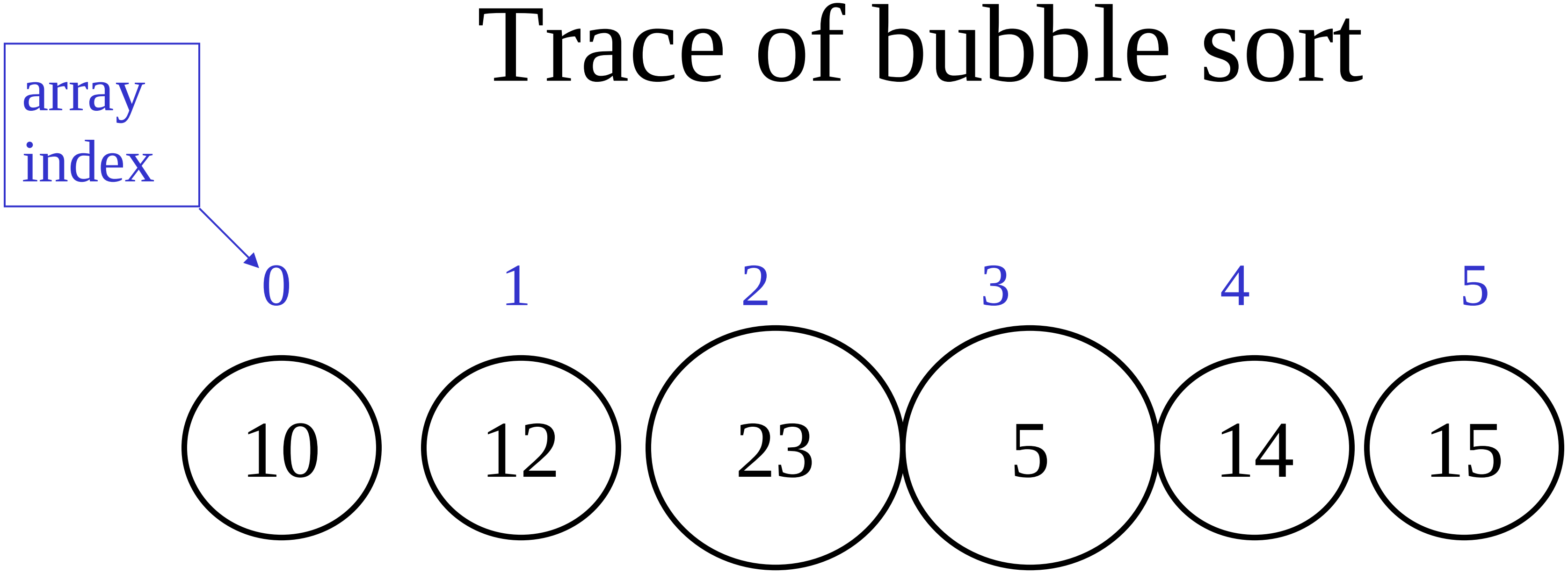


i = 5, first iteration of the outer loop

j = 2, arr[0]…arr[j] are all less than or equal to arr[j]

j = 2, comparing arr[2] and arr[3]

Trace of bubble sort

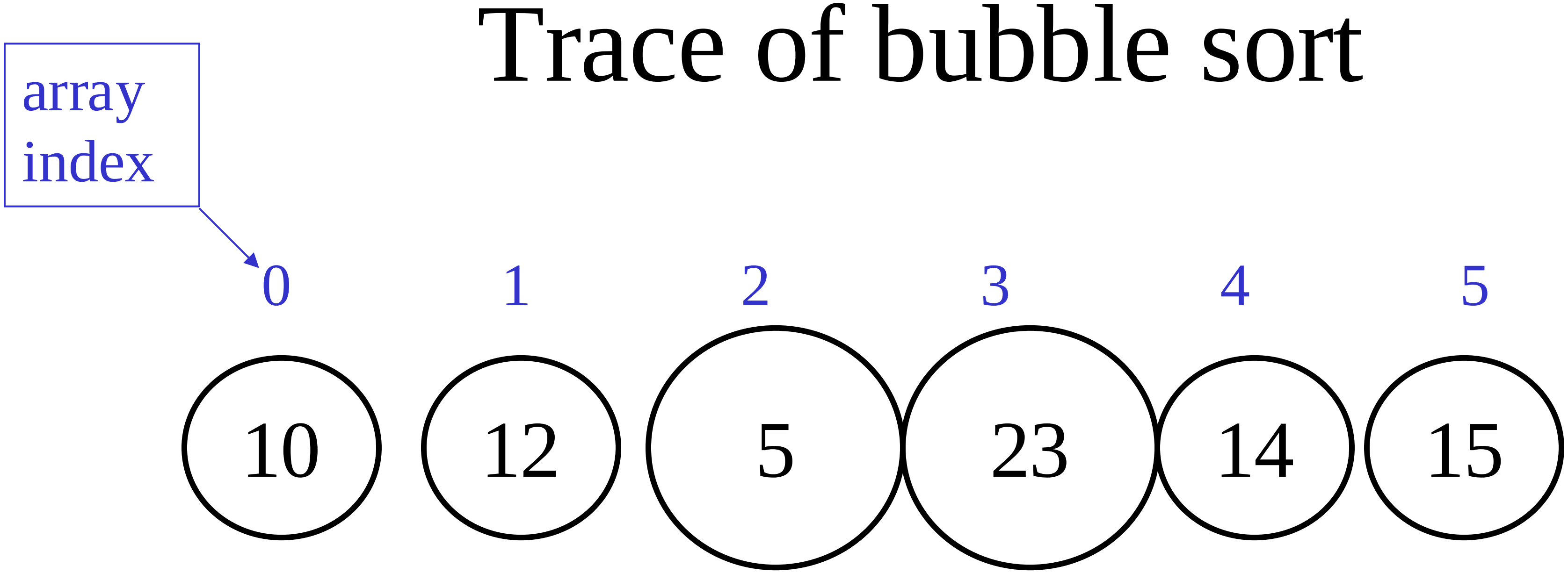


SWAP!

i = 5, first iteration of the outer loop

j = 2, comparing arr[2] and arr[3]

Trace of bubble sort



array index

0 1 2 3 4 5

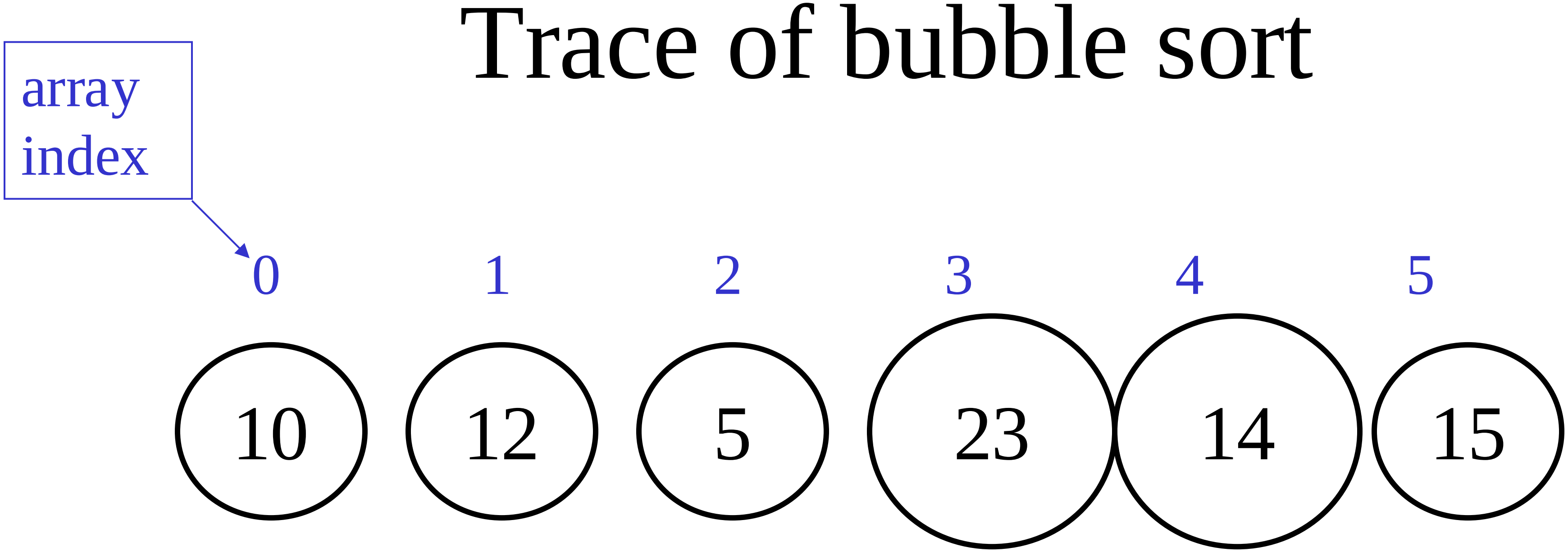
10 12 5 23 14 15

SWAP!

i = 5, first iteration of the outer loop

j = 2, comparing arr[2] and arr[3]

Trace of bubble sort



array index

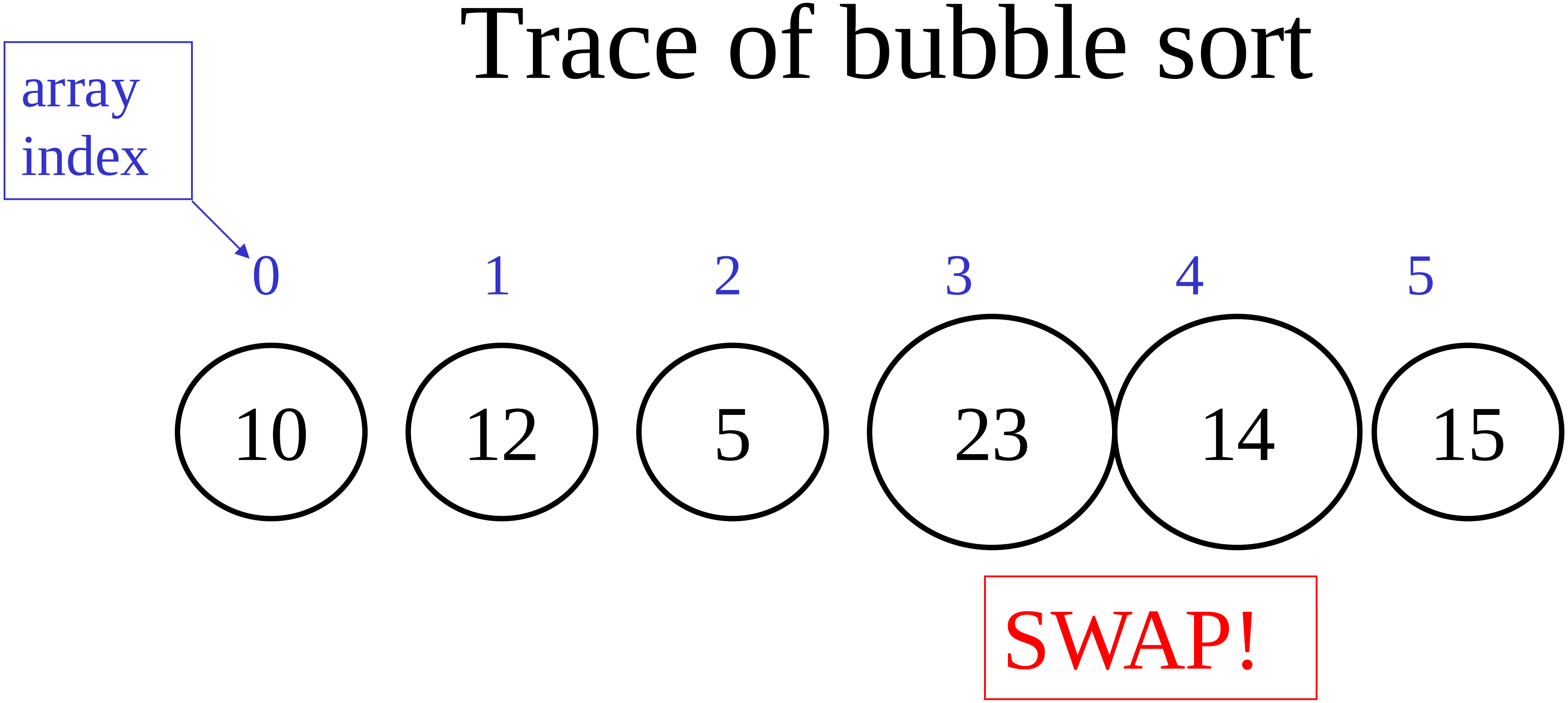
0	1	2	3	4	5
10	12	5	23	14	15

i = 5, first iteration of the outer loop

j = 3, arr[0]…arr[j] are all less than or equal to arr[j]

j = 3, comparing arr[3] and arr[4]

Trace of bubble sort



array index

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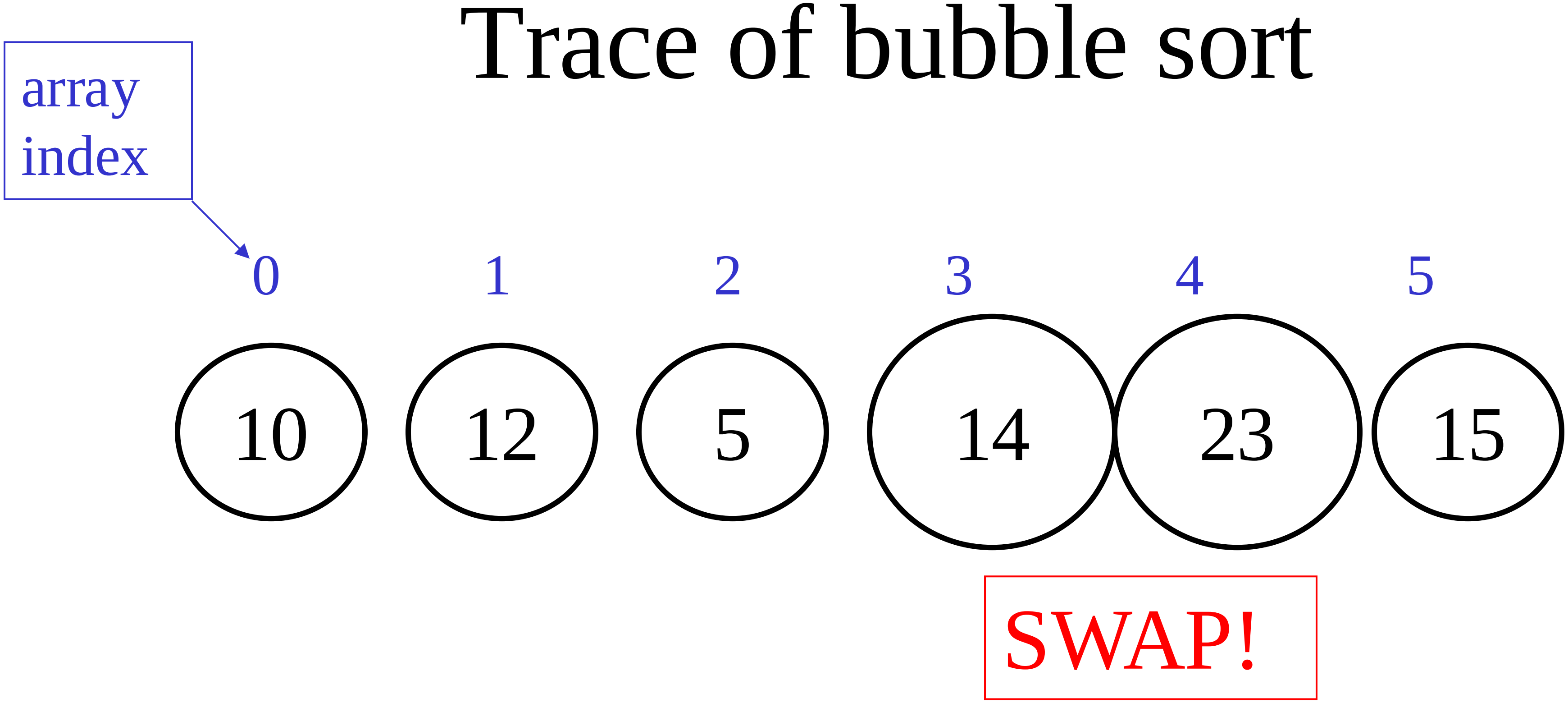
SWAP!

i = 5, first iteration of the outer loop

j = 3, comparing arr[3] and arr[4]

Trace of bubble sort



array index

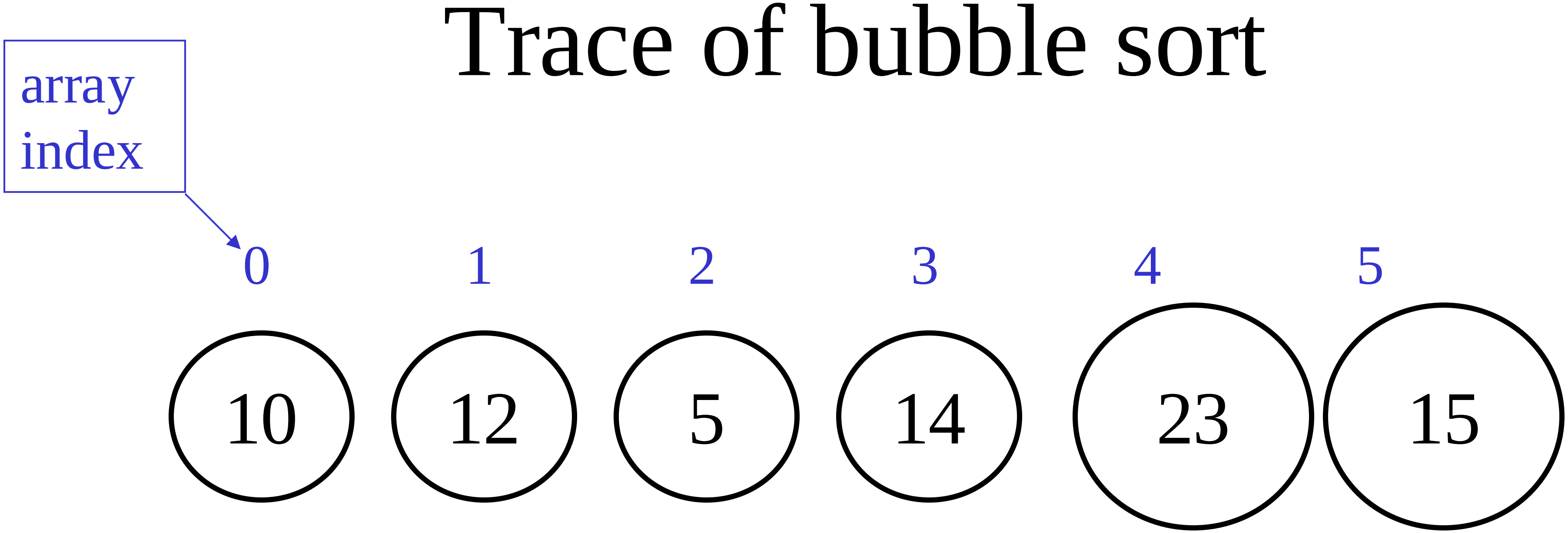
0	1	2	3	4	5
10	12	5	14	23	15

SWAP!

i = 5, first iteration of the outer loop

j = 3, comparing arr[3] and arr[4]

Trace of bubble sort



array
index

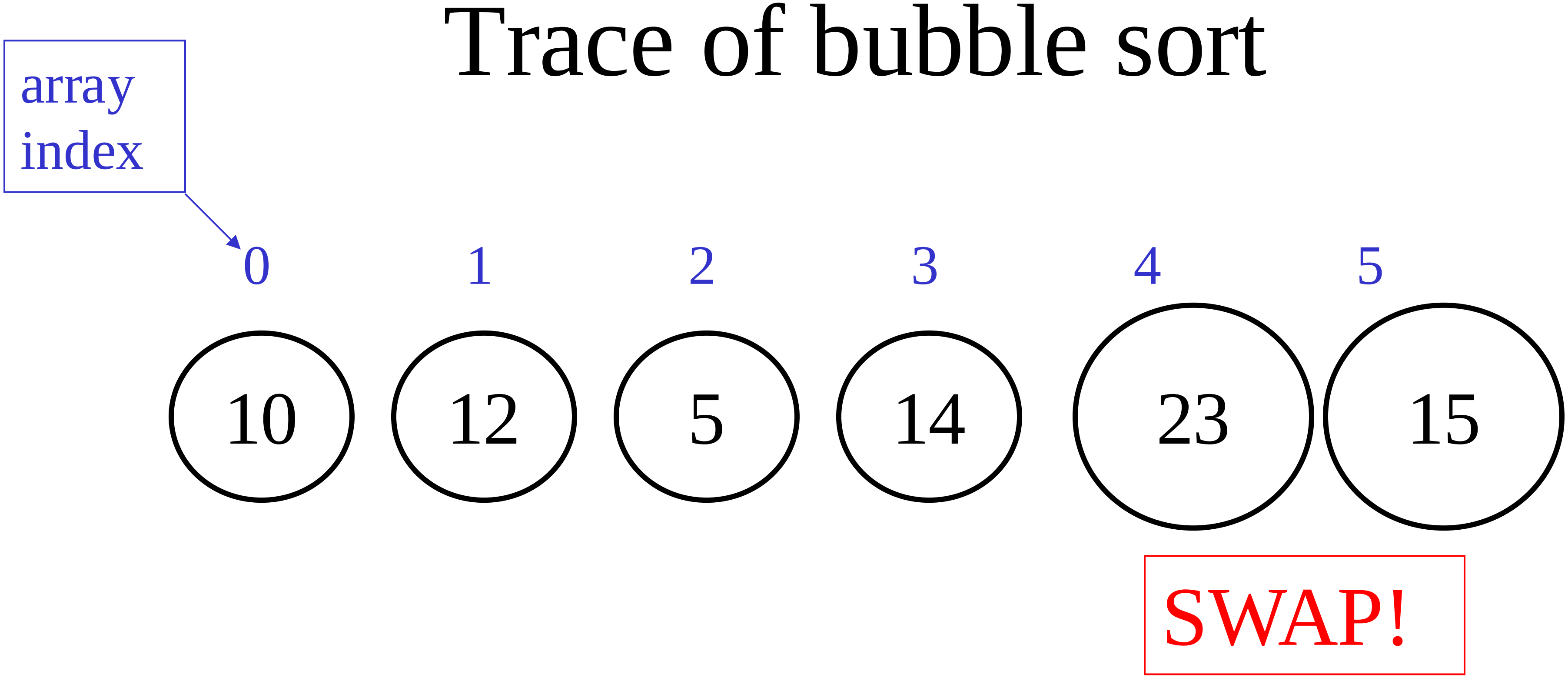
0	1	2	3	4	5
10	12	5	14	23	15

i = 5, first iteration of the outer loop

j = 4, arr[0]…arr[j] are all less than or equal to arr[j]

j = 4, comparing arr[4] and arr[5]

Trace of bubble sort



array index

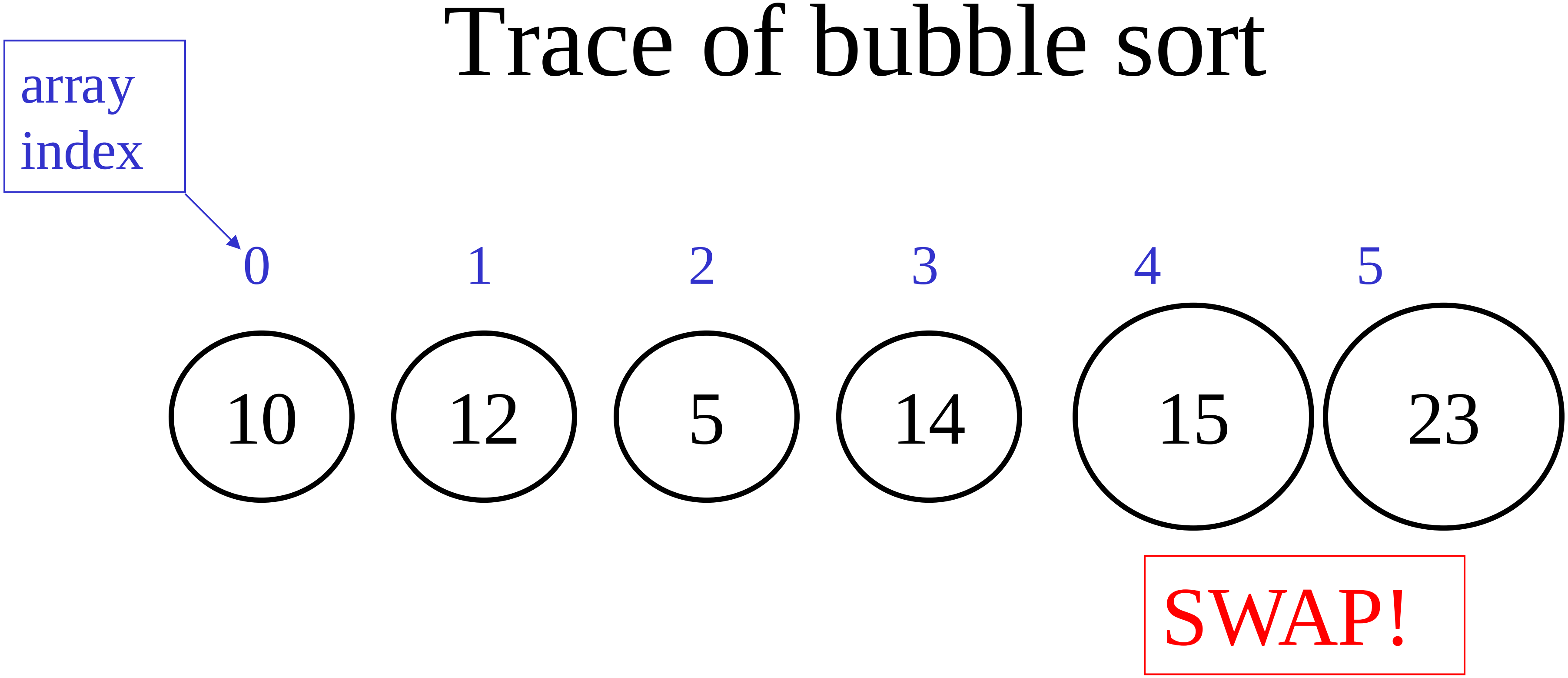
0	1	2	3	4	5
10	12	5	14	23	15

SWAP!

i = 5, first iteration of the outer loop

j = 4, comparing arr[4] and arr[5]

Trace of bubble sort



array index

0 1 2 3 4 5

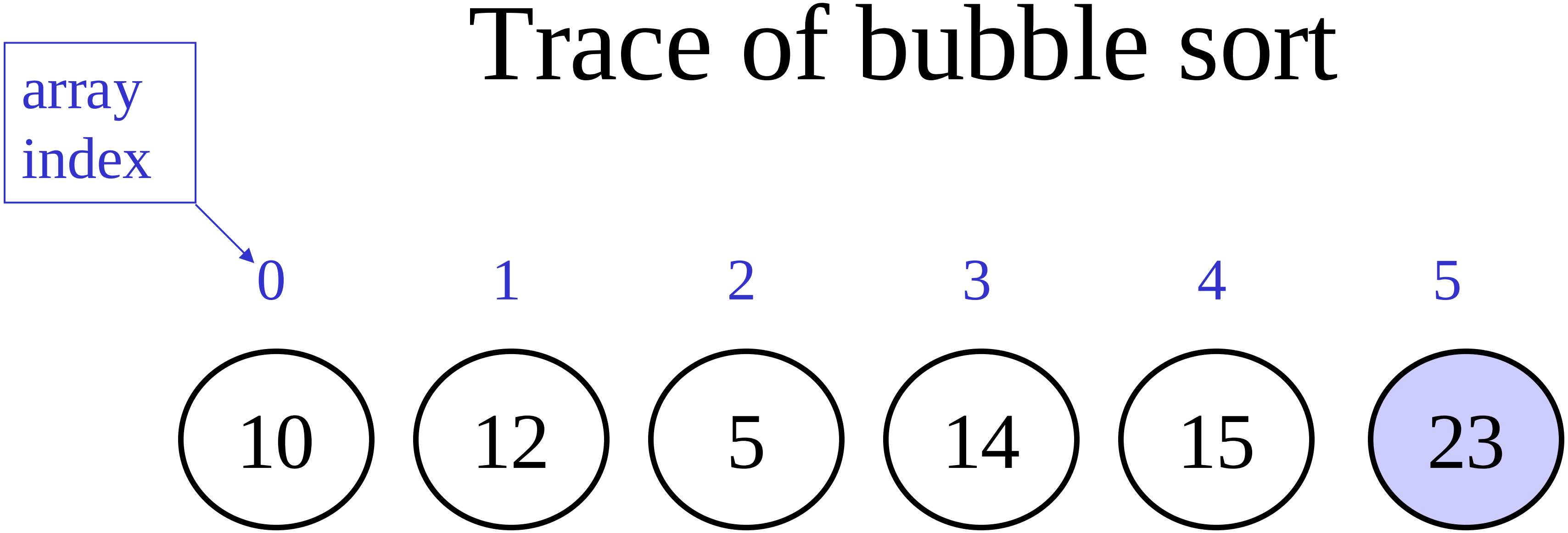
10 12 5 14 15 23

SWAP!

i = 5, first iteration of the outer loop

j = 4, comparing arr[4] and arr[5]

Trace of bubble sort

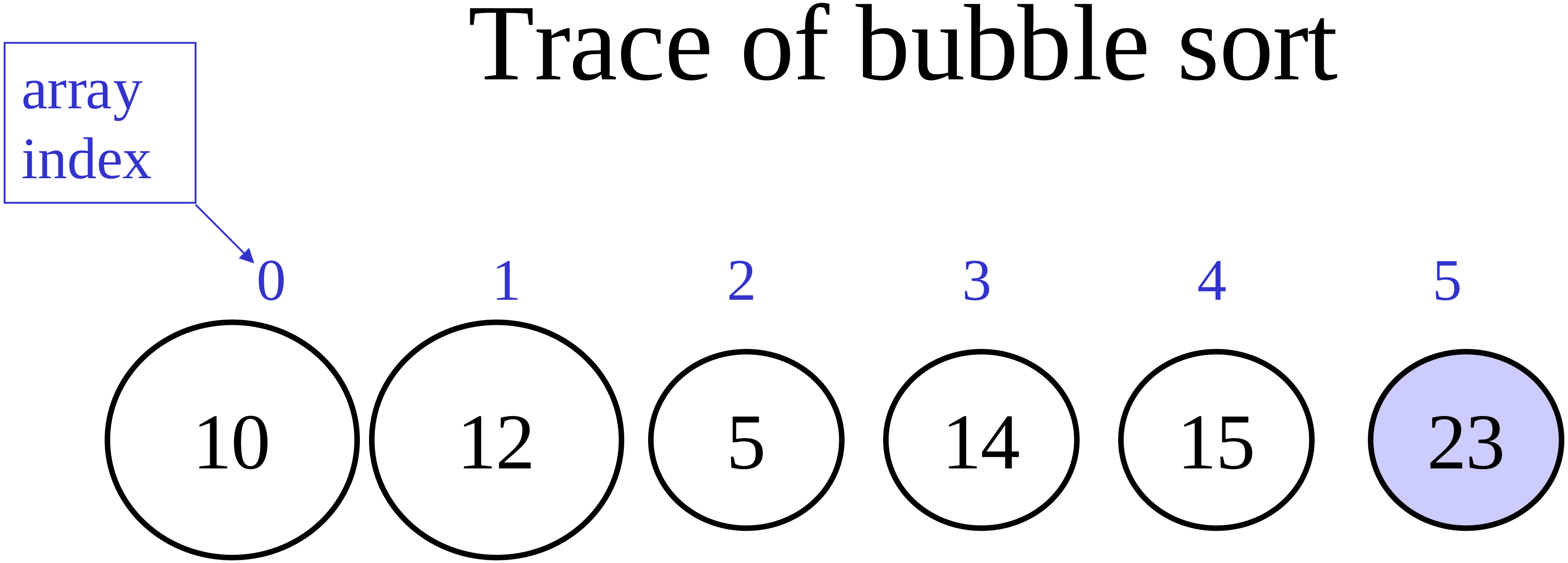


i = 5, first iteration of the outer loop

inner loop finished

largest element in position 5, positions 0-4 unsorted

Trace of bubble sort



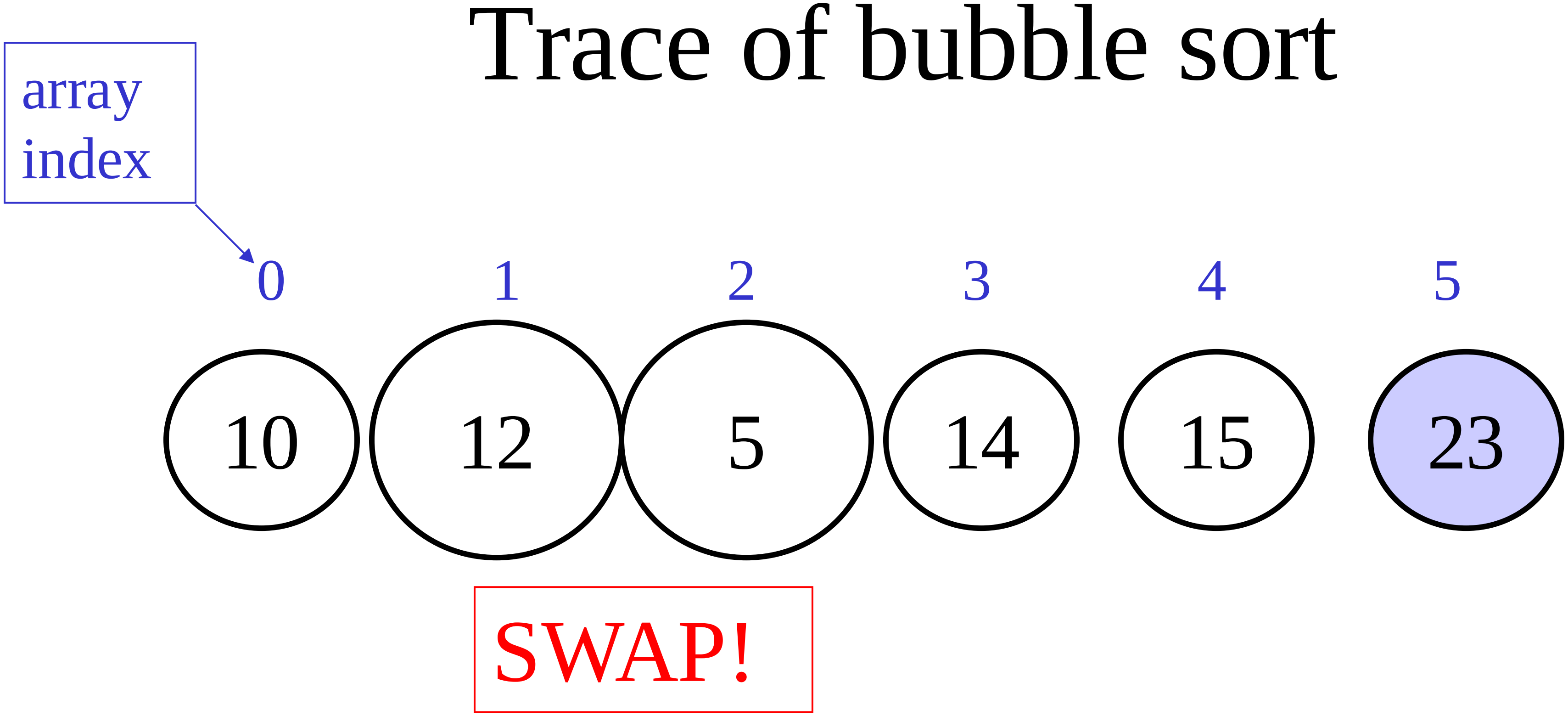
i = 4, second iteration of the outer loop

largest element in position 5, positions 0-4 unsorted

j = 0, arr[0]…arr[j] are all less than or equal to arr[j]

j = 0, comparing arr[0] with arr[1]

Trace of bubble sort

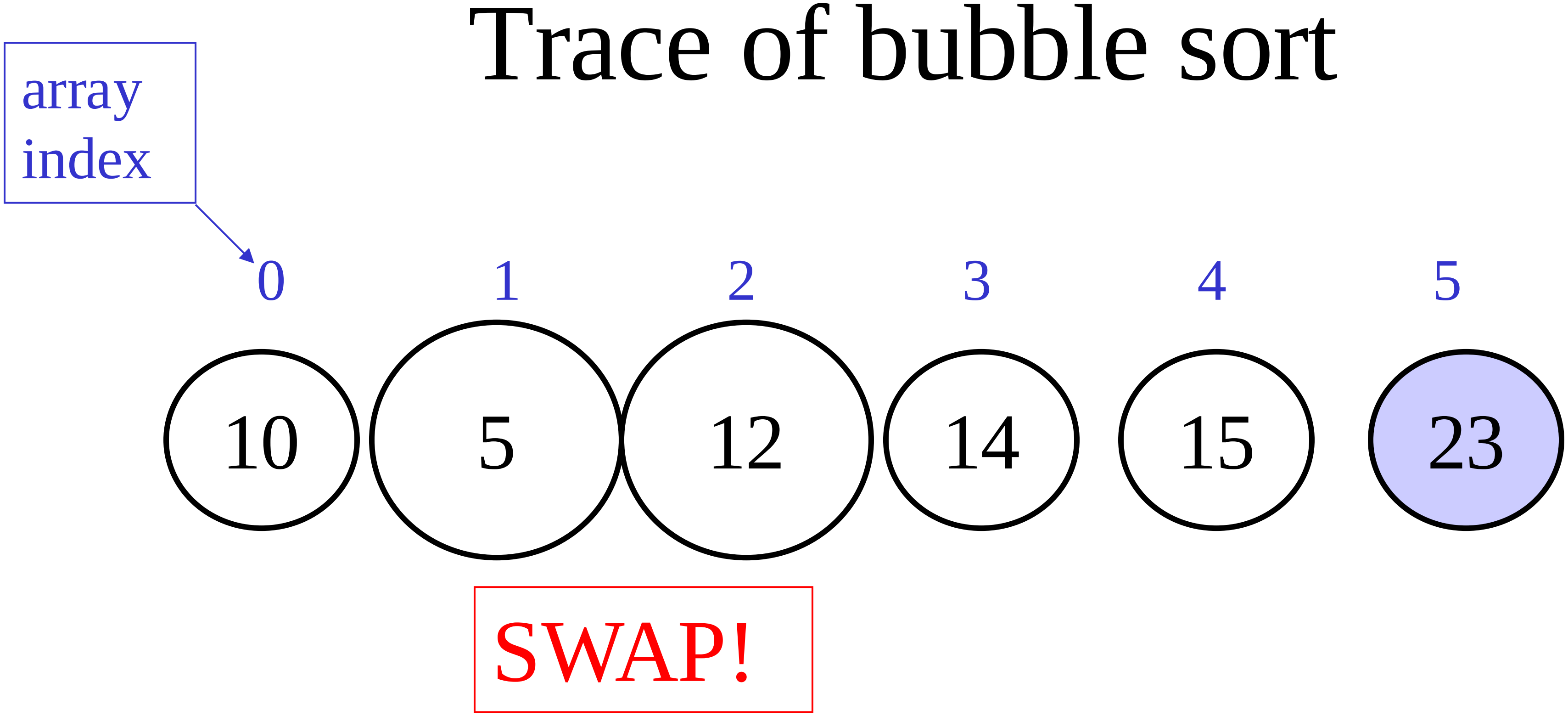


i = 4, second iteration of the outer loop

j = 1, arr[0]…arr[j] are all less than or equal to arr[j]

j = 1, comparing arr[1] with arr[2]

Trace of bubble sort



array index

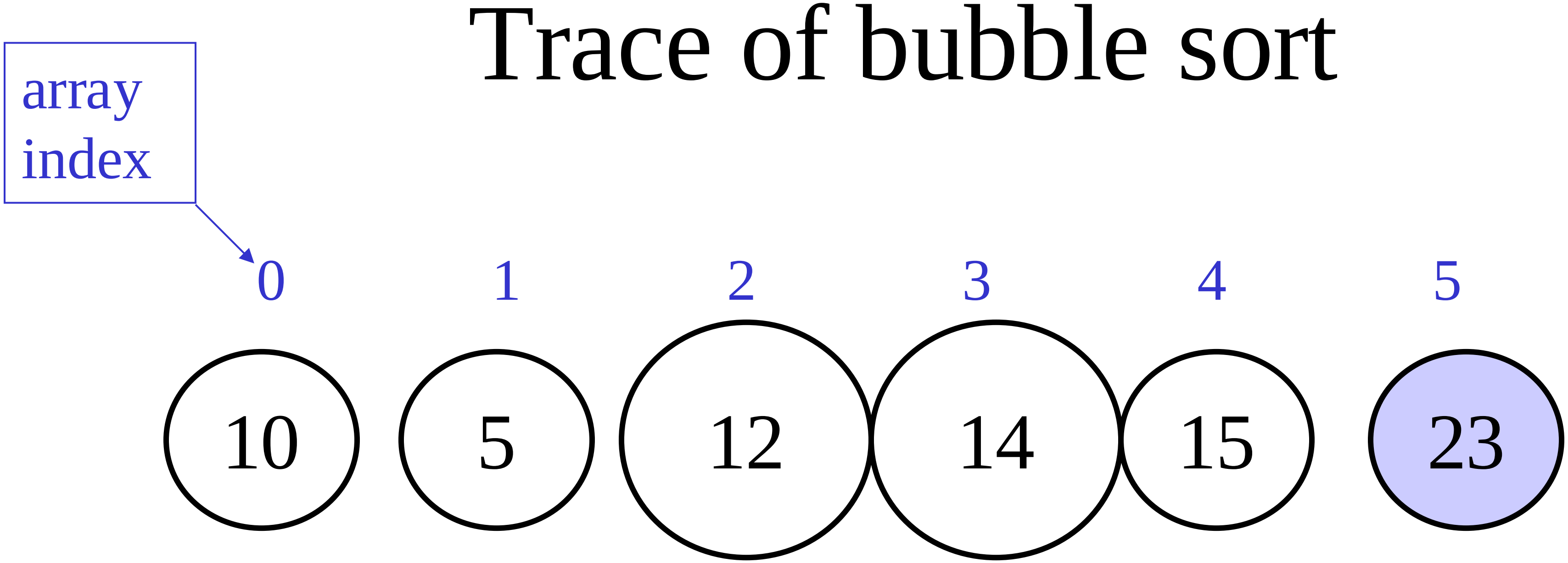
0	1	2	3	4	5
10	5	12	14	15	23

SWAP!

i = 4, second iteration of the outer loop

j = 1, comparing arr[1] with arr[2]

Trace of bubble sort

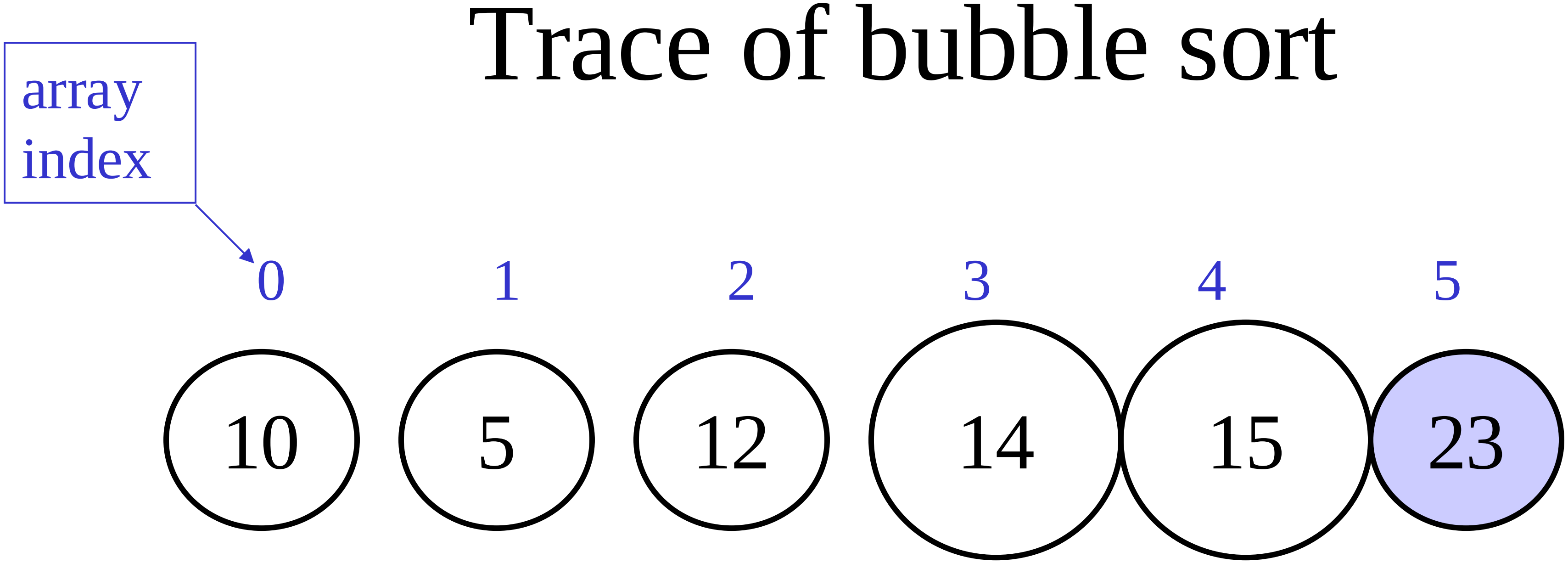


i = 4, second iteration of the outer loop

j = 2, arr[0]…arr[j] are all less than or equal to arr[j]

j = 2, comparing arr[2] with arr[3]

Trace of bubble sort



array index

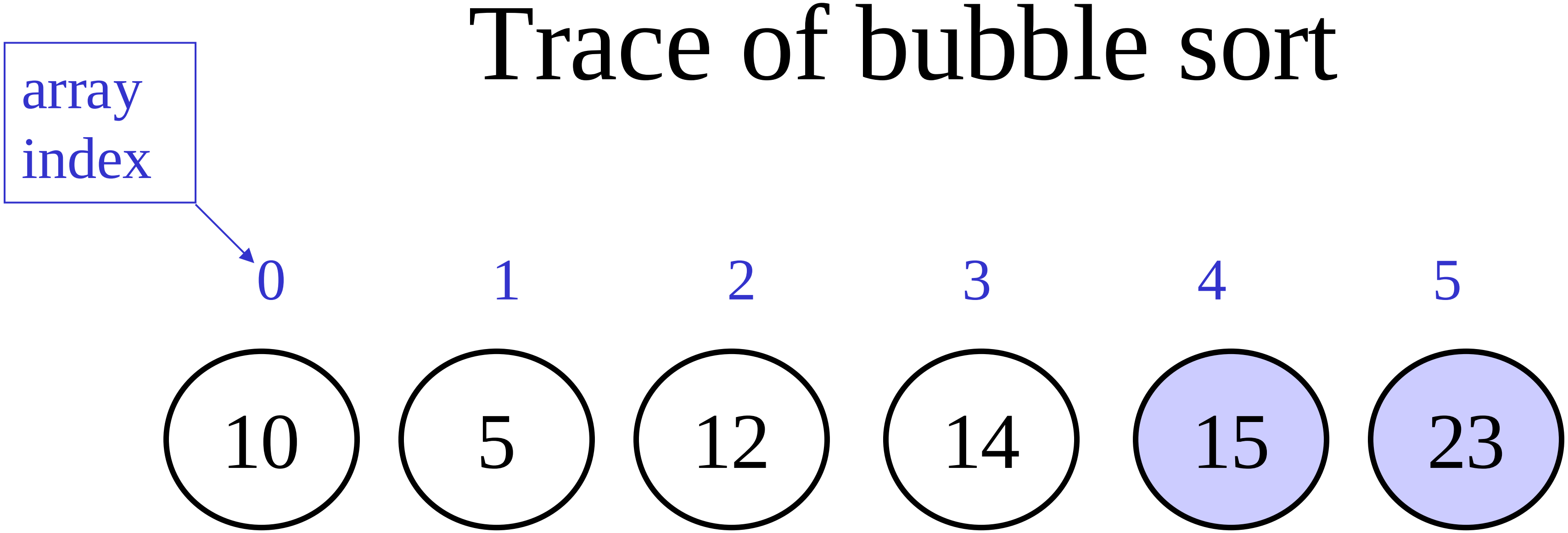
0	1	2	3	4	5
10	5	12	14	15	23

i = 4, second iteration of the outer loop

j = 3, arr[0]…arr[j] are all less than or equal to arr[j]

j = 3, comparing arr[3] with arr[4]

Trace of bubble sort



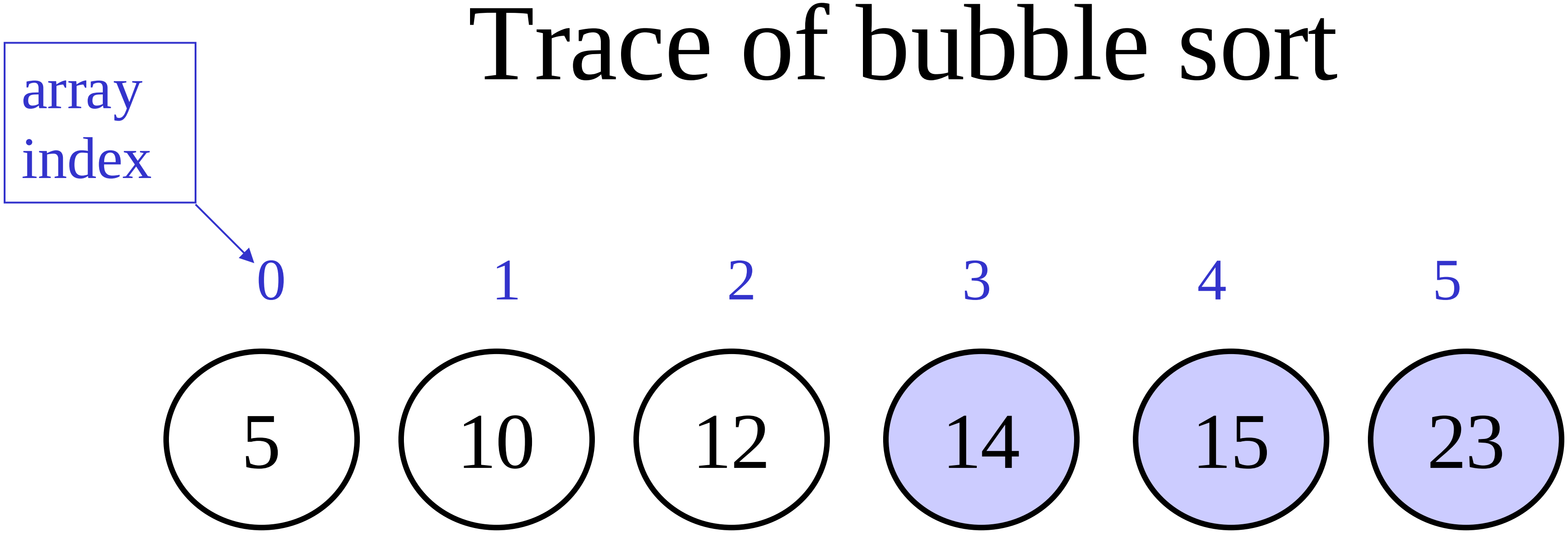
array index

0	1	2	3	4	5
10	5	12	14	15	23

i = 4, second iteration of the outer loop

inner loop finished, second largest element in position 4, positions 0-3 unsorted, positions 4 and 5 are sorted.

Trace of bubble sort

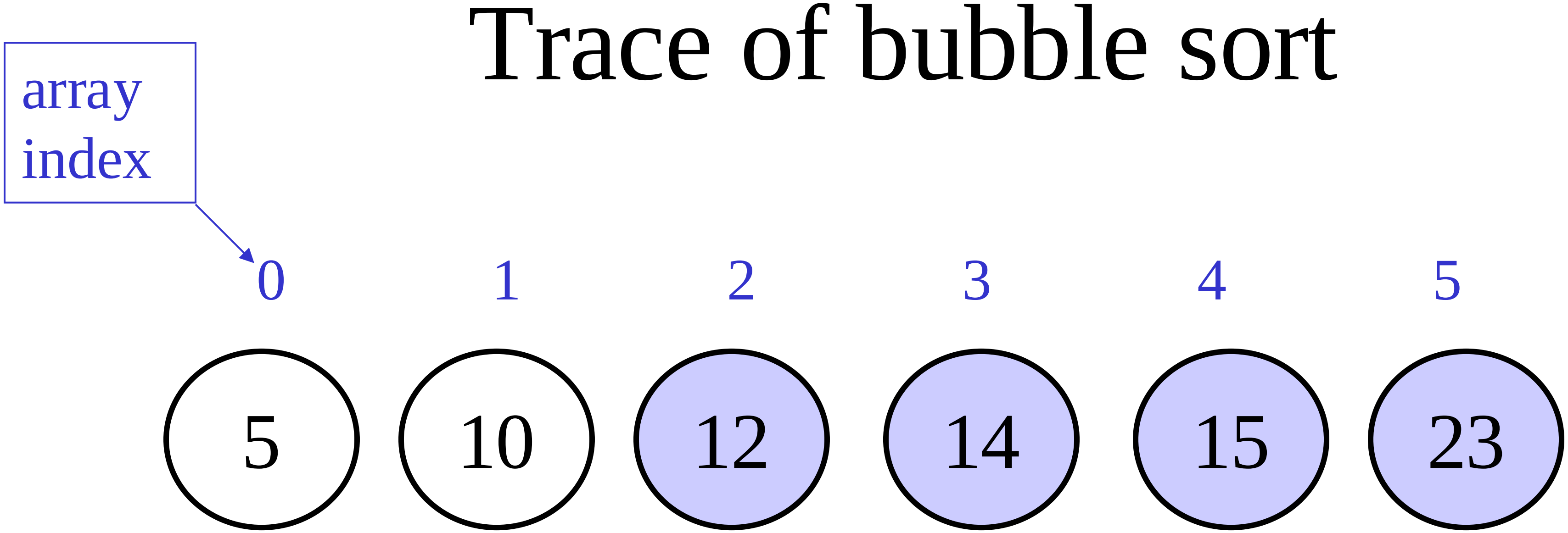


array index

0	1	2	3	4	5
5	10	12	14	15	23

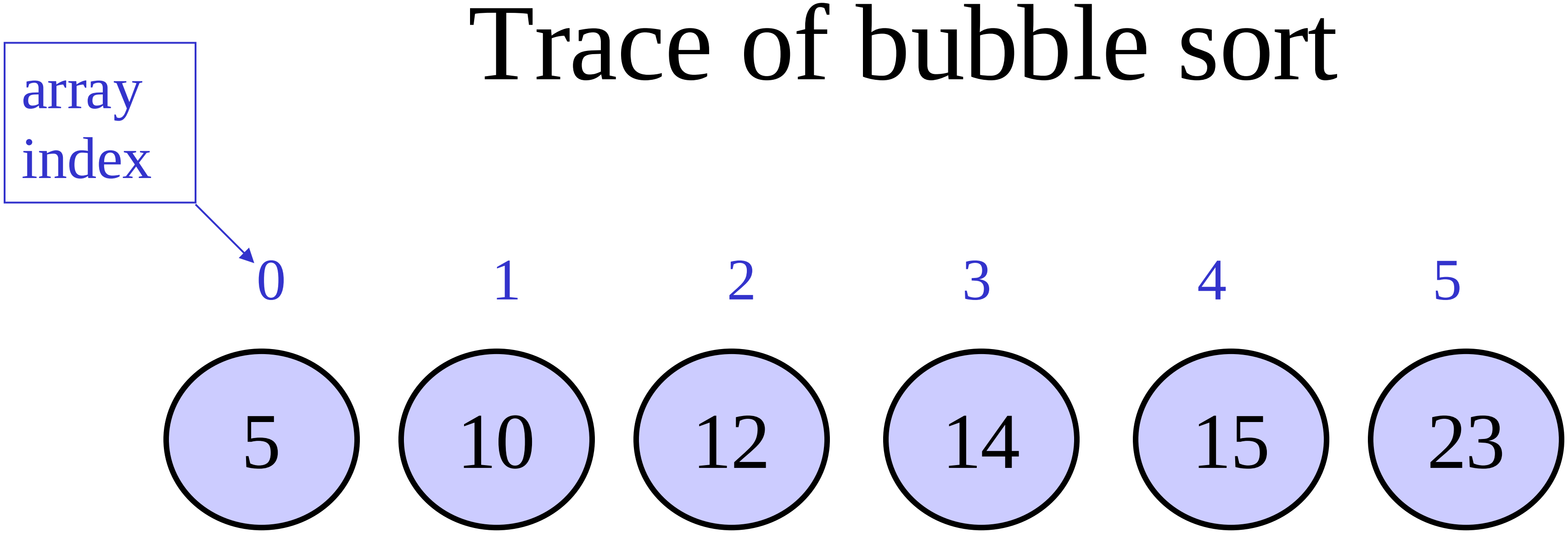
After third iteration…

Trace of bubble sort



After fourth iteration…

Trace of bubble sort



array index

0	1	2	3	4	5
5	10	12	14	15	23

After fifth iteration…

Complexity of bubble sort

- " For an array of size n, in the worst case: 1st passage through the inner loop: $n - 1$ comparisons and $n - 1$ swaps
- ...
- (n-1)th passage through the inner loop: one comparison and one swap.
- All together: $c\ ((n-1)\ +\ (n-2)\ +\ \ldots\ +\ 1),$ where c is the time required to do one comparison, one swap, check the inner loop condition and increment j.
- We also spend constant time k declaring $i, j, temp$ and initialising i. Outer loop is executed $n - 1$ times, suppose the cost of checking the loop condition and decrementing i is c_1.

Complexity of bubble sort

$c\ ((n-1) + (n-2) + \dots + 1) + k + c_1(n-1)$

$(n-1) + (n-2) + \dots + 1 = n(n-1)/2$

So our function equals

$c\ n*(n-1)/2 + k + c_1(n-1) = 1/2c\ (n^2-n) + c_1(n-1) + k$

Hence the time complexity is $O(n^2)$.

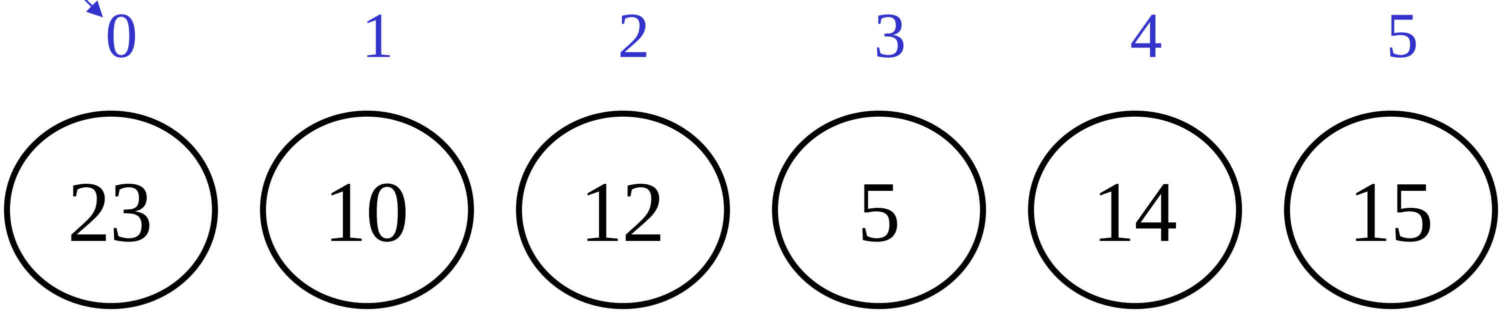
Selection sort

```
void  selectionSort(int[] arr){
  int i, j, temp, pos_greatest;
  for( i = arr.length-1; i > 0; i--){
     pos_greatest = 0;
     for(j = 0; j <= i; j++){
       if( arr[j] > arr[pos_greatest])
          pos_greatest =  j;
     }//end inner for loop
     temp = arr[i];
     arr[i] = arr[pos_greatest];
     arr[pos_greatest] = temp;
}//end outer for loop}//end selection sort
```

compare the current element to the largest seen so far; if it is larger, remember its index

swap the largest element to the end of range

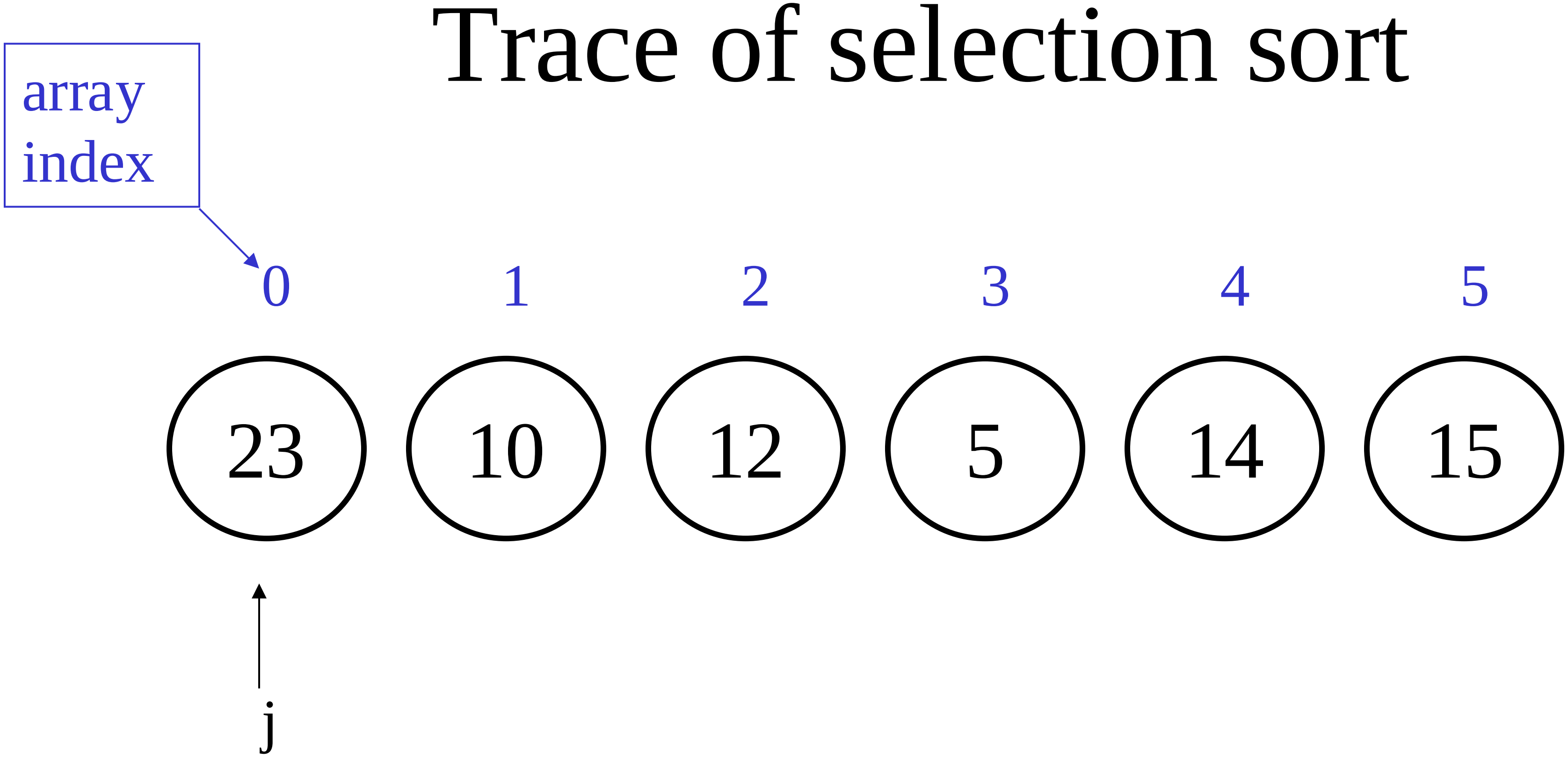
Trace of selection sort



array index

i = 5, first iteration of the outer loop

Trace of selection sort



array index

0	1	2	3	4	5
23	10	12	5	14	15

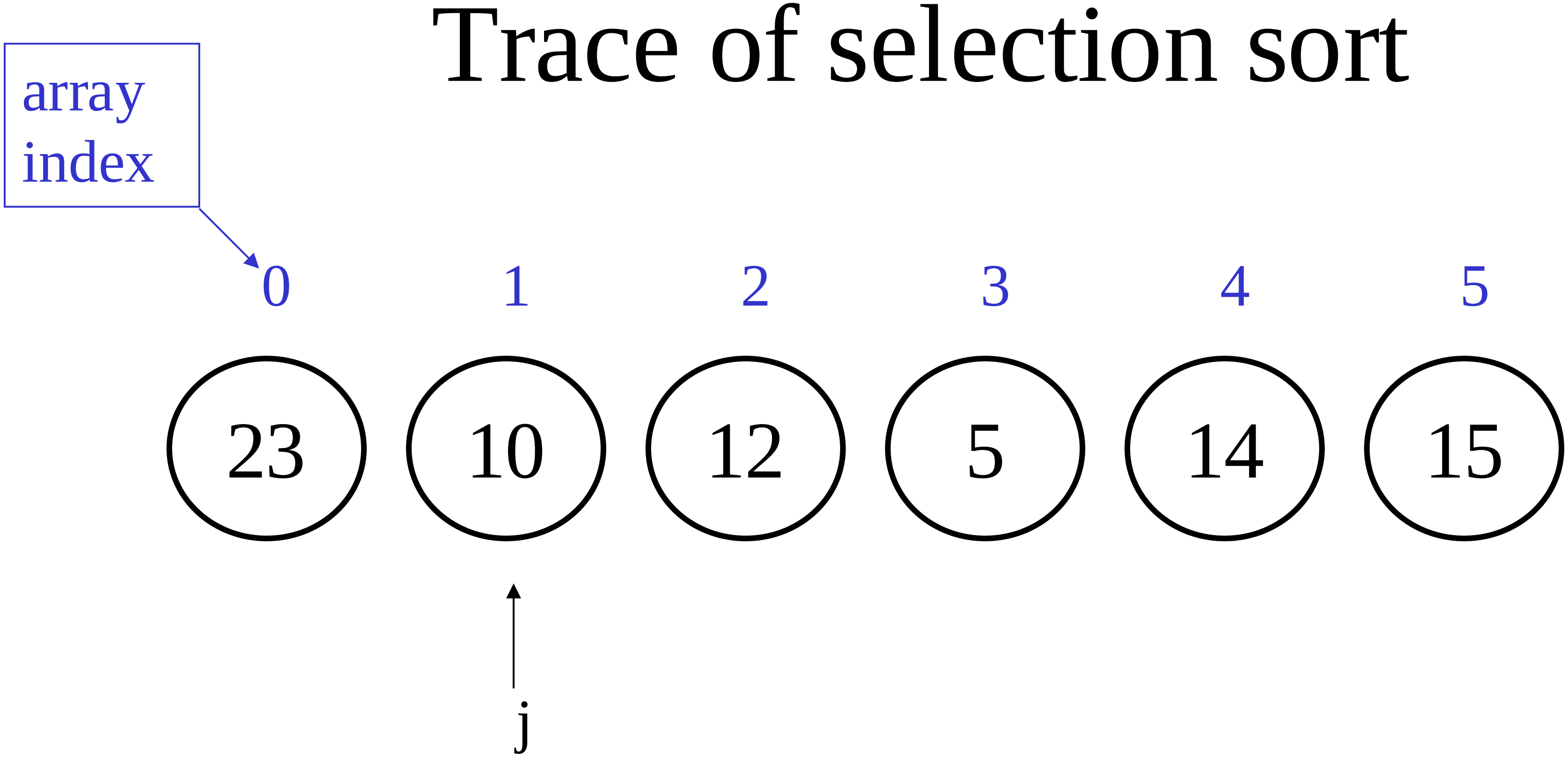
j

i = 5, first iteration of the outer loop

j = 0, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 0, pos_greatest = 0

Trace of selection sort

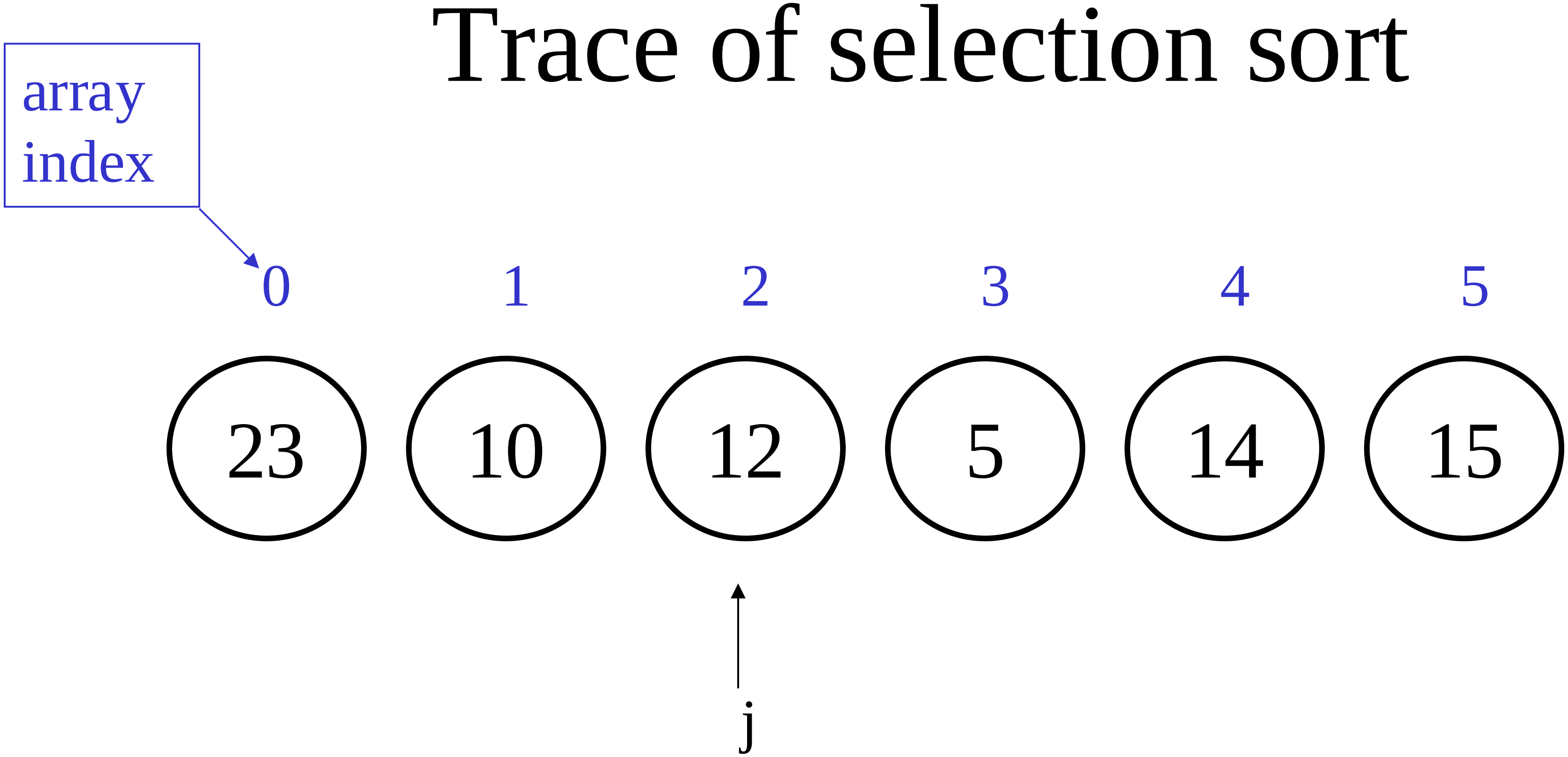


i = 5, first iteration of the outer loop

j = 1, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 1, pos_greatest = 0

Trace of selection sort

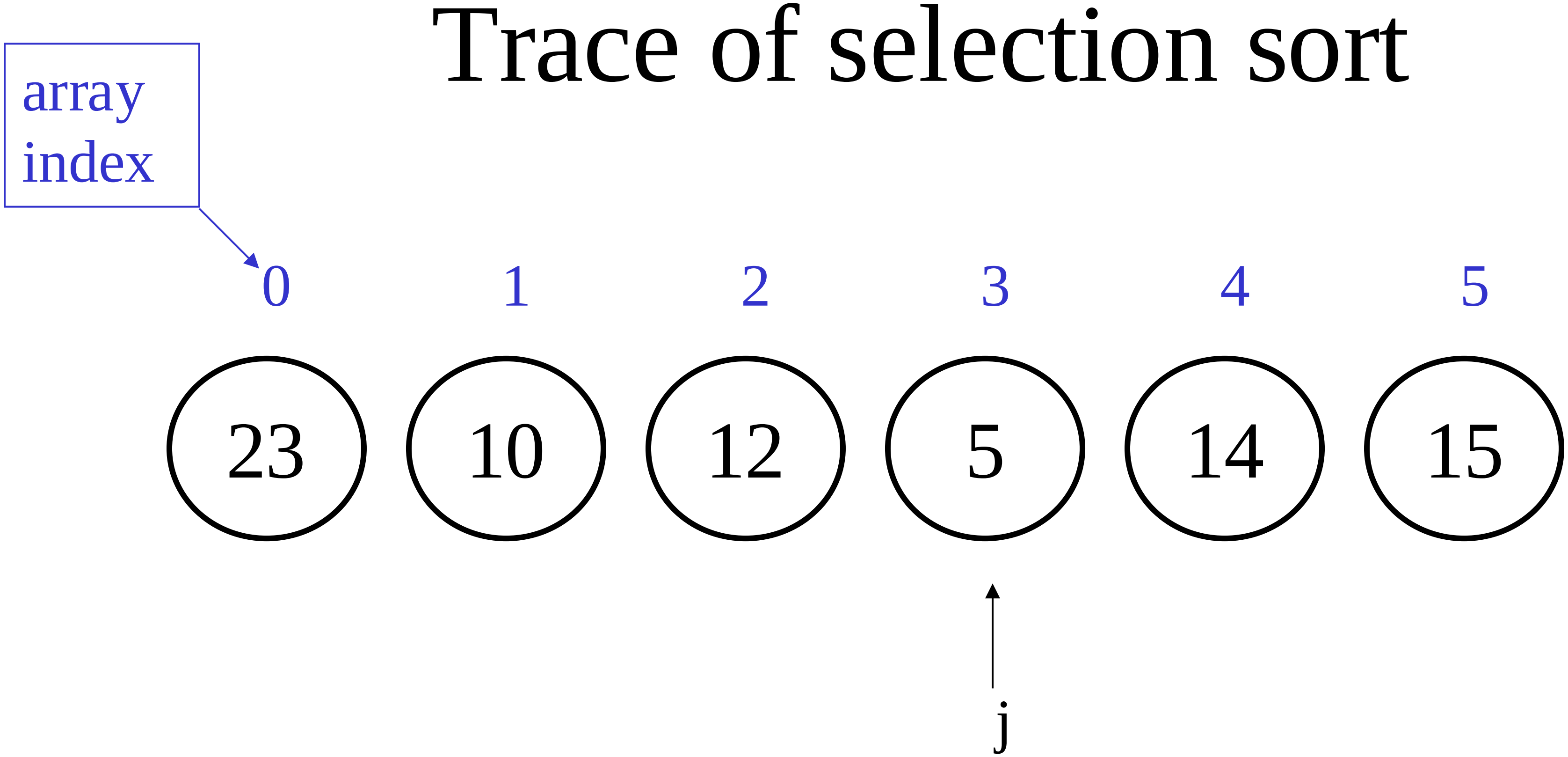


i = 5, first iteration of the outer loop

j = 2, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 2, pos_greatest = 0

Trace of selection sort



array index

0	1	2	3	4	5
23	10	12	5	14	15

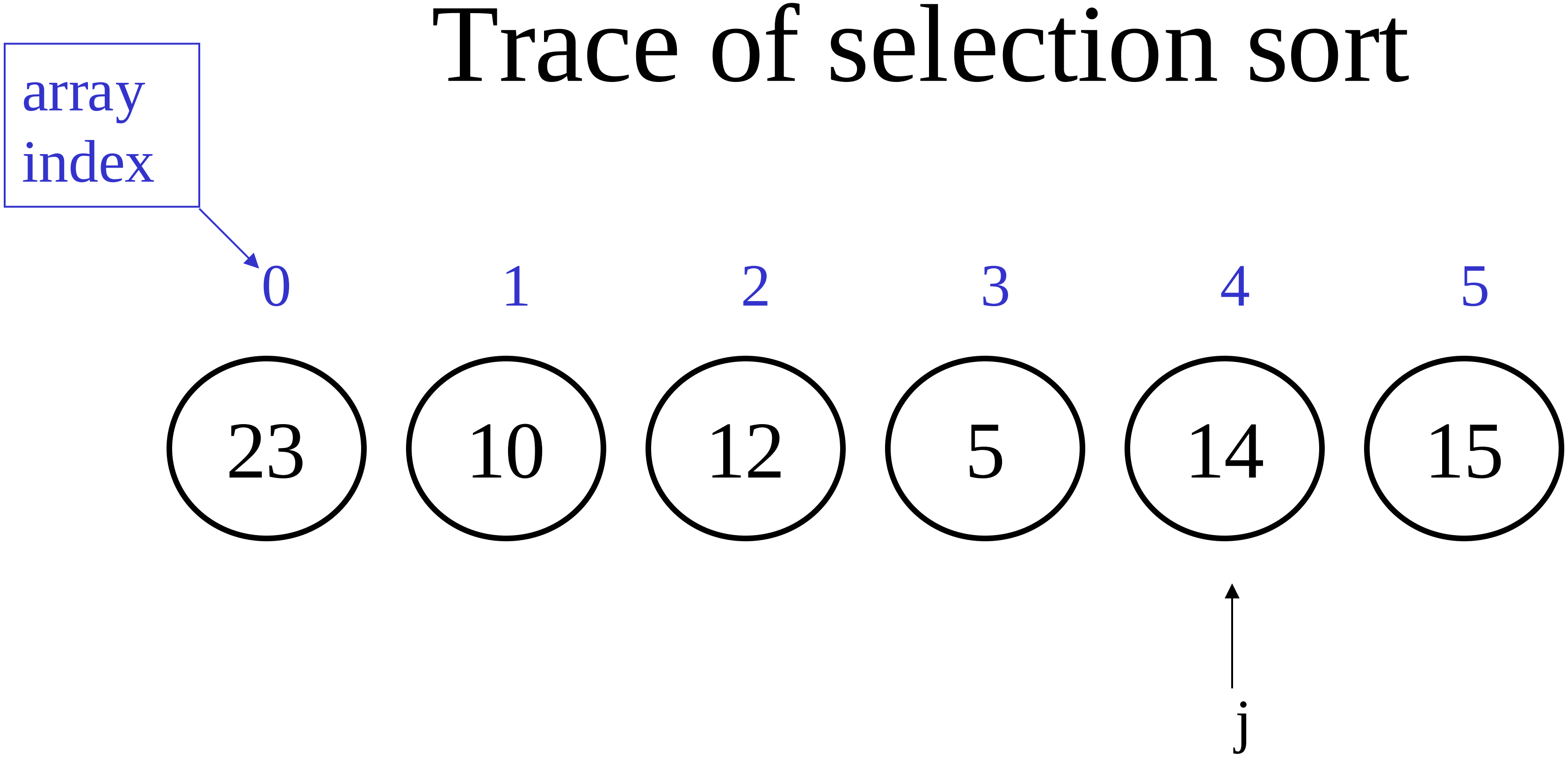
j

i = 5, first iteration of the outer loop

j = 3, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 3, pos_greatest = 0

Trace of selection sort

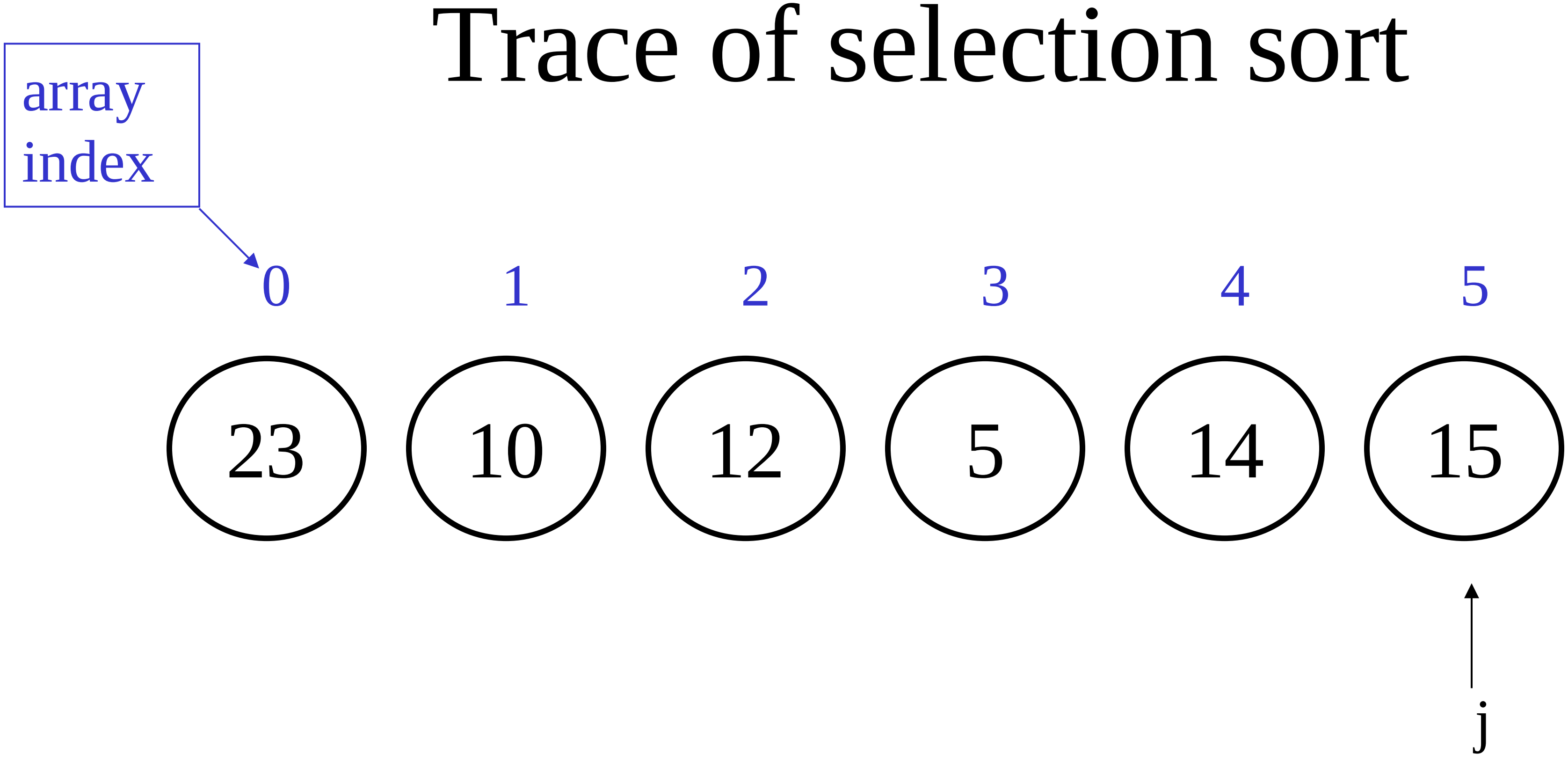


i = 5, first iteration of the outer loop

j = 4, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 4, pos_greatest = 0

Trace of selection sort

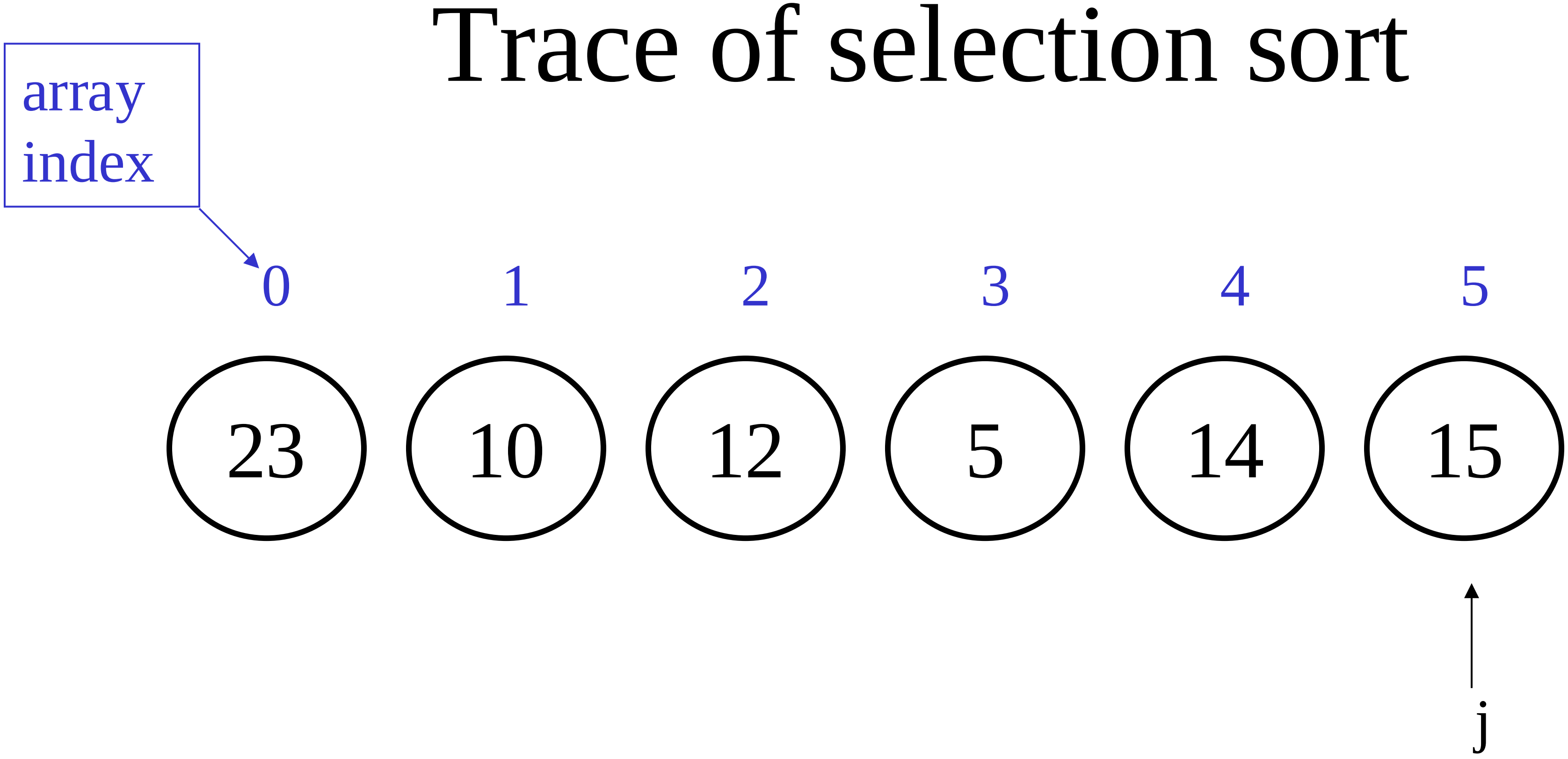


i = 5, first iteration of the outer loop

j = 5, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 5, pos_greatest = 0

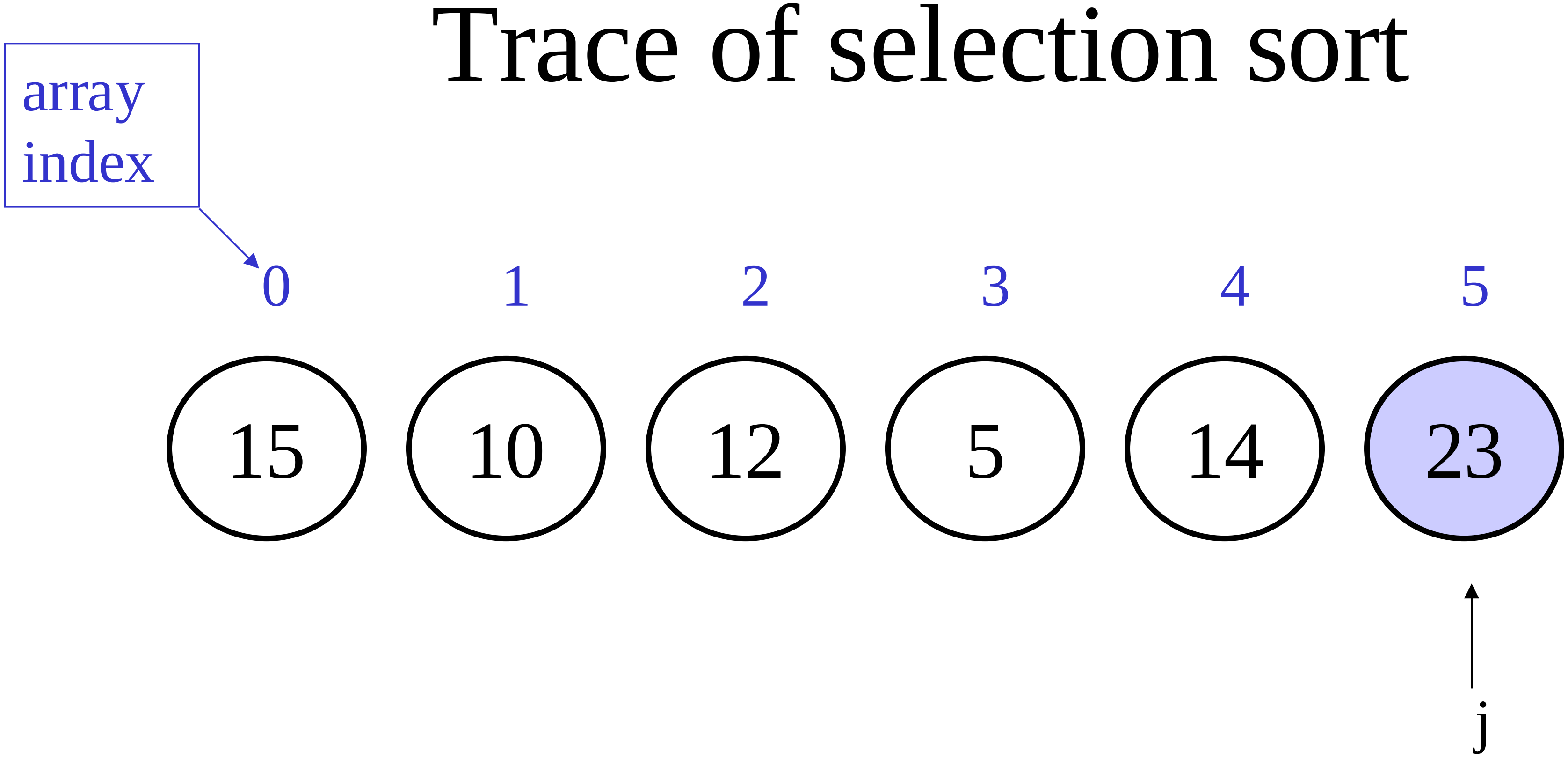
Trace of selection sort



i = 5, first iteration of the outer loop

swap element at pos_greatest to 5

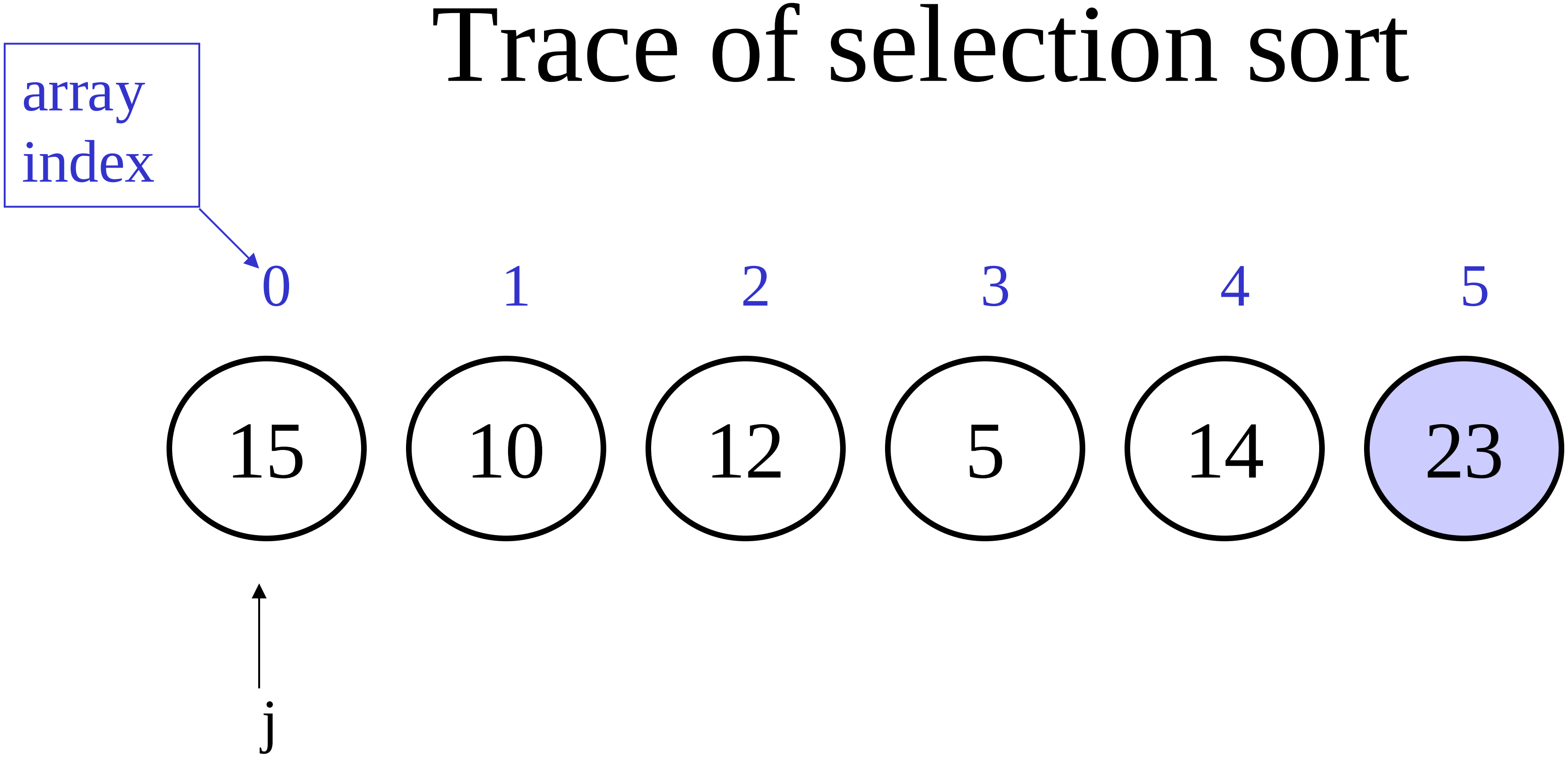
Trace of selection sort



i = 5, first iteration of the outer loop

swap element at pos_greatest to 5

Trace of selection sort

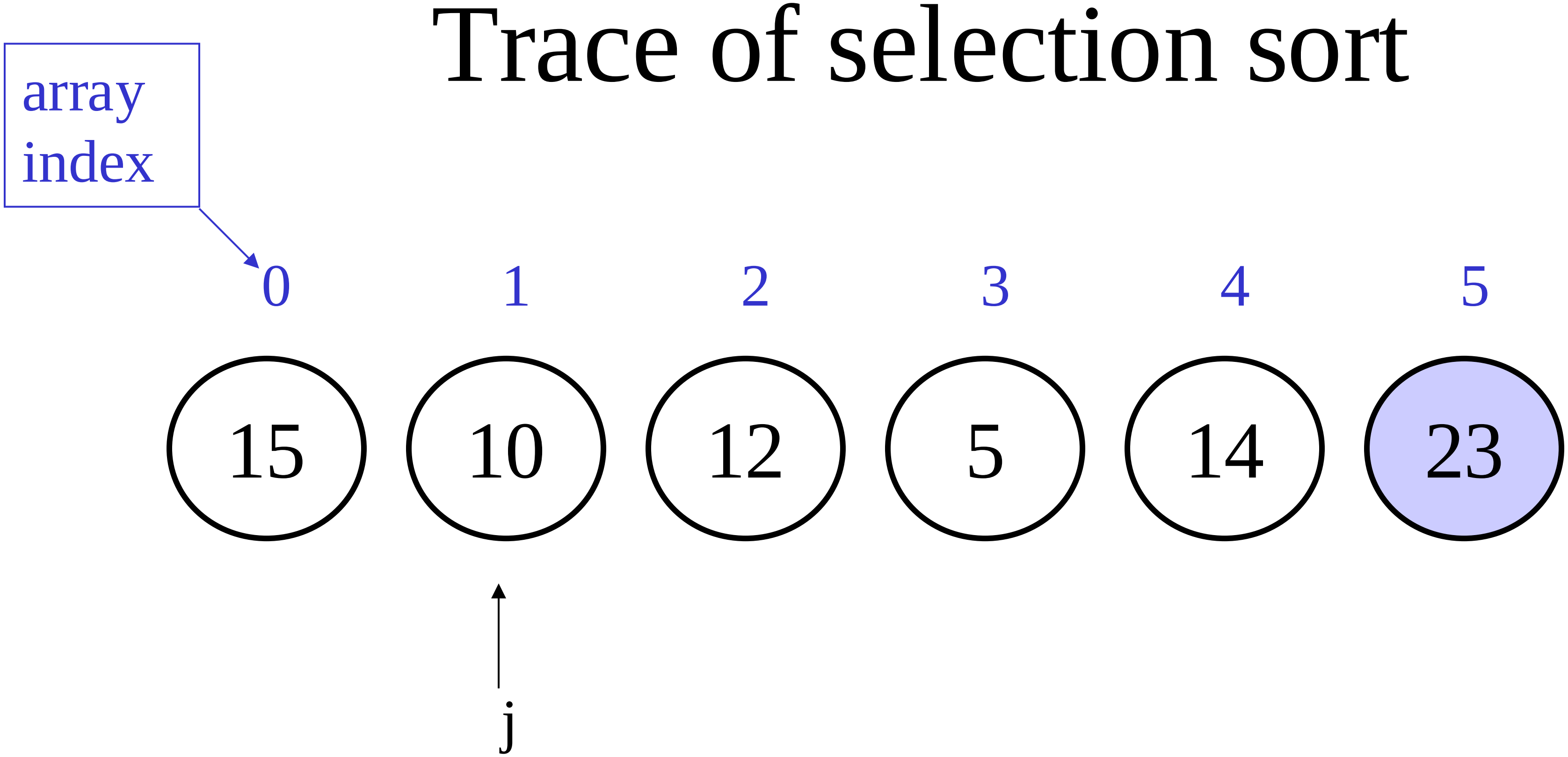


i = 4, second iteration of the outer loop

j = 0, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 0, pos_greatest = 0

Trace of selection sort

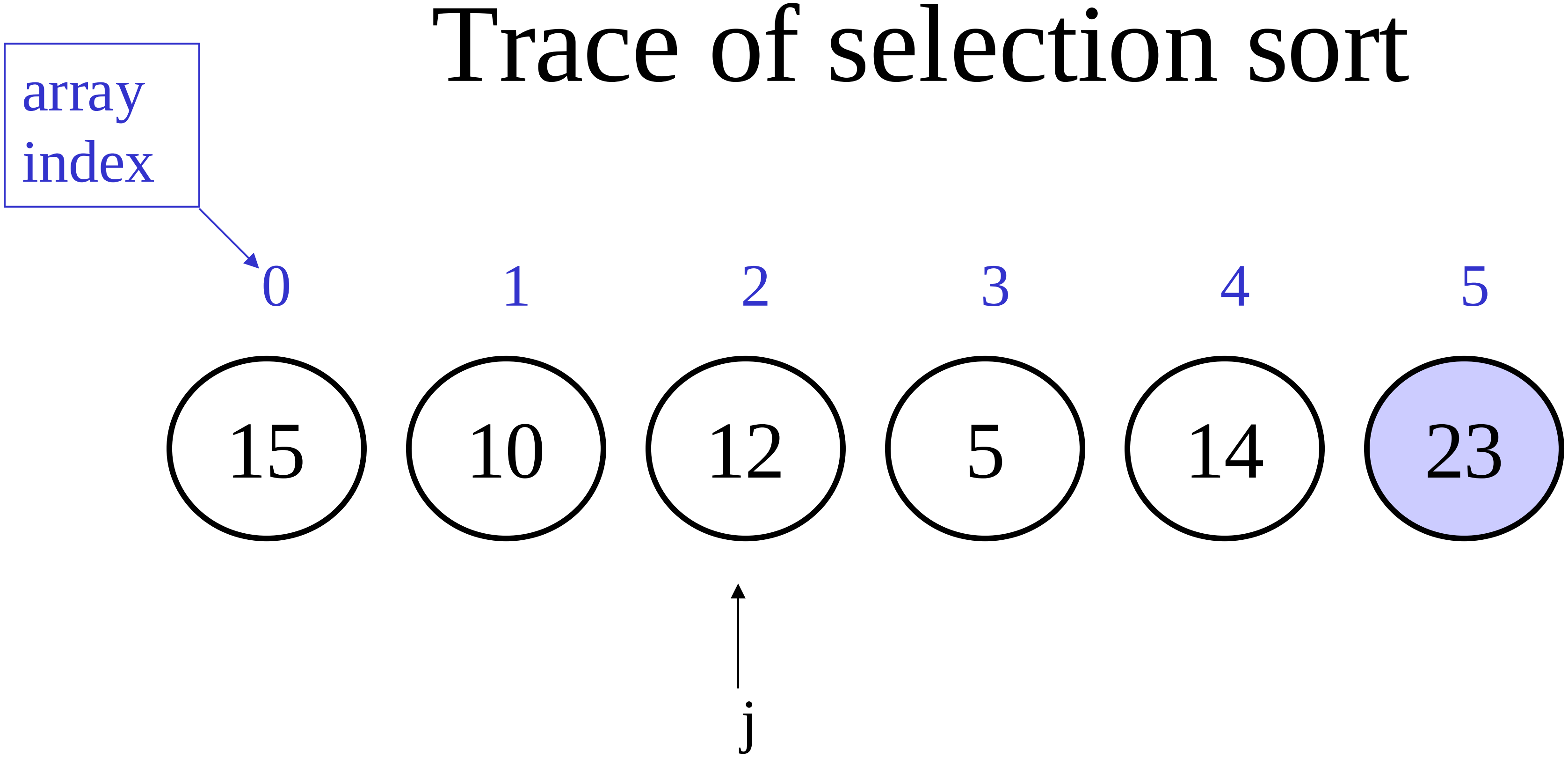


i = 4, second iteration of the outer loop

j = 1, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 1, pos_greatest = 0

Trace of selection sort

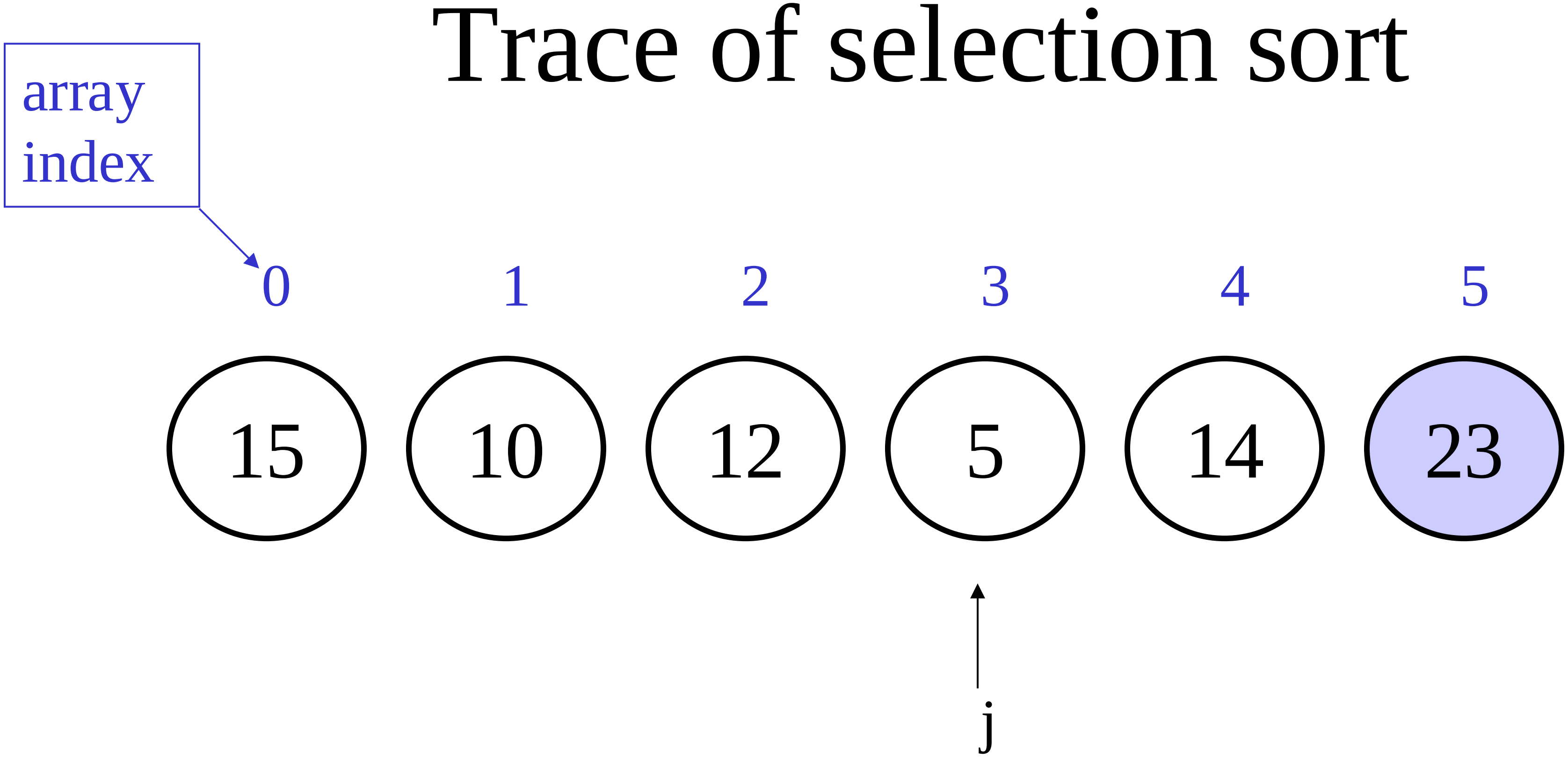


i = 4, second iteration of the outer loop

j = 2, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 2, pos_greatest = 0

Trace of selection sort

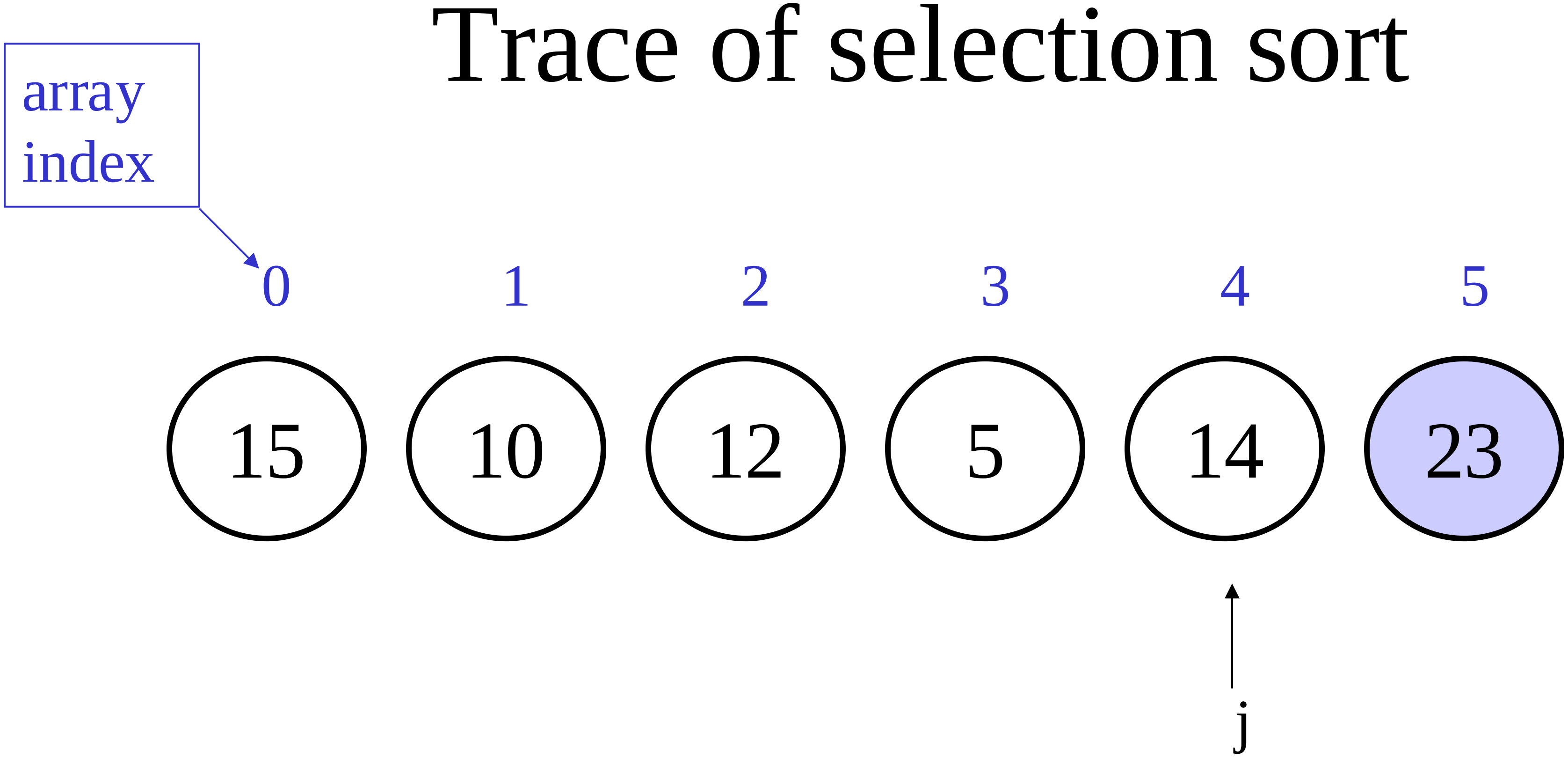


i = 4, second iteration of the outer loop

j = 3, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 3, pos_greatest = 0

Trace of selection sort

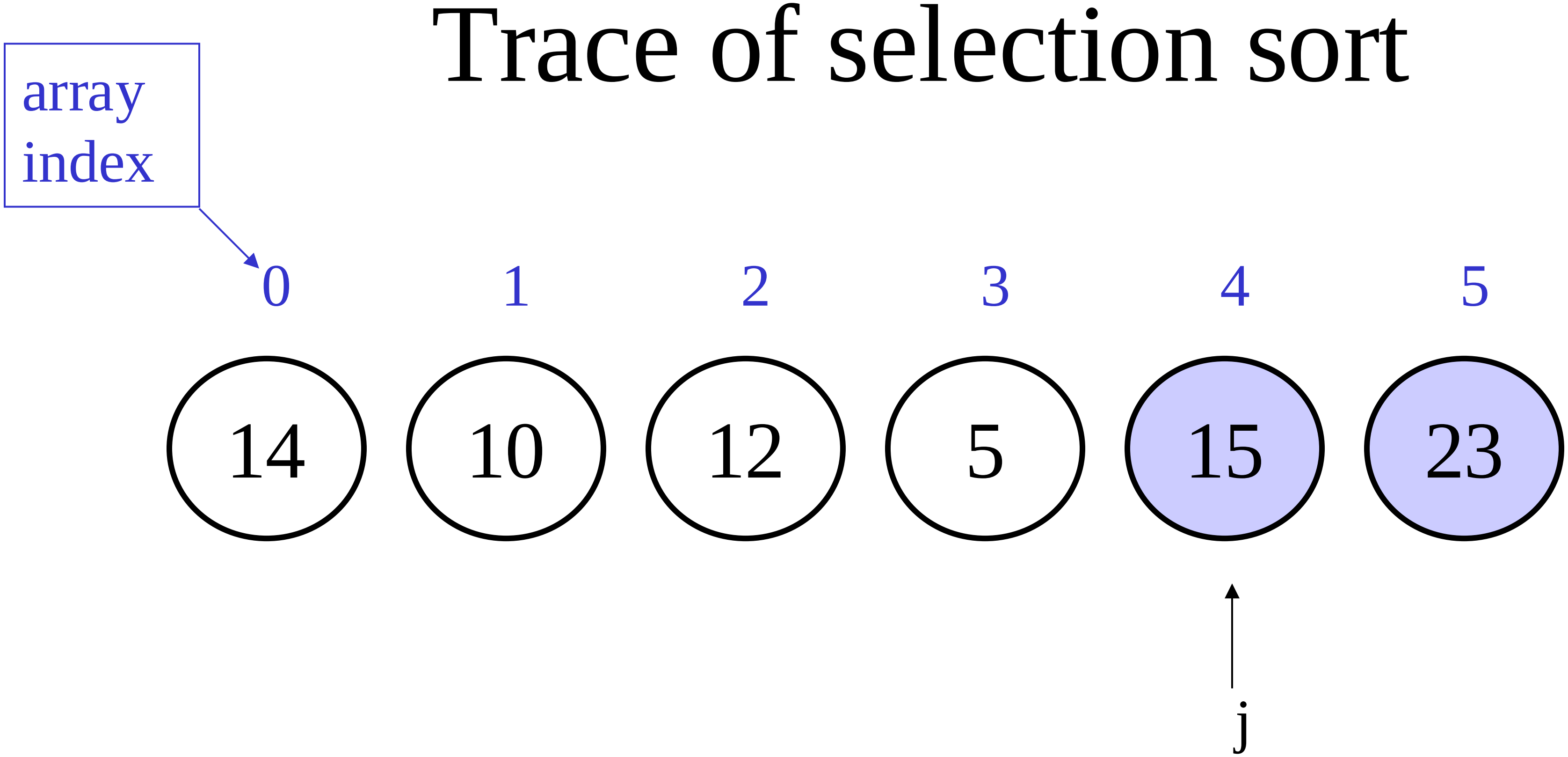


i = 4, second iteration of the outer loop

j = 4, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 4, pos_greatest = 0

Trace of selection sort



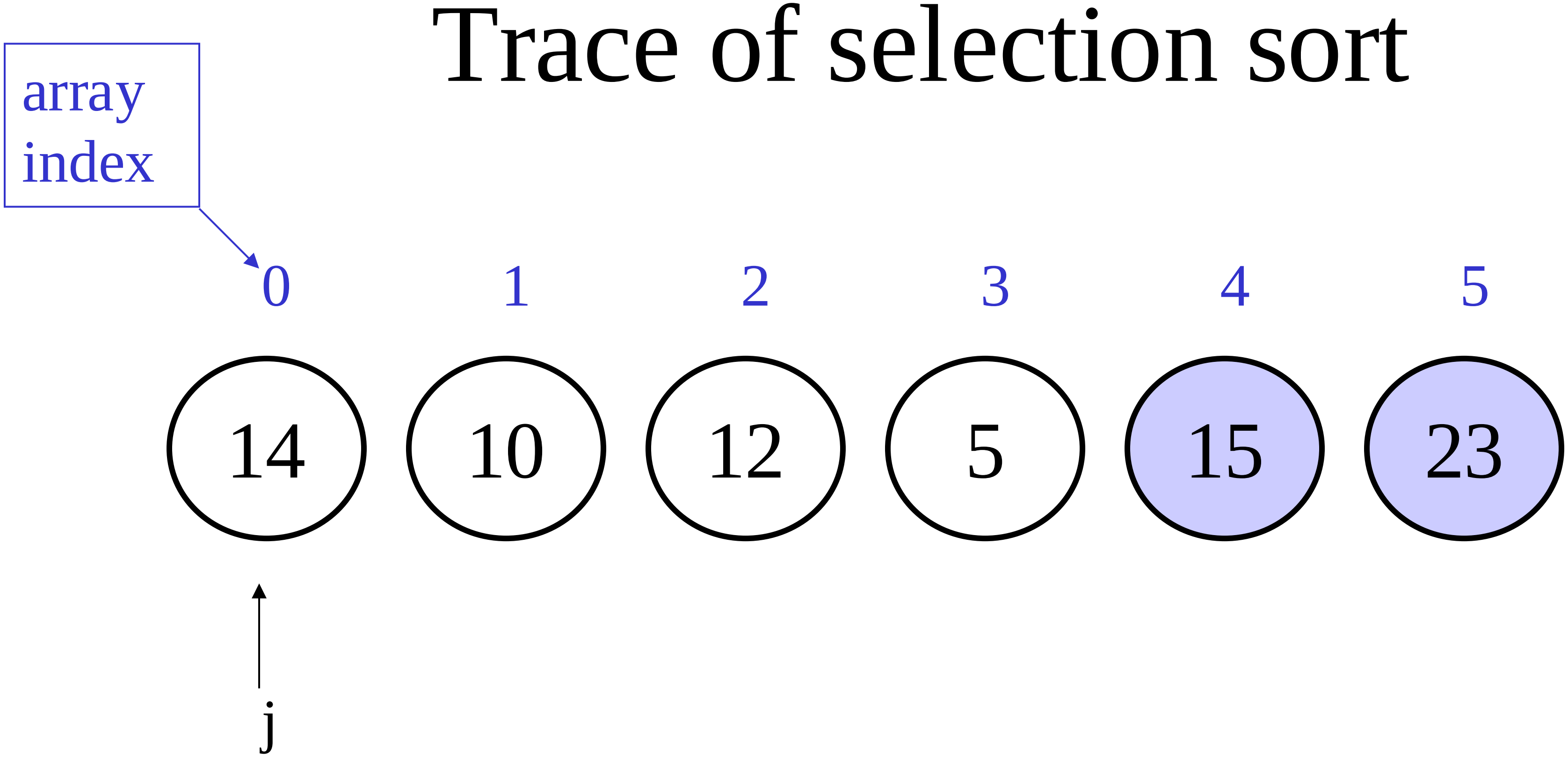
array index

0	1	2	3	4	5
14	10	12	5	15	23

i = 4, second iteration of the outer loop

Swap element at pos_greatest and 4

Trace of selection sort

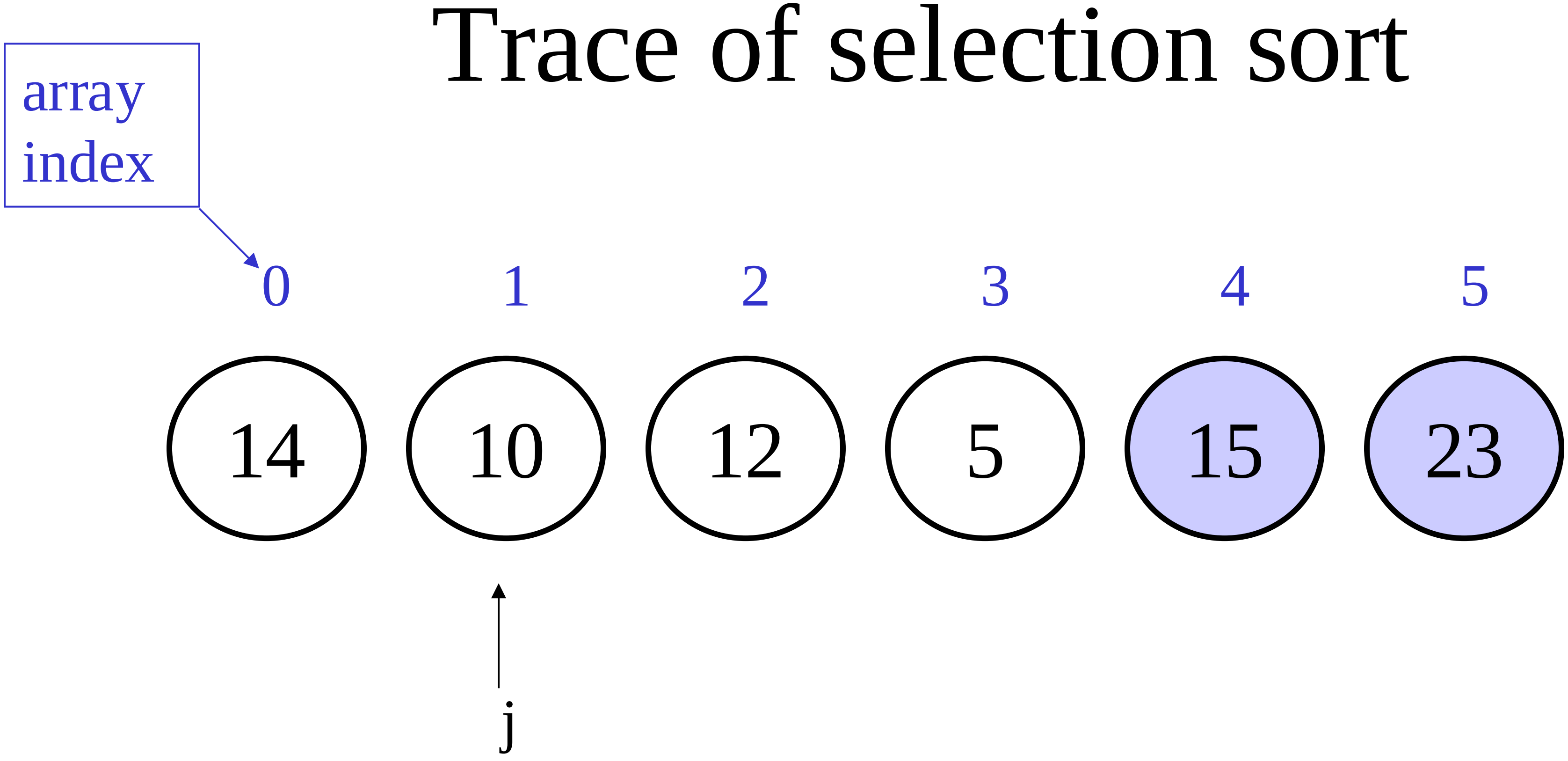


i = 3, third iteration of the outer loop

j = 0, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 0, pos_greatest = 0

Trace of selection sort

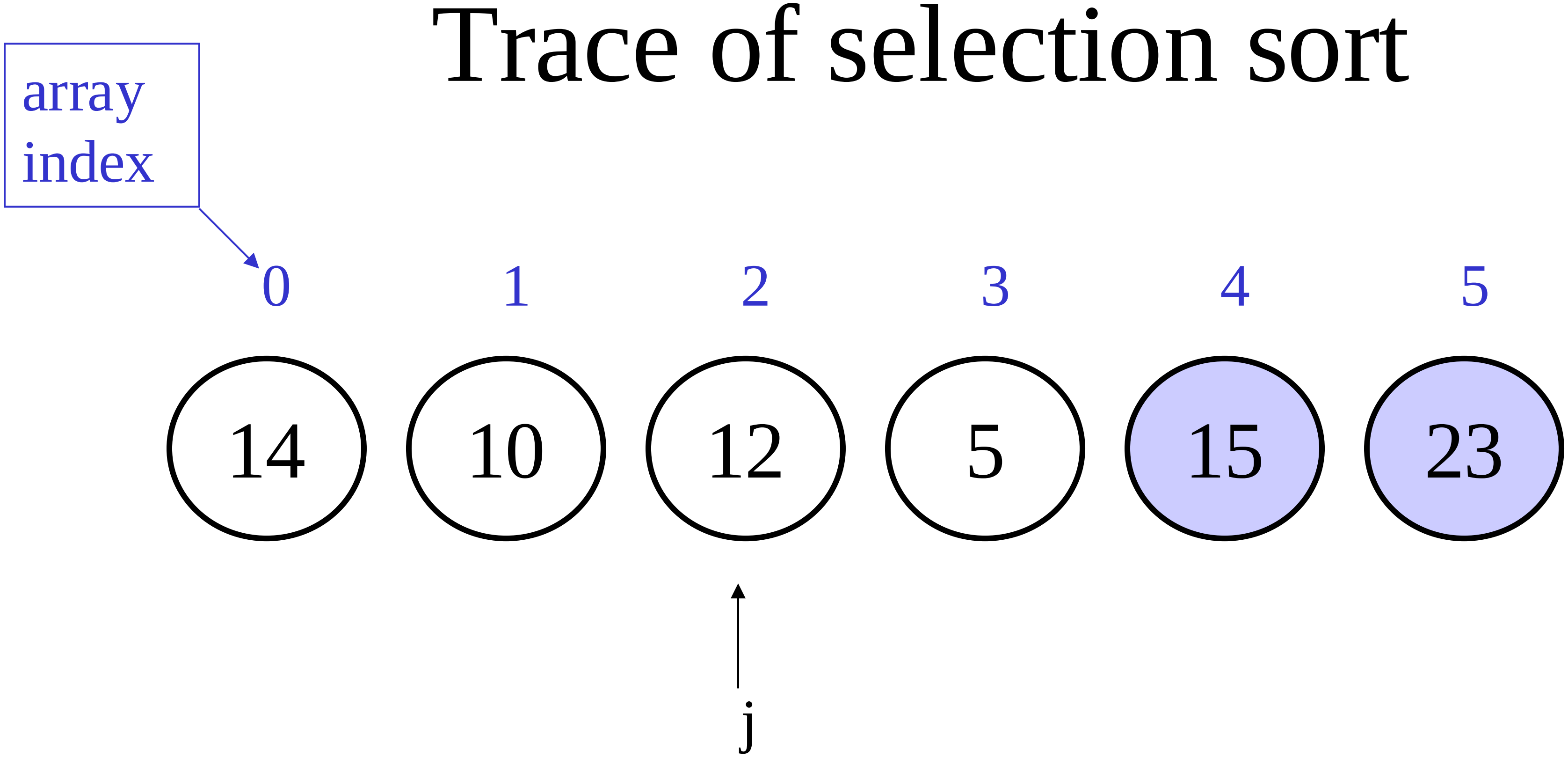


i = 3, third iteration of the outer loop

j = 1, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 1, pos_greatest = 0

Trace of selection sort



array index

0 1 2 3 4 5

14 10 12 5 15 23

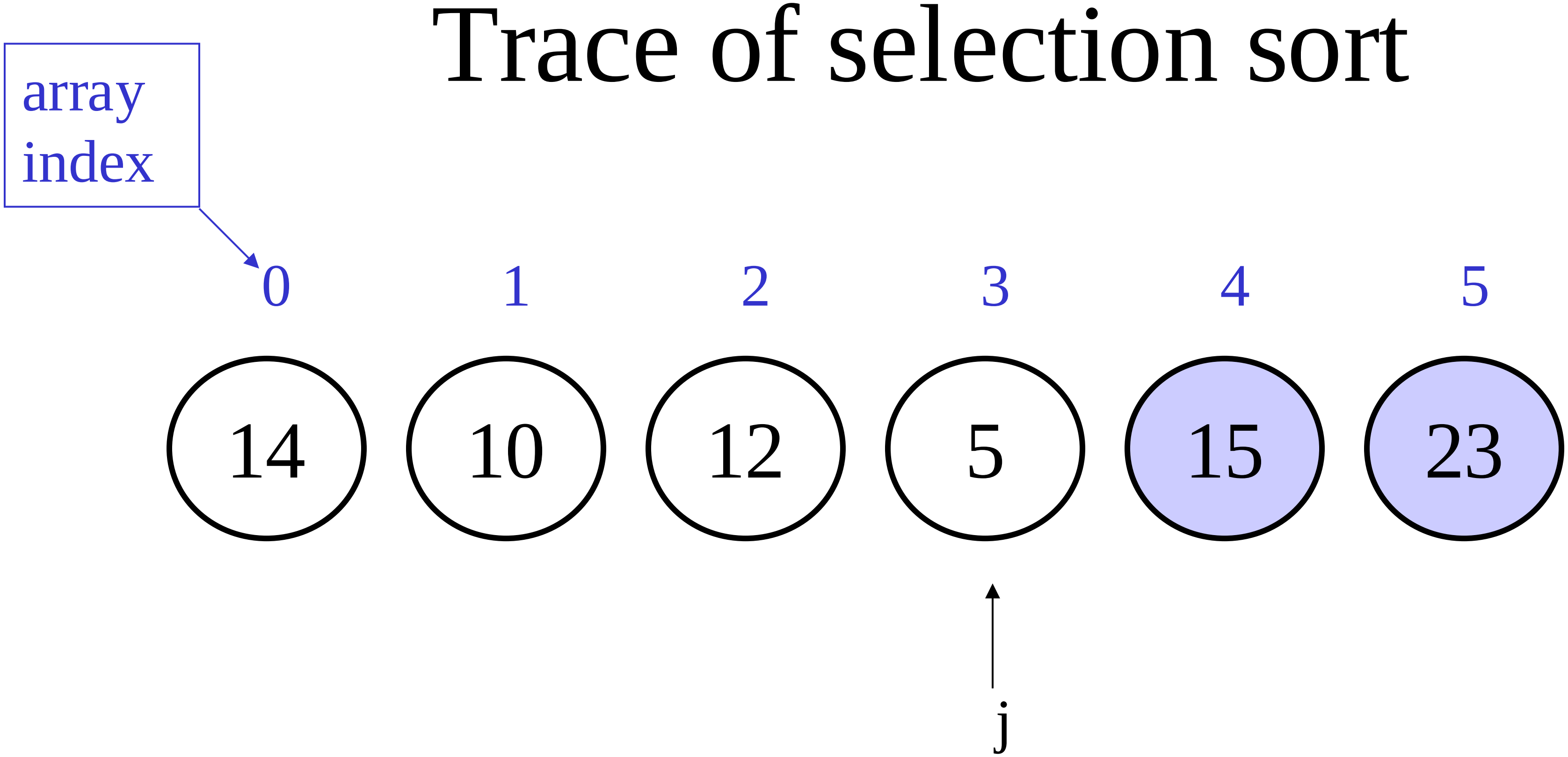
j

i = 3, third iteration of the outer loop

j = 2, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 2, pos_greatest = 0

Trace of selection sort

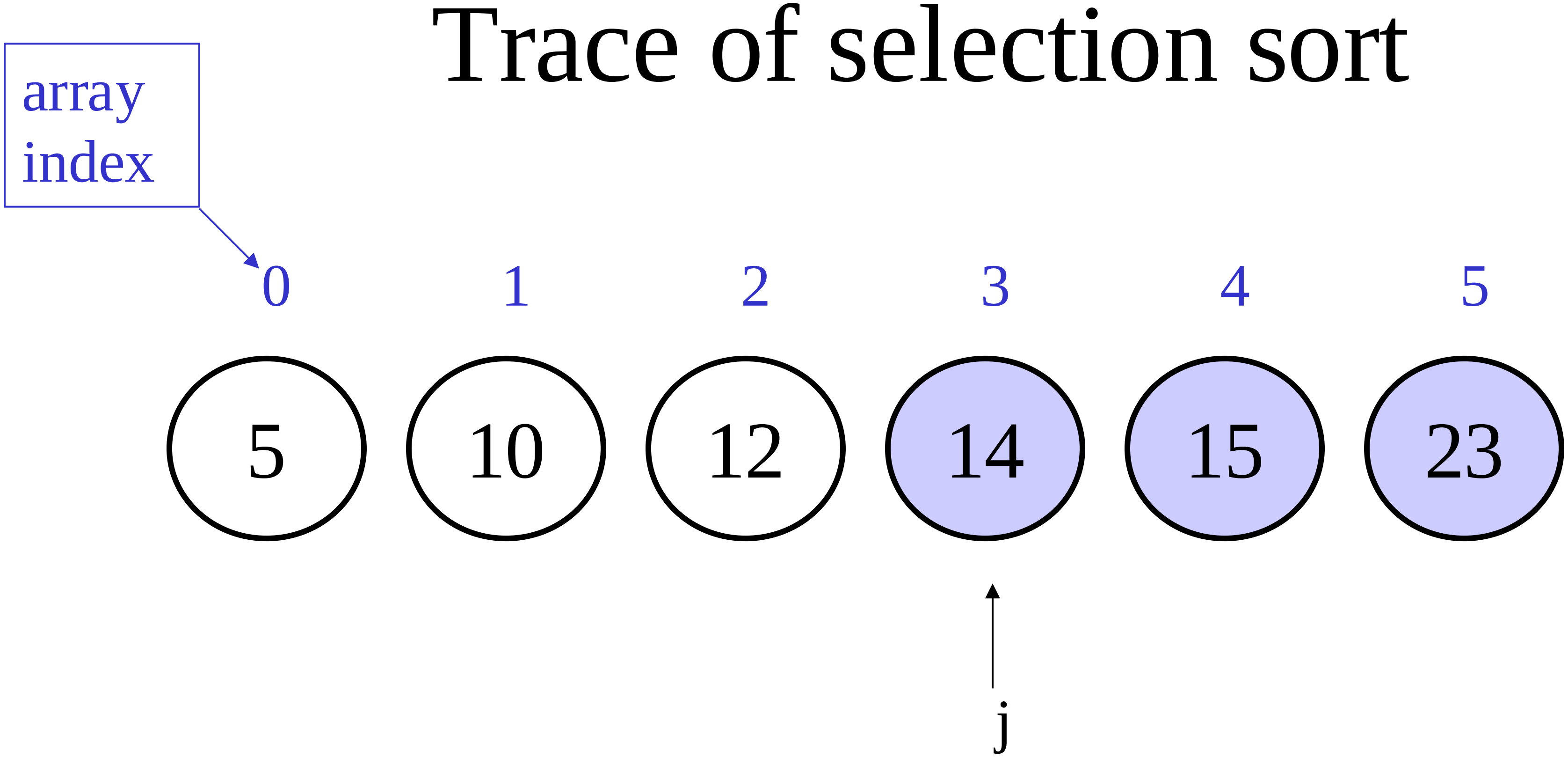


i = 3, third iteration of the outer loop

j = 3, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 3, pos_greatest = 0

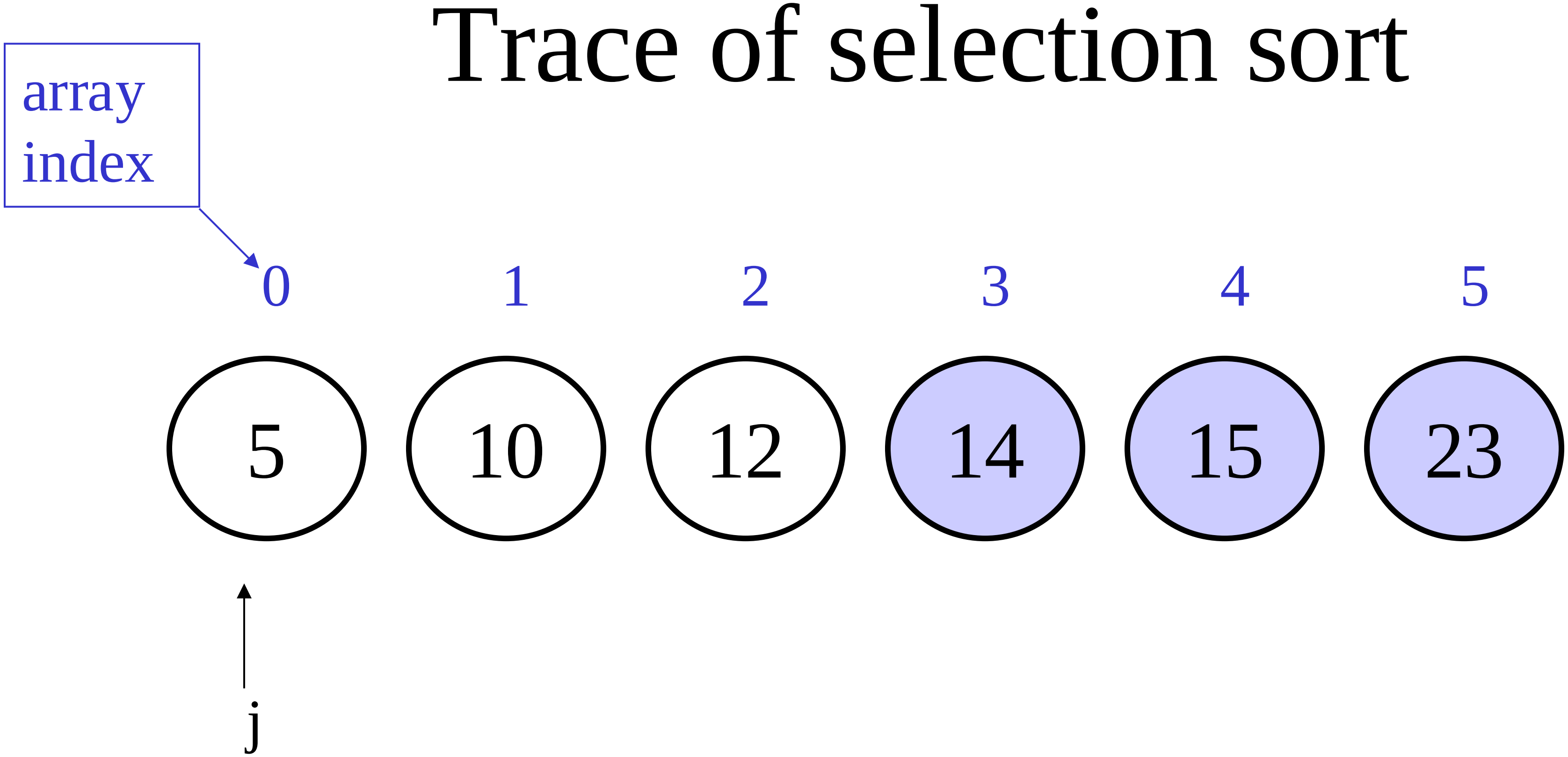
Trace of selection sort



i = 3, third iteration of the outer loop

swap elements at pos_greatest and 3

Trace of selection sort



array index

0 1 2 3 4 5

5 10 12 14 15 23

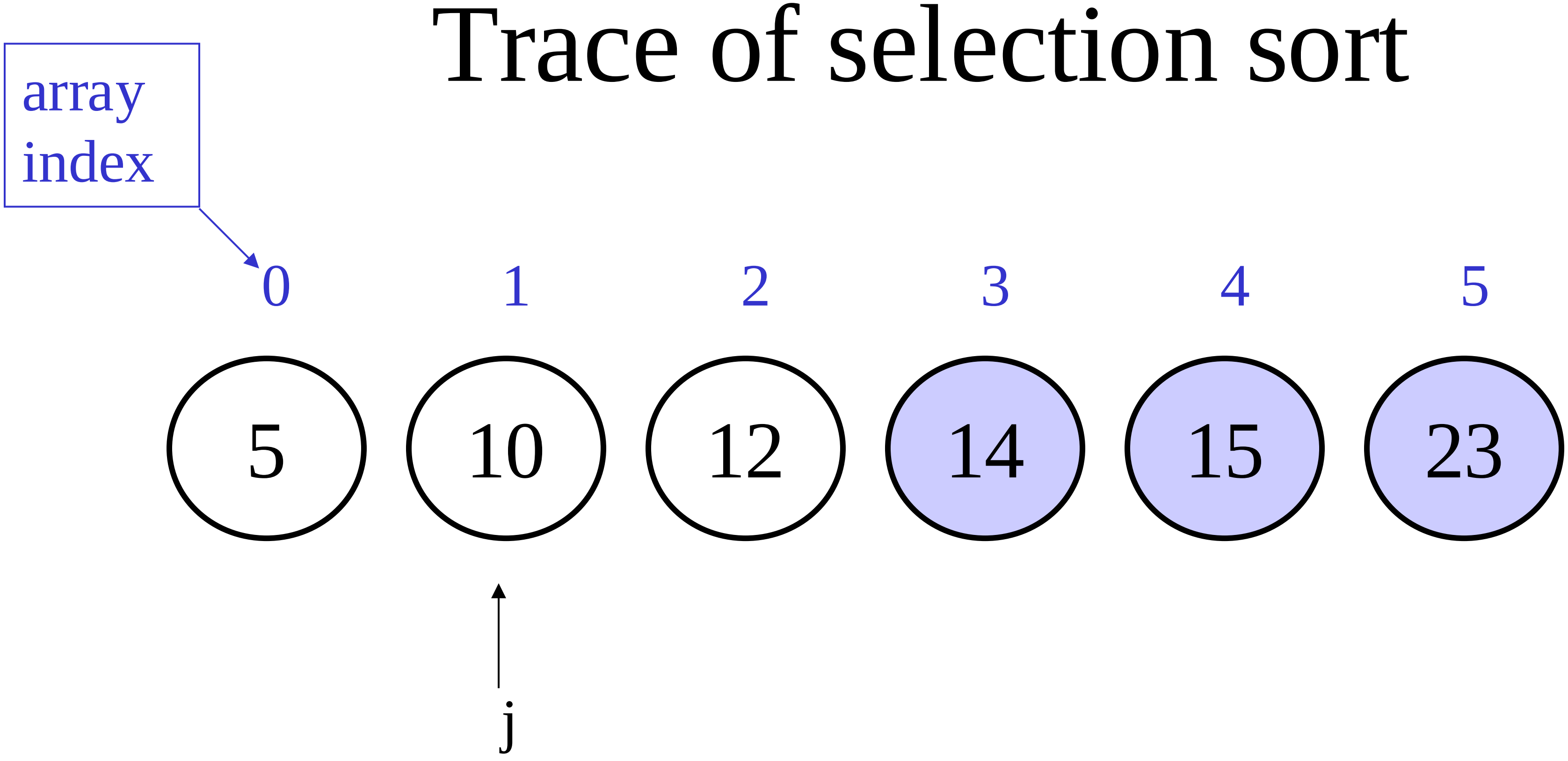
j

i = 2, fourth iteration of the outer loop

j = 0, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 0, pos_greatest = 0

Trace of selection sort

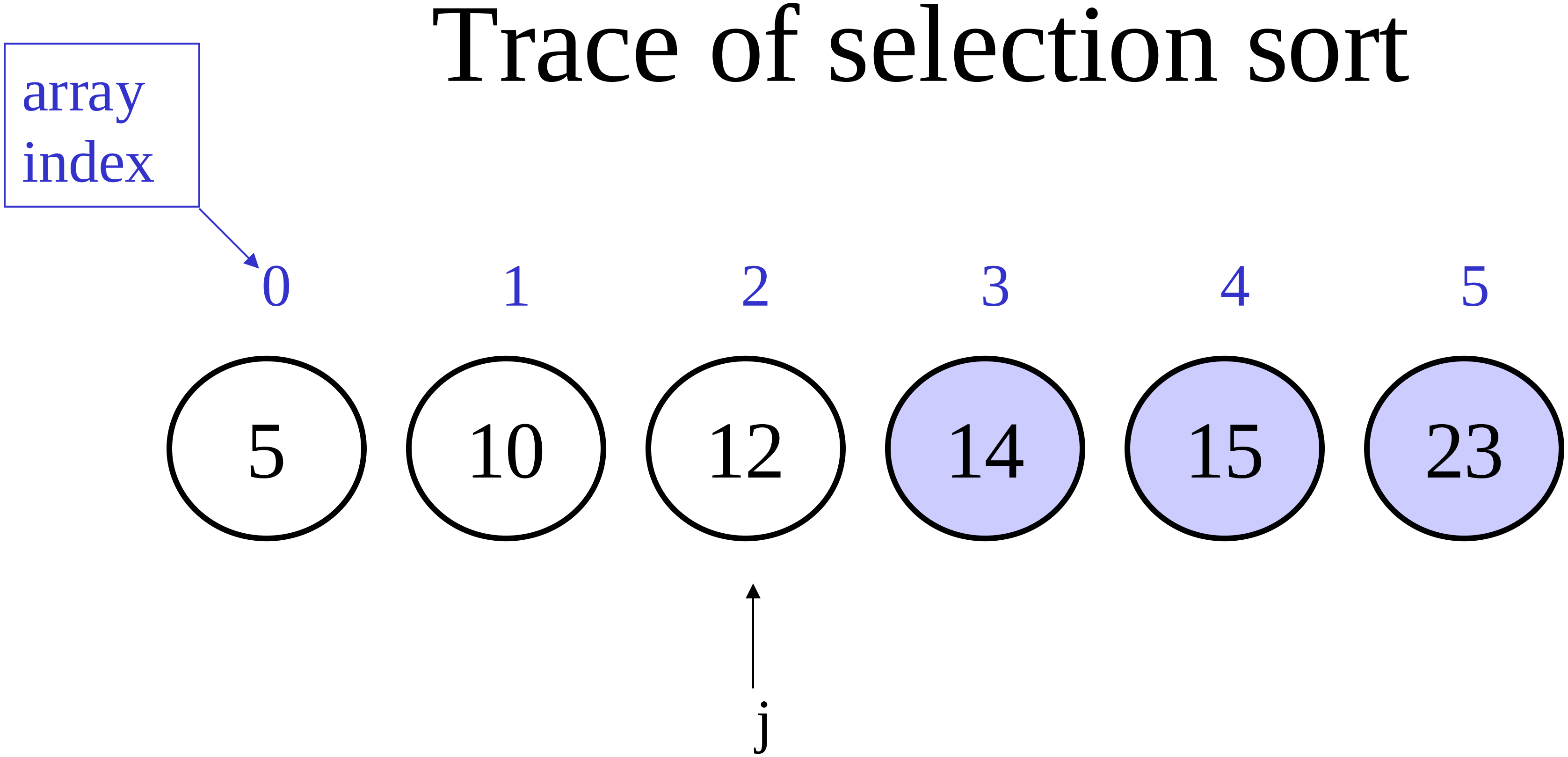


i = 2, fourth iteration of the outer loop

j = 1, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 1, pos_greatest = 1 (changed!)

Trace of selection sort

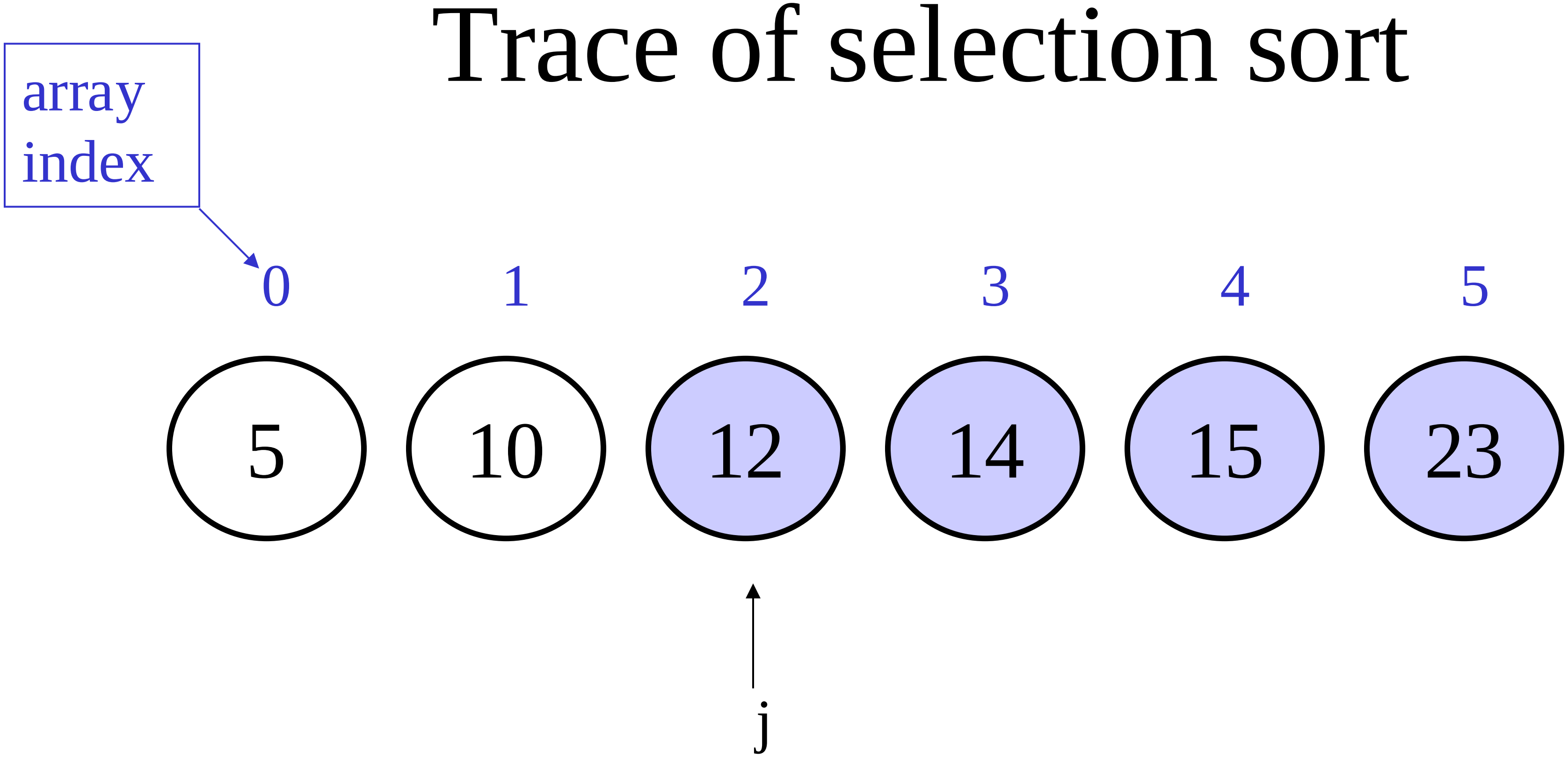


i = 2, fourth iteration of the outer loop

j = 2, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 2, pos_greatest = 2 (changed again!)

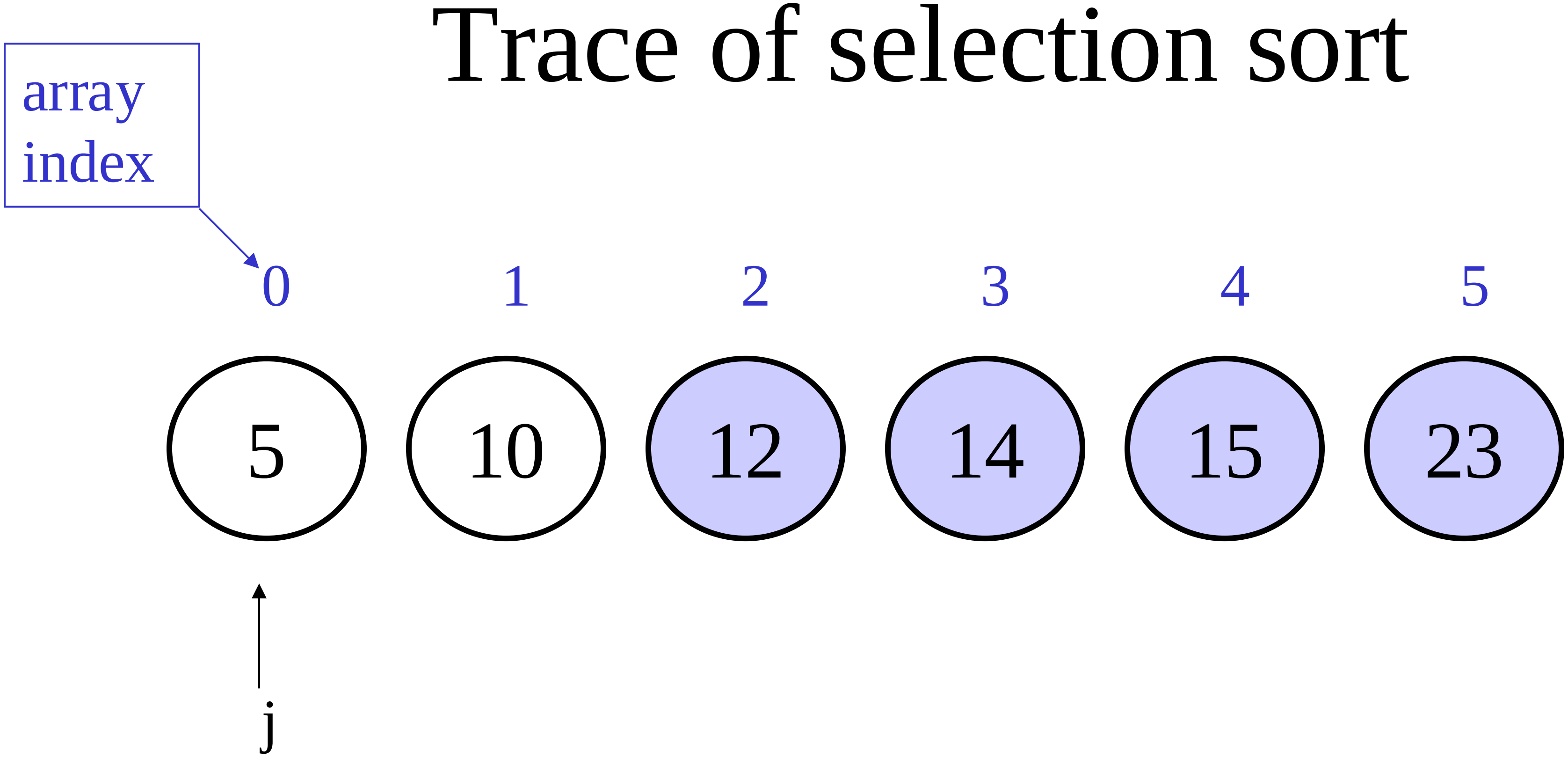
Trace of selection sort



i = 2, fourth iteration of the outer loop

swap elements at pos_greatest and 2 (element 12 with itself…)

Trace of selection sort



array index

0	1	2	3	4	5
5	10	12	14	15	23

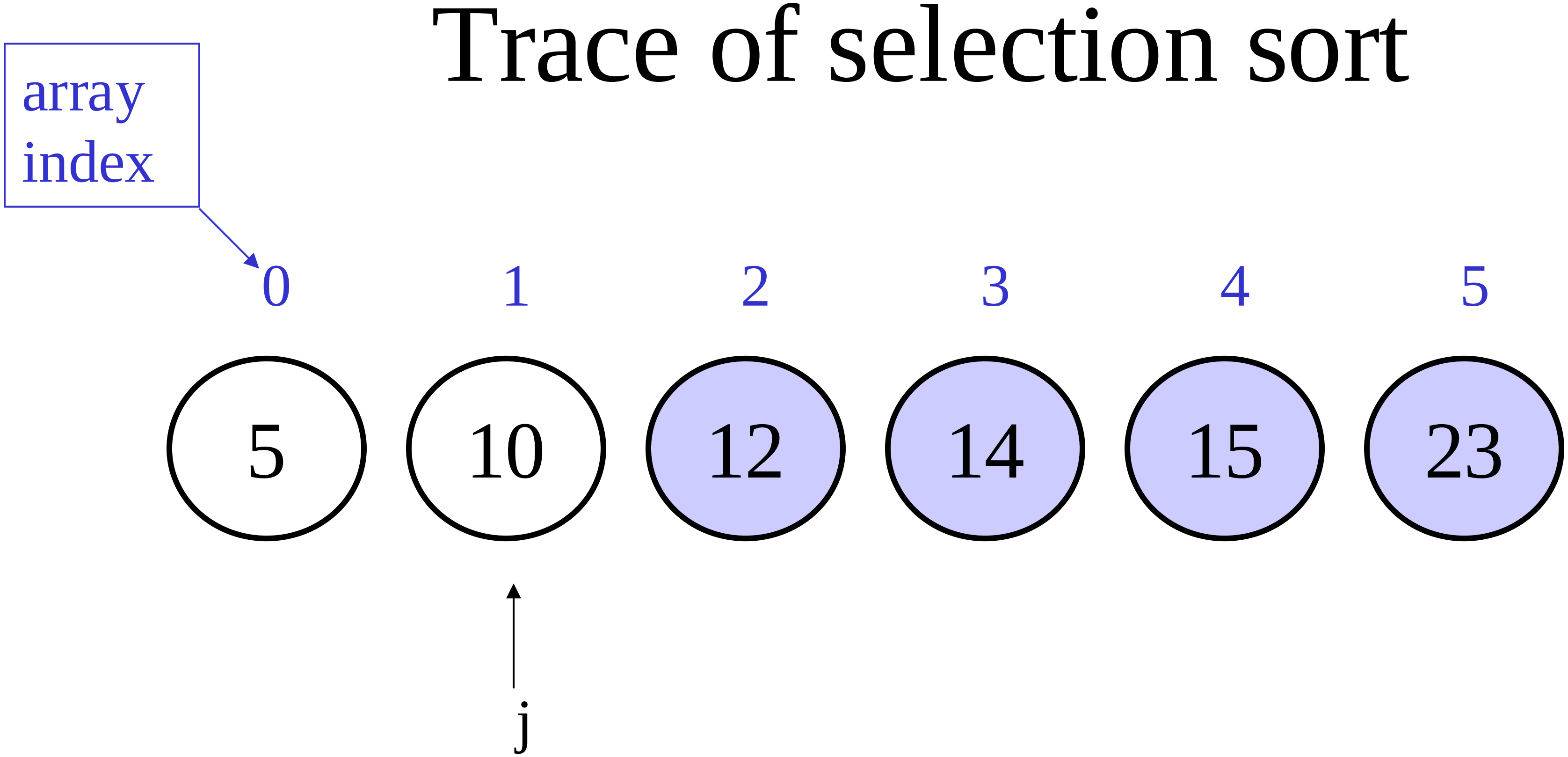
j

i = 1, fifth iteration of the outer loop

j = 0, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 0, pos_greatest = 0

Trace of selection sort

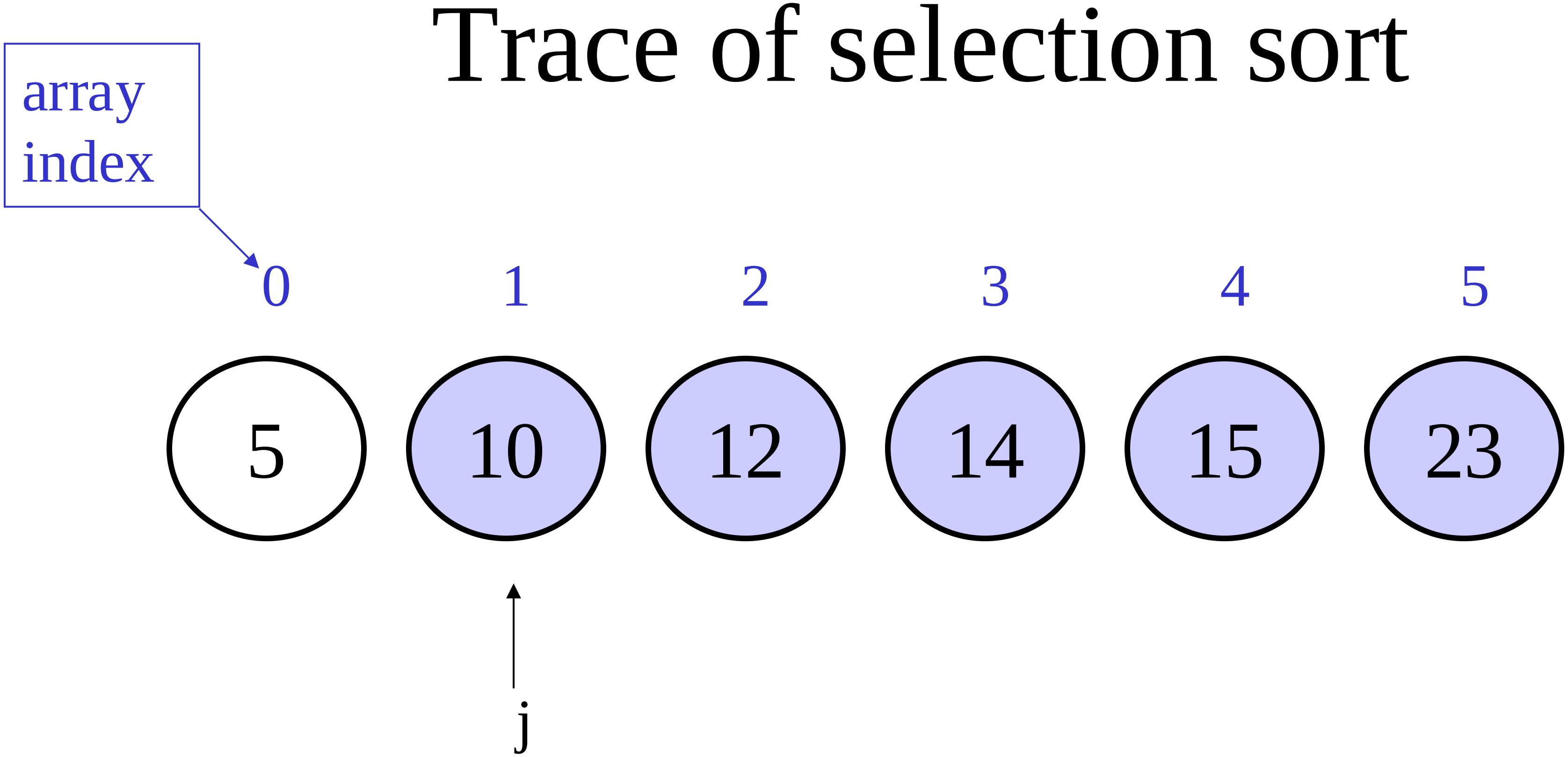


i = 1, fifth iteration of the outer loop

j = 1, arr[0]…arr[j-1] are all less than or equal to arr[pos_greatest]

j = 1, pos_greatest = 1 (changed)

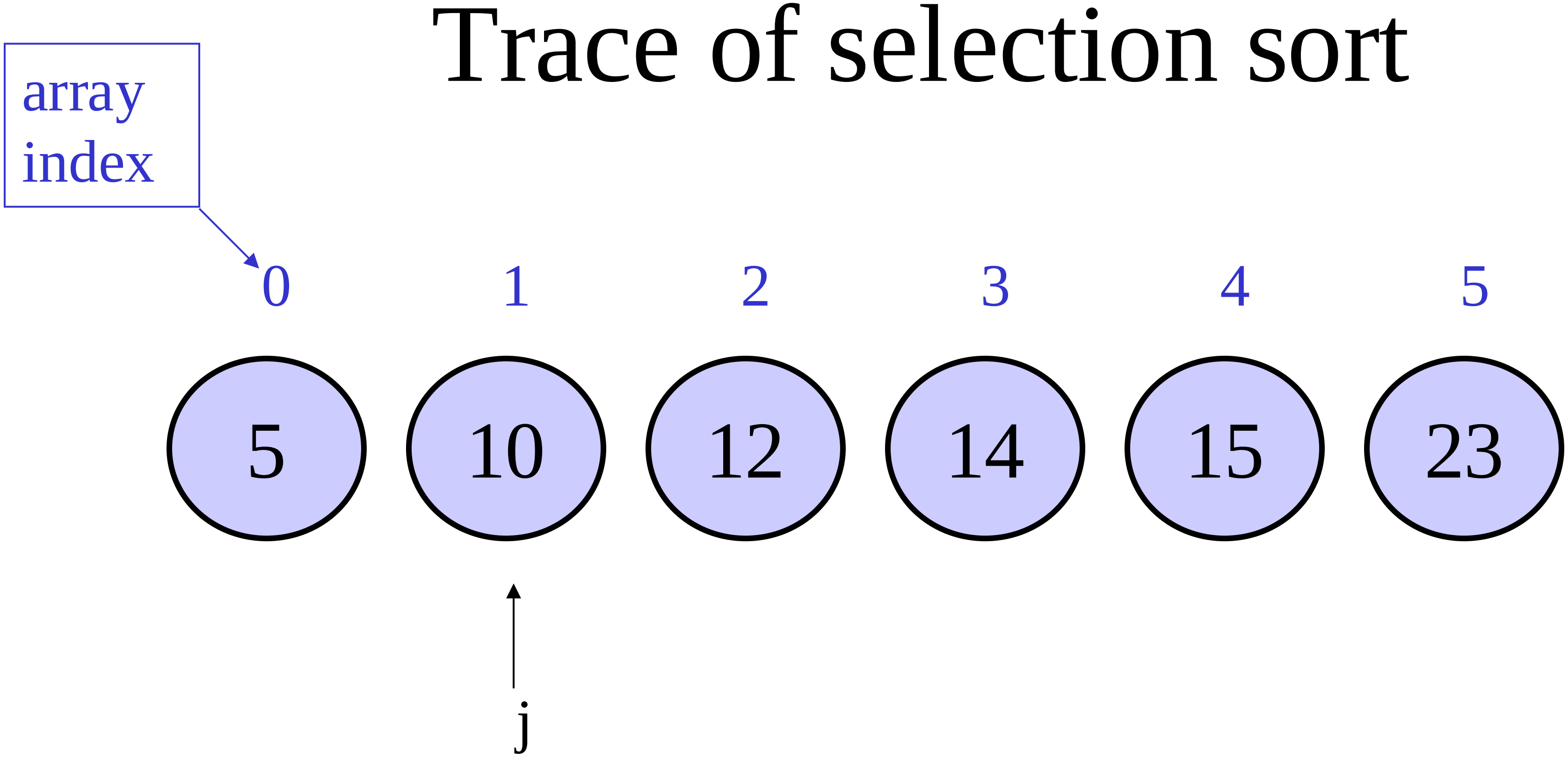
Trace of selection sort



i = 1, fifth iteration of the outer loop

swap element at pos_greatest with element at position 1
(10 with itself)

Trace of selection sort



i = 1, fifth iteration of the outer loop

done

Complexity of selection sort

- Same number of iterations
- Same number of comparisons in the worst case
- Fewer swaps (one for each outer loop)
- Also $O(n^2)$

Insertion sort

A visualisation video:

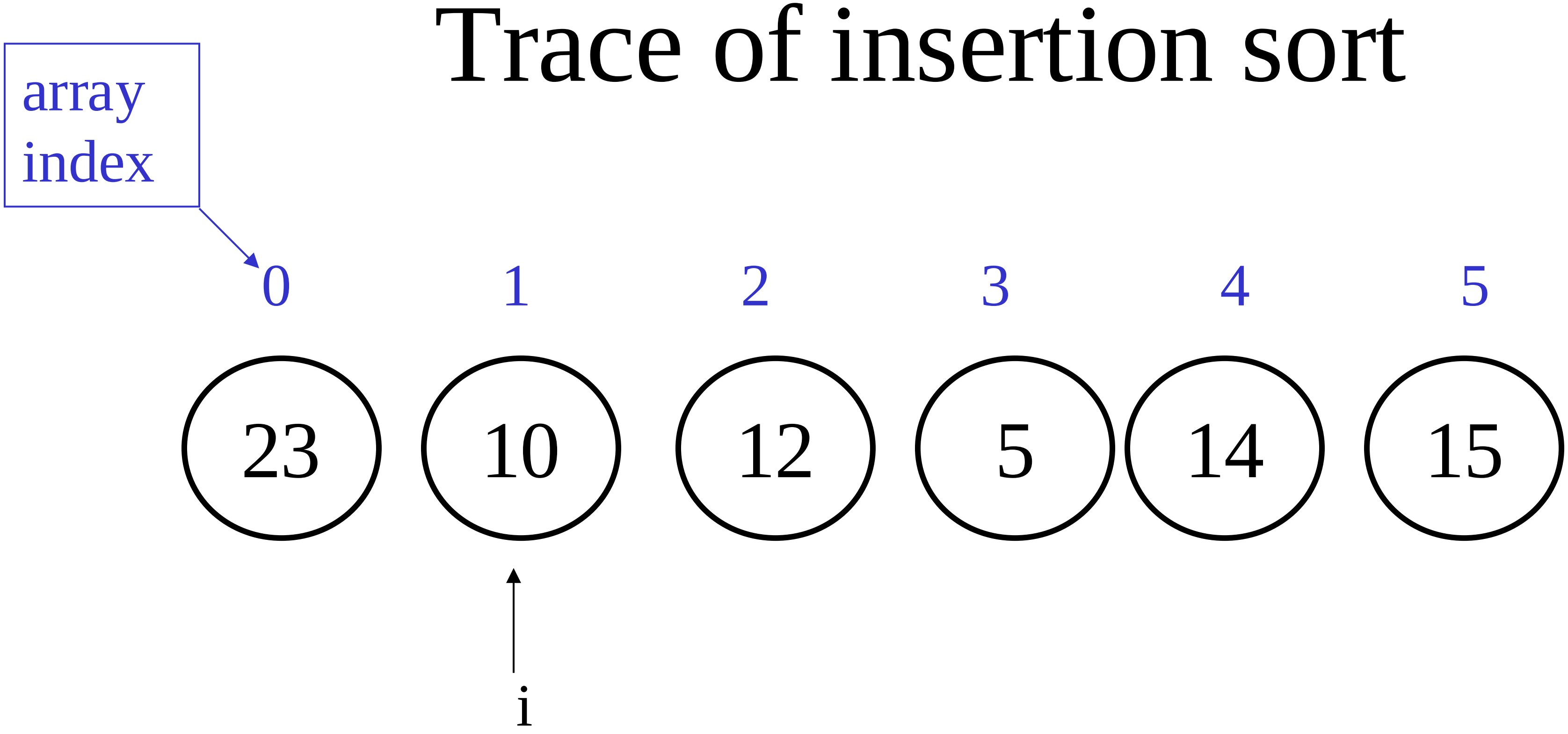
https://www.youtube.com/watch?v=gSdLGSM--dw

Insertion sort

```
void insertionSort(int[] arr){
  int i,j,temp;
  for(i=1; i < arr.length; i++){
    temp = arr[i];
    j = i; // range 0 to i-1 is sorted
    while(j >= 1 && arr[j-1] > temp){
      arr[j] = arr[j-1];
      j--;
    }
    arr[j] = temp;
  } // end outer for loop
}  // end insertion sort
```

Find a place to insert temp in the sorted range; as you are looking, shift elements in the sorted range to the right

Trace of insertion sort



array index

0 1 2 3 4 5

23 10 12 5 14 15

i

i = 1, arr[0] … arr[i-1] are sorted.

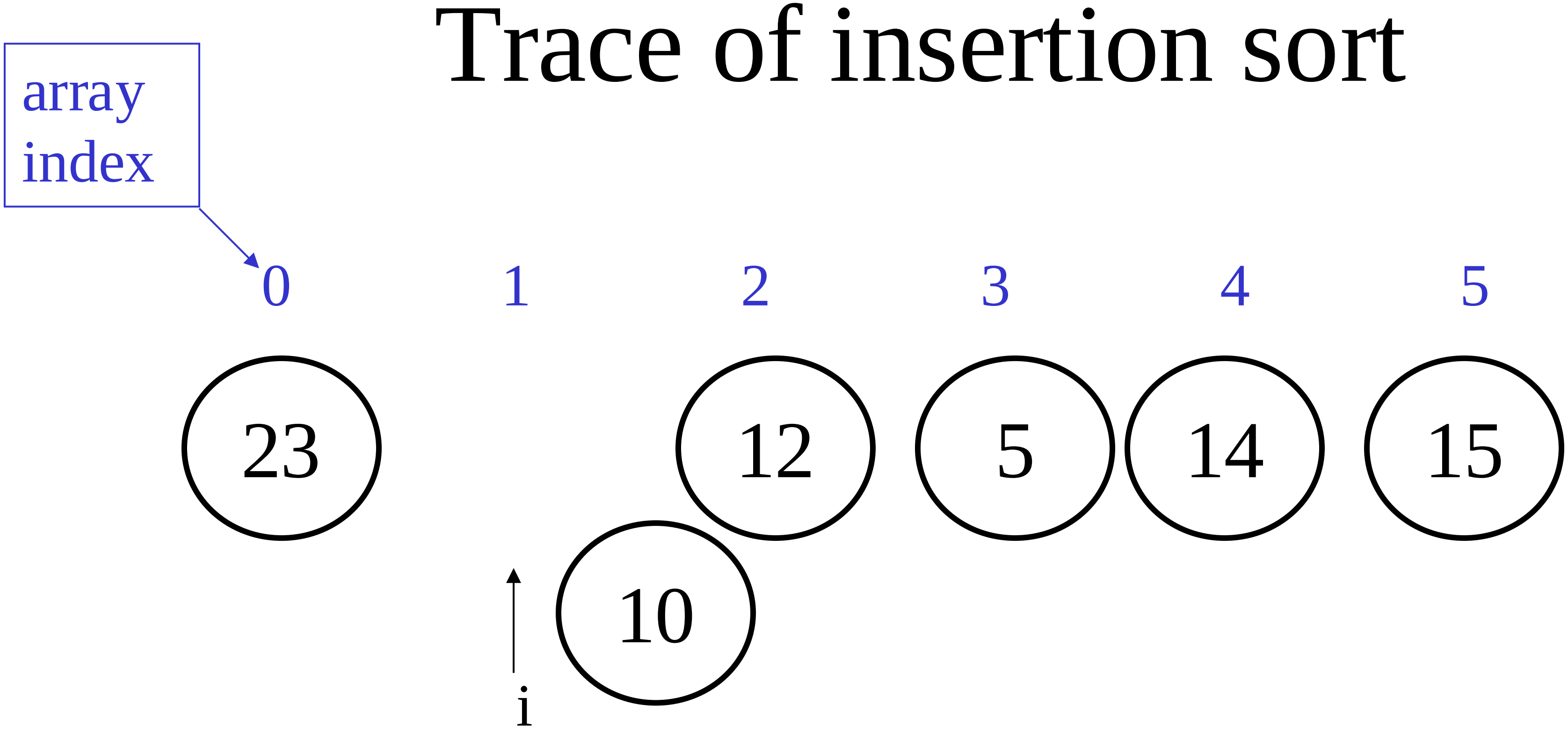
i = 1, first iteration of the outer loop

temp = 10; j = 1;

j = 1, arr[j]…arr[i] are all greater than or equal to temp

arr[j-1] > 10

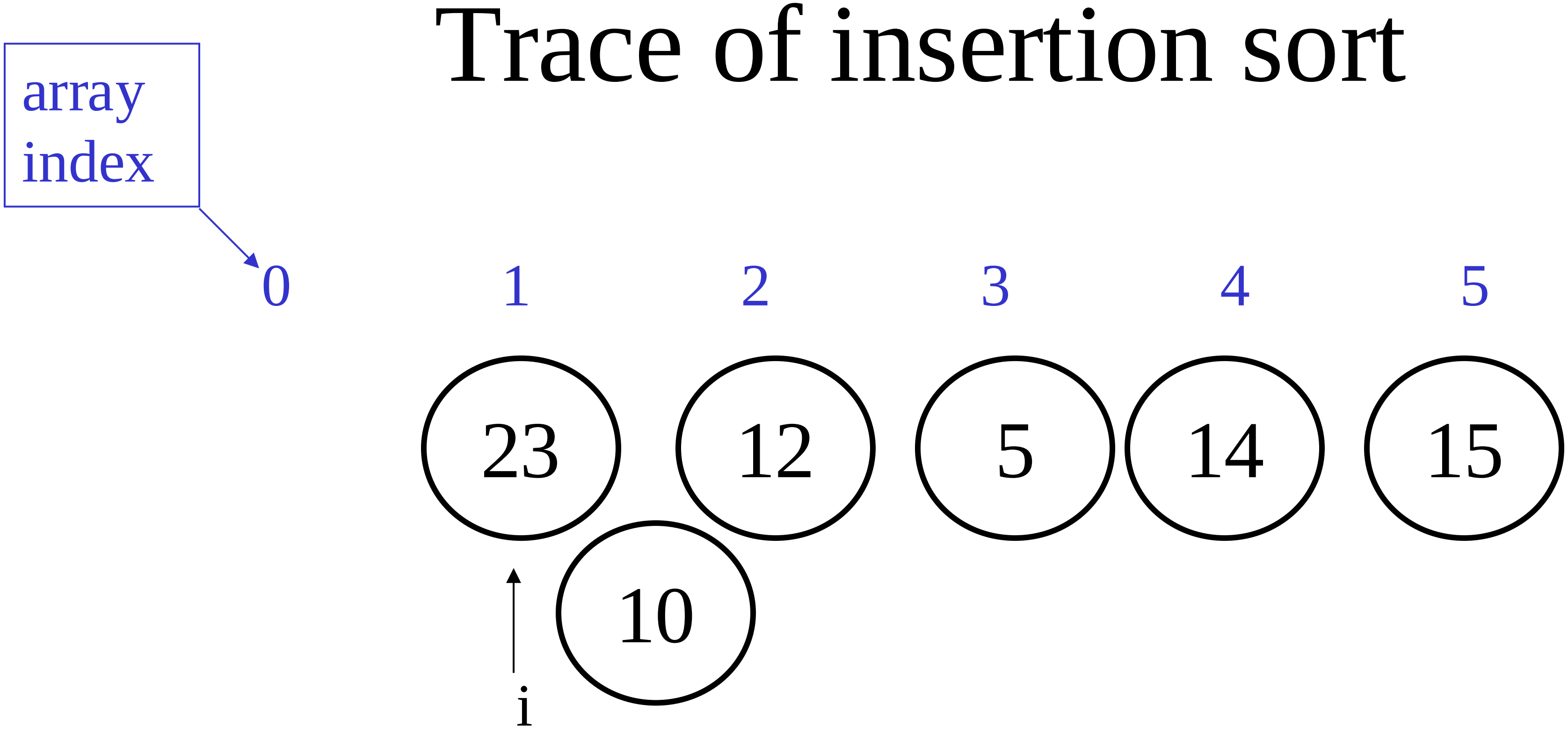
Trace of insertion sort



i = 1, first iteration of the outer loop

temp = 10; j = 1; arr[j-1] > 10

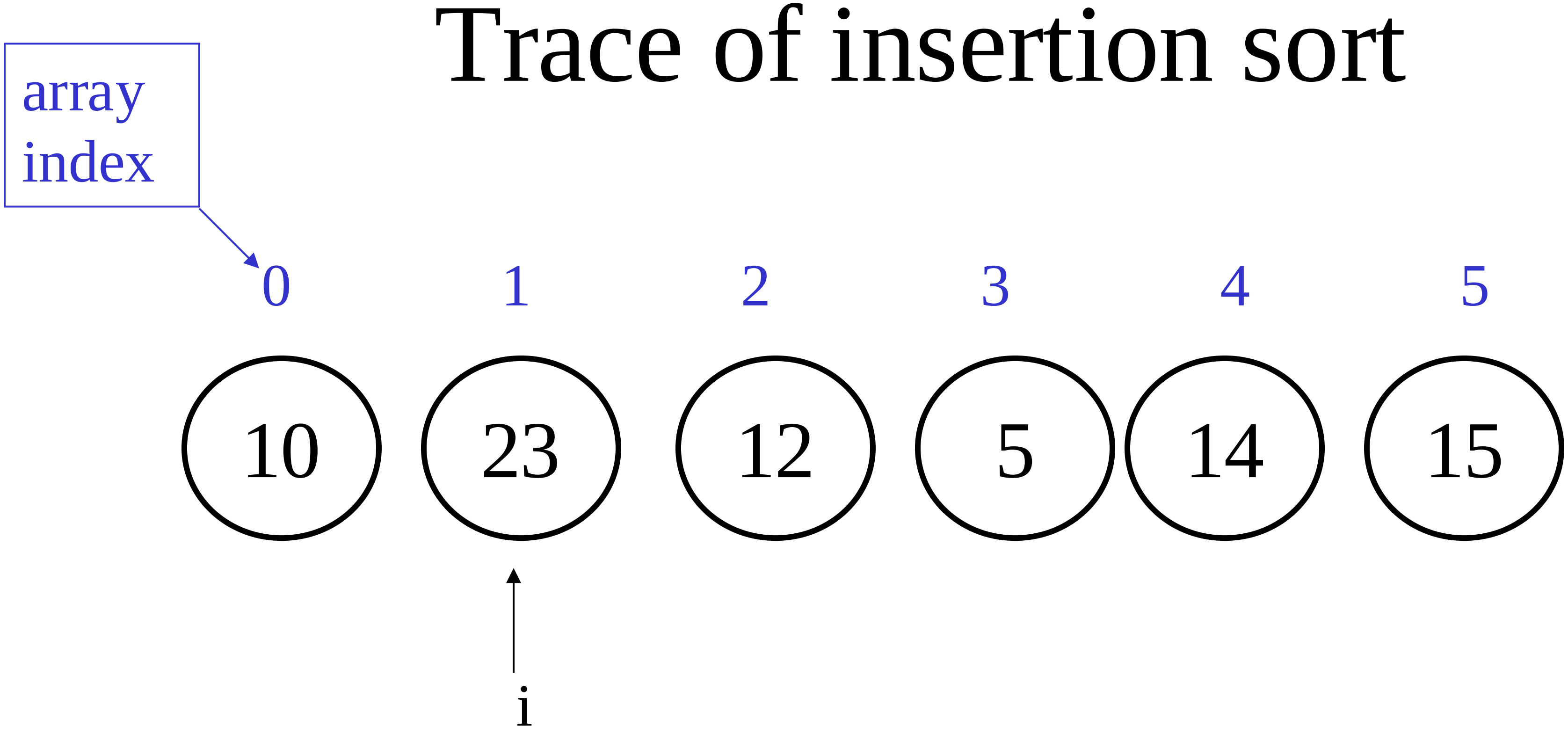
Trace of insertion sort



i = 1, first iteration of the outer loop

arr[j] = arr[j-1]

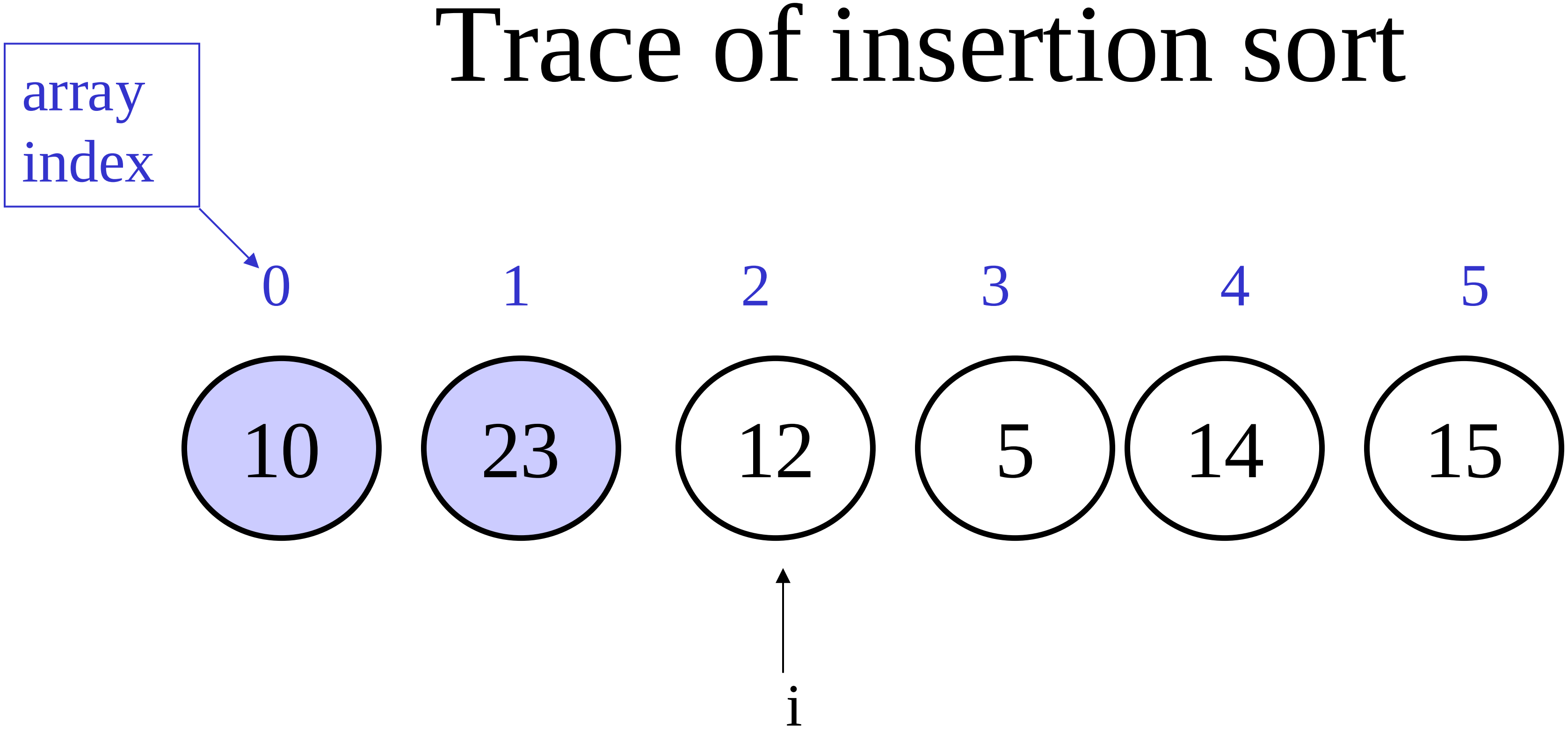
Trace of insertion sort



i = 1, first iteration of the outer loop

arr[j] = temp

Trace of insertion sort



i = 2, arr[0] … arr[i-1] are sorted.

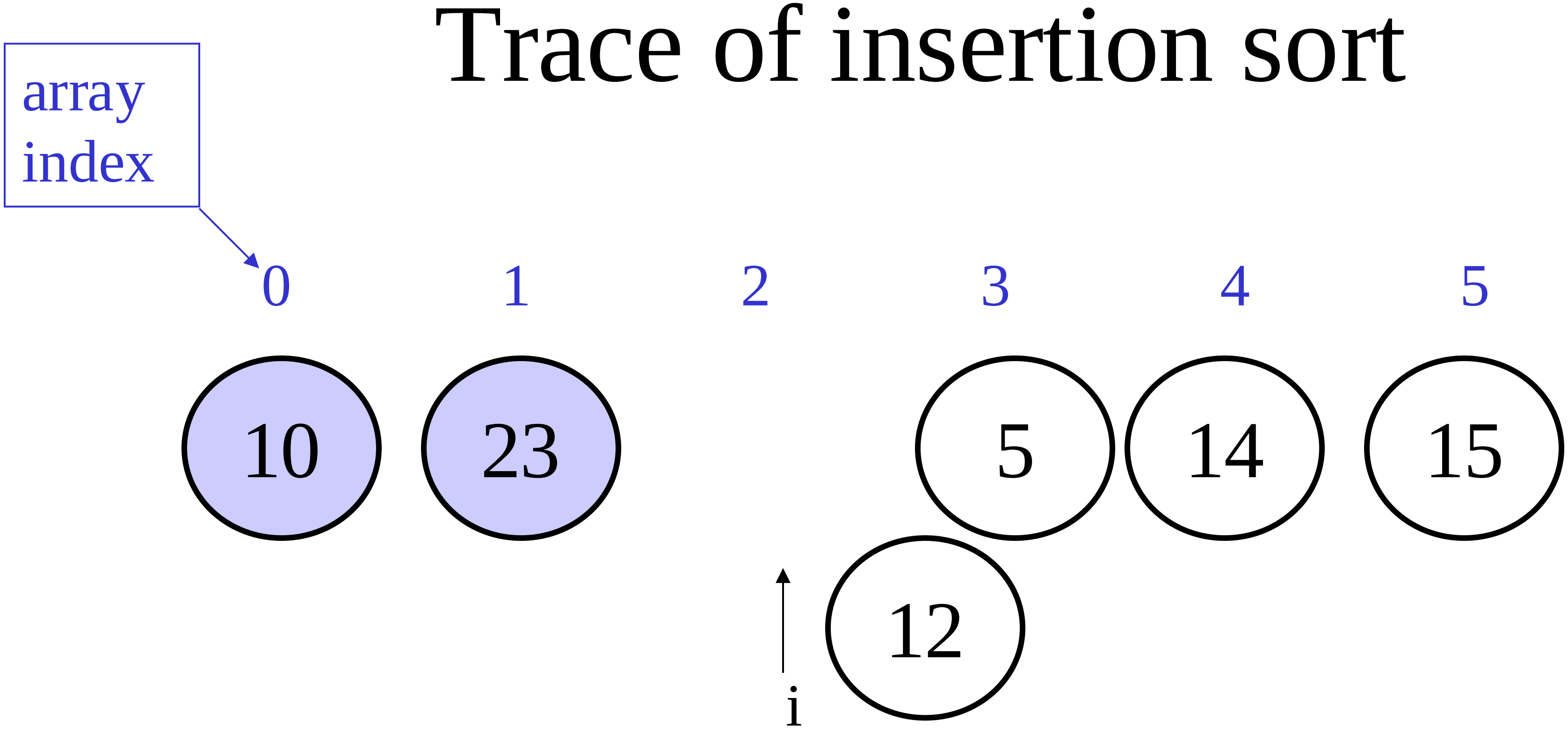
i = 2, second iteration of the outer loop

temp = 12;

j = 2, arr[j]…arr[i] are all greater than or equal to temp

arr[j-1] > temp

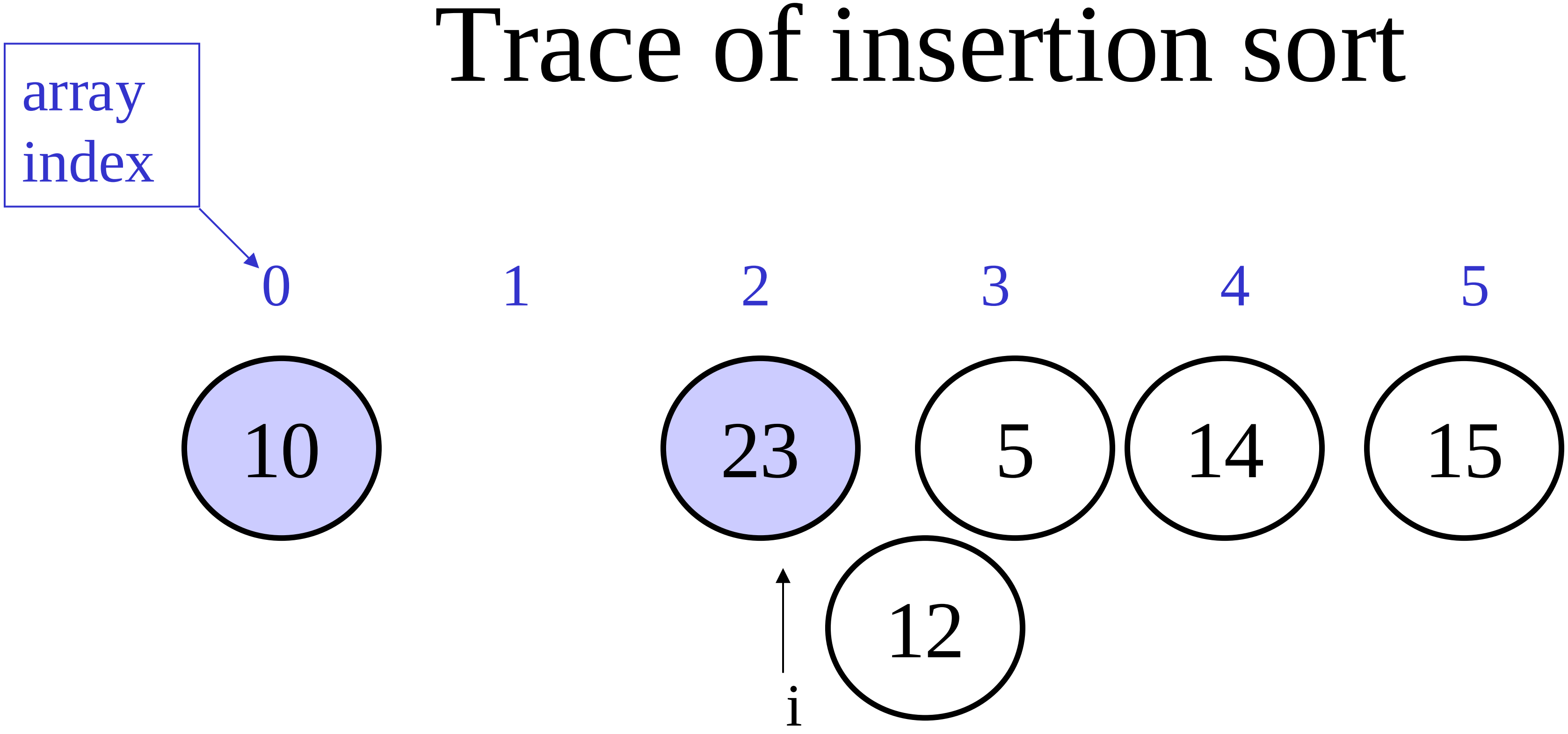
Trace of insertion sort



i = 2, second iteration of the outer loop

temp = 12; arr[j-1] > temp

Trace of insertion sort

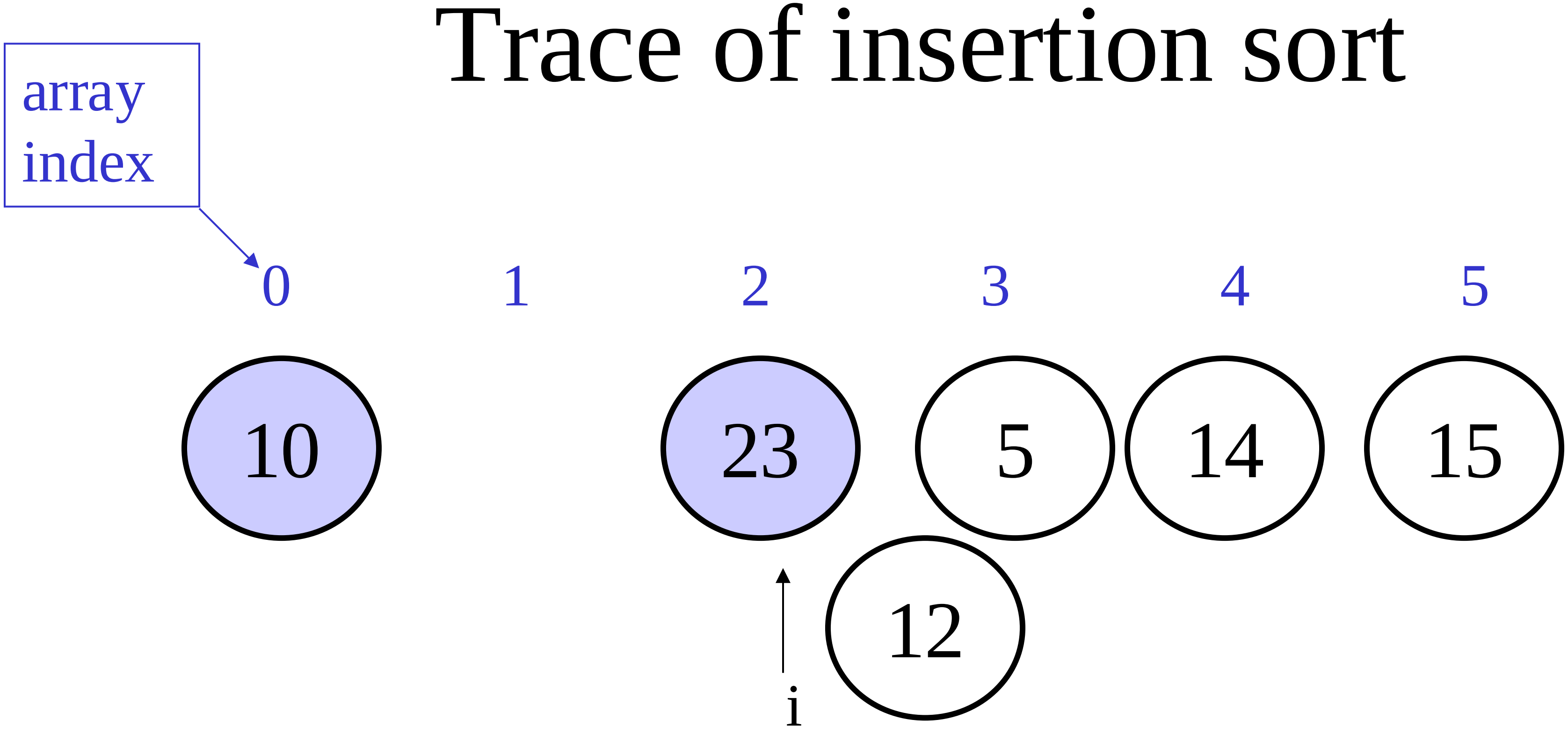


i = 2, second iteration of the outer loop

arr[j-1] = arr[j]

j = j-1

Trace of insertion sort

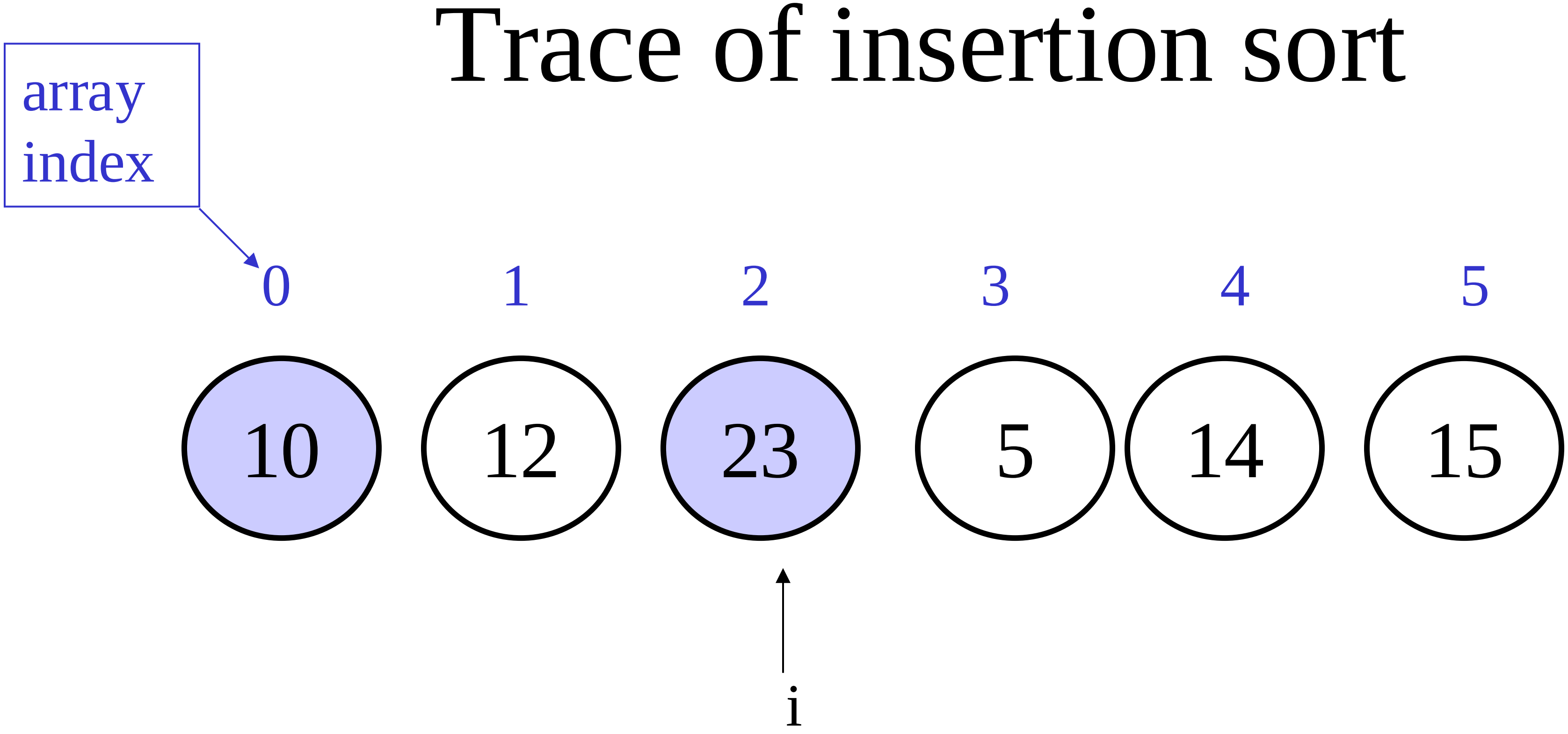


i = 2, second iteration of the outer loop

j = 1, arr[j]…arr[i] are all greater than or equal to temp

arr[j-1] < temp

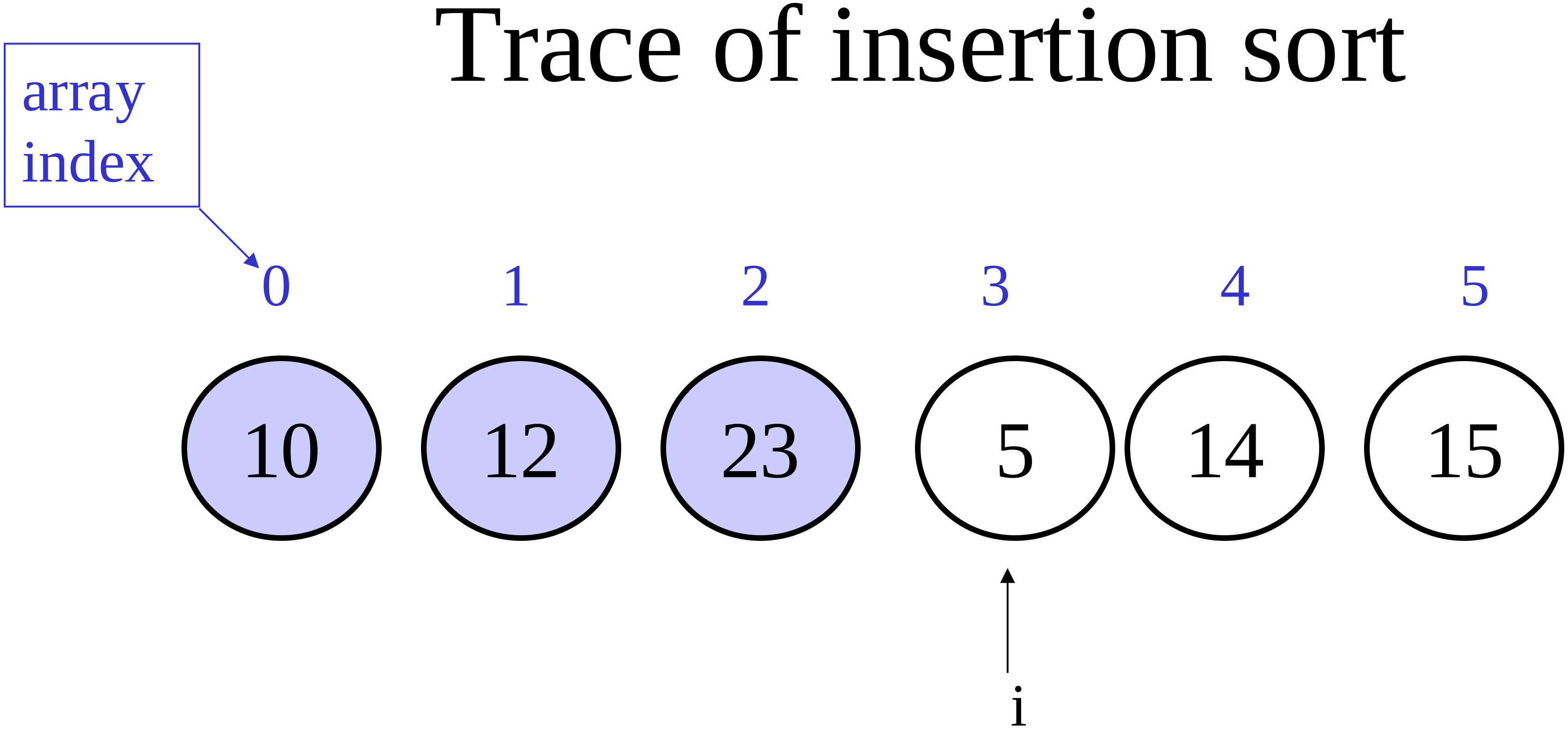
Trace of insertion sort



i = 2, second iteration of the outer loop

arr[j] = temp

Trace of insertion sort



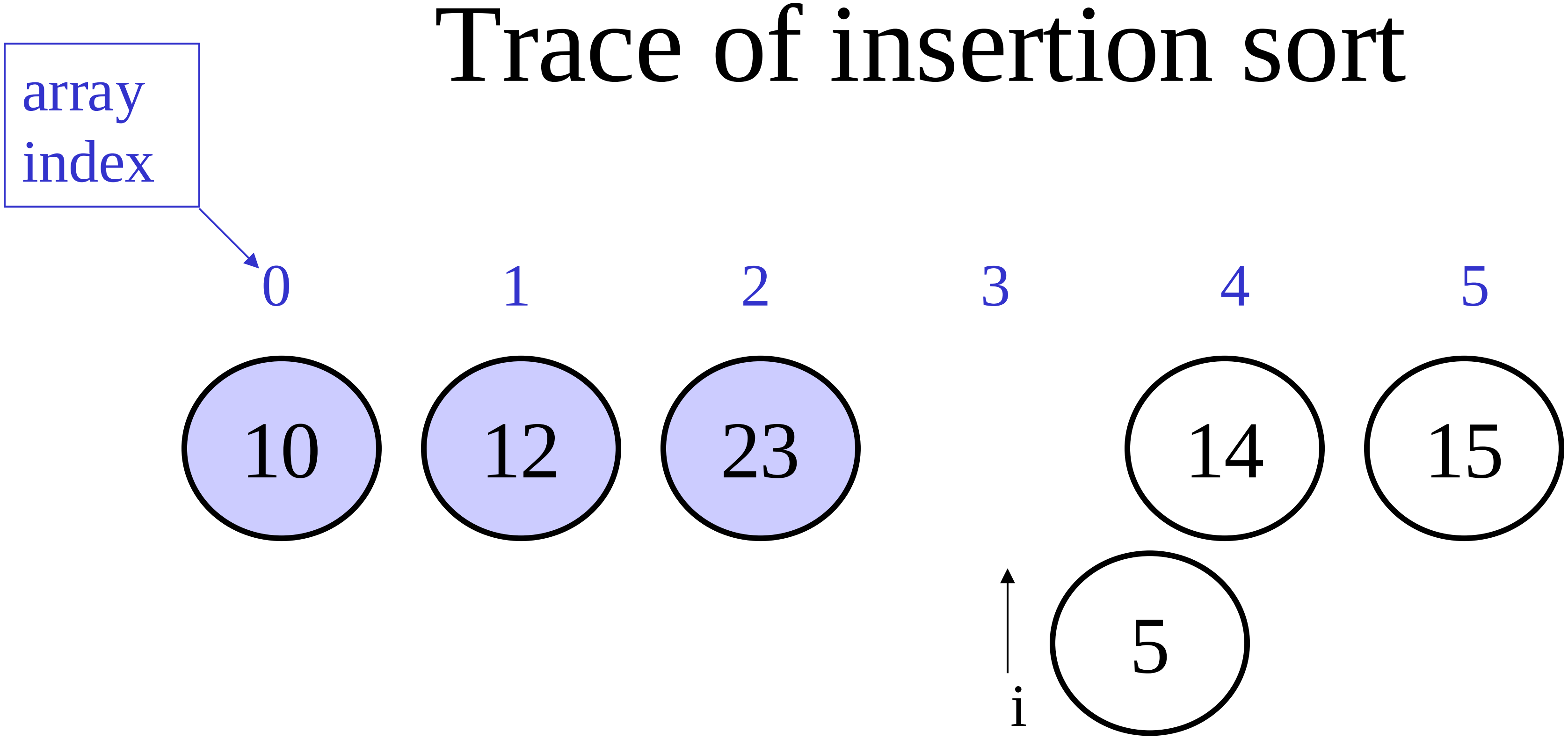
i = 3, arr[0] … arr[i-1] are sorted.

i = 3, third iteration of the outer loop

temp = 5

j = 3, arr[j]…arr[i] are all greater than or equal to temp

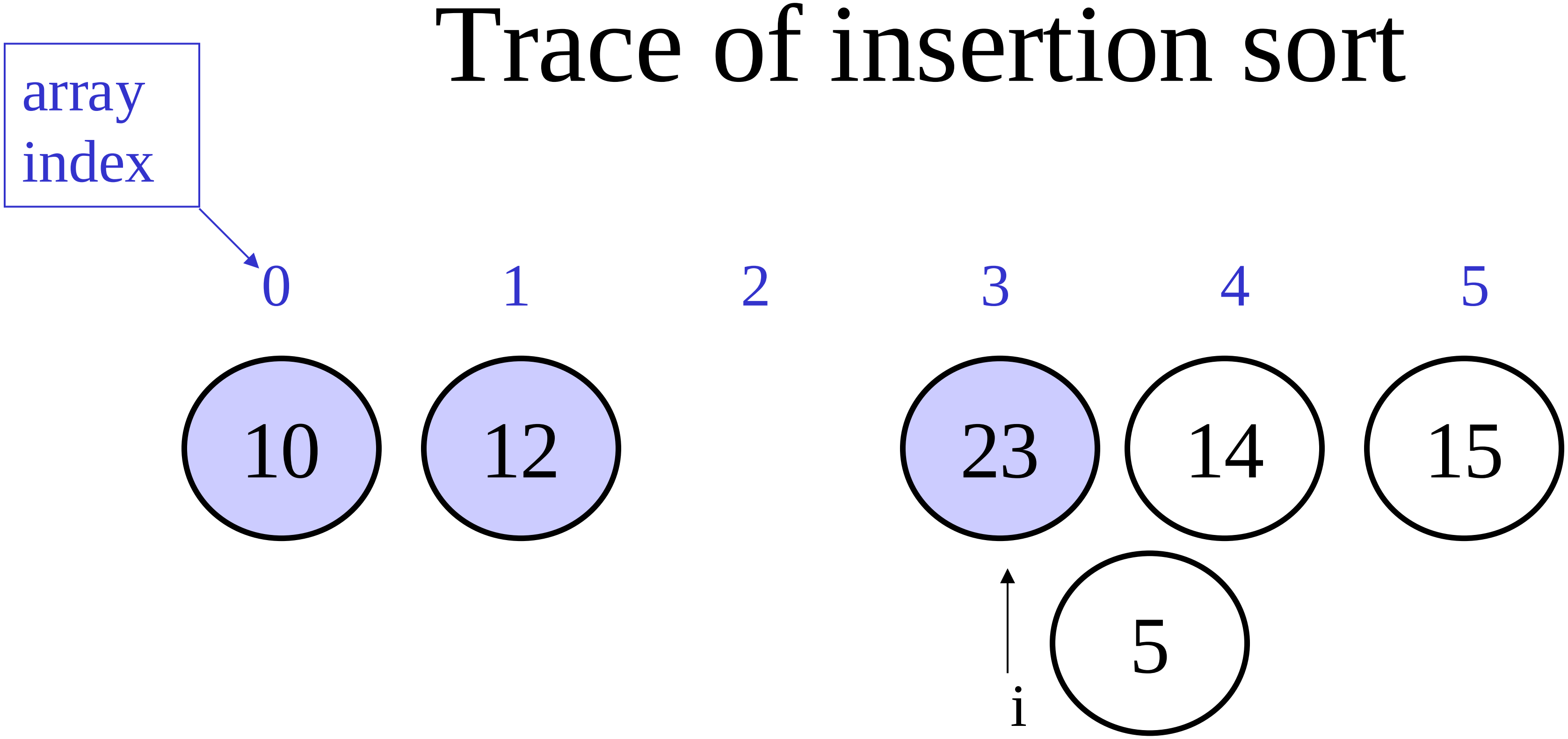
Trace of insertion sort



i = 3, third iteration of the outer loop

arr[j-1] > temp

Trace of insertion sort

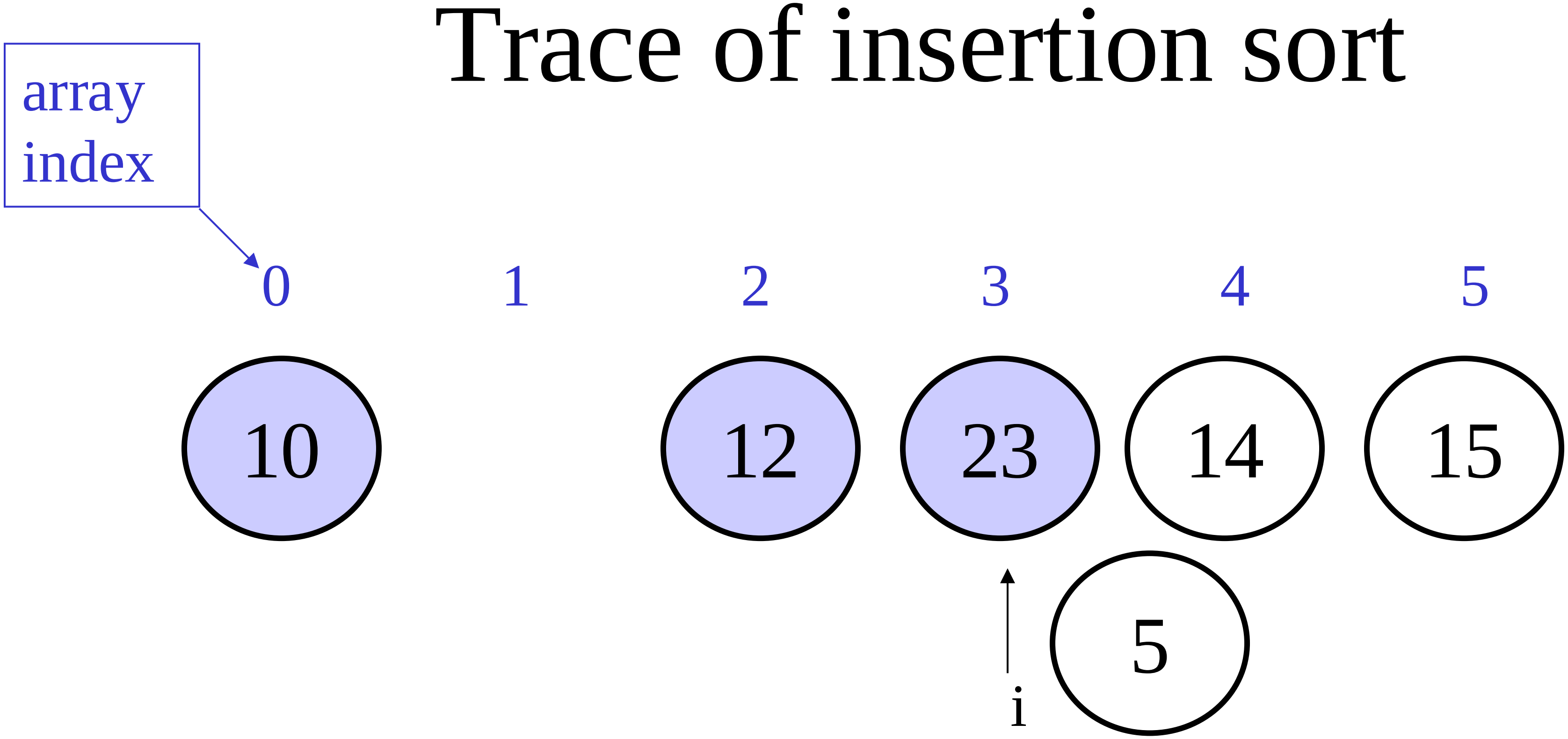


i = 3, third iteration of the outer loop

j = 2, arr[j]…arr[i] are all greater than or equal to temp

arr[j-1] > temp

Trace of insertion sort

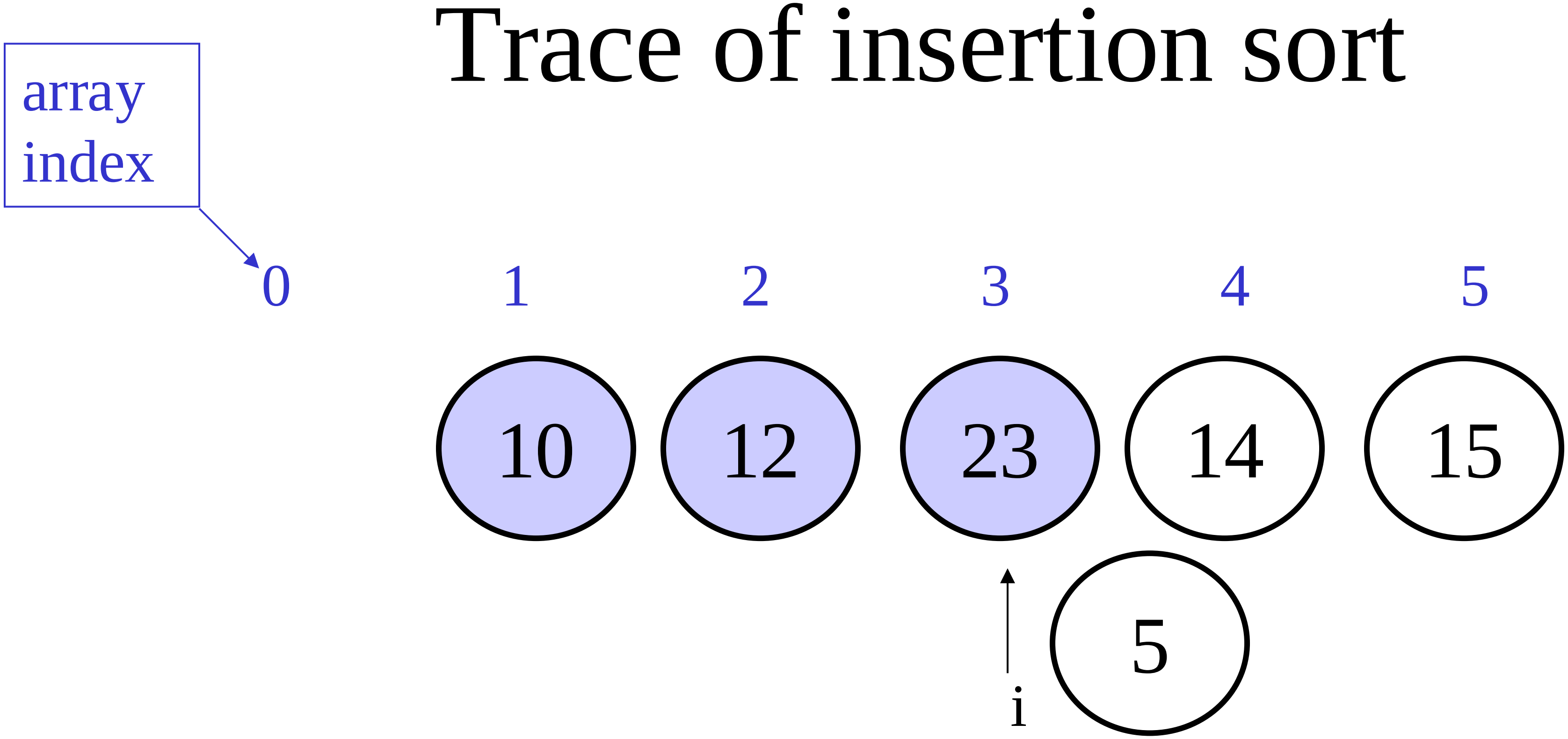


i = 3, third iteration of the outer loop

j = 1, arr[j]…arr[i] are all greater than or equal to temp

arr[j-1] > temp

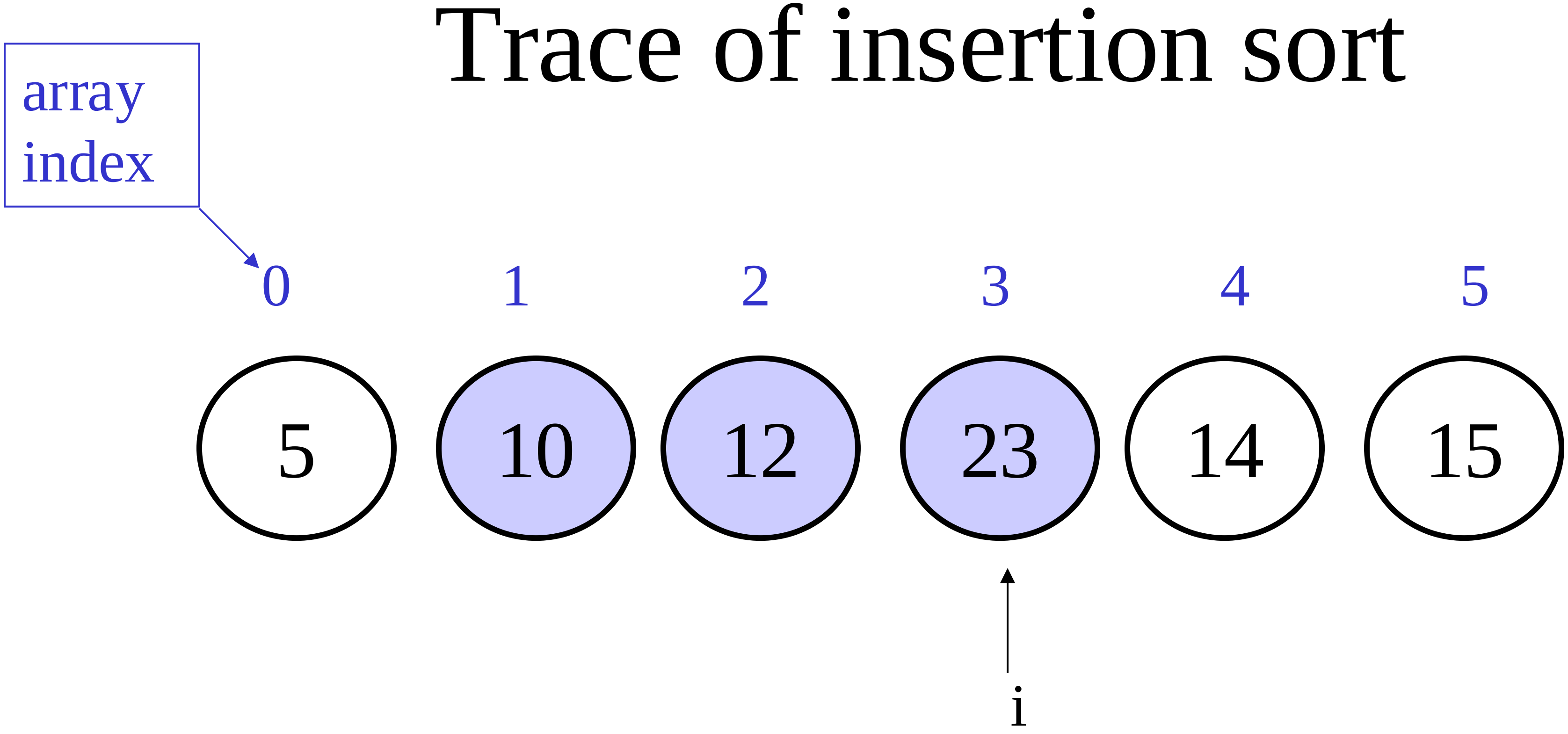
Trace of insertion sort



i = 3, third iteration of the outer loop

j=0

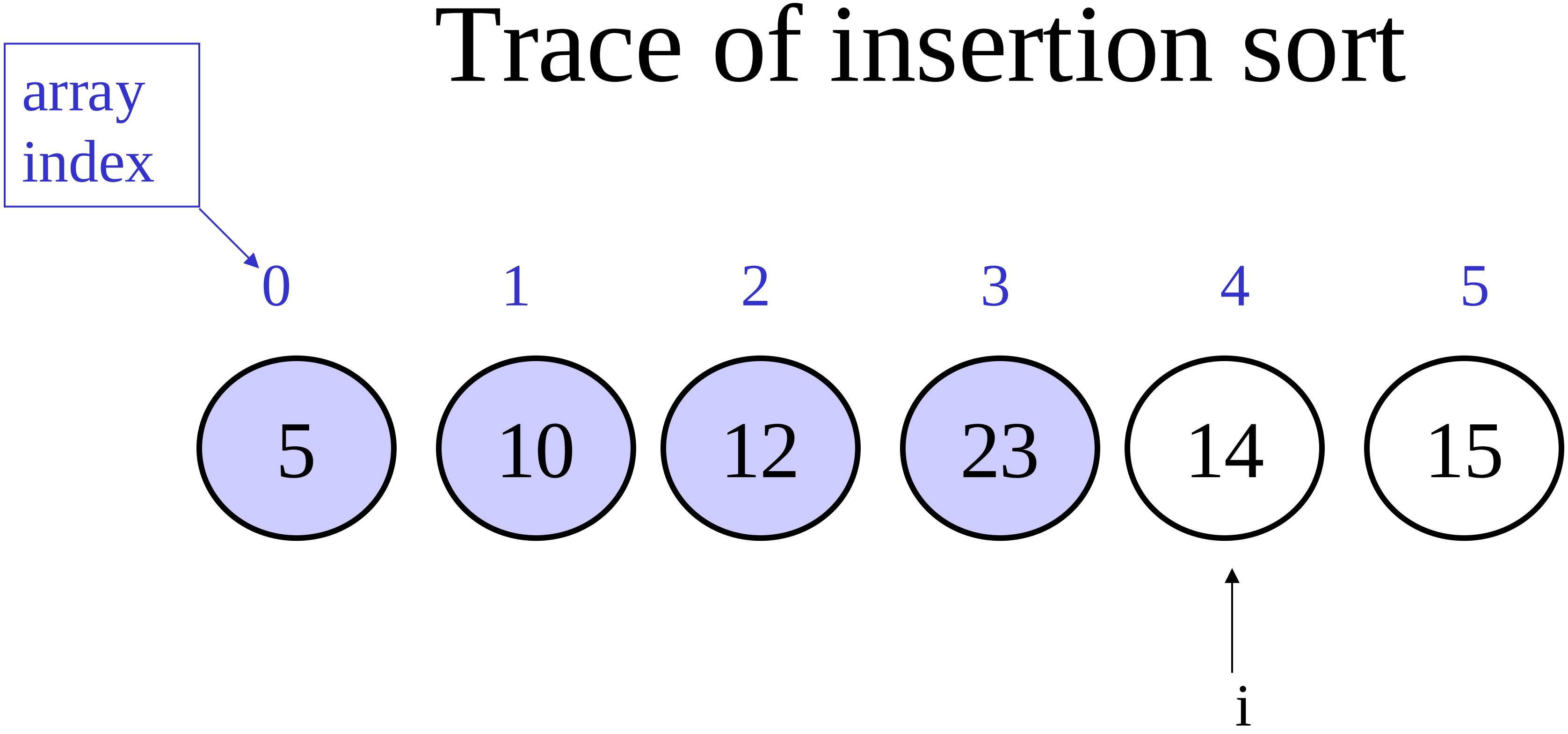
Trace of insertion sort



i = 3, third iteration of the outer loop

arr[j] = temp

Trace of insertion sort



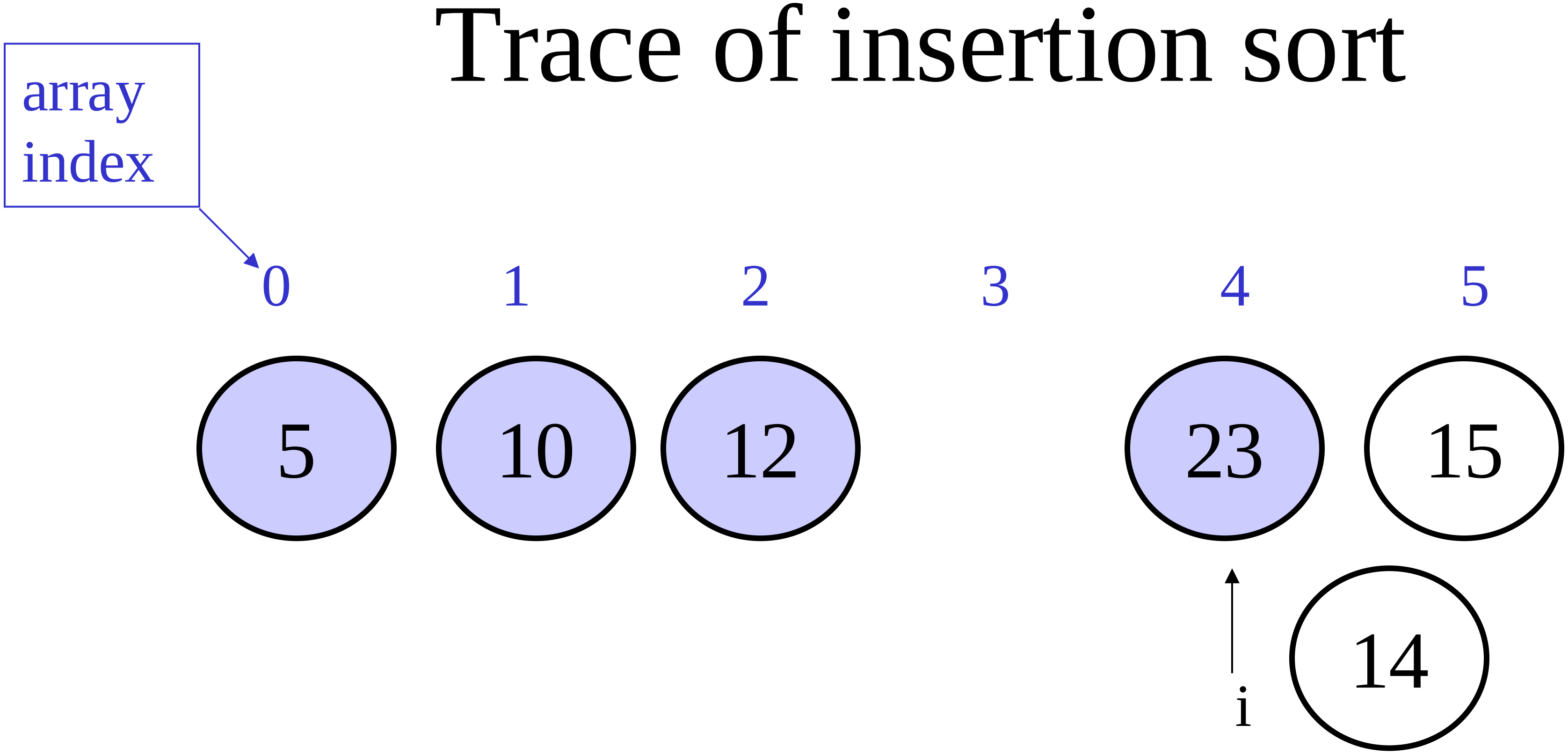
i = 4, arr[0] … arr[i-1] are sorted.

i = 4, fourth iteration of the outer loop

temp = 14

j = 4, arr[j]…arr[i] are all greater than or equal to temp

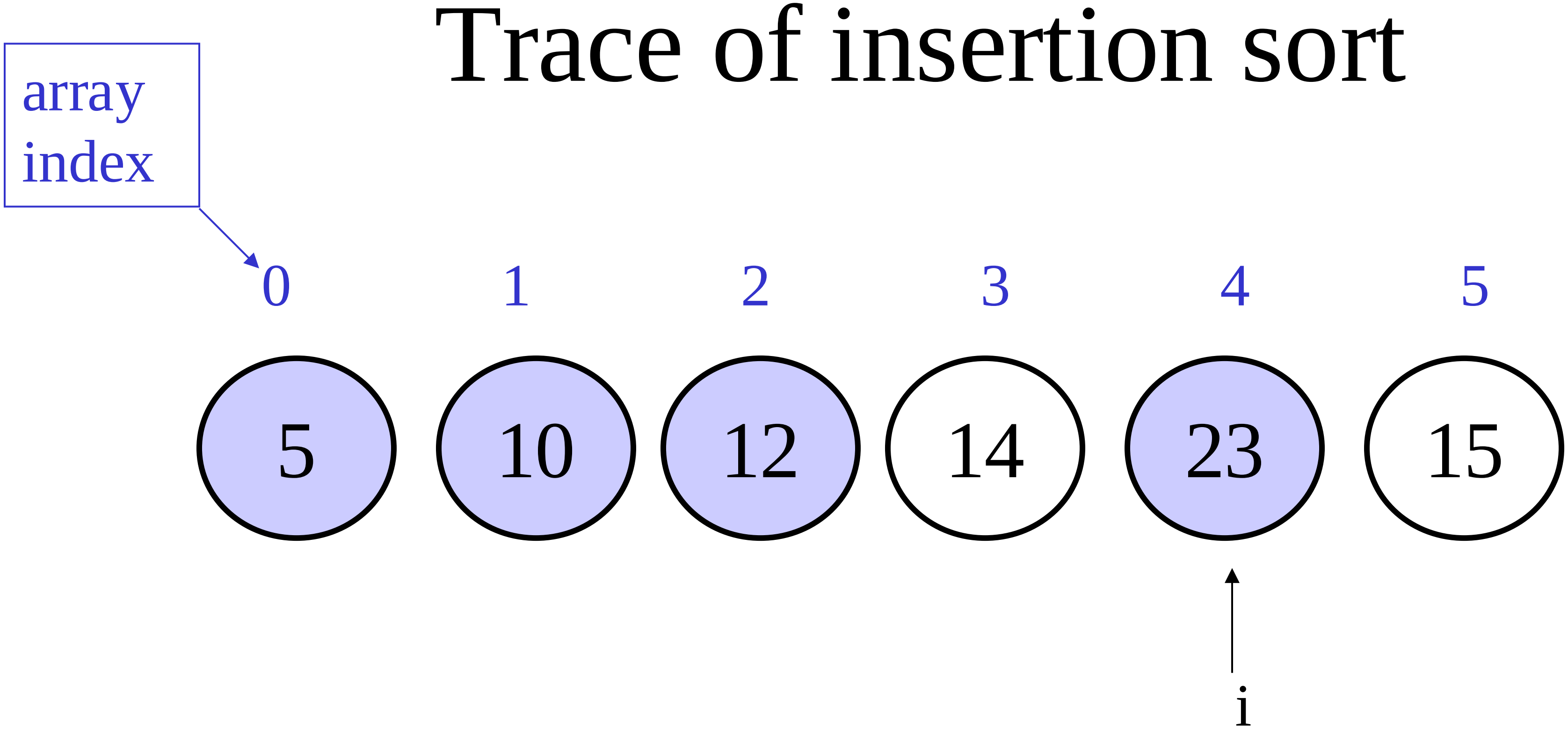
Trace of insertion sort



i = 4, fourth iteration of the outer loop

arr[j-1] > temp

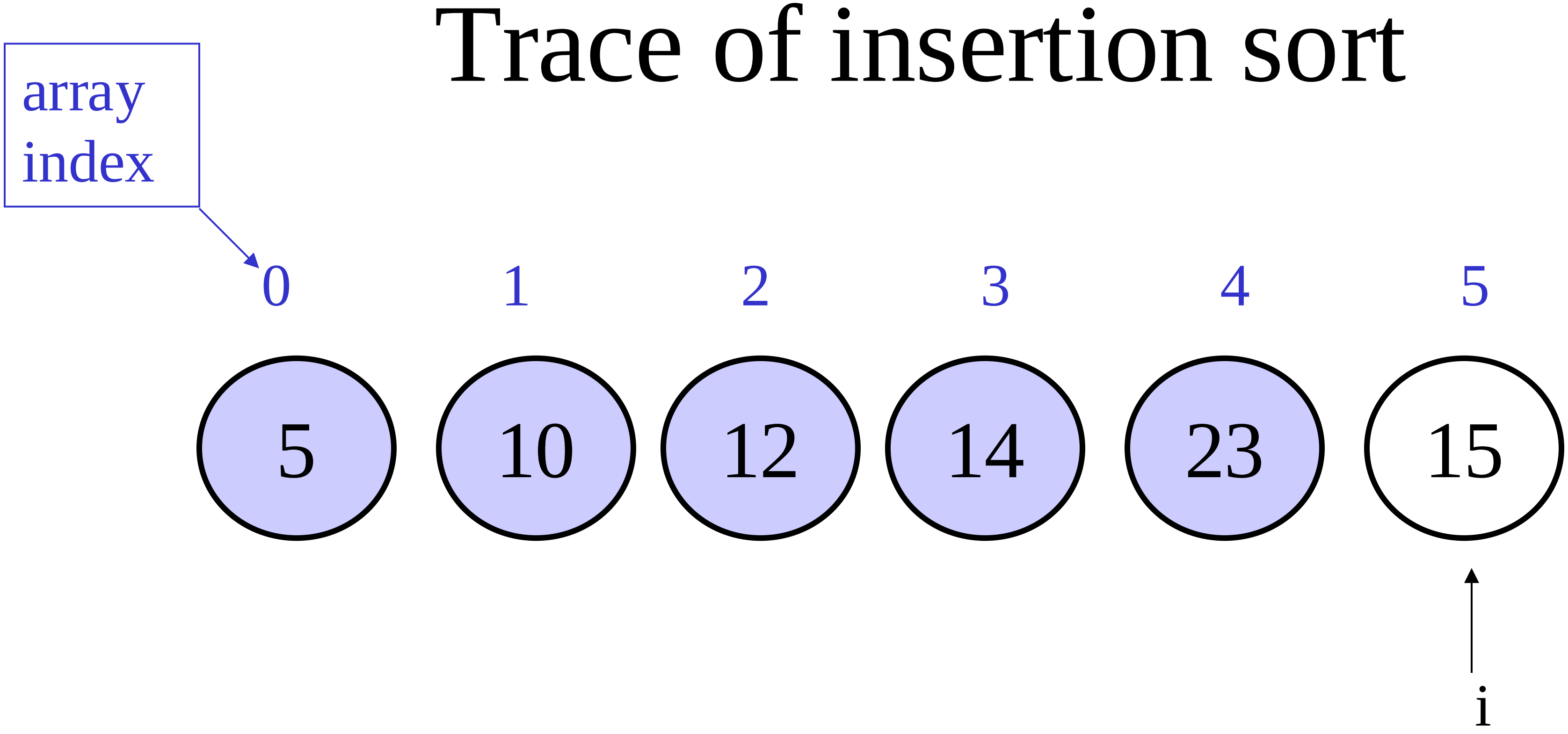
Trace of insertion sort



i = 4, fourth iteration of the outer loop

arr[j] = temp

Trace of insertion sort

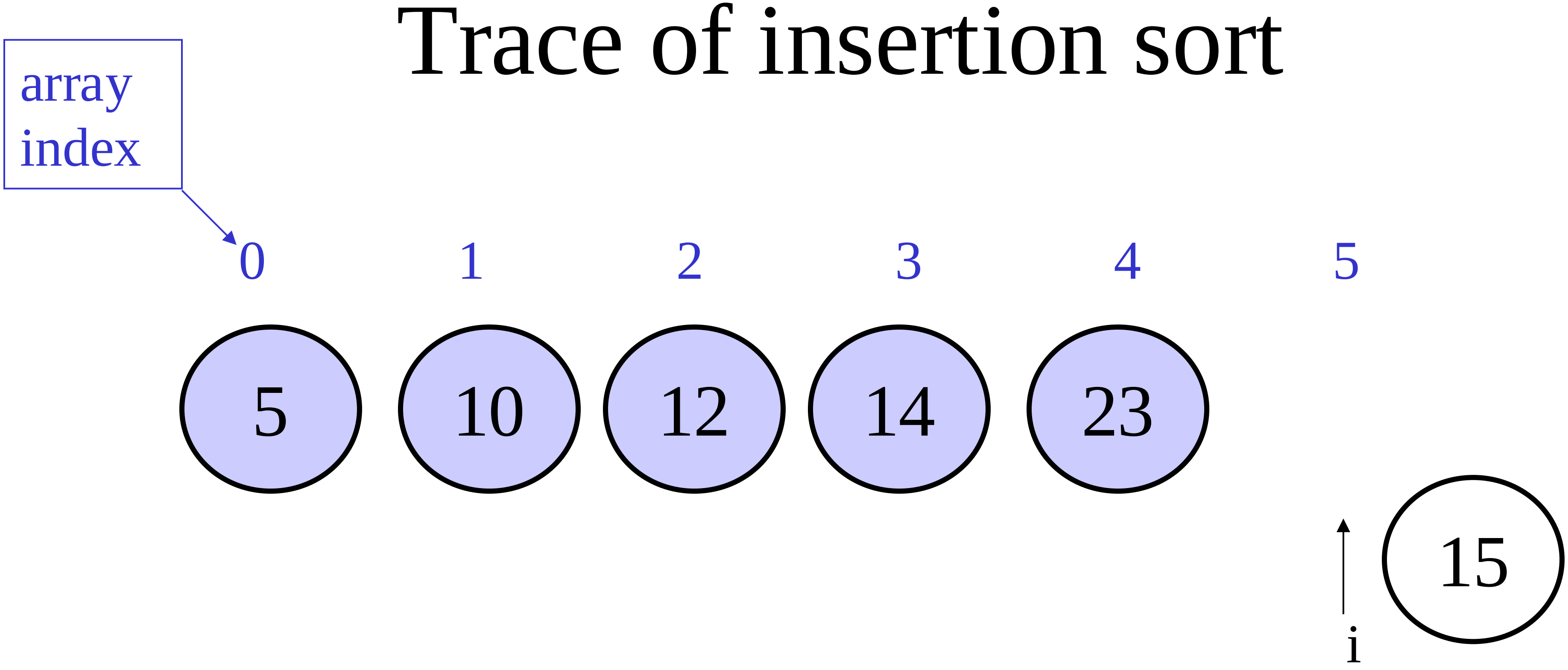


0	1	2	3	4	5
5	10	12	14	23	15

i = 5, fifth iteration of the outer loop

temp = 15

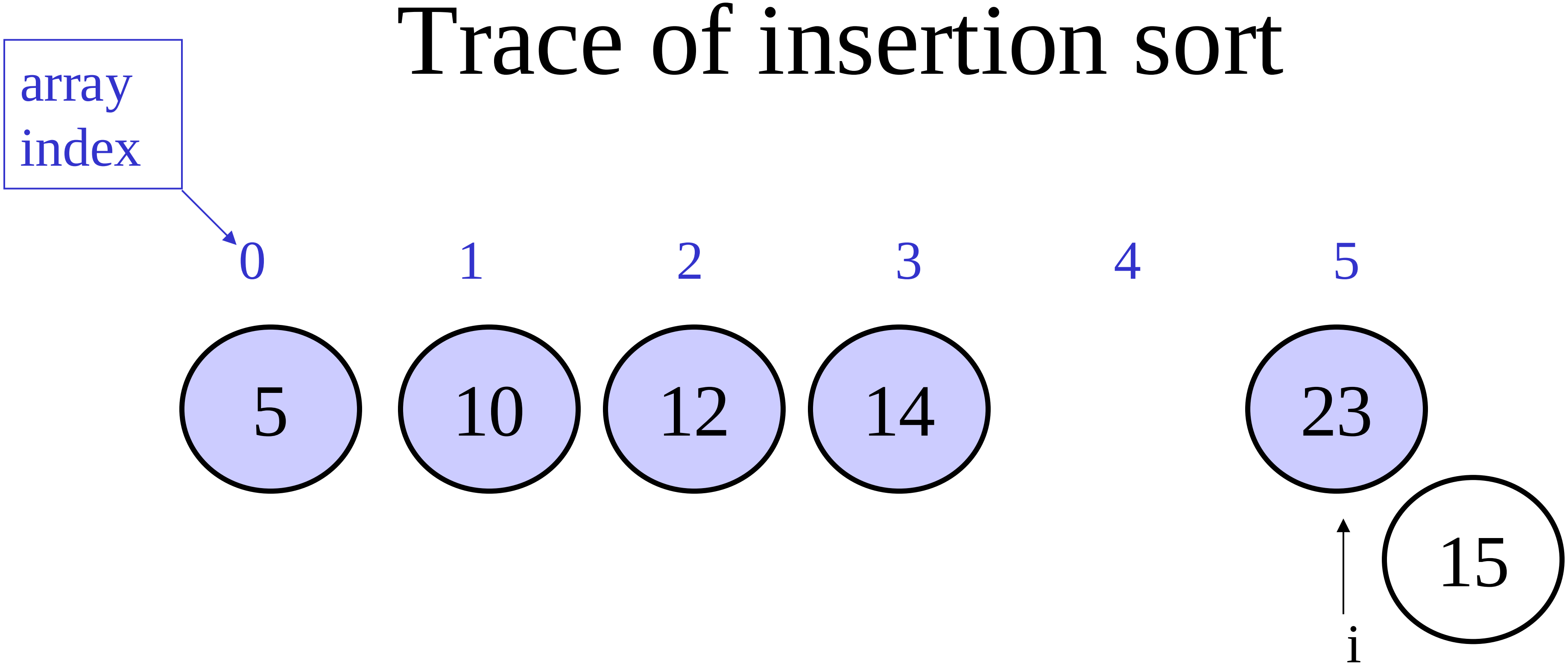
Trace of insertion sort



i = 5, fifth iteration of the outer loop

arr[j-1] > temp

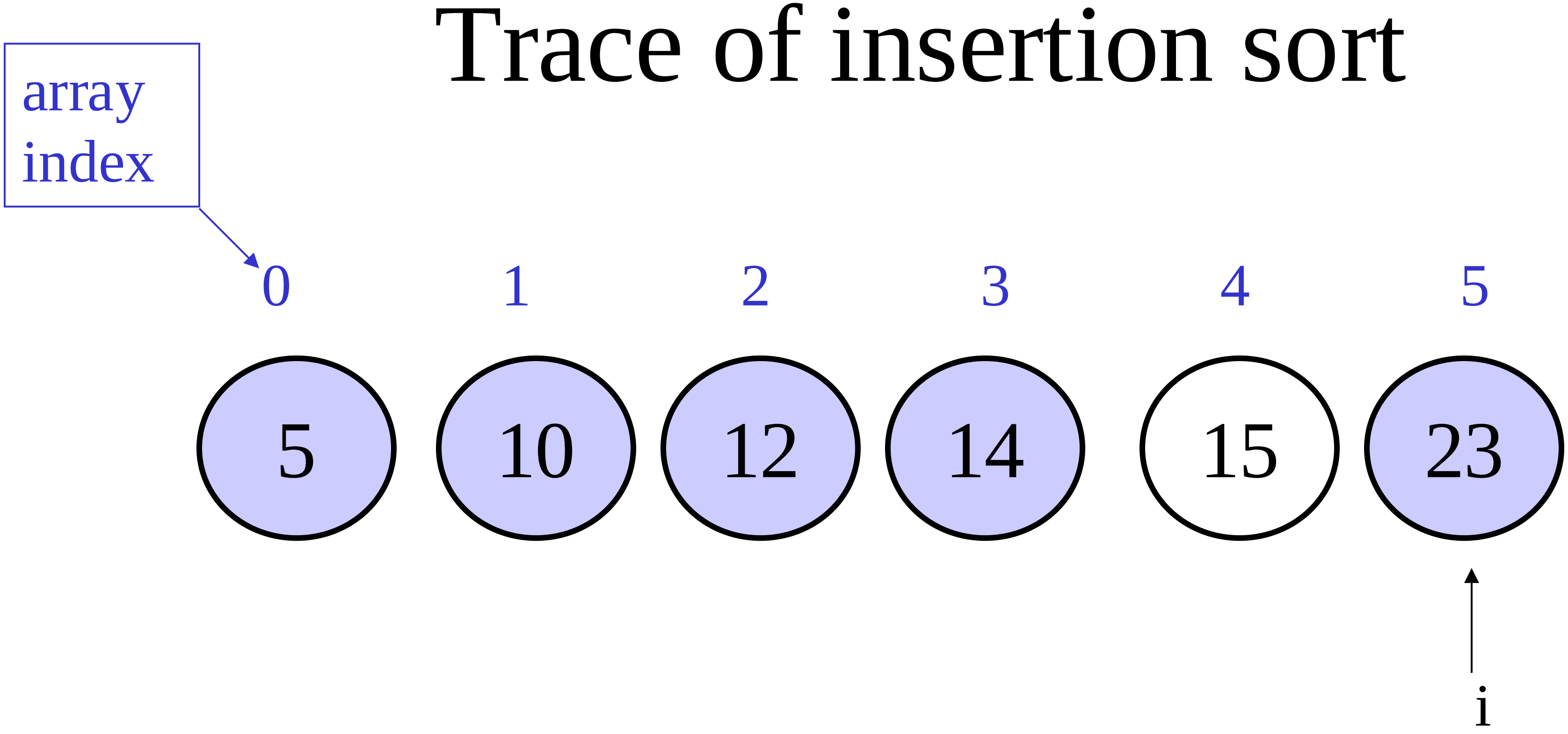
Trace of insertion sort



i = 5, fifth iteration of the outer loop

arr[j-1] > temp

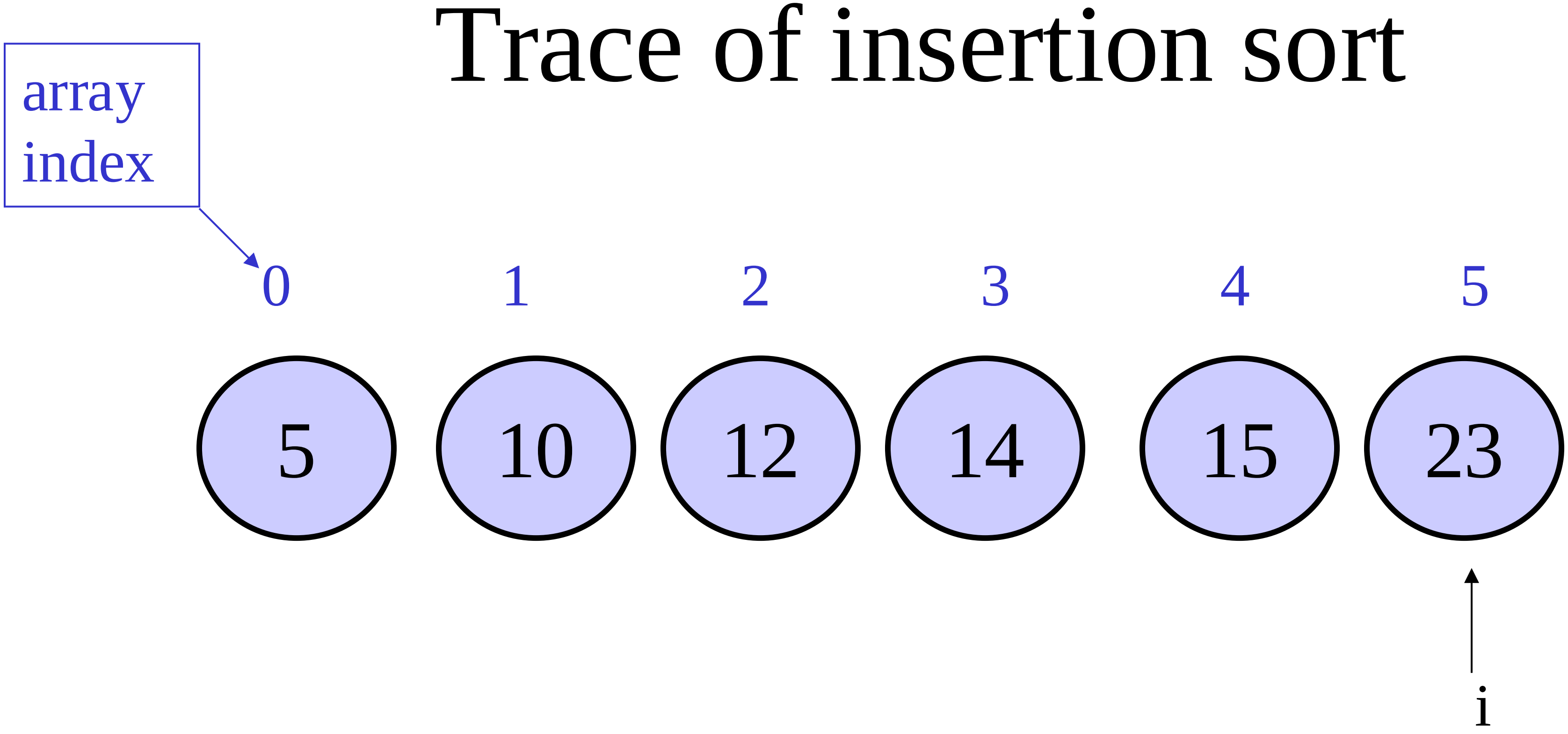
Trace of insertion sort



i = 5, fifth iteration of the outer loop

arr[j] = temp

Trace of insertion sort



i = 5, fifth iteration of the outer loop

arr[j] = temp

Complexity of insertion sort

- In the worst case, it has to make n*(n-1)/2 comparisons and shifts to the right.
- In the worst case, the time complexity is $O(n^2)$.
- In the best case, the array is already sorted, no shifts.
- In the best case, the time complexity is $O(n)$.