AE2ACE: Algorithms Correctness and Efficiency

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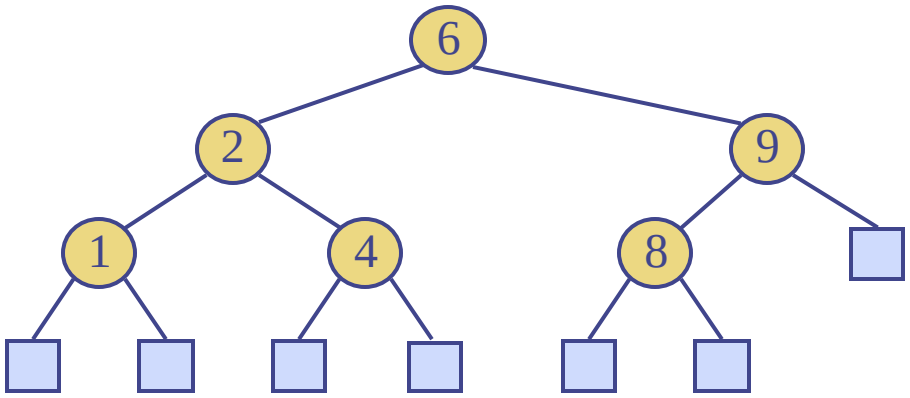
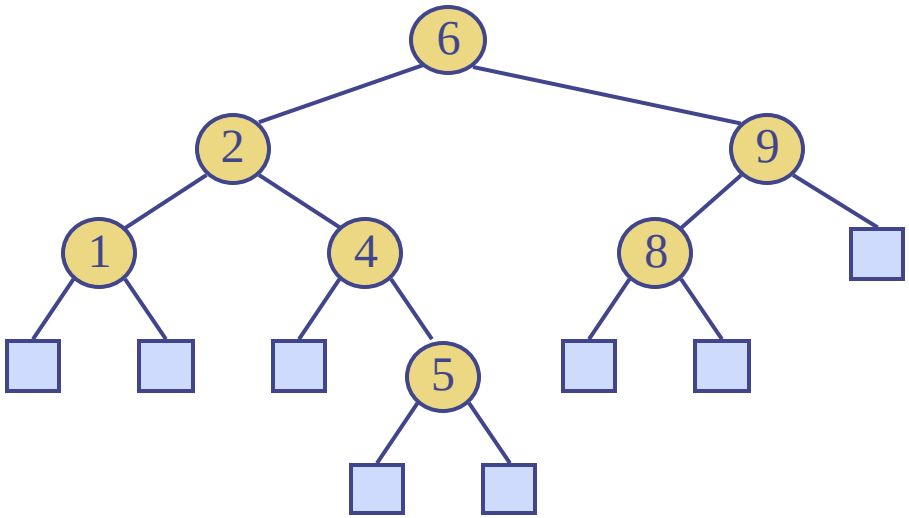
Exercise 1

What is a binary search tree? Explain and draw figures to show the process of inserting the key 5 into the following binary search tree.

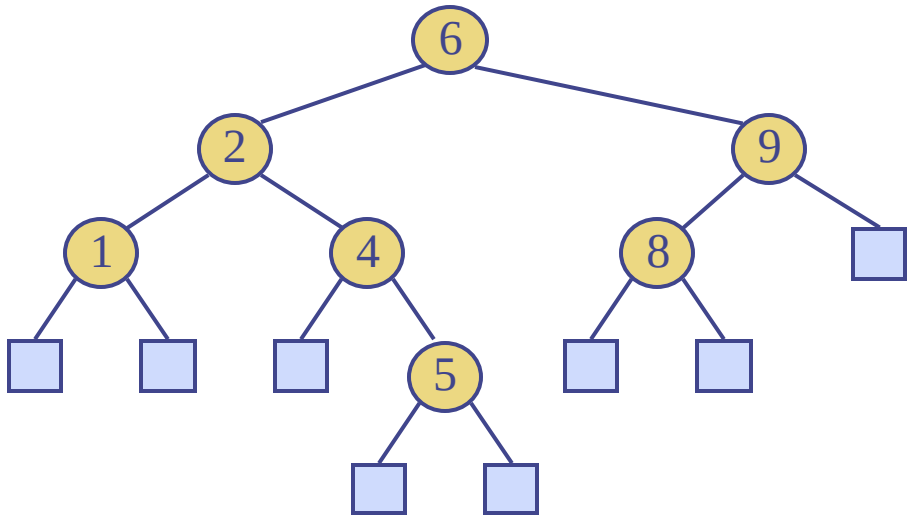


Exercise 1: Partial Answer



Exercise 2

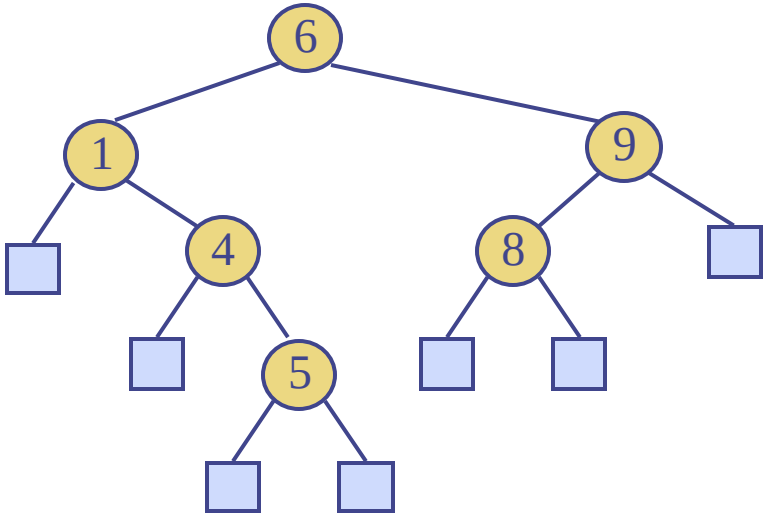
Explain and draw figures to show the process of removing the key 2 from the following binary search tree.



Exercise 2

Explain and draw figures to show the process of removing the key 2 from the following binary search tree.

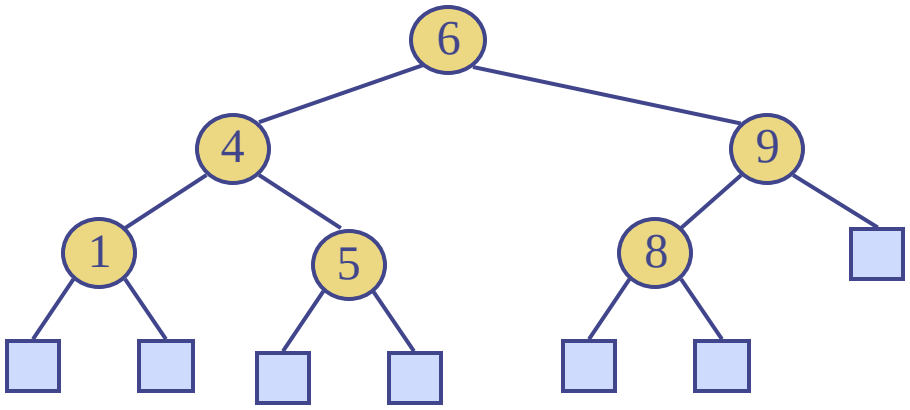
Partial answer 1:



Exercise 2

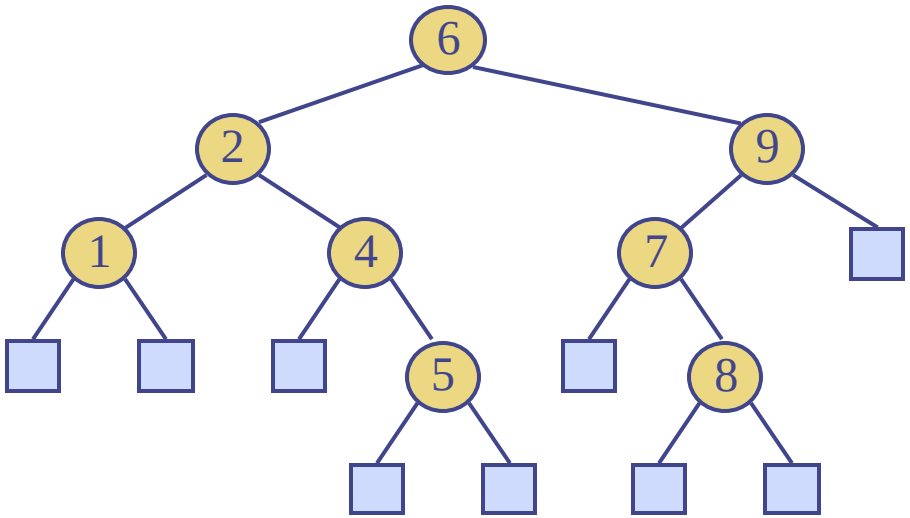
Explain and draw figures to show the process of removing the key 2 from the following binary search tree.

Partial answer 2:



Exercise 3

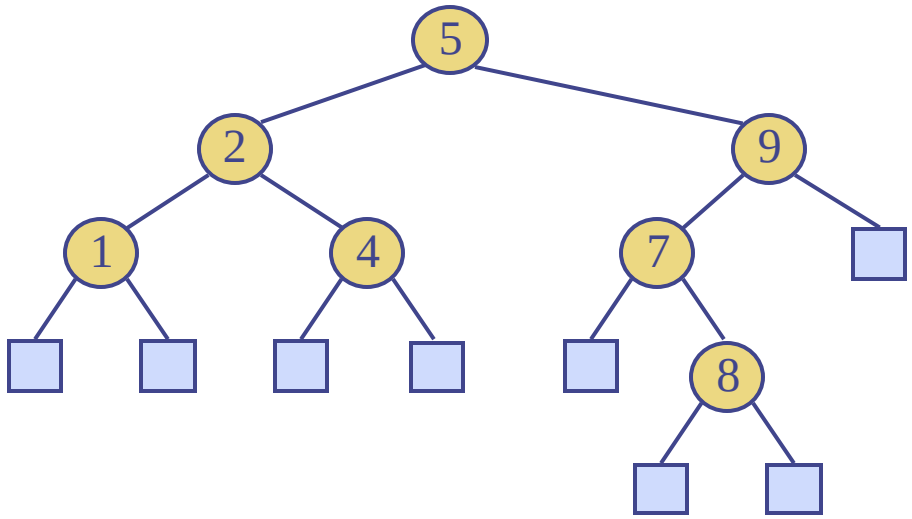
Explain and draw figures to show the process of removing the key 6 from the following binary search tree.



Exercise 3

Explain and draw figures to show the process of removing the key 6 from the following binary search tree.

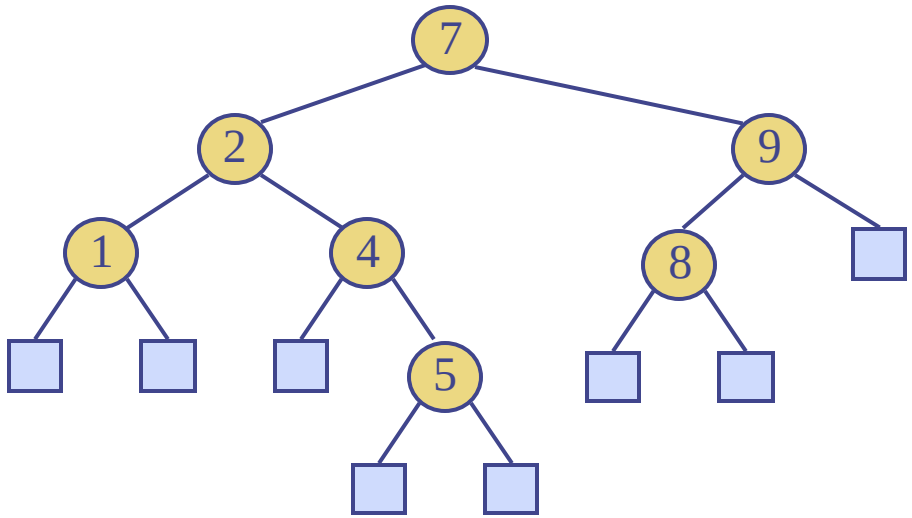
Partial answer 1:



Exercise 3

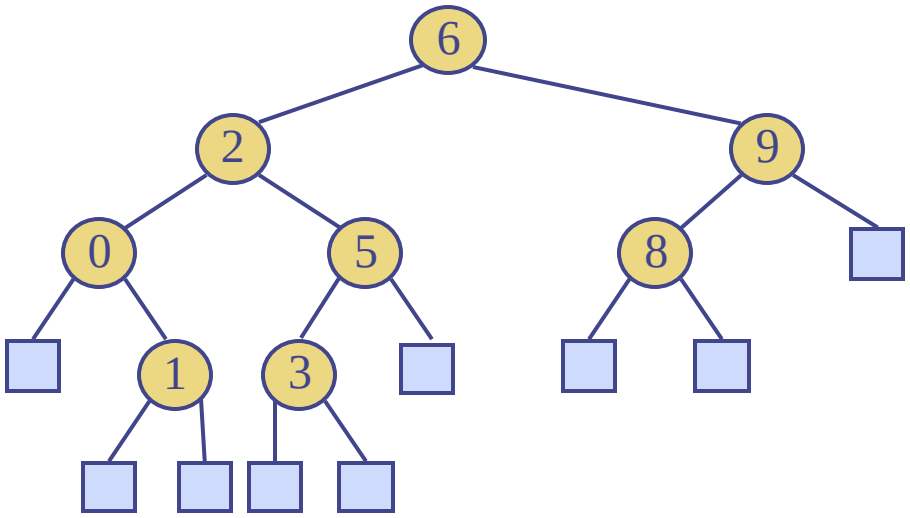
Explain and draw figures to show the process of removing the key 6 from the following binary search tree.

Partial answer 2:



Exercise 4

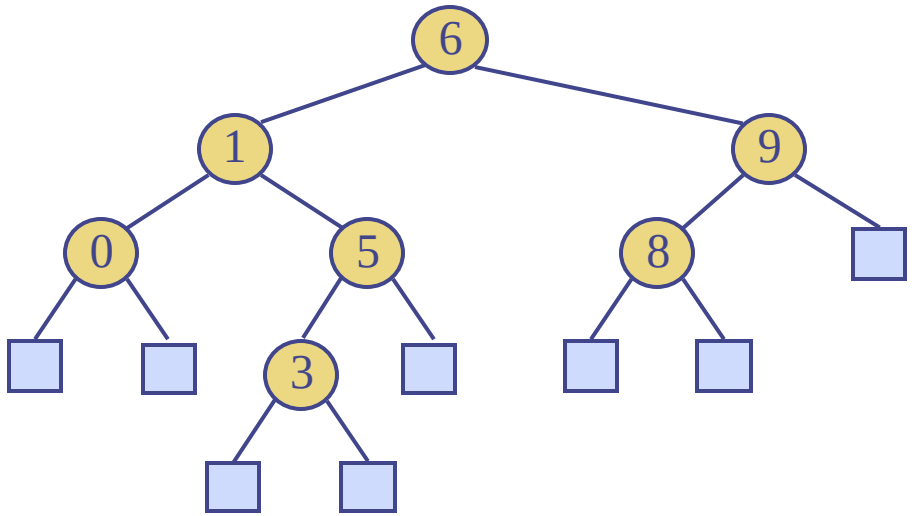
Explain and draw figures to show the process of removing the key 2 from the following binary search tree.



Exercise 4

Explain and draw figures to show the process of removing the key 2 from the following binary search tree.

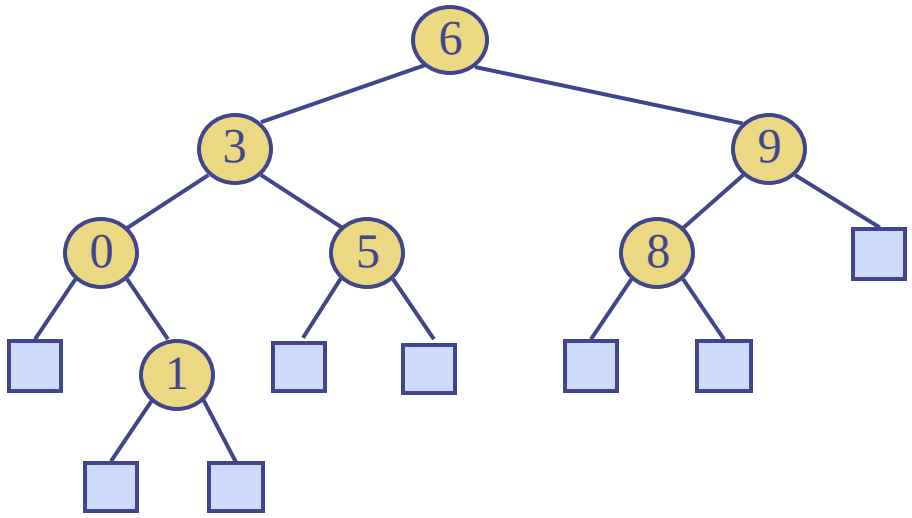
Partial answer 1:



Exercise 4

Explain and draw figures to show the process of removing the key 2 from the following binary search tree.

Partial answer 2:



从根节点开始；按照 key 比较大小，不断向左或右子树递归；
找到一个空的位置（None），插入新节点；
如果存在相同 key，可选择更新值（不改变结构）。

1. **最坏情况（不平衡）**：BST 退化成链表（如插入
排序后的一串递增元素）；
必须比较 n 次才能插入到底部；**时间复杂度**：O(n)

2. **平均/最优情况（平衡）**：每次都将树高度维持为对数级；
插入沿着从根到底的路径进行，最多比较约 \log_2 n 次；**时间复杂度**：O(\log n)

Exercise 5

Analyze the time complexity of inserting an entry into a binary search tree using the big-Oh notation. The time complexity of the main steps involved in the insertion process should be presented in the answer.

Hint: read the textbook.

情况	高度 h	插入时间
最坏情况	$h = n$	$O(n)$
平衡 BST	$h = \log n$	$O(\log n)$