The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, SPRING SEMESTER 2020-2021

Languages and Computation (AE2LAC, COMP2049)

Time allowed: 2 Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL questions

Total Marks: 75.

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: None.
INFORMATION FOR INVIGILATORS: Collect both the exam papers and the answer booklets at the end of the exam.

Question 1:

[**overall 25 marks**]

(a) True or False? (No explanation is needed.) [5 Marks]

(i) Every non-deterministic finite automaton (NFA) must have at least one final state.
(ii) Deterministic finite automata (DFAs) are special cases of NFAs.
(iii) For every regular language R and context-free language C, the language $R \cap C$ is regular.
(iv) Any language $L \subseteq \{0, 1\}^*$ which satisfies the pumping property of regular languages is a regular language.
(v) If L is a context-free language, then L^* is also context-free.

For the rest of this question, consider the grammar $G = (V, T, \langle \text{ number } \rangle, P)$, in which, the set of terminals is given by:

$$T = \{+, -, 0, 1, \mathsf{e}, \mathbf{.}\}$$

with the following production rules:

$$\begin{aligned}
\langle \text{ number } \rangle &\to \langle \text{ sign } \rangle \langle \text{ digits } \rangle \langle \text{ rest } \rangle \\
\langle \text{ sign } \rangle &\to + \mid - \mid \lambda \\
\langle \text{ digits } \rangle &\to \langle \text{ digit } \rangle \langle \text{ digits } \rangle \mid \langle \text{ digit } \rangle \\
\langle \text{ rest } \rangle &\to \langle \text{ exponent } \rangle \mid \mathbf{.} \langle \text{ frac } \rangle \\
\langle \text{ frac } \rangle &\to \langle \text{ digits } \rangle \langle \text{ exponent } \rangle \\
\langle \text{ exponent } \rangle &\to \lambda \mid \mathsf{e} \langle \text{ sign } \rangle \langle \text{ digits } \rangle \\
\langle \text{ digit } \rangle &\to 0 \mid 1
\end{aligned}$$

(b) True or False? (No explanation is needed.) [4 Marks]

(i) $\lambda \in L(G)$.
(ii) G is a context-free grammar (CFG).
(iii) G is a regular grammar.
(iv) G is a left-linear grammar.

(c) Draw the transition graph of a non-deterministic finite automaton M such that $L(M) = L(G)$. [6 Marks]

(d) Draw the transition graph of a deterministic finite automaton M' such that $L(M') = L(G)$. To make the result easier to read, the following do not need to be drawn:

- The trap state;
- Any transition into the trap state.

Thus, any missing transition will be assumed to lead to the trap state. *All the other transitions must be drawn.*

[10 Marks]

Question 2:

[**overall 25 marks**]

(a) Consider the language $L = \{a^n b^m \mid n \leq 3, m < 3\}$:

(i) Write a regular expression r such that $L = L(r)$. [2 Marks]

(ii) Write a regular expression r' for the complement of L, i. e., such that $\overline{L} = L(r')$. [3 Marks]

(iii) Write a regular expression r'' for the reverse of the complement of L, i. e., such that $(\overline{L})^R = L(r'')$. [2 Marks]

(b) Write a clear statement of the pumping lemma for regular languages. [4 Marks]

(c) Consider the language $L := \{ab^n c^m \mid m \geq n \geq 0\}$ over the alphabet $\Sigma = \{a, b, c\}$. Prove that L is not a regular language. [6 Marks]

(d) Is it true that for all languages L_1, L_2, and L_3 over the alphabet $\Sigma = \{0, 1\}$, the following equation holds?

$$L_1(L_2 \cap L_3) = L_1 L_2 \cap L_1 L_3.$$

If yes, then a complete proof must be presented. If not, then a clear counterexample must be provided. [8 Marks]

Question 3:

[**overall 25 marks**]

(a) What is meant when a context-free grammar $G = (V, \Sigma, S, P)$ is said to be *ambiguous*? [2 Marks]

(b) Consider the context-free grammar $G = (V, \Sigma, S, P)$ with the following productions:

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow aA \mid \lambda \\ B &\rightarrow ab \mid bB \mid \lambda \end{aligned}$$

(i) Write a rightmost derivation for the string $abab$. [3 Marks]

(ii) Demonstrate that G is an ambiguous grammar. [4 Marks]

(iii) Write a regular expression r such that $L(r) = L(G)$. [3 Marks]

(c) Let $G = (V, \Sigma, \langle \text{STMT} \rangle, P)$ be the grammar in which:

- $\Sigma = \{\texttt{if}, \text{condition}, \text{then}, \texttt{else}, \text{a}\}$
- $V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$

and the production rules P are given as follows:

$$\begin{aligned} \langle \text{STMT} \rangle &\rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\ \langle \text{IF-THEN} \rangle &\rightarrow \texttt{if}\ \text{condition then}\ \langle \text{STMT} \rangle \\ \langle \text{IF-THEN-ELSE} \rangle &\rightarrow \texttt{if}\ \text{condition then}\ \langle \text{STMT} \rangle\ \texttt{else}\ \langle \text{STMT} \rangle \\ \langle \text{ASSIGN} \rangle &\rightarrow \texttt{a} \end{aligned}$$

(i) Show that G is an ambiguous grammar. [8 Marks]

(ii) Write the production rules for a grammar $G' = (V', \Sigma', S', P')$ which is unambiguous and generates the same language as G. The components V', Σ', S', P' must be specified clearly. The proof that G and G' are equivalent is not needed. [5 Marks]