The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, SPRING SEMESTER 2018–2019

LANGUAGES AND COMPUTATION

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer ALL THREE questions

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject-specific translation directories are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

ADDITIONAL MATERIAL: none

INFORMATION FOR INVIGILATORS: none

Question 1

The following questions are multiple choice. There is at least one correct answer, but there may be several. To get all the marks you have to list all correct answers and none of the incorrect ones. 1 mistake results in 3 marks, 2 mistakes result in 1 mark, 3 or more mistakes result in zero marks.

(a) Which of the following statements are correct?

(i) $\epsilon \in \Sigma^*$, where $\Sigma = \{a, b, c\}$

(ii) $\epsilon \in \emptyset^*$

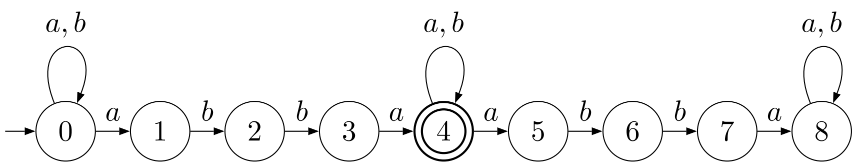
(iii) The empty word ϵ belongs to all languages.

(iv) The empty word ϵ is the only word in the empty language $\emptyset$.

(v) $\{a^i b^k \mid i, k \in \mathbb{N}, i + k \leq 42\}$ is a finite language.

(5)

(b) Consider the following finite automaton A over $\Sigma = \{a, b\}$:



Which of the following statements about A are correct?

(i) The automaton A is a Deterministic Finite Automaton (DFA).

(ii) $\epsilon \in L(A)$

(iii) $ababbaba \in L(A)$

(iv) The language accepted by the automaton A is all words over Σ that contain the letter sequence $abba$ at least once.

(v) The language accepted by the automaton A is all words over Σ that contains the letter sequence $abba$ exactly once.

(5)

(c) Consider the following set W of words:

$$W = \{\epsilon,\ ab,\ cab,\ abab\}$$

Which of the following regular expressions denote a language that contains *all* words in W? (But not necessarily *only* the words in W: it is OK if the language denoted by the regular expression contains *more* words.)

(i) $(\epsilon + \mathbf{ab} + \mathbf{c})(\epsilon + \mathbf{ab})$
(ii) $(\epsilon + \mathbf{ab} + \mathbf{c})(\emptyset + \mathbf{ab})$
(iii) $(\epsilon + \mathbf{ab} + \mathbf{c})^*$
(iv) $(\mathbf{ab} + \mathbf{c})^*$
(v) $(\mathbf{ab})^* + \mathbf{c}^*$

(5)

(d) Consider the following Context-Free Grammar (CFG) G:

$$\begin{array}{rcl} S & \to & X \mid XY \\ X & \to & aXb \mid aYb \\ Y & \to & bYc \mid \epsilon \end{array}$$

where S, X, Y are nonterminal symbols, S is the start symbol, and a, b, c are terminal symbols.

Which of the following statements about the language $L(G)$ generated by G are correct?

(i) $\epsilon \in L(G)$
(ii) $aaabbbcc \in L(G)$
(iii) $aabbbbcc \in L(G)$
(iv) $\{a^i b^i b^k c^k \mid i, k \in \mathbb{N},\ i > 0\} = L(G)$
(v) The following CFG is equivalent to G above:

$$\begin{array}{rcl} S & \to & \epsilon \mid XY \\ X & \to & aXb \mid ab \\ Y & \to & bYc \mid \epsilon \end{array}$$

(5)

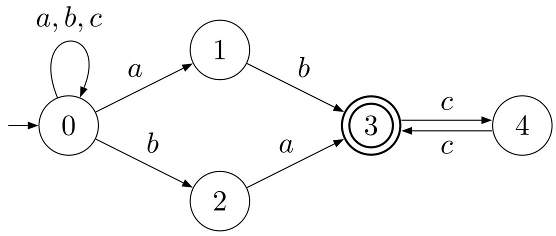
(e) Which of the following problems are $\mathcal{NP}$-complete?

(i) The Halting Problem for Turing Machines.
(ii) The Satisfiability Problem for propositional formulas.
(iii) The Normalization Problem for λ-calculus terms.
(iv) The Problem of whether a finite automaton is non-deterministic.
(v) The Subset Sum Problem.

(5)

Question 2

(a) Given the following NFA N over the alphabet $\Sigma = \{a, b, c\}$, construct a DFA $D(N)$ that accepts the same language as N by applying the *subset construction*:



To save work, consider only the *reachable* part of $D(N)$. Clearly show your calculations in a state-transition table. Do not forget to indicate the initial state and the final states of the resulting DFA $D(N)$ in the table. It suffices to give the complete state-transition table as the final answer. (10)

(b) The following is a context-free grammar (CFG) for Boolean expressions:

$$
\begin{array}{rcl}
E & \rightarrow & E \wedge E \\
& | & E \vee E \\
& | & \neg E \\
& | & (E) \\
& | & t \\
& | & f
\end{array}
$$

E is a nonterminal and the start symbol, $\wedge$, $\vee$, $\neg$, (,), t, and f are terminals.

Show that this grammar is ambiguous. (5)

(c) Construct an equivalent but *unambiguous* version of the context-free grammar for Boolean expressions given in (b) above by making it reflect the following conventions for *operator precedence* and *associativity*:

Operators	Precedence	Associativity
$\neg$	highest	N/A
$\wedge$	medium	left
$\vee$	lowest	left

(Note that $\neg$ is not associative as it is not a binary operator.) In the usual way, parentheses for grouping should have the highest precedence. For example, $t \vee \neg f \wedge t$ should be parsed similarly to $t \vee ((\neg f) \wedge t)$. (10)

Question 3

(a) Write the λ-calculus representation of some values and functions.

- The Church Numerals $\overline{0}$, $\overline{1}$, $\overline{2}$;
- The exponential function, that is a term `exp` such that, for every pair of natural numbers n and m, $\mathtt{exp}\,\overline{n}\,\overline{m} \rightsquigarrow^* \overline{n^m}$.
- The Boolean values `true` and `false`;
- The *conditional operator* `if`, such that, for every pair of terms u and v, we have:

$$\mathtt{if}\,\mathtt{true}\,u\,v \rightsquigarrow^* u \qquad \mathtt{if}\,\mathtt{false}\,u\,v \rightsquigarrow^* v$$

(8)

(b) For three λ-terms u, v, w, we define the term representing the triple as: $\langle u, v, w\rangle = \lambda x.x\,u\,v\,w$ (we assume x does not occur free in u, v, or w).

- Define projection functions `first`, `second`, `third` such that:

$$\mathtt{first}\,\langle u, v, w\rangle \rightsquigarrow^* u, \quad \mathtt{second}\,\langle u, v, w\rangle \rightsquigarrow^* v, \quad \mathtt{third}\,\langle u, v, w\rangle \rightsquigarrow^* w;$$

- Define a term `addf` such that: $\mathtt{addf}\,\langle u, v, w\rangle \rightsquigarrow^* \langle v, w, u + w\rangle$. (You can assume that a term `plus` that performs addition of Church numerals is given.)
- Finally define a λ-term `ffib` computing the following function (a generalisation of Fibonacci): given an initial triple of numbers a, b, and c, let the sequence x_i be defined by

$$x_0 = a, \quad x_1 = b, \quad x_2 = c, \quad x_n = x_{n-3} + x_{n-1} \text{ for } n > 2.$$

  Define the term `ffib` so that: $\mathtt{ffib}\,\langle a, b, c\rangle\,\overline{i} \rightsquigarrow^* x_i$.
- What does $\mathtt{ffib}\,\langle \overline{0}, \overline{1}, \overline{2}\rangle\,\overline{10}$ reduce to?

(10)

(c) Give a brief explanation if *the Church-Turing Thesis*. (7)