COMP1046 Mathematics for Computer Scientists 2023: Coursework 2

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1 Instructions

- This coursework has a total of 10 marks and represents 10% of assessment for the COMP1046 module.
- Answer all the questions below.
- Show all your working and use the same notation that is used in the lecture notes. The examples and exercises in the lectures and tutorials are a model for how you should present your answers.
- This must be your own individual work so do not work with other students on this coursework.
- You must submit your work as a PDF document on Moodle. You can use Word or Latex to type up your answer and save as PDF. Alternatively, you can handwrite your answer on A4 paper and scan to PDF. Do not use a mobile phone to take a photo of your coursework and submit the photo, since the quality is often poor.
- If your submitted coursework is illegible, you will receive a zero mark for the parts that cannot be read clearly. You will not be offered a second chance to resubmit, so ensure the good quality of your submission.
- Submit by the given deadline **23:59 on 4th December 2023**.
  You will lose 10% marks for each day your submission is late.

Please see the next page for the questions.

2 Questions

Consider this system of linear equations in variables w, x, y and z:

$$\begin{array}{lcl} 2w - x + 2y & = & 9 \\ w - x + y + 2z & = & 2 \\ 4w + 2x - 3y & = & 1 \\ 3w - 2x & = & 4 \end{array}$$

Q1. Express this system of linear equations in matrix notation as a complete matrix. (1 mark)

Q2. Show whether this system of linear equations is compatible or incompatible. (1 mark)

Q3. Use Cramer's Method to find the solution for x. (2 marks)
Note: 1 mark will be given for showing an efficient solution; i.e. involving the least calculations.

Consider the following matrices,

$$\mathbf{X} = \begin{pmatrix} a & b & c & d & 2 \\ 2 & 1 & 1 & 2 & 3 \\ 1 & e & f & g & h \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -10 \\ 20 \\ 50 \\ 0 \\ 50 \end{pmatrix}$$

where $\mathbf{X}$ is constructed taking a, b, c, d, e, f, g, h as the 8 consecutive digits in **your** student ID. For example, if my student ID is 20120790, then $a = 2$, $b = 0$, $c = 1$, $d = 2$, $e = 0$, $f = 7$, $g = 9$ and $h = 0$.

Q4. Compute $\mathbf{A} = \mathbf{X}^T\mathbf{X} + 10\mathbf{I}$. (1 mark)

Q4. Consider a system of linear equations in 5 variables x_1, x_2, x_3, x_4 and x_5, represented by the complete matrix $(\mathbf{A}|\mathbf{b})$. Use Gaussian Elimination as described in Lecture 5 to find solutions for this system.

- You may use a calculator or Excel or write your own program to help you.
- Show all steps of the Gaussian Elimination process in your answer, following the style given in Lecture 5.
- Your solution for x_1, x_2, x_3, x_4 and x_5 can be expressed as fractions or decimal form to 3 decimal places.

- If the system cannot be solved, explain why.
- Note that there are several variations on the Gaussian Elimination algorithm. For this exercise, you **must** use the version taught in Lecture 5.

(5 marks)