AE2ADS: Algorithms Data Structures and Efficiency

Lecturer: Heshan Du
Email: Heshan.Du@nottingham.edu.cn

University of Nottingham Ningbo China

Big-Oh

Let $f(n)$ and $g(n)$ be functions mapping positive integers to positive real numbers.

We say that $f(n)$ is $O(g(n))$, if there exist a real constant $c > 0$ and an integer constant $n_0 \geq 1$ such that for every $n \geq n_0$,

$$f(n) \leq cg(n).$$

Exercise 1

Prove that:

- $n^2 + 1$ is $O(n^2)$
- $(n - 3)^2$ is $O(n^2)$

Exercise 2

Given that $f(n) = n + 3$, if n is even; $f(n) = n^2 + 5$, if n is odd, state the Big-Oh behaviour of $f(n)$, and prove it.

Big-Omega

Let $f(n)$ and $g(n)$ be functions mapping positive integers to positive real numbers.

We say that $f(n)$ is $\Omega(g(n))$, if there exist a real constant $c > 0$ and an integer constant $n_0 \geq 1$ such that for every $n \geq n_0$,

$$f(n) \geq cg(n).$$

Big-Theta

Let $f(n)$ and $g(n)$ be functions mapping positive integers to positive real numbers.

We say that $f(n)$ is $\Theta(g(n))$, if there are real constants $c' > 0$, $c'' > 0$, and an integer constant $n_0 \geq 1$ such that for every $n \geq n_0$,

$$c'g(n) \leq f(n) \leq c''g(n).$$

Little-Oh

Let $f(n)$ and $g(n)$ be functions mapping positive integers to positive real numbers.

We say that $f(n)$ is $o(g(n))$, if for every real constant $c > 0$, there exists an integer constant $n_0 \geq 1$ such that for every $n \geq n_0$,

$$f(n) < cg(n).$$

Exercise 3

- Prove or disprove that:
  1. 5 is $\Omega(1)$
  2. $2n + 1$ is $\Omega(n)$
  3. 5 is $\mathrm{o}(1)$
  4. 5 is $\mathrm{o}(n)$
  5. $n^2 - 5n$ is $\Theta(n^2)$
  6. n^2 is $\Omega(n)$
  7. 1 is $\mathrm{o}(\log n)$
  8. $n \log n$ is $\mathrm{o}(n^2)$

Exercise 4

Given $f(n) = n^2$ if n is even, $f(n) = n$ if n is odd. Find the big-Oh and big-Omega behaviors of $f(n)$.

More Exercises

M. T. Goodrich, R. Tamassia and M. H. Goldwasser, *Data Structures and Algorithms in Java*, 6th Edition, 2014.

- Chapter 4. Analysis Tools