Languages and Computation (COMP 2049) Lab 06

Proving Non-Regularity

(1) Consider the alphabet $\Sigma = \{a, b\}$ and the following language over Σ:

$$L_1 := \{a^n b^k \mid n, k \in \mathbb{N} \land n > k\}.$$

(a) Using the pumping lemma for regular languages, prove that L_1 is not regular.

(b) Demonstrate that L_1 is context-free by presenting the productions of a grammar G for which $L_1 = L(G)$.

(2) Using the pumping lemma for regular languages, prove that the following language is not regular:

$$L_2 := \{xx \mid x \in \{a, b\}^*\}.$$

Remark: The language L_2 is sometimes refered to as the copy language, denoted as XX or WW. Not only is this language not regular, it is not even context-free.

(3) Consider the alphabet $\Sigma = \{a, b, c\}$ and the following language over Σ:

$$L_3 := \{a^j b^k c^k \mid j \geq 1 \land k \geq 0\} \cup \{b^j c^k \mid j \geq 0 \land k \geq 0\}.$$

In simple terms, for every string $x \in L_3$

- If there is at least one a in the string x, then the number of b's and c's must be the same.
- If there is no a in the string x, then the number of b's and c's can be anything.

(a) Prove that the language L_3 satisfies the conclusions of the pumping lemma for regular languages.

(b) Using the pigeonhole principle, prove that the language L_3 is not regular.

Remark: This exercise demonstrates the following points:

- Although every regular language must have the pumping property, the converse is not true, i. e., there are languages that do have the pumping property, but are not regular.
- The pigeonhole principle is stronger than the pumping lemma.