Question 1

1.	$\exists x(F(x) \land \forall y(G(y) \to P(x,y)))$	Premise (given)
2.	$F(x) \land \forall y(G(y) \to P(x,y))$	Existential Instantiation from (1)
3.	$\forall y(G(y) \to P(x,y))$	Simplification from (2)
4.	$F(x)$	Simplification from (2)
5.	$G(y) \to P(x,y)$	Universal Instantiation from (3)
6.	$\neg G(y) \lor P(x,y)$	Law 20 from (5)
7.	$\forall x(F(x) \to \forall y(H(y) \to \neg P(x,y)))$	Premise (given)
8.	$F(x) \to \forall y(H(y) \to \neg P(x,y))$	Universal Instantiation from (7)
9.	$F(x)$	Simplification from (8)
10.	$\forall y(H(y) \to \neg P(x,y))$	Simplification from (8)
11.	$H(y) \to \neg P(x,y)$	Universal Instantiation from (10)
12.	$\neg H(y) \lor \neg P(x,y)$	Law 20 from (11)
13.	$G(y) \to \neg H(y)$	Resolution from (6),(12)
14.	$\forall x(G(y) \to \neg H(y))$	Universal Generalization from (13)

Thus, I conclude that $\forall x(G(x) \to \neg H(x))$ is ture.

Question 2

I split task into two subtasks.
Firstly, suppose $x \in A - (A \cap B)$,
Therefore, $x \in A \wedge x \notin A \cap B($ by definition of difference).
If $x \notin A \cap B, then x \notin B$(by definition of intersection).
Since $x \in A$ and $x \notin B$,$x \in A \cup B$ but $x \notin B$
Therefore, $x \in (A \cup B) - B$(by definition of difference).
Secondly, suppose $x \in (A \cup B) - B$.
Therefore, $x \in A \cup B$ and $x \notin B$(by definition of difference).
If, $x \in A \cup B$, but $x \notin B$, then $x \in A$(by deffinition of union).
Since, $x \in A$and $x \notin B$, $x \notin A \cap B$(by defination of intersection).
Therefore, $x \in A - (A \cap B)$(by definition of difference).
Finally, we prove that $A - (A \cap B) = (A \cup B) - B$.

Question 3

Injective(one-to-one):
Let $\forall(x_1, y_1), (x_2, y_2) \in \mathbb{R} \times \mathbb{R}$.
Suppose $F(x_1, y_1) = F(x_2, y_2)$,then$(x_1 + y_1, x_1 - y_1) = (x_2 + y_2, x_2 - y_2)$.
Since,$(x_1 + y_1, x_1 - y_1) = (x_2 + y_2, x_2 - y_2)$, then $x_1 + y_1 = x_2 + y_2$, $x_1 - y_1 = x_2 - y_2$.
Therefore, $x_1 = x_2, y_1 = y_2$. Therefore, the function is Injective(one-to-one).
Surjective(onto):
For every element $(a, b) \in \mathbb{R} \times \mathbb{R}$, that$(x + y, x - y) = (a, b)$, with $(x + y, x - y) \in \mathbb{R} \times \mathbb{R}$.
Thus, the function is Surjective(onto).
Finally, I conclude F is a one-to-one correspondence.

Question 4

1. Since $x - x = 0$, for every natural numbers, $a + bi = 0$.
Therefore, R is reflexive.

2. Since $x - y = a + bi$, then $y - x = -a - bi$.
However, a, b are natural numbers, $-a, -b$ are not natural numbers.
For example, $x - y = 3 + 4i, y - x = -3 - 4i$, $-3, -4 \notin N$.
Therefore, R is not symmetric.

3. For$(x, y) \in R$, if $(x, y) \in R \wedge (y, x) \in R$ is ture, then $x - y = a + bi = y - x = -a - bi$.
$a, b, -a, -b \in N$, when $a = b = -a = -b$, existing $a = b = -a = -b = 0$.
Therefore, R is antisymmetric for some natural numbers.

4. Suppose $(x, y) \in R$ and $(y, z) \in R$,then $x - y = a + bi, y - z = c + di$.
Since $x - y = a + bi, y - z = c + di$, then $x - z = (a + c) + (b + d)i$.
Since, a, b, c, d are natural numbers, then $a + c, b + d$ are natural numbers.
Therefore, R is transitive.

Question 5

$$a_n = \begin{cases} 1 & , \quad if 0 \leq n \leq 3 \\ a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4} & , \quad if n \geq 4. \end{cases}$$

Since $a_n \equiv 1(mod\, 3)$, then$\frac{a_n - 1}{3} = 0$
For $0 \leq n \leq 3$, $a_n = 1$, then $\frac{1-1}{3} = 0$
Therefore, $a_n \equiv 1(mod\, 3)$ for $0 \leq n \geq 3$
For $n \geq 4$, $a_4 = a_3 + a_2 + a_1 + a_0 = 1 + 1 + 1 + 1 = 4$;
$a_5 = a_4 + a_3 + a_2 + a_1 + = 7$;
$a_6 = a_5 + a_4 + a_3 + a_2 = 13$;
$a_7 = a_6 + a_5 + a_4 + a_3 = 25$;
$\vdots$
$a_n = a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4}$
For $n \geq 4$, let $b_1 = a_4 = 4, b_2 = a_5 = 7, b_3 = a_6 = 13, b_4 = a_7 = 25, \cdots, b_n = a_n(n \geq 4)$
Since $b_1 = a_4 = 4, b_2 = a_5 = 7, b_3 = a_6 = 13, b_4 = a_7 = 25, \cdots, b_n = a_n(n \geq 4)$, then $b_n = 4 + 3(k-1) = 3k + 1$
Since $b_n = 3k + 1$, then $a_n = 3k + 1, (n \geq 4)$
Therefore, $\frac{a_n - 1}{3} = \frac{3k+1-1}{3} = k, (n \geq 4)$
Therefore, $a_n \equiv 1(mod\, 3)$ for $n \geq 4$
Thus, above all $a_n \equiv 1(mod\, 3)$ for all $n \geq 0$

Question 6

Let $A = \{The\,student\,passed\,the\,exam\,on\,the\,first\,attempt\}$
$B = \{The\,student\,passed\,the\,exam\,on\,the\,second\,attempt\}$
$P(A) = \frac{2}{3}$
$P(B) = \frac{1}{3}$

(a)

$C = \{The\,student\,passed\,at\,least\,one\,exam\}$

$$\begin{aligned} P(C) &= 1 - P(\overline{C}) \\ &= 1 - P(\overline{AB}) \\ &= 1 - P(\overline{A})P(\overline{A}\overline{B}) \\ &= 1 - [1 - P(A)][1 - P(B|\overline{A}) \\ &= 1 - (1 - \frac{2}{3}) \cdot (1 - \frac{1}{3}) \\ &= \frac{7}{9} \end{aligned}$$

(b)

$$\begin{aligned} P(A|B) &= \frac{P(AB)}{P(B)} \\ &= \frac{P(A)P(B|A)}{P(A)P(B|A) + P(\overline{A})P(B|\overline{A})} \\ &= \frac{\frac{2}{3} \times \frac{2}{3}}{\frac{2}{3} \times \frac{2}{3} + (1 - \frac{2}{3}) \times \frac{1}{3}} \\ &= \frac{\frac{4}{9}}{\frac{4}{9} + \frac{1}{9}} \\ &= \frac{\frac{4}{9}}{\frac{5}{9}} \\ &= \frac{4}{5} \end{aligned}$$

(c)

$$\begin{aligned} E &= 1 \times P(A) + 2 \times P(B) \\ &= 1 \times \frac{2}{3} + 2 \times \frac{1}{3} \\ &= \frac{2}{3} + \frac{2}{3} \\ &= \frac{4}{3} \end{aligned}$$

Appendix: LaTeX Symbol List

You may find the following symbol list useful.

Greek Letters

α	`\alpha`
β	`\beta`
χ	`\chi`
δ	`\delta`
ϵ	`\epsilon`
η	`\eta`
γ	`\gamma`
ι	`\iota`
κ	`\kappa`
λ	`\lambda`
μ	`\mu`
ν	`\nu`
ω	`\omega`
ϕ	`\phi`
π	`\pi`
ψ	`\psi`
ρ	`\rho`
σ	`\sigma`
τ	`\tau`
θ	`\theta`
υ	`\upsilon`
ξ	`\xi`
ζ	`\zeta`
Δ	`\Delta`
Γ	`\Gamma`
Λ	`\Lambda`
Ω	`\Omega`
Φ	`\Phi`
Π	`\Pi`
Ψ	`\Psi`
Σ	`\Sigma`
Θ	`\Theta`
Υ	`\Upsilon`
Ξ	`\Xi`

Math Symbols

$\times$	`\times`
$\div$	`\div`
$\sim$	`\sim`
$\neq$	`\neq`
$\equiv$	`\equiv`
$\ge$	`\ge`
$\le$	`\le`
∞	`\infty`
$\sum$	`\sum`
$\prod$	`\prod`
$\neg$	`\neg`
$\wedge$	`\wedge`
$\vee$	`\vee`
$\rightarrow$	`\rightarrow`
$\leftrightarrow$	`\leftrightarrow`
$\forall$	`\forall`
$\exists$	`\exists`
$\nexists$	`\nexists`
$\in$	`\in`
$\notin$	`\notin`
$\subset$	`\subset`
$\not\subset$	`\not\subset`
$\subseteq$	`\subseteq`
$\nsubseteq$	`\nsubseteq`
$\emptyset$	`\emptyset`
$\cup$	`\cup`
$\cap$	`\cap`
$\bigcup$	`\bigcup`
$\bigcap$	`\bigcap`
$\circ$	`\circ`
$\cdot$	`\cdot`
$\cdots$	`\cdots`
$\ldots$	`\ldots`

Math Constructs

$\&$	`\&`
$\%$	`\%`
$\{$	`\{`
$\}$	`\}`
$($	`(`
$)$	`)`
$\big($	`\big(`
$\big)$	`\big)`
$\Big($	`\Big(`
$\Big)$	`\Big)`
a^x	a^x
a_x	a_x
$\frac{abc}{xyz}$	`\frac`{abc}{xyz}
$\sqrt{abc}$	`\sqrt`{abc}
$\sqrt[n]{abc}$	`\sqrt[n]`{abc}
$\overline{abc}$	`\overline`{abc}
$\mathcal{P}(X)$	`\mathcal`{P}(X)
$\bigcup_{i=1}^{n}$	`\bigcup`_{i=1}^{n}

Mathcal letters `\mathcal`{A}: $\mathcal{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$
Mathbb letters `\mathbb`{A}: $\mathbb{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$
Mathfrak letters `\mathfrak`{A}: $\mathfrak{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$
Bold letters `\textbf`{A}: **ABCDEFGHIJKLMNOPQRSTUVWXYZ**
Bold italic letters `\pmb`{A}: ***ABCDEFGHIJKLMNOPQRSTUVWXYZ***