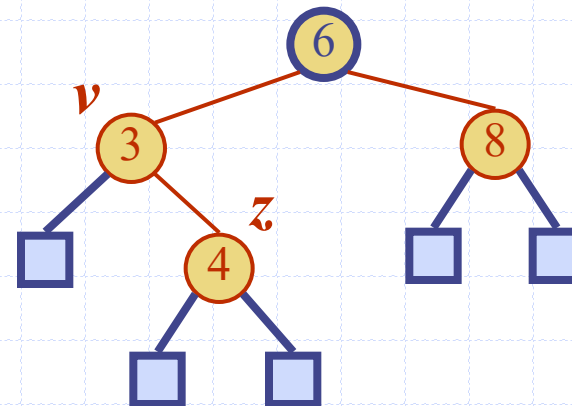


Presentation for use with the textbook **Data Structures and Algorithms in Java, 6<sup>th</sup> edition**, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

# Balanced Search Trees



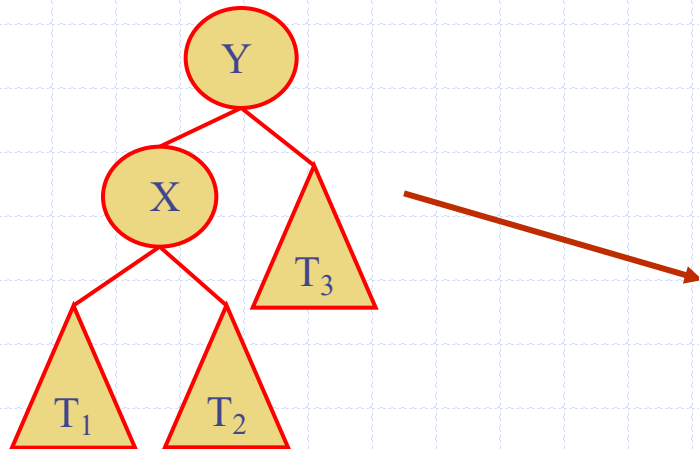
# Reading

**M. T. Goodrich, R. Tamassia and M. H. Goldwasser,**  
*Data Structures and Algorithms in Java*, 6th Edition,  
2014.

- Chapter 11. Search Tree Structures
- Sections 11.1-11.2
- pp. 423-442

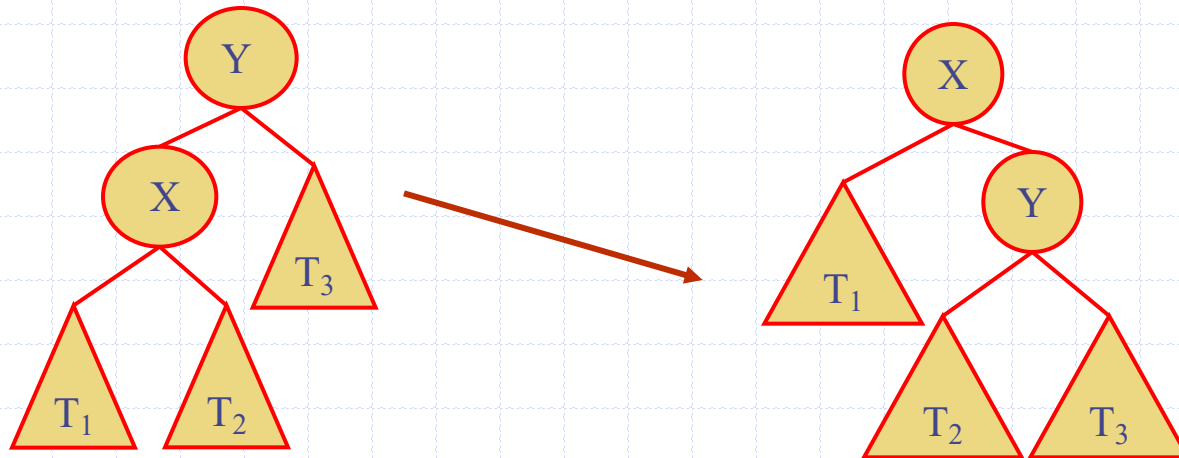
# Rotation

- ◆ The primary operation to rebalance a binary search tree is known as a *rotation*.
- ◆ During a rotation, we “rotate” a child to be above its parent.



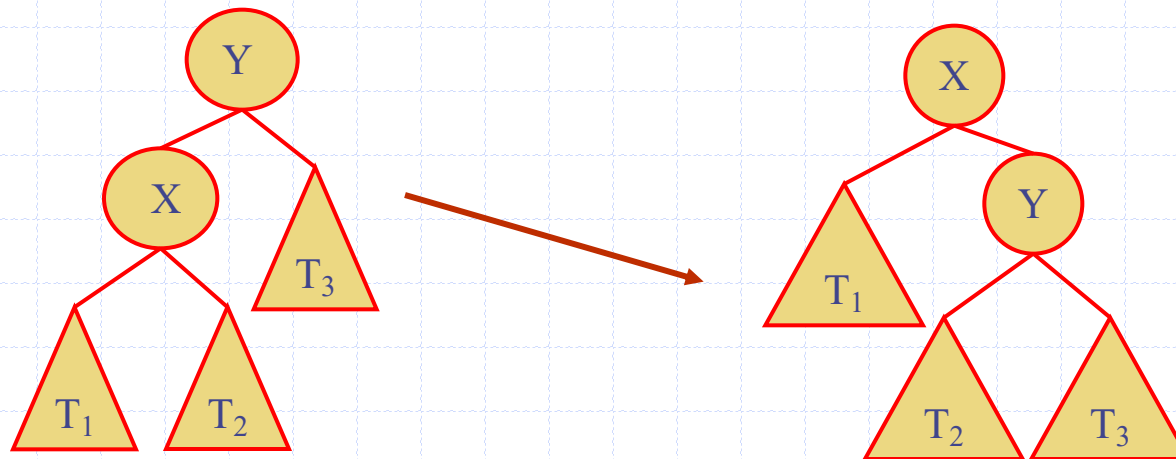
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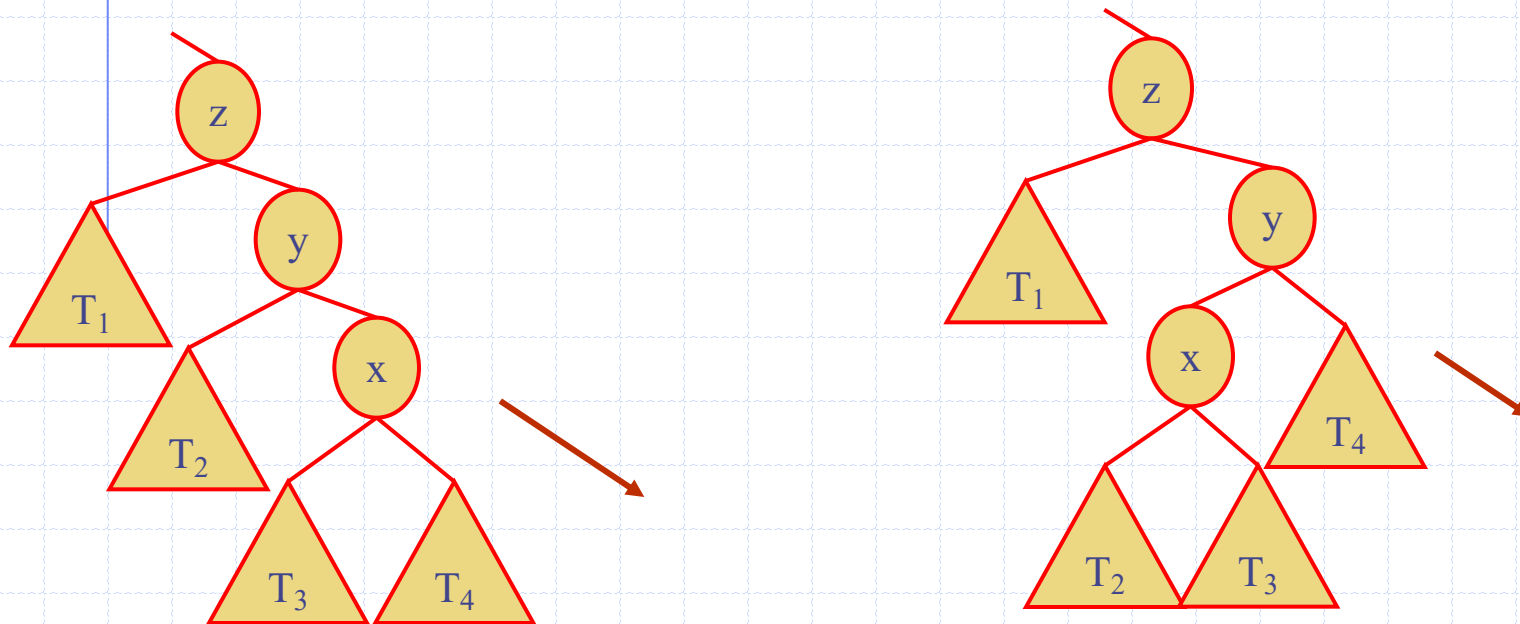


# Rotation

- ◆ In the context of a tree-balancing algorithm, a rotation allows the *shape* of a tree to be modified while maintaining the search-tree property.
- ◆ This operation can be used to *avoid highly unbalanced tree configurations*.
- ◆ Check how the depth of each node in subtree T1 and T3 was changed by the “rotation”.



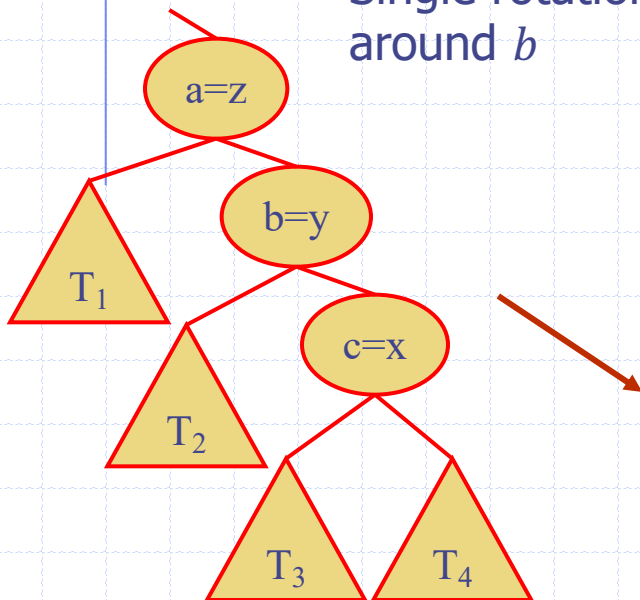
# Trinode Restructuring



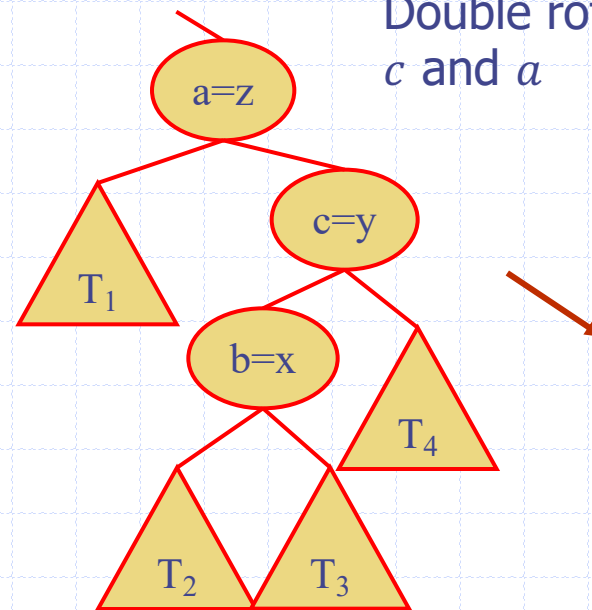
# Trinode Restructuring

- ◆ Let  $(a, b, c)$  be the inorder listing of  $x, y, z$
- ◆ Perform the rotations needed to make  $b$  the topmost node of the three

Single rotation  
around  $b$

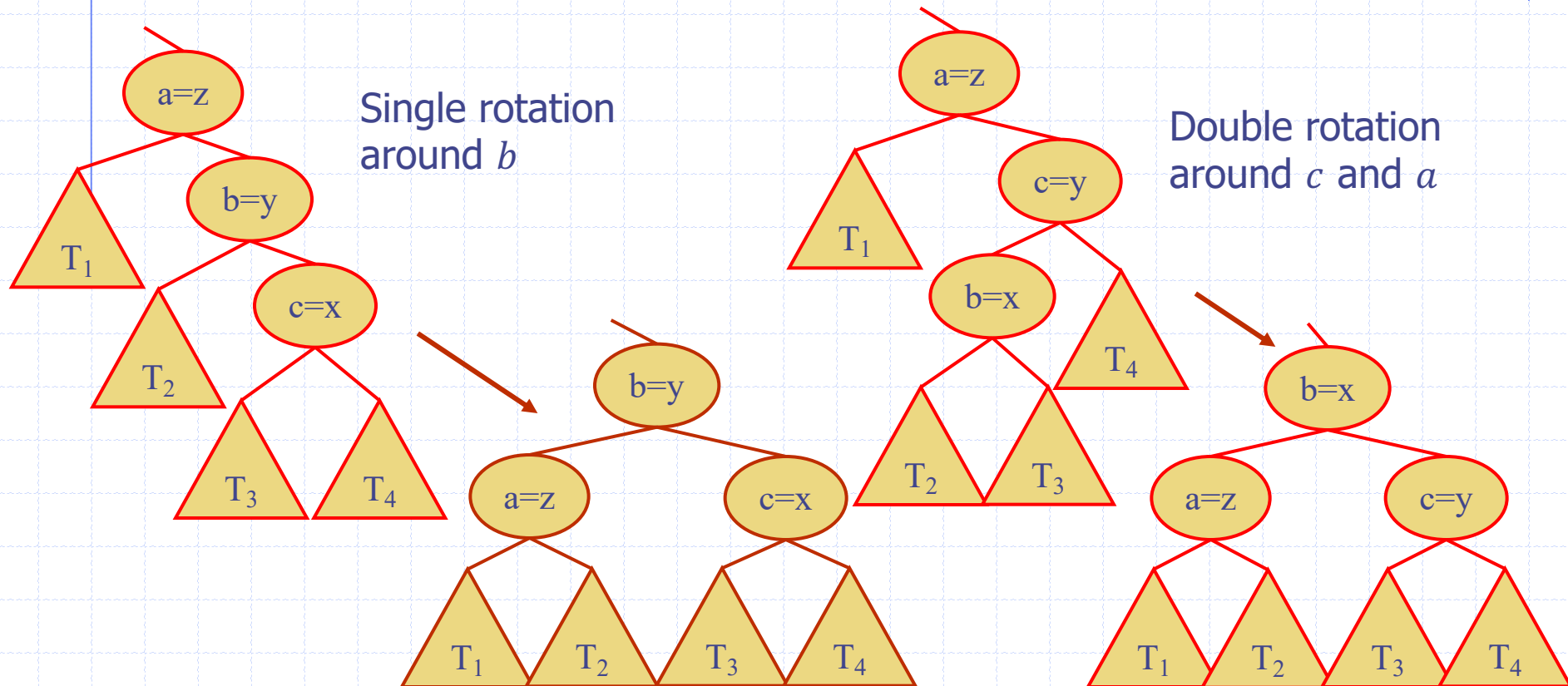


Double rotation around  
 $c$  and  $a$



# Trinode Restructuring

- ◆ Let  $(a, b, c)$  be the inorder listing of  $x, y, z$
- ◆ Perform the rotations needed to make  $b$  the topmost node of the three





# Restructure

**Algorithm**  $\text{restructure}(x)$ :

**Input:** A position  $x$  of a binary search tree  $T$  that has both a parent  $y$  and a grandparent  $z$

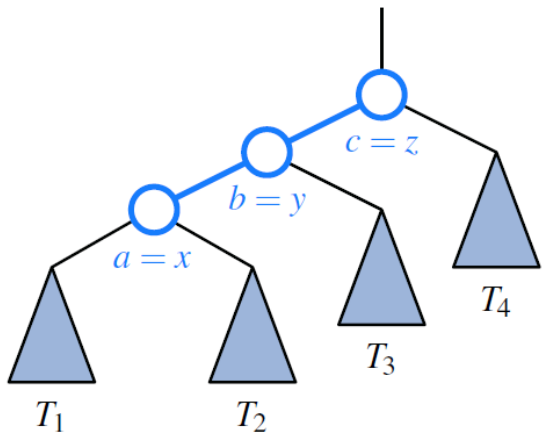
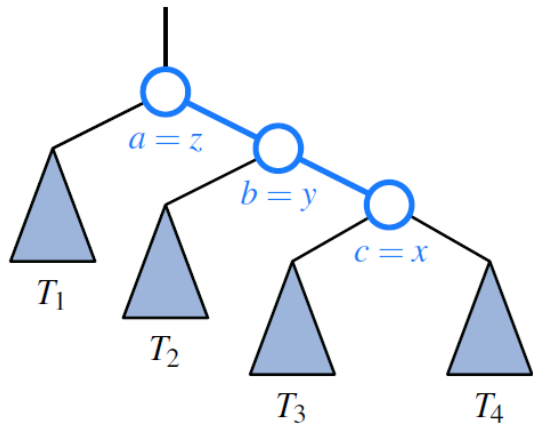
**Output:** Tree  $T$  after a trinode restructuring (which corresponds to a single or double rotation) involving positions  $x$ ,  $y$ , and  $z$

- 1: Let  $(a, b, c)$  be a left-to-right (inorder) listing of the positions  $x$ ,  $y$ , and  $z$ , and let  $(T_1, T_2, T_3, T_4)$  be a left-to-right (inorder) listing of the four subtrees of  $x$ ,  $y$ , and  $z$  not rooted at  $x$ ,  $y$ , or  $z$ .
- 2: Replace the subtree rooted at  $z$  with a new subtree rooted at  $b$ .
- 3: Let  $a$  be the left child of  $b$  and let  $T_1$  and  $T_2$  be the left and right subtrees of  $a$ , respectively.
- 4: Let  $c$  be the right child of  $b$  and let  $T_3$  and  $T_4$  be the left and right subtrees of  $c$ , respectively.

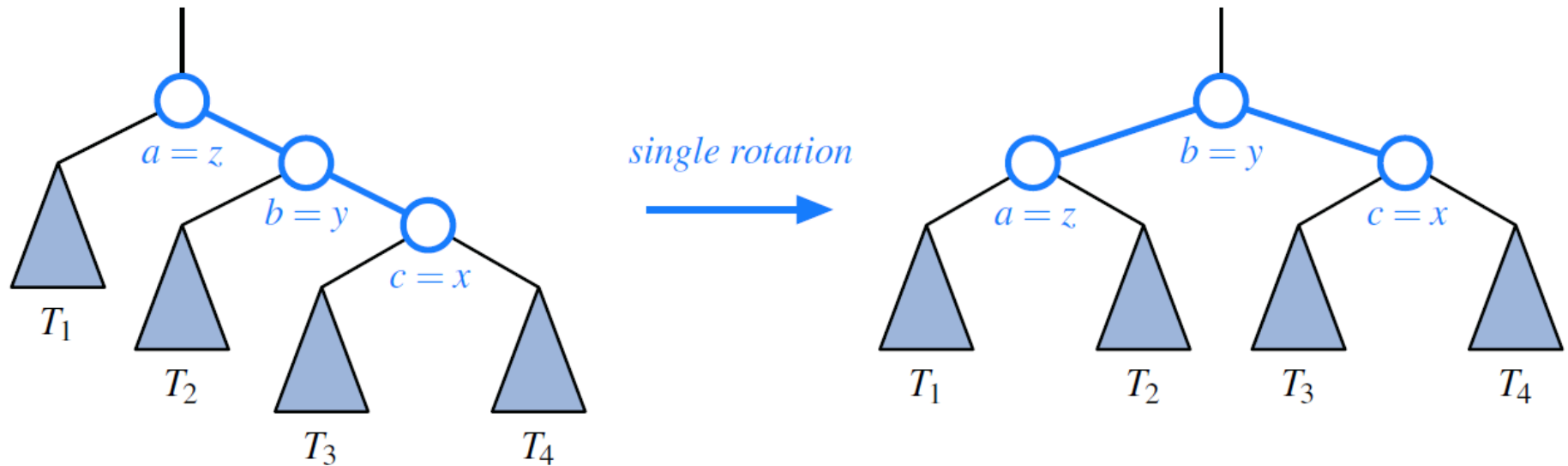
**Code Fragment 11.7:** The trinode restructuring operation in a binary search tree.

# Restructuring (as Single Rotations)

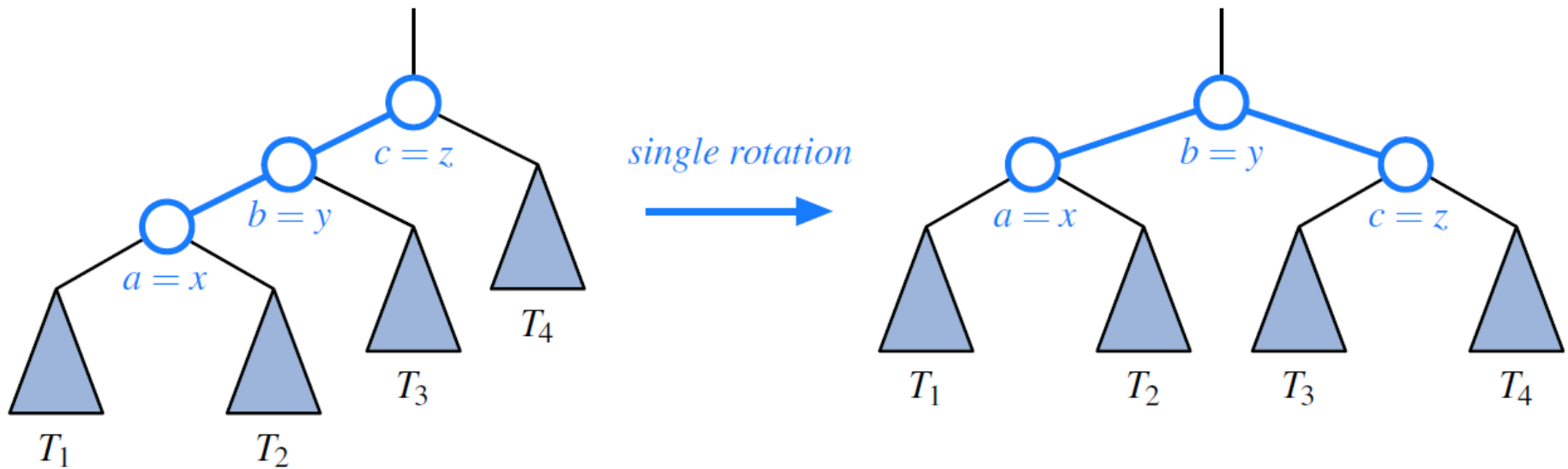
## ◆ Single Rotations:



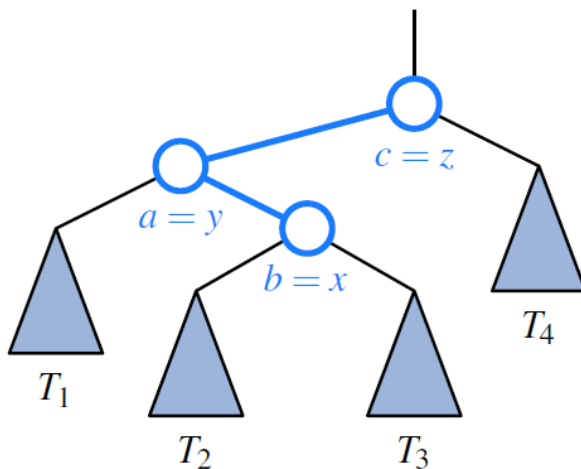
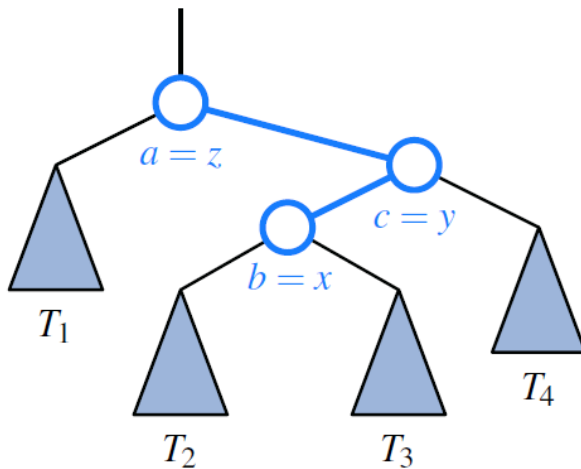
# Restructuring (as Single Rotations)



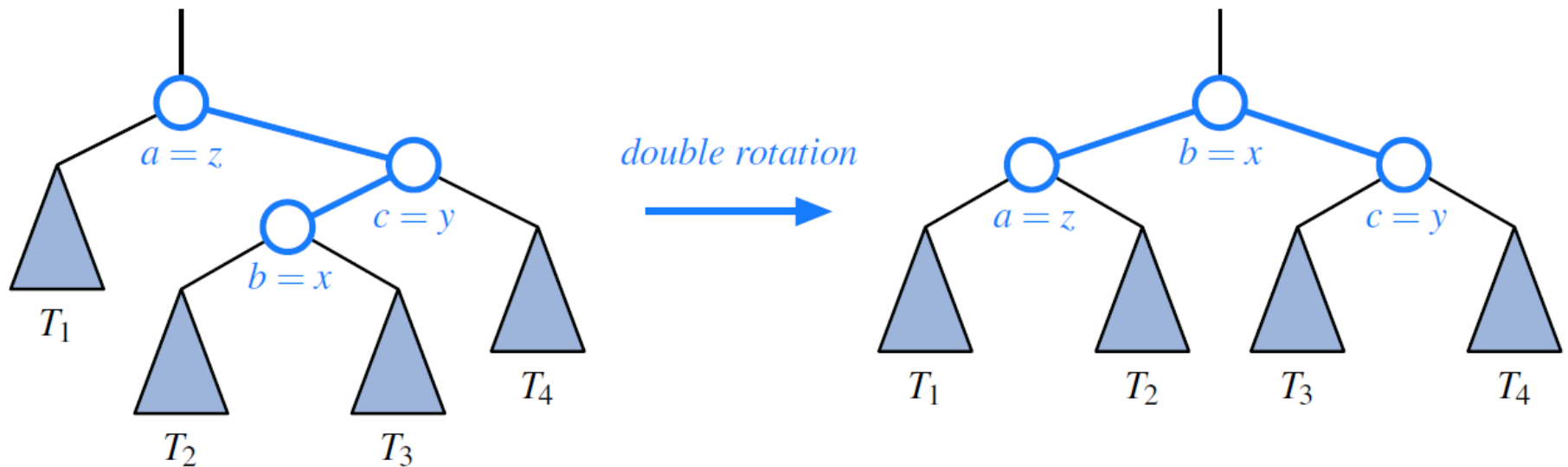
# Restructuring (as Single Rotations)



# Restructuring (as Double Rotations)



# Restructuring (as Double Rotations)



# Restructuring (as Double Rotations)

