Mathematics for Computer Scientists COMP1046 Coursework 1

- Deadline for submission: 23:59, 2 November 2023 (Thursday).
- Submit as a PDF file via Moodle. Use student ID as your file name.
- You may type your answer in Word or LaTeX. Handwritten solutions are **NOT** accepted.
- If you wish to type your answer in LaTeX, a LaTeX template is provided as a `.tex` file with a symbol list in it.
- The full mark for this coursework is 100.
- Late submission penalty: 5% deducted for each day late.
- Please show working process using the steps we used in the lectures.

1. Assume the following two statements are true:

$$\exists x(F(x) \land \forall y(G(y) \to P(x,y))), \quad \forall x(F(x) \to \forall y(H(y) \to \neg P(x,y)))$$

for x and y in the same domain.

Prove that $\forall x(G(x) \to \neg H(x))$ is true.

Please use the format of "Step-Reason" in your proof.

(20 marks)

2. For all sets A, B, prove that

$$A - (A \cap B) = (A \cup B) - B$$

Please indicate the reason for each of your step.

(13 marks)

3. Define $F : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ by the rule $F(x,y) = (x+y, x-y)$ for all $(x,y) \in \mathbb{R} \times \mathbb{R}$. Is F a one-to-one correspondence? Prove or give a counterexample.

(15 marks)

4. Let $\mathbf{C}$ be the set of complex numbers. A relation R is defined on $\mathbf{C}$ as follows:

$$R = \{(x,y) | x, y \in \mathbf{C}, x - y = a + bi, a, b \in \mathbf{N}\}$$

Determine whether this relation is reflexive, symmetric, antisymmetric, and transitive. Justify your answer.

(15 marks)

5. We define the sequence of numbers

$$a_n = \begin{cases} 1, & \text{if } 0 \leq n \leq 3 \\ a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4}, & \text{if } n \geq 4. \end{cases}$$

Prove that $a_n \equiv 1 \pmod 3$ for all $n \geq 0$.

(15 marks)

6. A student takes two consecutive exams in the same course and the probability of passing the first exam is $\frac{2}{3}$. If he passes the first exam, the probability of passing the second exam is also $\frac{2}{3}$. If he fails the first exam, the probability of passing the second exam is $\frac{1}{3}$.

   (a) If he passes at least once, he can obtain a certain qualification, calculate the probability of him obtaining the qualification.

   (b) If it is known that he has passed the second exam, calculate the probability of him passing the first exam.

   (c) Suppose that if the student passes the first exam, he does not need to take the second exam. What is the expected number of exams he would take?

   Please show your working steps in your answer.

(22 marks)

— END OF COMP1046 COURSEWORK 1 —