The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, SPRING SEMESTER 2021-2022

Languages and Computation (AE2LAC, COMP2049)

Time allowed: 2 Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL questions

Total Marks: 100.

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: None.
INFORMATION FOR INVIGILATORS: Collect both the exam papers and the answer booklets at the end of the exam.

Question 1:

[**overall 25 marks**]

(a) True or False? (No explanation is required.) [6 Marks]

(i) Every non-deterministic finite automaton M accepts the empty string, i. e., $\lambda \in L(M)$.

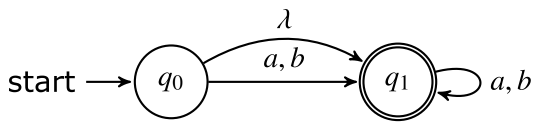
(ii) For every non-deterministic finite automaton M, there exists a context-free grammar G which accepts the complement of the language $L(M)$, i. e., $L(G) = \overline{L(M)}$.

(iii) For any pair of context-free languages L_1 and L_2, the intersection $L_1 \cap L_2$ is a regular language.

(iv) The syntax of the Python programming language is specified entirely using left-linear grammars.

(v) In terms of computational power and languages that they accept, deterministic Turing machines are strictly more powerful than non-deterministic pushdown automata.

(vi) Consider the following transition graph of a non-deterministic finite automaton (NFA) M with alphabet $\Sigma = \{a, b\}$:



Then $\overline{L(M)} = \{\lambda\}$.

(b) Consider the alphabet $\Sigma = \{0, 1\}$. In this question, we interpret every string $w \in \Sigma^*$ as the binary representation of a natural number. For simplicity, we ignore the leading zeros. For instance, the strings 0011 and 11 both represent the natural number 3.

(i) Draw the transition graph of a deterministic finite automaton (DFA) M_1 which accepts the set $\{2k \mid k \in \mathbb{N}\}$ of even natural numbers in binary format. [3 Marks]

(ii) Draw the transition graph of a DFA M_2 which accepts the set $\{3k + 2 \mid k \in \mathbb{N}\}$ of natural numbers that have remainder 2 when divided by 3, in binary format. [3 Marks]

(iii) Design a left-linear grammar G such that $L(G) = L(M_2)$. You must write down all the steps taken to obtain G, and specify what the start variable of the grammar is.

[8 Marks]

(iv) Draw the transition graph of a DFA M_3 that accepts the set of even natural numbers which do not have remainder 2 when divided by 3, in binary format, i. e., $L(M_3) = L(M_1) - L(M_2)$. [5 Marks]

Question 2:

[**overall 25 marks**]

(a) Consider the NFA (non-deterministic finite automaton) M over the alphabet $\Sigma = \{0, 1\}$ given by the following transition table:

	0	1
q_0	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_1\}$	$\{q_1, q_2\}$
$\underline{\boldsymbol{q_2}}$	$\emptyset$	$\{q_2\}$

Note that q_2 is the only accepting state.

(i) Draw the transition *diagram* of M. [3 Marks]

(ii) Draw the transition diagram for a *deterministic* finite automaton M_1 which accepts the same language as M. [6 Marks]

(iii) Write down a regular expression for the language accepted by these two automata. [4 Marks]

(b) Consider the alphabet $\Sigma = \{a, b, c\}$, and the following languages over Σ:

- $L_1 = \{a^p b^q c^r \mid p \geq 1, q \leq 1, r \geq 2\}$.
- $L_2 = \{a^p b c^r \mid p \geq 0, r \leq 2\}$.
- $L_3 = \{a^p b c^r \mid p \geq 0, r < p\}$.

For each of the languages L_1, L_2, and L_3, write down whether it is regular or not. Furthermore:

- If the language is regular, you must justify your answer by writing down a regular expression which denotes that language.
- If the language is not regular, you must present a proof. [12 Marks]

Question 3:

[**overall 25 marks**]

(a) What is meant when a context-free grammar $G = (V, \Sigma, S, P)$ is said to be *ambiguous*? [3 Marks]

(b) Consider the context-free grammar $G = (V, \Sigma, S, P)$ with the following productions:

$$S \rightarrow a \mid S\,a \mid bS\,S \mid S\,S\,b \mid S\,bS$$

(i) Write a rightmost derivation for the string $abaa$. [4 Marks]

(ii) Demonstrate that G is an ambiguous grammar. [6 Marks]

(c) Consider the pushdown automaton (PDA) $M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$ with:

- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{0, 1\}$
- $z = 0$
- $F = \{q_3\}$

with initial state q_0 and the following transitions:

$$\begin{aligned}
\delta(q_0, a, 0) &= \{(q_1, 10)\} \\
\delta(q_0, \lambda, 0) &= \{(q_3, \lambda)\} \\
\delta(q_1, a, 1) &= \{(q_1, 11)\} \\
\delta(q_1, b, 1) &= \{(q_2, \lambda)\} \\
\delta(q_2, b, 1) &= \{(q_2, \lambda)\} \\
\delta(q_2, \lambda, 0) &= \{(q_3, \lambda)\}
\end{aligned}$$

(i) Draw the transition graph of the PDA M. [3 Marks]

(ii) Write down the complete trace of instantaneous descriptions starting from $(q_0, abb, 0)$. [2 Marks]

(iii) Write down the complete trace of instantaneous descriptions starting from $(q_0, aabb, 0)$. [4 Marks]

(iv) Write down a simple description of the language $L(M)$. [3 Marks]

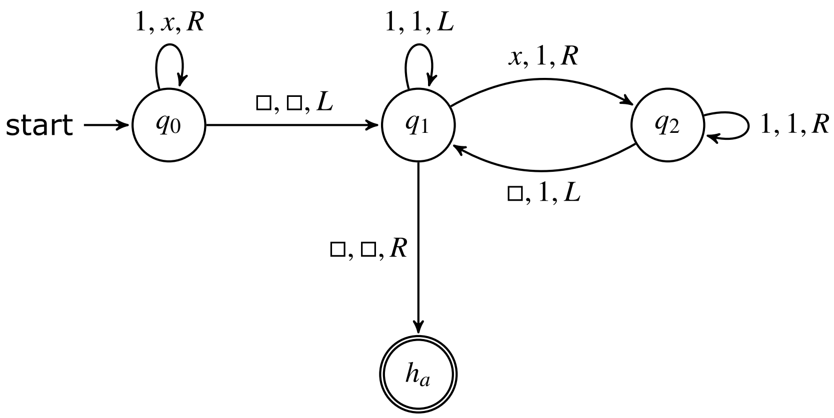
Question 4:

[**overall 25 marks**]

(a) Write down a clear statement of the Church-Turing thesis. [3 Marks]

(b) Explain briefly why the Church-Turing thesis is referred to as a '*thesis*' and not (say) a 'theorem' or a 'lemma'. [3 Marks]

(c) Consider the Turing Machine T with input alphabet $\Sigma = \{1\}$, and tape alphabet $\Gamma = \{1, x, \square\}$, given by the following transition diagram, in which $\square$ represents the blank symbol, h_a is the only accepting state of the machine, and q_0 is the start state:



On each of the following input strings, answer:

- Whether the machine halts or not;
- If it halts, in which of the states it halts;
- What the final configuration of the machine is.

(i) 11 [3 Marks]

(ii) 1111 [4 Marks]

Let $f_T : \{1\}^* \to \{1, x\}^*$ be the partial function computed by T.

(iii) What is the domain of f_T? [2 Marks]

(iv) What is the range of f_T? [2 Marks]

(v) Which familiar operation is carried out by f_T? [2 Marks]

(vi) Write a pseudo-code that explains how the machine T computes the function f_T. [6 Marks]