Languages and Computation (COMP 2049) Lab 08
Closure Properties of Context-Free Languages

In the previous lab, we learned that the class of context-free languages (CFLs) is closed under union, concatenation, and star-closure. Through the following exercises, we will learn that the class of CFLs is not closed under intersection or complementation.

Consider the following languages over the alphabet $\Sigma := \{a, b, c\}$:

- $L_1 := \{a^n b^n c^m \mid m \geq 0, n \geq 0\}$,
- $L_2 := \{a^m b^n c^n \mid m \geq 0, n \geq 0\}$.

In the lecture slides for Chapter 5 (Context-free languages) you may find two context-free grammars (CFGs) that generate these two languages.

(a) Draw the transition graphs of two pushdown automata (PDAs) M_1 and M_2 such that $L_1 = L(M_1)$ and $L_2 = L(M_2)$.

(b) Implement M_1 and M_2 in JFLAP and experiment with them.

It can be proven that the language $A^n B^n C^n := \{a^n b^n c^n \mid n \geq 0\}$ is not context-free. We did not discuss this in our lectures, but for those who are interested, you may find a proof in [LR23, Example 8.1], which uses the Pumping Lemma for Context-Free Languages [LR23, Theorem 8.1].

(c) Using the fact that $A^n B^n C^n$ is not context-free, prove that the class of CFLs is not closed under intersection.

(d) Prove that the class of CFLs is not closed under complementation. (Hint. Use De Morgan's laws.)

References

[LR23] Linz, P. and Rodger, S. H. An Introduction to Formal Languages and Automata. 7th ed. Jones & Bartlett Learning, 2023.