Languages and Computation (COMP 2049) Lab 03

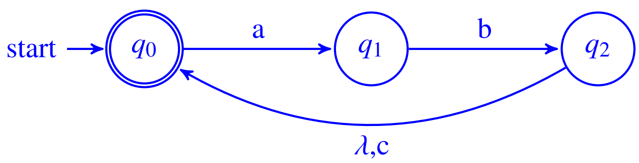
Non-deterministic Finite Automata, Regular Languages, Regular Expressions

(1) Construct a non-deterministic finite automaton (NFA) that accepts the language $\{ab, abc\}^*$.

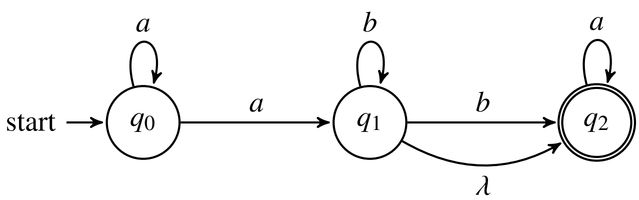
- Try to use as few states as possible. It is indeed possible to construct one with only three states.
- Use JFLAP to test your design.

Solution

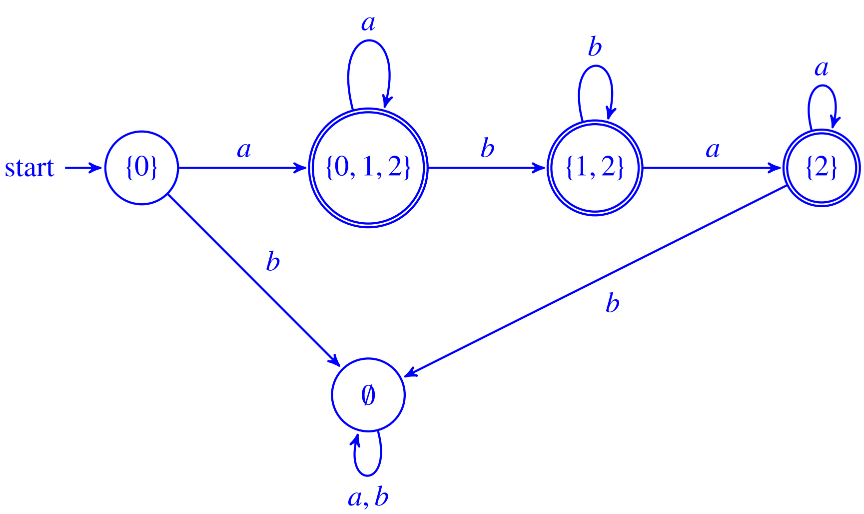
The following NFA accepts the language $\{ab, abc\}^*$ using only three states:



(2) Convert the following NFA into an equivalent deterministic finite automaton (DFA):



Solution



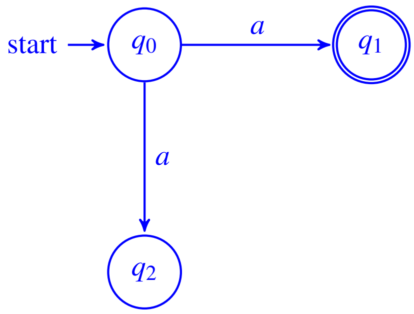
(3) Is it true that for every non-deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$, the complement $\overline{L(M)}$ satisfies the following?

$$\overline{L(M)} = \{w \in \Sigma^* \mid \delta^*(q_0, w) \cap (Q - F) \neq \emptyset\}$$

If yes, then you must write down a proof. If not, then you must present a counterexample.

Solution

False. For instance, consider the following NFA:



It is easy to see that $L(M) = \{a\}$. If we take $w = a$, then:

- $Q = \{q_0, q_1, q_2\}$
- $F = \{q_1\}$
- $(Q - F) = \{q_0, q_2\}$
- $\delta^*(q_0, w) = \delta^*(q_0, a) = \{q_1, q_2\}$

Therefore, $\delta^*(q_0, w) \cap (Q - F) = \{q_2\} \neq \emptyset$. Nonetheless, we know that $a \in L(M)$. Hence, $a \notin \overline{L(M)}$.

Remark: Remember that, as we have discussed in our lectures, for a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$, the following is indeed true:

$$\overline{L(M)} = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in (Q - F)\}.$$

(4) What languages are denoted by the expressions $r_1 = (\emptyset^*)^*$ and $r_2 = a\emptyset$?

Solution

$L((\emptyset^*)^*) = \{\lambda\}$ and $L(a\emptyset) = \emptyset$.

(5) Consider the language:

$$L = \{a^n b^m \mid n < 3, m \leq 3\}.$$

(a) Write down a regular expression r such that $L = L(r)$.

(b) Write down a regular expression r' for the complement of L, i.e., such that $\overline{L} = L(r')$.

Solution

(a) $r_1 = (\lambda + a + aa)(\lambda + b + bb + bbb)$.

(b) For the complement, we need to consider the following two cases:

   (i) The string is of the form $a^n b^m$, with $n \geq 3$ or $m > 3$;

   (ii) Those strings which are not of the form $a^n b^m$ at all, i. e., there is a 'b' followed by an 'a'.

   Thus, we obtain:

$$r_2 = aaaa^*b^* + a^*bbbbb^* + (a + b)^*ba(a + b)^*$$

(6) Optional self study: The syntax that we use in this module for regular expressions is suitable for educational purposes, but quite restrictive for practical applications. A good source for finding common regular expressions is:

https://regexlib.com/

Try to find regular expressions for:

(a) Integers in hexadecimal notation;

(b) Floating-point numbers.

Solution

Self study!