Recurrence

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Reading

- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein, *Introduction to Algorithms*, Third Edition, 2009.
  - Chapter 4.
- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein, *Introduction to Algorithms*, Fourth Edition, 2022.
  - Chapter 4.

Exercise 1

Solve the following recurrences. Note that $T(1)$ is assumed to be 1.

1. $T(n) = T(n-1)$
2. $T(n) = T(n-1) + 1$
3. $T(n) = T(n-1) + n$
4. $T(n) = 2T(n-1)$

Exercise 2

Solve the following recurrences.

1. $T(n) = 2T(n/2)$
2. $T(n) = 2T(n/4)$
3. $T(n) = 2T(n/2) + 1$
4. $T(n) = 2T(n/2) + n^2$

Exercise 3

Solve the following recurrences.

1. $T(n) = 2T(n/2) + n + 1$
2. $T(n) = 2T(n/2) + n \log n$
3. $T(n) = 2T(n/2) + n(\log n)^2$
4. $T(n) = 4T(n/2) + n \log n$

Exercise 4

Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

Exercise 5

Using the master method, we can show that the solution to the recurrence $T(n) = 4T(n/3) + n$ is $T(n) = \Theta(n^{\log_3 4})$. Show that a substitution proof with the assumption $T(n) \leq cn^{\log_3 4}$ fails. Then show how to substract off a lower-order term to make a substitution proof work.

Exercise 6

Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm, whose running time is in $\Theta(n^{\log 7})$. His algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size $n/4 \times n/4$, and the divide and combine steps together will take $\Theta(n^2)$ time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates a subproblems, then the recurrence for the running time $T(n)$ becomes $T(n) = aT(n/4) + \Theta(n^2)$. What is the largest integer value of a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?