The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, SPRING SEMESTER 2022–2023

Languages and Computation

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer QUESTION ONE and any THREE other questions

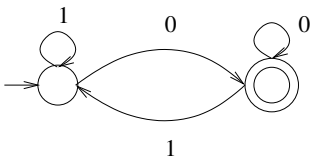
No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a dictionary to translate between that language and English provided that neither language is the subject of this examination.

No electronic devices capable of storing and retrieving text may be used.

Question 1: (Compulsory) The following questions are multiple choice. There is at least one correct choice but there may be several. To get all the marks you have to list all the correct answers and none of the wrong ones.

a. Given the following finite automaton A_1 over $\Sigma = \{0, 1\}$.



Which of the following statements about A_1 are correct?

(i) A_1 is a Deterministic Finite Automaton (DFA).

(ii) A_1 is a Nondeterministic Finite Automaton (NFA) but not a Deterministic Finite Automaton (DFA).

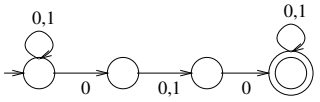
(iii) $00 \in L(A_1)$

(iv) A_1 accepts all words which end with 0.

(v) A_1 accepts all words which contain a 0.

[5 marks]

b. Given the following finite automaton A_2 over $\Sigma = \{0, 1\}$



Which of the following statements about A_2 are correct?

(i) A_2 is a Deterministic Finite Automaton (DFA).

(ii) A_2 is a Nondeterministic Finite Automaton (NFA) but not a Deterministic Finite Automaton (DFA).

(iii) $00 \in L(A_2)$

(iv) A_2 accepts all words containing two zeros.

(v) A_2 accepts all words with an even number of zeros.

[5 marks]

Continued on next page

c. Which of the following statements are correct?

(i) A language is a set of words.
(ii) A word is a set of symbols.
(iii) There are languages which cannot be recognized by a Nondeterministic Finite Automaton (NFA).
(iv) There are languages which cannot be recognized by a Pushdown Automaton (PDA).
(v) There are languages which cannot by recognized by a Turing Machine (TM).

[5 marks]

d. Given the following Context-Free Grammar (CFG)

$$G = (\{S, T\}, \{\texttt{(}, \texttt{)}, \texttt{a}, \texttt{b}\}, S, P)$$

with productions P:

$$S \to \texttt{(}T\texttt{)} \mid \texttt{a}$$
$$T \to \texttt{(}S\texttt{)} \mid \texttt{b}$$

Which of the following words are in the language of G (i.e. are elements of $L(G)$)?

(i) `(b)`
(ii) `((b))`
(iii) `(b)((a))`
(iv) `(())`
(v) `(((b)))`

[5 marks]

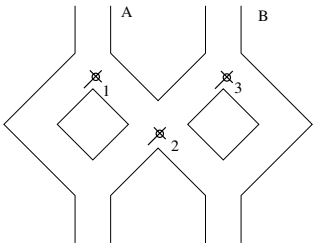
e. Which of the following statements about the Halting Problem are correct.

(i) The Halting Problem is undecidable.
(ii) The Halting Problem is semi-decidable.
(iii) The Halting Problem is the problem that Turing Machines can get stuck.
(iv) There is no Turing Machine which decides the Halting Problem.
(v) A Turing machine which never halts has got the Halting Problem.

[5 marks]

Question 2

a. The task is to model a simple marble-rolling toy by a Deterministic Finite Automaton DFA. It is initially in the following state:



You can drop in marbles at A or B, always one at a time. There are 3 levers numbered 1,2,3. Each time a marble rolls over a lever it changes its direction such that the next marble will go the other way. E.g. if we throw in a marble at A, it will go left at lever 1,but also change the lever such that the next marble goes right. We say that the toy accepts a sequence of marbles if all the levers are in the same position, e.g. it accepts the empty word but also `BA` because then all levers are pointing the other way and also `BAAB` because they are back in the initial position.

Construct a DFA over $\Sigma = \{\mathtt{A}, \mathtt{B}\}$ which models the marble toy, i.e. its states correspond to states of the marble toy and which accepts a word if the marble toy accepts the corresponding sequence of marble drops. [10 marks]

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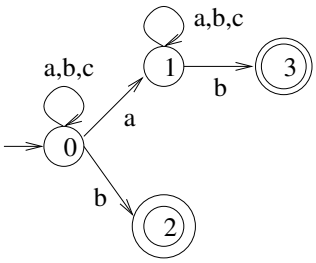
b. Given the following nondeterministic finite automaton (NFA) A over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$



Apply the subset construction to derive a deterministic finite automaton $D(A)$ which recognizes the same language. To save space you may restrict yourself to the reachable part of $D(A)$, i.e. states which are not reachable from the initial state may be left out.

[10 marks]

c. Construct a deterministic finite automaton with only two states which recognizes the same language as the automaton from the previous question (2 b).

[5 marks]

Question 3

a. Give Regular Expressions for the following languages over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$:

(i) Words that contain no $\mathtt{a}$s or $\mathtt{b}$s.

(ii) Words that only contain $\mathtt{a}$s or $\mathtt{b}$s.

(iii) Words that contain an even number of$\mathtt{b}$s.

(iv) Words that contain the sequence $\mathtt{aa}$.

(v) Words that do not contain the sequence $\mathtt{aa}$.

[10 marks]

b. Let $\Sigma = \{0, 1\}$, consider the following languages $L_1, L_2, L_3, L_4, L_5 \subseteq \Sigma^*$:

L_1 The language of words which contains the same number of zeros and ones.

$$L_1 = \{w \in \Sigma^* \mid \#_0(w) = \#_1(w)\}$$

L_2 The language of words containing the same number of zeros and ones modulo 2.

$$L_2 = \{w \in \Sigma^* \mid \#_0(w) \equiv \#_1(w) \mod 2\}$$

L_3 The finite language given by

$$L_3 = \{0, 00, 000\}$$

L_4 The complement of L_3, i.e. the language of all words which are not in L_3

$$L_4 = \{w \in \Sigma^* \mid w \notin L_3\}$$

L_5 The language of repeated words, i.e.

$$L_5 = \{ww \mid w \in \Sigma^*\}$$

(i) Which of the languages L_1, L_2, L_3, L_4, L_5 are regular? [5 marks]

(ii) Provide evidence for your answer to (i) by either

- Constructing a DFA, NFA or a Regular Expression defining the language, or

- Sketch an argument using the pumping lemma, showing that the language cannot be regular.

[10 marks]

Question 4

The following context-free grammar G defines the syntax of regular expressions over the alphabet $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.

$G = (\{E, S\}, \{\emptyset, \mathtt{+}, \mathtt{(}, \mathtt{)}, \mathtt{*}, \mathtt{a}, \mathtt{b}, \mathtt{c}\}, E, P)$ with P given by:

$$E \to \emptyset \mid S \mid E\mathtt{+}E \mid EE \mid E\mathtt{*} \mid \mathtt{(}E\mathtt{)}$$
$$S \to \mathtt{a} \mid \mathtt{b} \mid \mathtt{c}$$

a. Give a derivation and a parse tree for $\mathtt{a(b+c)}^*$. [7 marks]

b. Show that the grammar is ambiguous by exhibiting two different parse trees for $\mathtt{a+bc}$. [8 marks]

c. Suggest an alternative grammar for the same language which is not ambiguous. The grammar should reflect the conventions on how to read regular expressions (i.e. $*$ binds more then sequencing which binds more than $+$). [10 marks]

Question 5

Given the following Pushdown Automaton (PDA) P

$$P = (Q = \{q_0, q_1\}, \Sigma = \{\mathtt{a}, \mathtt{b}\}, \Gamma = \{a, b, \#\}, \delta, q_0, Z_0 = \#, F = \{q_0\})$$

where δ is given by the following equations:

$$\begin{array}{lcl}
\delta(q_0, \mathtt{a}, \#) & = & \{(q_1, a\#)\} \\
\delta(q_0, \mathtt{b}, \#) & = & \{(q_1, b\#)\} \\
\delta(q_1, \mathtt{a}, b) & = & \{(q_1, \epsilon)\} \\
\delta(q_1, \mathtt{a}, a) & = & \{(q_1, aa)\} \\
\delta(q_1, \mathtt{b}, a) & = & \{(q_1, \epsilon)\} \\
\delta(q_1, \mathtt{b}, b) & = & \{(q_1, bb)\} \\
\delta(q_1, \epsilon, \#) & = & \{(q_0, \#)\} \\
\delta(q, w, z) & = & \{\} \qquad \text{everywhere else}
\end{array}$$

a. Construct sequences of Instantaneous Descriptions (IDs) for the words

$$\mathtt{bba}, \mathtt{baab}, \epsilon$$

Which of these words are accepted (using acceptance by final state)?

[12 marks]

b. Describe the language accepted by P in one sentence.

[5 marks]

c. What does it mean for a PDA to be deterministic? Is P deterministic?

[8 marks]

Question 6

Write a short essay on the Chomsky Hierarchy addressing each of the questions below. Try to be as concise as possible. Address the following questions:

- What is the Chomsky Hierarchy? [4 marks]
- What are the levels of the Chomsky Hierarchy? What machines and what formalisms (e.g. grammars) correspond to the different levels? [8 marks]
- Give examples of languages which are on a certain level but not on the previous one. [9 marks]
- Where do the decidable languages fit it? Why do they not correspond to a proper level of the hierarchy? [4 marks]