The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, SPRING SEMESTER 2016–2017

LANGUAGES AND COMPUTATION

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer ALL THREE questions

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject-specific translation directories are not permitted.

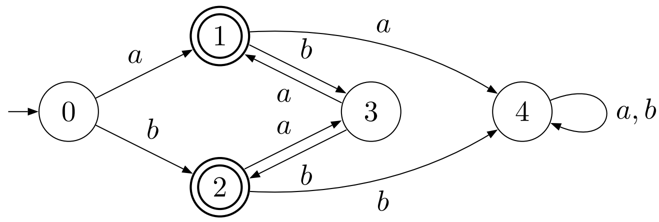
No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

Question 1

The following questions are multiple choice. There is at least one correct answer, but there may be several. To get all the marks you have to list all correct answers and none of the incorrect ones. 1 mistake results in 3 marks, 2 mistakes result in 1 mark, 3 or more mistakes result in zero marks.

(a) Consider the following finite automaton A over $\Sigma = \{a, b\}$:



Which of the following statements about A are correct?

(i) The automaton A is a Deterministic Finite Automaton (DFA).

(ii) $\epsilon \in L(A)$

(iii) $baaba \in L(A)$

(iv) All words accepted by A contain one more a than b or one more b than a.

(v) The automaton A accepts all words over Σ that contain one more a than b or one more b than a.

(5)

(b) Consider the following set W of words:

$$W = \{\epsilon,\ ab,\ cab,\ abab\}$$

Which of the following regular expressions denote a language that contains *all* words in W? (But not necessarily *only* the words in W: the language denoted by the regular expression is allowed to contain *more* words.)

(i) $(\epsilon + \mathbf{ab} + \mathbf{c})(\epsilon + \mathbf{ab})$

(ii) $(\epsilon + \mathbf{ab} + \mathbf{c})(\emptyset + \mathbf{ab})$

(iii) $(\epsilon + \mathbf{ab} + \mathbf{c})^*$

(iv) $(\mathbf{ab} + \mathbf{c})^*$

(v) $(\mathbf{ab})^* + \mathbf{c}^*$

(5)

(c) Consider the following Context-Free Grammar (CFG) G:

$$\begin{array}{lcl} S & \rightarrow & XY \\ X & \rightarrow & aXb \mid \epsilon \\ Y & \rightarrow & bYc \mid \epsilon \end{array}$$

where S, X, Y are nonterminal symbols, S is the start symbol, and a, b, c are terminal symbols.

Which of the following statements about the language $L(G)$ generated by G are correct?

(i) $\epsilon \in L(G)$

(ii) $abbc \in L(G)$

(iii) $bcab \in L(G)$

(iv) $aaabbbbbbccc \in L(G)$

(v) $L(G) = \{a^n b^{2n} c^n \mid n \in \mathbb{N}\}$ (where $\mathbb{N} = \{0, 1, 2, \ldots\}$)

(5)

(d) Which of the following properties of a problem P imply that P is undecidable?

(i) P is reducible to the Halting Problem.

(ii) The Halting Problem is reducible to P.

(iii) P is recursively enumerable but not recursive.

(iv) The complement of P is recursively enumerable.

(v) There is no Turing Machine that solves P.

(5)

(e) Which of the following statements about the λ-calculus are true?

(i) Every λ-term has a normal form.

(ii) Every λ-term has a fixed point.

(iii) Every computable function can be represented by a λ-term.

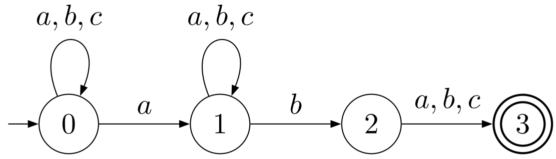
(iv) The normalization property of λ-terms is a decidable problem.

(v) The set of functions representable by λ-terms is the same as those computable by Turing Machines.

(5)

Question 2

(a) Given the following Nondeterministic Finite Automaton (NFA) N over the alphabet $\Sigma = \{a,\, b,\, c\}$, construct a Deterministic Finite Automaton (DFA) $D(N)$ equivalent to N by applying the *subset construction*:



Show your calculations in a state-transition *table*. Consider only the *reachable* part of $D(N)$. Then draw the transition *diagram* for the resulting DFA $D(N)$. Indicate the initial state and the final states both in the transition table and the final transition diagram. (12)

(b) Consider the following Context-Free Grammar (CFG):

$$\begin{aligned} S &\rightarrow SpA \mid A \\ A &\rightarrow BmA \mid B \\ B &\rightarrow a \mid b \mid c \mid lSr \end{aligned}$$

S, A, and B are nonterminals, a, b, c, l, m, p, and r are terminals, and S is the start symbol.

Draw the derivation tree according to this grammar for the word $amlapbpcrma$. (5)

(c) Construct an *unambiguous* Context-Free Grammar (CFG) for regular expressions over the alphabet $\Sigma = \{a, b, c\}$ (with the syntax defined in the lecture notes). To ensure your grammar is unambiguous, it should reflect the precedence and associativity for the regular expression constructs as specified by the following table:

Operators	Precedence	Associativity
$*$	highest	n/a
concatenation	medium	left
$+$	lowest	left

For example,

$$(\mathbf{a}(\epsilon + \mathbf{b}))^* + \emptyset$$

is a valid regular expression, whereas both

$$(\mathbf{a}$$

(because the parentheses are not balanced) and

$$\mathbf{a}+$$

(because $+$ is a binary operator) are not. (8)

Question 3

(a) Write the λ-terms that represent the following values:

- The Boolean values true and false;
- The *exclusive or* function xor, such that

$$\begin{array}{rr} \mathsf{xor\,true\,true} \rightsquigarrow^* \mathsf{false}, & \mathsf{xor\,false\,true} \rightsquigarrow^* \mathsf{true}, \\ \mathsf{xor\,true\,false} \rightsquigarrow^* \mathsf{true}, & \mathsf{xor\,false\,false} \rightsquigarrow^* \mathsf{false}; \end{array}$$

- For every couple of terms a and b, a term $\langle a, b\rangle$ representing the pair, such that

$$\langle a, b\rangle\,\mathsf{true} \rightsquigarrow^* a, \qquad \langle a, b\rangle\,\mathsf{false} \rightsquigarrow^* b;$$

- The Church Numeral $\overline{3}$.

(8)

(b) Consider the following λ-terms: $\mathsf{xor\text{-}pair}$ is a function on pairs of Booleans, $\mathsf{xor\text{-}fun}$ a function from Church Numerals to pairs of Booleans:

$$\begin{array}{l} \mathsf{xor\text{-}pair} = \lambda p.\langle p\,\mathsf{false}, \mathsf{xor}\,(p\,\mathsf{true})\,(p\,\mathsf{false})\rangle \\ \mathsf{xor\text{-}fun} = \lambda n.n\,\mathsf{xor\text{-}pair}\,\langle \mathsf{true}, \mathsf{true}\rangle \end{array}$$

What values do the following terms reduce to?

$$\begin{array}{ll} \mathsf{xor\text{-}pair}\,\langle \mathsf{true}, \mathsf{true}\rangle \rightsquigarrow^* ? & \mathsf{xor\text{-}pair}\,\langle \mathsf{false}, \mathsf{true}\rangle \rightsquigarrow^* ? \\ \mathsf{xor\text{-}pair}\,\langle \mathsf{true}, \mathsf{false}\rangle \rightsquigarrow^* ? & \mathsf{xor\text{-}pair}\,\langle \mathsf{false}, \mathsf{false}\rangle \rightsquigarrow^* ? \end{array}$$

Show the steps of reduction in the computation of ($\mathsf{xor\text{-}fun}\,\overline{3}$). [You can use the previous reductions and those from part (a) as single steps.]

Give an informal definition of what $\mathsf{xor\text{-}fun}$ does: for which numbers n does ($\mathsf{xor\text{-}fun}\,\overline{n}$) $\rightsquigarrow^*$ $\langle \mathsf{true}, \mathsf{true}\rangle$? (10)

(c) In the context of complexity theory, explain what it means for a decision problem to belong to the classes: $\mathcal{P}$, $\mathcal{NP}$, $\mathcal{NP}$-complete. (7)