The University of Nottingham Ningbo China

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, Spring SEMESTER 2023-2024

Languages and Computation (AE2LAC, COMP2049)

Time allowed: **TWO Hours**

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL Questions

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

ADDITIONAL MATERIAL: None.

INFORMATION FOR INVIGILATORS: Exam papers must be collected at the end of the exam.

Question ONE [30 marks]

a) Ture or False? (No explanation is required) **[7 marks]**

(i) Every Language L over the singleton alphabet {b} is regular.

(ii) If the language L(G) generated by a grammar G is regular, then G is either left-linear or right-linear.

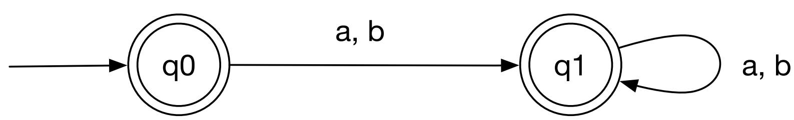
(iii) If a language L does satisfy the pumping lemma, then L is regular.

(iv) If a deterministic Turing machine M has only finitely many states, then it is equivalent to a deterministic pushdown automaton (DPDA).

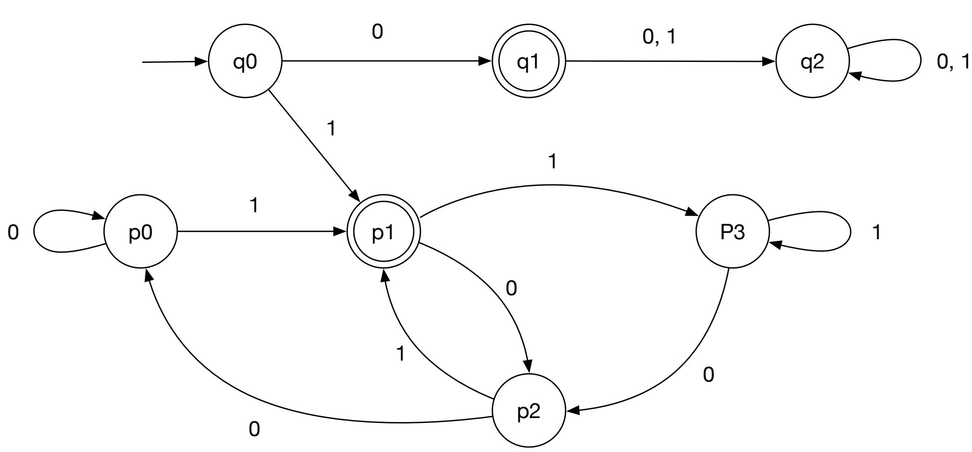
(v) The syntax of the Haskell programming language is not specified entirely using left-linear grammars.

(vi) Every context-free language (CFL) over the alphabet Σ = {a, b} can be written as the intersection of an infinite family of regular languages.

(vii) The following is the transition graph of a deterministic finite automaton *M* with Σ = {a, b}. Is it true that $\overline{L(M)} = \{\lambda\}$?



b) Consider the finite automaton M over the alphabet Σ = {0, 1} depicted by the following diagram:



(i) Is the empty string accepted by M? **[1 mark]**

(ii) Is the word 11101 accepted by M? **[1 mark]**

(iii) For what values of k will the string 10^k1 be accepted by M? **[3 marks]**

(iv) Describe the language L(M) of the above automaton M either in mathematical notation or in plain text. **[5 marks]**

c) Write down a clear statement of the pumping lemma for regular language. **[6 marks]**

d) Consider the language $L = \{ab^nc^m \mid m > n \geq 0\}$ over the alphabet $\Sigma = \{a, b, c\}$. Prove that L is not a regular language. **[7 marks]**

Question TWO [35 marks]

a) Write down a clear statement of the Church-Turing thesis. **[3 marks]**

b) Prove the Halting problem is undecidable. **[5 marks]**

c) Consider the following non-deterministic finite automaton (NFA) M over $\Sigma = \{a, b\}$ where q0 is the initial state and q1 is the final state.

	a	b
q0	$\emptyset$	{q1}
q1	{q2, q3}	$\emptyset$
q2	{q1}	$\emptyset$
q3	{q4}	$\emptyset$
q4	$\emptyset$	{q1}

(i) Draw the transition graph of M. **[3 marks]**

(ii) Write a regular expression for the language L(M). **[3 marks]**

(iii) Write a regular expression for the language $L(M)^R$. **[3 marks]**

(iv) Draw a transition diagram for a DFA M_1 which satisfies $L(M_1) = L(M)$. **[10 marks]**

d) Is it true that for all languages L_1, L_2, and L_3 over the $\Sigma = \{a, b\}$, the following equation holds?

$$L_1 (L_2 \cap L_3) = L_1L_2 \cap L_1L_3$$

If it is true, then provide a complete proof, otherwise, you need to provide a counterexample. **[8 marks]**

Question THREE [35 marks]

a) What is meant when a context-free grammar G = (V, Σ, S, P) is said to be ambiguous? **[3 marks]**

b) Consider the context-free grammar G = (V, Σ, S, P) with the following productions:

$$S \rightarrow AB$$

$$A \rightarrow aA \mid \lambda$$

$$B \rightarrow ab \mid bB \mid \lambda$$

(i) Write a rightmost derivation for the string abbab. **[3 marks]**

(ii) Demonstrate that G is an ambiguous grammar. **[4 marks]**

(iii) Write a regular expression r such that L(r) = L(G). **[5 marks]**

c) Let G = (V, Σ, <STMT>, P) be the grammar in which:

- Σ = {*if*, *condition*, *then*, *else*, *a*}
- V = {<STMT>, <IF-THEN>, <IF-THEN-ELSE>, <ASSIGN>}

And the production rules P are given as follows:

<STMT> → <ASSIGN> | <IF-THEN> | <IF-THEN-ELSE>

<IF-THEN> → *if condition then* <STMT>

<IF-THEN-ELSE> → *if condition then* <STMT> *else* <STMT>

<ASSIGN> → *a*

(i) Show that G is an ambiguous grammar. **[12 marks]**

(ii) Write the production rules for a grammar G′ = (V′, Σ′, S′, P′) which is unambiguous and generates the same language as G. Each component in G′ must be clearly specified. The proof that G and G′ are equivalent is not needed for this question. **[8 marks]**