The University of Nottingham Ningbo China

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, FULL YEAR, 2021–2022

ALGORITHMS CORRECTNESS AND EFFICIENCY

Time allowed TWO hours

Candidates may complete the front covers of their answer books and sign their desk cards but must NOT write anything else until the start of the examination period is announced.

Answer all FOUR questions. The total mark is 100.

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject-specific translation directories are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

Question 1 This question is about mathematical methods for analysing algorithm correctness and efficiency. (25 marks)

(a) Explain the meaning of 'partial correctness' and 'total correctness' of algorithms. Give an example to illustrate the difference between 'partial correctness' and 'total correctness'. (4 marks)

(b) Recall that the Assignment rule in the proof calculus is:

$$\frac{}{\{\psi[E/x]\}\ x = E\ \{\psi\}}\ \text{Assignment}$$

Given the program statement: $x = 2y - 1$. Use the Assignment rule to derive the precondition corresponding to each of the following post-conditions (it is not necessary to simplify the precondition):

(i) $x = y + 1$

(ii) $x * x = x + 1$

(iii) $\forall x.(x > 0)$

(iv) $\exists y.(x > y)$

(4 marks)

(c) The following program implements a sorting algorithm:

```
1   public static void sort(int[] arr) {
2        int i, j, temp;
3        for(i = 1; i < arr.length; i++){
4             temp = arr[i];
5             j = i;
6             while (j >= 1 && arr[j-1] > temp){
7                  arr[j] = arr[j-1];
8                  j--;
9             }
10            arr[j] = temp;
11       }
12  }
```

(i) Trace the program above for the array $arr = [5, 2, 3, 1]$. Describe how the content of the array change after each iteration of the outer loop. (3 marks)

(ii) The partial correctness of the program above can be proved by proving the loop invariant of the inner for loop and the loop invariant of the outer for loop. What is the loop invariant of the inner for loop? What is the loop invariant of the outer for loop? You may write your answer using either logical expressions or as pseudo-code. (4 marks)

(iii) Prove the loop invariant of the inner for loop by mathematical induction. (4 marks)

(iv) Does the program above terminate? Provide a Yes/No/Depends answer. Justify your answer briefly.

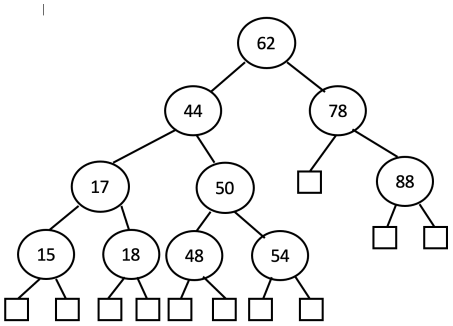
(2 marks)

(d) Solve the recurrence $T(n) = 4T(n/3) + n \log n$ using the master theorem. Assume that $T(n)$ is constant for sufficiently small n. Note that $\log_3 4 \approx n^{1.262}$. (4 marks)

Question 2 This question is about search tree structures. (25 marks)

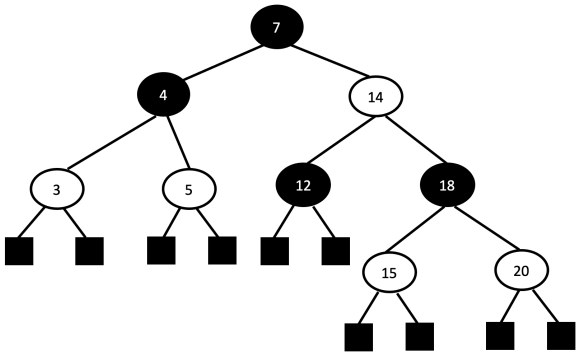
(a) What is a binary search tree? What is an AVL tree? (3 marks)

(b) Explain and draw figures to show the process of deleting the key 78 from the following AVL tree. Make sure that the resulting tree is still an AVL tree. (6 marks)



(c) What is a red-black tree? Describe the four properties that a red-black tree should satisfy. (4 marks)

(d) Explain and draw figures to show the process of inserting the key 17 into the following red-black tree. Make sure that the resulting tree is still a red-black tree. (6 marks)



(e) Suppose we implement the Sorted Map ADT using a red-black tree. Describe the main steps of *floorEntry*(*Key k*) using pseudocode. What is the time complexity of *floorEntry*(*Key k*)? Denote it using Big-Oh notation. (6 marks)

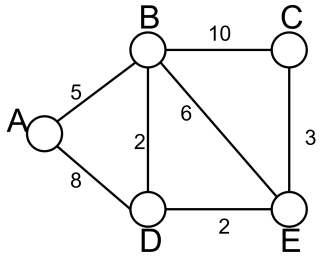
```
Entry floorEntry(Key k): returns the entry with the
greatest key value less than or equal to the given key k;
returns null if no such entry exists.
```

Question 3 This question is about sorting algorithms, heaps and graphs. (27 marks)

(a) Write down the pseudo-code of Quick-Sort, given the input sequence S. (8 marks)

(b) What is the expected running time of Quick-Sort in terms of Big-Oh notation? You need to provide detailed steps on the time analysis. (8 marks)

(c) Given the weighted graph shown below,



(i) Starting from Node A, Use the **Depth-first Search** to trace the graph. Show what nodes are in the stack for each step. (5 marks)

(ii) Find the shortest distances from node A to all nodes, and their summation. (3 marks)

(iii) Find the sum of the weights of all edges of the minimum spanning tree. (3 marks)

Question 4 This question is about string matching, trie and dynamic programming. (23 marks)

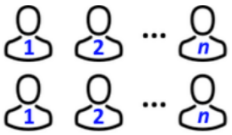
(a) Given the following string:
What a lovely day!
How many character matches are needed to find the pattern "day" from the string using the Boyer-Moore Algorithm? (4 marks)

(b) Construct the suffix trie for "banana". (5 marks)

(c) Given matrix chain-product $A_1 * A_2 * A_3 * A_4$ and the size of matrices shown below, find its optimal parenthesization, and the minimum number of scalar multiplications. (6 marks)

Matrix	Size
A_1	20×2
A_2	2×10
A_3	10×10
A_4	10×40

(d) Demid has decided to hold a basketball exercise session. $2n$ students have come to Demid's exercise session, and he lined up them into two rows of the same size. (There are exactly n people in each row). Students are numbered from 1 to n in each row in order from left to right. Now Demid wants to choose a team to play basketball. He will



choose players from left to right, and the index of each chosen player (excluding the first one taken) will be strictly greater than the index of the previously chosen player. To avoid giving preference to one of the rows, Demid chooses students in such a way that no consecutively chosen students belong to the same row. The first student can be chosen among all $2n$ students (there are no additional constraints), and a team can consist of any number of students.

Demid thinks, that in order to compose a perfect team, he should choose students in such a way that the total height of all chosen students is maximum possible. Help Demid to find the maximum possible total height of players in a team he can choose.

Input
The first line of the input contains a single integer n, $1 \leq n \leq 10^5$, the number of students in each row.
The second line of the input contains n integer $h_{1,1}, h_{1,2}, \ldots, h_{1,n}, 1 \leq h_{1,i} \leq 10^9$, where $h_{1,i}$ is the height of the i-th student in the first row.
The third line of the input contains n integer $h_{2,1}, h_{2,2}, \ldots, h_{2,n}, 1 \leq h_{2,i} \leq 10^9$, where $h_{2,i}$ is the height of the i-th student in the second row.

Output: Print a single integer, the maximum possible total height of players in a team Demid can choose.

(i) Given the following input:
5
9 3 5 7 3
5 8 1 4 5
Find the output. (4 marks)

(ii) Given the following input:
3
1 2 9
10 1 1
Find the output. (4 marks)