Tutorial 6

Exercise 1

- Solve $T(n)= T(n-1)$
- $T(n) = T(n-1)$
- $T(n-1) = T(n-2)$
- $T(n-2) = T(n-3)$
- …
- $T(2) = T(1)$
- Since $T(1) =1$, we have that $T(n) =1$.

Ex 1

- Solve $T(n) = T(n-1)+1$
- $T(n) = T(n-1)+1$
- $T(n-1) = T(n-2)+1$, $T(n) = T(n-2)+2$
- $T(n-2) = T(n-3) +1$, $T(n) = T(n-3) + 3$
- …
- $T(2) = T(1) +1$, $T(n) = T(1) + n-1$
- Since $T(1) = 1$, we have $T(n) = n$.

Ex 1

- Solve $T(n) = T(n-1)+n$
- $T(n) = T(n-1)+n$
- $T(n-1) = T(n-2) + (n-1)$, $T(n) = T(n-2) + (n-1)+n$
- $T(n-2) = T(n-3) + (n-2)$, $T(n) = T(n-3) + (n-2) + (n-1) +n$
- …
- $T(2) = T(1)+2$, $T(n) = T(1)+2+3 + \ldots + n$
- Since $T(1) = 1$, we have $T(n) = (1+n)n/2$.

Ex 2

- Solve $T(n) = 2T\left(\frac{n}{2}\right) + n^2$
- a = 2, b = 2, $f(n) = n^2$
- $n^{\log_b a} = n$
- $f(n) = \Omega(n^{1+\epsilon}), when\ \epsilon = 1$.
- f(n) is polynomially larger than the watershed function
- Now we check the regularity condition of case 3, $af\left(\frac{n}{b}\right) \leq cf(n)$, this is, $2\left(\frac{\mathrm{n}}{2}\right)^2 = \frac{\mathrm{n}^2}{2} \leq cn^2$. So we let c=0.5 <1.
- Case 3 applies. $T(n) = \Theta(n^2)$.

Ex 3

- Solve $T(n) = T\left(\frac{n}{2}\right) + n(\log n)^2$ [This is an additional one, not ex3.3]
- a = 1, b = 2, f(n) = $n(\log n)^2$
- $n^{\log_b a} = n^{\log_2 1} = n^0 = 1$
- $f(n) = \Omega(n^{0+\epsilon}), where\ \epsilon = 1.$
- f(n) is polynomially larger than the watershed function.
- Now we check the regularity condition. $af\left(\frac{n}{b}\right) \leq cf(n)$, this is, $1\left(\frac{\mathrm{n}}{2}\right)\left(\log\frac{n}{2}\right)^2 = \frac{n}{2}(\log n - \log 2)^2 = \frac{n}{2}(\log n - 1)^2 \leq cn(\log n)^2$. So we let c= 0.5<1.
- Case 3 applies. T(n) =$\Theta(n(\log n)^2)$.

Ex 3

- Solve $T(n) = 2T\left(\frac{n}{2}\right) + n(\log n)^2$ [This is the ex 3.3]
- a = 2, b = 2, f(n) = $n(\log n)^2$
- $n^{\log_b a} = n^{\log_2 2} = n^1 = n$
- $f(n) = \Theta(n(\log n)^2)$
- Case 2 applies, hence we have $T(n) = \Theta(n(\log n)^3)$.